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Entanglement entropy and proton-neutron interactions

Calvin W. Johnson

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ESNT workshop on PN Pairing + Quartets, Saclay, Sept 2019

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Two related questions about the structure of atomic nuclei:

1) Is there a **simple** picture in which to *understand* nuclear properties?

2) Is there an **efficient** scheme in which to *model* nuclear structure for applications (e.g., dark matter cross-sections, $0\nu\beta\beta$ matrix elements, ν-A scattering, etc.)?

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Two related questions about the structure of atomic nuclei:

1) Is there a **si** *tand* nuclear properties?

2) Is there an *el* nuclear structure for a ss-sections, $0\nu\beta\beta$ matrix ele s, ν-A scattering, etc.)?

What do we mean by "simple"?

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Ideally, "simple" means a few degrees of freedom describe many behaviors qualitatively / quantitatively

...mic nuclei:

1) Is there a si... *and* nuclear properties?

What do we mean by "simple"?

2) Is there an ... *el* nuclear structure for a... s-sections, 0νββ matrix ele... , ν-A scattering, etc.)?

Some simple pictures:

- Single-particle/ (deformed) mean-field;
- Single-quasiparticle (HFB);
- pair condensates (both like particles, and p-n);
- quartets;
- Group theoretical frameworks (“simple” = dominated by few irreps);
- ….

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Simplicity is defined relative to a "complete" picture, which here is *configuration-interaction*

$$\hat{\mathbf{H}}\left|\Psi\right\rangle = E\left|\Psi\right\rangle$$

$$\left|\Psi\right\rangle = \sum_{\alpha} c_{\alpha}\left|\alpha\right\rangle \qquad H_{\alpha\beta} = \left\langle\alpha\right|\hat{\mathbf{H}}\left|\beta\right\rangle$$

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha} \quad \text{if} \quad \left\langle\alpha|\beta\right\rangle = \delta_{\alpha\beta}$$

The basis states here are shell-model Slater determinants

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Sim…

pic…

The interacting shell model is useful here because it is flexible and is a superset of many of these approximations

$$|\Psi\rangle \ldots \sum_{\alpha} \ldots$$

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha} \quad \text{if} \quad \langle \alpha | \beta \rangle = \delta_{\ldots}$$

The basis states here are shell-model Slater determinants

The BIGSTICK shell-model code is public!

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Download from:
github.com/cwjsdsu/BigstickPublick

Manual at arXiv:1801.08432

Links to BIGSTICK and other free, open-source many-body codes available through

fribtheoryalliance.org

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Despite advances, it is easy to get to model spaces beyond our reach:

sd shell: max dimension 93,000. *Can be done in a few minutes on a laptop.*

pf shell: ^{48}Cr, dim 2 million, ~10 minutes on laptop
^{52}Fe, dim 110 million, a few hours on modest workstation
^{56}Ni, dim 1 billion, 1 day on advanced workstation
^{60}Zn, dim 2 billion, < 1 hour on supercomputer

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Despite advances, it is easy to get to model spaces beyond our reach:

shells between 50 and 82 ($0g_{7/2}$ 2s1d $0h_{11/2}$)

^{128}Te: dim 13 million (laptop)

^{127}I: dim 1.3 billion (small supercomputer)

^{128}Xe: dim 9.3 billion (supercomputer)

^{129}Cs: dim 50 billion (haven't tried!)

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Can we come up with an alternate ("simpler") approach?

How most shell-model codes represent the basis:
Proton-neutron factorization

$$|\Psi\rangle = \sum_{\mu\nu} c_{\mu\nu} |p_\mu\rangle |n_\nu\rangle$$

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$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

BIGSTICK is an M-scheme code, meaning total J_z fixed

We have a constraint: $M_p + M_n = M$

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BIGSTICK is an M-scheme code, meaning total J_z fixed

We have a constraint: $M_p + M_n = M$

$$\left|\Psi, M\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu, M_p\right\rangle \left|n_\nu, M_n = M - M_p\right\rangle$$

FACTORIZATION

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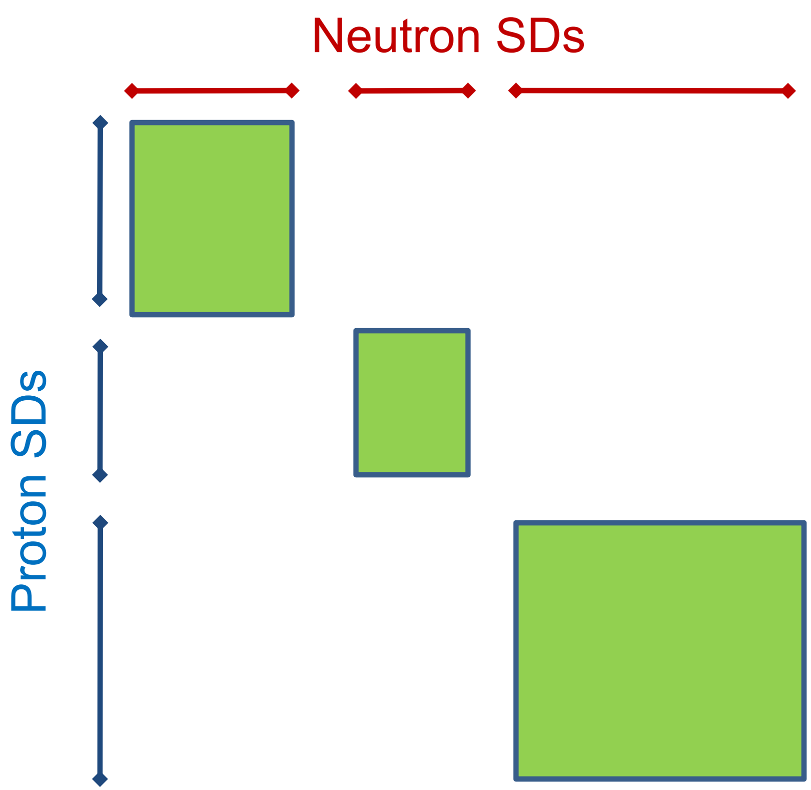
$$|\alpha\rangle = |\alpha_p\rangle \times |\alpha_n\rangle$$



This leads to a block structure for construction of the basis

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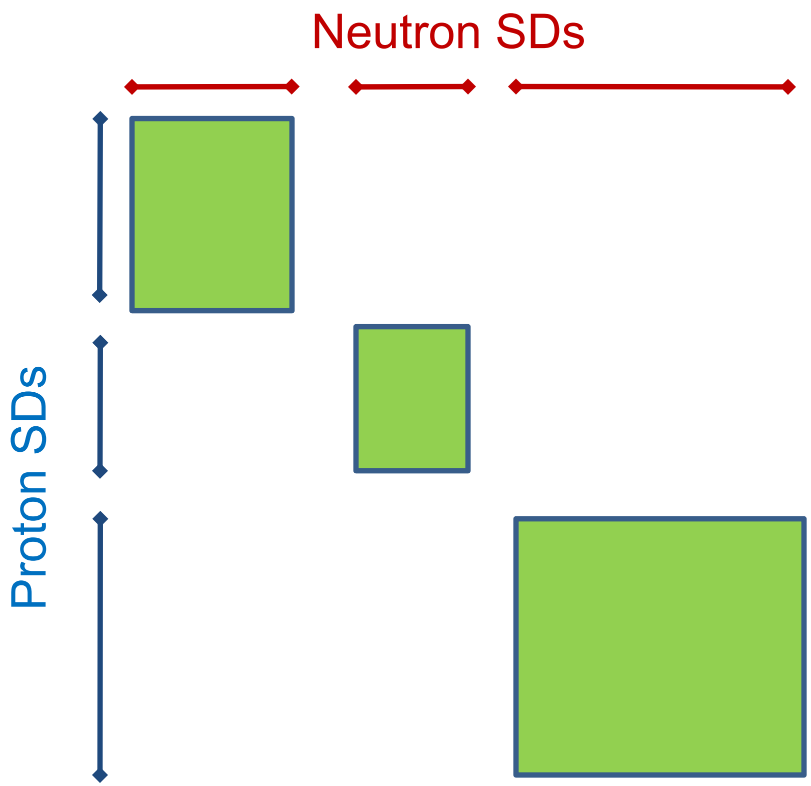
How most shell-model codes represent the basis: Proton-neutron factorization

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

BIGSTICK exploits this
for efficient representation
of basis and Hamiltonian

Factorization

$$|\alpha\rangle = |\alpha_p\rangle \times |\alpha_n\rangle$$



Example N = Z nuclei

Nuclide	Basis dim	# pSDs (=#nSDs)
^{20}Ne	640	66
^{24}Mg	28,503	495
^{28}Si	93,710	924
^{48}Cr	1,963,461	4895
^{52}Fe	109,954,620	38,760
^{56}Ni	1,087,455,228	125,970

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For fast calculation these are typically bit strings, or "*occupation representation of Slater determinants*"

$$|\Psi\rangle = \sum_{\mu\nu} c_{\mu\nu} |p_\mu\rangle |n_\nu\rangle$$

$$|01101000\ldots\rangle |10010100\ldots\rangle$$

Alternate approach for medium/nuclei:
Proton-neutron factorization

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_{\mu}\right\rangle \left|n_{\nu}\right\rangle$$

Can we truncate for just a few components?

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Alternate approach for medium/nuclei:
Proton-neutron factorization

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

$$\left(a_1\left|010110...\right\rangle + a_2\left|110010...\right\rangle + a_3\left|001011...\right\rangle + ...\right)$$

No longer single "Slater determinants" but
linear combinations…

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Alternate approach for medium/nuclei:
Proton-neutron factorization

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

Can we truncate for just a few components?

Priori work by Papenbrock, Juodagalvis, Dean, Phys. Rev. C **69**, 024312 (2004), but focused on N =Z

Alternate approach for medium/nuclei:
Proton-neutron factorization

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

Can we truncate for just a few components?

(Alternate idea: truncated in each configuration/ partition, e.g. Liao et al Phys. Rev. C 90, 024306)

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Example application:

shells between 50 and 82 ($0g_{7/2}$ 2s1d $0h_{11/2}$)

^{129}Cs: M-scheme dim 50 billion (haven't tried!)

Proton dimension: 14,677
Neutron dimension: 646,430

Example application:

^{129}Cs: M-scheme dim 50 billion (haven't tried!)

Proton dimension: 14,677
Neutron dimension: 646,430

The idea is to solve proton and neutron problems separately and then couple together a few "select" states

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So we want to ask:

Can the wave function *can* be well-approximated by just a few select proton and neutron states?

These would not be single Slater determinants but linear combinations

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So we want to ask:

Can the wave function *can* be well-approximated by just a few select proton and neutron states?

In other words, is it "simple" in terms of coupling between proton and neutron building blocks

My tool for investigation:
The *entanglement entropy*

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_{\mu}\right\rangle \left|n_{\nu}\right\rangle$$

Let any wavefunction have two components (i.e., proton and neutron components)
"*bipartite*"

Find the singular-value-decomposition eigenvalues of $c_{\mu\nu}$

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My tool for investigation:
The *entanglement entropy*

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

Find the singular-value-decomposition eigenvalues of $c_{\mu\nu}$:

Find eigenvalues λ_i of $\rho_{\mu\mu'} = \sum_\nu c_{\mu\nu} c_{\mu'\nu}$

$$S = -\sum_i \lambda_i \ln \lambda_i = -tr \rho \ln \rho$$

The *entanglement entropy*

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_\mu\right\rangle \left|n_\nu\right\rangle$$

$$S = -\sum_i \lambda_i \ln \lambda_i = -tr\rho \ln \rho$$

The *entanglement entropy* measures how correlated ('entangled') the two sectors are. S=0 means uncorrelated.

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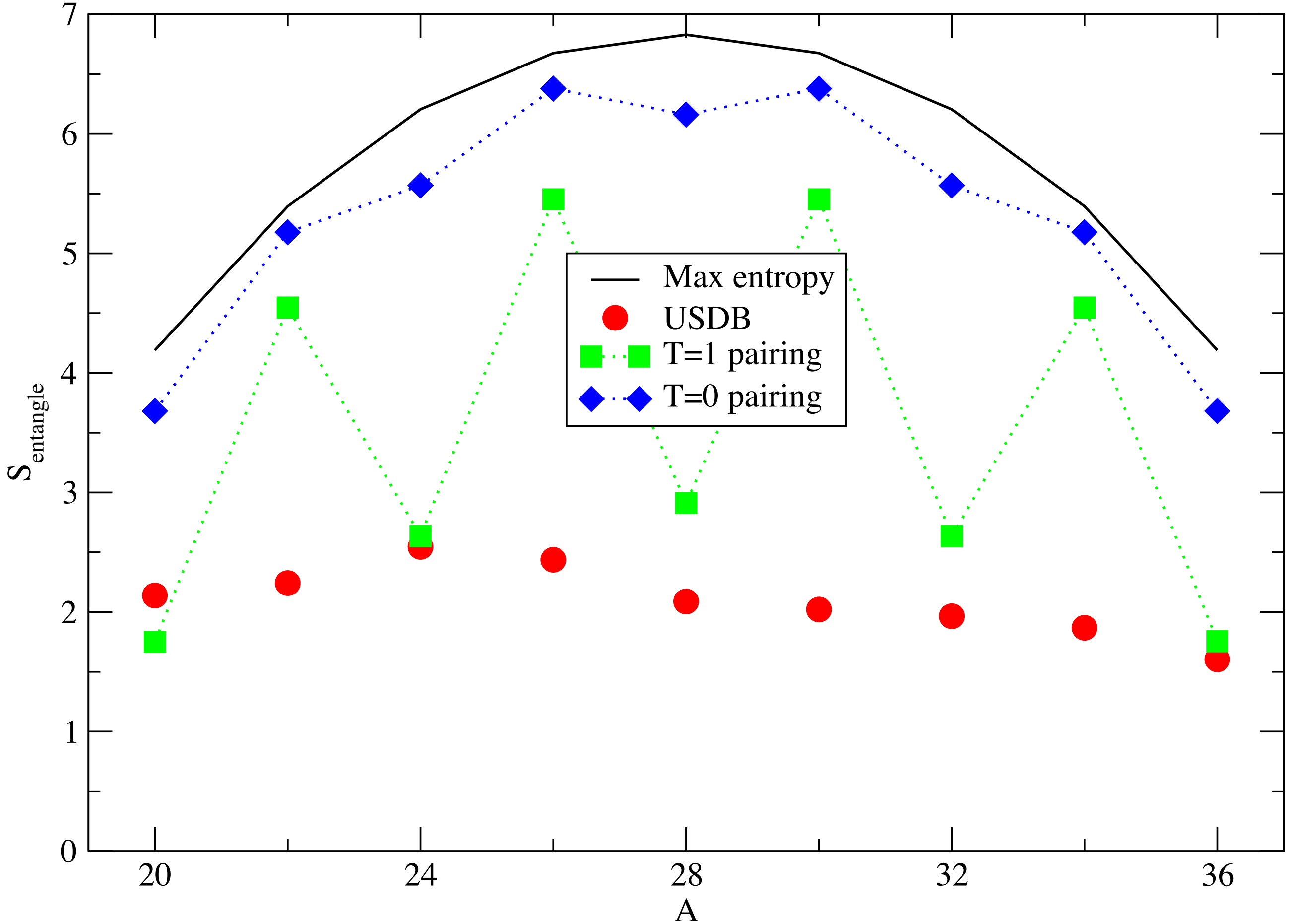
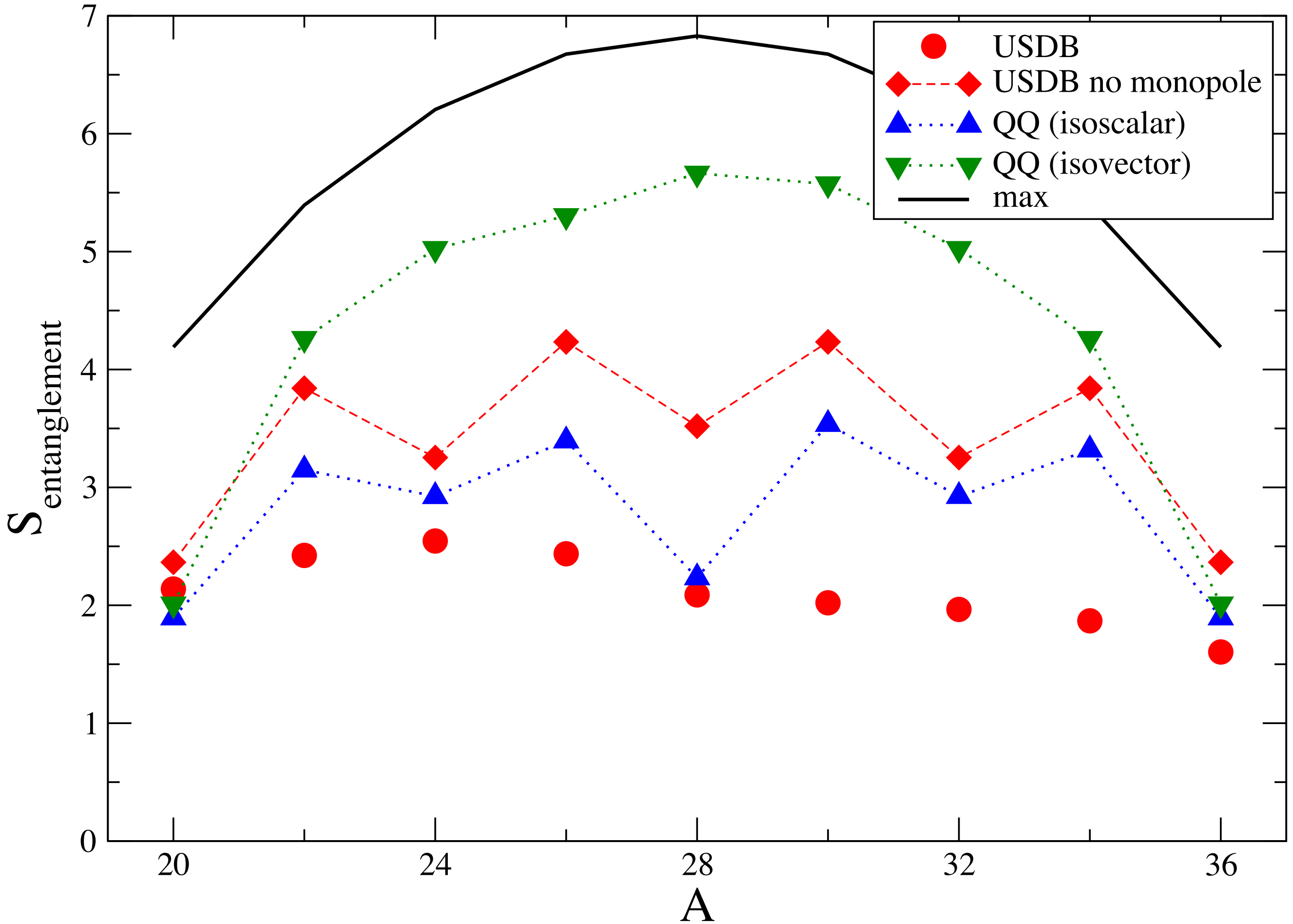
Now let's turn to nuclei, with

