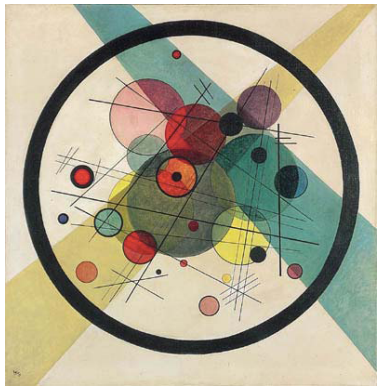
Pairing modes and pair transfer reactions

Andrea Vitturi (Padova)



ESNT Workshop Pairing Saclay 2018

- Definition of collectivity for pairing modes (correlation energies, pair-creation matrix elements, coherent wave functions, etc)
- Dynamical tests of pairing modes (pair transfer, two-particle break-up, two-particle knock-out, two-particle decay, etc)
- Reaction mechanism for two-particle transfer process
- Space correlations
- Q-value effects
- Weakly-bound systems, coupling to break-up channels and effects of continuum. One-dimensional case
- Pairing vibrations. Shape coexistence. Phase transitions
- Alpha-transfer processes and correlations with two-neutron and two-proton transfers

The essential quantity to characterize the system from the pairing point of view is given by the "pairing response", namely all the T_0 values of the square of the matrix element of the pair creation (or removal) operator (for T=1 pairs)

$$P^+ = \sum_j [a^+_j a^+_j]_{00} \qquad \text{(and similarly for } P^-\text{)}$$

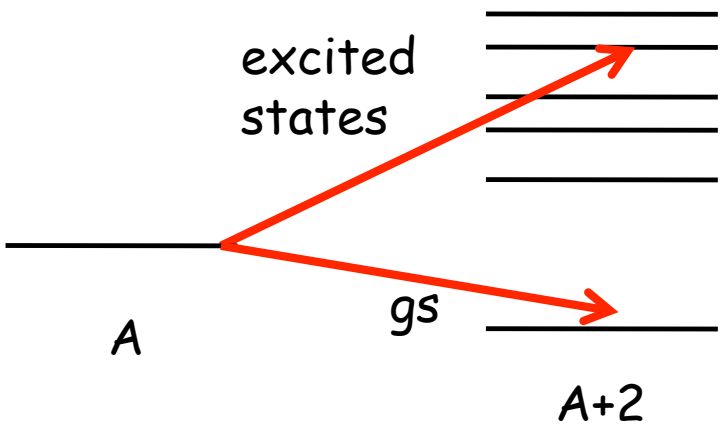
connecting the ground state of nucleus N with all 0+ states of nucleus A+2 (or A-2). It is often assumed that the cross section for two-particle transfer just scale with T_0.

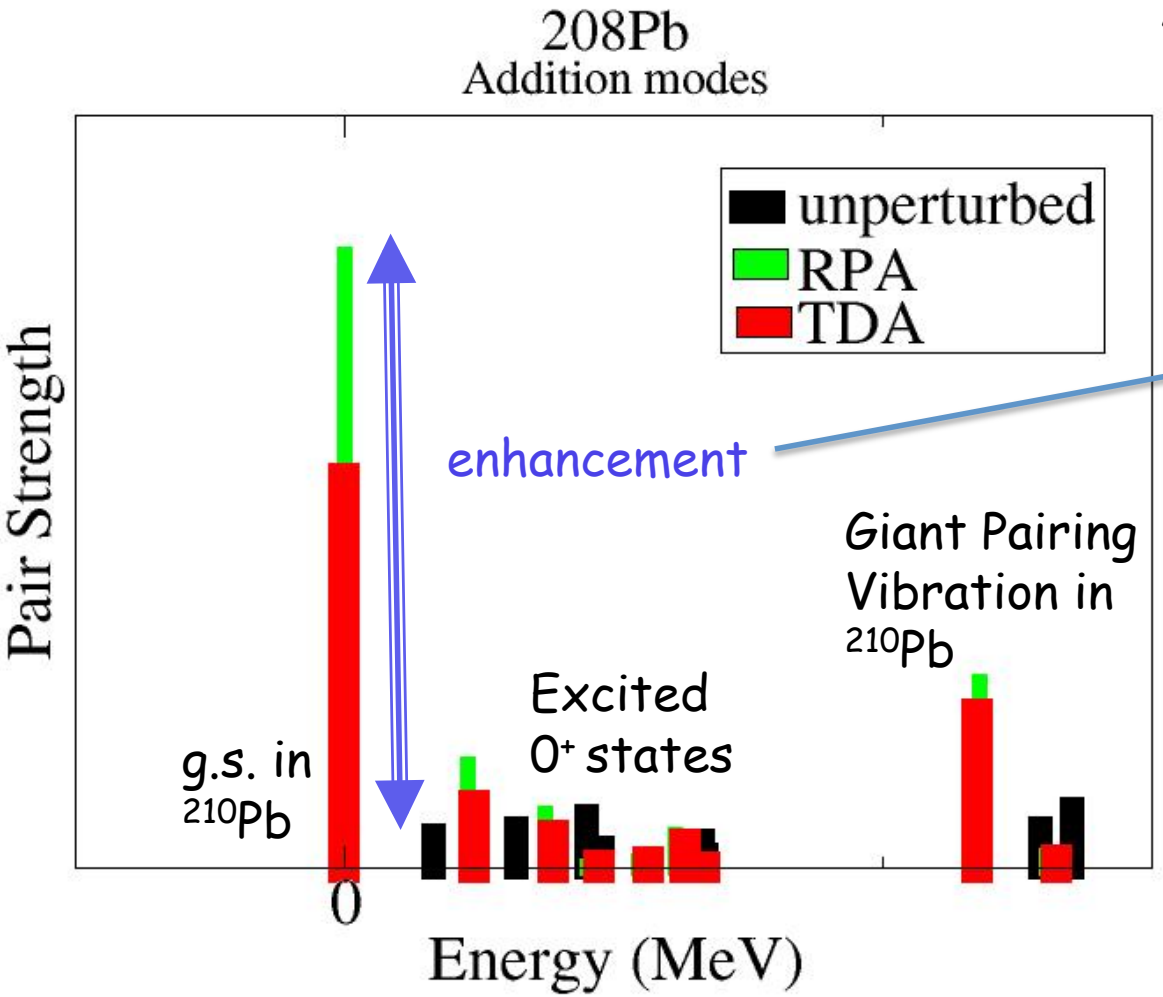
The traditional way to define and measure the collectivity of pairing modes is to compare with single-particle pair transition densities and matrix elements to define some "pairing" single-particle units and therefore "pairing" enhancement factors.

Similarly for the T=0 pairs, leading to 1+ states

The introduction of the concept of "collectivity" is fundamental if we want, for example, to compare T=0 and T=1 pairing

Typical "pairing" response



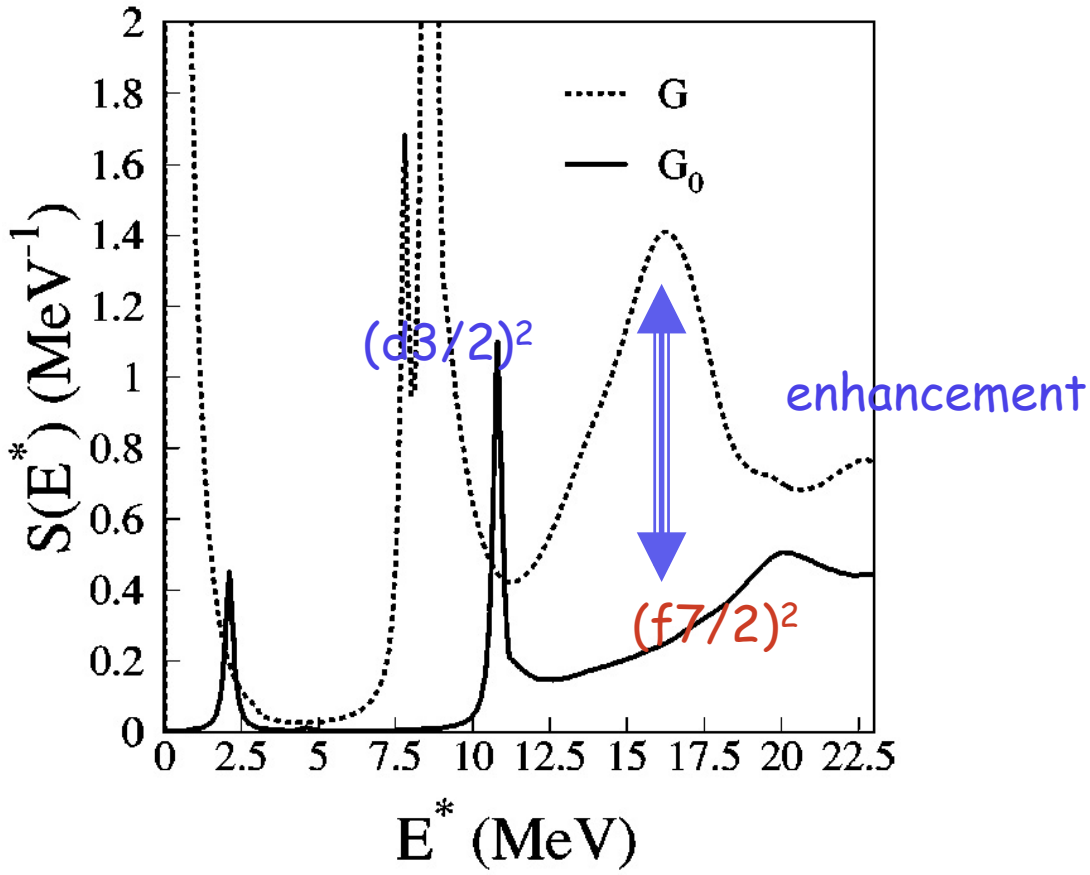


possible measure of collectivity

Sofia, Dasso, Vitturi

Pair strength function

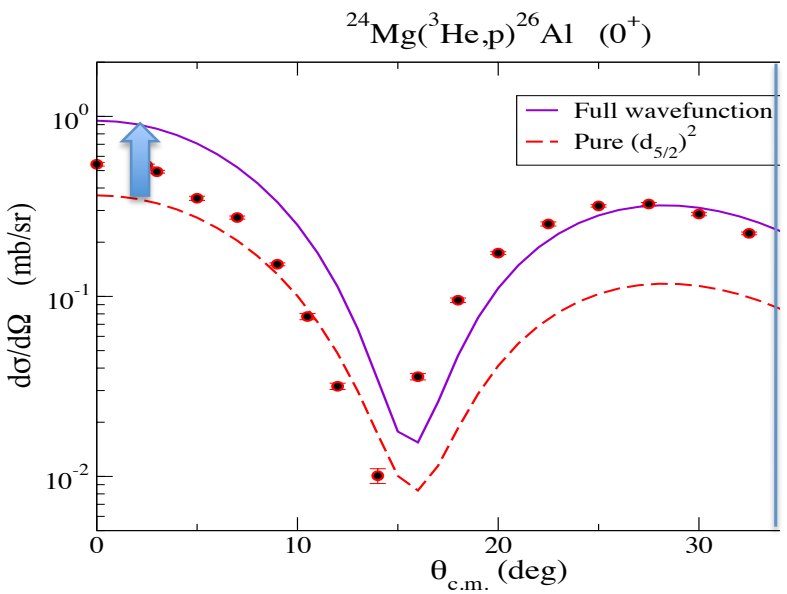
^{22}O

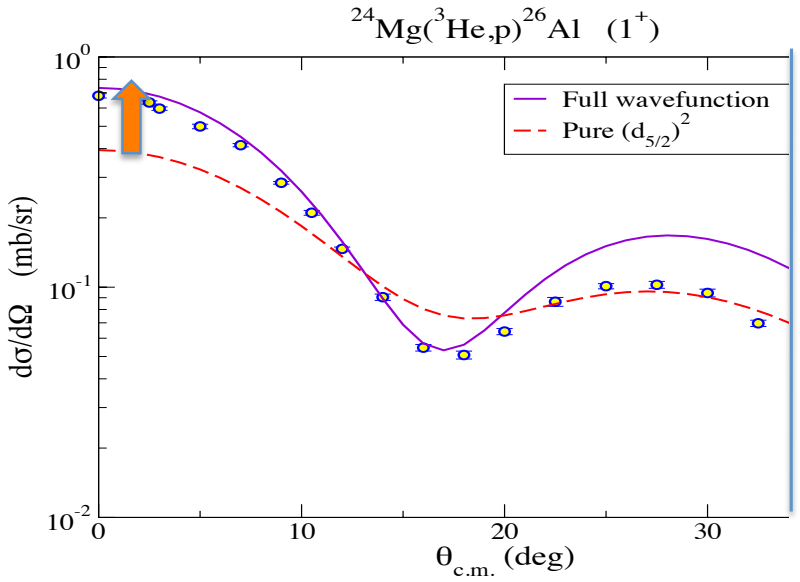


Khan, Sandulescu, Van Giai, Grasso

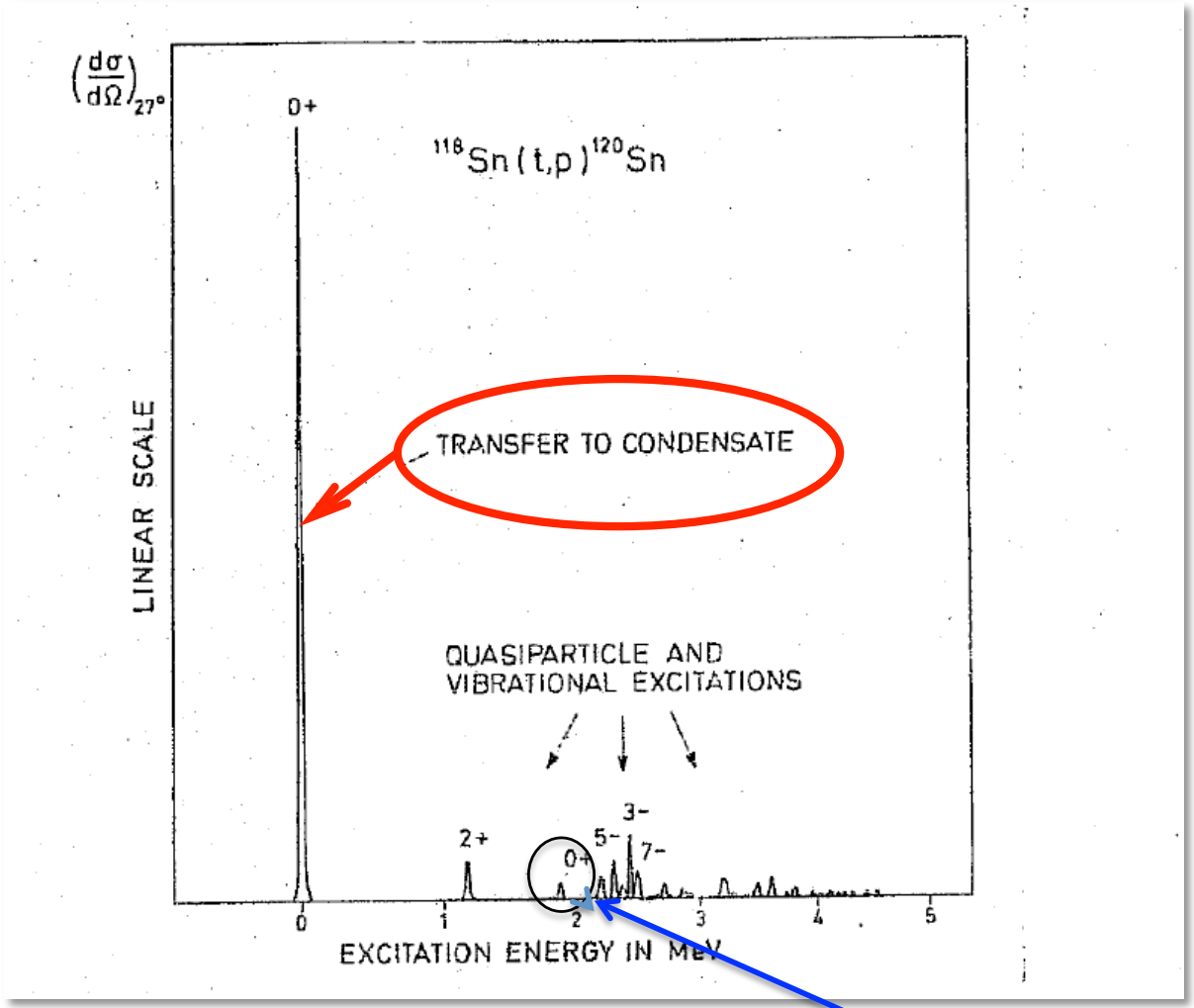
Comparing enhancement factors for 0+ and 1+ states

(from just cross sections)





An example of a "superfluid" nucleus (pairing rotations)



Practically **all** pairing strength goes to the ground state

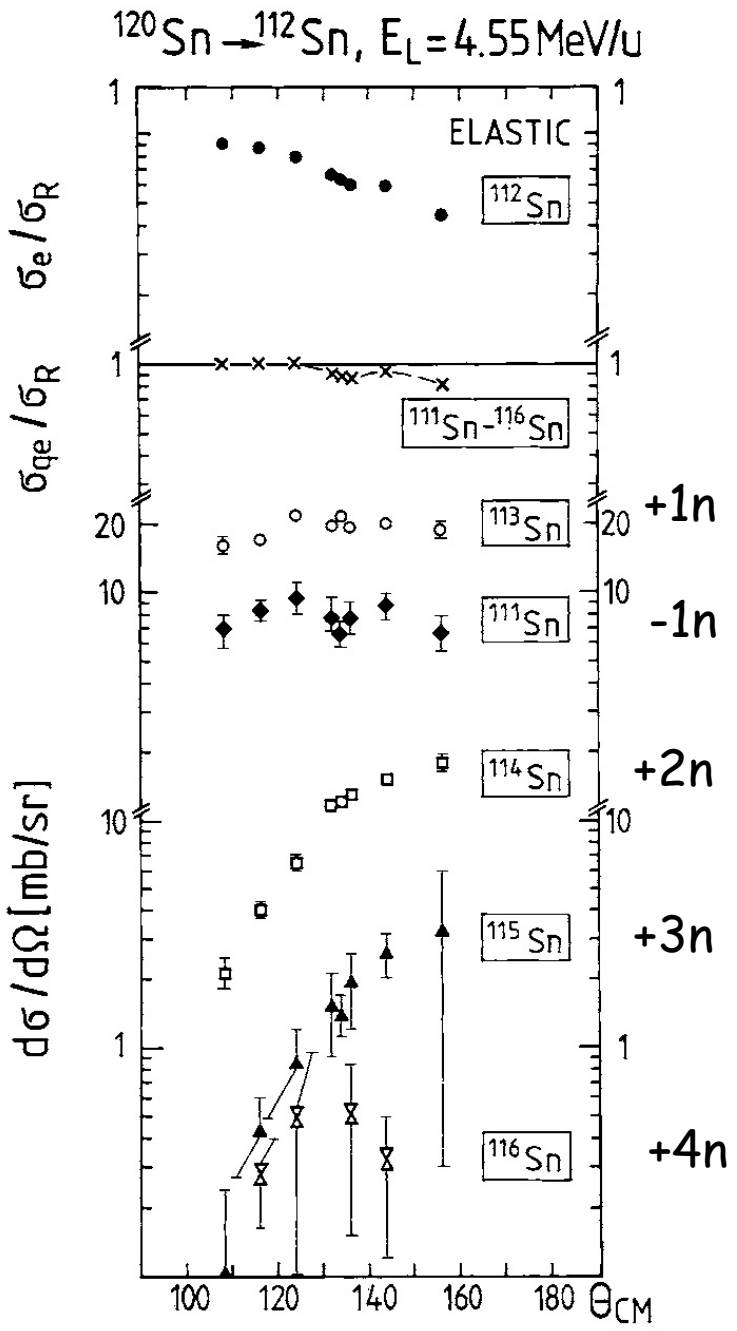
J.H.Bjerregaard *et al.* NPA 110 **1** (1968)

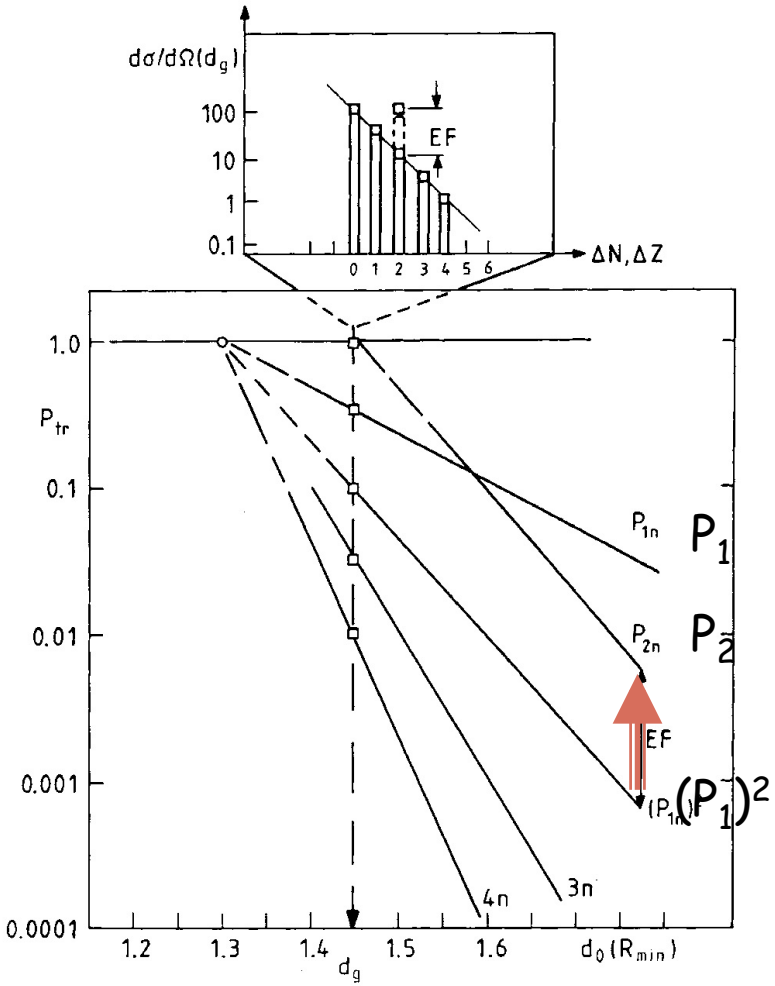
excited 0+ state

The "experimental" way of extracting the collectivity of pairing modes is from the enhancement of the two-particle cross section in heavy-ion induced reactions with respect to the square of probability of transferring just one particle.
This assumes that the transfer cross section is proportional to the square of the pair-creation operator, besides other assumptions on the reaction dynamics ……

The classical example:
Sn+Sn
(superfluid on superfluid)

