A trial for the general description of shell and cluster structures

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workshop on pn pairing & quartetting at ESNT-Saclay

Nuclear structure

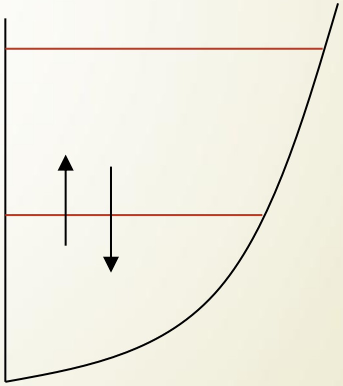
- Shell aspect
- Cluster aspect

Nuclear structure

- Shell aspect:
  single particle motion of each nucleon
- Cluster aspect

jj-coupling shell structure

- Each nucleon performs independent particle motion in this potential with good j = | **L**+**S** | , where the spin-orbit effect is important



Taken from ``The Birth of Venus" by Sandro Botticelli

Nuclear structure

- Shell aspects

  single particle motion of each nucleon
- Cluster aspects

  weakly interacting states of

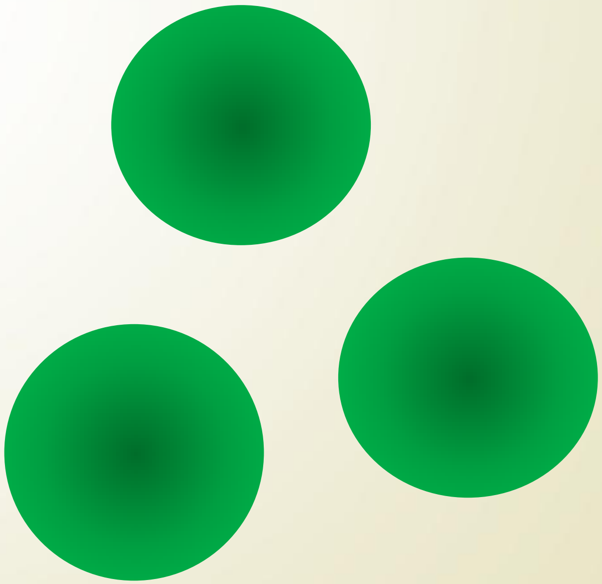
  strongly bound subsystem

a-cluster structure

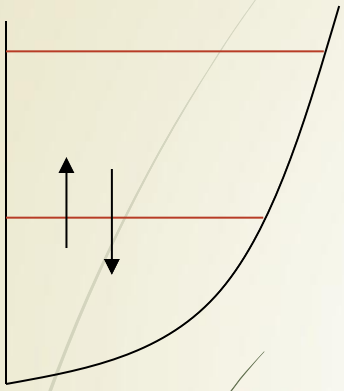
- ^{4}He is strongly bound (B.E. 28.3 MeV) (tensor effect)

→These can be subunits of nuclear systems

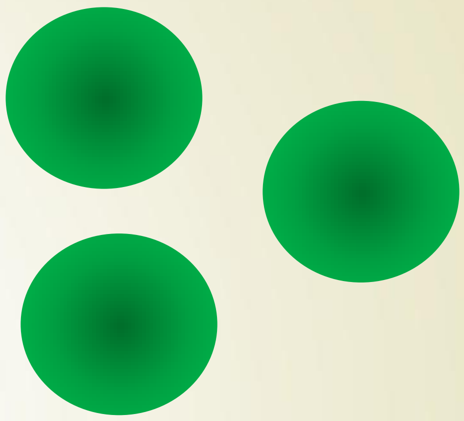
``Haystacks" by Claude Monet



Shell model side



Cluster side



Big computational challenge including modern *ab initio* ones

Cluster states contain large principal quantum numbers of the harmonic oscillator w.f.

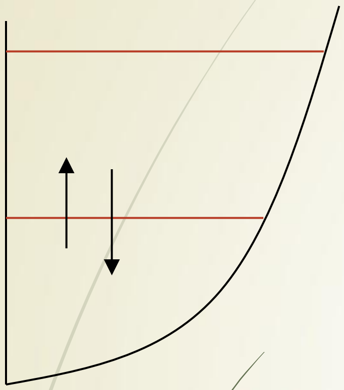
Second 0^+ state of ^{12}C (Hoyle state)

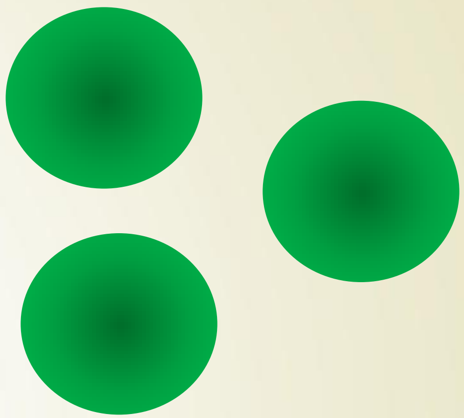
N	8	10	12	14	16	18	20	22
Probability	0.00	0.11	0.12	0.12	0.10	0.08	0.07	0.06

Y. Suzuki, K. Arai, Y. Ogawa, and K. Varga, Phys. Rev. C **54** 2073 (1996)

Shell model side

Cluster side





Big computational challenge

including modern *ab initio* ones

Our strategy

My dream 1 (rather easy)

To show that cluster model wave functions can be transformed into important configurations of the jj-coupling shell model, at least into the lowest configuration

MD wave function with complex Gaussian centers may be promising (there was not so explicit proof)

My dream 2 (final goal)

To obtain the overlap between
the solution of molecular dynamics or
cluster model |MD, CM >
and
arbitrary configuration of the jj-coupling
shell model |jj (arbitrary) >,

< MD, CM | jj (arbitrary) >

our model can be a bridge
between the cluster model
and
jj-coupling shell model

Brink's wave function (1965)

$$\Psi = P[\mathcal{A}(\Phi_1(\mathbf{r}_1)\,\Phi_2(\mathbf{r}_2)\cdot\cdot\cdot\cdot)]$$

P: Angular momentum and parity projection

***A*: Antisymmetrizer**

$$\Phi_i(r) = \exp[-\nu(r_i - R_i)^2]\chi_i$$

Gaussian-center parameter **spin-isospin**

α cluster is expressed as four nucleons (p,p,n,n) sharing the same R value

Similarity between shell model wave functions and cluster wave functions

$$\Phi_1 = \exp[\,-\nu(r - X)^2\,]\,\chi$$

$$\Phi_2 = \exp[\,-\nu(r + X)^2\,]\,\chi$$

$$A\,[\Phi_1\Phi_2] \propto A\,[(\Phi_1+\Phi_2)(\Phi_1 - \Phi_2)]$$

At X → 0

$$(\Phi_1+\Phi_2) \rightarrow \exp[\,-\nu r^2]$$

$$(\Phi_1 - \Phi_2)\,/\,|X| \rightarrow r\,\exp[\,-\nu r^2]$$

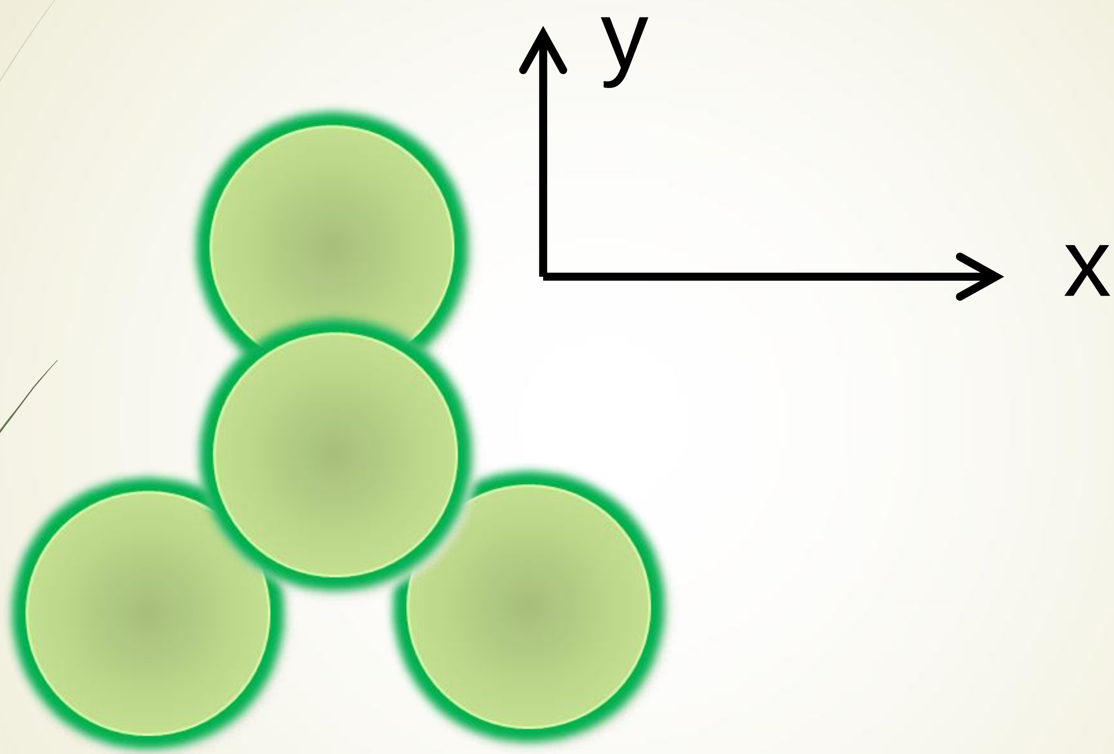
Cluster (local Gaussian) wave function coincides with the lowest shell-model wave function at X → 0

$$\exp[-(x-X)^2] = \sum X^n H_n(x) \exp(-x^2) / n!$$

- Local Gaussian corresponds to the coherent state of many higher orbits of the shell-model

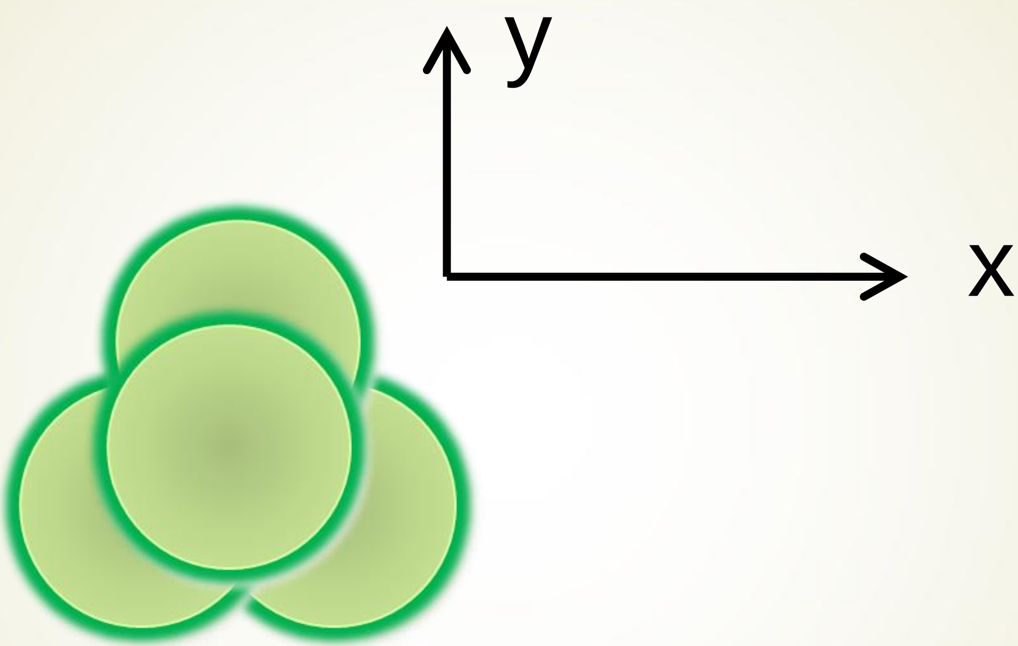
Cluster model partially covers the model space of the shell model

