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Espace de Structure Nucléaire Théorique
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Shape coexistence in $N \approx Z$ nuclei around $A \approx 70$

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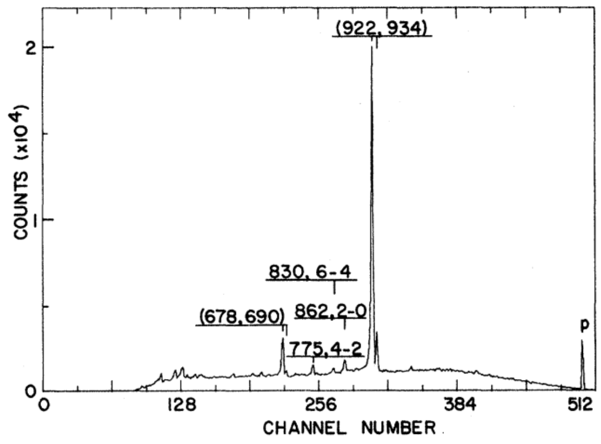
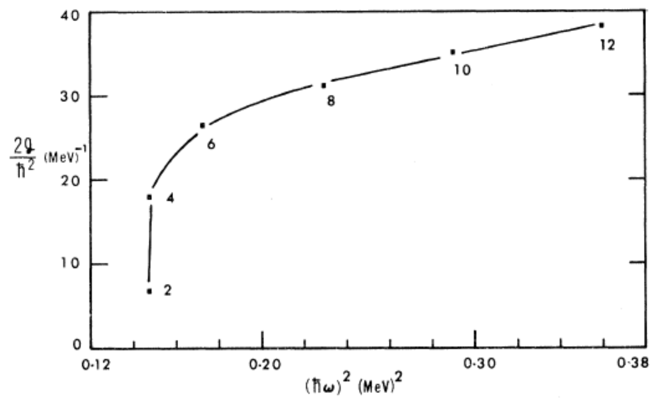
ESNT Workshop
Shape Coexistence and Electric Monopole Transitions in Atomic Nuclei
Saclay
23.-27. October 2017

Some history...

- low-lying 0_2^+ states in ^{70}Ge and ^{72}Ge interpreted as shape coexistence between spherical vibrational and deformed rotational states
  K. W. C. Stewart, B. Castel, Lett. Nuovo Cimento 4, 589 (1970)

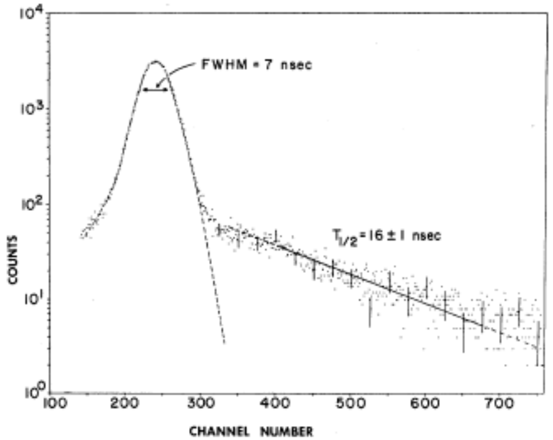
- Observation of a low-lying 0_2^+ state in ^{72}Se by in-beam conversion electron spectroscopy
  W.G. Wyckoff, J.E. Draper, PRC 8, 796 (1973)





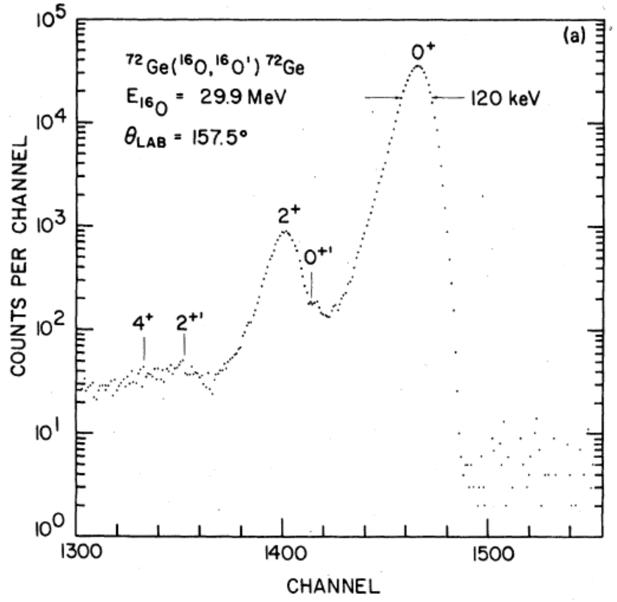
- distortion of ground-state band at low spin in ^{72}Se explained by shape coexistence and mixing
  J.H. Hamilton et al., PRL 32, 239 (1974)

- lifetime measurement for 0_2^+ state in ^{72}Se $\Rightarrow \rho^2(E0)$
  A.V. Ramayya et al, PRC12, 1360 (1975)

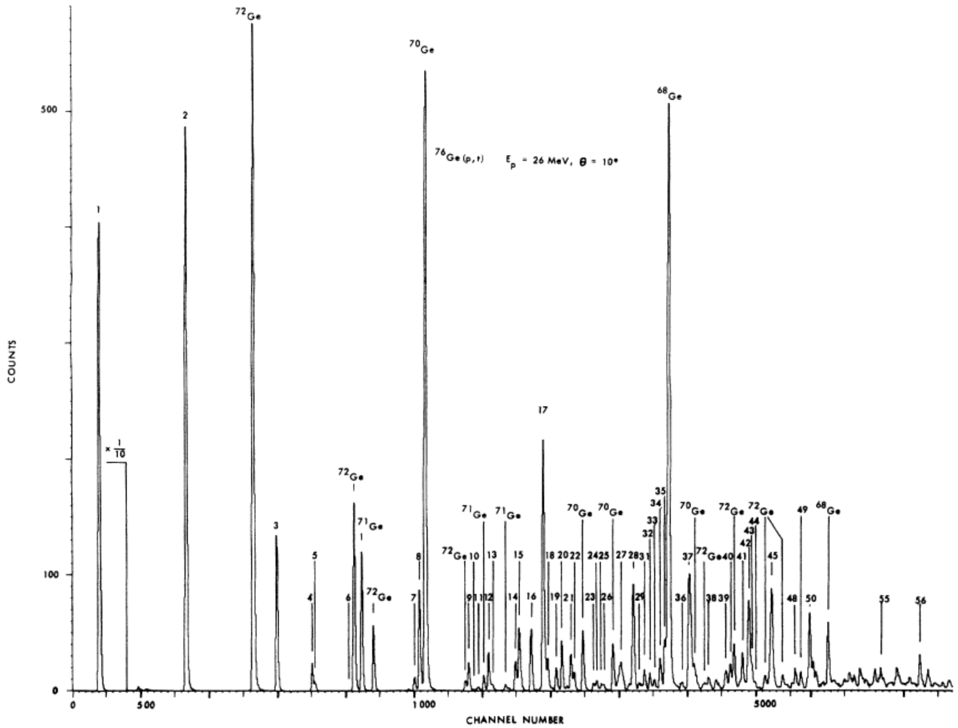


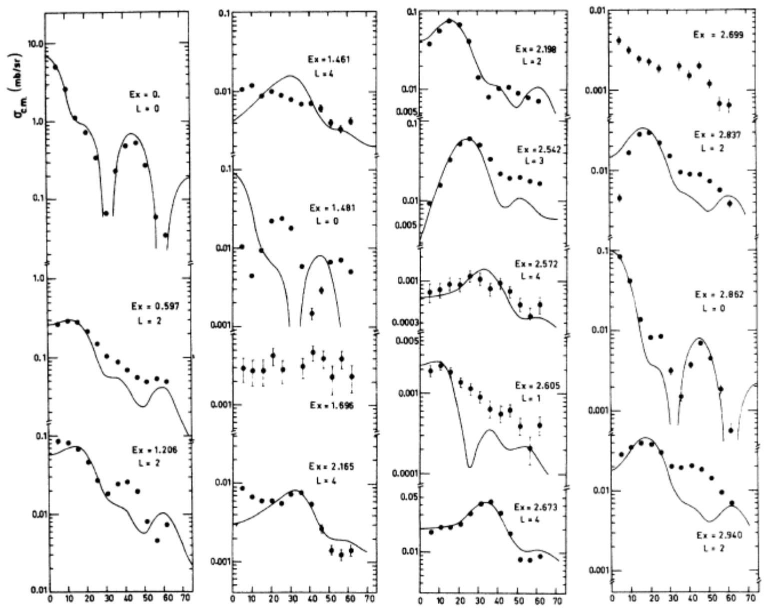
Experimental results for Ge isotopes

- Coulomb excitation of stable Ge isotopes $B(E2)$ values and $Q_s(2_1^+)$ (reorientation)
  R.Lecomte et al., PRC 22, 1530 (1980)



