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# CRITICAL HADRONIZATION

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## ABSTRACT

Monte Carlo calculations in lattice gauge theories suggest the existence of a phase transition in QCD, between a hadron gas and a quark gluon plasma. We examine the theoretical consequences of the assumption that this transition is a critical phenomenon, called critical hadronization. The main idea is that if hadronization is critical, then it provides a new domain of application to the renormalization group equation (RGE) of the underlying local field theory, that is QCD.

The theoretical framework is provided by the topological expansion of QCD interpreted as a dual topological unitarization (DTU) procedure or as a regularization/renormalization procedure.

At the planar level, which concentrates all the non linear character of unitarity, QCD is <u>regularized</u> in the so called dual parton model, which is a broad resonance dual model (BRDM) involving string like quarks, gluons and mesons and satisfying short range order scaling and confinement.

Criticality of the hadron-parton transition is insured by the degeneracy of all planar trajectories and couplings (which allows to constrain all the dimensionned parameters) and by the existence of a cylindrical pomeron with intercept one, which, through absorption corrections breaks short range order and induces long range correlations.

The summation over all higher topologies, that is over all multiple pomeron absorption corrections is shown to be equivalent to the <u>renormalization</u> of the dual parton model (that is to the removal of the transverse cut off provided by a non vanishing Regge trajectory slope in the BRDM).

As calculational consequences of the proposed scheme we show quantitatively how the RGE of QCD can be used to estimate the rise of hadronic total cross sections and to estimate the EMC effect.

## INTRODUCTION

The main explorations of the non perturbative regime of QCD are made by means of lattice quantization[1]. This quantization provides a specific regularization scheme which allows to renormalize the theory by analogy with a classical statistical problem in the neighborhood of a critical point [2]. This method allows to have an insight in the strong coupling regime of QCD. Confinement naturally appears in this regime in terms of linearly rising potentials at large distances. Progresses have been met in introducing fermions on the lattice, in numerical methods, and in analytic approximations. A very important result obtained by lattice quantization is the prediction of a phase transition in QCD[3]. The two phases involved in this transition are a hadron gas at low temperature and energy density and a quark gluon plasma (also called a "quagma") at high temperature and energy density. This transition is called deconfinement if one comes from the hadron phase and hadronization if one comes from the parton phase. The quagma phase would be a genuine new state of matter which appears extremely interesting to study. It could have been formed in the very early universe (at a time of about $10^{-5}$ sec.). One can expect the formation of this phase in ultra-relativistic heavy ion collisions (at about 200 GeV per nucleon)[4].

Our purpose in the present paper is to study the theoretical consequences on the description of high energy hadron production of the assumption that this phase transition is a critical phenomenon. In the time development of any high energy reaction involving hadrons there is a short time in which only hard processes occur. The description of these processes is (more or less) under control by means of perturbative QCD [5,6,7]. The part of the high energy reaction which is difficult to describe is the conversion into hadrons of the partons which have been produced in the hard part of the reaction. This is the reason why we are interested in the phase transition of QCD as a hadronization mechanism.

The main idea in the present paper is that if hadronization is a critical phenomenon, it should be possible to describe it in terms of the renormalization group equations (RGE) of QCD. What one has learnt from lattice quantization is that the regularization/renormalization procedure in QCD is equivalent to a critical phase transition in a statistical mechanical system [2]. Conversely, if high energy reactions proceed through critical hadronization, the renormalization group equations of the underlying local field theory, that is QCD, should be relevant also in this non perturbative process. One sees that the criticality of hadronization may have far reaching consequences : up to now RGE of QCD have been used only to establish asymptotic freedom (that is the logarithmic decrease of the running coupling at small distance, which is indeed a very important result) ; now, if our conjecture makes sense it would mean that the RGE, related to the ultraviolet behaviour of RCD provide some information about hadronization which is related to the infrared behaviour. It would be a step forward in the program of establishing QCD as the complete theory of strong interactions.

On a phenomenological level, the Saclay approach [8] to confinement has allowed to improve the understanding of the connection between soft and hard processes. This approach is based on a correspondence between QCD and the Dual Topological Unitarization(DTU) scheme [9]. The quarks of the duality diagrams which occur in DTU are identified with the quarks of a QCD inspired parton model. More precisely, the planar approximation of DTU can be interpreted in terms of a genuine quark parton model, the so called dual parton model. Higher topology corrections are associated with scaling violations and gluon cascading. This approach has met some phenomenological successes. It is currently being explored by other groups [11,8]. It is actually very well suited for our present purpose for several reasons.

i) The correspondence between QCD and DTU is precisely interpreted in terms of the equivalence of two "bases" or two "vacua" to describe hadronic reactions. Now vacuum degeneracy is a specific property of critical transitions.

ii) DTU is a procedure to implement unitarity in a dual string dynamics. At the zeroth order DTU reduces to a narrow resonance dual model (NRDM). Now it is well known [12] that a NRDM can reproduce at the limit of vanishing trajectory slope ($\alpha'$) a local field theory. It has been shown that under some conditions one can build dual string actions which reproduce a Yang Mills Lagrangian at the $\alpha' \rightarrow 0$ limit. It is also well known that Reggeization implies a gaussian transverse cut off in dual string theory. One thus sees that duality can provide a specific regularization scheme for QCD, analogous to lattice regularization.

iii) As an iterative unitarization procedure, DTU leads to a topological expansion of S matrix elements. Basically, the correspondence between DTU and QCD consists on identifying, term by term this topological expansion with the topological expansion of QCD proposed by 'tHooft (13). Since the topological expansion of QCD supposedly exhausts the full content of QCD, and if the zeroth order approximation of DTU, that is a NRDM is a regularization of QCD, then the summation over all higher topologies in DTU is nothing but the renormalization implied by this specific regularization.

iv) The assimilation of DTU as a regularization/renormalization procedure is also strongly suggested by the way how it implements unitarity. All non linear constraints of unitarity are concentrated in the planar topology, in terms of the so-called planar bootstrap constraints, whereas higher topologies are introduced, perturbatively, through linear equations. This method is strikingly similar to the resummation technique used in a renormalizable field theory : non linear RGE allow to define renormalized vertices and propagators with which one can build generalized ladder diagrams which in turn can be evaluated at the leading logarithm approximation, say (5).

v) At the planar level of DTU, the basic properties of the NRDM are preserved : local duality, exchange degeneracy, and short range order in the rapidity distributions of multiple production. Very important new properties appear : non vanishing widths for resonances, factorization in the sense of a quark parton model with confined quarks (the so-called dual parton model). At the first iteration, that is when considering the shadow scattering of the NRDM one obtains, apart from the planar reggeon, a new contribution, the cylindrical pomeron. This contribution corresponds to a Regge singularity, with intercept equal to one, and with vacuum quantum numbers. This feature is the most important for our purpose. Indeed the occurence of a Regge singularity with intercept one signals the possibility of the breacking of short range order, and thus of long range correlations which are characteristic of critical phase transitions.

vi) Because of the topology of the dual pomeron, summing over multi pomeron corrections is equivalent to sum over higher topologies. The basic ingredient to sum over pomeron corrections is provided by the celebrated Abramowski Gribov Kanchelli (14) (AGK) cutting rules, which allow to define cross sections for cutting a given number of pomerons and reggeons. In the framework of critical phenomena these cross sections can be interpreted as the production cross sections of fluctuating distributions. Some renormalization group techniques could hopefully help in averaging these fluctuations.

The purpose of the subsequent sections of the paper is to provide this qualitative overview with a quantitative and calculational content. The first section will be devoted to the lowest topology and to the criticality of the hadronization. One will specify the NRDM corresponding to the regularization of QCD. This NRDM involves three types of open strings : a meson string with flavor and anti-flavor at its ends, a quark string with flavor and color at its ends, and a gluon string with color and anti-color at its ends. Criticality is implied by planar unitarity that is the summation over all planar internal color and flavor loops. Indeed on the one hand planar unitarity implies the degeneracy of the three Regge trajectories of meson, quark and gluon strings (with important consequences on the values of the dimensioned parameters) and on the other hand the existence of a cylindrical pomeron with intercept one. Planar unitarity also guarantees confinement and it provides the theoretical basis of the dual parton model.

The second section is devoted to multi pomeron expansion and to renormalization. We discuss the calorimetric interpretation of the cross-section for cutting a given number of reggeons and pomerons. On the explicit example of deep inelastic scattering we show how the summation of the multi pomeron expansion is equivalent to the renormalization of the NRDM regularization of QCD.

The third section is devoted to the calculational consequences of the proposed scheme. According to the new interpretation we discuss some results obtained previously such as the estimate of the rise of high energy hadronic [15] total cross section or the estimate of the EMC effect[16].

# I. PLANAR UNITARITY AND CRITICALITY

## 1. The topological expansion of QCD

The starting point of the correspondence between QCD and DTU is the topological expansion of QCD. 'tHooft [13] has shown that one can rearrange the perturbation expansion of QCD in a way which exhibits the topological properties of the two dimension manifold on which one can draw the Feynman diagrams. One writes the n point Green's function as :

$$A_n\ (\{p_i\}) = g^n \sum_{b,h,l,w} (g^2)^{b-1}\ (g^2)^{2h}\ (g^2 N_c)^l\ (g^2 N_f)^w\ \tilde{A}^n_{b,h,l,w}\ (\{p_i\}) \qquad (1)$$

where $\{p_i\}$ denotes the set of external four momenta, $N_c$ is the number of colours, $N_f$ the number of flavors. b,h,l and w are topological indices. b is the number of boundaries, that is the number of closed loops to which external particles are attached, h is the number of handles allowing internal propagators to cross each other ; l is the number of closed color loops and w the number of closed flavor loops. The reduced amplitude $\tilde{A}^n_{b,h,l,w}$ contains the full dynamical information. To make more explicit the topological properties it is useful to use a notation for quark and gluon propagators which exhibits their color and flavor content (see fig. 1). This notation, when it is completed to include mesons, will be useful to identify the NRDM regularization of QCD.

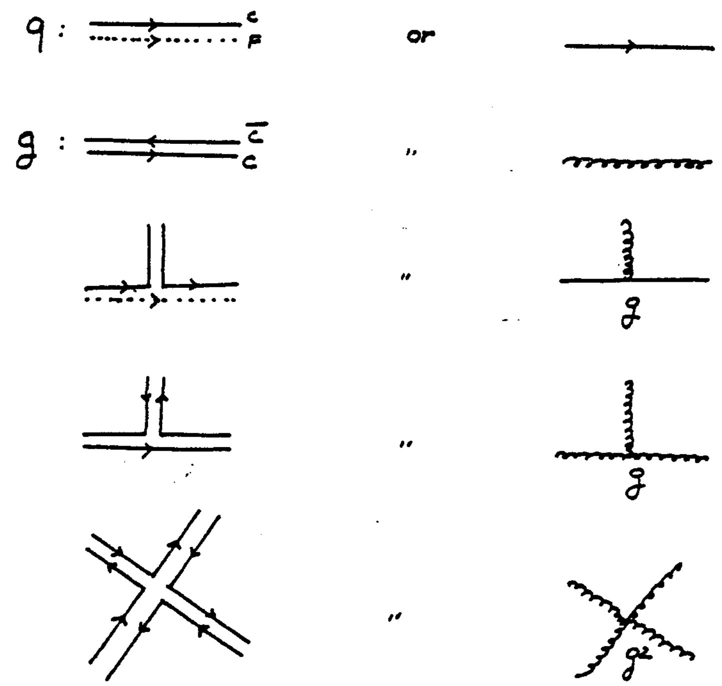
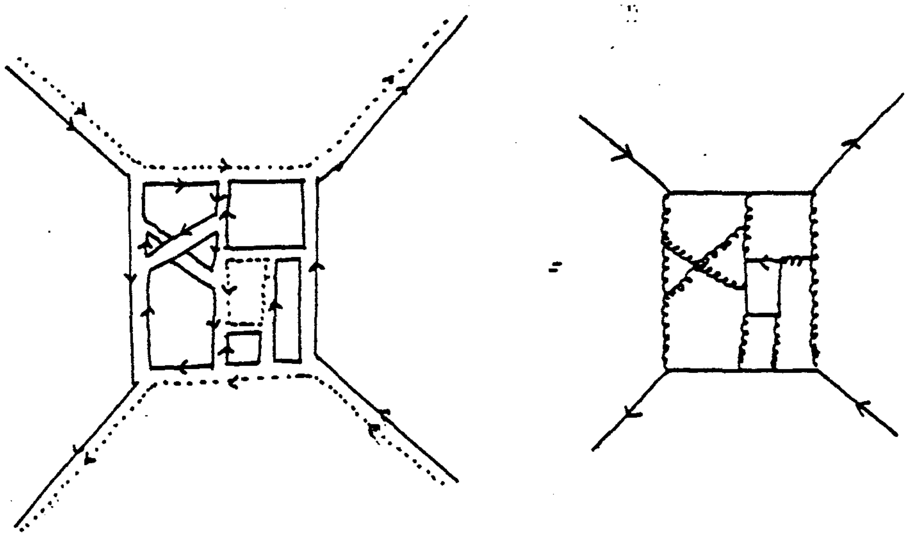


Fig. 1 : Two line representation of quark and gluon propagators. We use an axial or planar gauge which is free of Fadeev Popov Ghosts.