^{16}O



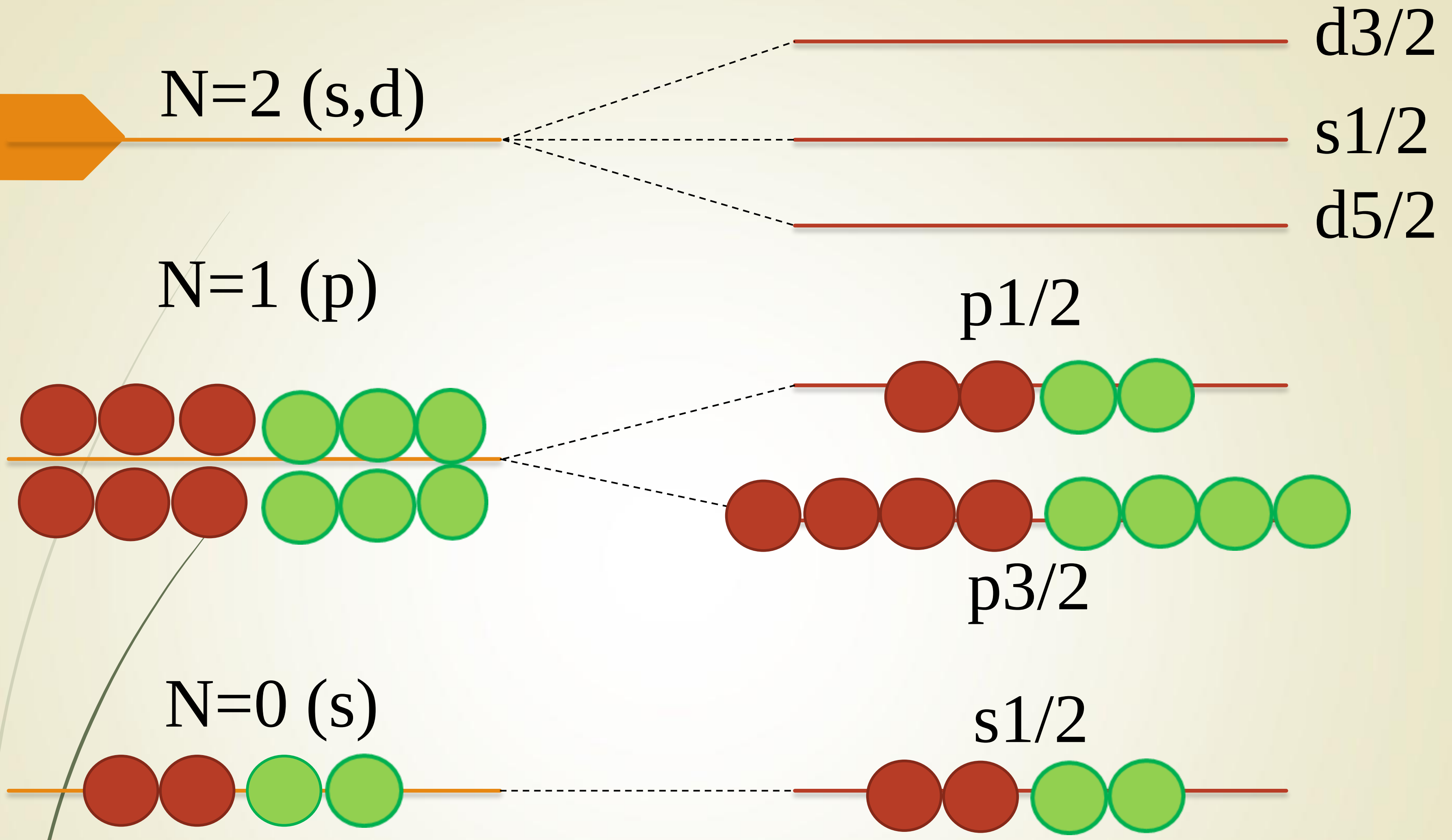
Cluster model partially covers the model space of the shell model

^{16}O



Elliott SU(3) limit

$$(s)^4(p_x)^4(p_y)^4(p_z)^4 = (s_{1/2})^4(p_{3/2})^8(p_{1/2})^4$$



Three dimensional
Harmonic oscillator

jj coupling
Shell Model

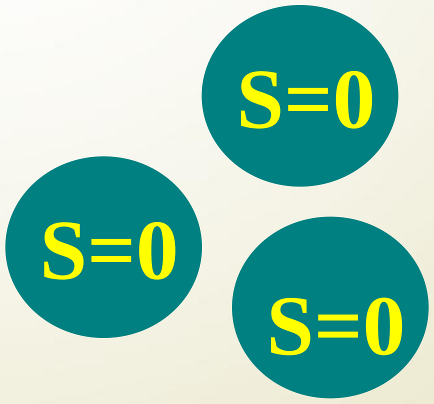
a-cluster model

Each ^{4}He:

$(0s)^4$ configuration at some localized position

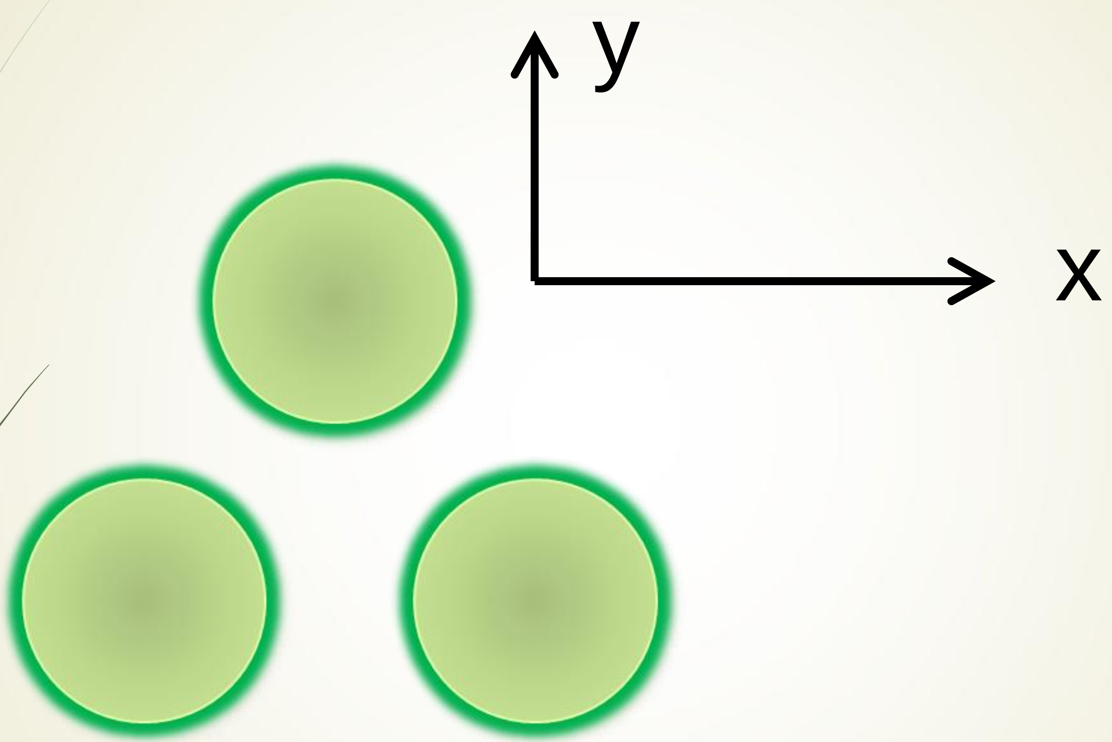
Non-central interactions between nucleons (spin-orbit, tensor) do not contribute

Most important effect is missing
and renormalized
in a different place



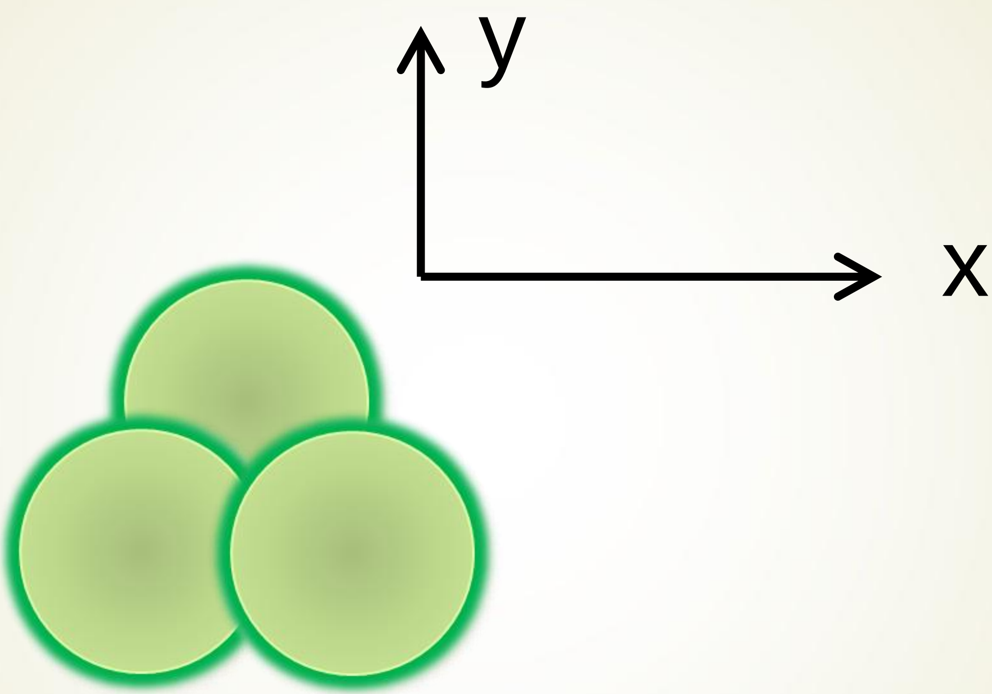
Cluster model partially covers the model space of the shell model

^{12}C

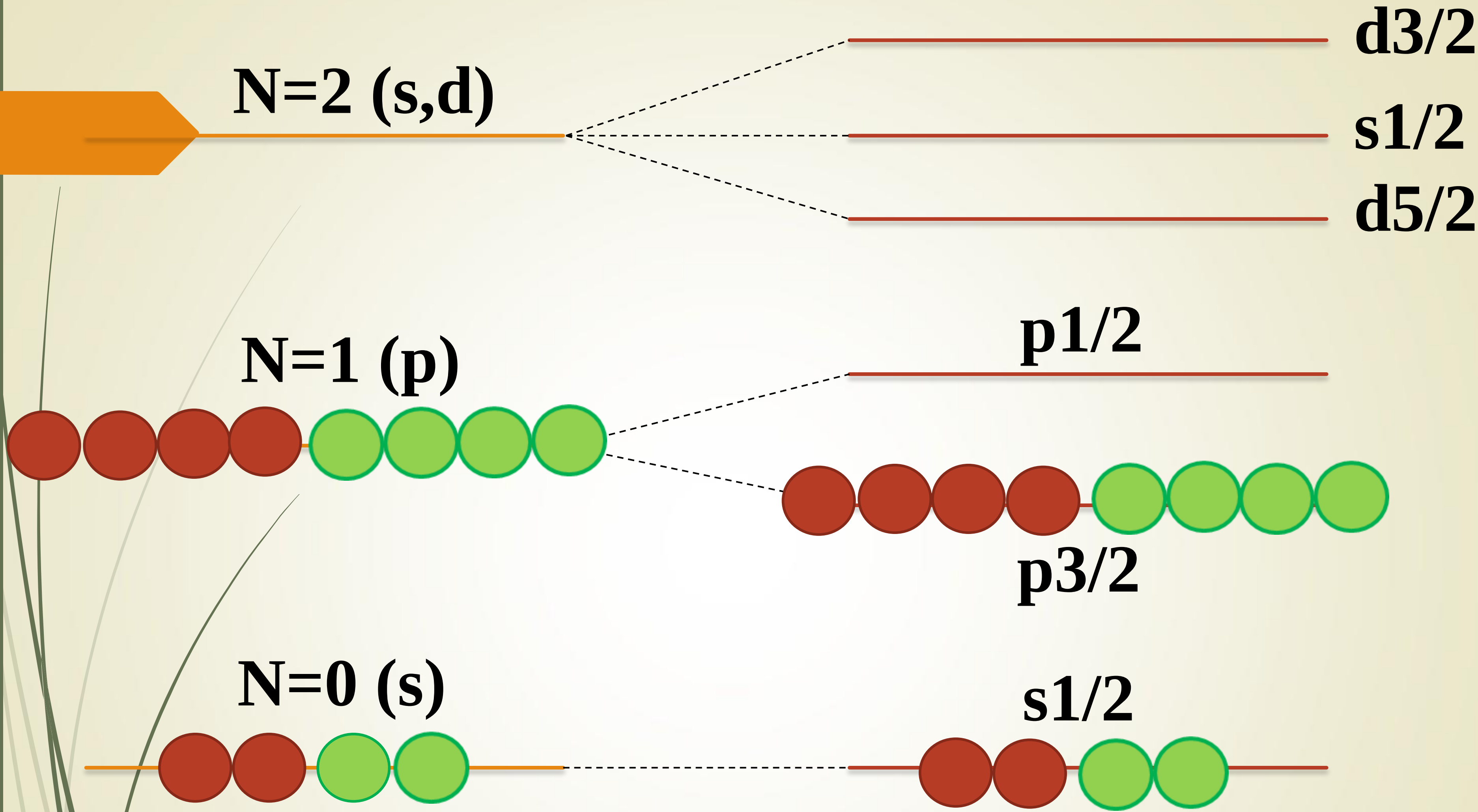


Cluster model partially covers the model space of the shell model

^{12}C



This is $(\mathbf{s})^4(\mathbf{p}_x)^4(\mathbf{p}_y)^4$, but not $(\mathbf{s}_{1/2})^4(\mathbf{p}_{3/2})^8$



Three dimensional Harmonic oscillator

jj coupling Shell Model

Is there simple transformation from cluster model to jj-coupling shell model?

