Particle-number projection in HFB and BCS
From heavy nuclei to neutron matter

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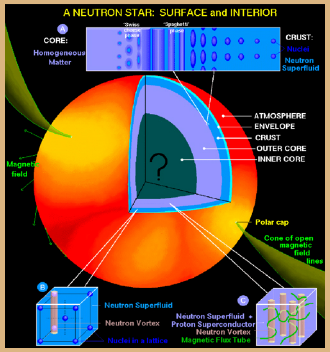
UNIVERSITY of GUELPH

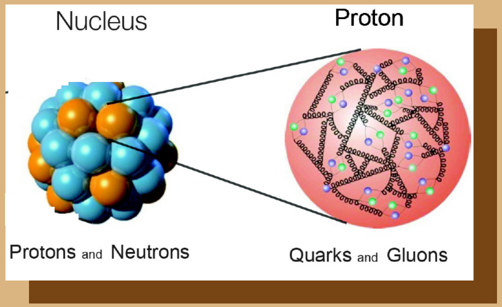
Recent advances on proton-neutron pairing Workshop
ESNT Saclay
September 3, 2019

Systems

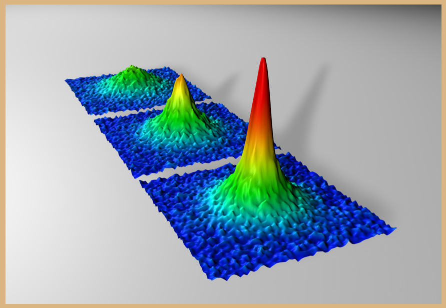


Credit: Dany Page

Neutron stars



Nuclei

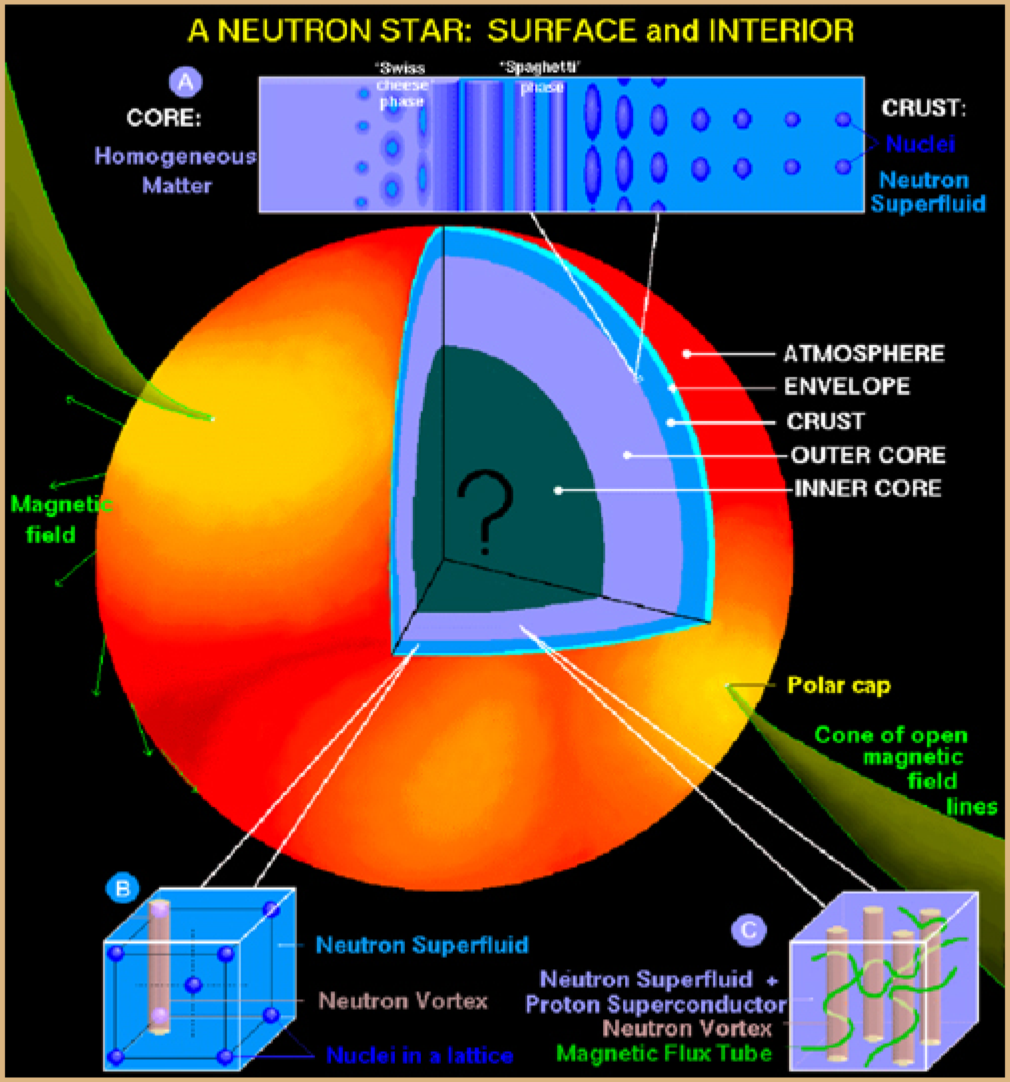


Credit: University of Colorado

Ultracold atomic gases

Systems: neutron stars

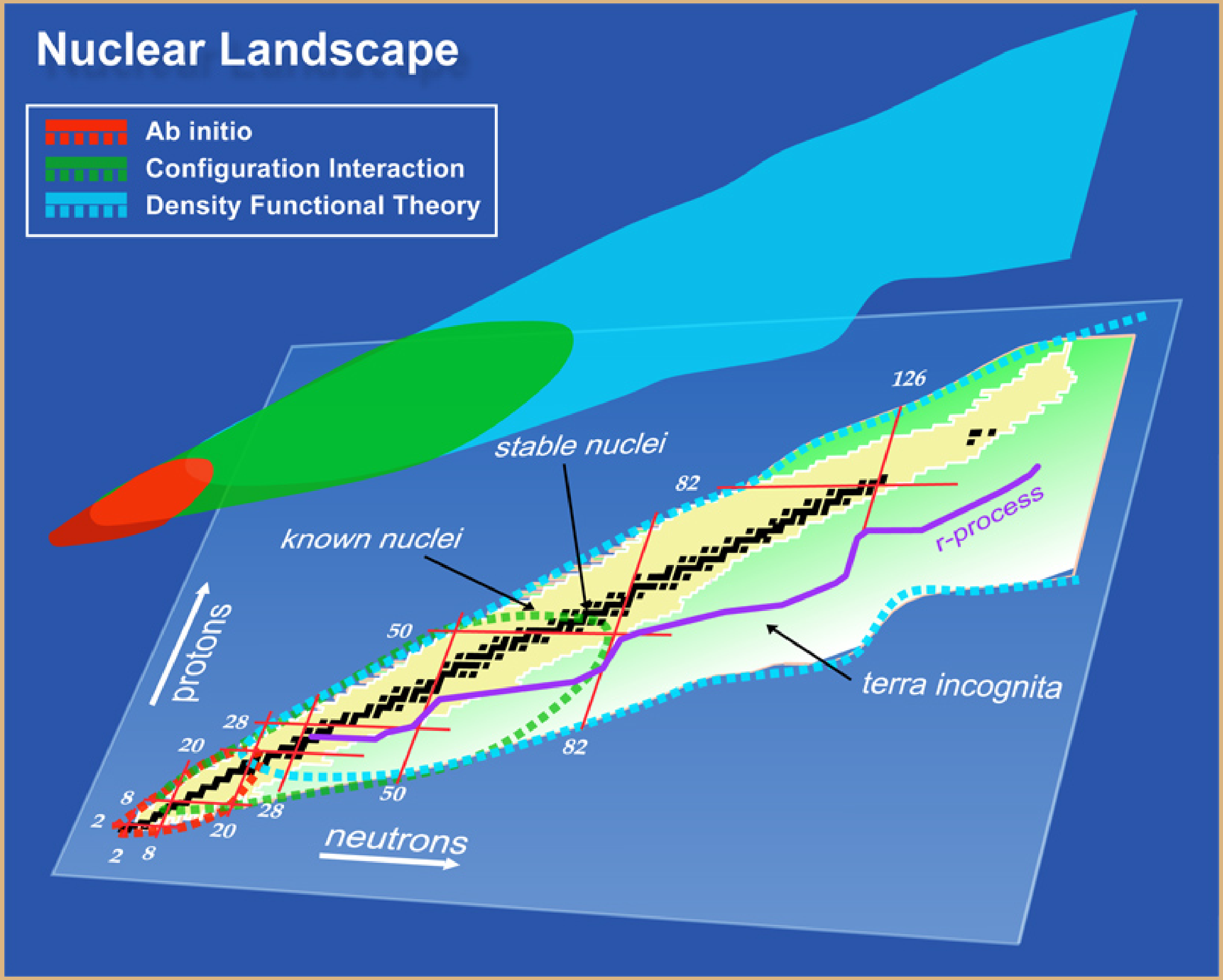
Neutron stars as ultra-dense matter laboratories



Credit: Dany Page

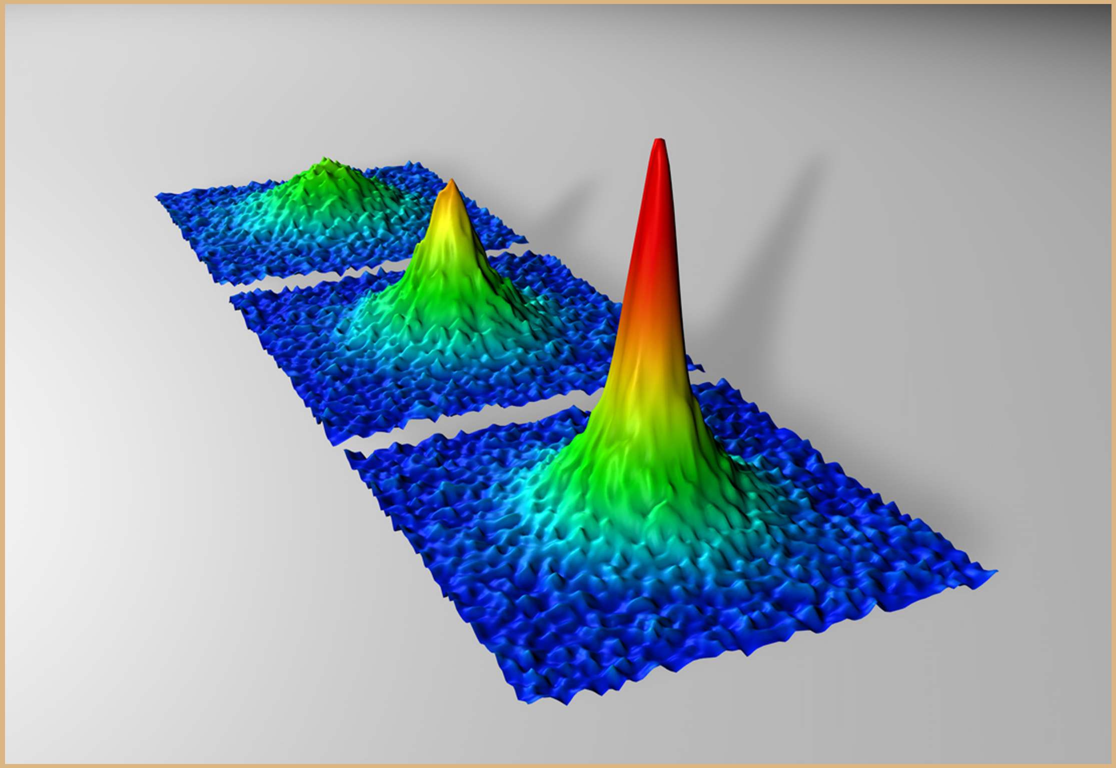
- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, i.e., not experimentally accessible
- Can we describe neutron-star matter *from first principles*?
- What does *first* mean?

Systems: nuclei



- Experimental facilities continue to push the envelope
- Using complicated many-body methods we can try to "build nuclei from scratch"
- No universal theoretical method exists (yet?)
- Regions of overlap between different methods are crucial
- Goal is to work to study nuclei from first principles (when possible)

Systems: cold atoms



Credit: University of Colorado

- Starting in the 1990s, it became possible to experimentally probe degenerate bosonic atoms (beyond ^4He)
- Starting in the 2000s, the same happened for fermionic atoms (beyond ^3He)
- These are very cold and strongly interacting (as well as strongly correlated)
- Can be used to simulate other systems, investigating pairing, polarization, polaron physics, many species, reduced dimensionality

Key questions

1. **What is the nature of the nuclear force that binds protons and neutrons into stable and rare isotopes?**

FRIB: Opening New Frontiers in Nuclear Science (2012), also LRP and NRC dec.

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2. **What is the origin of simple patterns in complex nuclei?**

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Key questions

1. **What is the nature of the nuclear force that binds protons and neutrons into stable and rare isotopes?**

2. **What is the origin of simple patterns in complex nuclei?**

3. **How did visible matter come into being and how does it evolve?**

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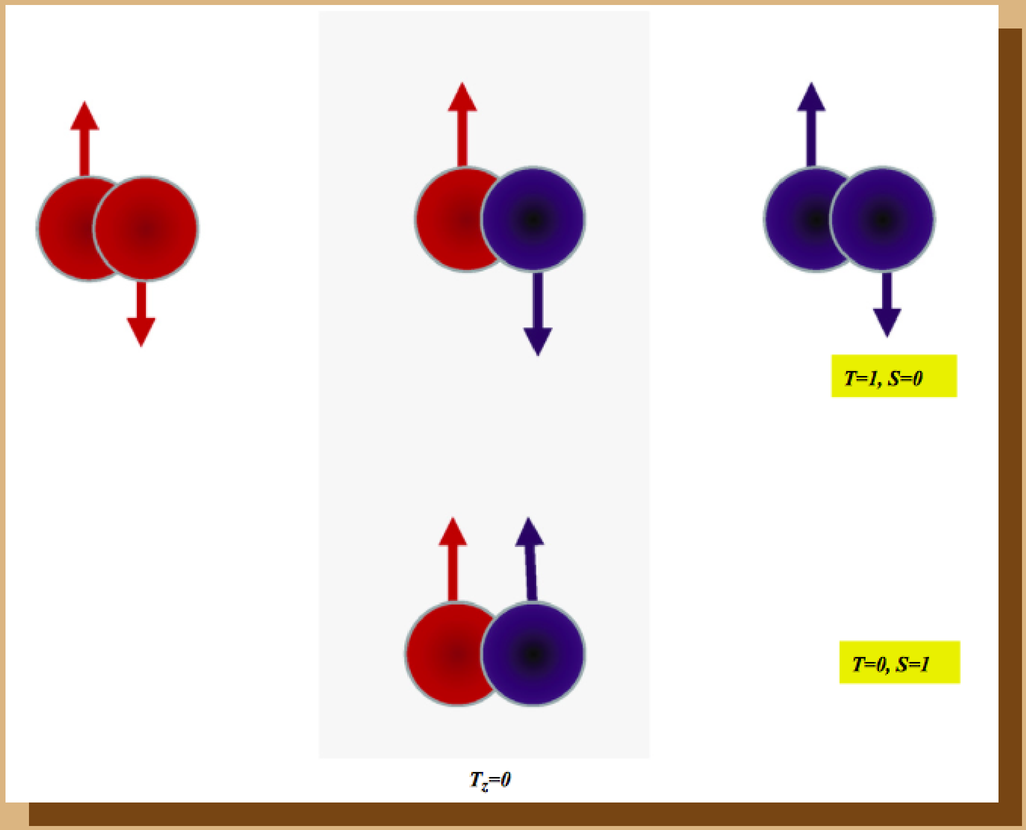
Types of pairs in nuclei



S. Frauendorf and A. O. Macchiavelli, Prog. Nucl. Part. Phys. **78**, 24 (2014)

Deuteron-like pairing

central to this workshop

- *np* interaction stronger than *nn* and *pp* interaction (*there is no bound dineutron in vacuum*)
- However, known nuclei exhibit *nn* and *pp* pairing.

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- However, known nuclei exhibit *nn* and *pp* pairing.

