Proton-neutron pairing and quartetting in odd-odd N=Z nuclei

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Outline of this talk

1. Introduction

2. Isovector-isoscalar pairing correlations in N=Z even-even nuclei

- isoscalar contribution to the quartets' structure;
- competition between isovector and isoscalar pairing in realistic calculations;
- application for N=Z nuclei with axially deformed symmetry.

3. Generalization of the isovector-isoscalar quartet model for odd-odd nuclei

- proton-neutron pairing and quartetting in odd-odd nuclei;
- competition between isovector and isoscalar pairing in odd-odd nuclei.

4. Conclusions and perspectives

Alpha-like quartet condensation offers very accurate results for isovector pairing correlations in even-even N=Z and N>Z nuclei (errors < 1%).

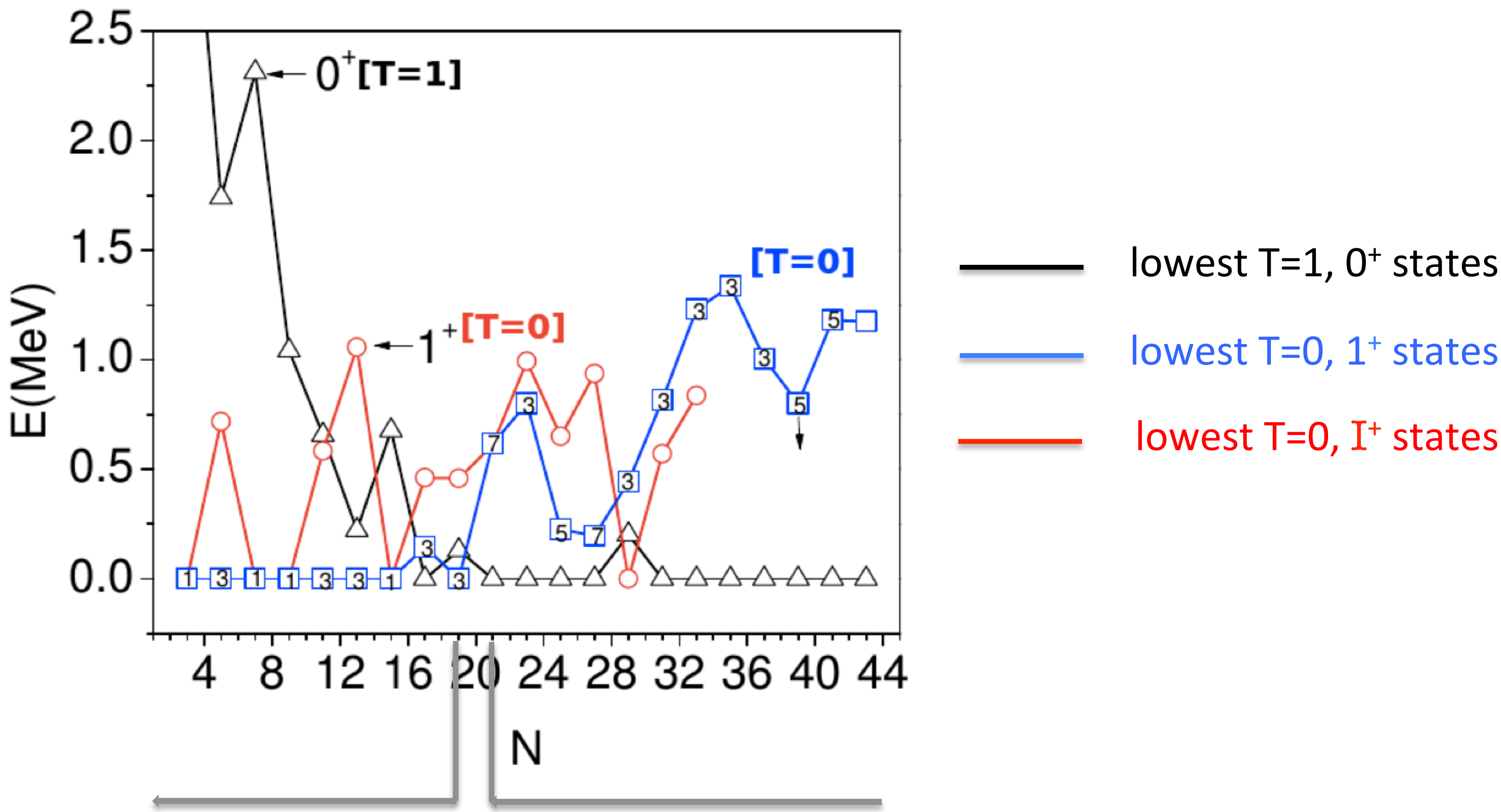
What about the isoscalar proton-neutron pairing?