Yes. We introduced

``Antisymmetrized Quasi Cluster Model (AQCM)”

We can transform cluster model to jj-coupling shell model and include the spin-orbit effect by introducing only one parameter, which enables us to discuss the competition of these two different features

How we can include the spin-orbit contribution?

spatial part of the single particle wave function

$$\exp[-\nu(\mathbf{r} - \mathrm{R}i)^2]$$

In the Brink's model,
4 nucleons share the same **R**i value in each α cluster

The spin-orbit interaction: **(r x p) • s**

r → Gaussian center parameter **R**i

p → imaginary part of **R**i

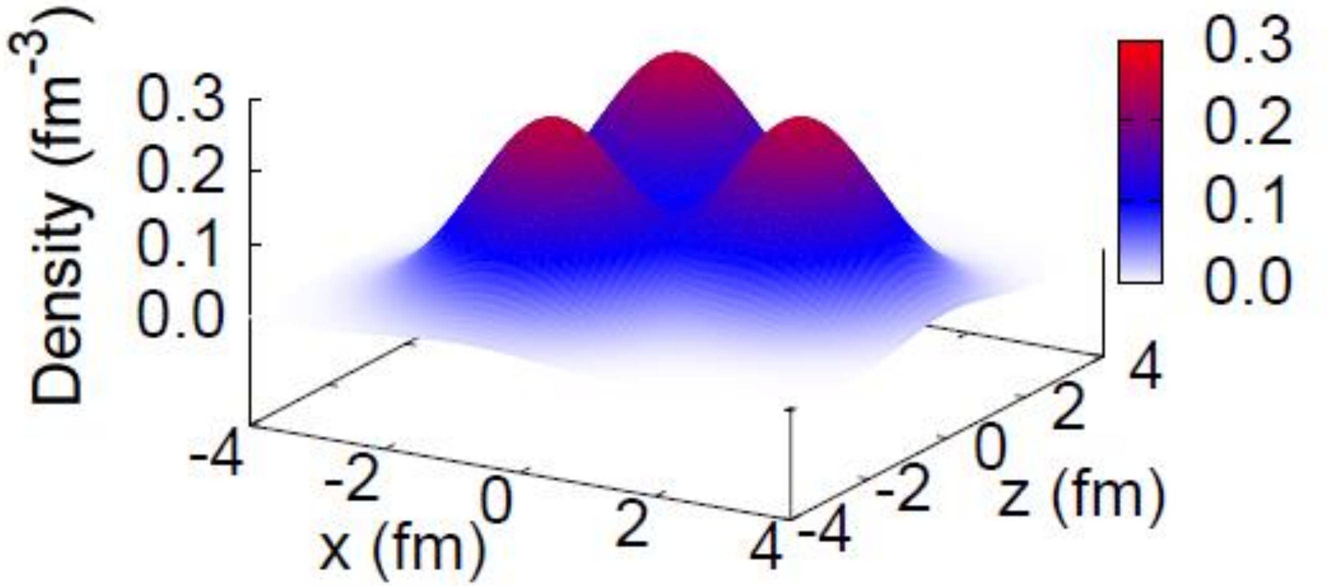
(r x p) • s = (s x r) • p

For the nucleons in α cluster:

Ri → **R**i + i Λ (**e**_spin x **R**i)   **quasi cluster**

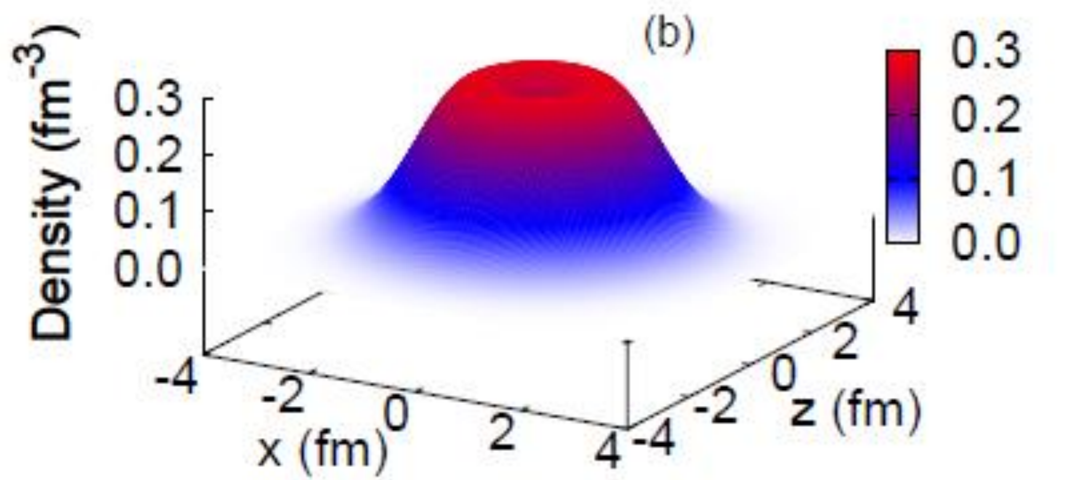
N. Itagaki, H. Masui, M. Ito, and S. Aoyama, Phys. Rev. C **71** 064307 (2005).

R = 3 fm
Λ = 0



^{12}C

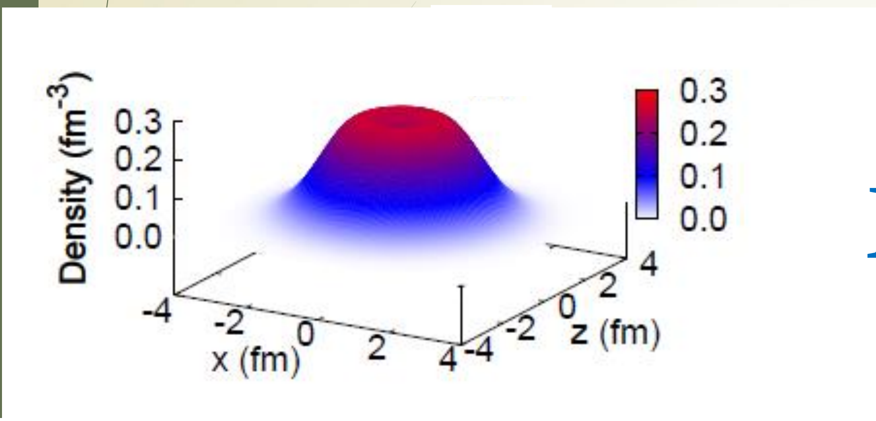
R = 0.01 fm
Λ = 0



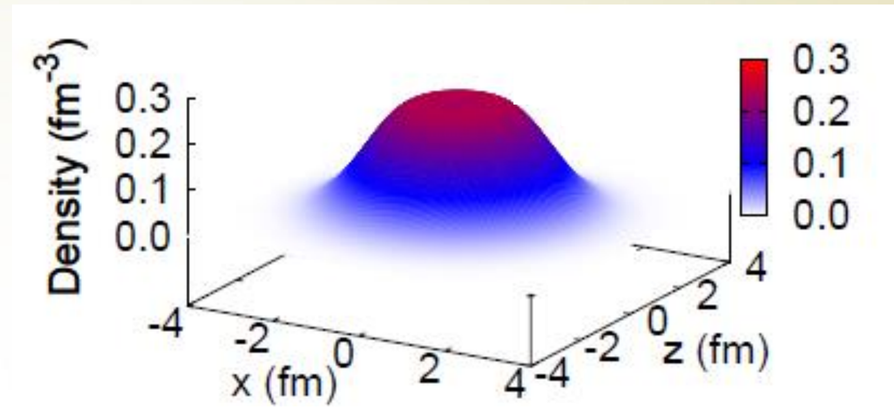
N. Itagaki, H. Matsuno, and T. Suhara
Prog. Theor. Exp. Phys. **2016**, 093D01 (2016).

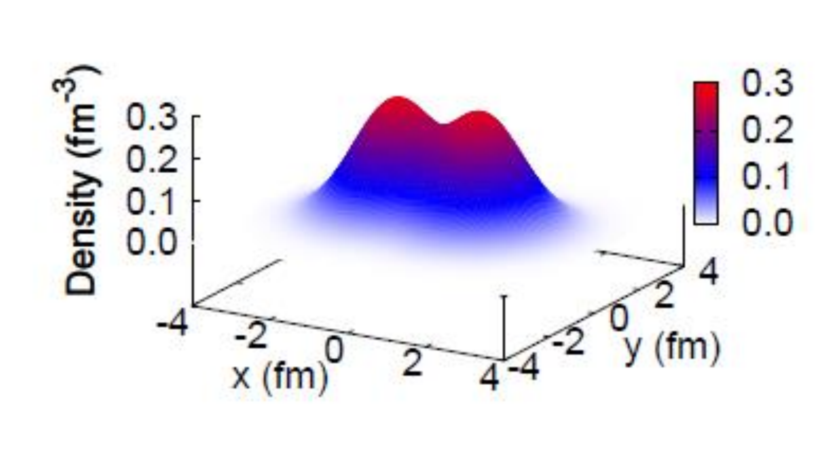
^{12}C R = 0.01 fm

Λ = 0 **Λ = 1**

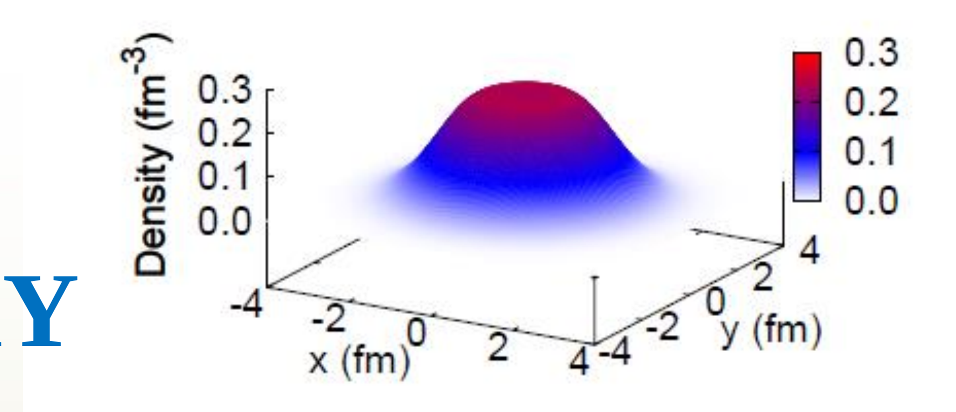


XZ





XY



N. Itagaki, H. Matsuno, and T. Suhara
Prog. Theor. Exp. Phys. **2016**, 093D01 (2016).

jj-coupling shell state

R → 0

Λ → 1

SU(3) limit (3 dim. Harmonic Oscillator)

