Proton Neutron Pairing Correlations
within a mean field picture

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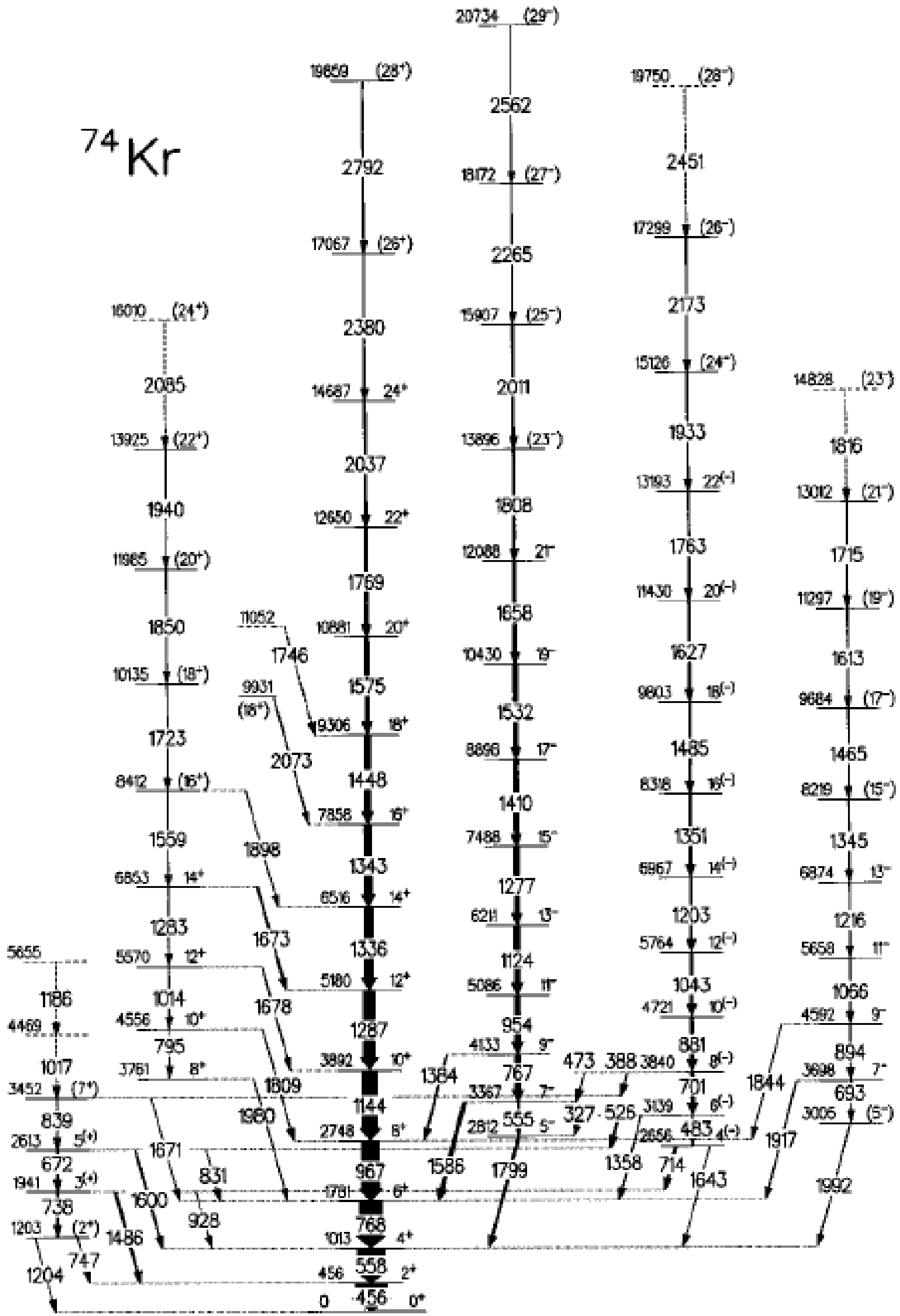
- The constant gap T=0 and T=1 pairing model and model properties
- The effect on binding energies
- Rotational states within a T=0 and T=1 model description
- T=0 pair scattering
- Symmetry breaking of the T=1 pairing interaction and calculations of T=1 rotational states in odd-odd nuclei

Spectrum 74Kr



Systematics of even-even $T_z = 1$ nuclei in the $A = 80$ region: High-spin rotational bands in ^{74}Kr, ^{78}Sr, and ^{82}Zr

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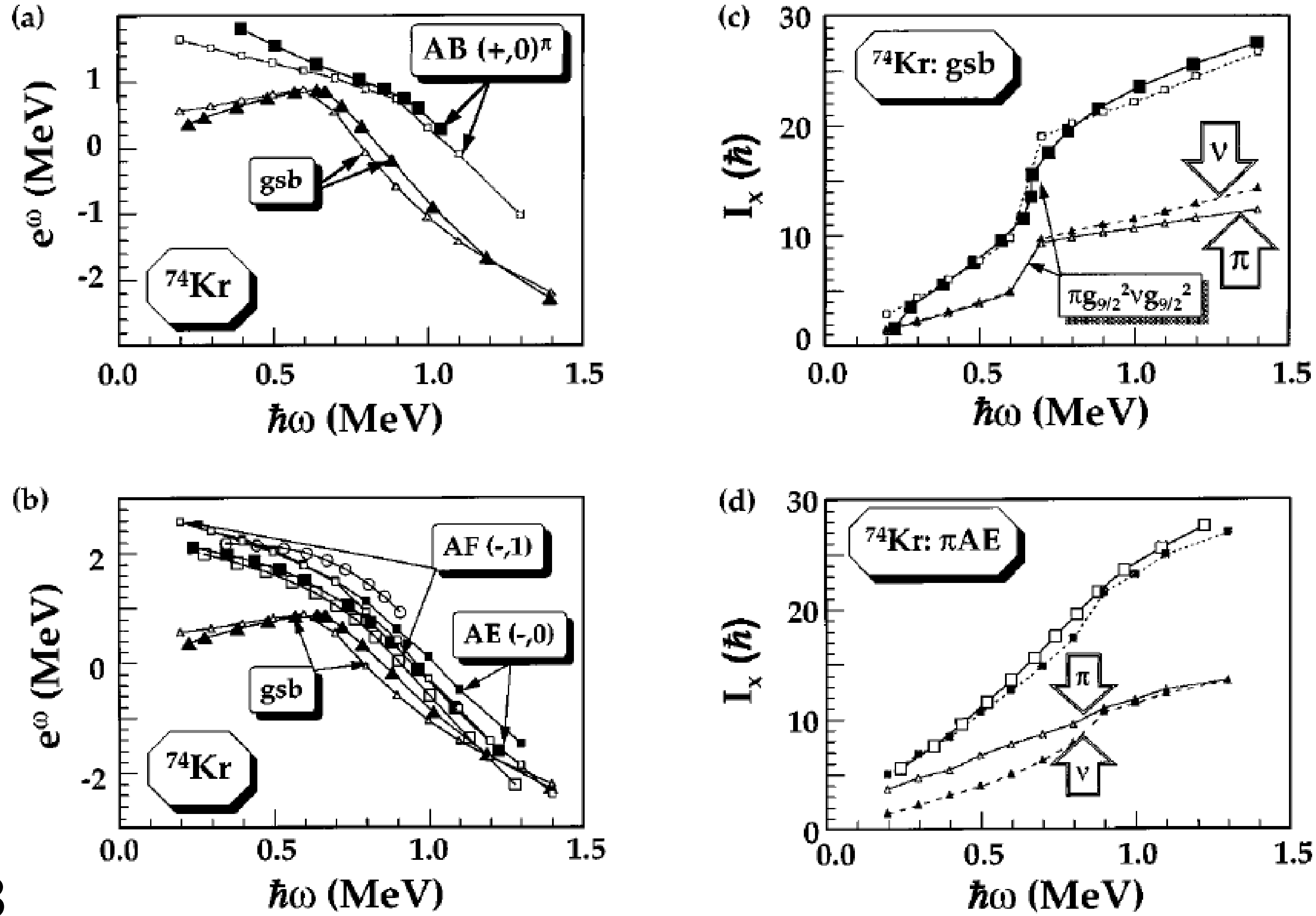
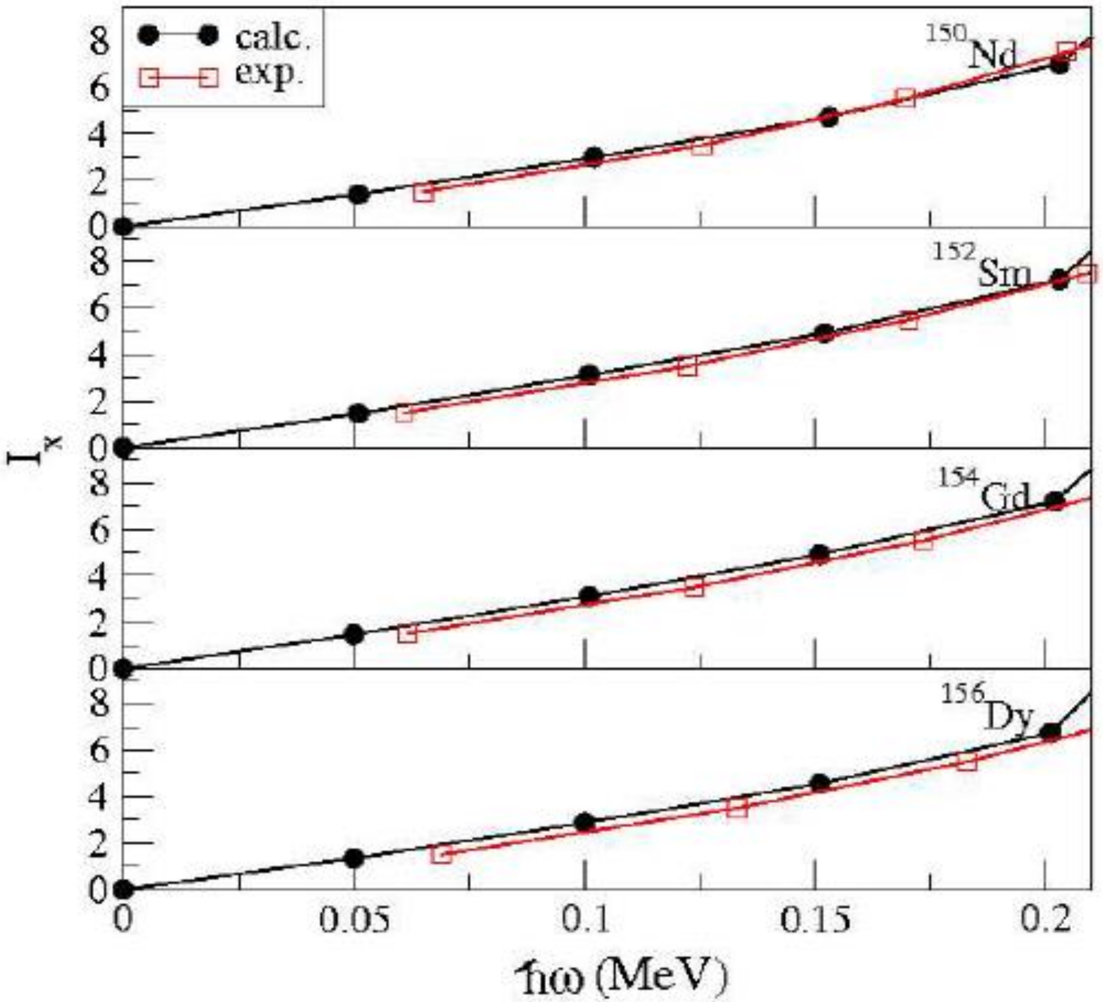


PRC 56(1996),98

N=90 isotones – comparison between experiment and extended TRS model calculations

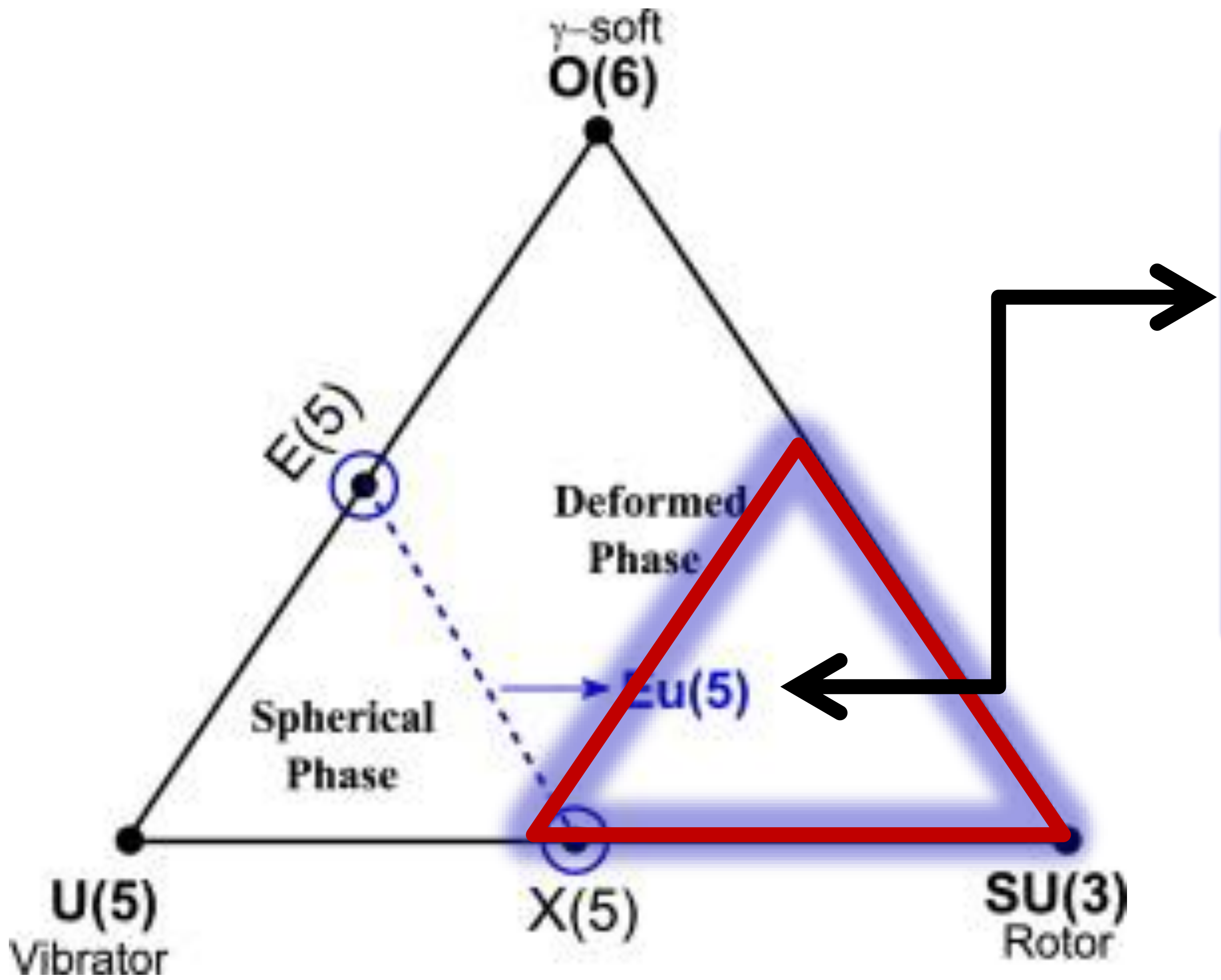


$$E(4_1^+)/E(2_1^+) = 3,33\,\frac{J^{(2)}}{J^{(4)}}$$

TABLE I: The calculated and experimental $E(4_1^+)/E(2_1^+)$ ratio for the N=90 isotones.