Possible answer I:

- Isospin polarization discourages spin-triplet pairing: look at N=Z nuclei

A. L. Goodman, Phys. Rev. C **58**, R3051 (1998)

A. O. Macchiavelli *et al.*, Phys. Rev. C **61**, 041303(R) (2000)

R. Chasman, , Phys. Lett. B **524**, 81 (2002)

Possible answer II:

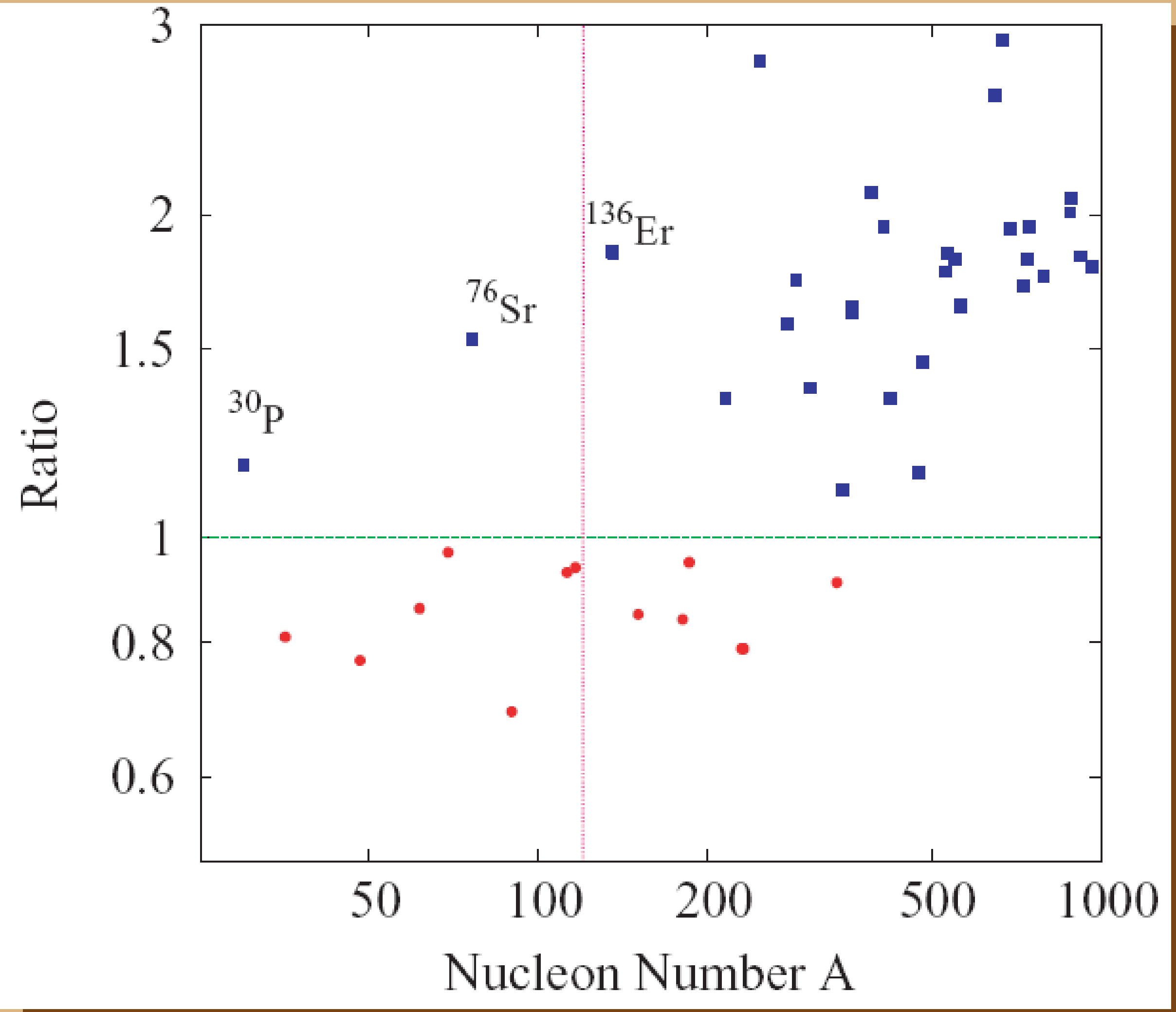
- Spin-orbit field interferes with spin-triplet pairing more: look at heavy nuclei

A. Poves and G. Martinez-Pinedo, Phys. Lett. B **430**, 203 (1998)

G. F. Bertsch and Y. L. Luo, Phys. Rev. C **81**, 064320 (2010)

S. Baroni, A. O. Macchiavelli, A. Schwenk, Phys. Rev. C 8**1**, 064308 (2010)

Motivation: Model for *N*=*Z*



Correlation energies

- Larger than one: spin-triplet
- Less than one: spin-singlet
- Vertical line: proton drip
- Spin-orbit influence mitigated for nuclei that are unrealistically large

G. F. Bertsch and Y. L. Luo, Phys. Rev. C **81**, 064320 (2010)

Hamiltonian

$$\hat{H} = \sum_i \langle i|H_{sp}|j\rangle a_i^\dagger a_j + \sum_{i>j,k>l} \langle ij|v|kl\rangle a_i^\dagger a_j^\dagger a_l a_k$$

- H_{sp} : kinetic + potential well + spin-orbit
- $\langle ij|v|kl\rangle$: contact pairing interaction in 6 channels

$$\langle ij|v|kl\rangle = \sum_{\alpha}^{6} v_\alpha \langle ij|\delta^{(3)}(\mathbf{r}-\mathbf{r}')P_{L=0}P_\alpha|kl\rangle$$

where v_s and v_t are fit (and varied)

α	1	2	3	4	5	6
(S, S_z)	(0,0)	(0,0)	(0,0)	(1,1)	(1,0)	(1,-1)
(T, T_z)	(1,1)	(1,0)	(1,-1)	(0,0)	(0,0)	(0,0)

Traditional HFB

- Applying the Bogoliubov U and V we go to the quasiparticle representation

- The ordinary and anomalous densities are:

  $\rho = V^* V^t$ and $\kappa = V^* U^t$

- Hartree-Fock-Bogoliubov equations:

$$\begin{bmatrix} h & \Delta \\ -\Delta^* & -h^* \end{bmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = \begin{pmatrix} U_k \\ V_k \end{pmatrix} E_k$$

  where $h = \varepsilon + \Gamma - \lambda$ and the interaction is buried inside Γ and Δ

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where $h = \varepsilon + \Gamma - \lambda$ and the interaction is buried inside Γ and Δ

$$\Gamma_{ij} = \sum_{kl} \bar{v}_{iljk} \rho_{kl}$$

$$\Delta_{ij} = \frac{1}{2} \sum_{kl} \bar{v}_{ijkl} \kappa_{kl}$$

Traditional HFB

End goal (for now) is

$$H^{00} = Tr\left(\varepsilon\rho + \frac{1}{2}\Gamma\rho - \frac{1}{2}\Delta\kappa^*\right)$$

HFB + gradient method

- We want to apply numerous constraining fields:

  $$H' = H - \sum_i \lambda_i Q_i$$

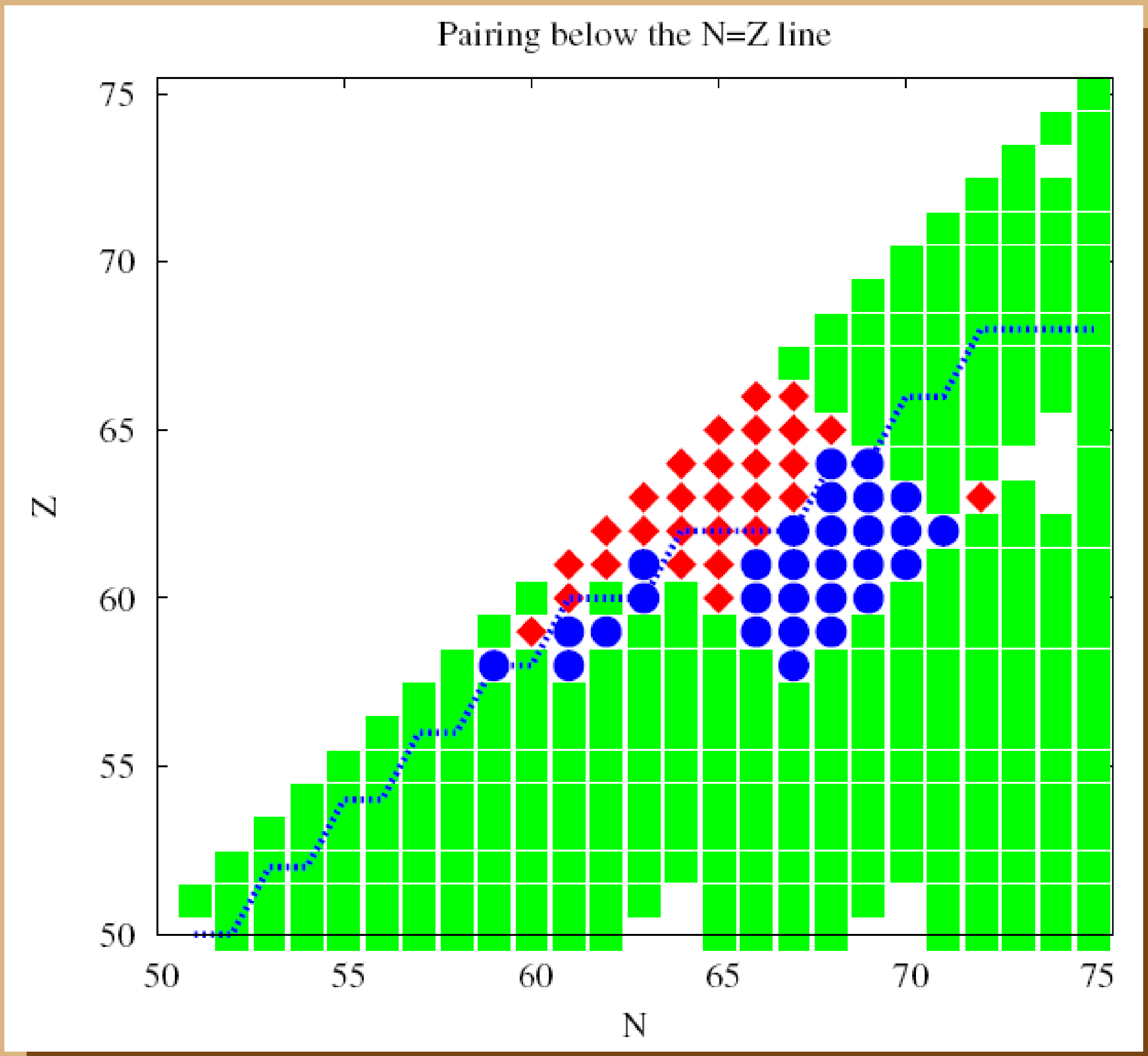
  The Q_i are neutron and proton numbers (and possibly 6 more quantities, 1 per channel). Constraining pairing to zero gives us normal system

- Using the Thouless matrix Z the energy can be expanded:

  $$(H')^{00}_{new} \approx (H')^{00} - \mathrm{Tr}((H')^{20} Z) - \mathrm{Tr}((H')^{11} Z^2)$$

- More information:
  → L. M. Robledo and G. F. Bertsch, Phys. Rev. C **84**, 014312 (2011)
  → P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer)

Pairing in heavy nuclei (A~130)

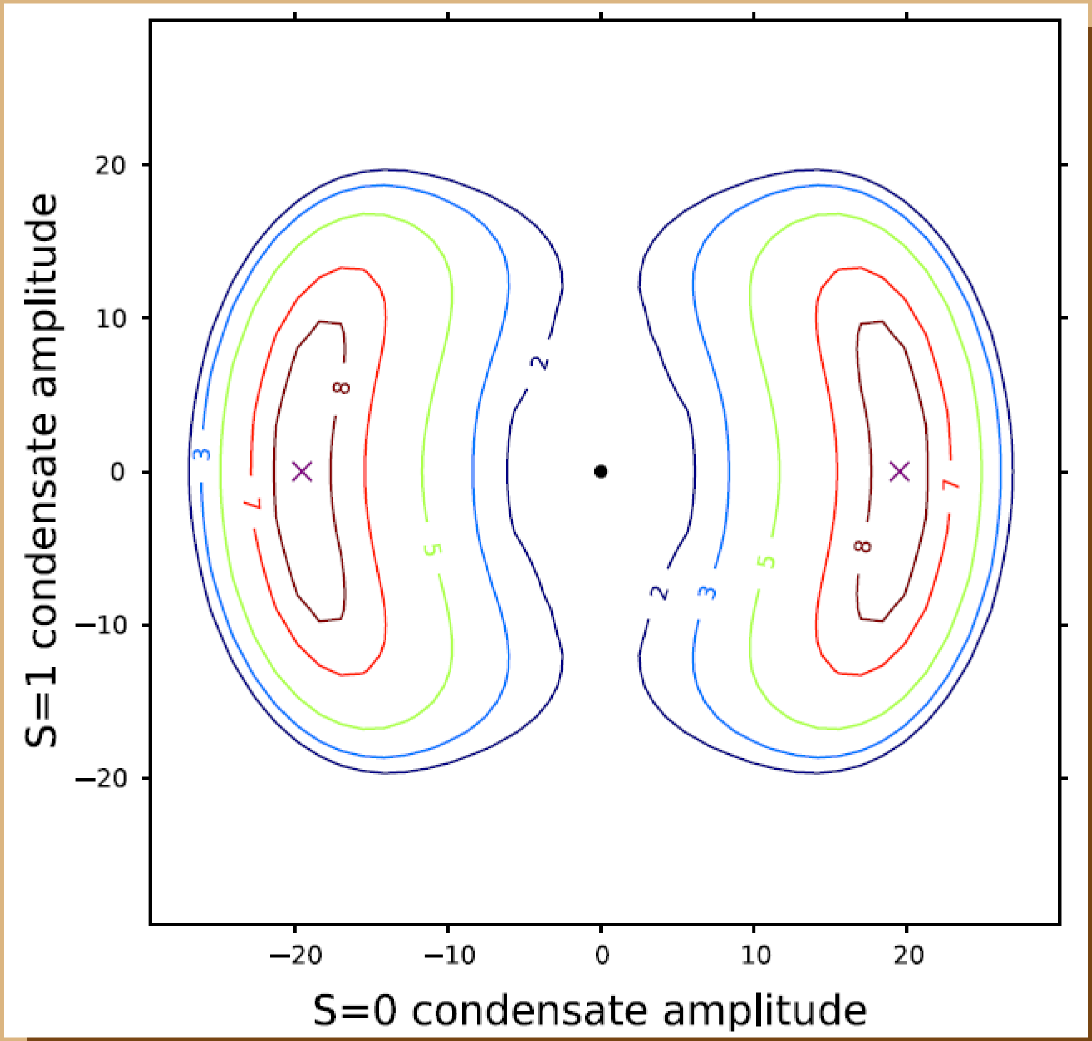


Correlation energies

- Blue line: proton drip
- Green: spin-singlet
- Red: spin-triplet
- Blue: mixed-spin
- Spin-triplet pairing persists off N=Z line
- Mixed-spin pairing appears to be energetically stable (note: no deformation)

A. Gezerlis, G. F. Bertsch, and Y. L. Luo, Phys. Rev. Lett. **106**, 252502 (2011)

Energy contour for spin-singlet

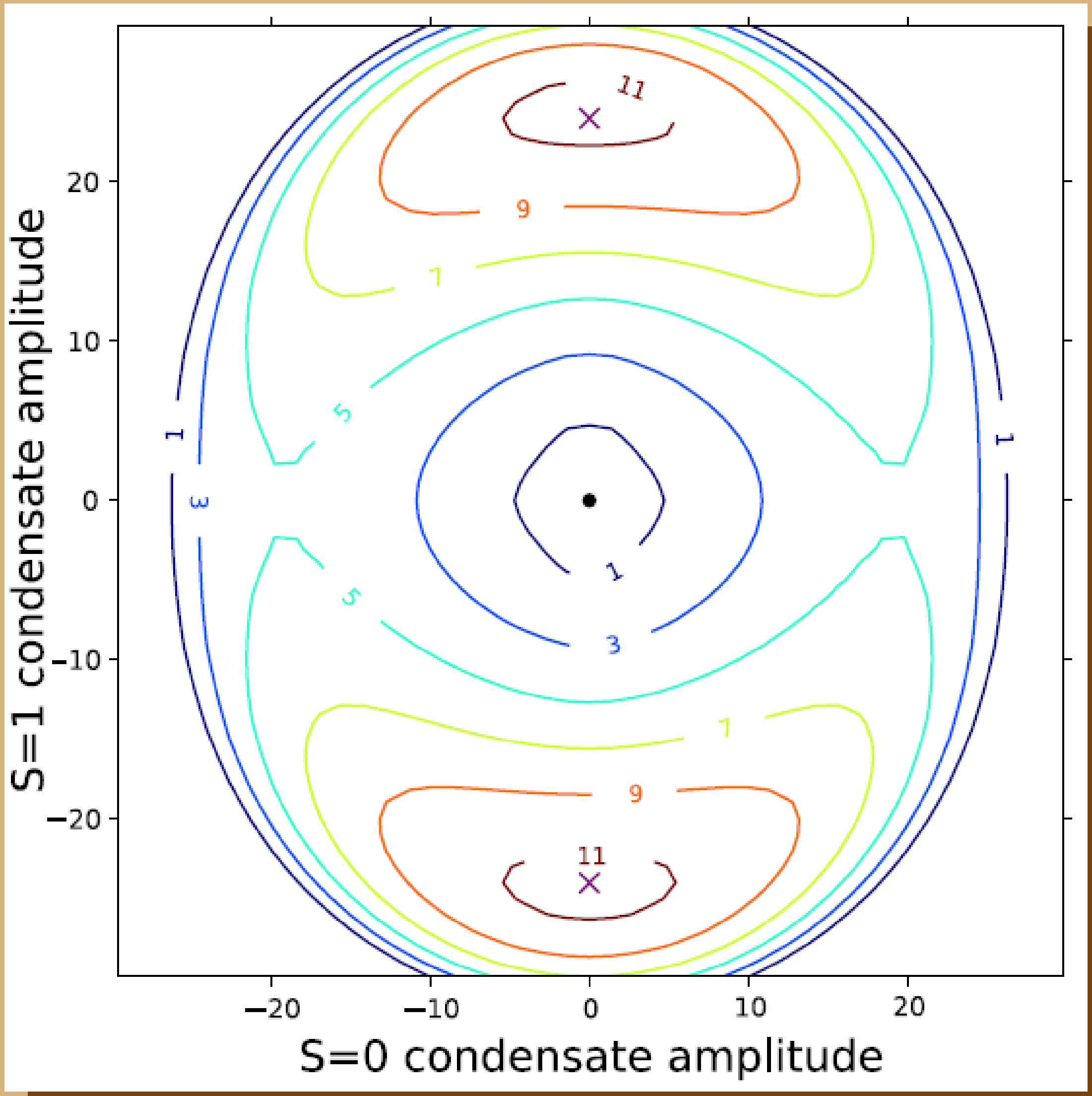


$^{132}_{60}\mathrm{Nd}$

- Dot: uncorrelated
- X: unconstrained
- Energy surface elongated in vertical direction, so is soft with respect to forming a spin-triplet condensate