Does exists such kind of pairing in N=Z nuclei?

How it competes with the isovector pairing?

Proton-neutron pairing in odd-odd nuclei

S. Frauendorf and A. O. Macchiavelli, Progr. Part. Nucl. Phys. 78, 24 (2014)

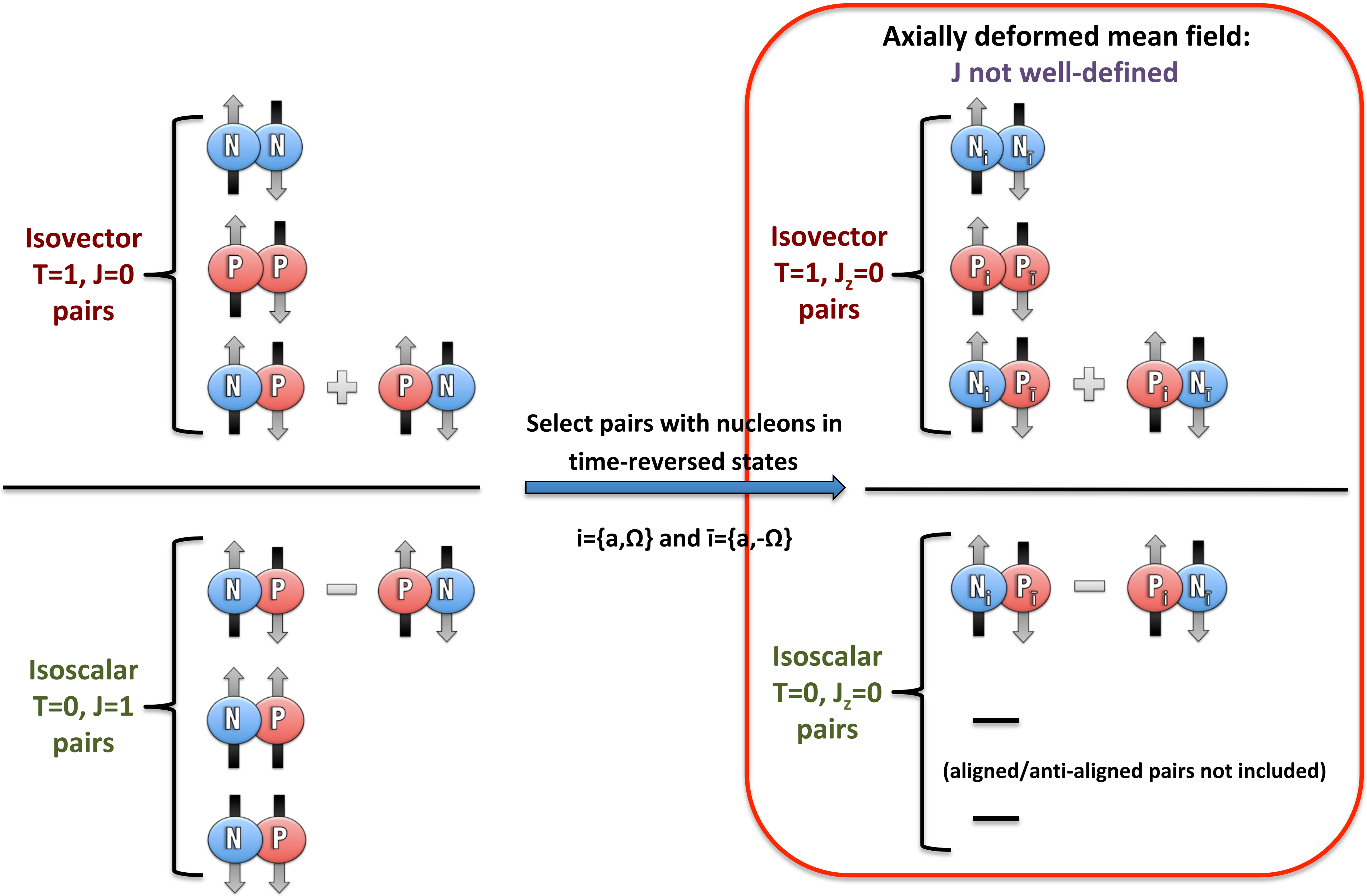


A<40
T=0 ground state

A≥40
T=1 ground state

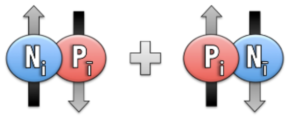
Competition between T=1 and T=0 pairing !