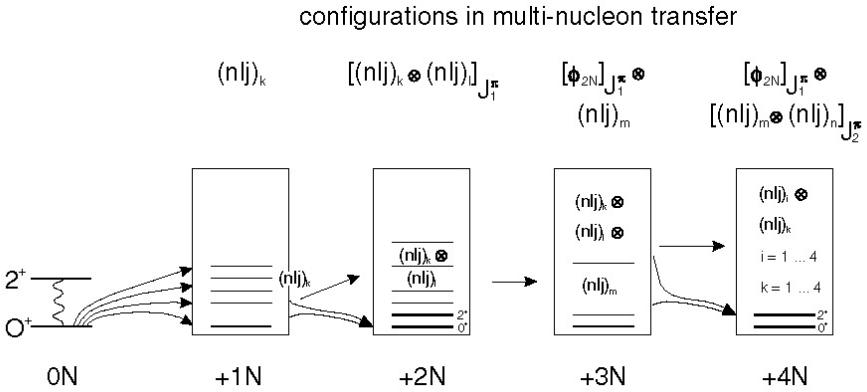
Von Oertzen, Bohlen etal

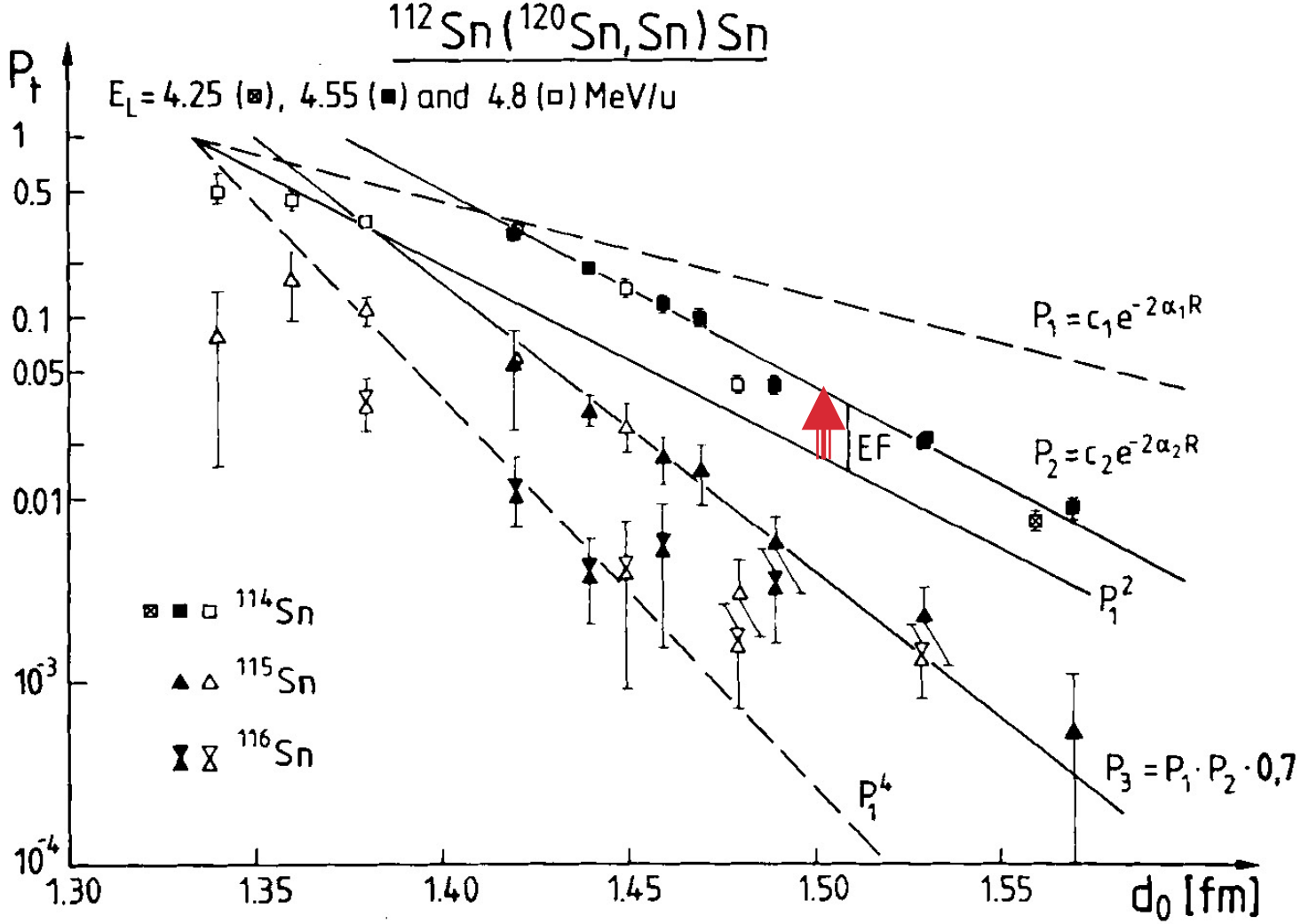




distance of closest approach

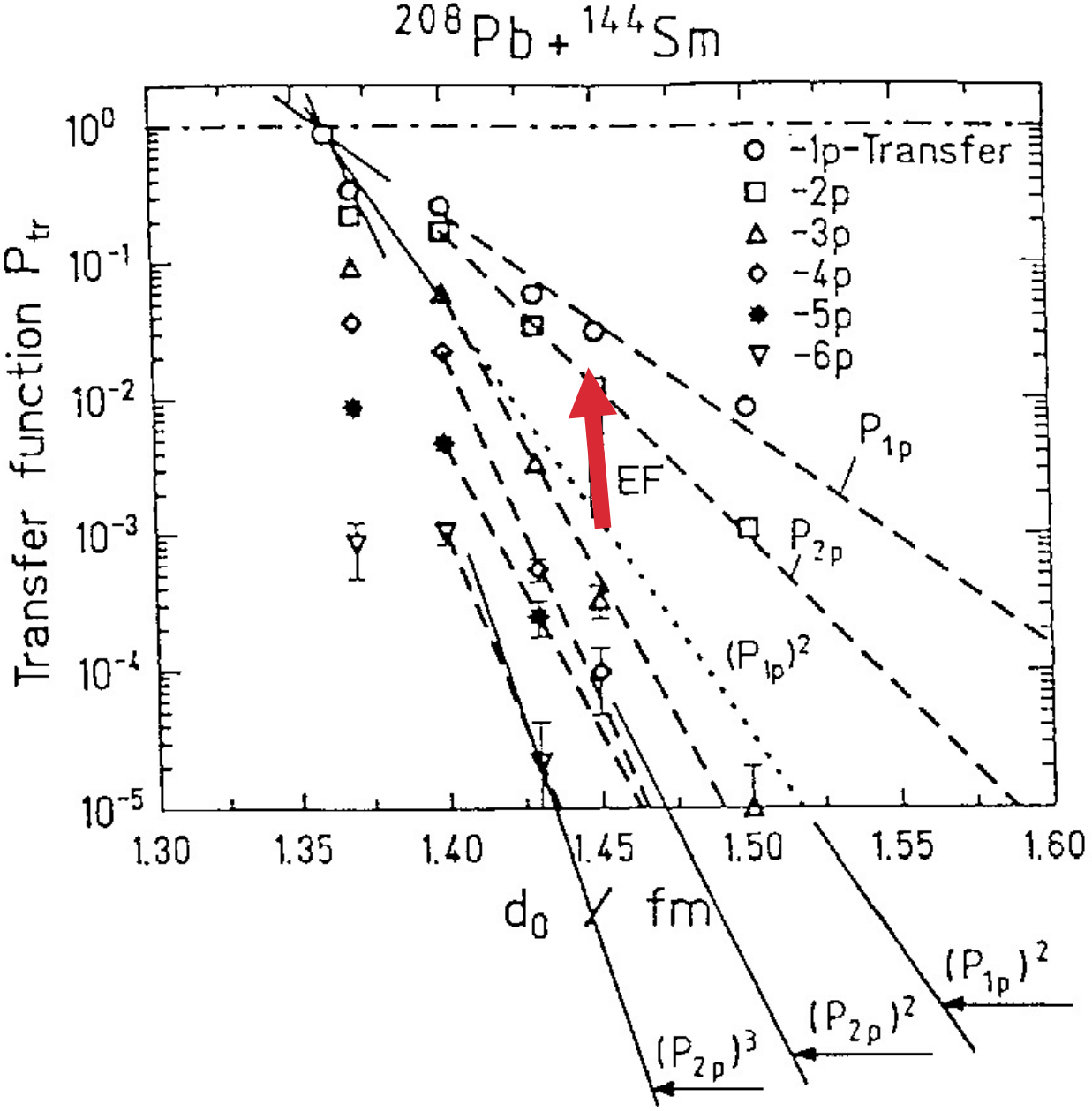
A way to define a pairing "enhancement" factor, by plotting transfer probabilities not as function of the scattering angle, but as function of the distance of closest approach of the corresponding classical trajectory





General problem: how separate the contribution of 0+ states?

Proton transfer



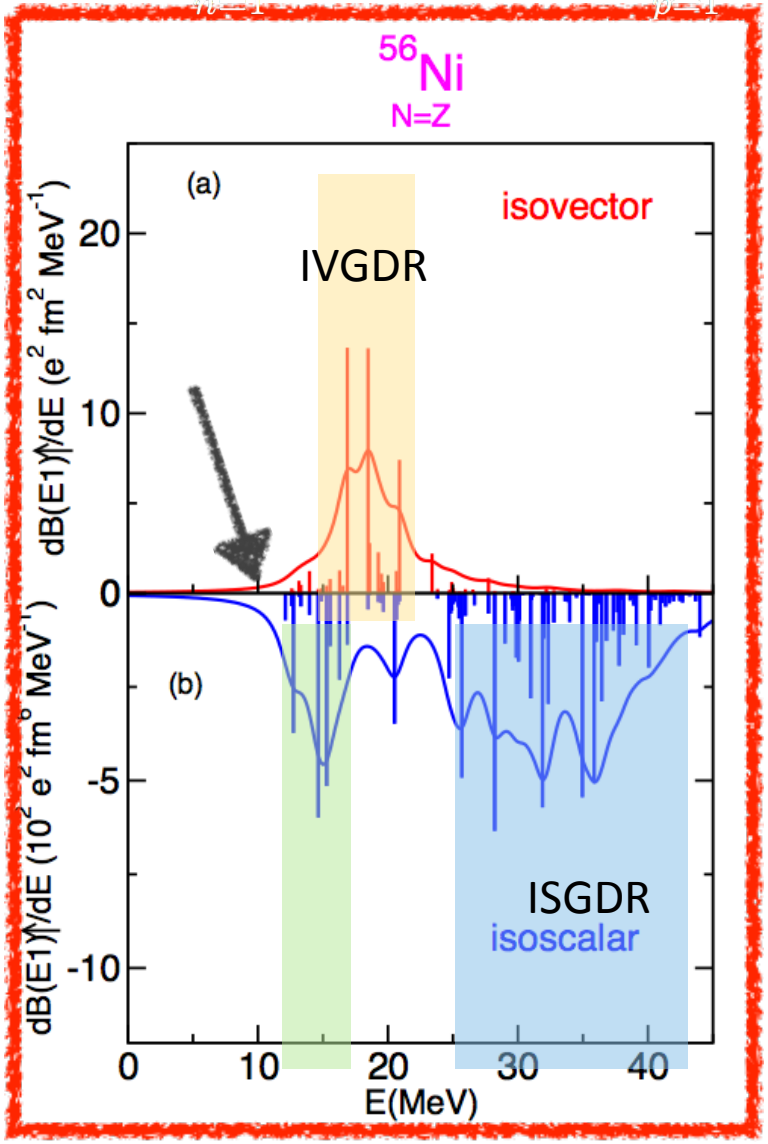
NEED FOR CONSTRUCTIVE INTERFERENCE

To display "collectivity" with respect to a definite operator one needs that the matrix elements of this operator shows "constructive" interference over the different components.

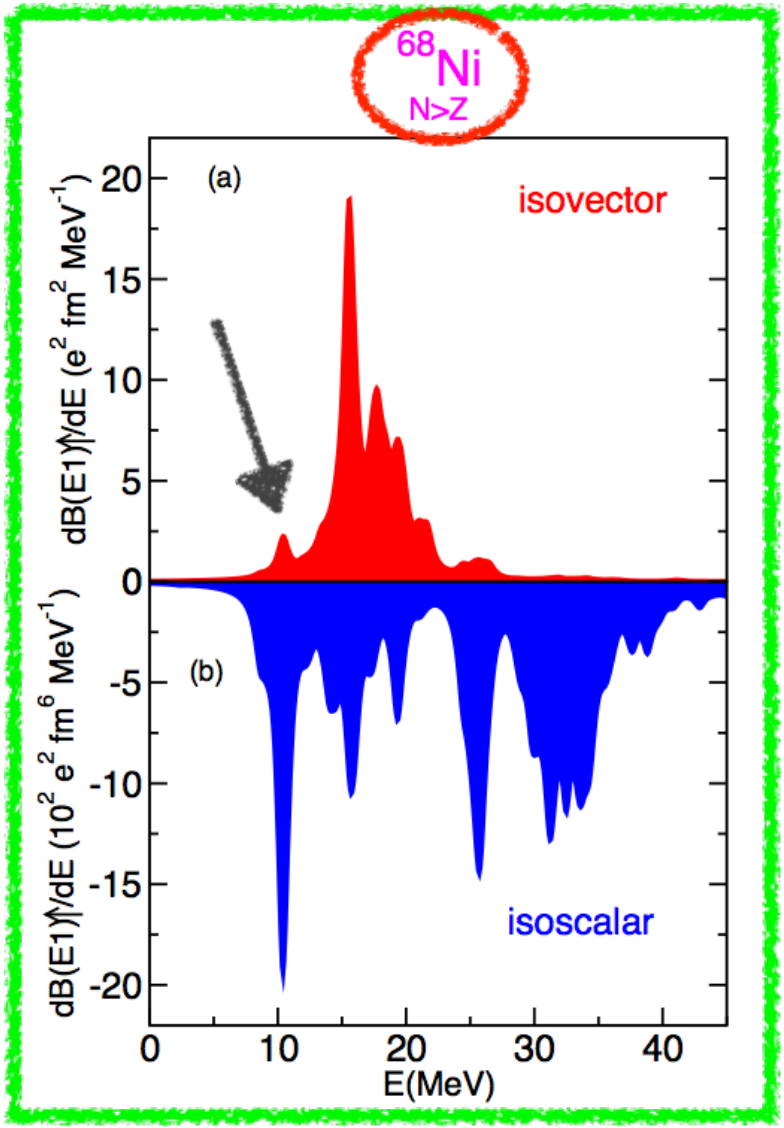
An example taken from a different mode: the case of the "so-called" pygmy dipole state (PDR)
Structure calculation: HF+ discrete RPA (with SGII interaction)

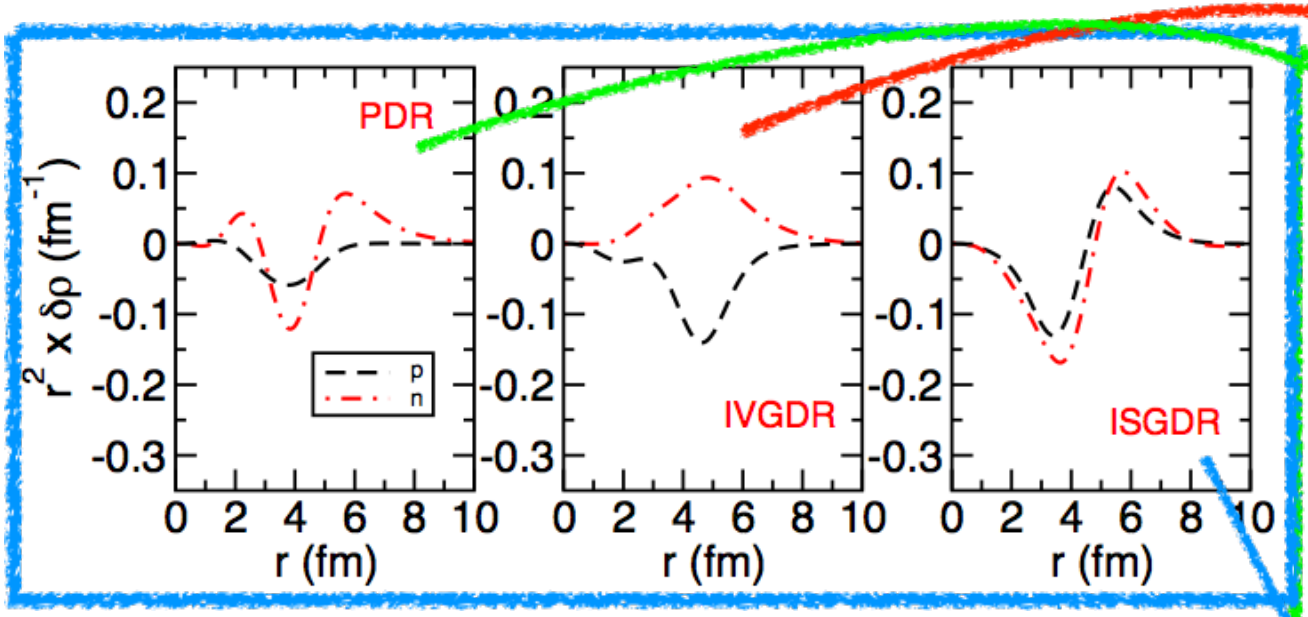
Import aspects: coherence plus shape of the transition density to characterize the nature of the state

$$O_{1M}^{(IV)} = 2\frac{Z}{A}\sum_{n=1}^{N} r_n Y_{1M}(\hat{r}_n) - 2\frac{N}{A}\sum_{p=1}^{Z} r_p Y_{1M}(\hat{r}_p)$$

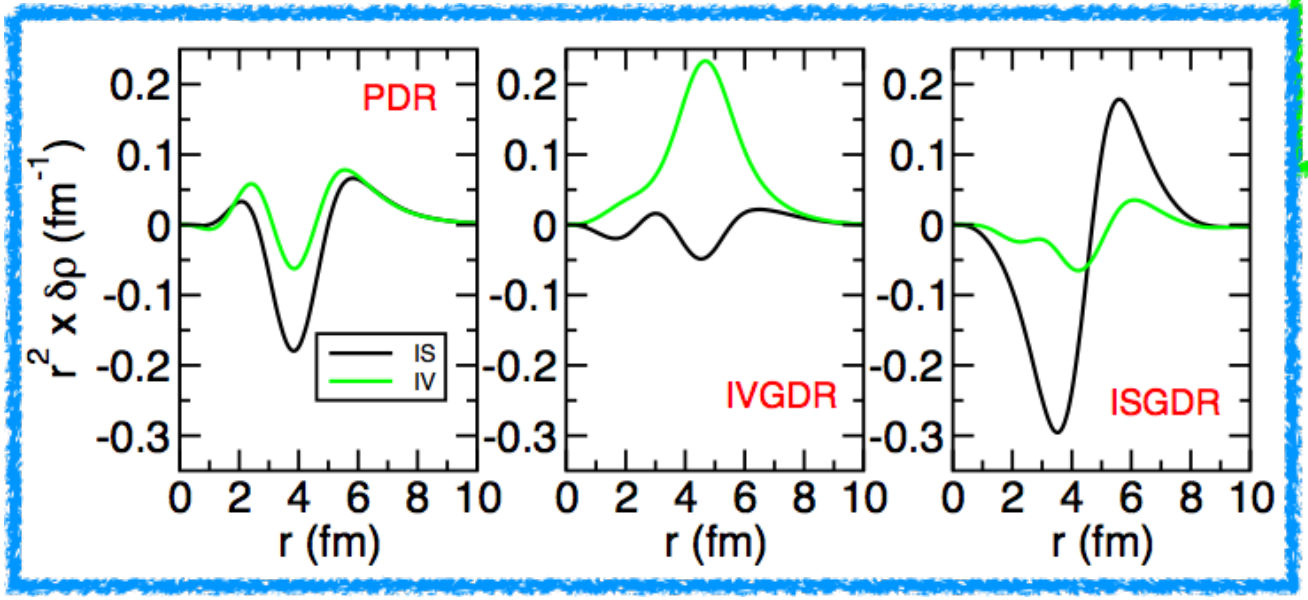


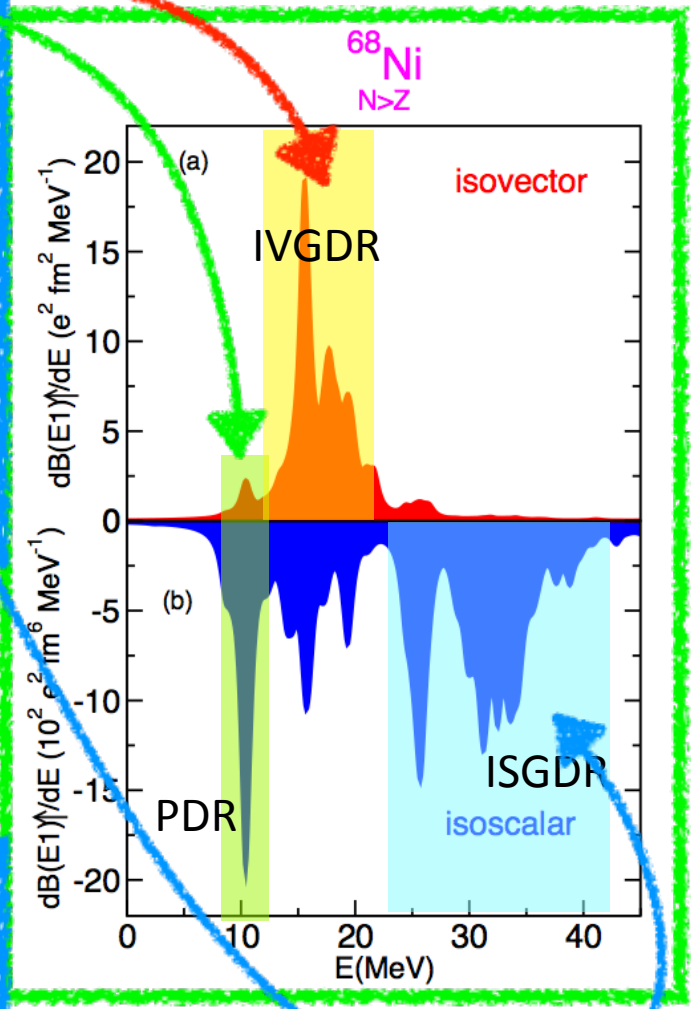
$$O_{1M}^{(IS)} = \sum_{i=1}^{A}(r_i^3 - \frac{5}{3} < r^2 > r_i)Y_{1M}(\hat{r}_i)$$





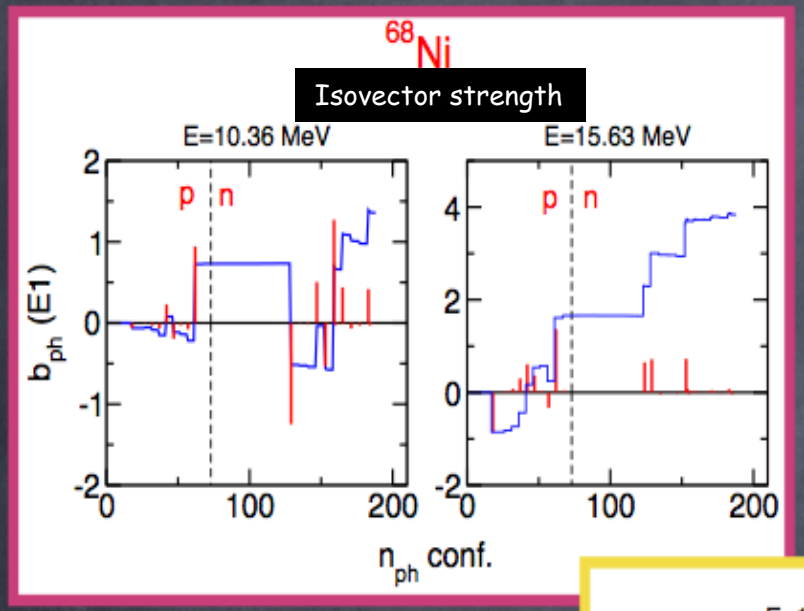
$$\delta\rho^{\nu} = \frac{1}{\sqrt{4\pi}} \sum_{ph} (-)^{j_p + l_p + \frac{1}{2}} \frac{\hat{j}_p \hat{j}_h}{\hat{\lambda}} < j_h \tfrac{1}{2}\ j_p\ -\tfrac{1}{2} | \lambda 0 > \delta(\lambda + l_p + l_h, even)$$
$$\cdot \left[X^{\nu}_{ph} - Y^{\nu}_{ph} \right] R_{l_p j_p}(r) R_{l_h j_h}(r)$$





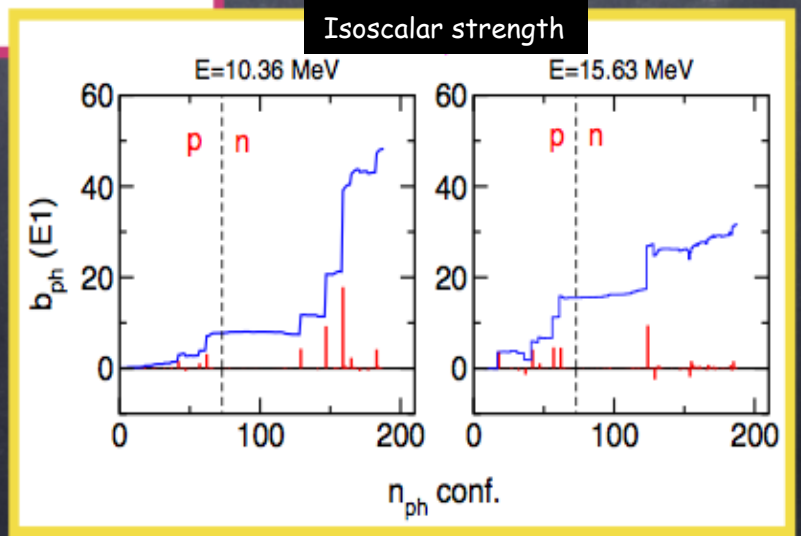
Are these states collective or not collective?
It depends with respect to which operator

$$B(E\lambda;0\rightarrow\nu)=\left|\sum_{ph} b_{ph}(E\lambda)\right|^2=\left|\sum_{ph}\left(X^{\nu}_{ph}-Y^{\nu}_{ph}\right)T^{\lambda}_{ph}\right|^2$$



collective

non collective



Basic point to discuss: how reactions involving pair of nucleons (e.g. two-particle transfer, two-particle break-up, two-particle knock-out, two-particle emission) can provide clear signature on the properties (and on the "phase") of nuclear systems.

This dynamical source of information should be complementary (but as important) to the one associated to other properties (as energy spectra and correlation energies or electromagnetic transition rates, for example).

Clearly the probabilities for such processes must be influenced by the particle-particle correlations and by the "collectivity" of the tested modes, and these will depend on the specific "phase" of the system. So, for example, they will be sensitive to any change in the status of the system, for example along an isotope chain.