To illustrate the definition of topological indices we show in fig. 2 a specific contribution to the $Q\bar{Q} \to Q\bar{Q}$ Green's functions.



$n=4$ ; $b=1$ ; $h=1$ ; $\ell=4$ ; $w=1$.

Fig. 2 : a specific contribution in the topological expansion of the $Q\bar{Q} \to Q\bar{Q}$ Green's function. The values of the topological indices are given.

On this example one can see the effect of non planarity : because of the existence of one handle, three independant gluon loops give rise to a color factor $N_c$ (one closed color loop) instead of $N_c^3$ in the absence of a handle (three closed color loops). This remark is at the origin of the conjectures of 'tHooft [13] and of Veneziano [17] about confinement. 'tHooft considers the limit in which $N_c \to \infty$ with $g^2N_c$ and $N_f$ fixed, whereas Veneziano considers the limit in which $N_c$ and $N_f$ go to infinity with $g^2N_c$ and $g^2N_f$ fixed. In these limits the contributions which would survive for the process of fig. 2 are shown in fig.3.

We focuss on the Veneziano's limit since it is the one which is the basis of the equivalence between QCD and DTU. In this limit it is clear that the summation over l and w cannot be performed perturbatively since $g^2N_c$ and $g^2N_f$ are not necessarily small. Once the effect of these summations is known or assumed, the topological expansion of (eq. 1) reduces to a perturbative expansion in h, the number of handles.

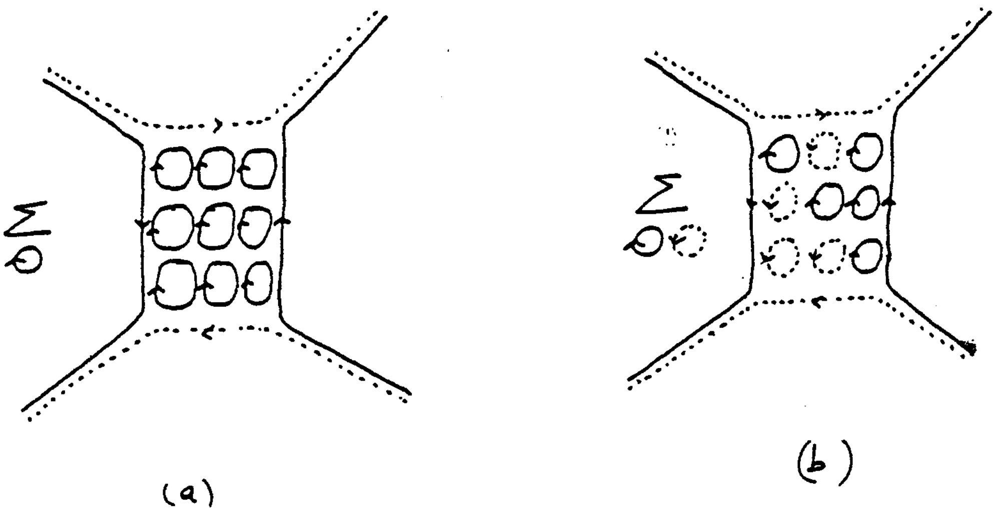



Fig. 3 : $Q\bar{Q} \to Q\bar{Q}$ amplitude in the 'tHooft limit (a) and in the Veneziano limit (b).

(The summation over b is necessarily finite, since for a given n, the number of boundaries cannot exceed the number of external particles). Now, such an expansion is precisely the one which occurs in the dual topological unitarization of a narrow resonance dual model. In order to identify the NRDM whose DTU is equivalent to the topological expansion of QCD we first concentrate on the planar topology.

## 2. Narrow Resonance Dual Regularization of QCD

We show on fig. 4, four types of cutting the configuration which survives at the Veneziano's limit. These cuttings exhibit the planar "renormalization" (insertion of closed color and flavor loops) of four open string propagators : the quark (flavor-color), the anti-quark (anti-color-anti flavor), the gluon (color - anticolor) and the meson (flavor - antiflavor). The NRDM we are looking for is provided by the three diagrams (i.e. without planar "renormalization") of the string action involving these four open strings.

Although the quark and gluon have the same representation in the NRDM as in QCD, they have different properties. In the NRDM quarks and gluons lie on linear Regge trajectory. Duality reflects the equivalence of two descriptions : in a two body reaction, say, the exchange of t-channel

Regge poles is equivalent to the formation of S-channel resonances - see fig. 5.

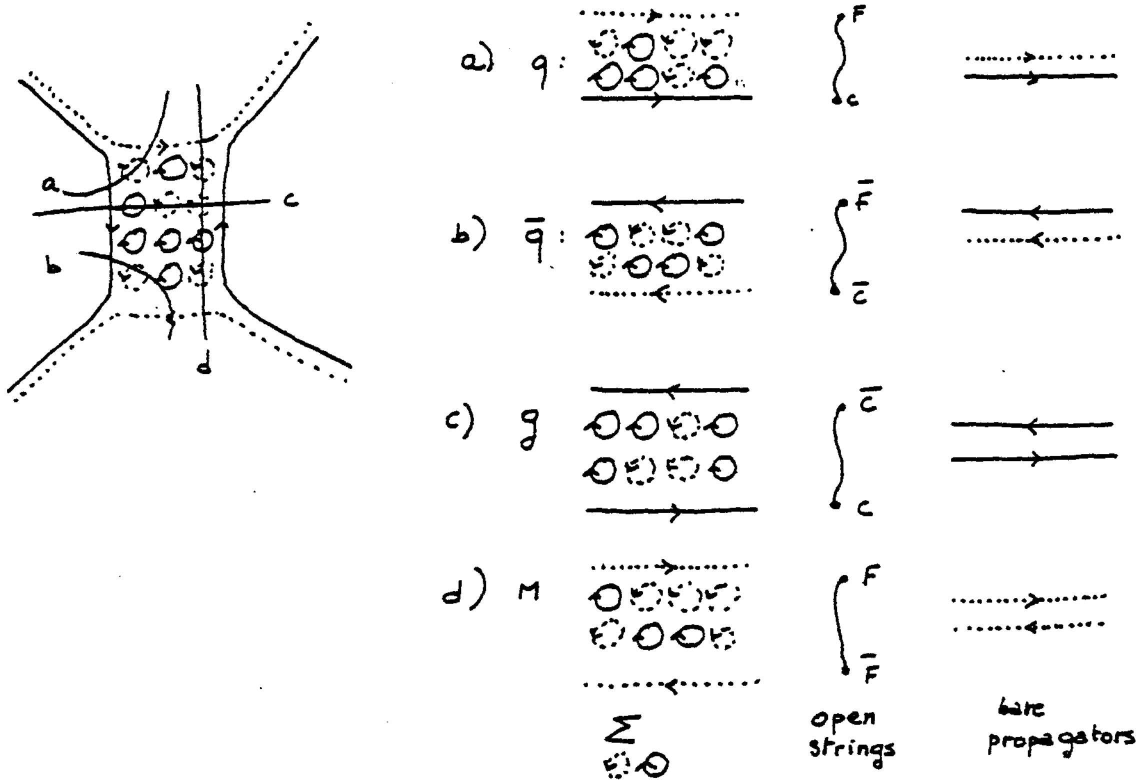



Fig. 4 : Four cuttings of the configuration of fig. 3b, exhibiting planar renormalized open string propagators. The corresponding bare propagators are those of the NRDM regularizing QCD.

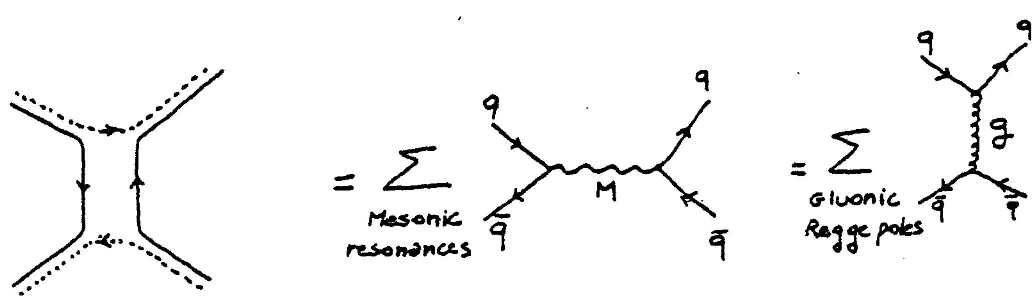



Fig. 5 : Duality properties of the $Q\bar{Q} \rightarrow Q\bar{Q}$ amplitude in the NRDM

In the NRDM the $Q\bar{Q} \rightarrow Q\bar{Q}$ amplitude would be written in terms of the Veneziano's ansatz [18]. The spin of the quarks can be taken into account by writing the invariant amplitudes (S,V,T,A,P). Flavor and color symmetries are imposed as <u>global</u> symmetries by means of the Chan Patton factors [19]. For instance the vector invariant amplitude in the t-channel (corresponding to fig. 5) would be equal to

$$V_t(s,t) = \{CF\} \frac{\Gamma(-\alpha_M(S) \quad \Gamma(-\alpha_g(t))}{\Gamma(-\alpha_M(s) - \alpha_g(t))} \qquad (2)$$

where $\{CF\}$denotes the Chan Patton factors for color and flavor symmetries $\alpha_M(S)$ is the S channel mesonic trajectory and $\alpha_g(t)$ the t-channel gluonic trajectory.

The main problem which one could enconter in writing the NRDM is to put in the same string action bosonic and fermionic strings. Now this problem has been solved by Neveu and Ramon [20]. Actually, it is precisely the Neveu Ramon NRDM which can reproduce QCD at the $\alpha' \rightarrow 0$ limit.

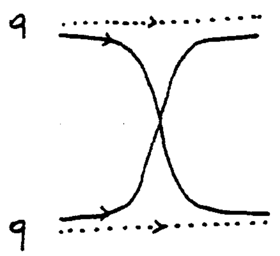
In order to constrain further the NRDM, that is to establish relations between trajectories we make some simplifying but natural assumptions.

i) We neglect heavy flavors, including strangeness. So we consider only u and d quarks.

ii) We want not to break isospin invariance nor color symetry, so we assume all the quark trajectories to be degenerate, all the mesonic trajectories to be degenerate and all the gluonic trajectories to be degenerate too.

iii) The simplest situation would be that all trajectories and couplings are degenerate. We shall see later on that such a degeneracy, which is fundamental for criticality is implied by planar unitarity.

Before going to planar renormalization, one more comment is in order about the spectrum of the NRDM. If we perform s-u crossing on the amplitude of fig. 5 we obtain a $QQ \rightarrow QQ$ amplitude which is exotic in the s-channel. Exoticity in the s channel is insured in the dual framework by means of exchange degeneracy [21]. This means that apart from the 8 degenerate negative signature gluon trajectories we have a singlet (corresponding to the trace in the color group) positive signature trajectory which is exchange degenerate with the gluon trajectories. The sum of the colored and singlet contributions amounts to a purely real contribution in the exotic channel and to a contribution with a rotating phase in the $Q\bar{Q}$ channel. In fig. 6 we show the duality diagram corresponding to the exotic s channel. 'tHooft [13] had remarked that with the two line notation for quarks and gluons one was led to introduce a color singlet gluon which one has to remove to recover standard QCD. In our scheme the color singlet partner of the gluons is a positive signature trajectory, the first materialization of which is a spin 2 particle. When we renormalize we have to make sure that this spurious trajectory is actually removed.




$$V(t,u) = \frac{\Gamma(-\alpha_g(t))\ \Gamma(-\alpha_M(u))}{\Gamma(-\alpha_g(t) - \alpha_M(u))}$$

Fig. 6 : NRDM behaviour in the exotic QQ channel.

