Signatures for proton-neutron pairs in $N \approx Z$ nuclei

S. Frauendorf



Department of Physics University of Notre Dame

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Review

Overview of neutron-proton pairing

S. Frauendorf^a, A.O. Macchiavelli^{b,*}

^a Department of Physics, University Notre Dame, Notre Dame, IN 46556, United States
 ^b Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, United States

Resume



From Monday, 19 September, 2016 - 09:00 to Friday, 23 September, 2016 - 13:00

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Which channel? ²H has ³S₁ ground state

"Pairing" : presence of many correlated pairs of the same type Analogy: pair condensate of infinite systems Difference: strong fluctuations of the condensate parameter Δ .



instead of phase transition smooth cross-over

HFB->static equilibrium QRPA->harmonic oscillations **Problem: critical regime** Shell model describes the crossover, ruler for correlation strength needed

mean field value "condensate"+pairing vibrations





Which are suitable indicators of the correlations?

- Spin orbit vs. short range attraction: What can be qualitatively be expected?
- Mean field predictions
- Mean field signals: symmetry breaking and pair- and iso-rotational bands quasiparticle spectra
- Experimental binding energies and odd-odd spectra
- Rotation
- Shell model calculations: mean field signatures, pair correlation measures
- Pair transfer, β -decay, charge exchange reactions
- Quarteting vs. pairing



Fig. 2. The experimental interaction matrix elements E_J between two nucleons in *j*-orbitals forming a T = 0 pair (left panel) and a T = 1 pair (right panel). The angle between the angular momenta \vec{j} of the two nucleons is denoted by θ_{12} . A scaling factor E is applied such that different *j*-orbitals fall on the same curve. For each *j*-shell, the first point to the left corresponds to J = 1 and the last to the right to J = 2j in the case T = 0, and the first point to the left corresponds to J = 0 in the case T = 1. *Source:* From [6].



The spin-orbit splitting not important for the T=1 pairing.



The spin-orbit splitting attenuates the T=0 pairing. 10



j, m_i

No pair scattering: angular momentum conserved. They do not generate a condensate.

Mean field calculations

The HFB equations $\beta^{+} = Uc^{+} + V\overline{c}$, pairs: $\left[c^{+}\overline{c}^{+}\right]_{TM_{T}JM}$

$$\begin{bmatrix} \varepsilon - \lambda + \Gamma & \Delta \\ \overline{\Delta} & -(\varepsilon - \lambda + \overline{\Gamma}) \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} = E \begin{bmatrix} U \\ V \end{bmatrix}$$

 $\Gamma_1 = Tr_2(v_{12}\rho_2), \quad \Delta_1 = Tr_2(\tilde{v}_{12}\kappa_2)$ The T=0 and T=1 pairfields usually appear as separate solutions. T=0 for

²⁰Ne ²⁴Mg ²⁸Si ³²S ³⁶Ar

HFB Yale-Shakin G-Matrix

A.L. Goodman,Adv. Nucl. Phys.11 (1979) 263.



A. L. Goodman, PRC60, 014311 (1999)

T=0 (α , α) field: p and n in identical orbitals ???



Evidence for the presence of the pair fields in energies

Spontaneous symmetry breaking -> pair rotational bands

T=1, J=0 and T=0, J=1 Cooper pairs assume good isospin, subtract Coulomb energy $\langle v_C \rangle$



S.F., J Sheikh NPA 645, 509 (1999)

S.F., J Sheikh NPA 645, 509 (1999)

Deformed nucleus



Isovector pair field



rotation in ordinary space rotational energy:

rotation in abstract isospace

isorotational energy:

$$E(I) = + \frac{I(I+1)}{2\theta}$$

$$E(I) = \langle H \rangle + \frac{T(T+1)}{2\theta_{iso}}$$

Limit of strong symmetry breaking: Wigner X=1 ("large deformation" in isovector space) The experimental X often close to 1, but not as close as for ordinary rotation. Weak deformation. p-n condensates generate pair-rotational bands: Regular sequence of ground states include the odd-odd nuclei





Lowest levels in odd-odd nuclei near N=Z

D. Jenkins et al., PRC65, 064307 (2002).

19



T=1 pair gap + isorotational energy account for the N \approx Z binding energies



T=0 rotational states have the same structure for all directions of the IV pair field .

