北京大学物理学院
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# Many-body perturbation theory calculations of closed-shell nuclei

**Furong Xu**

**i) MBPT calculations for closed-shell nuclei, with $N^3LO$(NN) (and also LQCD)**

**ii) HF-RPA calculations for nuclear giant resonances, with $NNLO_{sat}$ (NN+NNN)**

**iii) Gamow shell model for resonance and continuum in weakly bound nuclei, with CD Bonn (NN)**

**Using Similarity Renormalization Group (SRG), Low-momentum $V_{low\ k}$**

*Many-body perturbation theories in modern quantum chemistry and nuclear physics,*
**March 26-30, 2018, CEA, Saclay, France**

**For nuclear structure calculations: there are two fundamental problems**

**1) nuclear forces; 2) many-body methods**

# *Ab initio* nuclear structure calculations

**1. Starting from realistic nuclear forces**

**2. Renormalization (softening) to speed up convergence**

**3. *Ab-initio* methods to treat many-body problems**

$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

$$H_{\rm int} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am}$$

$$\vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

# 1. Realistic nuclear forces

**Yukawa nuclear force by $\pi$ meson exchange** $(m_\pi \approx 200 m_e)$

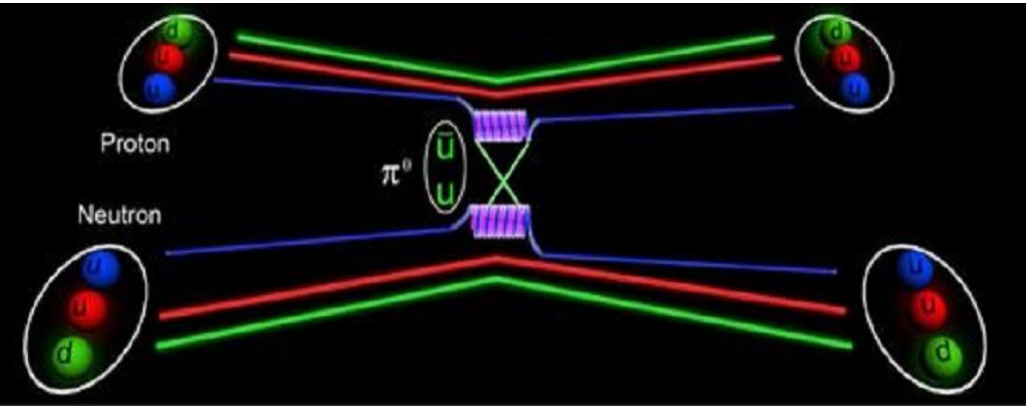
$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-\frac{m_\pi c}{\hbar}r}}{r}$$

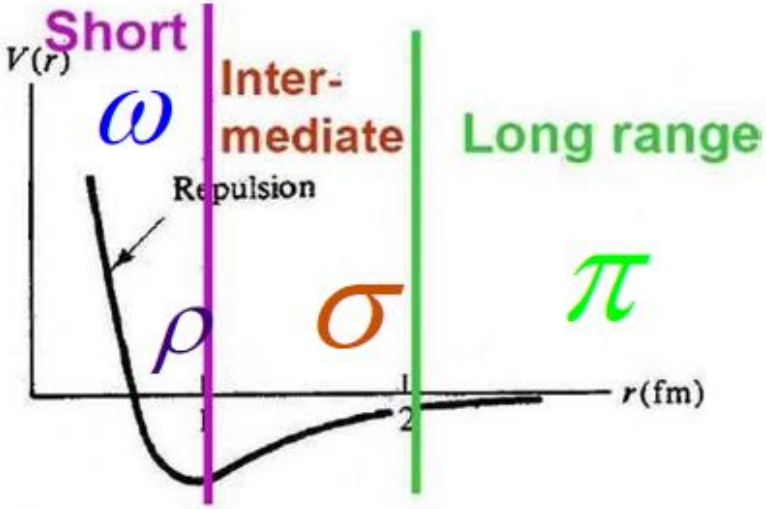
**Electromagnetic force has an infinite range!**

$$V(r) \sim \frac{1}{r}$$

**Finite range:**
**larger mass, longer range**

**Long-range attractive， no short-range repulsive**




**Nuclear force is not a fundamental interaction, but an effective force!**

QCD=quarks + gluons (symmetries: spin, isospin, parity, chiral symmetry broken spontaneously)

**QCD-based nuclear EFT, Weinberg (1990's)**

Chiral EFT=nucleons+pions (symmetries: spin, isospin, parity, chiral symmetry broken spontaneously)

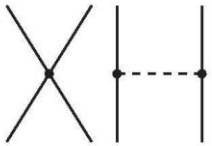
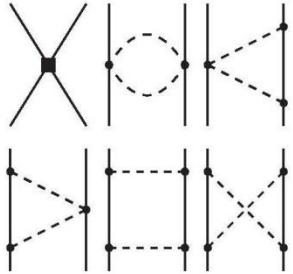
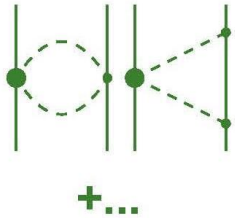
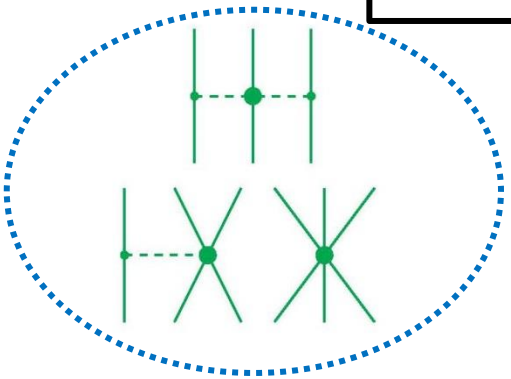
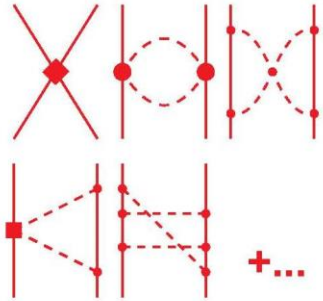
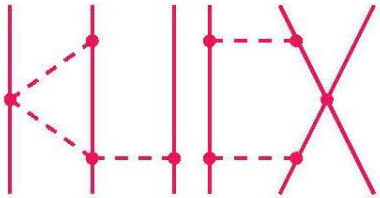
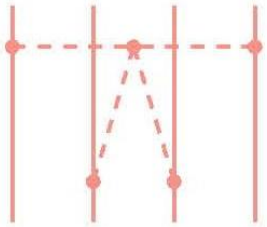
**At low energy, the effective degrees of freedom are nucleon and pion, rather than quark and gluon!**

- QCD at low energy is strong. Perturbation is inapplicable !
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual color forces (similar to van der Waals forces)

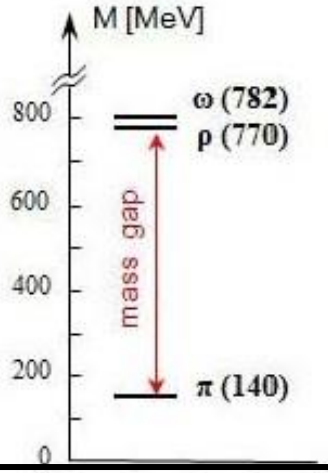
Nuclear force


Residual force of the QCD strong interaction outside the nucleon

**From QCD to nuclear physics via Chiral EFT**

| | | 2N forces | 3N forces | 4N forces |
|---|---|---|---|---|
| Leading Order | $Q^0$ LO |  | | |
| Next-to Leading Order | $Q^2$ NLO |  | | |
| Next-to-Next-to Leading Order | $Q^3$ $N^2LO$ |  +... |  | |
| Next-to-Next-to-Next-to Leading Order | $Q^4$ $N^3LO$ |  +... |  +... |  +... |

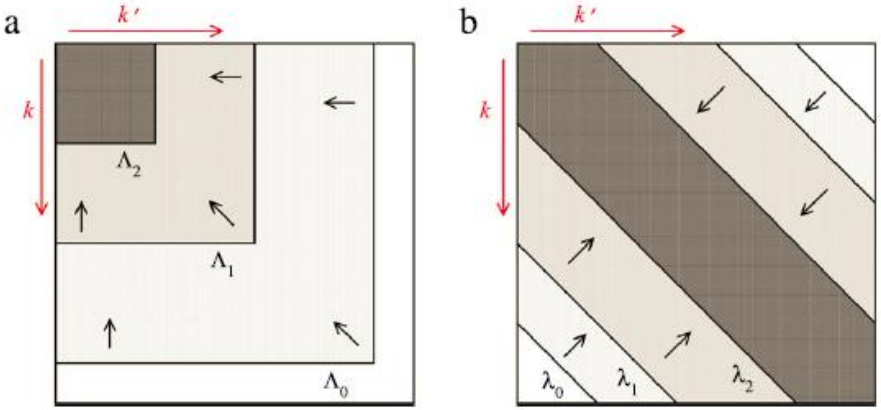
**Chiral EFT**



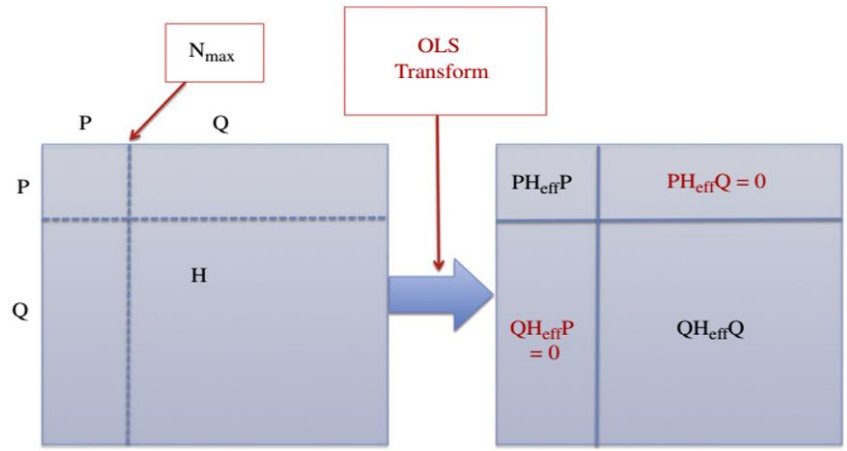

Power counting : $Q / \Lambda$

# 2. Renormalization

S.K. Bogner et al. / Progress in Particle and Nuclear Physics 65 (2010) 94–147

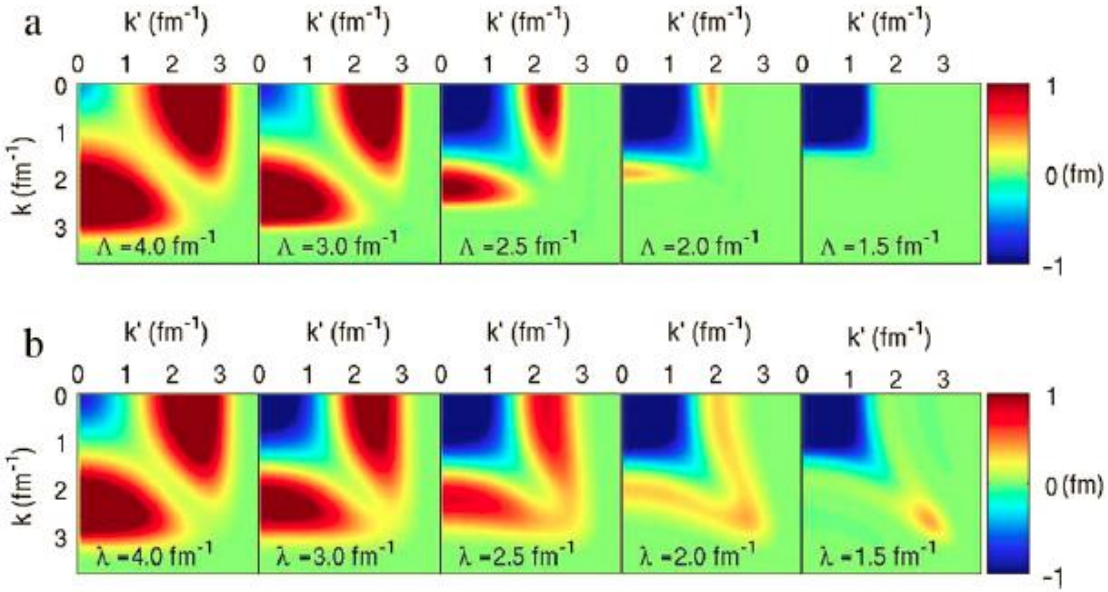



$V_{low\text{-}k}$ SRG




$V_{low\text{-}k}$

SRG




**Evolutions of renormalizations for $N^3LO$**

# 3.1 MBPT calculations for closed-shell nuclei

$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} \quad ; \qquad H_{\rm int} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am} \quad , \qquad \vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

**a) First we performed spherical HF calculation (in HO basis)**

**b) The HF state is chosen as a reference state, which limits to closed-shell nuclei.**

**c) In the HF basis, we make MBPT: for energy up to 3rd-order perturbation corrections, and for radius up to 2nd-order corrections, in the *j-j* scheme:**

$$H_0 = \sum_{l_1 l_2} h^{HF}_{l_1 l_2} a^{\dagger}_{l_1} a_{l_2}$$

$$\hat{H} = \hat{H}_0 + (\hat{H} - \hat{H}_0) = \hat{H}_0 + \hat{V}$$

The exact solutions of the A-nucleon system are,

$$\hat{H}\Psi_n = E_n \Psi_n, \qquad n = 0, 1, 2, ...$$

The zero-order part is,

$$\hat{H}_0 \Phi_n = E_n^{(0)} \Phi_n, \qquad n = 0, 1, 2, ...$$

## For the ground state:

Perturbation (MBPT)

$$\chi_0 = \Psi_0 - \Phi_0$$

$$\Delta E = E_0 - E_0^{(0)}$$

$$\Psi_0 = \sum_{m=0}^{\infty} \left[\hat{R}_0(E_0^{(0)})(\hat{V} - \Delta E)\right]^m \Phi_0$$

$$\Delta E = \sum_{m=0}^{\infty} \langle \Phi_0 | \hat{V} \left[\hat{R}_0(E_0^{(0)})(\hat{V} - \Delta E)\right]^m | \Phi_0 \rangle$$

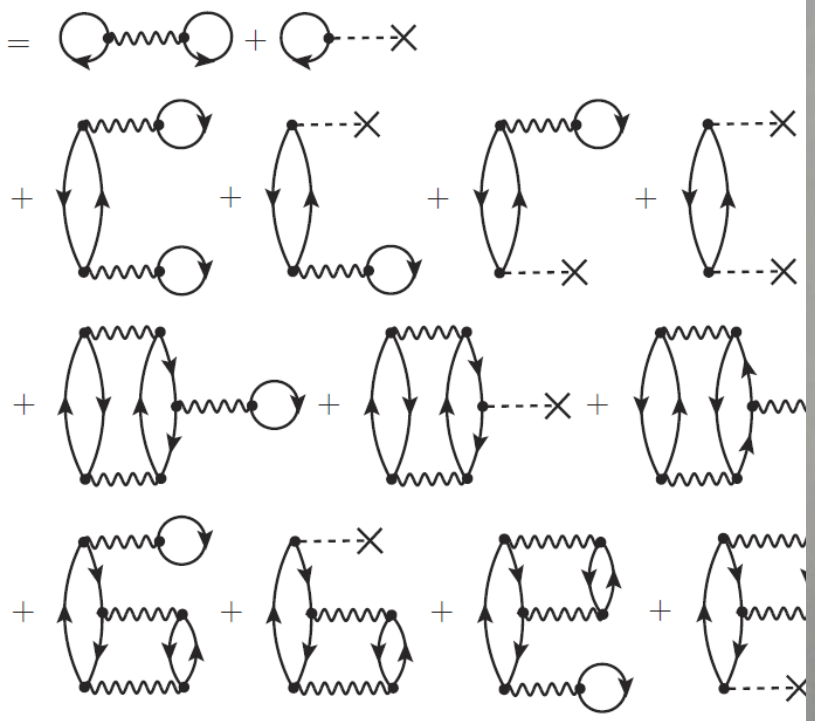
where $\hat{R}_0 = \sum_{i \neq 0} \frac{|\Phi_i\rangle\langle\Phi_i|}{E_0^{(0)} - E_i^{(0)}}$ is called the resolvent of $\hat{H}_0$