cluster state

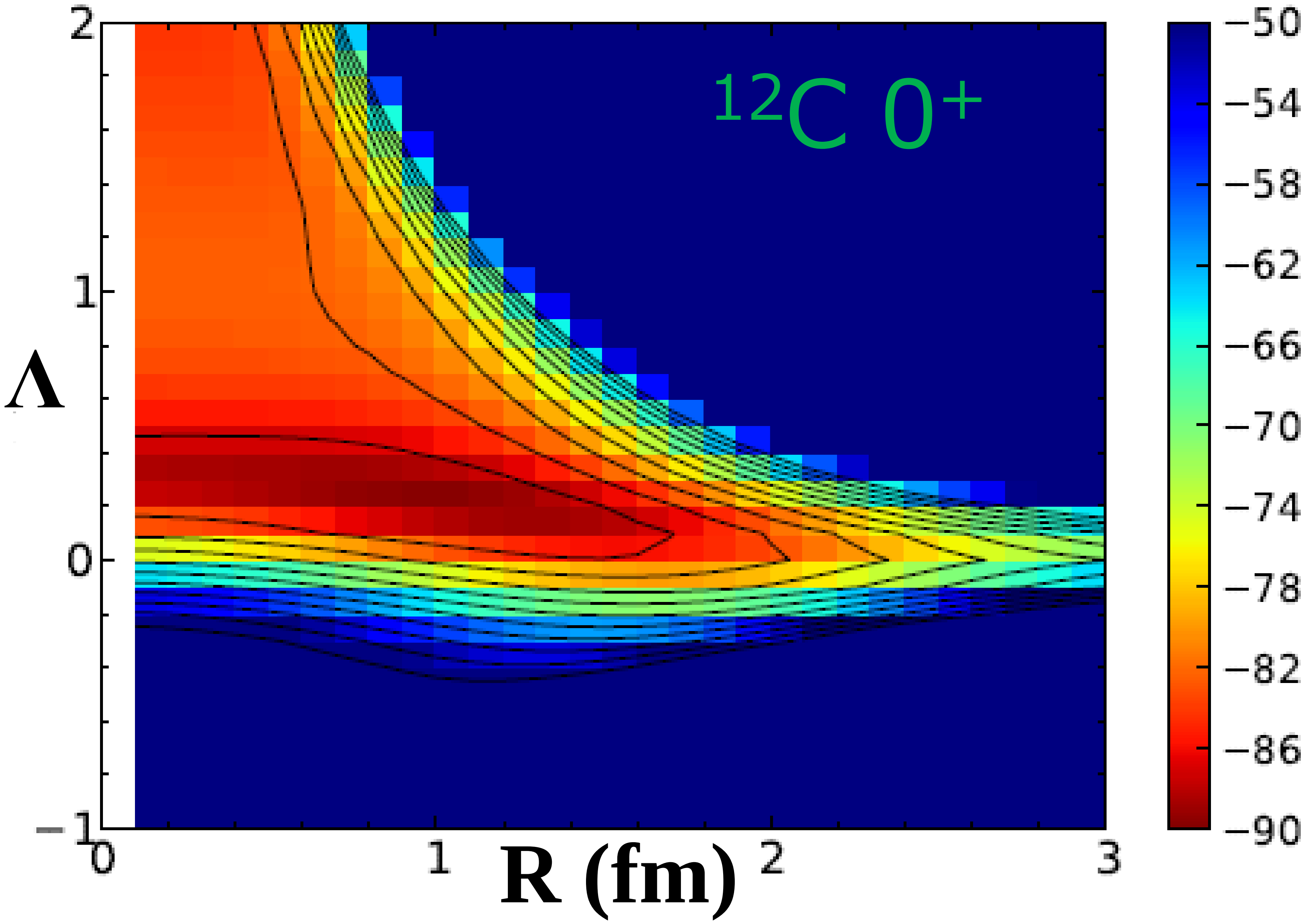
Large R

Zero Λ

increasing Λ ← zero Λ, zero R ← decreasing R

Spin-orbit contribution can be taken into account

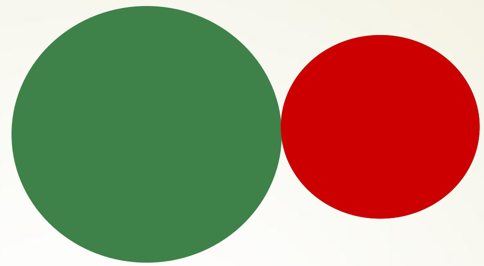
T. Suhara, N. Itagaki, J. Cseh, and M. Ploszajczak, Phys. Rev. C **87** 054334 (2013).



T. Suhara, N. Itagaki, J. Cseh, and M. Ploszajczak, Phys. Rev. C **87** 054334 (2013).

Mathematical explanation of AQCM -- ^{20}Ne case as an example

^{20}Ne case

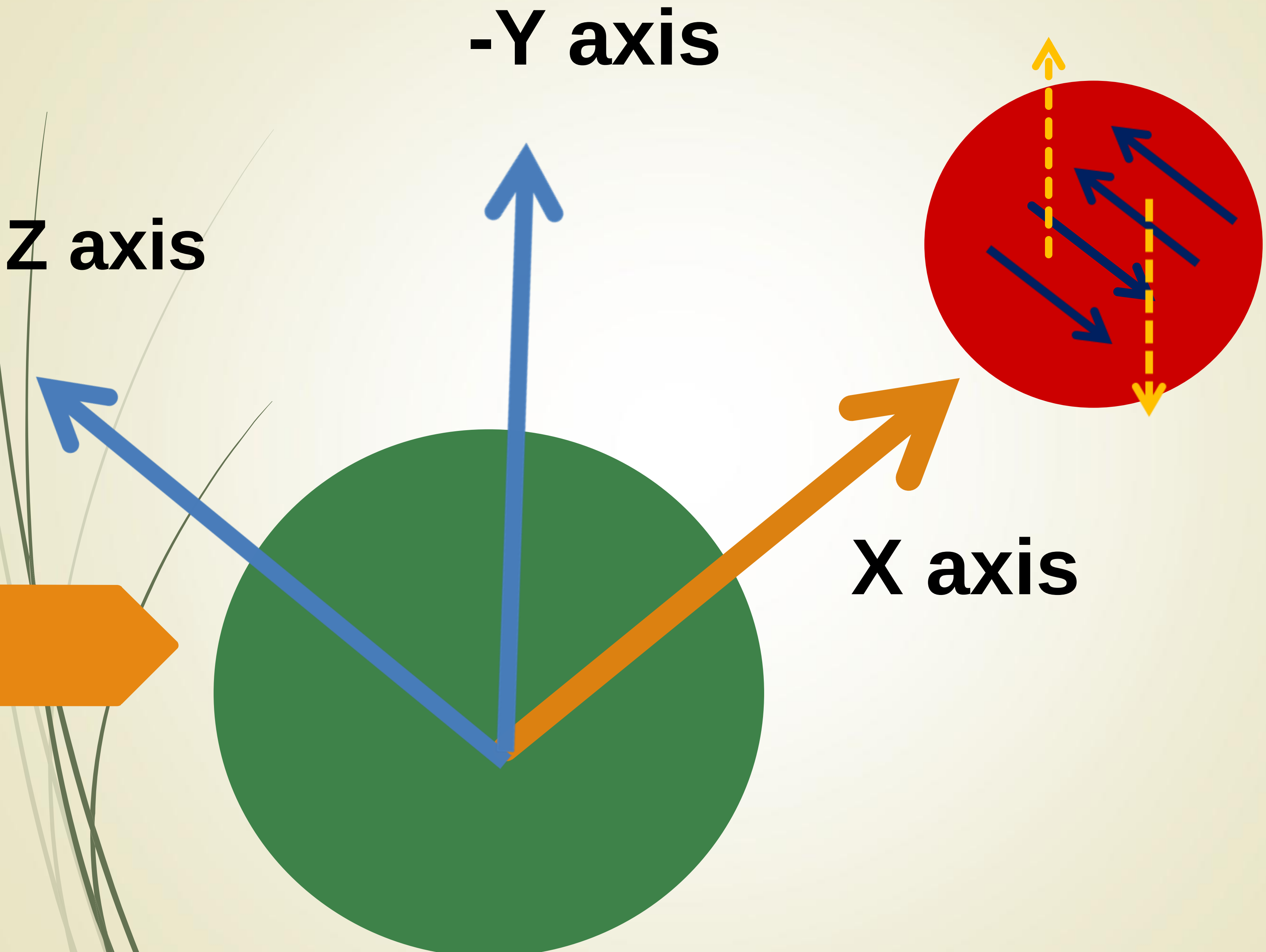


Cluster model – ^{16}O+alpha model

Present model – ^{16}O+quasi cluster

Simplified modeling of cluster-shell competition in ^{20}Ne and ^{24}Mg

N. Itagaki, M. Ploszajczak, and J. Cseh, Phys. Rev. C **83** 014302 (2011).


-Y axis
Z axis
X axis

Single particle wave function of nucleons in quasi cluster (spin

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp[-\nu(\vec{r} - \vec{\zeta}_i/\sqrt{\nu})^2]$$

$$\vec{\zeta}/\sqrt{\nu} = R(\vec{e}_x + i\Lambda\vec{e}_y)$$

Quasi cluster is along x
Spin direction is along z
Momentum is along y

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp[-\nu\vec{r}^2 - \vec{\zeta}^2 + 2\nu\vec{r}\cdot\vec{\zeta}/\sqrt{\nu}]$$

the cross term can be Taylor expanded as:

$$\exp[2\nu\vec{r}\cdot\vec{\zeta}/\sqrt{\nu}] = 1 + \sum_{k=1}^{\infty}\frac{1}{k!}(2\nu R(x + i\Lambda y))^k$$

For $\Lambda = 1$, one finds:

$$\exp[2\nu\vec{r}\cdot\vec{\zeta}/\sqrt{\nu}] = 1 + \sum_{k=1}^{\infty}\frac{1}{k!}\frac{1}{s_k}(2\nu r R)^k Y_{kk}(\Omega)$$

For $\Lambda = 1$

the single particle wave function in the quasi cluster becomes

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \{1 + s_1^{-1} 2\nu r_i R Y_{11}(\Omega_i)$$
$$+ (1/2!) s_2^{-1} (2\nu r_i R)^2 Y_{22}(\Omega_i)$$
$$+ (1/3!) s_3^{-1} (2\nu r_i R)^3 Y_{33}(\Omega_i)$$
$$+ \cdots + (1/n!) s_n^{-1} (2\nu r_i R)^n Y_{nn}(\Omega_i)$$
$$+ \cdots \} \exp[-\nu r_i^2].$$

for the spin-up nucleon (complex conjugate for spin-down)

As a result,
s1/2+p3/2+d5/2+f7/2+………………..

For $\Lambda = 1$

the single particle wave function
in the quasi cluster becomes

$$
\begin{aligned}
\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \{1 \ &+ \ s_1^{-1} 2\nu r_i R Y_{11}(\Omega_i) \\
&+ \ (1/2!) s_2^{-1} (2\nu r_i R)^2 Y_{22}(\Omega_i) \\
&+ \ (1/3!) s_3^{-1} (2\nu r_i R)^3 Y_{33}(\Omega_i) \\
&+ \ \cdots + (1/n!) s_n^{-1} (2\nu r_i R)^n Y_{nn}(\Omega_i) \\
&+ \ \cdots \} \exp[-\nu r_i^2].
\end{aligned}
$$

for the spin-up nucleon **(complex conjugate for spin-down)**

As a result, blocked by ^{16}O

s1/2+p3/2+d5/2+f7/2+……………….