Nucleus	Calculated $E(4_1^+)/E(2_1^+)$	Experimental $E(4_1^+)/E(2_1^+)$
^{150}Nd	2.90	2.927
^{152}Sm	3.04	3.009
^{154}Gd	3.12	3.015
^{156}Dy	3.07	2.934

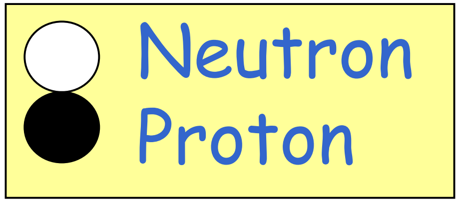
E Ganioğlu, R Wyss, P Magierski
Physical Review C 89 (1), 014311; 2014



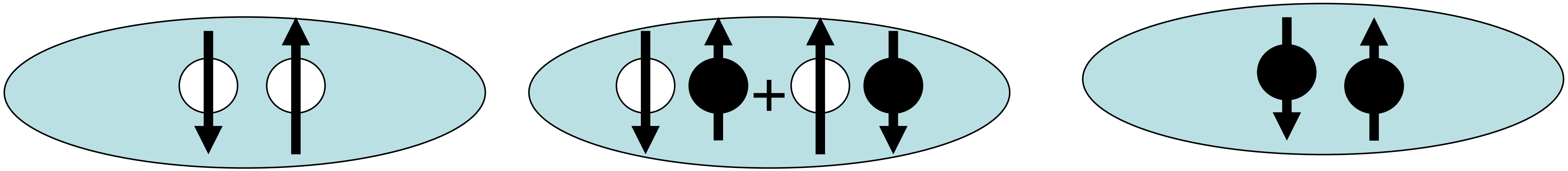
Validity of independent quasi particle motion like extended TRS calculations

Structure of Nucleonic Pairs

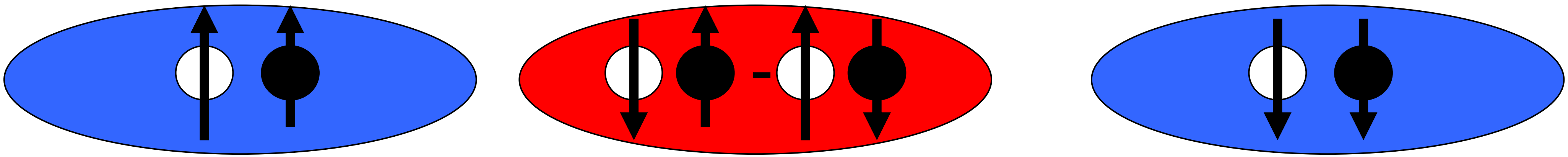
- N=Z → (almost) identical wavefunctions
- particle particle interaction between pairs with identical orbits
- Pauli Principle



Isovector Pairs T=1, S=0



Isoscalar Pairs T=0, S=1



Generalised pairing interaction

- Start from a basis in which signature α is a good quantum number:
  $R_x(\pi)|\phi_j\rangle = +/- \mathbf{i}\,|\phi_j\rangle = e^{i\alpha}|\phi_j\rangle, \alpha = +/- 1/2$,
- The standard pairing interaction scatters pairs in opposite signature orbits,

$$\alpha\overline{\alpha} \leftrightarrow \alpha'\overline{\alpha'} \qquad P^{\dagger}_{1\,\pm 1} = \sum_{i>0} a^{\dagger}_{i\,{n \atop p}} a^{\dagger}_{\tilde{i}\,{n \atop p}}$$

- All possible couplings need to be present: T=1 nn, pp, **and** T=1 np

$$P^{\dagger}_{1\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^{\dagger}_{i\,n} a^{\dagger}_{\tilde{i}\,p} + a^{\dagger}_{i\,p} a^{\dagger}_{\tilde{i}\,n})$$

- For the T=0 pairingg, ***two*** different couplings are possible:
  a) a T=0 np pair scatters between orbits of opposite signature,

$$\alpha\overline{\alpha} \leftrightarrow \alpha'\overline{\alpha'} \qquad P^{\dagger}_{1\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^{\dagger}_{i\,n} a^{\dagger}_{\tilde{i}\,p} - a^{\dagger}_{i\,p} a^{\dagger}_{\tilde{i}\,n})$$

- b)a T=0 np pair scatters between orbits of the same signature,

$$\alpha\alpha \leftrightarrow \alpha'\alpha' \qquad \tilde{P}^{\dagger}_{0\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^{\dagger}_{i\,n} a^{\dagger}_{i\,p} + a^{\dagger}_{\tilde{i}\,p} a^{\dagger}_{\tilde{i}\,n})$$

Is pairing a L=0 interaction?

- Depends whom you are asking

- Shell model: yes and very relevant for T=1 interaction.
- For T=0, there is a dominant J=1 component – but what is the L=?
  In singel j-shell, e.g. j=d5/2 ratio (L=0/L=2) = 6/5
  d3/2 ratio (L=0/L=2) = 5/24
- The L=0, T=0 coupling arises strongest between spin orbit partners, e.g. d3/2 and d5/2 etc, since in order to have S=1, the individual l couple l_m and l_{-m} .
  It becomes quenched by spin-orbit $(\sigma\tau\ \sigma\tau)$

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For mean field, definitively not – L is not determined-
For a simple seniority interaction GP^+P all multipoles can be present.

Investigate the generalised pairing hamiltonian

$$\hat{H}^{\omega_\tau} = \hat{h}_{sp} - G_{t=1}\hat{\boldsymbol{P}}_1^\dagger\hat{\boldsymbol{P}}_1 - G_{t=0}\hat{\boldsymbol{P}}_0^\dagger\hat{\boldsymbol{P}}_0 - \vec{\boldsymbol{\omega}}_\tau\hat{\vec{\boldsymbol{t}}},$$

$$h_{\alpha\beta} = e_\alpha\delta_{\alpha\beta} - \omega j^{(x)}_{\alpha\beta} + \Gamma_{\alpha\beta},$$

Employ approximate number projection via L.N.

$$\hat{\mathcal{H}}^\omega = \hat{H}^\omega - \sum_\tau \lambda_\tau^{(1)}\Delta\hat{N}_\tau - \sum_{\tau\tau'}\lambda_{\tau\tau'}^{(2)}\Delta\hat{N}_\tau\Delta\hat{N}_{\tau'}$$

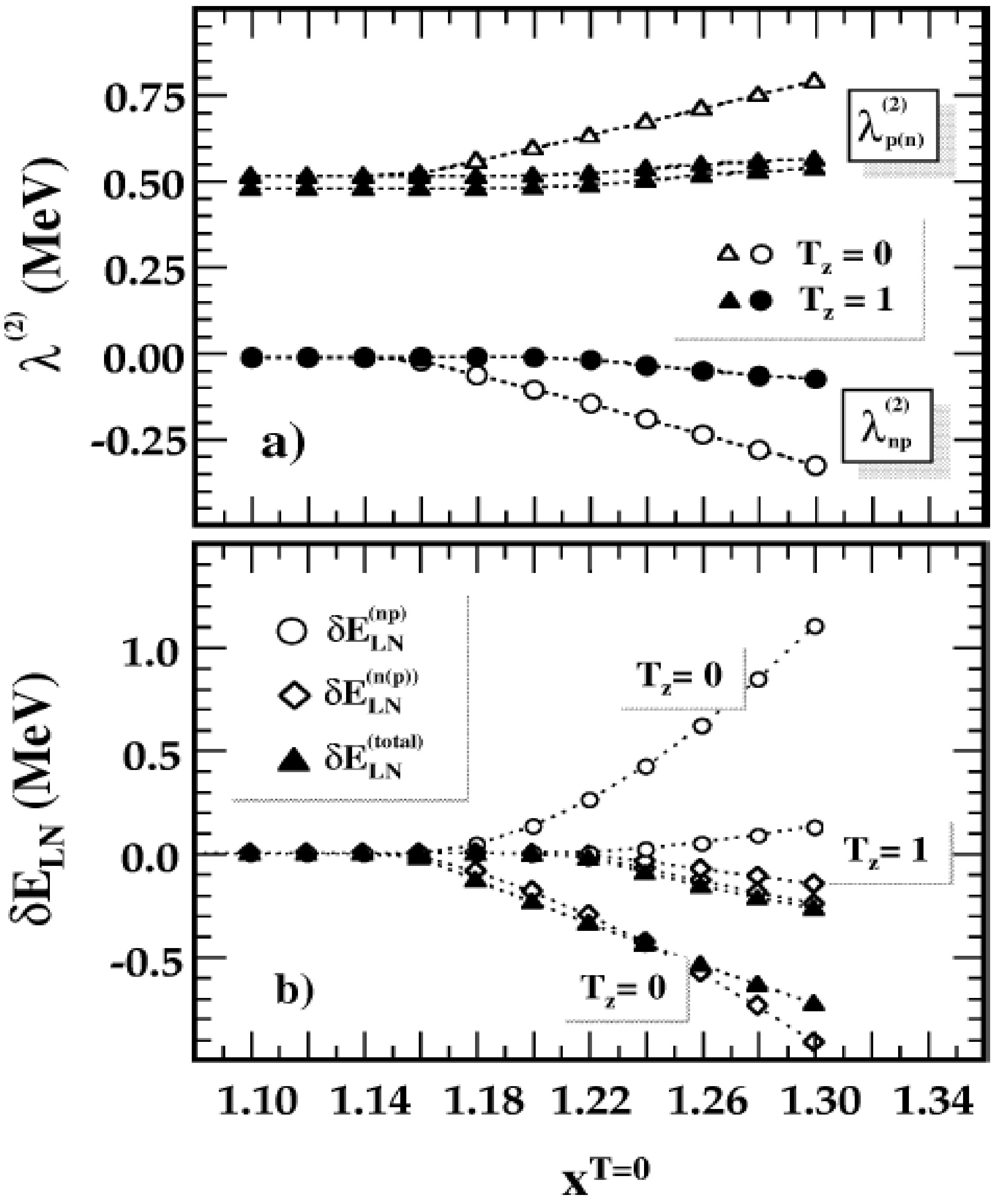
Investigate the BCS- and HFB solution as a function of strength

-BCS $G^{\mathbf{T=0}}/G^{\mathbf{T=1}}$ =? and HFB $G^{\mathbf{T=0}}/G^{\mathbf{T=1}}$ =?

Lipking Nogami corrections:

$$\langle \mathrm{LN} | \Delta \widehat{N}_\tau | \mathrm{LN} \rangle = 0,$$

$$\langle LN | \Delta \widehat{N}_\tau \Delta \widehat{N}_{\tau'} | \mathrm{LN} \rangle = 0$$

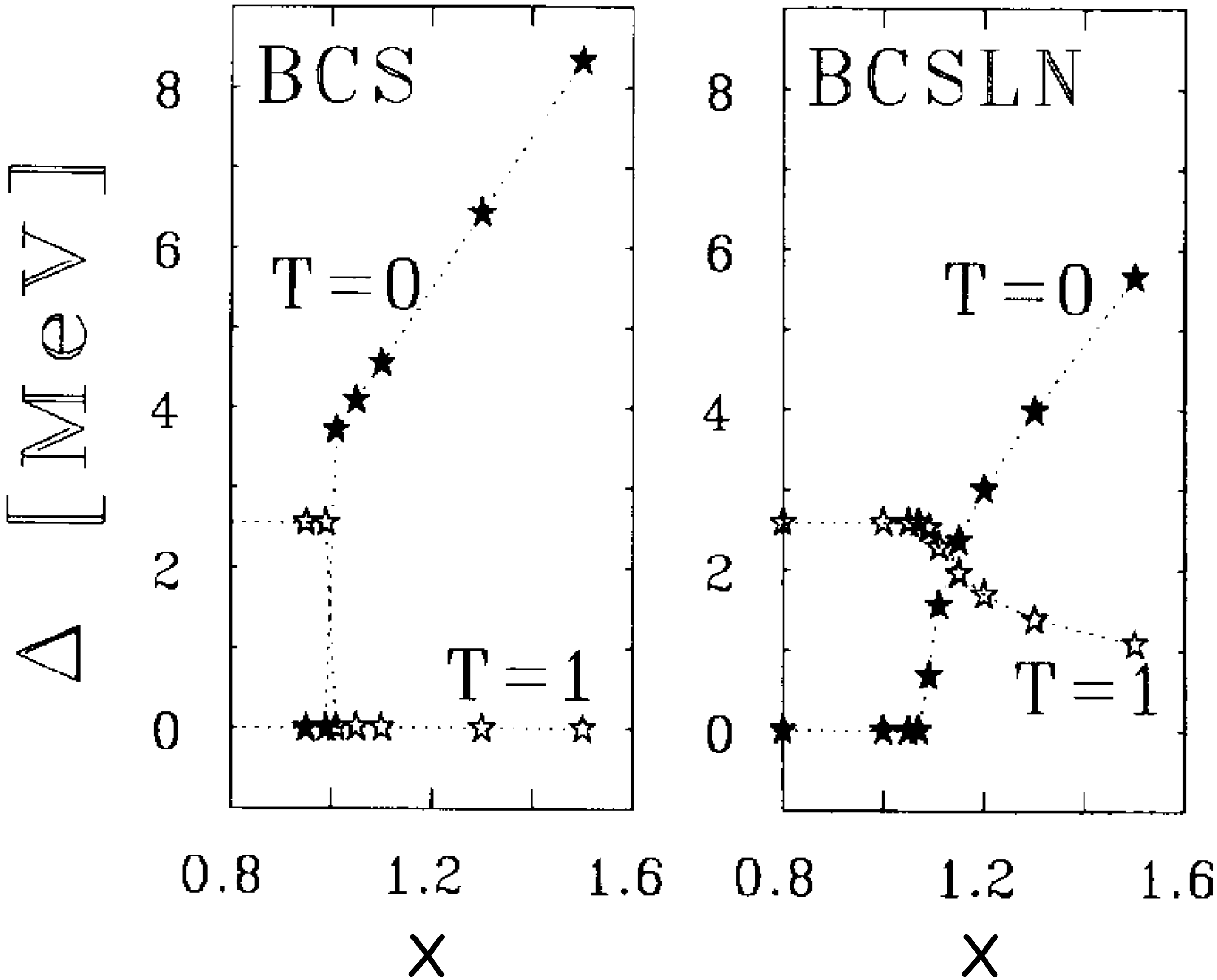


Intensity T=0/T=1 ; Resultats (1)

48Cr Calculation

1) meanfield = W.S.

2) X= $\tilde{G}^{t=0}$ / $G^{t=1}$

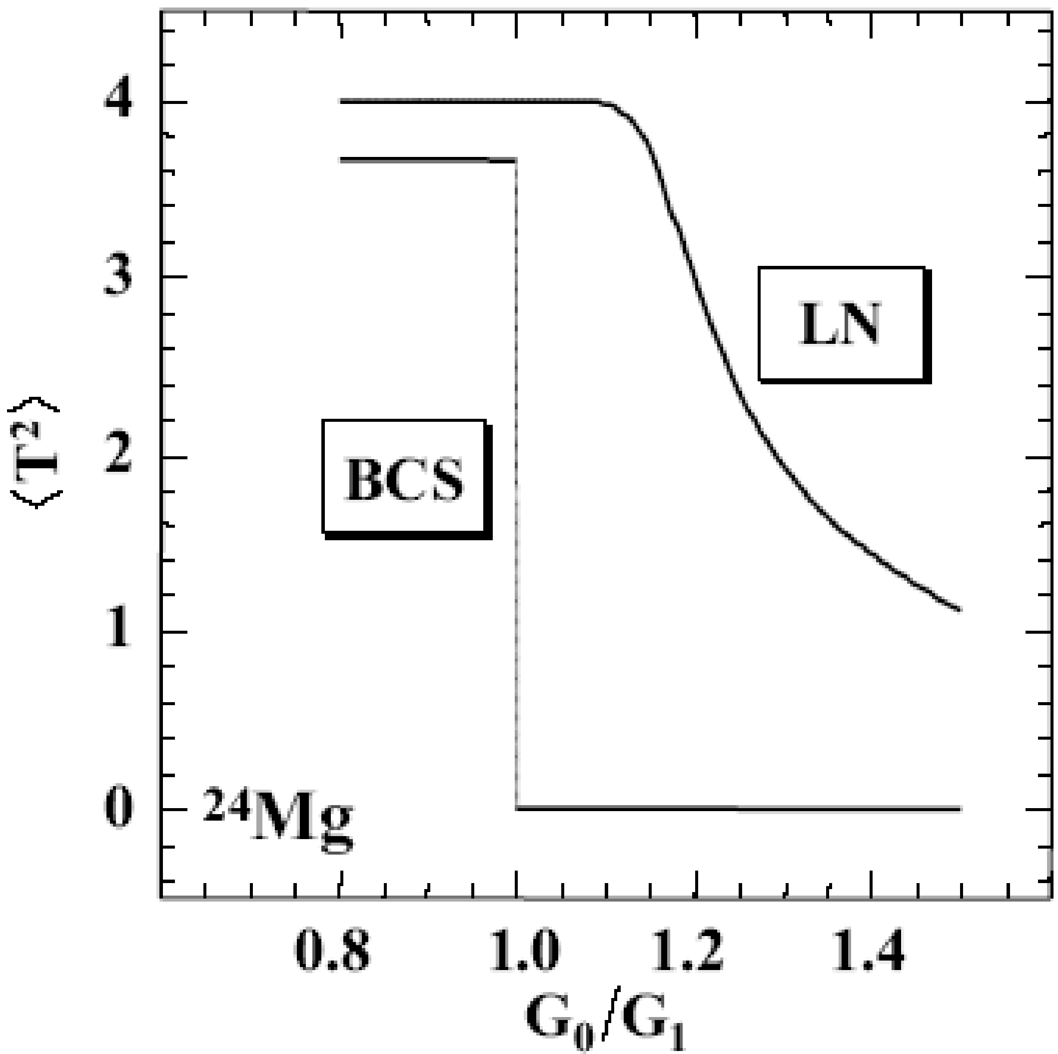


Incompatible?