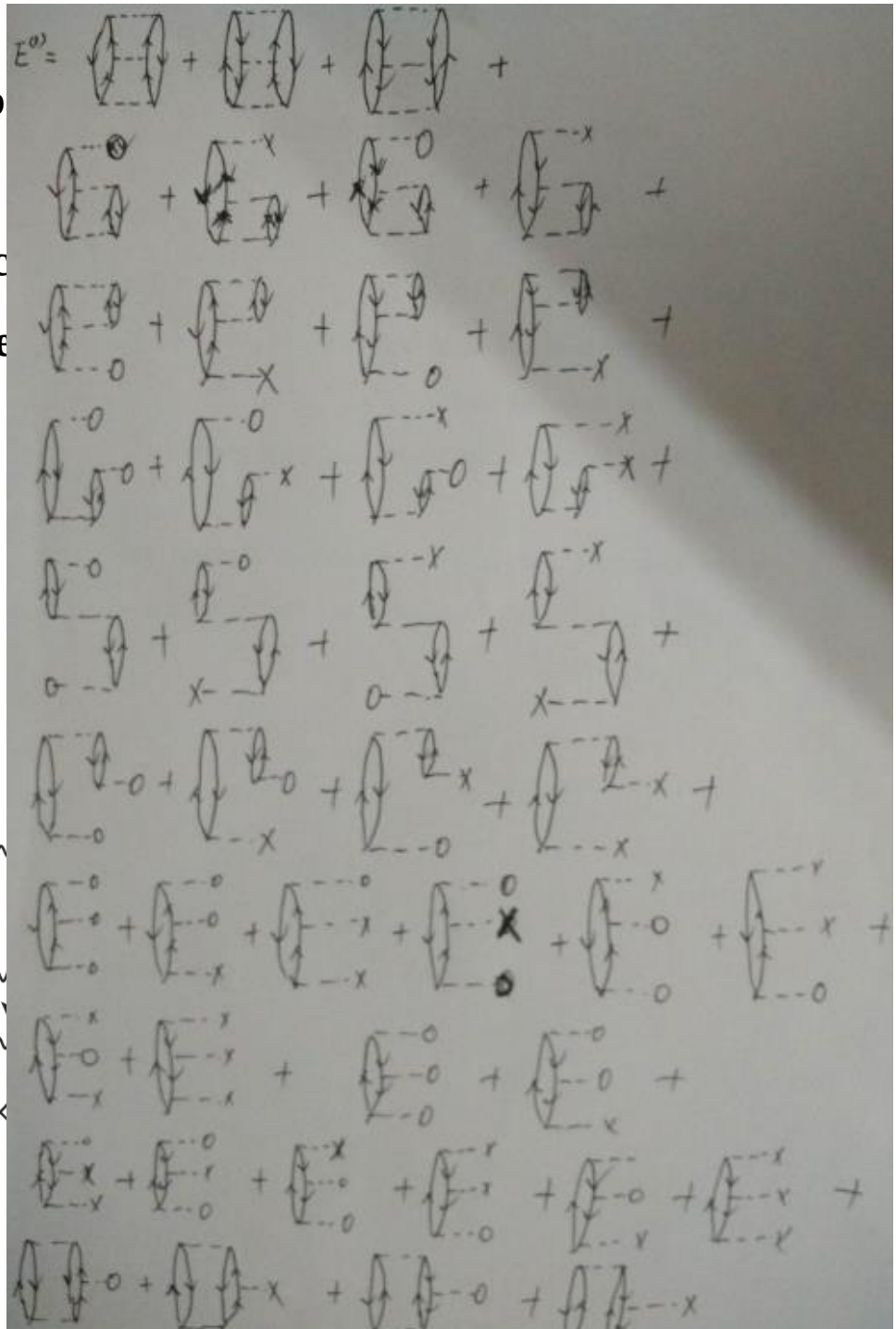
**Rayleigh-Schrodinger method**

**Advantages in HF basis, compared with HO b**

1) **faster convergence;**
2) **some-type perturbation diagrams are canc**
3) **In HO basis, calculations could be $\hbar\omega$ depe**

$E_0 - E_0^{(0)} =$





$$E_0 = E_0^{(0)} + E_0^{(1)} + E_0^{(2)} + E_0^{(3)} + \dots$$

$$\boxed{\text{HF energy} = \langle \phi_0 | H | \phi_0 \rangle}$$

$$E_0^{(1)} = \langle \Phi_0 | \hat{V} | \Phi_0 \rangle$$

$$E_0^{(2)} = \langle \Phi_0 | \hat{V} \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

$$E_0^{(3)} = \langle \Phi_0 | \hat{V} \hat{R}_0 (\hat{V} - \langle \Phi_0 | \hat{V} | \Phi_0 \rangle) \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

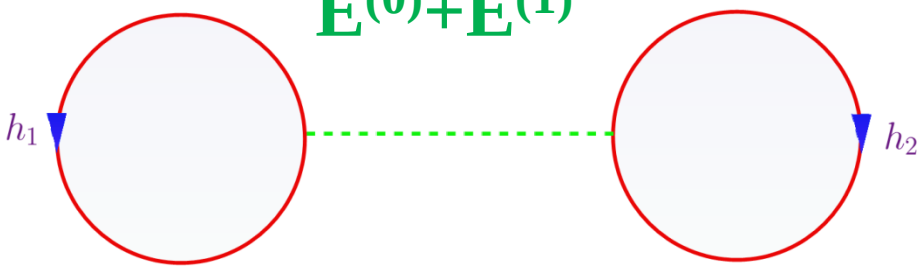
$$\Psi_0 = \underbrace{\Phi_0}_{\text{HF}} + \Psi_0^{(1)} + \Psi_0^{(2)} + \dots$$

$$\Psi_0^{(1)} = \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

$$\Psi_0^{(2)} = \hat{R}_0 (\hat{V} - E_0^{(1)}) \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

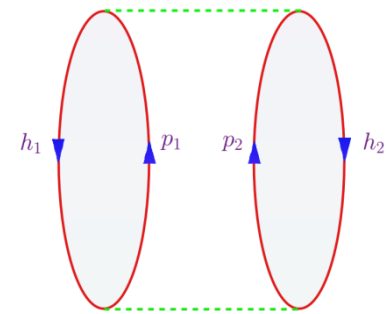
# Anti-Symmetrized Goldstone (ASG) diagram expansion

$E^{(0)}+E^{(1)}$



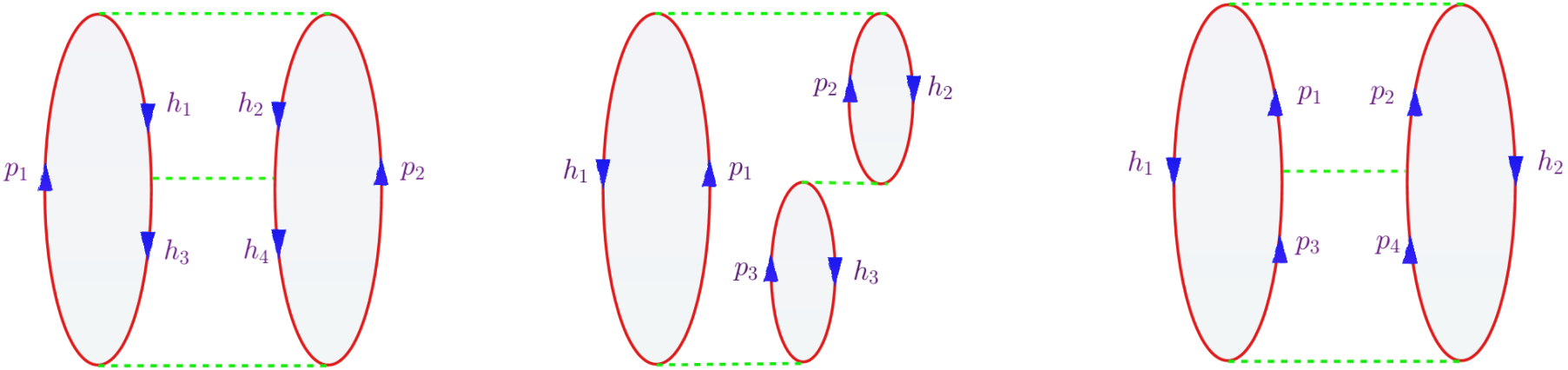

**HF energy**

$E^{(2)}$




$$\frac{1}{4}\sum_{h_1,h_2<\varepsilon_F}\sum_{p_1,p_2>\varepsilon_F}\frac{|\langle p_1p_2|\hat{\mathrm{H}}_{int}|h_1h_2\rangle|^2}{\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2}}$$

$E^{(3)}$

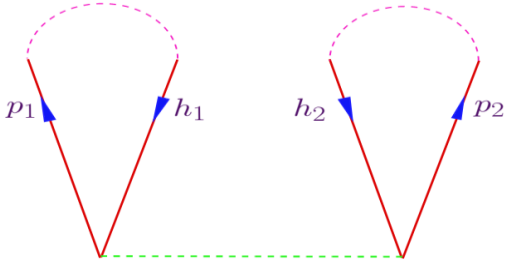



$$\textbf{2p4h} = \frac{1}{8}\sum_{h_1,h_2,h_3,h_4<\varepsilon_F}\sum_{p_1,p_2>\varepsilon_F}\frac{\langle p_1p_2|\hat{\mathrm{H}}_{int}|h_3h_4\rangle\langle h_3h_4|\hat{\mathrm{H}}_{int}|h_1h_2\rangle\langle h_1h_2|\hat{\mathrm{H}}_{int}|p_1p_2\rangle}{(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2})(\varepsilon_{h_3}+\varepsilon_{h_4}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

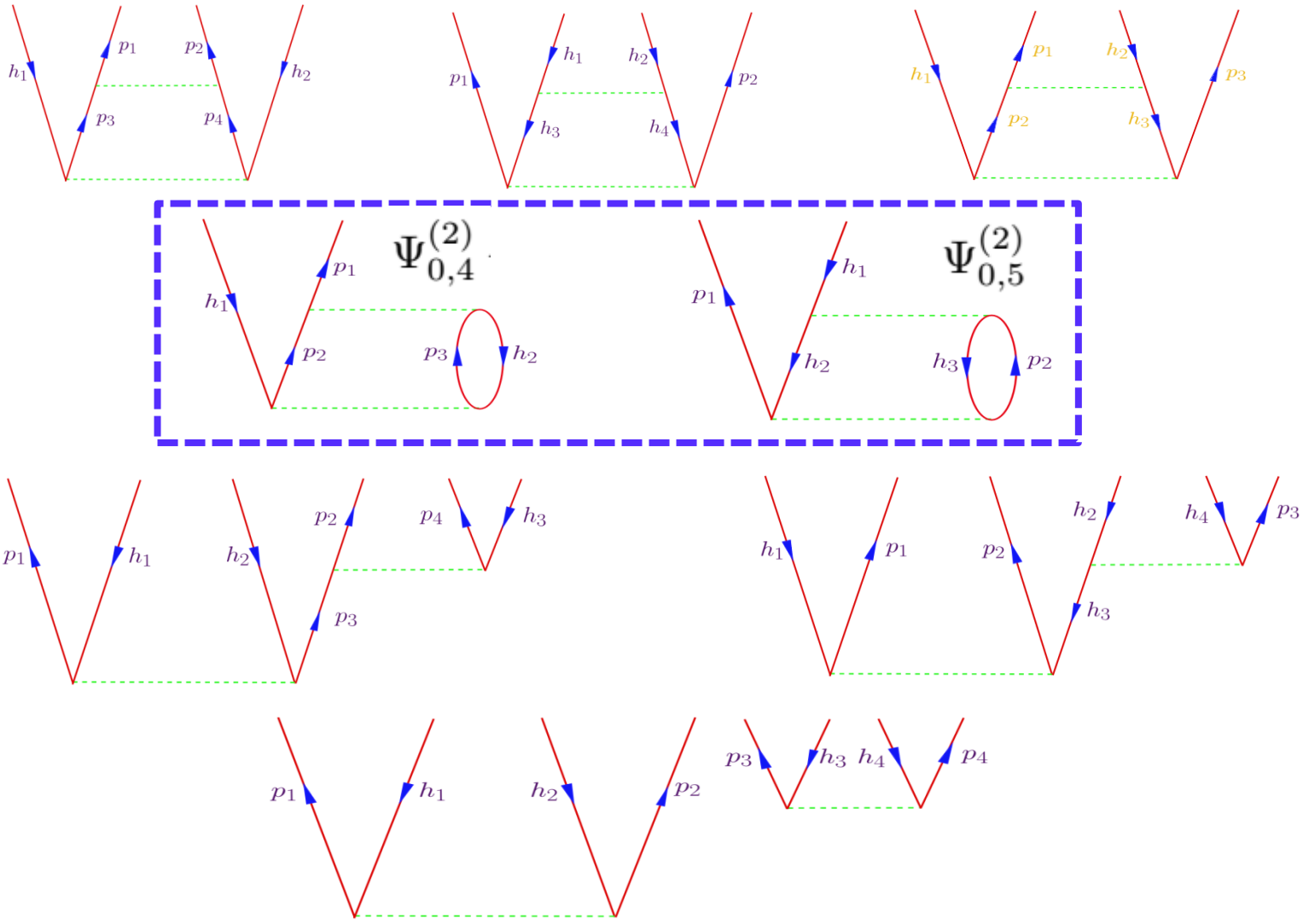
# ASG diagrams for wave functions

$$\Psi_0' = \Phi_0 + \Psi_0^{(1)} + \Psi_{0,4}^{(2)} + \Psi_{0,5}^{(2)}$$

$\psi^{(1)}$



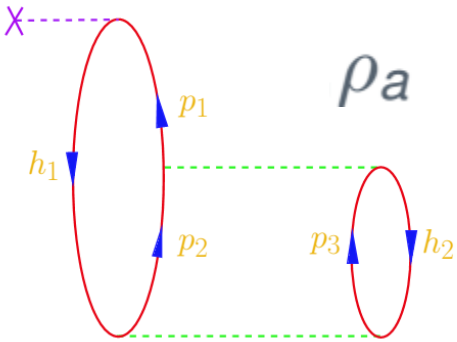

$\psi^{(2)}$



$$\rho(\vec{r}) = \langle\Phi_0|\rho(\vec{r})|\Phi_0\rangle + \underbrace{\langle\Phi_0|\rho(\vec{r})|\Phi_0\rangle\langle\Psi_0^{(1)}|\Psi_0^{(1)}\rangle}_{2^{nd}\text{ order}} + \underbrace{2\rho_a + 2\rho_b + \rho_{c1} + \rho_{c2}}_{2^{nd}\text{ order}} + \dots$$

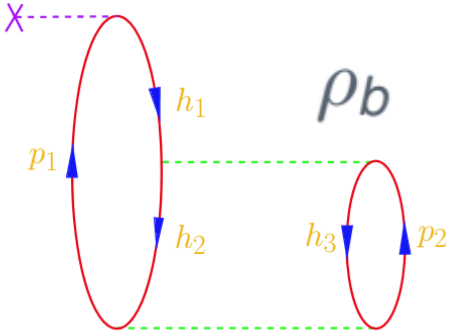
**HF**

$\rho_a$



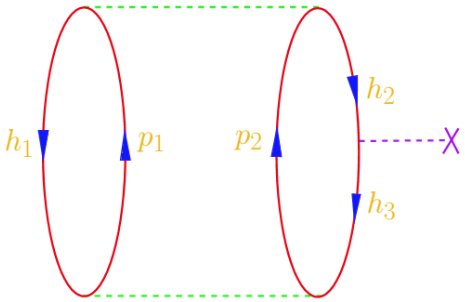

$$\frac{1}{2}\sum_{h_1,h_2<\varepsilon_F}\sum_{p_1,p_2,p_3>\varepsilon_F}\frac{\langle h_1 h_2|\hat{H}|p_2 p_3\rangle\langle p_2 p_3|\hat{H}|p_1 h_2\rangle\langle h_1|\rho|p_1\rangle}{(\varepsilon_{h_1}-\varepsilon_{p_1})(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_2}-\varepsilon_{p_3})}$$

