Correlations between charge radii, E0 transitions and M1 strength

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Shape coexistence and E0, ESNT, Saclay, October 2017

Correlations between charge radii, E0 transitions and M1 strength

The E0 operator

Charge radii and E0 transitions

Application in the rare-earth nuclei

Correlations between

- Charge radii and E0 transitions (Wood et al.)
- Charge radii and summed M1 strength (Heyde et al.)
- E0 transitions and summed M1 strength

S. Zerguine *et al.*, Phys. Rev. Lett. **101** (2008) 022502
S. Zerguine *et al.*, Phys. Rev. C **85** (2012) 034331

Electric monopole (E0) transitions

The probability for an E0 transition to occur is given by $P=\Omega\rho^2$ with Ω and ρ^2 electronic and nuclear factors.

The nuclear factor is the matrix element

$$\rho = \sum_{k\in\text{protons}}^{Z} \langle \text{f} | \left(\frac{r_k}{R}\right)^2 - \sigma \left(\frac{r_k}{R}\right)^4 + \cdots | \text{i} \rangle \quad \left(R = r_0 A^{1/3}, r_0 = 1.2 \text{ fm}\right)$$

Higher-order terms are usually not considered, σ=0, and hence contact is made with the charge radius.

E.L. Church & J. Weneser, Phys. Rev. **103** (1956) 1035

Coexistence or collective?

Origin of E0 transitions in nuclei:

Mixing of coexisting configurations with different shapes (Heyde & Wood);

Between β-vibrational states in the geometric collective model (Reiner).

In a geometric framework E0 strength should rise in the transition from spherical to deformed $\Rightarrow$ Link with phase transitions in nuclei (von Brentano *et al.*).

Hypothesis: collective EO

Charge-radius and E0 operators

Definition of a "charge radius operator":

$$\langle \mathrm{s}|\hat{T}(r^2)|\mathrm{s}\rangle \equiv \langle r^2 \rangle_{\mathrm{s}} = \frac{1}{Z}\sum_{k\in\mathrm{protons}}^{Z} \langle \mathrm{s}|r_k^2|\mathrm{s}\rangle \Rightarrow \hat{T}(r^2) = \frac{1}{Z}\sum_{k\in\mathrm{protons}}^{Z} r_k^2$$

Definition of an "E0 transition operator" (for σ=0):

$$\rho \equiv \frac{\langle \mathrm{f}|\hat{T}(\mathrm{E0})|\mathrm{i}\rangle}{eR^2} \Rightarrow \hat{T}(\mathrm{E0}) = e\sum_{k\in\mathrm{protons}}^{Z} r_k^2$$

Hence we find the following (standard) relation:

$$\hat{T}(\mathrm{E0}) = eZ\,\hat{T}(r^2)$$

Effective charges

Addition of neutrons produces a change in the charge radius $\Rightarrow$ need for effective charges.

Generalized operators:

$$\langle r^2 \rangle_{\mathrm{s}} = \frac{1}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z} \sum_{k=1}^{A} \langle \mathrm{s} | e_k r_k^2 | \mathrm{s} \rangle \Rightarrow \hat{T}(r^2) = \frac{1}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z} \sum_{\mathrm{k}=1}^{A} e_k r_k^2$$

$$\hat{T}(\mathrm{E0}) = \sum_{k=1}^{A} e_k r_k^2$$

Generalized (non-standard) relation:

$$\hat{T}(\mathrm{E0}) = (e_{\mathrm{n}}N + e_{\mathrm{p}}Z)\hat{T}(r^2)$$

E0 transitions in nuclear models

Nuclear shell model: E0 transitions between states in a single oscillator shell vanish.

Geometric collective model: Strong E0 transitions occur between β- and ground-state band.

Interacting boson model: Can be used to test the relation between charge radii and E0 transitions.

Operators in the IBM

The charge radius operator:

$$\hat{T}\left(r^2\right) = \left\langle r^2 \right\rangle_{\text{core}} + \alpha N_{\text{b}} + \frac{\eta}{N_{\text{b}}} \hat{n}_d$$

The E0 operator:

$$\hat{T}(\text{E0}) = \left(e_{\text{n}} N + e_{\text{p}} Z\right) \frac{\eta}{N_{\text{b}}} \hat{n}_d$$

The M1 operator:

$$\hat{T}(\text{M1}) = \sqrt{\frac{3}{4\pi}} \left(g_\nu \hat{L}_\nu + g_\pi \hat{L}_\pi\right)$$

F. Iachello and A. Arima, The Interacting Boson Model

Application to rare-earth nuclei

Application to even-even nuclei with Z=58-74.

Procedure:

- *Determine IBM hamiltonian from spectra with special care to the spherical-to-deformed transitional region.*
- *Determine coefficients α and η in $T(r^2)$ from isotope and isomer shifts.*
- *Calculate $\rho(E0)$ values.*

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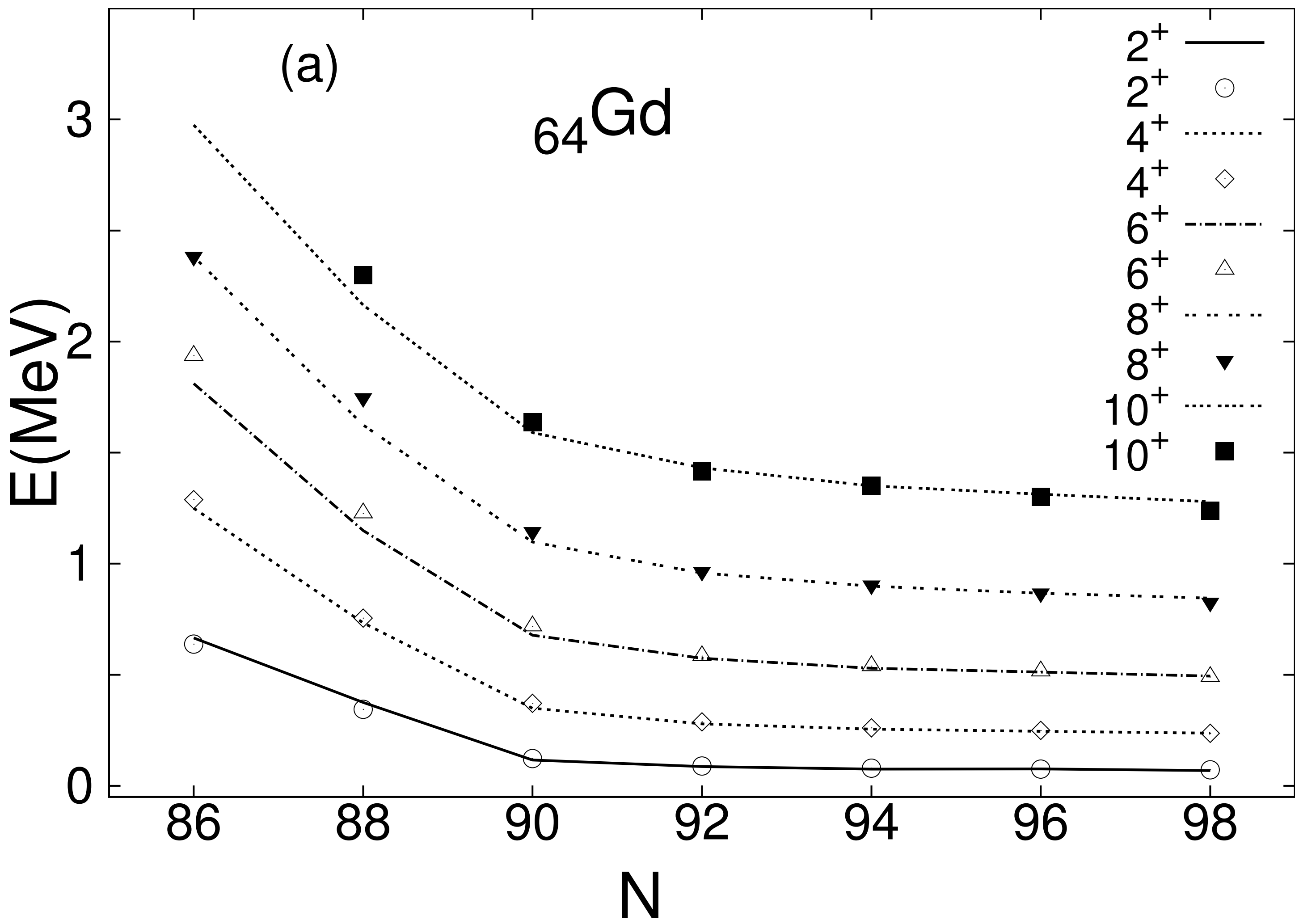
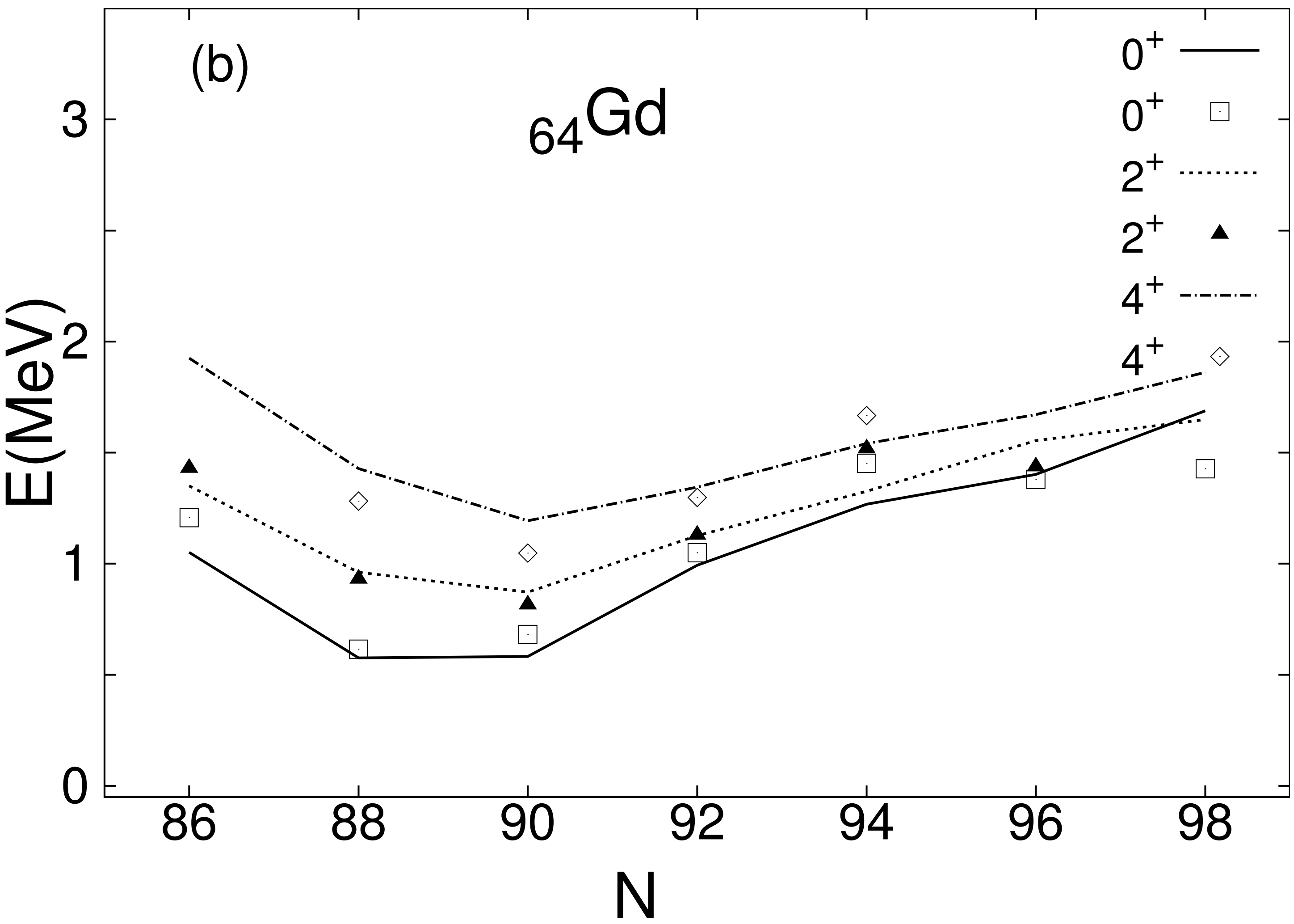
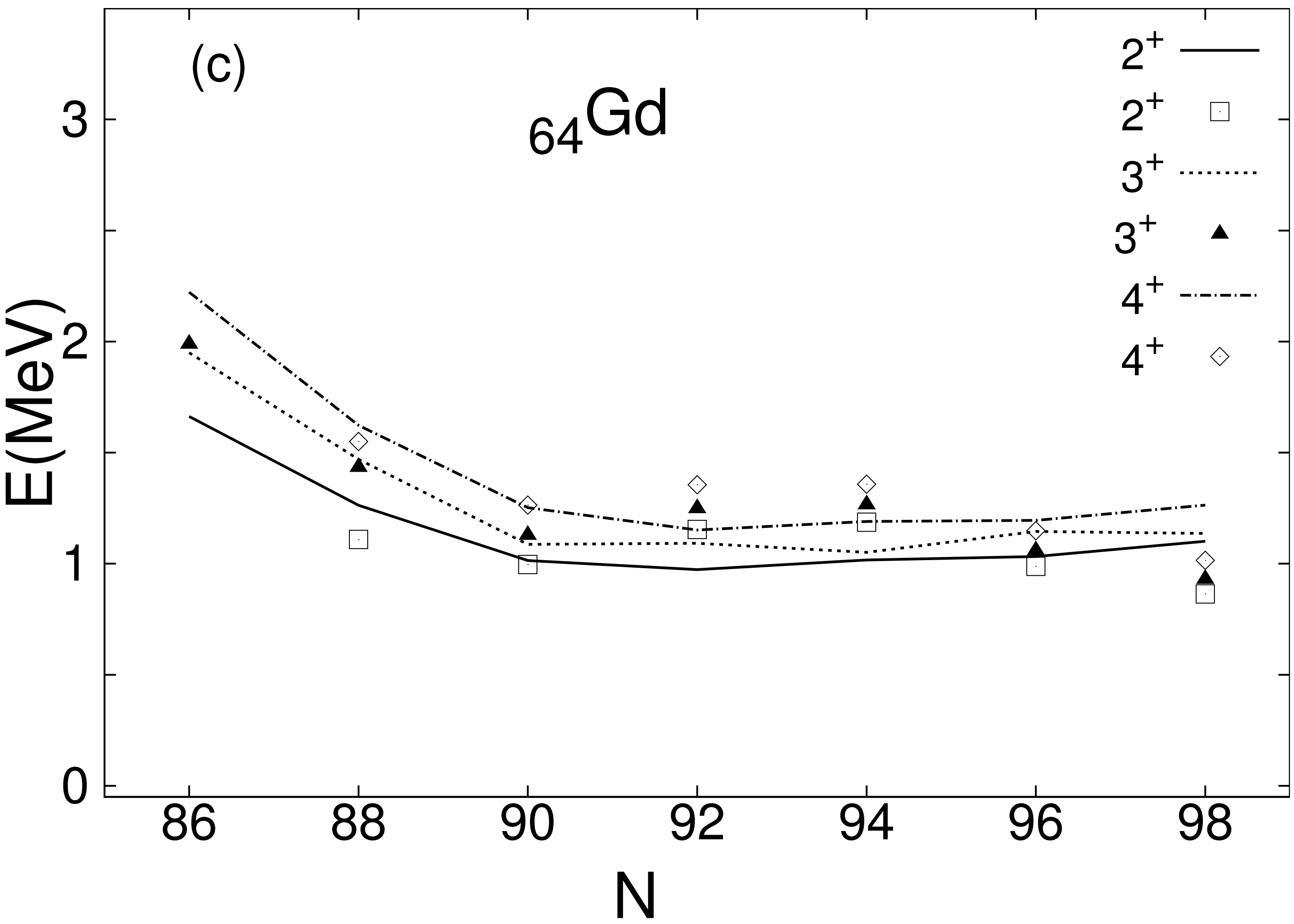
Example: gadolinium isotopes