To calculate the rotational spectra one can use the y- direction of the condensate, which has no pn-component. T=0 condensate generates pair-rotational bands: Regular sequence of ground states include the odd-odd nuclei



The experimental T=0 odd-odd states do not join a pair-rotational band

 $\frac{J(J+1)}{2\theta} \quad \theta \text{ large,}$ cranking, Shell Model



FIG. 1. (Color online) Quasiparticle energies in ⁴⁸Cr for the f-shell space. Red circles, spin-singlet; blue squares, spin-triplet with condensate in the $S_z = 0$ channel. Lines are drawn to guide the eye.

A. Gezerlis, G. F. Bertsch, and Y. L. Luo, PRL 106, 252502(2011) 23



J=2j or 2Ω (axial) the j of p and n are parallel "spin-alignment" J=1 the j of p and n are almost antiparallel

Low-lying states in odd-odd N=Z nuclei

PHYSICAL REVIEW C, VOLUME 60, 064310

Quasideuteron configurations in odd-odd N=Z nuclei

A. F. Lisetskiy,¹ R. V. Jolos,^{1,2} N. Pietralla,¹ and P. von Brentano¹ ¹Institut für Kernphysik, Universität zu Köln, D-50937 Köln, Germany

²Bogoliubov Theoretical Laboratory, Joint Institute for Nuclear Research, 141980 Dubna, Russia



- The p-n isovector pairing has to be as strong as the pp and nn pairing for symmetry reasons. No additional parameter to adjust.
- Mean field calculations predict a T=1 pair field for $40 \le A \le 100$
- Binding energies and low energy spectra are consistent with absence of a T=0 field
- T=0 interaction aligns the spins of the lowest qp and qn in the first T=0 states of oo nuclei. Not a pair field.
- There may be room for dynamical T=0 pair correlations.

Going far proton-rich





Fig. 22. Contour plots of the correlation energy in three different A = 132 nuclei as a function of the amplitudes of the isoscalar pair field κ^0 (S = 1 axis) and the isovector fields $\kappa_n = \kappa_p$ (S = 0 axis). Left panel: ¹³²₆₀Nd₇₂ with dominating isovector pairing; middle panel: ¹³²₆₆Dy₆₆ with dominating isoscalar pairing; right panel: ¹³²₆₄Gd₆₈ with a mixture of both pairing types. The numbers show correlation energies in MeV. In all three cases, the maximum is marked by an X.

- More detailed calculations needed to specify for the region the signals for pn- pair correlations in the binding energies and excitation spectra
- Check them experimentally

Shell model studies

–Determine the strength of the pair correlations: Pair counting operators $N(TM_T) = P_{TM_T}^+ P_{TM_T}$

-Test simple model for pairing (quarteting)

-Control the strength of the pair correlations $G_{T=0} / G_{T=1}$ and study the consquences for observables. Strong (staic) isovector correlations









T=1 and T=0 pairing in a simple model Hamiltonian

$$H = h_{nilsson} - G_{v} \sum_{M_{T}} P_{M_{T}}^{+} P_{M_{T}} - G_{S} D^{+} D$$

$$P_{-1}^{+} = \sum_{i} c_{pi}^{+} c_{p\bar{i}}^{+} \quad P_{0}^{+} = \frac{1}{\sqrt{2}} \sum_{i} c_{pi}^{+} c_{n\bar{i}}^{+} - c_{p\bar{i}}^{+} c_{ni}^{+} \quad P_{1}^{+} = \sum_{i} c_{ni}^{+} c_{n\bar{i}}^{+}$$

$$D^{+} = \frac{1}{\sqrt{2}} \sum_{i} c_{pi}^{+} c_{n\bar{i}}^{+} + c_{p\bar{i}}^{+} c_{ni}^{+}$$