$\rho_b$



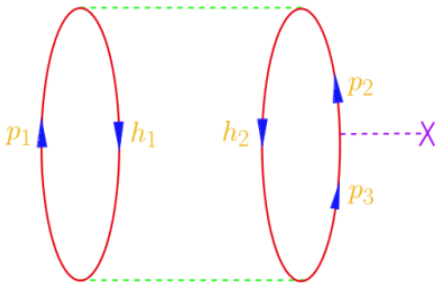

$$-\frac{1}{2}\sum_{h_1,h_2,h_3<\varepsilon_F}\sum_{p_1,p_2>\varepsilon_F}\frac{\langle p_1 p_2|\hat{H}|h_2 h_3\rangle\langle h_2 h_3|\hat{H}|h_1 p_2\rangle\langle h_1|\rho|p_1\rangle}{(\varepsilon_{h_1}-\varepsilon_{p_1})(\varepsilon_{h_2}+\varepsilon_{h_3}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

$\rho_{c1}$




$$-\frac{1}{2}\sum_{h_1,h_2,h_3<\varepsilon_F}\sum_{p_1,p_2>\varepsilon_F}\frac{\langle h_1 h_2|\hat{H}|p_1 p_2\rangle\langle p_1 p_2|\hat{H}|h_1 h_3\rangle\langle h_3|\rho|h_2\rangle}{(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2})(\varepsilon_{h_1}+\varepsilon_{h_3}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

$\rho_{c2}$




$$\frac{1}{2}\sum_{h_1,h_2<\varepsilon_F}\sum_{p_1,p_2,p_3>\varepsilon_F}\frac{\langle p_1 p_3|\hat{H}|h_1 h_2\rangle\langle h_1 h_2|\hat{H}|p_1 p_2\rangle\langle p_2|\rho|p_3\rangle}{(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_3})(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

# Root-mean-square radius calculation

$$\hat{r}_{\mathrm{m}}^2 = \frac{1}{A}\sum_{i=1}^{A}(\vec{r}_i - \vec{r}_0)^2 = \frac{1}{A^2}\sum_{i<j}^{A}(\vec{r}_i - \vec{r}_j)^2 = \left(1-\frac{1}{A}\right)\left(\sum_{i=1}^{A}\vec{r}_i^{\,2}/A\right) - \frac{2}{A^2}\left(\sum_{i<j}^{A}\vec{r}_i\cdot\vec{r}_j\right)$$

$$\approx \left(1-\frac{1}{A}\right)\left(\sum_{i=1}^{A}\vec{r}_i^{\,2}/A\right) \qquad \vec{r}_0 = \frac{1}{A}\sum_{i=1}^{A}\vec{r}_i$$

$$\hat{r}_{\mathrm{pp}}^2 = \frac{1}{Z}\sum_{i=1}^{Z}(\vec{r}_i - \vec{r}_0)^2 \approx \underbrace{\left(1-\frac{1}{A}\right)}_{\text{c.m. effect}}\boxed{\left(\sum_{i=1}^{Z}\vec{r}_i^{\,2}/Z\right)} \longrightarrow \langle R_{\mathrm{pp}}^2\rangle = \frac{\int r^2\rho_{\mathrm{p}}(\vec{r})d^3r}{\int \rho_{\mathrm{p}}(\vec{r})d^3r}$$

$$\langle r_{\mathrm{ch}}^2\rangle = \langle r_{\mathrm{pp}}^2\rangle + R_{\mathrm{p}}^2 + \frac{N}{Z}R_{\mathrm{n}}^2 + \frac{3\hbar^2}{4m_{\mathrm{p}}^2c^2}$$

A. Ekstron, *et al.*, PRC 91, 051301(R) (2015).

$R_{\mathrm{p}} = 0.8775(51)$ fm $\quad \frac{3\hbar^2}{4m_{\mathrm{p}}^2c^2} \approx 0.033\ \mathrm{fm}^2, \quad R_{\mathrm{n}}^2 = -0.1149(27)\ \mathrm{fm}^2$

$$\Delta r_{\mathrm{c.m.}} = \left[\left(1-\frac{1}{A}\right)\langle R_{\mathrm{pp}}^2\rangle\right]^{1/2} - \langle R_{\mathrm{pp}}^2\rangle^{1/2}$$

Approximate center-of-mass correction for charge radius (point-proton)

# NCSM

**S.K. Bogner *et al.*, arXiv0708.3754v2 (2007)**

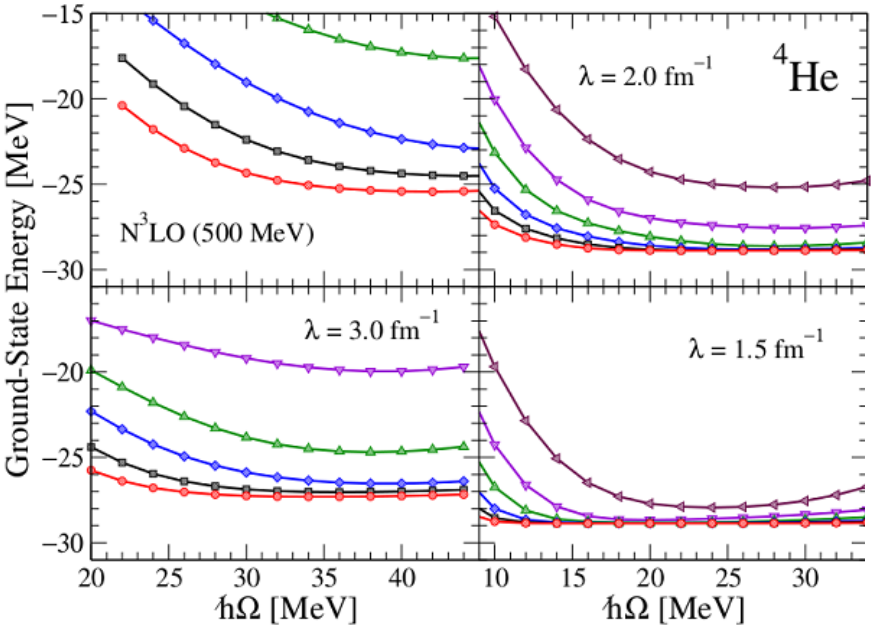


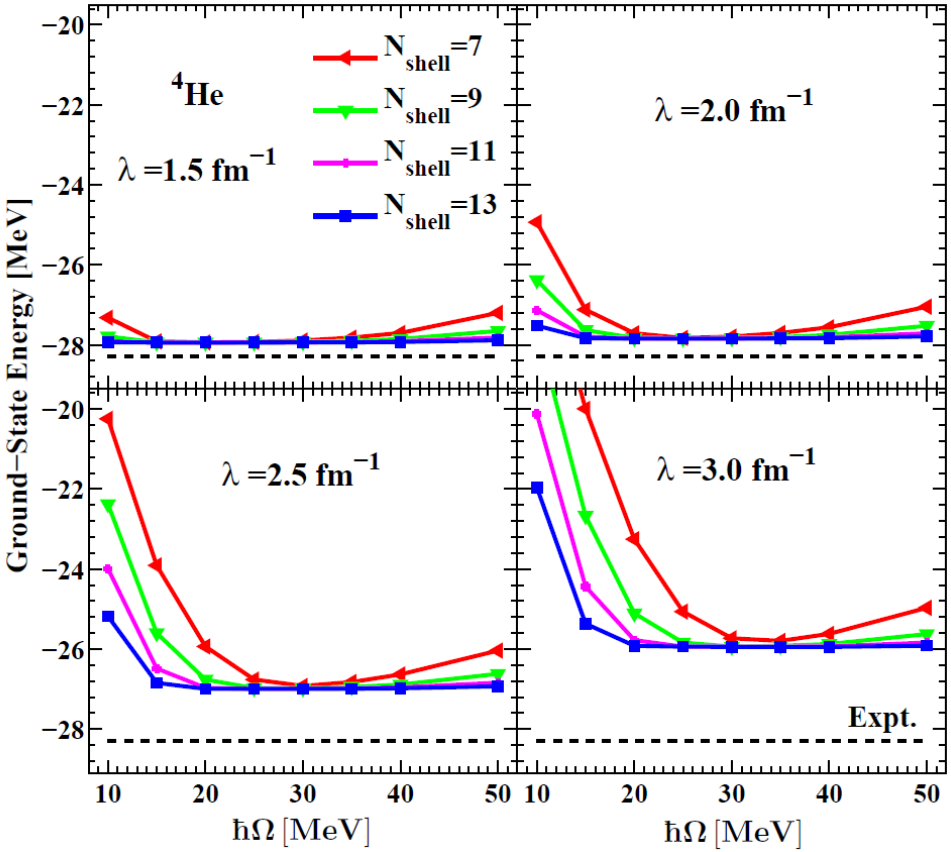

Fig. 3. Ground-state energy of $^4$He as a function of $\hbar\Omega$ at four different value ($\infty$, 3, 2, 1.5 fm$^{-1}$). The initial potential is the 500 MeV N$^3$LO NN-only pot from Ref. [13]. The legend from Fig. 1 applies here.

# $^4$He

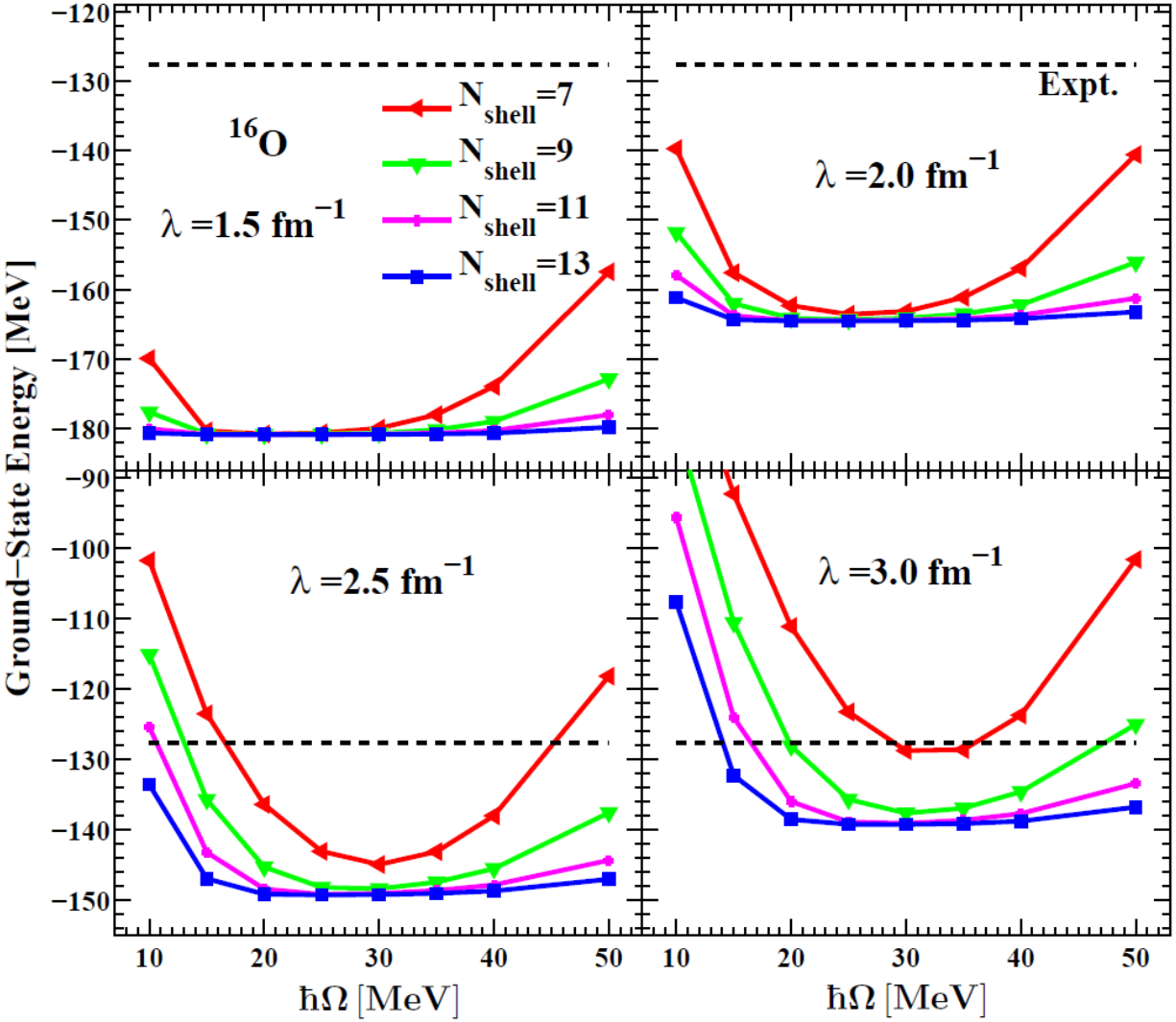
# Our MBPT calculations

**B.S. Hu, F.R. Xu, Z.H. Sun, J.P. Vary, T. Li, PRC 94, 014303 (2016)**

## N$^3$LO+SRG without 3NF

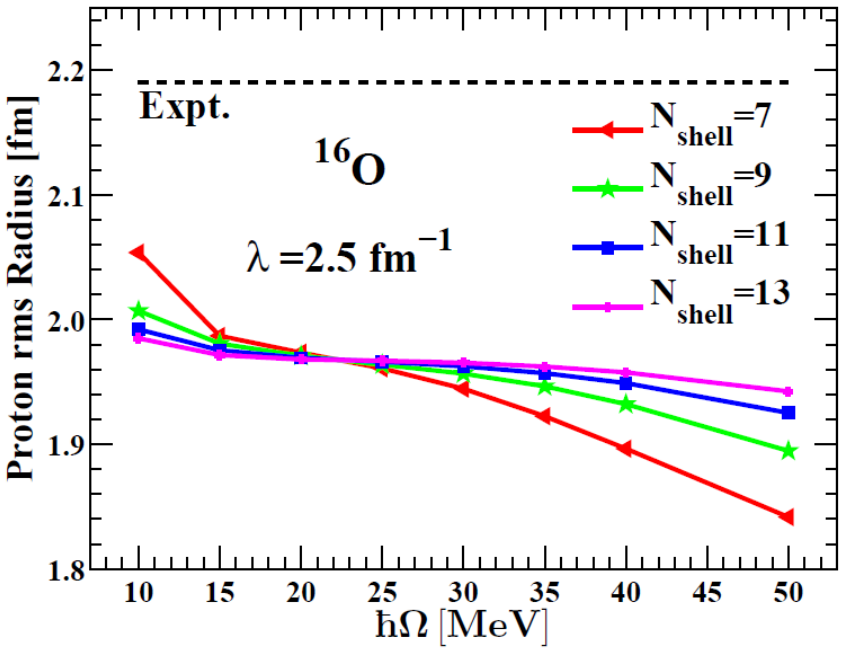


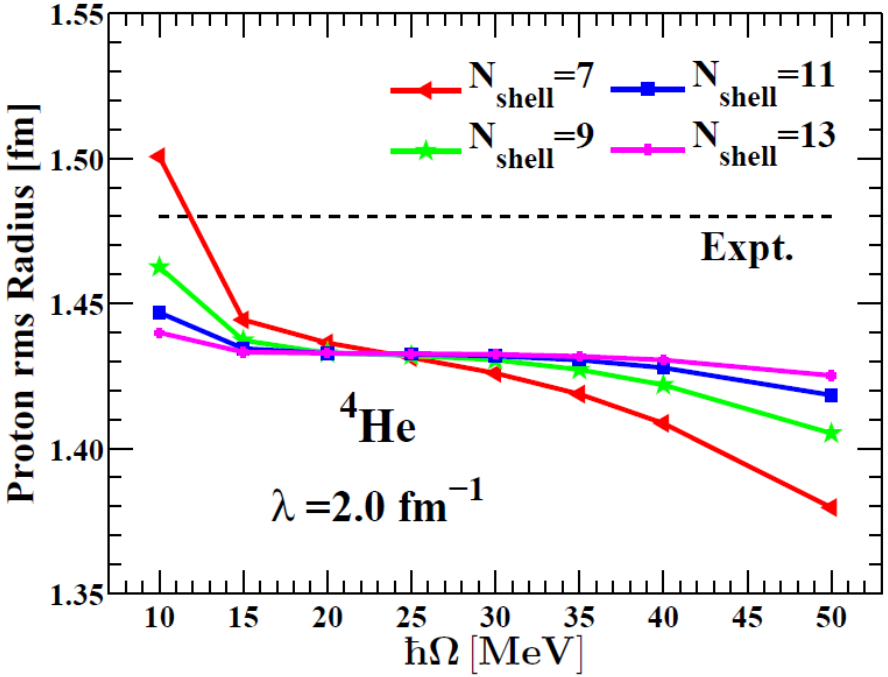
# Our MBPT: N$^3$LO+SRG without 3NF




$^{16}$O

# Our MBPT calculations with N$^3$LO+SRG: convergence in radius



# R. Roth *et al*. (2006) PRC 73, 044312

## AV18, UCOM, corrections to 3rd order in energy, 2nd order in radius

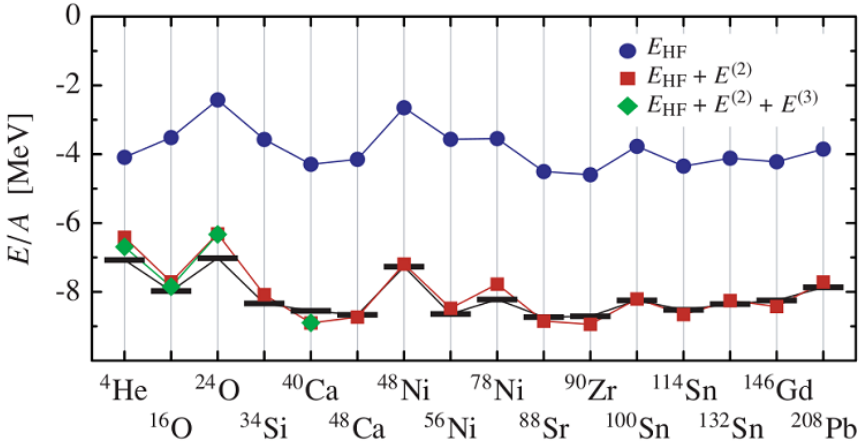



FIG. 5. (Color online) Ground-state energies for selected closed-shell nuclei in HF approximation and with added second- and third-order MBPT corrections. The correlated AV18 potential with $I_\vartheta = 0.09\,\mathrm{fm}^3$ was used. The bars indicate the experimental binding energies [31].

