Proton-neutron pairing and alpha-like quartetting in nuclei

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Proton-neutron pairing & alpha-like quartetting

the biginning

SOVIET PHYSICS JETP VOLUME 11, NUMBER 3 SEPTEMBER, 1960

SUPERFLUIDITY OF LIGHT NUCLEI

V. B. BELYAEV, B. N. ZAKHAR'EV, and V. G. SOLOV'EV

Joint Institute of Nuclear Research

Submitted to JETP editor October 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 952-954 (March, 1960)

"we must take into consideration the quadruple correlation of alpha-particle-like nucleons in addition to pair correlations; these new correlations evidently play a very important role and somewhat mask the effect of pair correlations"

pioneering studies on pn pairing & alpha correlations

V. G. Soloviev NP18 (1960)

B. Bremond and J. G. Valatin NP41(1963)

B. H. Flowers and M. Vijicic,NPA49(1963)

A. Arima and V. Gillet, Ann. Phys. 66 (1971)

J. Eichler and M. Yamamura, NPA182(1972)

....

J. Dobes And S. Pittel PRC57(1998)

R. Chasman, PLB577(2003)

R. A. Senkov and V. Zelevinski (2011)

alpha-like quartet = collective state of two neutrons and two protons coupled to T=0 and J=0

why alpha-like quartets for pairing ?

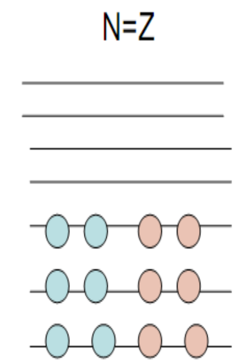
they are the simplest structures which conserve exactly the isospin and spin

&

provide accurate descriptions of pn pairing Hamiltonians !

Isospin conservation and quarteting: T=1 pairing

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + g \sum_{ij,\tau} P_{i,\tau}^{+} P_{j,\tau}$$



$$P_{i0}^{+} \propto \nu_i^{+}\pi_{\bar{i}}^{+} + \pi_i^{+}\nu_{\bar{i}}^{+} \qquad P_{i1}^{+} \propto \nu_i^{+}\nu_{\bar{i}}^{+} \qquad P_{i-1}^{+} \propto \pi_i^{+}\pi_{\bar{i}}^{+}$$

collective pn pairs

$$\Gamma_{\pi\nu}^{+} = \sum_i x_i (\nu_i^{+}\pi_{\bar{i}}^{+} + \pi_i^{+}\nu_{\bar{i}}^{+})$$

pair condensate

$$(\Gamma_{\nu\pi}^{+})^{\frac{N+Z}{2}} \,|->$$

no well-defined isospin !

collective quartet

$$Q^{+} = \sum_{ij,\tau} x_{ij} [P_{i\tau}^{+} P_{j\tau'}^{+}]^{T=0} \propto \sum_{ij\tau} x_{ij} (P_{\nu\nu,i}^{+} P_{\pi\pi,j}^{+} + P_{\pi\pi,i}^{+} P_{\nu\nu,j}^{+} - P_{\nu\pi,i}^{+} P_{\nu\pi,j}^{+})$$

quartet condensate

$$|QCM> = Q^{+n_q} \,|->$$

(has T=0, J=0)

Quartet condensation and Cooper pairs

$$|QCM>=Q^{+n_q}|->$$

$$Q^+=\sum_{ij}x_{ij}[P^+_{i\tau}P^+_{j\tau'}]^{T=0}$$

$$Q^+=2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi}-\Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi}$$

$$\Gamma^+_\tau=\sum_i x_i P^+_{i,\tau}$$

collective pairs

entangled collective pairs !

$$|QCM>=(2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi}-\Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi})^{n_q}|->$$

'coherent' mixing of condenstates formed by nn, pp and pn pairs

calculations

$$\delta_x<QCM|H|QCM>=0$$

method of reccurence relations

(24 non-linear coupled equations !)

Quartet condensation versus pair condensation

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V^{T=1}_{J=0}(i,j) \sum_t P^+_{it} P_{jt}$$

pairing forces extracted from SM interactions

	SM	QCM	PBCS1	PBCS0
		$(Q^+)^{n_q}$	$(\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi})^{n_q}$	$(\Gamma^{+2}_{\nu\pi})^{n_q}$
^{20}Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
^{24}Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
^{28}Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
^{32}S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
^{44}Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
^{104}Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
^{108}Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
^{112}Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

Conclusions

- *T=1 pairing is accurately described by quartets, not by pairs*
- *there is not a pure condensate of isovector pn pairs in N=Z nuclei*

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Quartet condensation versus isospin-projected BCS

$$H = \sum_i \varepsilon_i \left(N_i^{(\nu)} + N_i^{\pi} \right) - g \sum_{ij,\tau} P_{i,\tau}^{+} P_{j,\tau}$$

$$|QCM> \equiv (Q^+)^{n_q} |->$$

$$|PBCS(N,T)\rangle = \hat{P}_T \hat{P}_N |BCS\rangle$$

$$E_{corr} = E_0 - E$$

^{52}Fe

Exact value: 8.29 MeV

PBCS(N,T): 7.63 MeV (8%) (Chen et al , Nucl. Phys.A 1978)

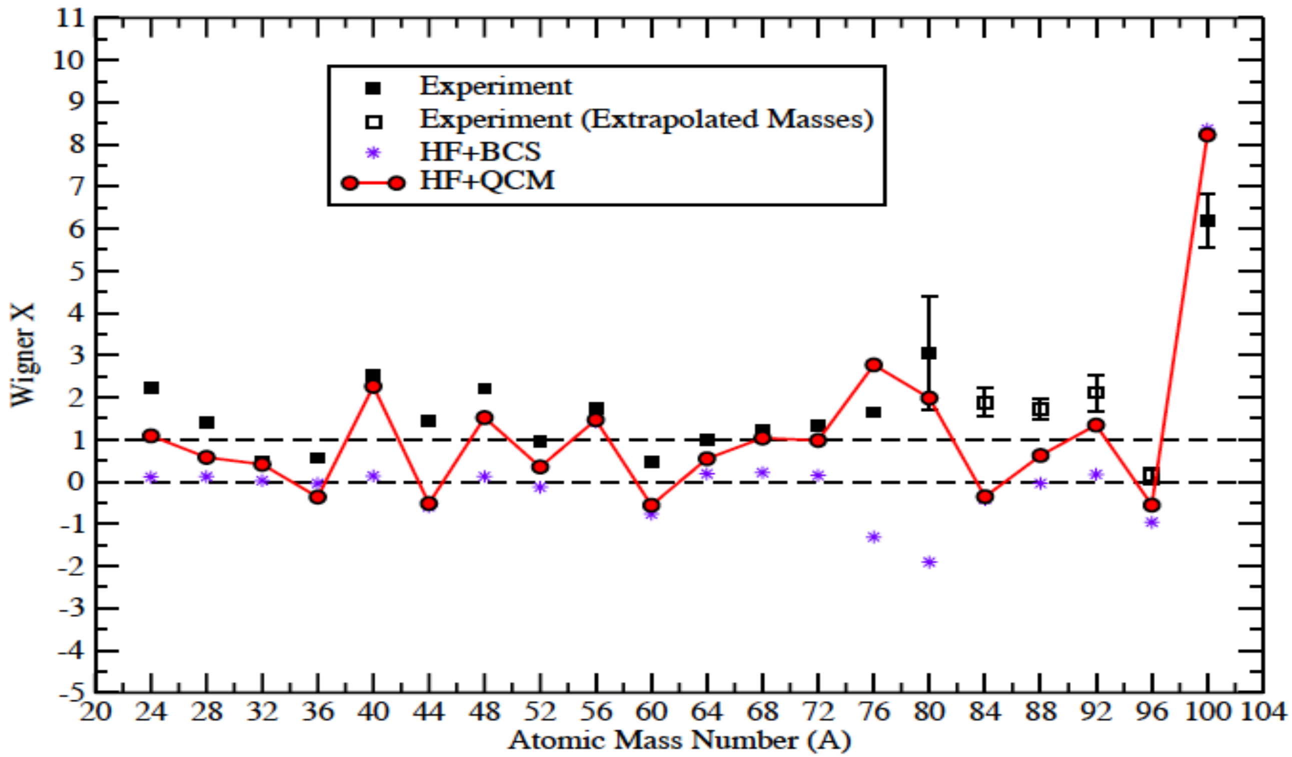
QCM: 8.25 MeV (0.5%)

QCM state describes additional quartet-type correlations

Isovector pairing in QCM: Wigner energy

$$E(N,Z) = E(N=Z) + a_s \frac{(N-Z)^2}{A} + a_W \frac{|N-Z|}{A} + \delta E_{shell} + \delta E_P$$ (no Coulomb)

$$E(N,Z) = E(N=Z) + \frac{T_z(T_z+X)}{2\Theta} \qquad T_z = 0,2,4$$



BCS fails to describe the Wigner energy

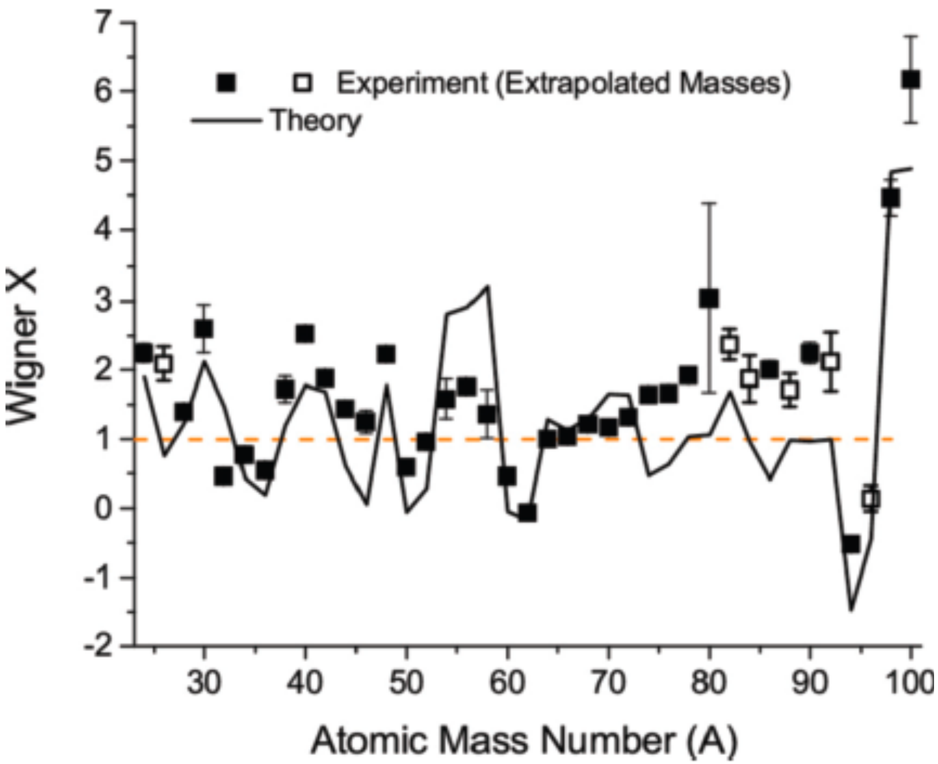
T=1 pairing, when treated accurately, is able to describe well the Wigner !