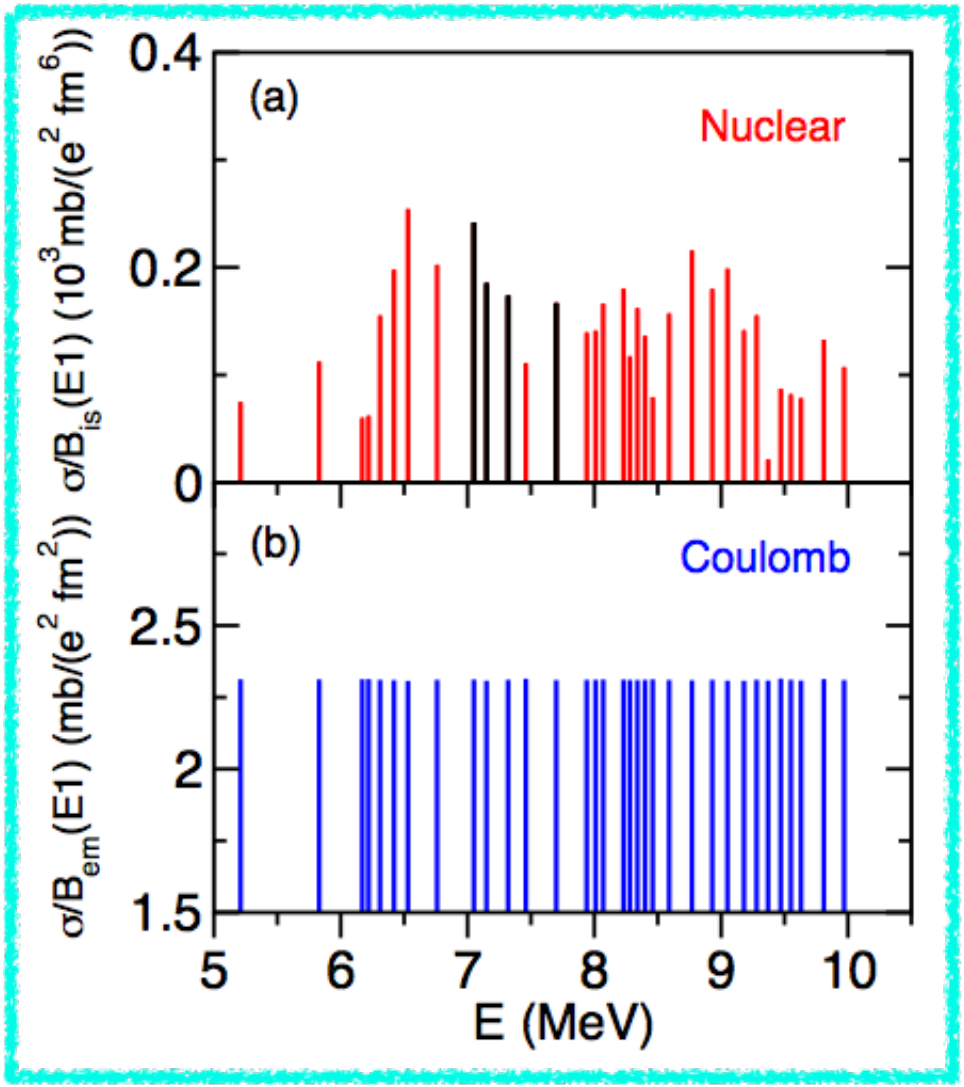
Unfortunately, at variance, for example, from low-energy one-step Coulomb excitation, where the excitation probability is directly proportional to the B(Eλ) values, the reaction mechanism associated with the simplest of the listed processes, i.e. pair transfer, is rather complicated and the possibility of extracting spectroscopic information on the pairing field is not obvious. The situation is actually more complicated even with respect to other processes (as inelastic nuclear excitation) that may need to be treated microscopically, but where the reaction mechanism is somehow well established.

To clarify the concept we consider again the case of dipole states (described within HF+RPA) and calculate the cross sections to inelastically excite these states via Coulomb or nuclear field in the reaction alpha+124Sn (in DWBA).
These cross sections are then compared with the corresponding isovector and isoscalar B(E1) values

Ratios between cross sections and B(E1)'

The relation between the isoscalar response and the inelastic excitation cross section due to an isoscalar probe it is not so evident.

For pure Coulomb excitation the relation between the inelastic cross section and the $B_{em}(E1)$ is clear: they are proportional.



OBS: For this comparison the energies of all the states have been put to zero in order to eliminate the contributions due to the Q-value effect (to be discussed later).

But if the qualitative behavior may be clear, the quantitative aspects require a proper treatment of the reaction mechanism. All approaches, ranging from macroscopic to semi-microscopic and to fully microscopic, try to reduce the actual complexity of the problem, which is a four-body scattering (the two cores plus the two transferred particles), to more tractable frameworks.

Two models are most popular:
A, Successive single-particle transfer
B. Cluster transfer

A

Sequential two-step process: each step transfers one particle

Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A +1) nuclei, due to the correlations in initial and final wave functions

Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions

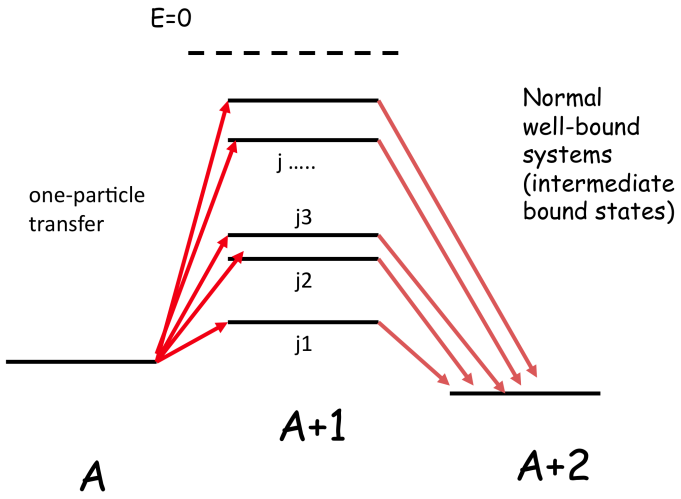
Expansion to second-order in the transfer potential

Simultaneus + Sequential + not-orthogonality

(first-order) (second-order) (second-order)

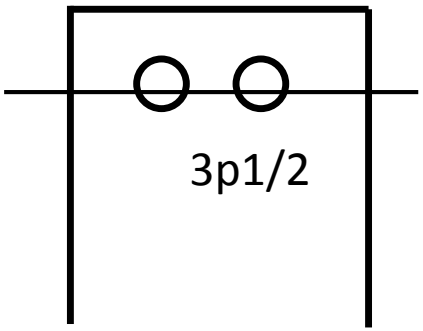
this is not the cluster contribution

these two terms approximately cancel each other

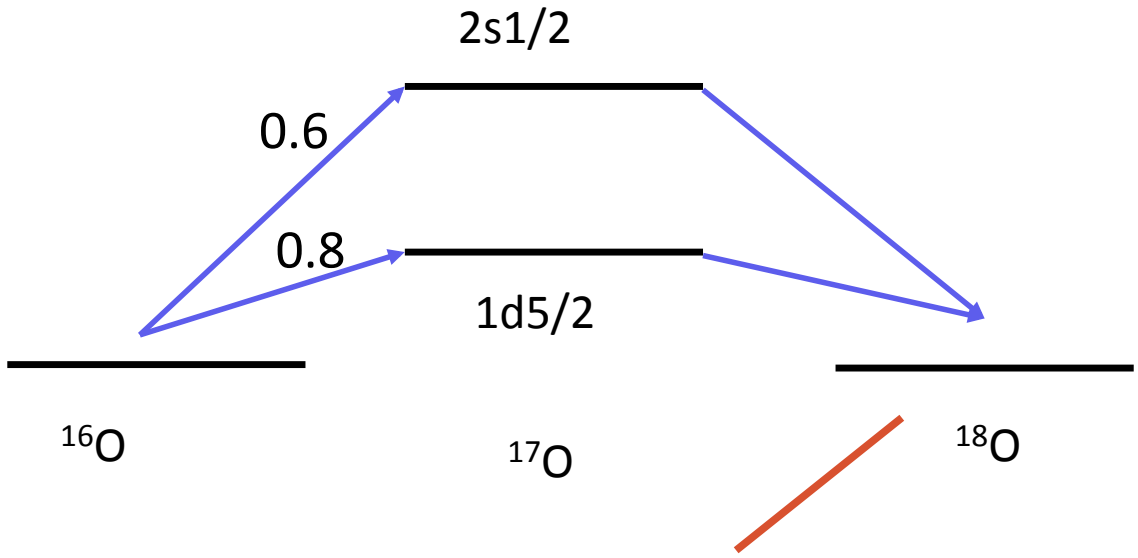


Example with just two components

$^{208}Pb(^{16}O,^{18}O)^{206}Pb$



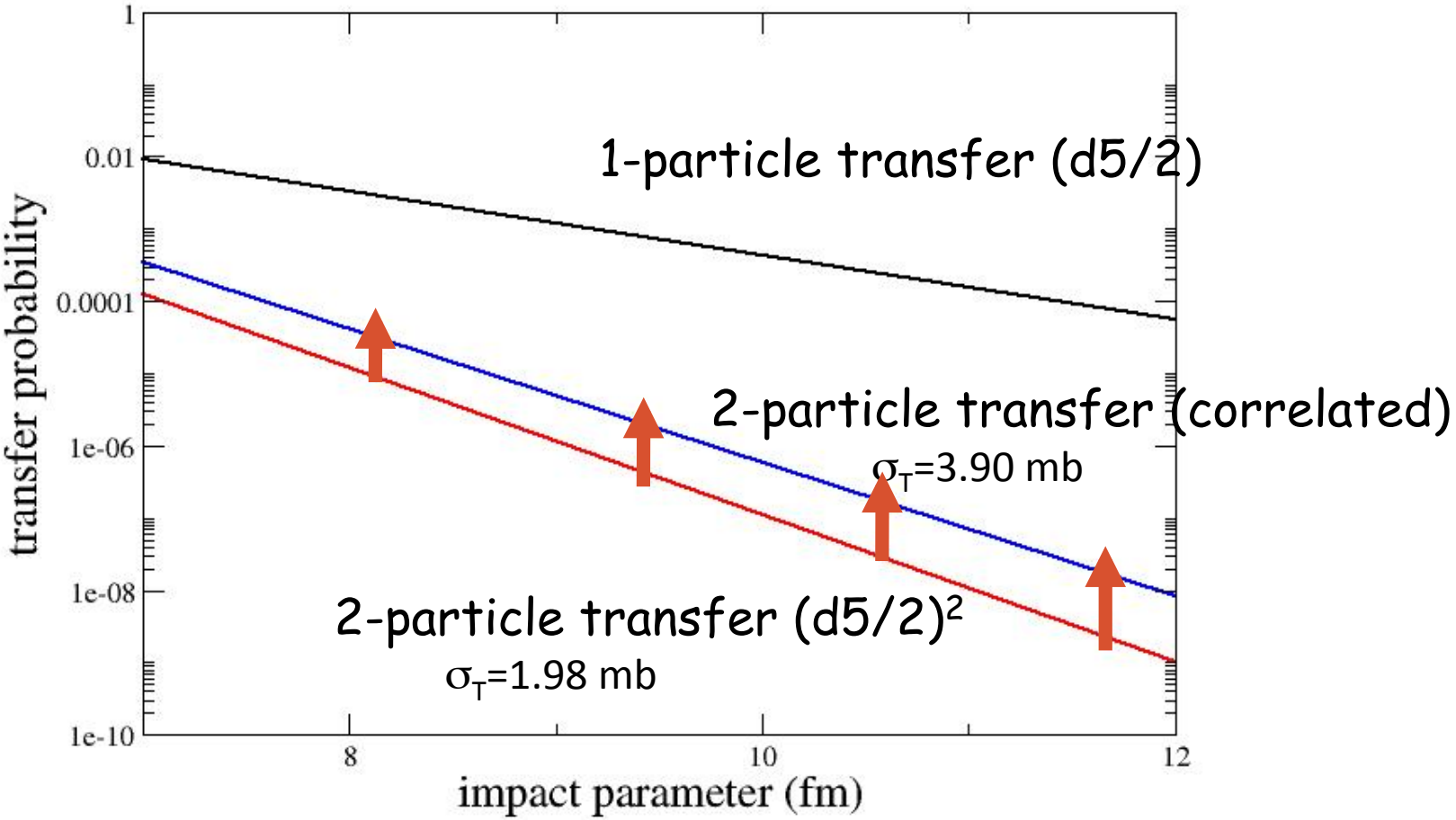
^{208}Pb



$0.8\ (1d5/2)^2 + 0.6(2s1/2)^2$

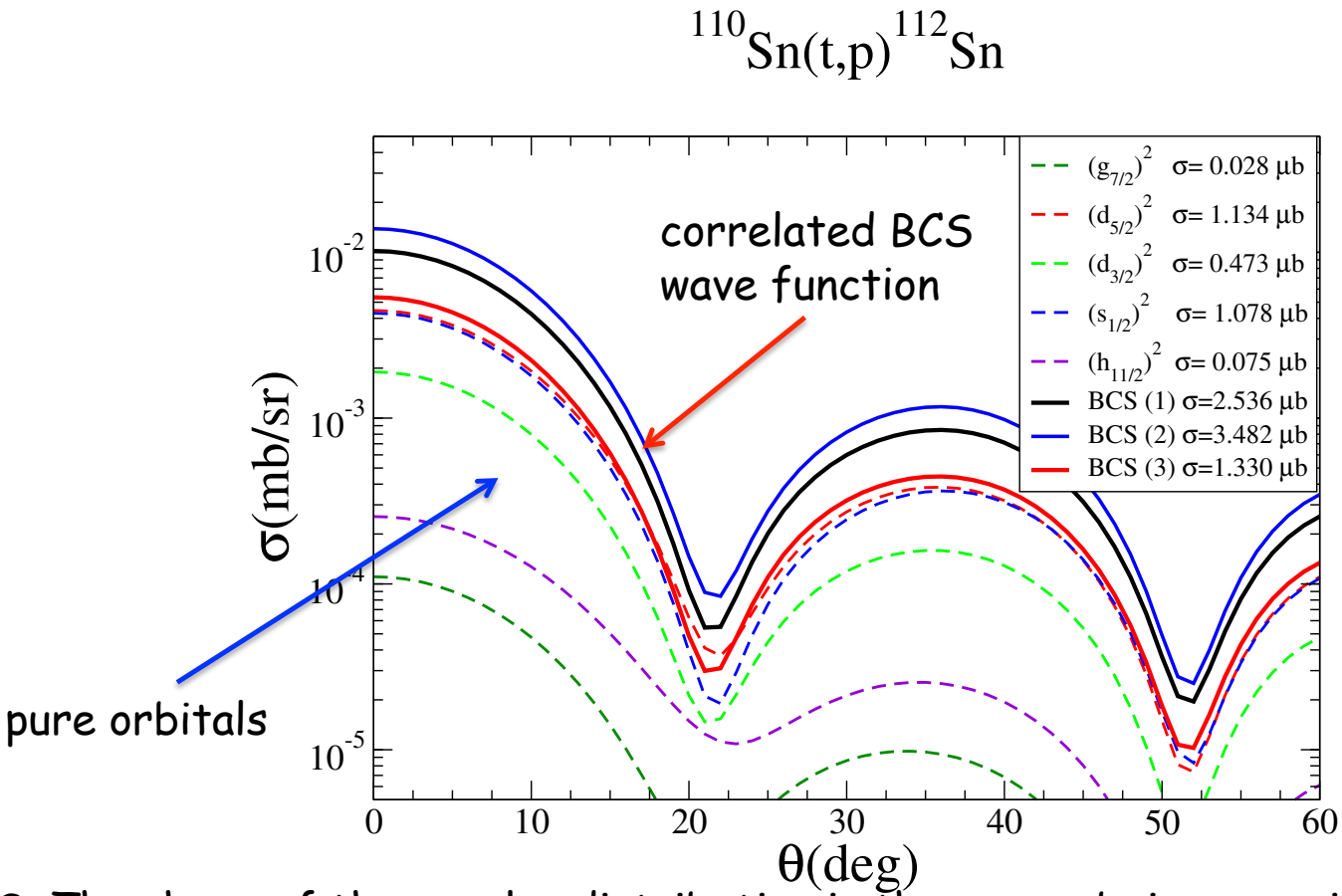
Example of calculation

$^{208}Pb(^{16}O,^{17,18}O)^{207,206}Pb$



In this case the pairing enhancement factor in the cross section is about a factor 2

The transfer probabilities vary strongly with the involved orbital. In addition whether the final wave function only involves a "pure" orbital, or whether it is correlated



OBS: The shape of the angular distribution is the same, being associated with the L=0 transfer

Does the two-particle transfer cross in superfluid systems simply scale with the pairing gap value Δ ?

The two-particle transfer probability is enhanced by the pairing correlation but does NOT scale simply with the value of the pairing gap Δ. In fact, if we alter the sequence of single-particle levels, and change the value of the pairing strength G to obtain the same Δ, the cross section will be different, due to different interplay of the single-particle states

	BCS 1		BCS 2		BCS 3	
	ϵ_i(MeV)	B_i	ϵ_i(MeV)	B_i	ϵ_i(MeV)	B_i
$0g_{7/2}$	-0.027	0.75	-0.027	1.15	-2.027	0.64
$1d_{5/2}$	0.882	1.13	-0.118	0.57	0.882	1.02
$2s_{1/2}$	1.330	0.53	-0.670	0.33	1.330	0.59
$0h_{11/2}$	2.507	0.79	4.507	0.61	5.507	0.46
$2d_{3/2}$	2.905	0.39	2.905	0.26	2.905	0.27
σ (mb)	2.5		3.4		1.3	

We consider the same case as before, i.e. the transfer of two neutrons from ^{110}Sn to ^{112}Sn (0+; gs) using the reactions

(14C,12C) or (18O,16O)

In addition to the information on the target, we need now to specify on which orbit the particle are transferred in the projectile

In the (14C,12C) the two neutrons are assumed to be picked-up from the p1/2 shell.

In the (18O,16O) from the pure d5/2 shell, or from a combination of $(d5/2)^2$ and $(s1/2)^2$

Resulting two-particle transfer total cross sections: the ranking of the different orbitals is different with different projectiles

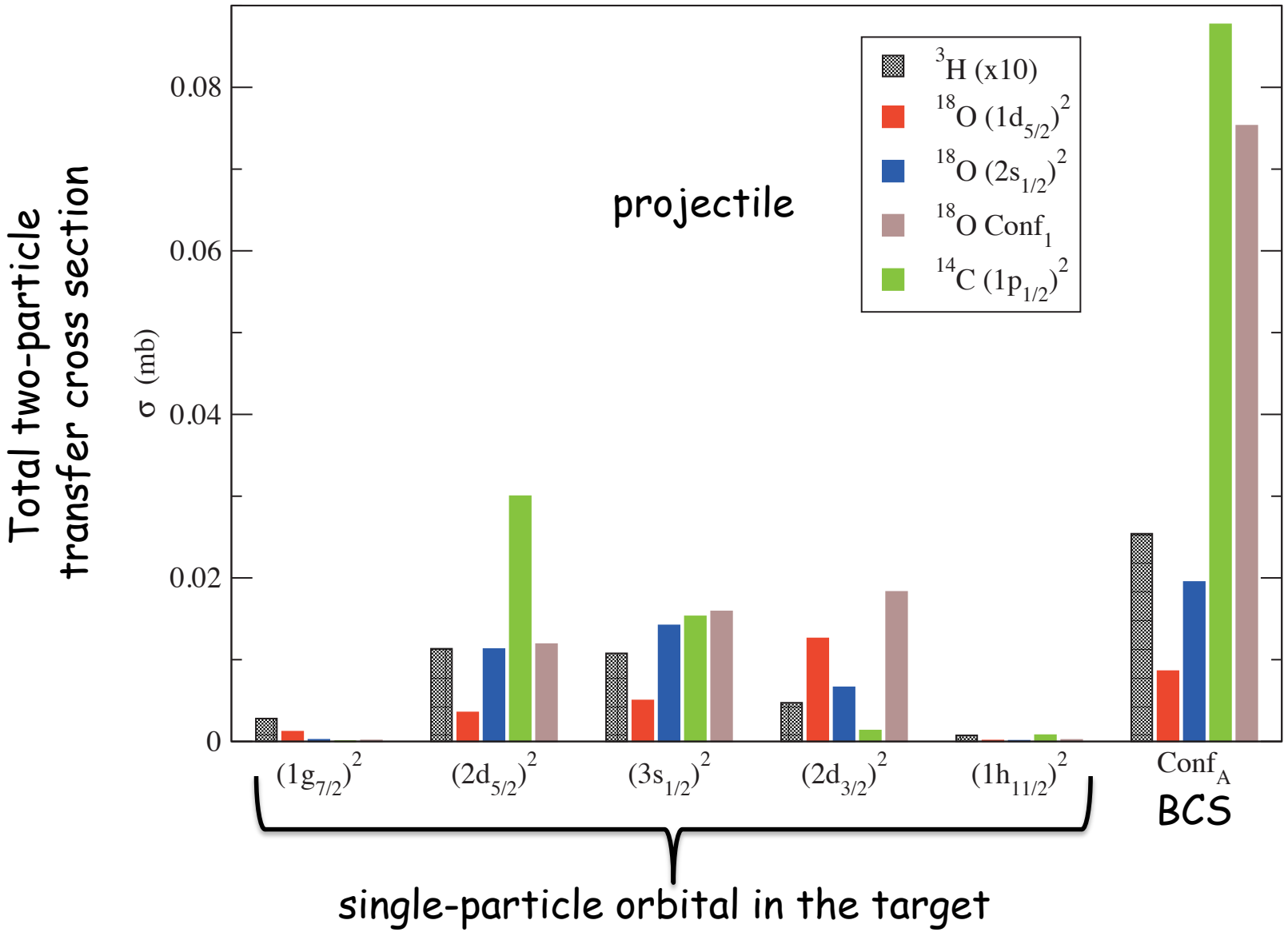
projectile

single-particle orbital in the target

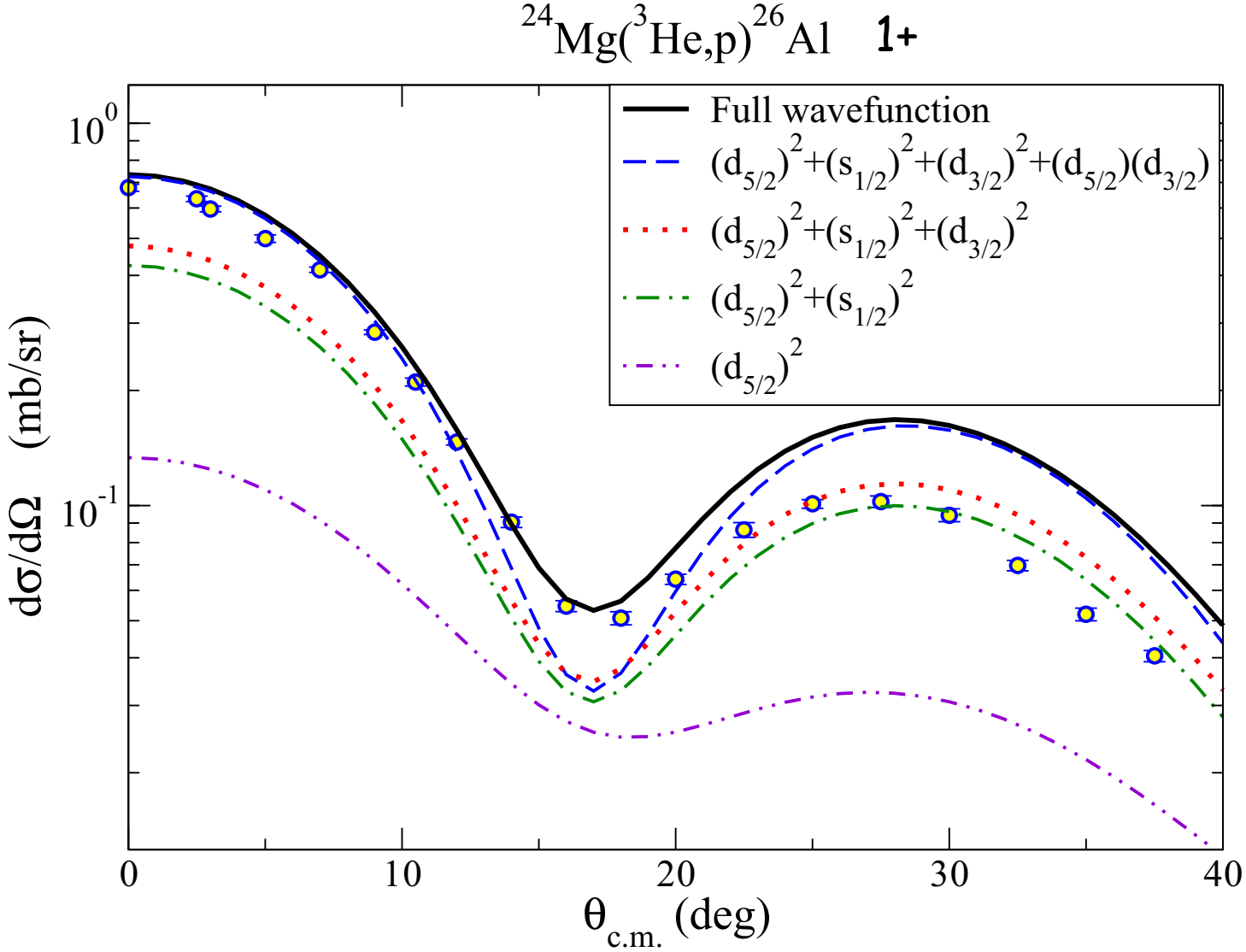
^{112}Sn	(t,p)	(^{14}C,^{12}C)				(^{18}O,^{16}O)		
		$(0p_{1/2})^2$	$(1s_{1/2})^2$	$(0p_{3/2})^2$	$(0d_{5/2})^2$	$(0d_{5/2})^2$	$(1s_{1/2})^2$	Conf_1
$(0g_{7/2})^2$	2.80E-5	1.73E-5	1.19E-4	7.09E-4	9.00E-4	1.19E-3	2.01E-4	1.24E-3
$(1d_{5/2})^2$	1.13E-3	3.00E-2	4.71E-3	5.54E-3	1.18E-3	3.55E-3	1.13E-2	1.19E-2
$(2s_{1/2})^2$	1.08E-3	1.53E-2	5.38E-3	7.05E-3	1.16E-3	5.02E-3	1.42E-2	1.59E-2
$(1d_{3/2})^2$	4.73E-4	1.34E-3	2.79E-3	9.87E-3	4.14E-3	1.26E-2	6.62E-3	1.83E-2
$(0h_{11/2})^2$	7.50E-5	7.77E-4	5.29E-5	1.05E-4	7.65E-5	1.10E-4	9.06E-5	1.88E-4
Conf_A	2.54E-3	8.77E-2	2.26E-2	3.77E-2	1.21E-2	8.60E-3	1.95E-2	7.53E-2