For $\Lambda = 1$

the single particle wave function in the quasi cluster becomes

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \{1 + s_1^{-1} 2\nu r_i R Y_{11}(\Omega_i)$$
$$+ (1/2!) s_2^{-1} (2\nu r_i R)^2 Y_{22}(\Omega_i)$$
$$+ (1/3!) s_3^{-1} (2\nu r_i R)^3 Y_{33}(\Omega_i)$$
$$+ \cdots + (1/n!) s_n^{-1} (2\nu r_i R)^n Y_{nn}(\Omega_i)$$
$$+ \cdots \} \exp[-\nu r_i^2].$$

for the spin-up nucleon (complex conjugate for spin-down)

As a result, blocked by ^{16}O

s1/2+p3/2+d5/2+f7/2+………… R → 0

$$V_{ls} = V_0(e^{-d_1 r^2} - e^{-d_2 r^2})P(^3\mathrm{O})\vec{L}\cdot\vec{S}$$

G3RS interaction

^{20}Ne

Levels after GCM

(R and Λ are two GCM parameters)

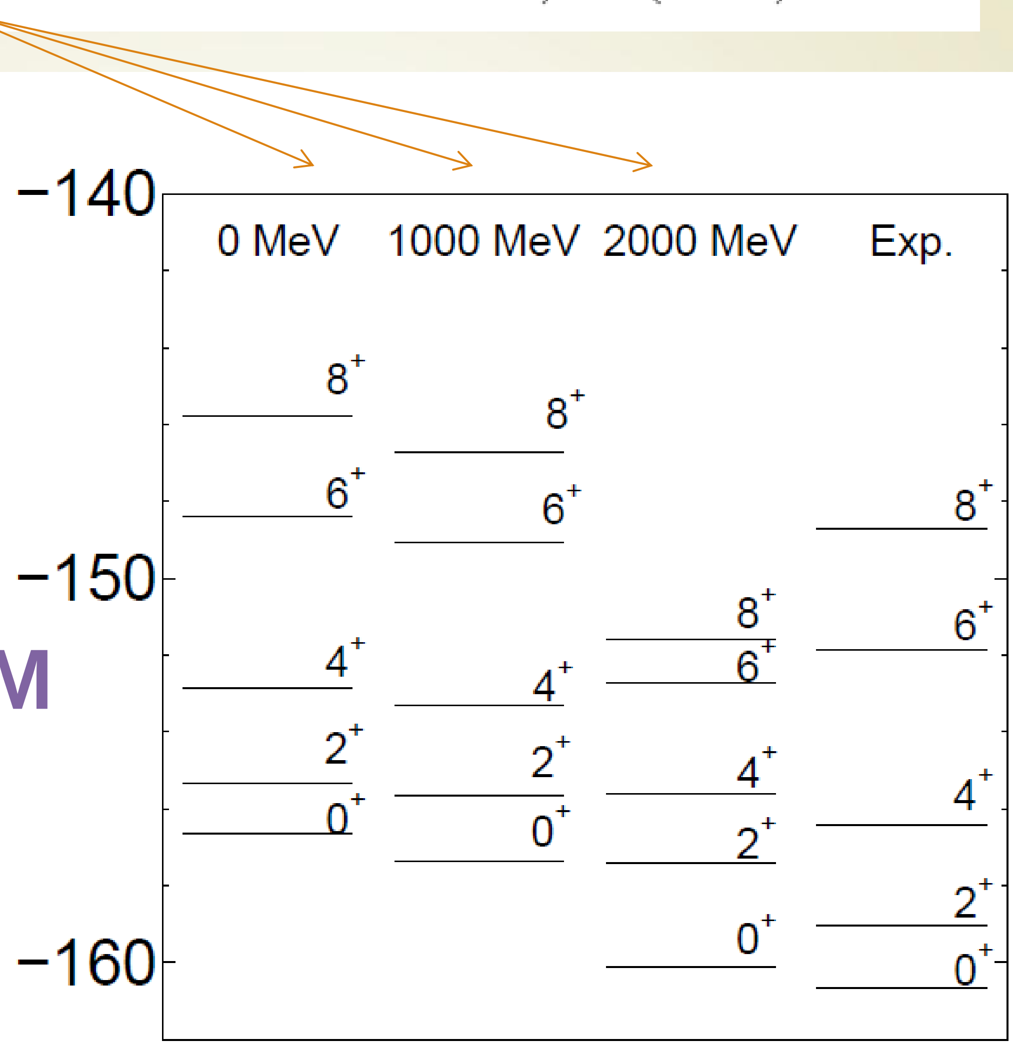


FIG. 1: The GCM calculations of yrast levels in ^{20}Ne for three different values of the strength of the spin-orbit interaction: $V_0 = 0, 1000, 2000$ MeV, are compared with the experimental data (Exp.). For more details, see the description in the text.

Last nucleons are in spin-orbit favored orbits

- Cluster breaking is large

Last nucleons are in spin-orbit unfavored orbits (described by negative Λ)

- Cluster breaking is not large (state dependence of the SU3 symmetry)

We can describe the lowest configurations of jj-coupling shell model starting with the cluster model

But final goal is to describe arbitral configuration of the jj-coupling shell model starting with the cluster model

Effects of cluster–shell competition and BCS-like pairing in ^{12}C

H. Matsuno and N. Itagaki
Prog. Theor. Exp. Phys. **2017**, 123D05 (2017).

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The antisymmetrized quasi-cluster model (AQCM) was proposed to describe α-cluster and jj-coupling shell models on the same footing. In this model, the cluster–shell transition is characterized by two parameters, R representing the distance between α clusters and Λ describing the breaking of α clusters, and the contribution of the spin–orbit interaction, very important in the jj-coupling shell model, can be taken into account starting with the α-cluster model wave function. Not only the closure configurations of the major shells but also the subclosure configurations of the jj-coupling shell model can be described starting with the α-cluster model wave functions; however, the particle–hole excitations of single particles have not been fully established yet. In this study we show that the framework of AQCM can be extended even to the states with the character of single-particle excitations. For ^{12}C, two-particle–two-hole (2p2h) excitations from the subclosure configuration of $0p_{3/2}$ corresponding to a BCS-like pairing are described, and these shell model states are coupled with the three α-cluster model wave functions. The correlation energy from the optimal configuration can be estimated not only in the cluster part but also in the shell model part. We try to pave the way to establish a generalized description of the nuclear structure.

Lowest configuration $(p3/2)^8$

proton p3/2 → p1/2

neutron p3/2 → p1/2

proton-neutron (T=0) pairing

proton $(p3/2)^2$ → $(p1/2)^2$

neutron $(p3/2)^2$ → $(p1/2)^2$

proton $(p3/2)^2$ → $(d5/2)^2$

neutron $(p3/2)^2$ → $(d5/2)^2$

proton or neutron (T=1) pairing

Table 3. 0^+ energies (E (MeV)) and principle quantum numbers (N) of ^{12}C calculated using the shell (shell), cluster (cluster) model basis states. The values for the mixed model space, subclosure configuration of $0p_{3/2}$, and cluster model basis states are shown in the column "*pn*–0p0h+cluster". The values for the full model space, shell, and cluster basis states are shown in the column "shell+cluster".

	shell		cluster		*pn*–0p0h+cluster		shell+cluster	
	E	N	E	N	E	N	E	N
0_1^+	−86.9	8.00	−89.1	11.22	−91.8	9.40	−92.6	9.15
0_2^+	−58.9	8.01	−79.1	20.01	−83.2	13.82	−83.4	14.00

	pn–0p0h	*pn*–$p_{1/2}$–2p2h	*pp*–$p_{1/2}$–2p2h	*nn*–$p_{1/2}$–2p2h	*pp*–$d_{5/2}$–2p2h	*nn*–$d_{5/2}$–2p2h
0_1^+	4.21×10^{-1}	3.96×10^{-2}	6.78×10^{-2}	6.86×10^{-2}	9.26×10^{-4}	9.64×10^{-4}
0_2^+	3.28×10^{-1}	1.10×10^{-4}	4.30×10^{-4}	5.27×10^{-4}	8.41×10^{-4}	7.92×10^{-4}

If we just would like to know the effect of the pairing, not necessary to do this

We should superpose the states, where Λ (R) value of two particles is continuously changed

But if we have analytic forms, we can directly compare MD and jj-coupling shell model

Tohsaki interaction

Akihiro Tohsaki, Phys. Rev. C **49**, 1814 (1994)

$$\hat{V}_{central} = \frac{1}{2}\sum_{ij} V_{ij}^{(2)} + \frac{1}{6}\sum_{ijk} V_{ijk}^{(3)},$$

where $V_{ij}^{(2)}$ and $V_{ijk}^{(3)}$ consist of three terms,

$$V_{ij}^{(2)} = \sum_{\alpha=1}^{3} V_{\alpha}^{(2)} \exp[-(\vec{r}_i - \vec{r}_j)^2/\mu_{\alpha}^2](W_{\alpha}^{(2)} + M_{\alpha}^{(2)} P^r)_{ij},$$

$$V_{ijk}^{(3)} = \sum_{\alpha=1}^{3} V_{\alpha}^{(3)} \exp[-(\vec{r}_i - \vec{r}_j)^2/\mu_{\alpha}^2 - (\vec{r}_i - \vec{r}_k)^2/\mu_{\alpha}^2] \times (W_{\alpha}^{(3)} + M_{\alpha}^{(3)} P^r)_{ij}(W_{\alpha}^{(3)} + M_{\alpha}^{(3)} P^r)_{ik}.$$

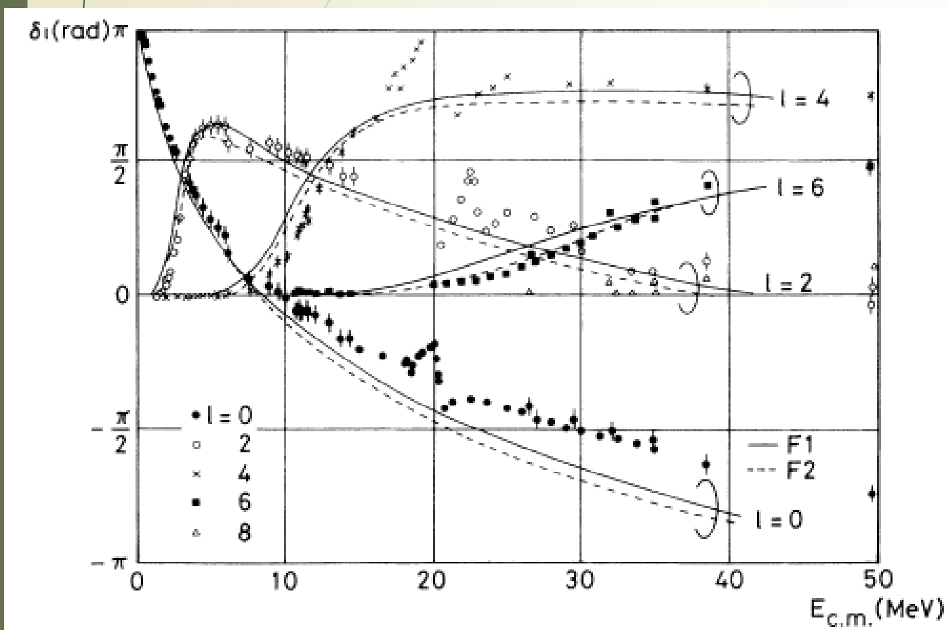
TABLE I: Parameter set for the two-body part of the Tohsaki interaction (F1 parameterization in Ref. [5]) together with the strengths of the three-body interaction.