D. Negrea and N. S, PRC90 (2014)

Wigner energy: comparison with earlier calculations

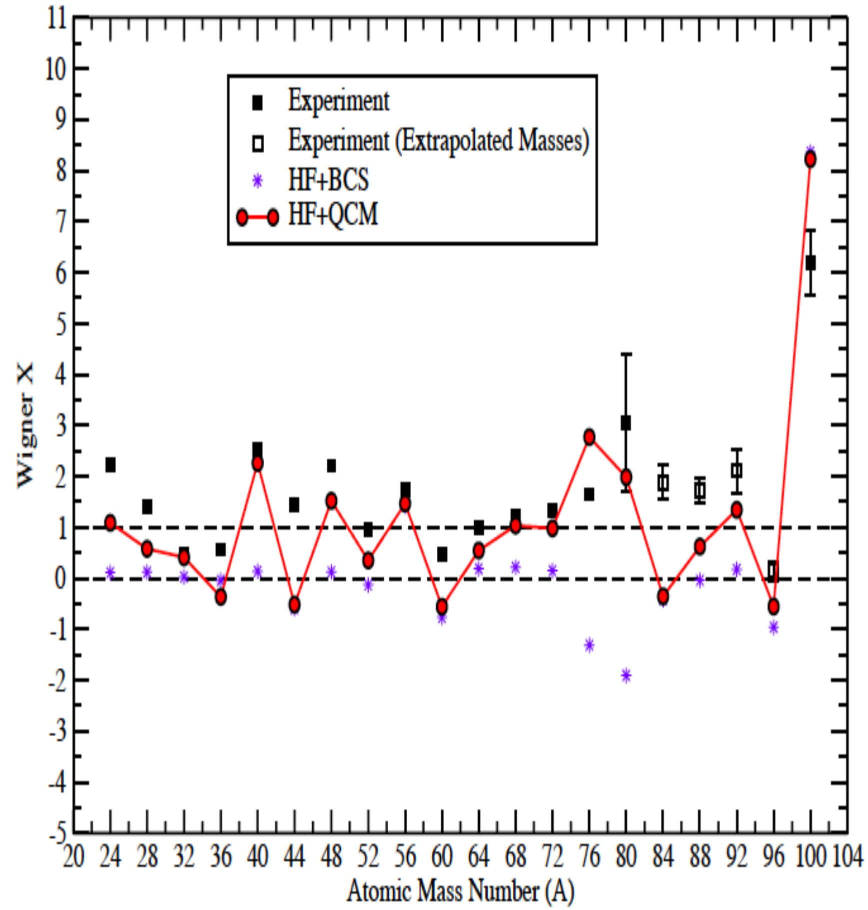
Bentley & Frauendorf PRC(2013)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}^+_{k,\tau} \hat{P}_{k',\tau} + C\vec{T}\cdot\vec{T}$$



Negrea & Sandulescu PRC(2014)

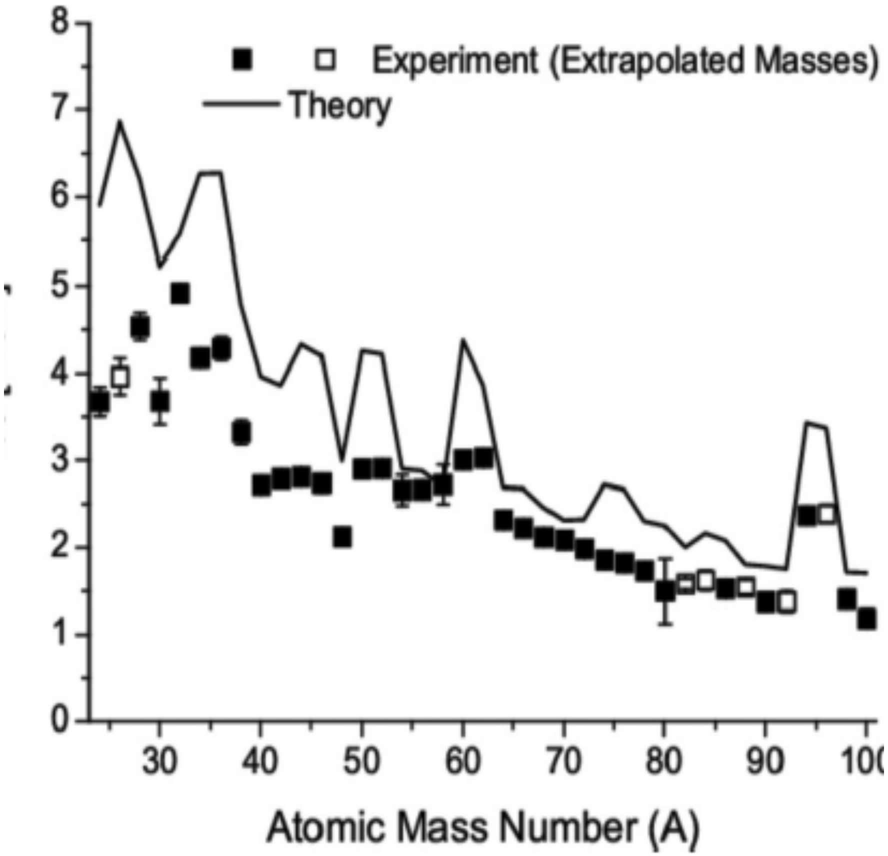
$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}^+_{k,\tau} \hat{P}_{k',\tau}$$



Symmetry energy: comparison with earlier calculations

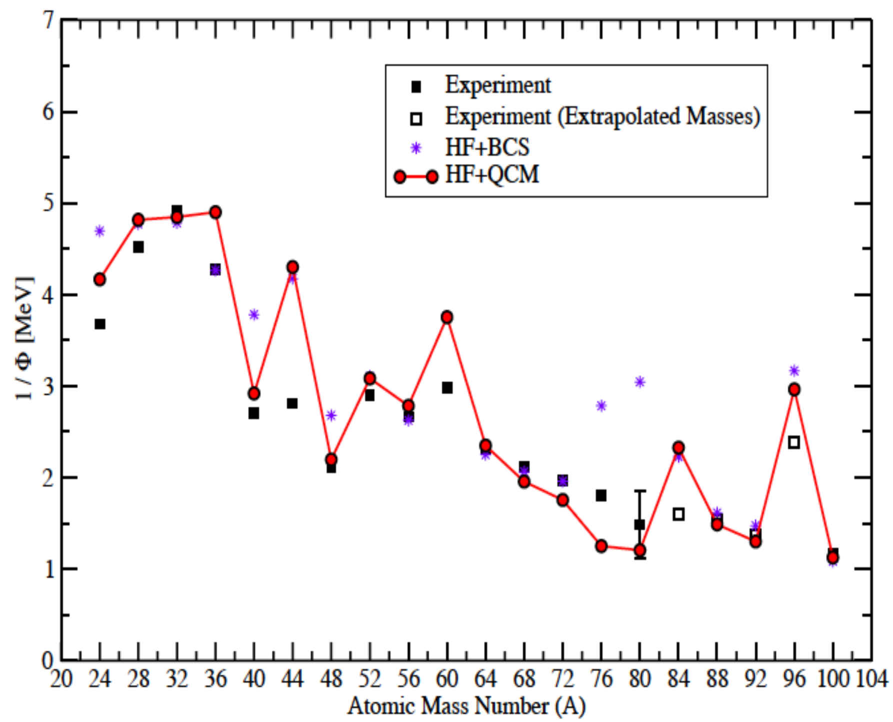
Bentley & Frauendorf PRC(2013)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}^+_{k,\tau} \hat{P}_{k',\tau} + C\vec{T}\cdot\vec{T}$$



Negrea & Sandulescu PRC(2014)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}^+_{k,\tau} \hat{P}_{k',\tau}$$