$$\left|\Psi\right\rangle = \sum_{\mu\nu} c_{\mu\nu} \left|p_{\mu}\right\rangle \left|n_{\nu}\right\rangle$$



Z=N nuclei
in *sd* shell



Z=N nuclei
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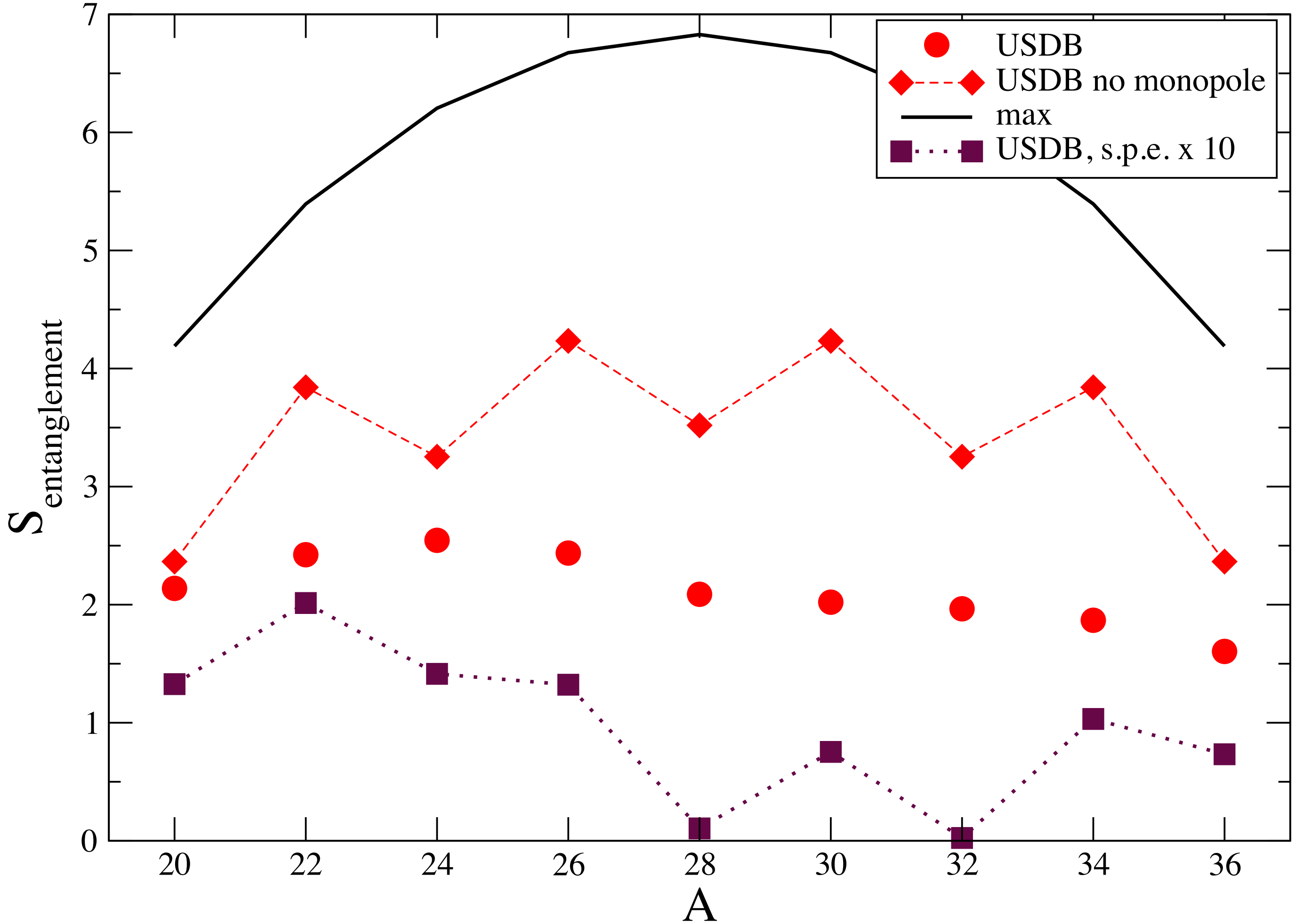

Z=N nuclei
in *sd* shell

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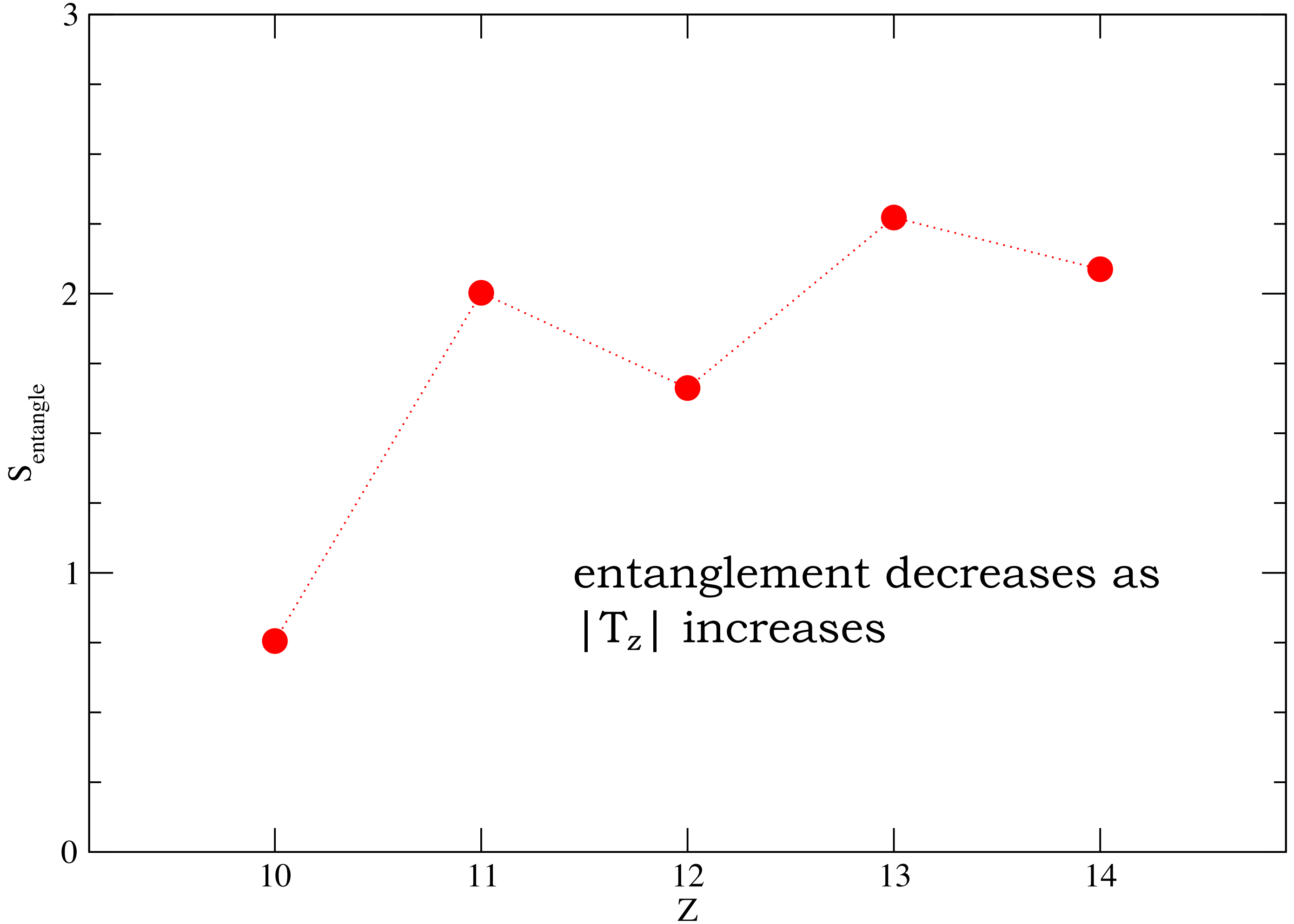
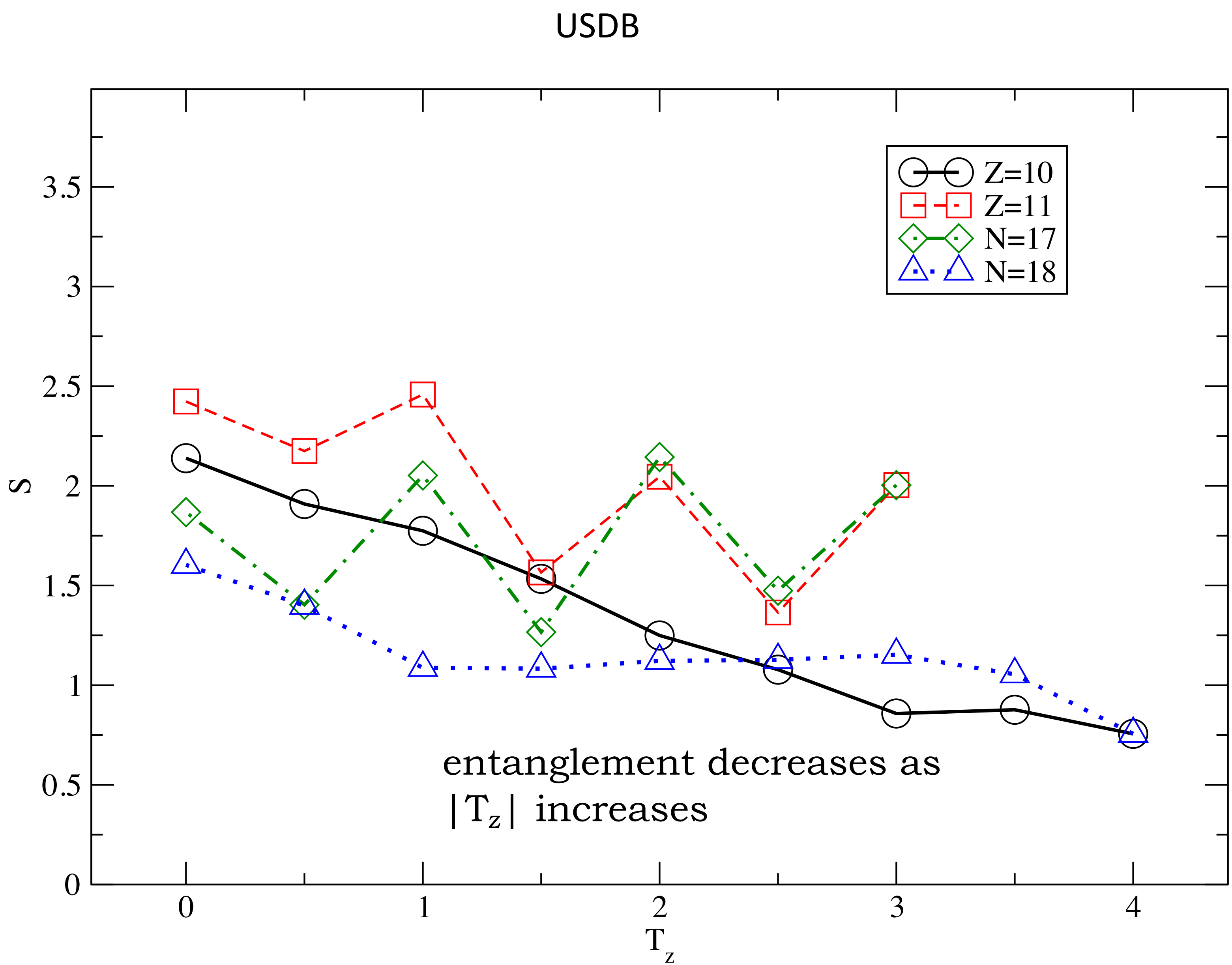
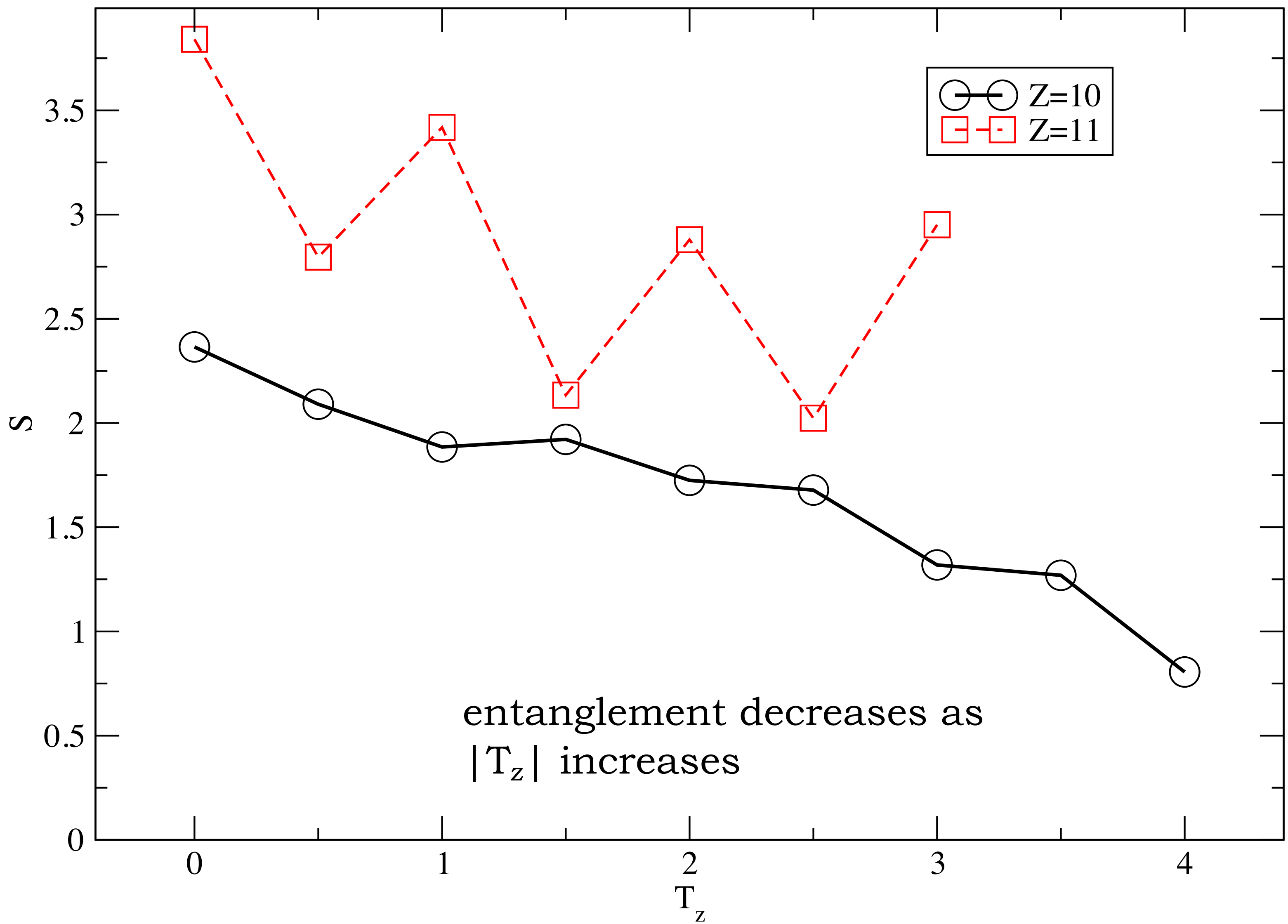
Now let's follow
as isospin increases

