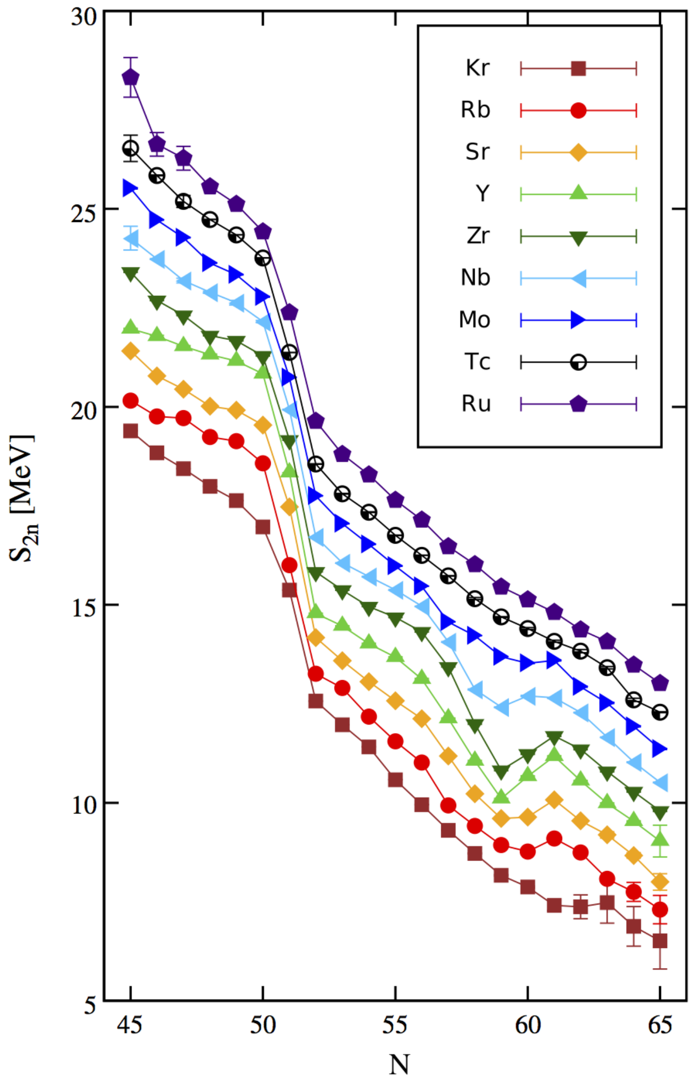
Shape coexistence in the Sr-Zr mass region around A $\sim$ 100

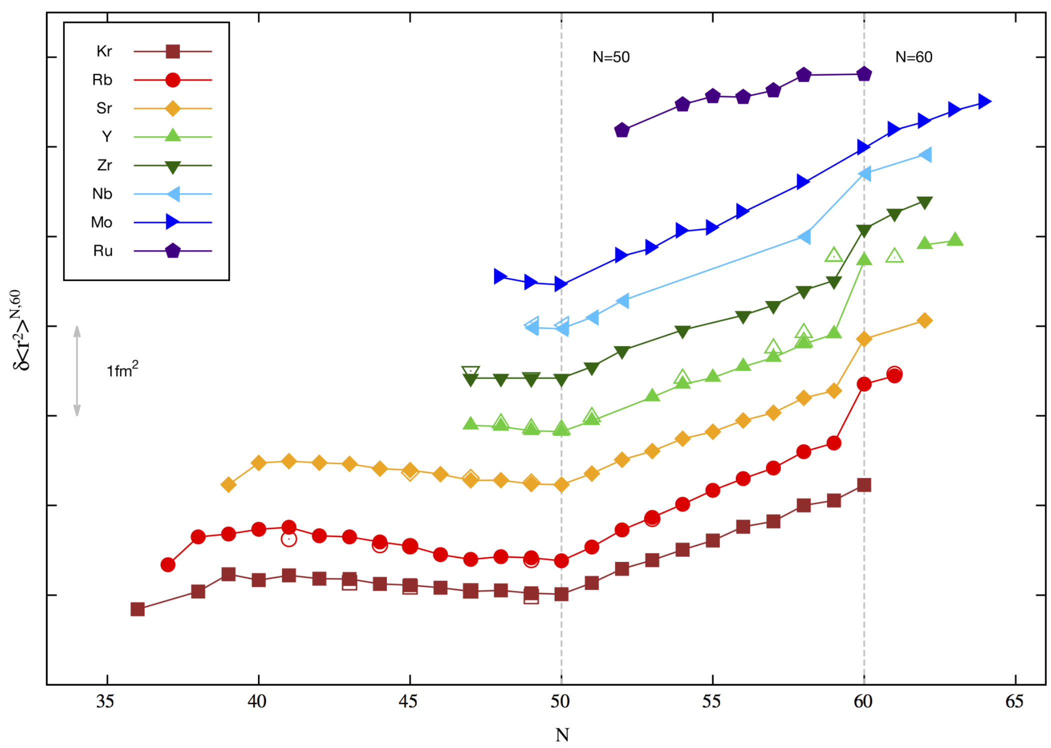
E. Clément[1], M. Zielińska[2]

[1] GANIL, Caen, France; [2] CEA Saclay, France;

Shape transition at N=60

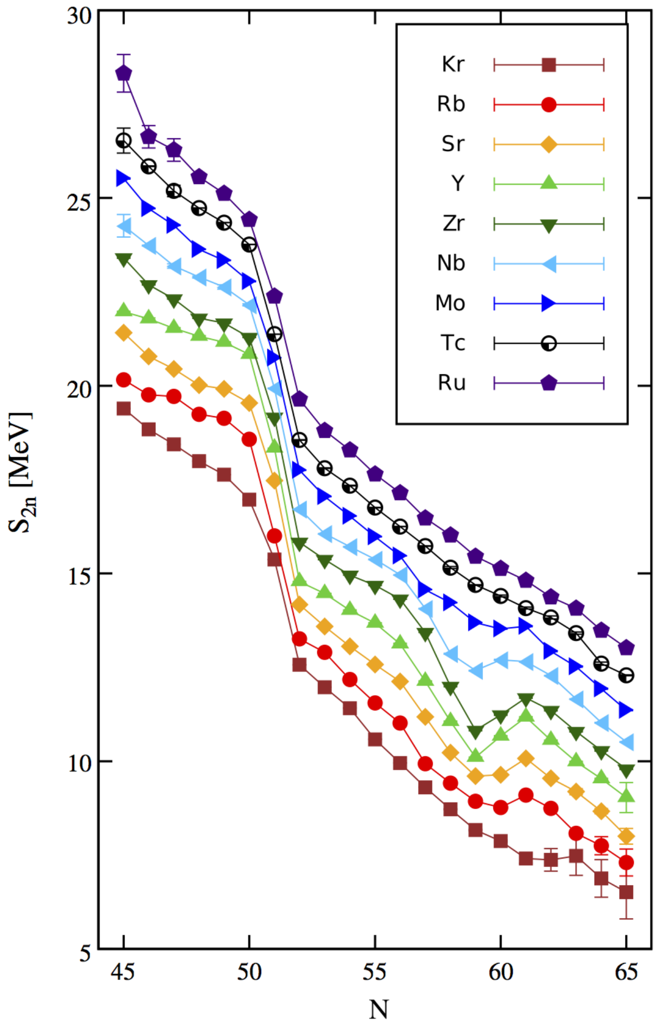


- dramatic change of the ground state structure observed at N = 58, 60 for Rb, Sr, Y, Zr
- considerable theoretical and experimental effort in this mass region

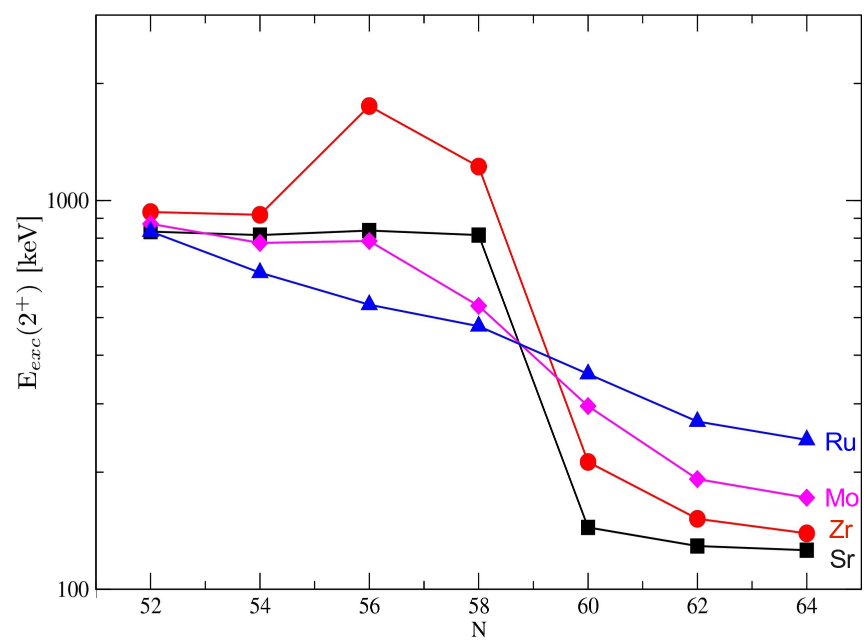


P. Campbell *et al.,* Prog. Part. Nucl. Phys. 86 (2016) 127

Shape transition at N=60

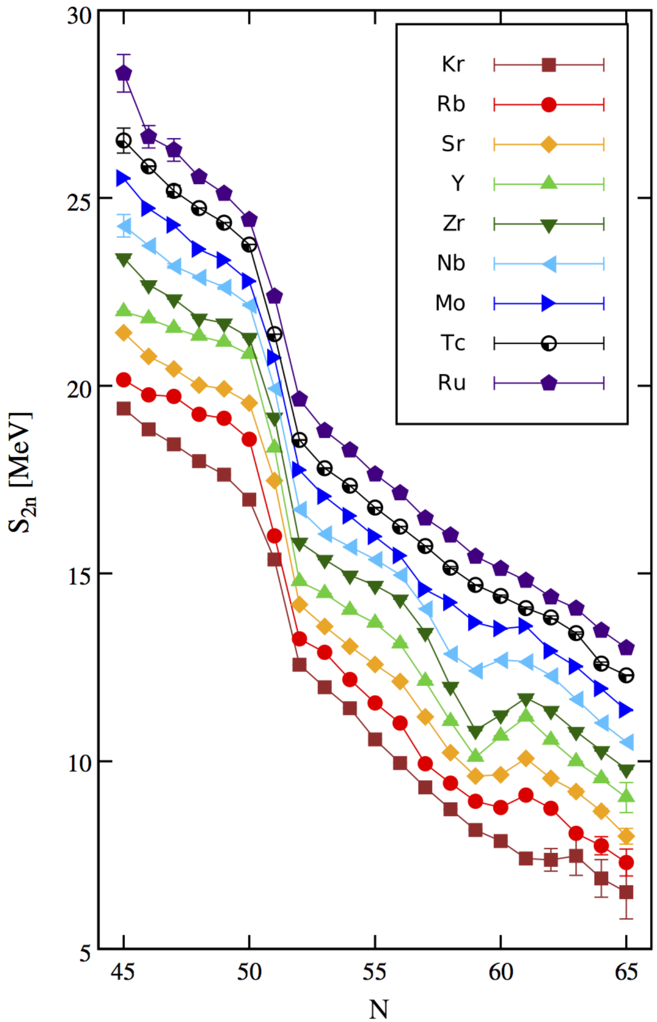


- dramatic change of the ground state structure observed at N = 58, 60 for Rb, Sr, Y, Zr
- onset of deformation at N=60 confirmed by 2^+ energies and transition probabilities

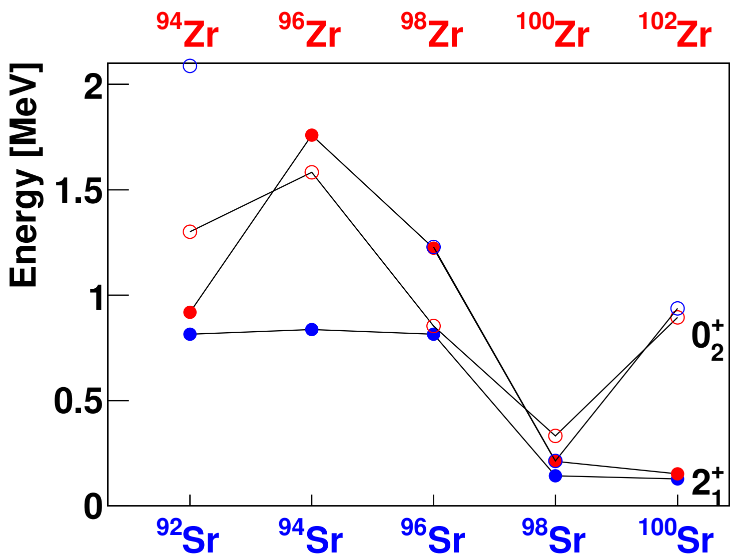


P. Campbell *et al.,* Prog. Part. Nucl. Phys. 86 (2016) 127

Shape transition at N=60 and shape coexistence around ^{100}Zr

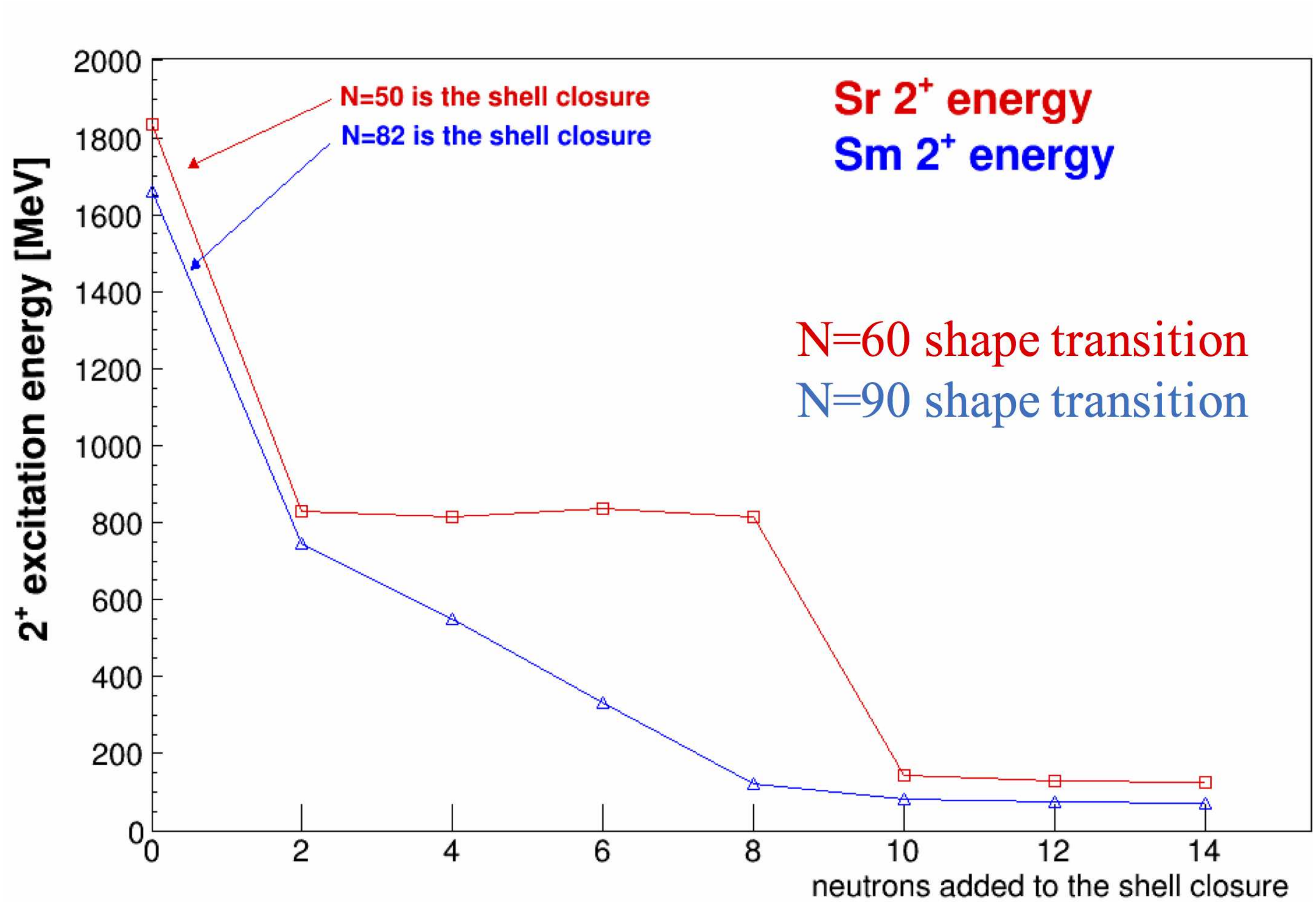


- dramatic change of the ground state structure observed at N = 58, 60 for Rb, Sr, Y, Zr
- onset of deformation at N=60 confirmed by 2^+ energies and transition probabilities
- low-lying 0^+ states observed in N=58,60 Zr, Sr