Isoscalar and isovector pairs in N=Z nuclei

(T=1,J=0) pairs

$$P^+_{i,T_z} = [a_i^+ a_i^+]^{T=1,J=0}_{T_z}$$

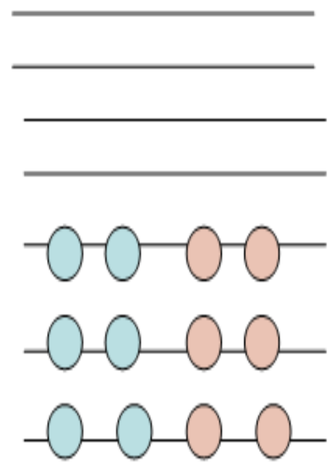
$$\Gamma^+_\tau = \sum_i x_i P^+_{i,\tau}$$

pair condensate

$$(\Gamma^+_{\nu\pi})^{2n_q} |->$$

not well-defined isospin

N=Z



(T=0,J=1) pairs

$$D^+_{ij,J_z} = [a_i^+ a_j^+]^{J=1,T=0}_{J_z}$$

$$\Delta^+_0 = \sum y_i D^+_{i,0}$$

pair condensate

$$(\Delta^+_0)^{2n_q} |0\rangle$$

not well-defined total spin

Quartetting for isovector (J=0) and isoscalar (J=1) pairing

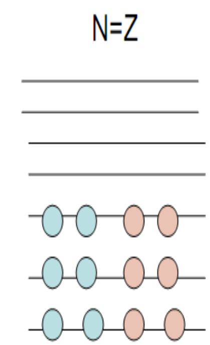
$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(ij,kl) \sum_{\sigma} D_{ij\sigma}^{+} D_{kl\sigma}$$

isovector

$$P_{i,T_z}^{+} = [a_i^{+} a_i^{+}]_{T_z}^{T=1,J=0}$$

isoscalar

$$D_{ij,J_z}^{+} = [a_i^{+} a_j^{+}]_{J_z}^{J=1,T=0}$$



collective quartets

$$Q_{iv}^{+} = \sum_{i,j} x_{ij} [P_i^{+} P_j^{+}]^{T=0}$$

$$Q_{is}^{+} = \sum_{i,j} y_{ij,kl} [D_{ij}^{+} D_{kl}^{+}]^{J=0}$$

generalised quartet

$$Q^{+} = Q_{iv}^{+} + Q_{is}^{+}$$

ground state

$$| QCM > = Q^{+n_q} | - >$$

superposition of T=0 and T=1 quartets

M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

Quartet condensation versus pair condensation for isovector & isoscalar pairing

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

$(Q^{+})^{n_q} |->$ $\quad (\Gamma_{\nu\nu}^{+}\Gamma_{\pi\pi}^{+})^{n_q} |->$ $\quad (\Gamma_{\nu\pi}^{+})^{2n_q} |->$ $\quad (\Delta_0^{+})^{2n_q} |0\rangle$

	QCM	PBC1	$PBCS0_{iv}$	$PBCS0_{is}$
^{20}Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
^{24}Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
^{28}Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
^{44}Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
^{48}Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
^{52}Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
^{104}Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
^{108}Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
^{112}Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- quartet condensation wins over Cooper pair condensates
- T=1 and T=0 pairing correlations always coexist in quartets

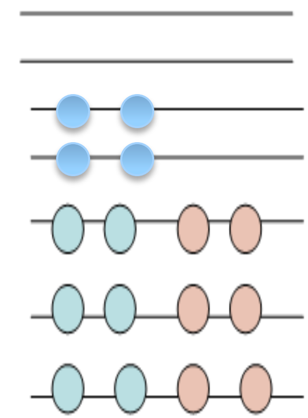
M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

Isovector-isoscalar pairing and quartetting for N>Z nuclei

nuclei with $N\text{-}Z=2n_N$

ansatz

- all protons are correlated in alpha-like quartets
- neutrons in excess form a pair condensate



$$|QCM\rangle = (\tilde{\Gamma}^{+}_{\nu\nu})^{n_N} (Q^{+}_{T=1} + \Delta_0^{+2})^{n_q} |-\rangle$$

$$Q^{+}_{T=1} = 2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+2}_{\nu\pi} \qquad \Delta_0^{+} = \sum y_i D^{+}_{i,0}$$

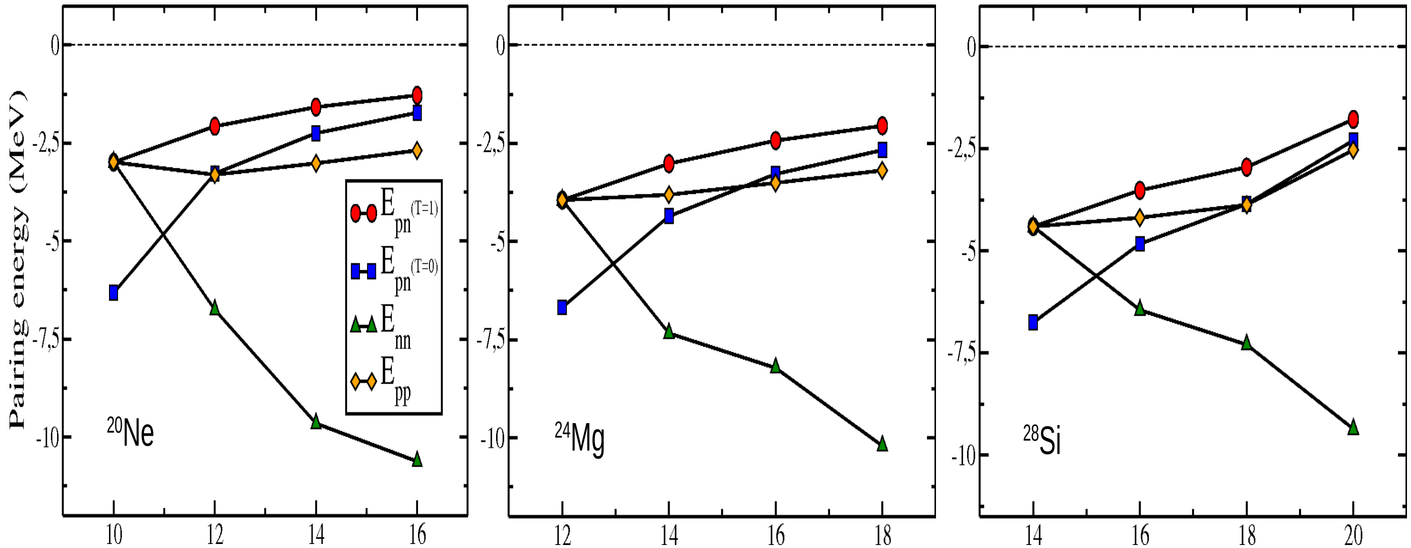
how fast are suppressed the pn correlations away of N=Z ?

Isovector-isoscalar pairing and quartetting for N>Z nuclei

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij,\tau} P^+_{i\tau} P_{j\tau} + g_{T=0} \sum_{ij} D^+_{i0} D_{j0}$$

$$g_{T=0} = 1.5\ g_{T=1}$$

$$|QCM\rangle = (\tilde{\Gamma}^+_{\nu\nu})^{n_N} (Q^+_{T=1} + \Delta_0^{+2})^{n_q} |-\rangle \qquad \Delta_0^+ = \sum y_i D^+_{i,0}$$



pn pairing and quartet correlations survive in N > Z nuclei !

D. Negrea, P. Buganu, D. Gambacurta, N. S., arXiv (2018), submitted

Quartet correlations for general two-body forces ?

Quartet correlations for general two-body forces

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

$$|QCM\rangle = Q^{+n_q}|-\rangle \qquad Q^+ = \sum_{ii',jj',JT} x_{ii',jj'} [A^+_{ii'JT} A^+_{jj'JT}]^{0,0}$$

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$E_{corr}(QM)$	$\langle SM\|QCM\rangle$
^{20}Ne	24.77	24.77	24.77	1
^{24}Mg	55.70	53.04 (4.77%)	53.24 (4.41%)	0.85
^{28}Si	88.75	86.52 (2.52%)	87.12 (1.84%)	0.86
^{32}S	122.51	122.02 (0.40%)	122.29 (0.18%)	0.98

$$E(n_q) = n_q \times E(1) + \frac{n_q(n_q-1)}{2} \times V(n_q),$$

the interaction between the quartets is small compared to their binding energies

quartets acts as weakly interacting building blocks

M. Sambataro and N. S., EPJ A53 (2017) 43

How to identify the transition to a quartet condensate ?

Long-range correlations of superfluidity-type and density matrix

Penrose (1951) , Penrose and Onsager (1956), **C. N. Yang (1962)**

n-body long-range correlations ⟺ a large eigenvalue of n-body density

<u>Example: pair condensation</u>

$$\rho^{(2)}(r_1,r_2;r_1',r_2') = \langle\Phi_0^{(N)}|\hat{\Psi}^+(r_1)\hat{\Psi}^+(r_2)\hat{\Psi}(r_2')\hat{\Psi}(r_1')|\Phi_0^{(N)}\rangle$$

$$\rho^{(2)}(r_1,r_2;r_1',r_2') = \lambda_0\phi_0^*(r_1,r_2)\phi_0(r_1',r_2') + \sum_{n>0}\lambda_n\phi_n^*(r_1,r_2)\phi_n(r_1',r_2')$$

long-range correlations: $\lambda_0 >> \lambda_{n\neq 0}$ ("off-diagonal long-range order")

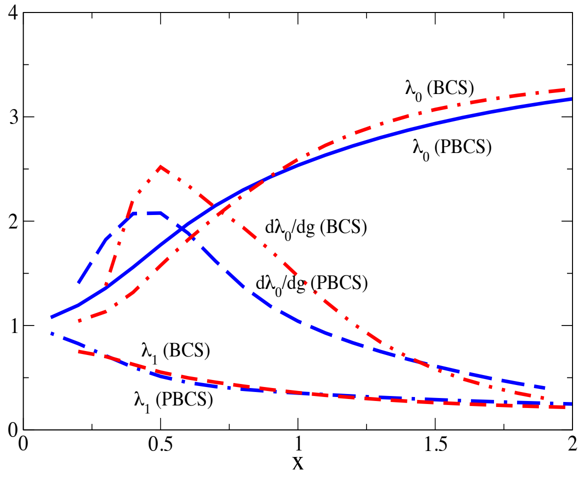
λ_0 - associated to the number of "condensed" pairs