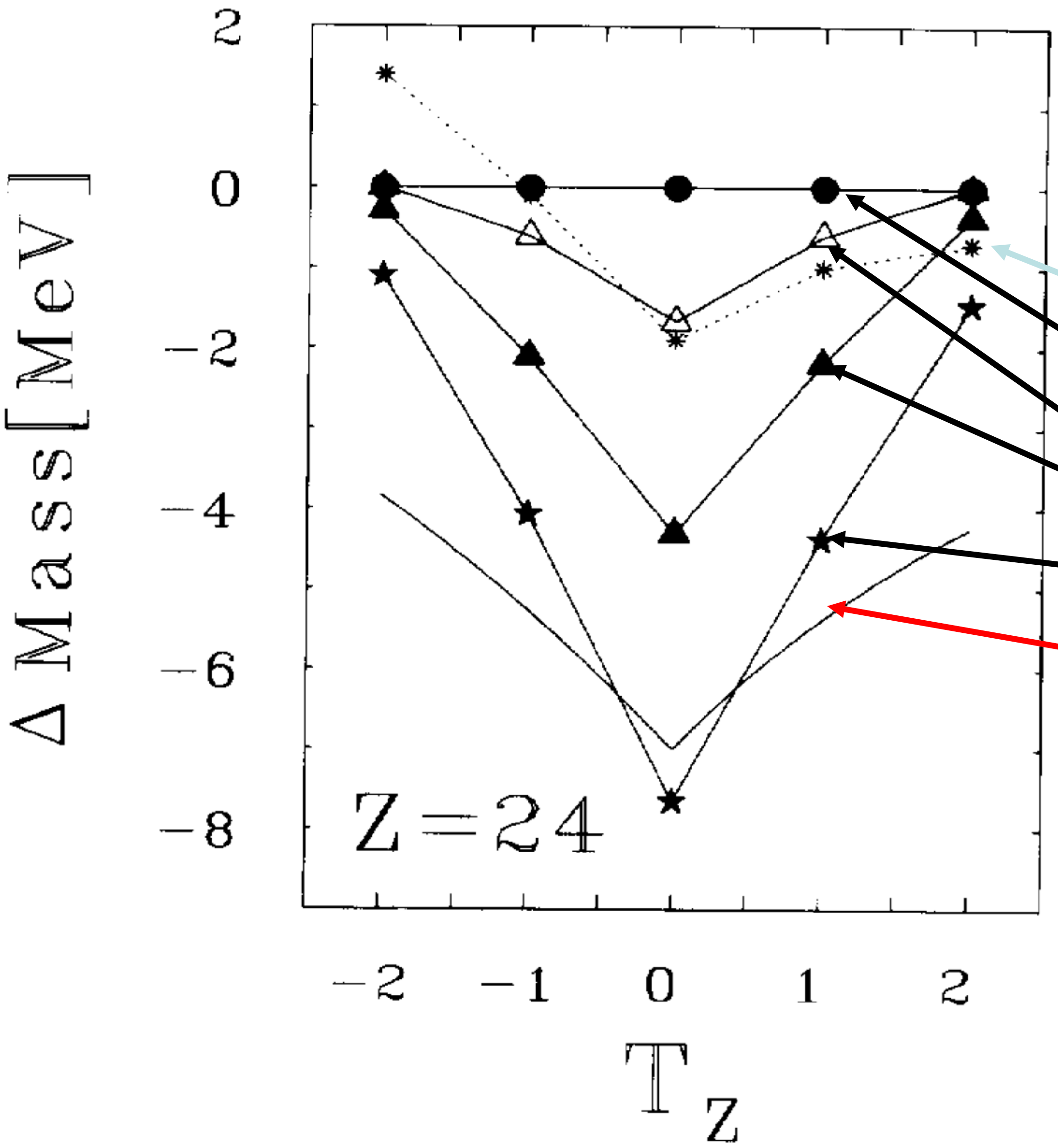
-T=1 with T=0

Iso spin mixing due to T=1 pairing interaction

- T=1 pairing violates iso-spin – resulting in deformation in iso-space
- T=0 pairing restores iso spin (scalar in iso space)
- We need iso spin breaking to calculate iso spin excited states.



X=Intensity T=0/T=1



Gain in binding energy
E(T=0+1)- E(T=1)

Extended Thomas-Fermi
X=1.1
X=1.2
X=1.3
X=1.4
Wigner term(Myers-Swiatecki)

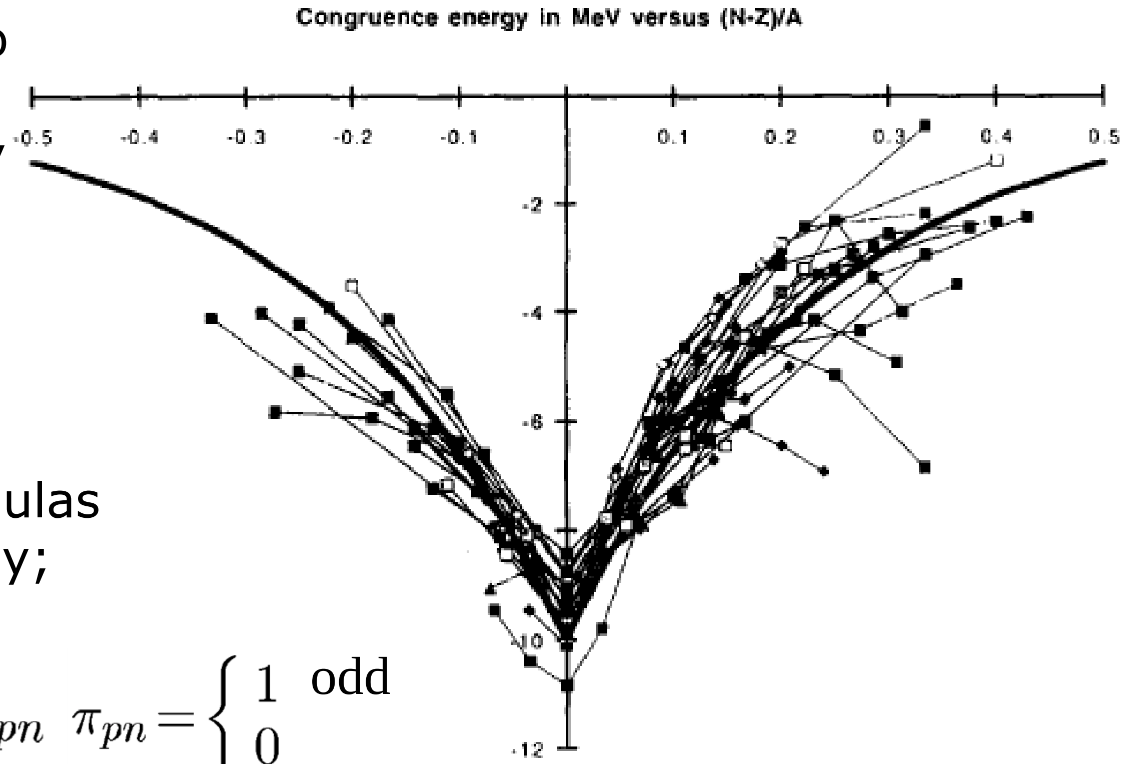
X= $\tilde{G}^{t=0} / G^{t=1}$ ~1.2

W.Satula, R.Wyss

PLB 393 (1997) 1

Mass excess due to Wigner energy

Mass defect with respect to the Thomas Fermi model. The fitted curve is given by $C(I)=10e^{(-4.2|I|)}$ /MeV, I= N-Z = 1/2 Tz (Myers Swiatecki, NPA612 (1997),249



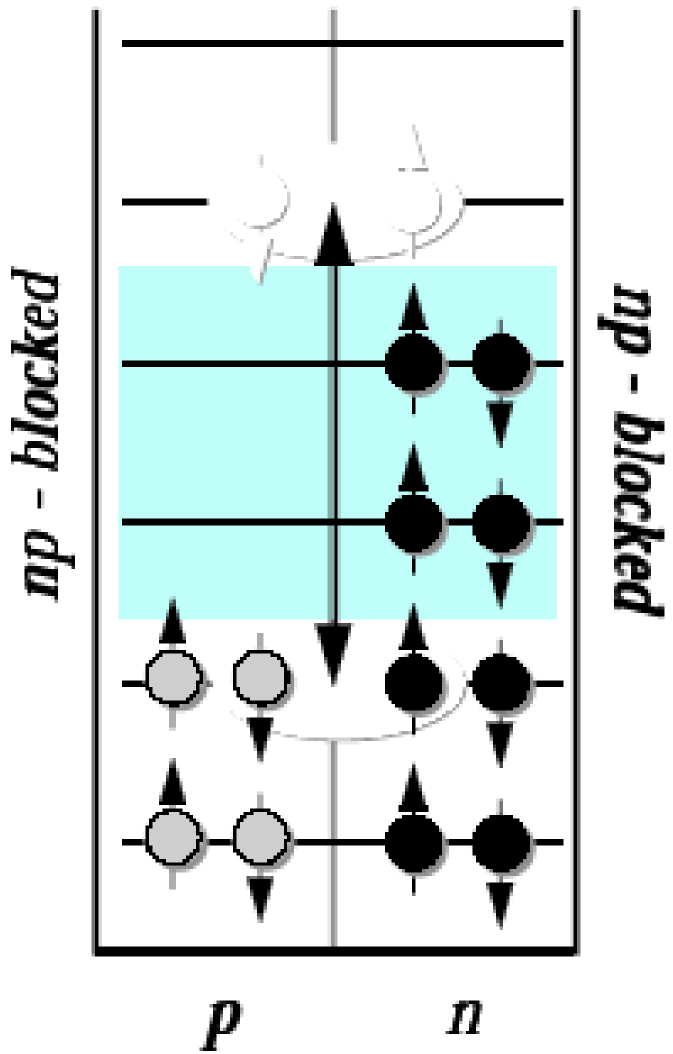
In semiempirical massformulas one adds the Wigner energy;

$$E_W = W(A)|N-Z| + d(A)\delta_{NZ}\pi_{pn} \quad \pi_{pn} = \begin{cases} 1 & \text{odd} \\ 0 & \text{even} \end{cases}$$

W(A)~47/A(MeV)

N=Z nuclei appear to be more bound, o-o have a repulsive term

Generalised blocking effect:
T=0 pairng correlations present only in N~Z nuclei



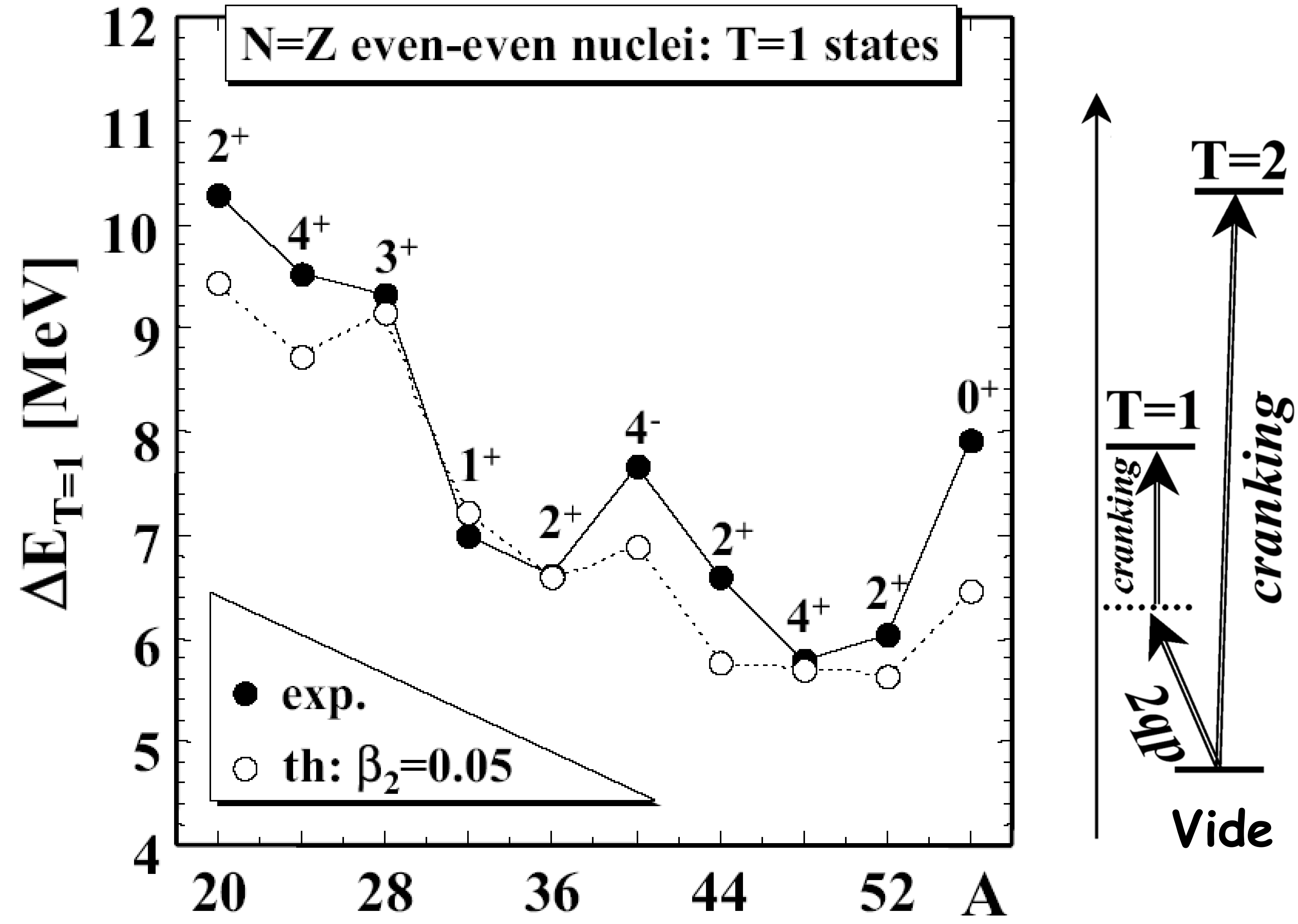
Interpretation of the Wigner Energy as due to RPA Correlations

Kai Neergard

In a schematic model with equidistant fourfold degenerate single-nucleon levels, a conventional isovector pairing force and a symmetry force, the RPA correlation energy rises almost linearly with the isospin T, thus producing a Wigner term in accordance with the empirical proportionality of the symmetry energy to $T(T+1)$.