## 3. Planar unitarity and the broad resonance dual model

Dual topological unitarization is an iterative procedure [9] to unitarize a NRDM, with three basic rules.

i) the zeroth order approximation is the NRDM itself

ii) the $n^{th}$ iteration is computed as the shadow scattering of the $(n-1)^{th}$ iteration

iii) at each step one neglects interferences. The neglected contributions correspond to higher topologies which are recovered in higher iterations.

Veneziano has shown [17] that this procedure leads to a topological expansion of amplitudes which can be put in a one to one correspondence with the topological expansion of QCD provided that the sum over l and w (eq. 1) is performed non-perturbatively.

We focuss now on the "planar renormalization" of the NRDM, that is effect of the insertion of all possible internal, planar color and flavor loops. Obviously this insertion does not change the topology of the duality diagrams representing the amplitudes. We thus expect Regge behaviour, exchange degeneracy and duality to be preserved by the insertions. The first important change is that resonances acquire non vanishing widths. One could thus call broad resonance dual model (BRDM) the dual model which emerges from the planar renormalization. It is interesting to note that there exists an ansatz for an analytic expression of scattering amplitudes in a BRDM, it is the so called DAMA [22] ansatz (Dual Amplitude with Mandelstam Analyticity). This ansatz allows to build amplitudes with crossing symmetry, duality, non linear trajectories, that is broad resonances, for any given number of external particles, and as a by product with Mandelstam analyticity (non vanishing double spectral functions in the domains implied by unitarity). It is also interesting to note that going from the NRDM to the BRDM weakens the transverse cutoff : in DAMA the transverse cutoff is exponential whereas it is gaussian in the NRDM. This last feature enforces our interpretation of DTU as a regularization/renormalization procedure. Planar unitarity is the beginning of renormalization.

## 4. Planar unitarity and vacuum degeneracy

The most important property of the BRDM is that it concentrates all the non linear constraints of unitarity. Since the BRDM is obtained by the summation over all planar color and flavor loops, any amplitude is left invariant by the removal or the insertion of any finite number of color or flavor loops. We call planar bootstrap equations the equations resulting from this invariance. In fig. 7 we show the graphical representation of some planar bootstrap equations.

Planar bootstrap implies vacuum degeneracy. The equation of fig. 7 a implies that a gluon loop and quark loop contribute the same amount to the gluon propagator (of course these equations are true only in regularized QCD !).

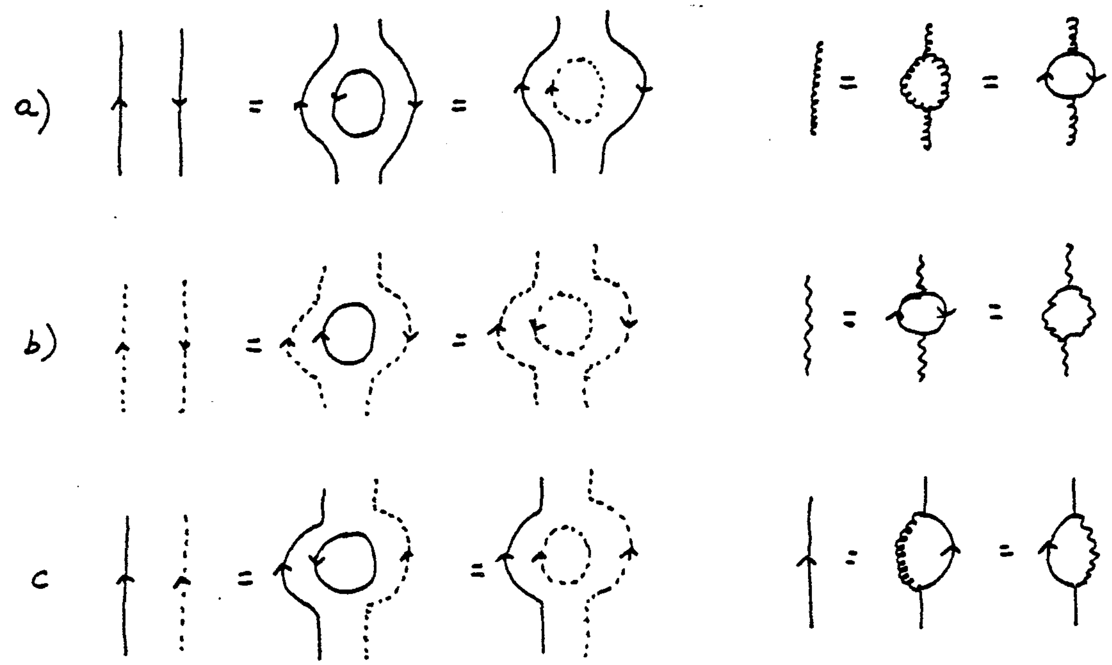



fig. 7 : planar bootstrap equations for the gluon (a) meson (b) and quark (c) propagators.

For the energy dependence to coincide we infer :

$$\alpha_g(t) = \alpha_Q(t) \qquad (3)$$

From fig. 7 b we obtain :

$$\alpha_Q(t) = \alpha_M(t) \qquad (4)$$

and from fig. 7 c :

$$\alpha_g(t) = \alpha_M(t) \qquad (5)$$

In the same way we obtain that all possible three-particle vertices are equal.

These degeneracy equations are important since they relate the unphysical trajectories (the quark and gluon ones) to the physical trajectory, the meson one. The dominant planar mesonic Regge trajectory is indeed known with accuracy : it is the $\rho$ trajectory for which we know two materializations : the $\rho$ (770) $I^G J^P = 1^+1^-$ and the g (1690) $I^G J^P = 1^+3^-$ . We also know the behaviour of this trajectory in the negative t region from the analysis of high energy reactions involving $\rho$ exchange like $\pi^- p \to \pi^0 n$. The intercept of the $\rho$ trajectory is about 0.5. Actually Lovelace [23] has derived a useful theoretical constraint on the $\rho$ trajectory by demanding the existence of an Adler zero [24] in the $\pi\pi$ amplitude :

$$\alpha_\rho (m^2_\pi) = \frac{1}{2} \qquad (6)$$

From eq. (3) (4) (5) and (6) we obtain very interesting consequences.

i) $\alpha_g (m^2_g) = 1$ (spin of the gluon)

So we have $m_g = m_\rho$ (7)

This mass is in excellent agreement with the mass given to the gluon in the analysis of the radiative $\psi$ decay [25] .

ii) $\alpha_Q (m^2_Q) = \frac{1}{2}$ (spin of the quark).

So we have $m_Q = m_\pi$ (8)

So we note that in our regularization scheme the quark is given a smaller mass than the gluon. The gluon can thus "decay" into a $Q\bar{Q}$ pair. The quarks thus produced have a transverse mass

$$m_\perp = \frac{m_\rho}{2} = 385 \text{ MeV} \qquad (9)$$

which is good agreement with the primordial transverse momentum observed in various high energy reactions [26] .

iii) if we demand the quark propagator to have a pole at $p^2 = m^2_\pi$ we see from the equation of fig. 7 c that the coupling squared must have this pole since the loops are not singular. So $m^2_\pi$ is the first infrared pole of the QCD couplings. So we infer

$$\Lambda_{QCD} = m_\pi = 140 \text{ MeV} \qquad (10)$$

an equation which is in excellent agreement with the world average value of $\Lambda_{QCD}$ (27) and which, to our knowledge had never been written before.

## 5. Planar unitarity, reggeons and pomeron

The resolution of all coupled planar bootstrap equations would be a very difficult task. Fortunately, by means of approximation methods one can extract from them some useful informations. We consider here the multi-reggeon approximation method (28) which consists on considering only the dominant contributions in multiproduction reactions, i.e. the configurations where all $P'_\perp$s are limited and the rapidities are ordered (this method is strinkingly similar with the leading logarithm method in standard QCD)

Since all trajectories are degenerate we shall forget the distinction between flavor and color lines (except when we need it for some theoretical arguments). We thus deal with a BRDM with $N = N_c = N_f$ ( = 5 in our simplified version) degrees of freedom. One finds, as shown in fig. 8 two configurations contributing to the n particle production cross section in the multi-reggeon approximation. According to the DTU procedure the shadow scattering of these contributions leads to the planar reggeon and cylindrical pomeron.

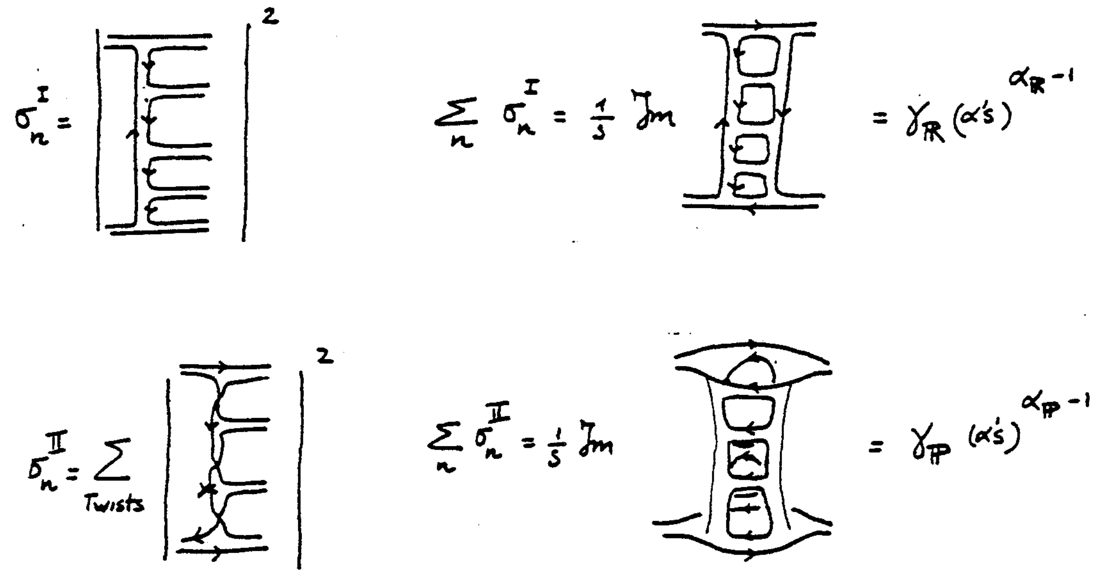



Fig. 8 - Building the planar reggeon ($\alpha_{\mathbb{R}}$) and cylindrical pomeron ($\alpha_{\mathbb{P}}$) from the shadow scattering on n particle multiproduction cross sections in the multi-reggeon approximation

Analytically one finds for $\sigma_n^I$ and $\sigma_n^{II}$ Poisson distributions [28] :

$$\sigma_n^I = \frac{(g^2 N \log \alpha' s)^n}{n!} (\alpha' s)^{2\alpha_o - 2} \frac{1}{N} \quad (11)$$

$$\alpha_n^{II} = \frac{(g^2 N \operatorname{Log} \alpha' s)^n}{n\,!} (\alpha' s)^{2\alpha_o - 2} \frac{2^{n-1} - N}{N^2} \quad (12)$$

where $\alpha_o$ and $\alpha'$ are the intercept and the slope of the input planar trajectory. The factor $2^{n-1}$ in (12) is due to the summation over all possible twists. The summations over n can be performed and we obtain the following output trajectories and residues :

$$\alpha_{IR} = 2\alpha_o - 1 + g^2 N \quad (13\ a)$$

$$\alpha_{IP} = 2\alpha_o - 1 + 2\, g^2 N \quad (13\ b)$$

$$\frac{\gamma_{IP}}{\gamma_{IR}} = \frac{1}{2N} \quad (13\ c)$$

Planar bootstrap obviously implies

$$\alpha_{IR} = \alpha_o \quad (14)$$

which has three important consequences.

i) in a multi-reggeon chain, the intercept of the input trajectory determines a correlation length in rapidity : the exchange of the Regge trajectory between two subsequent particles in the chain induces in the amplitude a factor

$$(\alpha' s_{ij})^{\alpha_o - 1} \propto e^{(\alpha_o - 1)\ |y_i - y_j|} \quad (15)$$

The correlation length in rapidity is thus equal to

$$\Delta = \log \frac{\bar{s}}{m_\perp^2} = \frac{1}{1 - \alpha_o} \quad (16)$$

with $\alpha_o \simeq 0.5$ one recovers the celebrated correlation length of two units of rapidity. It is very interesting to interpret $\bar{s}$ which one obtains by exponentiation of this correlation length : it is a cut off in the squared cluster mass. A cluster with a higher squared mass would split into lighter clusters. $\bar{s}$ provides a cut off on the transverse masses of the produced particles and thus on their transverse momenta. With $\alpha_o = 0.5$, $m_\perp = 0.385$ we obtain $\bar{s} = 1.1$ Gev$^2$ which is in excellent agreement with the inverse of the slope of the meson trajectory,

$$\alpha'p = 0.9 \text{ Gev}^{-2} \quad *$$

From equations (13 a), (14) and (16) we obtain

$$g^2N = \frac{1}{\log \frac{\bar{s}}{m_\perp^2}} \qquad (17)$$

which guarantees that when we renormalize, that is when we let $\bar{s}$ go to infinity like $Q^2$ we recover asymptotic freedom.

ii) Taking (14) into account we find from (13b) that

$$\alpha_{\mathbb{P}} = 1 \qquad (18)$$

whatever is the value of $\alpha_o$. This is may be the most important result of DTU : planar unitarity implies the existence of a pomeron with intercept one.