Example: gadolinium isotopes



Example: gadolinium isotopes



Isotope shifts

Isotopes shifts depend on the coefficients α and η:

$$\Delta\langle r^2\rangle \equiv \langle r^2\rangle^{(A+2)}_{0_1^+} - \langle r^2\rangle^{(A)}_{0_1^+} = |\alpha| + \frac{\eta}{N_\mathrm{b}}\left(\langle \hat{n}_d\rangle^{(A+2)}_{0_1^+} - \langle \hat{n}_d\rangle^{(A)}_{0_1^+}\right)$$

Estimate of parameters

Average increase of the charge radius with particle number:

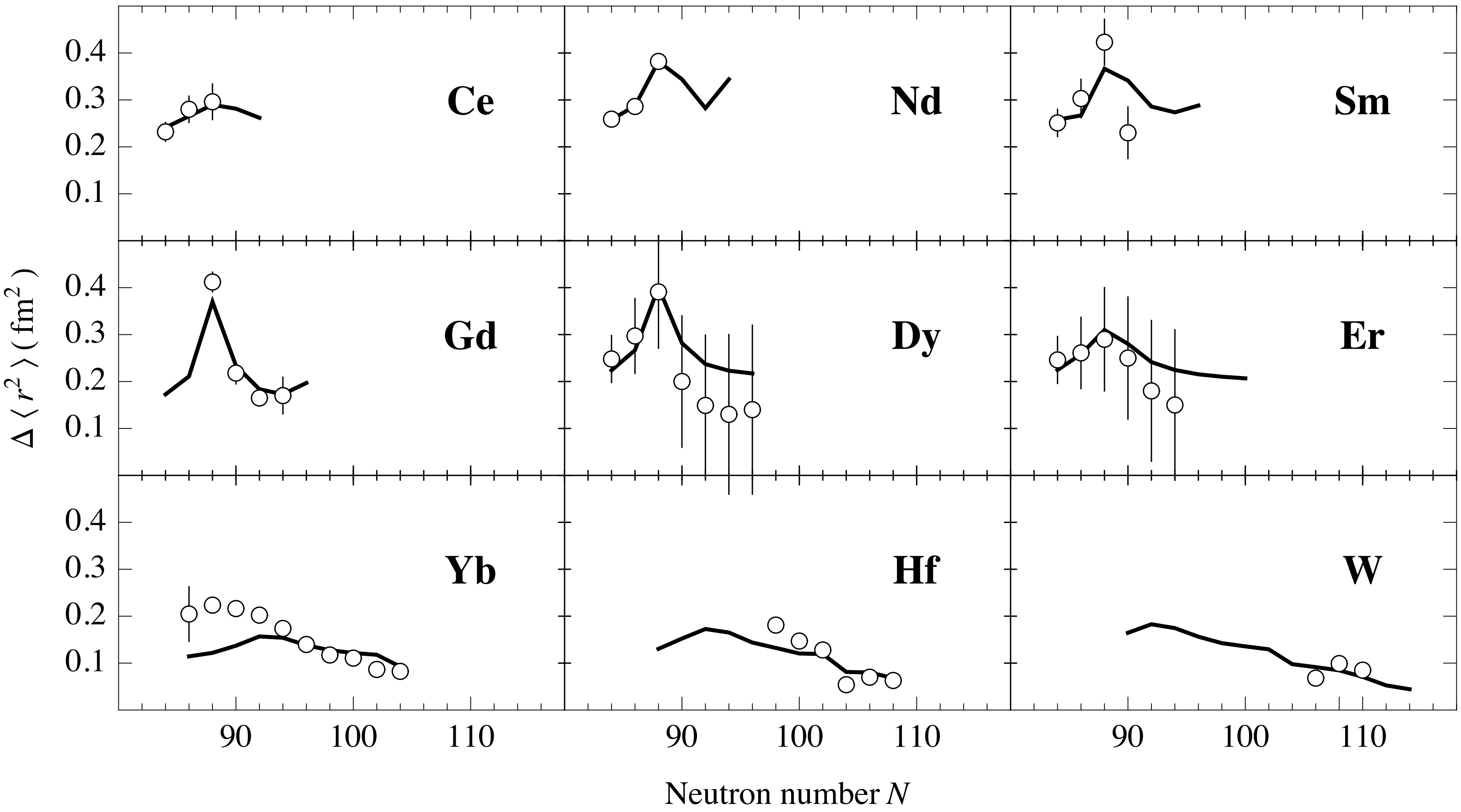
$$\langle r^2 \rangle_{\text{av}} \approx \frac{3}{5} r_0^2 A^{2/3} \Rightarrow |\alpha| \approx \frac{4}{5} r_0^2 A^{-1/3} \sim 0.2\ \text{fm}^2$$

Increase of charge radius due to deformation:

$$\langle r^2 \rangle_{\text{def}} \approx \frac{3}{4\pi} \beta^2 r_0^2 A^{2/3}$$

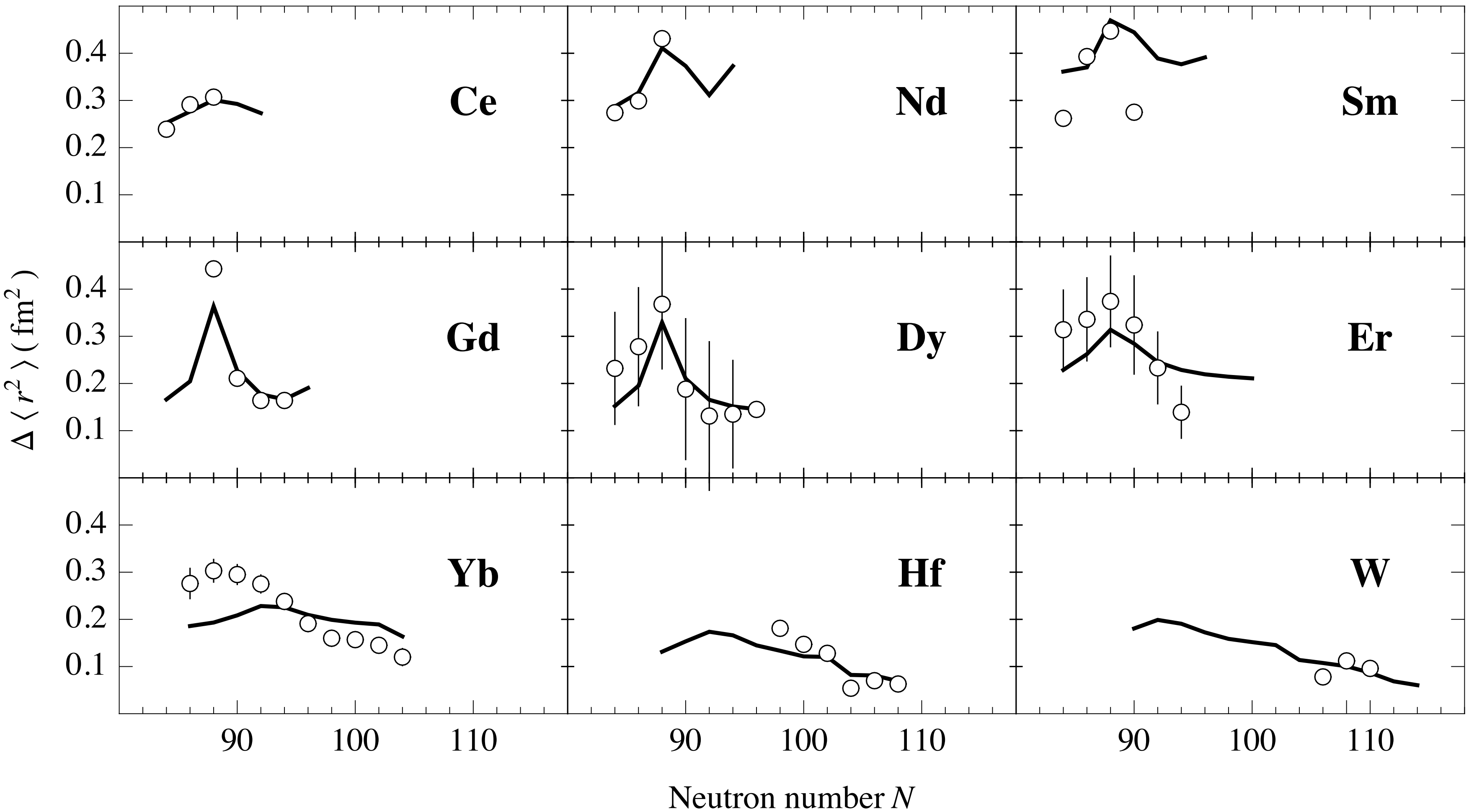
$$\Rightarrow \eta \approx \frac{4}{3}\left(1 + \overline{\beta}^2\right) r_0^2 N_b^2 A^{-4/3} \sim 0.25 - 0.75\ \text{fm}^2$$