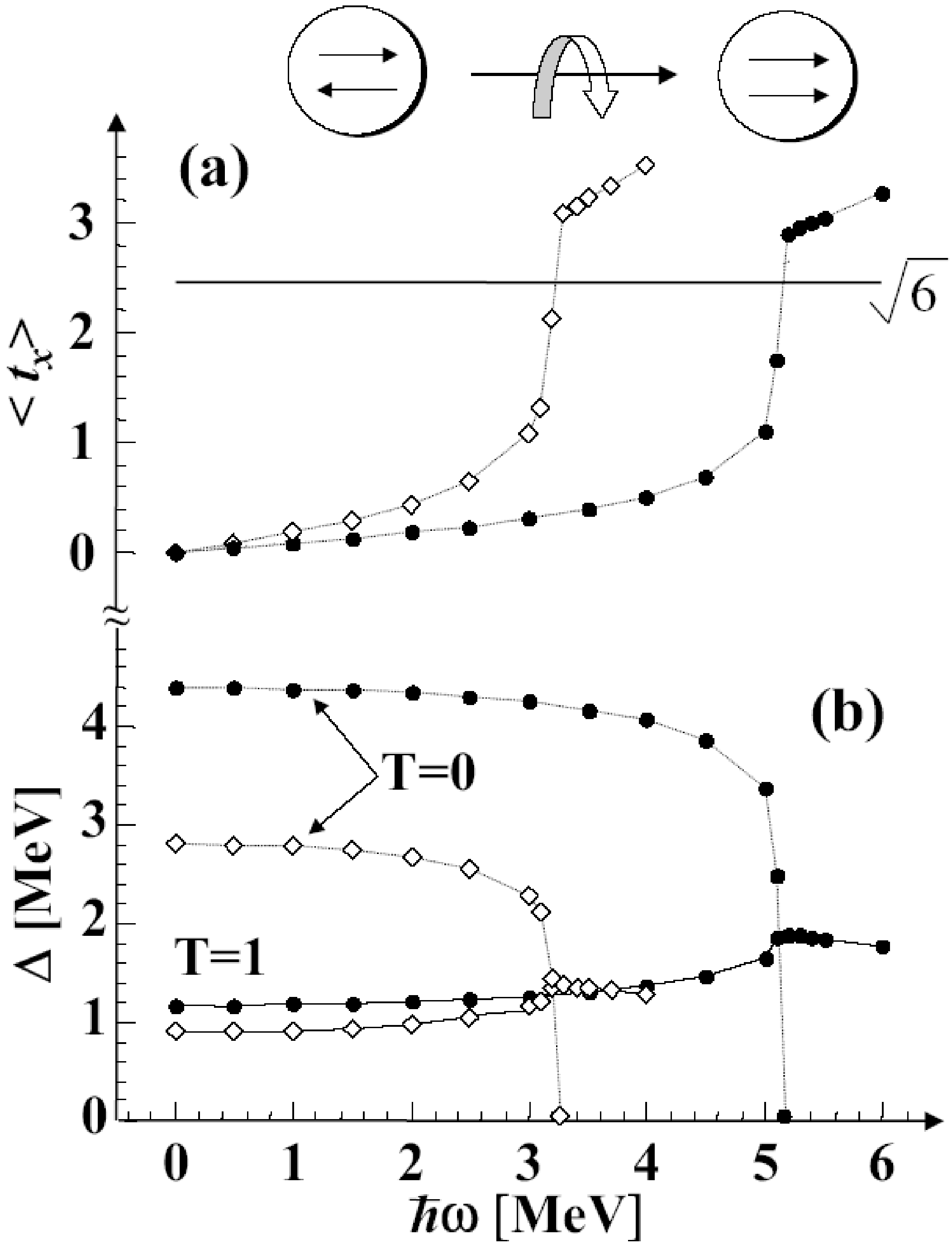
PLB 537 (2002), 287
PLB 572(2003), 159

Replace LD formula (N-Z)^2 with T(T+1)


N=Z even-even nuclei: T=1 states
ΔE_T=1 [MeV]
12
11
10
9
8
7
6
5
4
20
28
36
44
52
A
2+
4+
3+
1+
2+
4-
2+
4+
2+
0+
exp.
th: β2=0.05
T=2
T=1
cranking
cranking
2qp
Vide

The alignment in iso space *tx* and the respons of T=1 and T=0 pair field. Calculations for 24Mg and 48Cr.

Meissner effect in isospace!



Moments of inertia and T=0 Pairing Correlations

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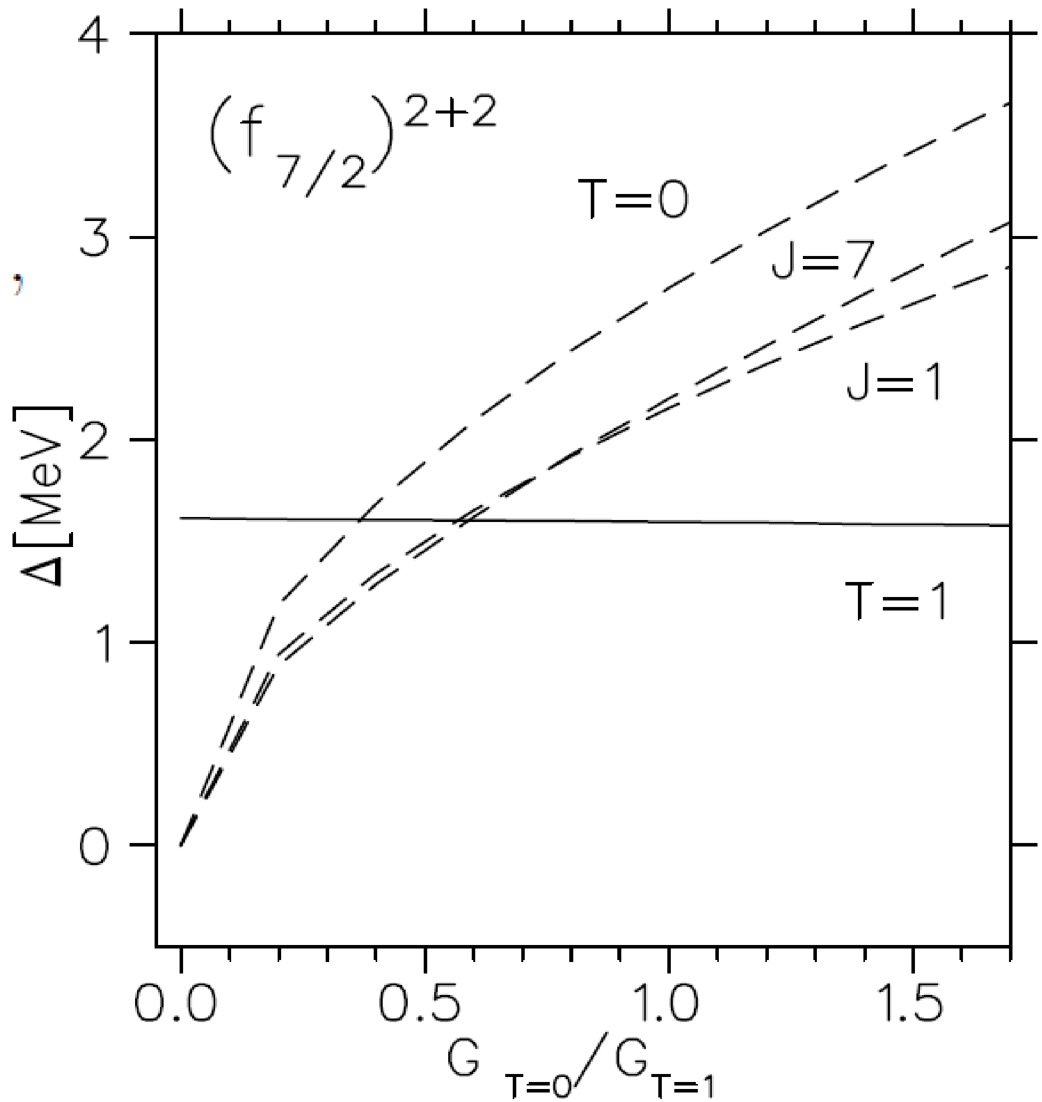
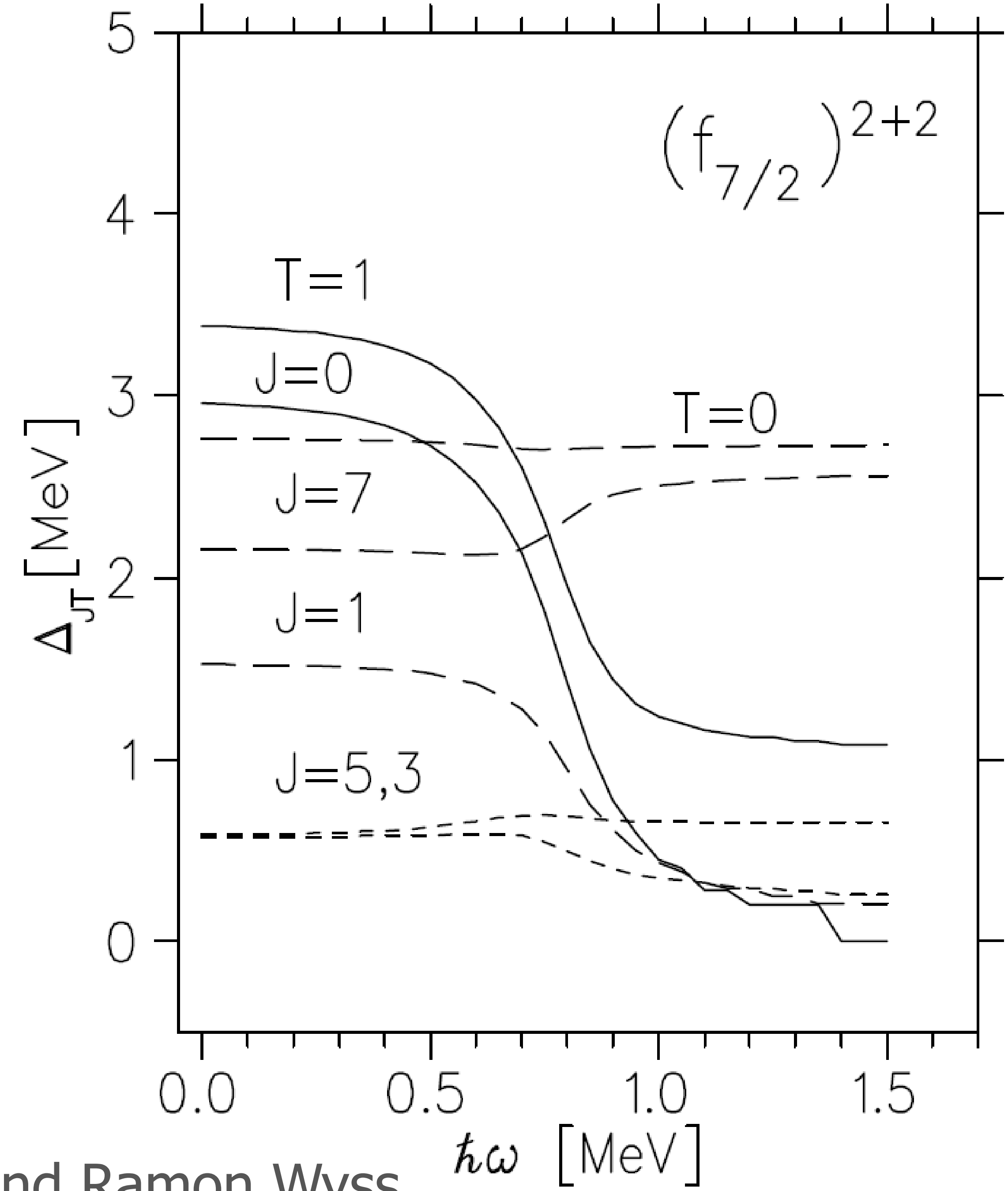
Exact solutions in single j-shell model

$$V_2 = \frac{1}{2} \sum_{JMTT_z} E_{JT} A^\dagger_{JM;TT_z} A_{JM;TT_z},$$

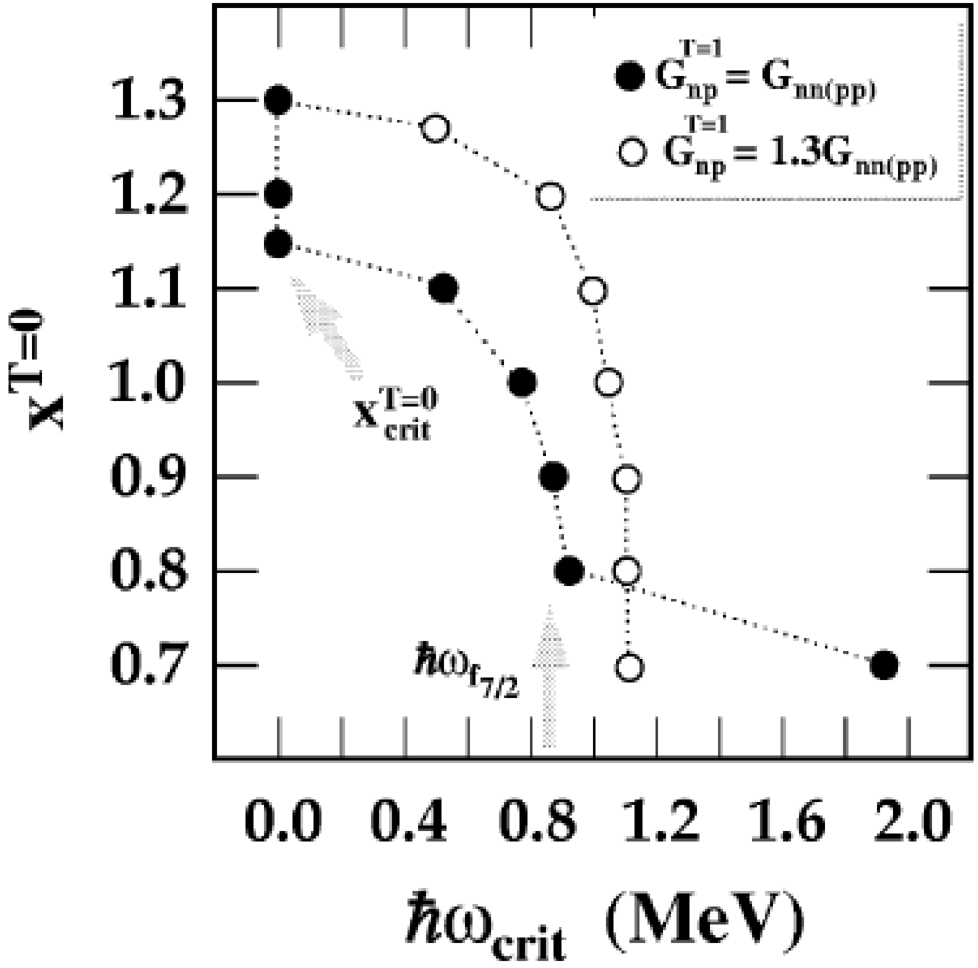
$$E_{pair} = \frac{1}{2} \sum_{JT} \frac{\Delta_{JT} \Delta^*_{JT}}{E_{JT}}$$





Javid A. Sheikh and Ramon Wyss
Phys. Rev. C **62**, 051302(R)