- (p,t) and (t,p) pair transfer reactions cross sections and angular distributions
  F. Guilbault et al., PRC 16, 1840 (1977)

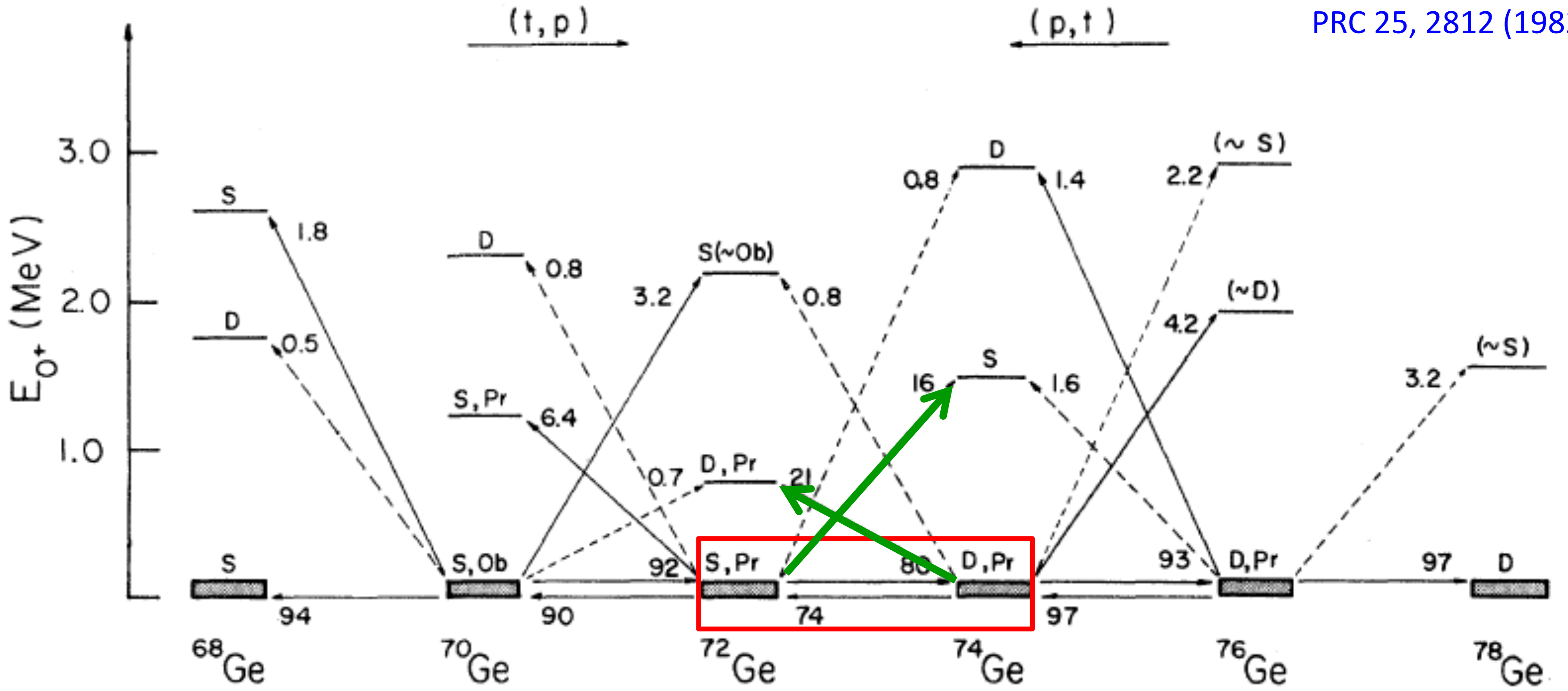




Systematics for Ge isotopes

- combining quadrupole moments and transfer cross sections

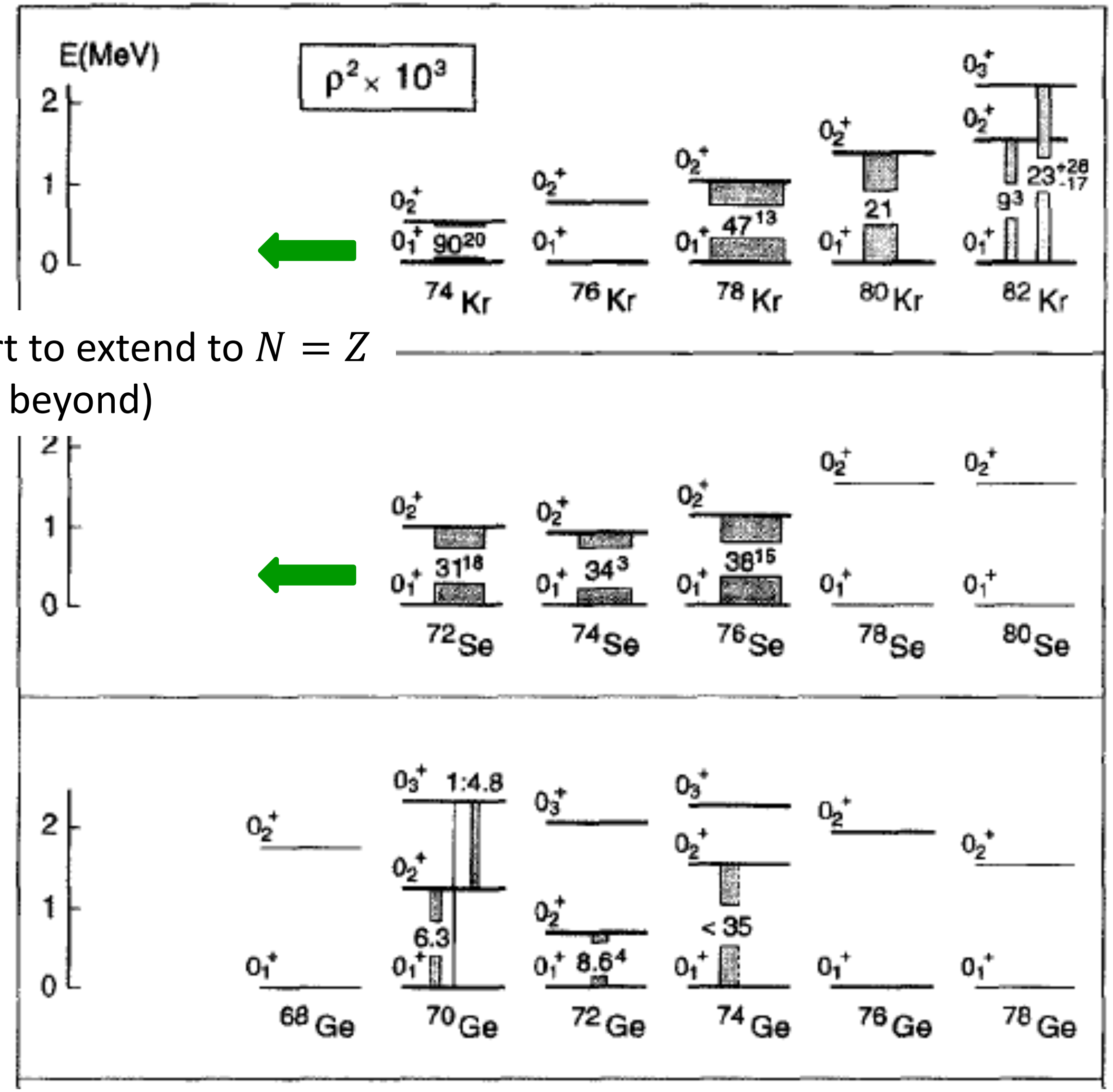
R. Lecomte et al.,
PRC 25, 2812 (1982)



- transition from spherical to deformed ground state with increasing N
- reduced cross section for $0_1^+ \rightarrow 0_1^+$ for ^{72}Ge ⇆ ^{74}Ge
- enhanced cross section for $0_1^+ \rightarrow 0_2^+$ where configurations cross

Electric monopole strengths

- low-lying 0_2^+ states and strong $E0$ transitions observed throughout the region



effort to extend to $N = Z$ (and beyond)