A. Gezerlis, G. F. Bertsch, and Y. L. Luo, Phys. Rev. Lett. **106**, 252502 (2011)

Energy contour for spin-triplet

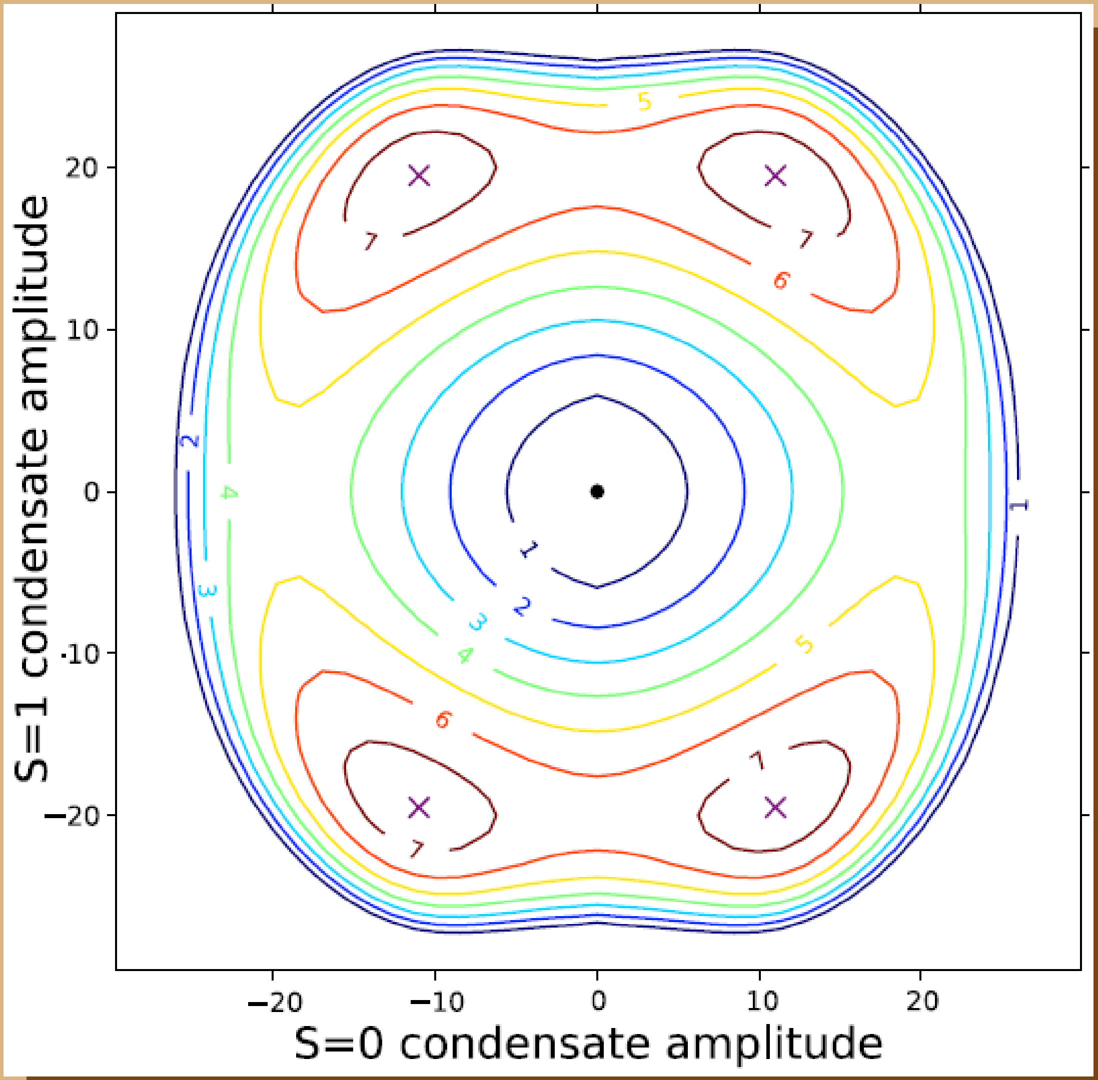


$^{132}_{66}\mathrm{Dy}$

- Dot: uncorrelated
- X: unconstrained
- What happens when one goes between this case and the previous one?

A. Gezerlis, G. F. Bertsch, and Y. L. Luo, Phys. Rev. Lett. **106**, 252502 (2011)

Energy contour for mixed-spin



$^{132}_{64}\mathrm{Gd}$

- Dot: uncorrelated
- X: unconstrained
- Smooth transition. Dependent on the presence of spin-orbit splitting. Prediction.

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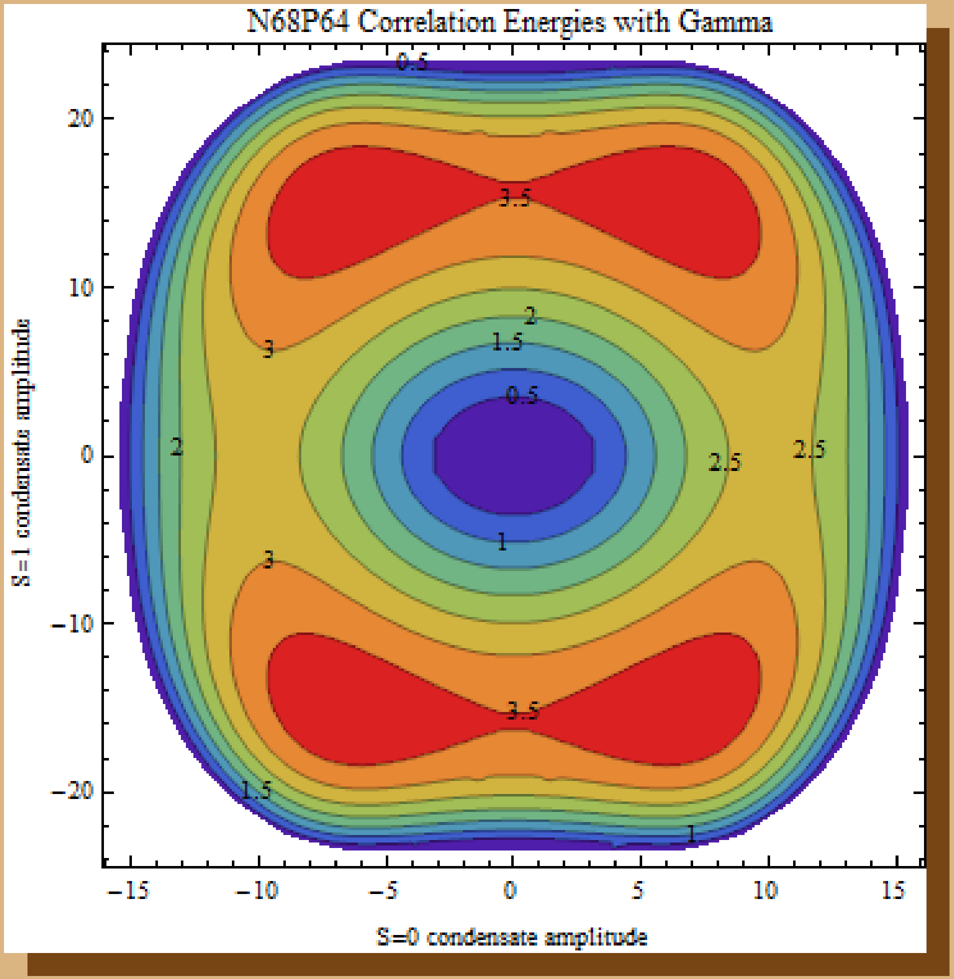

$$H^{00} = Tr\left(\varepsilon\rho + \frac{1}{2}\Gamma\rho - \frac{1}{2}\Delta\kappa^*\right)$$

- Fixed up Γ term (Hartree-Fock contribs)
- Shown is the prototypical mixed-spin case
- Effect weakened
- Rich structure yet to be determined

B. Bulthuis and A. Gezerlis, Phys. Rev. C **93**, 014312 (2016)

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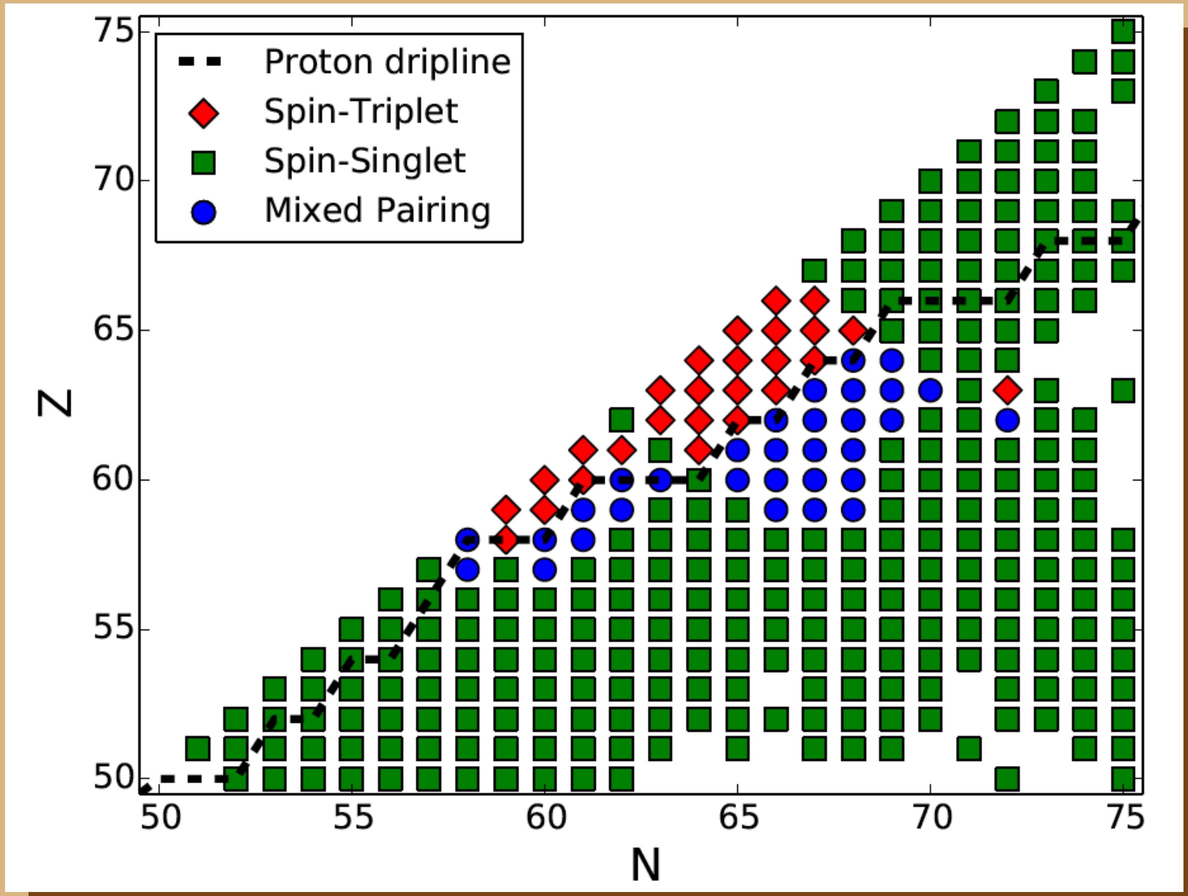

Again, fixed up Γ term

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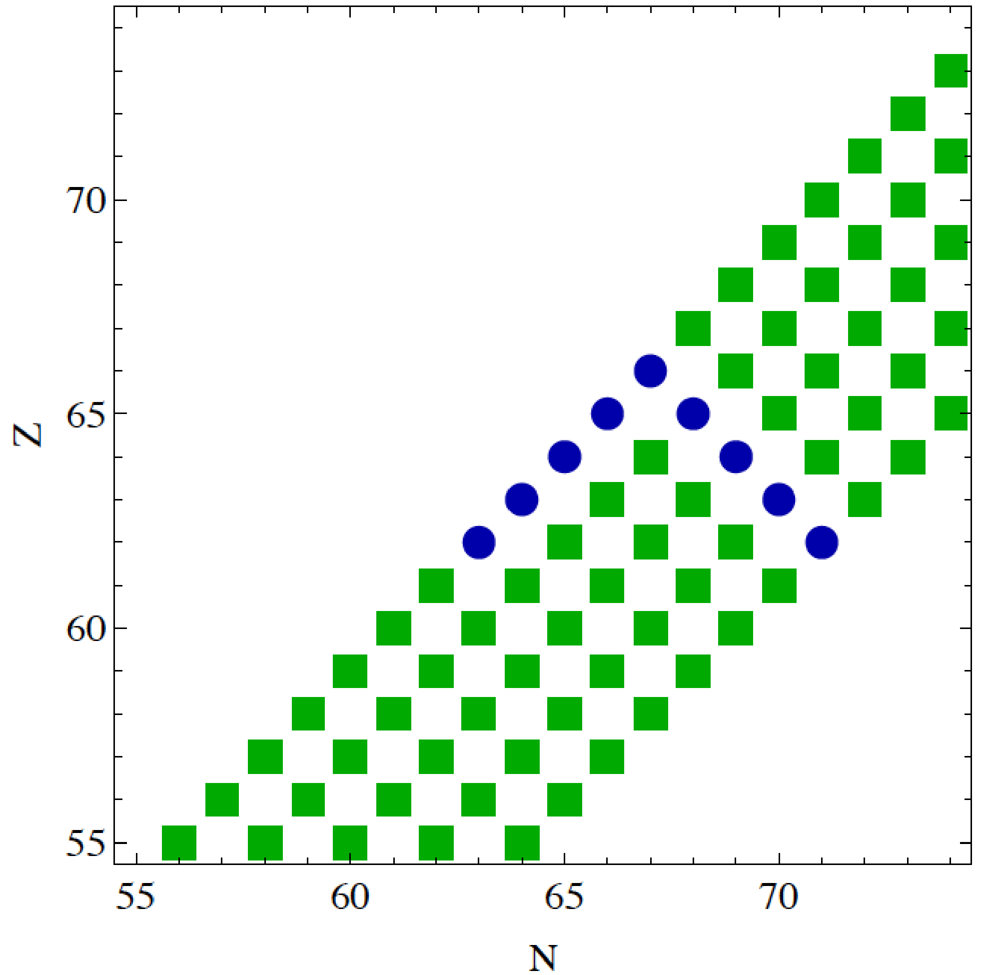
Experimental signatures?