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* We note, by the way, another relation which enforces once more our scheme. In lattice quantization one obtains confinement in terms of a linearly rising potential. The rate of increase is related to the string tension Kogut[(1)] quotes the prediction obtained from Monte Carlo method for the string tension in SU(3) QCD :

$$K = (220 \pm 60)\ \Lambda_L$$

where $\Lambda_L = \Lambda_{QCD}/83.5$, which leads to

$$K = (2.5 \pm 1.0)\ \Lambda_{QCD}$$

Now, Artru[(29)] related the string tension to $\alpha'$ by

$$\alpha' = \frac{J}{m^2} = \frac{1}{2\pi K^2}$$

with $\Lambda_{QCD} = m_\pi$ and $\alpha' = 0.9$ GeV$^{-2}$ we obtain $K = 420$ MeV $= 3\ \Lambda_{QCD}$

Indeed this result has been obtained with the help of some wild approximations, but it turns out that with various approximation schemes [30,31] one obtains the same or a very similar result. From the discussion of the preceeding paragraph we see that the correlation length in rapidity can increase to infinity as soon as one has a Regge singularity with intercept one. Now long range correlations are specific of critical transitions. So, to assume that (18) is an exact result is essentially equivalent to assume that hadronization is critical.

The pomeron can be exchanged between any type of incoming particles : partons as well as hadrons. If the intercept of the pomeron is one, the strength of the interaction through pomeron exchange is independent of the distance in rapidity of the two interacting particles. So, two colored partons can compensate their color through pomeron exchange, without exchanging much of four momentum whatever is their separation in rapidity. This is called soft confinement [5] or soft bleaching a confinement or hadronization mechanism which is widely agreed upon. It is interesting to note that such a mechanism is exactly what one expects in a critical phenomenon.

iii) eq (13 c) reflects that since the pomeron topology involves two boundaries instead of 1 for the reggeon topology, the residue of the pomeron is suppressed by a factor $\frac{1}{N}$ with respect to the residue of the reggeon.

Eq (12) shows that the planar pole exists in the pomeron propagator with a negative residue, in such a way that the planar pole disappears in the full singlet sector (planar reggeon + pomeron). This phenomenon, known in standard DTU as "$f_0$ promotion" [32], is very useful for our purpose : as we said above, in order to renormalize it is necessary to get rid of the spurious singlet gluonic trajectory which we have introduced to regularize. Summing over all virtual pomerons may do the job.

## 6. Planar unitarity, confinement and the dual parton model

We now answer a question which probably has upset the reader from the beginning of this paper : if gauge symmetry is broken by the regularization (the gluon is massive), if quarks and gluons are so similar to mesons, why are they confined ? It is not enough to answer that the phase transition is a deconfinement transition. We have to make sure that for any process with a color singlet initial state, the description in terms of quarks and gluons can be completely replaced by a description involving only hadrons. This requirement is the way how confinement is formulated in the Saclay approach [(8)].

It is easy to see that planar unitarity insures confinement in this sense. Consider the $e^+e^-$ annihilation into hadrons, at the planar approximation. We assume that the virtual photon couples in a point like way to the flavor line. Planar unitarity allows us to remove all the internal flavor loops and to replace them by a single color loop (see fig. 9). We thus obtain the confinement equation

$$\sigma^{\text{planar}}_{e^+e^- \to \text{HADRONS}} = \sigma^{\text{planar}}_{e^+e^- \to Q\bar{Q}} \qquad (19)$$

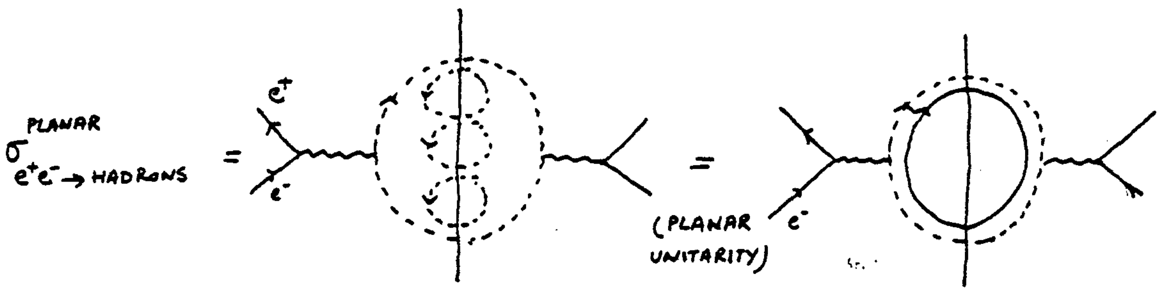



Fig. 9 : Planar unitarity and confinement in $e^+e^-$ annihilation into hadrons.

which can easily be generalized to any process involving planar configurations. Planar unitarity appears as the basis of the parton interpretation of DTU : the replacement, in the graph representing the inclusive summation, of the closed flavor loops by a color loop allows to replace an inclusive hadron production cross section by an exclusive parton production cross section and vice versa.

The dual parton model which thus emerges appears as a very convincing realization of the quark-parton model proposed by Feynmann[33]. What we call partons in our picture are the string like quarks and gluons defined above. They are string like fragments of string like hadrons. Without planar unitarity, which allows to treat on the same foot mesons and partons, the parton interpretation of duality would be impossible. In a NRDM, which is the tree diagram approximation of a classical string action, there is no parton structure. The end points of the string carry no momentum. They cannot be interpreted as partons. In our scheme, the parton which carries momentum is not the end point, but a fragment of the string.

We shall not review all the applications of the dual parton model and we refer the reader to the quoted literature. However, just to make the transition with the problem of restoring full unitarity by means of the inclusion of higher topologies we discuss here the parton interpretation of the configuration building the cylindrical pomeron.

For a meson-meson interaction, the summation over all possible twists in the multi-Reggeon chain leading to the cylindrical pomeron, induces the two-chain configurations shown in fig. 10 a.

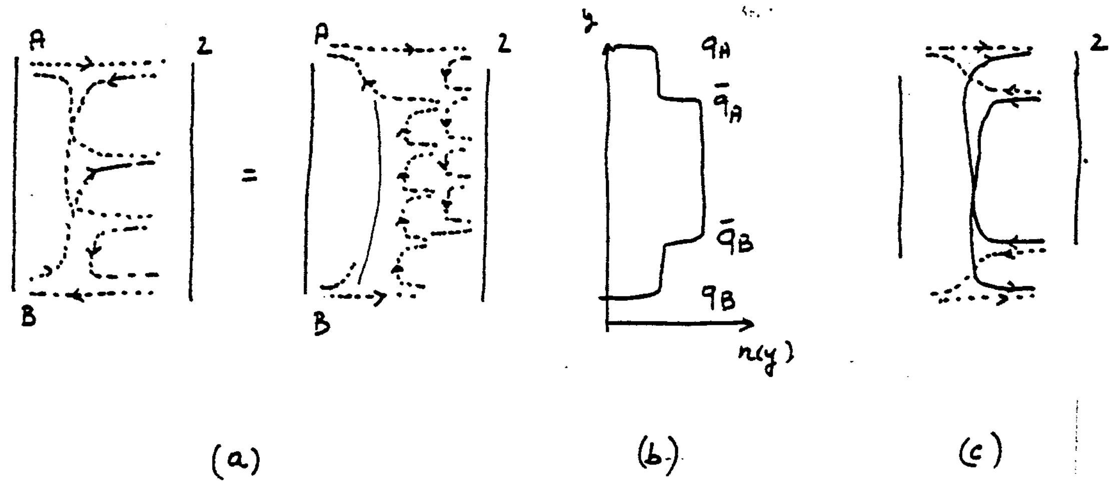



Fig. 10 (a) Two-chain configuration leading to cylindrical pomeron
(b) The corresponding rapidity distribution of multiplicity
(c) Parton interpretation

This result has been obtained under the assumption that higher topologies can be neglected. The question of what becomes the dual parton model when taking into account higher topology corrections is the subject of the next section.

## II. HIGHER TOPOLOGIES AND RENORMALIZATION

### 1. Space-time description of reggeon and pomeron exchange

The parton model relies on a space time description of high energy processes. The scaling property is based on the existence of two scales of time : a short one for the interaction at the level of partons and a long one either to extract partons from hadrons or to let the partons hadronize.

The fact that dual amplitudes show Regge behaviour already at the tree diagram approximation has obscured the space time description of dual processes. On the contrary Regge behaviour has been studied in the framework of a $\lambda\varphi^3$ theory. In such a theory Regge behaviour is satisfyed by sums of ladder diagrams obtained when computing the shadow scattering of multiperipheral contributions [35] - see fig. 12. A multipheripheral production processes involves two scales of time : the short one is related to the inverse mass of the exchanged particle in the multipheripheral chain, the long one

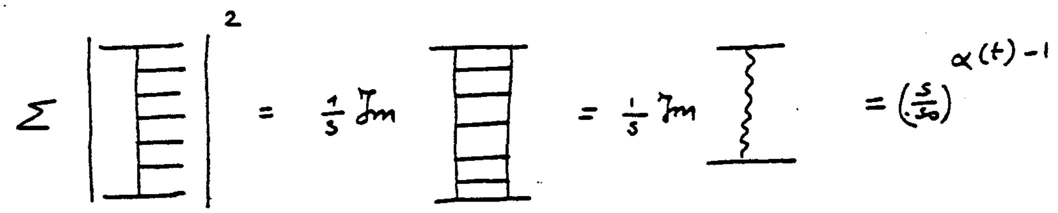



Fig. 12 - Multipheripheral dynamics and Regge behaviour in $\lambda\varphi^3$ theory.

is the full development time of the chain which is of the order of the total c.m. energy because of time dilatation. The existence of the two scales of time allows a parton interpretation of the multiperipheral dynamics. Indeed in the very crude $\lambda\varphi^3$ model there is only one particle which can be considered either as a hadron or as a parton. Also in this crude model a pomeron with intercept one does not emerge in a natural way.

## 2. Abramowski Gribov and Kanchelli (AGK) cutting rules

But these defects of the $\lambda\varphi^3$ model do not prevent to use this model to discuss theoretically the effects of multiple reggeon exchanges. Basically the reasoning which underlies the reggeon diagram technique or reggeon calculus developed by Abramowski Gribov and Kanchelli [14] goes as follows. If it takes a long time to interact through one reggeon exchange, a given parton in an incoming hadron can interact only once. A planar diagram with two subsequent reggeon interactions should be strongly suppressed. As a consequence the only important contribution involving 2 reggeon interactions must be non planar : each of the two incoming hadrons splits into two partons which interact at a time - see fig. 13.