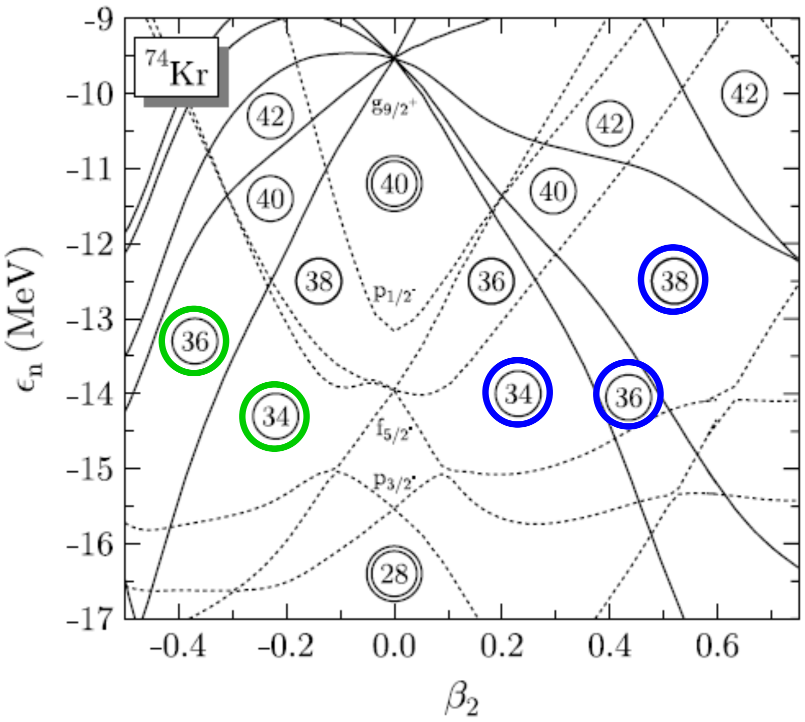
largest $\rho^2(E0)$ for ^{74}Kr

Ge and Se: min $E(0_2^+)$ and max $\rho^2(E0)$ for $Z \approx 40$.

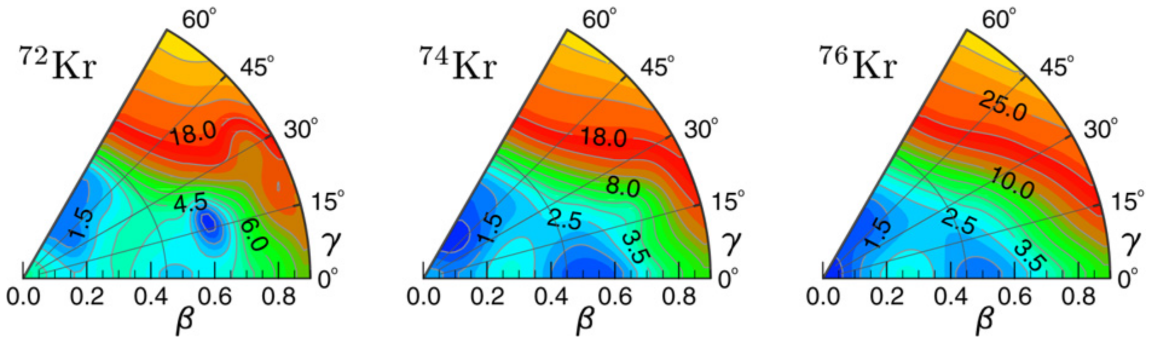
J.L. Wood et al.
Nucl. Phys. A 651, 323 (1999)

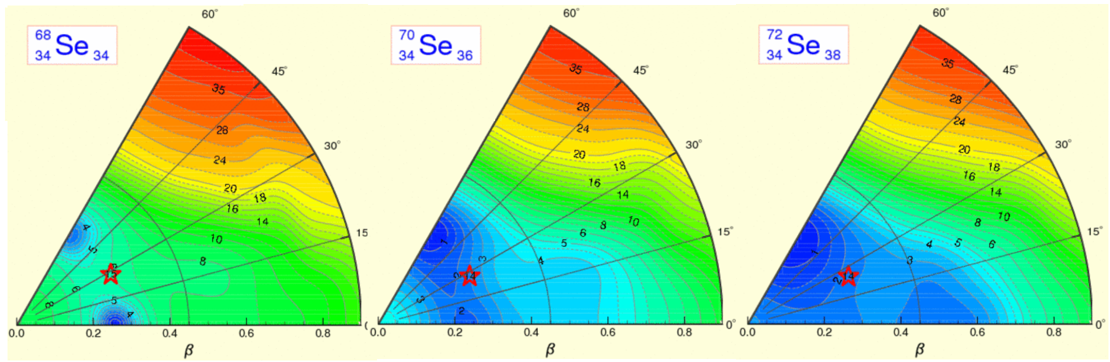
Deformed shell gaps and potential energies

- neutron single-particle energies (SLy6)
- potential energy surfaces (Gogny D1S)



M. Bender et al., PRC 74, 024312 (2006)



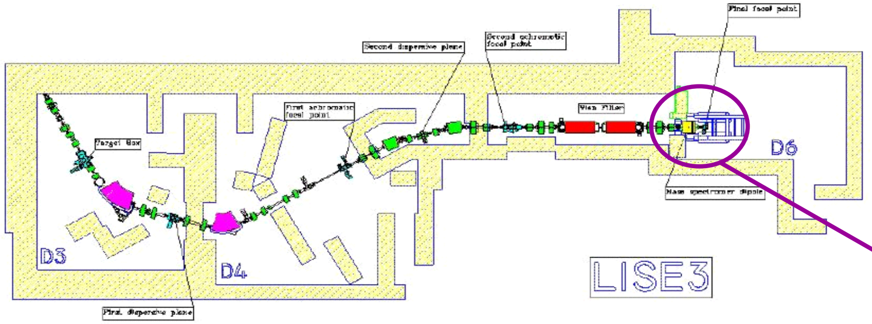


- competition between prolate and oblate shapes

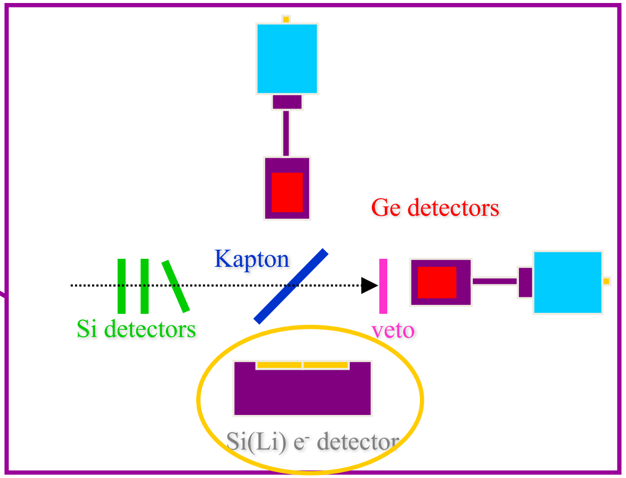
M. Girod et al., Phys. Lett. B 676, 39 (2009)
http://www-phynu.cea.fr/