BCS

Same results shown as histograms

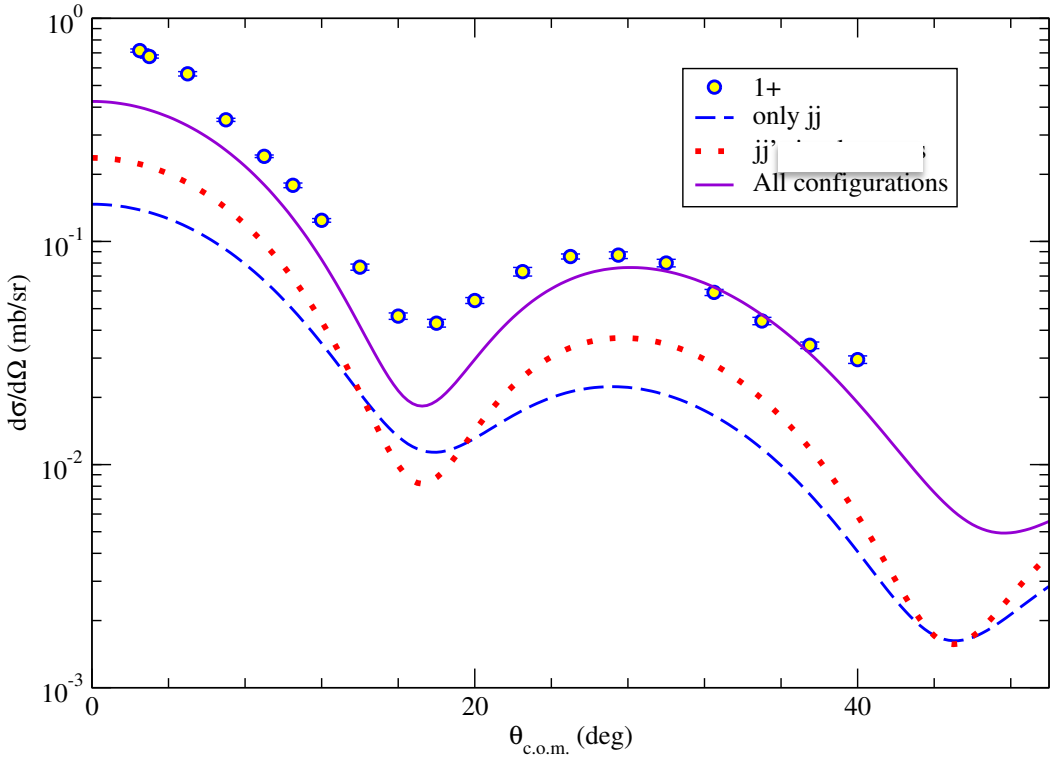


Another example of constructive interference over the intermediate states: "deuteron " transfer



Effect of contributions from L=0 and L=2 for 1+ states

^{24}Mg(^{3}He,p)^{26}Al



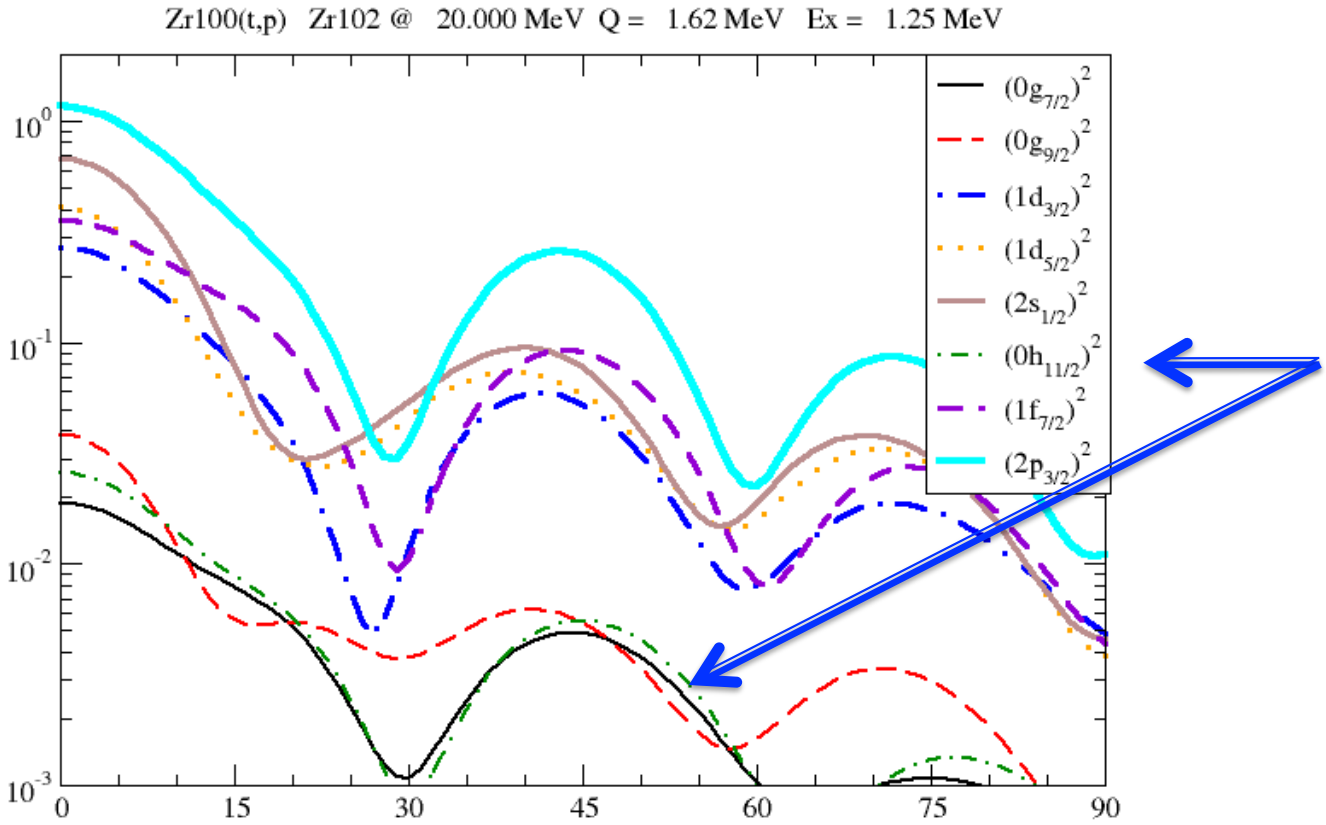
⇒ Components with $j_\pi \neq j_\nu$ increases $\sigma(1^+)$ reducing $\sigma(0^+)/\sigma(1^+)$

⇒ Small components (2s1/2)(1d3/2) and (1d5/2)(1d3/2) but they further increase the cross section

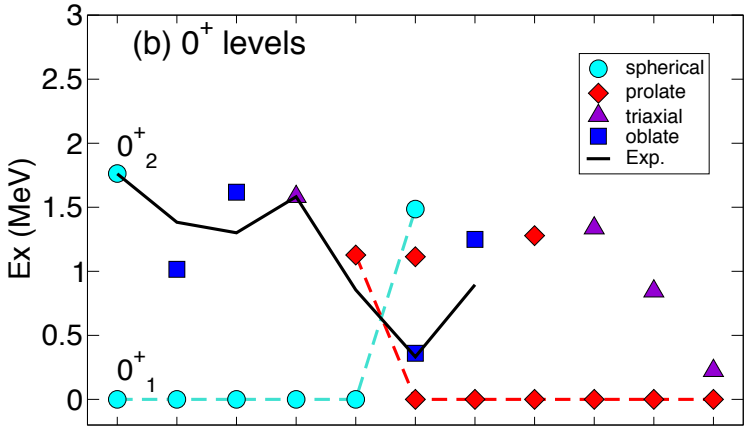
One more example of dependence on specific orbitals and consequent strong variation of the cross section along an isotope chain

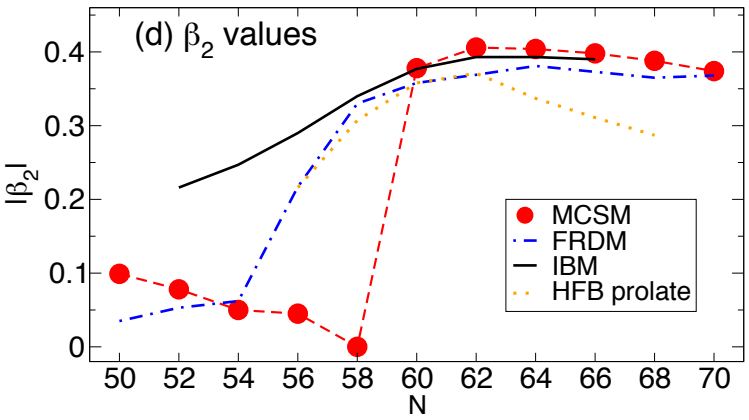
^{100}Zr (t,p) ^{102}Zr

Cross sections for pure configurations

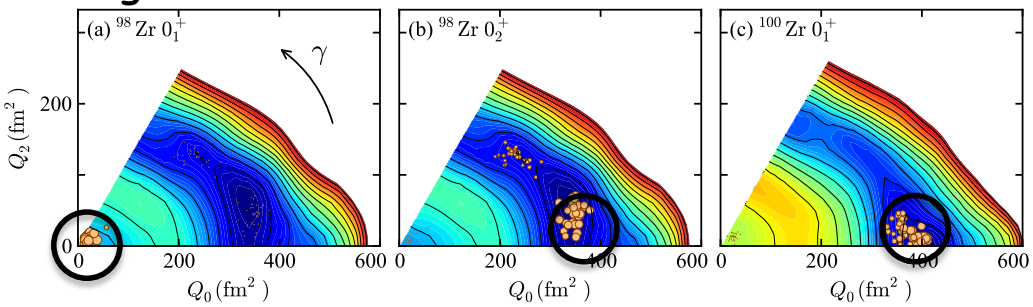


Shape phase transition in Zr isotopes between N=58 and 60





0_{gs} in ^{98}Zr 0_2 in ^{98}Zr 0_{gs} in ^{100}Zr

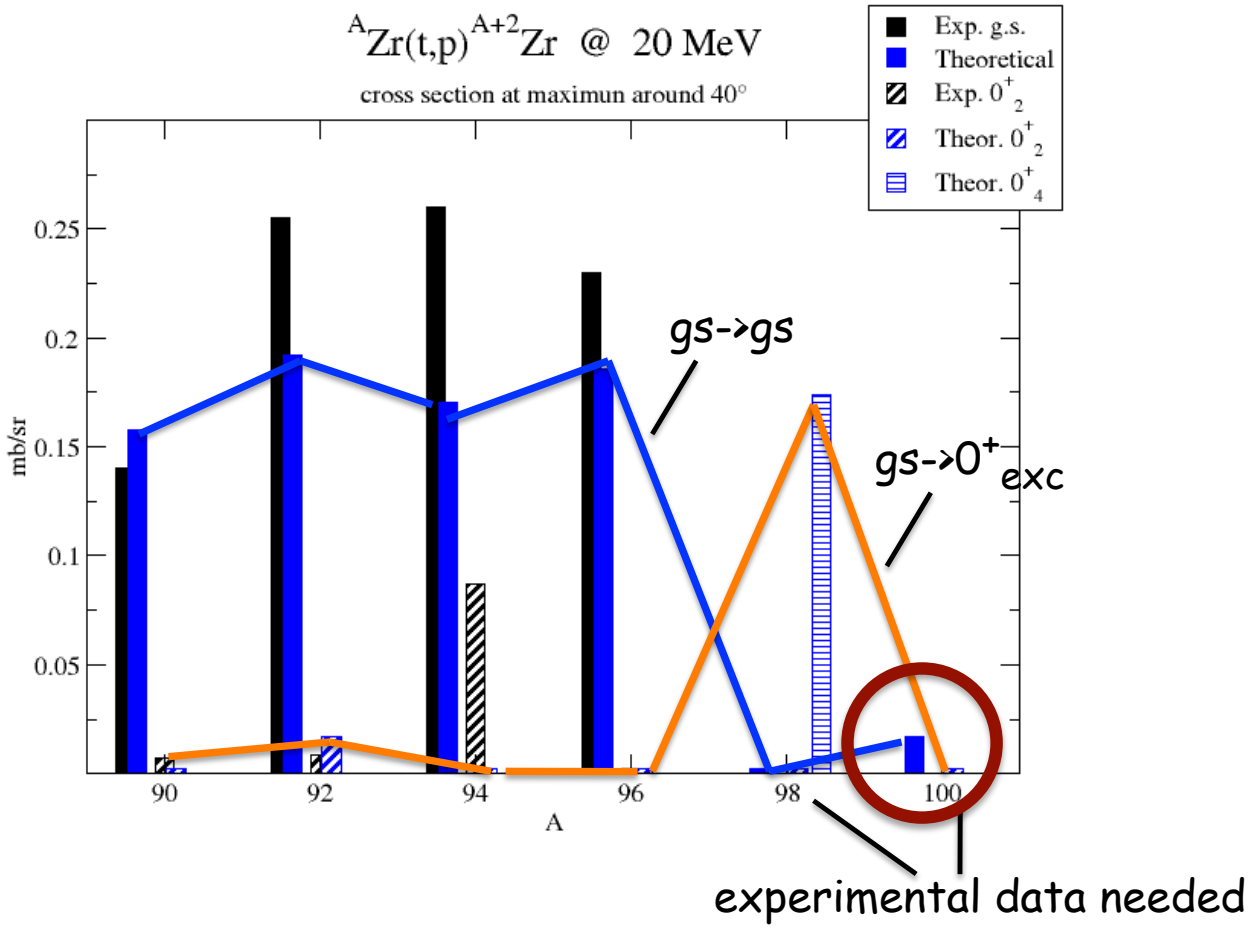


similar shape

Quantum Phase Transition in the Shape of Zr isotopes

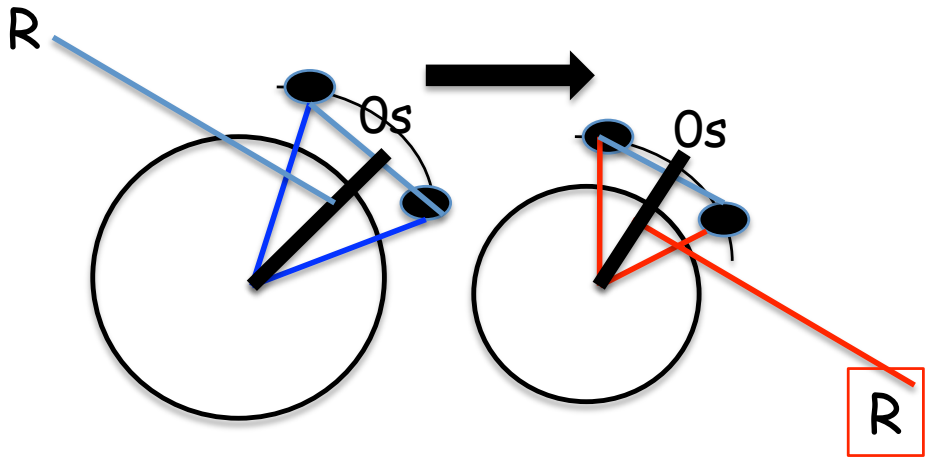
Tomoaki Togashi[1], Yusuke Tsunoda[1], Takaharu Otsuka[1,2,3,4] and Noritaka Shimizu[1]

Calculation of two-particle transfer reactions using:
sequential model for the reaction mechanism
one- and two-particle spectroscopic amplitudes from the Tokyo group



B

Cluster-transfer model (suggested by the close radial correlation of the pairs)



Initial and final cluster wave functions are obtained by taking the overlap between the two-particle wave functions and a 0s wave function for the relative motion

Also in this case the resulting cross section depends on the specific single-particle orbitals (via the Talmi-Moshinsky brackets), but the dependence is different from the one associated with the sequential transfer

The preference to either model may depend on the colliding systems and on kinematical conditions.

The proper approach will depend on the competition between the two colliding single-particle mean fields and the residual two-body interaction (for relatively weak interaction the mean fields will prevail, while in the other extreme of infinite pairing correlation the cluster structure will take over).

Space pair correlation

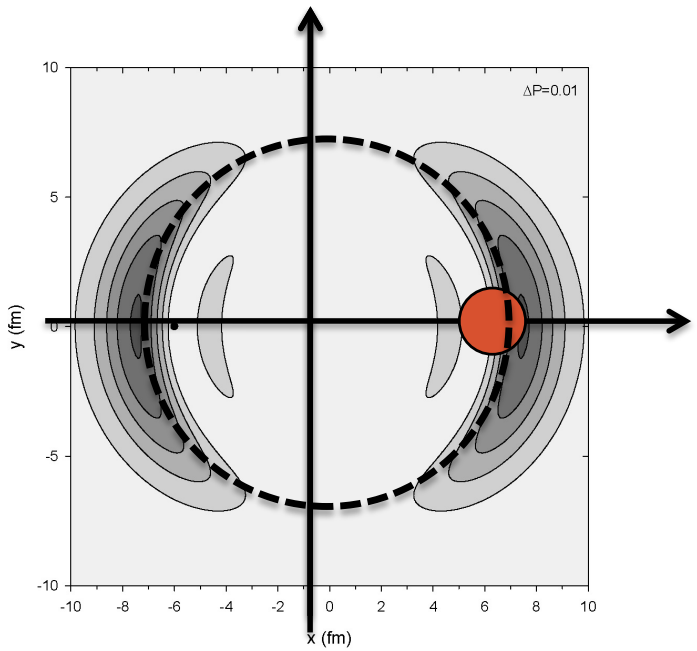
We conclude that the simple behavior of two-particle matrix elements in not the end of the story, because the reaction mechanism in a specific two-particle transfer reactions is sensitive to the radial behavior of the pair wave function, and therefore to the specific form of the single-particle states.

We have therefore to look at possible correlations in space and to the dependence of this correlation on the specific involved orbitals

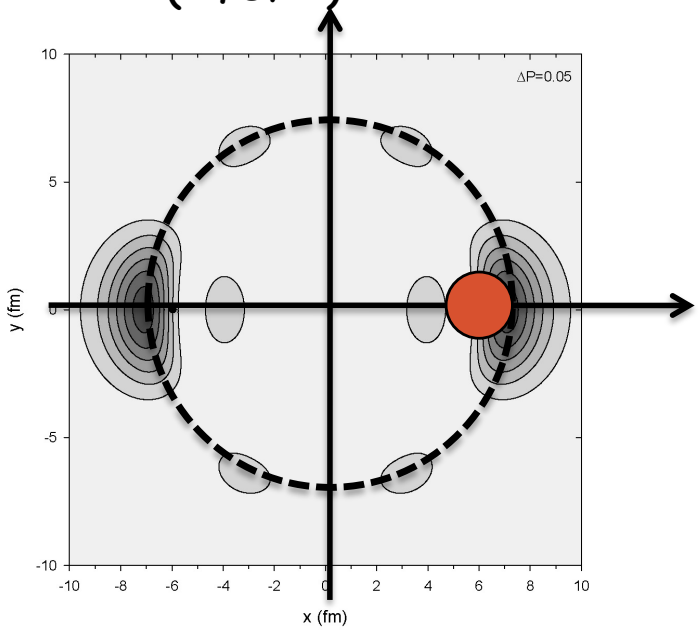
A simple way of displaying the space correlations in the pair: one puts one particle in one point and look at the probability of finding the other one in another point

^{206}Pb

$(3p1/2)^{-2}$

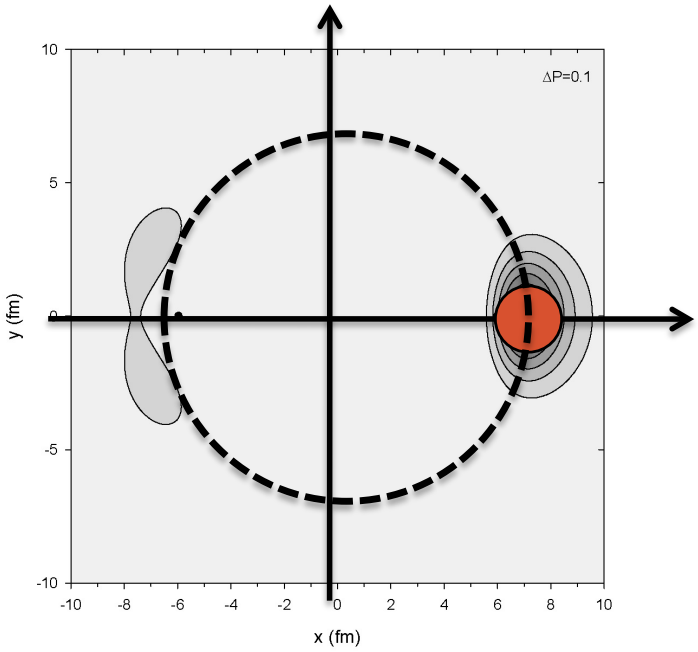


$(2f5/2)^{-2}$



Correlated ground state

$|\Psi(r_1,r_2)|^2$ as a function of r_2, for fixed r_1



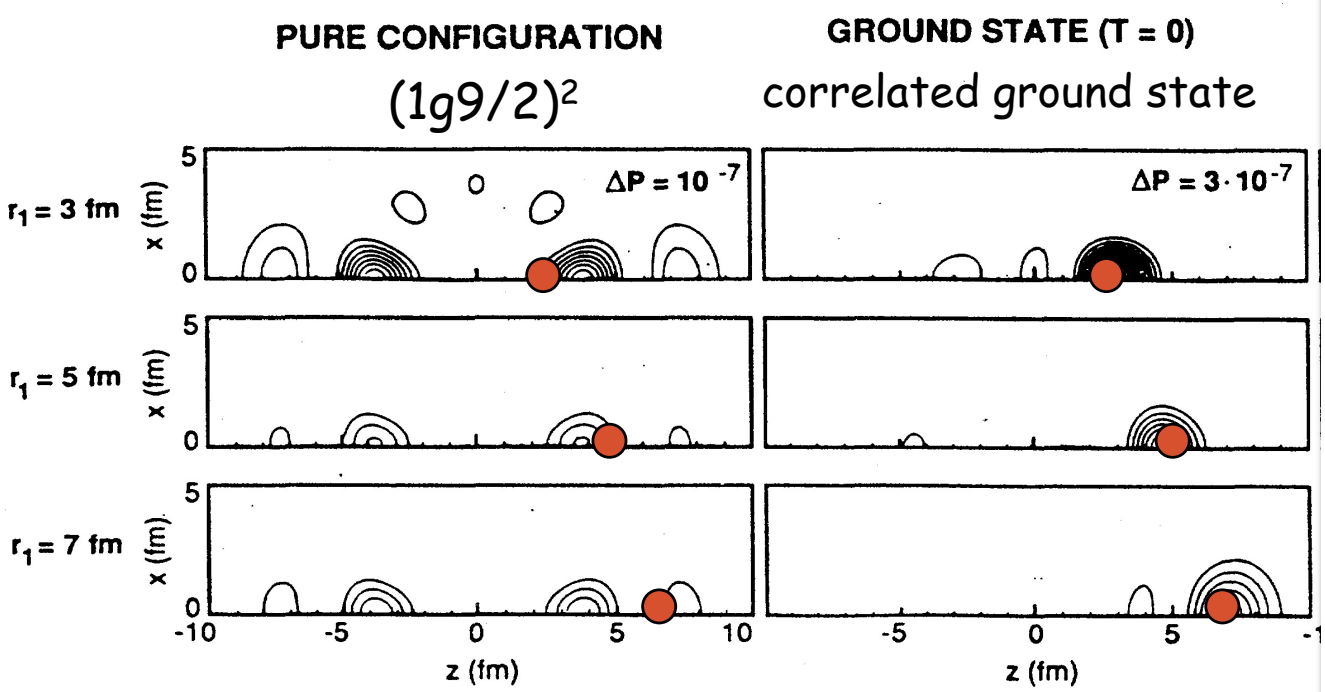
OBS: mixing of configurations with opposite parity

position of particle 1

particle-particle spatial correlations

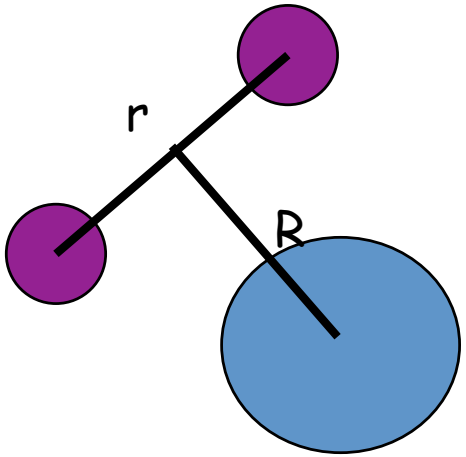
$|\Psi(r_1,r_2)|^2$ as a function of r_2, for fixed r_1