P. Campbell *et al.,* Prog. Part. Nucl. Phys. 86 (2016) 127

Unique shape transition

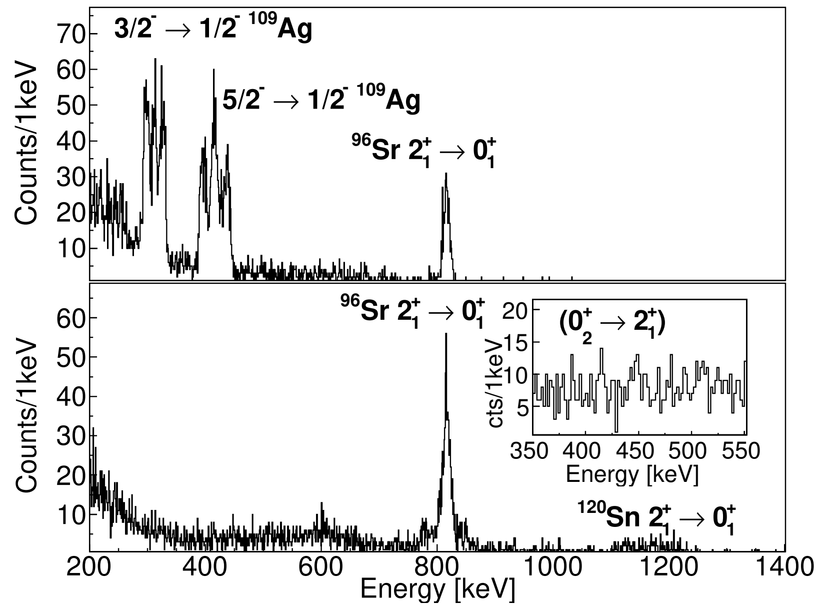


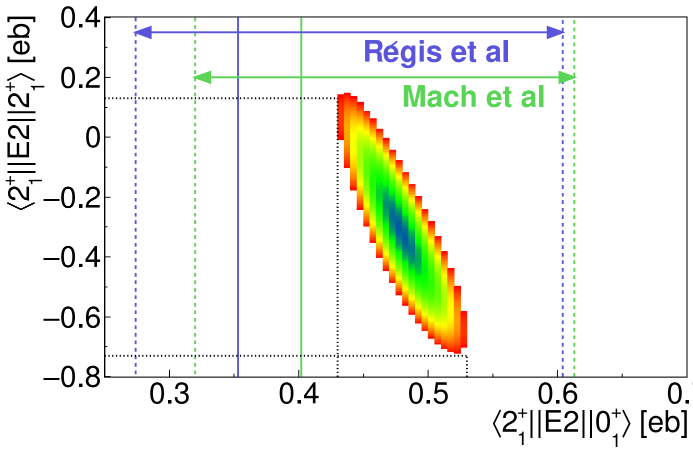
- most complete data set on both sides of the transition: ^{96,98}Sr

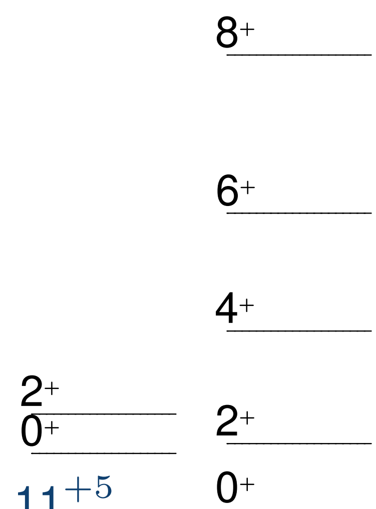
Deformation of ^{96}Sr

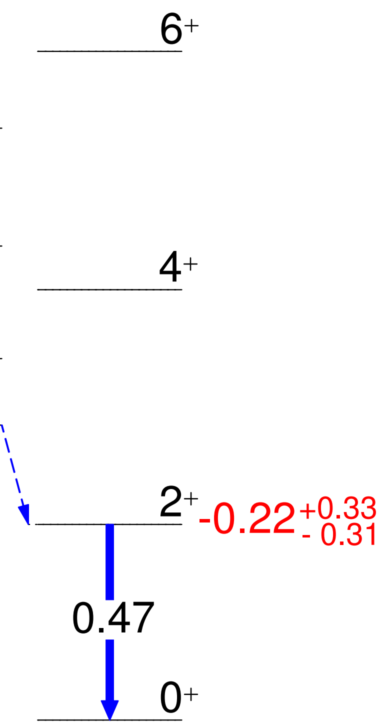
E. Clément, MZ *et al.* PRL 116, 022701 (2016)

Coulomb excitation at REX-ISOLDE: ^{96}Sr on ^{109}Ag, ^{120}Sn, ^{98}Sr on ^{60}Ni, ^{208}Pb









β (from Q_s) = 0.11^{+5}_{-4}

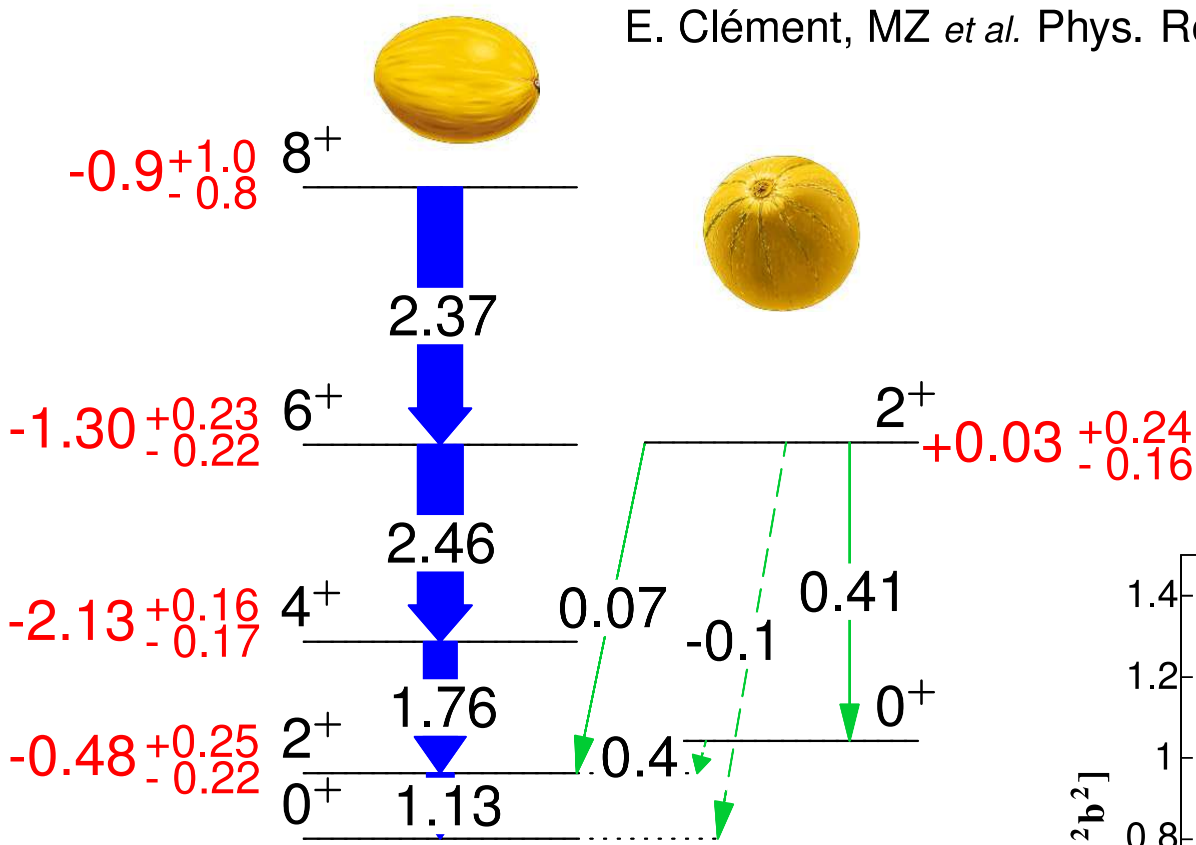
B(E2) in agreement

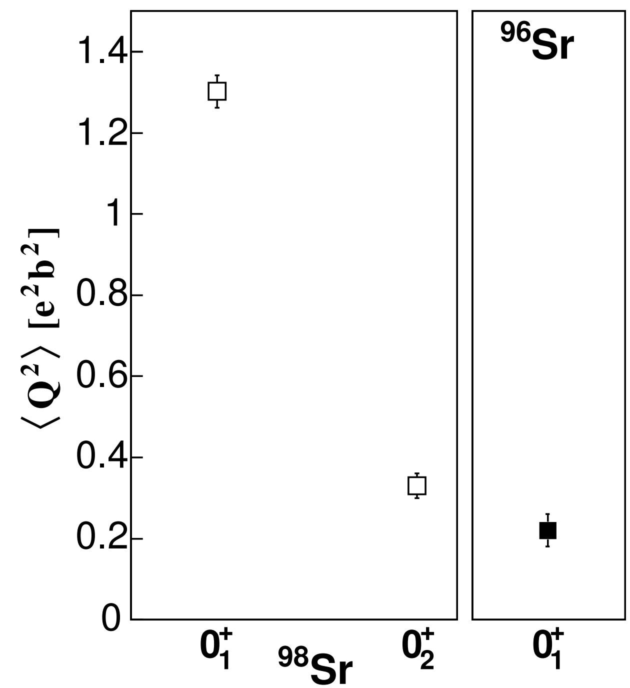
with lifetime but more precise

low deformation of gsb confirmed

^{98}Sr: quadrupole moments and transition probabilities

E. Clément, MZ *et al.* Phys. Rev. Lett. 116, 022701 (2016)

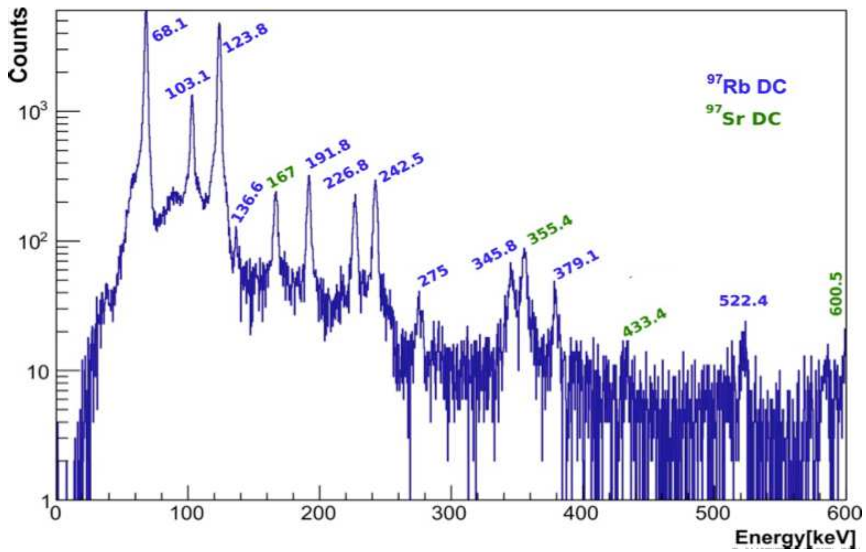


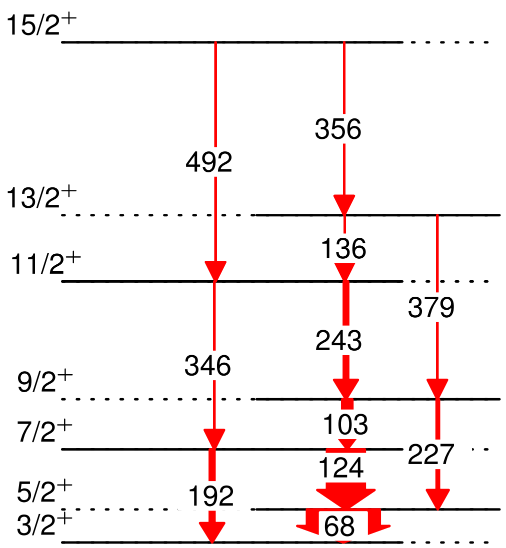


- well deformed prolate band ($\beta \geq 0.3$)
- low deformation of the excited band ($\beta < 0.1$)
- similar deformation of 0_1^+ in ^{96}Sr and 0_2^+ in ^{98}Sr

Identification of the southern border: deformation of N=60,62 ^{97,99}Rb

- identification of rotational bands in ^{97,99}Rb in low-energy Coulomb excitation at REX-ISOLDE

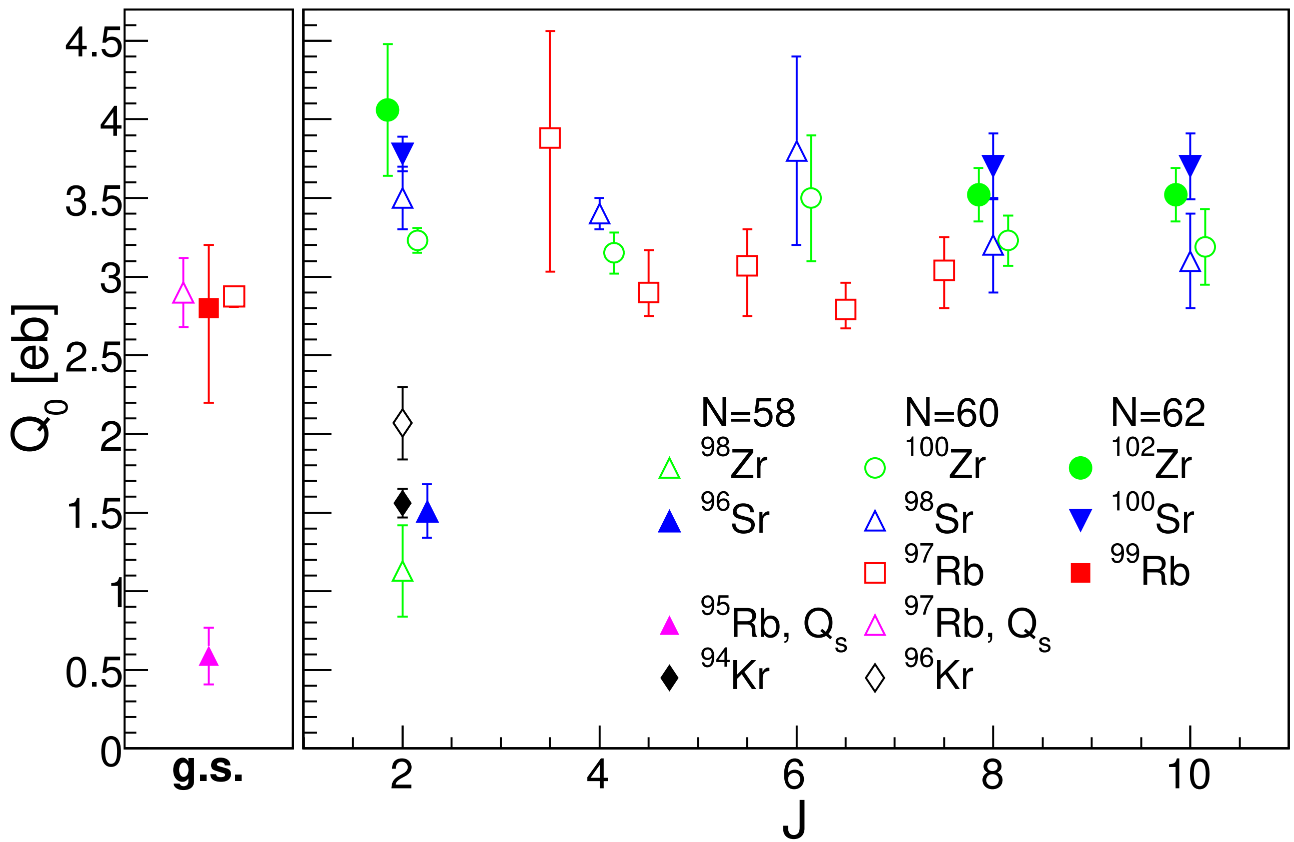




C. Sotty, MZ *et al.*, PRL 115 (2015) 172501

- extracted B(E2) values confirm strong constant deformation in gsb in ^{97,99}Rb
- B(M1)/B(E2) ratios in ^{97}Rb favour $3/2^{+}$[431] configuration of the ground state