- Pairing gaps:

$$\Delta_o^{(3)}(n) = E(n) - \tfrac{1}{2}\left[E(n-1) + E(n+1)\right]$$

  blue(=small) one unit off *N*=*Z* and a little below

- Two-particle transfer direct reaction cross sections

B. Bulthuis and A. Gezerlis, Phys. Rev. C **93**, 014312 (2016)

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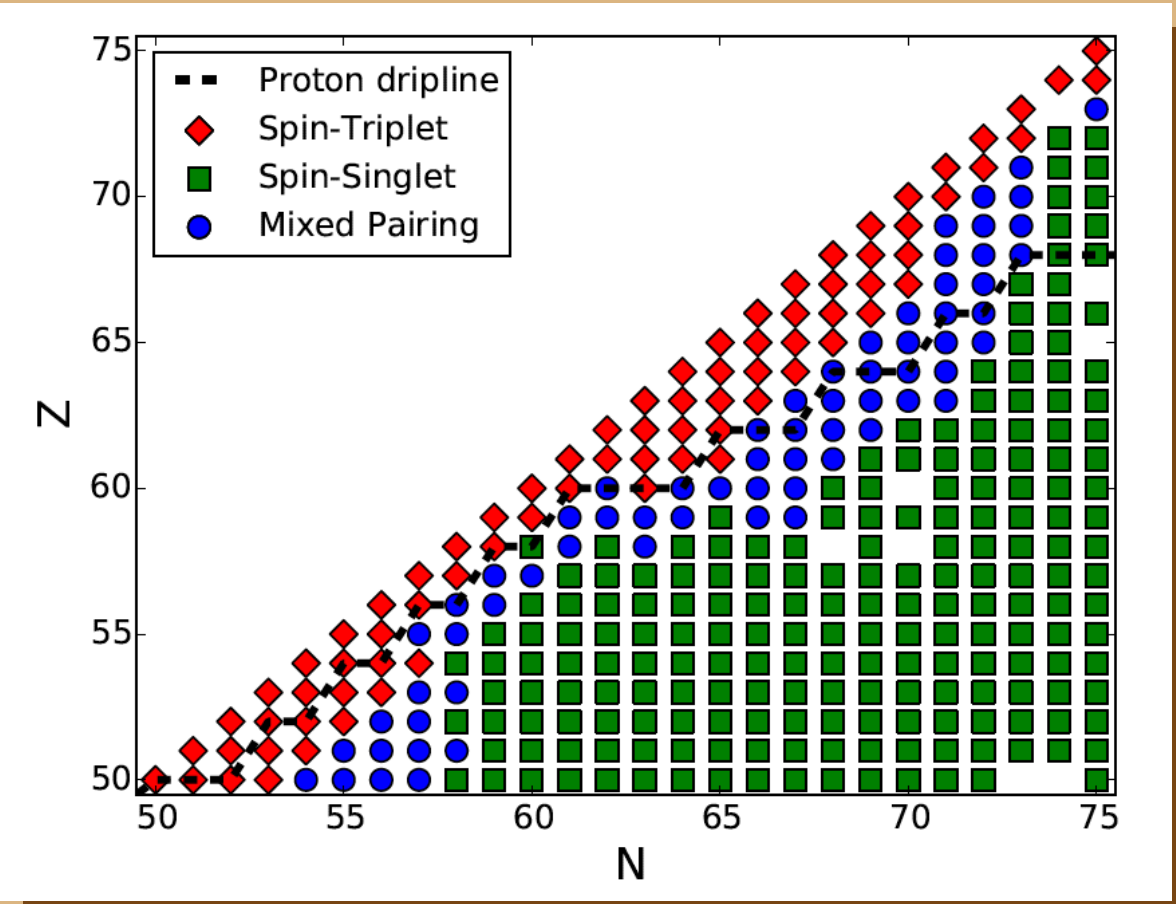
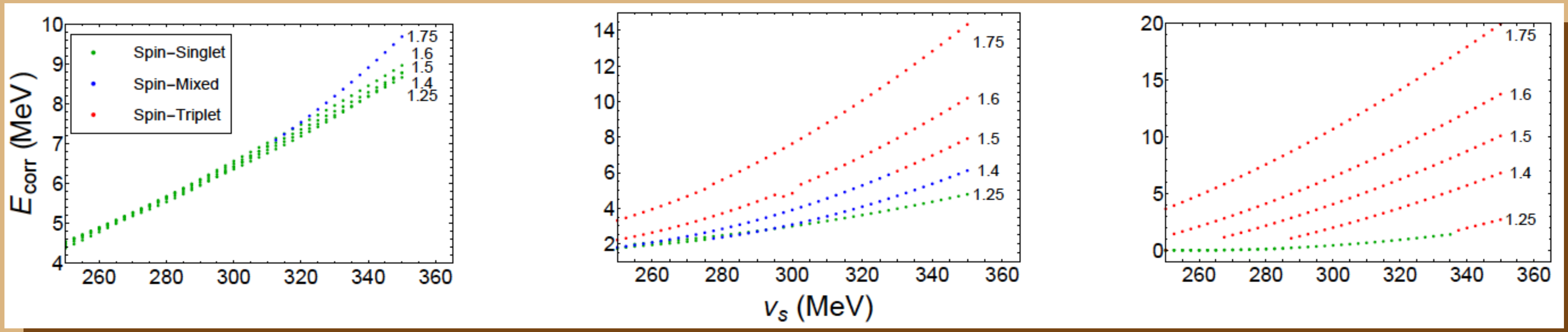


Turn spin-orbit off as a consistency check

B. Bulthuis and A. Gezerlis, Phys. Rev. C **93**, 014312 (2016)

Pairing in heavy nuclei (A~130)

Check to see that you're not using a "magic" value of v_t/v_s



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Symmetry restoration: particle number

$$\hat{P}^{(LTS)}_{NZ}(N_0, Z_0) = \int_0^{2\pi} \frac{d\varphi_N}{2\pi} e^{-iN_0\varphi_N} \int_0^{2\pi} \frac{d\varphi_Z}{2\pi} e^{-iZ_0\varphi_Z} \times \ e^{iR(\varphi_N,\varphi_Z)}$$

$$R(\varphi_N, \varphi_Z) = \mathbb{I}_{N_L} \otimes \begin{pmatrix} \varphi_N & 0 \\ 0 & \varphi_Z \end{pmatrix} \otimes \mathbb{I}_2$$

E. Rrapaj, A. O. Macchiavelli, and A. Gezerlis, Phys. Rev. C **99**, 014321 (2019)

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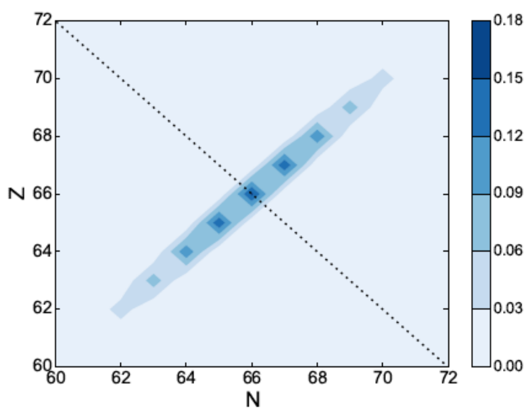

(a) $^{132}_{66}\mathrm{Dy}$

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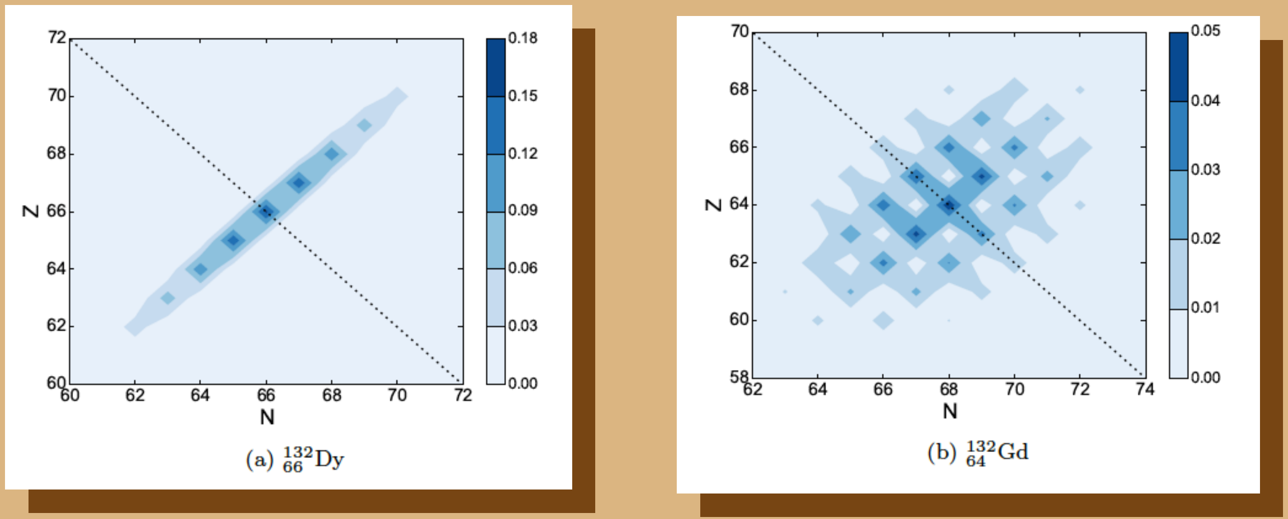

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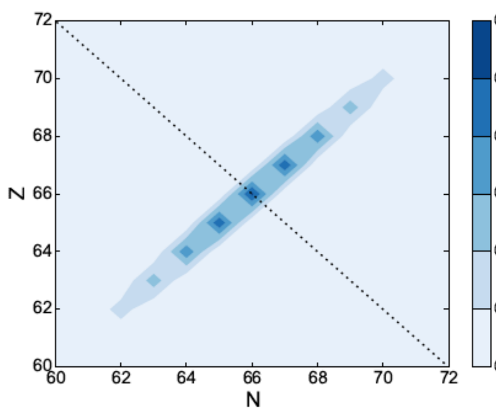

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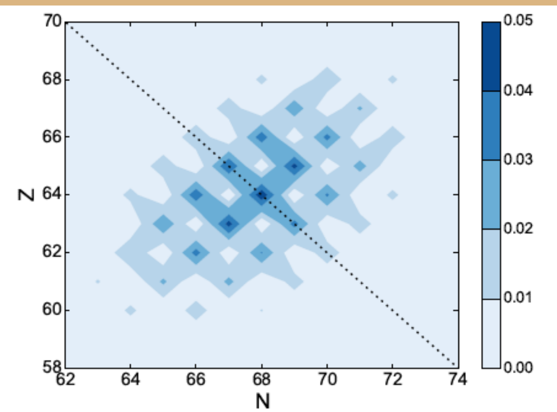

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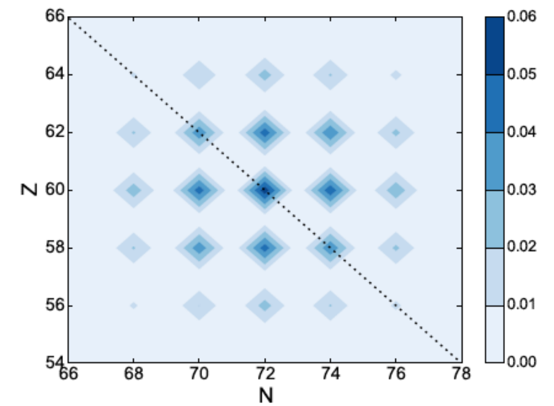

(c) $^{132}_{60}$Nd

E. Rrapaj, A. O. Macchiavelli, and A. Gezerlis, Phys. Rev. C **99**, 014321 (2019)

Symmetry restoration: spin

$$\hat{P}_J^{(LTS)}(I_0, m', m) = \frac{2I_0+1}{8\pi^2}\int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \sin(\beta)$$
$$\times \int_0^{2\pi} d\gamma e^{i(m'\alpha + m\gamma)} d^{(I_0)}_{m',m}(\beta) e^{i\alpha \hat{J}_z^{(LTS)}}$$
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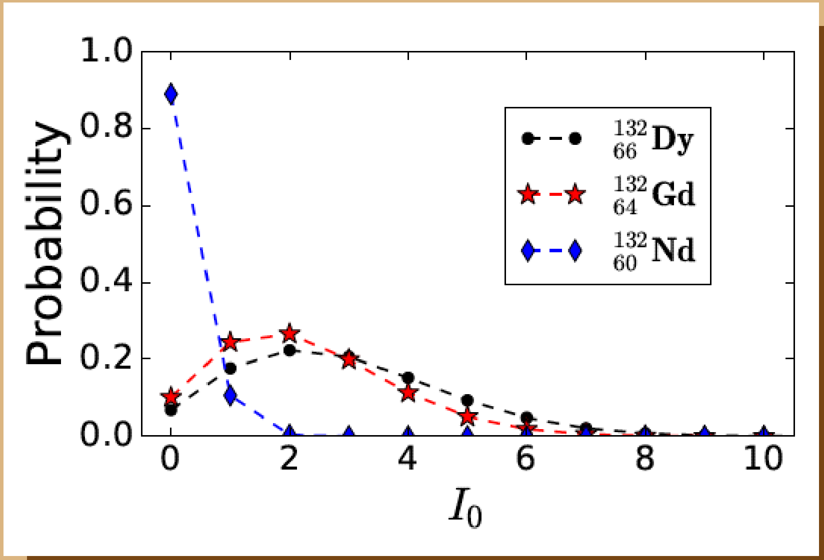

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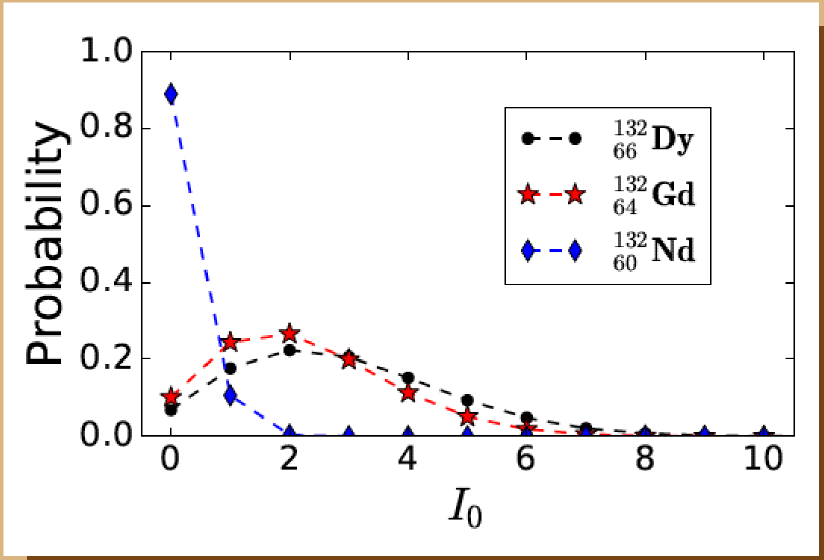

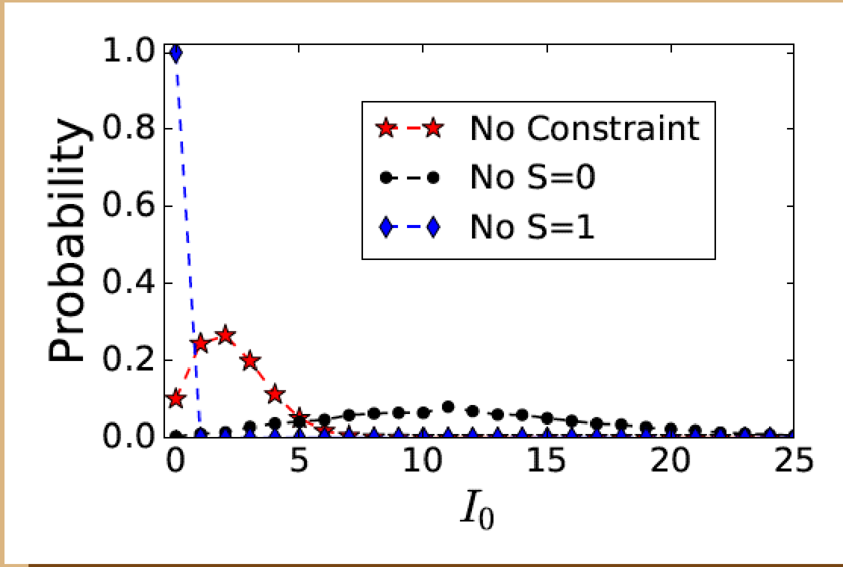

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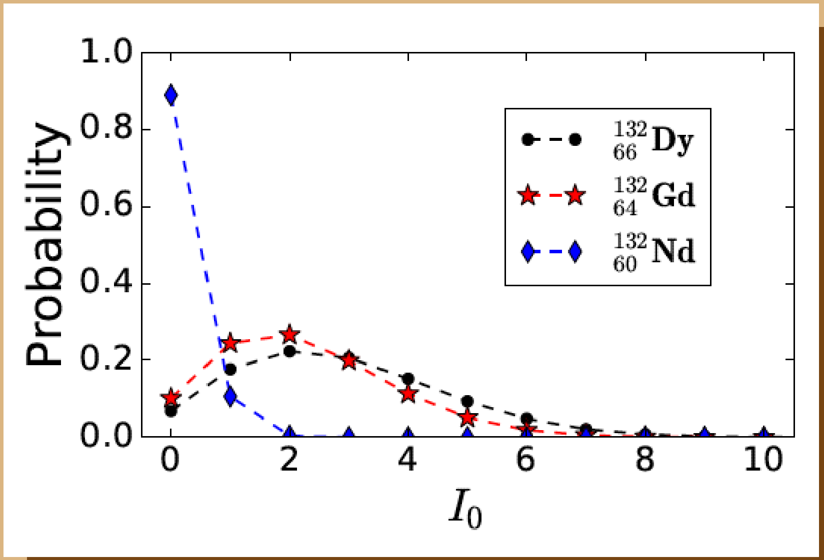
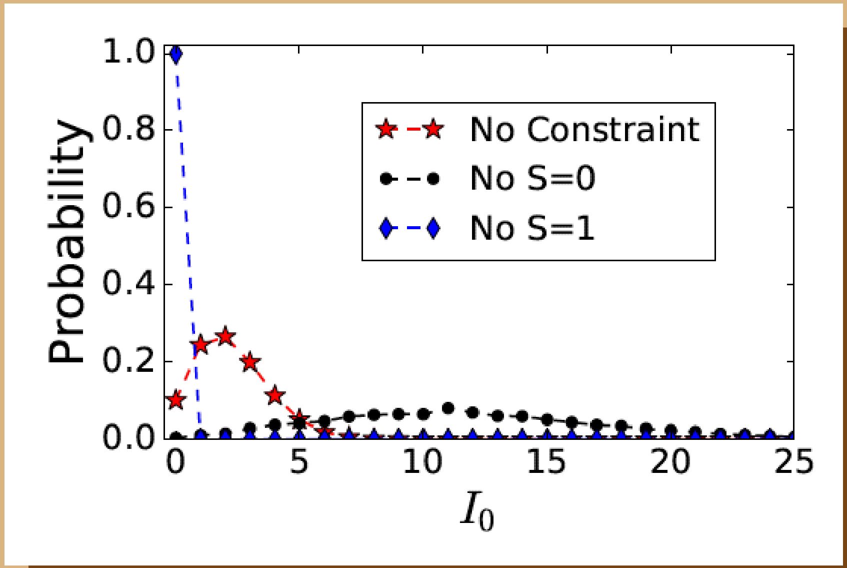
N.B. This is Projection After Variation (but the 5d integral can still get challenging)





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Symmetry restoration: pair transfer

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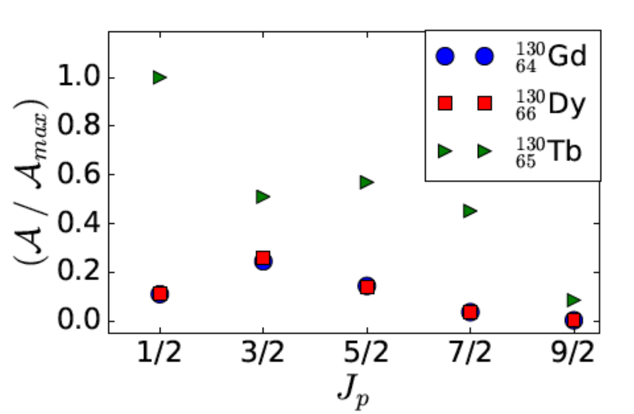

(a) Final isotope is $^{132}_{66}$Dy.