Isovector and isoscalar pairs in N=Z nuclei

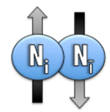


Pairing in even-even N=Z nuclei: axially deformed symmetry

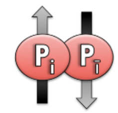
Isovector T=1, J_z=0 pairs



$$P^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}})$$

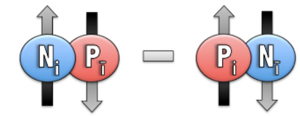


$$P^+_{i,1} = \nu^+_i \nu^+_{\bar{i}}$$



$$P^+_{i,-1} = \pi^+_i \pi^+_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$$

Hamiltonian

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}}_{\text{isoscalar}}$$

Isovector quartets

$$A^+ = \underbrace{2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0}_{\Gamma^+_t = \sum_i x_i P^+_{i,t}}$$

Isovector-isoscalar quartet

$$Q^+ = 2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0$$

Collective isoscalar pairs

$$\Delta^+_0 = \sum_i y_i D^+_{i,0}$$

$$\Downarrow$$

Quartet condensate

$$\boxed{|QCM\rangle = (Q^+)^{n_q} |0\rangle}$$

$n_q = (N+Z)/4$

(exact solution for a set of degenerate states)

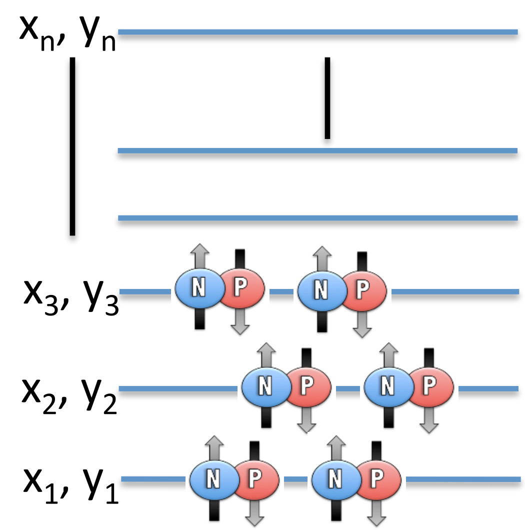
Calculation scheme

Hamiltonian: $$\hat{H} = \sum_{i,\tau=\pm1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}$$

Quartet condensate: $$|QCM\rangle = (Q^+)^{n_q}|0\rangle = (2\underbrace{\Gamma_1^+\Gamma_{-1}^+ - \Gamma_0^{+2}} + \underbrace{\Delta_0^{+2}})^{n_q}|0\rangle$$

$$\Gamma_t^+ = \sum_i x_i P^+_{i,t} \quad \Delta_0^+ = \sum_i y_i D^+_{i,0}$$

Unknown parameters: **mixing amplitudes x_i and y_i**



Minimization: $$\delta_{x,y}\underbrace{\langle\Psi|\hat{H}|\Psi\rangle} = 0$$

Constraint: $$\underbrace{\langle\Psi|\Psi\rangle} = 1$$

The method of recurrence relations

Auxiliary states: $$|n_1 n_2 n_3 n_4\rangle = \underset{\substack{\Uparrow \\ \text{nn}}}{(\Gamma_1^+)^{n_1}}\underset{\substack{\Uparrow \\ \text{pp}}}{(\Gamma_{-1}^+)^{n_2}}\underset{\substack{\Uparrow \\ \text{pn(T=1)}}}{(\Gamma_0^+)^{n_3}}\underset{\substack{\Uparrow \\ \text{pn(T=0)}}}{(\Delta_0^+)^{n_4}}|0\rangle$$

Competition between T=1 and T=0 pairing in realistic calculations

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j)\, D^{+}_{i,0} D_{j,0}$$

- s.p. states given by axially deformed Skyrme-HF calculations

$$V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = V_0^{T=\{0,1\}} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}} \quad \begin{cases} V_0^{T=1} = 465 \text{ MeV fm}^{-3} \\ V_0^{T=0}/V_0^{T=1} = 1.5 \end{cases}$$

(Bertsch et al.)