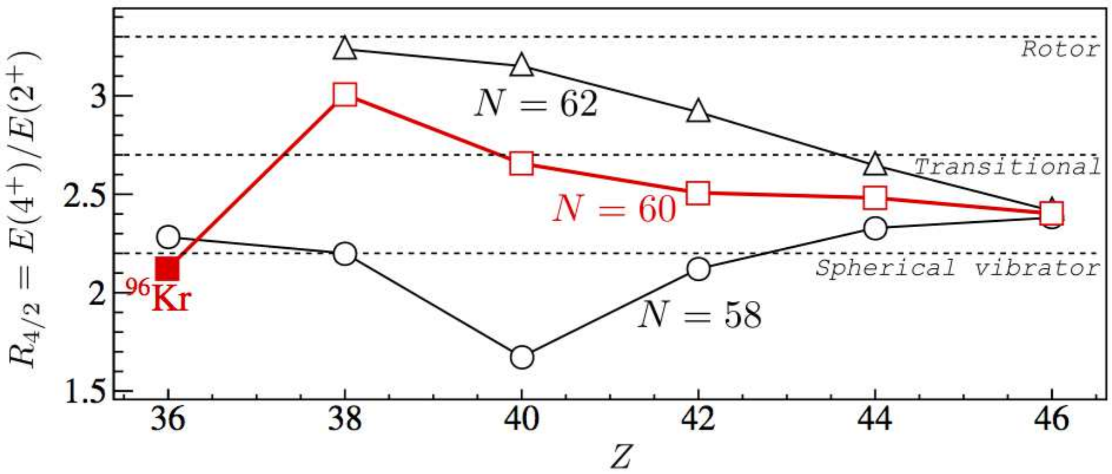
Transition probabilities and quadrupole moments in N=58,60,62 nuclei

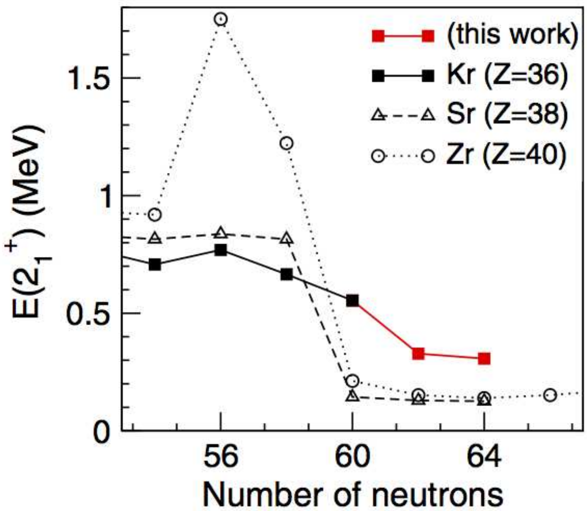


- visible reduction of Q_0 for N=60 ^{96}Kr – similar to what is observed for N=58 nuclei
- large deformation appears in ^{97}Rb and remains constant with increasing Z and N: Q_0 in ^{97,99}Rb similar to that of N=60,62 Zr and Sr nuclei
- Q_{sp} values from laser spectroscopy confirm a dramatic shape change at N=60 in Rb isotopes, deformation for ^{97}Rb consistent with Coulex results

Identification of the southern border: spectroscopy of ^{96,98,100}Kr

J. Dudouet *et al.* Phys. Rev. Lett. 118 (2017) 162501



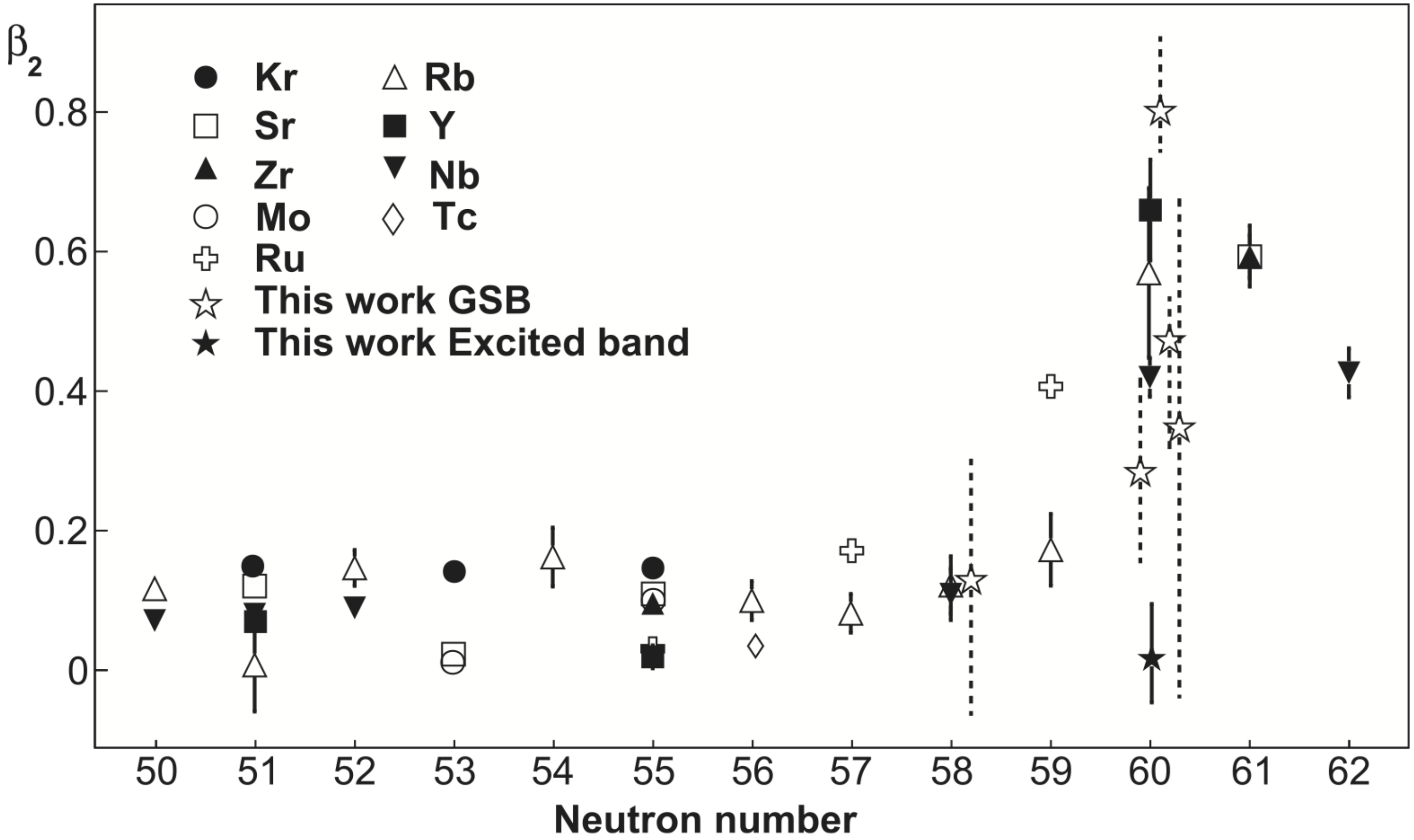


F. Flavigny *et al.* Phys. Rev. Lett. 118 (2017) 242501

- 4^+ state in ^{96}Kr behaves differently than in heavier N=60 nuclei
- 2^+ energies in ^{98,100}Kr suggest that the shape transition may be delayed to N=62

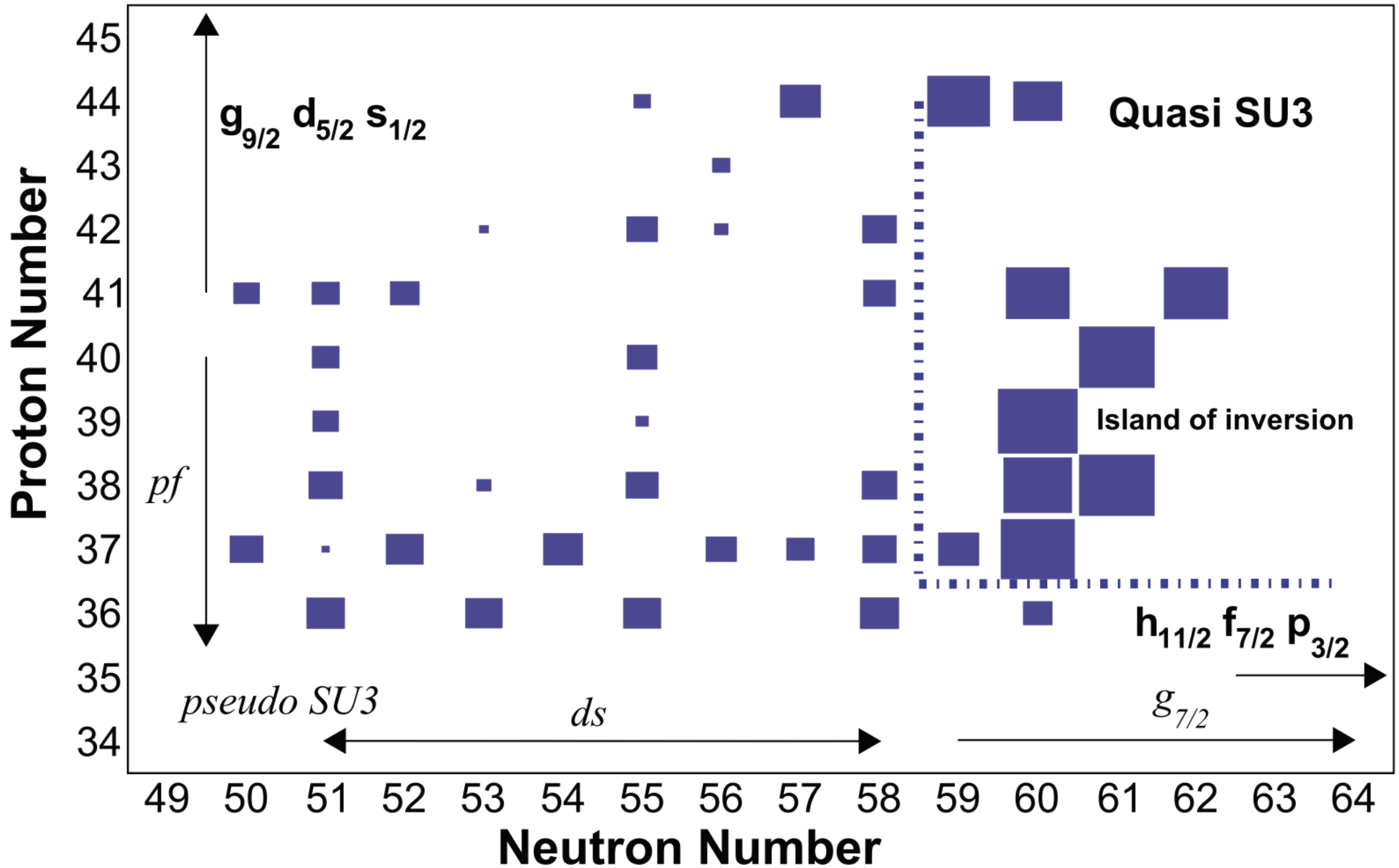
What happens at N=59?

- deformations obtained from Q_s for ground (laser spectroscopy) and excited states (Coulex) consistent



E. Clément and MZ, Phys. Scr. 92 (2017) 084002

Missing experimental information

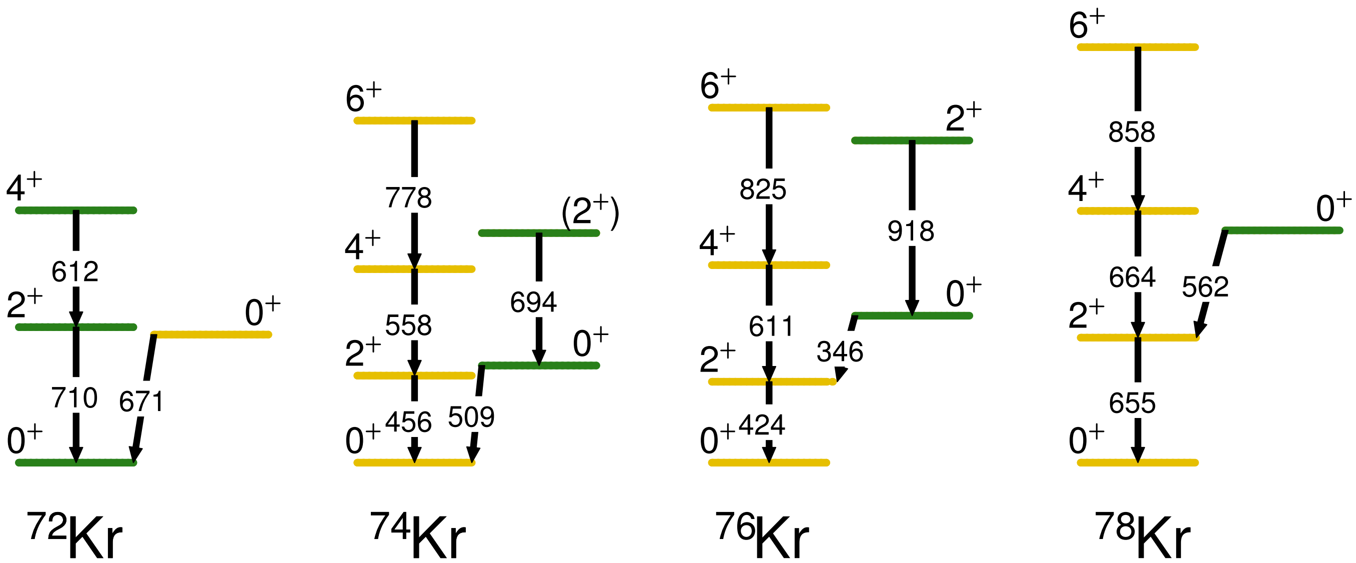


- in particular: g.s. deformation in ^{90,92}Rb, $^{91-93,95-98}$Y, $^{94-98,100}$Nb $^{99-101}$Mo
- 2^+ quadrupole moments in ^{88,90,92}Kr, ^{90,92,94}Sr and $^{92-100}$Zr