Eigenvalues of two-body density matrix for like-particle pairing

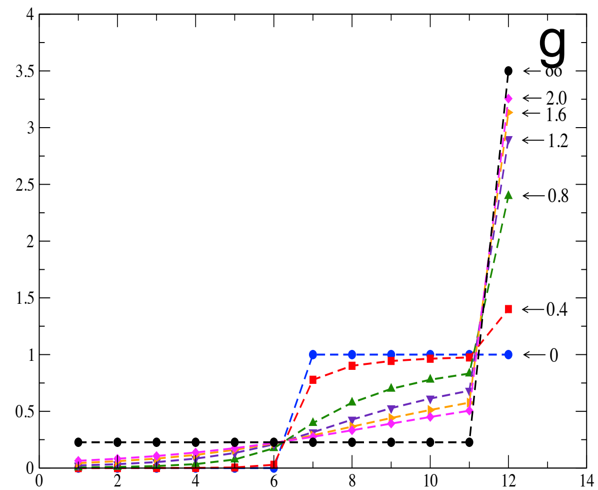
$$H = \sum_i \varepsilon_i N_i - k \sum_{ij} V(i,j) P_i^+ P_j$$ (k is a scaling factor)

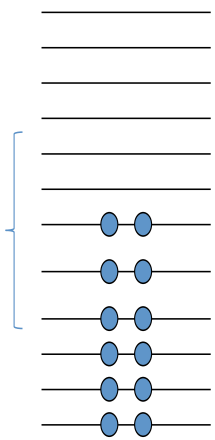
two-body density $\rho_{i,j}^{(2)} = \langle \Psi | P_i^+ P_j | \Psi \rangle$ $\quad P_i^+ = a_i^+ a_{\bar{i}}^+$

^{110}Sn



12 particles in 12 levels $\quad V(i,j) = g$



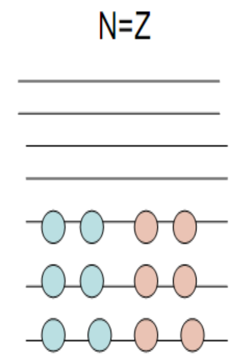


at physical strength : $\lambda_0 > 1 \; (\lambda_{n>0} < 1)$

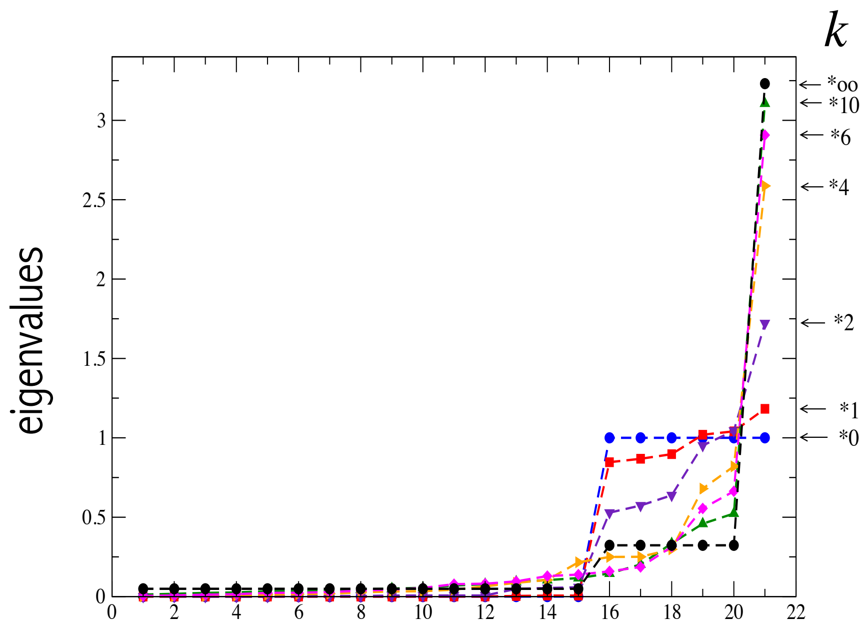
M. Sambataro and N. S, in preparation

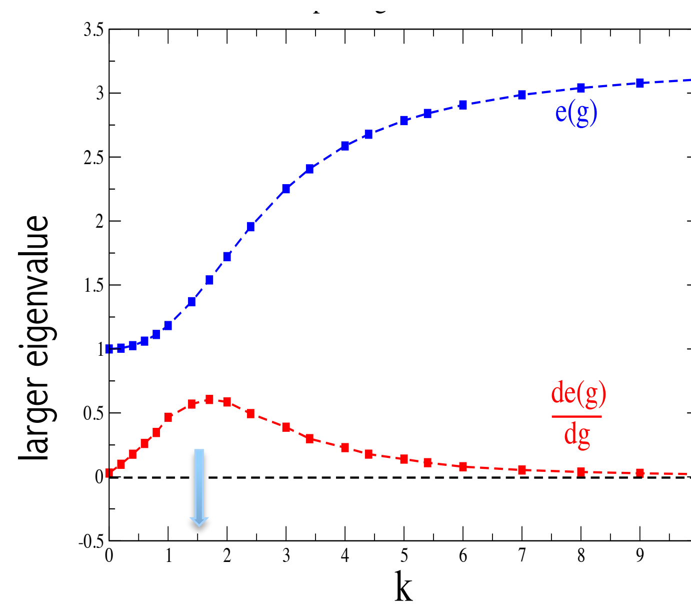
Eigenvalues of 4-body density matrix for T=1 pairing: ^{28}Si

$$H=\sum_i \varepsilon_i (N_i^{(\nu)}+N_i^{(\pi)}) - kg\sum_{ij}(P^{+}_{i,\nu\pi}P_{j,\nu\pi}+P^{+}_{i,\nu\nu}P_{j,\nu\nu}+P^{+}_{i,\pi\pi}P_{j,\pi\pi}) \qquad g=\frac{24}{A}$$



4-body density $\qquad \rho^{(4)}_{i,j} = <\Psi | q_i^{+} q_j | \Psi> \qquad q_i^{+} = (a^{+}_{i_1}a^{+}_{i_2}a^{+}_{i_3}a^{+}_{i4})^{T=0}$





in the physical region $\lambda_0^{(4)} > 1$

4-body density for general two-body forces: sd-shell nulcei

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

$$\rho^{(4)}_{i,j} = \langle SM | q_i^+ q_j | SM \rangle \qquad q_i^+ = (a^+_{i_1} a^+_{i_2} a^+_{i_3} a^+_{i4})^{T=0}$$

Largest 5 eigenvalues for sd-shell nuclei

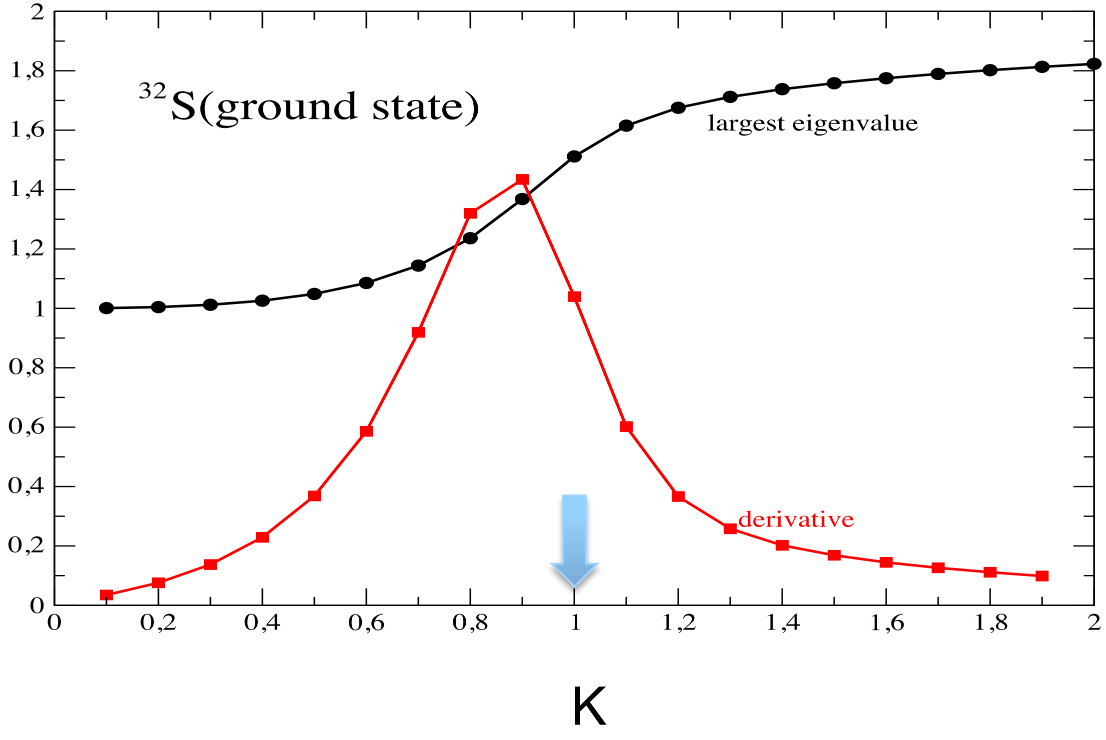
^{24}Mg	**1.18**	0.15	0.03	0.29	0.01
^{28}Si	**1.19**	0.47	0.27	0.20	0.12
^{32}S	**1.51**	0.83	0.74	0.59	0.53