- zero range delta interaction

$$|QCM\rangle = (A^{+} + \Delta_0^{+2})^{n_q}|0\rangle \quad |iv\rangle = (A^{+})^{n_q}|0\rangle \quad |is\rangle = (\Delta_0^{+2})^{n_q}|0\rangle$$

		Exact	|QCM>	$\|iv\rangle$	$\|is\rangle$	$\langle iv\|is\rangle$
^{16}O	^{20}Ne	11.38	11.38 (0.00%)	11.31 (0.62%)	10.92 (4.00%)	0.976
	^{24}Mg	19.32	19.31 (0.03%)	19.18 (0.74%)	18.93 (2.00%)	0.980
	^{28}Si	18.74	18.74 (0.01%)	18.71 (0.14%)	18.54 (1.07%)	0.992
^{40}Ca	^{44}Ti	7.095	7.094 (0.02%)	7.08 (0.18%)	6.30 (10.78%)	0.928
	^{48}Cr	12.78	12.76 (0.1%)	12.69 (0.67%)	12.22 (4.37%)	0.936
	^{52}Fe	16.39	16.34 (0.26%)	16.19 (1.17%)	15.62 (4.65%)	0.946
^{100}Sn	^{104}Te	4.53	4.52 (0.06%)	4.49 (0.82%)	4.02 (11.26%)	0.955
	^{108}Xe	8.08	8.03 (0.61%)	7.96 (1.45%)	6.75 (16.47%)	0.814
	^{112}Ba	9.36	9.27 (0.93%)	9.22 (1.43%)	7.50 (19.81%)	0.784

Correlation energies (MeV)

$$E_{corr} = E_0 - E$$

Conclusions:

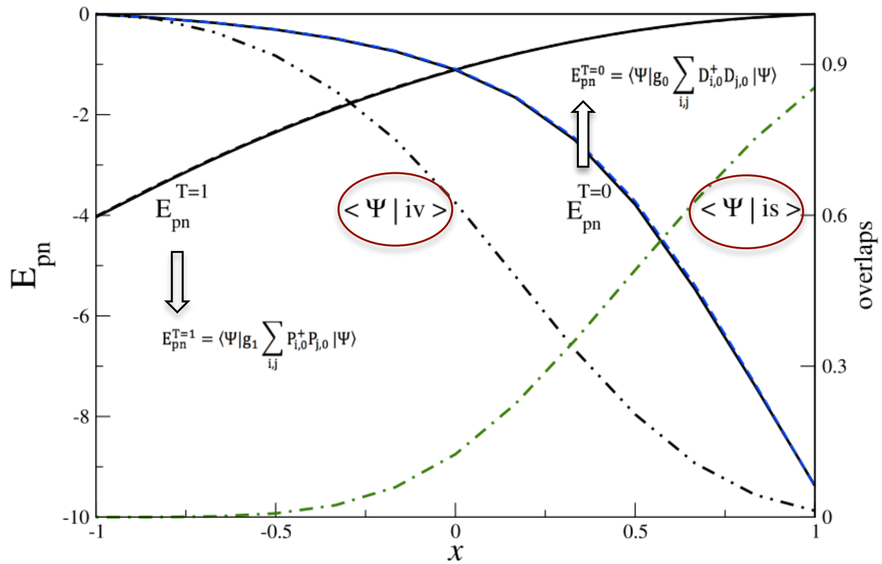
- QCM describes with very good precision the isoscalar-isovector pairing (errors under 1%);
- isovector pairing correlations are stronger than the isoscalar ones;
- isoscalar pairing ALWAYS coexist with the isovector pairing.

N. Sandulescu, D. N. and D. Gambacurta, Phys. Lett. B751 (2015), p. 348

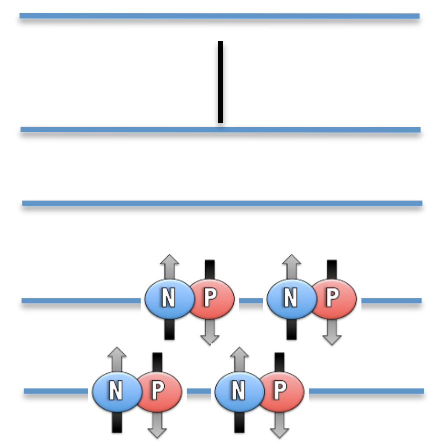
Evolution of the isovector and isoscalar proton-neutron pairing correlations

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + g_1 \sum_{i,j} \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + g_0 \sum_{i,j} D^{+}_{i,0} D_{j,0} \qquad \begin{aligned} g_1 &= g(1-x)/2 \\ g_0 &= g(1+x)/2 \end{aligned}$$

$$|\Psi\rangle = (A^{+} + \Delta_0^{+2})^{n_q}|0\rangle \qquad |iv\rangle = (A^{+})^{n_q}|0\rangle \qquad |is\rangle = (\Delta_0^{+2})^{n_q}|0\rangle$$



- 4 proton-neutron pairs;
- 10 equidistant levels.