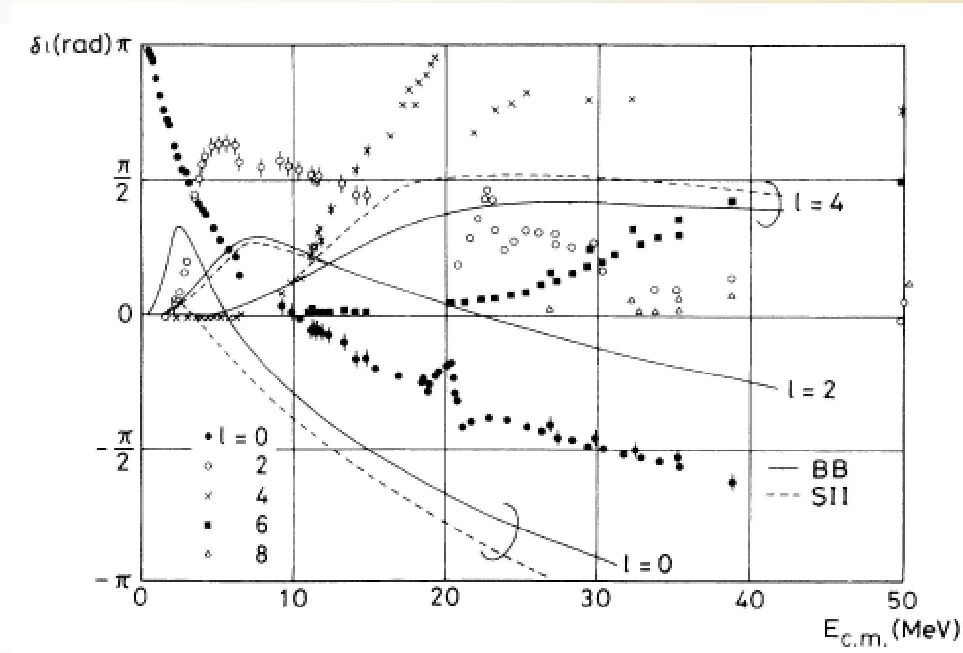
α	μ_α (fm)	$V_\alpha^{(2)}$ (MeV)	$V_\alpha^{(3)}$ (MeV)	$M_\alpha^{(2)}$	$W_\alpha^{(2)}$
1	2.5	−5.00	−0.31	0.75	0.25
2	1.8	−43.51	7.73	0.462	0.538
3	0.7	60.38	219.0	0.522	0.478

The advantage of Tohsaki interaction

- Saturation property is satisfied
- Binding energy and size of ^{4}He are reasonably reproduced
- ^{4}He- ^{4}He scattering phase shift is reproduced
- We can go to heavier regions

α-α scattering phase shift





Tohsaki F1, F2

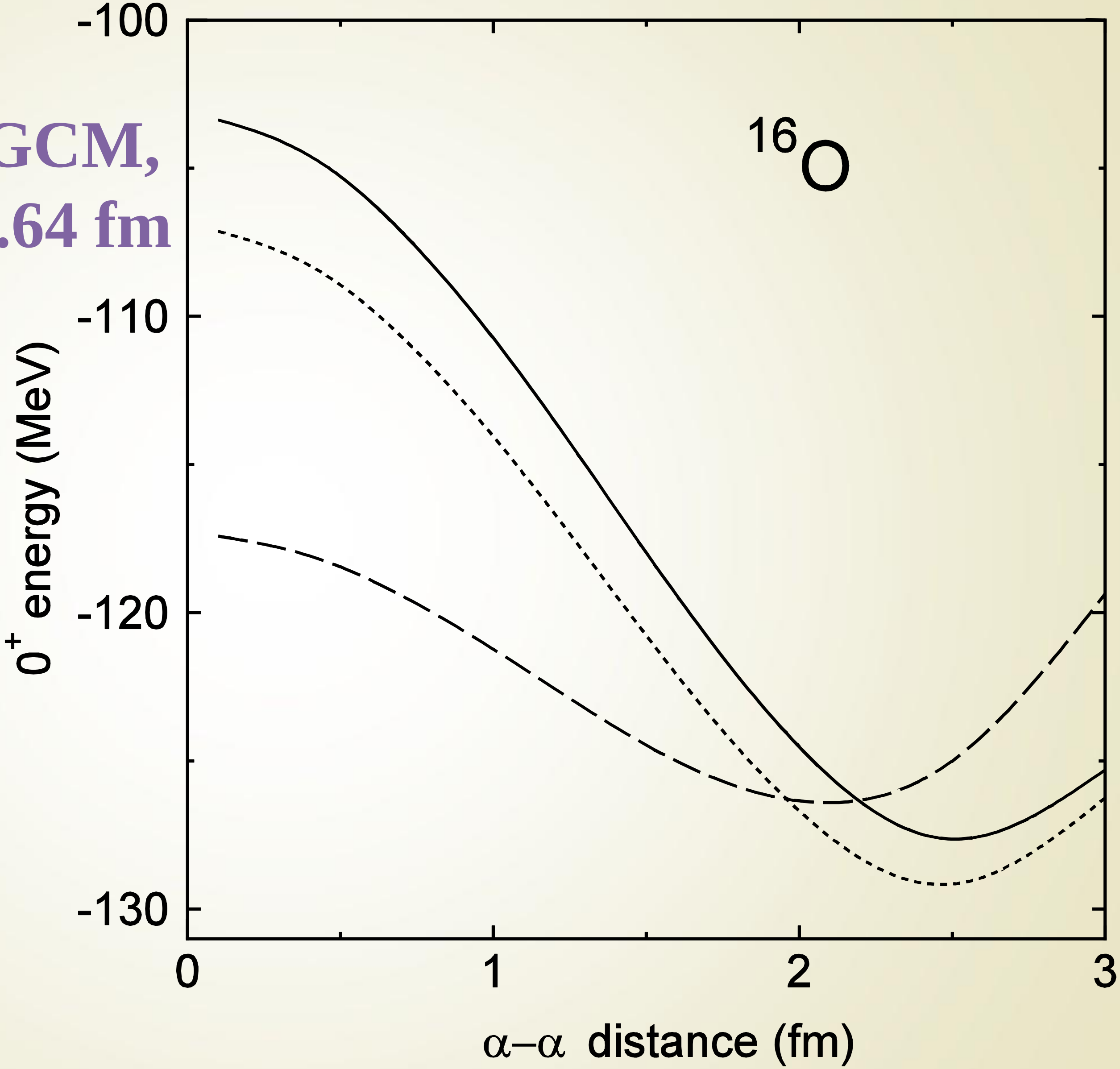
BB, SII

A. Tohsaki, Phys. Rev. C 49 1814 (1994)

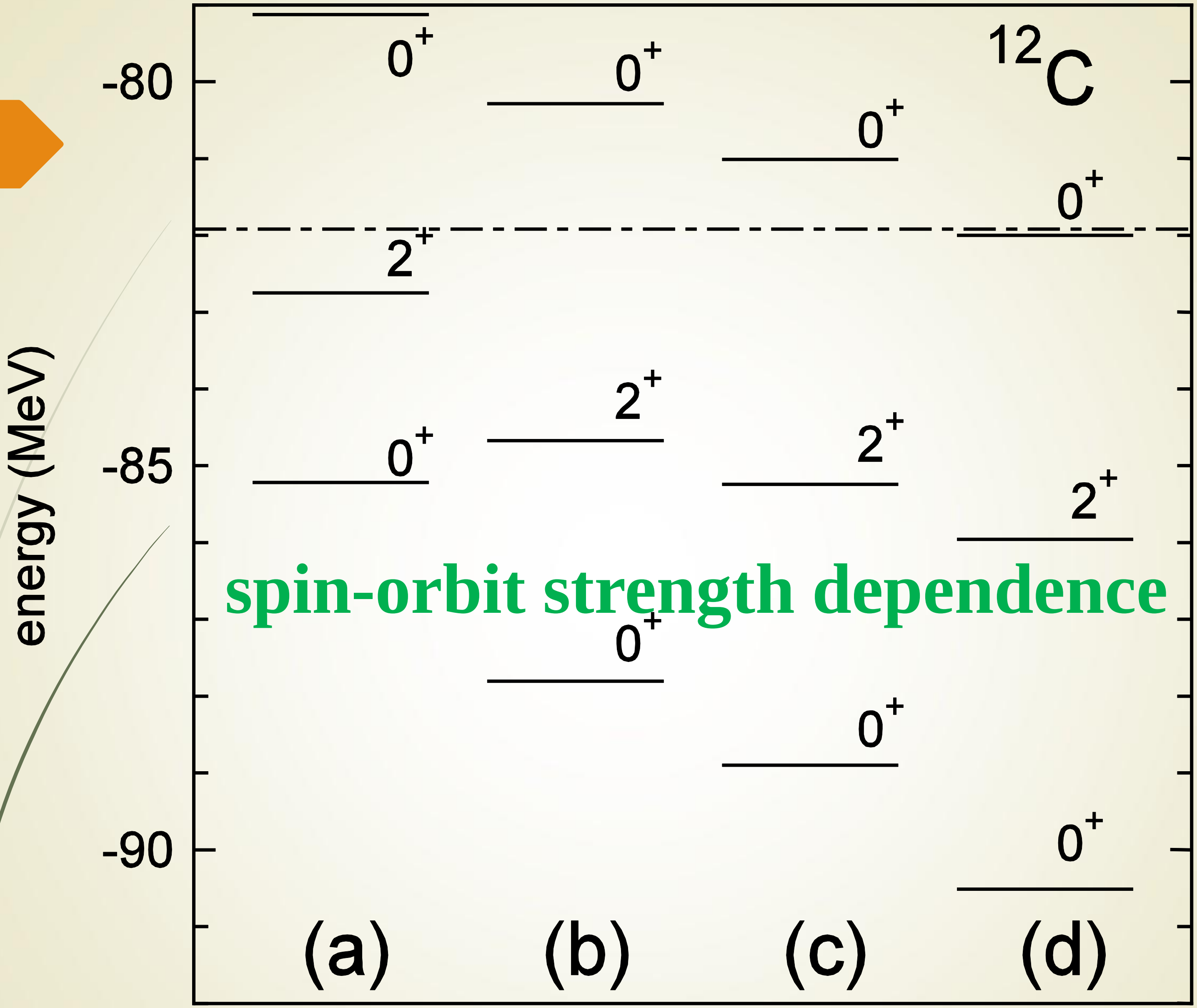
^{16}O tetrahedral 4α's

After performing GCM, charge radius → 2.64 fm (Exp. 2.69 fm)

F1—dotted
F1'—solid



N. Itagaki, Phys. Rev. C **94**, 064324 (2016).



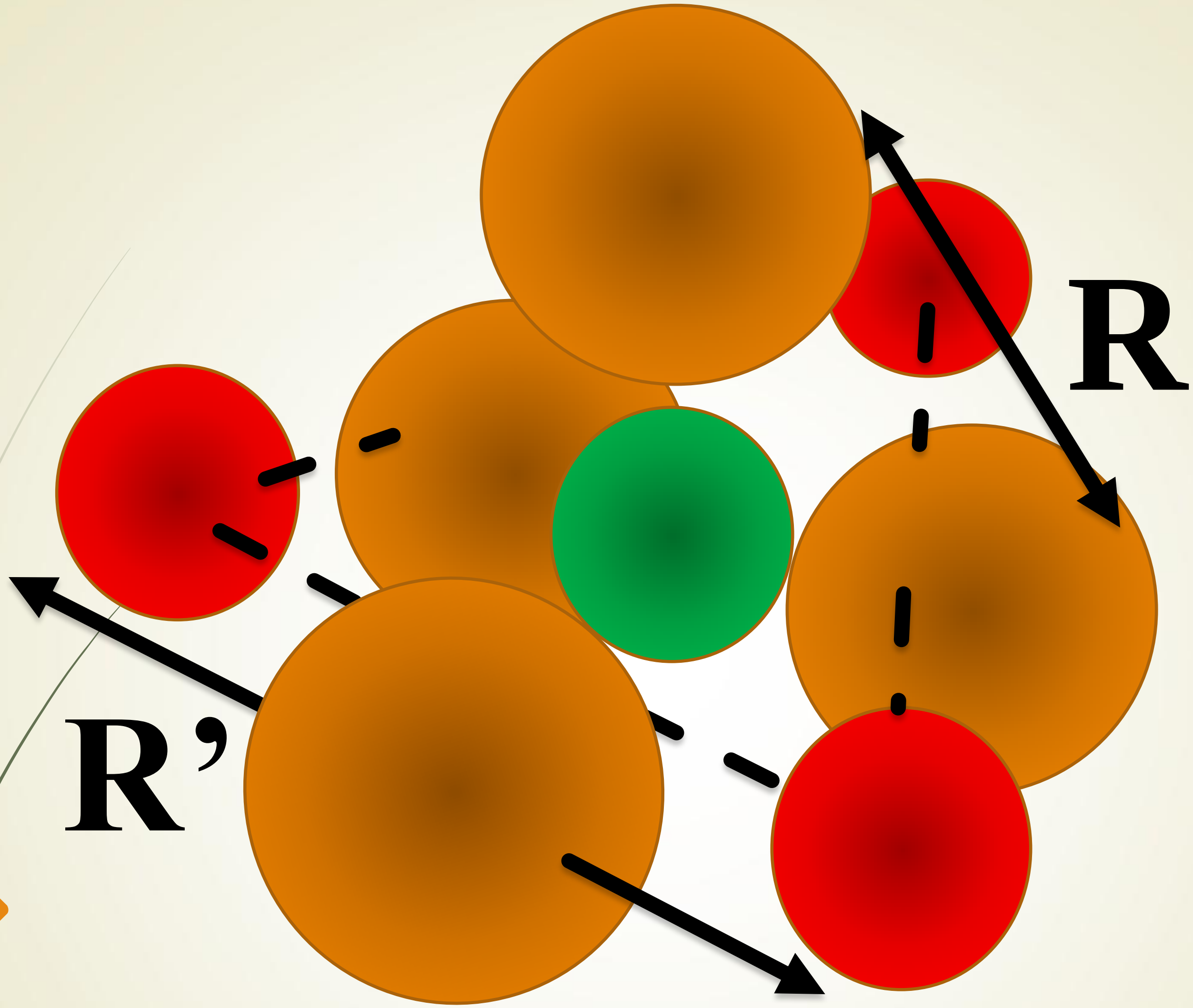
N. Itagaki, Phys. Rev. C **94**, 064324 (2016)