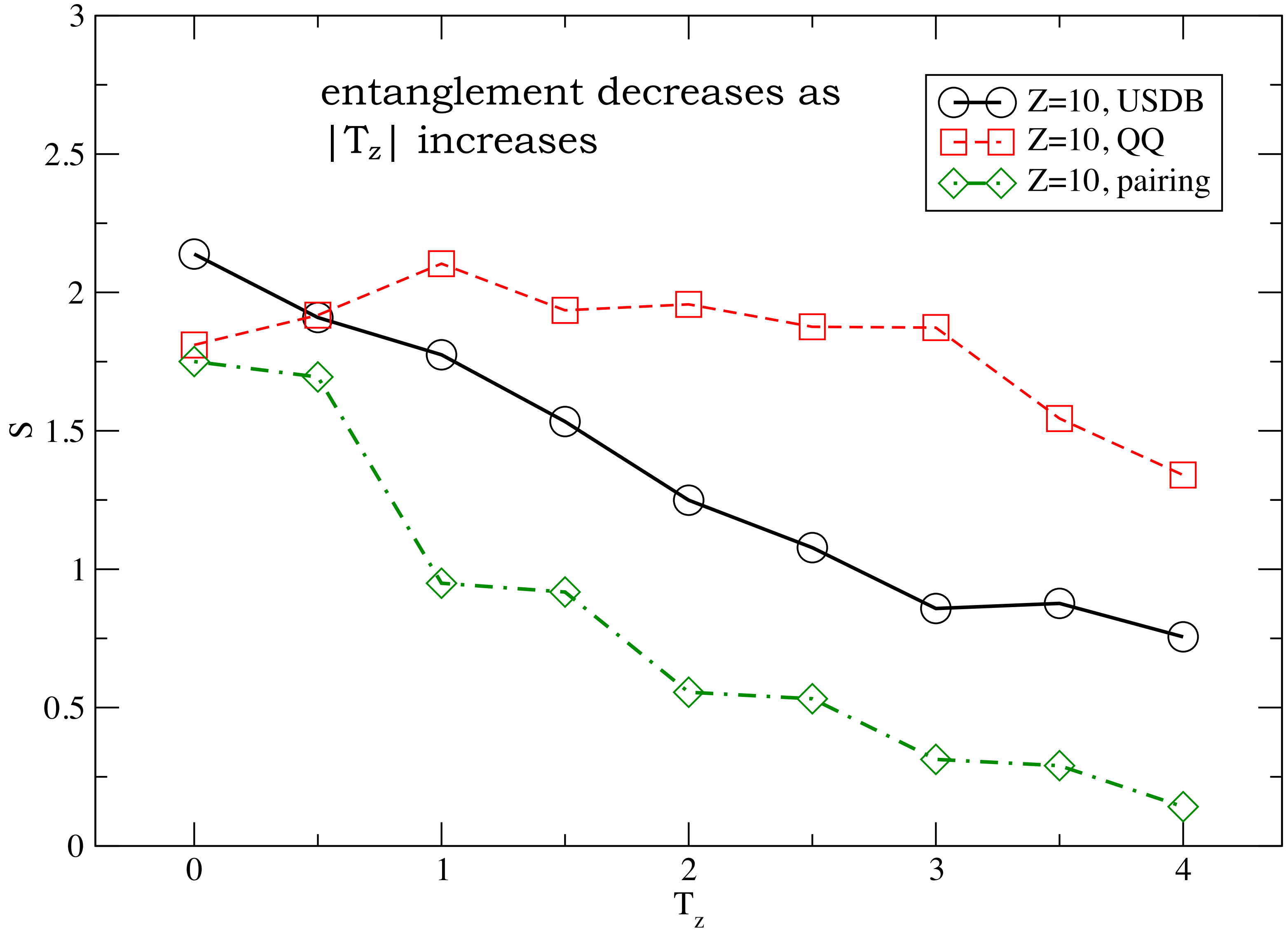


A=28 nuclei
in *sd* shell
with USDB

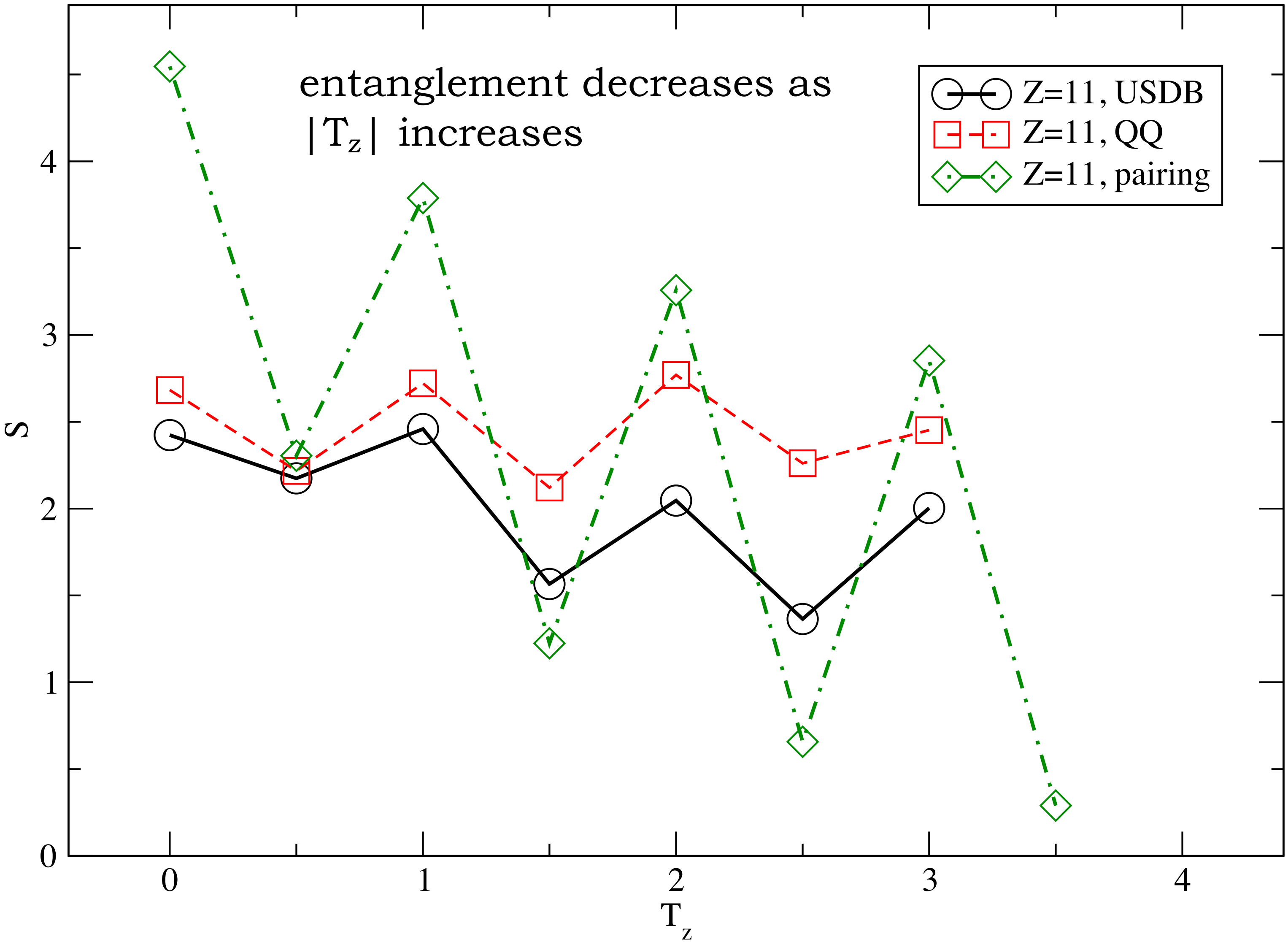

USDB
Z=10
Z=11
N=17
N=18
S
T_z
0
0.5
1
1.5
2
2.5
3
3.5
0
1
2
3
4
entanglement decreases as
|T_z| increases

USDB "traceless" = s.p.e, monopoles removed





entanglement decreases as
$|T_z|$ increases
Z=10, USDB
Z=10, QQ
Z=10, pairing
S
T_z
0
0.5
1
1.5
2
2.5
3
4



entanglement decreases as
|T_z| increases
Z=11, USDB
Z=11, QQ
Z=11, pairing
S
T_z
0
1
2
3
4

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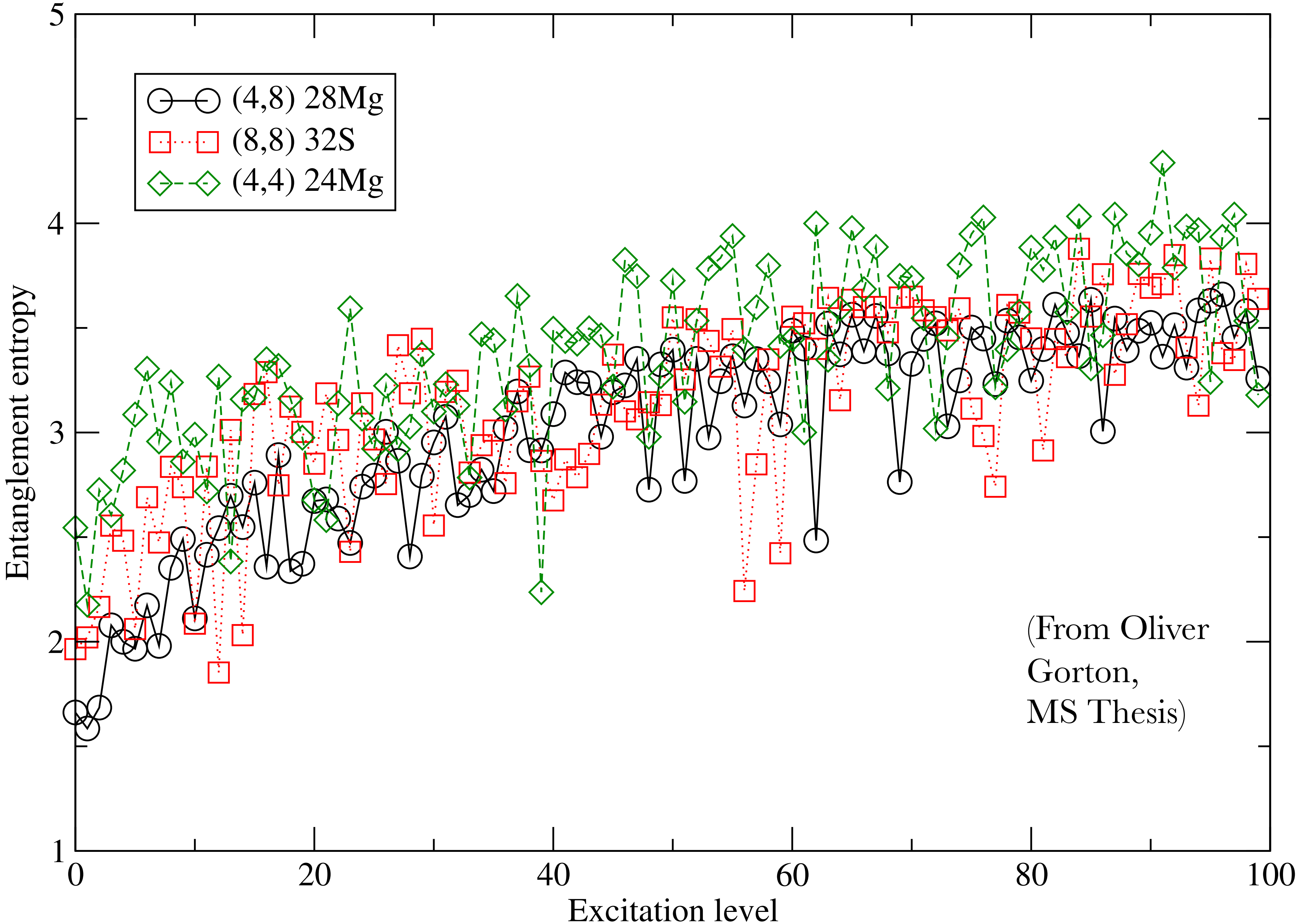
What about as we increase in excitation energy?



Entanglement entropy
Excitation level
(4,8) 28Mg
(8,8) 32S
(4,4) 24Mg
(From Oliver Gorton, MS Thesis)
1
2
3
4
5
0
20
40
60
80
100

What about other partitions?

Like pairing, or quartets?

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It would be very interesting to look at the entanglement entropy of, say, single-particles, or pairs, or quartets.

These are independent tests of our ideas about mean-field pictures, or various ‘condensates’

However it’s not so easy if one wants to do it rigorously.

Let

$$\left|\Psi\right\rangle = \sum_{\alpha,i} c_{i,\alpha} \hat{a}_i^\dagger \hat{b}_\alpha^\dagger \left|0\right\rangle$$

How to find $\rho_{ij} = \sum_{\alpha} c_{i,\alpha}^* c_{j,\alpha}$?

Especially if $\left[\hat{a}_i , \hat{b}_\alpha^\dagger\right] \neq 0$?