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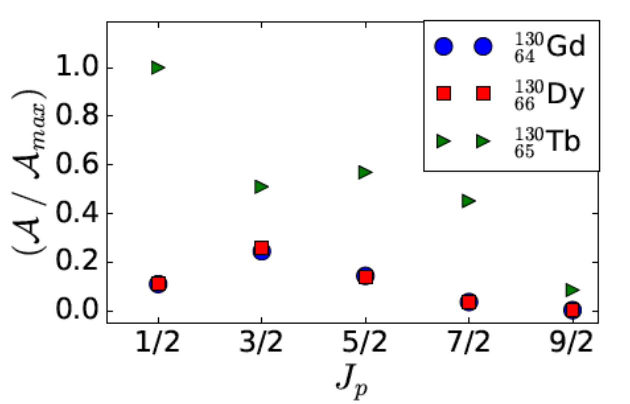
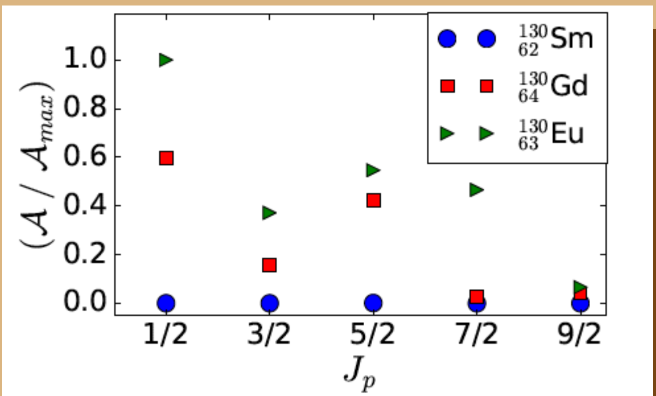


(a) Final isotope is $^{132}_{66}$Dy.



(b) Final isotope is $^{132}_{64}$Gd.

E. Rrapaj, A. O. Macchiavelli, and A. Gezerlis, Phys. Rev. C **99**, 014321 (2019)

Symmetry restoration: pair transfer

$$\mathcal{A}^{(J_p)}_{\Phi_i,\Phi_f}(I_i, I_f) = \left| \frac{\langle \Phi_f | \hat{\mathscr{P}}(I_f, J_p) | \Phi_i \rangle}{\mathcal{N}_i \mathcal{N}_f} \right|$$

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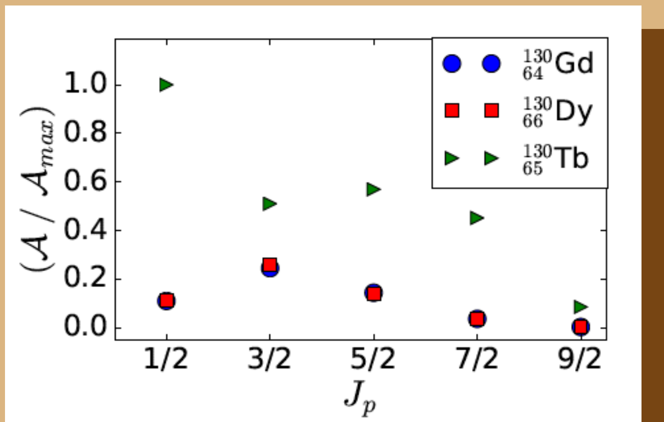

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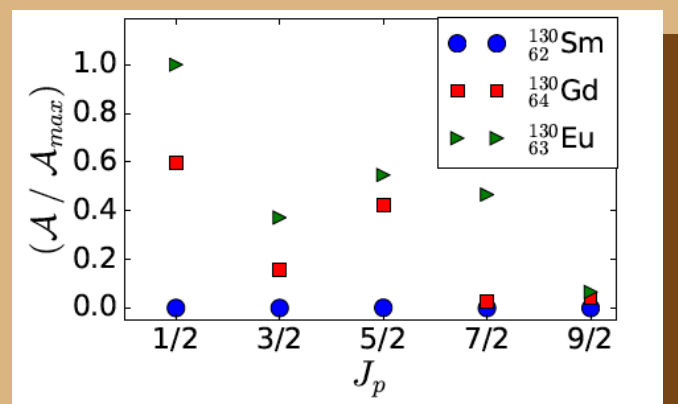
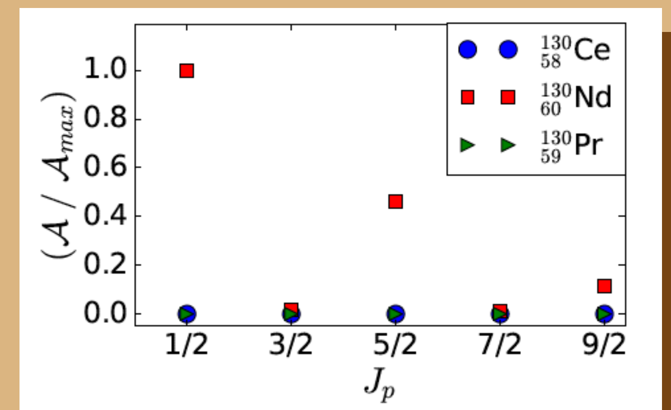


(b) Final isotope is $^{132}_{64}$Gd.



(c) Final isotope is $^{132}_{60}$Nd.

E. Rrapaj, A. O. Macchiavelli, and A. Gezerlis, Phys. Rev. C **99**, 014321 (2019)

Approaching the thermodynamic limit with BCS in neutron matter

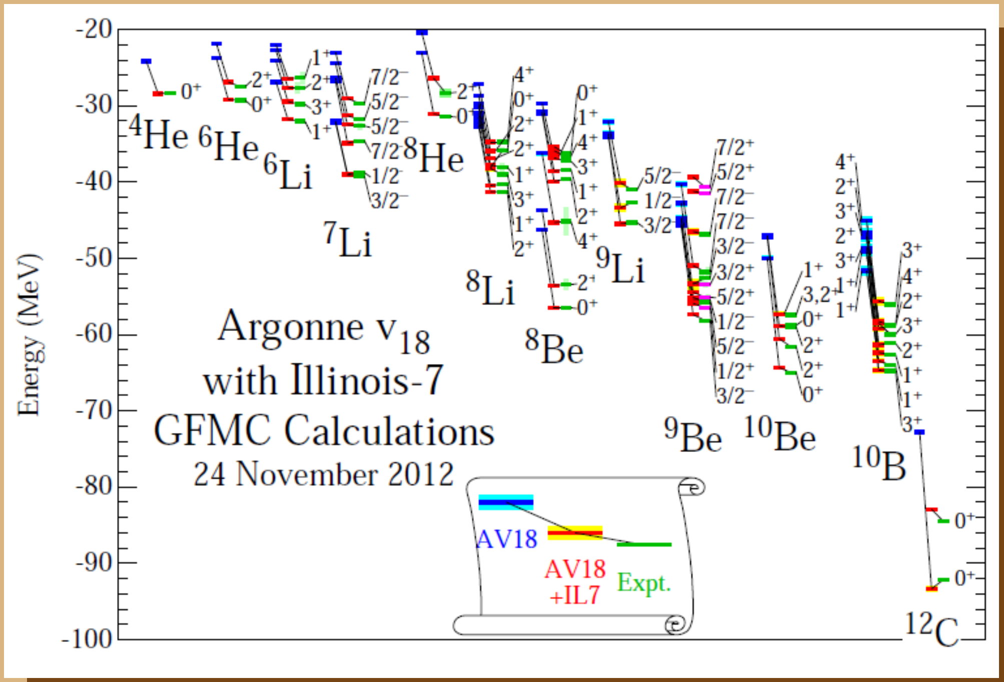
Two complementary approaches

Quantum Monte Carlo

- Microscopic
- Computationally demanding (3N particle coordinates + spins)
- Limited to smallish N

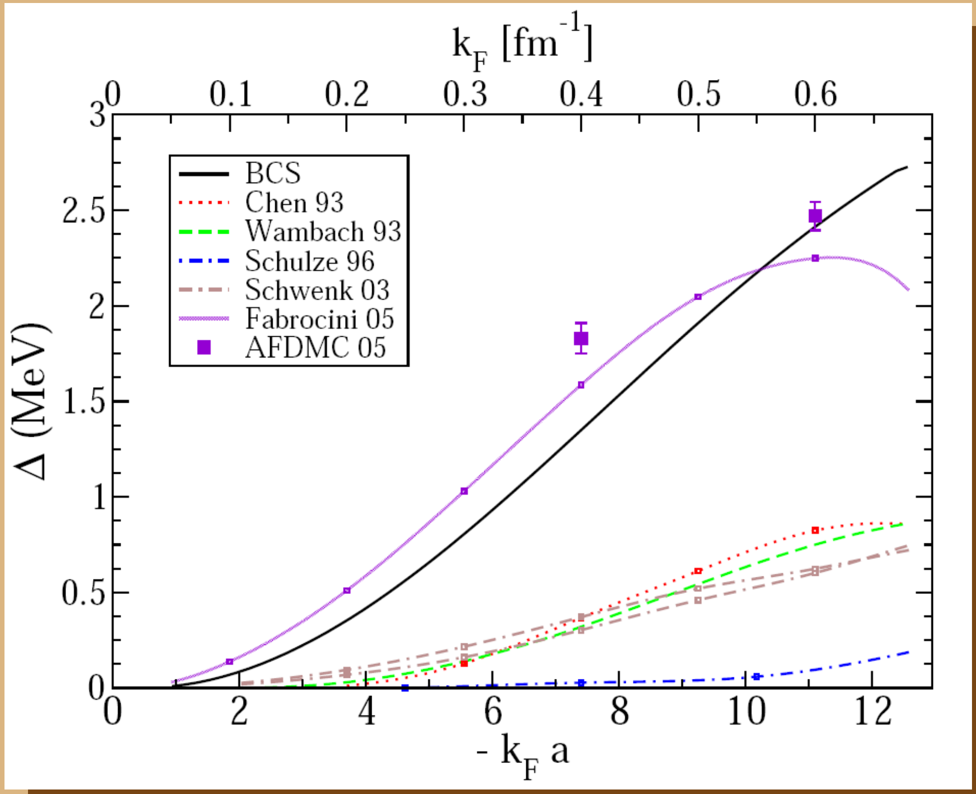
$$\Psi(\tau \rightarrow \infty) = \lim_{\tau \rightarrow \infty} e^{-(\mathcal{H}-E_T)\tau}\Psi_V$$

$$\rightarrow \alpha_0 e^{-(E_0-E_T)\tau}\Psi_0$$



Credit: Steve Pieper

Two complementary approaches



Mean-field theory

- More phenomenological
- Easier to implement, allowing quicker access to qualitative insights
- Can do any large N, including thermodynamic limit

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k}|V|\mathbf{k}'\rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}$$

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Research Strategies

i) Use QMC as a benchmark with which to compare mean-field results

ii) Constrain mean-field-like theory with QMC, then use former to make predictions

Start from strong pairing

Cold atoms to the rescue

***Theoretical* many-body problem formulated by George Bertsch more than 15 years ago:**

"What is the ground-state energy of a gas of spin-1/2 particles with infinite scattering length, zero range interaction?"

$$E = \xi E_{FG} \quad E_{FG} = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m}$$

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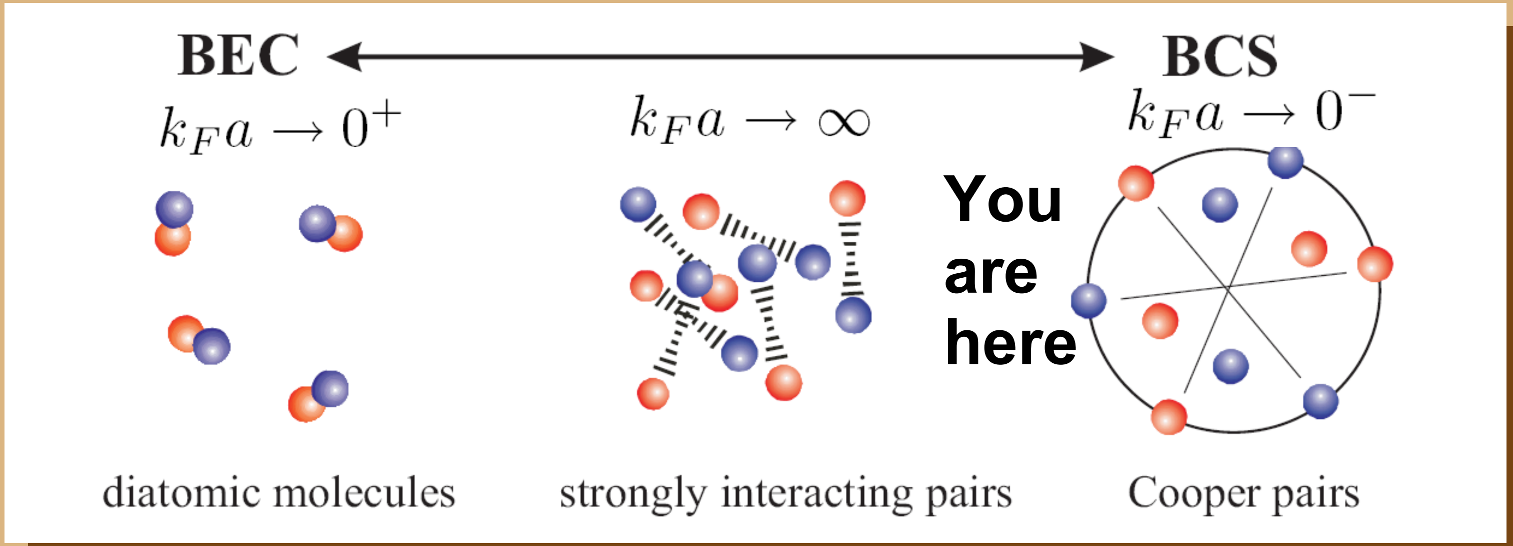
Now within *direct* experimental reach!



Credit: Ph.D. Thesis of Cindy Regal

What do cold atoms have to do with nuclear physics?

Connections

Difficulties with ab initio description of neutron stars:

- No direct experiment
  (though gravitational-wave studies of NS-NS mergers promising)
- No such thing as “one true interaction”
- Many-body problem difficult, due to strong correlations

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Connections

Difficulties with ab initio description of neutron stars:

- No direct experiment: **tune atomic interactions**
- No such thing as “one true interaction”**: probe universal regime**
- Many-body problem difficult, due to strong correlations**: carry out non-perturbative calculations for both systems and compare**

