UNIVERSIDAD
DE GRANADA

Mirror nuclei in the mass region $A = 20 - 50$: effect of tensor interaction and pairing

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Work in colaboration with

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Outline

Why including a tensor term in the nuclear effective interaction?

The method: HF+BCS+QRPA

Low-lying excitations in mirror nuclei

Conclusions

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Overview

Introduction

- Otsuka *et al.* showed that shell evolution cannot be studied without tensor force
  Phys. Rev. Lett. **95**, 232502 (2005)
- They proposed a new parametrization for the Gogny force including a tensor-isospin term $\rightarrow$ GT2
  Phys. Rev. Lett. **97**, 162501 (2006)

μ (fm)	W (MeV)	B (MeV)	H (MeV)	M (MeV)
0.7	2311	−3480	2962	−2800
1.2	−339	388	−370	260
W_0=160 MeV fm^5		$x_0 = 1$		$\alpha = 1/3$

Introduction

In addition, GT2 interaction includes a tensor-isospin term

$$\boxed{v_{\mathrm{T}} = F_{\mathrm{T}}\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, 3\left(\frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r}_{12})(\boldsymbol{\sigma}_2 \cdot \mathbf{r}_{12})}{(\mathbf{r}_{12})^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right) f_{\mathrm{G}}(r)}$$

$f_{\mathrm{G}}(r) \Rightarrow$ a gaussian function with a range of 1.2 fm.

$F_{\mathrm{T}} = 50.795$ MeV $\Rightarrow$ to reproduce the volume integral of the AV8′

"The present work is focused on Hartree-Fock (HF) calculation, and pairing issues are left open for Hartree-Fock- Bogoliubov calculation to be done. Thus, there may be certain rooms for refinement of the parameters, but the GT2 interaction appears to be good enough for the present purpose,..." (from **Phys. Rev. Lett. 97, 162501 (2006))**

The tensor interaction

(M. A. and M. Grasso, Phys. Rev. C88, 054328 (2013))

- We have proposed different types of finite range tensor interactions onto D1S and D1M Gogny parametrizations, adding a term such as:

$$\boxed{V_{\mathrm{T}}(1,2) = \left[V_{\mathrm{T1}} + V_{\mathrm{T2}}\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\right] S_{12}\, \exp\left[-(\mathbf{r}_1 - \mathbf{r}_2)^2/\mu_{\mathrm{T}}\right]}$$

- Different fits have been done in order to fix the free parameters in each case:
  1. Adding a tensor-isospin term ($V_{T1} = 0$), and modifying the strength of the spin-orbit term: D1ST.
  2. Adding a pure tensor and tensor-isospin terms ($V_{T1}, V_{T2} \neq 0$): D1ST2a, D1ST2b.
  3. Adding a pure tensor, tensor-isospin ($V_{T1}, V_{T2} \neq 0$) and modifying the spin-orbit term: D1ST2c, D1ST2d

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The tensor interaction

- In particular, in this presentation, we are going to use the interaction D1ST2a and D1MT2d.
- For fitting parameters in D1ST2a → neutron splitting $1f$ in ^{48}Ca and the energy of the first 0^- state in ^{16}O.
- For D1MTd → energy values of the first 0^- state for the nuclei ^{16}O, ^{40}Ca and ^{48}Ca.
- The parameters of the tensor and s.o. terms of the D1ST2a and D1MTd interactions are:

	V_{T1} (MeV)	V_{T2} (MeV)	μ_T (fm)	$W_{s.o.}$ (MeV $\cdot$ fm^5)
D1ST2a	-135	115	1.2	130
D1MTd	-230	180	1.0	115.36

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- With these interactions, we have studied:

1. Binding and single particle energies in HF approximation.
2. Excitation states with DRPA and CRPA approximations.
3. Pairing properties using a HF+BCS model.
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Overview

Our Hartree-Fock (HF) approximation

(G. Co' and A.M. Lallena, Nuovo Cimento A111 (1998) 527)

- We considere as effective nucleon-nucleon interaction a finite-range two-body force of the type:

$$\boxed{V(\vec{r_1},\vec{r_2}) = \sum_{p=1}^{6} V_p(\vec{r_1},\vec{r_2})O_p(1,2) + V_{\rm SO}(\vec{r_1},\vec{r_2}) + V_{\rm DD}(\vec{r_1},\vec{r_2}) + V_{\rm Coul}(\vec{r_1},\vec{r_2})}$$