8 levels diagonalization I. Bentley, S. F. PRC 88, 014322 (2013)

Micro-Macro for shell structure and deformation interpolated QRPA K. Neergard, I. Bentley, S. F., PRC 89, 034302 (2014) K. Neergard, NUCLEAR THEORY, Vol. 36 (2017) eds. M. Gaidarov, N. Minkov, Heron Press, Sofia, and private communication. 35







WS+shell correction +T=1 pair field+interpolated RPA Sn isotopes K. Neergard, to be published Pure isovector pairing approaches that exactly conserve isospin describe the binding and excitation energies in detail, including the Wigner X term and local fluctuations.

8 levels diagonalization I. Bentley, S. F. PRC 88, 014322 (2013)

Micro-Macro for shell structure and deformation interpolated QRPA K. Neergard, I. Bentley, S. F., PRC 89, 034302 (2014) K. Neergard, NUCLEAR THEORY, Vol. 36 (2017) eds. M. Gaidarov, N. Minkov, Heron Press, Sofia and private communicatio

No new parameters compared to standard N>>Z approach. Strength of T=1 interaction adjusted to ee-oo mass differences or eo mass differences.

T=1 and T=0 pairing in a simple model Hamiltonian

$$H = h_{nilsson} - G_{v} \sum_{M_{T}} P_{M_{T}}^{+} P_{M_{T}} - G_{S} D^{+} D$$

$$P_{-1}^{+} = \sum_{i} c_{pi}^{+} c_{p\bar{i}}^{+} \quad P_{0}^{+} = \frac{1}{\sqrt{2}} \sum_{i} c_{pi}^{+} c_{n\bar{i}}^{+} - c_{p\bar{i}}^{+} c_{ni}^{+} \quad P_{1}^{+} = \sum_{i} c_{ni}^{+} c_{n\bar{i}}^{+}$$

$$D^{+} = \frac{1}{\sqrt{2}} \sum_{i} c_{pi}^{+} c_{n\bar{i}}^{+} + c_{p\bar{i}}^{+} c_{ni}^{+}$$

8 levels diagonalization I. Bentley, S. F. PRC 88, 014322 (2013)

Switch on the the isoscalar interaction



There is room for dynamic isoscalar pair correlations.

Pair transfer strength

Collectively enhanced by the pair correlations Enhancement is the most direct signature.



Systematic relative measurements and within a given nucleus.

The results from the Two j-shells model



PHYSICAL REVIEW C 85, 034317 (2012)

Pair-transfer probability in open- and closed-shell Sn isotopes

M. Grasso,¹ D. Lacroix,² and A. Vitturi^{3,4}

¹Institut de Physique Nucléaire, IN2P3-CNRS, Université Paris-Sud, F-91406 Orsay Cedex, France ²Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DSM-CNRS/IN2P3, Boulevard Henri Becquerel, F-14076 Caen, France ³Dipartimento di Fisica G. Galilei, via Marzolo 8, I-35131 Padova, Italy ⁴Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Padova, via Marzolo 8, I-35131 Padova, Italy



Mean field approximation, surface delta interaction

- Only N-projected mean field or simple 1 or 2 shell model calculations on the market.
- Realistic Shell Model not yet applied to pair transfer.
- Measurement of absolute enhancement is difficult
- Ratio of IS/IV enhancement is easier and interesting because the IV strength is well established

(³He,p) Transfer Reactions



Measure the *np* transfer cross section to T=1 and T=0 states

Both absolute $\sigma(T=0)$ and $\sigma(T=1)$ and relative $\sigma(T=0) / \sigma(T=1)$ tell us about the character and strength of the correlations

Does a beyond- m. f. IV scenario account for experiment?