Cold atoms can help with all 3 problems

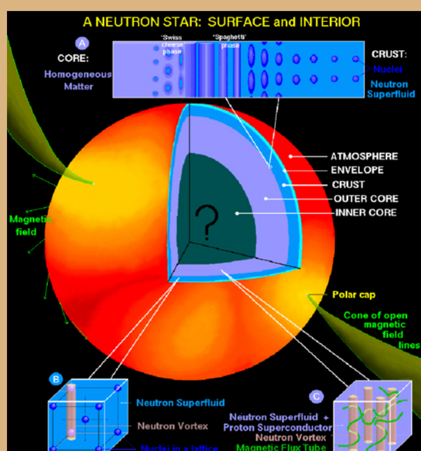
Connections

Neutron matter

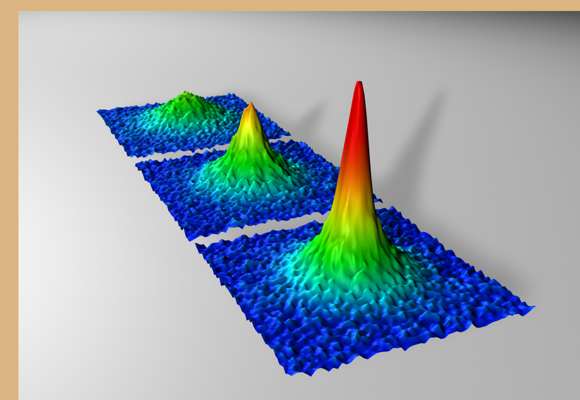
- MeV scale
- $O(10^{57})$ neutrons

Cold atoms

- peV scale
- O(10) or $O(10^5)$ atoms



Credit: Dany Page



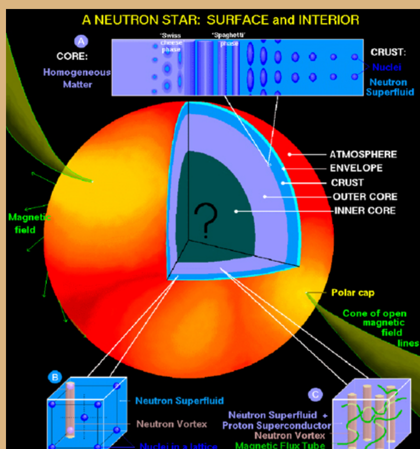
Credit: University of Colorado

- Very similar E/E_{FG} and Δ/E_F
- Weak to intermediate to strong coupling

Connections

Neutron matter

- MeV scale
- $O(10^{57})$ neutrons



Credit: Dany Page

Cold atoms

- peV scale
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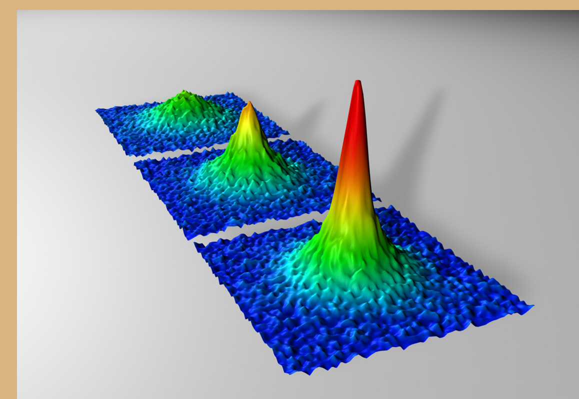
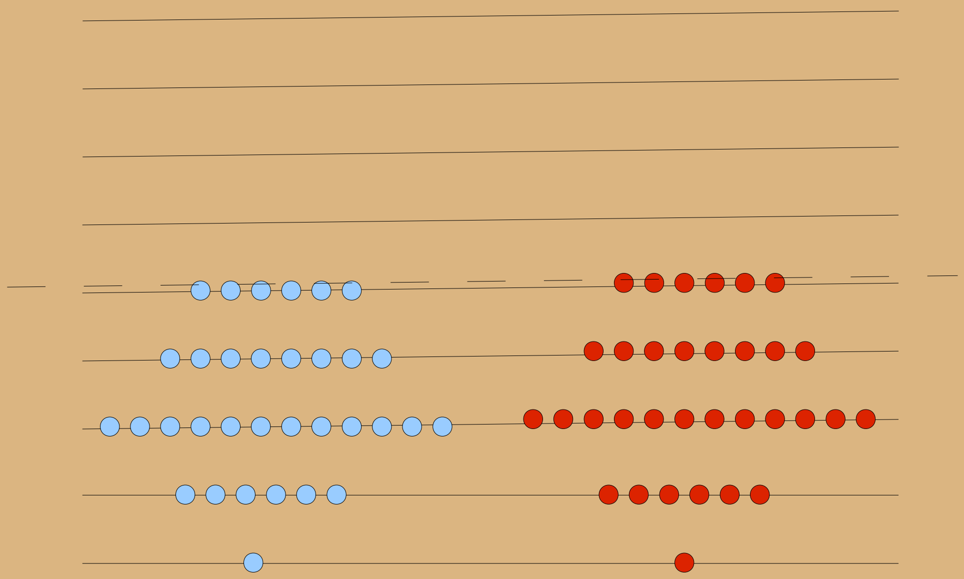


Credit: University of Colorado

A. Gezerlis, C. J. Pethick, and A. Schwenk
Pairing and superfluidity of nucleons in neutron stars
chapter in "Novel Superfluids: Volume 2"
(Oxford University Press, 2014)

Fermionic dictionary



Energy of a free Fermi gas: $E_{FG} = 3/5 N E_F$

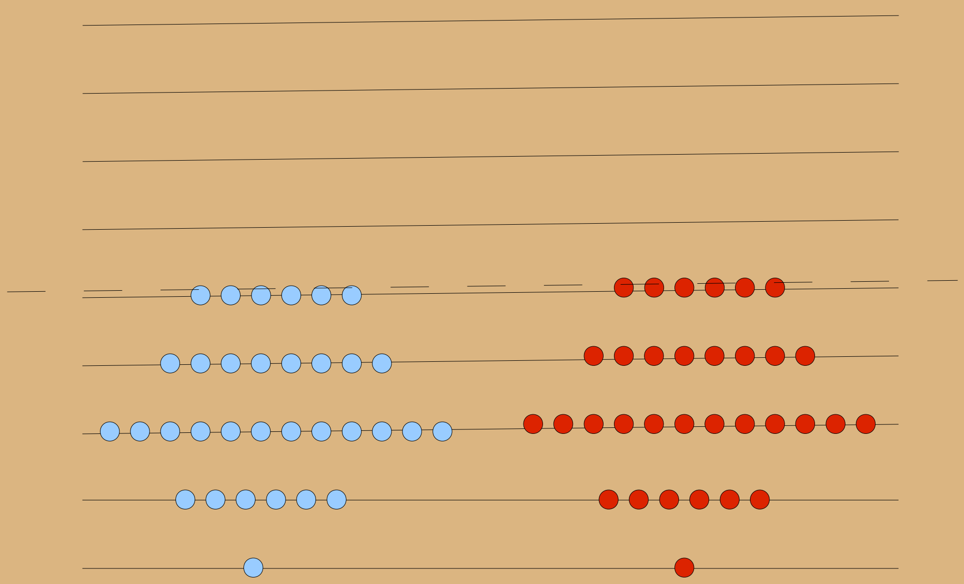
Fermi energy: $E_F = \hbar^2 k_F^2 / 2m$

Fermi wave number: k_F

Number density: $\rho = g k_F^3 / 6\pi^2$

Scattering length: a

Fermionic dictionary



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Fermi energy: $E_F = \hbar^2 k_F^2 / 2m$

Fermi wave number: k_F

Number density: $\rho = g k_F^3 / 6\pi^2$

Scattering length: a

In what follows, the dimensionless quantity $k_F a$ is called the "coupling"

Coupling

Weak coupling

- $k_F a \to 0$
- Studied for decades
- Experimentally difficult
- Pairing exponentially small
- Analytically known

Strong Coupling

- $k_F a \to \infty$
- More recent (2000s)
- Experimentally probed
- Pairing significant
- Non-perturbative

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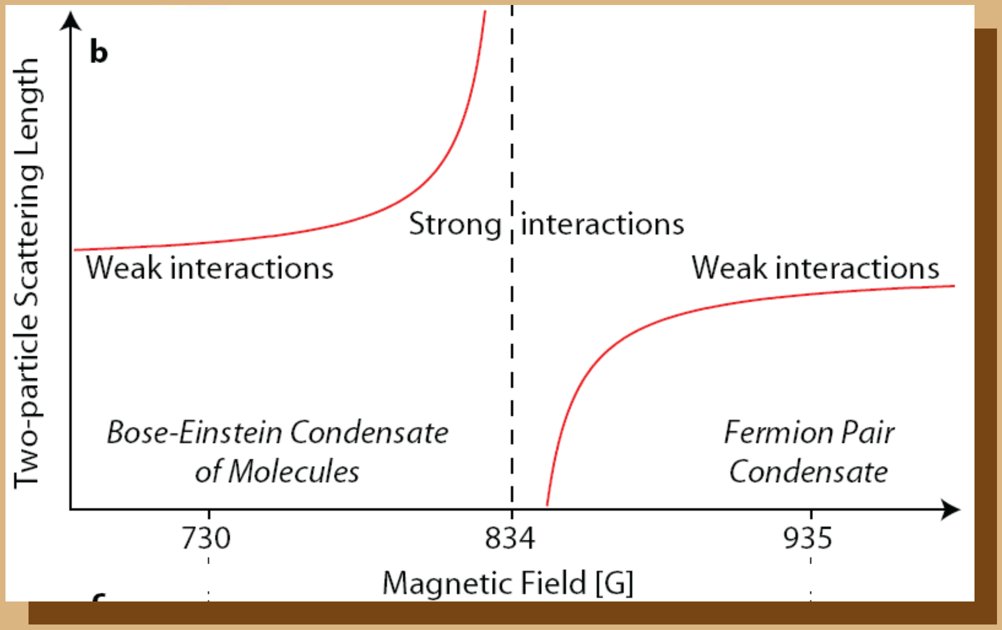
- Pairing significant
- Non-perturbative

Connection:

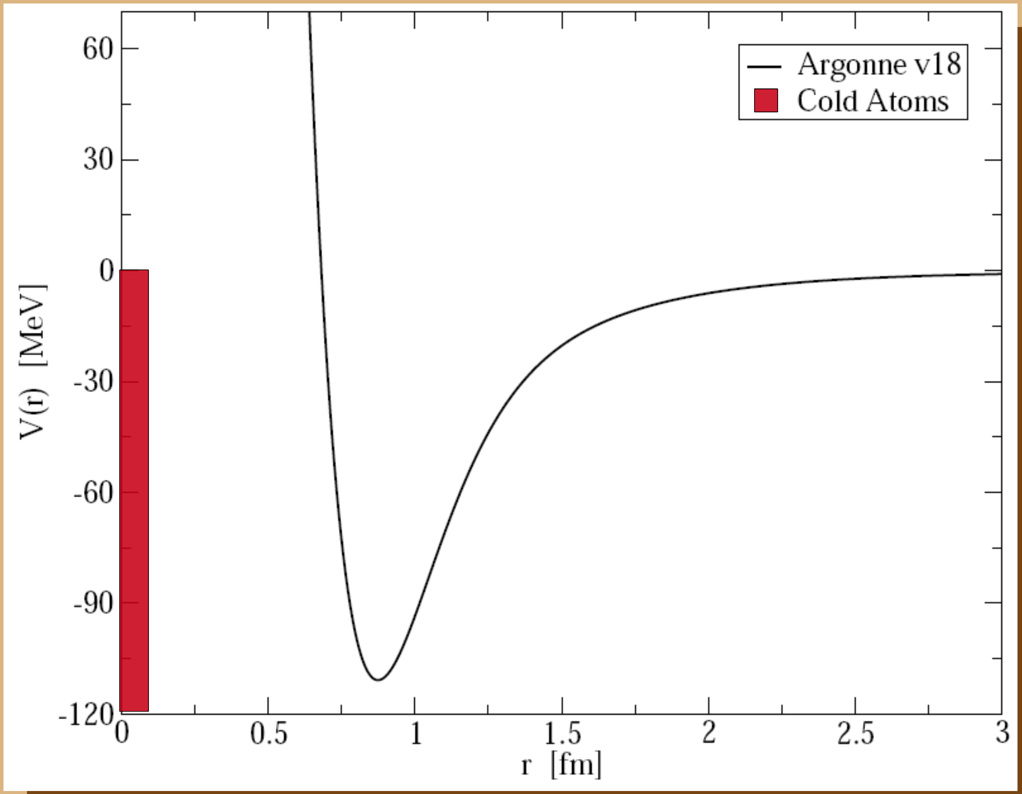
Using "Feshbach" resonances one can tune the coupling

Credit: Thesis of Martin Zwierlein



Hamiltonian: unity in diversity

$$\mathcal{H} = -\frac{\hbar^2}{2m}\sum_{k=1}^{N}\nabla_k^2 + \sum_{i<j'} v(r_{ij'})$$



Neutron matter

$^1S_0$ channel of AV18 – later AV4

a = -18.5 fm, r_e = 2.7 fm

Cold atoms

basically any well-behaved potential

a = tunable, r_e =tunable/infinitesimal

What do we know for sure?

Weak Coupling

Equation of state: $$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} k_F a + \frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a)^2$$

Pairing gap: $$\frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}} \Delta_{\mathrm{BCS}}$$

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Weak Coupling

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Pairing gap: $$\frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}} \Delta_{\mathrm{BCS}}$$

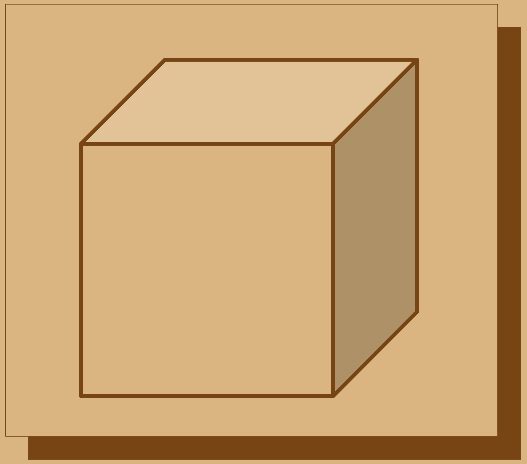
Strong Coupling

Mean-field BCS is easy but unreliable:

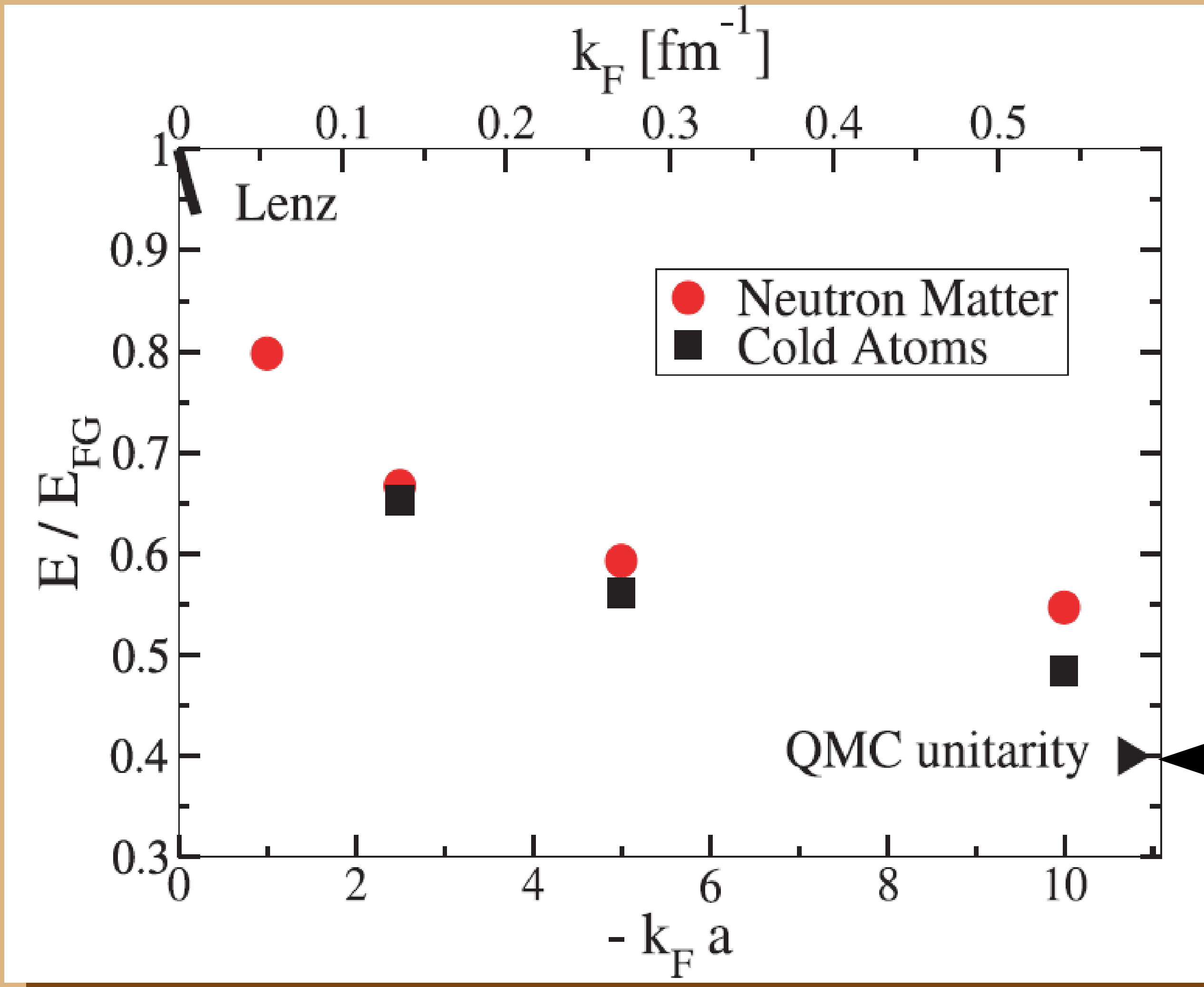
$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k}|V|\mathbf{k}'\rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}$$

Ab initio GFMC is difficult but accurate:

$$\Psi_V = \prod_{i<j} f(r_{ij})\ \mathcal{A}[\prod \phi(r_{ij})]$$



Equations of state: results



- Results identical at low density
- Range important at high density
- Duke and ENS experiments at unitarity (current QMC and MIT experiment are lower)