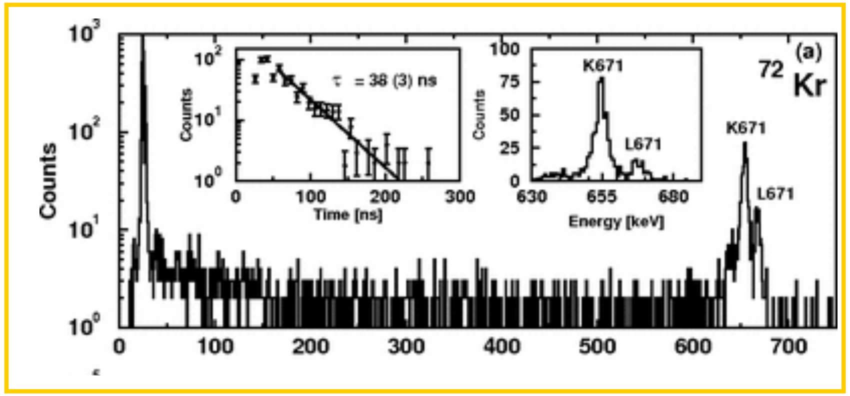
Shape isomers in ^{72}Kr and ^{74}Kr

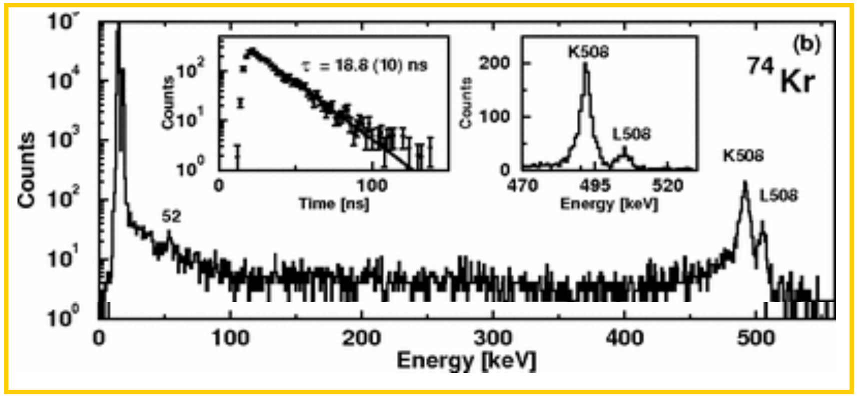
- isomer spectroscopy at GANIL / LISE

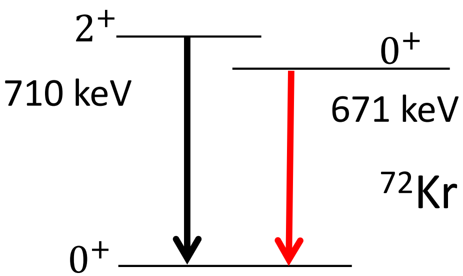


E. Bouchez et al.,
PRL 90, 082502 (2003)

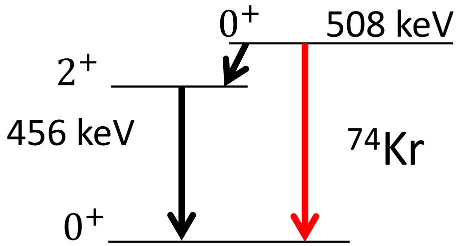








$\tau = 38(3)$ ns

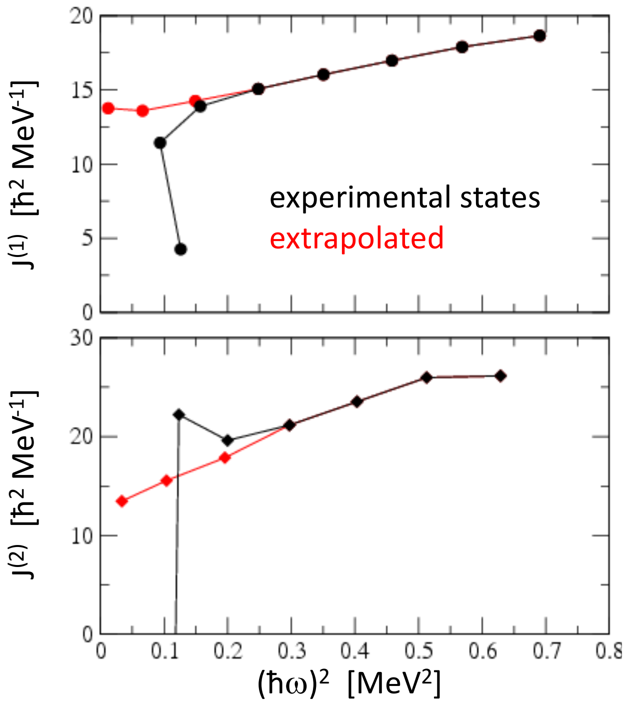


$\tau = 18.8(10)$ ns

$\rho^2(E0; 0_2^+ \to 0_1^+) = 72(6) \cdot 10^{-3}$

$\rho^2(E0; 0_2^+ \to 0_1^+) = 85(19) \cdot 10^{-3}$

Band mixing

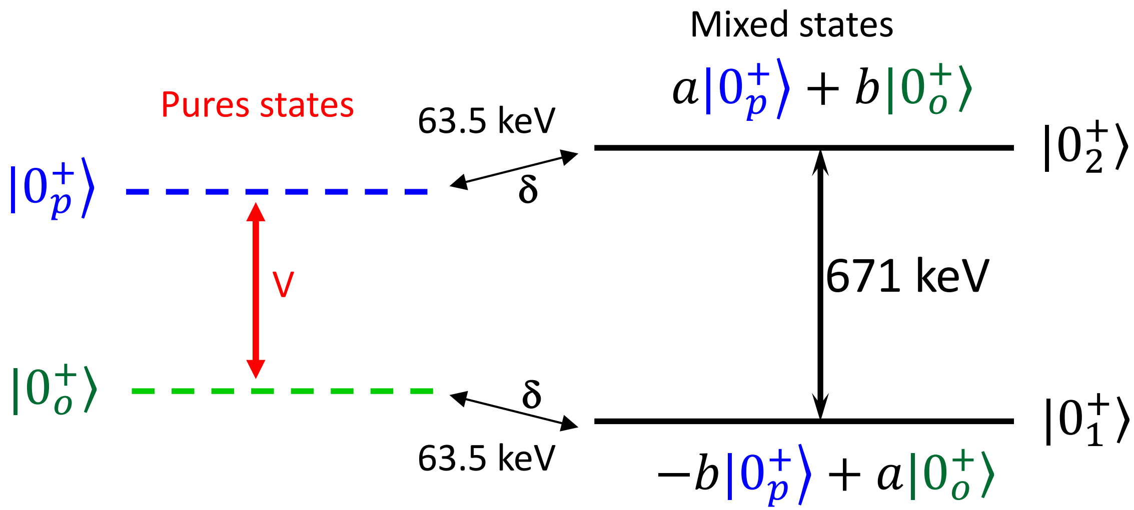


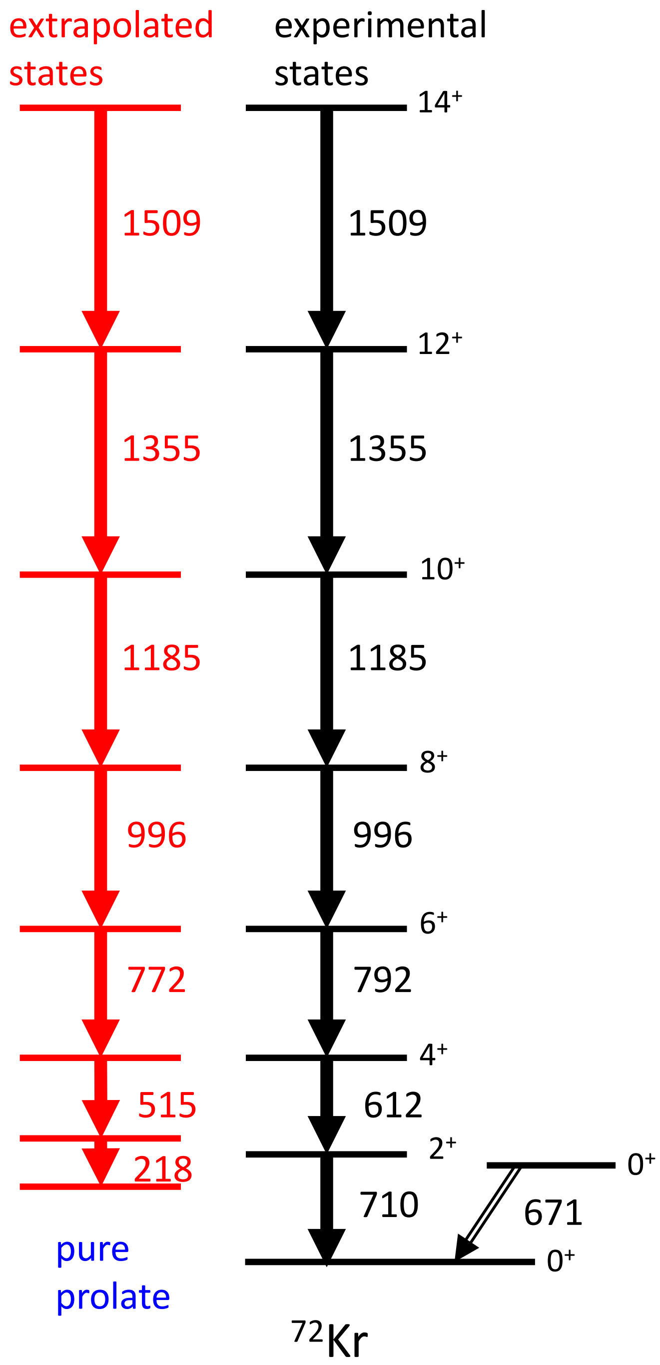
Regular rotational cascade at high spin:

$$E(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1)$$

Rotational band is distorted at low spin.
⇒ influence of mixing

- Interaction V
- mixing amplitudes a, b



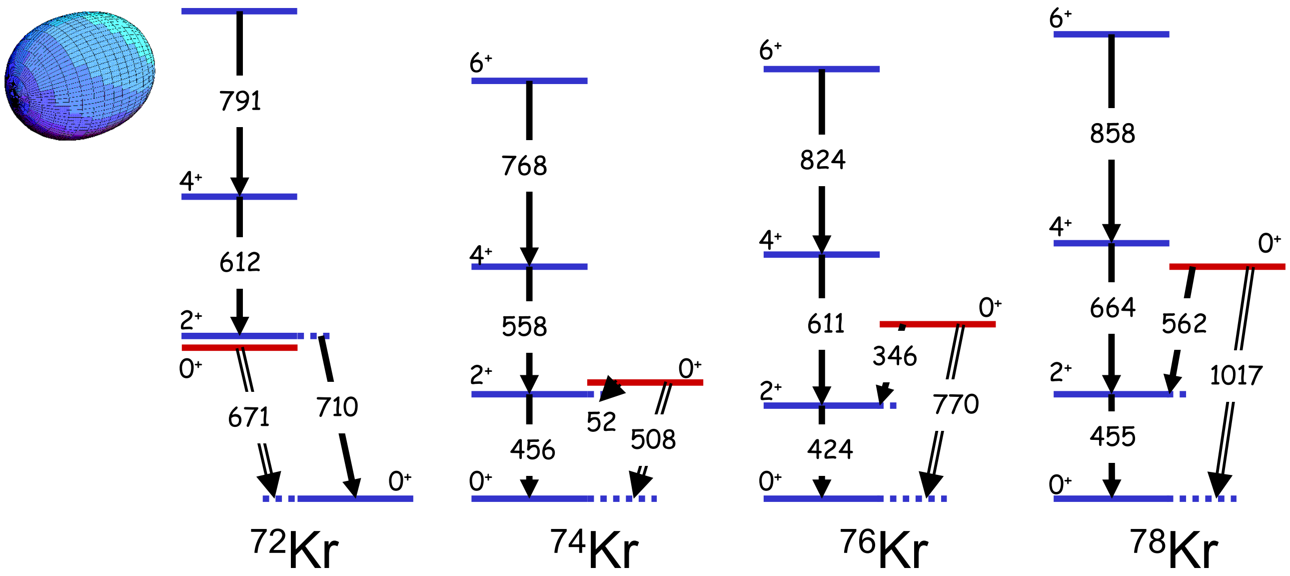


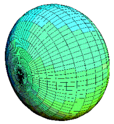
Systematics of the light krypton isotopes

- energy of excited 0^+
- E0 strengths $\rho^2(E0)$
- configuration mixing

- Inversion of ground state shape for ^{72}Kr

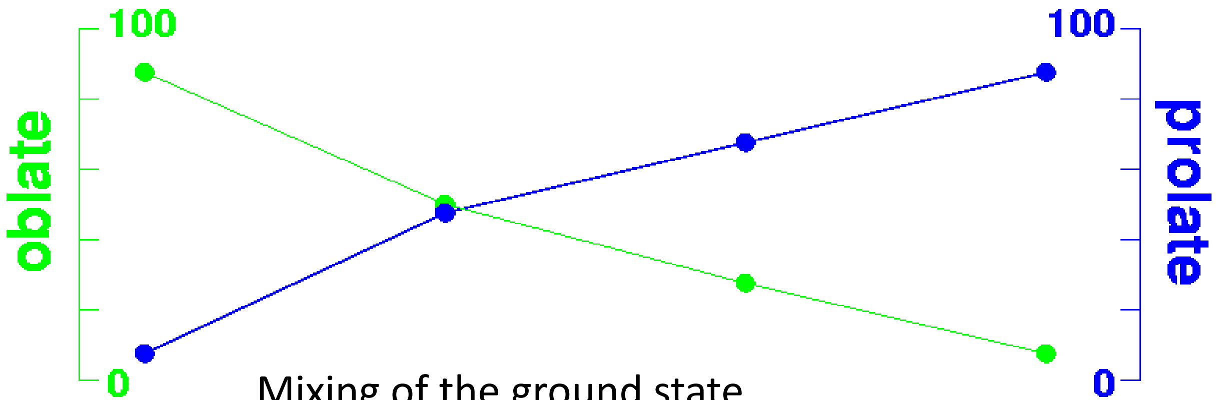
prolate





oblate

$\rho^2(E0)$ $70 \cdot 10^{-3}$ $91 \cdot 10^{-3}$ $79 \cdot 10^{-3}$ $47 \cdot 10^{-3}$



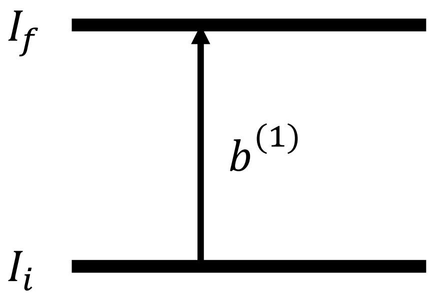
Mixing of the ground state
(from distortion of rotational bands)

- next step:
Coulomb excitation

E. Bouchez et. al.,
Phys. Rev. Lett. 90, 082502 (2003)

Second-order effects in Coulomb excitation

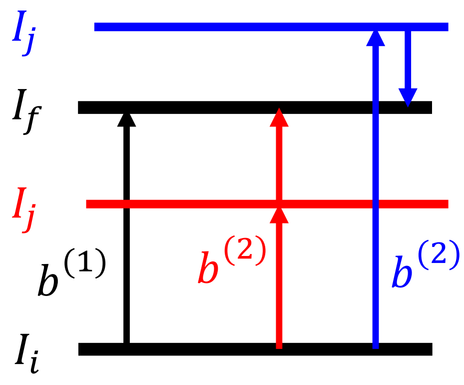
$$P_{i\to f} = \frac{1}{2I_i + 1}\left|b^{(1)}_{i\to f}\right|^2$$



1st order:

$$b^{(1)}_{i\to f} \propto \langle I_f|M(\sigma\lambda)|I_i\rangle$$

$$P_{i\to f} = \frac{1}{2I_i + 1}\left|b^{(1)}_{i\to f} + b^{(2)}_{i\to f}\right|^2$$



2nd order:

$$b^{(2)}_{i\to f} \propto \sum_j \langle I_f|M(\sigma\lambda)|I_j\rangle\langle I_j|M(\sigma\lambda)|I_i\rangle$$

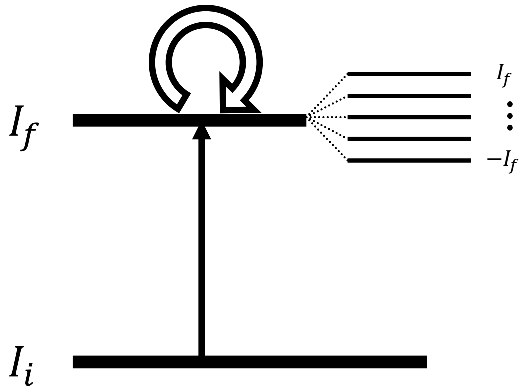
$P_{i\to f}$ depends on:

- transitional matrix element
  $B(\sigma\lambda; I_i \to I_f) \propto \left|\langle I_f|M(\sigma\lambda)|I_i\rangle\right|^2$
- diagonal matrix element
  $\langle I_f|M(\sigma\lambda)|I_f\rangle \propto Q_s(I_f)$

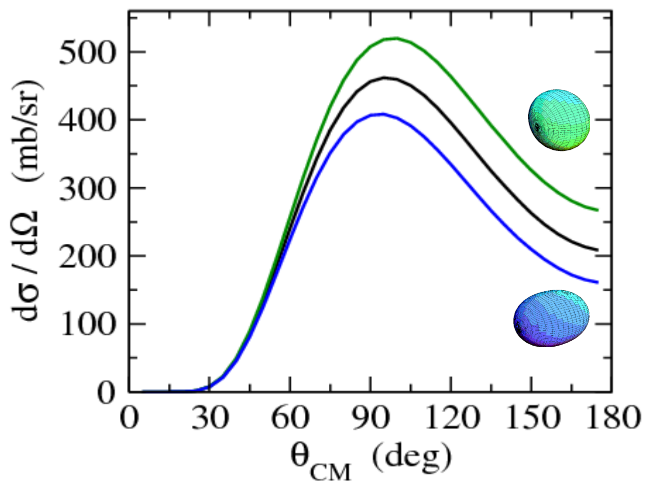
special case:
intermediate state
another magnetic substate

"reorientation effect"

$$b^{(2)}_{i\to f} \propto \langle I_f|M(\sigma\lambda)|I_f\rangle\langle I_f|M(\sigma\lambda)|I_i\rangle$$