Compute

$$\langle b_\alpha | \hat{a}_i | \Psi \rangle \quad \text{where} \quad | b_\alpha \rangle = \hat{b}_\alpha^\dagger | 0 \rangle$$

"spectroscopic factor"

but also need a density matrix

$$\langle b_\alpha | \hat{a}_i \hat{a}_j^\dagger | b_\beta \rangle$$

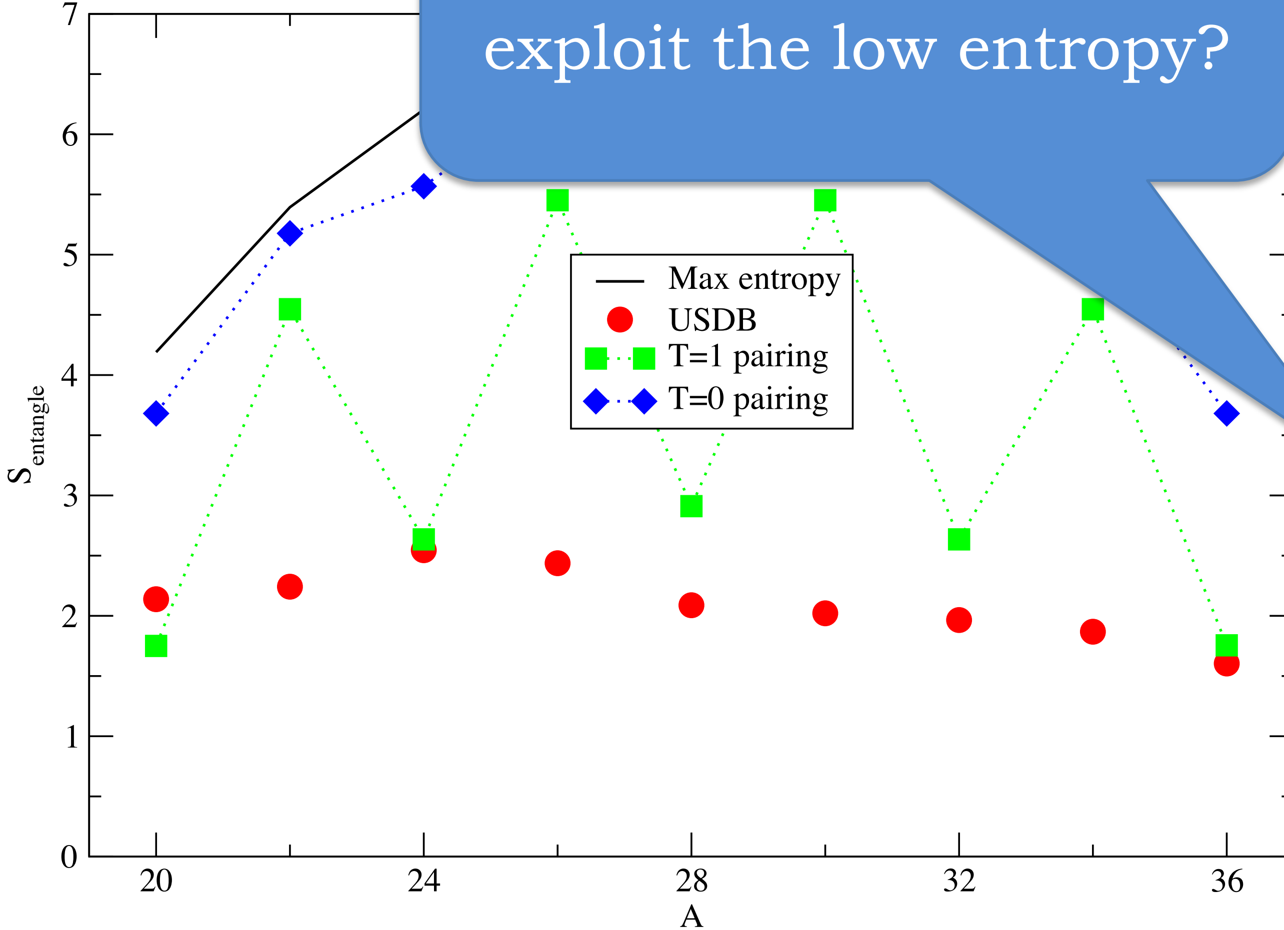
Then solve

$$\left\langle b_\alpha \middle| \hat{a}_i \middle| \Psi \right\rangle = \sum_{j,\beta} c_{j,\beta} \left\langle b_\alpha \middle| \hat{a}_i \hat{a}_j^\dagger \middle| b_\beta \right\rangle$$

Highly nontrivial, but possible in modest-sized systems (not yet done)

→ could lead to "how simple" a wave function looks like in terms of "pairing" or "quartet" bases

Back to this: can we exploit the low entropy?



Z=N nuclei

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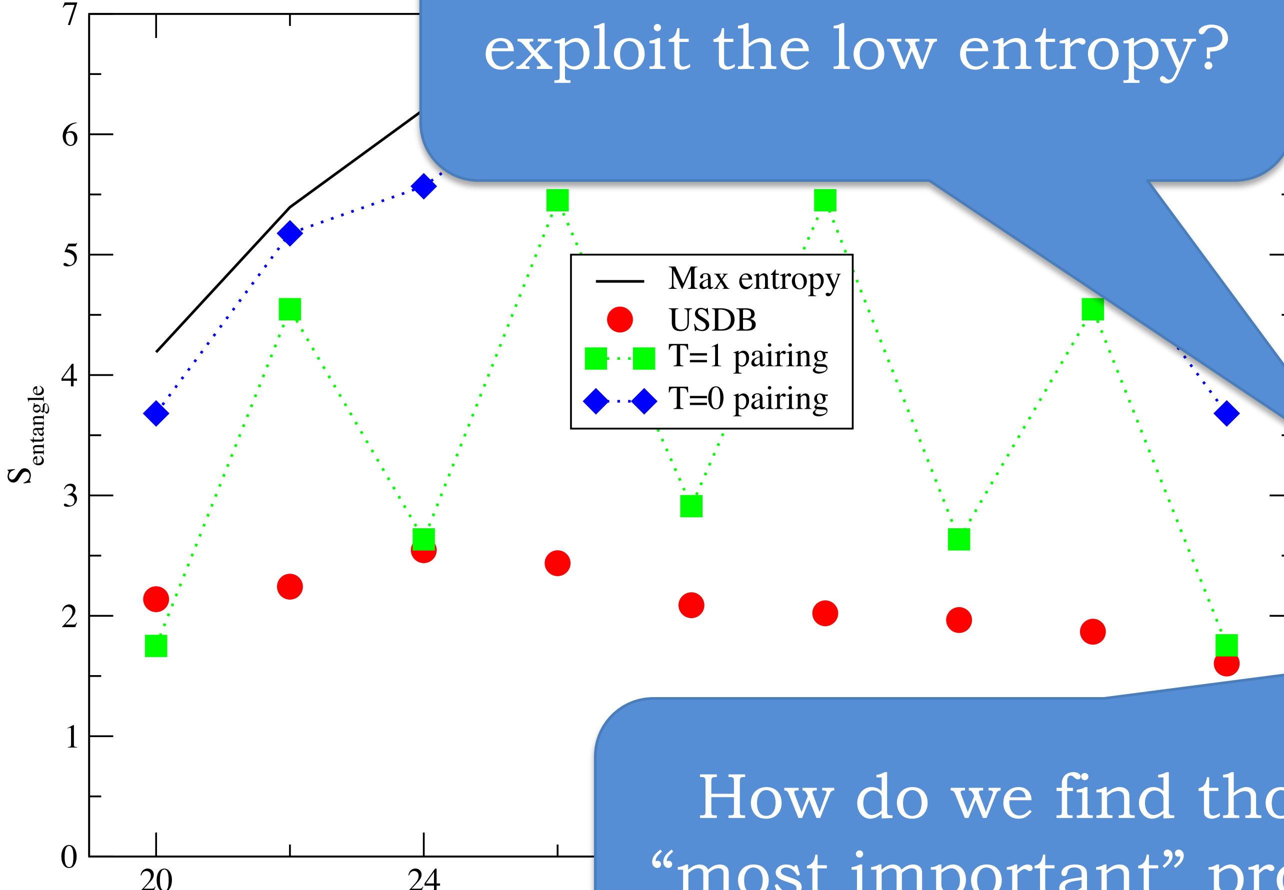
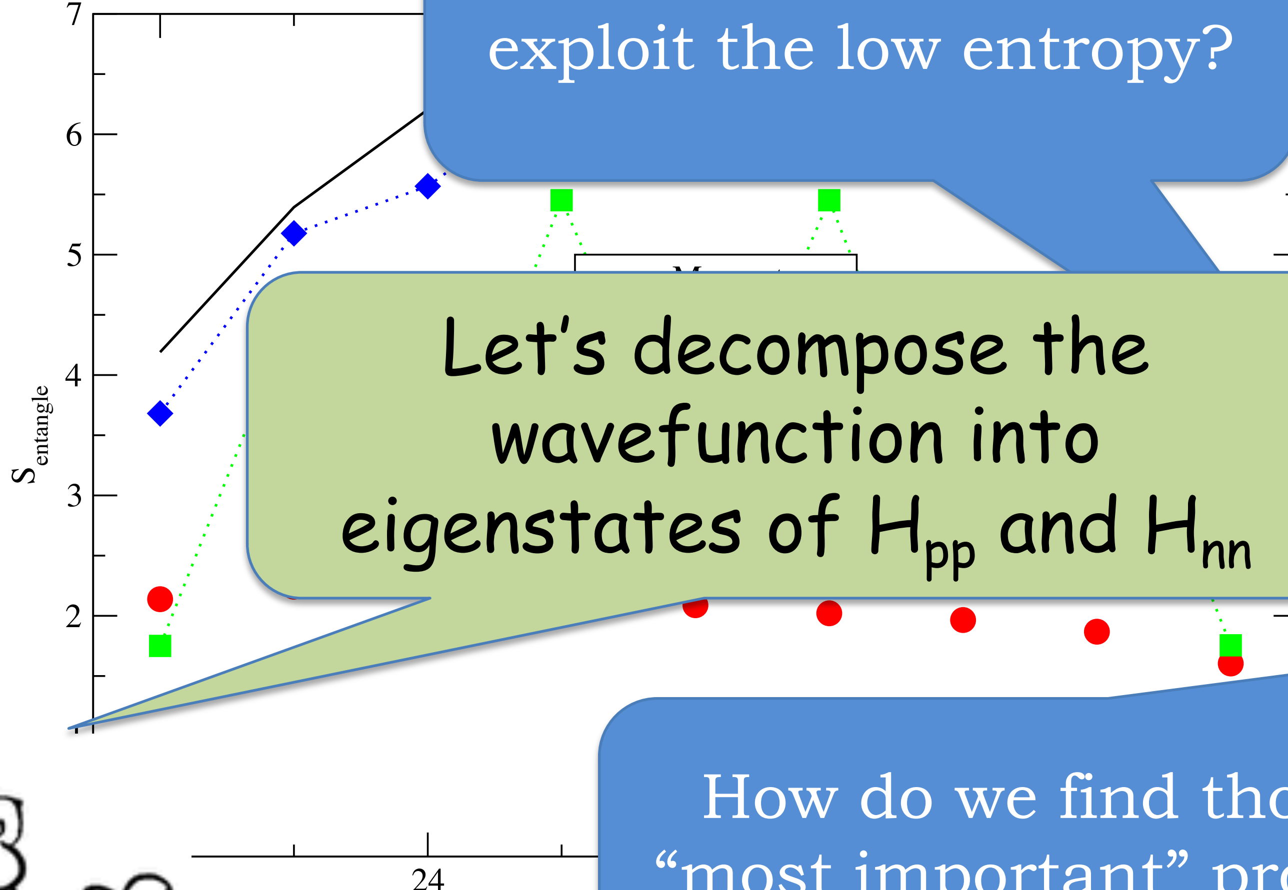


Z=N nuclei

How do we find those "most important" proton or neutron states?



Back to this: can we exploit the low entropy?

Let's decompose the wavefunction into eigenstates of H_{pp} and H_{nn}

Z=N nuclei

How do we find those "most important" proton or neutron states?

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Let's decompose the wavefunction into eigenstates of H_{pp} and H_{nn}

That is, take low-lying solutions of H_{pp} and H_{nn} and then project full solutions onto them

This will test if we can use the *low-lying* eigenstates of H_{pp} and H_{nn} as building blocks

Let's decompose the wavefunction into eigenstates of H_{pp} and H_{nn}

That is, take low-lying solutions of H_{pp} and H_{nn} and then project full solutions onto them

Test:

Solve $$\left(\mathbf{H}_{pp}+\mathbf{H}_{nn}+\mathbf{H}_{pn}\right)\left|\Psi_{full}\right\rangle = E\left|\Psi_{full}\right\rangle$$

then solve $$\mathbf{H}_{pp}\left|\Psi_{p}\right\rangle = E_{p}\left|\Psi_{p}\right\rangle \qquad \mathbf{H}_{nn}\left|\Psi_{n}\right\rangle = E_{n}\left|\Psi_{n}\right\rangle$$

Expand $$\left|\Psi_{full}\right\rangle = \sum_{p,n} c_{p,n}\left|\Psi_{p}\right\rangle \otimes \left|\Psi_{n}\right\rangle$$

and compute P(p) = $\left|\left\langle \Psi_{p} \middle| \Psi_{full}\right\rangle\right|^{2} = \sum_{n} C_{p,n}^{2}$

Test:

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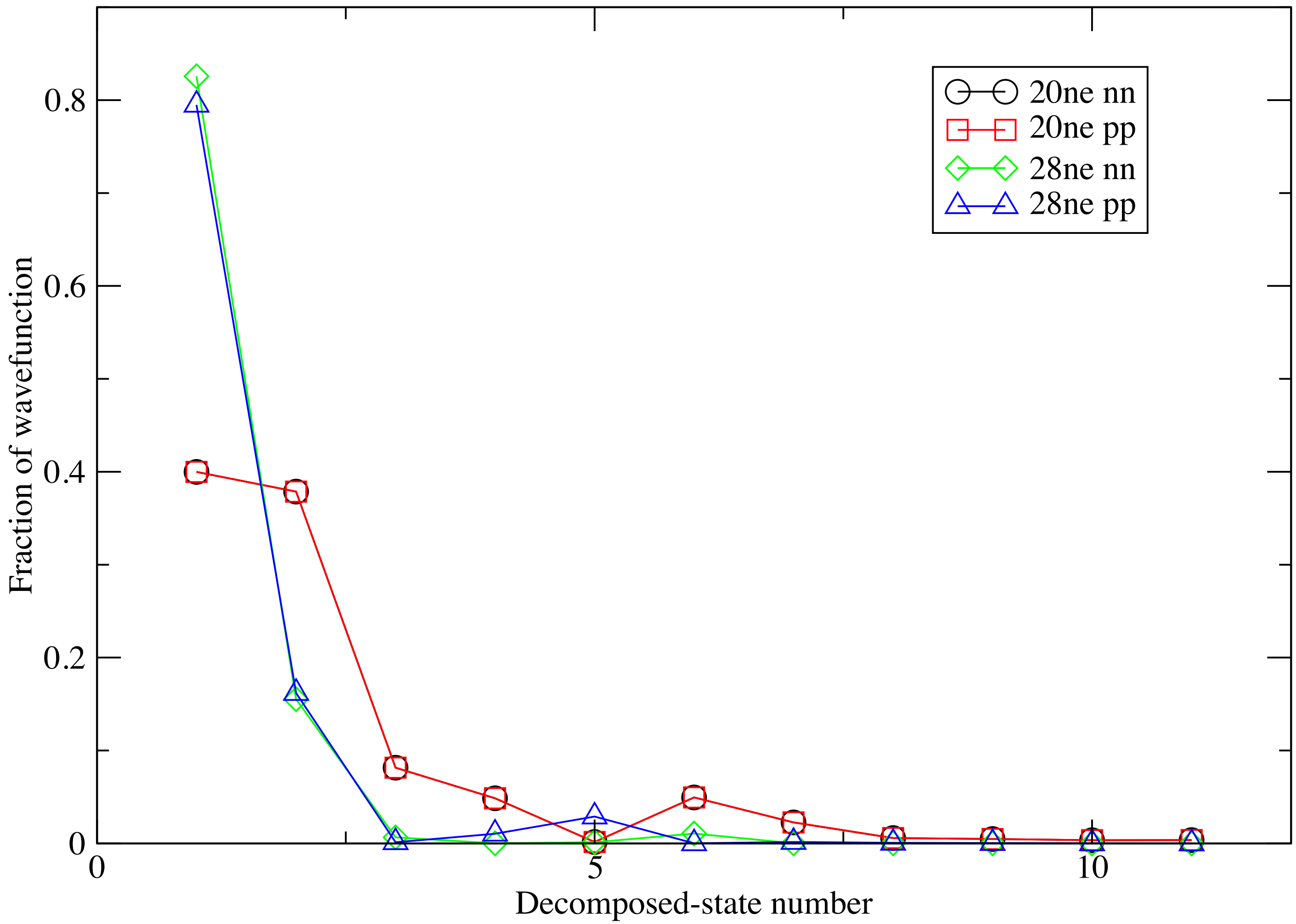
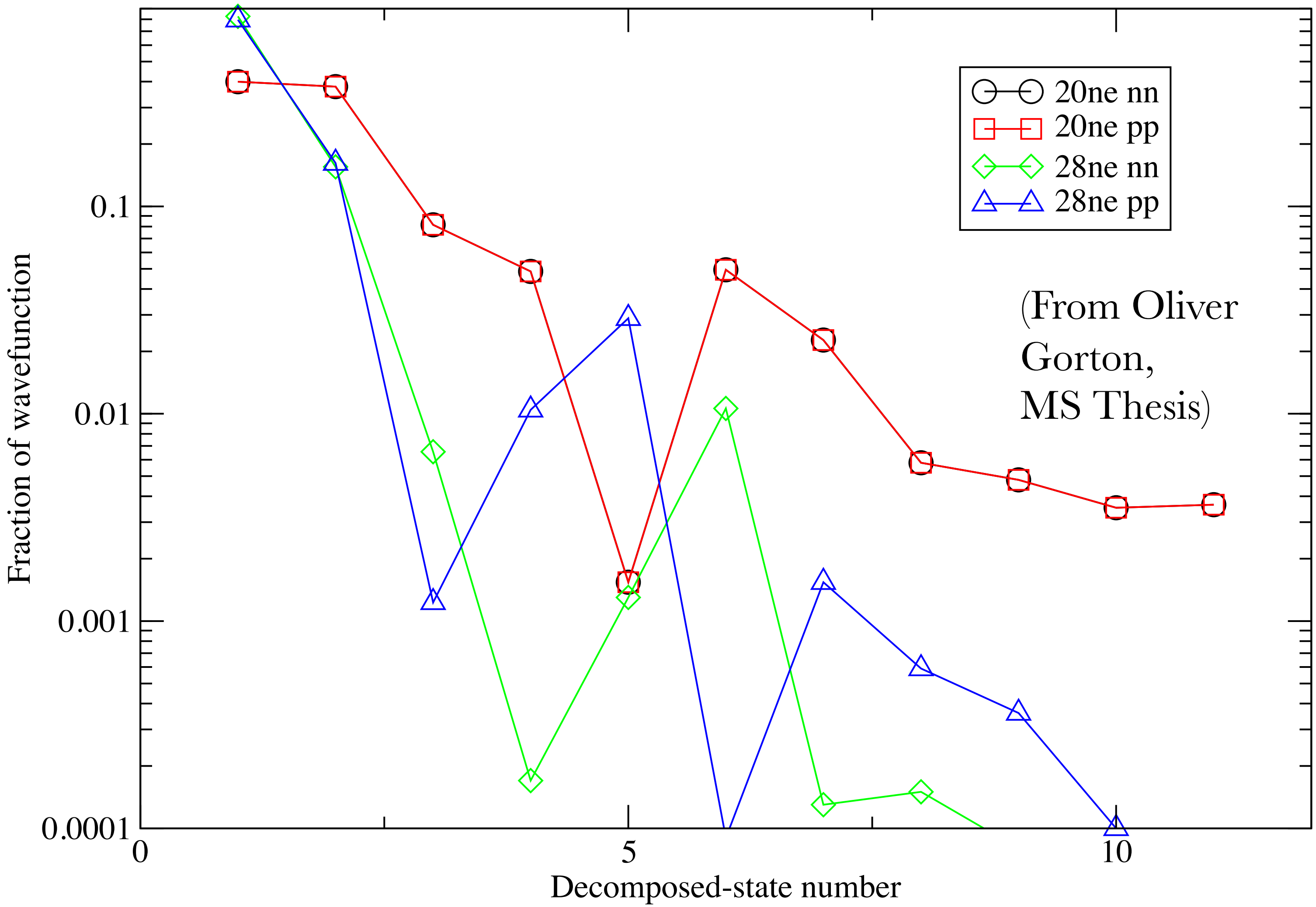
Solv