Isotope shifts



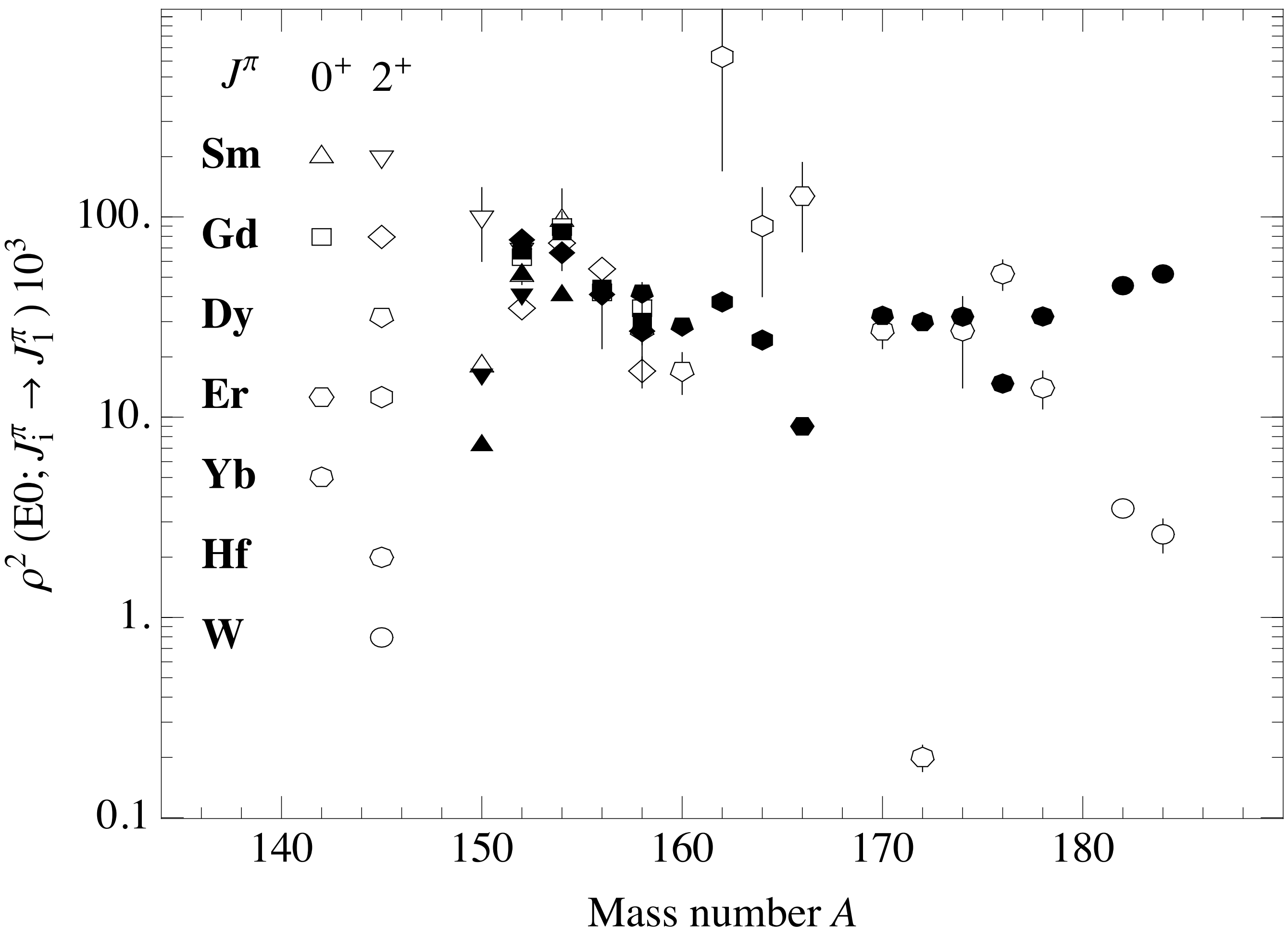
Data: compilation of many references

Isotope shifts



Data: I. Angeli, At. Data Nucl. Data Tables **60** (2004) 177

ρ^2 values



Data: Compilation B. Singh

ρ^2 values

Isotope	Transition	J	$\rho^2(E0) \times 10^3$				
			Th1[a]	Th2[b]	Th3[c]	Expt.[d]	
^{150}Sm	740 → 0	0	7	6		18	*2*
	1046 → 334	2	16	13		100	*40*
^{152}Sm	685 → 0	0	52	52	72	51	*5*
	811 → 122	2	41	41	77	69	*6*
	1023 → 366	4	29	29	84	88	*14*
	1083 → 0	0	2	2		0.7	*0.4*
	1083 → 685	0	47	47		22	*9*
^{154}Sm	1099 → 0	0	41	49		96	*42*
^{152}Gd	615 → 0	0	68	68		63	*14*
	931 → 344	2	77	77		35	*3*
^{154}Gd	681 → 0	0	84	102		89	*17*
	815 → 123	2	66	80		74	*9*
	1061 → 361	4	38	46		70	*7*
^{156}Gd	1049 → 0	0	44	64		42	*20*
	1129 → 89	2	41	59		55	*5*
^{158}Gd	1452 → 0	0	30	51		35	*12*
	1517 → 79	2	27	45		17	*3*
^{158}Dy	1086 → 99	2	42	70		27	*12*
^{160}Dy	1350 → 87	2	28	56		17	*4*
^{162}Er	1171 → 102	2	38	64		630	*460*
^{164}Er	1484 → 91	2	24	48		90	*50*
^{166}Er	1460 → 0	0	9	20		127	*60*
^{170}Yb	1229 → 0	0	32	72		27	*5*
^{172}Yb	1405 → 0	0	30	76		0.2	*0.03*
^{174}Hf	900 → 91	2	32	71		27	*13*
^{176}Hf	1227 → 89	2	15	38		52	*9*
^{178}Hf	1496 → 93	2	32	72		14	*3*
^{182}W	1257 → 100	2	45	77		3.5	*0.3*
^{184}W	1121 → 111	2	52	75		2.6	*0.5*

Data: Compilation B. Singh

Summed B(M1) strength

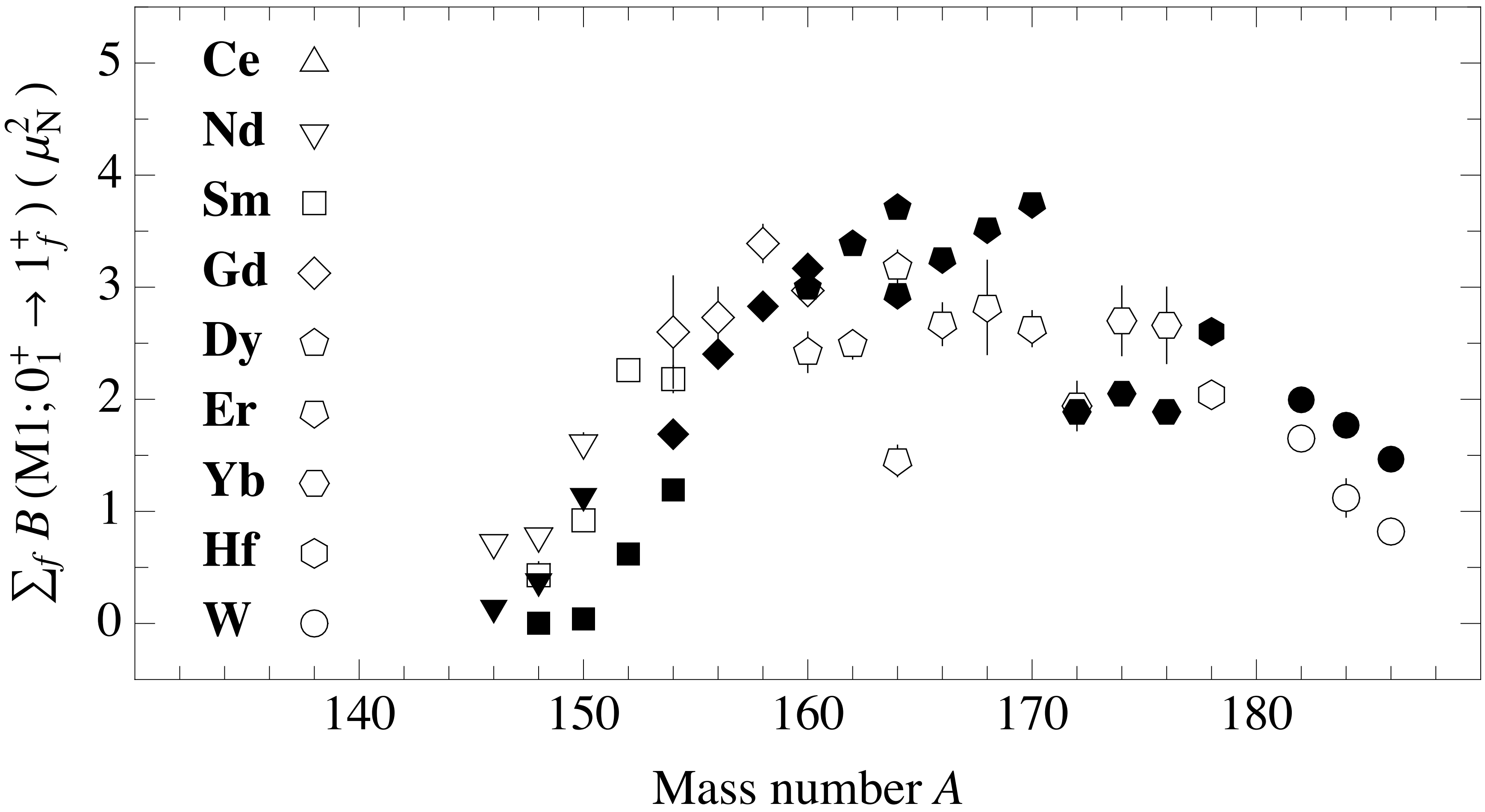
Ginocchio proved the following M1 sum rule:

$$\sum_f B\left(\mathrm{M1};0_1^+ \to 1_f^+\right) = \frac{3}{4\pi}\left(g_\nu - g_\pi\right)^2 \frac{6N_\nu N_\pi}{N_\mathrm{b}\left(N_\mathrm{b}-1\right)}\left\langle \hat{n}_d \right\rangle_{0_1^+}$$

Summed M1 strength to the scissors state is known in many rare-earth nuclei.

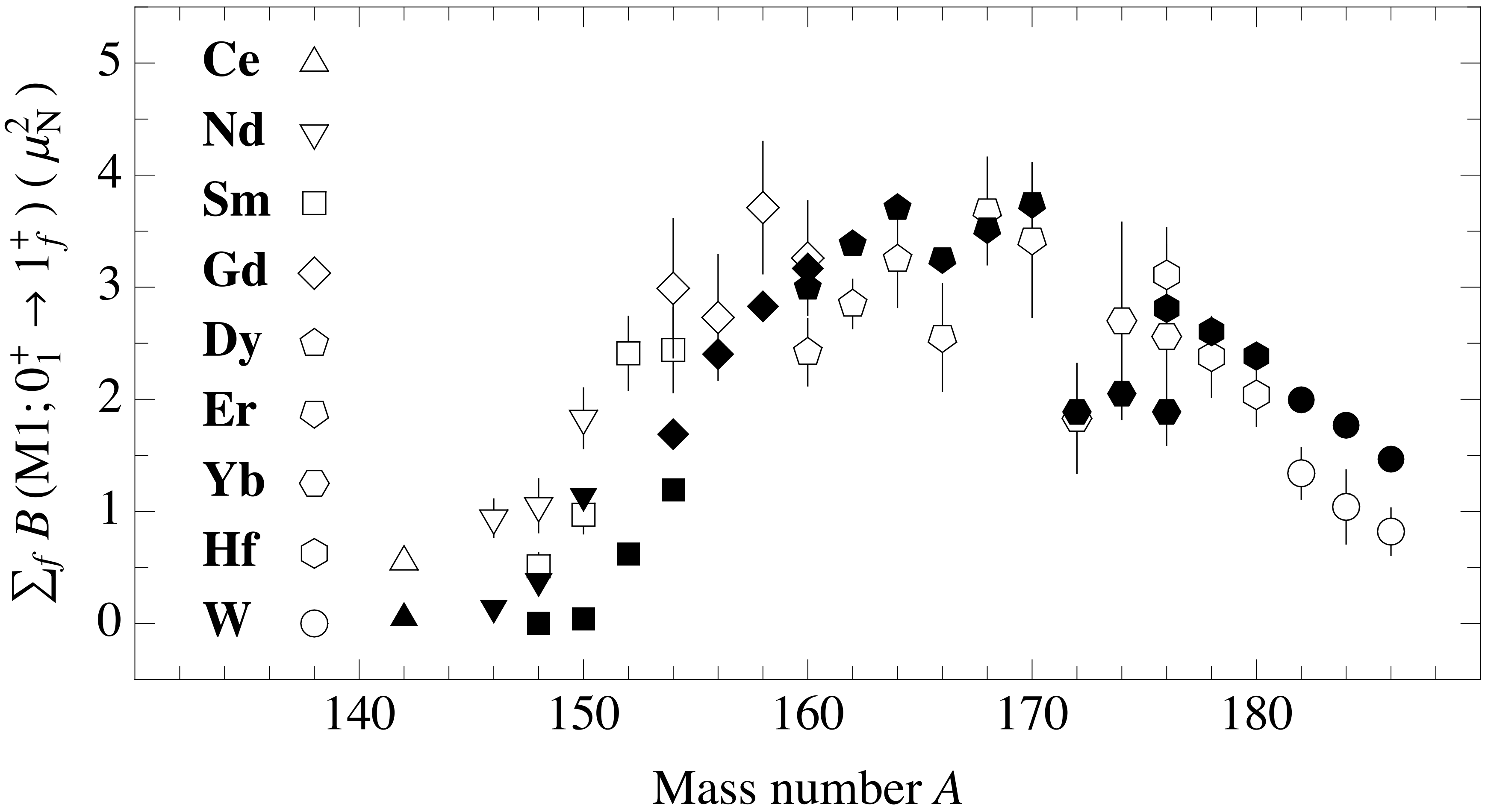
J.N. Ginocchio, Phys. Lett. B **265** (1991) 6

Summed B(M1) strength



N. Pietralla *et al.*, Phys. Rev. C **52** (1995) 2317(R)

Summed *B*(M1) strength



J. Enders *et al.*, Phys. Rev. C **71** (2005) 014306

Correlation S(M1)–<r^2>

Rewrite expressions for <r^2> and S(M1):

$$\Delta\left\langle r^2\right\rangle-|\alpha|\equiv\frac{\eta}{N_{\rm b}}\left(\left\langle\hat{n}_d\right\rangle_{0_1^+}^{(A+2)}-\left\langle\hat{n}_d\right\rangle_{0_1^+}^{(A)}\right)$$

$$\tilde{S}(\mathrm{M1})\equiv\frac{N_{\rm b}-1}{N_\nu}\sum_f B\left(\mathrm{M1};0_1^+\rightarrow 1_f^+\right)=\frac{9}{2\pi}\left(g_\nu-g_\pi\right)^2\frac{N_\pi}{N_{\rm b}}\left\langle\hat{n}_d\right\rangle_{0_1^+}$$