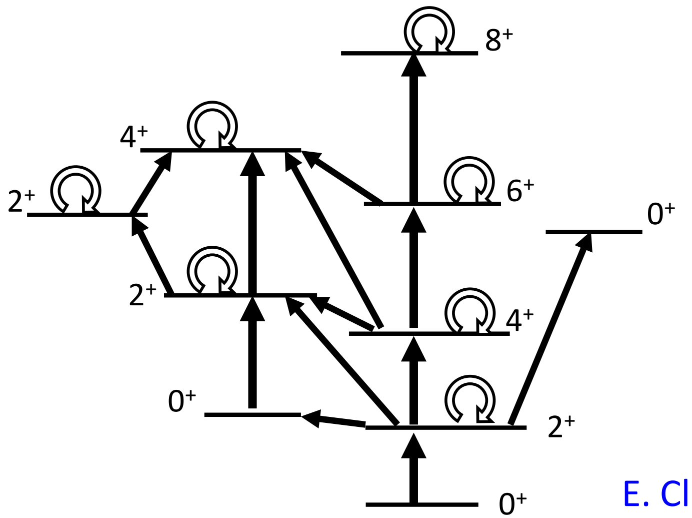


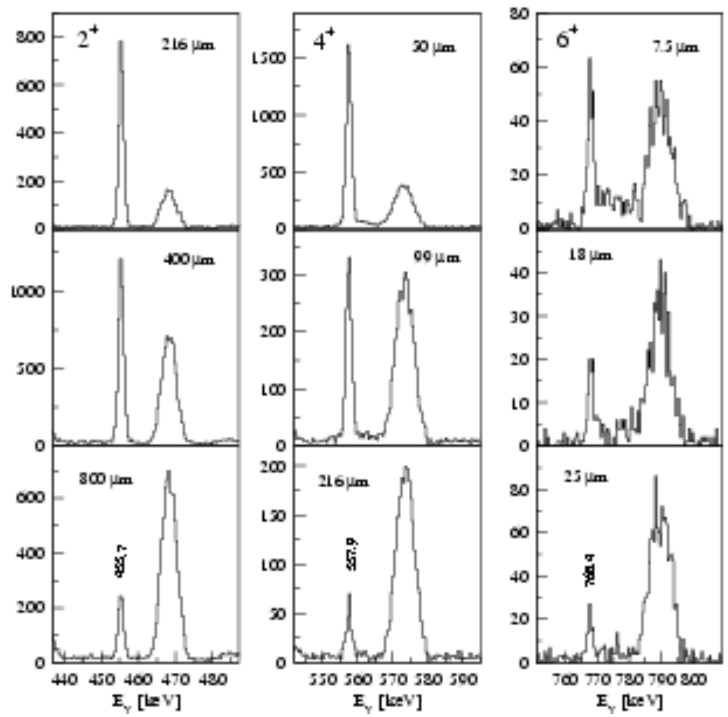
Coulomb excitation sensitive
to spectroscopic quadrupole moments



Multi-step Coulomb excitation of ^{74}Kr and ^{76}Kr at GANIL/SPIRAL

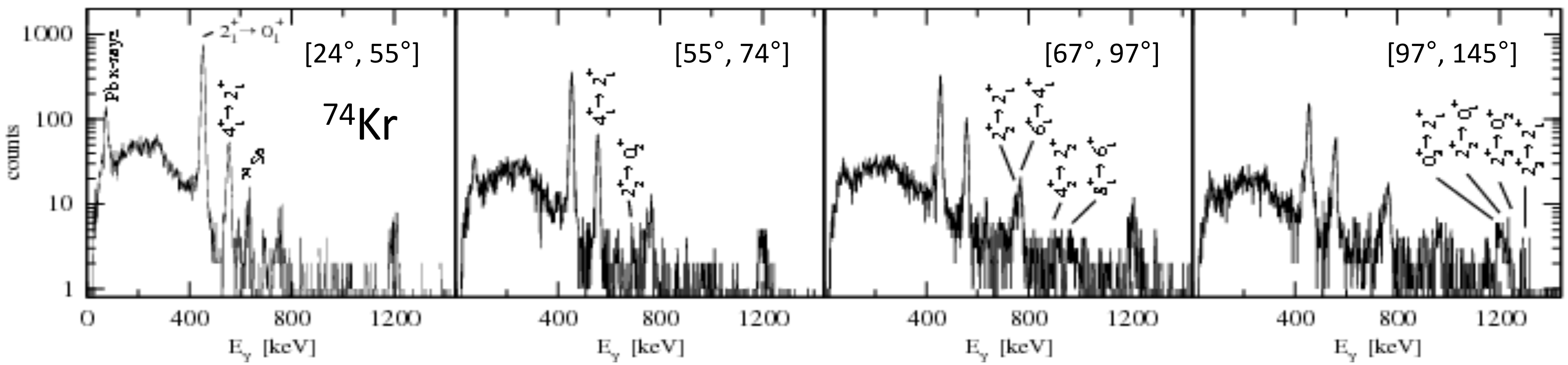
- γ yields as function of scattering angle: differential cross section
- least squares fit of ~ 30 matrix elements (transitional and diagonal)
- experimental spectroscopic data
  - lifetimes, branching and mixing ratios