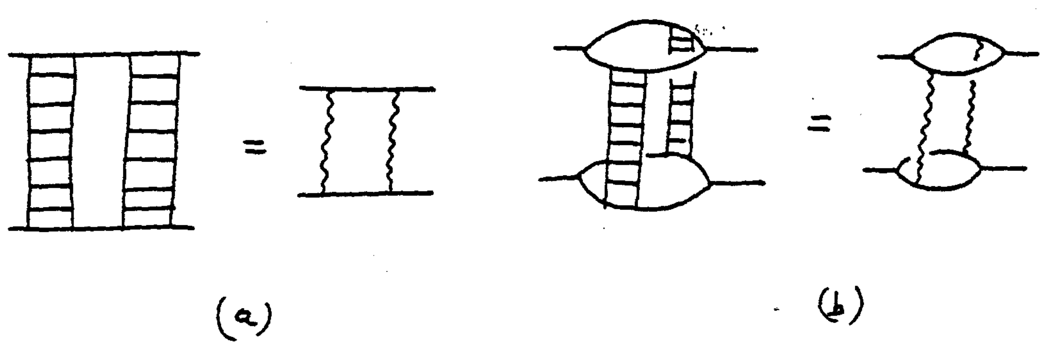


Fig. 13 (a) Vanishing planar two reggeon contribution
(b) Non vanishing, non planar two reggeon contribution.

It is possible to push further the argument. The contribution of fig. 13 b should be negative with respect to the one reggeon exchange contribution, since it is an absorption correction [36].

The AGK cutting rules precisely allow not only to determine the sign of this screening correction but to evaluate it. To evaluate a contribution like the one of fig. 13 b one has first to evaluate its absorptive part by cutting it in all the possible ways. In fig. 14 we sketch the argument according to which there are only three ways of cutting the diagram of fig. 13 b leading to non vanishing absorptive parts. The cutting of fig. 14 a

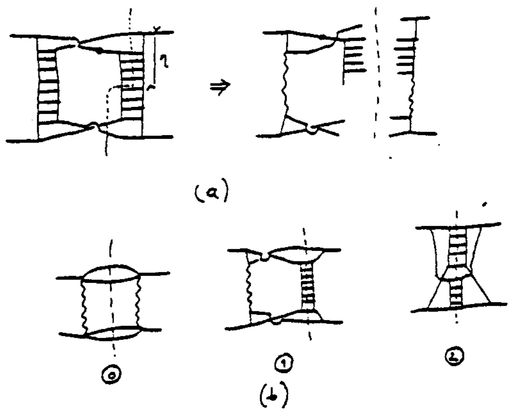


Fig. 14 (a) cutting leading to a vanishing absorptive part

(b) the three cuttings leading to non vanishing absorptive parts.

⓪ : none cut reggeon , ① : one cut reggeon ,

② : 2 cut reggeons.

(which is a partial cutting of a reggeon on a rapidity interval $\eta$ ) implies that the dotted propagator is connected to a "hanging" multiperipheral chain, and has thus a high virtualness (of the order of $m^2 e^{\eta}$ ). Now $\lambda\varphi^3$ is a super-renormalizable theory in which all high transverse momenta are cut-off. So we expect all cuttings which involve partial cutting of reggeons to give rise to vanishing absorptive parts. The three remaining cuttings shown in fig. 14 b lead to contributions which can be evaluated by taking the imaginary parts of the cut reggeons and by convoluting on shell amplitudes when reggeon loops are involved. It is a matter of combinatories to generalize these rules to any number of cut and non cut reggeons.

There are two possible analogies (which, actually, can be used together) for the problem of summing all the multiple reggeon exchange contributions[37].

i) A non relativistic field theoretical analog

The reggeon is represented by a complex field $\psi$ in two-space and one-time dimensions. Its propagator is of the form :

$$\frac{i}{E^2 - \alpha' k^2 - \Delta + i\varepsilon} \qquad (22)$$

where the squared energy $E^2 = 1 - J$ (J is the angular momentum in the t-channel) and $k^2 = -t$ is the square of the transverse momentum. $\Delta = 1 - \alpha(o)$ plays the role of squared mass. The coordinate space variables are $\vec{b}$ the 2-vector impact parameter for space and iy (y is the rapidity) for time. We note, by the way, that with these assignments, the limit $\alpha(o) \to 1$ is indeed an infrared limit. The theory, called a reggeon field theory (RFT), has a three-point coupling shown in fig. 15. The coupling constant is complex in connection with the phase of reggeon exchange amplitudes. It is pure imaginary for a pomeron with intercept one.

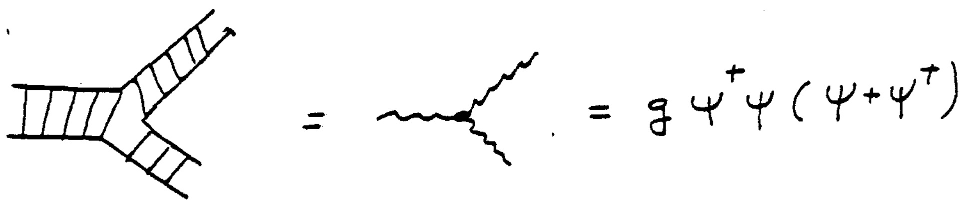



Fig. 15 - Three reggeon coupling in R.F.T.

ii) A phase transition analog

The diagram of fig. 13 b contribute to a Regge singularity which is not a pole but a branch point. The intercept of this branch point is $\alpha_c = 2\,\alpha\,(o) - 1$. The high energy behaviour of the two reggeon branch point is

$$\frac{s^{2\alpha(o) - 1}}{\log s} \qquad (23)$$

We thus see that if $\alpha(o) < 1$, all multiple reggeon exchanges give contributions which are negligible with respect to single reggeon exchange. If on the contrary $\alpha(o) > 1$ one expects multiple reggeon contributions to be more and more important and to be overwhelming with respect to the single reggeon exchange. Fig. 16 shows the fluid analogy corresponding to the two situations. $\alpha(o) < 1$ corresponds to a fluid with low density, a gas say, whereas $\alpha(o) > 1$ corresponds to a liquid with high density.

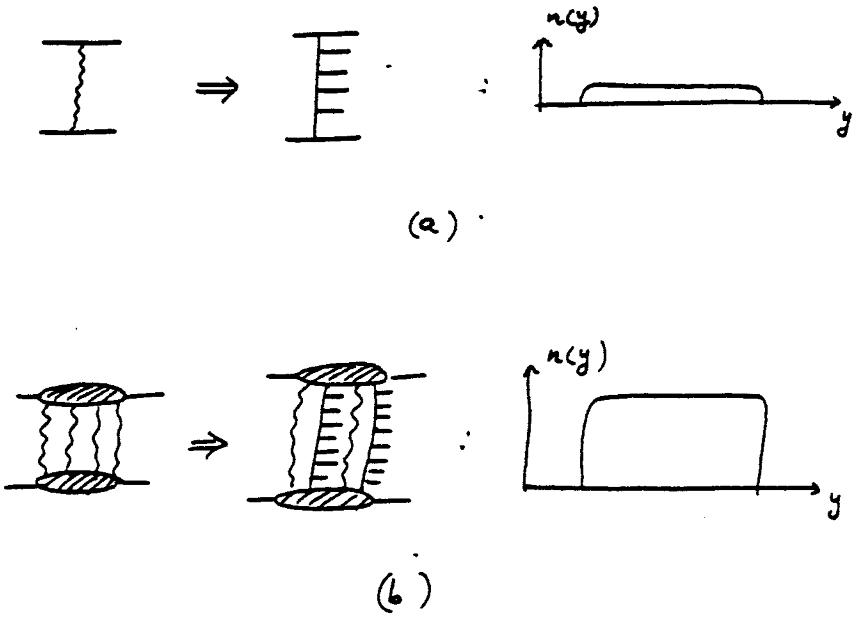



Fig. 16 (a) $\alpha(o) < 1$ gas analogy
(b) $\alpha(o) > 1$ liquid analogy

Indeed the interesting situation is when $\alpha(o) = 1$, in which case multiple reggeon exchanges and the single reggeon are comparable in amplitude. Such a situation can give rise, as we sketch in fig. 17, to fluctuating multiplicity distributions, typical of a critical phase transition.

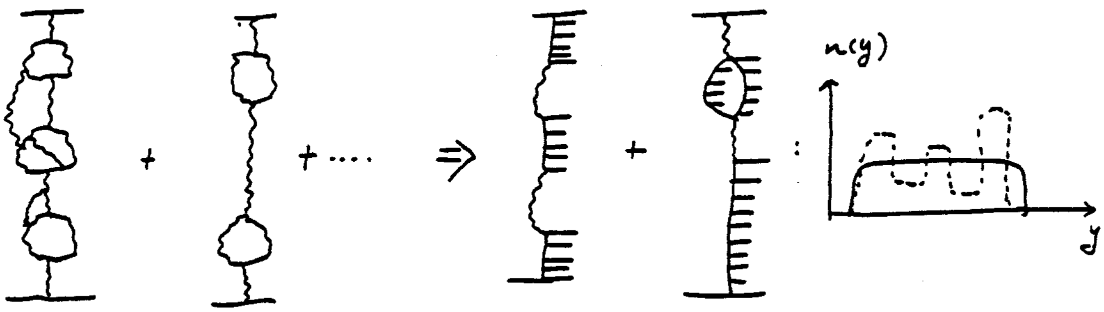



Fig. 17 - Fluctuating multiplicity distribution expected for $\alpha(o) = 1$

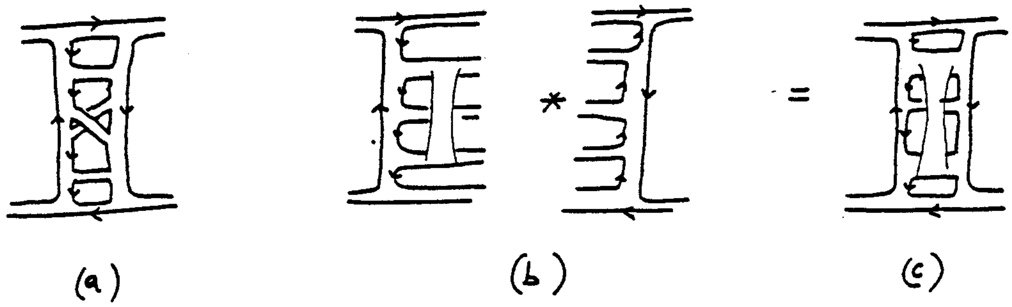
By using together the two analogies which we have just sketched one may hope to use some renormalization inspired techniques to compute "critical exponents". In the analogy which we have discussed a critical exponent would be related to the power of logs in the behaviour of the high energy total cross section (renormalized pomeron propagator). The hope was that, thanks to universality properties, this critical exponent is not too much dependent on the specific properties of the underlying local field theory.

One must recognize that the outcome of this program is not very convincing. In our opinion, this is due to the fact that, despite universality, the $\lambda\varphi^3$ model is a too crude model. It is not strictly renormalizable but super renormalizable ; there are no partons ; there is no bare pomeron with intercept one. We thus come back to the discussion of the topological expansion of QCD.

3. <u>DTU as a reggeon-pomeron calculus</u>

In contradistinction with the $\lambda\varphi^3$ model, our approach has good chances to lead to interesting results since the underlying local field theory, i.e. QCD , is strictly renormalizable, since there are hadrons and partons, and since the bare pomeron emerges , in a natural way, with intercept equal to one.

The cylindrical topology of the bare pomeron allows to understand why the summation over handles is equivalent to the summation over all multiple pomeron contributions. In fig. 18 we show how an interference which is neglected in DTU is recovered as a final state interaction correction.




<u>Fig. 18</u> (a) an interference leading to a one handle topology

(b) final state interaction through one cylindrical pomeron exchange leading to

(c) a one handle topology; when summing over all internal loops, topologies (a) and (c) are equivalent

AGK cutting rules apply to DTU since there is a transverse cut off in the driving term of the iteration. The DTU expansion can thus be reorganised in terms of cross sections for cutting a given number of reggeons and pomerons, the summation over all non cut reggeons and pomerons giving rise to what we call handle renormalization. For these cross sections one can apply the same procedure as the one defined in fig. 9 : one can replace the flavor loops corresponding to the mesons produced in the chains of cut pomerons or reggeons by color loops. The cross-section for cutting a given number of reggeons and pomerons which is an inclusive cross section is thus interpreted as an exclusive cross section for producing a given number of partons (quarks and gluons). As in fig. 10 the produced partons are associated to inhomogeneities in the distribution of produced hadrons. A good observable to use to define experimentally cross sections for cutting reggeons and pomerons in the transverse energy deposited in a calorimeter covering the full azimutal phase space : the higher is the number of cut reggeons or pomerons, the higher is the multiplicity, and the higher is, in average, the transverse momentum of produced hadrons. So our procedure to define exclusive cross section for producing partons appears as completely standard : partons are inhomogeneities in calorimetric distribution.