then s

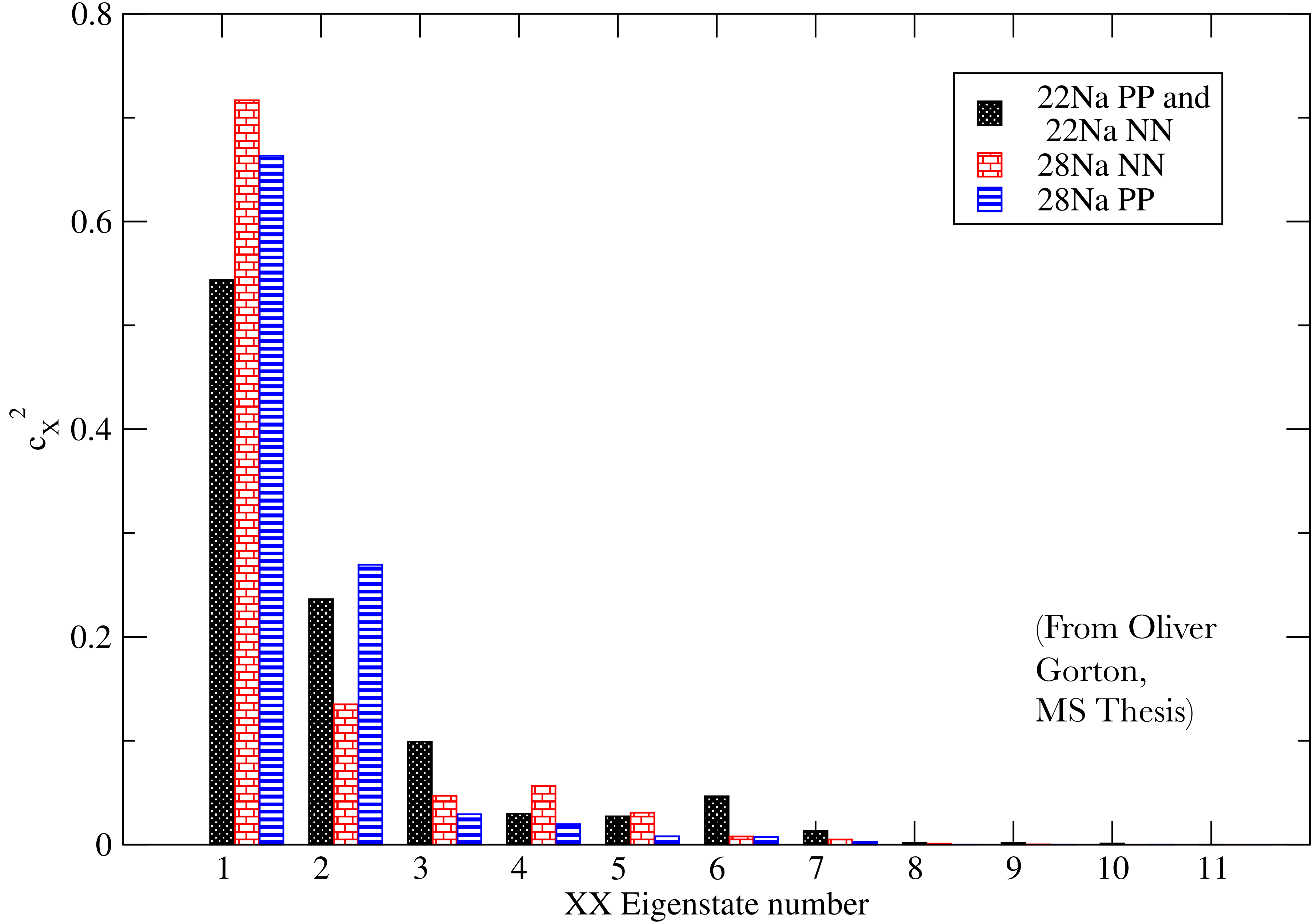
Expand $\left|\Psi_{full}\right\rangle = \sum_{p,n} c_{p,n} \left|\Psi_p\right\rangle \otimes \left|\Psi_n\right\rangle$

pute P(p) $= \left|\left\langle \Psi_p \middle| \Psi_{full} \right\rangle\right|^2 = \sum_n C_{p,n}^2$

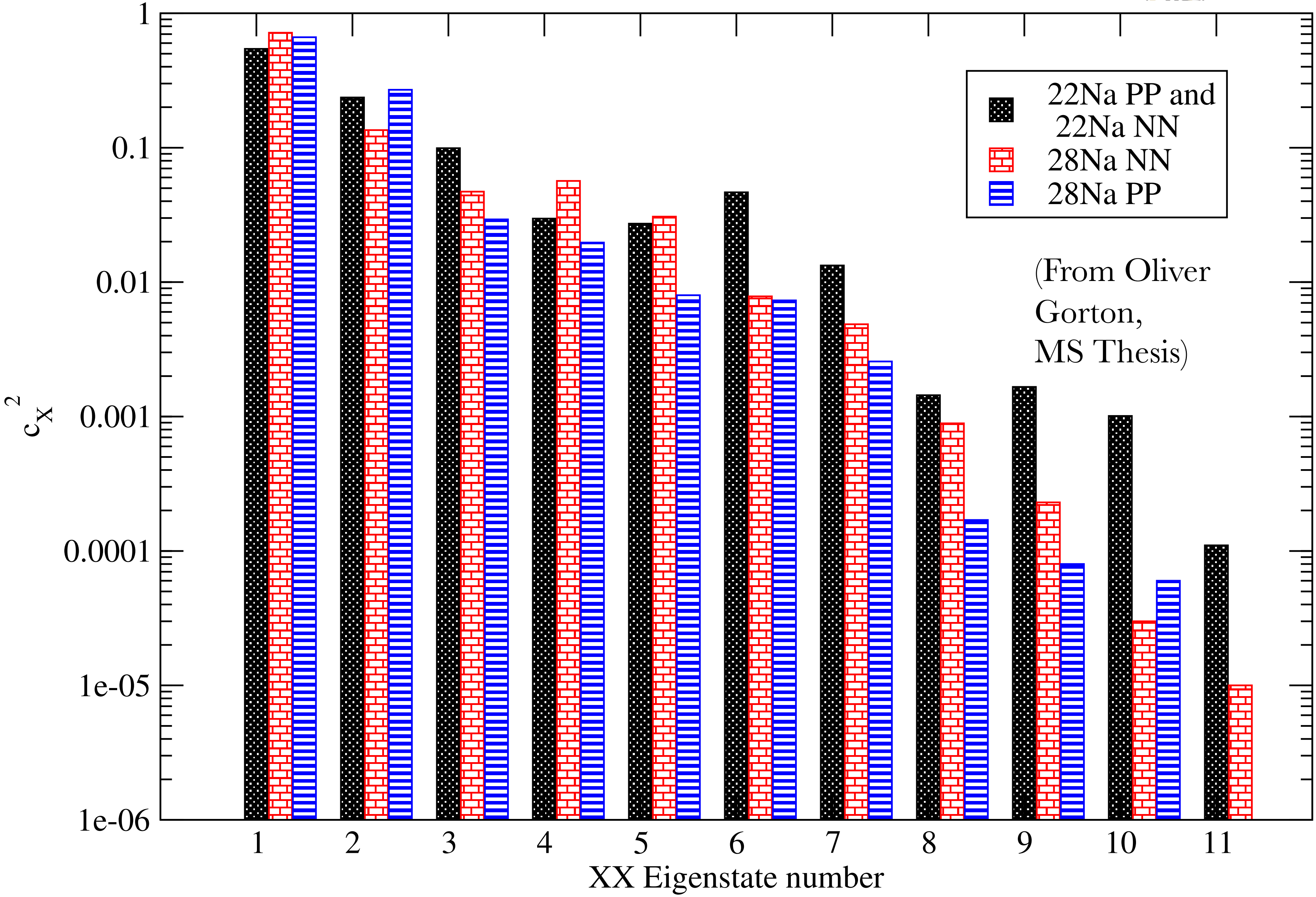

Fraction of wavefunction
Decomposed-state number
20ne nn
20ne pp
28ne nn
28ne pp
0
0.2
0.4
0.6
0.8
0
5
10



Fraction of wavefunction
Decomposed-state number
0.1
0.01
0.001
0.0001
0
5
10
20ne nn
20ne pp
28ne nn
28ne pp
(From Oliver Gorton, MS Thesis)



0.8
0.6
0.4
0.2
0
$c_X^{\ 2}$
1
2
3
4
5
6
7
8
9
10
11
XX Eigenstate number
22Na PP and
22Na NN
28Na NN
28Na PP
(From Oliver
Gorton,
MS Thesis)



ESNT workshop on PN Pairing + Quartets, Saclay, Sept 2019

We have written a code to take advantage of this (O. Gorton)

We want to find solutions to

$$\hat{H}\left|\Psi\right\rangle = E\left|\Psi\right\rangle \quad \text{where} \quad \hat{H} = \hat{H}_{pp} + \hat{H}_{nn} + \hat{H}_{p}$$

We solve $\hat{H}_{pp}\left|\Psi_p\right\rangle = E_p\left|\Psi_p\right\rangle \qquad \hat{H}_{nn}\left|\Psi_n\right\rangle = E_n\left|\Psi_n\right\rangle$

and choose certain $\left|\Psi_p\right\rangle\left|\Psi_n\right\rangle$ as basis for diagonalization;
our results with the entropy suggest we only need a few

PNISM = *proton-neutron interacting shell model*

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and choose certain $\left|\Psi_{p}\right\rangle\left|\Psi_{n}\right\rangle$ as basis for diagonalization;

our results with the entropy suggest we only need a few

Although BIGSTICK is an M-scheme code

$$\left|\Psi,M\right\rangle = \sum_{\mu\nu} c_{\mu\nu}\left|p_\mu,M_p\right\rangle\left|n_\nu,M_n = M - M_p\right\rangle$$

because $\mathbf{H}$ commutes with $\mathbf{J}^2$, the eigenstates have good J

$$\left|\Psi,JM\right\rangle = \sum_{\mu\nu} c_{\mu\nu}\left|p_\mu,M_p\right\rangle\left|n_\nu,M_n = M - M_p\right\rangle$$

This is true even if only protons or only neutrons

Using BIGSTICK we construct many-proton states of good J

$$\left|\Psi_p, J_p M\right\rangle = \sum_\mu c_\mu \left|p_\mu, M\right\rangle$$

and the same for many-neutron states; these we couple together in a *J-scheme* code with fixed *J* for basis:

$$\left|\Psi_J\right\rangle = \sum_{ab} c_{ab} \left[\left|\Psi_p a, J_p\right\rangle \otimes \left|\Psi_n b, J_n\right\rangle \right]_J$$

we find matrix elements of the Hamiltonian in basis of these states and diagonalize.

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Some "J-scheme" codes, such as NuShell(X), do this, but including *all* states.

...tes of good J

μ

and ...e for many-neutron states; these we couple to... in a *J-scheme* code with fixed *J* for basis:

$$\left|\Psi_J\right\rangle = \sum_{ab} c_{ab} \left[\left|\Psi_p a, J_p\right\rangle \otimes \left|\Psi_n b, J_n\right\rangle \right]_J$$

...rix elements of the Hamiltonian in basis of ... and diagonalize.

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Some "J-scheme" codes,
such as NuShell(X), do this,
but including *all* states.

…tes of good J

and…
to…

By truncating we hope to get
approximate solutions
in much larger spaces

…rix elements of the Hamiltonian in basis of
…and diagonalize.

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Technical details (if time allows)

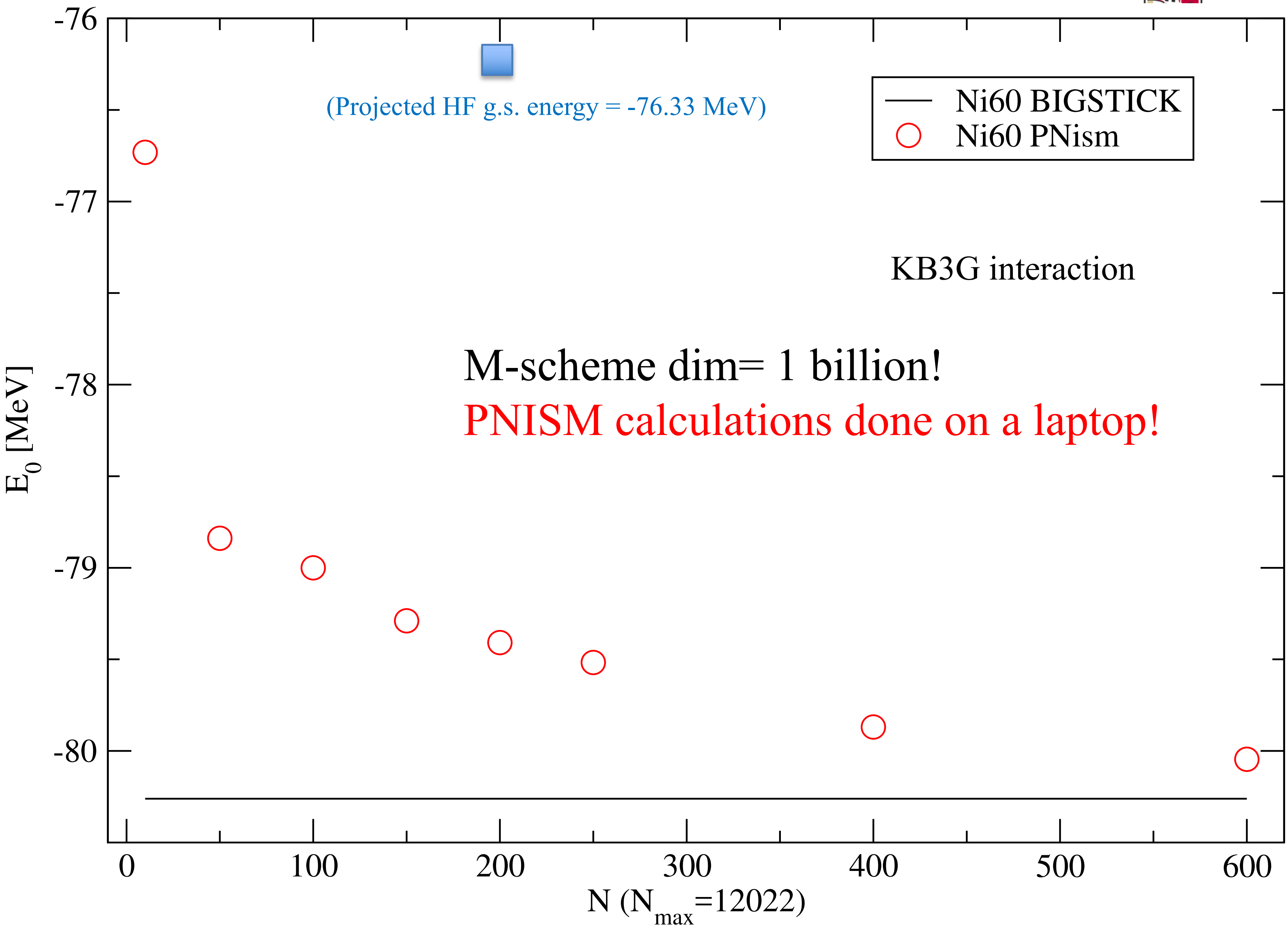
Let $\mathbf{H} = \mathbf{H}_{pp} + \mathbf{H}_{nn} + \mathbf{H}_{pn}$