Pairing energies:

- E(T=1) and E(T=0) follow very well the exact pairing energies (obtained by diagonalization);
- isovector and isoscalar pairing correlations coexist for any ratio between the strengths of the two pairing forces.

Overlaps:

- the overlaps show a smooth transition from a condensate of quartets to a condensate of pairs.

N. Sandulescu, D. N. and D. Gambacurta, Phys. Lett. B751 (2015), p. 348

How does the proton-neutron pairing affects the ground state of odd-odd N=Z nuclei?

⇓

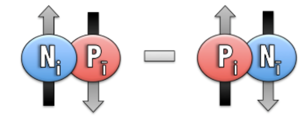
Generalization of QCM for odd-odd systems!

Pairing and quartetting in odd-odd N=Z nuclei

Isovector T=1, J_z=0 pairs

$$P^{+}_{i,0}=\frac{1}{\sqrt{2}}(\nu^{+}_{i}\pi^{+}_{\bar{i}}+\pi^{+}_{i}\nu^{+}_{\bar{i}}) \qquad P^{+}_{i,1}=\nu^{+}_{i}\nu^{+}_{\bar{i}} \qquad P^{+}_{i,-1}=\pi^{+}_{i}\pi^{+}_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^{+}_{i,0}=\frac{1}{\sqrt{2}}(\nu^{+}_{i}\pi^{+}_{\bar{i}}-\pi^{+}_{i}\nu^{+}_{\bar{i}})$$

Hamiltonian

$$\hat{H}=\sum_{i,\tau=\pm 1/2}\varepsilon_{i,\tau}N_{i,\tau}+\underbrace{\sum_{i,j}V^{T=1}(i,j)\sum_{t_z=-1,0,1}P^{+}_{i,t_z}P_{j,t_z}}_{\text{isovector}}+\underbrace{\sum_{i,j}V^{T=0}(i,j)D^{+}_{i,J_z=0}D_{j,J_z=0}}_{\text{isoscalar}}$$

$$\Delta^{+}_{0}=\textstyle\sum_i y_i\, D^{+}_{i,0} \qquad \underbrace{Q^{+}=2\Gamma^{+}_{1}\Gamma^{+}_{-1}-\Gamma^{+2}_{0}+\Delta^{+2}_{0}}_{\text{Isovector-isoscalar quartets}} \qquad \Gamma^{+}_{t}=\textstyle\sum_i x_i\, P^{+}_{i,t}$$

T=0 state: $\boxed{|is,QCM\rangle=\tilde{\Delta}^{+}_{0}(Q^{+})^{n_q}|0\rangle}$

(exact solution for degenerate states)

collective isoscalar odd pair $\tilde{\Delta}^{+}_{0}=\sum_i z_i\, D^{+}_{i,0}$

T=1 state: $\boxed{|iv,QCM\rangle=\tilde{\Gamma}^{+}_{0}(Q^{+})^{n_q}|0\rangle}$

(exact solution for degenerate states)

collective isovector odd pair $\tilde{\Gamma}^{+}_{0}=\sum_i z_i\, P^{+}_{i,0}$

Calculation scheme

Hamiltonian: $\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}$

Pair-quartet condensate:

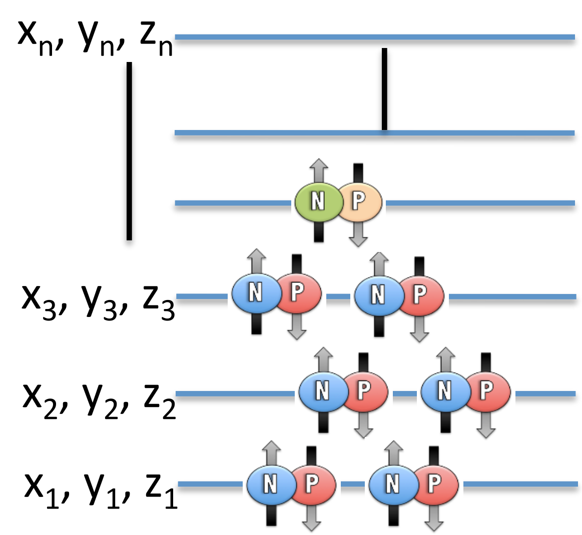
(T=0 state)

$|is, QCM\rangle = \tilde{\Delta}^+_0 (2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0)^{n_q} |0\rangle \qquad \tilde{\Delta}^+_0 = \Sigma_i z_i D^+_{i,0} \qquad \Gamma^+_t = \Sigma_i x_i P^+_{i,t}$