## 4. Handle renormalization in deep inelastic scattering

We show on an explicit example how works the topological definition of calorimetric cross-sections, and how handle renormalization (summation over all non cut pomerons) can be performed by means of the renormalization group equation of QCD. The example we choose is deep inelastic scattering in the small x region. The region in x is chosen in such a way that

i) The leading logarithm approximation (LLA) applies to solve the RGE, with no violation of unitarity that is

$$\mathrm{Log}\,\frac{1}{x} \leq \left( \mathrm{Log}\,\frac{Q^2}{m_\perp^2} \right)^2 / \log\log\frac{Q^2}{m_\perp^2} \qquad (24)$$

ii) $\mathrm{Log}\,\frac{1}{x}$ is large enough so that reggeon calculus is relevant ; that is

$$\mathrm{Log}\,\frac{1}{x} \geq \mathrm{Log}\,\frac{Q^2}{m_\perp^2} \qquad (25)$$

$W^2 = \frac{Q^2}{x}$ is the squared invariant mass of the hadronic system produced in the D.I.S. reaction

$$eA \to e' + X \qquad (26)$$

it is the squared c.m. energy in the $V^*A$ reaction - see fig. 19.

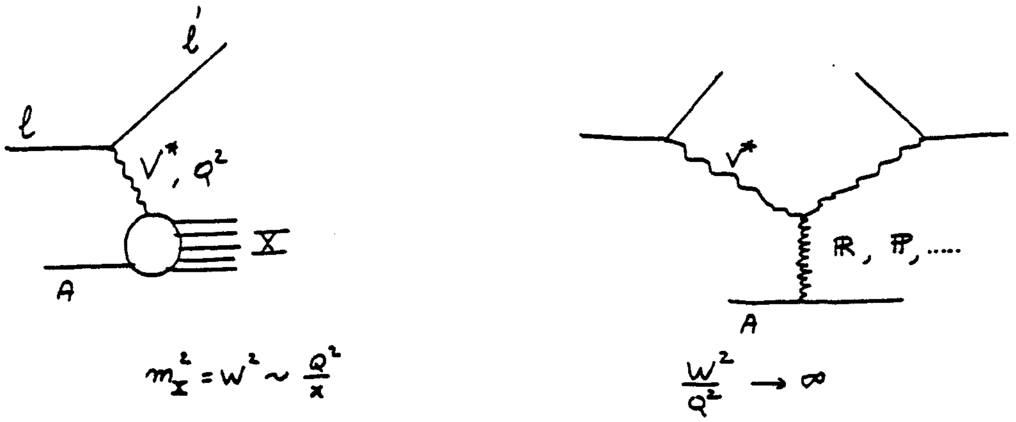
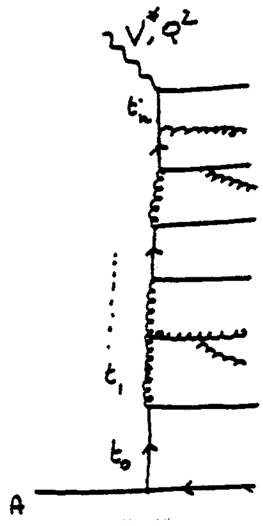



Fig. 19 - Regge model in D.I.S.
($V^*$ is a $\gamma^*$ , $W^*$ or $Z^*$ according to the type of leptons).

If (25) is satisfied one can analyze the $V^*A$ total cross section in terms of Regge pole exchanges. Applying DTU would lead to a reggeon pomeron calculus for this specific reaction.

On the other hand L.V. Gribov, E.M. Levin and M.G. Ryskin(6) have shown that when (24) is satisfied the LLA is valid, more precisely that the double logarithmic approximation ($\log \frac{Q^2}{m_\perp^2}$ and $\log \frac{1}{x}$ both large) works and that the Froissard bound for the $\gamma^*A$ total cross section is satisfied.

Let us recall the main results obtained at the LLA (38). In this approximation the dominant configurations are the ones shown in fig. 20. They involve three diagrams with strongly ordered virtualnesses. At the LLA one can neglect interferences and the cross section is represented in terms of generalized ladder diagrams that is ladder diagrams with renormalized vertices and propagators, evaluated in terms of the running coupling. It is interesting to note that, with the two line notation for quarks and gluons the non singlet ladder has the topology of a plane whereas the singlet ladder has the topology of a cylinder - see fig. 21.




$$\bar{s} \sim |t_0| \ll |t_1| \ll \cdots\cdots \ll |t_n| \ll Q^2$$

Fig. 20 - Strong ordering of virtualnesses in the LLA

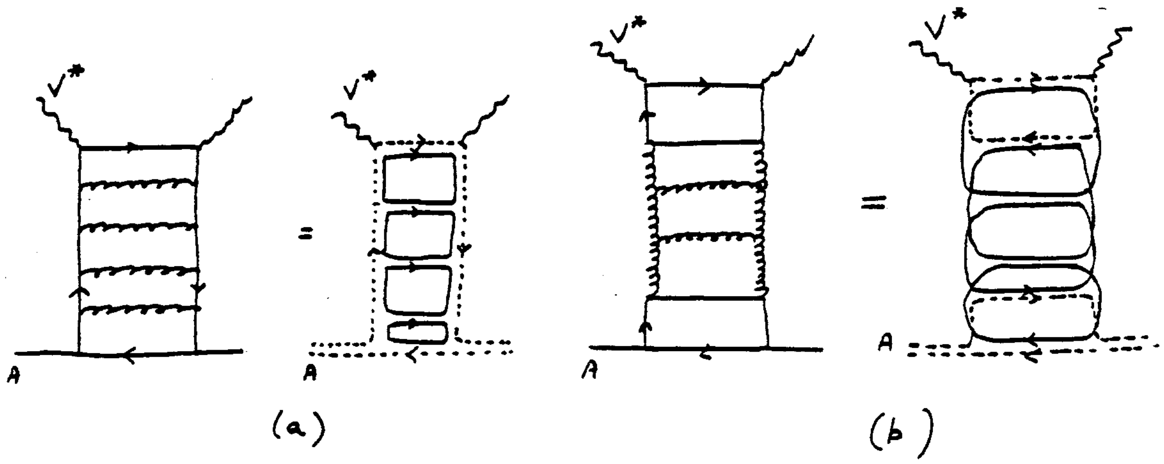



Fig. 21 - Topologies of the generalized latter diagrams in the LLA

a - non singlet ladder

b - singlet ladder.

The LLA is at the root of the so-called QCD improved parton model [38]. The strong ordering of virtualnesses shows that the highest momentum transfer $|t_n|$ is still very small as compared to $Q^2$. So $Q^2$ can be considered as a transverse cut off and one can continue to use a parton model language. The RGE of QCD, which are solved by means of the LLA, just tell us how the QCD improved parton model depends on the transverse cut off. This interpretation of the LLA solution of the RGE as a renormalization of the parton model is important for our purpose for it suggests that handle renormalization is nothing but the renormalization of the dual parton model.

In terms of DTU, which can be applied in the region defined by (24) and (25), the $V^*A$ total cross section can be written as the sum of all cross sections for cutting reggeons and pomerons in all the possible configurations. We show on fig. 22 an example of such a partial cross section. We consider, in the non singlet structure function, the cross section for cutting one pomeron and one reggeon.

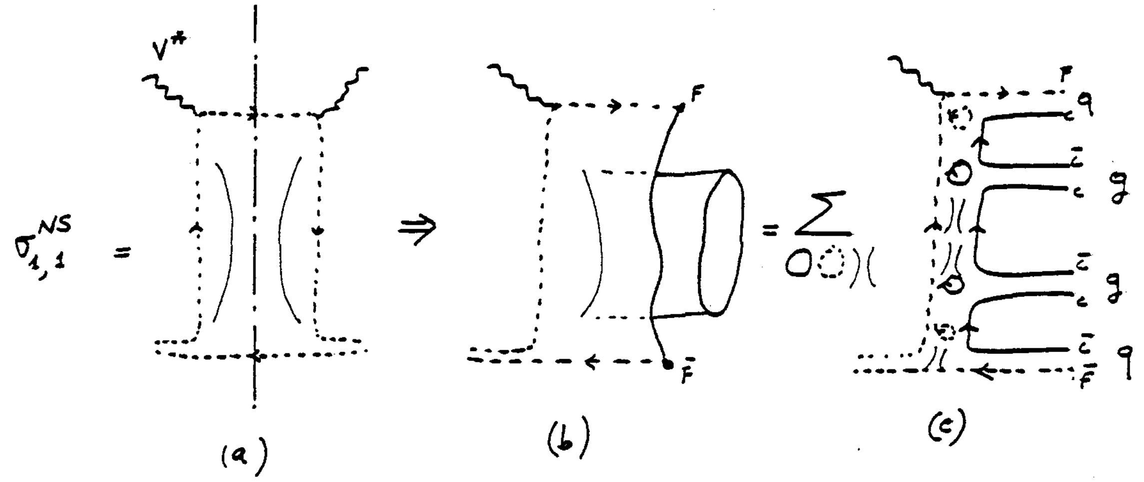


Fig. 22 -Cross section for cutting 1 reggeon and 1 pomeron in the non singlet structure function

(a) cutting one pomeron and one reggeon

(b) produced string configuration ; one mesonic open string plus one closed string

(c) interpretation as exclusive parton production cross section. All internal color loops, flavor loops and handles are summed over.

We can perform the same procedure for the sum over all cross sections for cutting all the possible configurations, namely

i) exhibit only the color lines associated with the produced partons

ii) swallow all the non cut pomerons (handles) in the renormalization of vertices and propagators.

It is clear that doing so we shall obtain for the structure functions the skeleton diagrams shown in fig. 21 a for the non singlet structure function and in fig. 21 b for the singlet structure function evaluated in the LLA.

This method to renormalize the dual parton model is strikingly similar to the renormalization group approach used to average fluctuations in a critical phenomenon [39]. If one performs DTU with $\bar{S}$ being fixed, it means that all fluctuations inside a rapidity interval $\Delta = \log\frac{\bar{S}}{m_\perp^2}$ are averaged. The other fluctuations extending on a larger scale in rapidity, and corresponding to the cutting of pomerons (like the one shown in fig. 22 b) have to be evaluated by means of the pomeron calculus. The idea which we propose here is to short circuit this calculation by letting $\bar{S}$ vary : if one increases $\bar{S}$ the transverse cut off becomes weaker and larger fluctuations are averaged. In the considered process, the largest value that $\bar{S}$ can take is $Q^2$. In this case all fluctuations are averaged and we can describe the interaction in terms of one single parton with maximal virtualness equal to $Q^2$. We thus recover the QCD improved parton model, for which the dependence in the transverse cut off is expressed in the RGE. If, to solve the RGE, we use the LLA with a running coupling, it means that the averaging over fluctuations is performed step by step from $\Delta$ to $\log\frac{Q^2}{m_\perp^2}$ . This is exactly how works the renormalization program in a critical phenomenon.

## III. CALCULATIONAL CONSEQUENCE

Indeed the interpretation of DTU as a renormalization procedure which we have just developped has no calculational consequence for deep inelastic scattering since we know that standard methods based on LLA are reliable in the considered region.

On the contrary in processes for which the only theoretical framework available is the pomeron calculus, our approach may allow to enlarge the domain of application of the RGE of QCD. We want here to discuss in terms of this approach, two results which have been obtained previously.

### 1. QCD and the rise of hadronic total cross sections (15)

The collider data have confirmed the rise of hadronic total cross sections. The most accurate measurements (including the ratio $\sigma_{el}/\sigma_{tot}$) (40) suggest that the asymptotic regime is not yet reached at the collider energy. The explanation of this rise remains an important theoretical challenge.

Pomeron calculus is the natural theoretical framework to describe high energy multi particle production in hadronic reactions. It is well suited for the description of multiplicity fluctuations, long range correlations, KNO scaling, etc... Some recent phenomenological analyses have led to good agreements with data [41]. However the total cross section itself remains difficult to approach in this framework.