Ratio between the cross sections for transfer of an IV pair and an IS pair from ee 0^+_1 to the 0^+_1 and the 1^+_1 48 states in the oo.

GT - transitions



enhancement



Fig. 50. Experimental (black color) and calculated (red color) single level B(GT) and accumulated B(GT) values for the β^+ decay ${}^{62}\text{Ge} \rightarrow {}^{62}\text{Ga}$: left panels with the Shell Model calculations using the KB3G interaction and right panels with the QRPA approach of Ref. [168]. Experimental uncertainty corridors are indicated in gray. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.) *Source:* From Ref. [172].

QRPA with isoscalar paring interaction with coupling constant $G_s=0.3 G_{critical}$ for onset of isoscalar condensate.

Other QRPA studies of β^+ decay, $\beta^+\beta^+$ decay, and β^- decay of neutron-rich nuclei require dynamical isoscalar pair correlation to reproduce data.

Charge exchange reactions A(³He,³H)A test the influence of pairing correlations on the GT matrix element in a different energy range



HFB+QRPA calculations Uncertainty: Competition of the GT resonance in pp channel with isoscalar pair correlations

Fig. 51. GT strength in ⁴⁸Cr, ⁵⁶Ni and ⁶⁴Ge by HFB + QRPA with the Skyrme interaction. Source: From Ref. [176].

Quarteting

Isospin conservation and quarteting

$$H = \sum_{i} \mathcal{E}_{i} (N_{i}^{(\nu)} + N_{i}^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^{+} P_{j,\tau}$$

 $P_{i1}^{+} \propto \nu_{i}^{+} \nu_{\bar{i}}^{+} \qquad P_{i-1}^{+} \propto \pi_{i}^{+} \pi_{\bar{i}}^{+} \qquad P_{i0}^{+} \propto \nu_{i}^{+} \pi_{\bar{i}}^{+} + \pi_{i}^{+} \nu_{\bar{i}}^{+}$

non-collective quartets

$$Q_{ij}^{+} = [P_{i\tau}^{+}P_{j\tau'}^{+}]^{T=0} \propto P_{vv,i}^{+}P_{\pi\pi,j}^{+} + P_{\pi\pi,i}^{+}P_{vv,j}^{+} - P_{v\pi,i}^{+}P_{v\pi,j}^{+}$$

collective quartet

$$Q^{+} = \sum_{ij} x_{ij} [P_{i\tau}^{+} P_{j\tau'}^{+}]^{T=0}$$

quartet condensate

$$|QCM>=Q^{+n_q}| ->$$
 (has T=0, J=0)

N=Z

Quartet condensation and Cooper pairs

$$|QCM> = Q^{+n_q}| - > \qquad Q^+ = \sum_{ij} x_{ij} [P_{i\tau}^+ P_{j\tau'}^+]^{T=0}$$

$$Q^{+} = 2\Gamma_{\nu\nu}^{+}\Gamma_{\pi\pi}^{+} - \Gamma_{\nu\pi}^{+}\Gamma_{\nu\pi}^{+} \qquad \Gamma_{\tau}^{+} = \sum_{i} x_{i}P_{i,\tau}^{+} \quad \text{collective pairs}$$

$$|QCM\rangle = (2\Gamma_{vv}^{+}\Gamma_{\pi\pi}^{+} - \Gamma_{v\pi}^{+}\Gamma_{v\pi}^{+})^{n_{q}}|-\rangle$$