there is one eigenvalue larger than 1

fingerprints of long-range quartet correlations

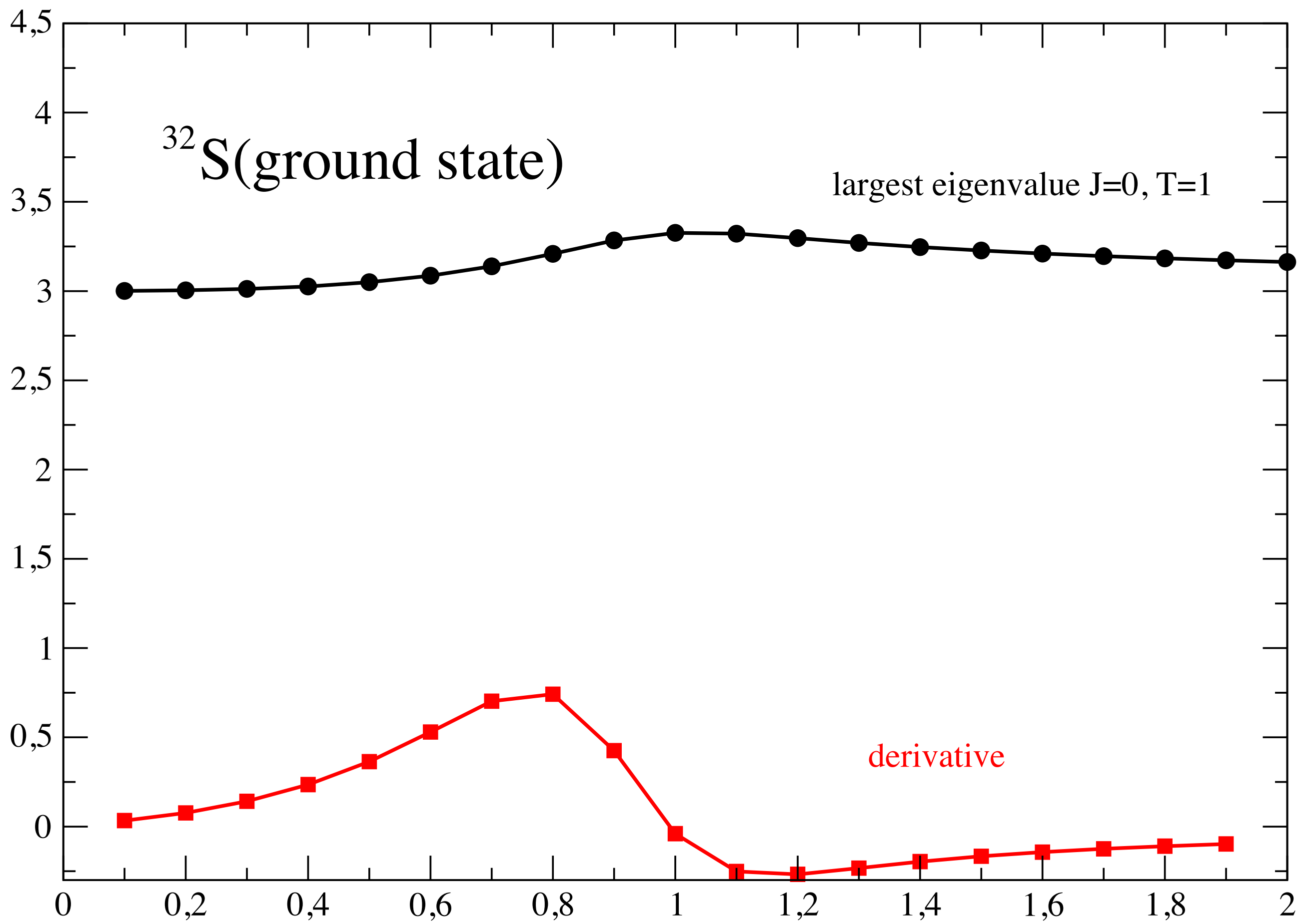
Evolution of the largest eigenvalue of 4-body density matrix: ^{32}S

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \quad k \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$



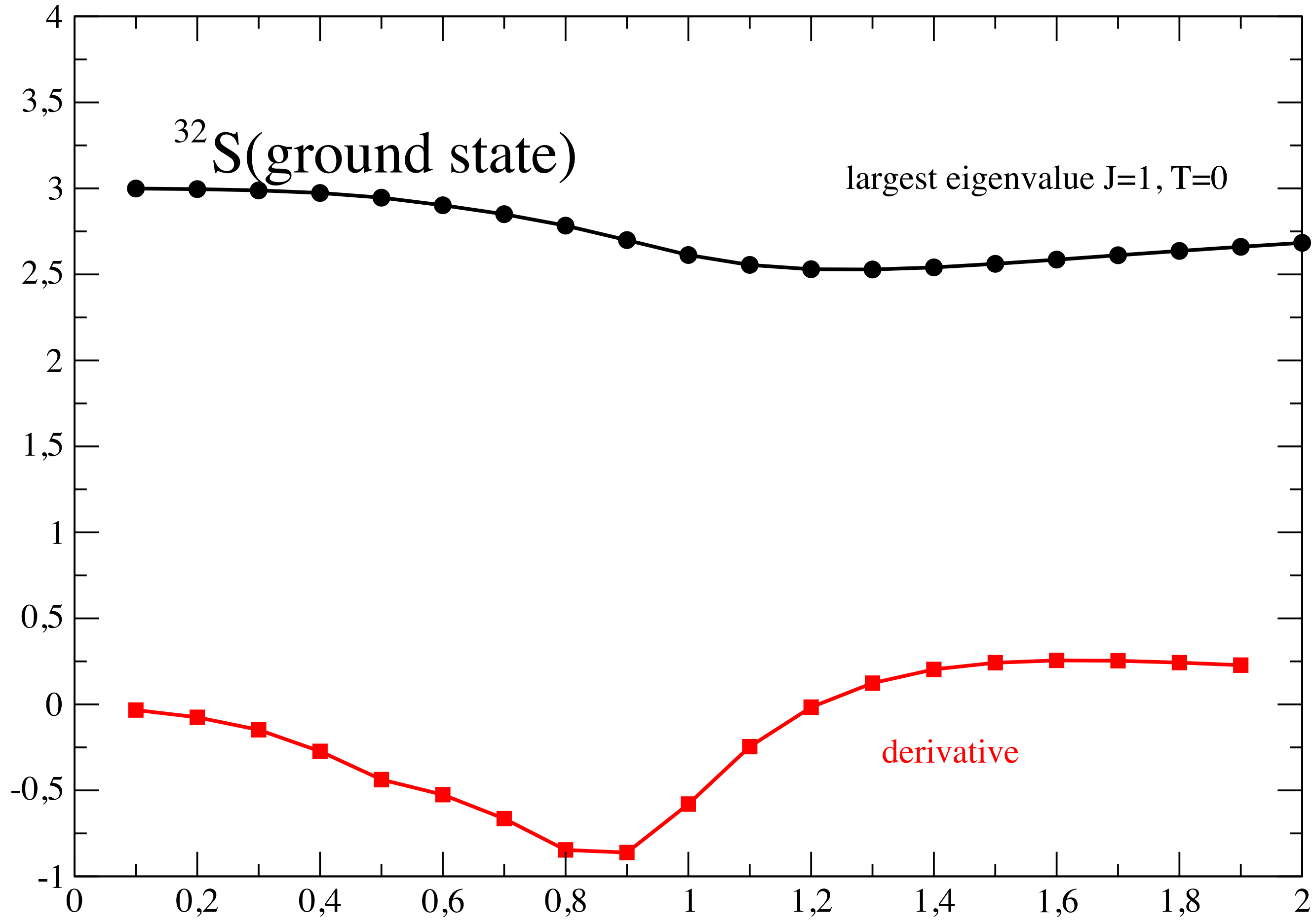
Indication of a fast transition towards a quartet condensate !

Evolution of the largest eigenvalue for 2-body density matrix: J=0,T=1



signature of weak long-range correlations !

Evolution of the largest eigenvalue for 2-body density matrix: J=1,T=0



no signature of long-range correlations !

Summary and Conclusions

Main message: *isovector and isoscalar pairing are accurately described by alpha-like quartets, not by Cooper pairs*

- *isovector pairing gives a significant contribution to Wigner energy*
- *isoscalar and isovector pairing always coexist in the ground state of N=Z nuclei*
- *quarteting appears to be a general feature in N=Z nuclei*
- *4-body density matrix indicates long-range correlations of "condensate" type*

pairing correlations are "masked" by quartetting ?!

Perspectives

- *testing the quartet condensation by alpha transfer reactions ?*
- *unified microscopic treatment of quartetting and clustering ?*

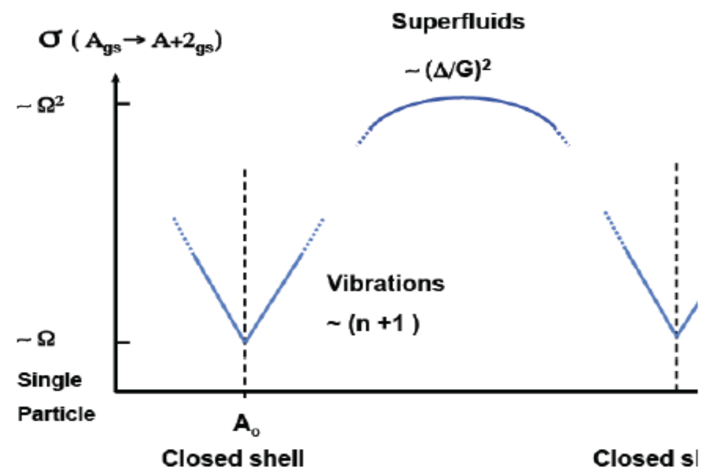
Testing alpha-like quartet condensation in N=Z nuclei ?

- test of pair condensation: *pair transfer*

$$<PBCS(A+2)|c_i^+ c_{\bar{i}}^+|PBCS(A)> \qquad |PBCS> = (\Gamma^+)^{N/2}|->$$

fingerprint of pairs condensation:

plateau in the two-neutron transfer cross section



- **test of quartet condensation:** *alpha particle transfer along N=Z line*

$$<QCM(A+4)|Q^+|QCM(A)> \qquad |QCM> \equiv (Q^+)^{n_q}|->$$

plateau in alpha transfer cross section ?