E. Clément and MZ, Phys. Scr. 92 (2017) 084002

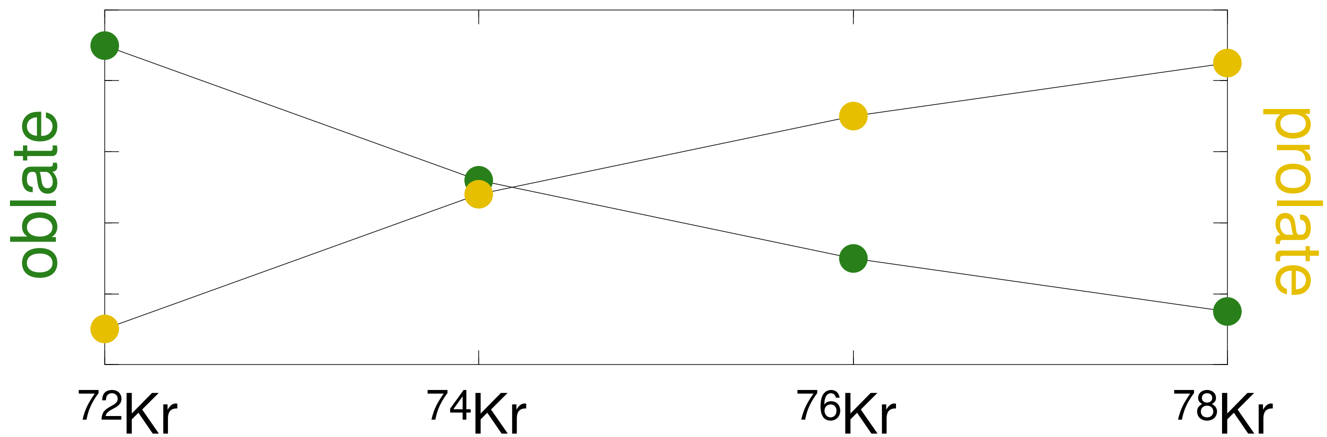
Shape coexistence: two-state mixing

Kr: E. Bouchez *et al.*
PRL 90 (2003) 082502



prolate

oblate



mixing of the g.s.
(from distortion
of rotational bands)

mixing amplitudes for ^{98}Sr (from ME): $\cos^2\theta_0$=0.87(1), $\cos^2\theta_2$=0.99(1)

sharp transition related to the very weak mixing in contrast to Kr and Hg

Shape coexistence: two-state mixing

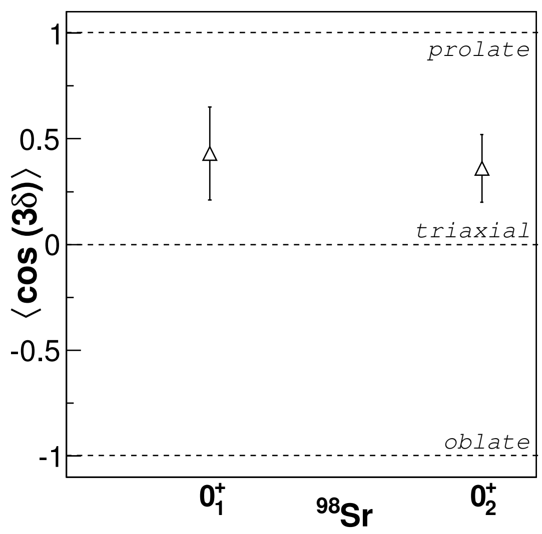
- four E2 matrix elements (including relative signs) needed to determine cos $^2\theta_0$ and cos $^2\theta_2$: $\langle\, 0_1^+ \| \,\mathrm{E2}\, \| \,2_1^+ \rangle$, $\langle\, 0_1^+ \| \,\mathrm{E2}\, \| \,2_2^+ \rangle$, $\langle\, 0_2^+ \| \,\mathrm{E2}\, \| \,2_1^+ \rangle$ and $\langle\, 0_2^+ \| \,\mathrm{E2}\, \| \,2_2^+ \rangle$
- but if cos $^2\theta_2 \approx 1$, cos $^2\theta_0$ is simply given by $\tan\theta_0 = \frac{\langle 0_2^+ | E2 | 2_1^+ \rangle}{\langle 0_1^+ | E2 | 2_1^+ \rangle}$

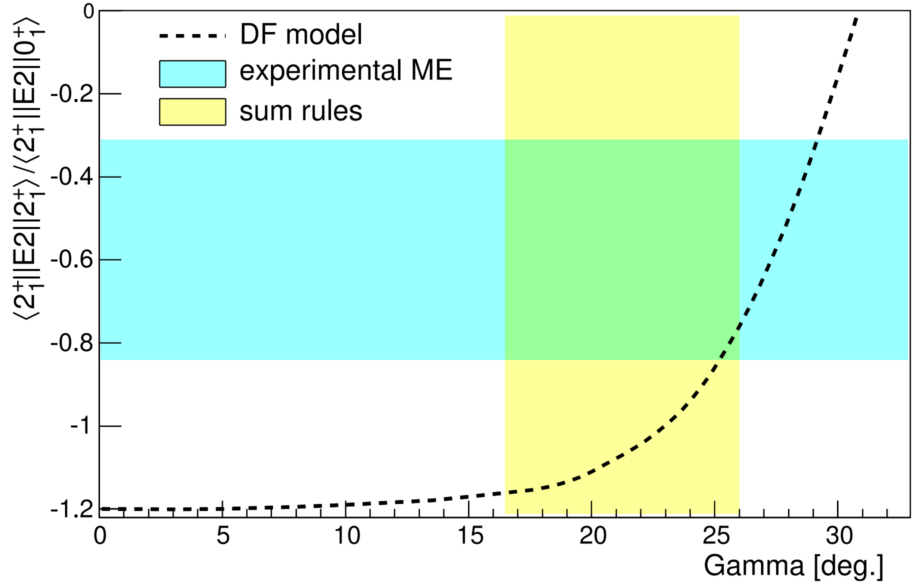
element	N=58	N=60
Pd	0.93	0.86
Ru	0.86	0.92
Mo	0.63	0.84
Zr	not measured	0.84
Sr	0.84	0.88
Kr	not measured	not measured

- low mixing both inside and north of the region of rapid shape change

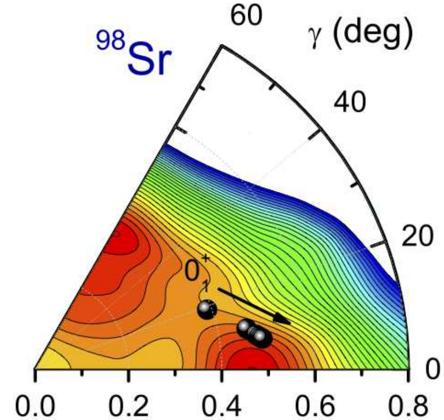
E. Clément and MZ, Phys. Scr. 92 (2017) 084002

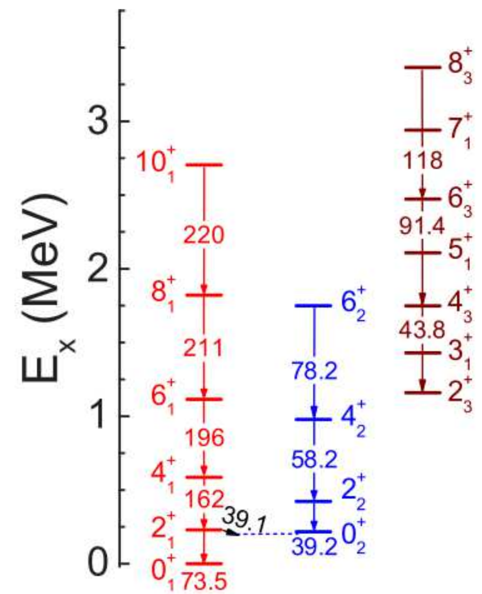
Triaxiality in ^{98}Sr





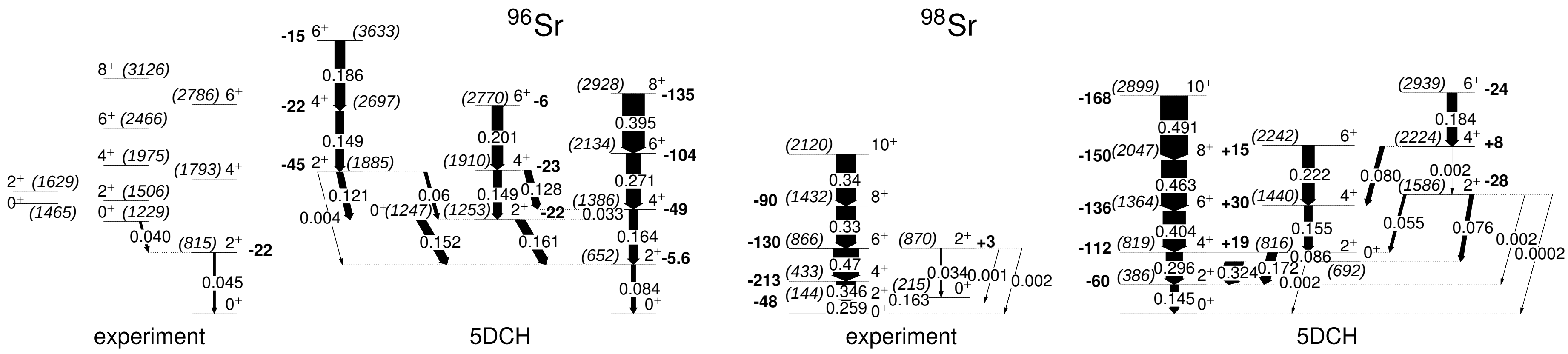
- gamma ≈ 25° would explain the reduction of $Q_s(2_1^+)$ in ^{98}Sr
- but where is the gamma band?





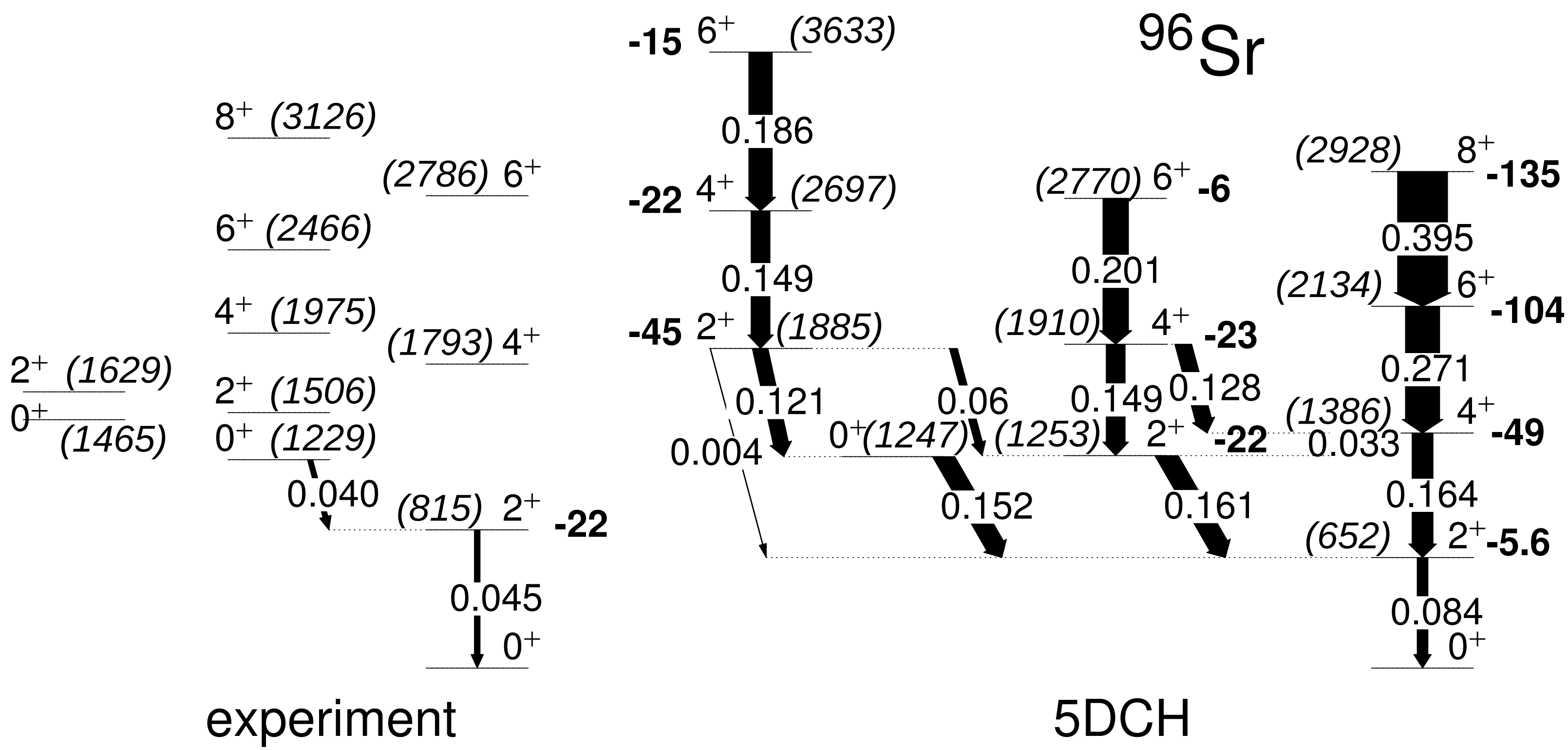
J. Xiang *et al.*, PRC 93, 054324 (2016), 5DCH with PC-PK1 interaction

Theoretical predictions for Sr isotopes



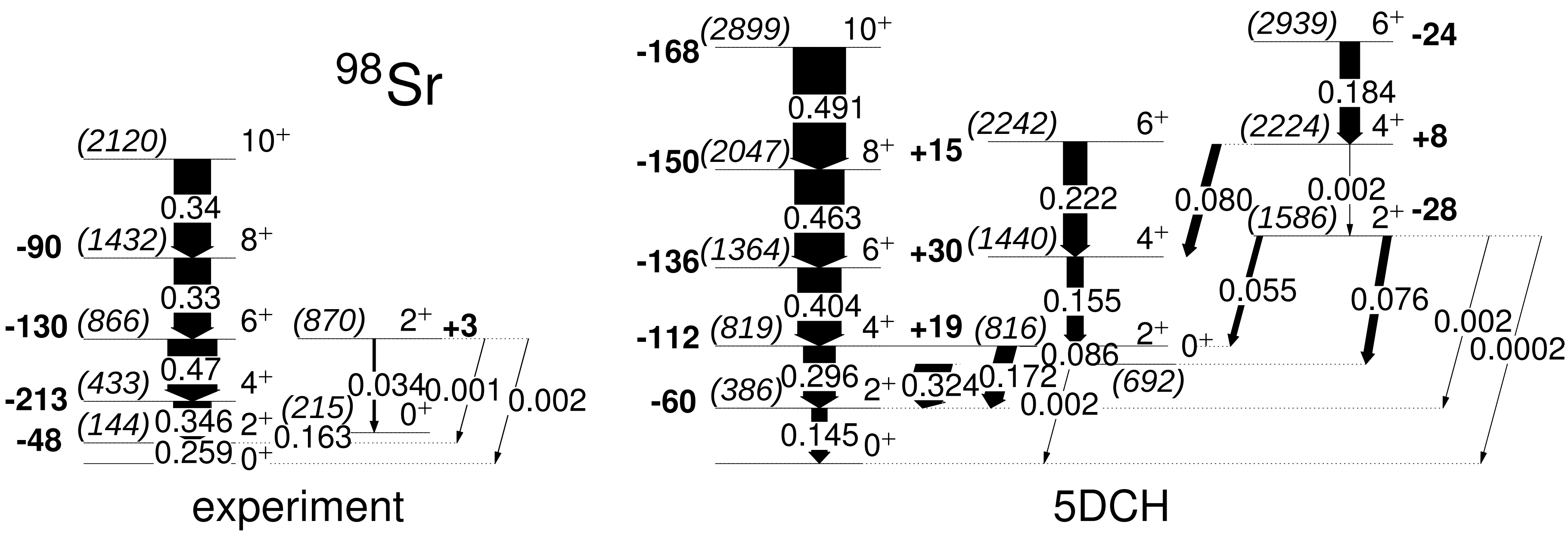
- beyond mean field calculations: GCM (GOA) D1S, (S. Péru, H. Goutte, J. Libert et al)
- first detailed calculation of transition probabilities on both sides of the N=60 shape transition
- shape change at N=60 and shape coexistence reproduced