$|iv, QCM\rangle = \tilde{\Gamma}^+_0 (2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0)^{n_q} |0\rangle \qquad \tilde{\Gamma}^+_0 = \Sigma_i z_i P^+_{i,0} \qquad \Delta^+_0 = \Sigma_i y_i D^+_{i,0}$

(T=1 state)

Unknown parameters: mixing amplitudes xi, yi and z_i



Minimization: $\delta_{x,y,z} \underbrace{\langle \Psi | \hat{H} | \Psi \rangle} = 0$

Constraint: $\underbrace{\langle \Psi | \Psi \rangle} = 1$

The method of recurrence relations

Auxiliary states:

$|n_1 n_2 n_3 n_4 n_5\rangle = (\Gamma^+_1)^{n_1} (\Gamma^+_{-1})^{n_2} (\Gamma^+_0)^{n_3} (\Delta^+_0)^{n_4} \tilde{\Delta}^+_0 |0\rangle$ (T=0 state)

$|m_1 m_2 m_3 m_4 m_5\rangle = (\Gamma^+_1)^{m_1} (\Gamma^+_{-1})^{m_2} (\Gamma^+_0)^{m_3} (\Delta^+_0)^{m_4} \tilde{\Gamma}^+_0 |0\rangle$ (T=1 state)

Strength of the pairing force in T=1 and T=0 channels

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$$

- s.p. states given by Skyrme-HF calculations for axially deformed m.f.

- zero range delta interaction $V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = V_0^{T=\{0,1\}} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$

Strength of the isovector pairing force

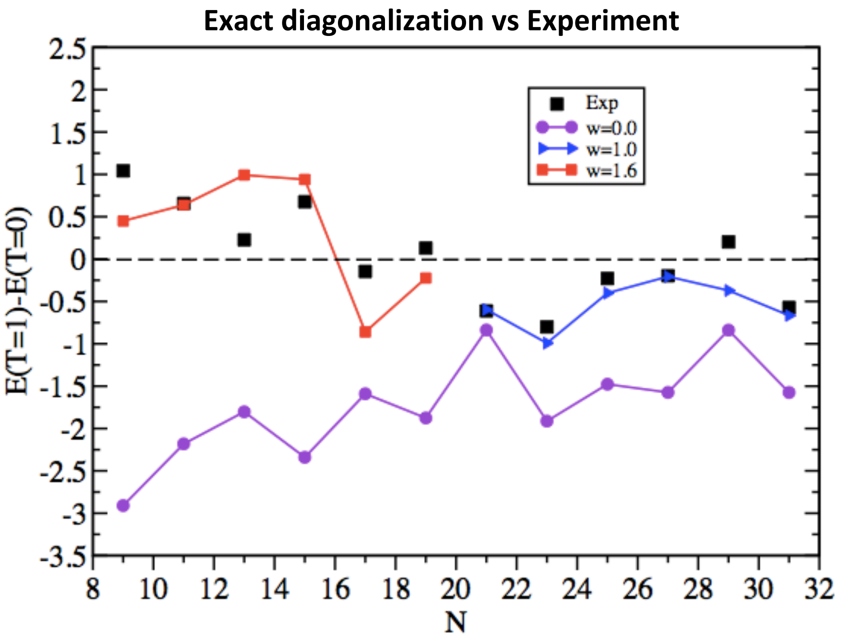
$$V_0^{T=1} = 465 \ \text{MeV fm}^{-3}$$

Strength of the isoscalar pairing force

$$V_0^{T=0} = w \cdot V_0^{T=1}$$

w = ?

A<40: w=1.6 A≥40: w=1.0



The structure of the lowest T=0 and T=1 states of odd-odd nuclei

Correlation energies (MeV): $E_{corr} = E_0 - E$

T=0 ground state

		Exact	$\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Delta}_0^+(Q_{iv}^+)^{n_q}$	$(\Delta_0^+)^{2n_q+1}$	$\tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

		Exact	$\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Gamma}_0^+(Q_{iv}^+)^{n_q}$	$\tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}$	$(\Gamma_0^+)^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

D.N., N. Sandulescu, D. Gambacurta, PTEP 2017, 073D05

Conclusions:

QCM describes well the low-lying states of odd-odd nuclei.

The pn pair condensates (isovector or isoscalar) are less accurate than the quartet condensates.

Isovector and isoscalar pairing correlations coexist in the even-even core.