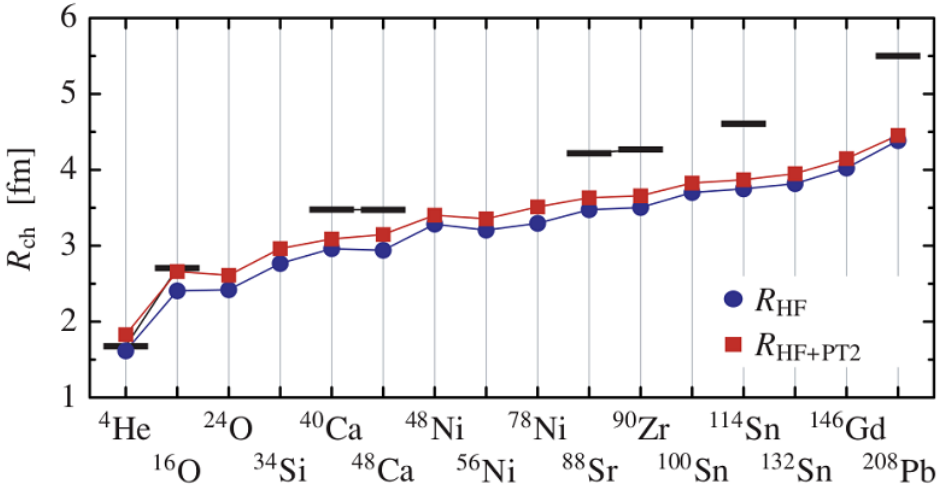



FIG. 8. (Color online) Charge radii for selected closed-shell nuclei in the HF approximation and with added second-order MBPT corrections. The correlated AV18 potential with $I_\vartheta = 0.09\,\mathrm{fm}^3$ was used. The bars indicate experimental charge radii [32].

## HF-MBPT calculations for $^4$He with N$^3$LO-SRG, $N_{shell}$=13, hΩ=35 MeV

**Binding energy**

| | SRG flow parameter $\lambda$ (fm$^{-1}$) | | | |
|---|---|---|---|---|
| | 1.5 | 2.0 | 2.5 | 3.0 |
| Expt. [60] | -28.296 | -28.296 | -28.296 | -28.296 |
| NCSM [61] | -28.20 | -28.41 | -27.43 | -26.80 |
| SHF | -25.754 | -21.864 | -15.854 | -10.278 |
| PT2 | -1.788 | -5.088 | -9.652 | -13.783 |
| PT3 | -0.391 | -0.899 | -1.523 | -1.953 |
| SHF+PT2+PT3 | -27.933 | -27.850 | -27.029 | -26.013 |

**$r_p$(NCSM)=1.418 fm with $N_{max}$=10**

**Point-proton rms radius**

| | SRG flow parameter $\lambda$ (fm$^{-1}$) | | | |
|---|---|---|---|---|
| | 1.5 | 2.0 | 2.5 | 3.0 |
| Expt. | 1.477 | 1.477 | 1.477 | 1.477 |
| SHF | 1.677 | 1.652 | 1.714 | 1.816 |
| PT2 | 0.007 | 0.001 | -0.021 | -0.065 |
| $\Delta r_{\mathrm{c.m.}}$ | -0.226 | -0.222 | -0.227 | -0.235 |
| SHF+PT2+$\Delta r_{\mathrm{c.m.}}$ | 1.458 | 1.431 | 1.466 | 1.516 |

## HF-MBPT calculations for $^{16}O$ with $N^3LO$-SRG, $N_{shell}$=13, $h\Omega$=35 MeV

**Binding energy**

| | SRG flow parameter $\lambda$ ($fm^{-1}$) | | | |
|---|---|---|---|---|
| | 1.5 | 2.0 | 2.5 | 3.0 |
| Expt. [60] | -127.619 | -127.619 | -127.619 | -127.619 |
| SHF | -169.968 | -133.169 | -85.173 | -44.102 |
| PT2 | -10.132 | -29.497 | -59.617 | -88.326 |
| PT3 | -0.794 | -1.931 | -4.630 | -7.339 |
| SHF+PT2+PT3 | -180.893 | -164.597 | -149.419 | -139.767 |

**3NF important!**

**Point-proton rms radius**

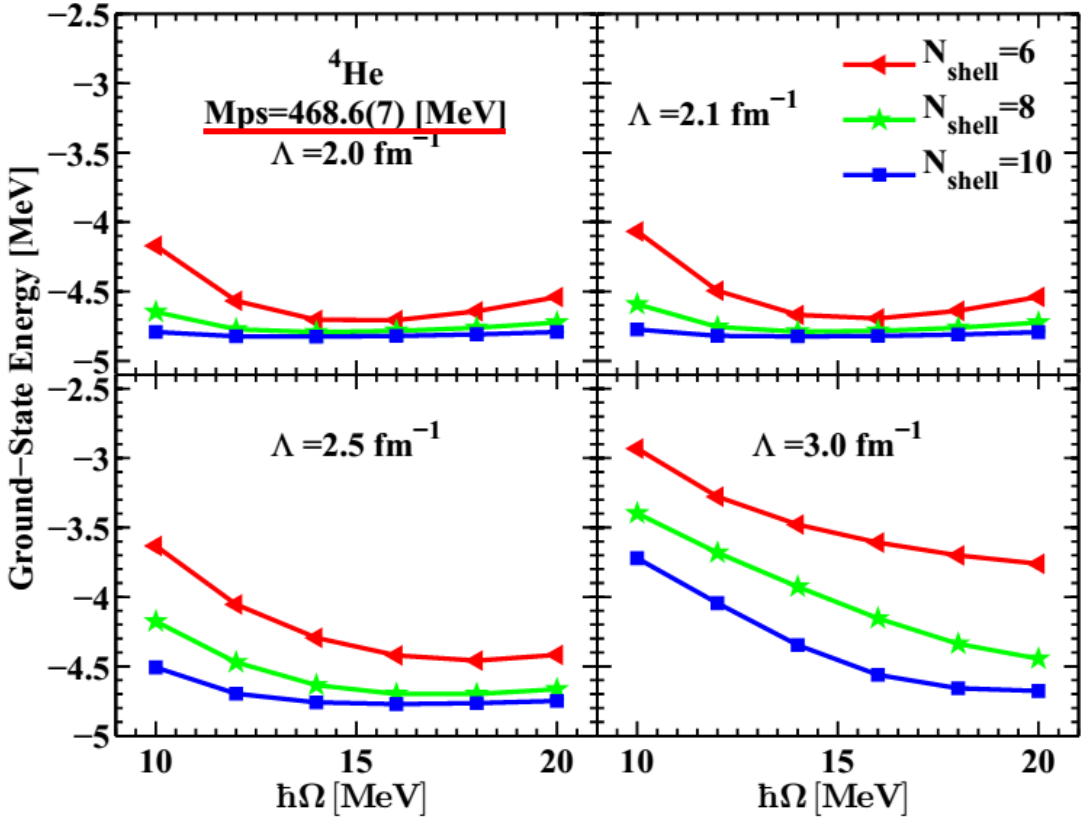
| | SRG flow parameter $\lambda$ ($fm^{-1}$) | | | |
|---|---|---|---|---|
| | 1.5 | 2.0 | 2.5 | 3.0 |
| Expt. | 2.581 | 2.581 | 2.581 | 2.581 |
| SHF | 2.098 | 2.096 | 2.201 | 2.345 |
| PT2 | 0.011 | 0.011 | -0.006 | -0.042 |
| $\Delta r_{c.m.}$ | -0.067 | -0.067 | -0.070 | -0.073 |
| SHF+PT2+$\Delta r_{c.m.}$ | 2.042 | 2.040 | 2.125 | 2.230 |

# LQCD ➡ MBPT calculations

**LQCD was provided by Aoki and Inoue**

**We renormalize it using $V_{low\text{-}k}$**

**Preliminary**




**$E^{expt}$ = -28.3 MeV**

FIG. 6. Ground-state energy of $^4He$ in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different $V_{lowk}$ interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

# LQCD + MBPT

**Preliminary**

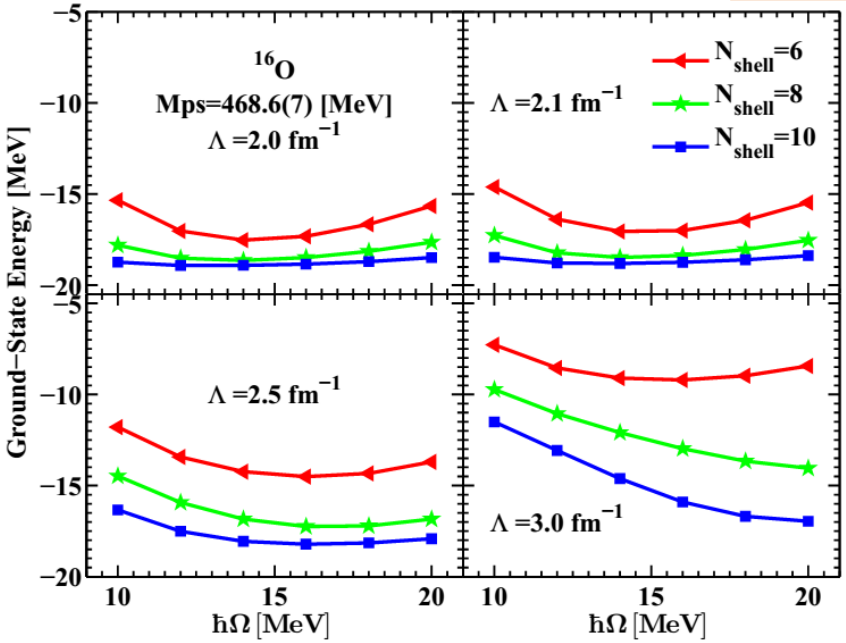



FIG. 7. Ground-state energy of $^{16}O$ in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different $V_{lowk}$ interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

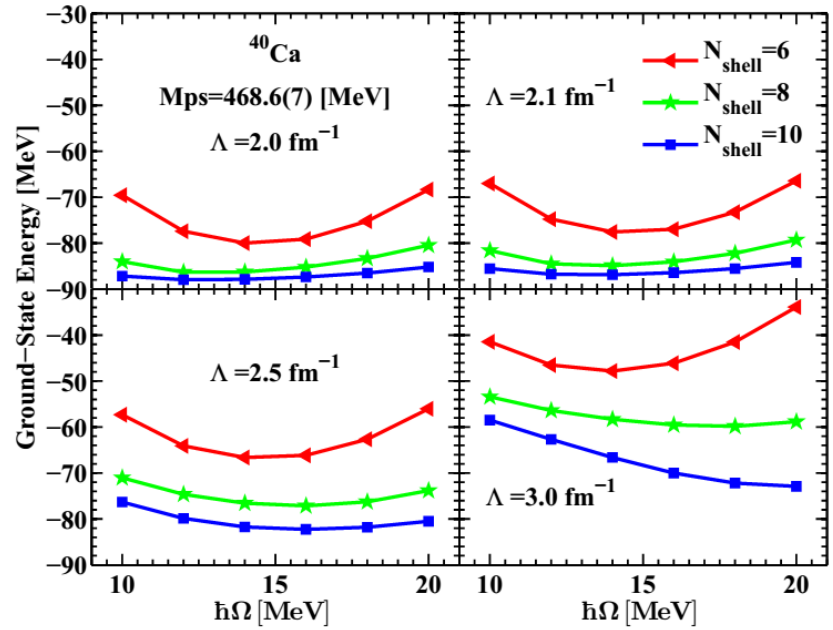
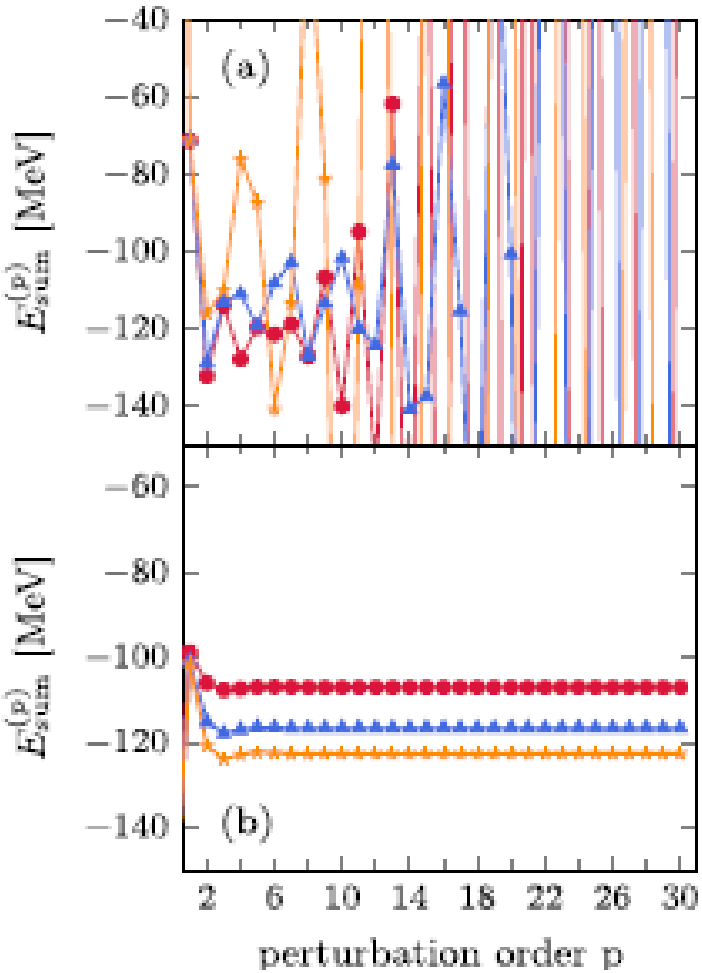



FIG. 8. Ground-state energy of $^{40}Ca$ in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different $V_{lowk}$ interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

$E^{expt}$ = -127.6 MeV

$E^{expt}$ = -342.0 MeV

A. Tichaia, J. Langhammera, S. Binder R. Roth, PLB 756 (2016) 283.




**MBPT using HO basis**

**MBPT using HF basis**

**Fig. 1.** Partial sums for the ground-state energy of $^{16}$O in the HO basis (a) and the HF basis (b) for the NN+3N-full interaction with $\alpha = 0.08\,\mathrm{fm}^4$ and model-space truncation parameters $N_{max} = 2$ (●), 4 (▲), and 6 (★). The corresponding energy corrections for each order are displayed in panels (c) and (d), respectively. All calculations are performed at frequency $\hbar\Omega = 24\,\mathrm{MeV}$.