Neutron addition mode: ground state of ^{210}Pb



Work done in collaboration with PierFrancesco Bortignon

position of particle 1



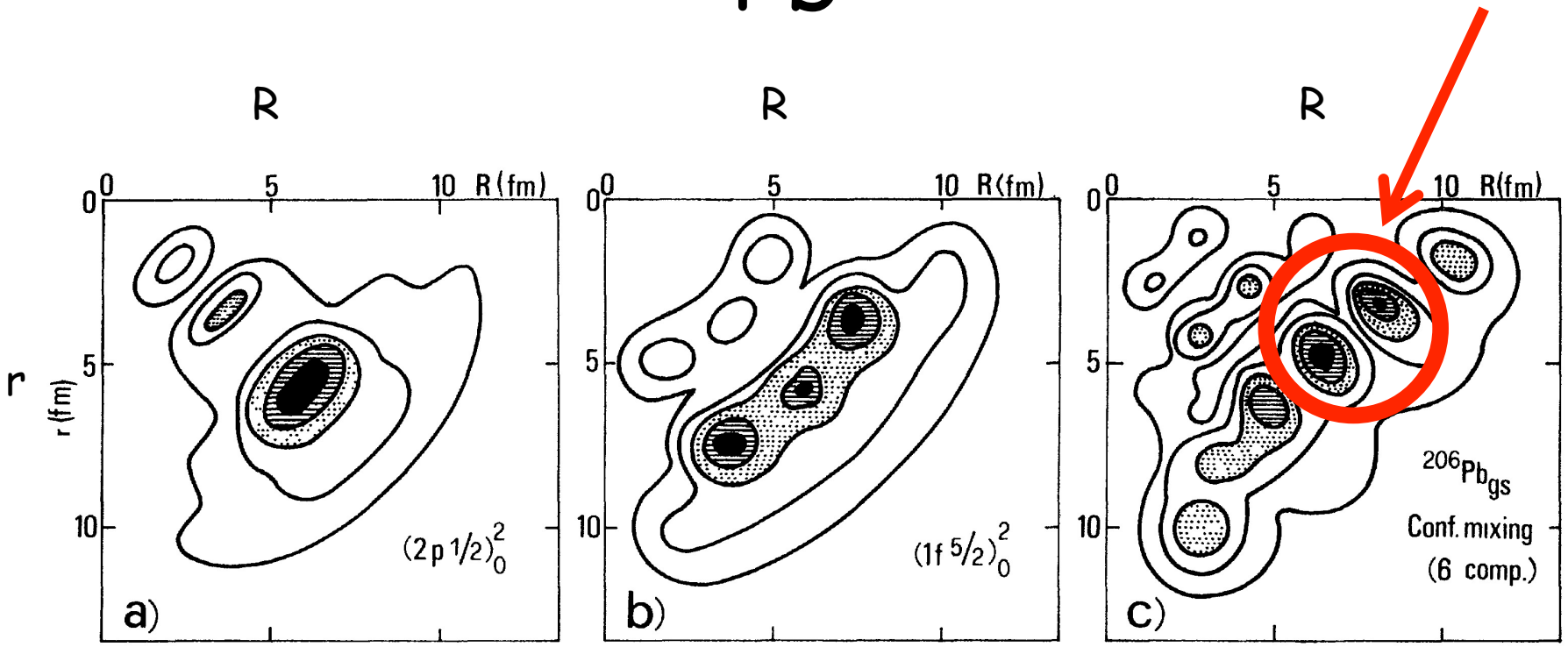
Another representation of the pair transition density in terms of R and r

Catara etal

$\delta\rho_p(R,r)$

^{206}Pb

larger R, smaller r



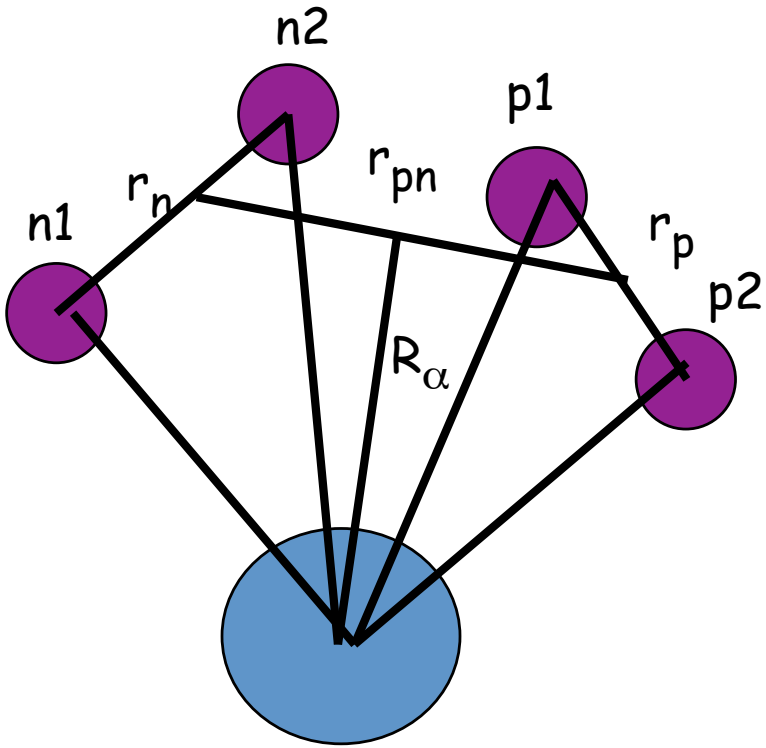
$(3p1/2)^{-2}$

$(2f5/2)^{-2}$

Correlated g.s. (RPA)

A natural extension: effect of pairing on alpha-clustering

Catara, Insolia, Maglione, A.V.

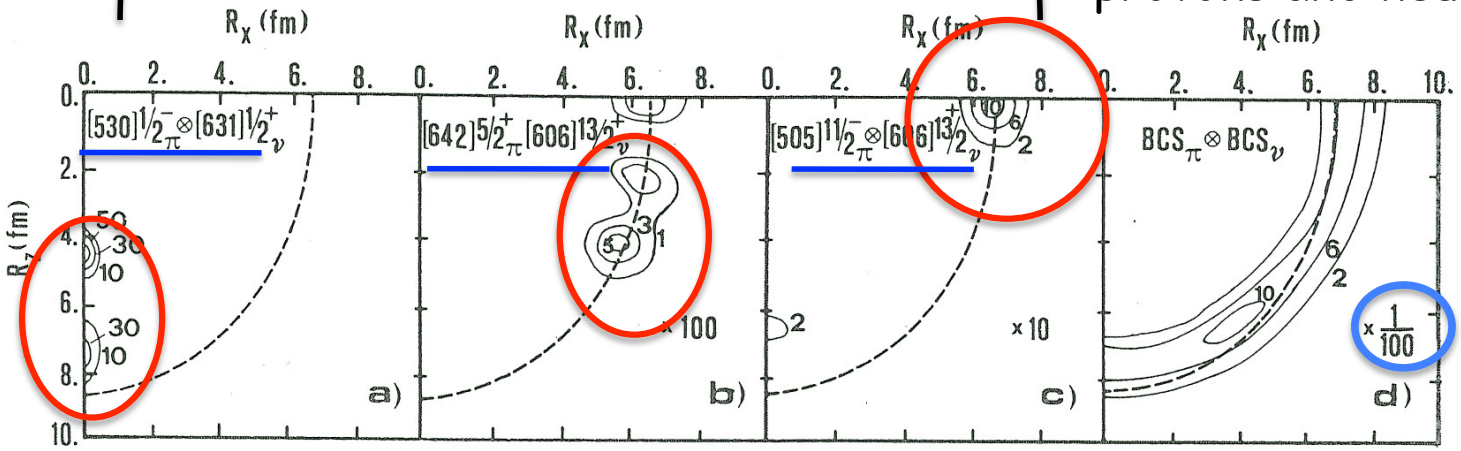


$$\Phi(Ra)=\langle\Phi(r1n)\Phi(r2n)\Phi(r1p)\Phi(r2p)\,|\,\Phi_{00}(rn)\Phi_{00}(rp)\Phi_{00}(rpn)\rangle$$

Alpha-cluster probabilities in ^{232}Th, displayed in the intrinsic frame by projecting over an alpha particle wave function

4 particles in pure (time-reversed) Nilsson orbits

Nilsson+BCS for both protons and neutrons

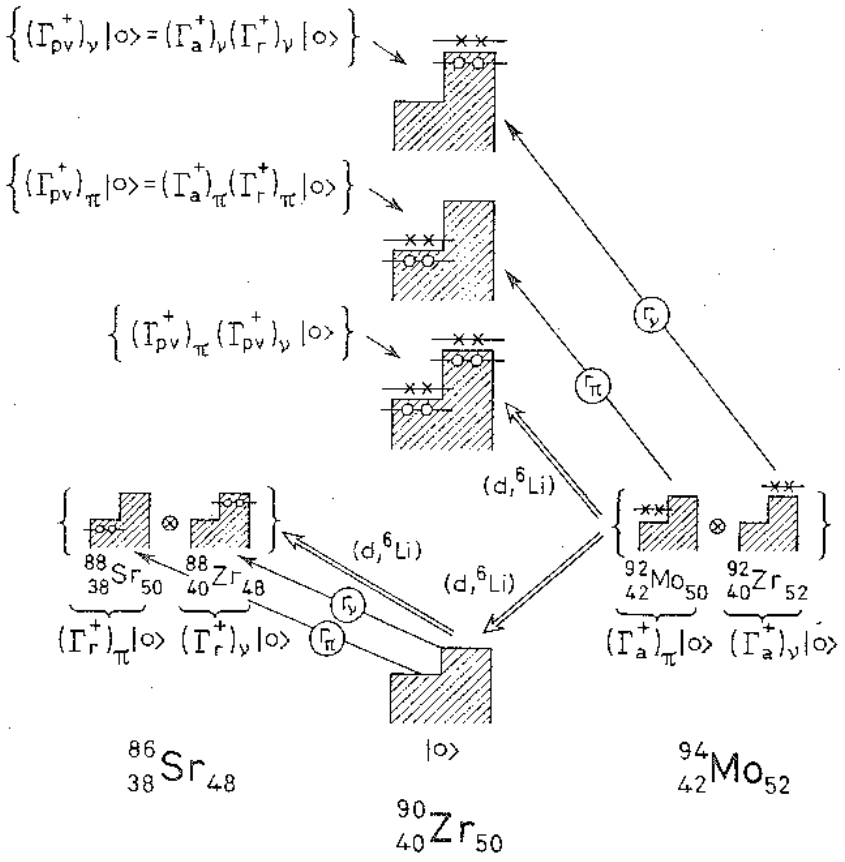


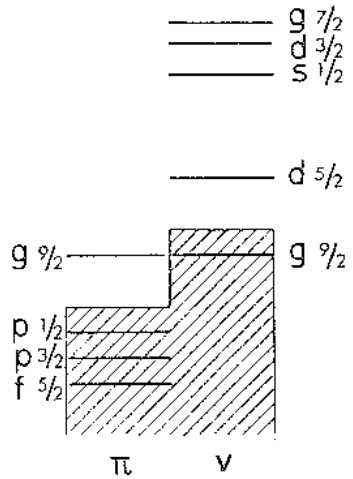
OBS: 1. Alpha-probability distributed over the entire surface

2. Total alpha spectroscopic factor Sα increases orders of magnitude (although still a factor 10 smaller than experiment)

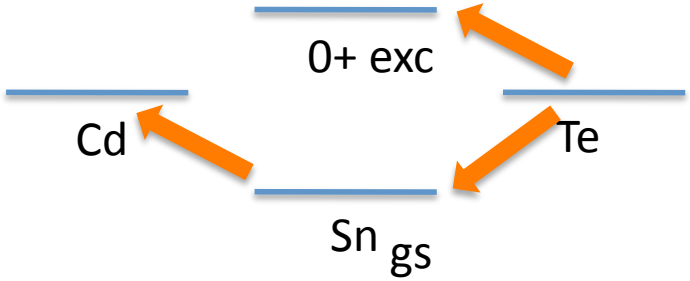
Need for neutron-proton interaction

Alpha clusters transfer in terms of transfer of correlated nn and pp pairs (and np pairs)





Isotope dependence of
Te(d,^{6}Li)Sn(gs)
Te(d,^{6}Li)Sn(0+ exc)
Sn(d,^{6}Li)Cd(gs)



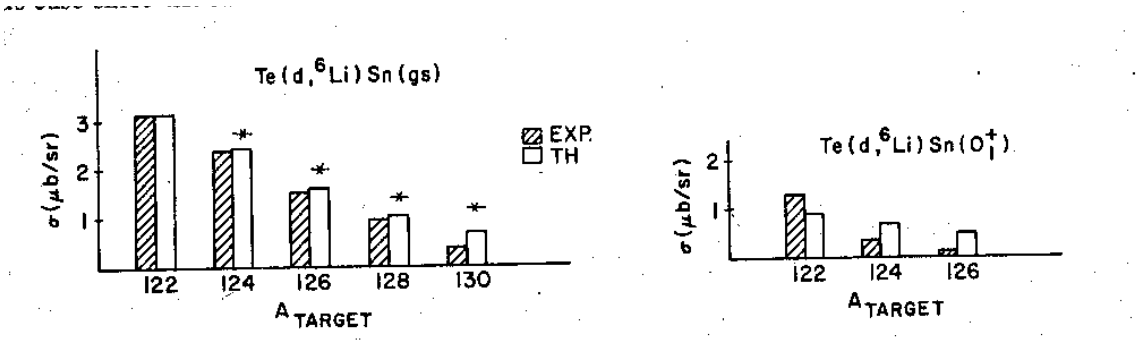
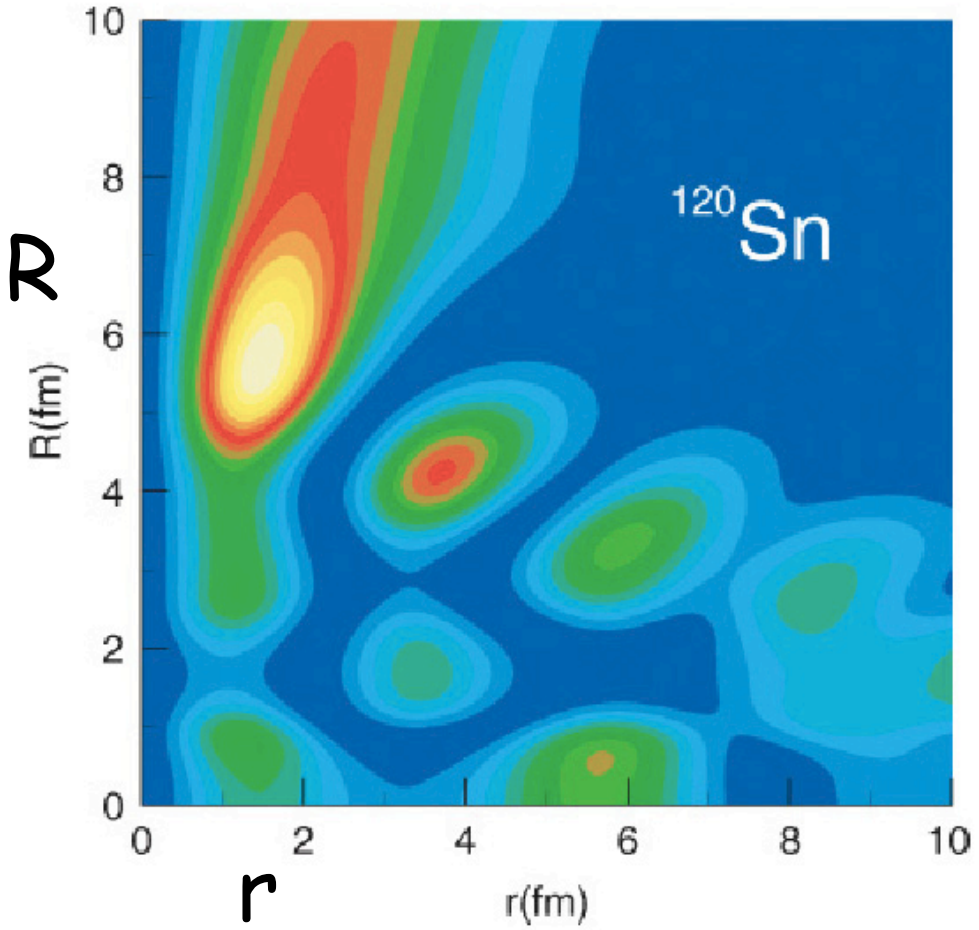


Table 2

		$B(j^2(0),A)$		
(ℓj)	$\sqrt{\sigma(j^2(0))}$	116	118	120
$s_{1/2}$	1.3	0.5	0.4	0.3
$d_{3/2}$	0.5	0.8	0.7	0.6
$h_{11/2}$	0.1	0.9	1.1	1.3
$^{A+4}$Sn(d,^{6}Li)^{A}Cd	Theory	1.0	0.7	0.5
	Exp.	1.0	0.6	0.4

Proton and neutron parts factorized

Neutrons: BCS
Protons: Cd hole pair (2h)
Sn gs closed shell Z=50
Te particle pair (2n)
Sn 0+ exc 2p-2h



$\delta\rho_p(R,r)$

HFB Cooper pairs

Pillet, Sandulescu, Schuck

A simple model to investigate the relative role of of the mean-field and the residual interaction

$$i\hbar\frac{d}{dt}\Psi(x_1,x_2,t) = \mathcal{H}(x_1,x_2,t)\Psi(x_1,x_2,t)$$

Two-neutron transfer

to test pairing interaction

Two-body wavefunction

Hamiltonian:

moving well

$$\mathcal{H}(x_1,x_2,t) = KIN + V_T(x_1,x_2) + V_P(x_1,x_2,t) + V_{int}(x_1,x_2)$$

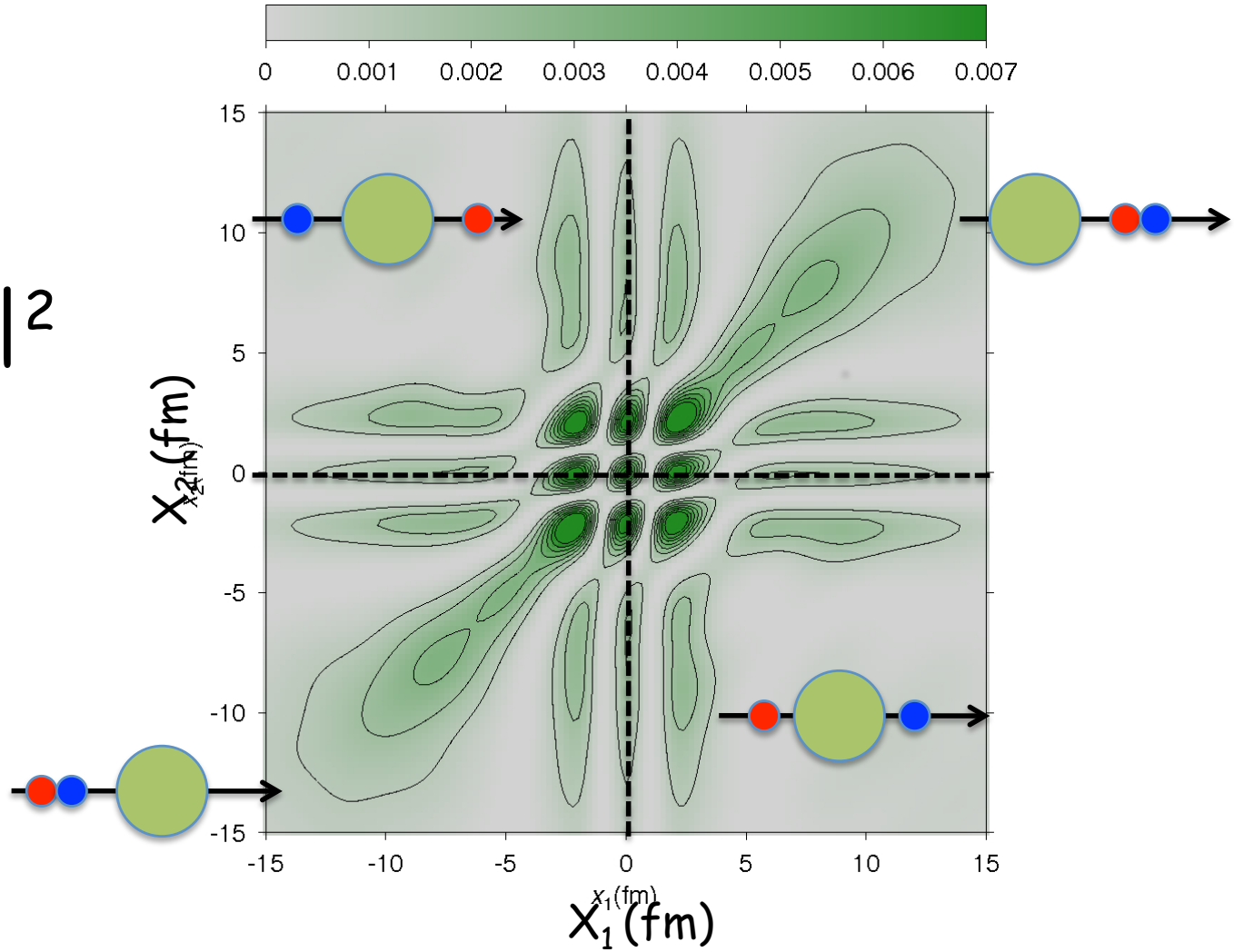
Density dependent residual interaction

$$V_{int}(x_1,x_2) = -V\left[\frac{\rho[(x_1+x_2)/2]}{\rho_0}\right]\delta(x_1-x_2)$$

(acting only when the two particles are both inside the same well)

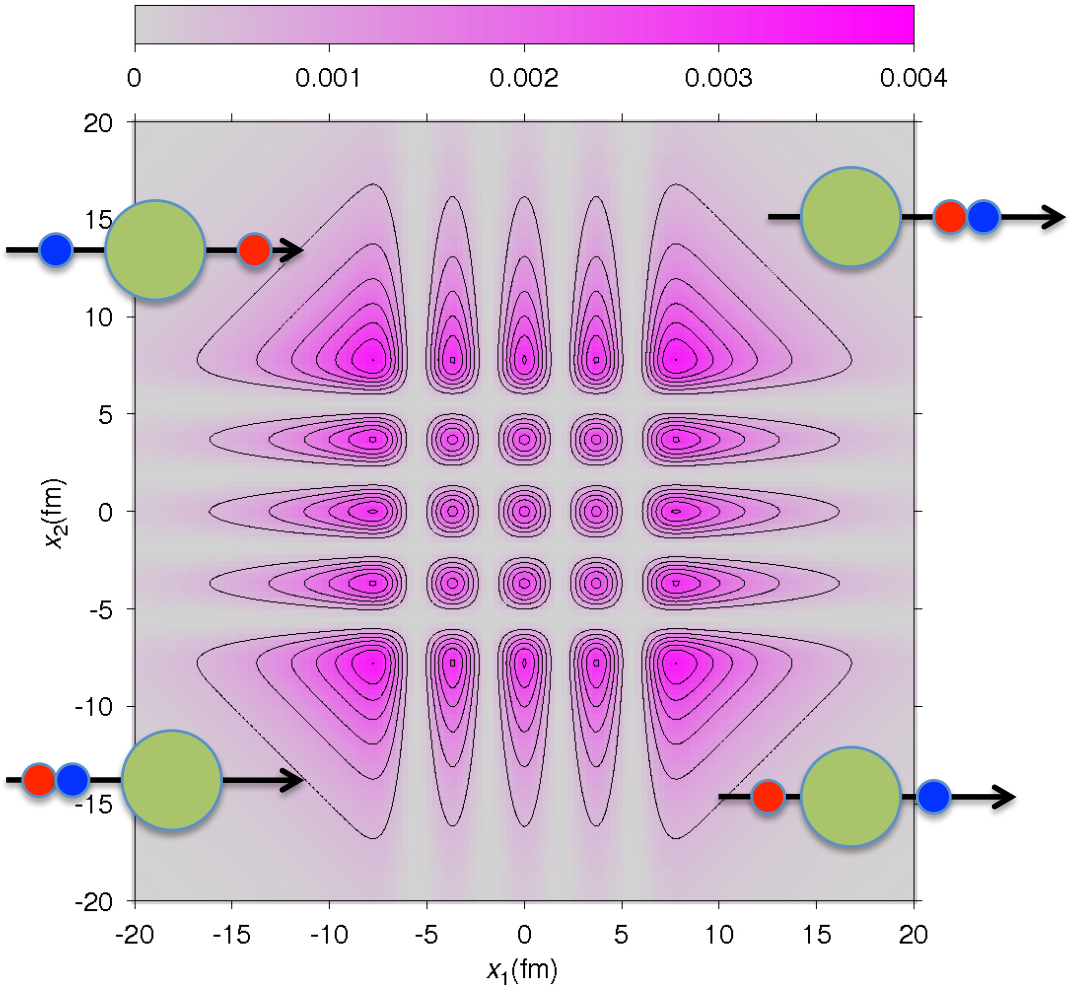
Two-particle correlated wave function: correlation clearly favors the situation with the two particles on the same side