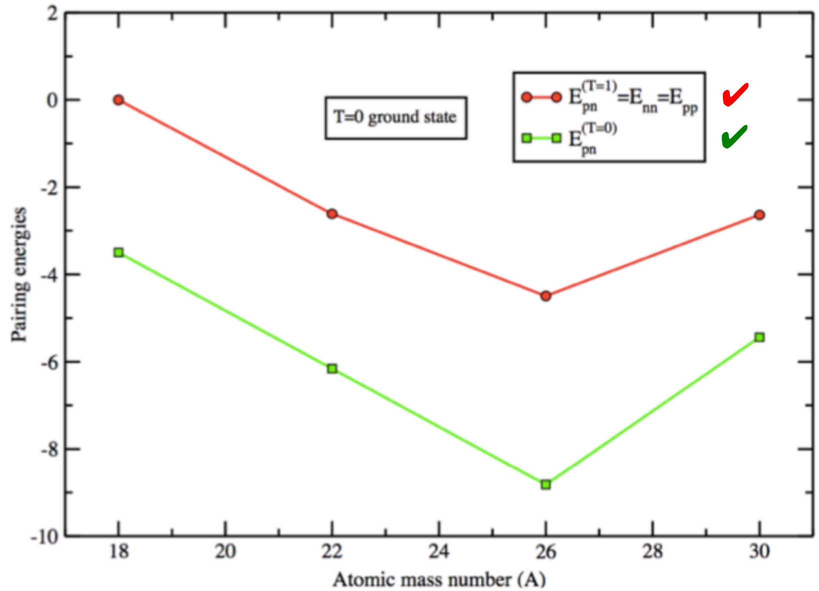
^{50}Mn	**Exact**	$\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Gamma}_0^+(Q_{iv}^+)^{n_q}$	$\tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}$	$(\Gamma_0^+)^{2n_q+1}$
$T = 1$	12.77	12.76 (0.07%)	12.75 (0.14%)	12.52 (2.02%)	12.62 (1.22%)
$T = 0$	12.37	12.36 (0.04%)	12.34 (0.24%)	12.18 (1.61%)	12.19 (1.48%)
	Exact	$\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Delta}_0^+(Q_{iv}^+)^{n_q}$	$(\Delta_0^+)^{2n_q+1}$	$\tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}$

^{54}Co	$T = 1$	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)
	$T = 0$	15.93	15.92 (0.04%)	15.89 (0.22%)	15.53 (2.56%)	15.66 (1.73%)

		g.s./e.s.	Exact	$\lvert iv; QCM\rangle / \lvert is; QCM\rangle$	$\lvert iv; Q_{iv}\rangle / \lvert is; Q_{iv}\rangle$	$\lvert iv; C_{is}\rangle / \lvert C_{is}\rangle$	$\lvert C_{iv}\rangle / \lvert is; C_{iv}\rangle$
^{16}O	^{18}F	T=0	3.365	3.365	3.365	3.365	3.365
		T=1	2.914	2.914	2.914	2.914	2.914
	^{22}Na	T=0	13.869	13.869 (0.00%)	13.859 (0.07%)	13.853 (0.12%)	13.848 (0.15%)
		T=1	13.230	13.226 (0.03%)	13.224 (0.05%)	12.974 (1.97%)	13.216 (0.11%)
	^{26}Al	T=0	22.058	22.052 (0.03%)	22.043 (0.07%)	21.941 (0.53%)	21.789 (1.24%)
		T=1	21.066	21.061 (0.02%)	21.051 (0.07%)	20.929 (0.66%)	20.980 (0.41%)
	^{30}P	T=0	12.655	12.599 (0.44%)	12.547 (0.86%)	11.955 (5.86%)	11.944 (5.95%)
		T=1	11.715	11.664 (0.44%)	11.620 (0.82%)	10.937 (7.11%)	10.955 (6.94%)
^{40}Ca	^{42}Sc	T=1	0.837	0.837	0.837	0.837	0.837
		T=0	0.241	0.241	0.241	0.241	0.241
	^{46}V	T=1	7.922	7.919 (0.04%)	7.914 (0.10%)	7.328 (8.11%)	7.758 (2.11%)
		T=0	6.930	6.929 (0.01%)	6.925 (0.07%)	6.729 (2.99%)	6.791 (2.05%)
	^{50}Mn	T=1	12.774	12.765 (0.07%)	12.756 (0.14%)	12.521 (2.02%)	12.620 (1.22%)
		T=0	12.372	12.367 (0.04%)	12.343 (0.24%)	12.176 (1.61%)	12.192 (1.48%)
	^{54}Co	T=1	16.138	16.116 (0.14%)	16.093 (0.28%)	15.667 (3.01%)	15.856 (1.78%)
		T=0	15.931	15.925 (0.04%)	15.896 (0.22%)	15.533 (2.56%)	15.660 (1.73%)
^{100}Sn	^{102}Sb	T=1	0.104	0.104	0.104	0.104	0.104
		T=0	0.039	0.039	0.039	0.039	0.039
	^{106}I	T=1	5.147	5.143 (0.08%)	5.135 (0.23%)	4.706 (9.37%)	4.925 (4.51%)
		T=0	4.525	4.523 (0.04%)	4.506 (0.42%)	4.196 (7.84%)	4.288 (5.53%)
	^{110}Cs	T=1	8.034	7.989 (0.56%)	7.974 (0.75%)	7.164 (12.14%)	7.589 (5.86%)
		T=0	7.096	7.064 (0.45%)	7.040 (0.80%)	6.472 (9.64%)	6.646 (6.77%)
	^{114}La	T=1	9.758	9.723 (0.36%)	9.687 (0.73%)	8.789 (11.03%)	9.273 (5.23%)
		T=0	8.954	8.929 (0.28%)	8.917 (0.42%)	8.311 (7.74%)	8.513 (5.18%)