BIGSTICK:
generate states $|a_p\rangle$, matrix elements $\langle a_p | \mathbf{H}_{pp} | a'_p \rangle$
and one body densities $\langle a_p | c^+_i c_j | a'_p \rangle$

generate states $|b_n\rangle$, matrix elements $\langle b_n | \mathbf{H}_{nn} | b'_n \rangle$
and one body densities $\langle b_n | c^+_i c_j | b'_n \rangle$

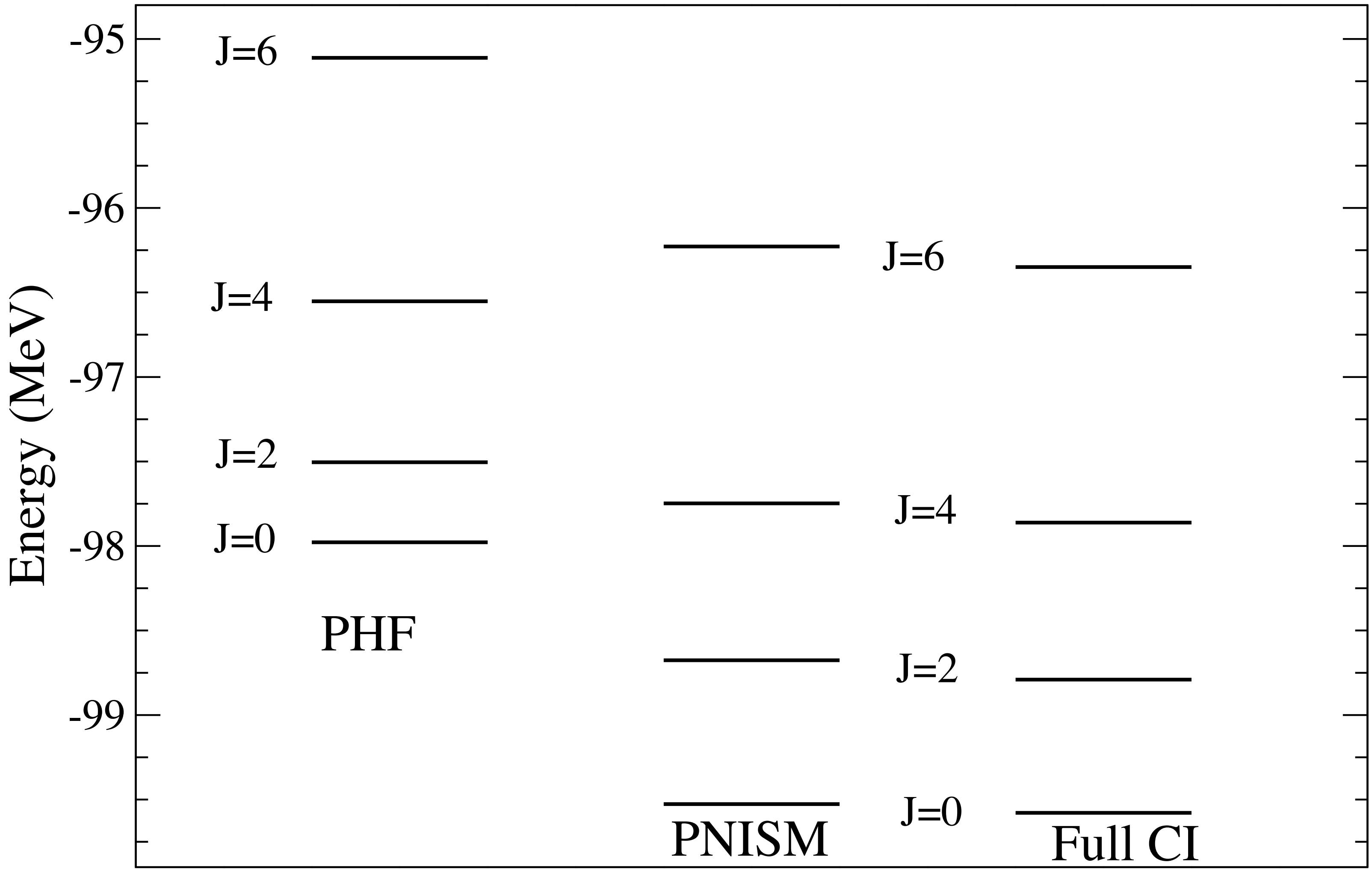
PNISM (*proton-neutron interacting shell model*)
read in the above and
generate matrix elements $\langle a_p , b_n | \mathbf{H}_{pn} | a'_p , b'_n \rangle$
using proton, neutron one-body densities

Diagonalize $\mathbf{H}_{pp} + \mathbf{H}_{nn} + \mathbf{H}_{pn}$ in truncated space.



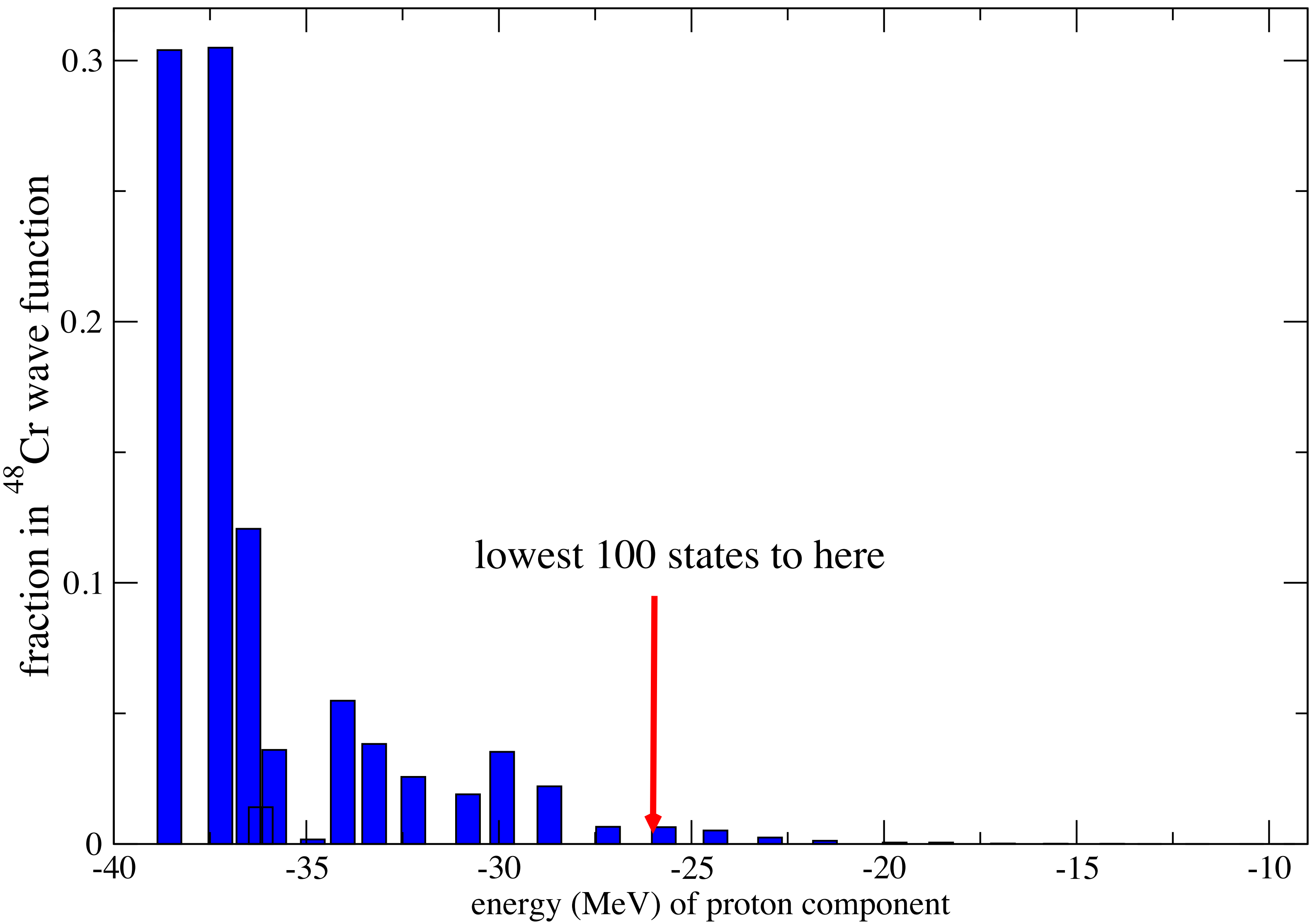
-76
-77
-78
-79
-80
E_0 [MeV]
0
100
200
300
400
500
600
N (N_{max}=12022)
(Projected HF g.s. energy = -76.33 MeV)
Ni60 BIGSTICK
Ni60 PNism
KB3G interaction
M-scheme dim= 1 billion!
PNISM calculations done on a laptop!

^{48}Cr, GX1A interaction



PNISM used 250 proton and 250 neutron states (out of 4845 each)

^{48}Cr, GX1A interaction



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We have yet to do applications, only "proof of principle."

Sample application:
shells between 50 and 82 ($0g_{7/2}$ 2s1d $0h_{11/2}$)

^{129}Cs: M-scheme dim 50 billion (haven't tried!)

Proton dimension: 14,677
Neutron dimension: 646,430

We have yet to do applications, only "proof of principle."

Crazy-difficult isotope:
shells between 50 and 82 ($0g_{7/2}$ 2s1d $0h_{11/2}$)

^{132}Nd: M-scheme dim 85 TRILLION

Proton dimension =Neutron dimension= 3.7 million

Summary:

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- We can use *entanglement entropy* to see how "simple" a wave function looks in some bipartite basis.

- In many shell-model codes, it is natural to look at entanglement between neutron and proton Slater determinants

- With realistic interactions, shell model wave functions look simpler (have lower entropy) than with many schematic interactions (pairing, QQ)

* Entropy is often systematically lower for $N \neq Z$

This is not only enlightening, it is useful:

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- Shell model codes are restricted in size of problem; how to go further?

- We build states using the “Weak-entanglement approximation” (WEA) for proton-neutron coupling.

* Looks promising—will try to apply to heavy nuclei

San Diego State University

Additional slides

It’s also important to know:

Computational burden is *not* primarily the dimension but is the # of nonzero Hamiltonian matrix elements.

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}$$

J-scheme matrices are smaller but much denser than M-scheme, and "symmetry-adapted" (i.e. SU(3)) matrices are smaller still.

example: ^{12}C $N_{max} = 8$

scheme	basis dim	# of nonzero matrix elements
M	0.6×10^9	5×10^{11}
J (J=4)	9×10^7	3×10^{13}
SU(3)	9×10^6	2×10^{12}

(truncated)

It's also important to know:

Computational burden is *not* primarily the dimension but is the # of nonzero Hamiltonian matrix elements.

BIGSTICK's factorization algorithm is less efficient for N_{max} calculations than for complete spaces.
e.g. ^{51}Cr (dim 28 million) requires 0.4 Gb but
^{12}C $N_{max} = 6$ dim 30 million requires 6 Gb!

to store the nonzero matrix elements would require ~ 150 Gb!

Example of entanglement entropy: good angular momentum

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Consider 2 spin-1/2 particles:

$$\left|\uparrow\uparrow\right\rangle, \left|\uparrow\downarrow\right\rangle, \left|\downarrow\uparrow\right\rangle, \left|\downarrow\downarrow\right\rangle$$

Example of entanglement entropy: good angular momentum

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Consider 2 spin-1/2 particles:

$$\left|\uparrow\uparrow\right\rangle, \left|\uparrow\downarrow\right\rangle, \left|\downarrow\uparrow\right\rangle, \left|\downarrow\downarrow\right\rangle$$

Consider total J=0 state: $\left|J=0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

then

$$\mathbf{C} = \begin{pmatrix} 0 & +\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

and

$$\rho_{\mu\mu'} = \sum_{\nu} c_{\mu\nu} c_{\mu'\nu}$$

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Example of entanglement entropy: good angular momentum

Consider total J=0 state: $\left|J=0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

then $\mathbf{c} = \begin{pmatrix} 0 & +\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$ and $\rho_{\mu\mu'} = \sum_{\nu} c_{\mu\nu} c_{\mu'\nu}$

or $\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

Note trace ρ = 1.

Example of entanglement entropy: good angular momentum

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Consider total J=0 state: $\left|J=0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

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or $\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

Then entropy $S = \ln 2$, which is the maximum.

Note trace ρ = 1.

Example of entanglement entropy: good angular momentum

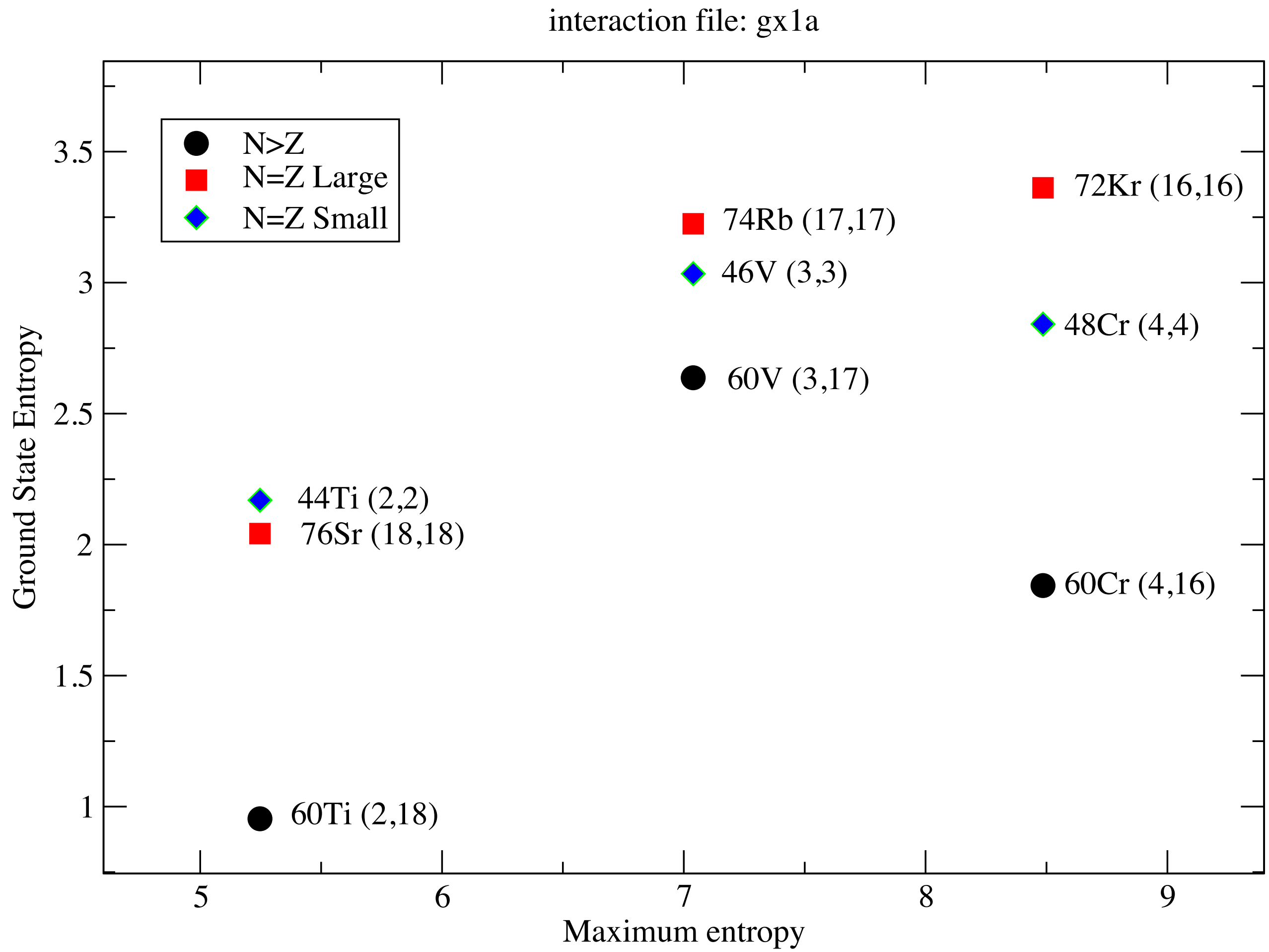
Conversely, $$\left|J=1, M=1\right\rangle = \left|\uparrow\uparrow\right\rangle$$

has $$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and $$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$ Then entropy $S = 0$.