# 3.2 HF-RPA calculations for multipole giant resonances

**i) Chiral EFT potential $NNLO_{sat}$ (NN + NNN)**

**ii) Hartree-Fock + Random Phase Approximation (RPA)**

**iii) Multipole giant resonances (monopole, dipole and quadrupole)**

$$H = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j=1}^{A} V_{ij} - \frac{(\sum_{i=1}^{A} \boldsymbol{p}_i)^2}{2mA} + \sum_{i<j<k=1}^{A} W_{ijk}$$

$$= (1 - \frac{1}{A}) \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j=1}^{A} (V_{ij} - \frac{\boldsymbol{p}_i \cdot \boldsymbol{p}_j}{mA}) + \sum_{i<j<k=1}^{A} W_{ijk}$$

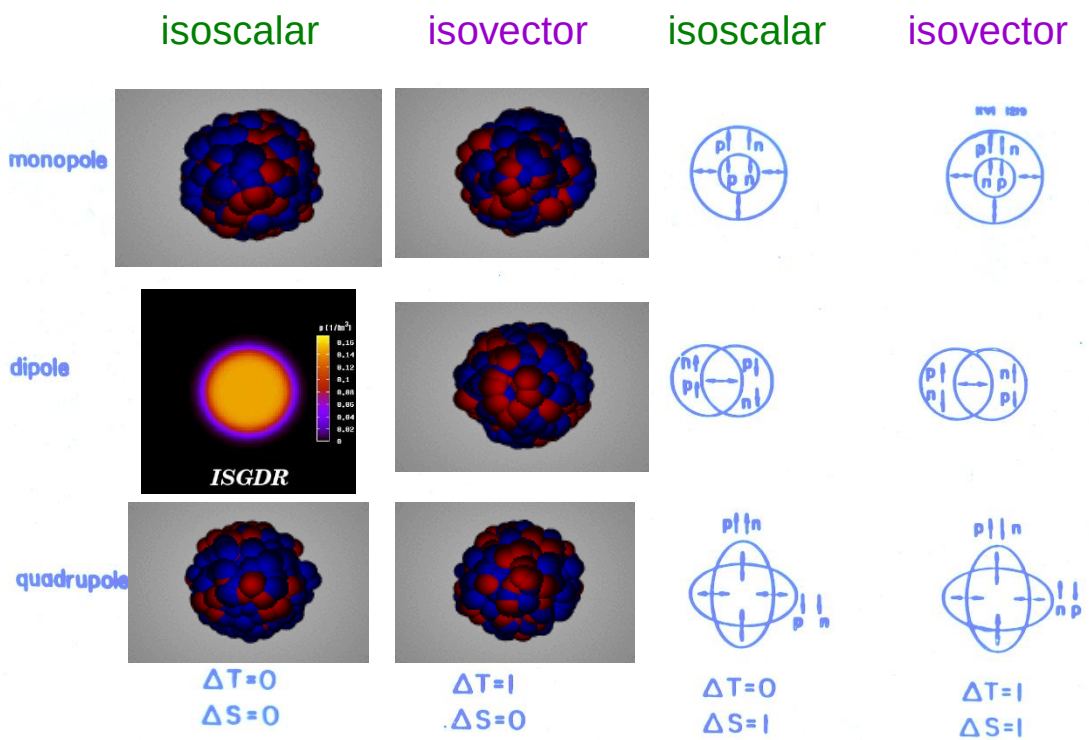
**Performing spherical HF**

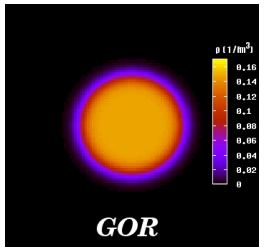
$$W_{\mathrm{NO}} = \frac{1}{6} \sum_{ijk} \langle ijk|W|ijk\rangle + \frac{1}{2} \sum_{ijps} \langle pij|W|sij\rangle \{a_p^\dagger a_s\} + \frac{1}{4} \sum_{ipqst} \langle pqi|W|sti\rangle \{a_p^\dagger a_q^\dagger a_t a_s\} +$$

$$\frac{1}{36} \sum_{pqrstu} \langle pqr|W|stu\rangle \{a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s\},$$

**NNN normal ordering for RPA calculation**

# Nuclear giant resonances

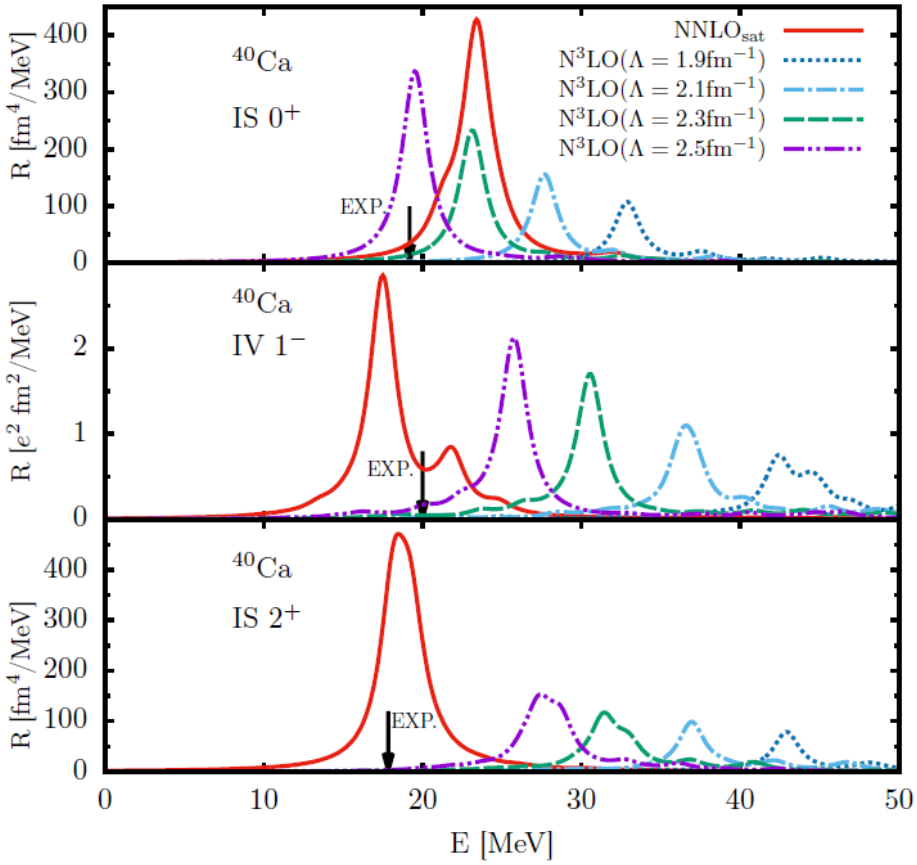



*animations courtesy M. Itoh and T. Aumann*

From U. Garg

# HF-RPA calculations with $N^3LO$ (NN) and $NNLO_{sat}$ (NN+NNN)

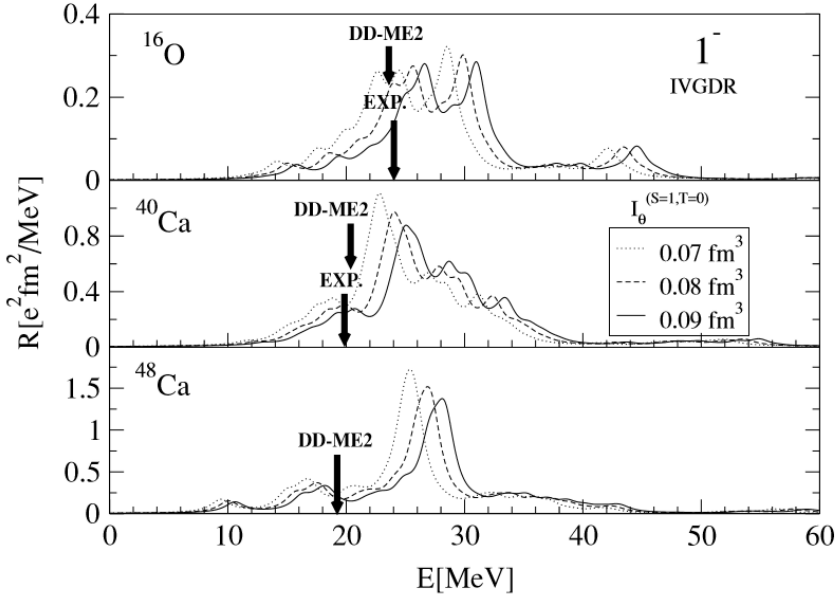


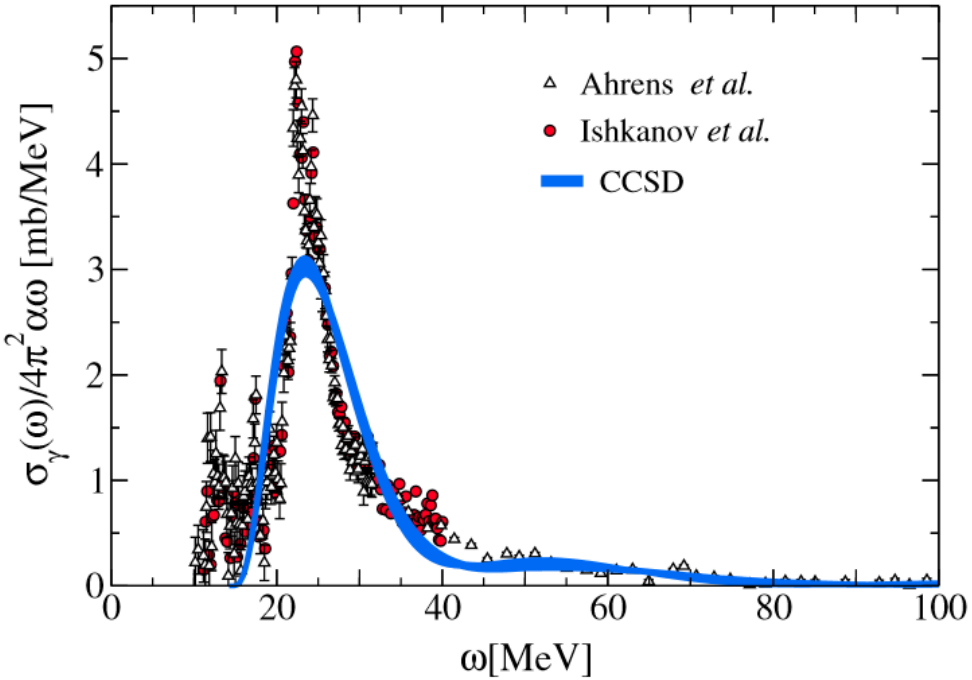

**$N^3LO$ (NN) without NNN calculations are sensitive to the momentum cutoff $\Lambda$ in $V_{low\text{-}k}$.**

**One cannot find a single $\Lambda$ value to describe simultaneously the different types of resonances.**

# AV18 UCOM RPA

**N. Paar, P. Papakonstantinou, H. Hergert, R. Roth, PRC 74, 014318 (2006)**

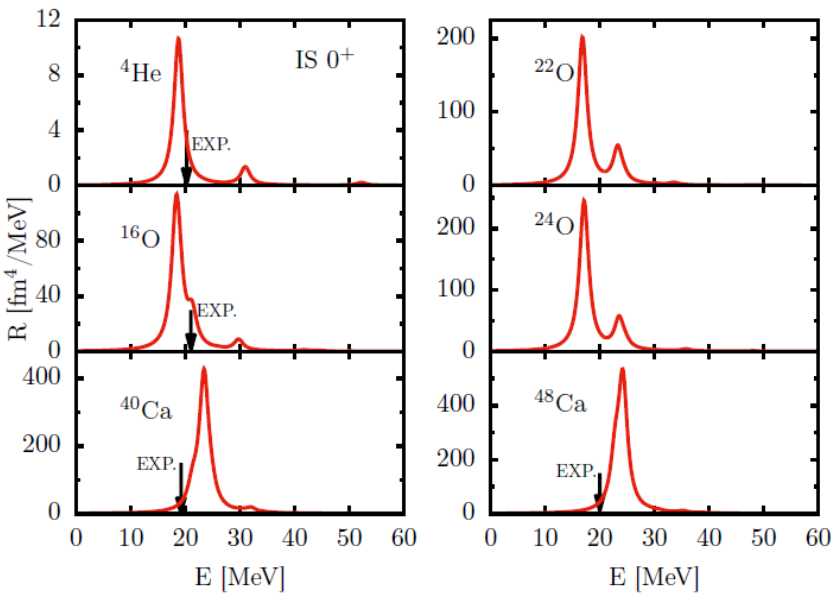


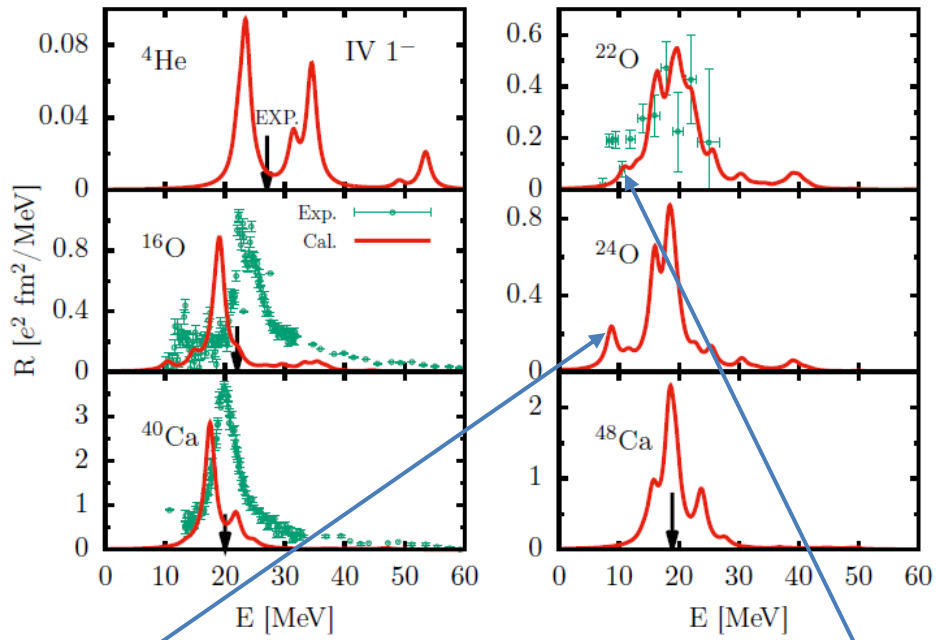

$^{16}$O, IVGDR , CCSD with bare N3LO(NN)
S. Bacca, N. Barnea, G. Hagen, G. Orlandini,
T. Papenbrock, PRL 111, 122502 (2013)

# NNLO$_{sat}$ HF-RPA

ISGMR




IVGDR




$^{24}$O: 8.7 MeV
Neutron excitations:

$87\%\ 1s_{1/2} \rightarrow 1p_{3/2}$

$5\%\ 0d_{5/2} \rightarrow 0f_{7/2}$

$^{22}$O: 10.8 MeV
Neutron excitations:

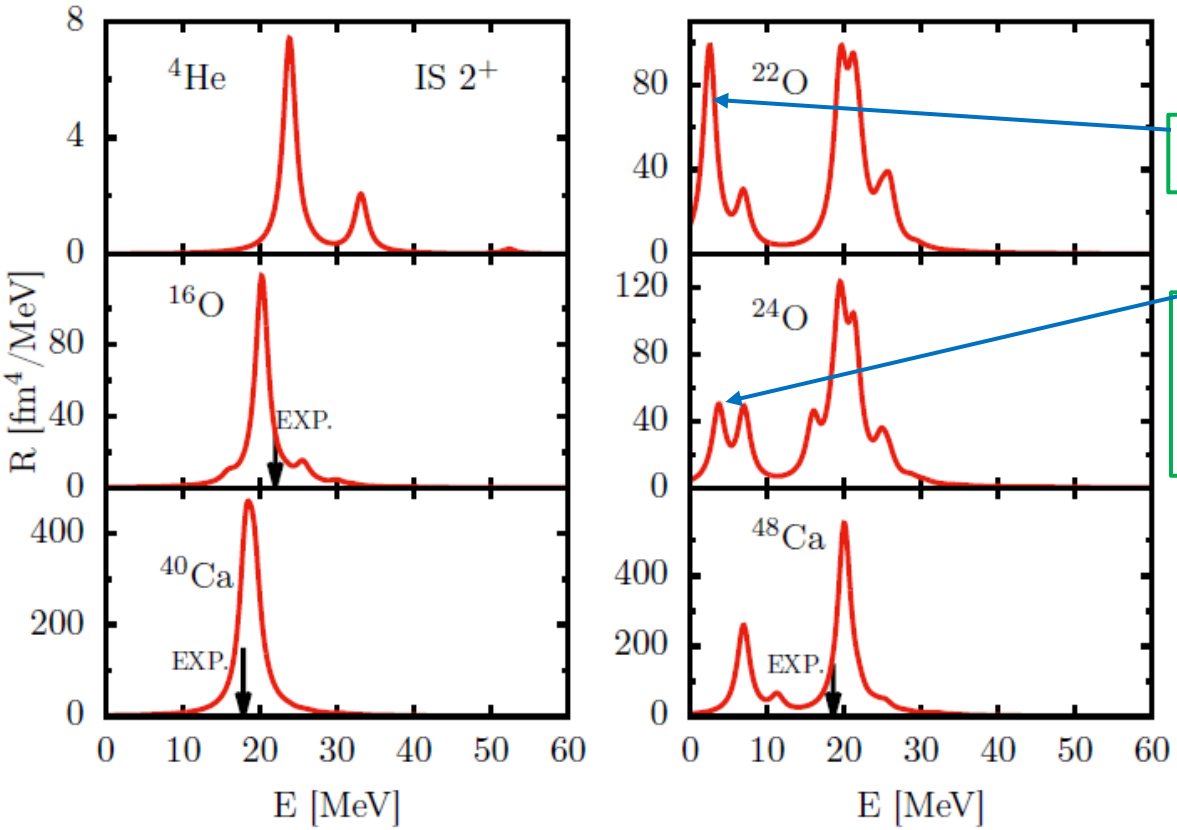
$62\%\ 0d_{5/2} \rightarrow 1p_{3/2}$

$15\%\ 0p_{1/2} \rightarrow 1s_{1/2}$

$8\%\ 0d_{5/2} \rightarrow 0f_{7/2}$

ISGQR

**NNLO$_{sat}$ HF-RPA**




$^{22}$O: $E_x$=2.5 MeV, 1ν $0d_{5/2} \rightarrow 1s_{1/2}$

$^{24}$O: $E_x$=3.7 MeV, 1ν $1s_{1/2} \rightarrow 0d_{3/2}$

$E_x$=7 MeV, 1ν $0d_{5/2} \rightarrow 0d_{3/2}$

**GSM**

$^{22}$O: $E(2_1^+)$=3.3 MeV, 1ν $0d_{5/2} \rightarrow 1s_{1/2}$

$^{24}$O: :$E(2_1^+)$=4.5 MeV, 1ν $1s_{1/2} \rightarrow 0d_{3/2}$

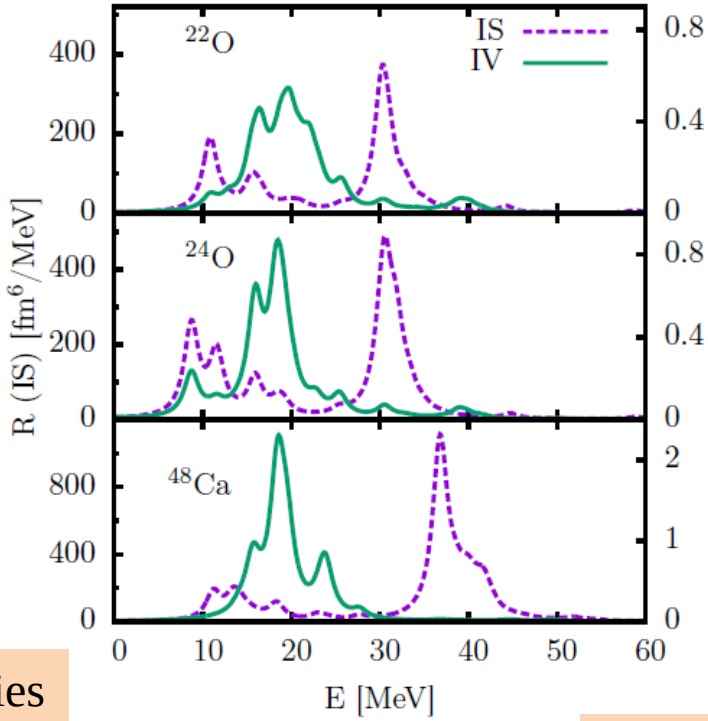
**Expt**

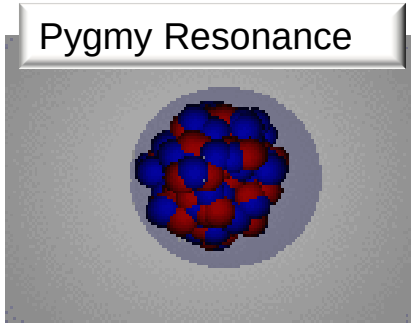
$^{22}$O: $E(2_1^+)$=3.2 MeV

$^{24}$O: :$E(2_1^+)$=4.7 MeV

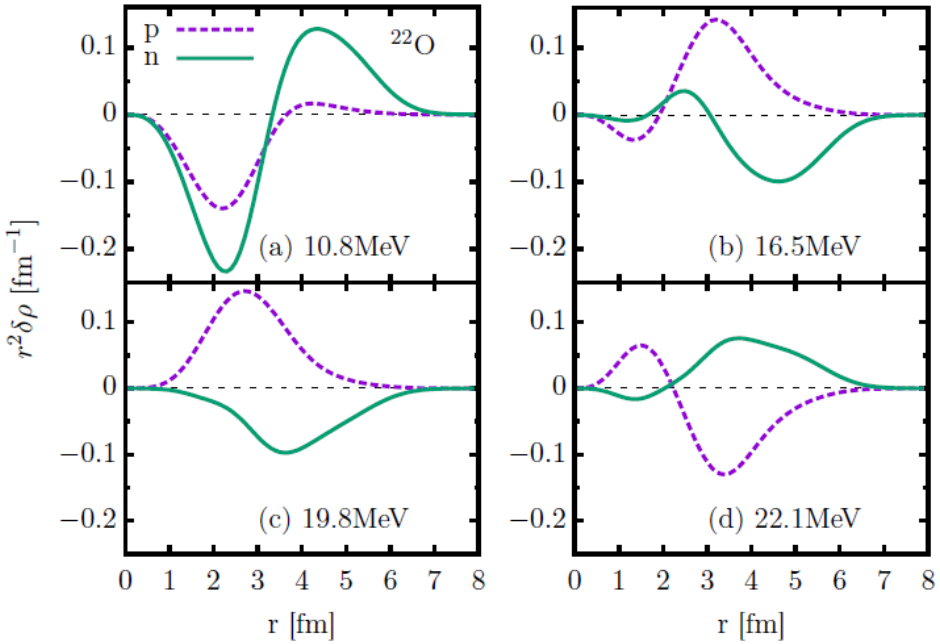
# NNLO$_{sat}$ HF-RPA

Dipole resonances

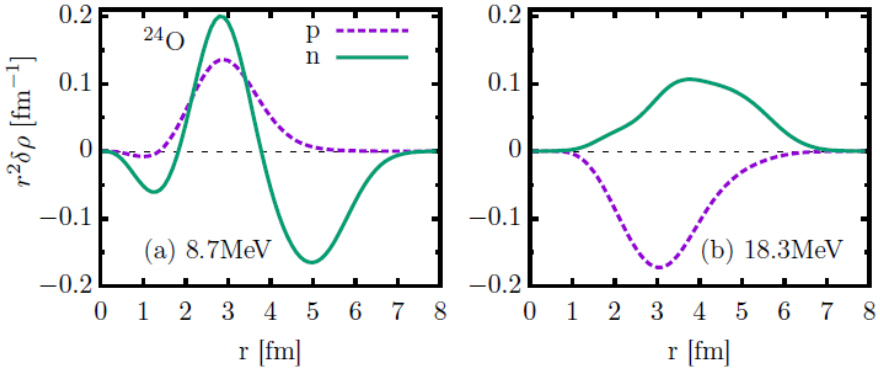



Transition densities



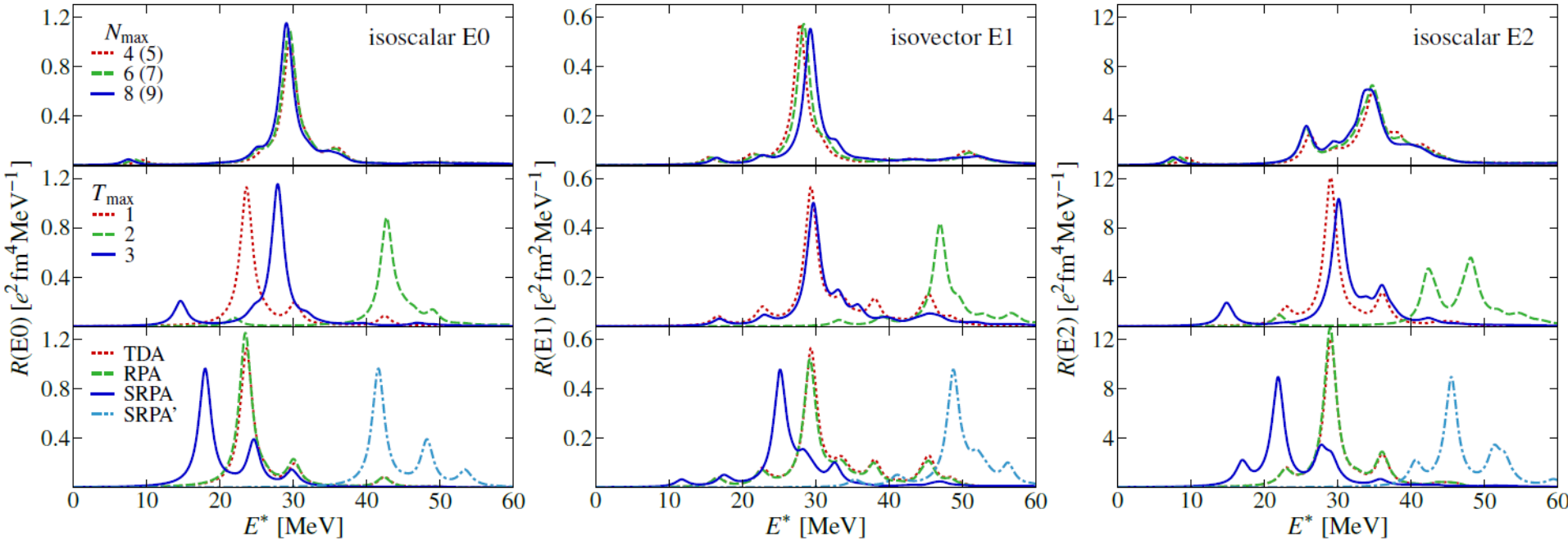

Transition densities




$$\delta\rho_\nu^J(r) = \sum_\mu \langle\nu| \sum_i \frac{\delta(r-r_i)}{r^2} Y_{J\mu}(\hat{\boldsymbol{r}}_i)|0\rangle$$

**3NFs (high-order forces) may be important for descriptions of nuclear giant resonances.**

**How about high-order many-body correlations? RPA considers only 1p1h correlation.**

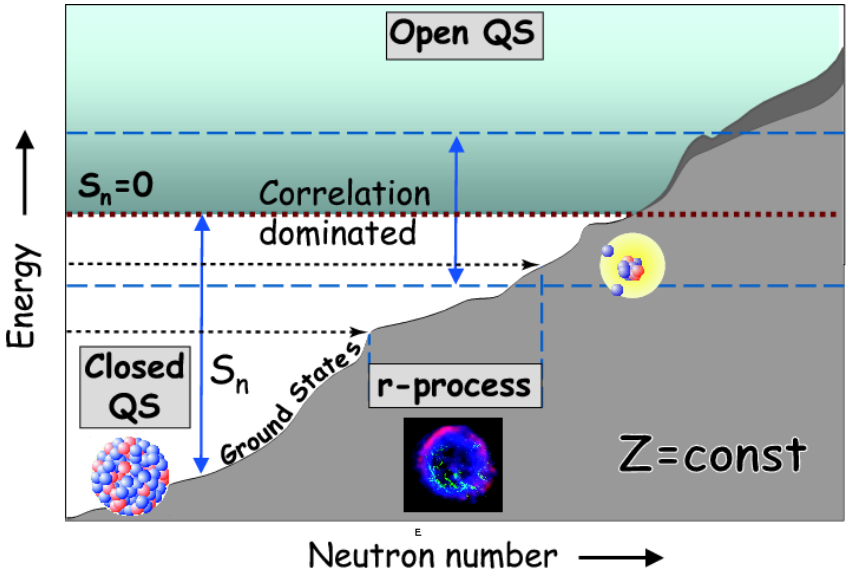



$^{16}$O, SRG N$^3$LO(NN)+N$^2$LO(NNN) NCSM in HF basis

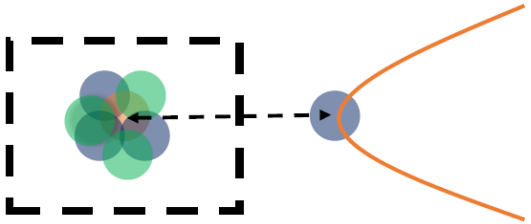
C. Stumpf, T. Wolfgruber, R. Roth, arXiv:1709.06840v1 [nucl-th] 20 Sep 2017

**RPA 1p1h approach seems good!**

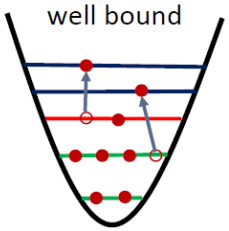
# 3.3 Gamow Shell Model (GSM) for resonance and continuum in weakly bound nuclei