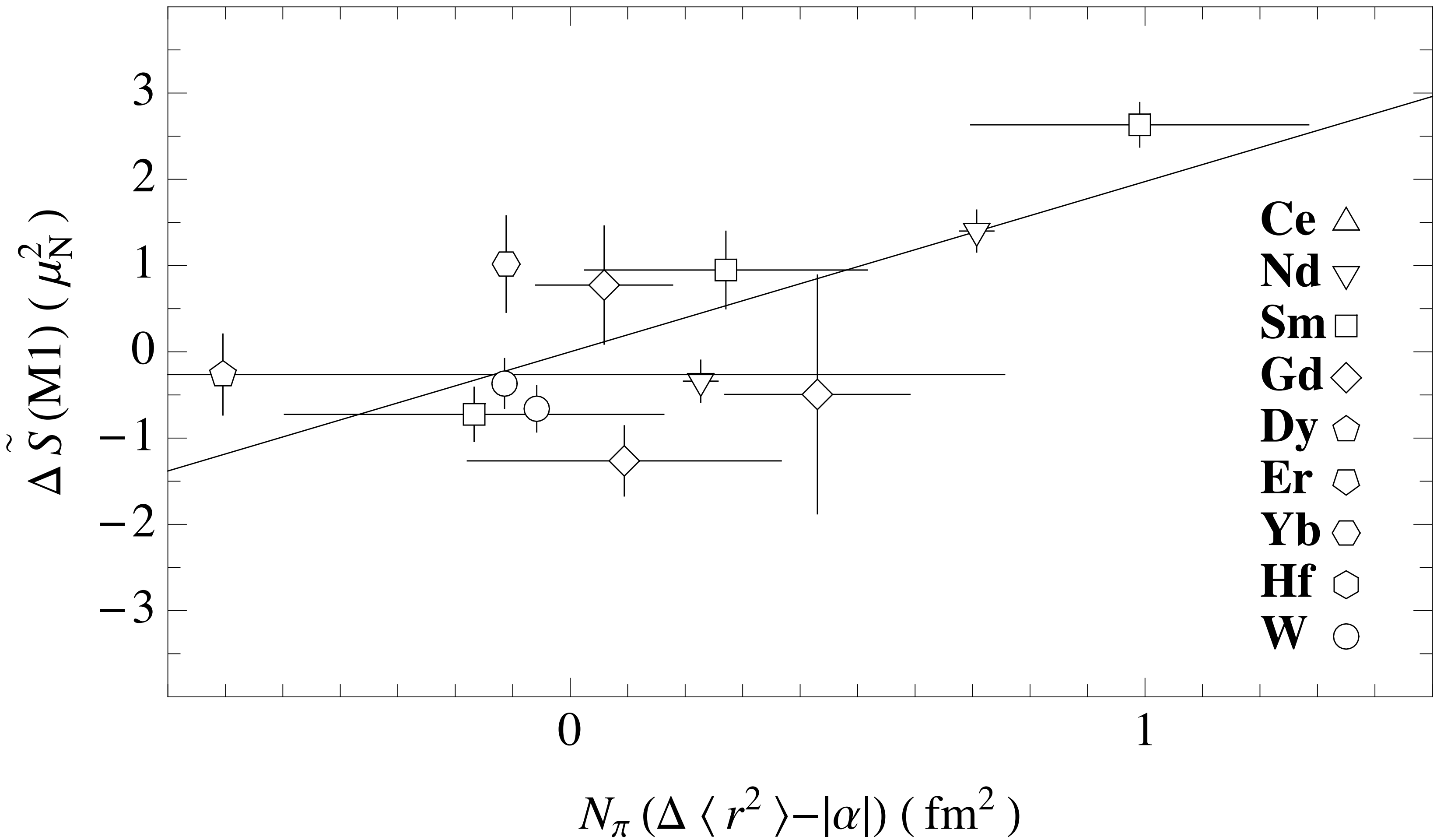
We obtain the relation

$$\Delta\tilde{S}(\mathrm{M1})\equiv\tilde{S}(\mathrm{M1})^{(A+2)}-\tilde{S}(\mathrm{M1})^{(A)}$$

$$=\frac{9}{2\pi}\frac{\left(g_\nu-g_\pi\right)^2}{\eta}N_\pi\left(\Delta\left\langle r^2\right\rangle-|\alpha|\right)$$

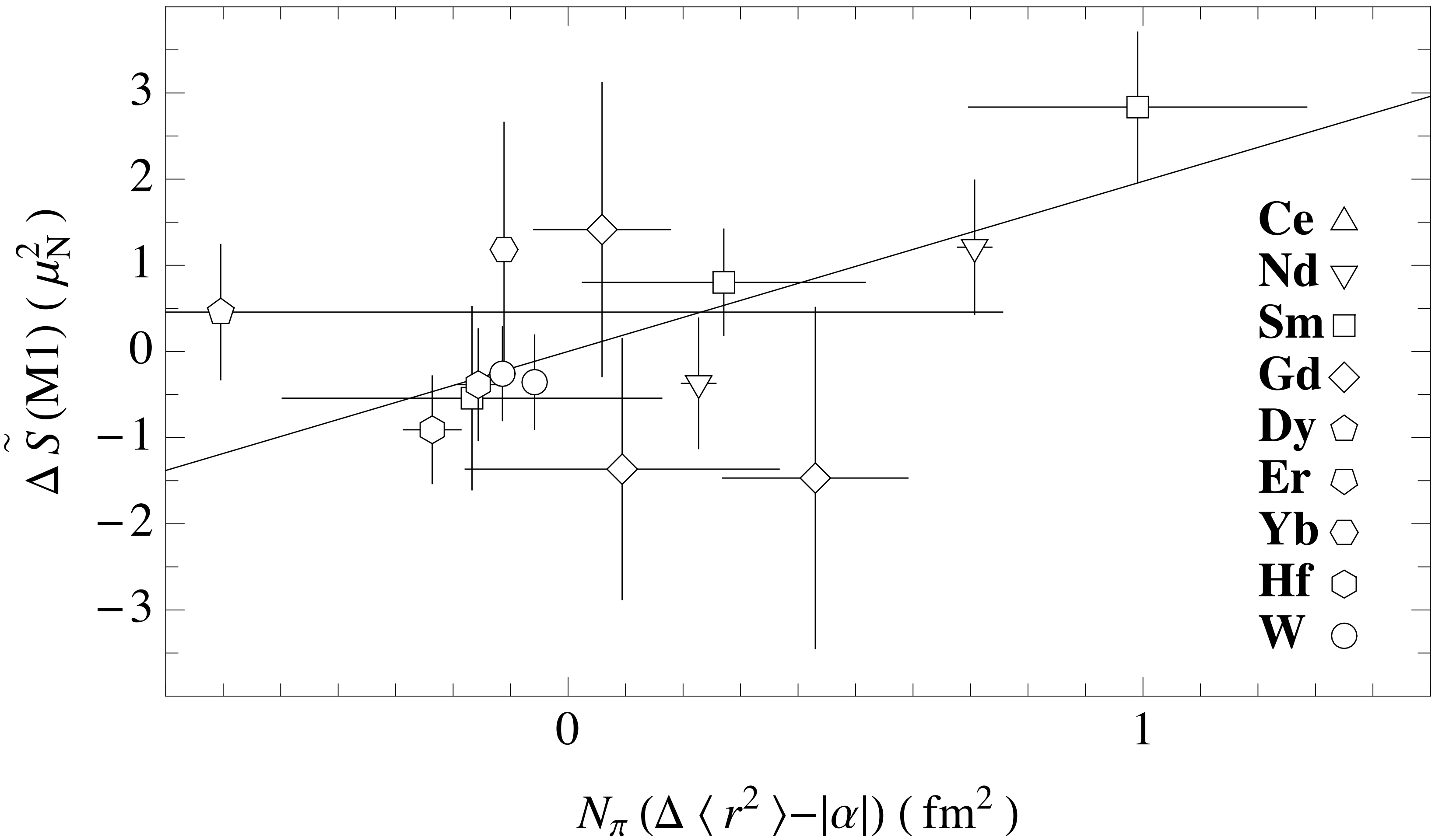
K. Heyde *et al.*, Phys. Lett. B **312** (1993) 267

Correlation ΔS(M1)-$\Delta \langle r^2 \rangle$



N. Pietralla *et al.*, Phys. Rev. C **52** (1995) 2317(R)

Correlation ΔS(M1)- $\Delta \langle r^2 \rangle$



J. Enders *et al.*, Phys. Rev. C **71** (2005) 014306

Correlation S(M1)- ρ (E0)

In well-deformed nuclei [SU(3)]:

$$\left\langle 0_1^+ \left| \hat{n}_d \right| 0_1^+ \right\rangle = \frac{4N_\mathrm{b}\left(N_\mathrm{b}-1\right)}{3\left(2N_\mathrm{b}-1\right)}$$

$$\left|\left\langle 0_\beta^+ \left| \hat{n}_d \right| 0_1^+ \right\rangle\right| = \left[\frac{8\left(N_\mathrm{b}-1\right)^2 N_\mathrm{b}\left(2N_\mathrm{b}+1\right)}{9\left(2N_\mathrm{b}-3\right)\left(2N_\mathrm{b}-1\right)^2}\right]^{1/2}$$

We obtain the relation (for large N_b)

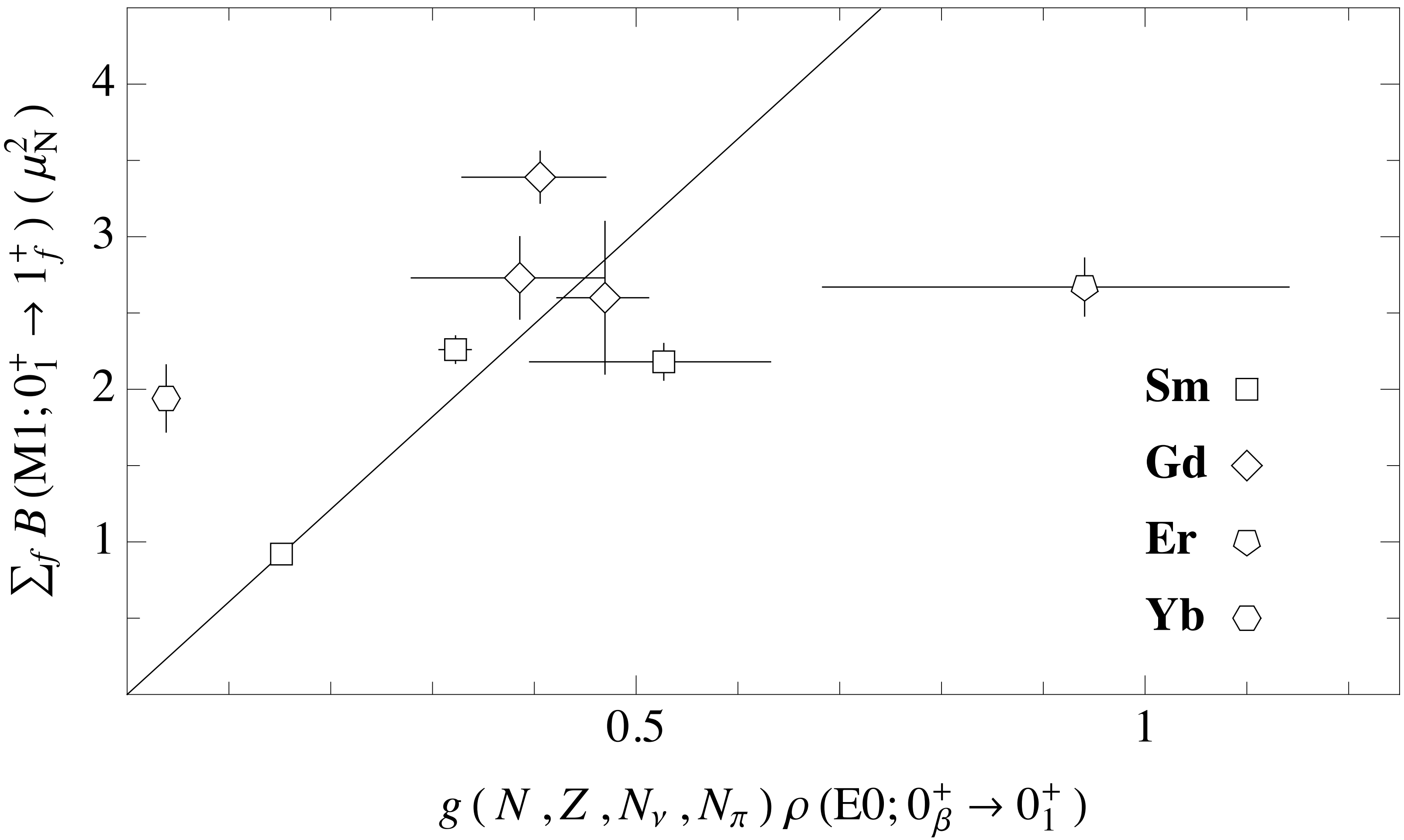
$$B\left(\mathrm{M1};0_1^+ \to 1_1^+\right) \approx \frac{9}{\pi}\left(g_\nu - g_\pi\right)^2 \frac{r_0^2}{\eta} g\left(N,Z,N_\nu,N_\pi\right)\rho\left(\mathrm{E0};0_\beta^+ \to 0_1^+\right)$$

with

$$g\left(N,Z,N_\nu,N_\pi\right) = \frac{e\left(N+Z\right)^{2/3}}{e_\mathrm{n}N + e_\mathrm{p}Z}\frac{N_\nu N_\pi}{\sqrt{2N_\mathrm{b}}}$$