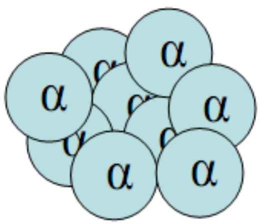
$$^{16}O \xrightarrow{\alpha} {}^{20}Ne \xrightarrow{\alpha} {}^{24}Mg \xrightarrow{\alpha} {}^{28}Si \xrightarrow{\alpha} {}^{32}S$$

experiments for heavier N=Z nuclei (ph-shell) ?

Alpha-like quartetting versus alpha clustering

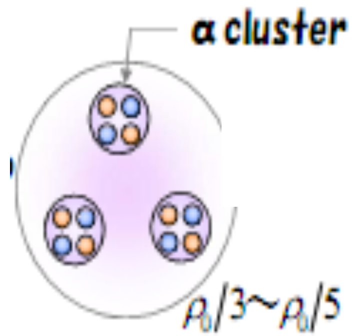
unified microscopic treatment ?

Quartetting



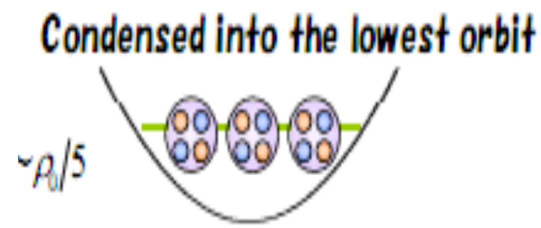
ground state

Alpha-clustering



excited states

Alpha condensation ?



Hoyle state in ^{12}C ?

Thanks for your attention !

Quartet condensation in the excited states ?

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

$$|0_n^+; QCM\rangle = (Q_n^+)^{n_q}|-\rangle \qquad Q_n^+ = \sum_{ii',jj',JT} x^{(n)}_{ii',jj'} [A^+_{ii'JT} A^+_{jj'JT}]^{0,0}$$

First excited 0^+

	$E_{0_1^+}(SM)$	$E_{0_1^+}(QCM)$	$\langle SM\|QCM\rangle$
^{20}Ne	−33.77 (6.7)	−33.77 (6.7)	1
^{24}Mg	−79.76 (7.34)	−76.97 (7.47)	0.70
^{28}Si	−131.00 (4.84)	−126.91 (6.71)	0.65
^{32}S	−178.98 (3.46)	−178.04 (3.92)	0.95

SM is a QCM state !?

Second excited 0^+

	$E_{0_2^+}(SM)$	$E_{0_2^+}(QCM)$
^{20}Ne	−28.56 (11.91)	−28.56 (11.91)
^{24}Mg	−77.43 (9.67)	−70.85 (13.59)
^{28}Si	−128.51 (7.33)	−120.64 (12.99)
^{32}S	−175.04 (7.4)	−170.84 (11.12)

superposition of many shell-model states: **cluster- type excitations** ?

M. Sambataro and N. S., EPJ A53 (2017) 43

Isovector (J=0) pairing versus isoscalar (J=1) pairing

$$|QM\rangle = \prod_{\nu=1}^{N_Q} Q_\nu^\dagger |0\rangle. \qquad Q_\nu^+ = Q_\nu^{+(iv)} + Q_\nu^{+(is)}$$

$$|is\rangle = \prod_{\nu=1}^{N_Q} Q_\nu^{\dagger(is)}|0\rangle \qquad |iv\rangle = \prod_{\nu=1}^{N_Q} Q_\nu^{\dagger(iv)}|0\rangle$$

	QM	iv	is	$< QM\|iv >$	$< QM\|is >$	$< iv\|is >$
^{20}Ne	15.985	14.402 (9.9%)	15.130 (5.35%)	0.884	0.953	0.843
^{24}Mg	28.625	23.269 (18.71%)	26.925 (5.94%)	0.650	0.910	0.336
^{28}Si	35.386	28.896 (18.34%)	33.377 (5.68%)	0.590	0.910	0.341
^{32}S	38.844	33.958 (12.58%)	37.881 (2.48%)	0.640	0.974	0.587
^{44}Ti	7.02	6.27 (10.6%)	4.92 (30%)	0.90	0.68	0.3
^{48}Cr	11.624	10.59 (8.9%)	7.38 (36.5%)	0.906	0.497	0.22
^{52}Fe	13.823	12.814 (7.3%)	9.98 (27.83%)	0.927	0.753	0.74
^{104}Te	3.147	3.041 (3.37%)	1.549 (50.78%)	0.978	0.489	0.314
^{108}Xe	5.495	5.240 (4.64%)	2.627 (52.19%)	0.958	0.354	0.234
^{112}Ba	7.035	6.614 (5.98%)	4.466 (36.52%)	0.939	0.375	0.376

T=1 and T=0 pairing correlations always coexist
&
difficult to disentangle

M. Sambataro, N.S. and C.W.Johnson, Phys. Lett. B740 (2015)137

Isovector (J=0) and isoscalar (J=1) pairing: alpha-like quartetting

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

pairing forces: from standard shell model interactions

$$|QM>== Q_1^+ Q_2^+ \dots Q_{n_q}^+ |->$$

$$|QM(l.o)>== Q^+ |QCM(n_q - 1)>$$

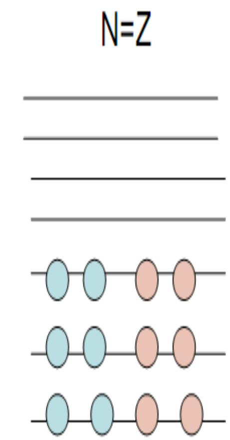
	N_Q	exact	QM	QM(l.o.)	|<QM|QM(l.o.)>|
^{24}Mg	2	-28.694	-28.626 (0.24%)	-28.592 (0.35%)	0.9993
^{28}Si	3	-35.600	-35.396 (0.57%)	-35.307 (0.82%)	0.9980
^{32}S	4	-38.965	-38.865 (0.25%)	-38.668 (0.76%)	0.9942
^{48}Cr	2	-11.649	-11.624 (0.21%)	-11.614 (0.30%)	0.9996
^{52}Fe	3	-13.887	-13.823 (0.46%)	-13.804 (0.60%)	0.9994
^{108}Xe	2	-5.505	-5.495 (0.18%)	-5.490 (0.27%)	0.9995
^{112}Ba	3	-7.059	-7.035 (0.34%)	-7.025 (0.48%)	0.9987

alpha-like quartets appear as relevant degrees of freedom in N=Z nuclei

M. Sambataro, N.S. and C.W.Johnson, Phys. Lett. B740 (2015)137

Isoscalar and isovector proton-neutron pairing in time-reversed states

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$



isovector

isoscalar

$$P^{+}_{i,0} = (\nu^{+}_{i} \pi^{+}_{\bar{i}} + \pi^{+}_{i} \nu^{+}_{\bar{i}})/\sqrt{2}. \qquad D^{+}_{i,0} = (\nu^{+}_{i} \pi^{+}_{\bar{i}} - \pi^{+}_{i} \nu^{+}_{\bar{i}})/\sqrt{2}$$

$$P^{+}_{i1} = \nu^{+}_{i} \nu^{+}_{\bar{i}} \qquad P^{+}_{i-1} = \pi^{+}_{i} \pi^{+}_{\bar{i}}$$

$$Q^{+}_{T=1} = \sum_{ij} x_i x_j [P^{+}_{i\tau} P^{+}_{j\tau'}]^{T=0} \qquad \Delta^{+}_{0} = \sum y_i D^{+}_{i,0}$$

ansatz for ground state

$$|\Psi\rangle = (Q^{+}_{T=1} + \Delta^{+2}_{0})^{n_q} |-\rangle$$

superposition of T=1 quartet condensates and T=0 pair condensates

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B751 (2015) 348

Competition between isovector and isoscalar pairing

pairing on top of deformed Skyrme-HF

$$V_{paring}^{T=\{0,1\}} = \mathrm{v}_0^{T=\{0,1\}} \delta(r_1 - r_2)\hat{P}_{S=\{0,1\}} \qquad \mathrm{v}_0^{T=0} = 1.5\ \mathrm{v}_0^{T=1}$$

$(Q_{T=1}^{+} + \Delta_0^{+2})^{n_q}$ $\quad (Q_{T=1}^{+})^{n_q}$ $\quad (\Delta_0^{+2})^{n_q}$

	exact	$\mid\Psi\rangle$	$\mid iv\rangle$	$\mid is\rangle$	$\langle iv \mid is\rangle$
^{20}Ne	11.38	11.38 (0.00%)	11.31 (0.62%)	10.92 (4.00%)	0.976
^{24}Mg	19.32	19.31 (0.03%)	19.18 (0.74%)	18.93 (2.00%)	0.980
^{28}Si	18.74	18.74 (0.01%)	18.71 (0.14%)	18.54 (1.07%)	0.992
^{44}Ti	7.095	7.094 (0.02%)	7.08 (0.18%)	6.30 (10.78%)	0.928
^{48}Cr	12.78	12.76 (0.1%)	12.69 (0.67%)	12.22 (4.37%)	0.936
^{52}Fe	16.39	16.34 (0.26%)	16.19 (1.17%)	15.62 (4.65%)	0.946
^{104}Te	4.53	4.52 (0.06%)	4.49 (0.82%)	4.02 (11.26%)	0.955
^{108}Xe	8.08	8.03 (0.61%)	7.96 (1.45%)	6.75 (16.47%)	0.814
^{112}Ba	9.36	9.27 (0.93%)	9.22 (1.43 %)	7.50 (19.81%)	0.784

isovector and isoscalar pairing **always coexist** together

large overlaps between |iv> and |is>

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B (2015)