Theoretical predictions for Sr isotopes



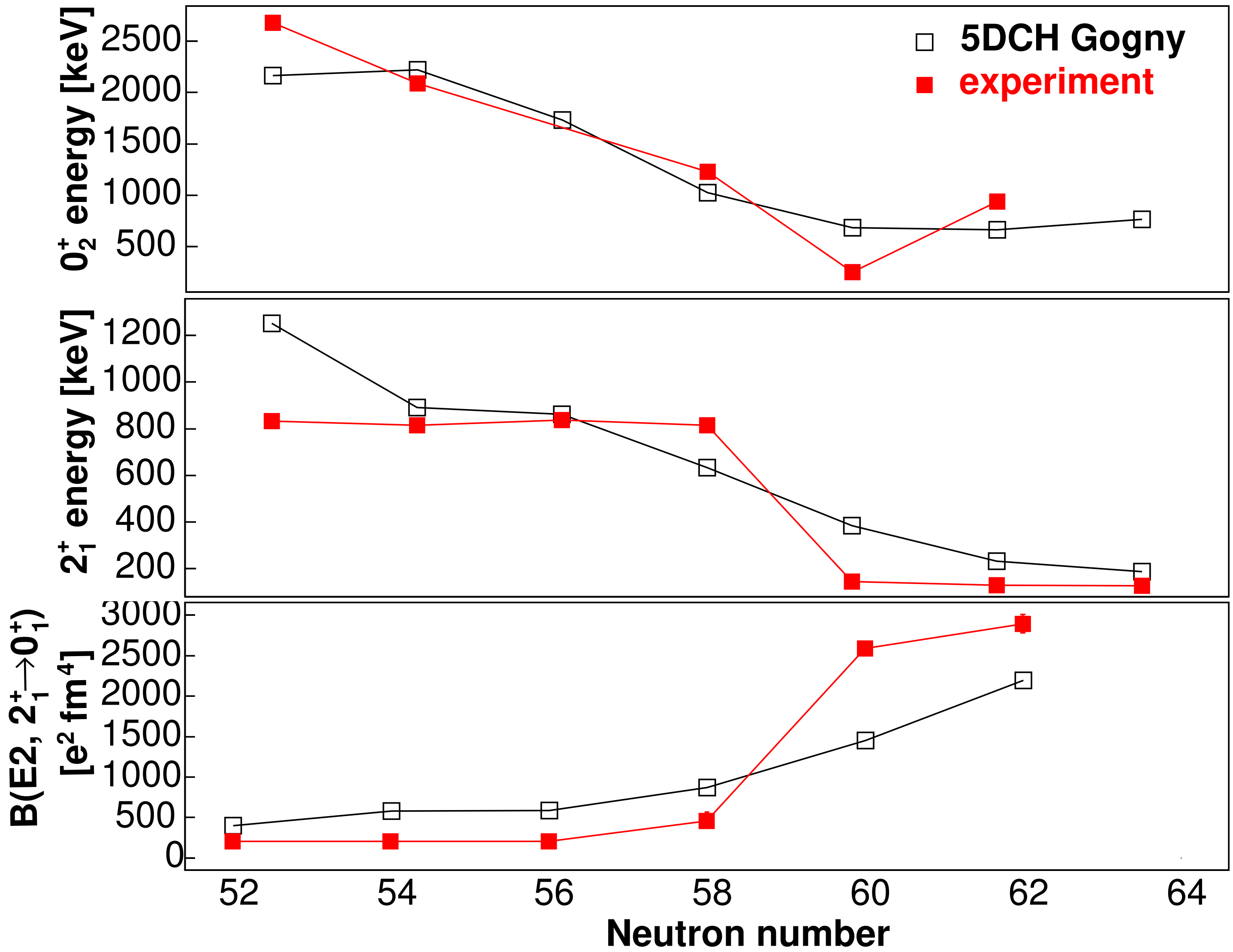
- collectivity in ground-state bands overestimated as well as mixing of the structures

Theoretical predictions for Sr isotopes



- collectivity in ground-state bands overestimated as well as mixing of the structures
- calculated K=2 band in ^{98}Sr has no experimental counterpart

Theoretical predictions for Sr isotopes



GCM(GOA) D1S vs experiment: smoother evolution of energies and transition probabilities than observed experimentally

E0 transition probabilities

^{96}Sr

I_1^π	I_2^π	$B(E2, I_1 \to I_2)$ (W.u.)		
		Eexperiment	5DCH (Gogny)	Excited VAMPIR
2_1^+	0_1^+	$17.3^{+4.0}_{-3.2}$	32	30
4_1^+	2_1^+		63	68
0_2^+	2_1^+	15.3(16) [10]	58	83
0_3^+	2_1^+	0.028(11) [11]		
2_2^+	2_1^+	>8.9 [10]	62	65
4_1^+	2_2^+		13	7
4_2^+	2_2^+		57	73
4_2^+	4_1^+		49	47
		$\rho^2(E0)$ $(\times 10^3)$		
0_2^+	0_1^+		106	66
0_3^+	0_1^+		22	
0_3^+	0_2^+	185(50) [13]	95	9

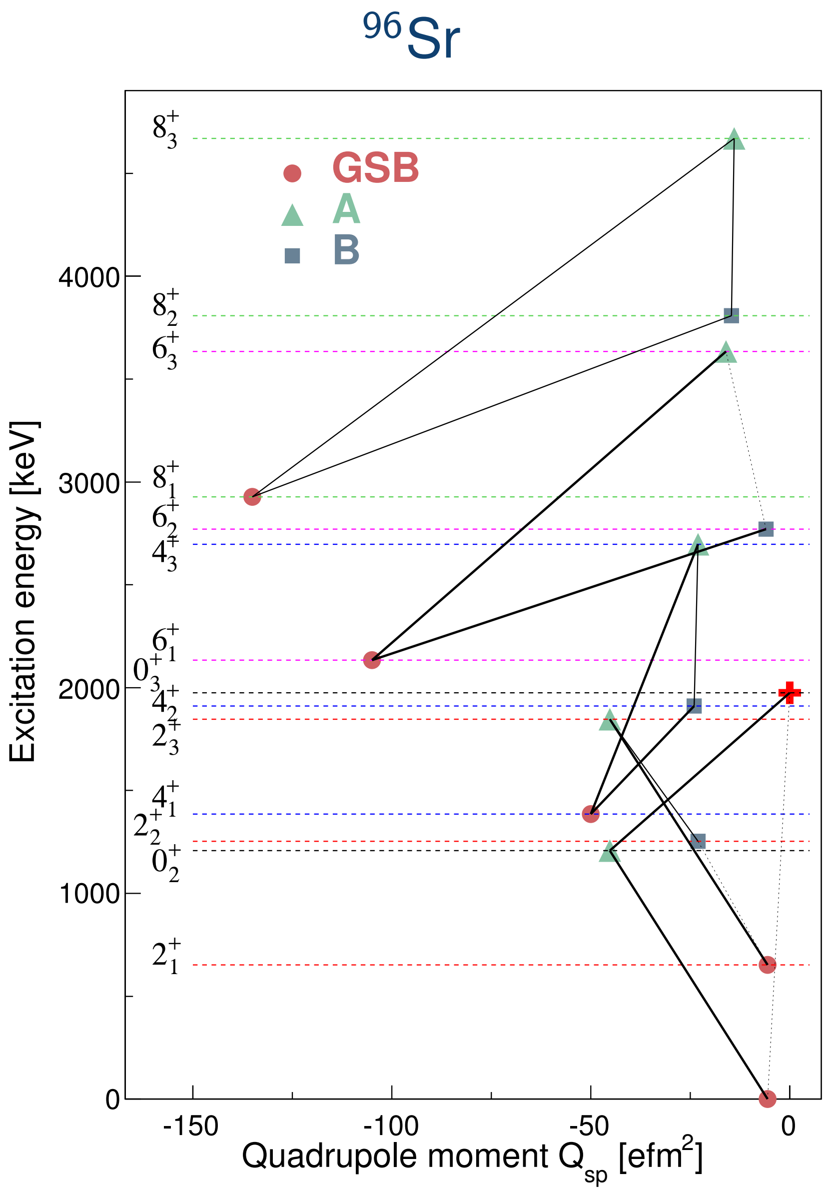
^{98}Sr

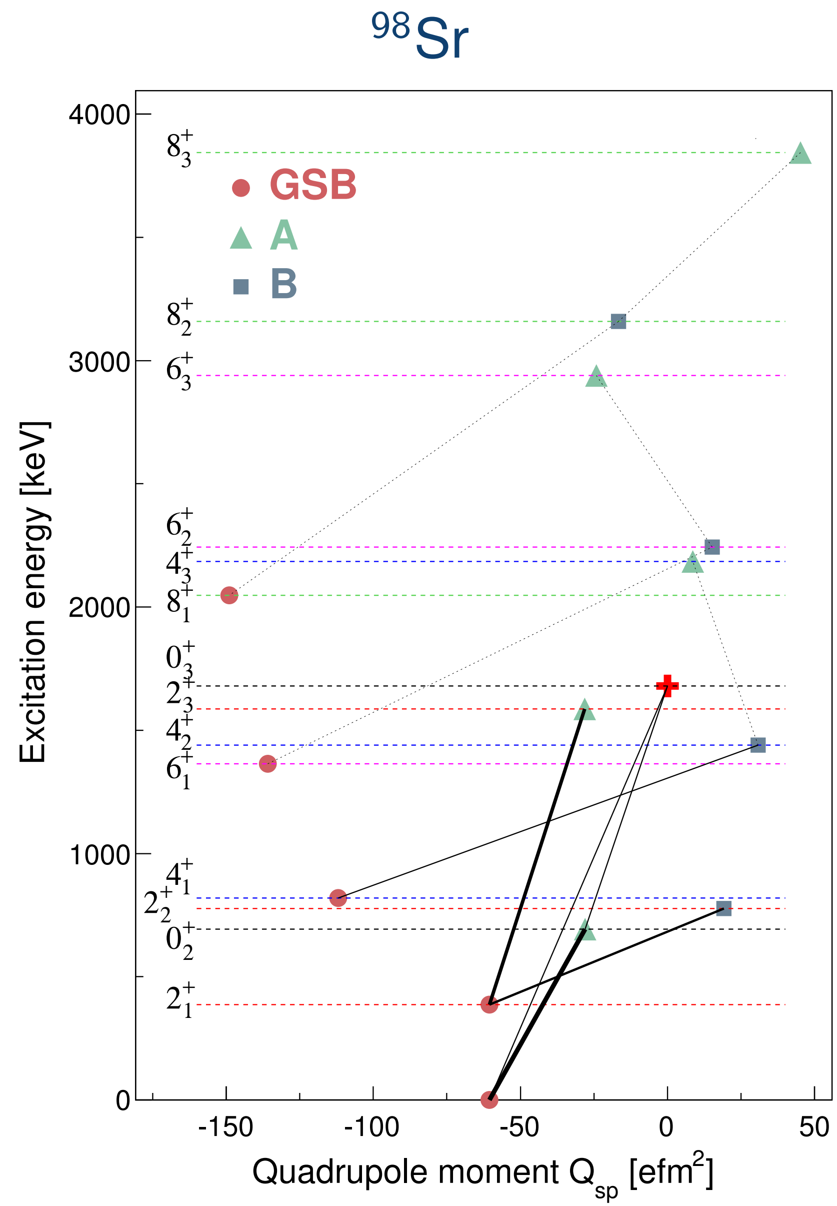
I_1^π	I_2^π	$B(E2, I_1 \to I_2)$ (W.u.)		
		Experiment	5DCH (Gogny)	5DCH (PC-PK1)
2_1^+	0_1^+	96 (3)	54	73.5
4_1^+	2_1^+	129^{+8}_{-7}	110	162
6_1^+	4_1^+	175^{+17}_{-14}	150	196
8_1^+	6_1^+	123^{+19}_{-14}	173	211
2_2^+	0_2^+	13 (2)	28	39.2
0_2^+	2_1^+	61 (5)	120	195[a]
2_2^+	0_1^+	0.77 (13)	0.07	
2_2^+	2_1^+	$0.61^{+0.22}_{-0.30}$	0.78	
2_2^+	4_1^+	4^{+4}_{-2}	19.4	
		$\rho^2(E0)$ $(\times 10^3)$		
0_2^+	0_1^+	53(5) [21]	179	117
0_3^+	0_1^+		40	
0_3^+	0_2^+		75	

- huge discrepancies and scarce experimental data

E. Clément, MZ et al, PRC 94 (2016) 054326

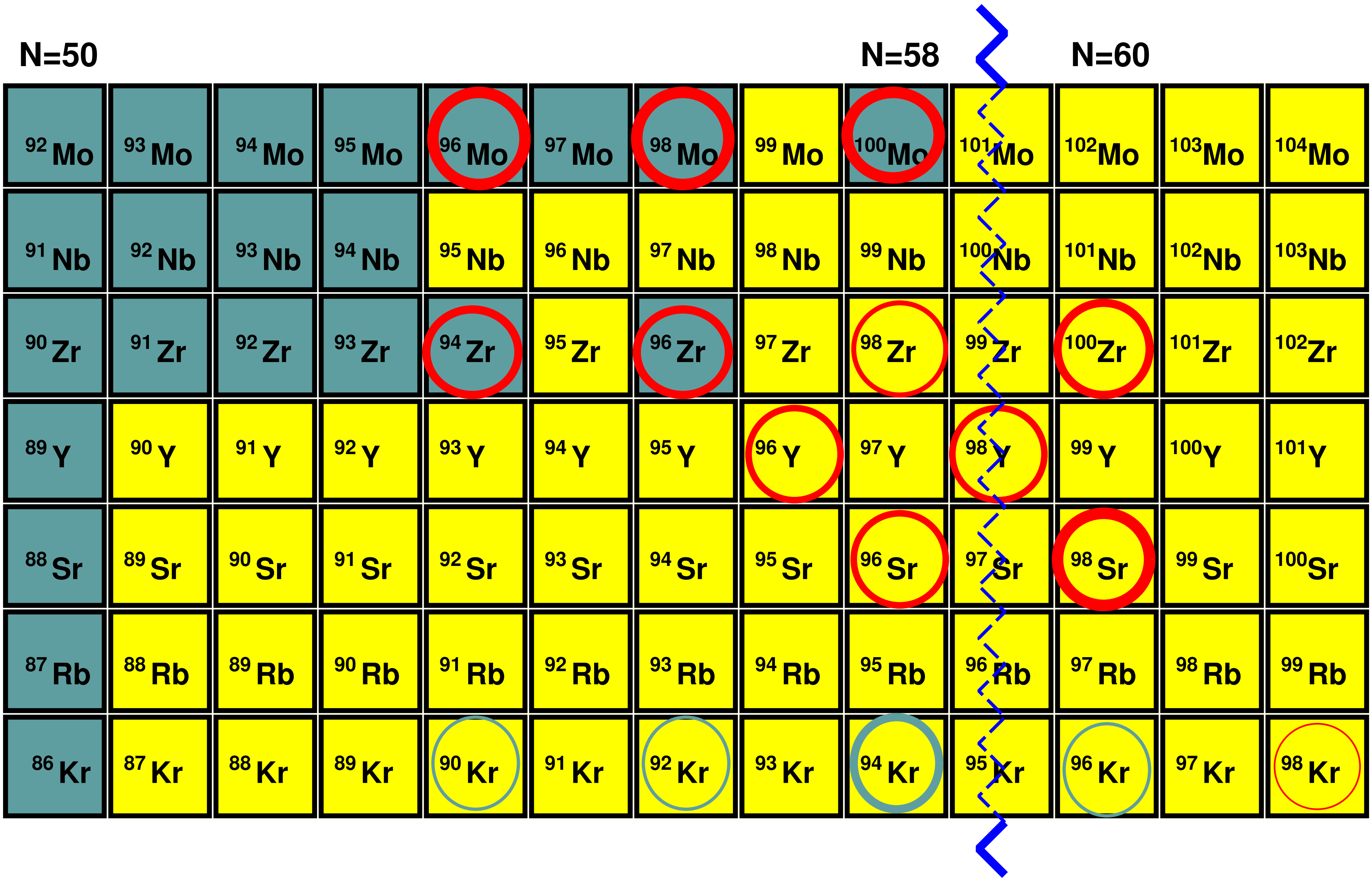
E0 transition probabilities: different predictions for ^{96}Sr and ^{98}Sr