T=1 and T=0 pairing energies

A<40

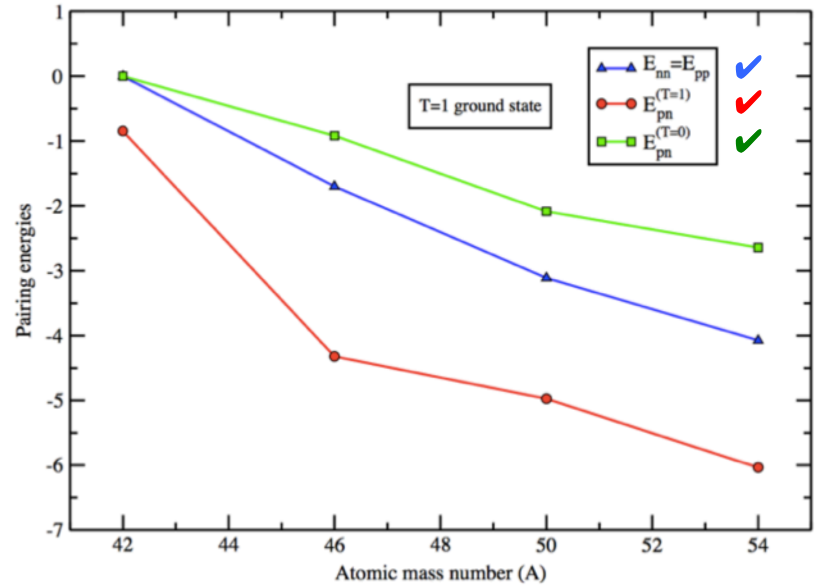


$$E_{t_z}^{(T=1)} = \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} \langle is;QCM|P_{i,t_z}^{+}P_{j,t_z}|is;QCM\rangle$$

$$E_{pn}^{(T=0)} = \sum_{i,j} V^{T=0}(i,j)\,\langle is;QCM|D_{i,0}^{+}D_{j,0}|is;QCM\rangle$$

Extra pairing energy in the T=0 channel: contribution from the odd T=0 pairs

A≥40



$$E_{t_z}^{(T=1)} = \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} \langle iv;QCM|P_{i,t_z}^{+}P_{j,t_z}|iv;QCM\rangle$$

$$E_{pn}^{(T=0)} = \sum_{i,j} V^{T=0}(i,j)\,\langle iv;QCM|D_{i,0}^{+}D_{j,0}|iv;QCM\rangle$$

Extra pairing energy in the T=1 channel: caused not only by the odd T=1 pairs

Main conclusions of this talk

QCM describes with good precision the isovector-isoscalar pairing.

Isoscalar pairing always coexist with the isovector pairing in N=Z even-even nuclei.

Isovector pairing correlations are stronger than the isoscalar ones.

QCM describes very well the low-lying states of odd-odd nuclei.

The isovector and isoscalar pairing correlations coexist also in odd-odd nuclei.

Perspectives

Total angular momentum restoration by standard projection techniques.

Generalization of the QCM formalism to N>Z odd-odd nuclei.

$$(\tilde{\Gamma}_1^+)^{n_N}\left[\tilde{\Gamma}_0^+(\mathrm{Q}_{\mathrm{iv}}^+ + \Delta_0^{+2})^{\mathrm{n_q}}\right]|0\rangle$$ **T=1 state**

$$(\tilde{\Gamma}_1^+)^{n_N}\left[\tilde{\Delta}_0^+(\mathrm{Q}_{\mathrm{iv}}^+ + \Delta_0^{+2})^{\mathrm{n_q}}\right]|0\rangle$$ **T=0 state**