In our opinion this difficulty is due to the fact that one always uses a pomeron calculus with a fixed transverse cut off. Now, one of the major experimental facts obtained at the collider [42] is that the cross section of processes in which at least one hadron has a transverse momentum $P_\perp > 2$ GeV is about 15 mb, i.e. it amounts to about 1/4 of the total cross section. Moreover the comparison between ISR and collider data shows that it is precisely this hard or semi hard component of the multiple production which is responsible for the rise of the total cross section. This observation suggests to use our modified pomeron calculus, with a running transverse cut off, to evaluate the rise of the total cross section.

The total inelastic cross section is expressed in terms of the total parton cross section :

$$\sigma_{AB}^{inel} = \int dZ_A \, dZ_B \, \psi_{A \to Q_A}(Z_A) \, \Psi_{B \to Q_B}(Z_B) \, \tilde{\sigma}_{Q_A Q_B}(\tilde{S} = Z_A Z_B S) \quad (27)$$

If A and B are nucleons (or antinucleons) the distributions of the constituent quarks $\psi_{A \to Q_A}$ and $\psi_{B \to Q_B}$ are such that eq. 27 can be approximated by

$$\sigma_{p\bar{p}}^{inel} = 9 \, \tilde{\sigma} \left(\frac{S}{9}\right) \quad (28)$$

which is nothing but the prediction of the additive quark model with quark kinematics.

The total $q\bar{q}$ cross section $\tilde{\sigma}$ is evaluated as an incoherent sum of partial cross sections of processes involving a maximal virtualness $\hat{t}$.

$$\tilde{\sigma}(\tilde{s}) = \int_{m_\perp^2}^{\tilde{s}} d\hat{t} \; \tilde{\sigma}_{\hat{t}} \quad (29)$$

If we are interested only in the rise of the total cross section we can split the integral of eq. 29 into two pieces

$$\tilde{\sigma}(\tilde{s}) = \int_{m_\perp^2}^{\bar{s}} d\hat{t}\, \tilde{\sigma}_{\hat{t}} + \int_{\bar{s}}^{\tilde{s}} d\hat{t}\, \tilde{\sigma}_{\hat{t}} \qquad (30)$$

the first part would correspond to the bare pomeron, leading to a flat contribution in the total cross section, the second part would lead to the rise of the total cross section

$$\Delta_s \tilde{\sigma}(\tilde{s}) = \int_{\bar{s}}^{\tilde{s}} d\hat{t}\, \tilde{\sigma}_{\hat{t}} \qquad (31)$$

For a fixed $\hat{t} > \bar{s}$ we can apply the same decomposition of $\tilde{\sigma}_{\hat{t}}$ as the one discussed in the previous section, and express $\tilde{\sigma}_{\hat{t}}$ as a sum of cross sections for cutting a given number of pomerons - see fig. 23.

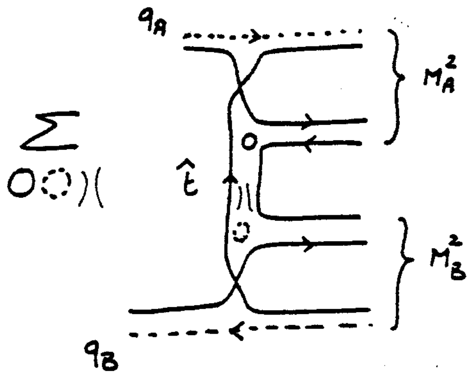



Fig. 23 - Exclusive parton cross section interpretation of the cross section for cutting one pomeron in $\tilde{\sigma}_{\hat{t}}$ .

To evaluate these cross sections it is necessary to perform the handle renormalization, that is to sum over all non cut pomerons. We use the RGE of QCD for this purpose : we average step by step fluctuations in larger and larger rapidity intervals from $\Delta = \log \bar{s}/m_\perp^2$ up to $\log \hat{t}/m_\perp^2$ . We thus must bound $M_A^2$ and $M_B^2$ by

$$\log \frac{M_A^2}{\hat{t}} \leq \log \frac{\hat{t}}{m_\perp^2}$$
$$\log \frac{M_B^2}{\hat{t}} \leq \log \frac{\hat{t}}{m_\perp^2} \qquad (32)$$

since higher rapidity intervals imply propagators with a virtualness higher than $\hat{t}$.

This step by step averaging is completely equivalent to a LLA calculation in a specific integration domain - see fig. 24.

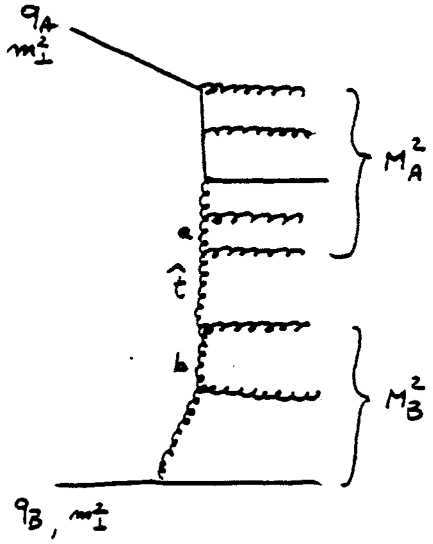



Fig. 24 - The LLA $q_A q_B$ cross section with a gluon exchange with virtualness $\hat{t}$ .

$$\tilde{\sigma}_{\hat{t}} = \frac{d\sigma_{ab}}{d\hat{t}} \iint_{\Delta_{\hat{t}}} \mathcal{D}_{q_A \to a} (x_a, \hat{t})\, \mathcal{D}_{q_B \to b} (x_b, \hat{t})\, dx_a\, dx_b \qquad (33)$$

where $\mathcal{D}_{q_A \to a} (x_a, \hat{t})$ (resp $\mathcal{D}_{q_B \to b} (x_b, \hat{t})$ ) is the structure function of parton a (resp b) in the constituent quark $q_A$ (resp $q_B$) with fraction of momentum $x_a$ (resp $x_b$) evaluated at the LLA with resolution $\hat{t}$, $\frac{d\sigma_{ab}}{d\hat{t}}$ is the differential cross section of ab scattering at the one gluon exchange approximation. The domain of integration $\Delta_{\hat{t}}$ is the intersection of two domains :

i) since $x_{a,b} = \frac{\hat{t}}{M^2_{A,B}}$ , Eq 32 gives

$$x_{a,b} \geq \frac{m_\perp^2}{\hat{t}} \qquad (34)$$

ii) $\hat{t} \leq \hat{s} = \tilde{s}\ x_a\ x_b$ (35)

(kinematics of the a b collision).

The interplay of these two constraints is crucial to determine the most sensitive value of $\hat{t}$. In ref (15) it is shown that this sensitive value is $\hat{t} \sim m_\perp^{4/3}\ \tilde{s}^{1/3}$ which corresponds to a semi hard process ($\hat{t} \to \infty$ but $\hat{t}/\tilde{s} \to 0$ at large $\tilde{s}$).

Constraint (34) was not well justified in ref (15) : in the space like region there is no kinematical restriction on $x_a$ and $x_b$ and with $\log \frac{1}{x} \leq \log Q^2/m_\perp^2$ one is far below the dangerous region in which unitarity could be violated (see eq. 24). The argument which we present here is new. The approach which we have developped consists in calculating each $\tilde{\sigma}_{\hat{t}}$ by means of a pomeron calculus with a transverse cut off equal to $\hat{t}$. We thus must not take into account parton cluster with too large squared invariant masses : violating constraint 34 would be equivalent to a partial cutting of a reggeon or a pomeron, giving rise to a vanishing absorptive part.

The rise of the $p\bar{p}$ total cross section has been quantitatively estimated in ref (15). It is interesting to note that, with the approach presented here, all dimensionned parameters are constrained theoretically

$m_\perp = \frac{m_\rho}{2} = 385$ meV (see eq. 9)

$\Lambda_{QCD} = m_\pi = 140$ MeV (see eq. 10)

$\bar{s} = 1.1\ GeV^2$ (see eq. 16)

On the other hand, one knows that in the LLA the variable

$$\xi = \log \frac{\log \frac{Q^2}{\Lambda^2}}{\log \frac{\mu^2}{\Lambda^2}} \qquad (36)$$

controls the amount of cascading [5]. The parameter $\mu^2$ is the reference $Q^2$ ($\alpha_s(\mu^2)$ is the "bare" coupling constant). Our approach leads to take for $\mu$, the value of the critical temperature of hadronization transition. It is interesting to note that with $\mu = T_c$ = 200 MeV (as suggested by Monte Carlo calculation [3]) the amount of cascading is consistent with the average momentum taken by gluons in nucleon structure functions at $Q^2 \sim \bar{S}$ . So, all dimensionned parameters can be fixed at values suggested by theoretical arguments. Doing so one obtains a rise of the total cross section of about 2.5 mb per unit of rapidity which is in good agreement with data. One can even, optimistically, use the proposed model to estimate the bare pomeron contribution (first piece of the $d\hat{t}$ integration in eq. 30). One obtains this way an excellent agreement with the total $p\bar{p}$ inelastic cross section from ISR to collider energies.

2. <u>The EMC effect</u> [16]

Apart from high energy hadronic collisions, there is another domain for which pomeron calculus provides the most suitable theoretical framework ; it is the domain of high energy reactions involving nuclei.

For instance, in a high energy hadron-nucleus interaction the AGK cutting rules [14] allow to decompose the total inelastic cross section [43] in a sum of cross sections of processes in which n nucleons of the target have been involved in an inelastic interaction, whereas the A-n remaining nucleons either are spectator or are involved in an elastic scattering. These cross sections generalize the cross sections for cutting a given number of pomerons : in the case of scatterings on a nuclear target, the pomerons which are cut are not the bare pomerons ; one has to take into account handle renormalization. So, the relevant cross section is the cross section $\sigma_n$ for cutting n renormalized pomerons or pomeron bundles. AGK cutting rules lead to a very simple form for $\sigma_n$ :

$$\sigma_n = \int d^2b \quad \binom{A}{n} \left[ \sigma_{in}^{hN} \; T(b) \right]^n \left[ 1 - \sigma_{in}^{hN} \; T(b) \right]^{A-n} \qquad (37)$$

where T(b) is the nuclear profile function and $\sigma_{in}^{hN}$ the inelastic hadron-nucleon cross section. This equation has a simple interpretation : $\sigma_{in}^{hN}$ T(b) is the probability that a nucleon with impact parameter b interacts inelastically with the projectile. The Bernouilli's binomial distribution one has in eq. 37 expresses the probability of n "successes" in A independent trials. From eq. 37 one can compute the total inelastic cross section. One finds :

$$\sigma_{in}^{hA} = \sum_{n=1}^{A} \sigma_n = \int d^2b \ \{1 - [1 - \sigma_{in}^{hN} \ T(b)]^A\} \qquad (38)$$

One can also evaluate $\bar{\nu}$ , the average number of wounded nucleons, that is <n> . One finds :

$$\sum_{n=1}^{A} n\sigma_n = \langle n\rangle \ \sigma_{in}^{hA} = \bar{\nu}\, \sigma_{in}^{hA} = A \ \sigma_{in}^{hA} \qquad (39)$$

Despite their simplicity the results summarized in eq. 37, 38 and 39 are far from trivial [(43)]. To understand them physically one needs a parton description of the inelastic interaction. One knows for instance that the projectile h is already far away from the target when the first inelastic interaction has completely taken place (remember that it takes a long time to exchange a reggeon or a pomeron). So, one does not understand eq. 37, except if one assumes a parton picture : the incoming hadron has to be thought as a beam of partons, among which the rapid ones go through the target without interactions, whereas the slow ones (the number of which is unlimited) may trigger, at a time, multiple interactions with the target nucleons. This picture is very important for the physical interpretation of the EMC effect [(44)] : the higher is n, the number of wounded nucleons, the higher is the number of radiated partons.

It is essentially this reasoning which has been applied in ref. (16) to elucidate the EMC effect. In deep inelastic scattering on a nuclear target, the role of the projectile is played by a virtual vector meson $V^*$ . Although it is reasonable to assume that the $V^*$ itself interacts only once, the probability that n target nucleons are wounded is of the same order of magnitude as in hadron nucleus collisions since slow partons are equally important for a $V^*$ or a hadron projectile.