Suppresion of isoscalar and isovector pairing by spin-orbit

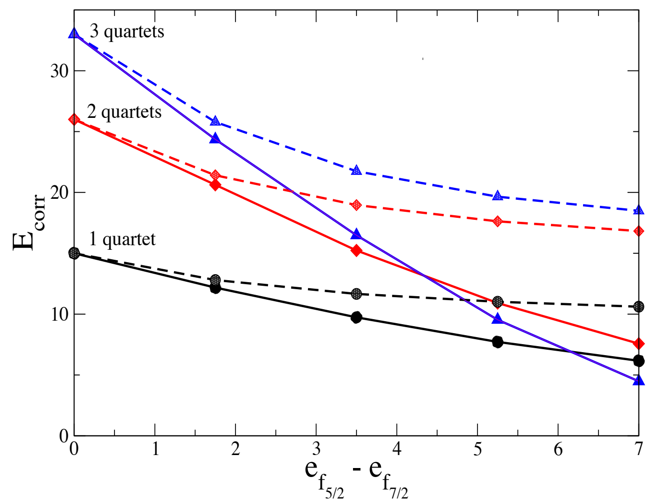


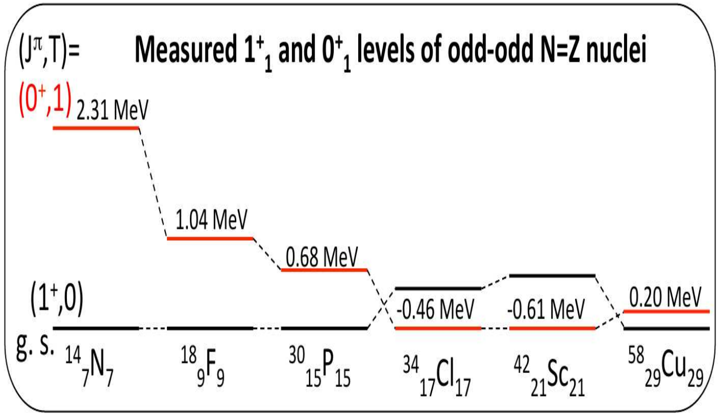
FIG. 1: Correlation energies (in MeV) provided by the QCM approach in corresponence with the Hamiltonian (1) for 1, 2 and 3 quartets moving in the orbits $f_{7/2}$ and $f_{5/2}$. Dashed lines refer to the isovector Hamiltonian (g_1=-1, g_0=0) while full lines refer to the isoscalar Hamiltonian (g_1=0, g_0=-1). On the horizontal axis we show the spin-orbit energy splitting between the two orbits (in MeV).

M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

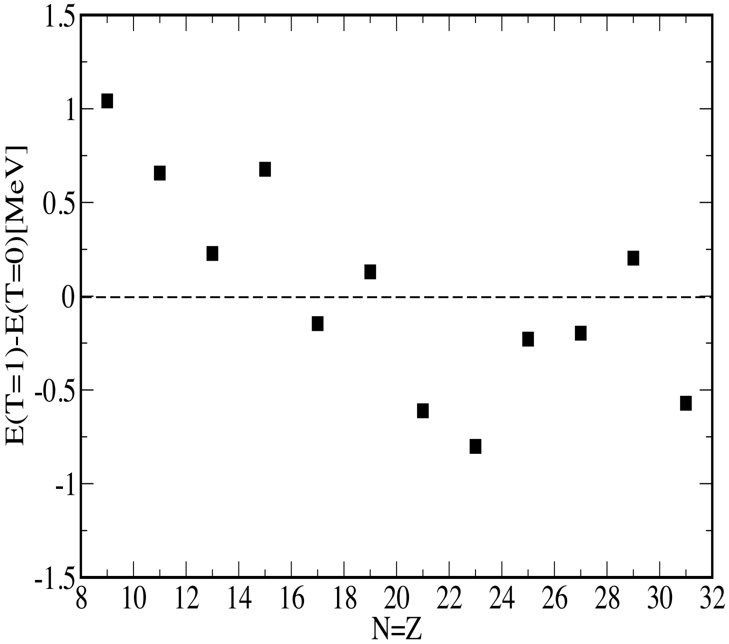
Isovector and isoscalar pairing in odd-odd N=Z

Low Lying States in Odd-Odd Z=N nuclei

- Below A = 34, the g.s. has T = 0, the T = 1, 0^+ state becomes progressively disfavoured.



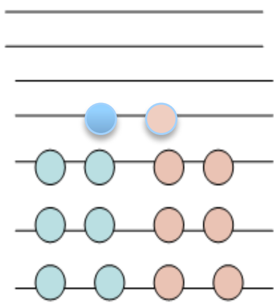
From Y. Tanimura, H. Sagawa, K. Hagino, PTEP, 053D02, (2014)



strong T=0 pairing for odd-odd N=Z nuclei with A< 40 ?

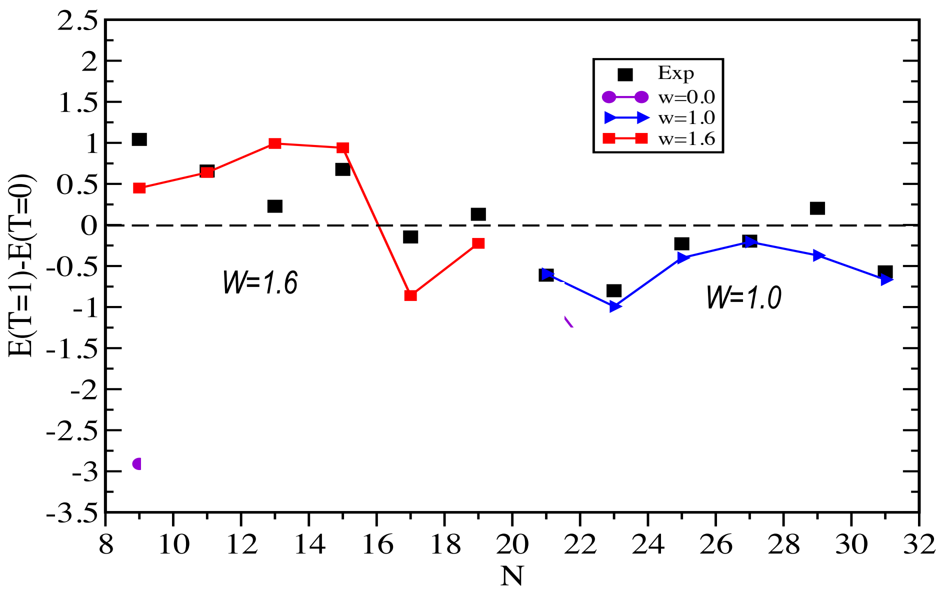
Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$



T=1 state $\quad | iv; QCM >= \tilde{\Gamma}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} \, |- >$

T=0 state $\quad | is; QCM >= \tilde{\Delta}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} \, |- >$



$$V^{T=\{0,1\}}_{paring} = V^{T=\{0,1\}}_{0} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}} \qquad w = \frac{V^{T=0}_{0}}{V^{T=1}_{0}}$$

what we can learn about
the structure of the states ?

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

The structure of lowest T=0 and T=1 states

T=0 ground state

		Exact	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$(\Delta^{+}_{\nu\pi})^{2n_q+1}$	$\tilde{\Delta}^{+}_{\nu\pi}(\Gamma^{+2}_{\nu\pi})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

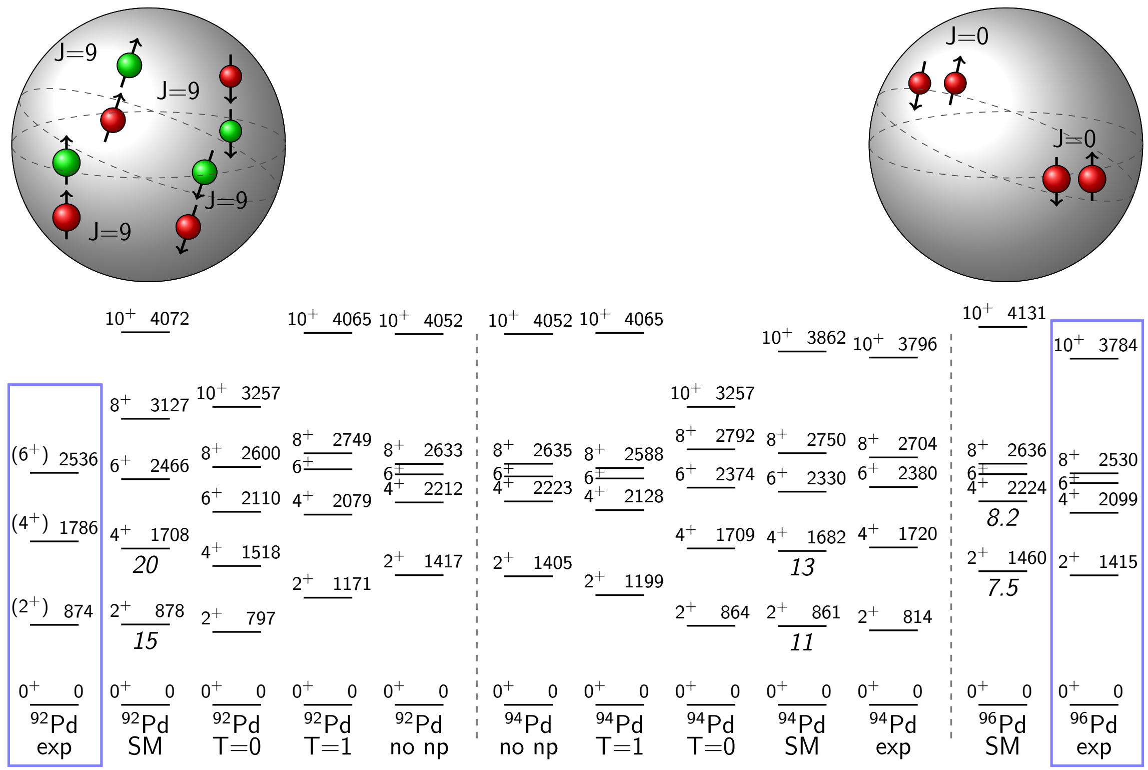
		Exact	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(\Delta^{+2}_{\nu\pi})^{n_q}$	$(\Gamma^{+}_{\nu\pi})^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