- $O_p(1,2)$ indicates $\mathbb{1}$, $\vec{\tau_1}\cdot\vec{\tau_2}$, $\vec{\sigma_1}\cdot\vec{\sigma_2}$, $\vec{\sigma_1}\cdot\vec{\sigma_2}\,\vec{\tau_1}\cdot\vec{\tau_2}$, S_{12}, $S_{12}\,\vec{\tau_1}\cdot\vec{\tau_2}$.
- $V_{\rm SO}$ and $V_{\rm DD}$, terms of zero-range.
- We solve, in coordinate space, a set of equations of the type:

$$-\frac{\hbar^2}{2m_k}\nabla_1^2\,\phi_k(\vec{r_1}) + U(\vec{r_1})\phi_k(\vec{r_1}) - \int {\rm d}^3 r_2 W(\vec{r_1},\vec{r_2})\phi_k(\vec{r_2}) = \epsilon_k\phi_k(\vec{r_1})$$

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Our Hartree-Fock (HF) approximation: Numerical procedure

JOURNAL OF COMPUTATIONAL PHYSICS 45, 374–389 (1982)

On the Numerical Integration of the Schrödinger Equation in the Finite-Difference Schemes

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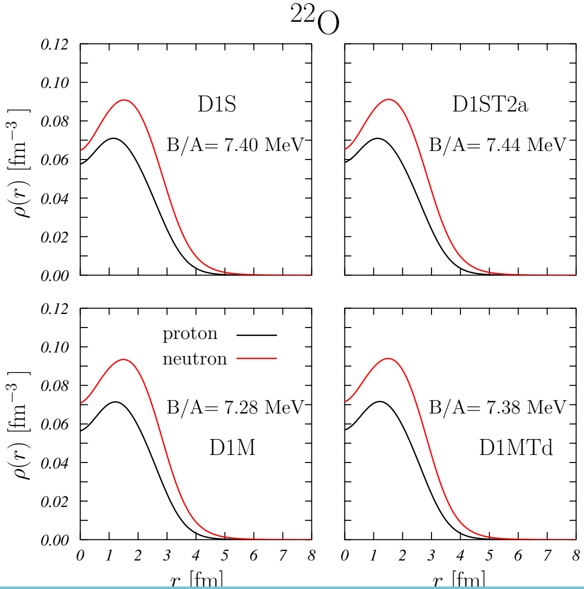
Received August 14, 1981

Formulae for the integration of the Schrödinger equation for bound states based on the scheme of central differences are generated from the Taylor expansion with the help of formal Padé approximants. These methods are studied in matrix form, and a limiting formula—the best for a given discretization—is obtained.

Our Hartree-Fock (HF) approximation: an example



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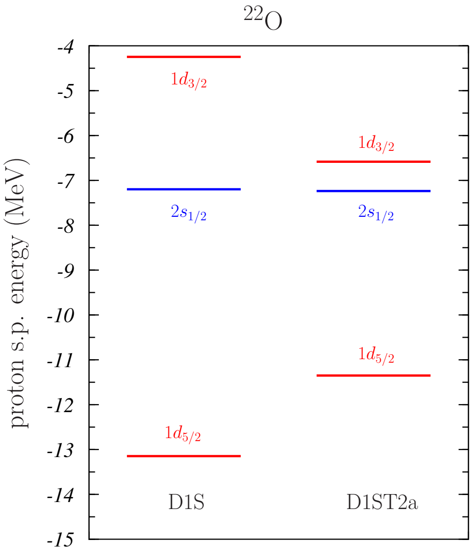

Our HF+BCS approximation

(M. A. *et al.*, J. Phys. G41 (2014) 025102)

- From the s.p. energies and wave functions obtained in HF we solve the BCS equations, considering for the pairing field the same effective nucleon-nucleon interaction as in the HF.
- As result, we have the occupation of the single particle levels, and we can calculate nuclear properties as:

1. Fluctuation of the particle number:

$$\langle(\Delta N)^2\rangle = \langle BCS|\hat{N}^2|BCS\rangle - \langle BCS|\hat{N}|BCS\rangle^2 = \sum_k (2j_k+1)\, u_k^2 v_k^2$$

2. Binding energy:

$$E = \frac{1}{2}\sum_k (2j_k+1)\left[v_k^2(\epsilon_k + \langle k|T|k\rangle) - u_k v_k \Delta_k\right] + R$$

R is the rearrangement term and Δ_k is defined as:

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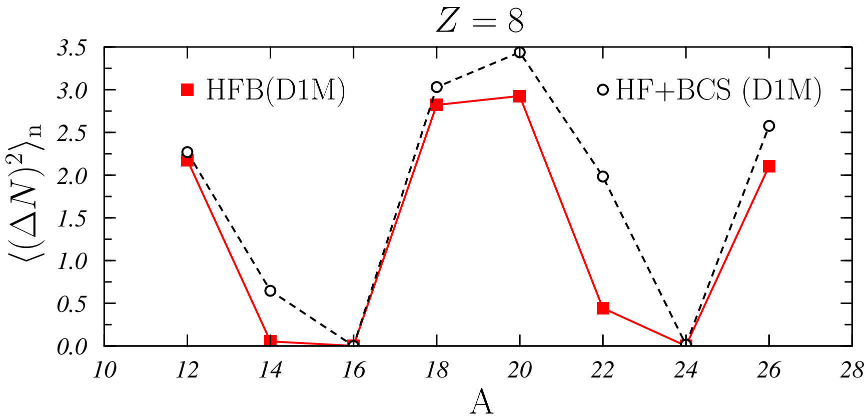

The QRPA approximation

(V. De Donno *et al.*, Phys. Rev. C95, 05439 (2017))

- A QRPA excited state $|k\rangle$ of angular momentum J, third component M, parity Π, and excitation energy ω_k, is described as a combination of quasi-particle excitations on top of the ground state $|0\rangle$:

$$|k\rangle \equiv |J^{\Pi}M;\omega_k\rangle = \sum_{\mu\leq\mu'} \left[X^{(k)}_{\mu\mu'}(J)\, A^{\dagger}_{\mu\mu'}(JM) + (-1)^{J+M+1}\, Y^{(k)}_{\mu\mu'}(J)\, A_{\mu\mu'}(J\ -M)\right]$$

- The QRPA amplitudes X and Y must verify the relation

$$\sum_{\mu\leq\mu'} \left\{[X^{(k)}_{\mu\mu'}(J)]^*\, X^{(k')}_{\mu\mu'}(J) - [Y^{(k)}_{\mu\mu'}(J)]^*\, Y^{(k')}_{\mu\mu'}(J)\right\} = \delta_{kk'}$$

The QRPA approximation

(V. De Donno *et al.*, Phys. Rev. C95, 05439 (2017)

- A QRPA excited state $|k\rangle$ of angular momentum J, third component M, parity Π, and excitation energy ω_k, is described as a combination of quasi-particle excitations on top of the ground state $|0\rangle$:

$$|k\rangle \equiv |J^{\Pi}M;\omega_k\rangle = \sum_{\mu\leq\mu'} \left[X^{(k)}_{\mu\mu'}(J)\, A^{\dagger}_{\mu\mu'}(JM) + (-1)^{J+M+1}\, Y^{(k)}_{\mu\mu'}(J)\, A_{\mu\mu'}(J-M)\right]$$

- The QRPA amplitudes X and Y must verify the relation

$$\sum_{\mu\leq\mu'} \left\{[X^{(k)}_{\mu\mu'}(J)]^*\, X^{(k')}_{\mu\mu'}(J) - [Y^{(k)}_{\mu\mu'}(J)]^*\, Y^{(k')}_{\mu\mu'}(J)\right\} = \delta_{kk'}$$

The QRPA approximation

- The definition of the quasi-particle pair creation and annihilation operators:

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\begin{aligned}
A^\dagger_{\mu\mu'}(JM) &= C_{\mu\mu'}(J) \sum_{m_\mu, m_{\mu'}} \langle j_\mu \, m_\mu \, j_{\mu'} \, m_{\mu'} | J \, M \rangle \, \alpha^\dagger_{\mu m_\mu} \, \alpha^\dagger_{\mu' m_{\mu'}} \\
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\end{aligned}
$$

where

$$
C_{\mu\mu'}(J) = \frac{\sqrt{1 + (-1)^J \delta_{\mu\mu'}}}{1 + \delta_{\mu\mu'}}
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- By using standard techniques the QRPA secular equations can be written as

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\begin{bmatrix} \mathcal{A}(J) & \mathcal{B}(J) \\ -\mathcal{B}^*(J) & -\mathcal{A}^*(J) \end{bmatrix} \begin{bmatrix} X^{(k)}(J) \\ Y^{(k)}(J) \end{bmatrix} = \omega_k \begin{bmatrix} X^{(k)}(J) \\ Y^{(k)}(J) \end{bmatrix}
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- The matrix elements of $\mathcal{A}$ is given by the expression:

$$\begin{aligned}\mathcal{A}_{[\mu\mu']J,[\nu\nu']J} &= (E_\mu + E_{\mu'})\,\delta_{\mu\nu}\,\delta_{\mu'\nu'} \\ &+ C_{\mu\mu'}(J)\,C_{\nu\nu'}(J)\Big\{F(\mu\mu'\nu\nu';J)\,(u_\mu\bar{v}_{\mu'}u_\nu\bar{v}_{\nu'} + \bar{v}\leftrightarrow u) \\ &- (-1)^{j_\nu+j_{\nu'}-J}\,F(\mu\mu'\nu'\nu;J)\,(u_\mu\bar{v}_{\mu'}\bar{v}_\nu u_{\nu'} + \bar{v}\leftrightarrow u) \\ &+ G(\mu\mu'\nu\nu';J)\,(u_\mu u_{\mu'}u_\nu u_{\nu'} + \bar{v}\leftrightarrow u)\Big\}\end{aligned}$$

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The QRPA approximation

- The secular QRPA equations are independent of the *m* quantum numbers because we are considering spherical nuclei.
- In the previous equations:

$$\bar{v}_\mu = (-1)^{l_\mu} v_\mu$$

- The F and G functions contain the matrix elements of the interaction; their expressions are:

$$F(\mu\mu'\nu\nu';J) = \sum_K (-1)^{j'_\mu+j_\nu+K} (2K+1) \left\{ \begin{array}{ccc} j_\mu & j_{\mu'} & J \\ j_\nu & j_{\nu'} & K \end{array} \right\} \langle \mu\nu';K|\overline{V}|\mu'\nu;K\rangle$$

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Convergence of calculations

- In HF calculation $\rightarrow R_{\text{box}} = 15$ fm.
- In BCS calculation, to determine the configuration space $\rightarrow E_{\text{cut}} = 10$ MeV.
- In QRPA calculation, the configuration space is selected considering the limits

E_p [MeV]	v_{cut}	E_{pp} [MeV]
70	10^{-4}	50

- The configuration space used in the QRPA calculations includes all the s.p. states with energy smaller than E_p.
- We neglect those pairs composed by quasi-particles whose energy is larger than E_{pp}.
- We further reduce the number of this type of pairs by omitting those where the product of the occupation probability v^2 of both states forming the pair is smaller than v_{cut}.
- Numerical limit of the QRPA matrix dimension $\rightarrow N = 4000$.

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2^+ and 4^+ excitations for mirror nuclei: O isotopes

Nucleus	excited state	D1S	D1ST2a	D1M	D1MTd	exp
^{18}O	2^+	2.409	2.615	2.324	2.691	1.982
	4^+	3.452	3.677	3.196	3.575	3.554
^{18}Ne	2^+	1.787	1.973	1.697	2.027	1.887
	4^+	2.703	2.912	2.467	2.811	3.338
^{20}O	2^+	2.342	2.509	2.233	2.554	1.67
	4^+	3.493	3.690	3.191	3.535	3.57
^{20}Mg	2^+	1.633	1.809	1.528	1.832	1.65
	4^+	2.627	2.825	2.363	2.685	3.70
^{22}O	2^+	2.839	2.936	2.447	2.716	3.199
	4^+	4.067	4.430	3.423	3.876	
^{22}Si	2^+	2.212	2.325	1.920	2.121	
	4^+	3.477	3.722	2.936	3.329	

2^+ and 4^+ excitations for mirror nuclei: Ca isotopes

Nucleus	excited state	D1S	D1ST2a	D1M	D1MTd	exp
^{42}Ca	2^+	2.150	2.374	2.090	2.462	1.525
	4^+	2.856	3.111	2.683	3.079	2.752
^{42}Ti	2^+	1.478	1.653	1.418	1.711	1.554
	4^+	2.079	2.281	1.921	2.237	2.676
^{46}Ca	2^+	2.062	2.259	1.997	2.323	1.346
	4^+	2.794	3.019	2.619	2.973	2.574
^{46}Fe	2^+	1.362	1.523	1.291	1.562	
	4^+	1.990	2.176	1.826	2.119	

2^+ and 4^+ excitations for mirror nuclei: Ni isotopes

Nucleus	excited state	D1S	D1ST2a	D1M	D1MTd	exp
^{52}Ni	2^+	1.624	1.397	1.566	1.502	1.397
	4^+	2.963	3.012	2.802	3.019	2.385
^{52}Cr	2^+	1.111	0.954	1.064	1.093	1.434
	4^+	2.346	2.403	2.176	2.378	2.369
^{54}Ni	2^+	1.671	1.319	1.581	1.396	1.392
	4^+	2.883	2.975	2.709	2.975	2.620
^{54}Fe	2^+	1.217	0.964	1.111	1.029	1.408
	4^+	2.305	2.401	2.119	2.341	2.538

2^+ and 4^+ excitations for mirror nuclei: Results

- Experimentally, the values of the excitation energies for the first 2^+ and 4^+ states are very similar for mirror nuclei.
- When the main excitation correspond to proton states (closed shell in neutrons), the excitation energies always decrease respect to the values for the corresponding mirror nucleus.
- The only reason to have different values theoretically → Coulomb?