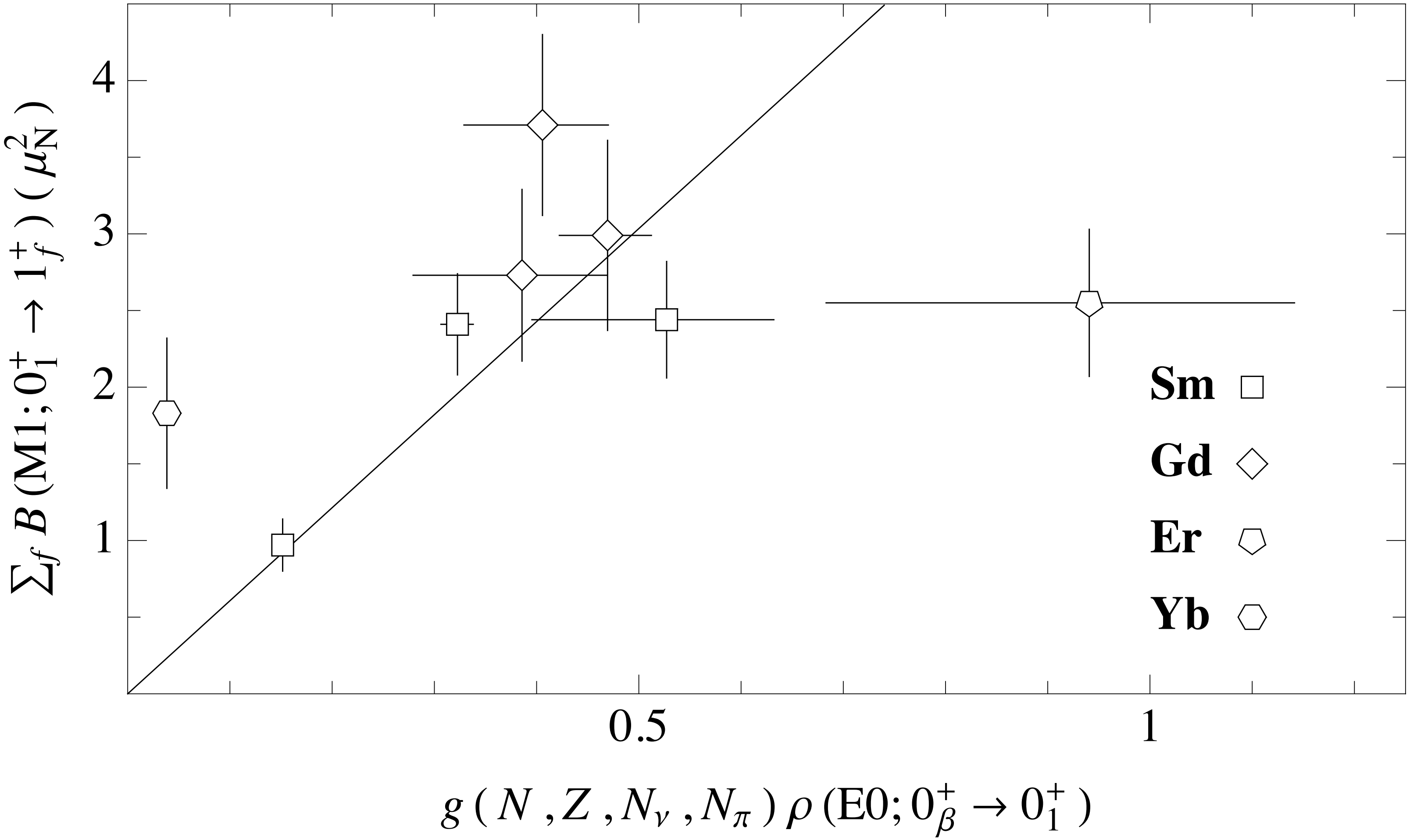
P. Van Isacker, Nucl. Data Sheets **120** (2014) 119

Correlation S(M1)- ρ (E0)



N. Pietralla *et al.*, Phys. Rev. C **52** (1995) 2317(R)

Correlation S(M1)- ρ (E0)



J. Enders *et al.*, Phys. Rev. C **71** (2005) 014306

Conclusions

Consistent treatment of charge radii and E0 transitions assuming the same effective charges.

An additional correlation exists with summed M1 strength. Which can be related to charge radii and E0 transitions.

Outlook:

S(M1)-ρ (E0) correlation for transitional nuclei.

Need for $\Delta \langle r^2 \rangle$ through shape transition.

Back-up

Charge radii and E0 transitions

Standard relation:

$$\hat{T}(\mathrm{E0}) = eZ\hat{T}\left(r^2\right)$$

Generalized relation depends on effective charges:

$$\hat{T}(\mathrm{E0}) = \left(e_{\mathrm{n}}N + e_{\mathrm{p}}Z\right)\hat{T}\left(r^2\right)$$

Because of these relations, a correlation can be established between nuclear charge radii and ρ (E0) values.

Effective charges from radii

Estimate with harmonic-oscillator wave functions:

$$\langle r^2 \rangle_{\mathrm{s}} = \frac{1}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z} \sum_{k=1}^{A} \langle \mathrm{s} | e_k r_k^2 | \mathrm{s} \rangle$$

$$= \frac{3^{4/3}}{4} \frac{b^2}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z} \left(e_{\mathrm{n}} N^{4/3} + e_{\mathrm{p}} Z^{4/3} \right)$$

$$= \frac{3\sqrt[3]{2}}{5} r_0^2 \frac{A^{1/3}\left(e_{\mathrm{n}} N^{4/3} + e_{\mathrm{p}} Z^{4/3} \right)}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z}$$

Fit for rare-earth nuclei (Z=58 to 74) gives:

r_0=1.24 fm, e_{n}=0.50e and e_{p}=e.

Energy spectra

The standard (1+2)-body IBM hamiltonian:

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P}_+ \hat{P}_- + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4$$

Constant parameters for a given isotopic chain except for the quadrupole strength:

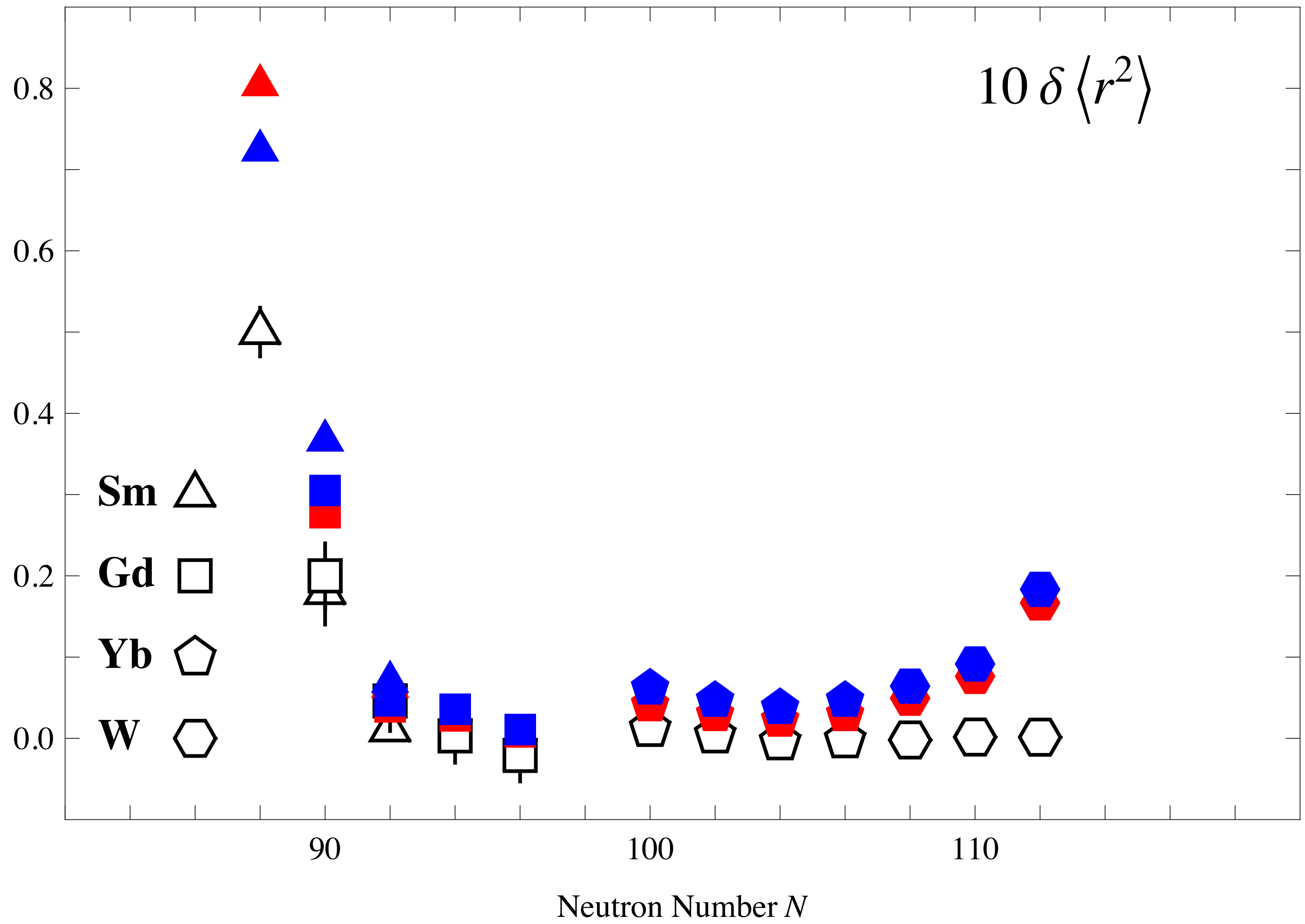
$$a_2 = a_2' + \frac{N_\nu N_\pi}{N_\nu + N_\pi} a_2''$$

Isomer shifts

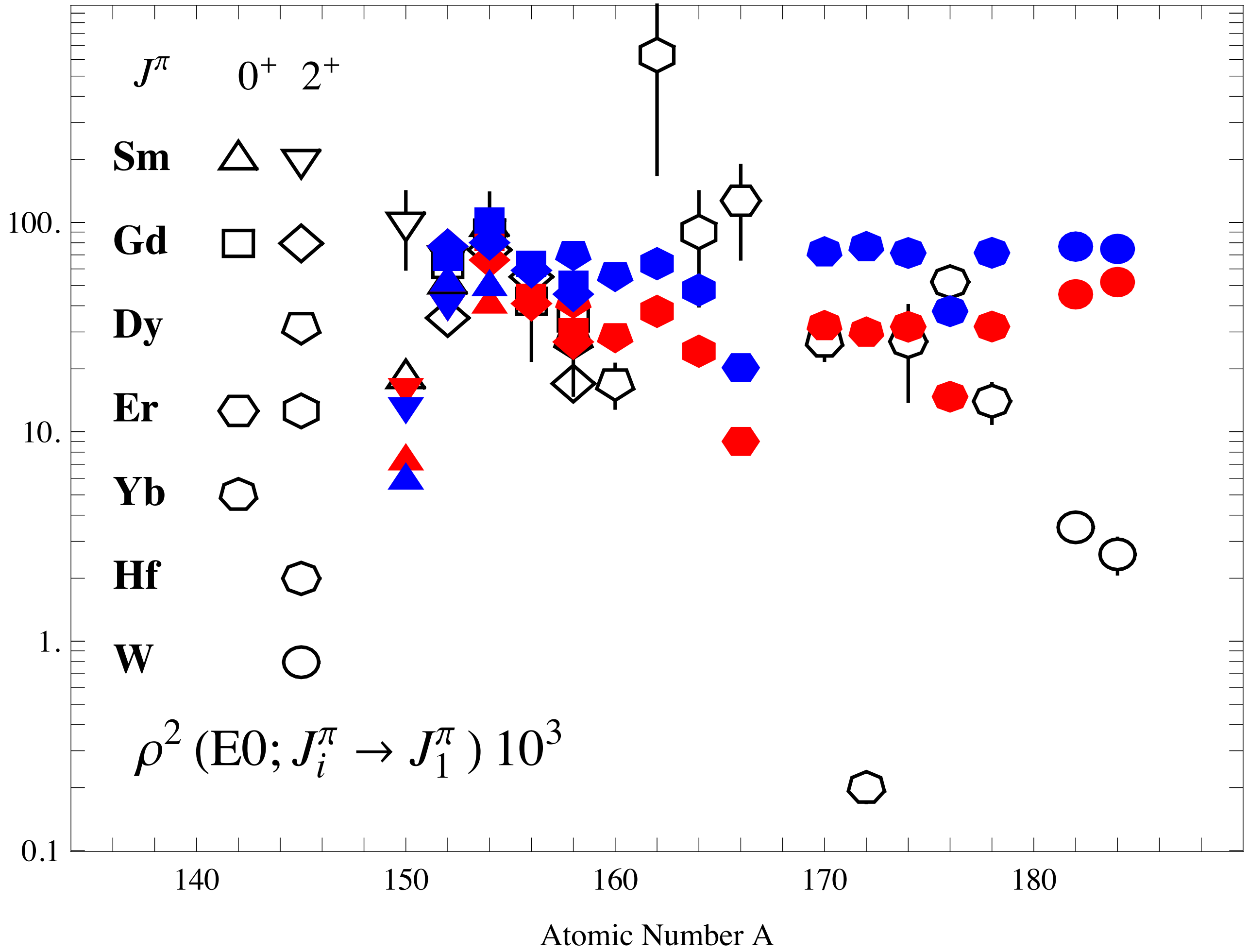
Isomer shifts depend on the coefficient η:

$$\delta\left\langle r^2\right\rangle \equiv \left\langle r^2\right\rangle_{2_1^+}^{(A)} - \left\langle r^2\right\rangle_{0_1^+}^{(A)} = \eta\left(\left\langle \frac{\hat{n}_d}{N_b}\right\rangle_{2_1^+}^{(A)} - \left\langle \frac{\hat{n}_d}{N_b}\right\rangle_{0_1^+}^{(A)}\right)$$