The idea is thus to perform a pomeron calculus in D.I.S. on a nuclear target and to use our renormalization group approach to sum it up. For the sake of simplicity we assume that the profile function T(b) is a flat function. We thus introduce the normalized n nucleon cluster distribution :

$$P(n,A) = \binom{A}{n} P(A)^{n-1} \left[1 - P(A)\right]^{A-n} \qquad (40)$$

which satisfies

$$\sum_{n=1}^{A} n\, P(n,A) = A \qquad (41)$$

provided that

$$0 \leq P(A) \leq 1 \qquad (42)$$

with P(n,A) we can write the structure function of the nucleus A as

$$F_2^A (x,Q^2) = \sum_{n=1}^{A} P(n,A)\, F_2^n (x,Q^2) \qquad (43)$$

where $F_2^n$ is the structure function of the n nucleon cluster [45].

For a non singlet quark $F_{2,NS}^n$ is constrained by

$$\int_0^1 dx\, F_{2,NS}^n (x,Q^2) = 3\, n \qquad (44)$$

so, from eq. 41, we obtain trivially

$$\int_0^1 dx\, F_{2,NS}^A (x, Q^2) = 3A \qquad (45)$$

The EMC effect is to be found in the n dependence of P(n,A) and of $F_2^n(x,Q^2)$.

i) The n dependence of P (n,A) is given by eq. 40. We constrain P(A) by the fact that the average number of wounded nucleons $\bar{\nu}$ is of the same order as in hadron-nucleus collisions

$$\bar{\nu} = \langle n \rangle = \sum_{n=1}^{A} \frac{n^2}{A} P(n,A) = 1 + (A-1)\, P(A) \qquad (46)$$

We take $\bar{\nu} = \lambda A^{1/3}$ with $\lambda$ between 0.7 and 1.0 [43].

ii) We apply our renormalization group approach to determine the n dependence of $F_2^n$ $(x, Q^2)$ in terms of a recursion relation.

We first note that if $Q^2$ and x/n are not too small, there is no screening effect, and $F_2^n$ $(x,Q^2)$ is, up to scaling violation effects, proportional to n. More precisely if

$$\log \frac{n}{x} \leq (\log \frac{Q^2}{m_\perp^2})^2 / \log \log \frac{Q^2}{m_\perp^2} \qquad (47)$$

which is the equivalent of condition (24) for DIS on a n nucleon cluster, one has :

$$F_2^{n+1}(x, Q^2) - F_2^n (x,Q^2) = \frac{F_2^n (x, Q^2)}{n} + E_n(x,Q^2) \qquad (48)$$

where $E_n(x,Q^2)$ is related to scaling violation effect. We verify that if $E_n( x,Q^2) = 0$ ,

$$F_2^N (x,Q^2) = n F_2 (x,Q^2) \qquad (48)$$

where $$F_2(x,Q^2) = F_2^{n=1} (x,Q^2)$$

and there is no EMC effect :

$$F_2^A (x,Q^2) = AF_2 (x,Q^2) \qquad (49)$$

To estimate $E_n(x,Q^2)$ we first recall the physical meaning of the Altarelli Parisi equation [38] : when one increments $\log Q^2$ one increments $\log W^2$, where $W^2 = Q^2/x$ is the maximal value that can take the transverse momentum of the probed quark. The corresponding increment of $F_2(x,Q^2)$ is the convolution of $F_2(x,Q^2)$ by the splitting function P. For the sake of simplicity we shall use the approximate form of the Altarelli-Parisi equation :

$$\left.\frac{d \log F_2(x,Q^2)}{d \log W^2}\right|_x = \frac{d \log F2 (x,Q^2)}{d \log Q^2} = a(x) \qquad (50)$$

where a(x) is independent of $Q^2$, at least in a large range of values, and can be extracted from data [46].

Let us increment n by one unit in $F_2^n$ $(x,Q^2)$, keeping x and $Q^2$ fixed. This increment has three consequences.

i) The relative momentum of the probed quark is decremented from x/n to x/n+1.

ii) $\log W_n^2 = \log \frac{nQ2}{x}$ is incremented by

$$\Delta \log W_n^2 \Big|_{x,Q^2} = \frac{1}{n} \qquad (51)$$

iii) There is one more target nucleon wounded. There is thus one more cut pomeron, and thus, according to the discussion of the previous section, one more gluon produced.

On the whole we can estimate the effect of the increment of n by one unit

$$\Delta n = 1 \quad \Rightarrow \quad \text{Log } F_2^n (x,Q^2) = \frac{F_2^{n+1}(x, Q^2) - F_2^n (x, Q^2)}{F_2^n \quad (x, Q^2)}$$

$$\Rightarrow \quad \log W_n^2 = \frac{1}{n}$$

$$\frac{\Delta\log F_2^n \quad (x, Q^2)}{\Delta\log W_n^2} \Bigg|_{x,\, Q^2} = a \, (\frac{x}{n}) \qquad (52)$$

leading to

$$E_n (x, Q^2) = \frac{F_2^n (x, Q^2) \quad a \, (\frac{x}{n})}{n} \qquad (53)$$

The estimate of the EMC effect, performed by means of this model with a(x) taken from data and $\overline{\nu}$ defined in eq. 46 equal to 0.9 $A^{1/3}$ (54) leads to a satisfactory agreement with data - see fig. 25.

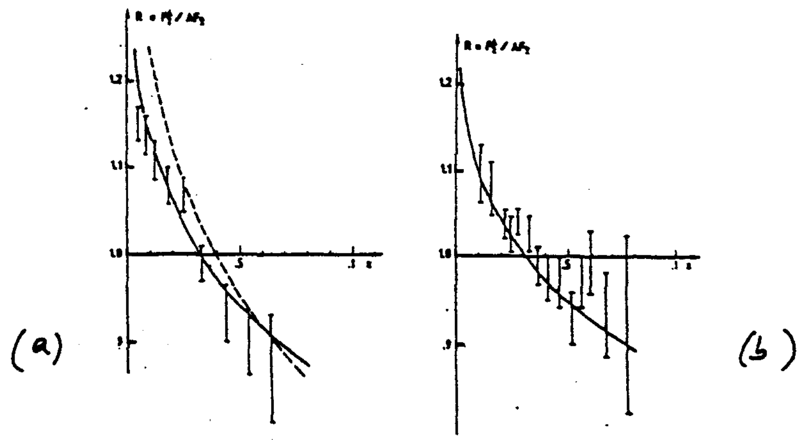



Fig. 25 - Estimate of the EMC effect - ref. (16)
a. full line : iron compared to data - dotted line : prediction for uranium
b. aluminium compared to data

## IV. CONCLUSION AND OUTLOOK

The results which we have just discussed are not new. What is new is their theoretical justification. Both results (the rise of the hadronic total cross section and the EMC effect) are obtained by means of the LLA. But what is intriguing is that LLA applies in such processes. The message of this paper is that LLA acts as a renormalization group approach applied to hadronization considered as a critical phenomenon. Indeed the effects which we have studied with our theoretical scheme have analogs in the framework of critical phenomena.

We would compare the rise of the hadronic total cross section with critical opalescence [47]. Critical opalescence is a phenomenon which occurs at the critical transition between steam and water : drops of liquid and bubbles of vapor are intermixed at all scales of length, producing a milky medium which refracts the light [39]. Now, as remarked by I.M.Dremin [48] one can relate the refractivity index of the quark-gluon medium to the real part of the forward scattering amplitude by

$$n(\omega) = 1 + \frac{2\pi N}{\omega^2} \text{ Re } F(\omega) \qquad (55)$$

Our estimate of the rise of the hadronic cross section clearly indicates a positive real part (actually the real part of the forward scattering amplitude is equal, to a good approximation to the logarithmic derivative of the total cross section) and thus a refractivity index greater than one.I.M. Dremin has argued that one can expect a relativistic parton travelling through this medium to emit a Cerenkov gluon·radiation. Such an effect would appear as a ring of particles with a given $\Theta$ (or pseudo rapidity) and isotropically distributed in azimut. In our approach, it would be a fluctuation in a high multiplicity event corresponding to the cutting of a large number of reggeons in a small interval of rapidity. It is interesting to note that such an effect has been observed by the UA5 collaboration [49] - see fig. 26.

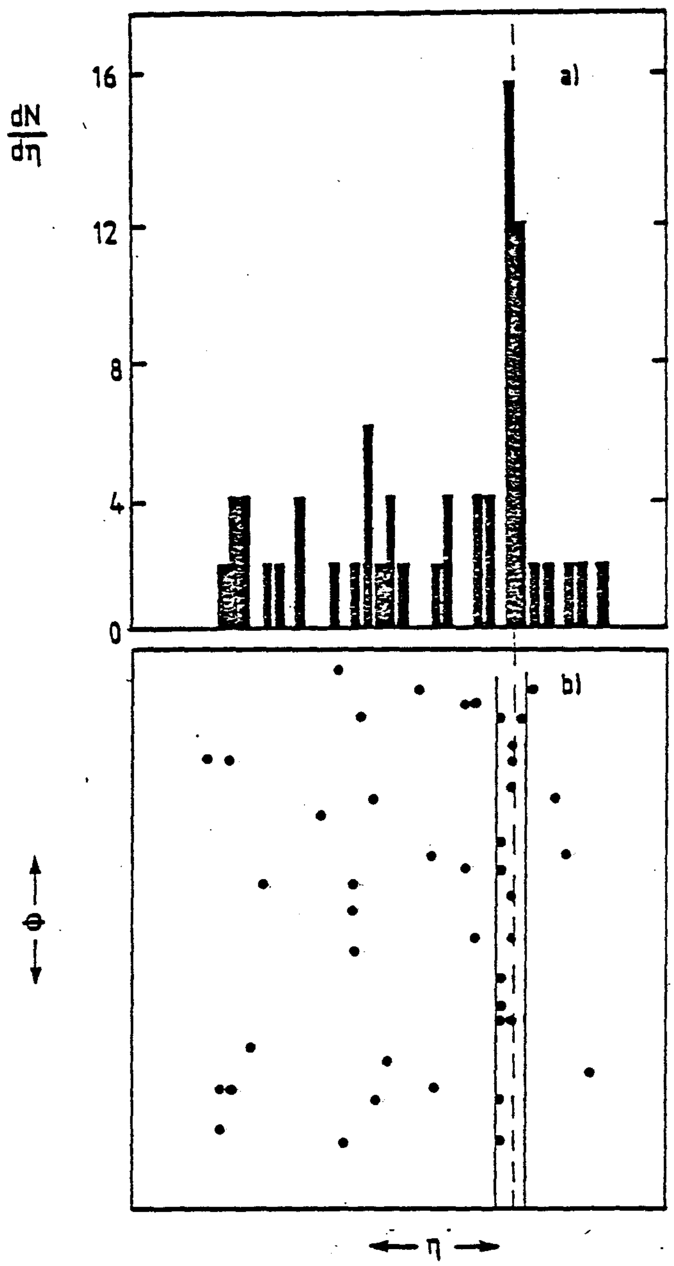



Fig. 26 - UA5 observation of a ring like event

For the EMC effect, the analog which we propose, is the Mossbauer effect. The n wounded nucleons are connected together by pomerons. They form a cluster which recoils as a whole, which increases the resolution . Here the pomeron plays the role of a phonon (the zero mass quasi particle associated with a vibration mode of a Mossbauer crystal).

Analogs can be very stimulating but they do not replace calculational methods.

The experimental study of ultra relativistic heavy ion collision to try and reproduce in a laboratory the conditions of the deconfinement phase transition, represents a challenge not only for experiment but also for theory. Up to now the only theoretical framework which has been used is provided by the thermodynamics of plasmas. However the experiment (if it is done) will be a scattering experiment. Indeed the number of produced particles is expected to be very large (a few thousands say). But, can one speak of thermal equilibrium for such transient processes as high energy hadronic or nuclear reactions ? How can one define and measure a temperature ? a pressure ?

We present our work as a step forward to the building of a theoretical framework suitable to the description of such experiments. Indeed the reggeon calculus is already known as the most suitable framework to describe high energy processes leading to high multiplicity events. This framework is already used to estimate various expectations for high energy ion collisions such as multiplicities, correlations [50] ...

The method which we propose would allow to use the most powerful tool provided by R.G.E. to disentangle the pomeron calculus. How the two examples discussed above have shown it, the reasoning which underlies over approach is difficult, but the calculation at the end of this reasoning is very simple (a standard LLA calculation).

We have begun to investigate the possibility of building a Monte Carlo generator of events, based on critical hadronization, suitable for any high energy reaction, including heavy ion collisions.

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