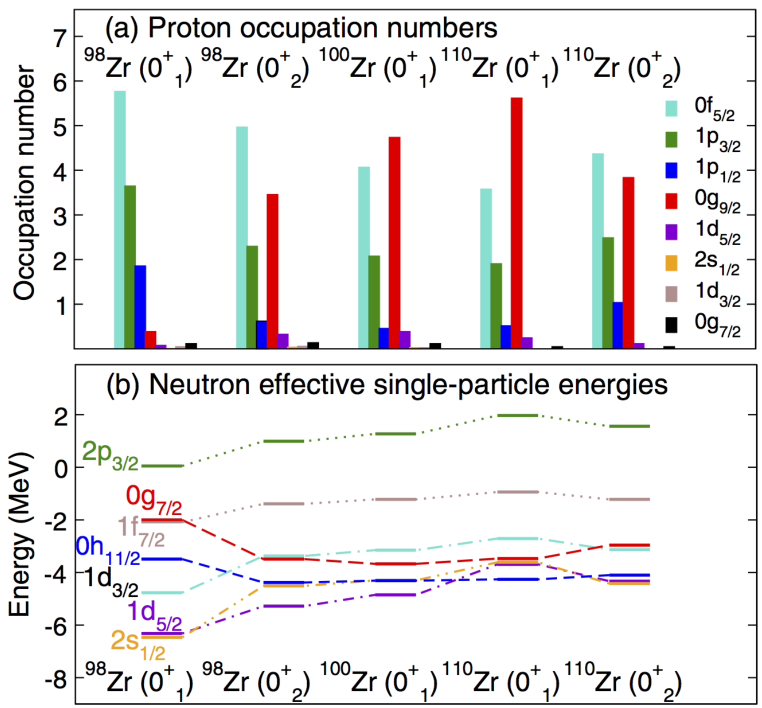


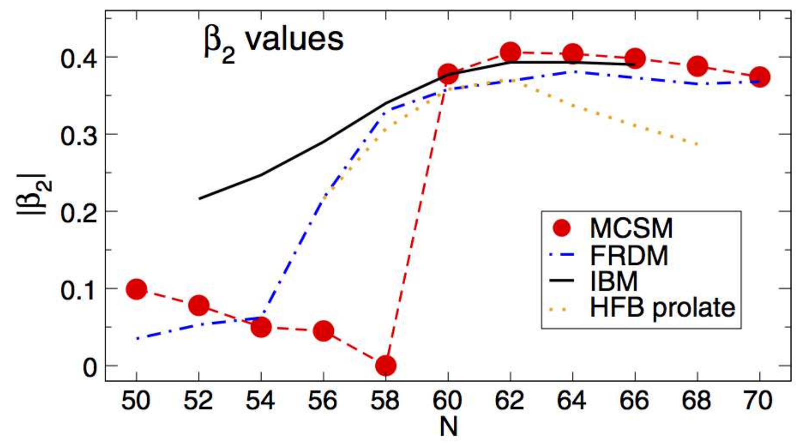
E. Clément, MZ et al, PRC 94 (2016) 054326

Shape coexistence around A $\sim$ 100: where we are?

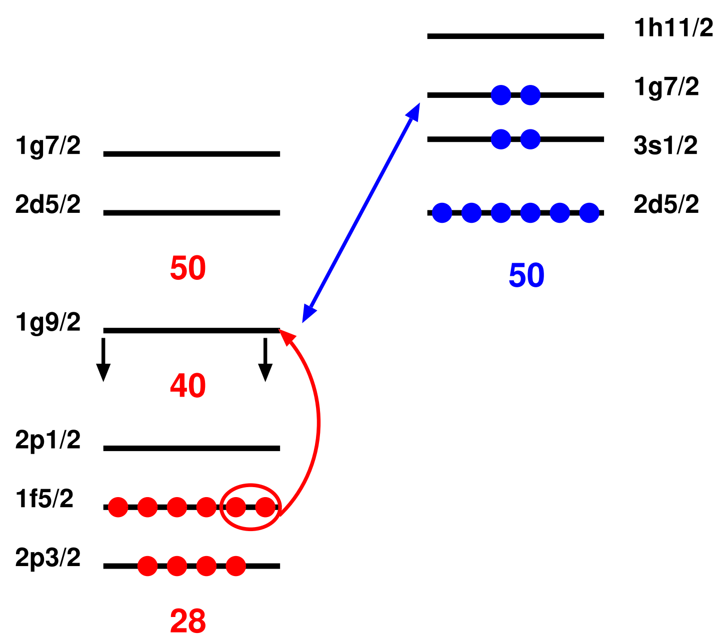


Shape coexistence and type-II shell evolution in Zr isotopes





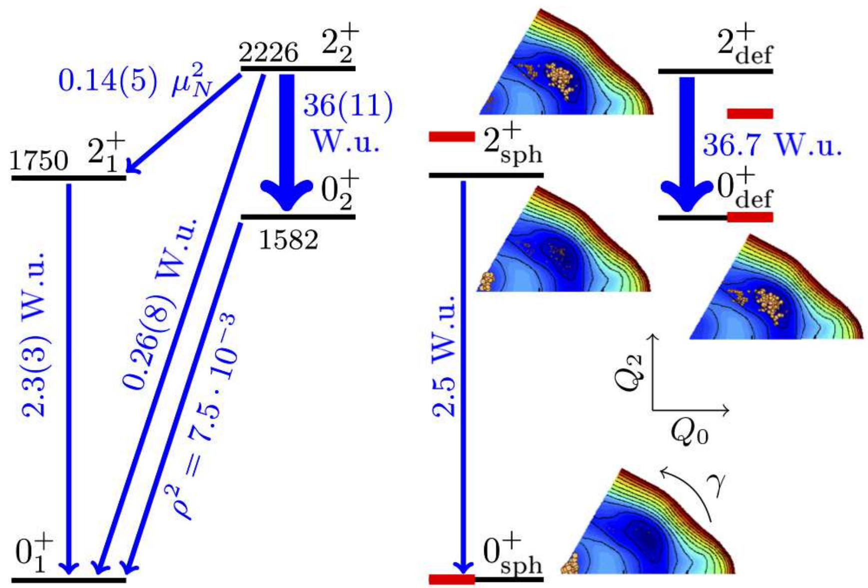
T. Togashi et al, PRL 117, 172502 (2016)



- p-n tensor interaction reduces the Z=40 gap when ν g7/2 is being filled
- 0_2^+ states created by 2p-2h (+ 4p-4h...) excitation across Z=40
- very different configurations and small mixing of 0_1^+ and 0_2^+

Shape coexistence in ^{96}Zr

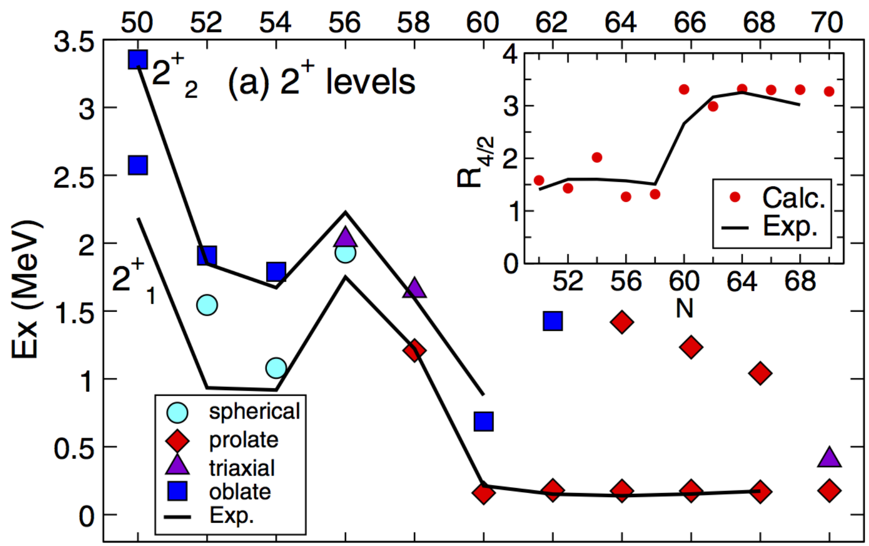
S. Kremer et al, PRL 117, (2017) 172503



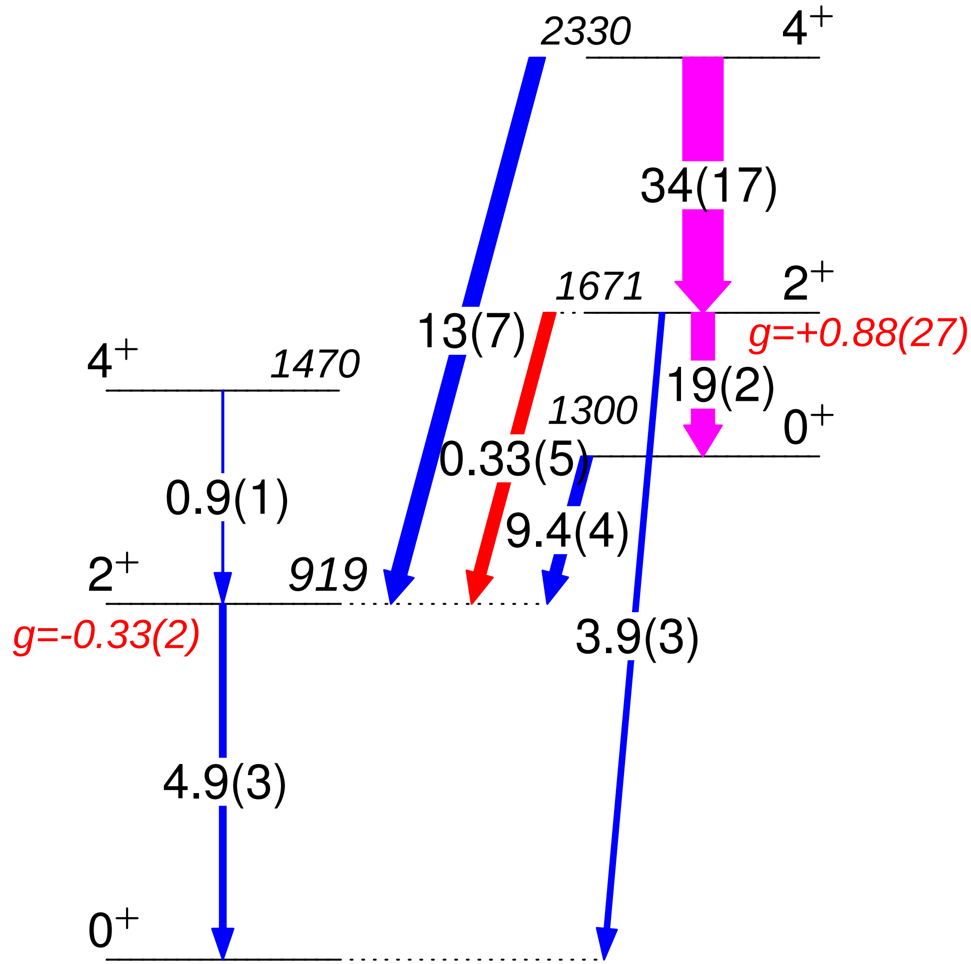
- measured B(E2; $2_2^+ \rightarrow 0_1^+$), combined with known branching and mixing ratios, yields transition strengths from the 2_2^+ state
- B(E2; $2_1^+ \rightarrow 0_1^+$) = 2.3(3) Wu vs B(E2; $2_2^+ \rightarrow 0_2^+$) = 36(11) Wu
  nearly spherical and a well-deformed structure ($\beta \approx 0.24$)
- very low mixing of coexisting structures: cos$^2\theta_0$=99.8%, cos$^2\theta_2$=97.5%,

Shape coexistence in ^{94}Zr

A. Chakraborty et al, PRL 110, 022504 (2013)



T. Togashi et al, PRL 117, 172502 (2016)



- observation of a strong $2^+_2 \rightarrow 0^+_2$ transition (19 W.u.) – deformed band built on 0^+_2
- shell model calculations suggest an oblate shape

Coulex experiment accepted at LNL

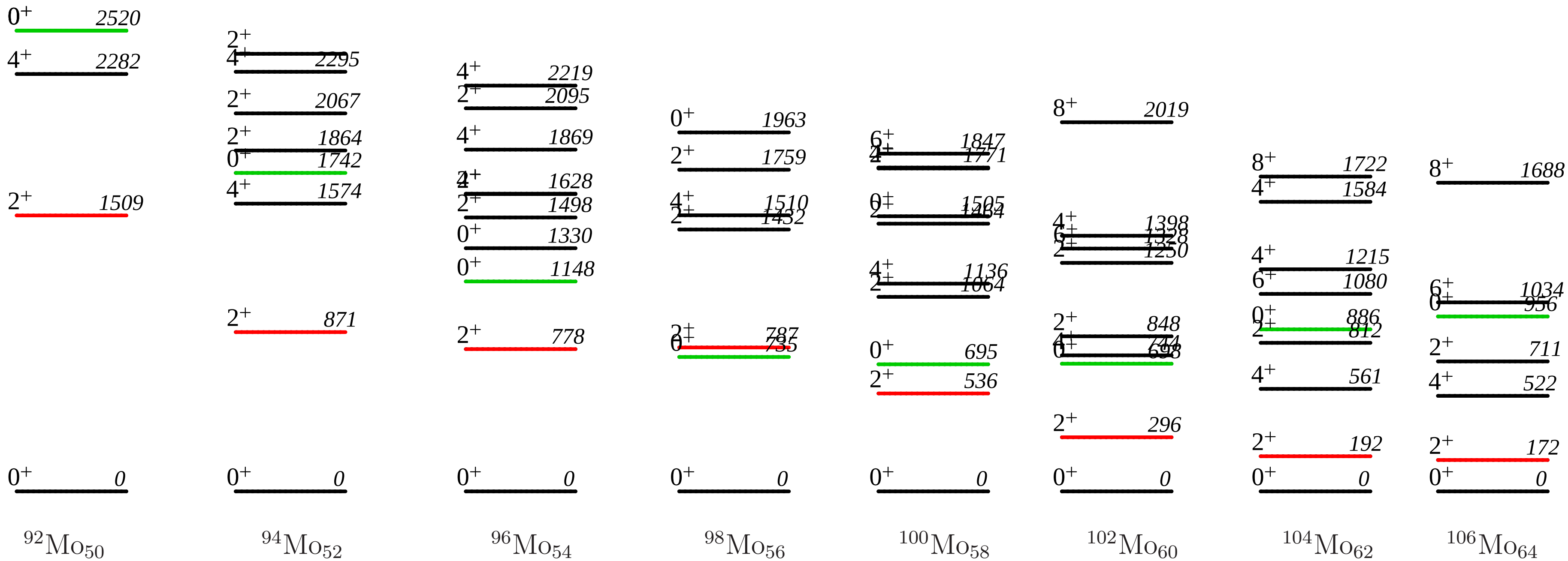
Coulomb excitation studies of $^{96-100}$Mo

MZ et al, Nucl. Phys. A 712 (2002)

K. Wrzosek-Lipska et al, PRC 86 (2012)

STABLE

EXOTIC



8.4 W.u. | 16 W.u. | 21 W.u. | 20 W.u. | 37 W.u. | 74 W.u. | 92 W.u. | 87 W.u.

JAEA Tokai, HIL Warsaw
^{96}Mo+^{208}Pb, ^{40}Ar,^{20}Ne+^{96}Mo

23 E2 ME's
(3 diagonal)

JAEA Tokai, HIL Warsaw
^{136}Xe,^{84}Kr, ^{40}Ar+^{98}Mo

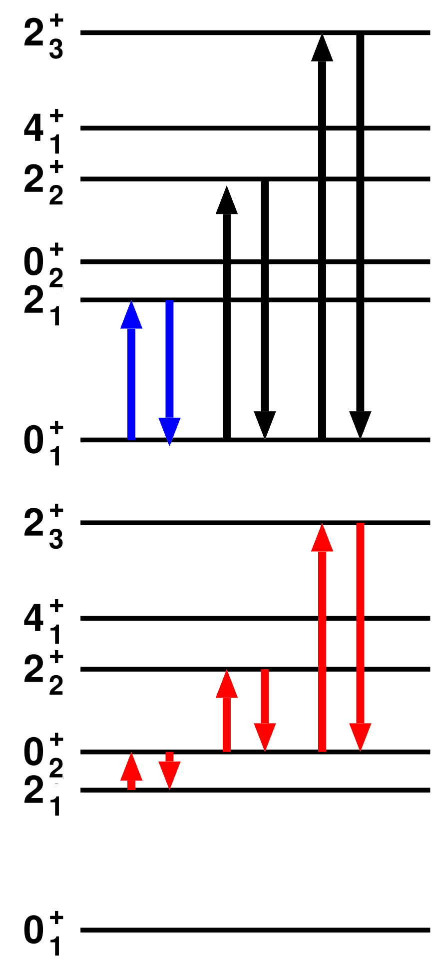
17 E2 ME's
(4 diagonal)