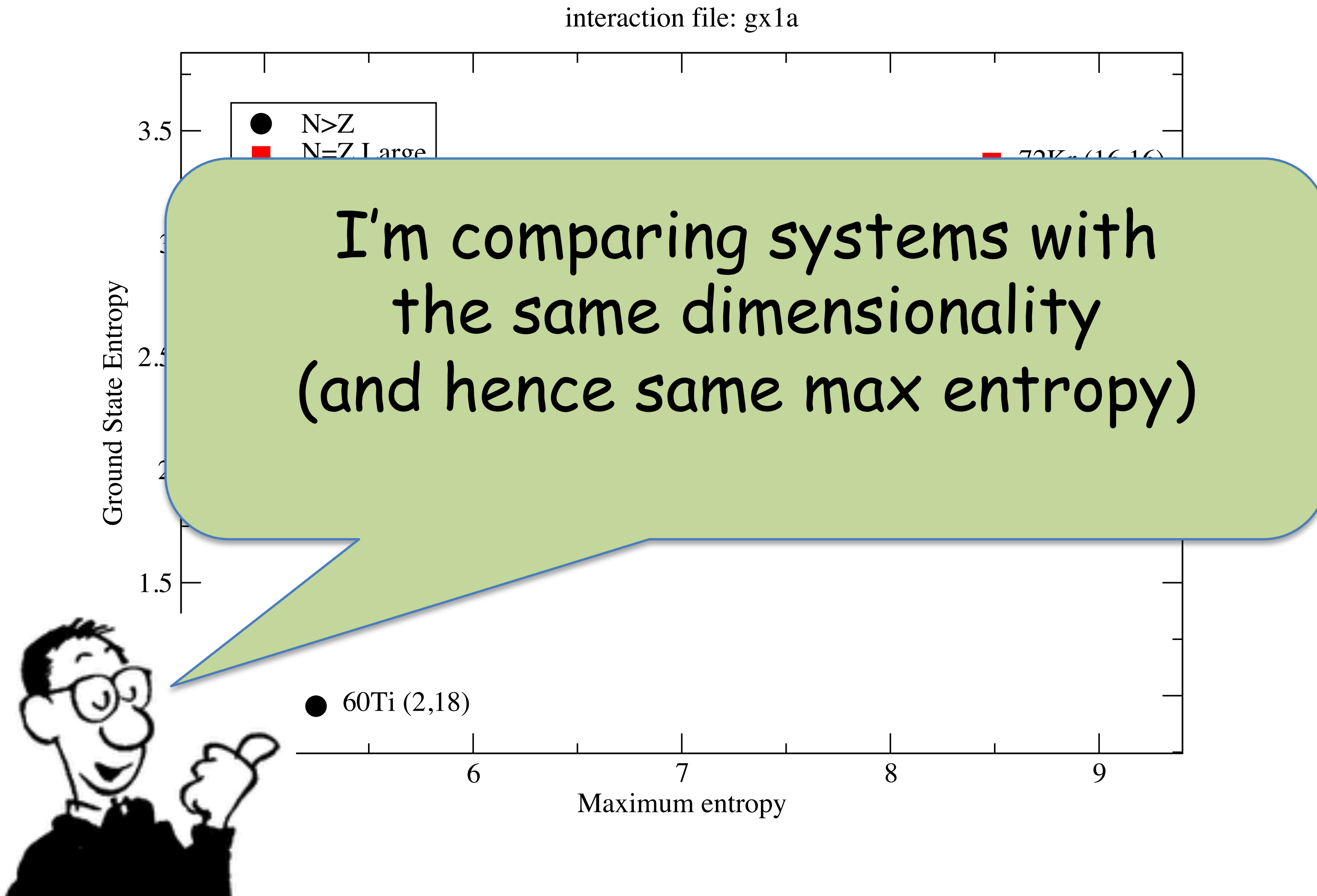
Note trace $\rho = 1$.

Entanglement entropy for ph conjugate triplets in the 40Ca-core space



(From Oliver Gorton, MS Student)

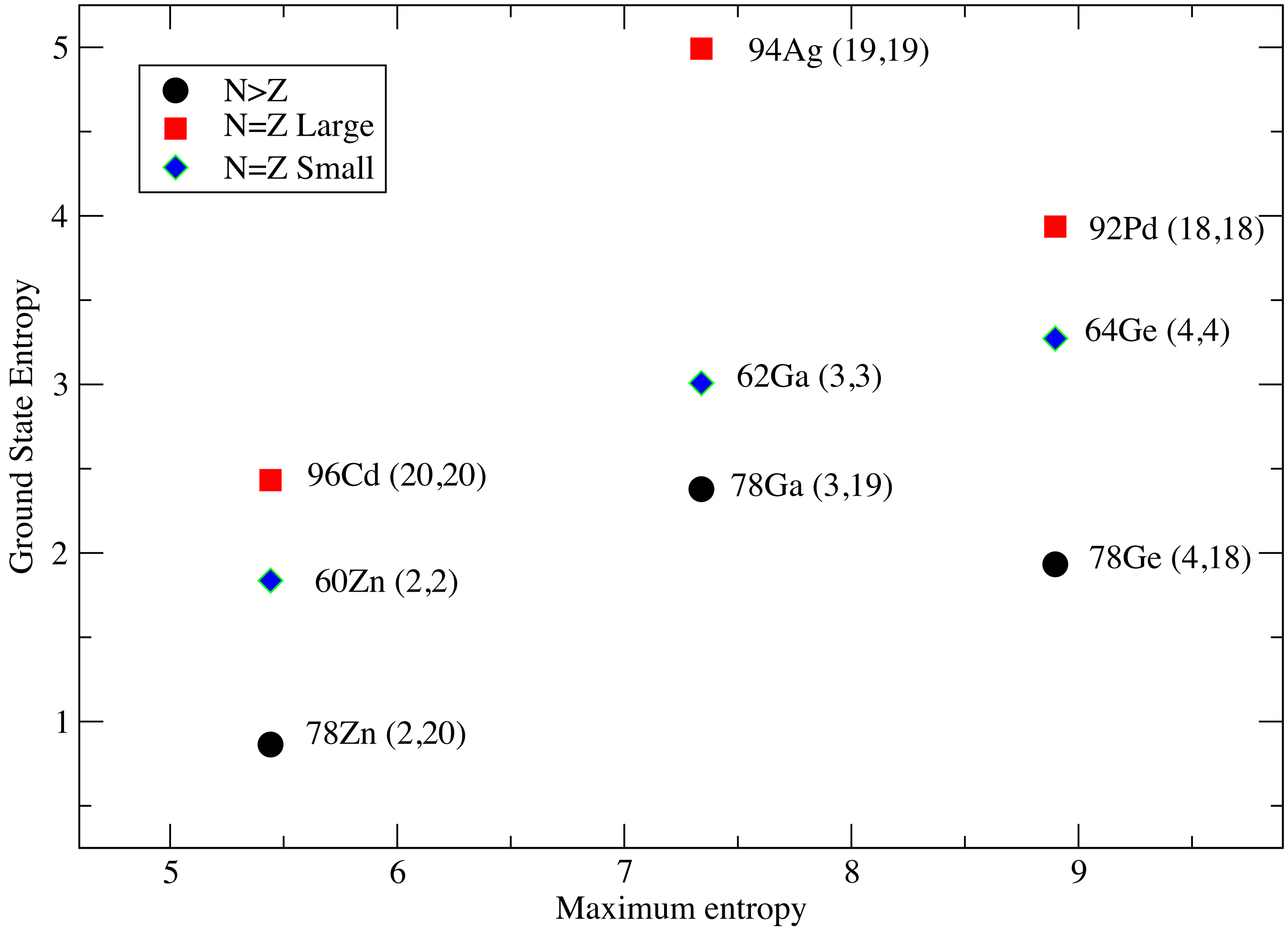
Entanglement entropy for ph conjugate triplets in the 40Ca-core space



om Oliver
rton,
Student)

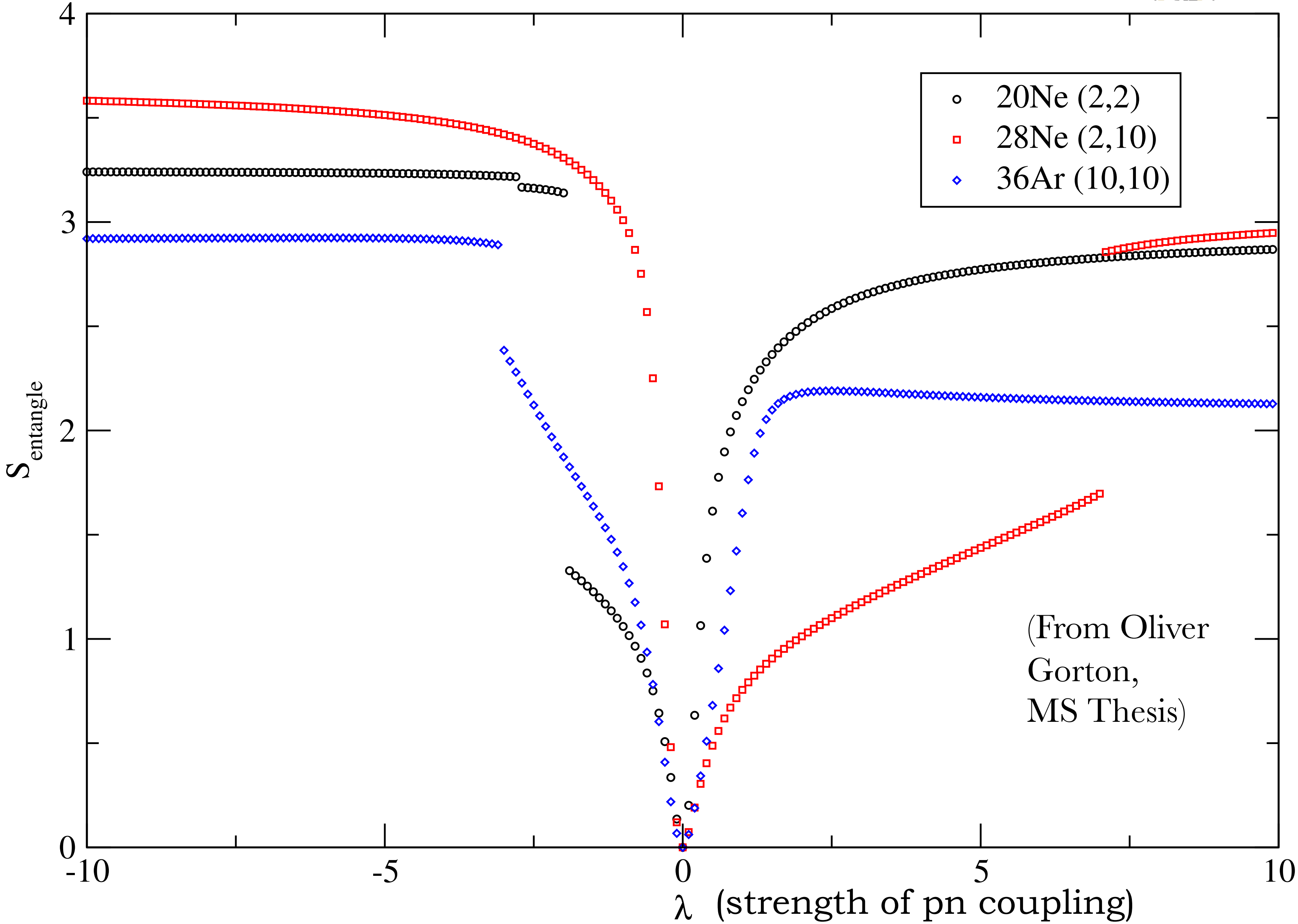
Entanglement entropy for ph conjugate triplets in the 56Ni-core space

interaction file: jun45

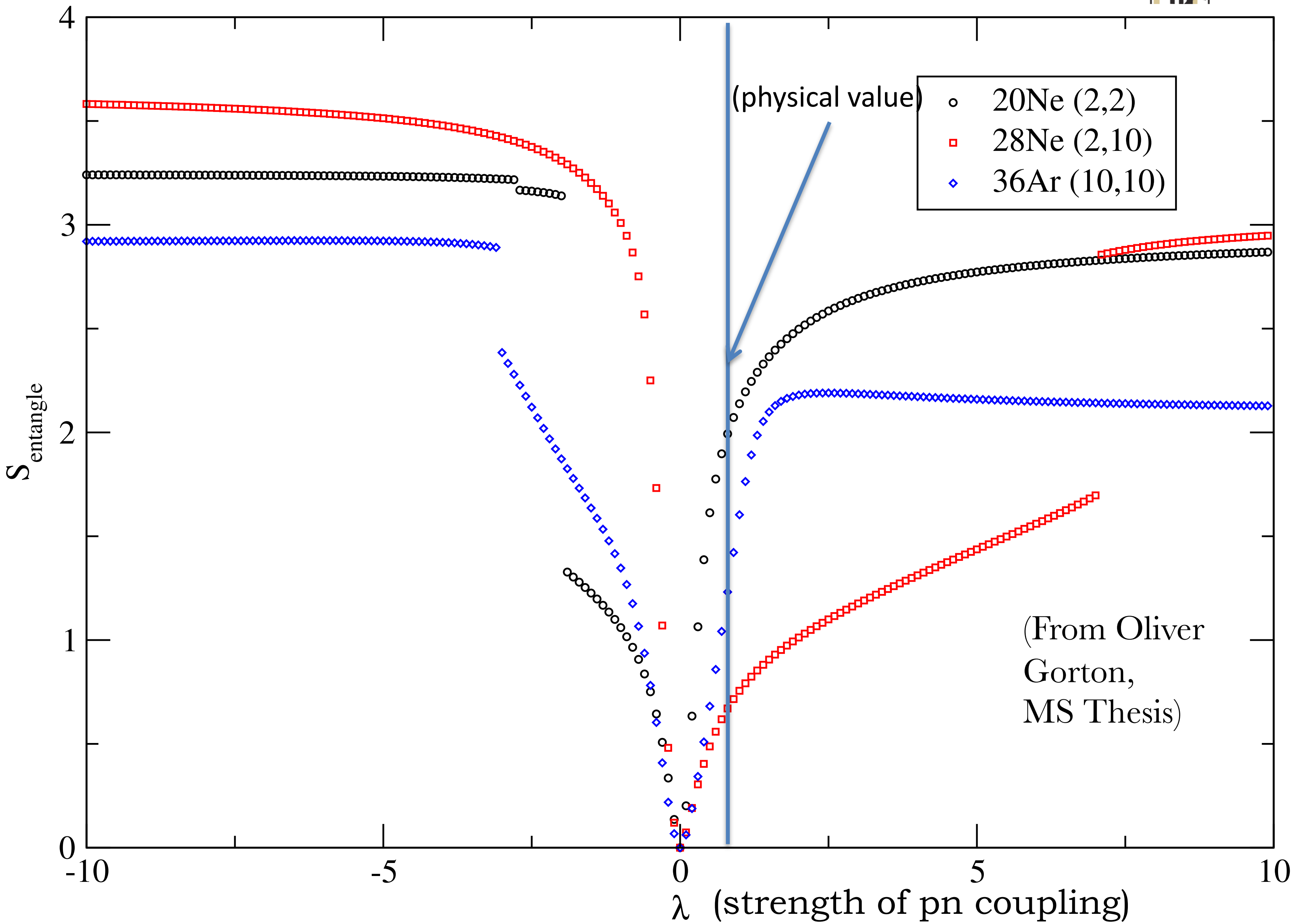


(From Oliver Gorton, MS Student)