Isomer shifts

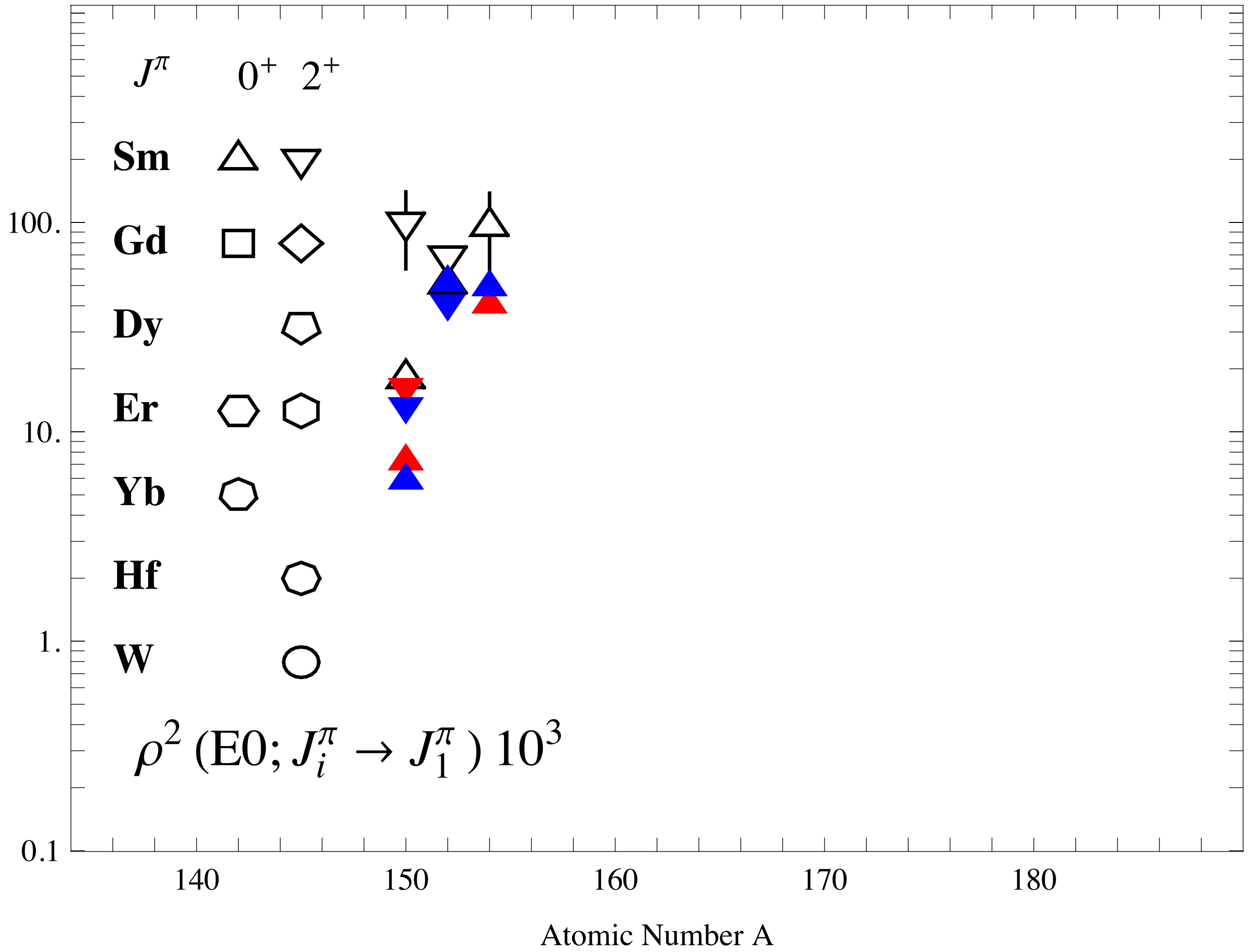


ρ^2 values

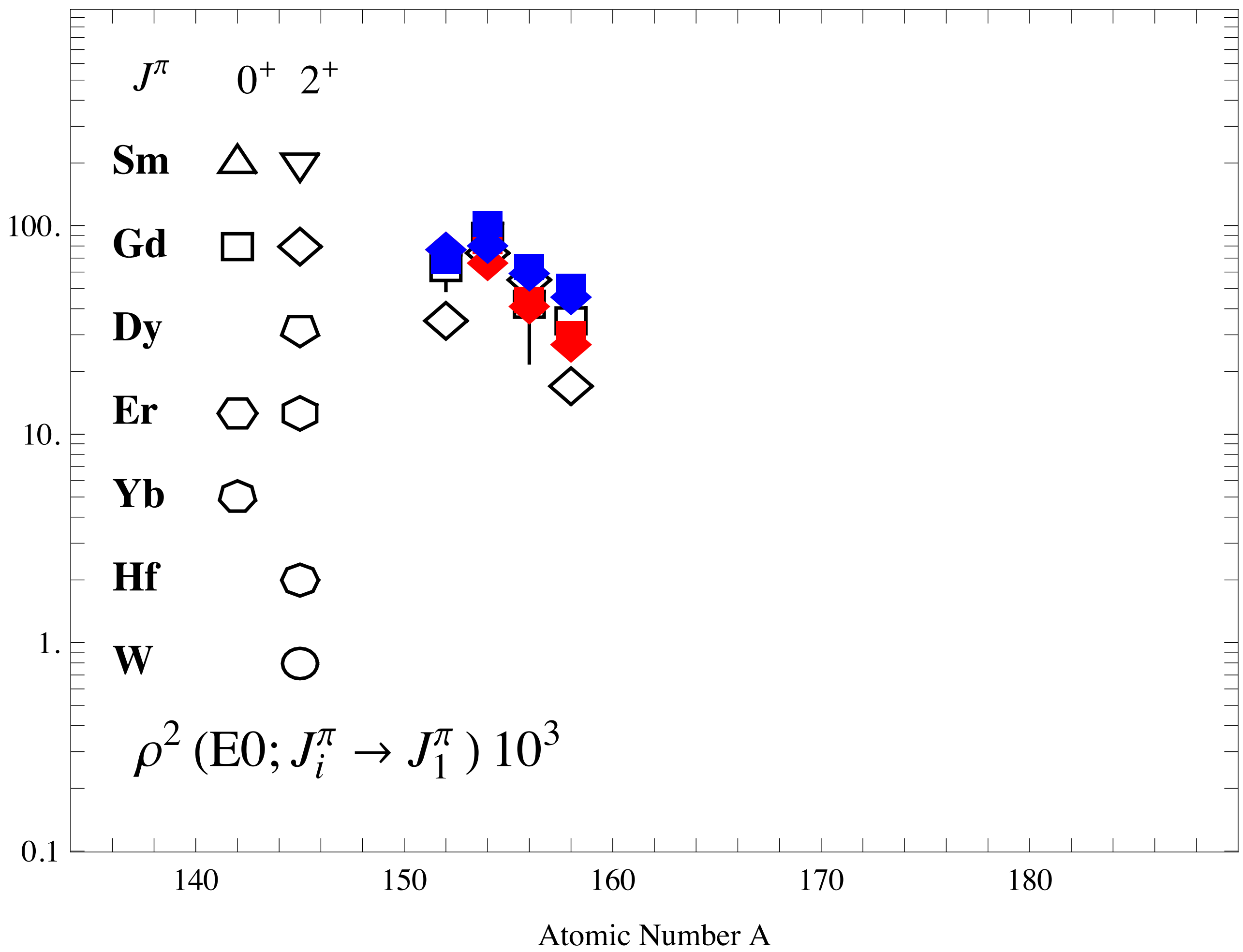


J^π 0^+ 2^+
Sm
Gd
Dy
Er
Yb
Hf
W
ρ^2 (E0; $J_i^\pi \rightarrow J_1^\pi$) 10^3
100.
10.
1.
0.1
140
150
160
170
180
Atomic Number A

ρ^2 values in samarium



ρ^2 values in gadolinium



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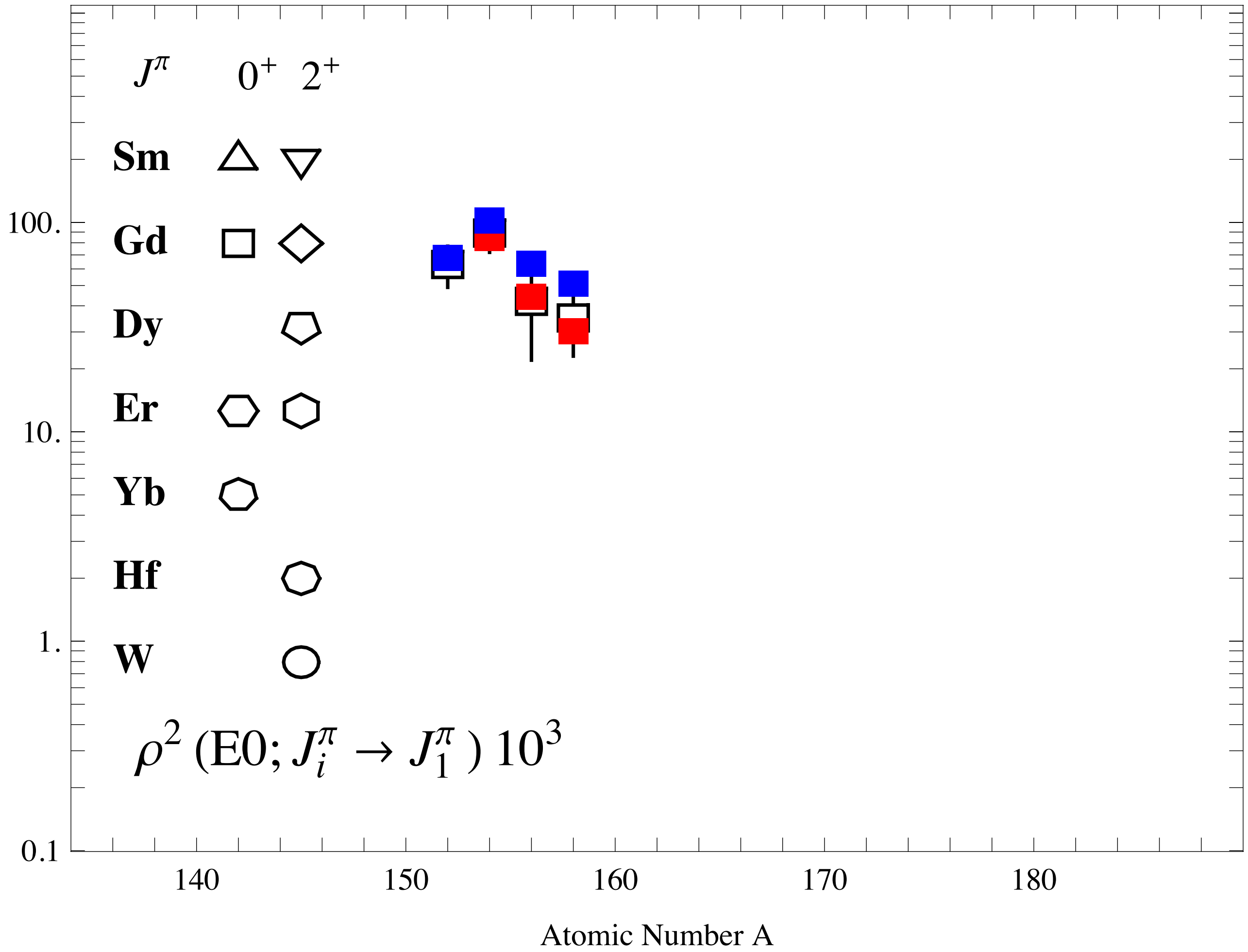