HIL Warsaw
^{40}Ar,^{32}S+^{100}Mo

20 E2 ME's
(4 diagonal)

Determination of $\langle Q^2 \rangle$: example of ^{100}Mo

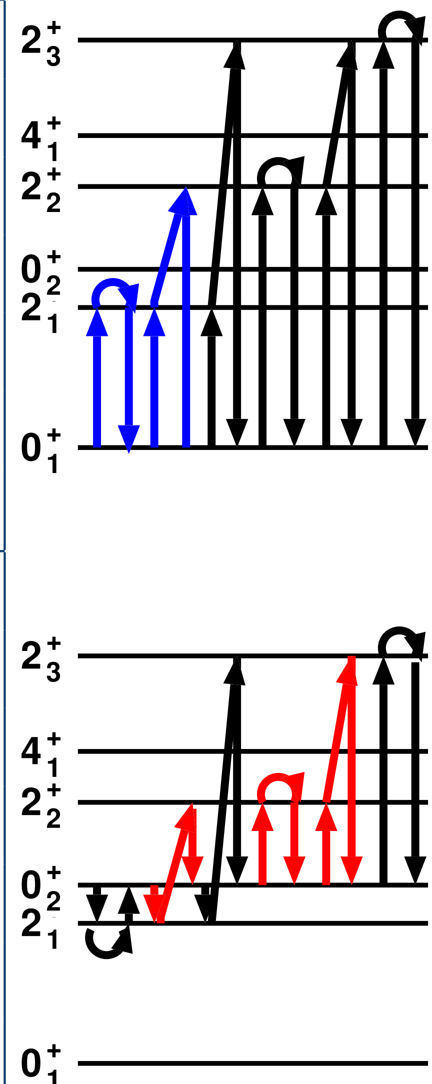
state	loop	contribution to $\langle Q^2 \rangle$ [e2b2]
0_1^+	$\langle 0_1^+ \Vert E2 \Vert 2_1^+ \rangle \langle 2_1^+ \Vert E2 \Vert 0_1^+ \rangle$	0.46
	$\langle 0_1^+ \Vert E2 \Vert 2_2^+ \rangle \langle 2_2^+ \Vert E2 \Vert 0_1^+ \rangle$	0.01
	$\langle 0_1^+ \Vert E2 \Vert 2_3^+ \rangle \langle 2_3^+ \Vert E2 \Vert 0_1^+ \rangle$	0.0002
	Total	0.48
0_2^+	$\langle 0_2^+ \Vert E2 \Vert 2_1^+ \rangle \langle 2_1^+ \Vert E2 \Vert 0_2^+ \rangle$	0.26
	$\langle 0_1^+ \Vert E2 \Vert 2_2^+ \rangle \langle 2_2^+ \Vert E2 \Vert 0_2^+ \rangle$	0.10
	$\langle 0_2^+ \Vert E2 \Vert 2_3^+ \rangle \langle 2_3^+ \Vert E2 \Vert 0_2^+ \rangle$	0.25
	Total	0.62



K. Wrzosek-Lipska et al, PRC 86 (2012)

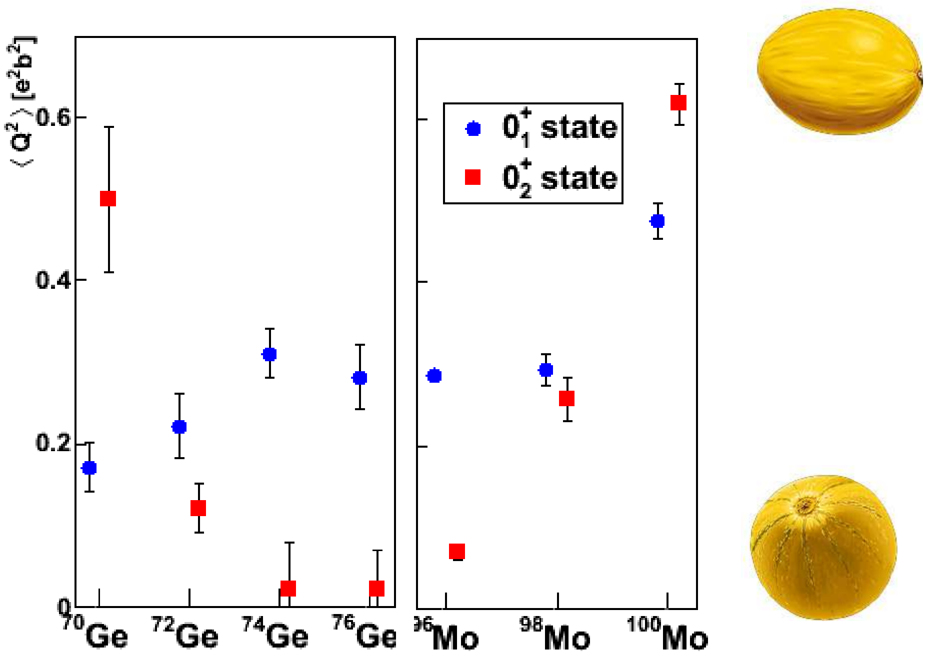
Determination of $\langle \cos 3\delta \rangle$: example of ^{100}Mo

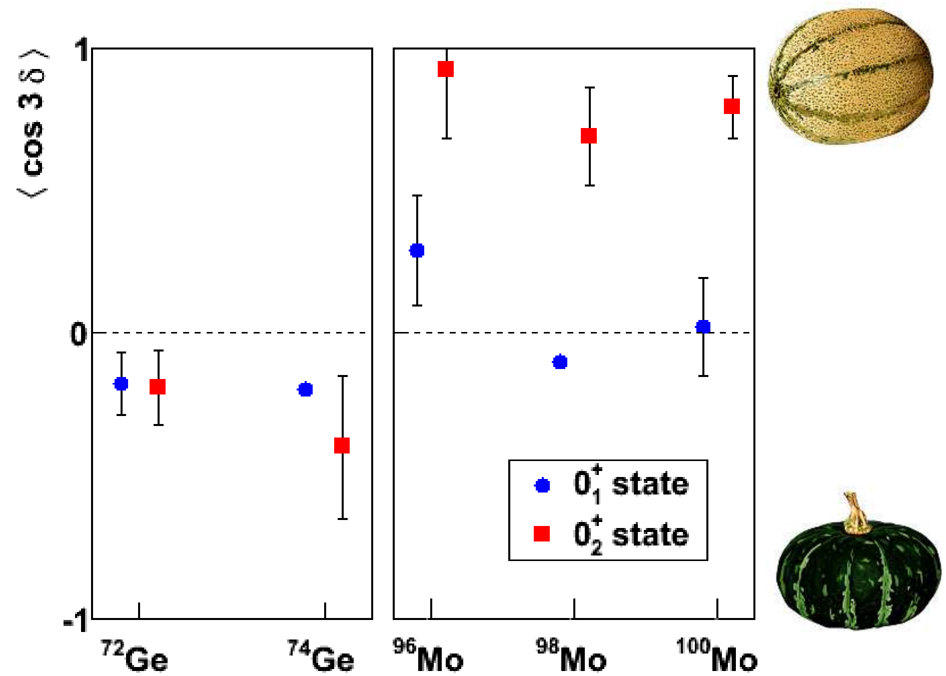
state	loop	contribution to $\langle Q^3 \cos 3\delta \rangle$
0_1^+	$\langle 0_1^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 0_1^+ \rangle$	-0.154
	$\langle 0_1^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 0_1^+ \rangle$	0.132
	$\langle 0_1^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 0_1^+ \rangle$	0.002
	$\langle 0_1^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 0_1^+ \rangle$	0.013
	$\langle 0_1^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 0_1^+ \rangle$	-0.001
	$\langle 0_1^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 0_1^+ \rangle$	-0.0001
	Total	-0.008
0_2^+	$\langle 0_2^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 0_2^+ \rangle$	-0.09
	$\langle 0_2^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 0_2^+ \rangle$	-0.31
	$\langle 0_2^+ \|\| E2 \|\| 2_1^+ \rangle \langle 2_1^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 0_2^+ \rangle$	-0.04
	$\langle 0_2^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 0_2^+ \rangle$	0.12
	$\langle 0_2^+ \|\| E2 \|\| 2_2^+ \rangle \langle 2_2^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 0_2^+ \rangle$	-0.13
	$\langle 0_2^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 2_3^+ \rangle \langle 2_3^+ \|\| E2 \|\| 0_2^+ \rangle$	-0.06
	Total	-0.51



Shape evolution of $^{96-100}$Mo

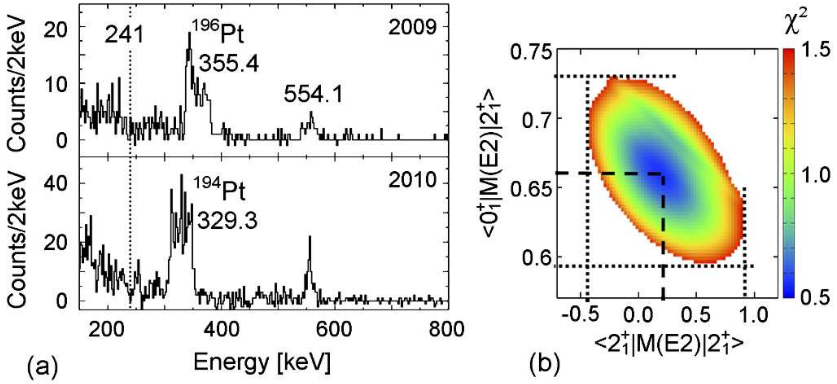
M. Zielińska et al, Nucl. Phys. A 712 (2002)
K. Wrzosek-Lipska et al, PRC 86 (2012)

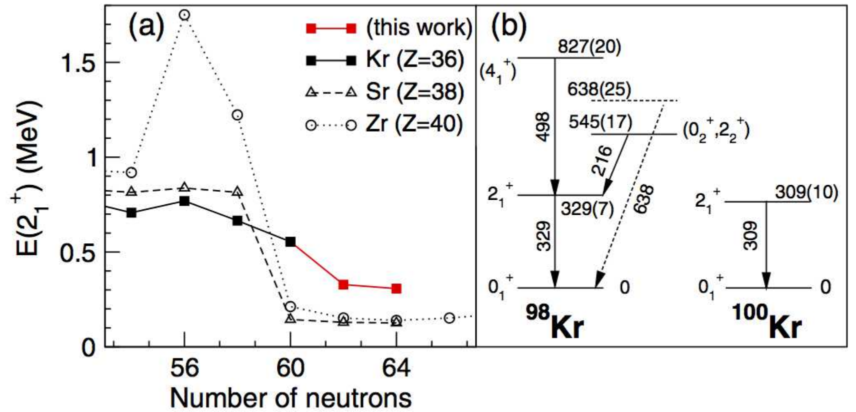




- Ge isotopes, ^{96}Mo: deformed ground states coexist with spherical 0_2^+
- $\langle \cos 3\delta \rangle$ for ground states of Mo isotopes corresponds to maximum triaxiality (probably γ softness); deformation of 0_2^+ increasing with N
- shape coexistence in ^{98}Mo manifested in a different average triaxiality of 0_1^+ and 0_2^+

Kr isotopes around N=60: where we are?





M. Albers *et al.* Phys. Rev. Lett. 108, 062701 (2012)

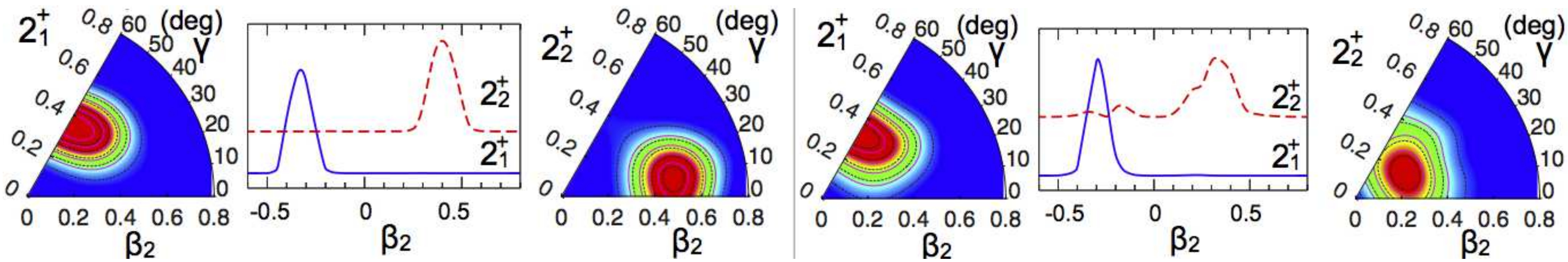
F. Flavigny *et al.* Phys. Rev. Lett. 118 (2017) 242501

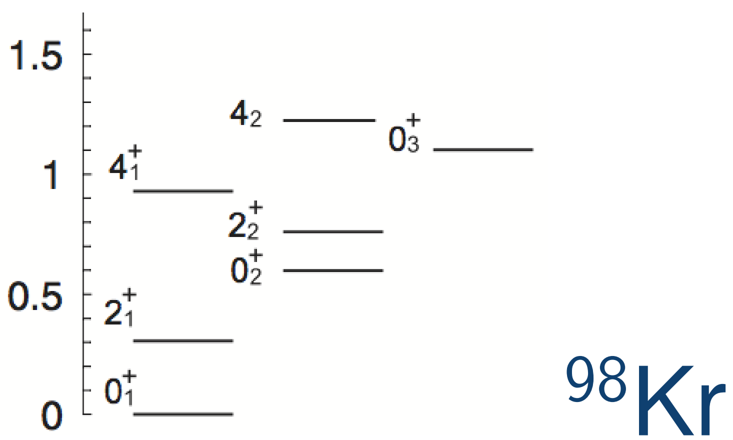
- Coulex of ^{96}Kr at REX-ISOLDE (2010): $7 \cdot 10^3$ pps
  - statistics not really sufficient to determine $Q_s(2_1^+)$

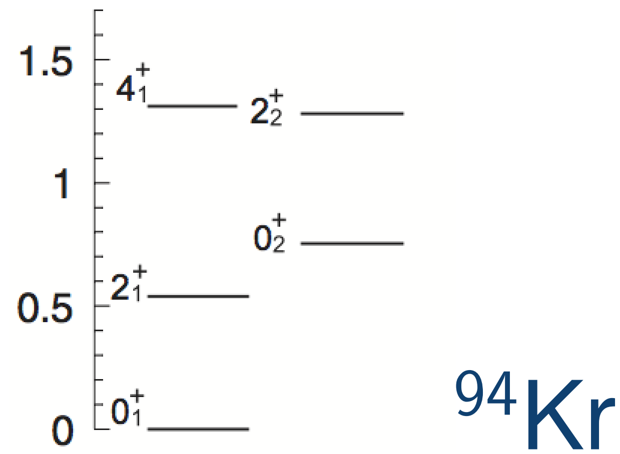
- at 10^5 pps population of non-yrast states via Coulex likely
- intensity expected at new generation ISOL facilities
  - LoI's for SPES: V. Modamio et al ($^{94-98}$Kr), K. Hadyńska-Klęk et al (^{90,92}Kr)

Shape coexistence in $^{94-98}$Kr: SPES LoI (V. Modamio et al)