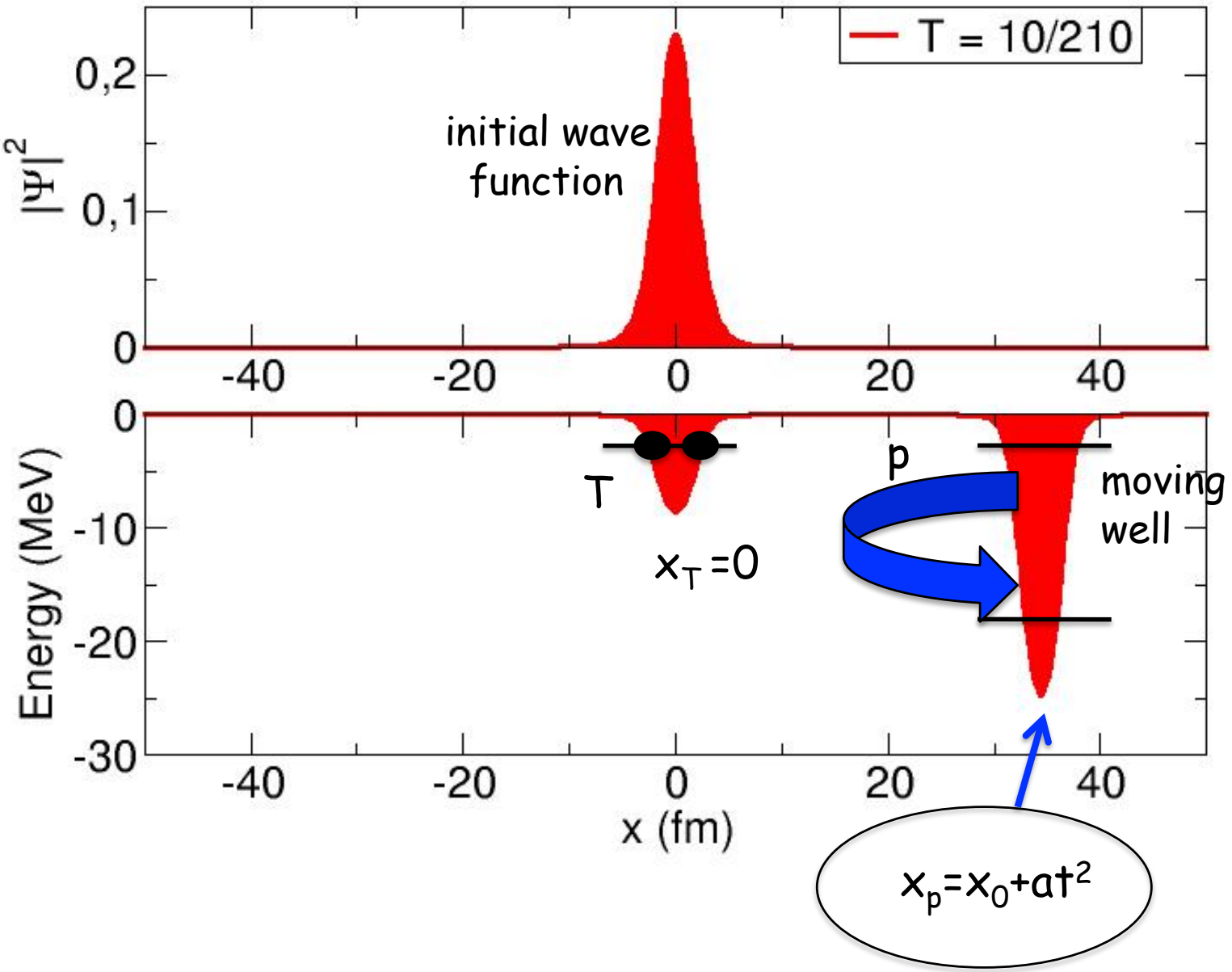
$|\Phi(x_1,x_2)|^2$



For comparison the situation with uncorrelated wave function

$|\Phi(x_1,x_2)|^2$




T = 10/210
initial wave function
$|\Psi|^2$
0,2
0,1
0
-40
-20
0
20
40
Energy (MeV)
-10
-20
-30
T
$x_T = 0$
p
moving well
x (fm)
$x_p = x_0 + at^2$

We can now follow the evolution in time of the two-particle wave function in the (x_1,x_2) plane, due to the action of the moving well

At the end of the process, from the final two-particle wave function, we can separate different final states:

1. elastic/inelastic (both particles still in the initial well)
2. one-particle transfer (one particle in the initial well and one in the moving one)
3. one-particle break-up (in particle in the continuum outside the wells and one in the initial or final well)
4. two-particle transfer (both particles in the moving well)
5. two-particle break-up (both particles outside the wells)

We first consider the case of uncorrelated systems
(no residual interaction).

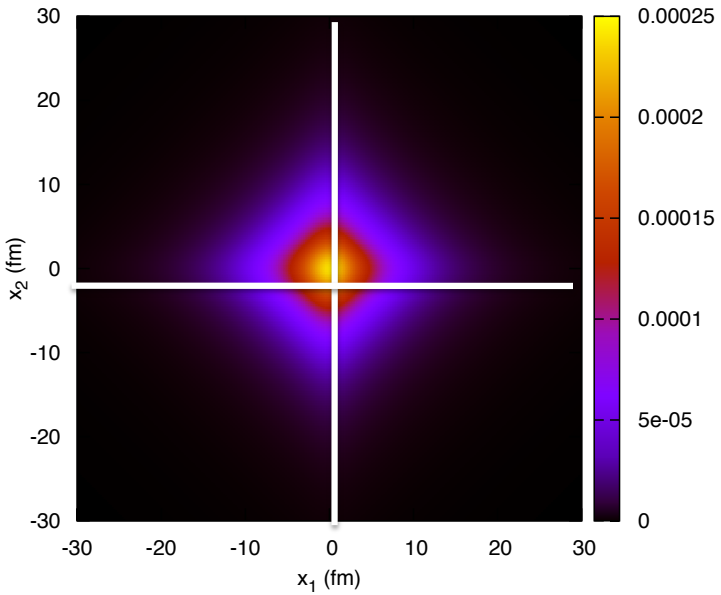
In this case the transfer process is induced by the mean-field of the moving well, and in terms of reaction mechanism the two-particle transfer can only be interpreted as obtained by the successive transfer of single particles.

In such a situation, in a perturbative approach, we expect a pair transfer probability

$$P_2 \sim (P_1)^2$$

Let us see what comes from the "exact" solution

Initial wave function

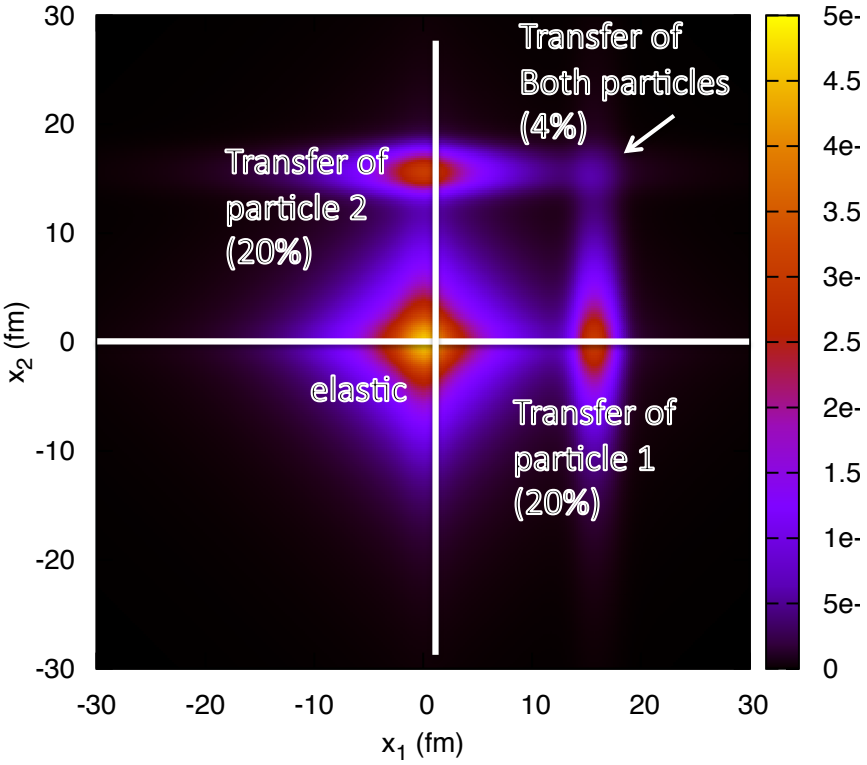


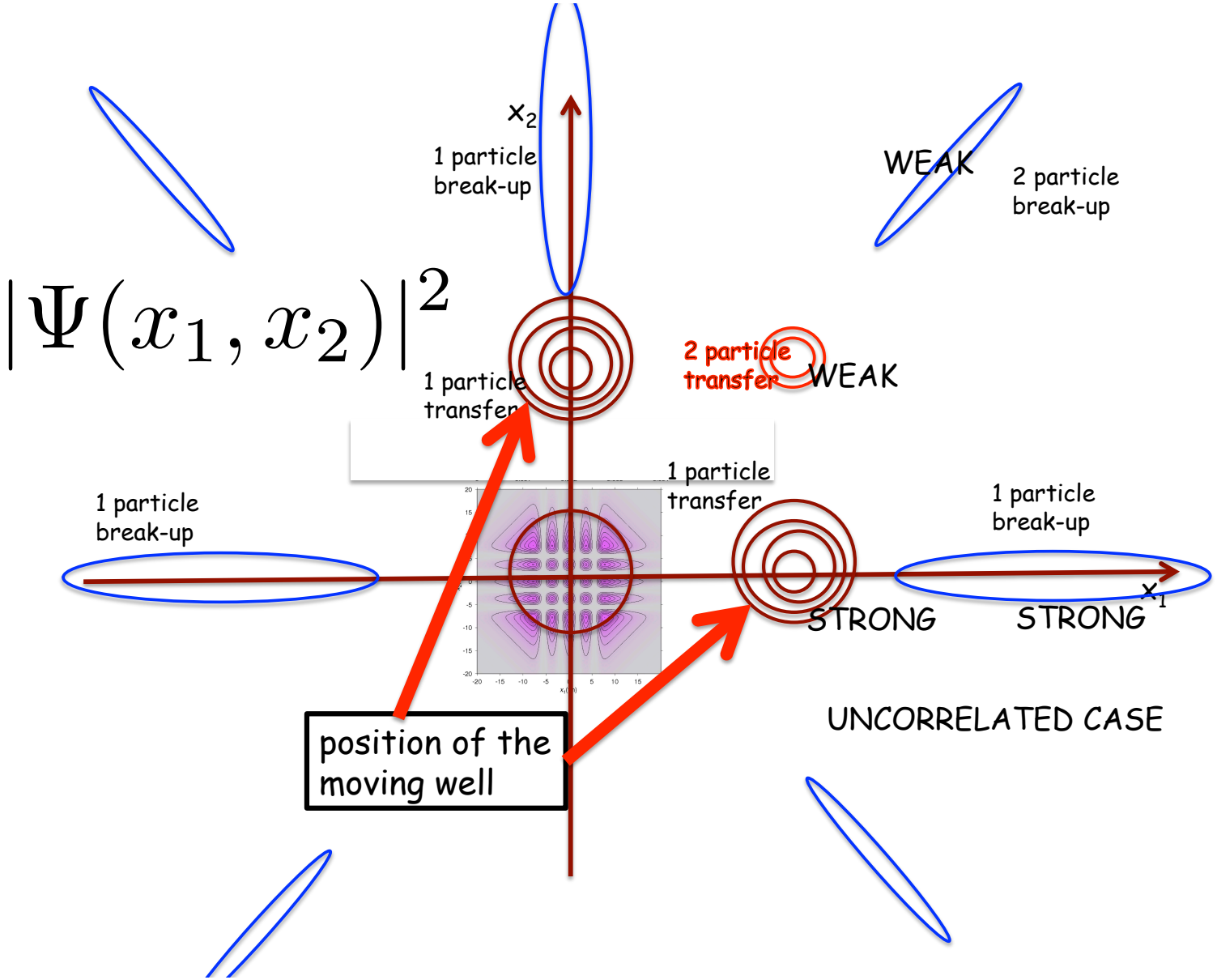
Perturbative estimate:

$P_2 = 0.2 \times 0.2 = 0.04$

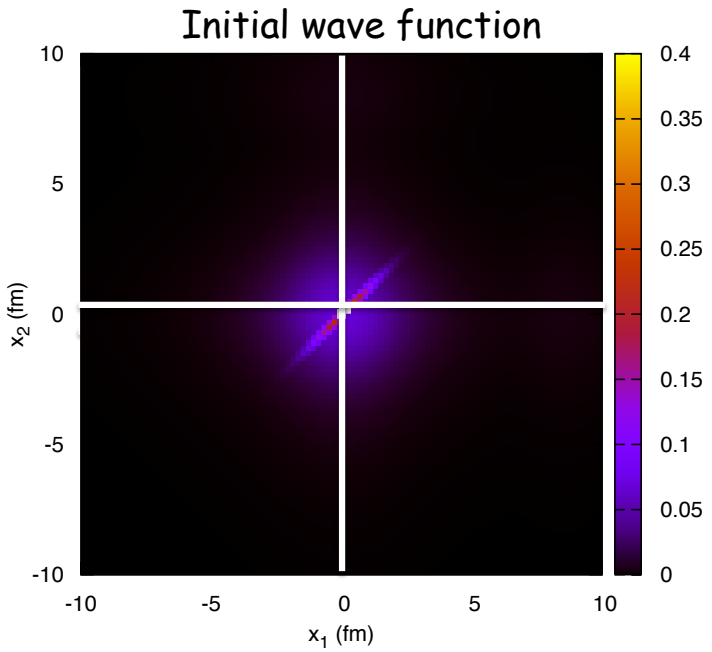
OK

Final wave function




$|\Psi(x_1, x_2)|^2$
x_2
1 particle break-up
WEAK
2 particle break-up
1 particle transfer
2 particle transfer
WEAK
1 particle transfer
1 particle break-up
1 particle break-up
x_1
STRONG
STRONG
position of the moving well
UNCORRELATED CASE

Let us see now the case of the "correlated" pair, due to the action of the residual pairing interaction (density dependent, therefore only acting when both particles are within a well, and not when the particles are "in the air").

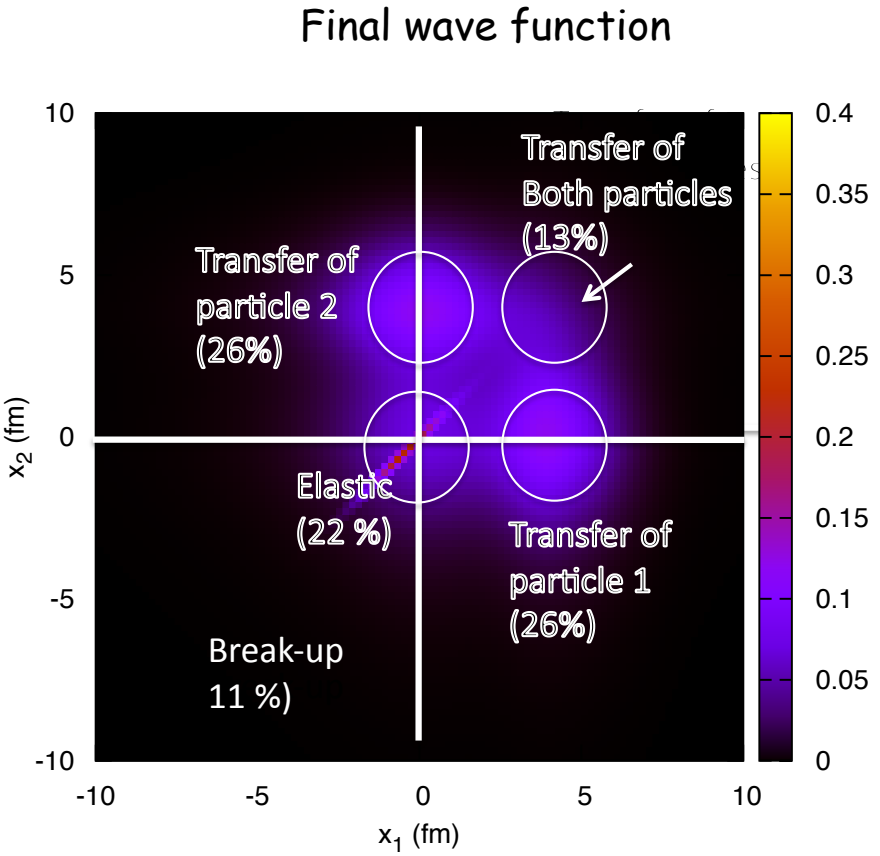


Perturbative uncorrelated estimate:

P_2 (uncorr) = 0.26 x 0.26 = 0.07

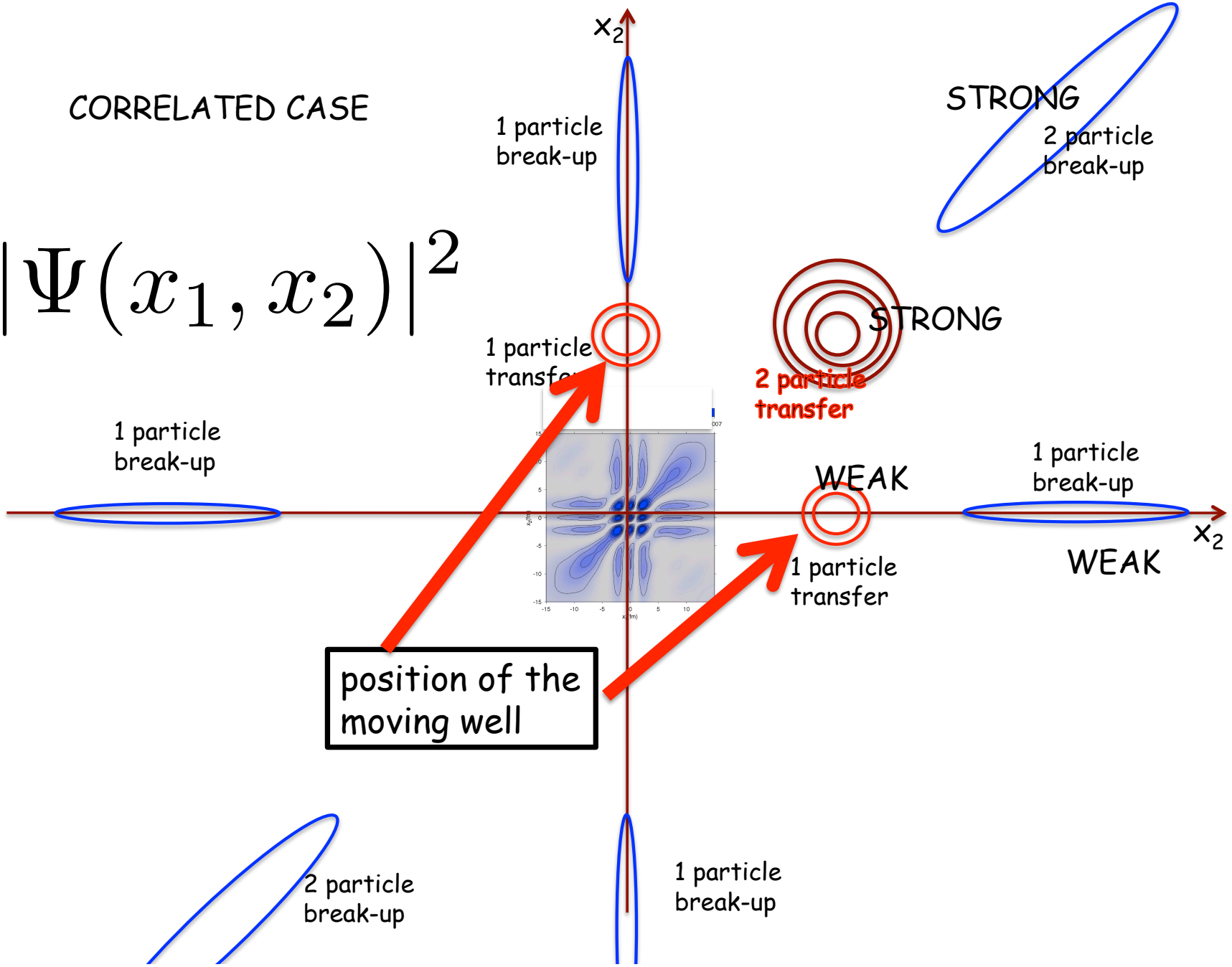
P_2 (corr) = 0.13 = 2 P_2 (uncorr)

Pairing enhancement factor: 2



CORRELATED CASE

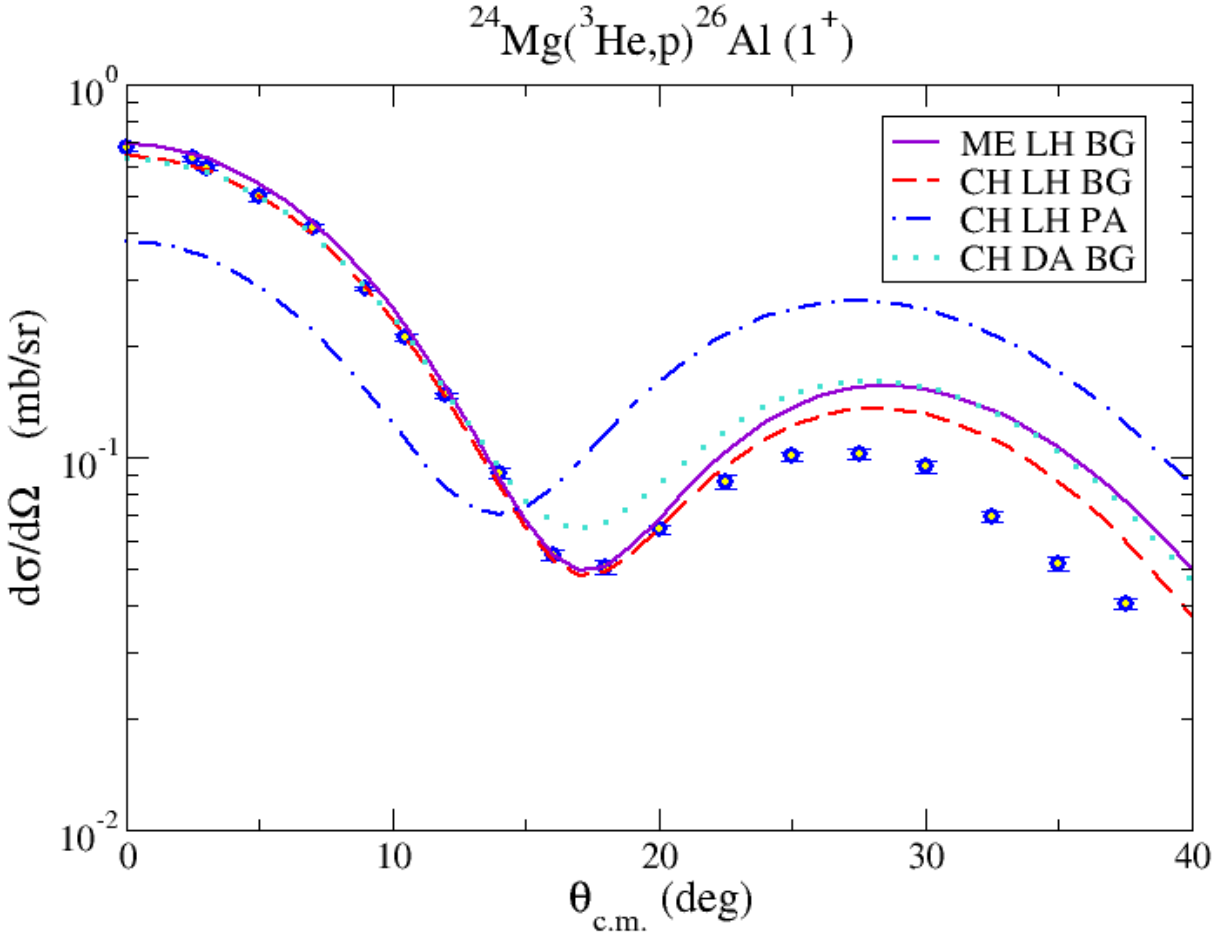
$$|\Psi(x_1, x_2)|^2$$

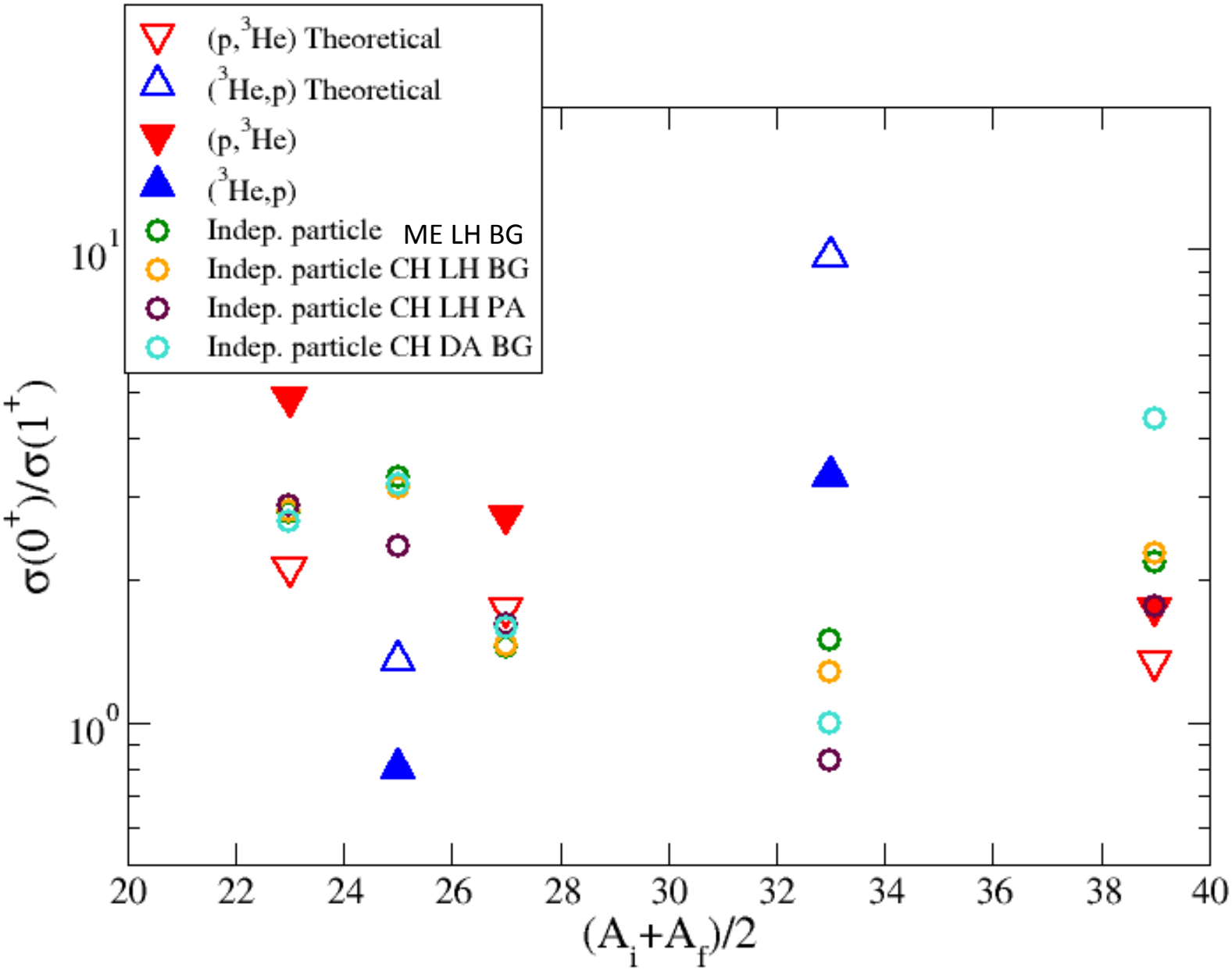


Additional points that make not trivial the correspondence between the pair-transfer cross section and the pairing matrix elements:
optical potentials
Q-value effects
continuum effects