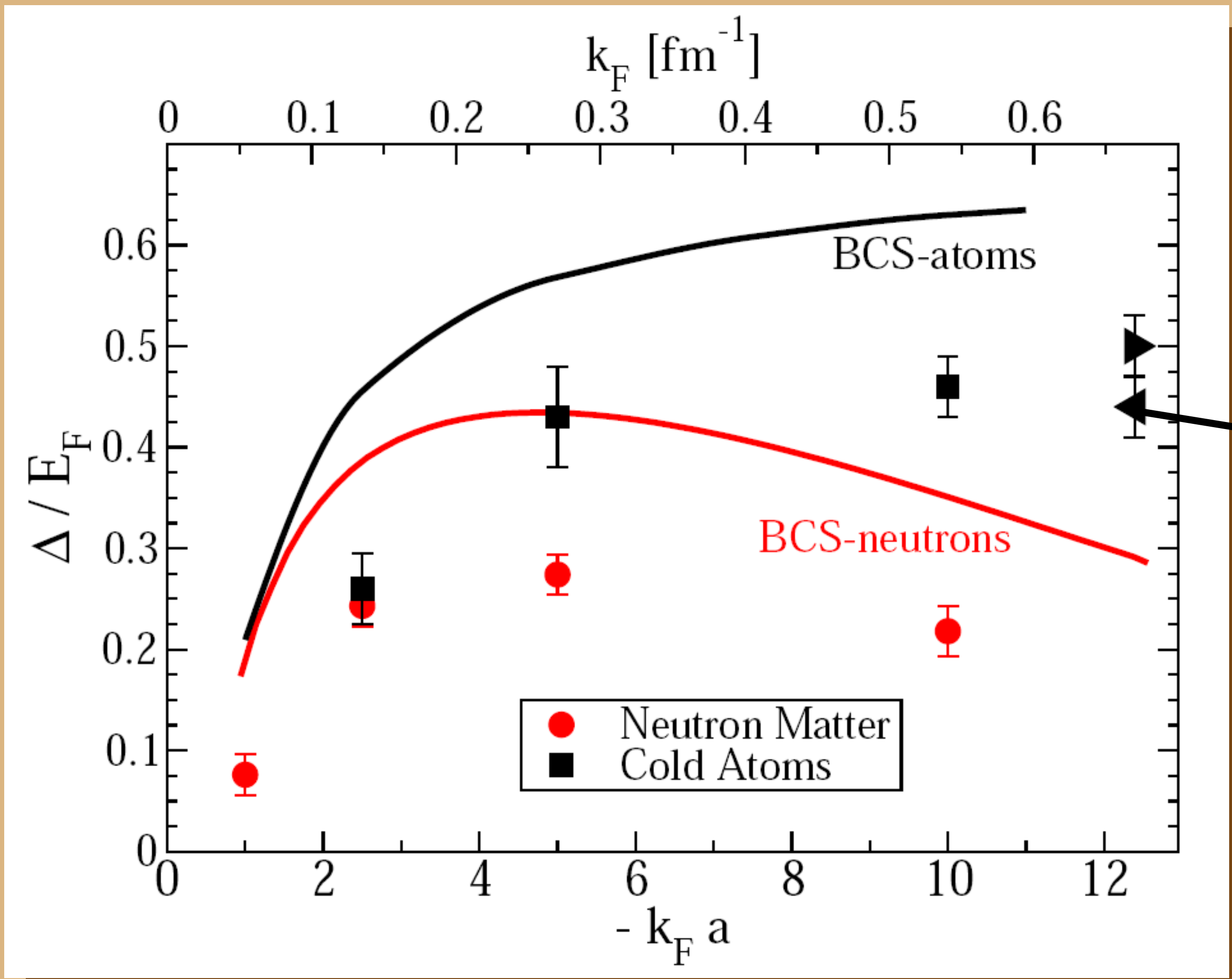
NEUTRONS

ATOMS

A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)

S. Gandolfi, A. Gezerlis, and J. Carlson, Ann. Rev. Nucl. Part. Sci. **65**, 303 (2015)

Pairing gaps: results



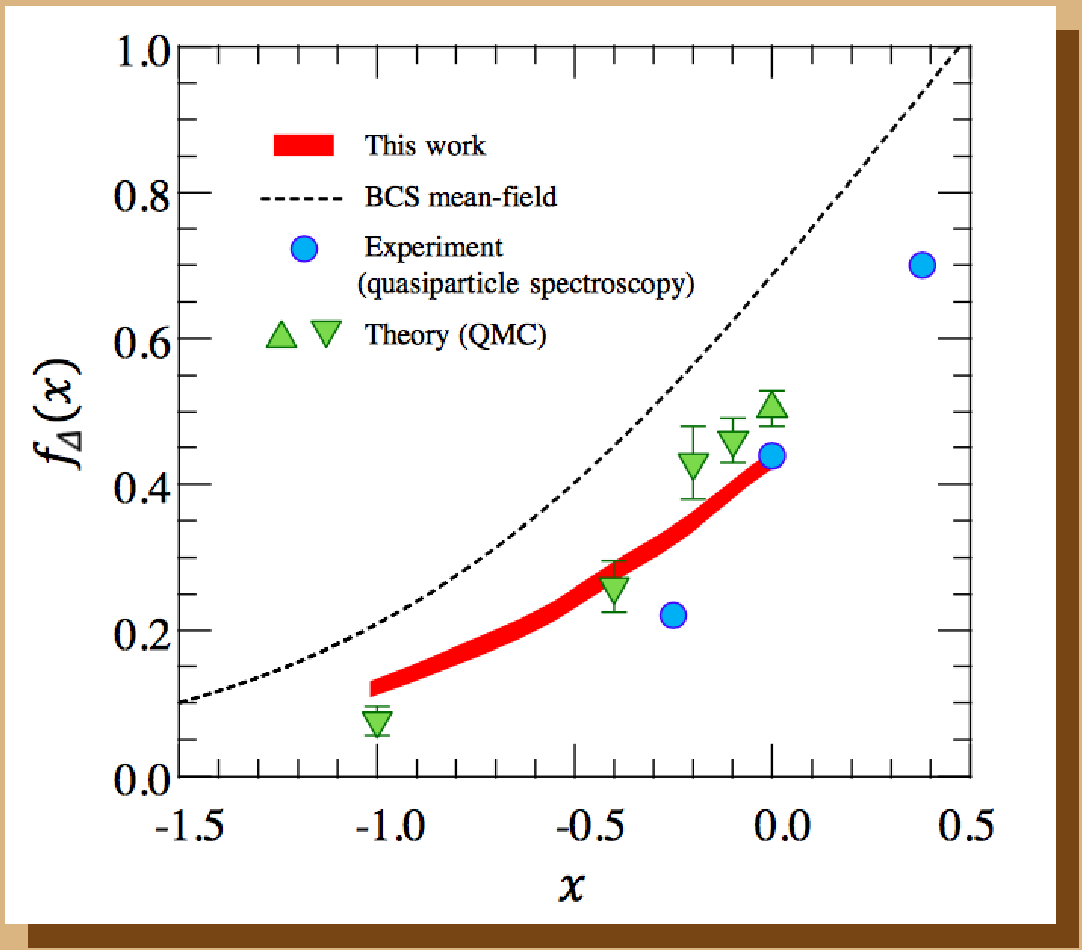
- Results identical at low density
- Range important at high density
- Two independent MIT experiments at unitarity

NEUTRONS

ATOMS

A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)
S. Gandolfi, A. Gezerlis, and J. Carlson, Ann. Rev. Nucl. Part. Sci. **65**, 303 (2015)

Experiment on cold-gas gaps away from unitarity

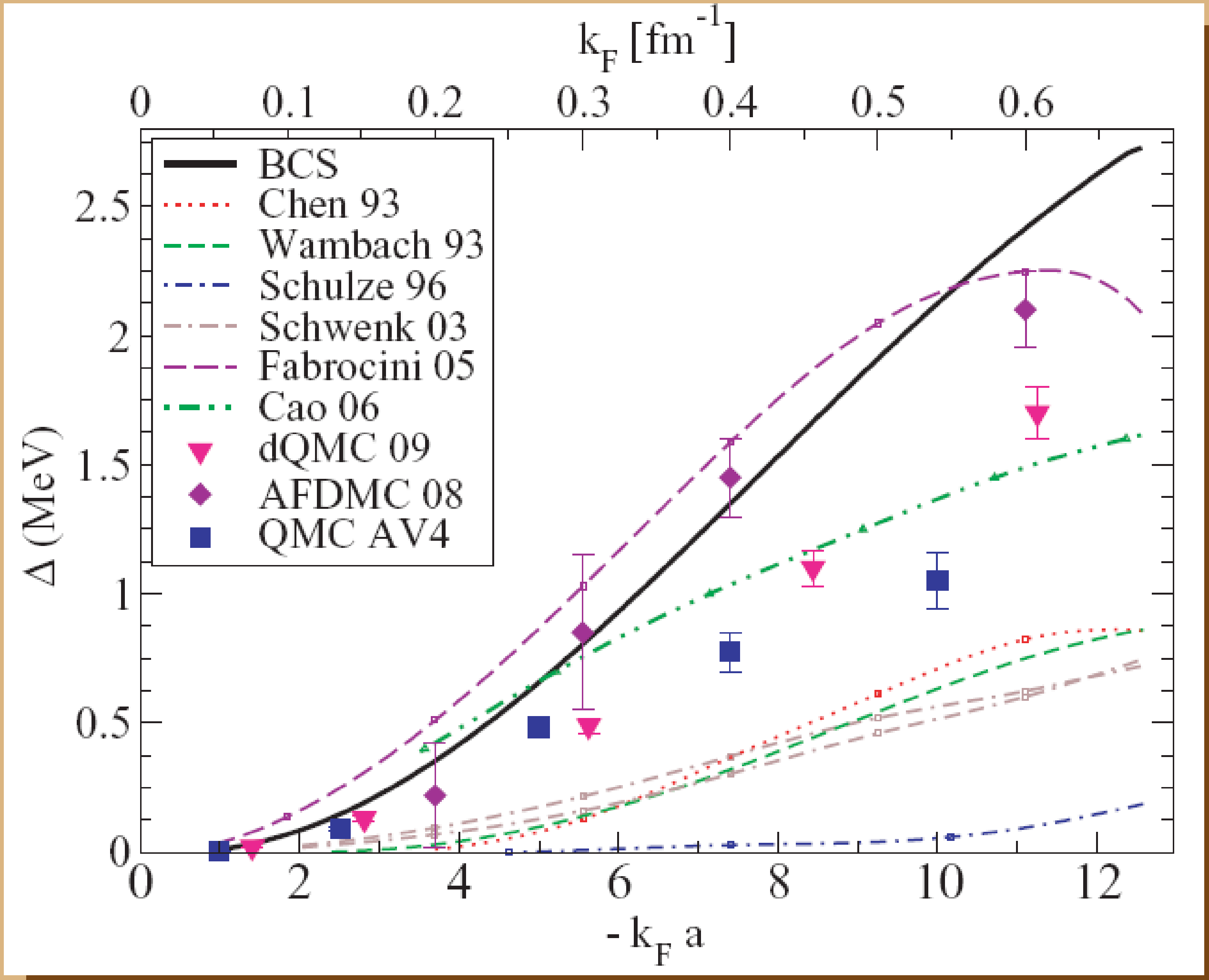


- New experiment at University of Tokyo
- ^{6}Li at $T/T_F < 0.06$
- Experimental extraction includes (some) beyond mean-field effects

M. Horikoshi et al, Phys. Rev. X **7**, 041004 (2017)

ATOMS

Pairing gaps: comparison



- Consistent suppression with respect to BCS; similar to Gorkov
- Disagreement with AFDMC studied extensively
- Emerging consensus

NEUTRONS

A. Gezerlis and J. Carlson, Phys. Rev. C **81**, 025803 (2010)
S. Gandolfi, A. Gezerlis, and J. Carlson, Ann. Rev. Nucl. Part. Sci. **65**, 303 (2015)

BCS in a box

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k}|V|\mathbf{k}'\rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}} \qquad \langle N \rangle = \sum_{\mathbf{k}} \left[1 - \frac{\xi(\mathbf{k})}{\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}} \right]$$

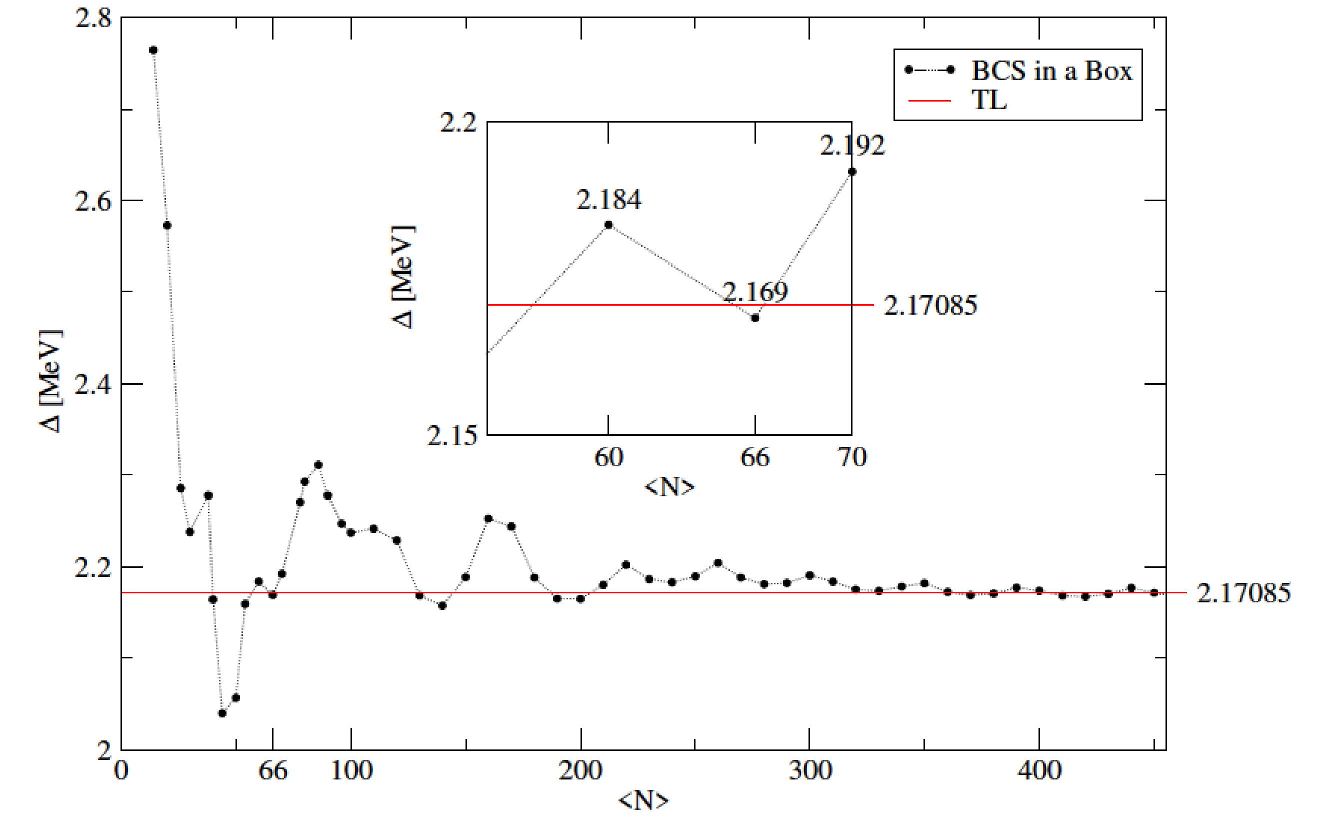
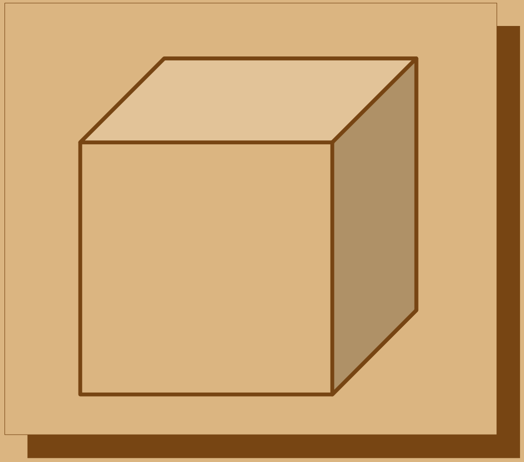


Figure by George Palkanoglou

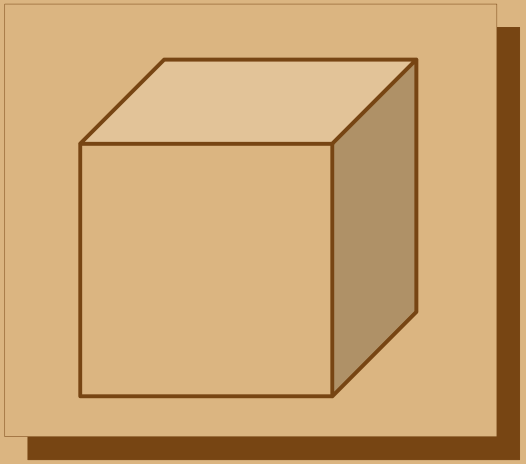


BCS in a box: symmetry restoration

$$|\psi_N\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi i} e^{-iM\phi} \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + e^{i\phi} v_{\mathbf{k}} \hat{p}^\dagger_{\mathbf{k}} \right) |0\rangle$$

$$E_{even}(N) = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} 2 v_{\mathbf{k}}^2 \frac{R_1^1(\mathbf{k})}{R_0^0} + \sum_{\mathbf{k}} V_{\mathbf{kk}} v_{\mathbf{k}}^2 \frac{R_1^1(\mathbf{k})}{R_0^0} + \sum_{\mathbf{k},\mathbf{l}|\mathbf{k}\neq\mathbf{l}} V_{\mathbf{kl}} u_{\mathbf{k}} u_{\mathbf{l}} v_{\mathbf{k}} v_{\mathbf{l}} \frac{R_1^2(\mathbf{kl})}{R_0^0}$$