48Cr, critical strength



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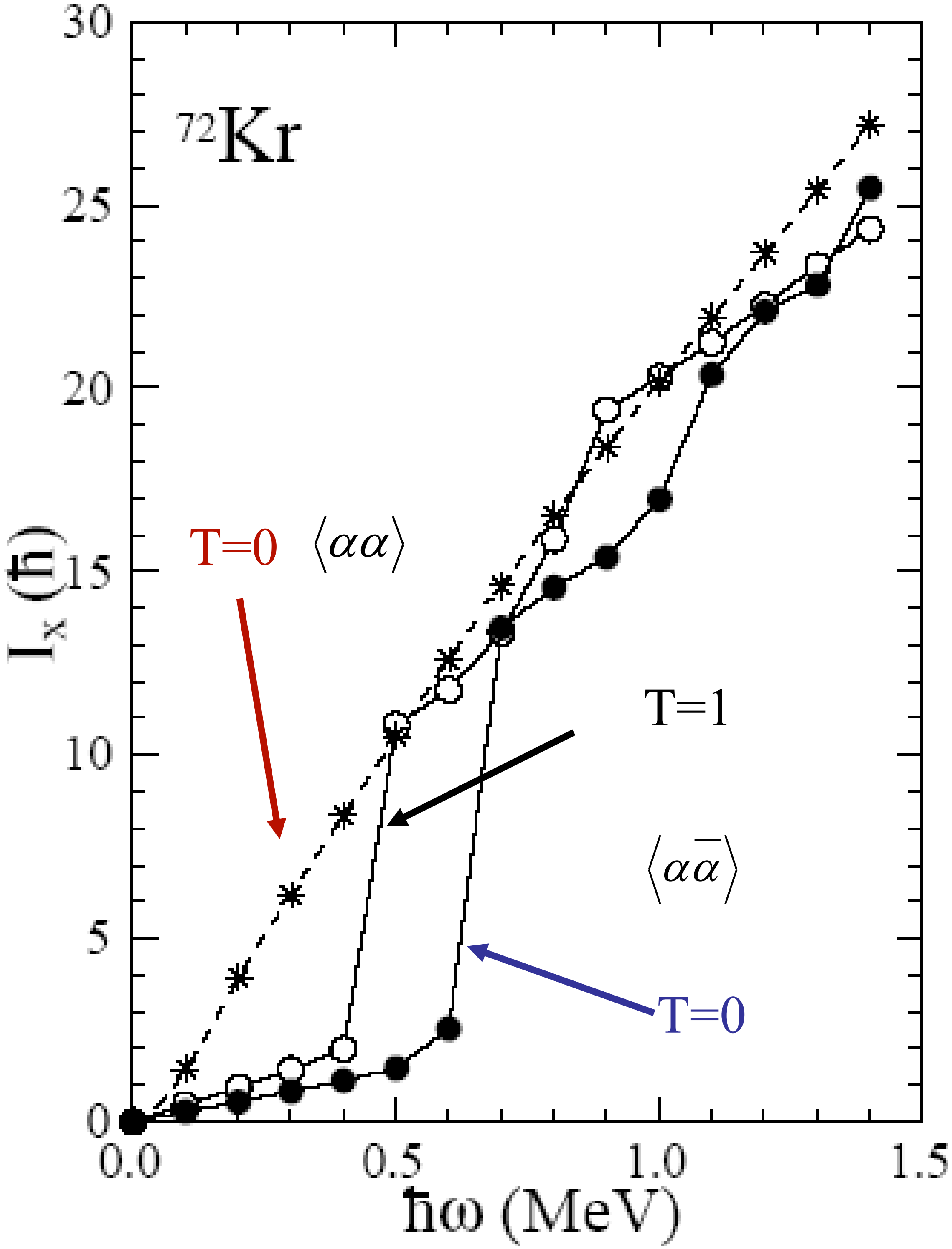
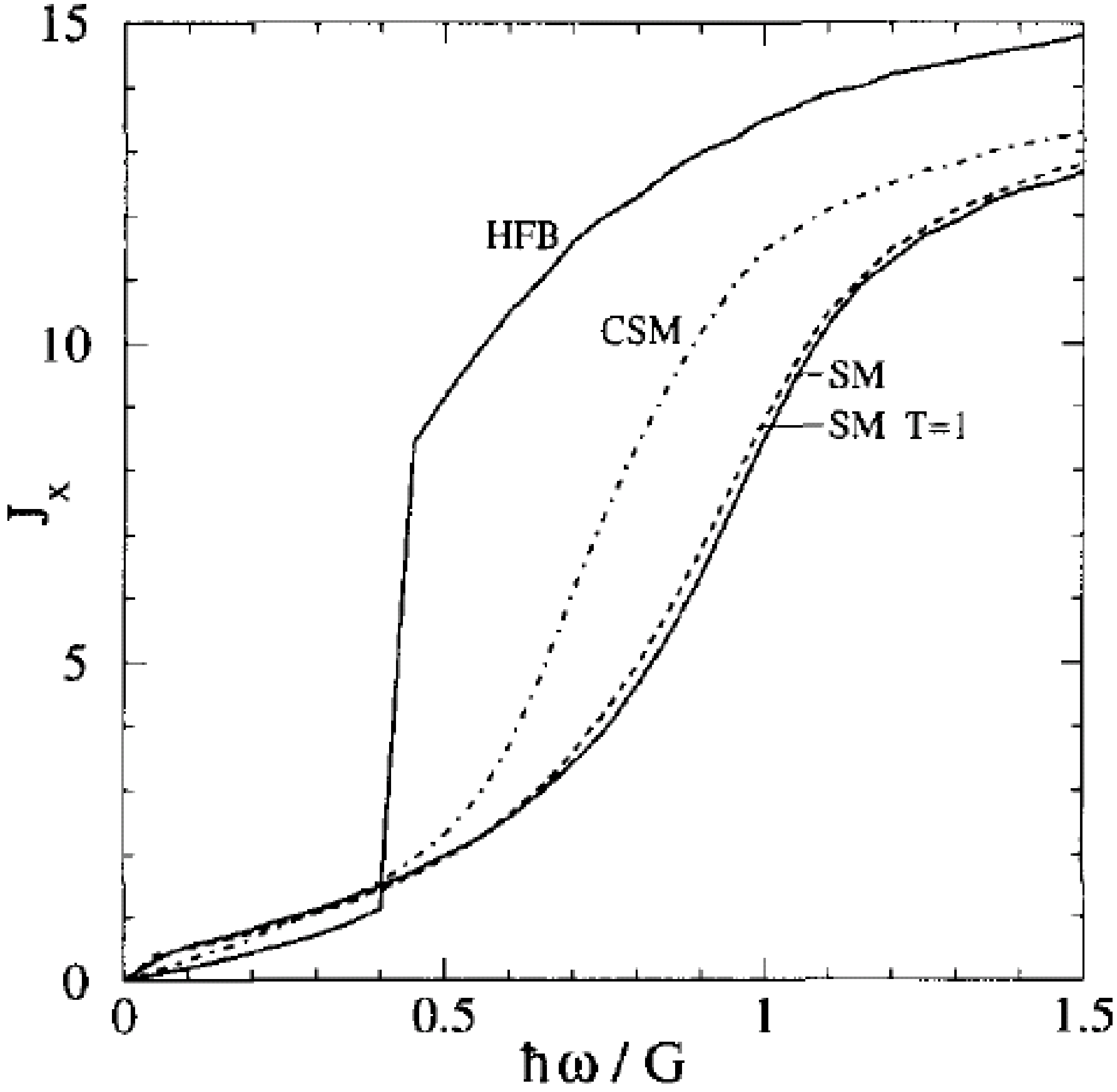
Generic features of the alignment in the presence of the different T=0 and T=1 pairing modes



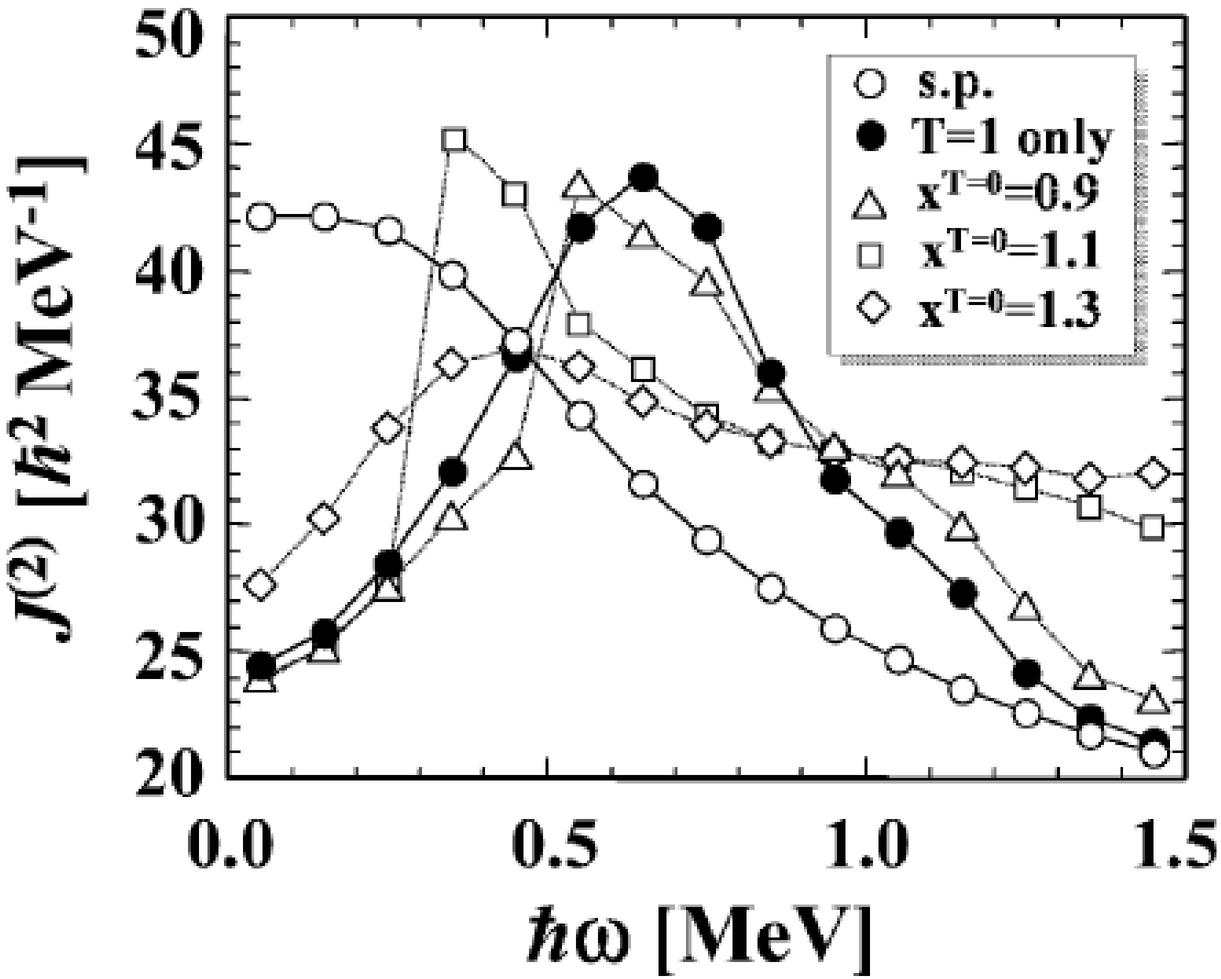
No effect on MoI from T=0 pairing in a single j-shell

S Frauendorf and J Sheikh, NPA645 (1999)509

single j-shell calculations with δ interaction



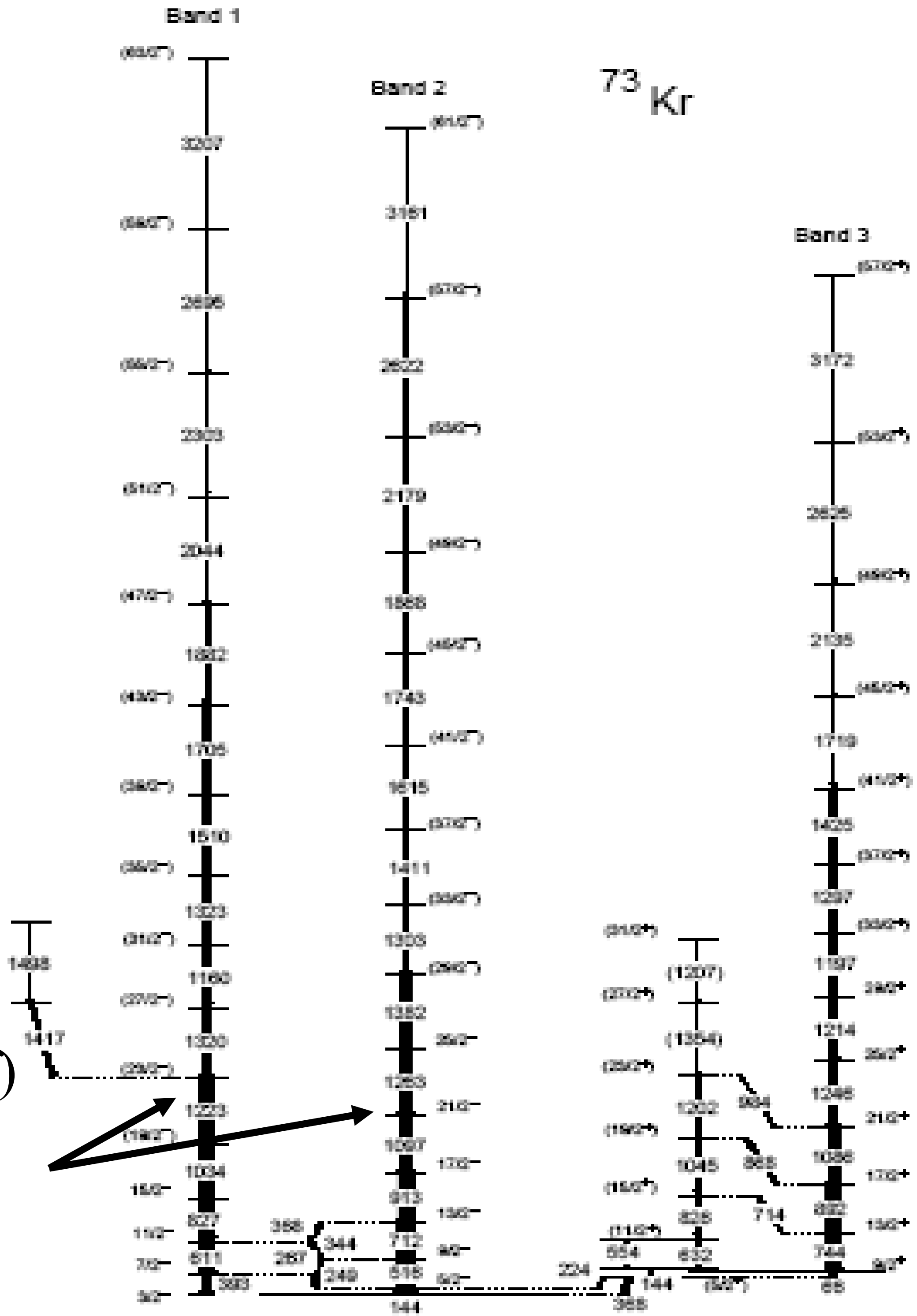
Effect of T=0 Pairing on MoI



Measuring Pair Transfer via Band Crossing at High Spins

Level-scheme of 73Kr

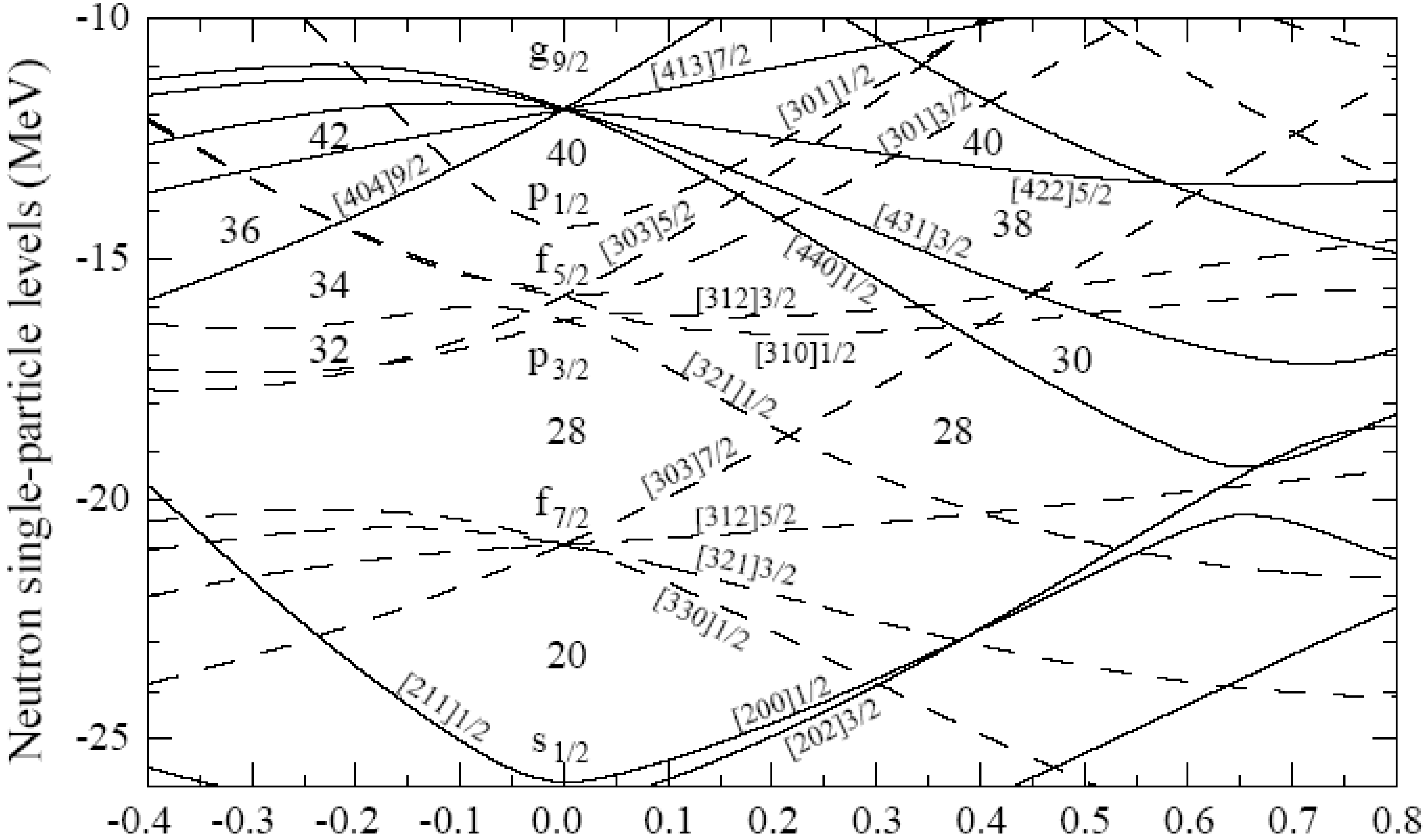
N.S.Kelsall et. al., Phys.Rev. C65, 044331 (2002)