Puzzle of neutron-rich O isotopes

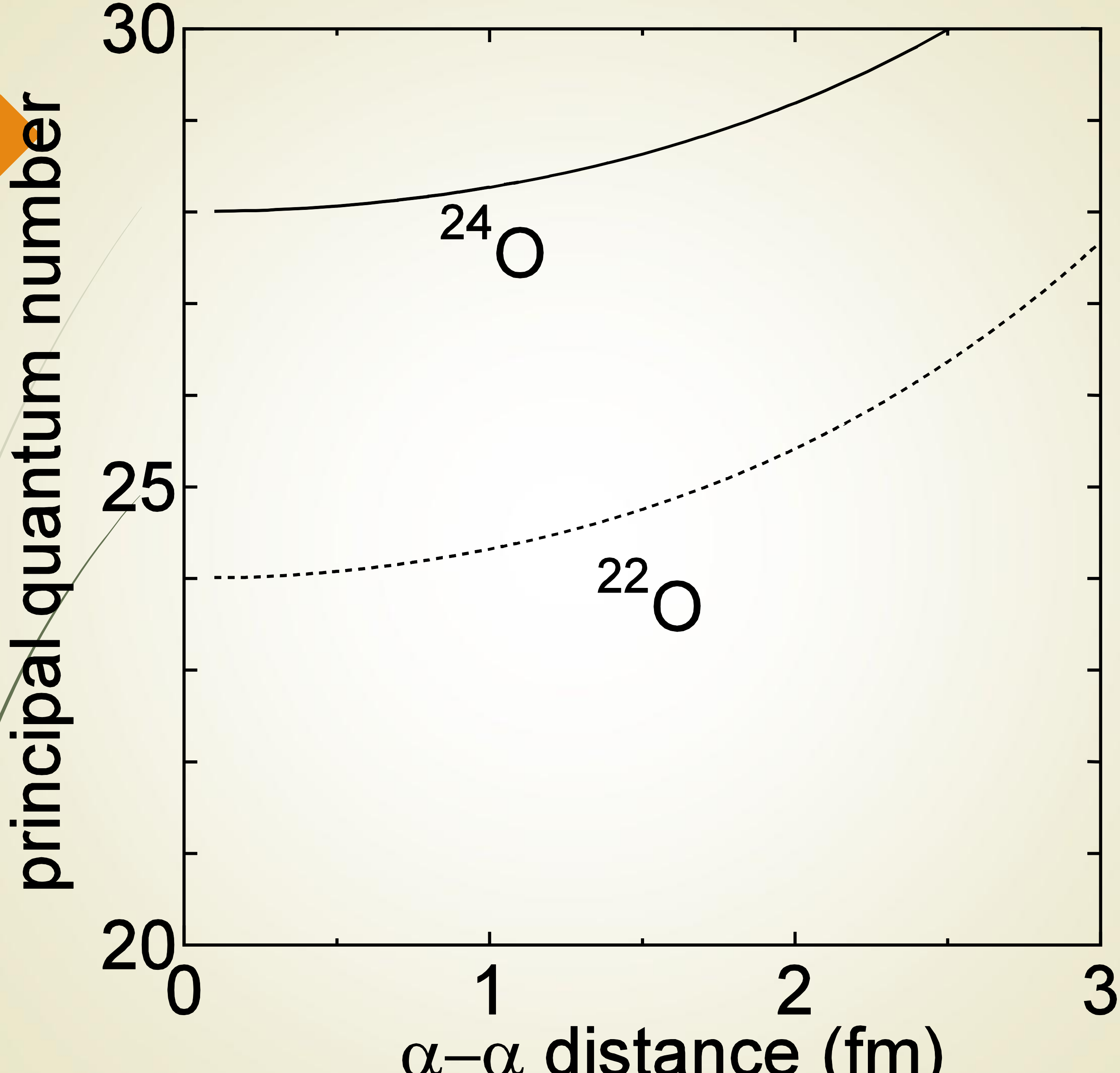
- ^{23}O and ^{24}O are enough bound (Sn = 4.19 MeV and 6.92 MeV, respectively), but they have very large radii.
- There must be non-trivial effect for the increase of the radii (not simple halo structure)
- Increase of ^{22}O size has been discussed

R. Kanungo, Phys. Rev. Lett. **88**, 142502 (2002)

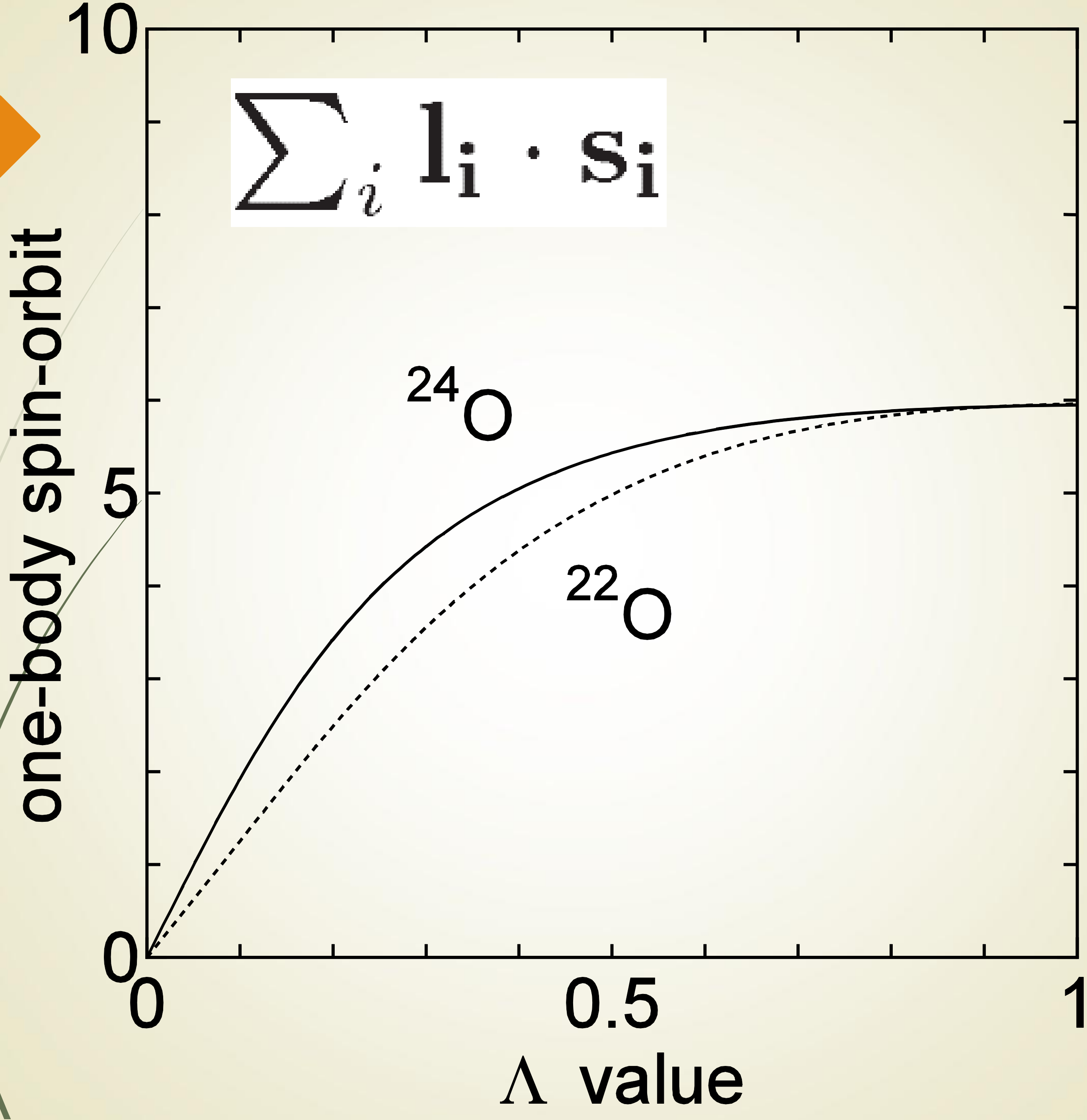
H. Masui, K. Kato, and K. Ikeda, Phys. Rev. C **73**, 034318 (2006).



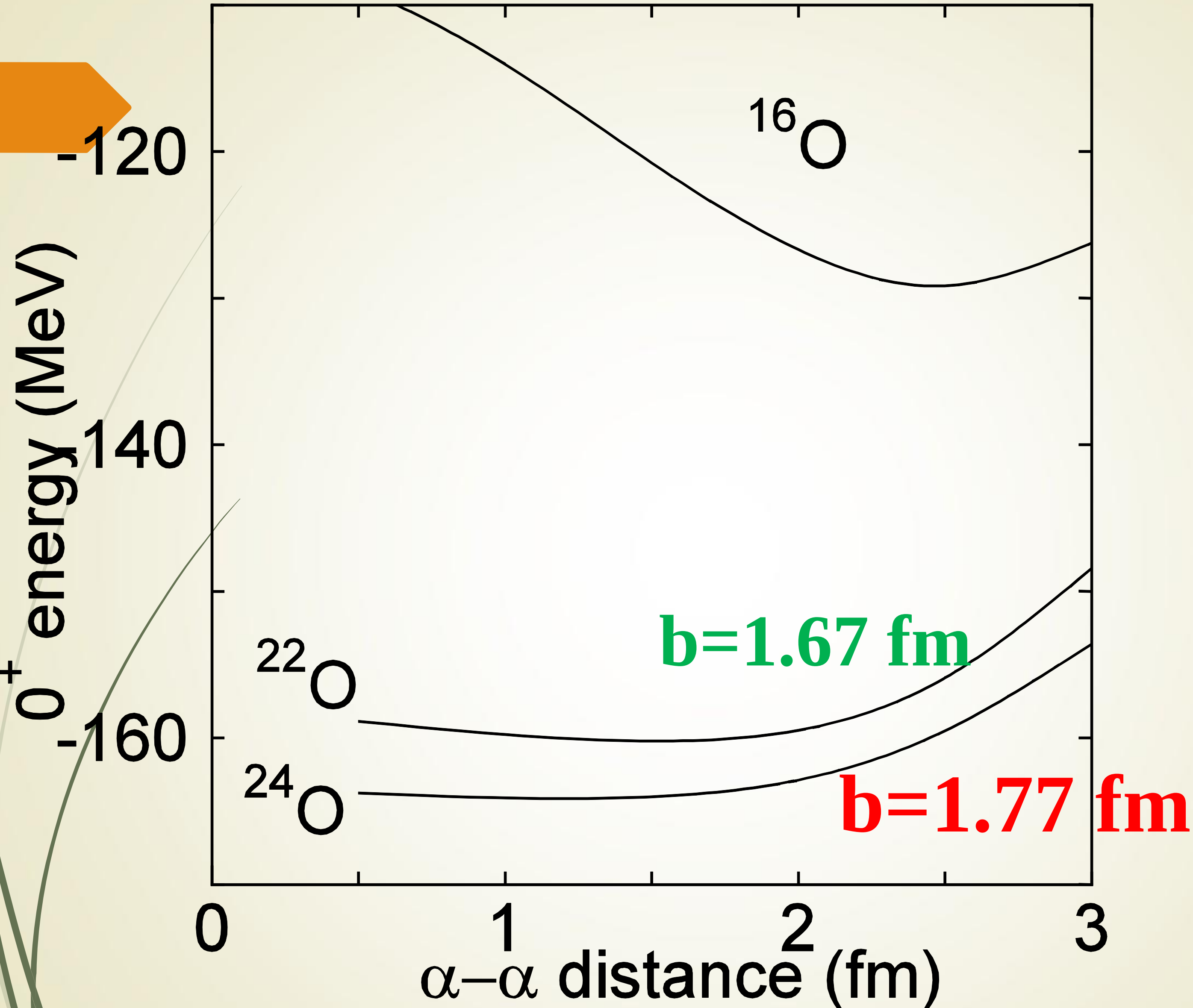
^{24}O



principal quantum number
30
25
20
0
1
2
3
α–α distance (fm)
24O
22O



10
5
0
0
0.5
1
one-body spin-orbit
Λ value
$\sum_i \mathbf{l_i} \cdot \mathbf{s_i}$
^{24}O
^{22}O