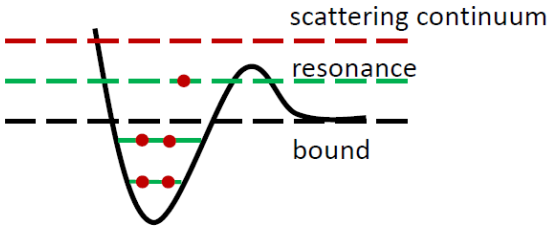

Closed quantum system

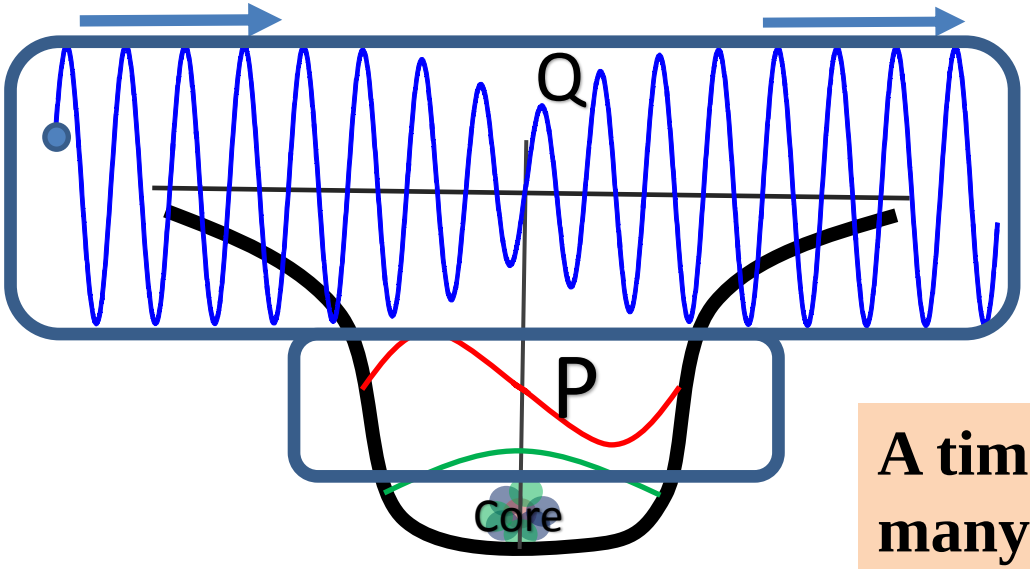


Open quantum system




HO basis




**A time-dependent many-body problem**

# Gamow Shell Model

## T. Berggren, Nucl. Phys. A109 (1968) 265

**Single-particle basis in complex-*k* plane describe bound, resonance and scattering on equal footing.**

**The radial wave function** $u(r)/r$

$$\frac{d^2u(k,r)}{dr^2} = \left[\frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}U(r) - k^2\right]u(k,r)$$

boundary conditions

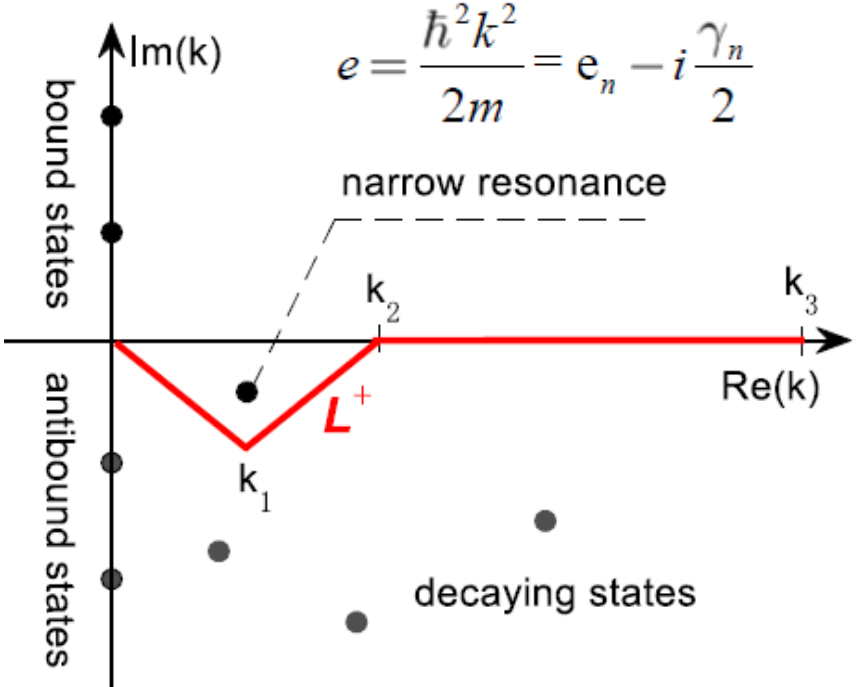
$$u(0) = 0,$$

$$u(a)O_l'(ka) - u'(a)O_l(ka) = 0$$

$$O_l(kr) \sim e^{i(kr - l\pi/2)}$$

**Outgoing solution at large distance**

$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

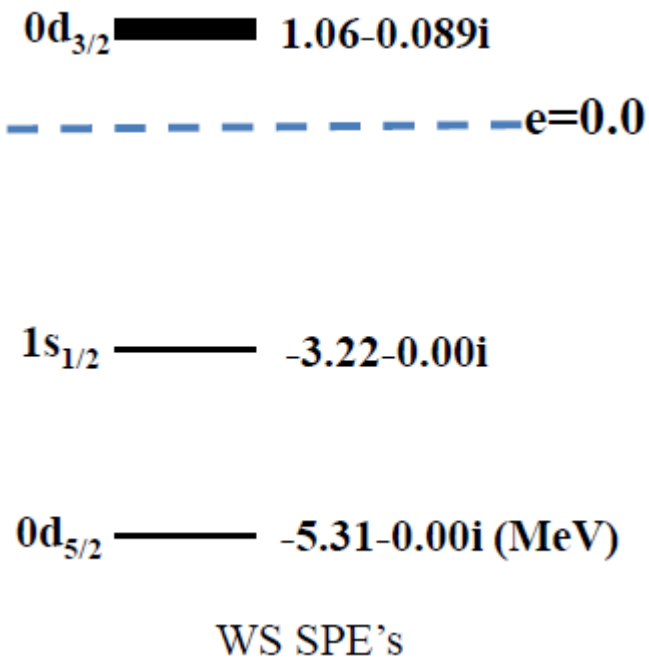



**Orthogonality and Completeness**

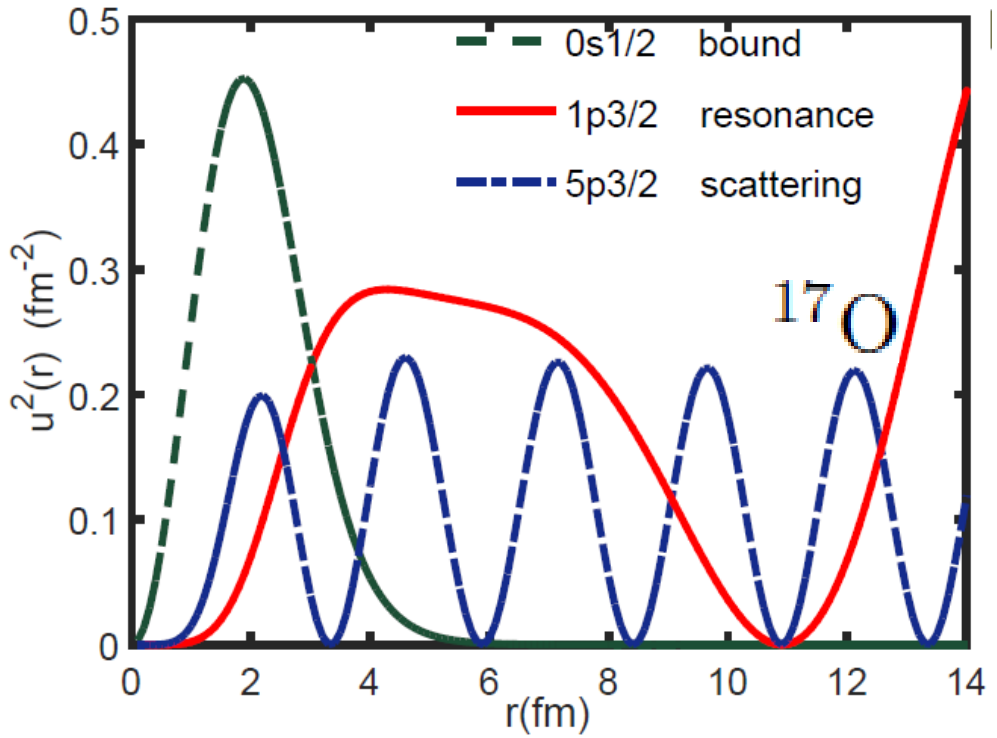
$$\delta(r - r') = \sum_n w_n(r, k_n)w_n(r', k_n) + \frac{1}{\pi}\int_{L^+} dq\, u(r,q)u(r',q)$$

**Discretized**

# Woods-Saxon potential, CD-Bonn, $^{16}$O core




$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$



# CGSM based on realistic nuclear forces

**Realistic nuclear forces ⇒ Gamow shell model calculations**

## Taking a doubly closed core

**Bare forces: Strong repulsion, slow convergence**

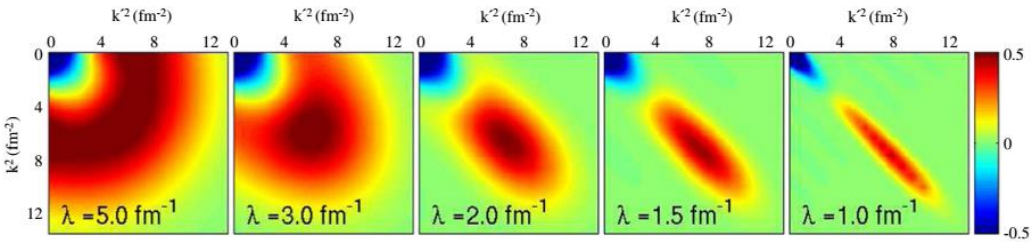
$V_{low\,k}$ or SRG

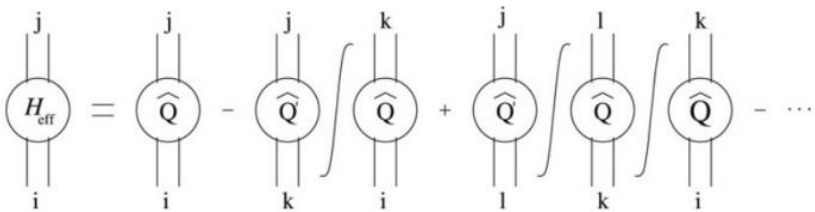
**To remove hard core, but still keep good descriptions of NN scattering phase shifts**

$$\langle \alpha_P | \bar{H}_{\rm eff} | \alpha_{P'} \rangle = \sum_{\alpha_{P''}} \sum_{\alpha_{P'''}} \sum_{kk'k''\in\mathcal{K}} \langle \alpha_P | \tilde{k}'' \rangle \langle \tilde{k}'' | \alpha_{P''} \rangle \langle \alpha_{P''} | \tilde{k} \rangle E_k \langle \tilde{k} | \alpha_{P'''} \rangle \langle \alpha_{P'''} | \tilde{k}' \rangle \langle \tilde{k}' | \alpha_{P'} \rangle$$

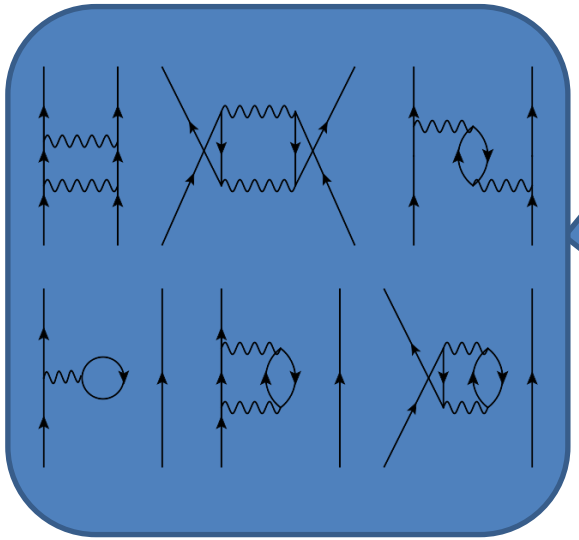
$$\frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T_{\rm rel}, H_\lambda], H_\lambda]$$

Q

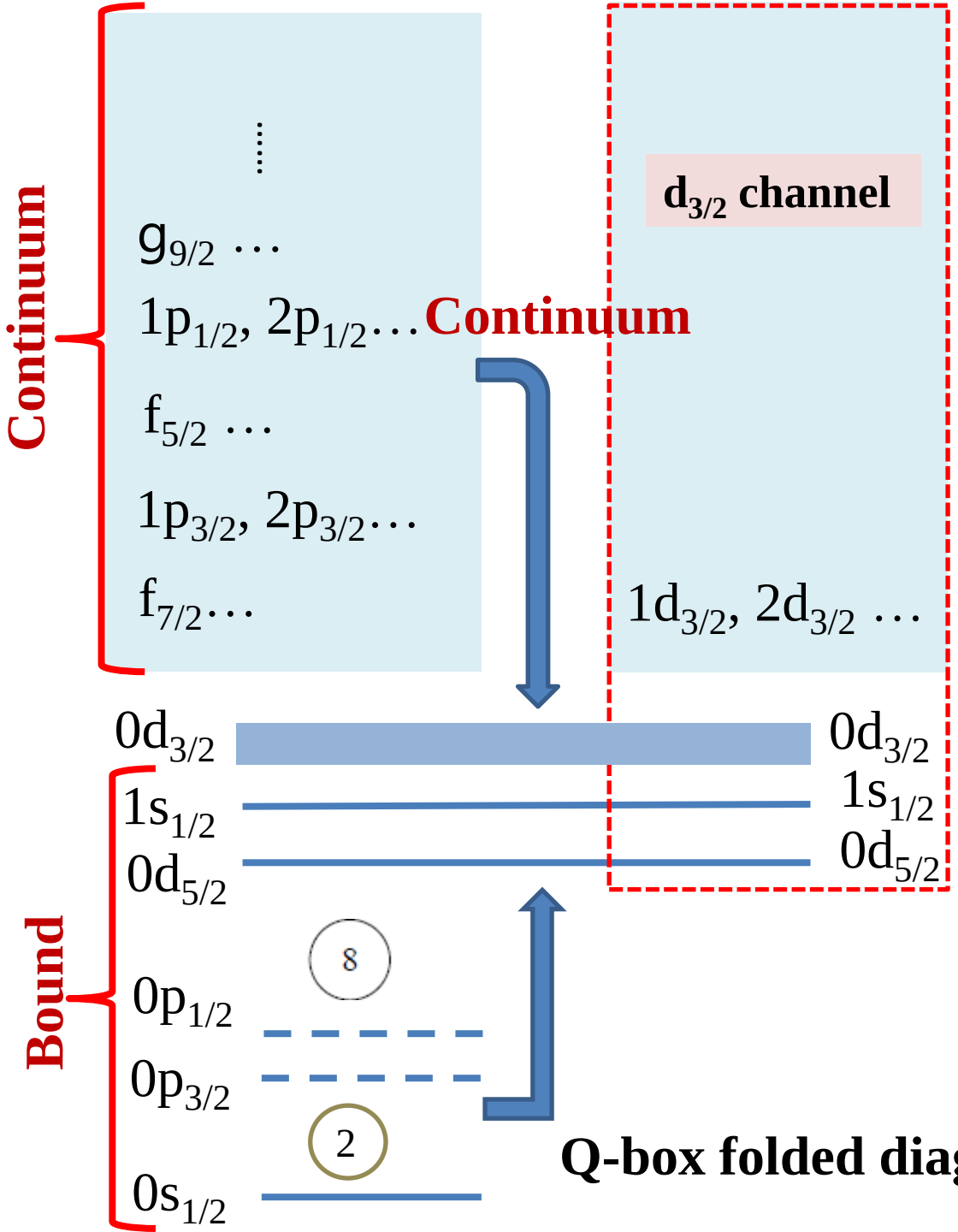



$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP$$



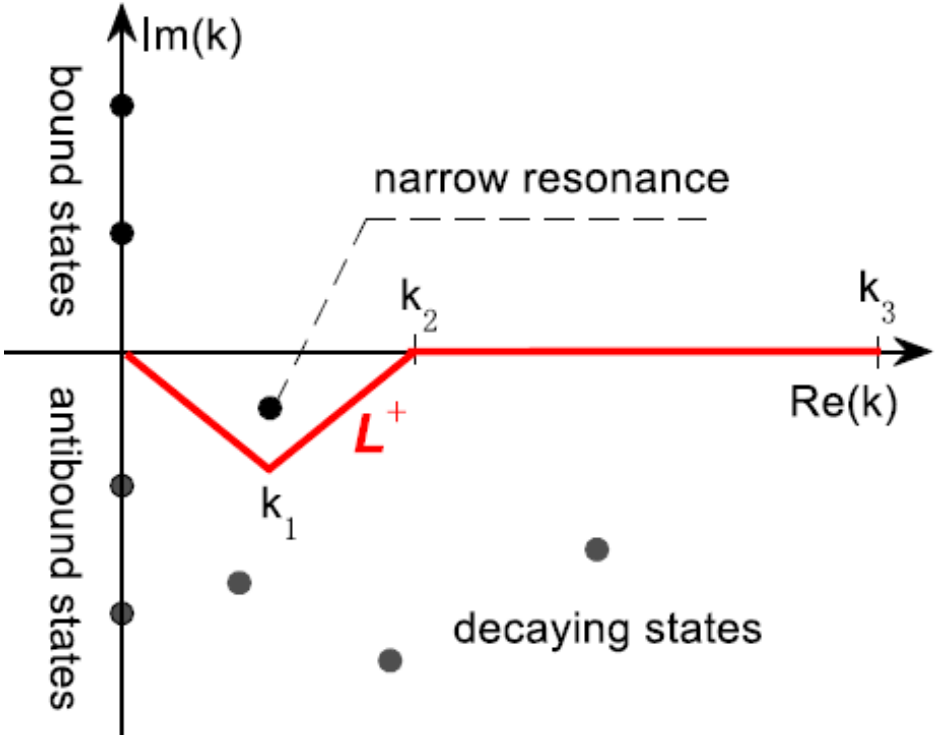
**Non-degenerate extended Kuo-Krenciglowa folded-diagram method (EKK) by Takayanagi, NPA 852, 61 (2011);**