conclusion

isovector correlations are stronger in both T=0 and T=1 low-lying states

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

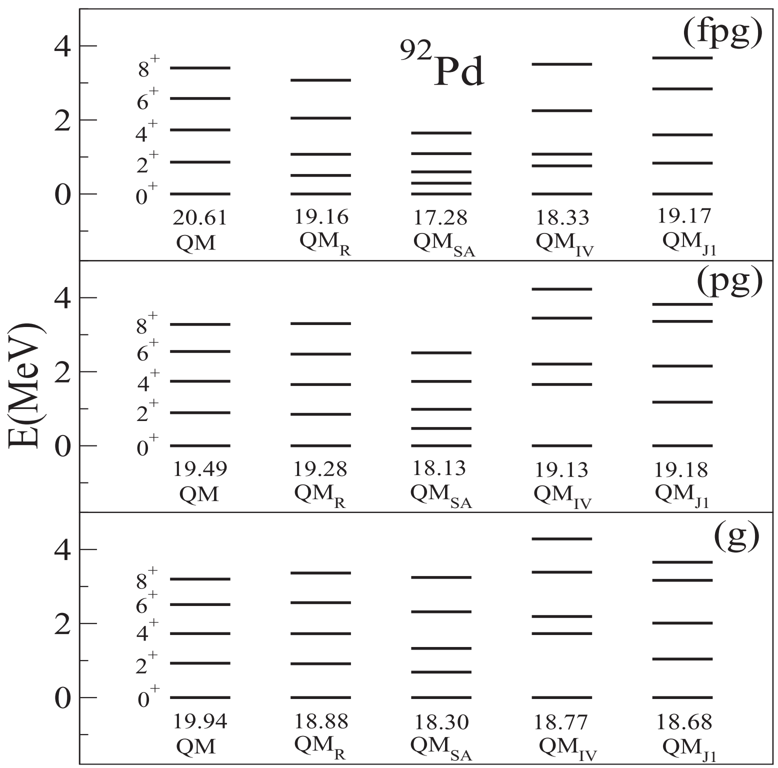
Spin-aligned J=9 pairs in ^{92}Pd ?

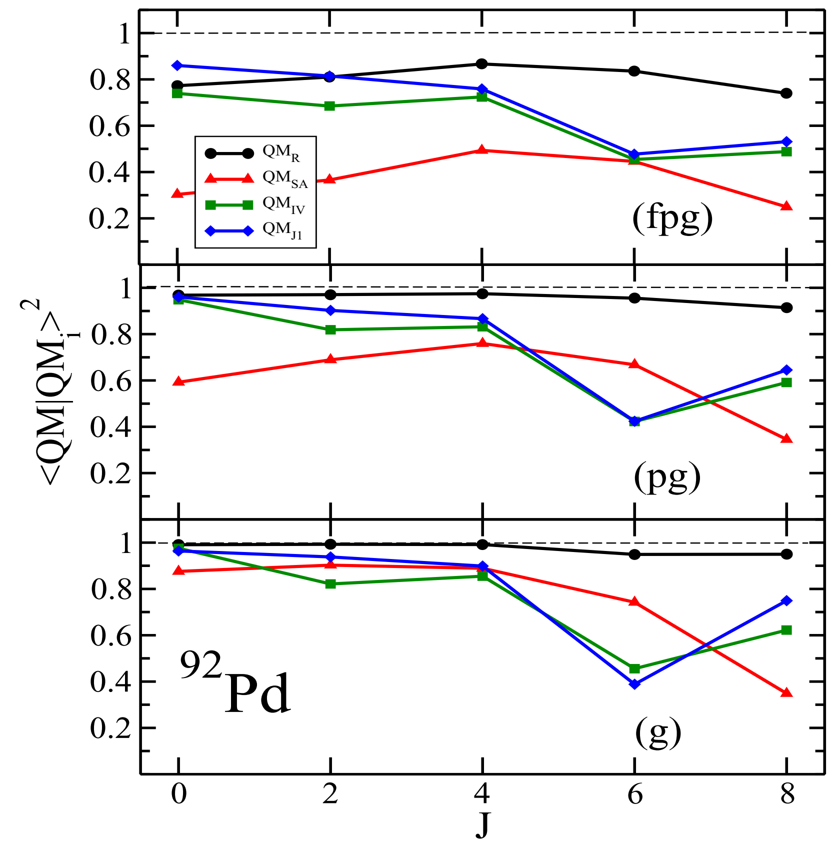


B. Cederwall et al, Nature 469 (2011)68

Role of spin-aligned pairs in ^{92}Pd

$$Q^+_{\alpha,JM,TT_z} = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C^{(\alpha)}_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2} \times [[a^+_{i_1} a^+_{j_1}]^{J_1 T_1} [a^+_{i_2} a^+_{j_2}]^{J_2 T_2}]^{JT}_{MT_z}, \qquad [Q^+_{\alpha_1,J',T'} \otimes Q^+_{\alpha_2,J'',T''}]^{J,T},$$



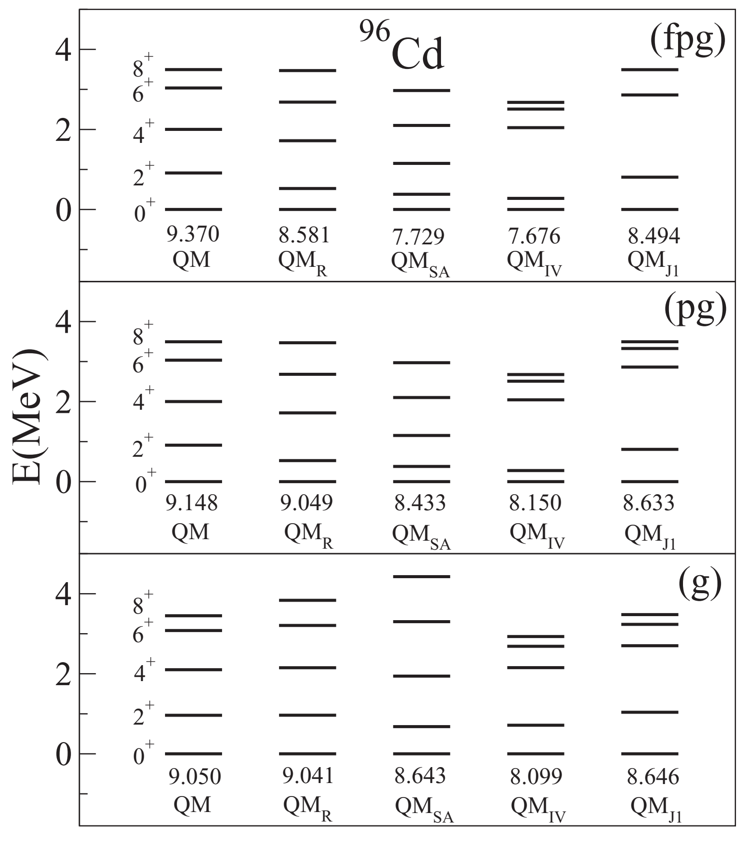


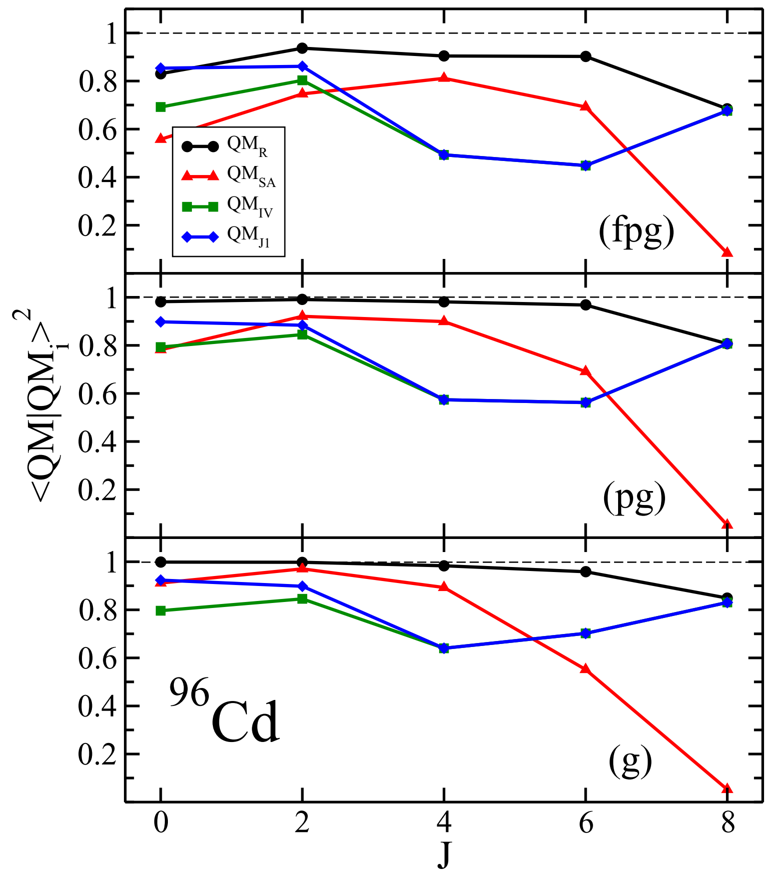
the structure of ^{92}Pd **is not** dominated by J=9 pairs

ground state is mainly built by J=0 and J=1 pairs

M. Sambataro and N. S, PRC91 (2015)

Role of spin-aligned pairs in ^{96}Cd





M. Sambataro and N. Sandulescu, PRC91 (2015)