0+ energy (MeV)
-120
140
-160
0
1
2
3
α–α distance (fm)
16O
22O
24O
b=1.67 fm
b=1.77 fm

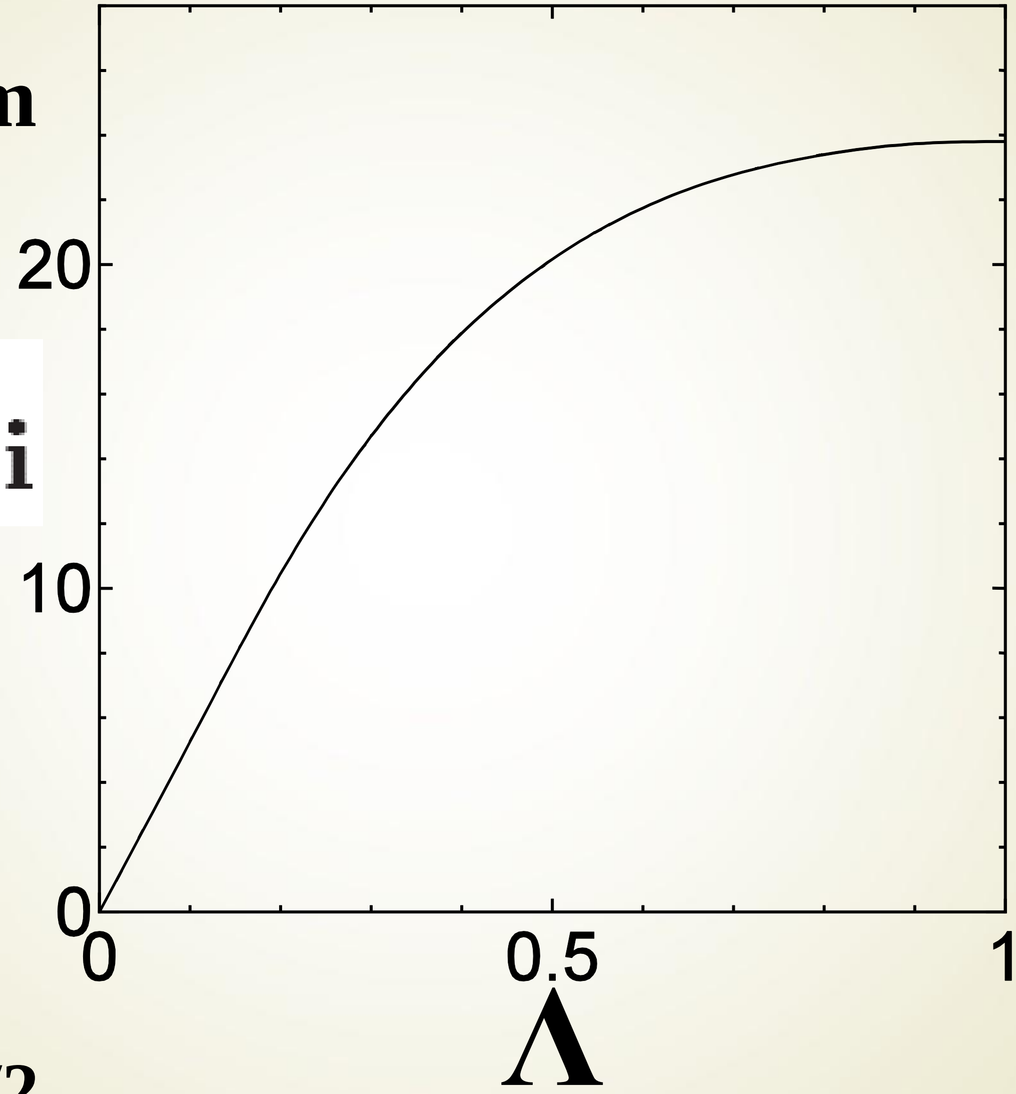
How we can generalize it?

Magic numbers 2, 8, 20 corresponding to the closure of 3 dimensional harmonic oscillator can be described by cluster models, but how about 28 and 50, typical jj-coupling ones?

N. Itagaki, H. Matsuno, and T. Suhara
Prog. Theor. Exp. Phys. **2016**, 093D01 (2016).

at R = 0.01 fm

$$\sum_i \mathbf{l_i} \cdot \mathbf{s_i}$$



^{56}Ni

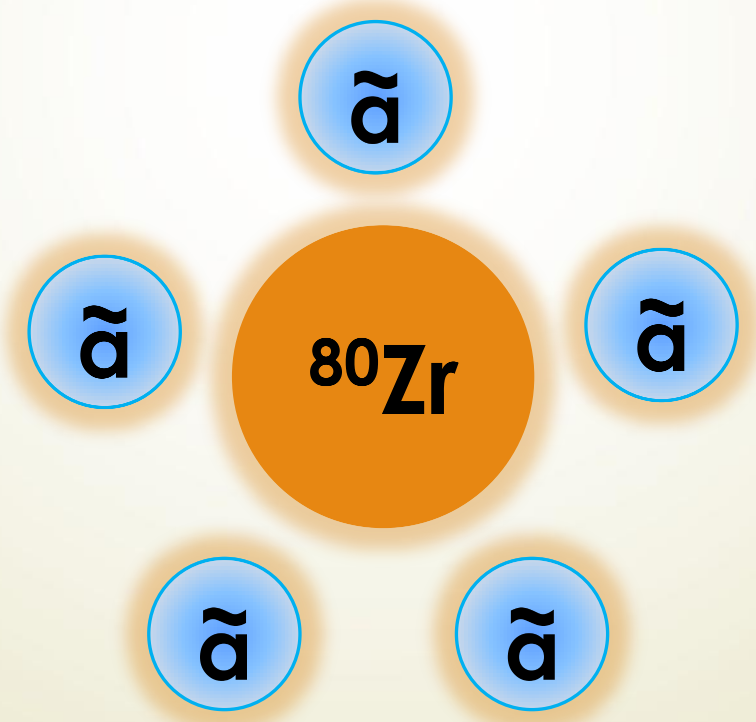
for $f_{7/2}$

j=7/2, l=3, s=1/2,

l*s= (j(j+1) - l(l+1) - s(s+1))/2 = (63/4 - 12 - 3/4) / 2 = 3/2

For even heavier nuclei, we can just replace the ^{40}Ca core with ^{80}Zr

For ^{100}Sn we add 5 quasi cluster around the ^{80}Zr core



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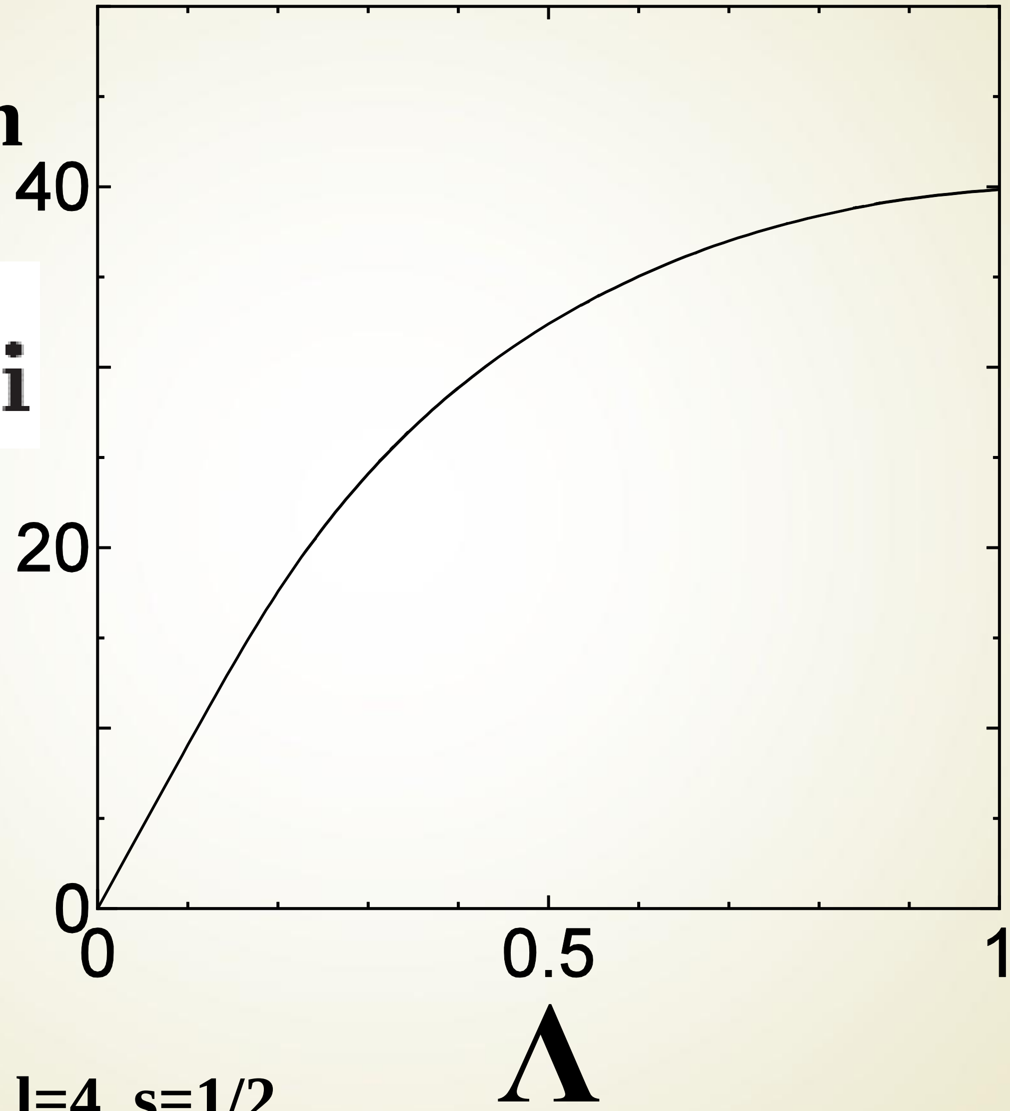
at R = 0.01 fm

$$\sum_i \mathbf{l_i} \cdot \mathbf{s_i}$$



^{100}Sn

for $g_{9/2}$

for j=9/2, l=4, s=1/2,
(j(j+1) - l(l+1) - s(s+1))/2 = (99/4 - 20 - 3/4) / 2 = 2

How we can utilize the present framework?

Microscopic description of the alpha decay

-- spatial correlation of four nucleons around the surface is difficult to be described in the shell model

-- if we assume the formation of alpha cluster around the surface based on the cluster model, we often overestimate the decay probability

We take into account both formation of alpha cluster around the surface and melting inside the nucleus

Summary

- Nuclear systems have characteristic features that non-central interactions play important roles
- Rank-1 non-central interaction,
  the spin-orbit interaction,
  creates the symmetry of the jj-coupling shell model
- Rank-2 non-central interaction,
  the tensor interaction,
  is important for the cluster structure in two senses
  (strong binding of the subsystems and
  enhancement of the relative distances between them)
- It is getting feasible to combine shell and cluster models and directly discuss the effects of non-central interactions