T. R. Rodriguez *et al.* Phys. Rev. C 90, 034306 (2014)



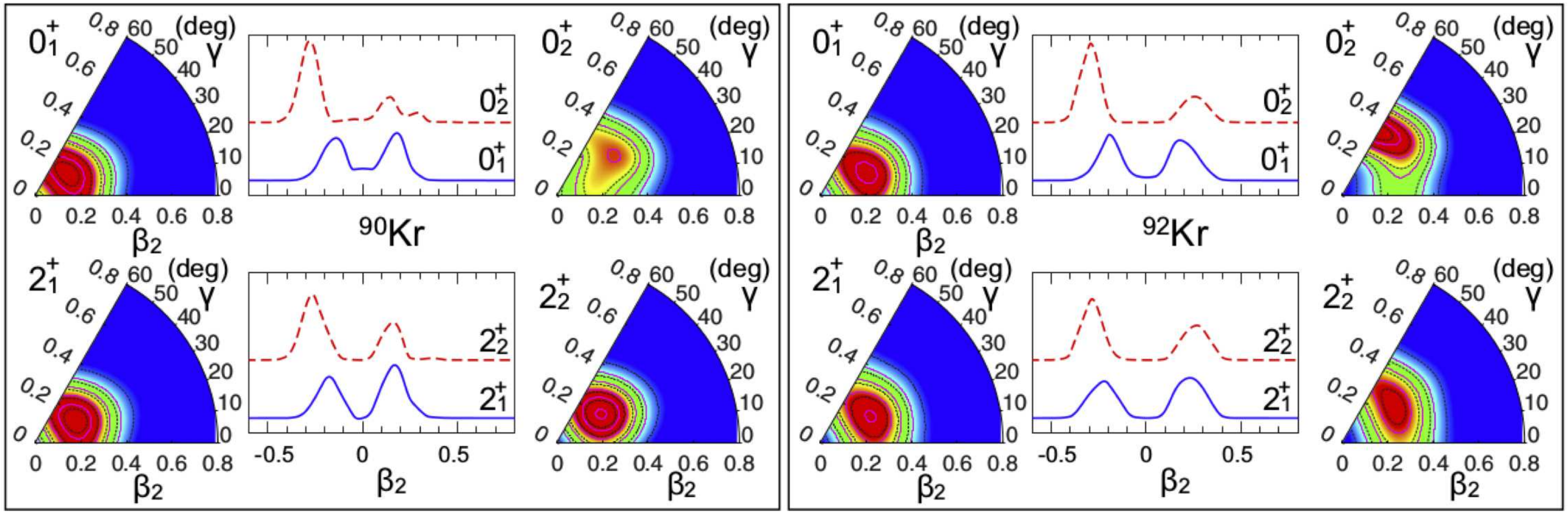




- coexistence of prolate-oblate (^{98}Kr) or oblate-triaxial shapes (^{94}Kr)
- 0_2^+ states predicted below 1 MeV – accessible in Coulex
- smooth evolution of measured 2_1^+ energies suggests mixing of $2_{1,2}^+$ states
- measurement of 0_2^+ decay will already give an estimate of mixing

Shapes of $^{90-92}$Kr: SPES LoI (K. Hadyńska-Klęk et al)

T. R. Rodriguez *et al.* Phys. Rev. C 90, 034306 (2014)

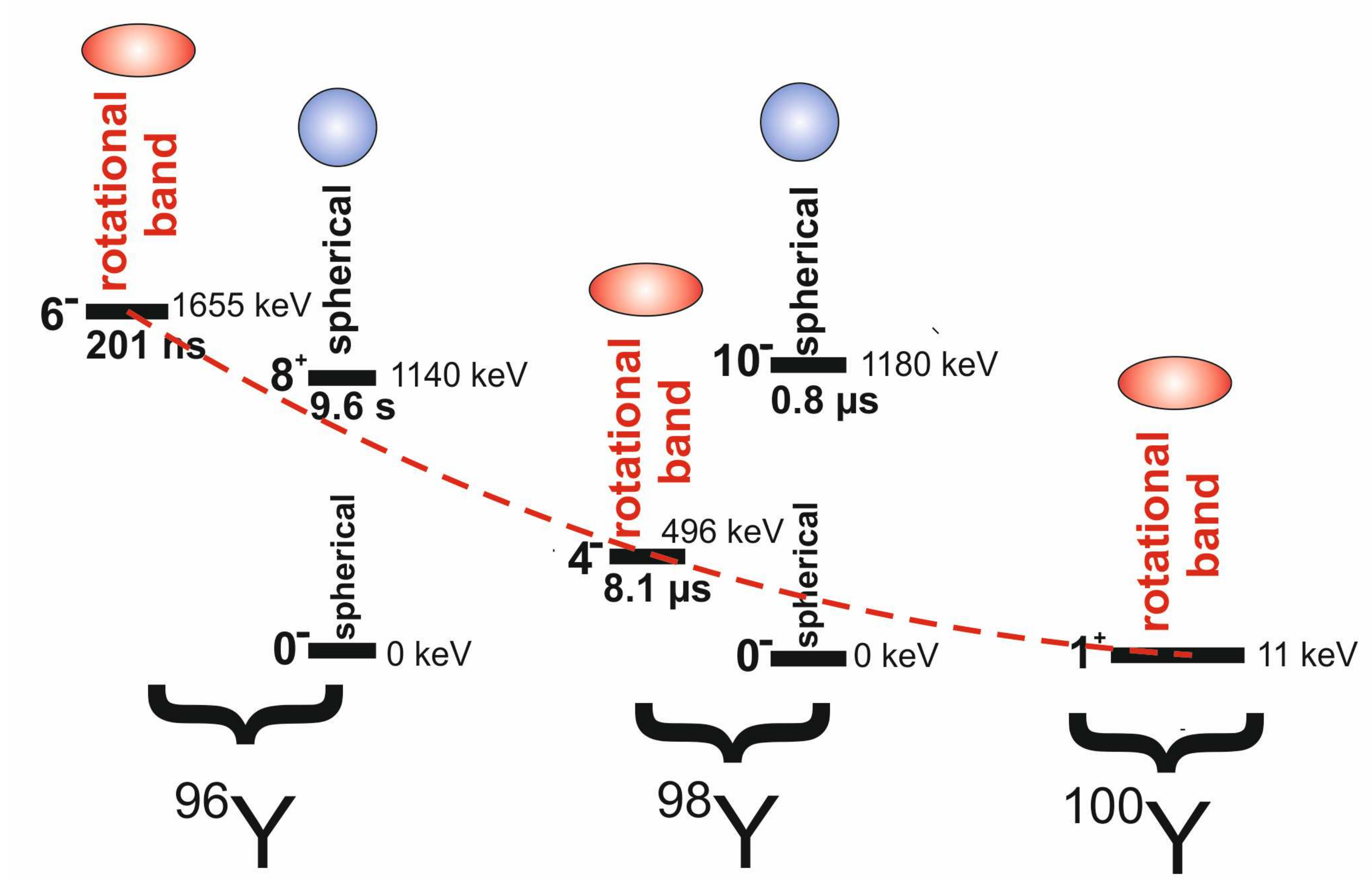


- first step: high-precision study of less exotic Kr isotopes
  - identification of predicted coexisting structures in $^{90-92}$Kr and precise measurement of their deformation
  - determination of their mixing via measurement of intra-band transition probabilities
  - study of the role of triaxiality

Outlook and open questions

- our understanding of the region has improved thanks to detailed spectroscopic studies and advances in theory
- detailed high-precision studies of stable nuclei ot those close to stability are important
- examples of missing pieces of the puzzle:
  - quadrupole moments of ground states in N=59 nuclei
  - shape coexistence and mixing in the Kr chain
  - quadrupole moments of excited states in the Zr chain
  - role of triaxiality in Sr isotopes

Example for odd-odd nuclei: rotational bands in Y isotopes



L. Iskra et al, EPL 117, 12001 (2017)

Quadrupole sum rules

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683

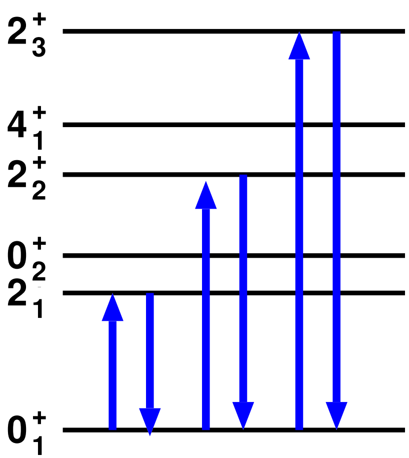
K. Kumar, PRL 28 (1972) 249

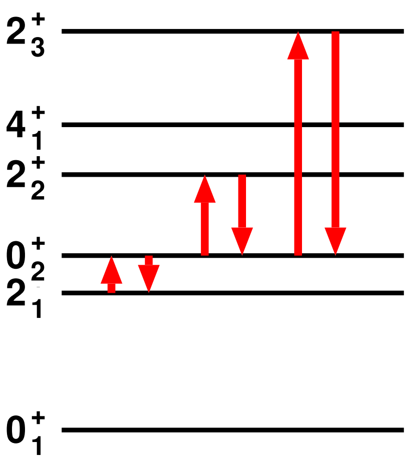
• electromagnetic multipole operators are spherical tensors – products of such operators coupled to angular momentum 0 are rotationally invariant

• in the intrinsic frame of the nucleus, the E2 operator may be expressed by 2 parameters related to charge distribution:

$$
\begin{aligned}
E(2,0) &= Q\cos\delta \\
E(2,2) = E(2,-2) &= \frac{Q}{\sqrt{2}}\sin\delta \\
E(2,1) = E(2,-1) &= 0
\end{aligned}
$$

$$
\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i|\,[\mathrm{E2}\times\mathrm{E2}]^0\,|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i\|\mathrm{E2}\|t\rangle\langle t\|\mathrm{E2}\|i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}
$$





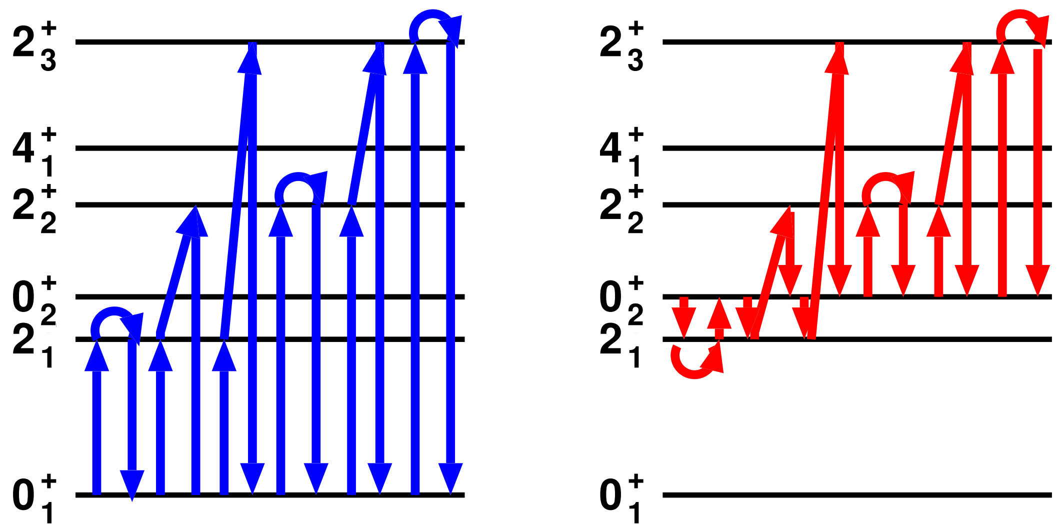
$\langle Q^2\rangle$: overall deformation parameter

Quadrupole sum rules: triaxiality

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986)
K. Kumar, PRL 28 (1972)

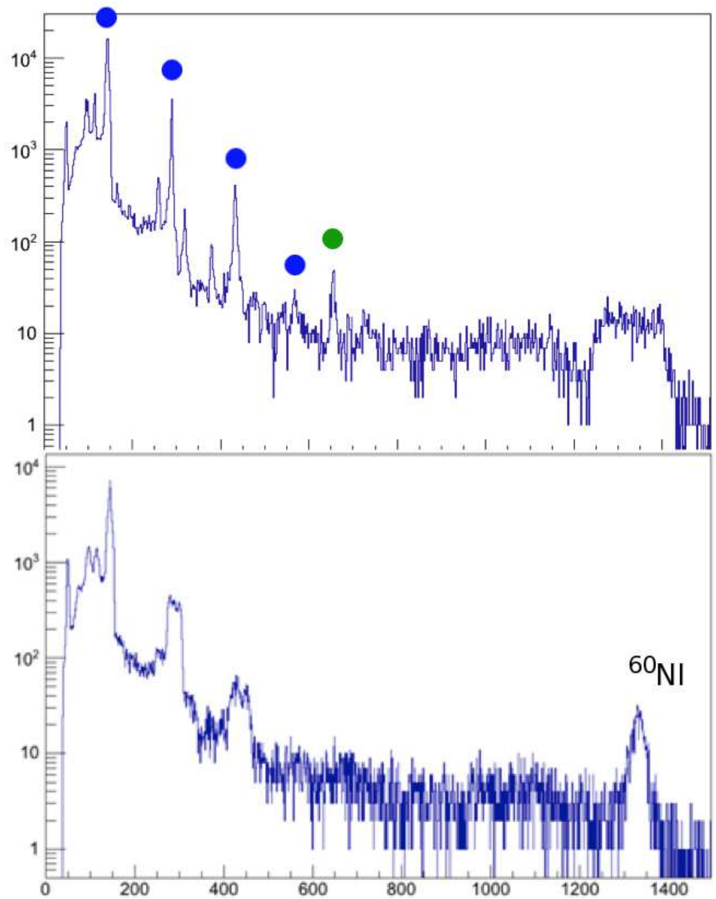
$$\sqrt{\frac{2}{35}}\langle \mathrm{Q}^3 \cos 3\delta \rangle = \langle i | \{[\mathrm{E2} \times \mathrm{E2}]^2 \times \mathrm{E2}\}^0 | i \rangle$$

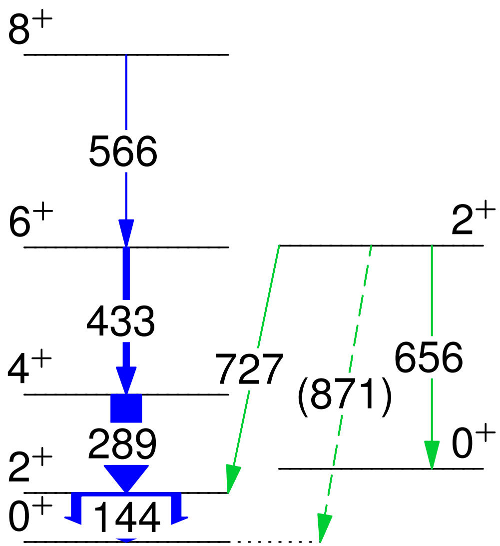
$$= \frac{1}{(2I_i+1)} \sum_{t,u} \langle i \| \mathrm{E2} \| u \rangle \langle u \| \mathrm{E2} \| t \rangle \langle t \| \mathrm{E2} \| i \rangle \left\{ \begin{array}{ccc} 2 & 2 & 2 \\ I_i & I_t & I_u \end{array} \right\}$$

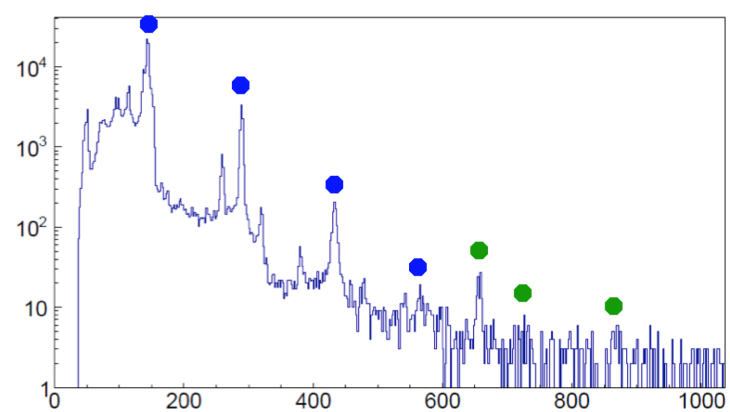


$\langle \cos 3\delta \rangle$: triaxiality parameter

Coulomb excitation of ^{98}Sr







- 2 targets differing in Z: ^{60}Ni and ^{208}Pb
- gsb populated up to 8^+
- good statistics: 4 subdivisions of CM angles for ^{208}Pb, 3 for ^{60}Ni