For the optical potentials, to show the sensitivity of the results on their choices, different options are used for the three relevant channels (proton, deuteron, 3He)

Legenda:
ME: Menet forprotons,
CH: Chapel Hill 89 for protons,
LH: Lohr-Haeberli for deuteron,
DA: Daehnick for deuteron,
BG: Bechetti-Greenless for 3He,
PA: Pang for 3He.


^{24}Mg(^{3}He,p)^{26}Al (1^{+})
ME LH BG
CH LH BG
CH LH PA
CH DA BG
dσ/dΩ (mb/sr)
θ$_{c.m.}$ (deg)
10^{0}
10^{-1}
10^{-2}
0
10
20
30
40


(p,3He) Theoretical
(3He,p) Theoretical
(p,3He)
(3He,p)
Indep. particle
Indep. particle CH LH BG
Indep. particle CH LH PA
Indep. particle CH DA BG
ME LH BG
σ(0+)/σ(1+)
(Ai+Af)/2
10^1
10^0
20
22
24
26
28
30
32
34
36
38
40

Q-value effect

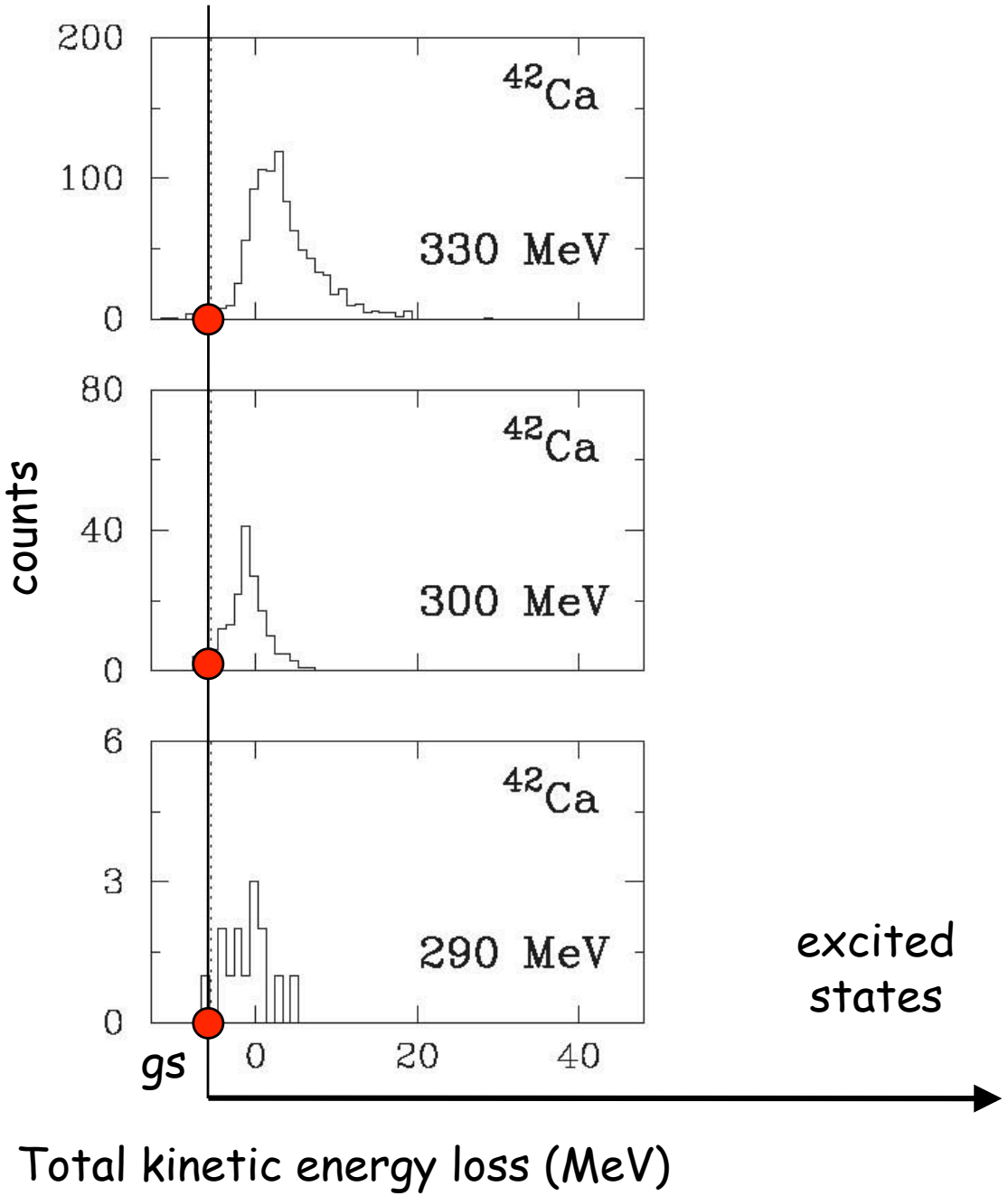
Keeping fixed any other parameter, the probability for populating a definite final channel depends on the Q-value of the reaction. The dependence (in first approximation a gaussian distribution centered in the optimum Q-value) is very strong in the case of heavy-ion induced reactions, weaker in the case of light ions.

The optimum Q-value depends on the angular momentum transfer and on the charge of the transferred particles. In the specific case of L=0 two-neutron transfer, the optimal Q-value is zero.

Experimental evidence

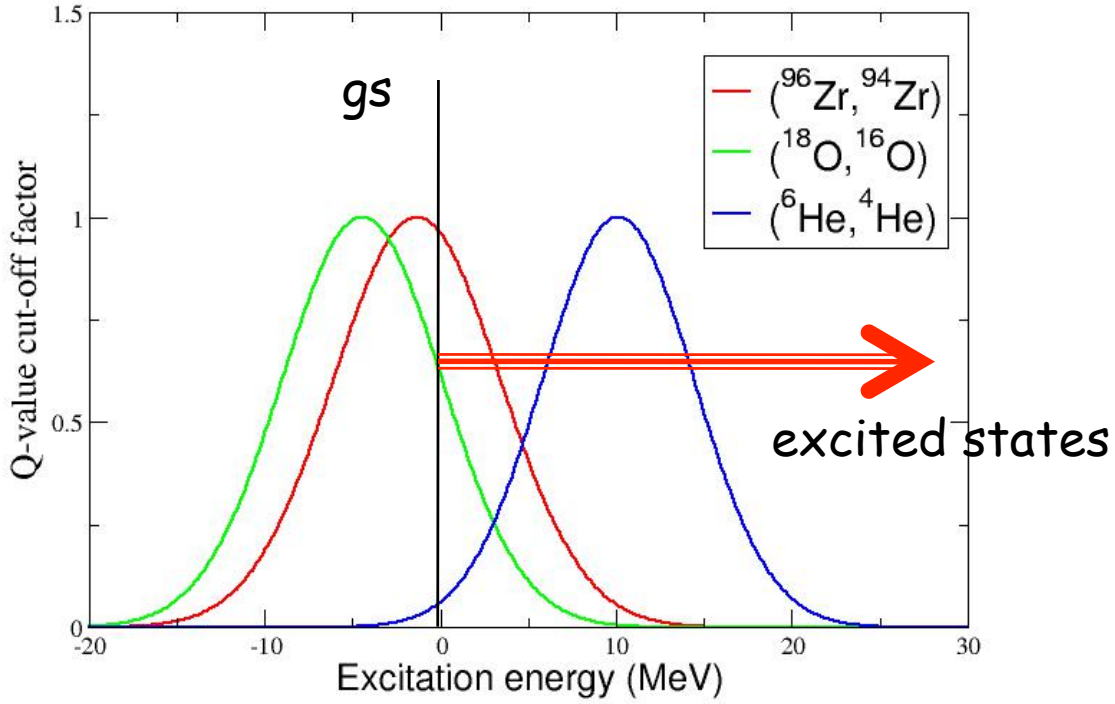
$^{96}Zr+^{40}Ca$

Selecting final
^{42}Ca mass partition

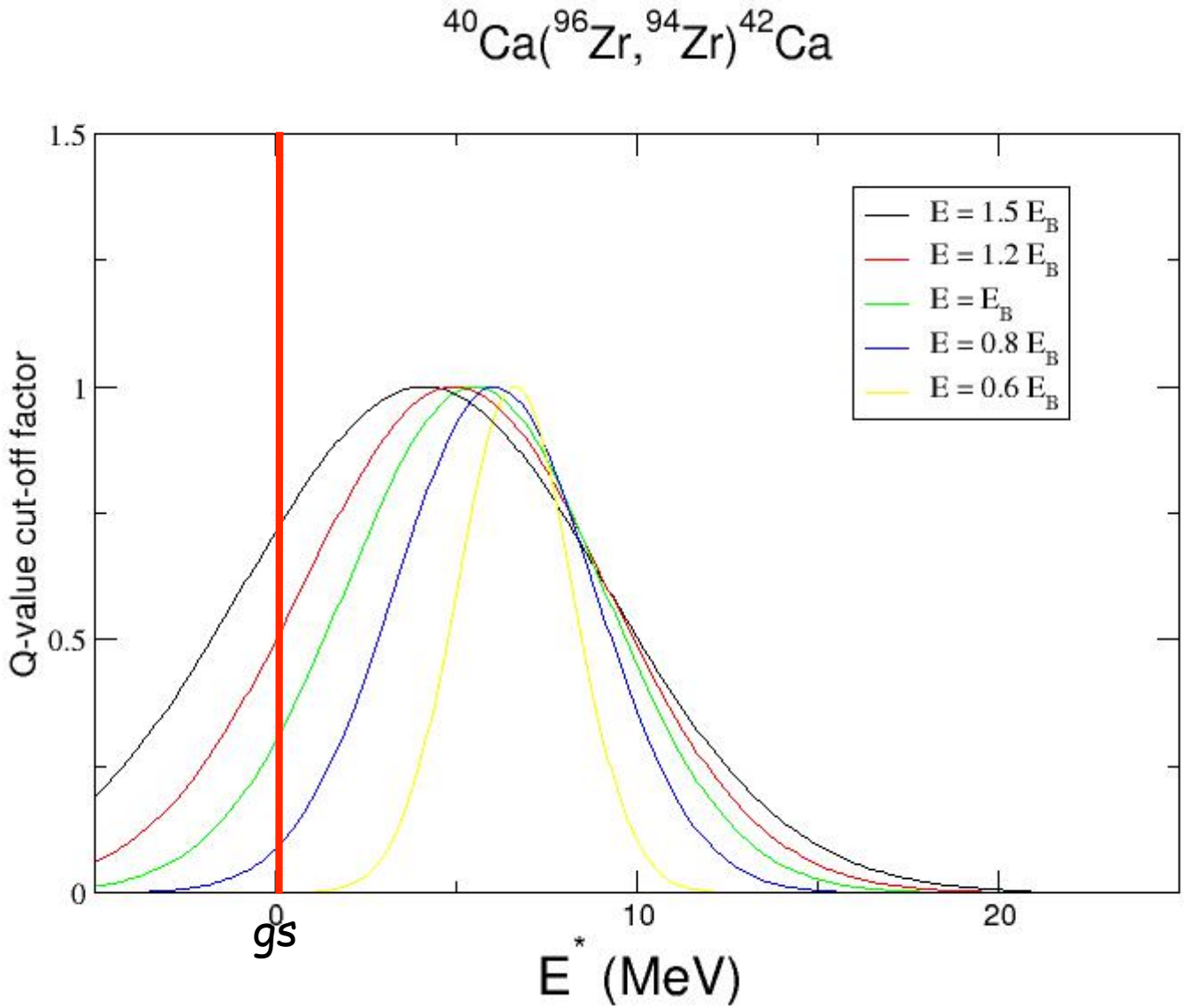


Playing with different combinations of projectile/target (having different Q_{gg}-value) one can favour different energy windows

Example: Target ^{208}Pb Final ^{210}Pb (at bombarding energy $E_{cm} = 1.2\ E_{barrier}$)

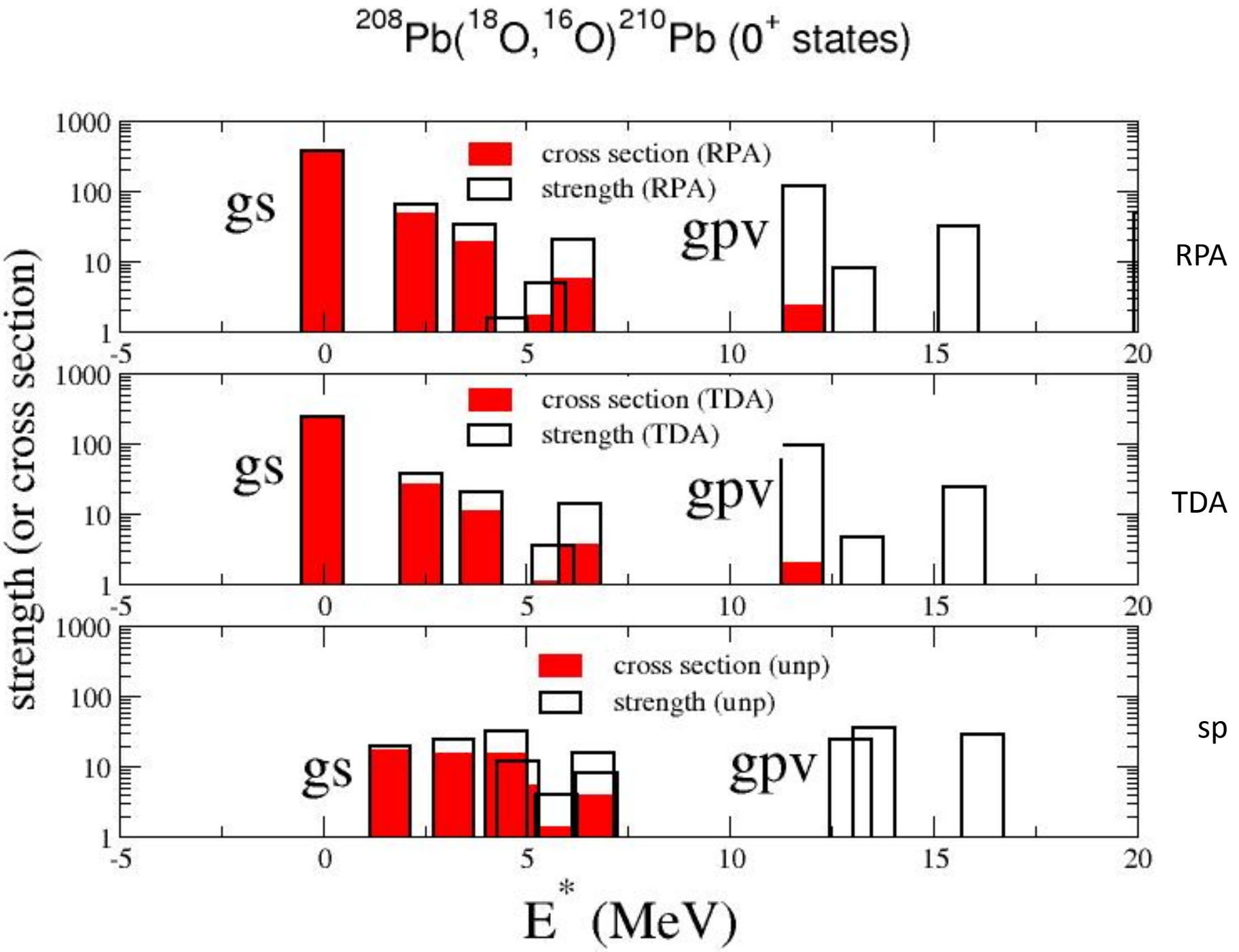


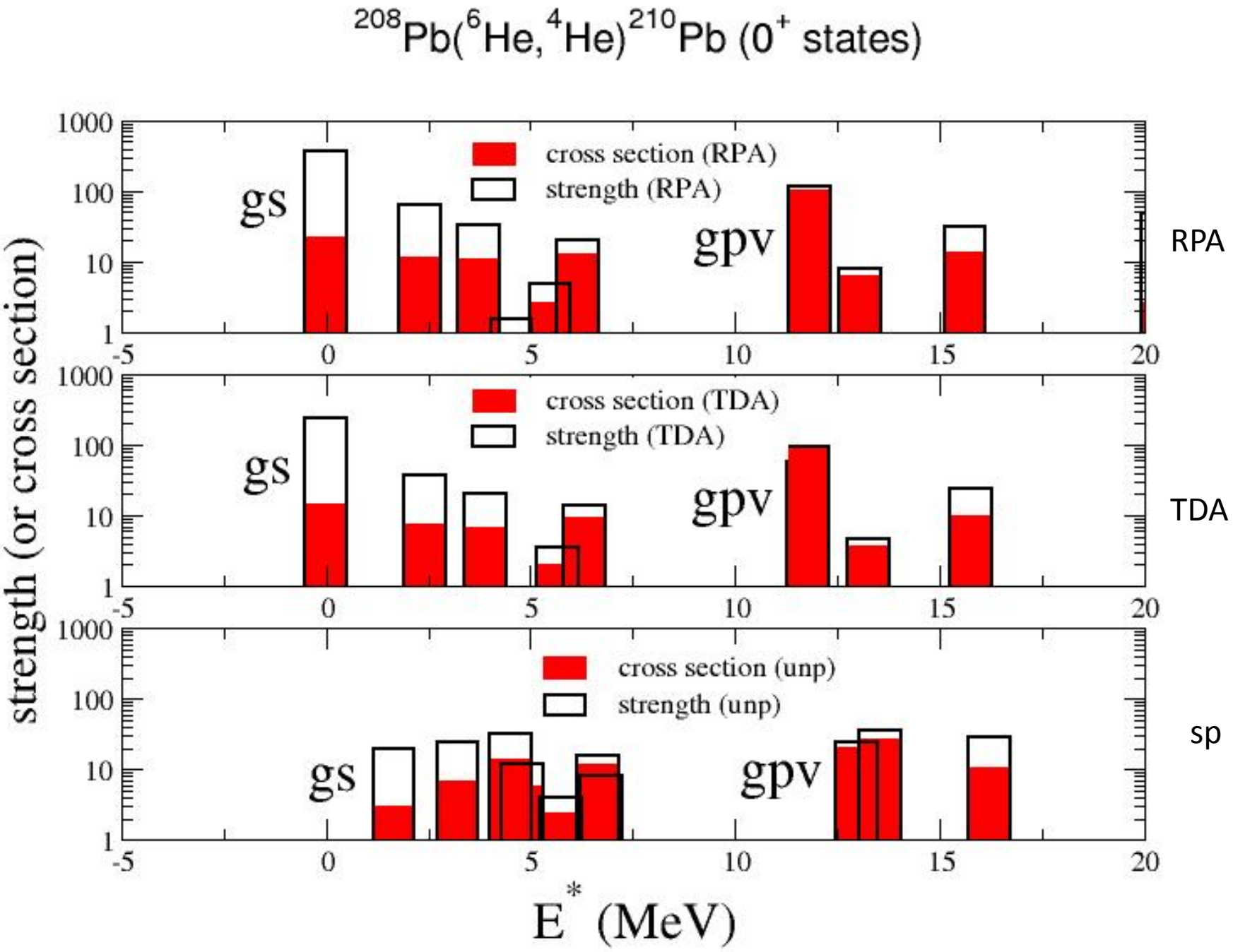
The width of the Q-value window increases
with the bombarding energy



The pairing strength is therefore modulated by the Q-value cut-off to yield the final two-particle cross section. And the modulation depends on the specific reaction.