**Model space**

**Q-box folded diagrams in complex-k basis**

**Convergence against discretization number N_L**

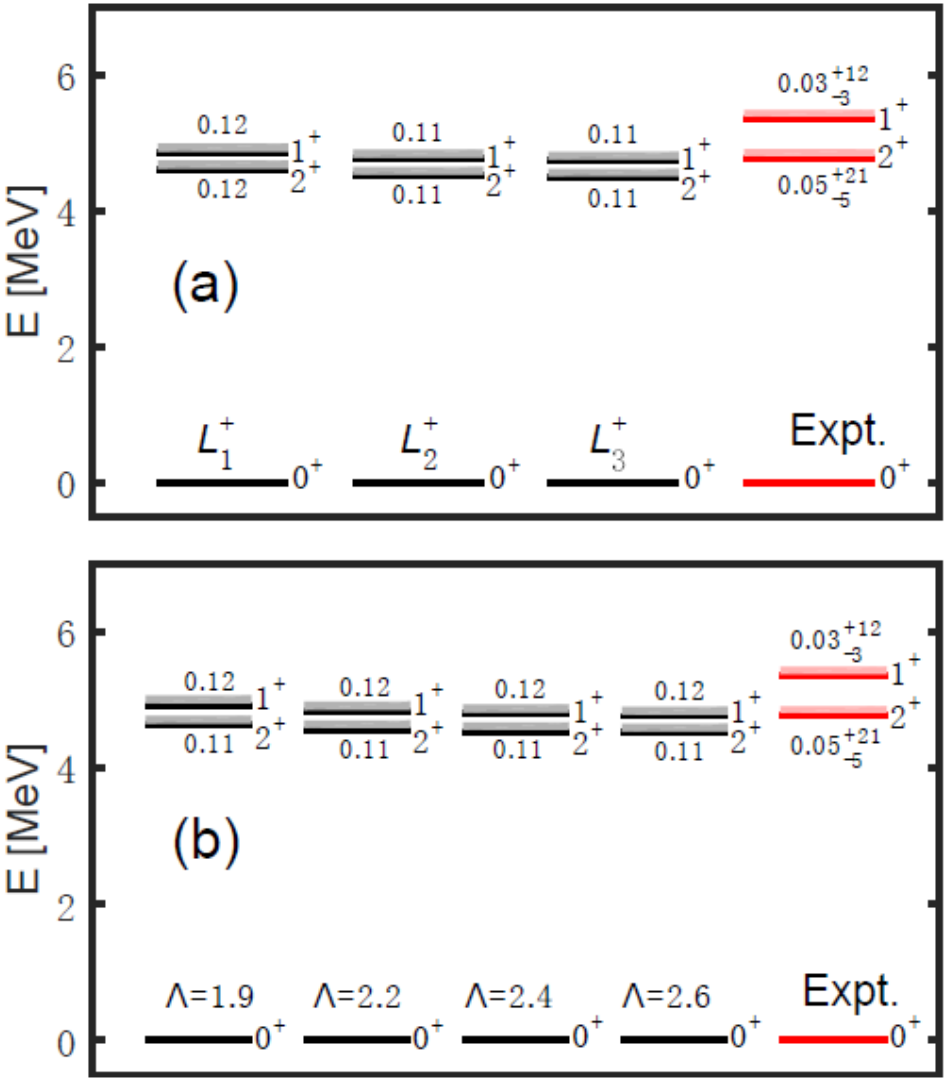


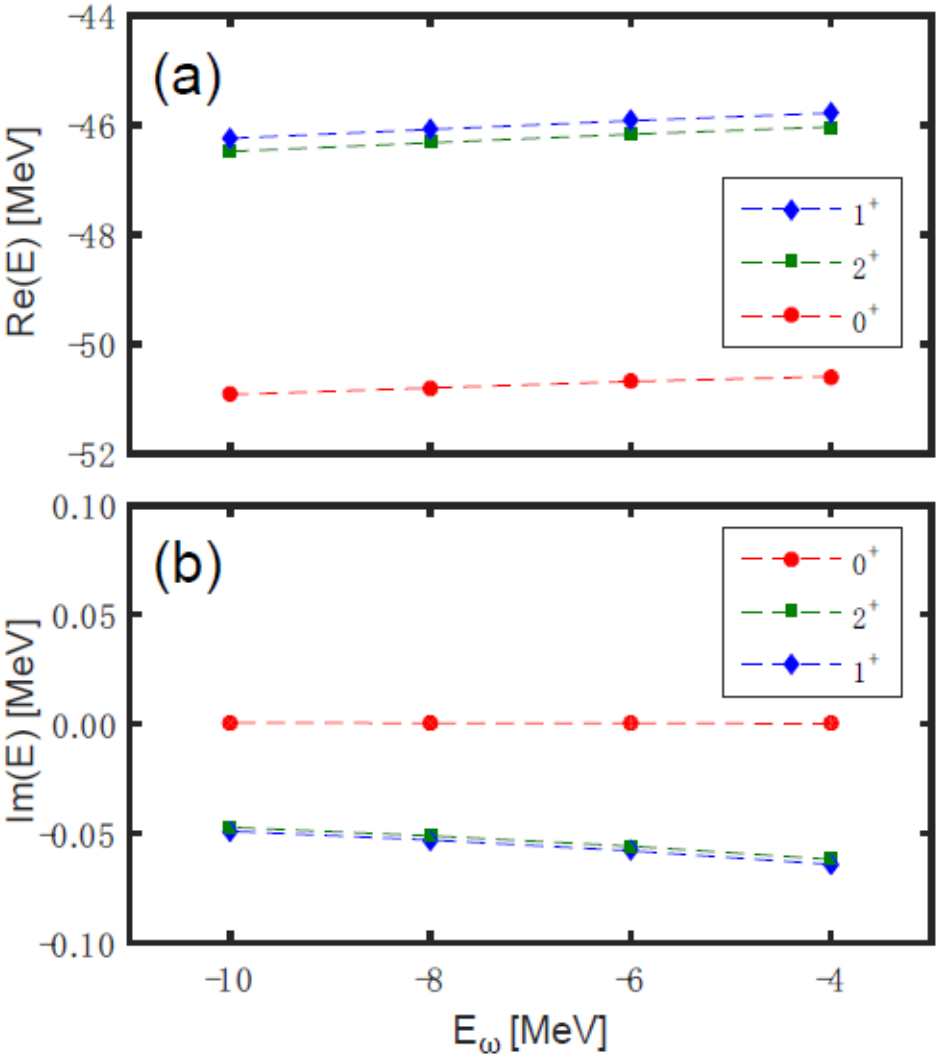

$\tilde{E}_n = E_n - i\Gamma/2$ $\quad$ $^{24}\mathrm{O}$ $\quad$ $\Lambda = 2.6\ \mathrm{fm}^{-1}$

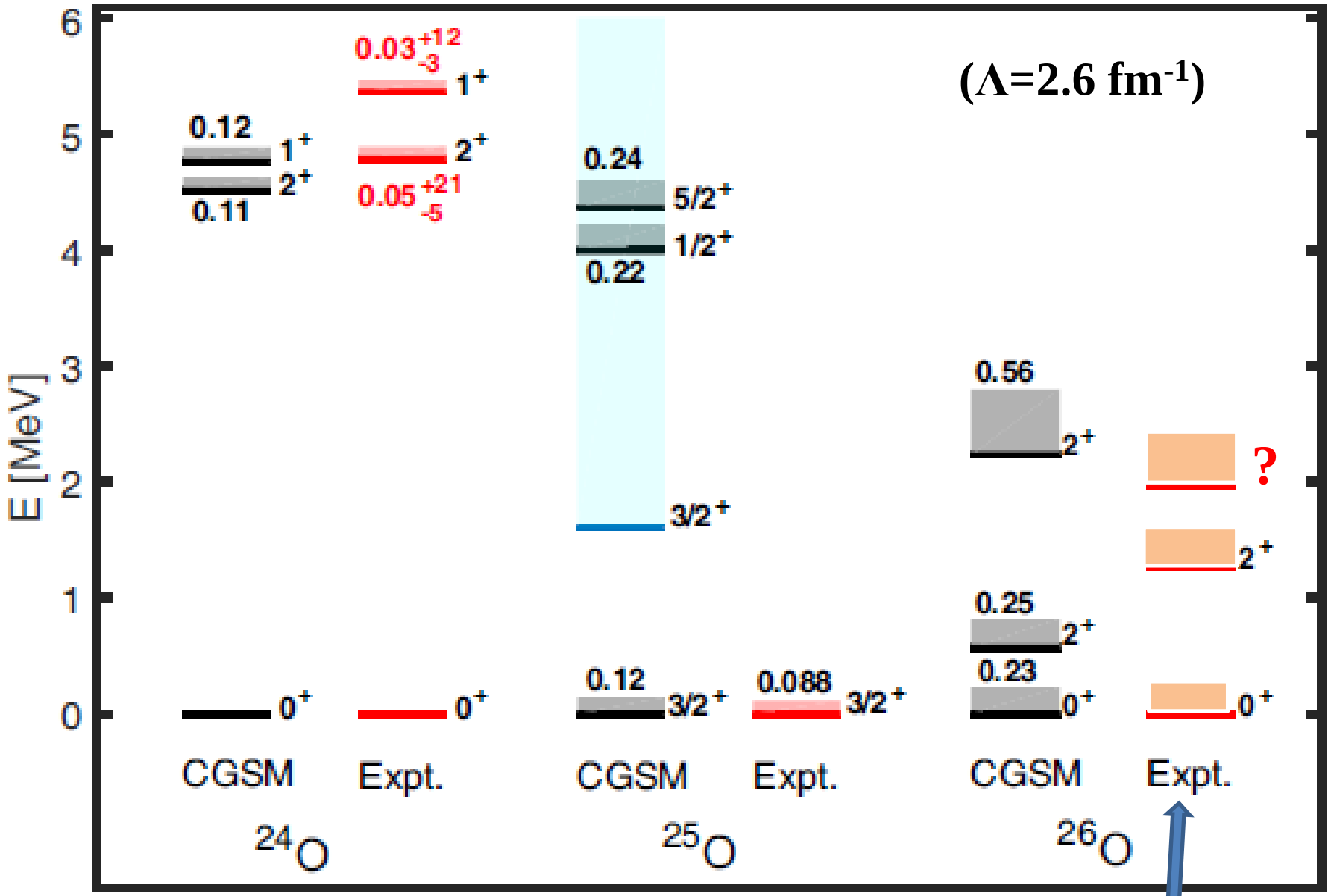
| $N_L$ | $0^+$ | $2^+$ | $1^+$ |
|---|---|---|---|
| 16 | $-50.642 + 0.013i$ | $-46.172 - 0.004i$ | $-45.922 - 0.009i$ |
| 18 | $-50.716 + 0.002i$ | $-46.262 - 0.046i$ | $-46.017 - 0.049i$ |
| 20 | $-50.711 - 0.001i$ | $-46.219 - 0.054i$ | $-45.976 - 0.056i$ |
| 22 | $-50.712 + 0.000i$ | $-46.218 - 0.053i$ | $-45.974 - 0.056i$ |

# Convergences of spectroscopic calculations

$^{24}O$



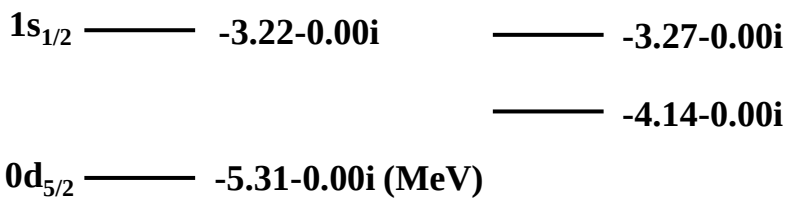






Y. Kondo *et al.*, PRL 116, 102503 (2016)

# Binding energies, one-neutron separation energies

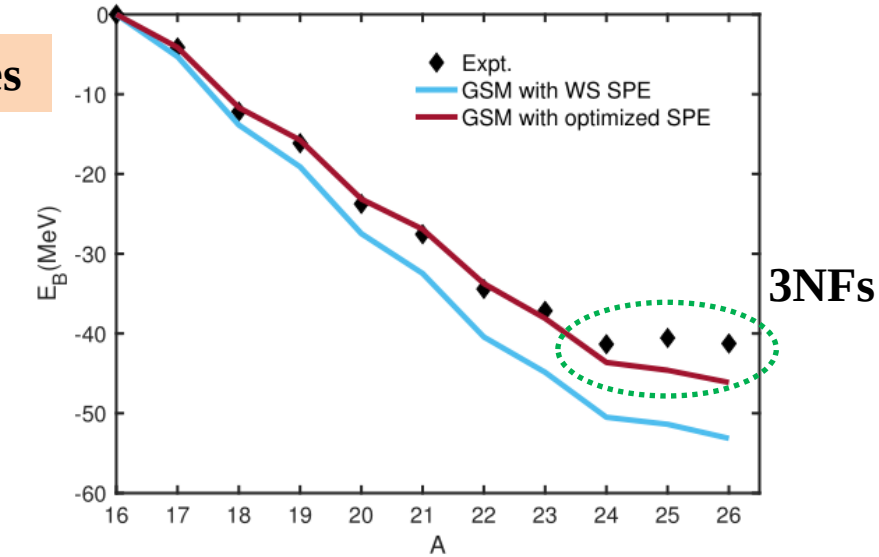
$$\tilde{e}_n = e_n - i\gamma_n/2$$

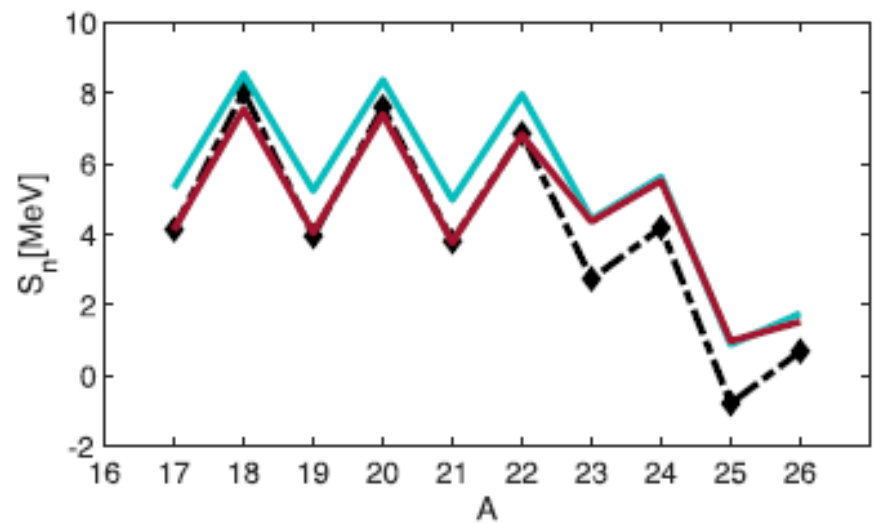




WS SPE's

Expt. SPE's

[Extracted from $^{17}$O,
by Michel *et al.*, PRC 67, 054311 (2003)]



# Summary

**1) $N^3LO$(NN) + Hartree-Fock + MBPT for closed-shell nuclei**

**2) $NNLO_{sat}$(NN+NNN) + Hartree-Fock + RPA for nuclear giant resonances**

**3. CD Bonn (NN) + $V_{low\text{-}k}$ + CGSM with resonance and continuum for weakly-bound nuclei**

# Thank you for your attention

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