K. Dietrich, H. J. Mang, and J. H. Pradal, Phys. Rev. **135**, 22 (1964)



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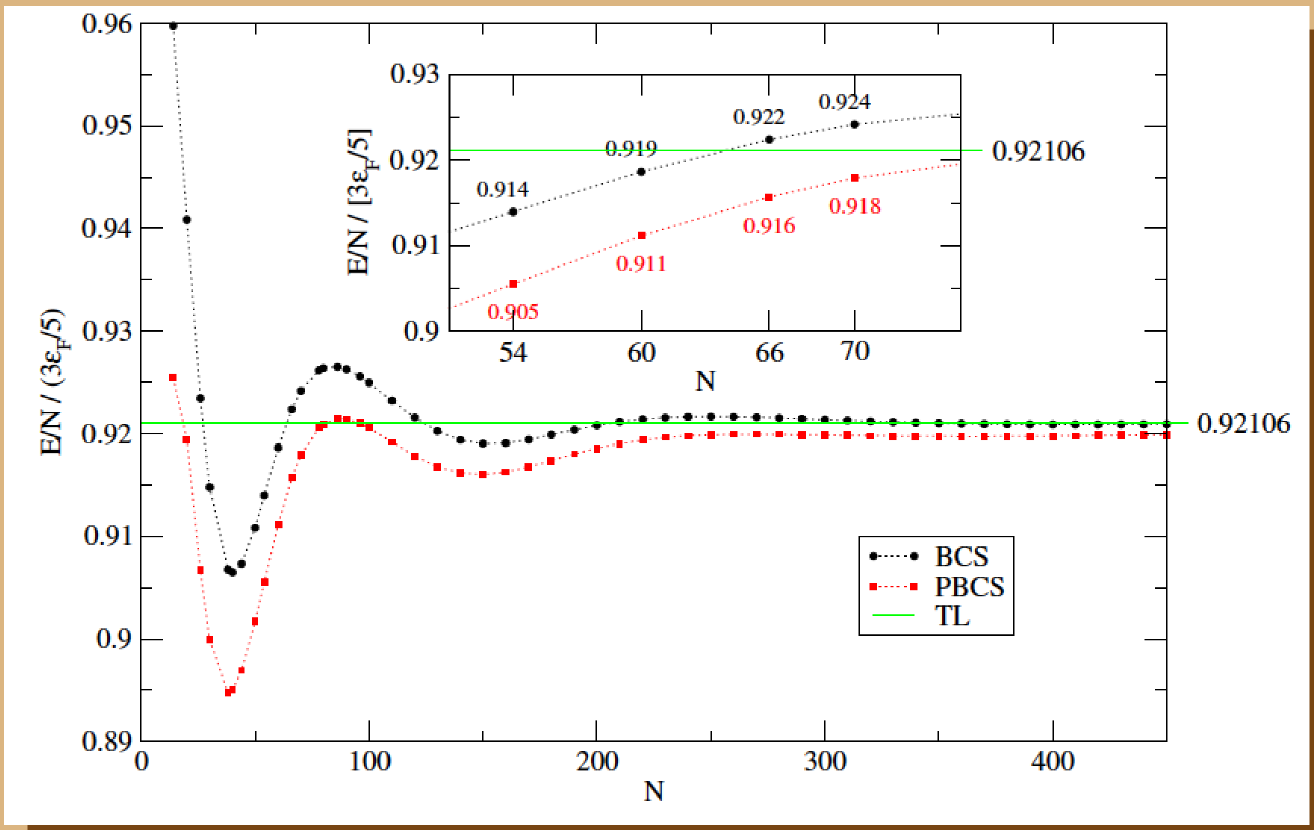
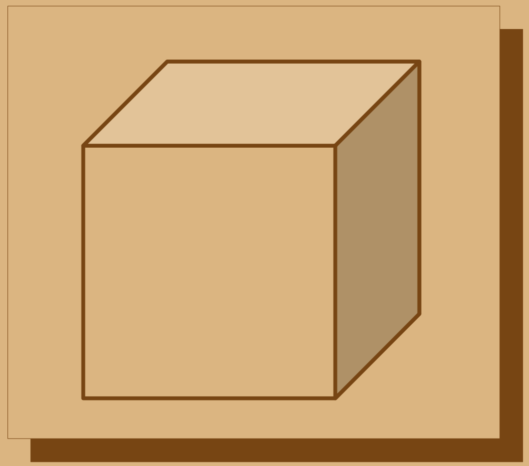
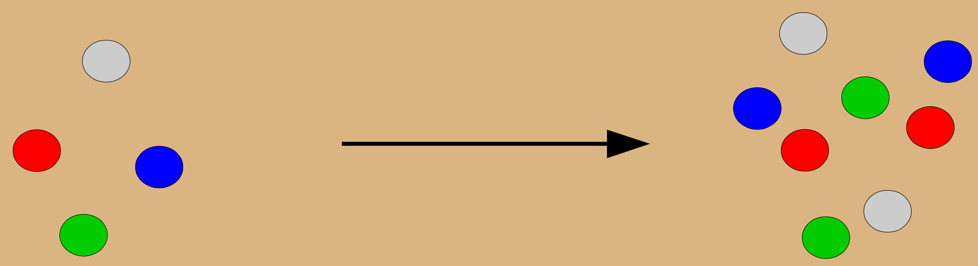


Figure by George Palkanoglou



Clustering in four-component unitary fermions

QMC for 4 species



QMC for 4 species

Motivation

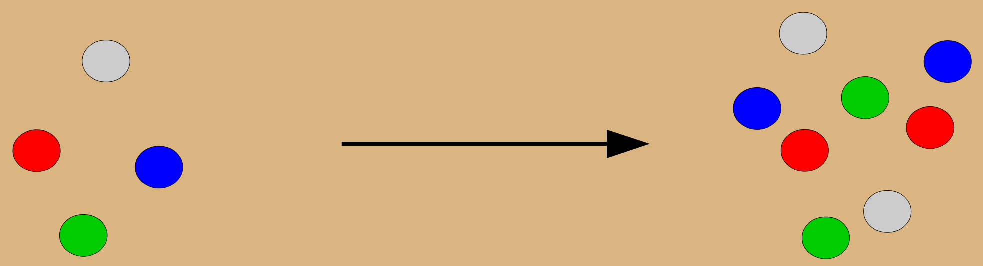
- Very successful cold Fermi atom experiments with few or many particles

- Nuclear physics around the unitary limit:
  S. Koenig, H. W. Griesshammer, H.-W. Hammer, U. van Kolck
  Phys. Rev. Lett. **118**, 202501 (2017)

- Unitary bosons from clusters to matter
  J. Carlson, S. Gandolfi, U. van Kolck, S. A. Vitiello
  Phys. Rev. Lett. **119**, 223002 (2017)

NUCLEONS

QMC for 4 species



Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

$$V_{i,j} = -V_2\mu^2\frac{2\hbar^2}{m}\exp[-(\mu r_{ij})^2/2]$$

$$V_{i,j,k} = V_3\left(\frac{\mu}{4}\right)^2\frac{2\hbar^2}{m}\exp[-(\mu R_{ijk}/4)^2/2]$$

QMC for 4 species

Three trial wave functions explored

$$\psi_T^A = f_J \left[\Phi_{BCS}^{I,II} \Phi_{BCS}^{III,IV} + \Phi_{BCS}^{I,III} \Phi_{BCS}^{II,IV} + \Phi_{BCS}^{I,IV} \Phi_{BCS}^{II,III} \right]$$

W. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis, *arXiv:1908.04288*

QMC for 4 species

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$$\begin{aligned}\psi_T^B =& \mathcal{A}\Big[e^{-\alpha\sum_{i=1,3,5,7}(r_i - r_{CM}^{1,3,5,7})^2}\times\\ & e^{-\alpha\sum_{j=2,4,6,8}(r_j - r_{CM}^{2,4,6,8})^2}\times\\ & e^{-\beta(r_{CM}^{1,3,5,7} - r_{CM}^{2,4,6,8})^2}(r_{CM}^{1,3,5,7} - r_{CM}^{2,4,6,8})^n\Big]\end{aligned}$$

W. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis, *arXiv:1908.04288*

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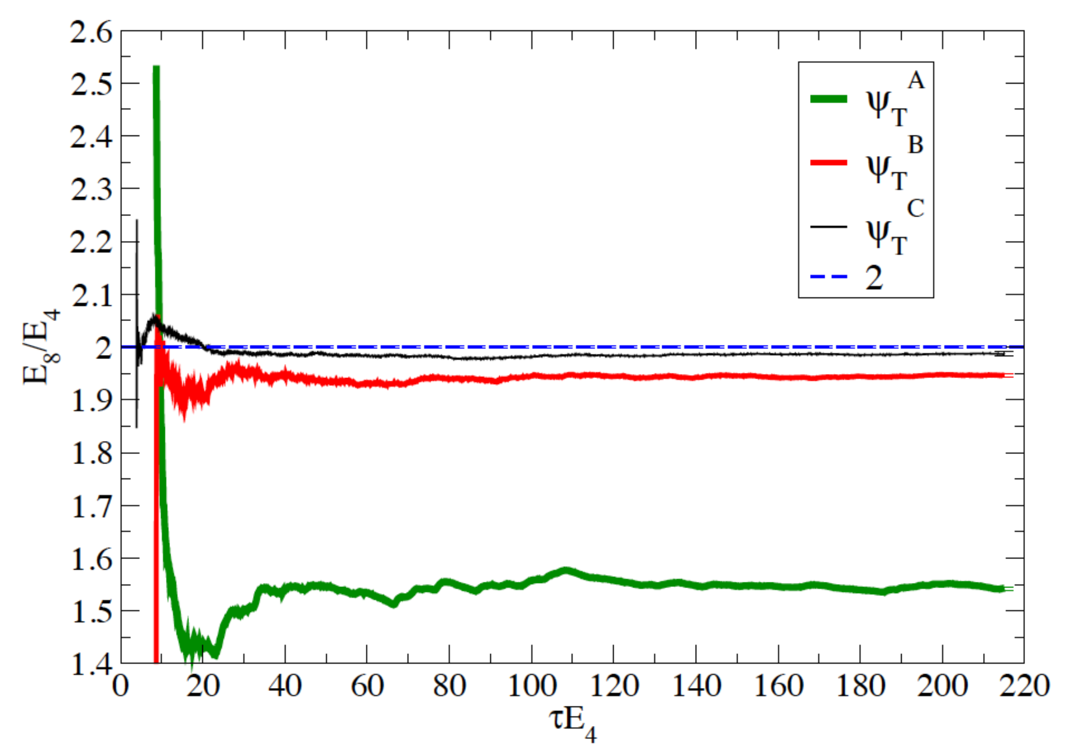
$$\begin{aligned}\psi_T^C = \mathcal{A}[F(r_{CM}^{1,3,5,7} - r_{CM}^{2,4,6,8}) \times f_J(r_1, r_3, r_5, r_7)\times \\ f_J(r_2, r_4, r_6, r_8) \times \prod_{\substack{n=1,3,5,7\\ m=2,4,6,8}} g(r_{nm})]\end{aligned}$$

W. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis, *arXiv:1908.04288*

QMC for 4 species: 8 particles



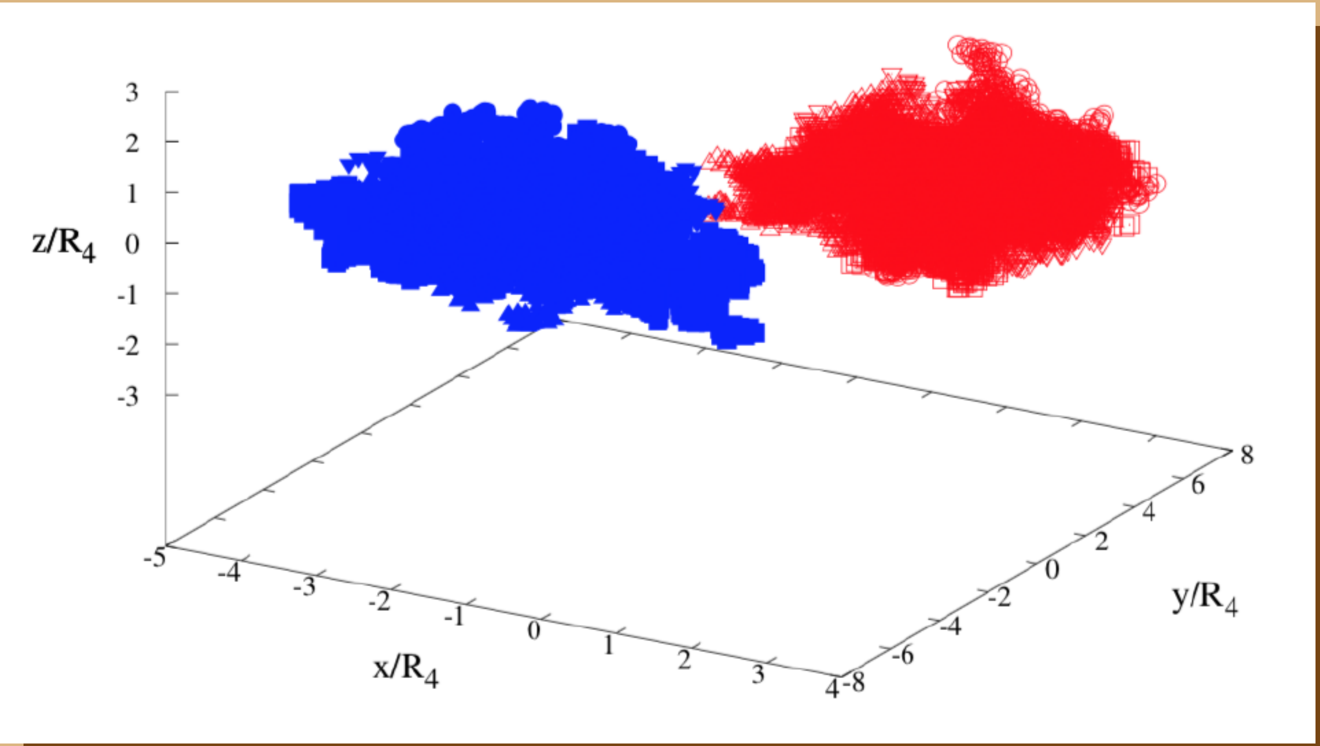
- Pionless EFT with NN+NNN
- Careful time-step extrapolation
- 8Be found to be (barely) bound wrt to α decay, already at LO

W. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis, *arXiv:1908.04288*

NUCLEONS

QMC for 4 species: 8 particles

The two clusters are communicating



W. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis, *arXiv:1908.04288*

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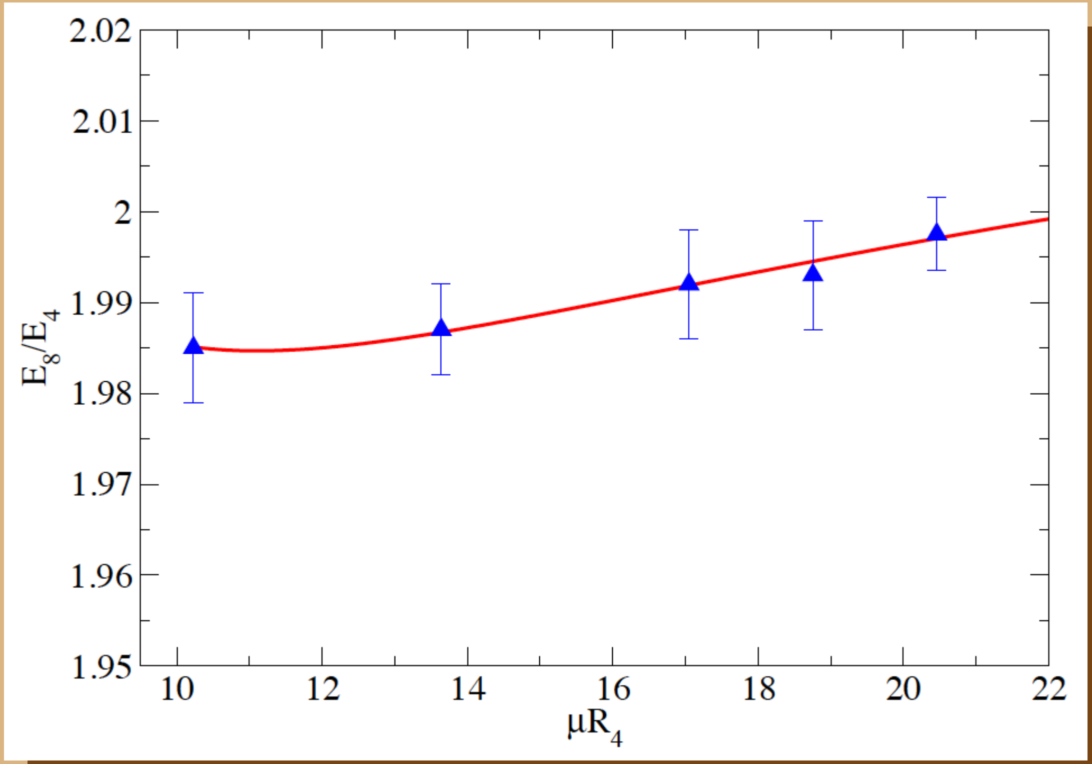
QMC for 4 species: 8 particles



- Clusters of unitary fermions
- Tested dependence on V_3 and μ
- Extrapolated to zero range

$$\frac{E_8}{E_4} = c_0 + \frac{c_1}{\mu R_4} + \frac{c_2}{(\mu R_4)^2}$$

finding $$\frac{E_8}{E_4} = 2.04 \pm 0.05$$

W. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis, *arXiv:1908.04288*

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Conclusions

- Lots of pairing physics insensitive to interaction details
- Rich connections between ultracold atomic gases and nuclear physics
- Ab initio and phenomenology are mutually beneficial

Acknowledgments

Collaborators

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- Brendan Bulthuis
- George Palkanoglou
- Bernard Ross
- Ermal Rrapaj
- Tash Zielinski

LBNL

- Augusto Macchiavelli

LANL

- Joe Carlson
- Stefano Gandolfi

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