'coherent' mixing of condenstates formed by nn, pp and pn pairs

$$|PBCS0 > \propto (\Gamma_{\nu\pi}^{+2})^{n_q}| - > |PBCS1 > \propto (\Gamma_{\nu\nu}^{+}\Gamma_{\pi\pi}^{+})^{n_q}| - >$$

calculations

$$\delta_x < QCM \mid H \mid QCM >= 0$$

method of recurence relations

Quartet condensation versus pair condensation

$$H = \sum_{i} \varepsilon_{i} N_{i} + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{t} P_{it}^{+} P_{jt}$$

pairing forces extracted from SM interactions

 $|QCM\rangle = (Q^{+})^{n_{q}}|-\rangle \qquad |PBCS1\rangle \propto (\Gamma_{vv}^{+}\Gamma_{\pi\pi}^{+})^{n_{q}}|-\rangle \qquad |PBCS0\rangle \propto (\Gamma_{v\pi}^{+2})^{n_{q}}|-\rangle$

	SM	QCM	PBCS1	PBCS0
²⁰ Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
²⁴ Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
28Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
32S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
44Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
48Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
⁵² Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
104 Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
¹⁰⁸ Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
112Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

Conclusions

- T=1 pairing is accurately described by quartets, not by pairs
- there is not a pure condensate of isovector pn pairs in N=Z nuclei States with good isospin always contain a mixture of $\Gamma \pi \pi$, Γvv , $\Gamma \pi v$. How different are $P_{TM=0}P_A|T=1$ MF> and |QCM>?

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Wigner energy: comparison with earlier calculations



Bentley & Frauendorf PRC(2013)

Negrea & Sandulescu PRC(2014)

Isovector (J=0) pairing versus isoscalar (J=1) pairing

$$\begin{split} |QM\rangle &= \prod_{\nu=1}^{N_Q} Q_{\nu}^{\dagger} |0\rangle. \qquad Q_{\nu}^{+} = Q_{\nu}^{+(iv)} + Q_{\nu}^{+(is)} \\ |is\rangle &= \prod_{\nu=1}^{N_Q} Q_{\nu}^{\dagger(is)} |0\rangle \qquad \qquad |iv\rangle = \prod_{\nu=1}^{N_Q} Q_{\nu}^{\dagger(iv)} |0\rangle \end{split}$$

	QM	iv	is	< QM iv >	< QM is >	< iv is >
20 Ne	15.985	14.402 (9.9%)	15.130 (5.35%)	0.884	0.953	0.843
^{24}Mg	28.625	23.269 (18.71%)	26.925~(5.94%)	0.650	0.910	0.336
^{28}Si	35.386	28.896~(18.34%)	33.377 (5.68%)	0.590	0.910	0.341
^{32}S	38.844	33.958 (12.58%)	37.881 (2.48%)	0.640	0.974	0.587
$^{44}\mathrm{Ti}$	7.02	6.27~(10.6%)	4.92 (30%)	0.90	0.68	0.3
^{48}Cr	11.624	10.59~(8.9%)	7.38~(36.5%)	0.906	0.497	0.22
52 Fe	13.823	12.814 (7.3%)	9.98 (27.83%)	0.927	0.753	0.74
$^{104}\mathrm{Te}$	3.147	3.041 (3.37%)	1.549~(50.78%)	0.978	0.489	0.314
108 Xe	5.495	5.240~(4.64%)	2.627 (52.19%)	0.958	0.354	0.234
^{112}Ba	7.035	6.614~(5.98%)	4.466 (36.52%)	0.939	0.375	0.376

T=1 and T=0 pairing correlations always coexist &

T=1 condensate+dynamical T=0

difficult to disentangle

T=1 correlations dominate, some T=0 correlations

M. Sambataro, N.S. and C.W.Johnson, Phys. Lett. B740 (2015)137

A<100: T=1 condensate combined with dynamical T=0 correlations

- The binding energies show iso-rotational pattern
- The N=Z odd-odd spectra have low density
- The rotational spectra can be quantitatively described by conventional mean-field without explicit pn-pairing, some indication for T=0 correlations
- Cross section for IV pair transfer larger than for IS pair transfer.
- Dynamical T=0 correlations->GT, M1