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Main configurations (D1S)

Nucleus	Excitation	Configuration	X	Y
^{18}O	2^+	$(2s_{1/2}, 1d_{5/2})_\nu$	-0.287381	-0.019379
		$(1d_{5/2}, 1d_{5/2})_\nu$	-0.948714	$+0.079155$
	4^+	$(1d_{5/2}, 1d_{5/2})_\nu$	$+0.999831$	-0.080058
^{18}Ne	2^+	$(2s_{1/2}, 1d_{5/2})_\pi$	$+0.256248$	$+0.031559$
		$(1d_{5/2}, 1d_{5/2})_\pi$	$+0.955319$	-0.011334
	4^+	$(1d_{5/2}, 1d_{5/2})_\pi$	$+0.997869$	-0.037098
^{42}Ca	2^+	$(1f_{7/2}, 1f_{7/2})_\nu$	-0.986666	$+0.057348$
	4^+	$(1f_{7/2}, 1f_{7/2})_\nu$	$+0.997532$	-0.052466
^{42}Ti	2^+	$(1f_{7/2}, 1f_{7/2})_\pi$	-0.989179	-0.006139
	4^+	$(1f_{7/2}, 1f_{7/2})_\pi$	-0.996897	$+0.011713$

Main excitations (D1S)

Nucleus	Excitation	Configuration	X	Y
^{52}Ni	2^+	$(2p_{3/2}, 1f_{7/2})_\pi$	-0.448755	-0.215238
		$(1f_{7/2}, 1f_{7/2})_\nu$	-0.864352	-0.088426
		$(2p_{3/2}, 1f_{7/2})_\nu$	-0.247448	-0.101419
	4^+	$(1f_{7/2}, 1f_{7/2})_\nu$	-0.977384	$+0.049847$
^{52}Cr	2^+	$(2p_{3/2}, 1f_{7/2})_\nu$	$+0.406814$	$+0.254743$
		$(1f_{7/2}, 1f_{7/2})_\pi$	$+0.931338$	$+0.204358$
		$(2p_{3/2}, 1f_{7/2})_\pi$	$+0.222564$	$+0.116904$
	4^+	$(1f_{7/2}, 1f_{7/2})_\pi$	-0.987416	$+0.005029$

Case of $A = 18$ and $A = 42$

- $1d_{5/2}$ s.p. states appear in the main configurations for ^{18}O and ^{18}Ne, and $1f_{7/2}$ in the case of ^{42}Ca and ^{42}Ti.
- With the D1S interaction, we have:

Nucleus	s.p state	s.p. energy [MeV]
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^{18}Ne	$(1d_{5/2})_\pi$	-1.910
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- We have studied the low lying excitations 2^+ and 4^+ for different mirror nuclei in the region $A \approx 20$ and $A \approx 50$.
- We have used a QRPA approximation on top of a HF+BCS calculation.
- The Gogny interaction (D1S and D1M parametrizations) and two versions with a tensor term (D1ST2a and D1MTd) has been considered as effective force in all the steps of the calculations.
- Tensor interaction slightly increases the values of the first excited states 2^+ for $Z, N = 8$ and $Z, N = 20$ and the opposite for $Z, N = 28$. For 4^+ state, we always observe a small increase.
- Experimentally, the energy of the 2^+ and 4^+ excitation states is very similar for mirror nuclei.
- Theoretically, we obtain for the mirror nucleus with closed shell in neutrons a lower excitation energy for both 2^+ and 4^+ states with respect to the values for the other nucleus, with closed shell in protons

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- Theoretically, we obtain for the mirror nucleus with closed shell in neutrons a lower excitation energy for both 2^+ and 4^+ states with respect to the values for the other nucleus, with closed shell in protons

Conclusions

- We have studied the low lying excitations 2^+ and 4^+ for different mirror nuclei in the region $A \approx 20$ and $A \approx 50$.
- We have used a QRPA approximation on top of a HF+BCS calculation.
- The Gogny interaction (D1S and D1M parametrizations) and two versions with a tensor term (D1ST2a and D1MTd) has been considered as effective force in all the steps of the calculations.
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