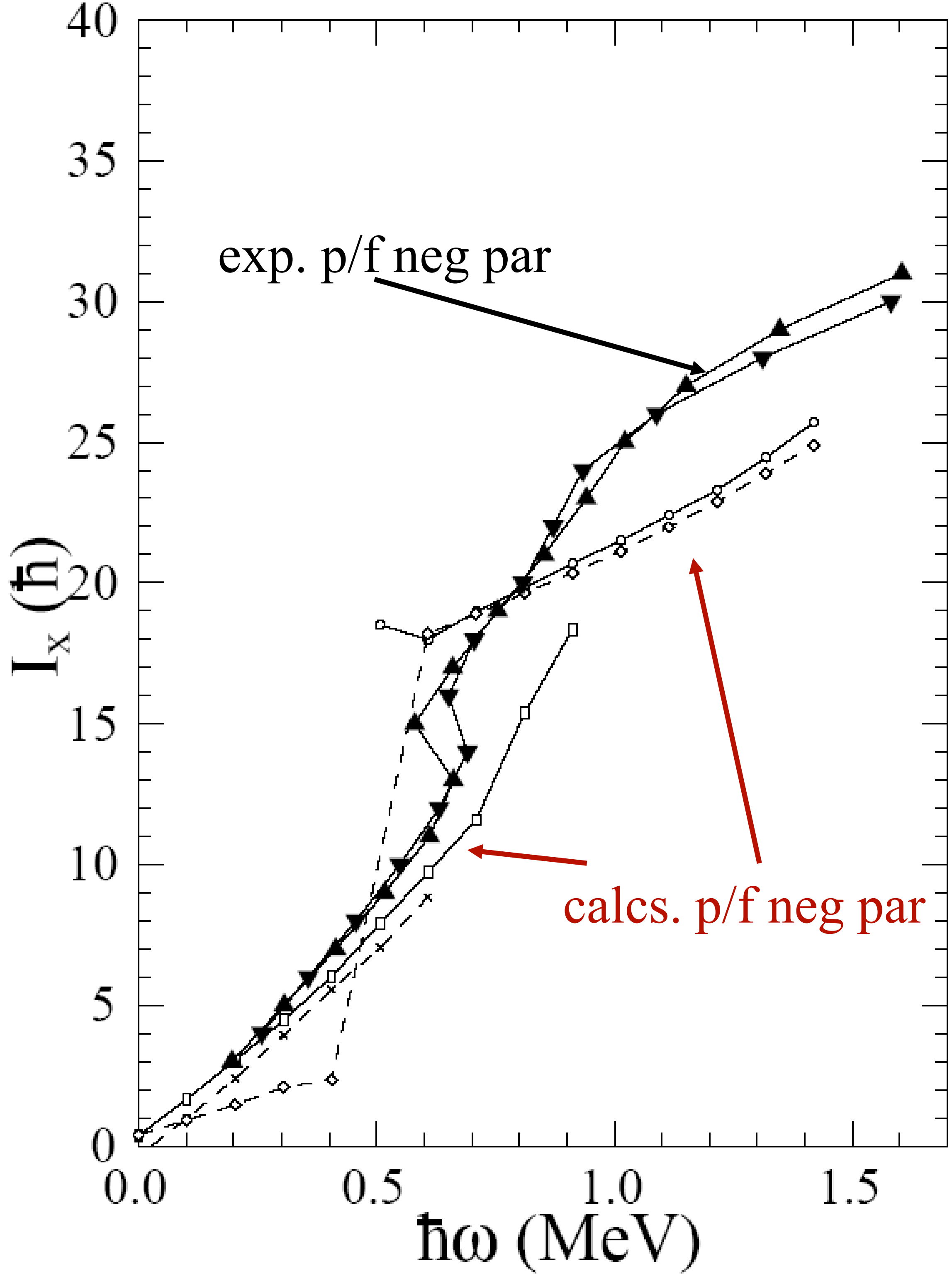


ν p3/2-f5/2 (p/f)
neg. par.

ν g9/2
pos. par.

WS sp diagramme

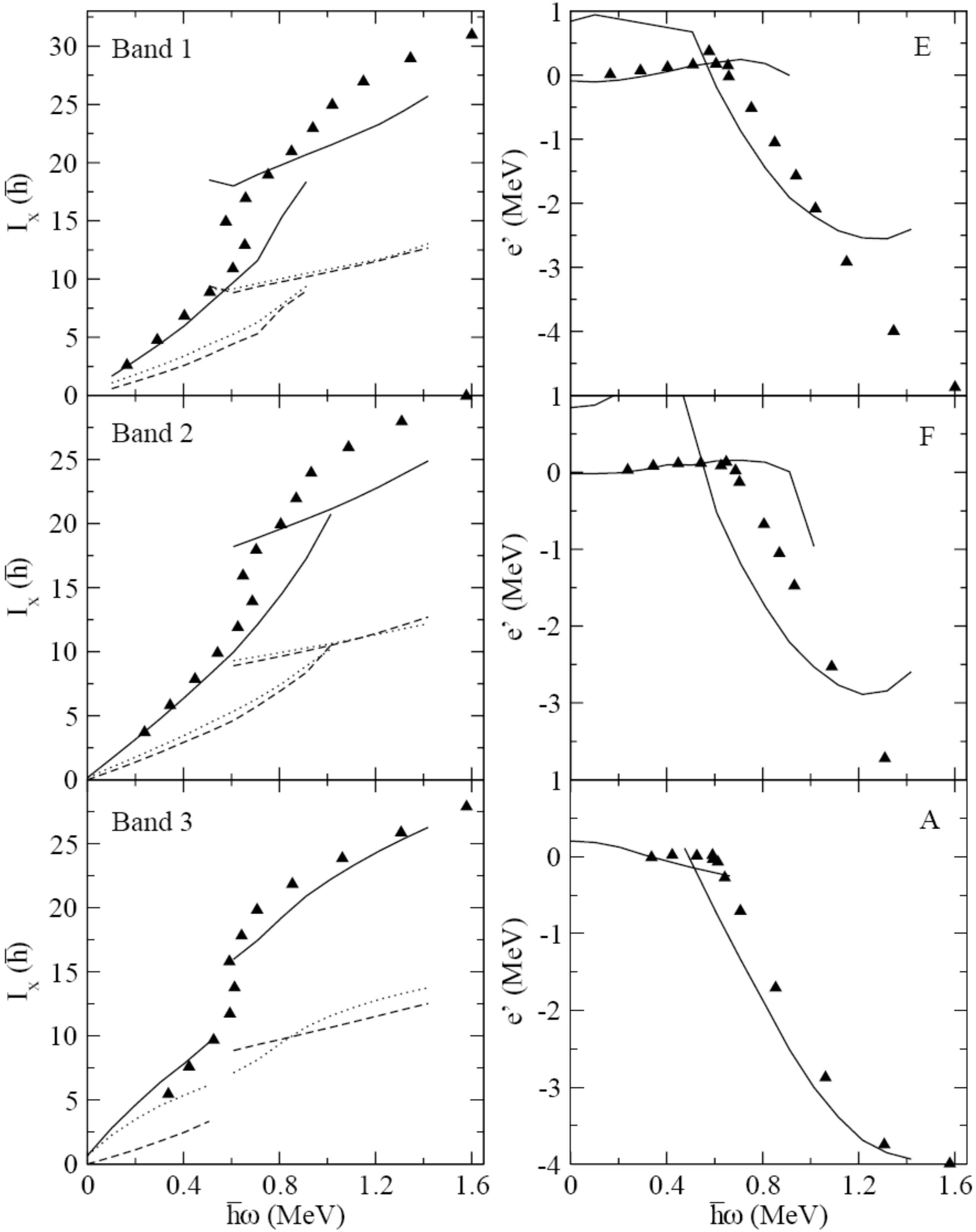



exp. p/f neg par
calcs. p/f neg par
I_x (ħ)
ħω (MeV)
0
5
10
15
20
25
30
35
40
0.0
0.5
1.0
1.5

Alignments and Routhians for 73Kr

good agreement for low spin for the two neg. par. bands – disagreement at high spins

good agreement for the pos.par. band (g9/2) over the entire spin range



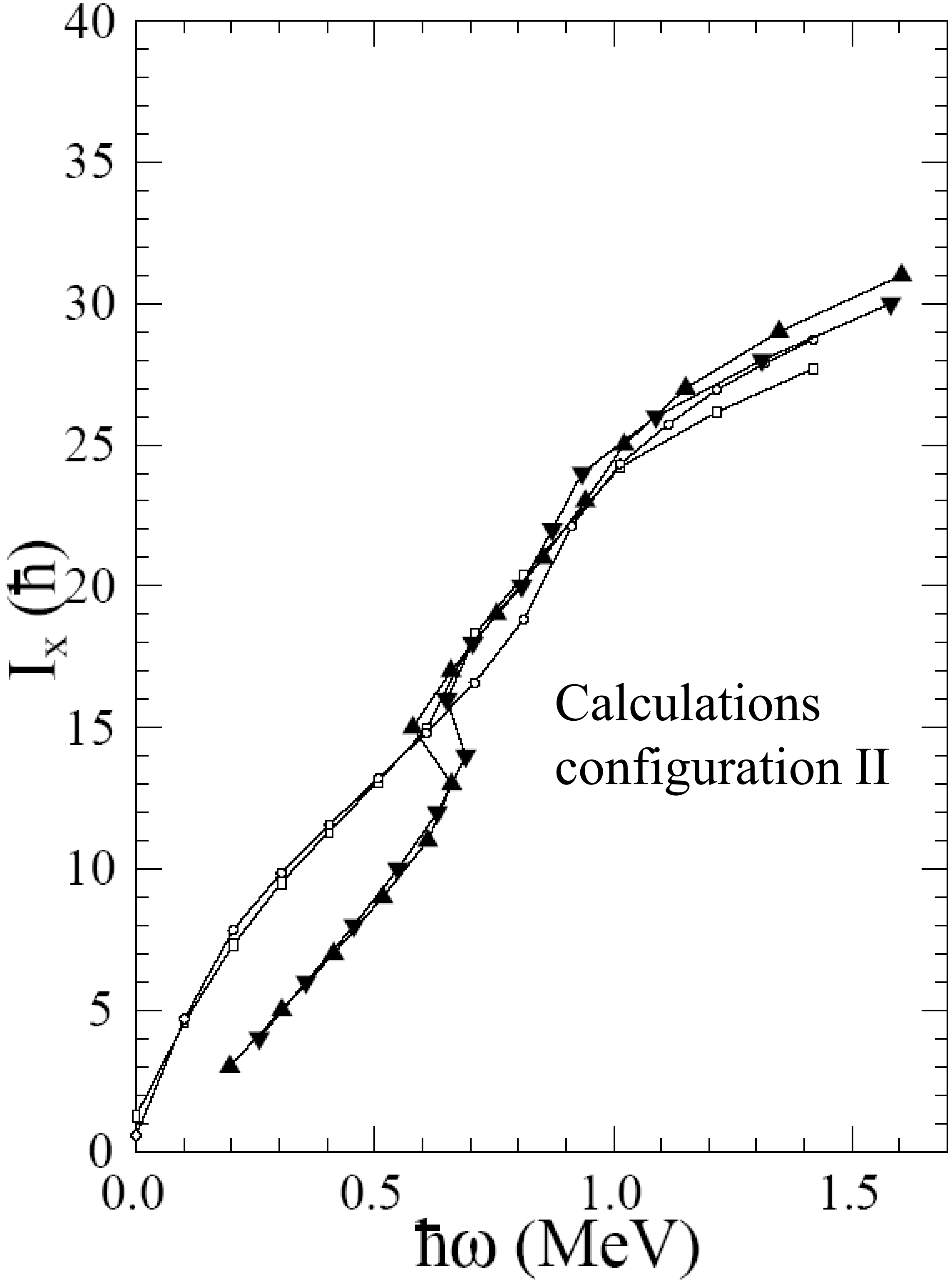
p/f
(π,α):(-, -1/2)

p/f
(π,α):(-,+1/2)

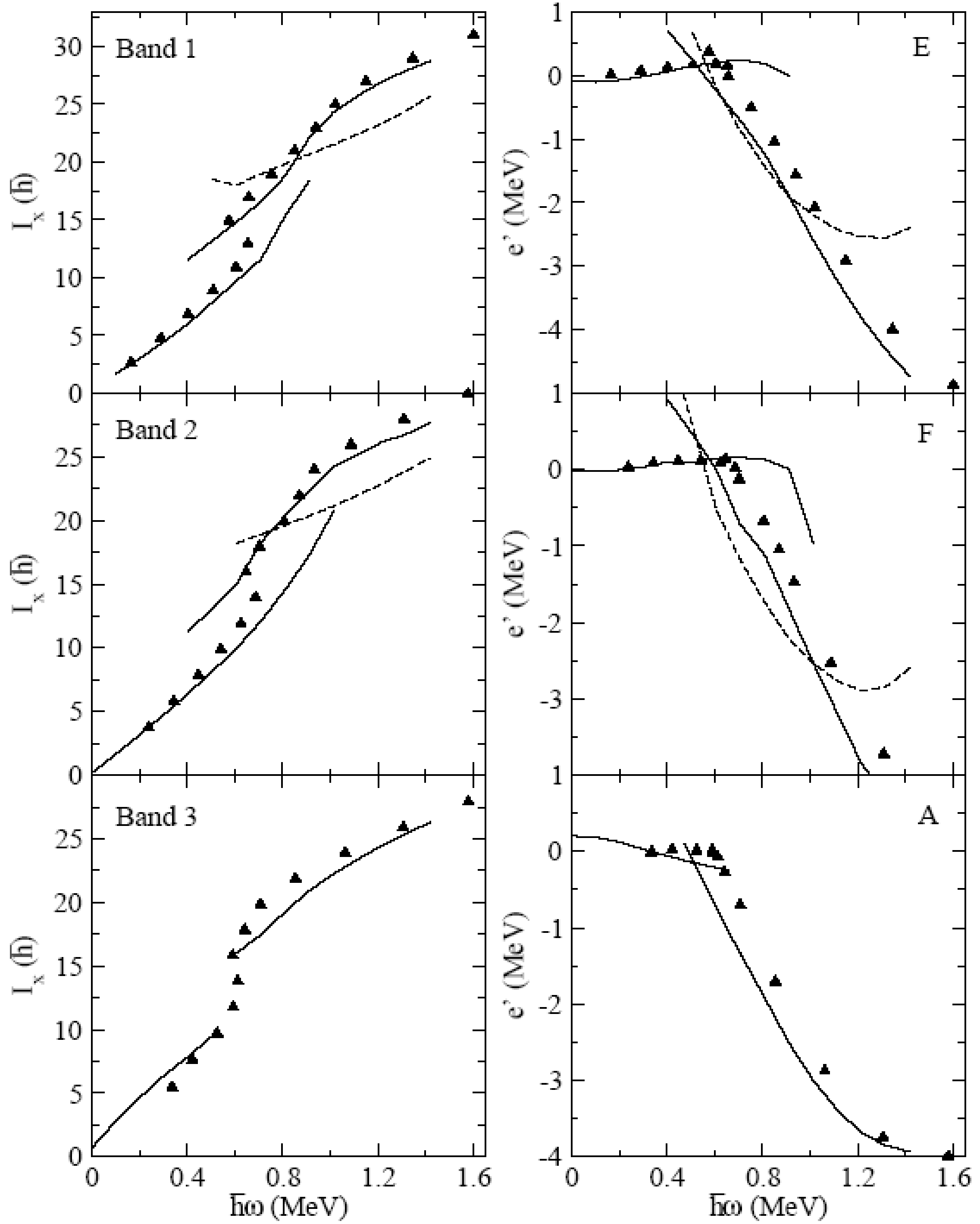
g9/2
(π,α)(+,+1/2)