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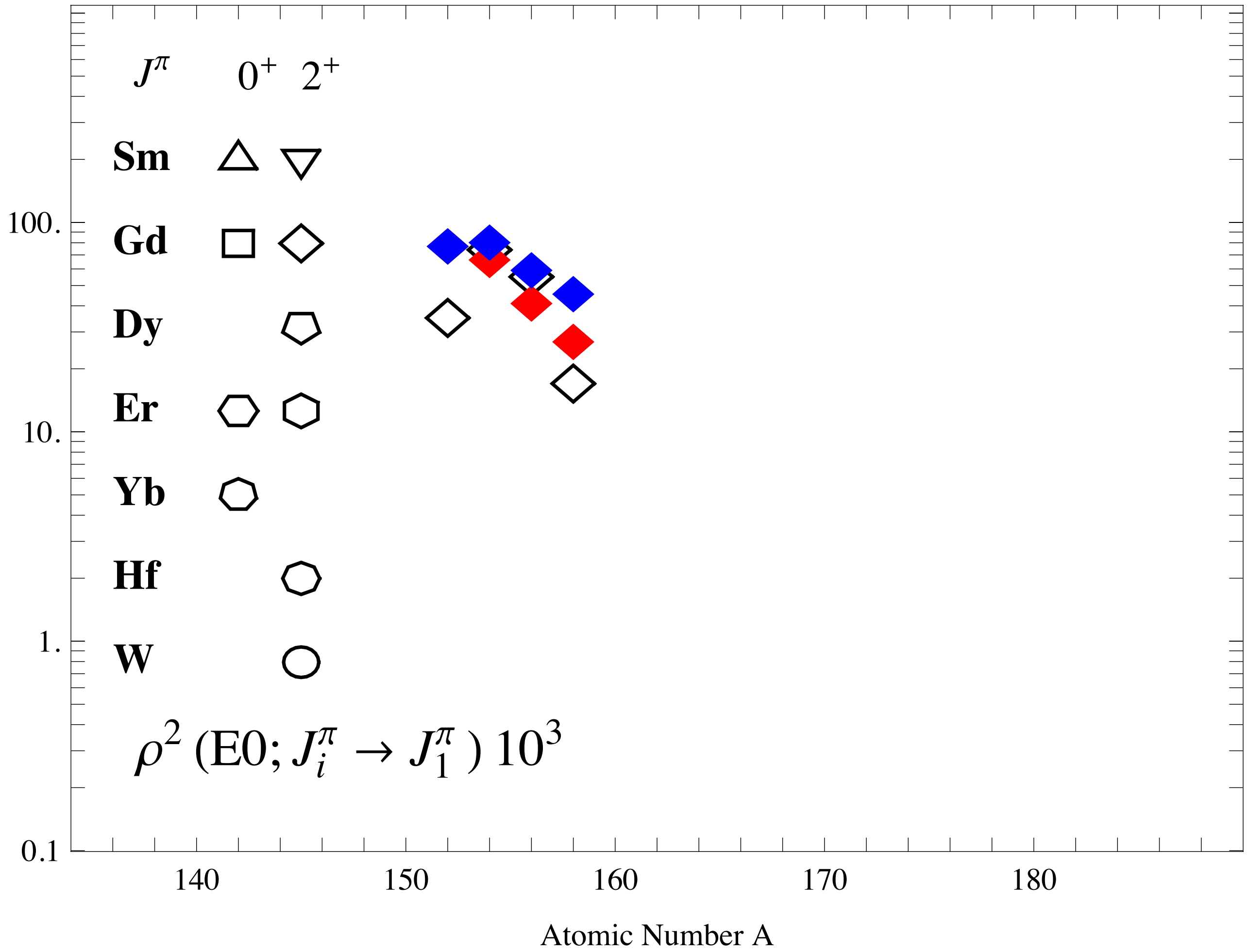
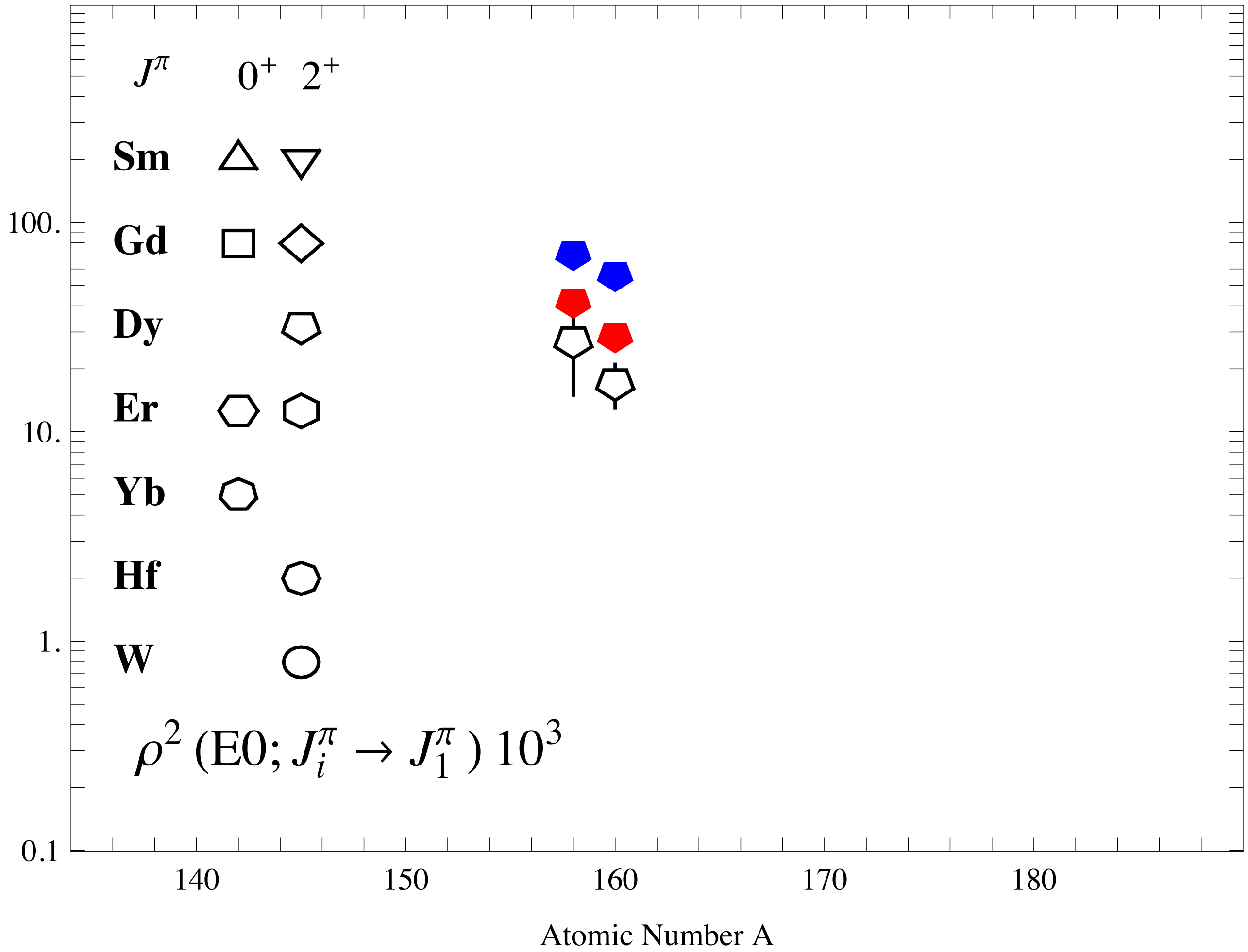
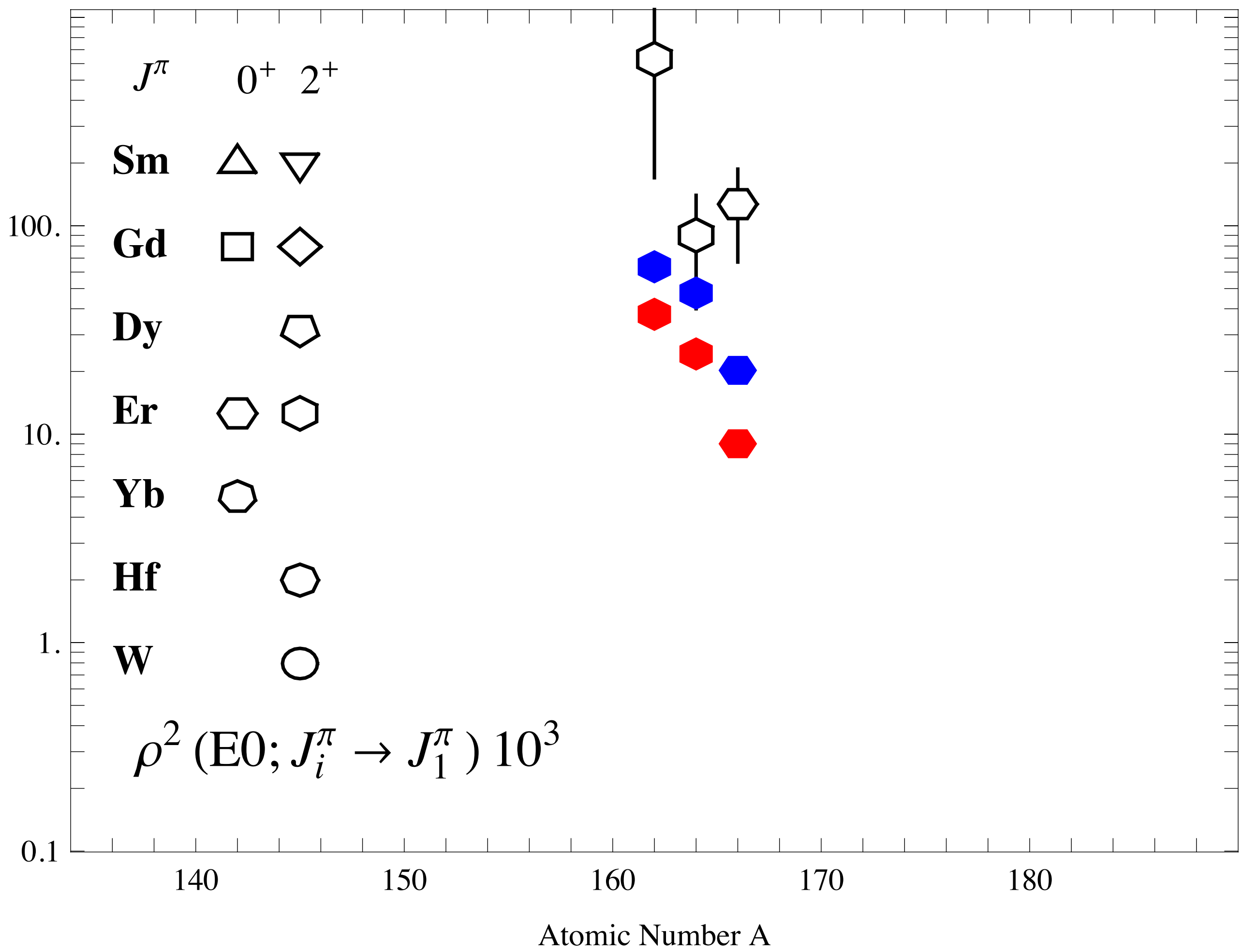