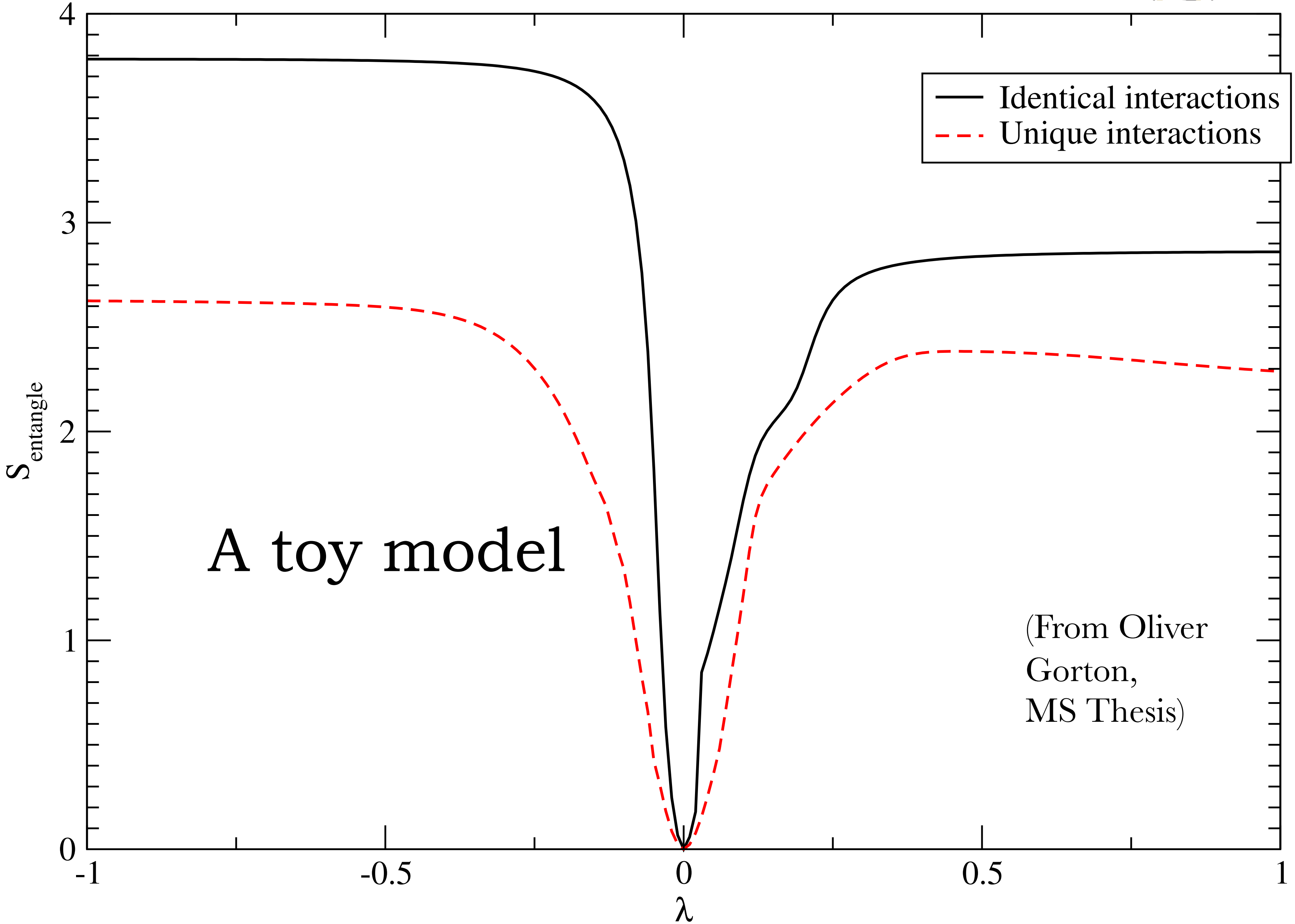
Let's see what happens as we change the strength of the proton-neutron coupling



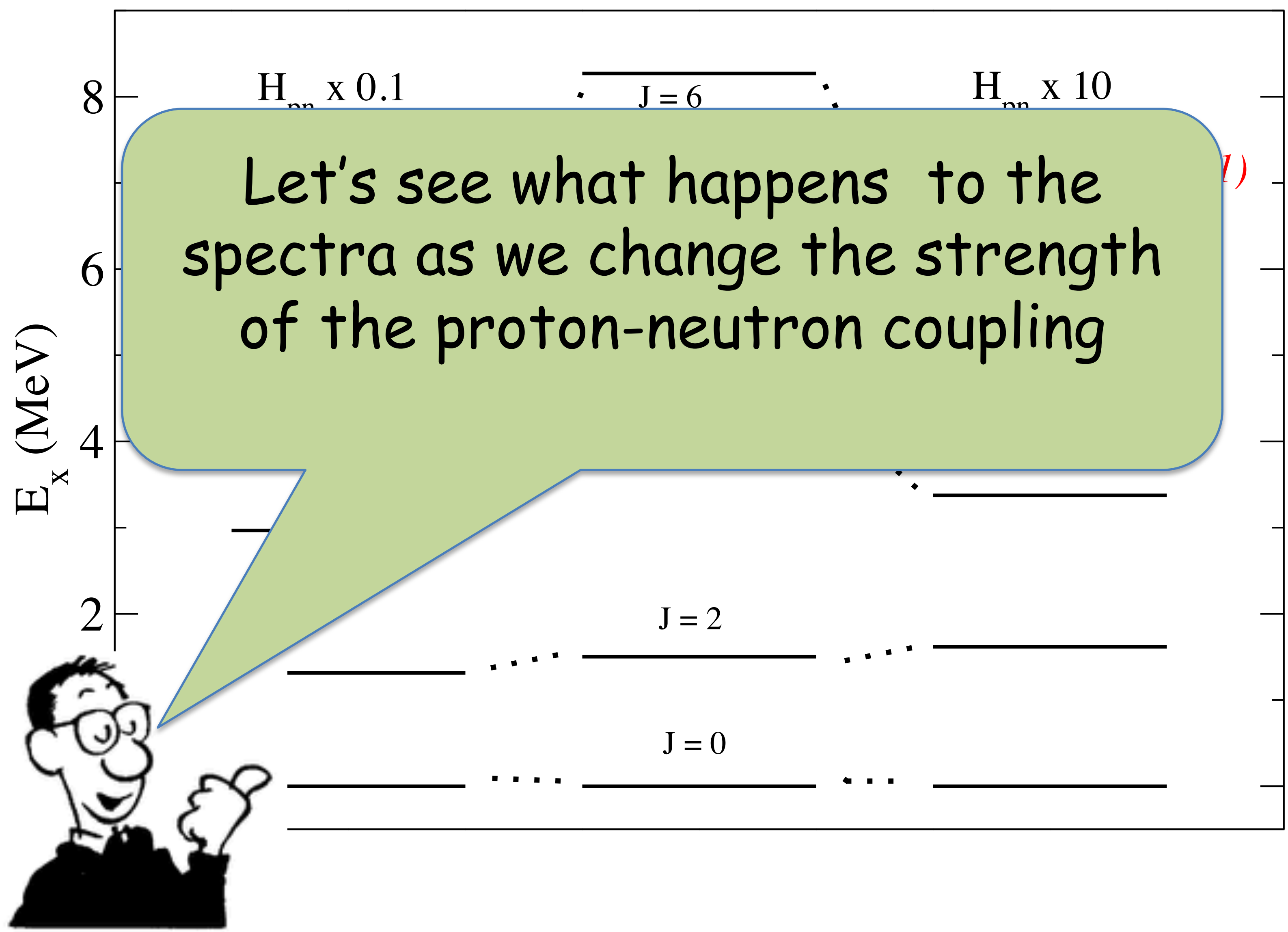
4
3
2
1
0
-10
-5
0
5
10
$S_{entangle}$
λ (strength of pn coupling)
20Ne (2,2)
28Ne (2,10)
36Ar (10,10)
(From Oliver Gorton, MS Thesis)



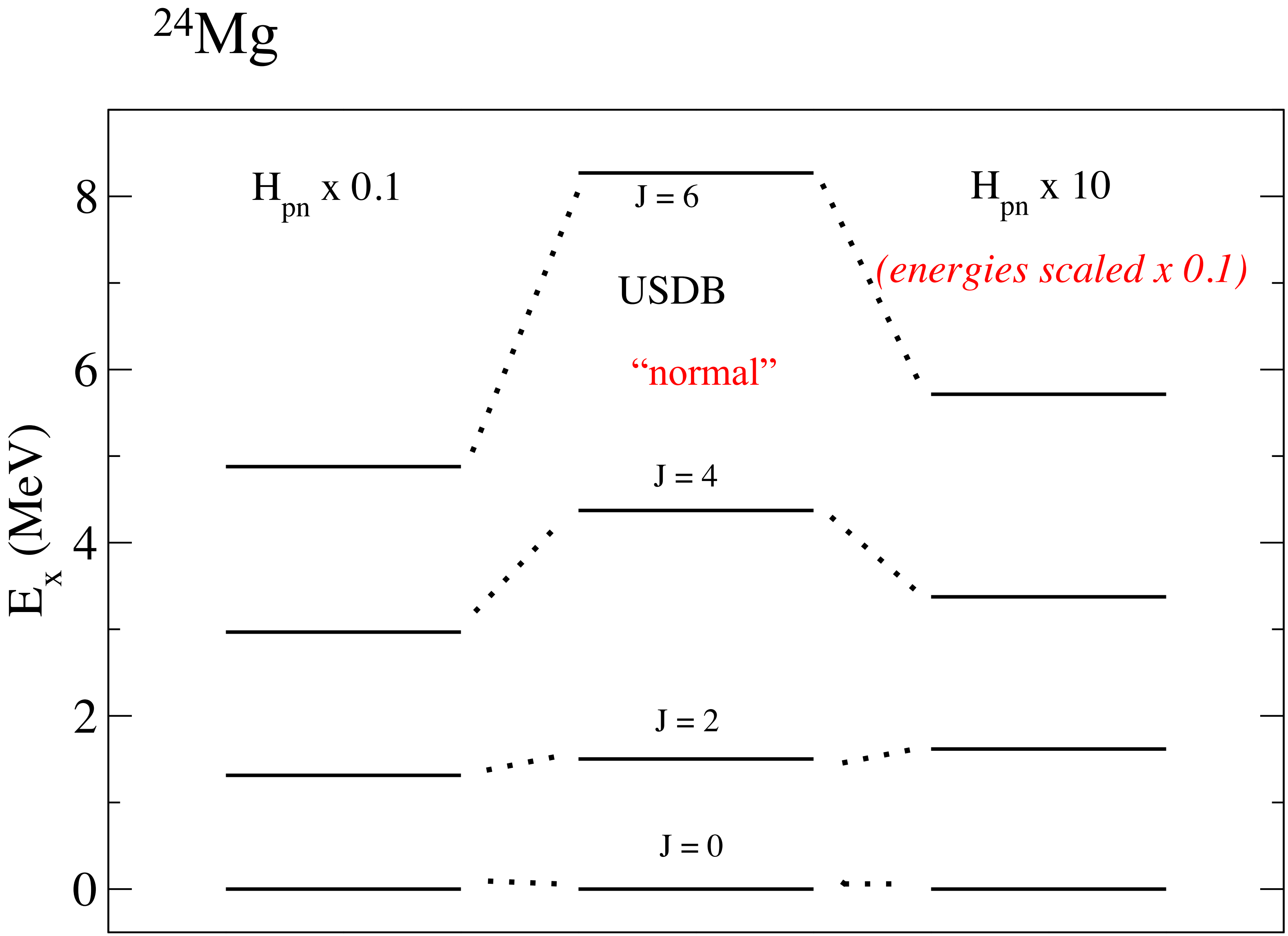
(physical value)
20Ne (2,2)
28Ne (2,10)
36Ar (10,10)
(From Oliver Gorton, MS Thesis)
$S_{entangle}$
λ (strength of pn coupling)


Identical interactions
Unique interactions
A toy model
(From Oliver Gorton, MS Thesis)
$S_{entangle}$
λ
0
1
2
3
4
-1
-0.5
0
0.5
1

^{24}Mg

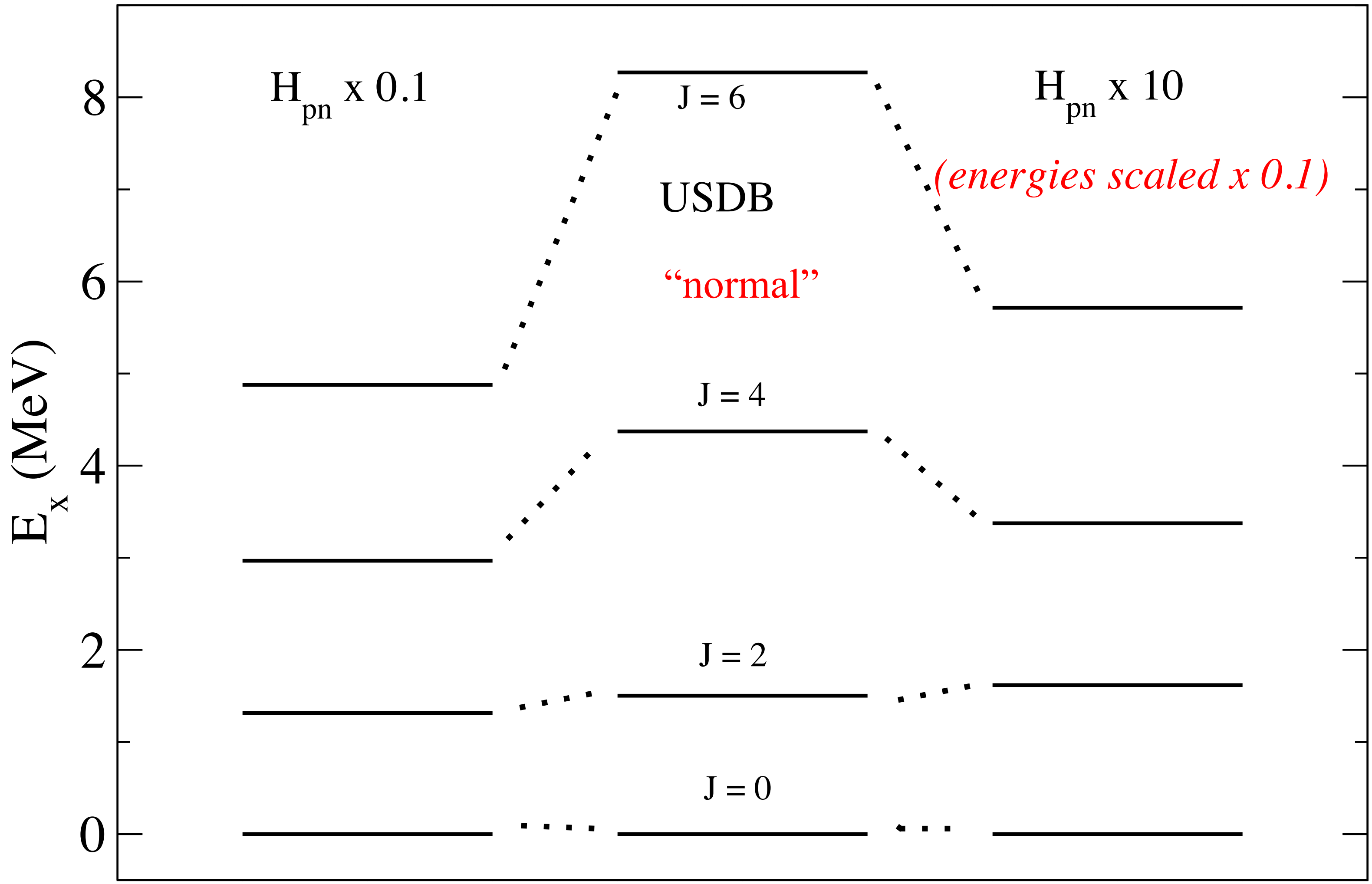


Let's see what happens to the spectra as we change the strength of the proton-neutron coupling
H_{pn} x 0.1
H_{pn} x 10
J = 6
J = 2
J = 0
E_x (MeV)
8
6
4
2



^{24}Mg
E_x (MeV)
0
2
4
6
8
H_{pn} x 0.1
USDB
"normal"
H_{pn} x 10
(energies scaled x 0.1)
J = 0
J = 2
J = 4
J = 6

^{24}Mg



Overlap probability between states ~ 40-60%