Assume an entire different configuration:
Move the neutron from neg. par. f/p orbit into g9/2 and make a 2qp proton excitation from a f/p orbit into g9/2

ν g9/2 pos par (+,+1/2)
Π:[f/p x g9/2] neg par (-,-/+1/2)



Alignment and Routhian for the new configuration



p/f
(π,α):(-, -1/2)

p/f
(π,α):(-, +1/2)

g9/2
(π,α)(+,+1/2)

T=1 scenario:

conf I $\alpha^{+}_{\nu(f/p)} \prod BCS_{\nu} > \prod BCS_{\pi} >$

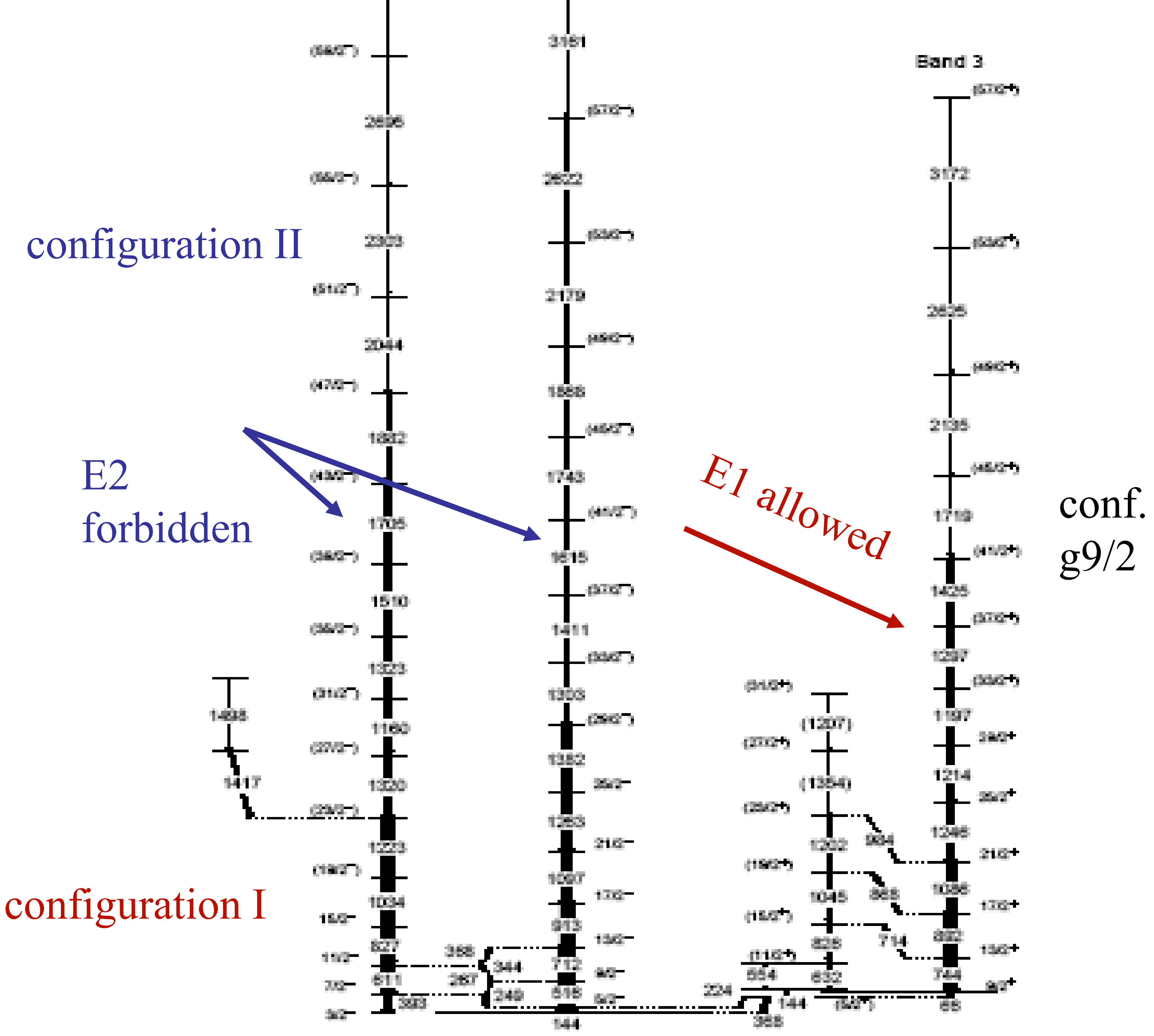
conf II $\alpha^{+}_{\nu(g9/2)} \prod BCS_{\nu} > \alpha^{+}_{\pi(g9/2)} \alpha^{+}_{\pi(fp)} \prod BCS_{\pi} >$

conf g9/2 $\alpha^{+}_{\nu(g9/2)} \prod BCS_{\nu} > \prod BCS_{\pi} >$

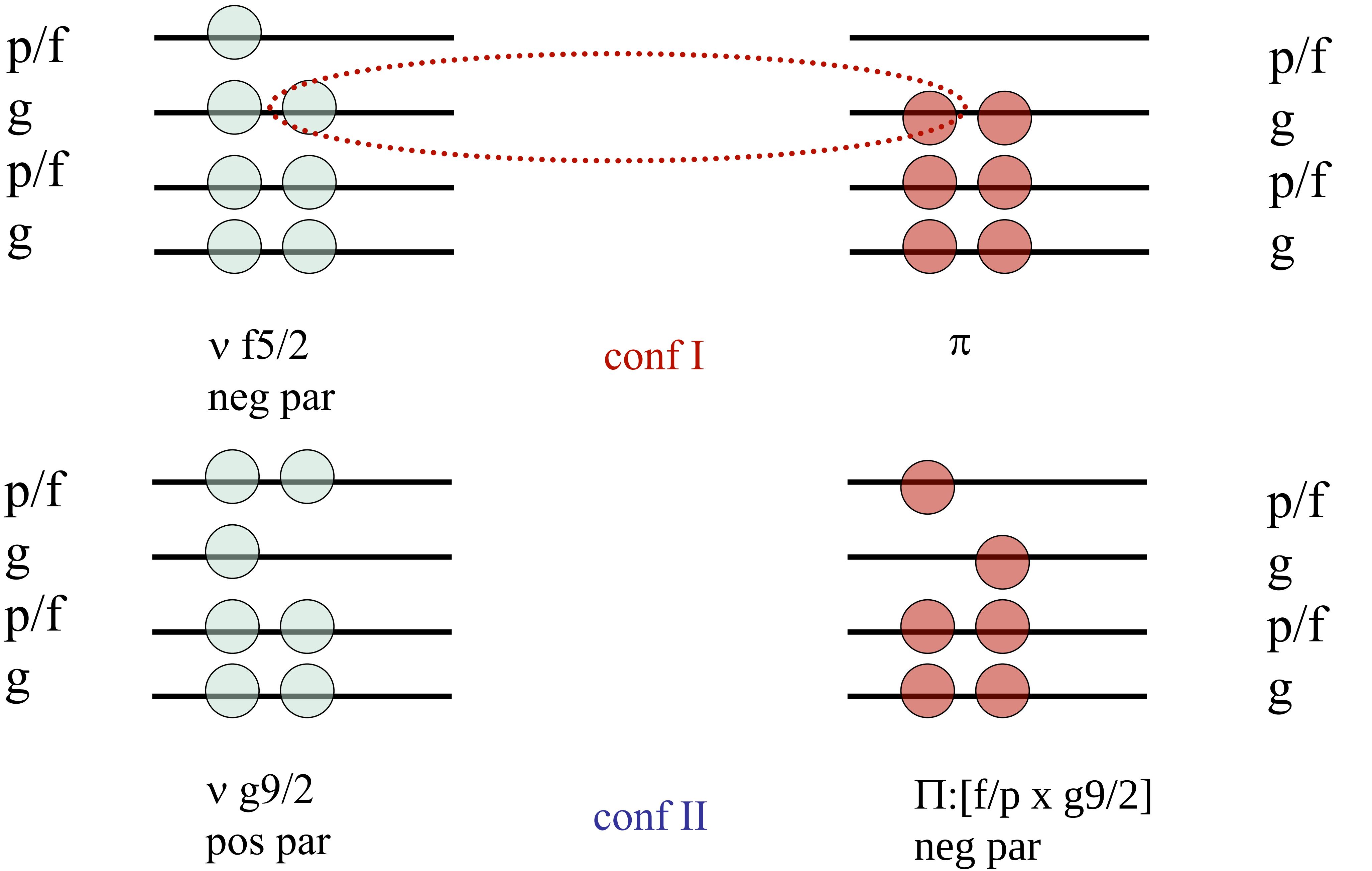
<conf I $|O(E2)|$ conf II > forbidden

<conf II $|O(E1)|$ conf g9/2> allowed

Level-scheme of 73Kr

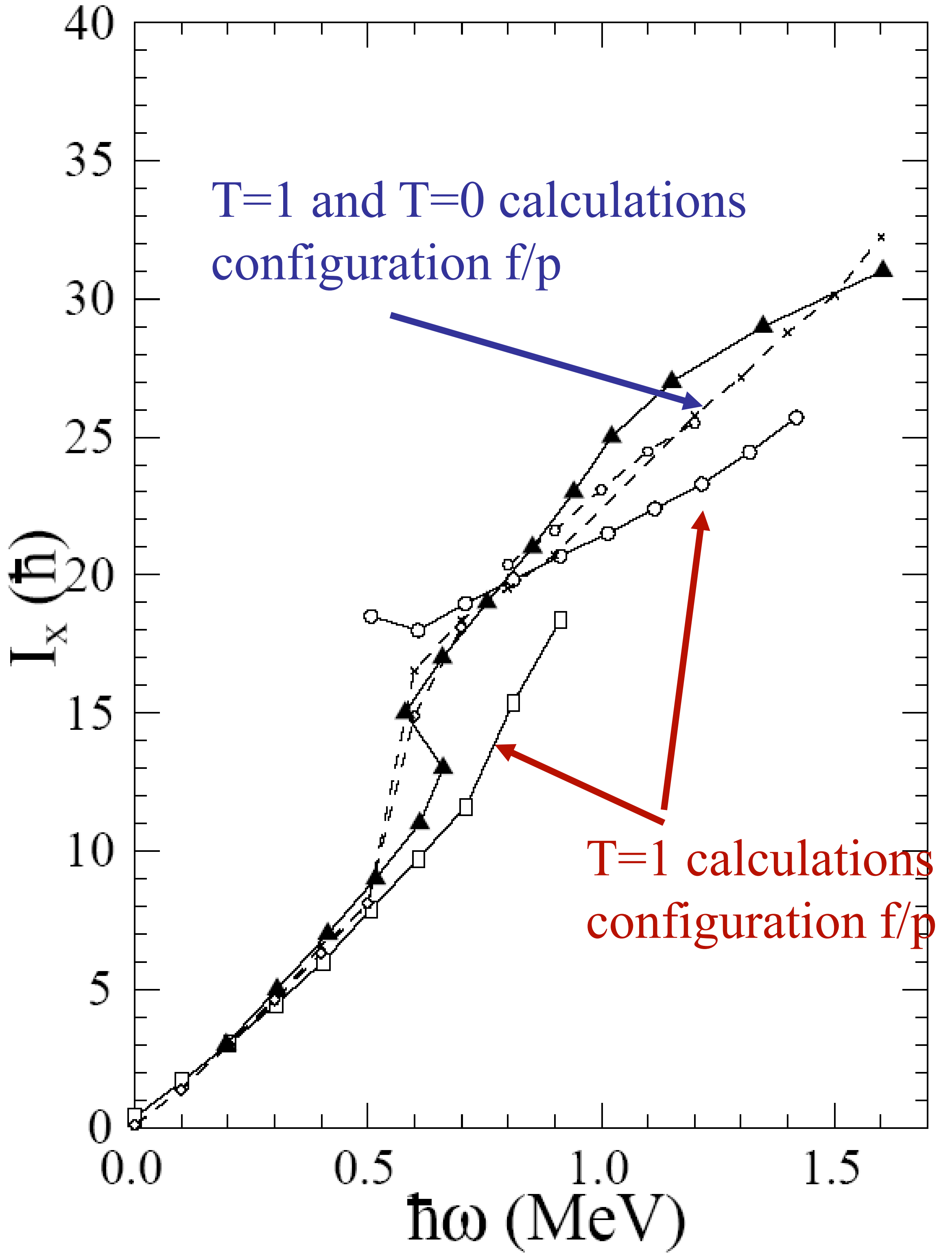


Scattering of a T=0 np pair

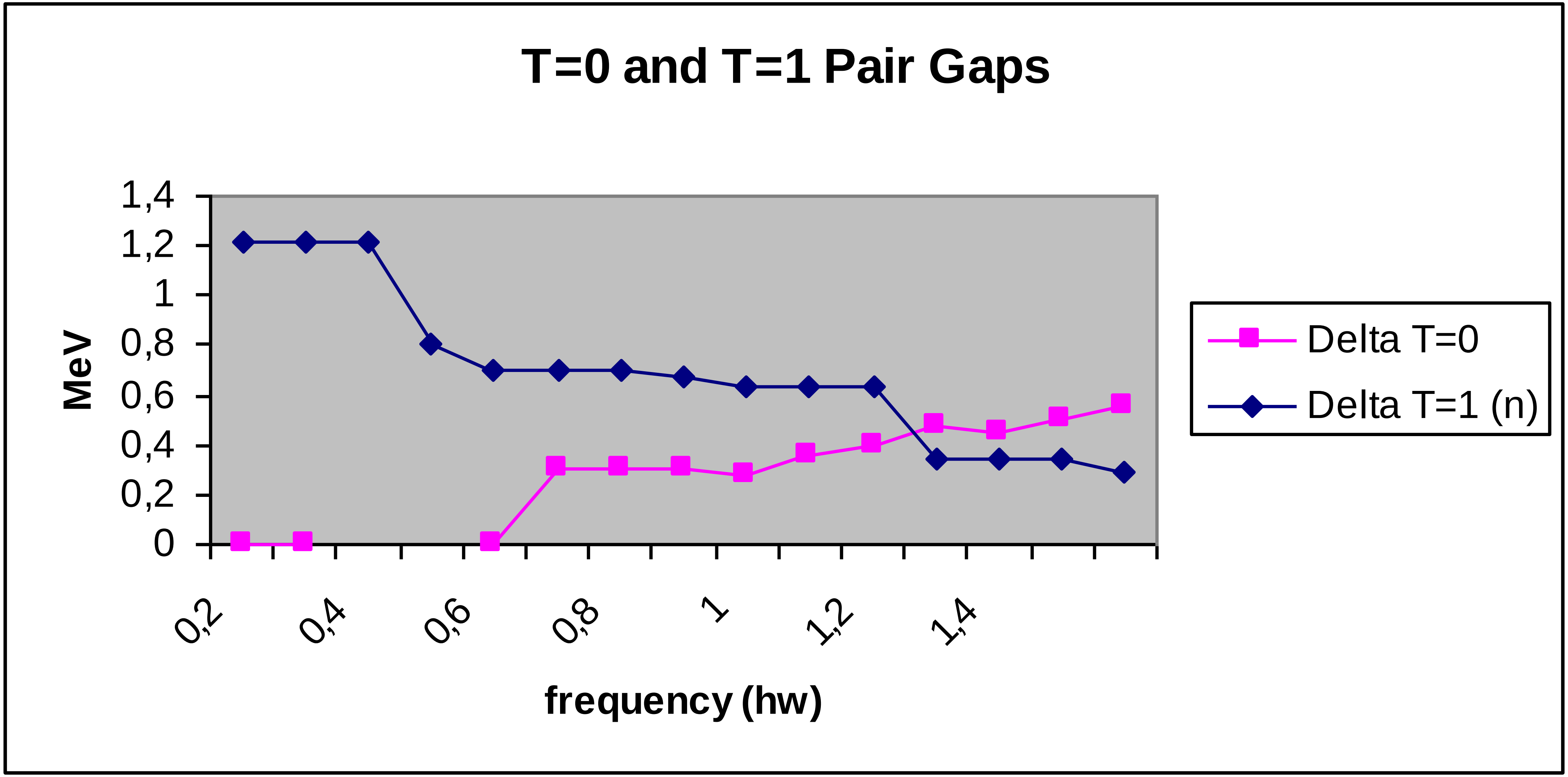


TRS calculations with T=0 and T=1 pairing

Same configuration blocked in both calculations – phase transition from T=1 to T=0 pairing