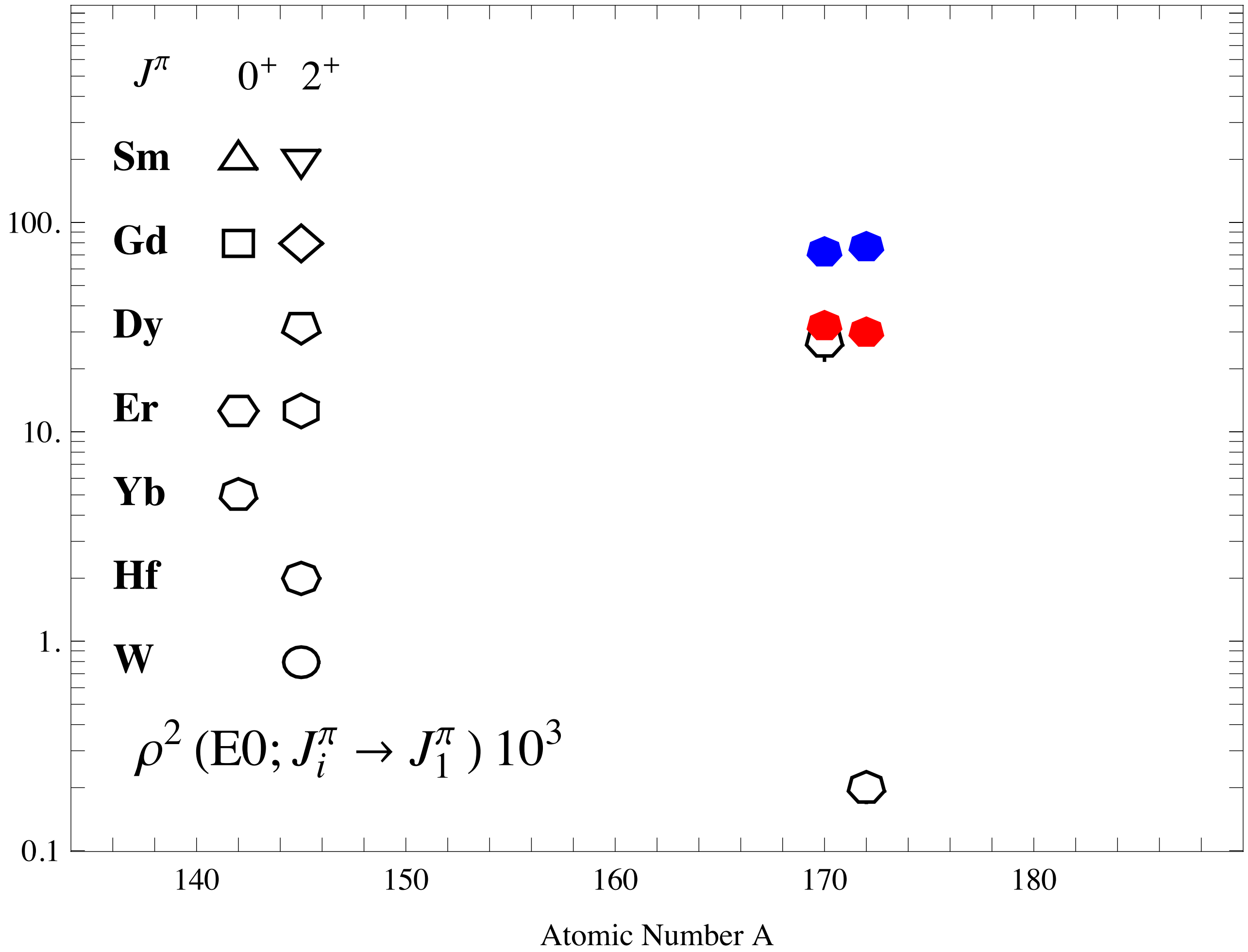
ρ^2 values in dysprosium



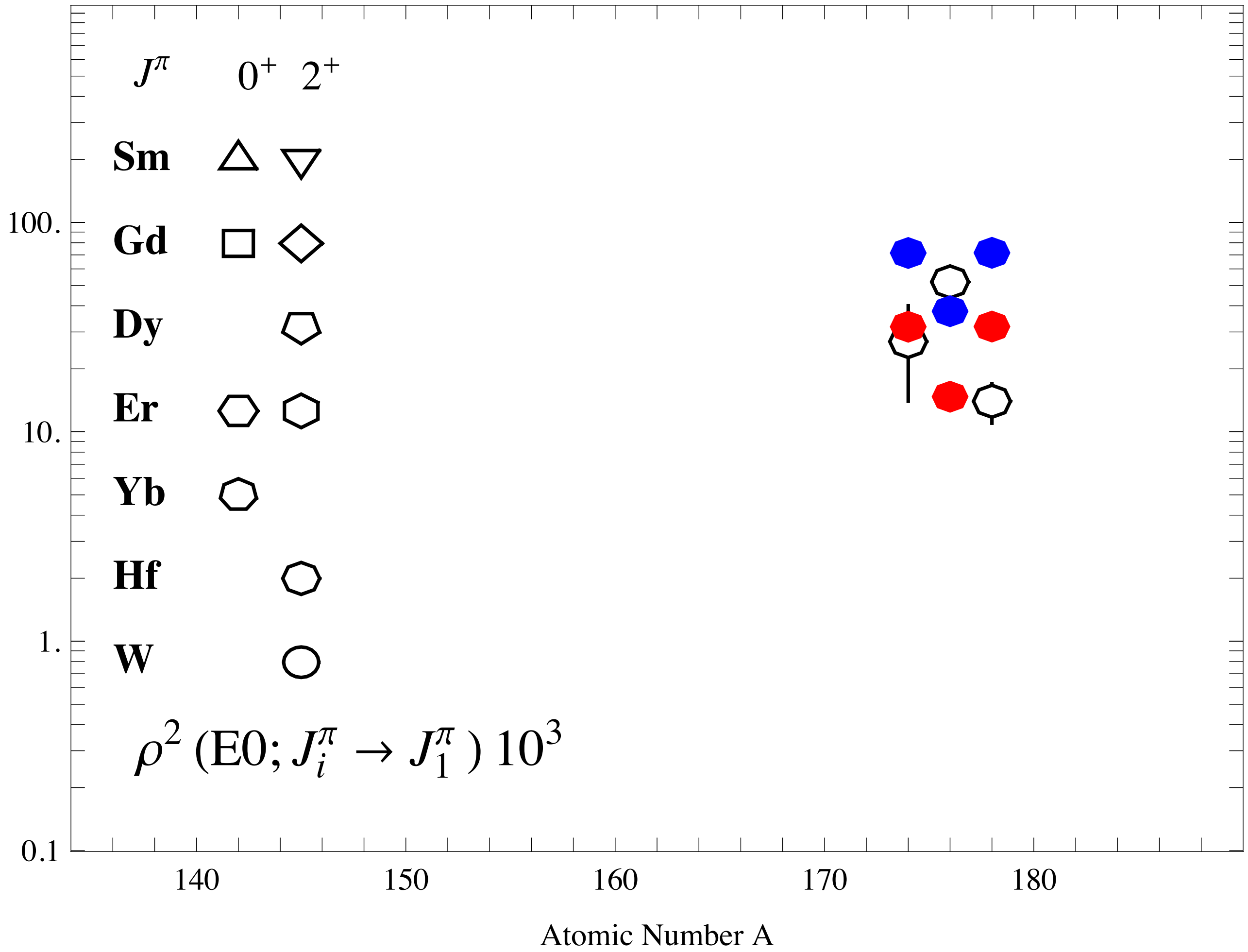
ρ^2 values in erbium



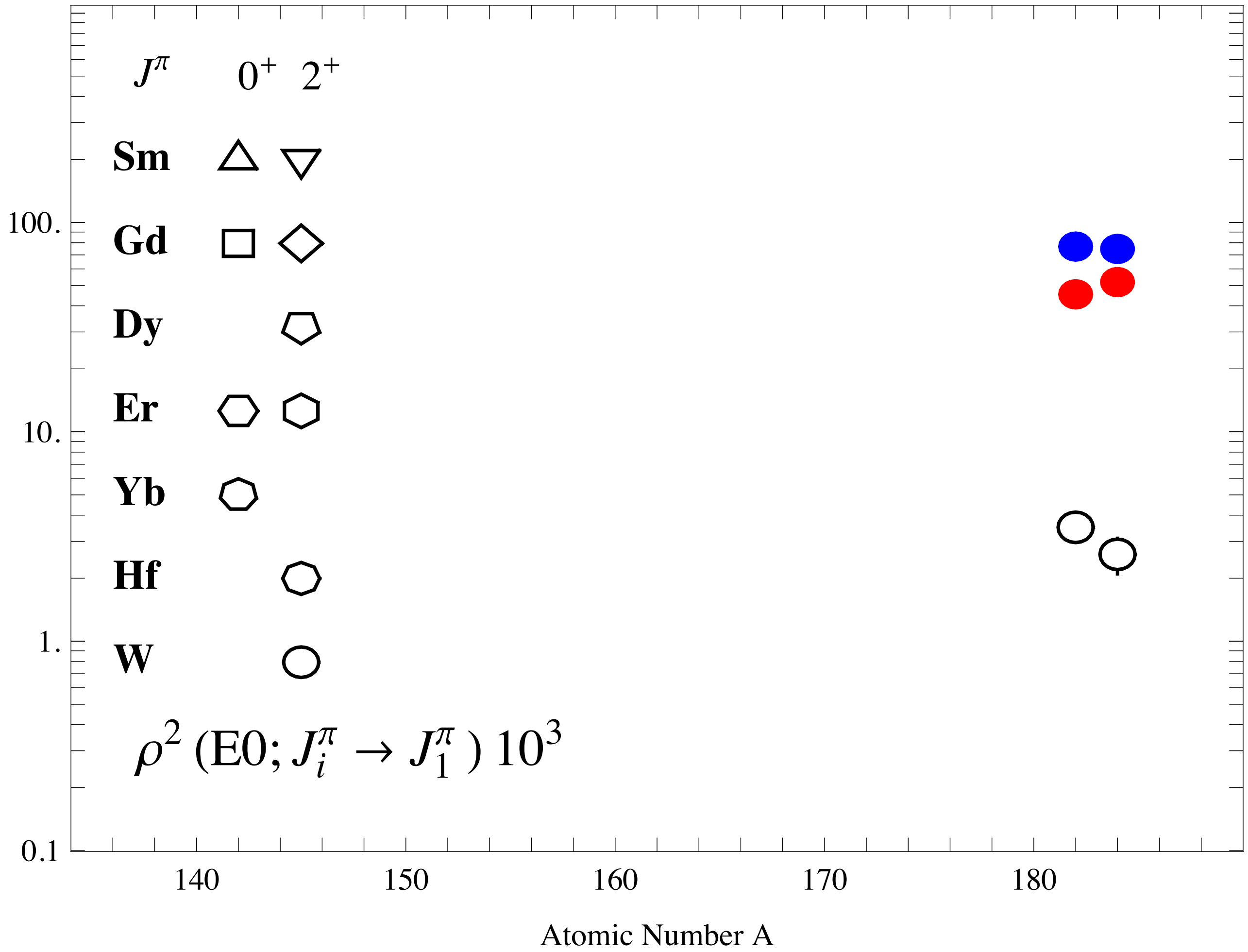
ρ^2 values in ytterbium



ρ^2 values in hafnium



ρ^2 values in tungsten



Influence of g boson

Spherical-to-deformed transitional hamiltonian in sdg-IBM-1:

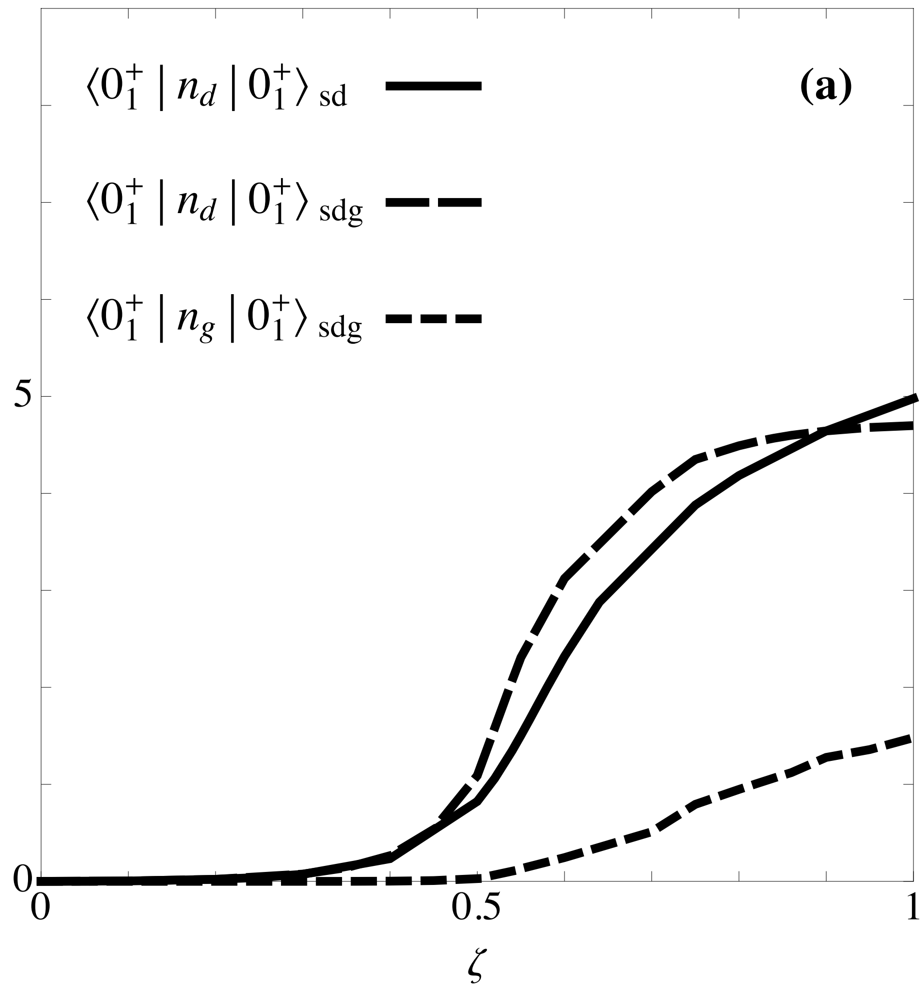
$$\hat{H} = c\left[(1-\varsigma)\left(\hat{n}_d + \lambda\hat{n}_g\right) - \frac{\varsigma}{4N_b} Q\cdot Q\right]$$

$$Q_\mu = \left[s^+ \times \tilde{d} + d^+ \times \tilde{s}\right]^{(2)}_\mu - \frac{11}{14}\left[d^+ \times \tilde{d}\right]^{(2)}_\mu$$

$$+\frac{9}{7}\left[d^+ \times \tilde{g} + g^+ \times \tilde{d}\right]^{(2)}_\mu - \frac{3}{14}\left[g^+ \times \tilde{g}\right]^{(2)}_\mu$$

Take λ =1.5 and let ξ vary from 0 (spherical) to 1 (deformed).

Effect of g boson on radii



Effect of g boson on E0s