And the pairing interaction, by modifying via the correlation energy the energies and the Q-value can in fact produce a reduction of the pair transfer cross section


^{208}Pb(^{18}O,^{16}O)^{210}Pb (0^{+} states)
strength (or cross section)
cross section (RPA)
strength (RPA)
gs
gpv
RPA
cross section (TDA)
strength (TDA)
gs
gpv
TDA
cross section (unp)
strength (unp)
gs
gpv
sp
1000
100
10
1
-5
0
5
10
15
20
E* (MeV)

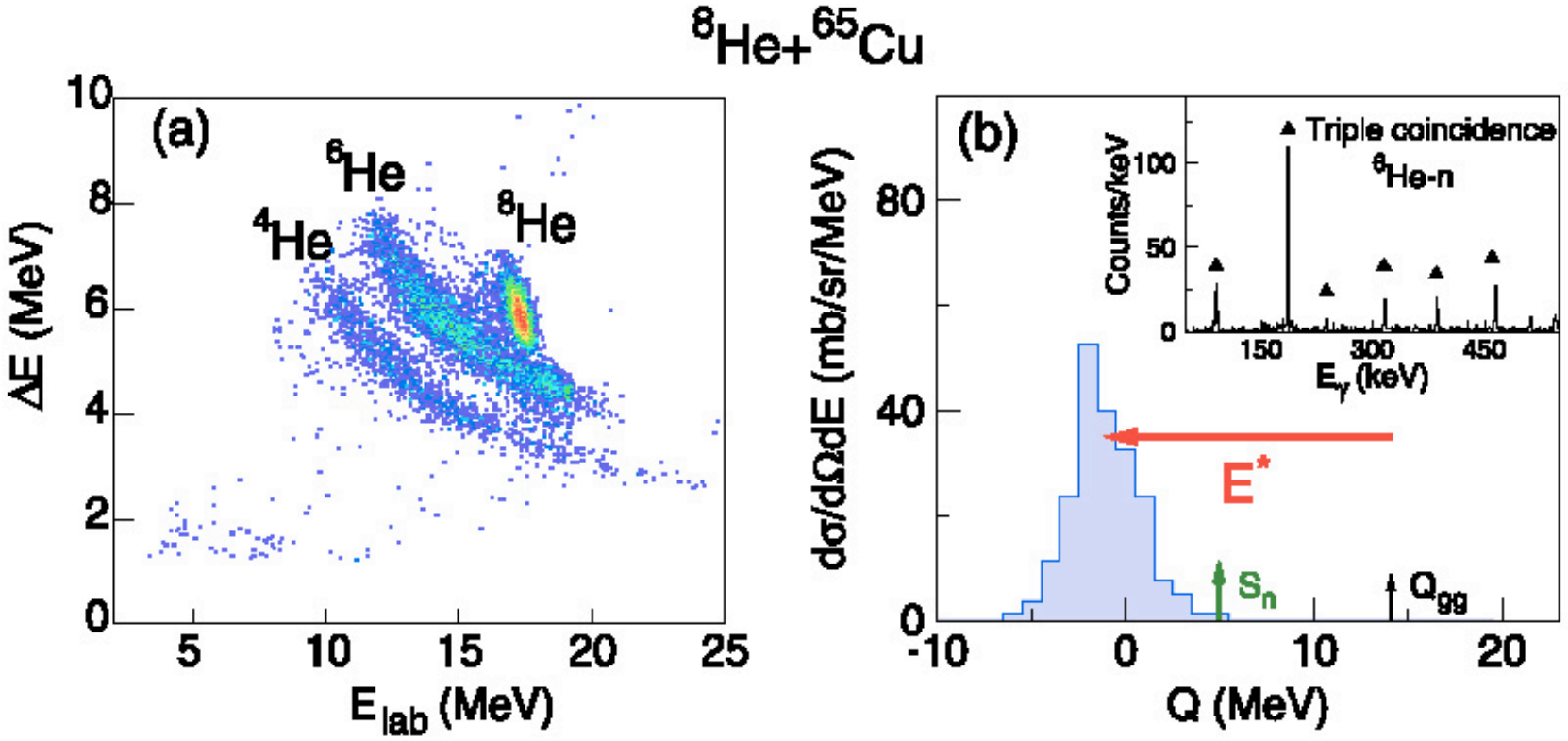

^{208}Pb(^{6}He,^{4}He)^{210}Pb (0^{+} states)
strength (or cross section)
cross section (RPA)
strength (RPA)
gs
gpv
RPA
cross section (TDA)
strength (TDA)
gs
gpv
TDA
cross section (unp)
strength (unp)
gs
gpv
sp
1000
100
10
1
-5
0
5
10
15
20
E* (MeV)

Basic problem:

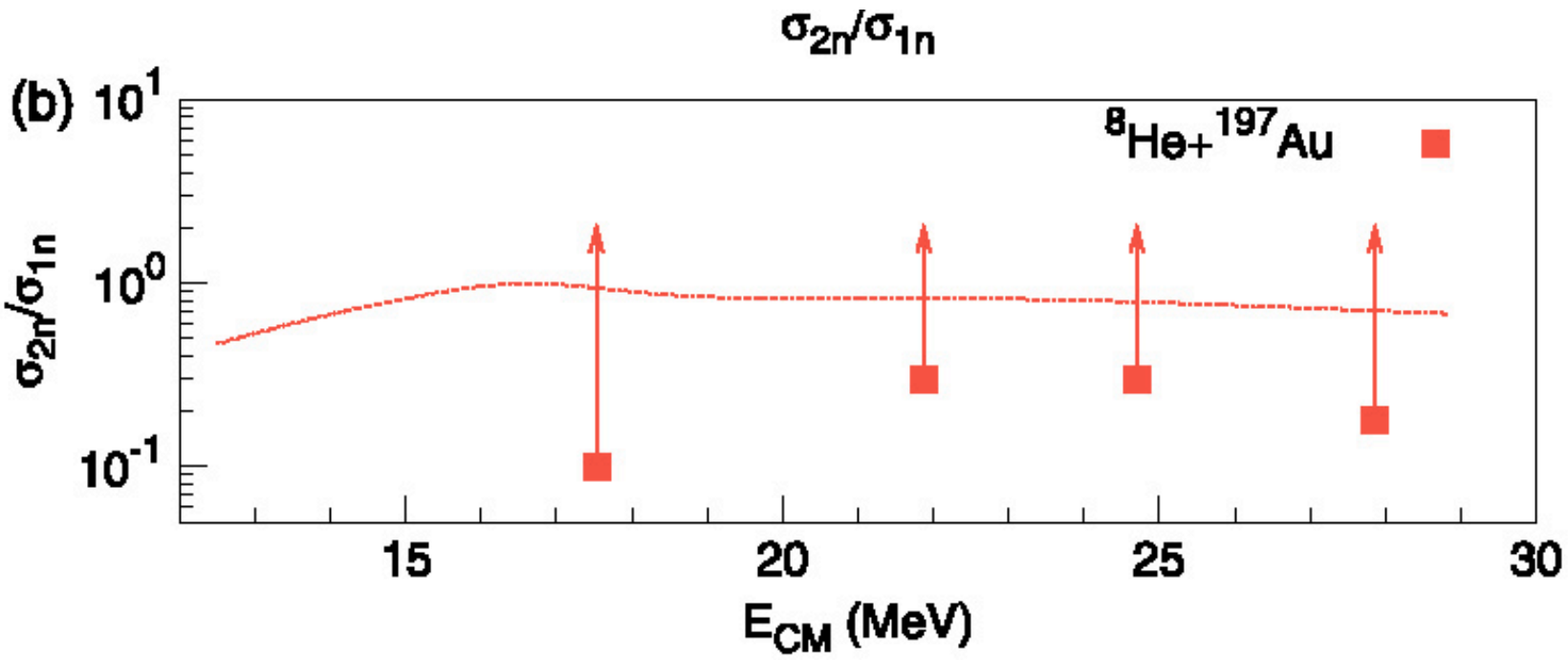
how is changed the picture as we move closer or even beyond the drip lines?

What is the effect of continuum states in the wave functions and of break-up channels in the reaction mechanism?

Data from GANIL, Navin etal, 2011



Extremely difficult to extract the fundamental $\sigma 2/\sigma 1$ ratio



^{11}Li+p -> ^{9}Li+t

Data from ISAC-2, TRIUMF

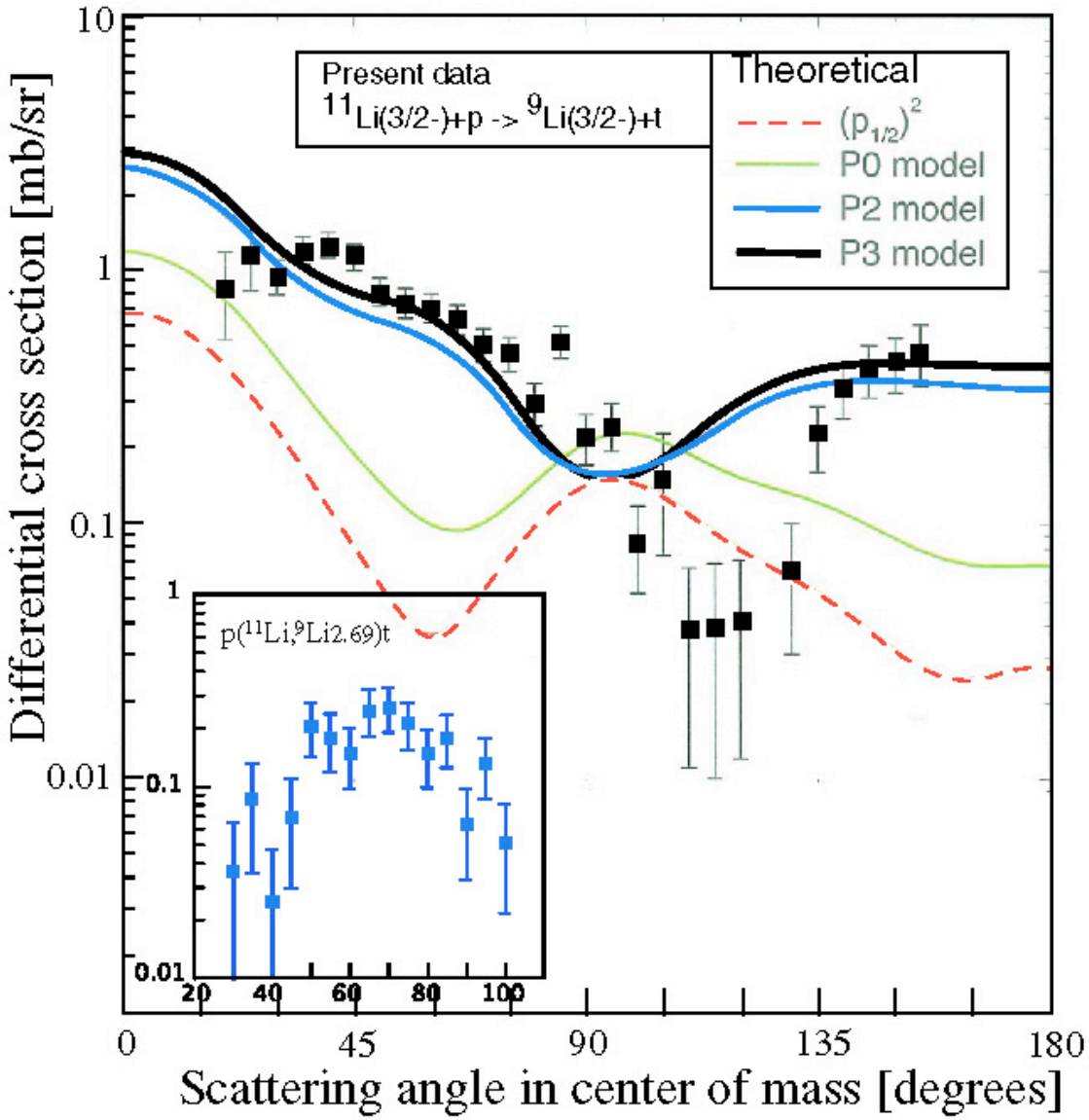
Tanihata, Thompson

Sensitivity to the pairing function in ^{11}Li

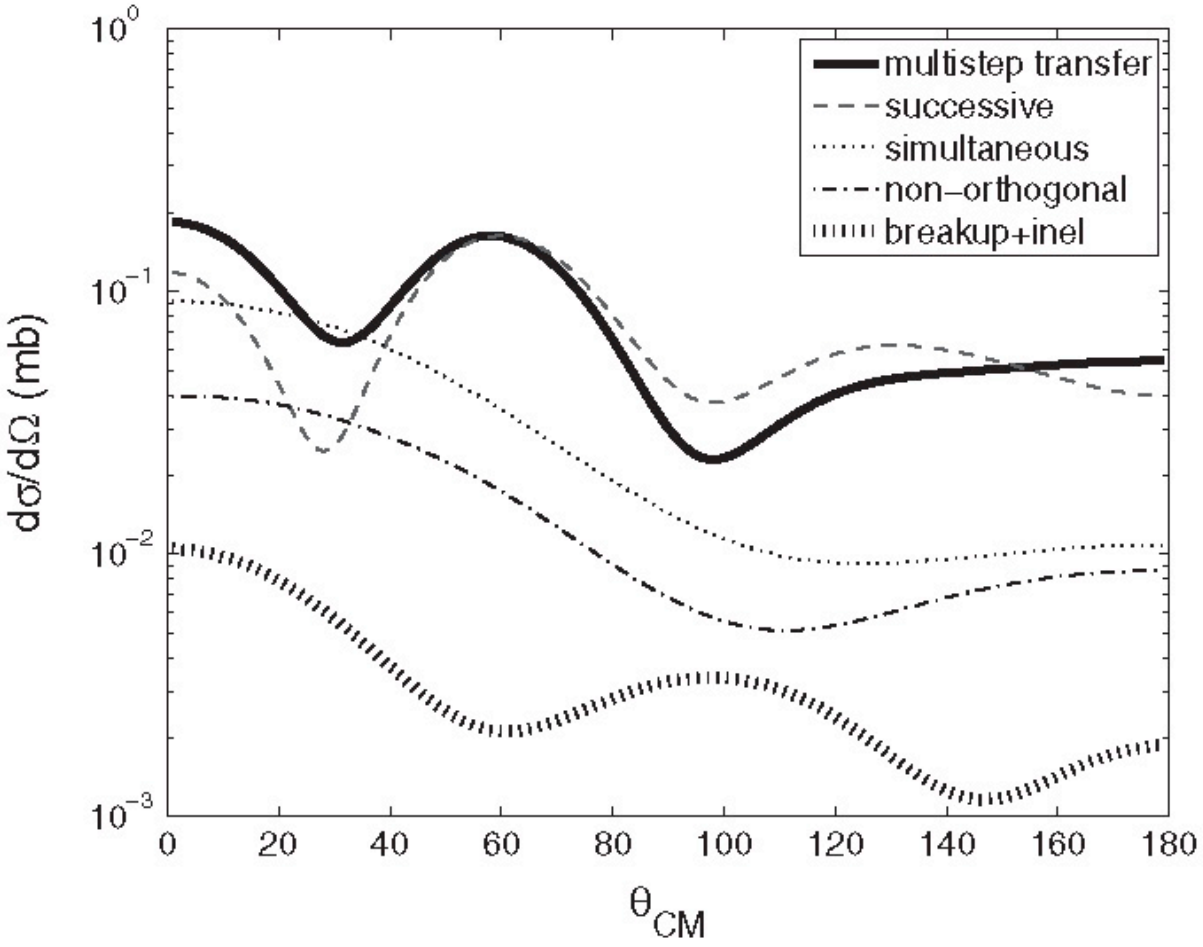
P0: 3% of $(s_{1/2})^2$

P2: 31% of $(s_{1/2})^2$

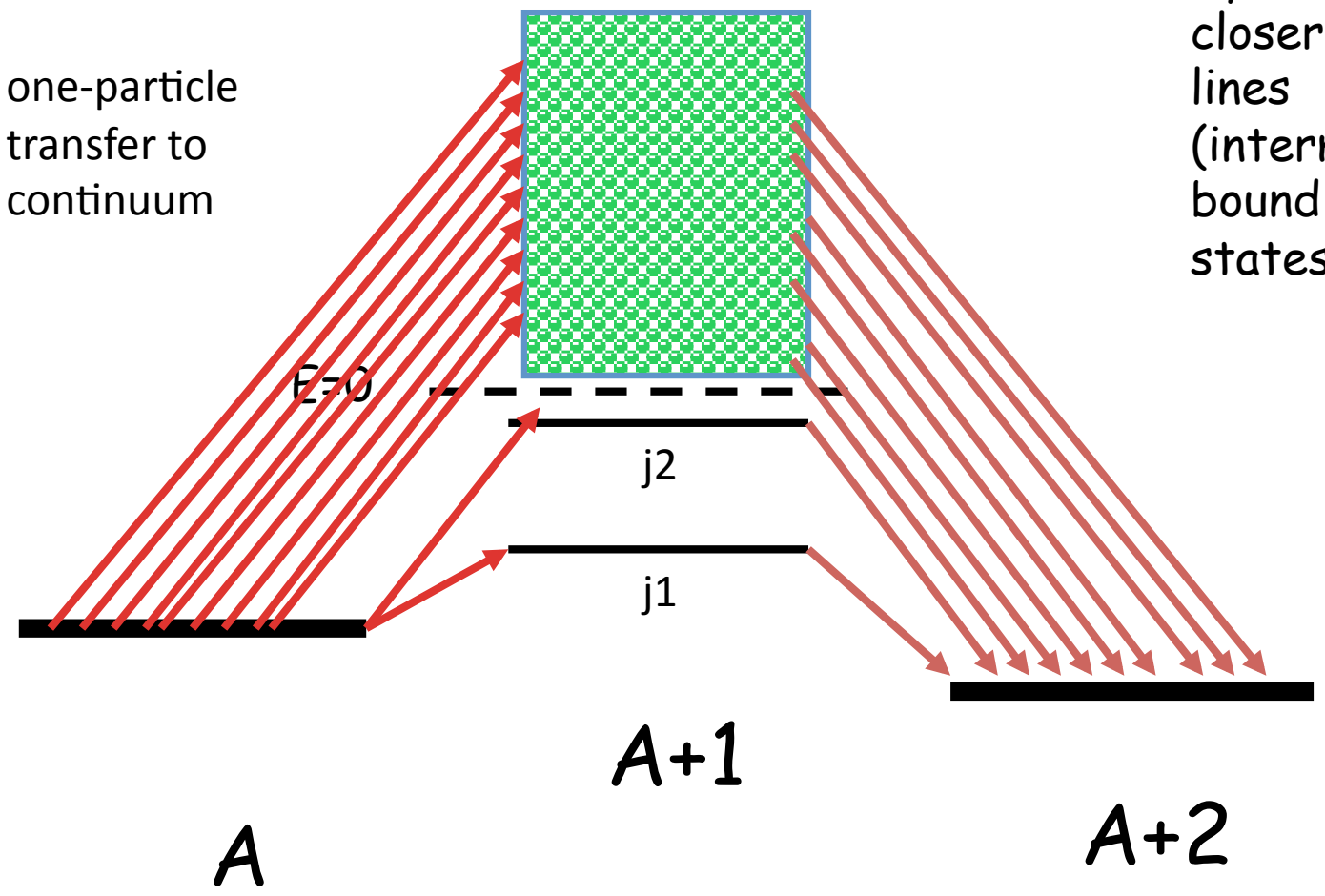
P3: 45% of $(s_{1/2})^2$



^{11}Li+p -> ^{9}Li+t

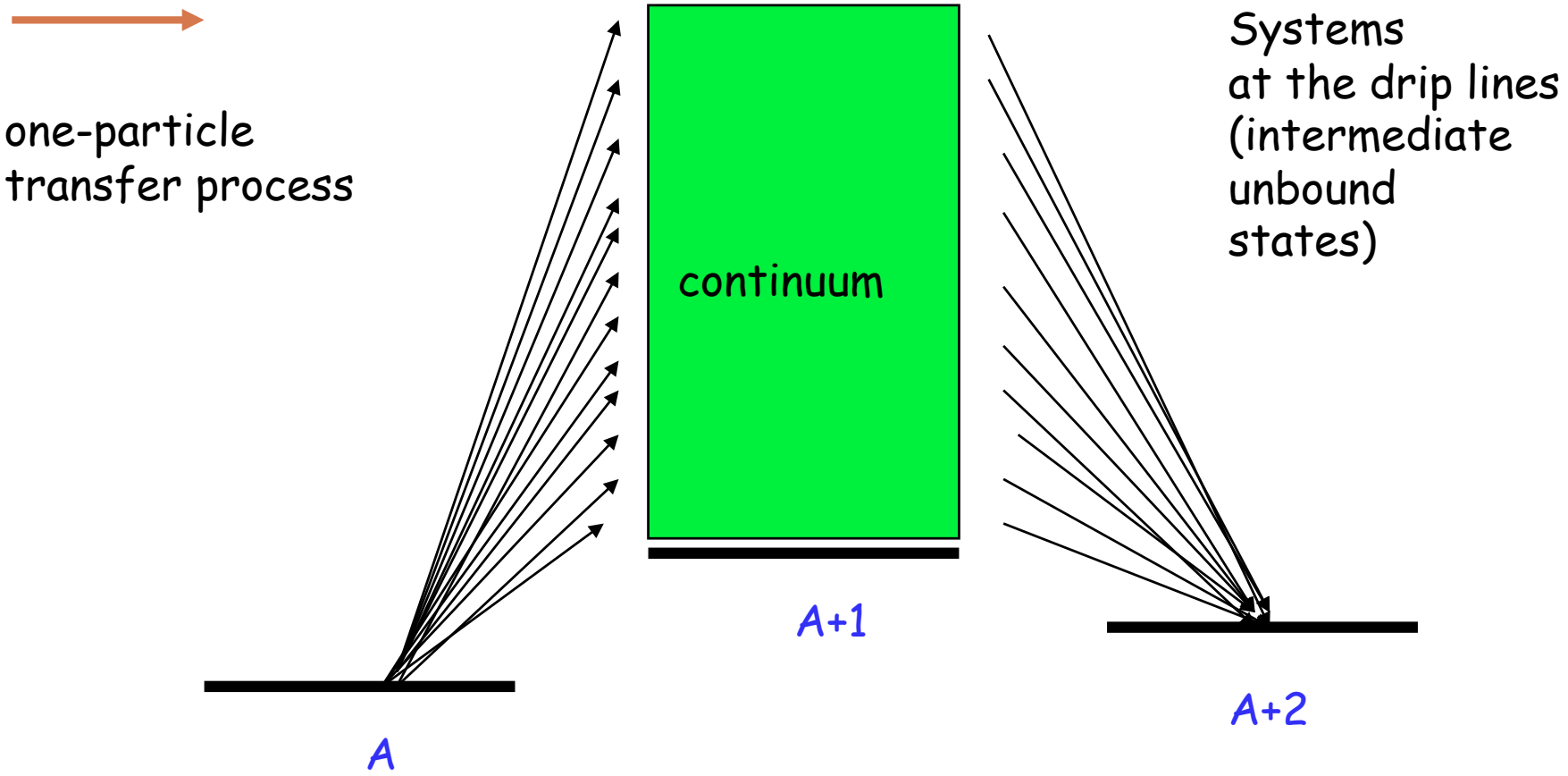


Potel etal, PRL, 2010



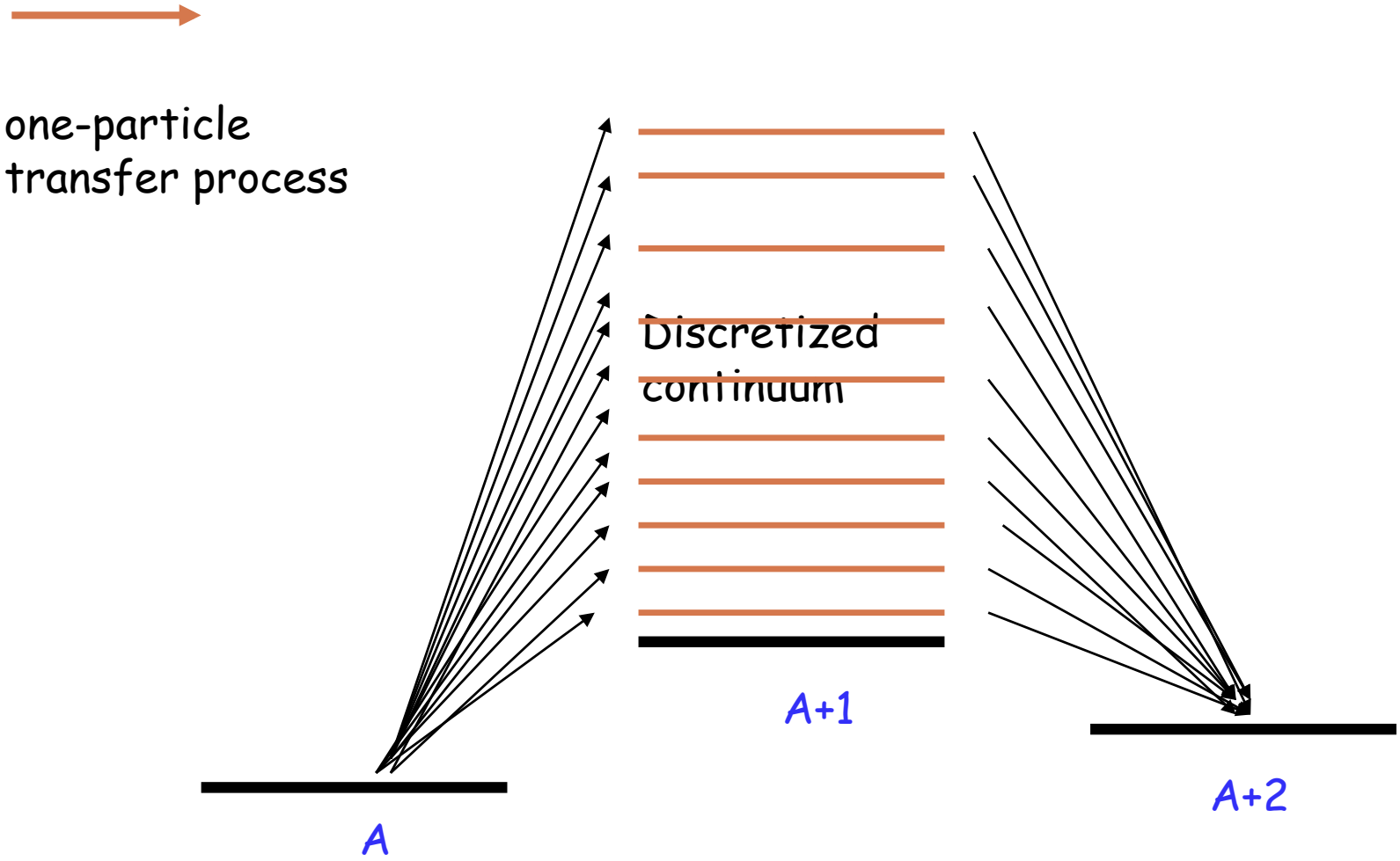
Example

$$|A=2\rangle = \{ \sum_i X_i [a_i^+ a_i^+]_0 + \int dE\, X(E)\, [a^+(E) a^+(E)]_0 \} \, |A\rangle$$



$$|A=2\rangle = \int dE\, X(E)\, [a^{+}(E)a^{+}(E)]_0\, |A\rangle$$

Two-particle trasfer will proceed mainly by
constructive interference of successive transfers
through the (unbound) continuum intermediate states



The integration over the continuum intermediate states can becomes feasible by continuum discretization:
but how many paths should we include? Thousands or few, for example only the resonant states?