Pairing gaps for 73Kr



Mass Differences

Mass difference in the presence of T=0 pairing:

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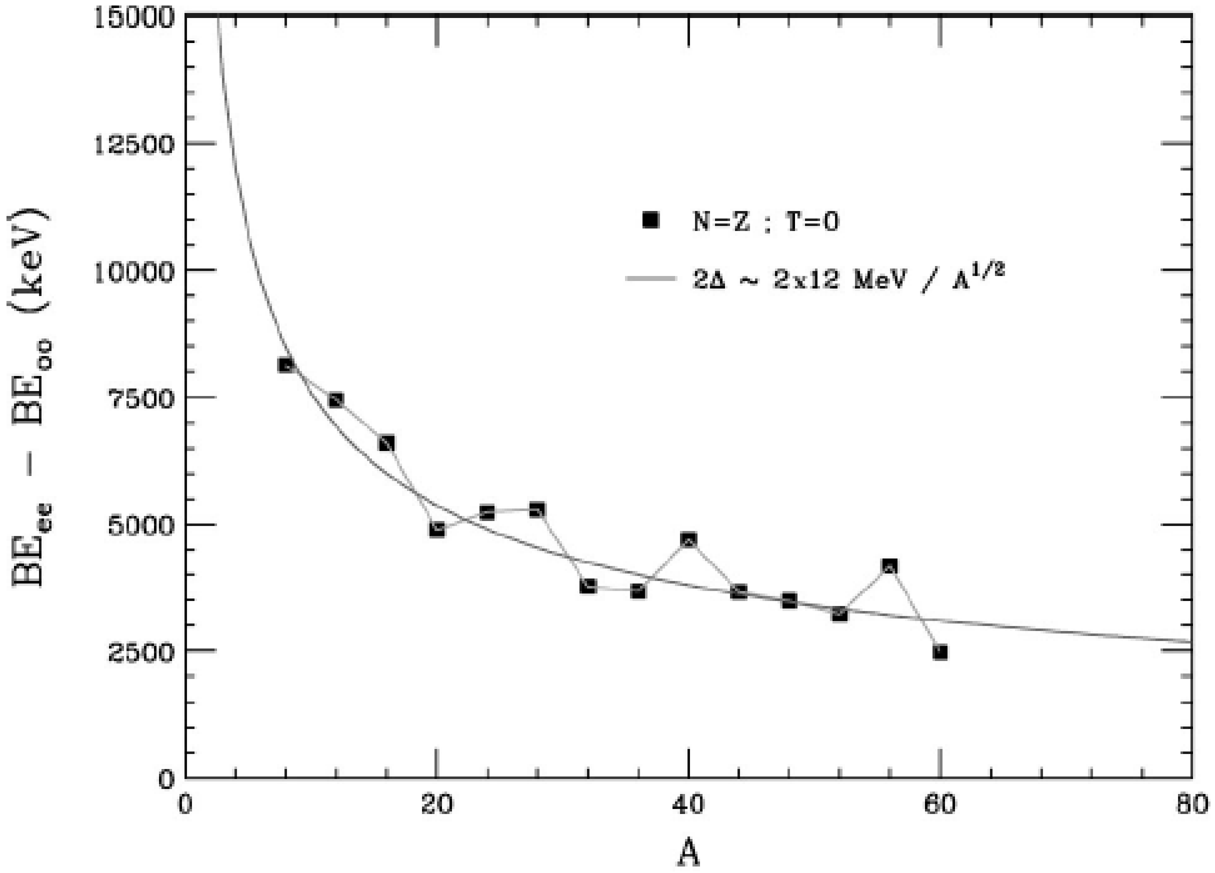
- Groundstate of e-e nucleus is a condensate of T=0 np-pairs
- Remove(add) one T=0 pair, go from e-e to o-o nucleus
- Hence, o-o nuclei as bound as e-e, no difference in B.E.

Energy difference between T=1 and T=0 o-o N=Z nuclei

- Energy difference decomposed in pairing energy and symmetry energy
- Symmetry energy assumed 75T(T+1)

$$BE_{ee}(N,Z) - \frac{(BE_{oo}(N-1,Z-1) + BE_{oo}(N+1,Z+1))}{2},$$



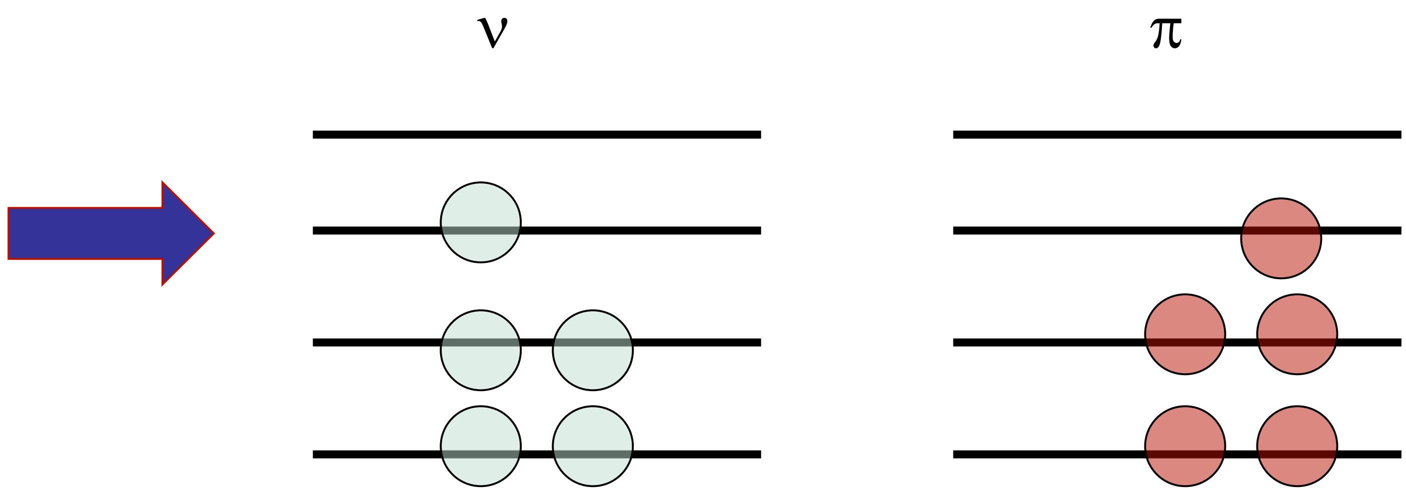
No trace of T=0 np-pairing?

All nuclei have as their ground state T=Tz
Only exception: odd-odd N=Z

A.O. Macchiavelli, PRC61, 041303R(2000)

Problem for T=0 scenario?

Odd-odd nucleus



odd-odd T=0 nucleus has always a given spin (odd spin) and parity (even).
This level is blocked for pairing correlations, irrespectively we deal with T=0 or T=1 pairing. The symmetry of the wavefunction of e-e nuclei is different from that of o-o

Massdifference in the presence of T=0 pairing

- T=0 states in o-o nuclei carry angular momentum
- cannot be described by the BCS-vacuum of e-e nuclei (time even)
- Correspond to 2qp excitations

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$$\Psi^{o-o}_{T=0} = \alpha^{+}_{1n}\alpha^{+}_{1p} | VAC_{BCS} >$$

$$| VAC_{BCS} > = \prod_{i>0} (u_i + v_{i1} c^{+}_{ip} c^{+}_{\bar{i}p} + v^{*}_{i2} c^{+}_{ip} c^{+}_{\bar{i}n})$$

$$(u_i + v_{i1} c^{+}_{in} c^{+}_{\bar{i}n} + v_{i2} c^{+}_{in} c^{+}_{\bar{i}p})$$

Odd-even mass differences

- B.E. (A) – ½ (B.E. (A+1) + B.E. (A-1))= $\Delta_{\mathbf{oe}}$
  $\Delta_{\mathbf{oe}} \sim \Delta_{BCS}$
- Where $\Delta_{BCS} = \overline{\text{sqrt}(\Delta^2_{T=1} + \Delta^2_{T=0})}$
- The last proton and neutron interact via the residual proton neutron interaction, $\delta_{\mathbf{np}}$
- B.E.($A_{\mathbf{ee}}$)- B.E.($A_{\mathbf{oo}}$) ~ 2 $\Delta_{\mathbf{BCS}}$- $\delta_{\mathbf{np}}$
- In N=Z nuclei, we expect $\delta_{\mathbf{np}}$ to be stronger (maximum overlap). Should be seen in experiment.

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Masses summary1

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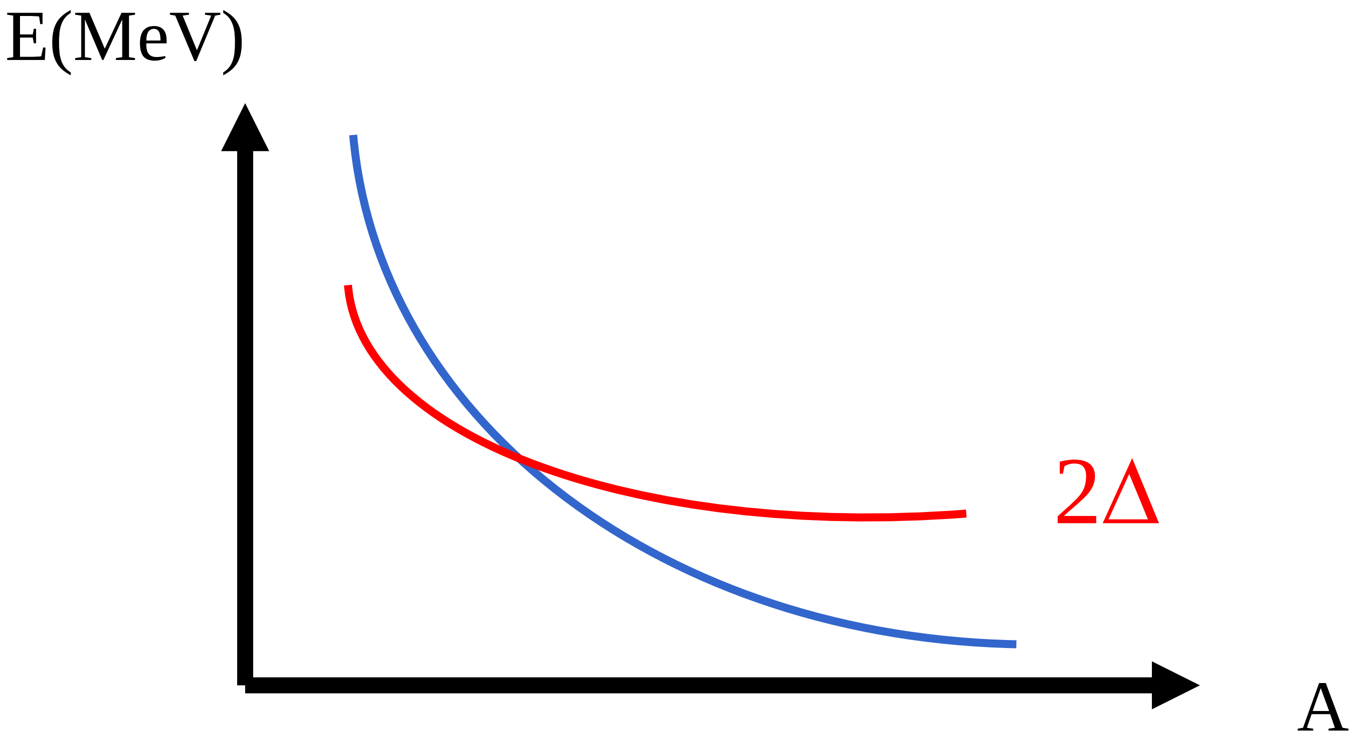
- o-o N=Z nuclei are less bound than N≠Z
- Indicative of larger gap (pairing correlations) in e-e N=Z nuclei

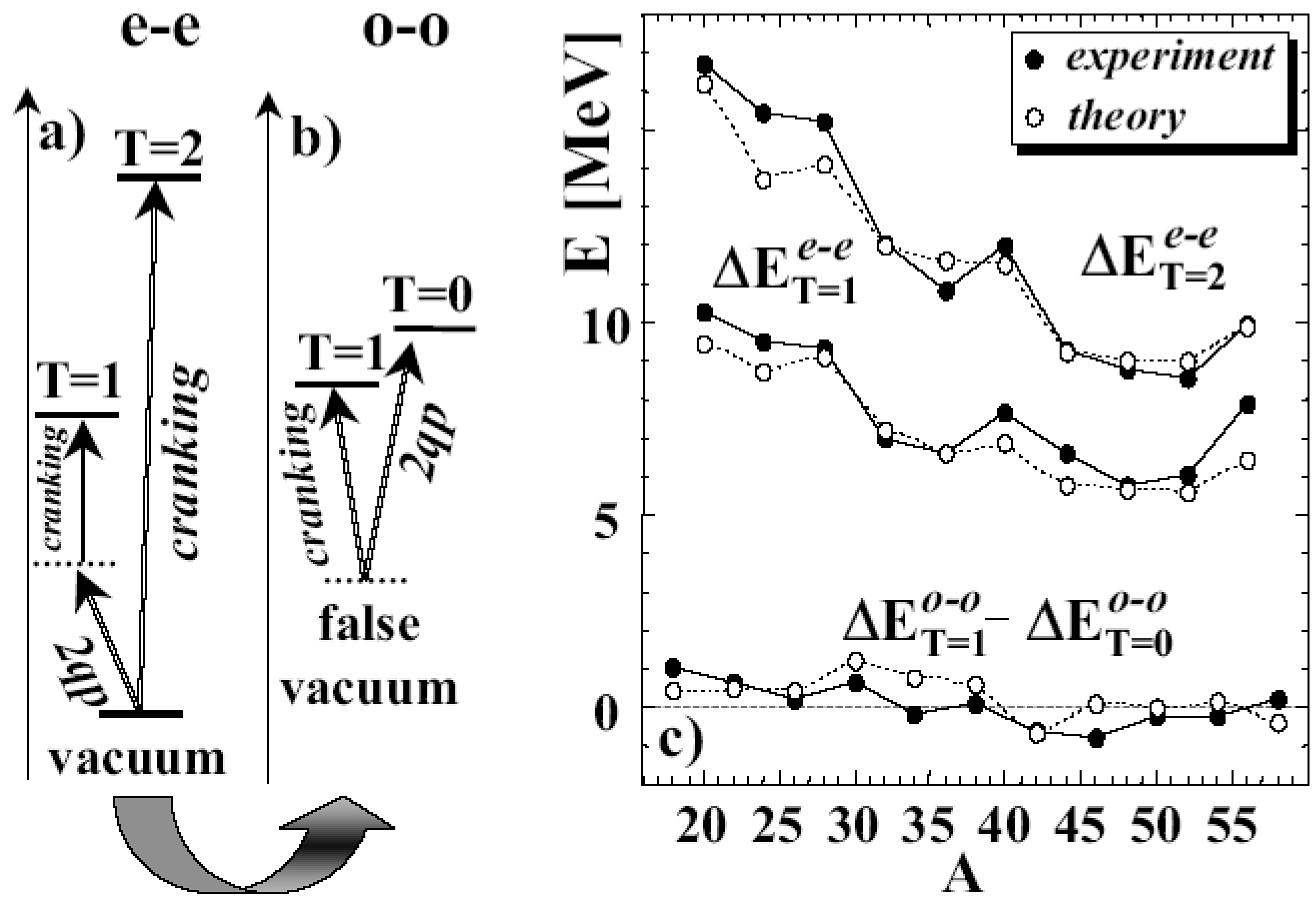
Comparison between T=0 and T=2

Competition between 2qp excitation and symmetry energy in o-o nuclei

T=0 states in o-o nuclei are 2qp excitations ~1/sqrt(A)

T=1states have larger symmetry energy ~ 1/A




e-e
o-o
a)
T=2
T=1
cranking
cranking
2qp
vacuum
b)
T=0
T=1
cranking
2qp
false
vacuum
E [MeV]
experiment
theory
$\Delta E^{e-e}_{T=1}$
$\Delta E^{e-e}_{T=2}$
$\Delta E^{o-o}_{T=1} - \Delta E^{o-o}_{T=0}$
10
5
0
c)
20 25 30 35 40 45 50 55
A

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