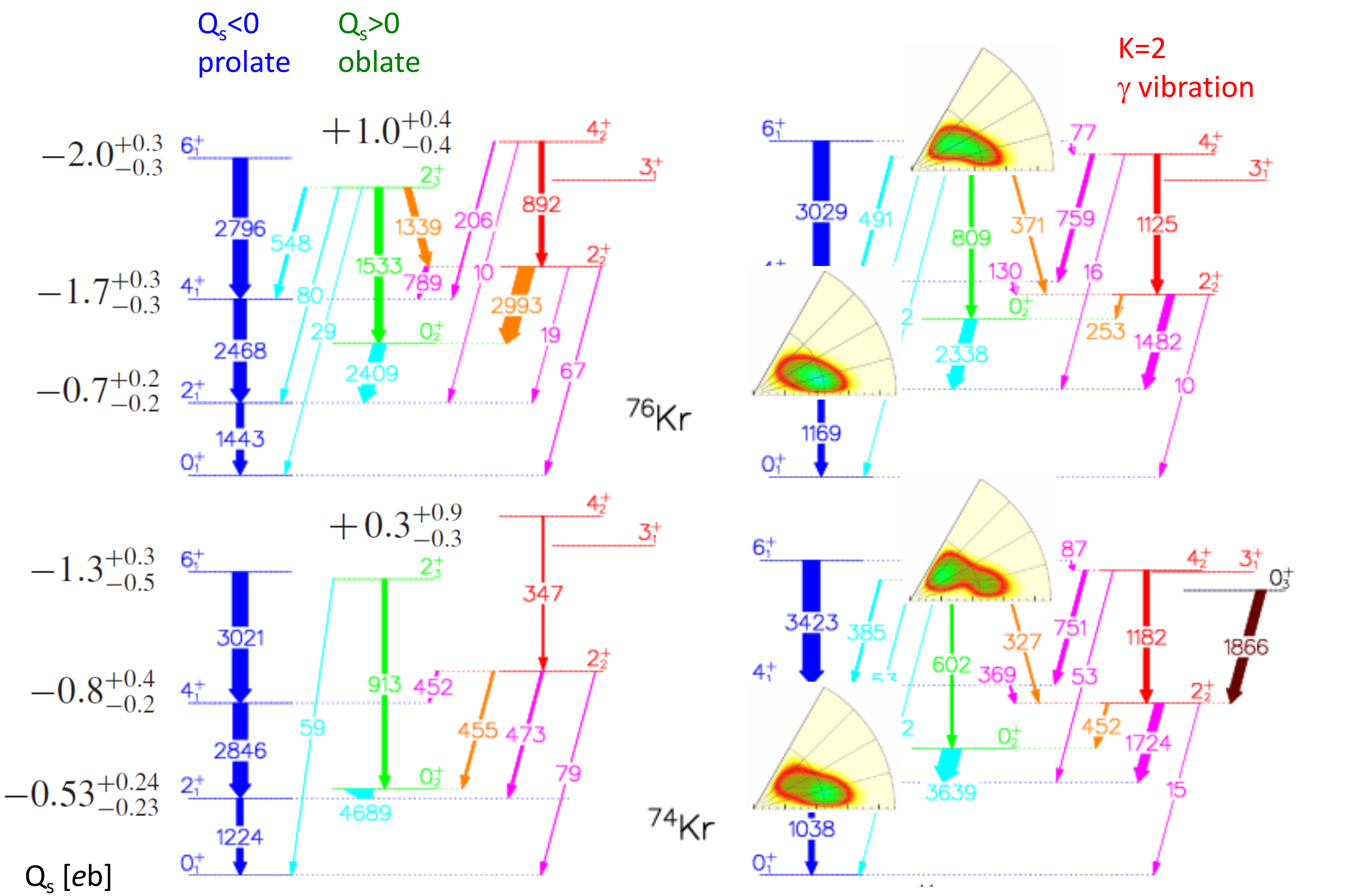


A. Görgen, Eur. Phys. J. A 26, 153 (2005)

E. Clément et al., PRC 75, 054313 (2007)



Prolate – oblate shape coexistence



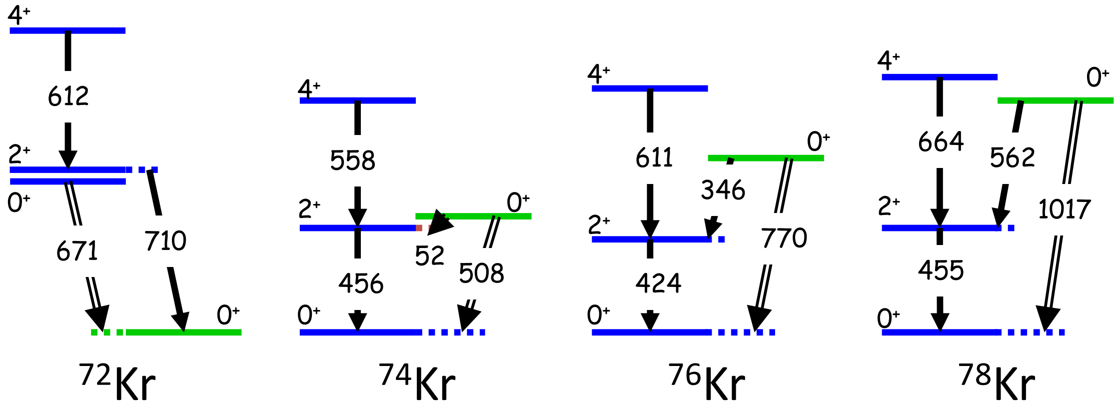
experimental B(E2;↓) [e^2fm^4]

E. Clément et al.,
PRC 75, 054313 (2007)

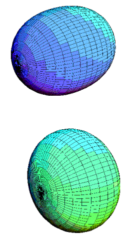
GCM (GOA) calculation with Gogny D1S
q_0, q_2: triaxial deformation

M. Girod et al., Phys. Lett. B 676, 39 (2009)

Shape transition in the light krypton isotopes

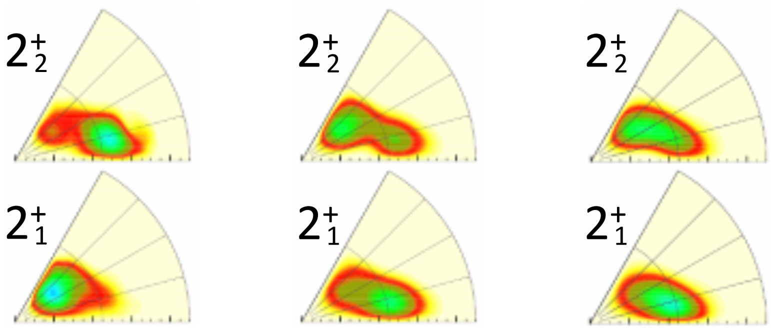


Experimental
(direct and indirect evidence)



prolate

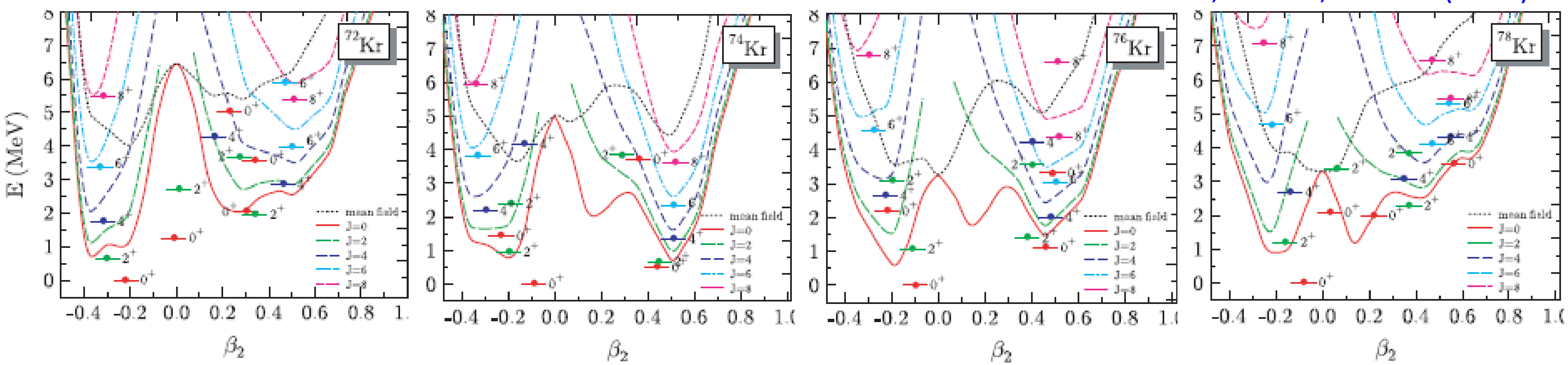
oblate



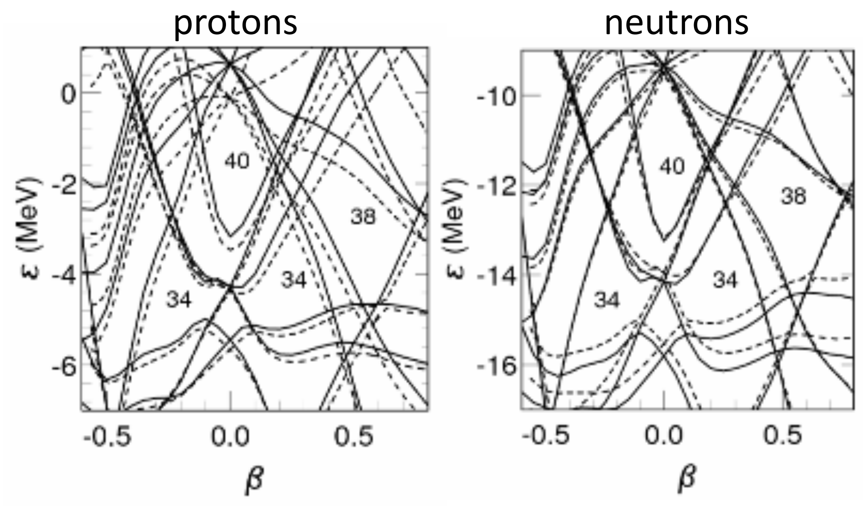
also mixing of K=0 and K=2 states

Gogny D1S 5D GCM (GOA):
M. Girod et al., PLB 676, 39 (2009)

Skyrme SLy6 GCM:
M. Bender et al., PRC 74, 024312 (2006)



Configuration mixing calculations



Difference #1: effective interaction

Skyrme SLy6 ⇔ **Gogny D1S**
Bender et al. Girod et al.

very similar single-particle energies
⇒ no big differences on the mean-field level

M. Girod et al., PLB 676, 39 (2009)

Difference #2: generator coordinates

axial quadrupole deformation q_0 ⇔ triaxial quadrupole deformation q_0, q_2
(exact GCM formalism)

Euler angles $\Omega=(\theta_1,\theta_2,\theta_3)$
⇒ 5-dimensional collective Hamiltonian
(Gaussian overlap approximation)

- good agreement for in-band B(E2) and quadrupole moments
- wrong ordering of states: oblate ground-state shape for ^{72}Kr → ^{78}Kr
- K=2 states outside model

- excellent agreement for excitation energies, B(E2), and quadrupole moments
- inversion of ground-state shape from prolate in ^{76}Kr to oblate in ^{72}Kr reproduced
- assignment of prolate, oblate, and K=2 states

⇒ triaxiality seems to be important to describe prolate-oblate shape coexistence in this region

Quadrupole sum rules

method to determine the **intrinsic shape** (also for 0^+)

rotate electric quadrupole tensor into principal axis frame:
⇒ only two non-zero quadrupole moments
⇒ two parameters (Q,δ) in analogy with Bohr's parameters (β,γ)

$$\mathcal{M}(E2,\mu=0) = Q\cos\delta$$
$$\mathcal{M}(E2,\mu=\pm1) = 0$$
$$\mathcal{M}(E2,\mu=\pm2) = \frac{1}{\sqrt{2}}Q\sin\delta$$

zero-coupled products of the E2 operators are rotationally invariant:

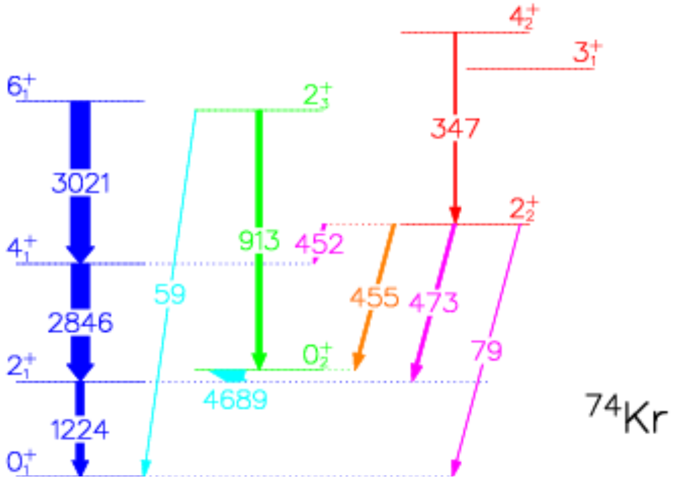
$$\langle s|[E2\times E2]_0|s\rangle = \frac{1}{\sqrt{5}}Q^2 = \frac{(-1)^{2s}}{\sqrt{2s+1}}\sum_t \langle s||E2||t\rangle\langle t||E2||s\rangle \left\{ \begin{array}{ccc} 2 & 2 & 0 \\ s & s & t \end{array} \right\}$$

$$\langle s|[[E2\times E2]_2\times E2]_0|s\rangle = -\sqrt{\frac{2}{35}}Q^3\cos(3\delta) = \frac{1}{2s+1}\sum_{tu}\langle s||E2||t\rangle\langle t||E2||u\rangle\langle u||E2||s\rangle \left\{ \begin{array}{ccc} 2 & 2 & 2 \\ s & t & u \end{array} \right\}$$

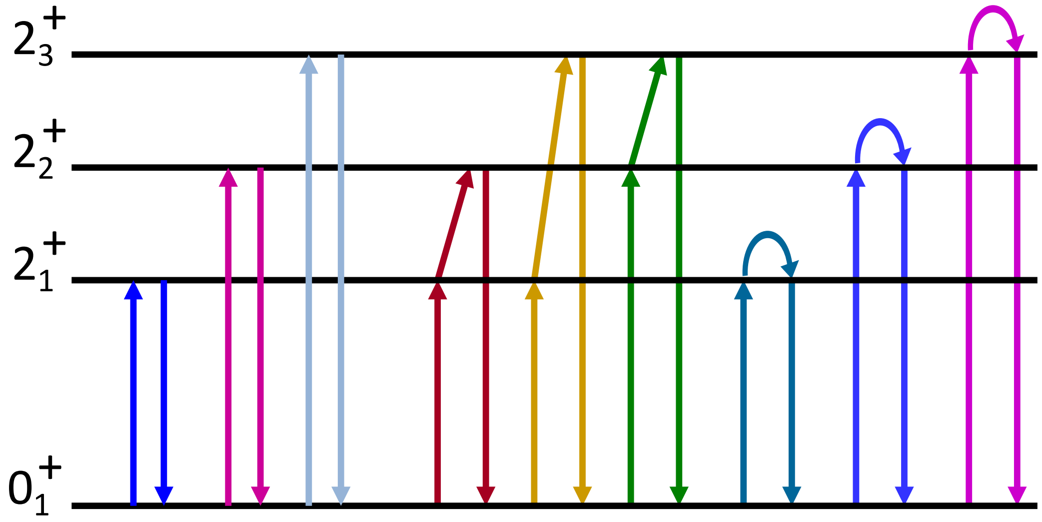
D. Cline, Ann. Rev. Nucl. Part. Sci. 36, 683 (1986)

- model-independent method to determine the intrinsic quadrupole shape from a set of E2 matrix elements

Example for ^{74}Kr, ^{76}Kr



experimental B(E2;↓) [e^2fm^4]

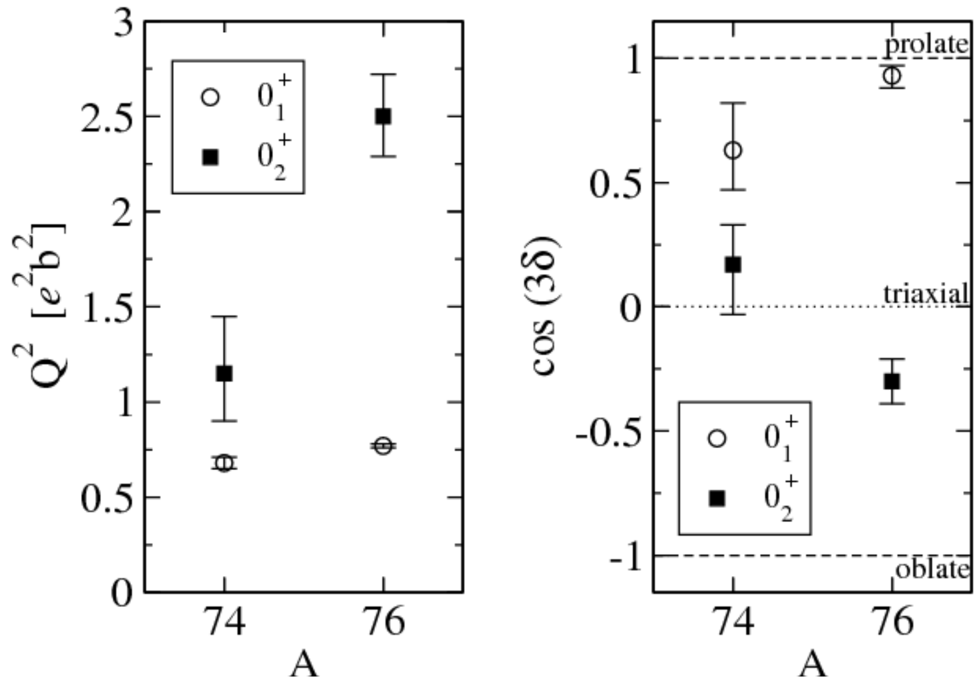


$$\sum_t \langle s||E2||t\rangle \langle t||E2||s\rangle \begin{Bmatrix} 2 & 2 & 0 \\ s & s & t \end{Bmatrix}$$

$$\sum_{tu} \langle s||E2||t\rangle \langle t||E2||u\rangle \langle u||E2||s\rangle \begin{Bmatrix} 2 & 2 & 2 \\ s & t & u \end{Bmatrix}$$

needs complete set of matrix elements

⇒ usually only feasible for 0^+ states



Matrix elements from Coulomb excitation

$$|0_1^+\rangle = a_0|0_p^+\rangle + b_0|0_o^+\rangle \qquad |2_1^+\rangle = a_2|0_p^+\rangle + b_2|0_o^+\rangle$$

$$|0_2^+\rangle = -b_0|0_p^+\rangle + a_0|0_o^+\rangle \qquad |2_2^+\rangle = -b_2|0_p^+\rangle + a_2|0_o^+\rangle$$

$$a_0^2 + b_0^2 = 1 \qquad a_2^2 + b_2^2 = 1$$

no transitions between intrinsic prolate and oblate states:

$$\langle I_p^+|E2|J_o^+\rangle = 0$$

4 equations:

$$\langle 2_1^+|E2|0_1^+\rangle = b_0 b_2\langle 2_o^+|E2|0_o^+\rangle + a_0 a_2\langle 2_p^+|E2|0_p^+\rangle$$
$$\langle 2_1^+|E2|0_2^+\rangle = a_0 b_2\langle 2_o^+|E2|0_o^+\rangle - a_2 b_0\langle 2_p^+|E2|0_p^+\rangle$$
$$\langle 2_2^+|E2|0_1^+\rangle = a_2 b_0\langle 2_o^+|E2|0_o^+\rangle - a_0 b_2\langle 2_p^+|E2|0_p^+\rangle$$
$$\langle 2_2^+|E2|0_2^+\rangle = a_0 a_2\langle 2_o^+|E2|0_o^+\rangle + b_0 b_2\langle 2_p^+|E2|0_p^+\rangle$$

4 unknowns:

2 mixing amplitudes: a_0, a_2

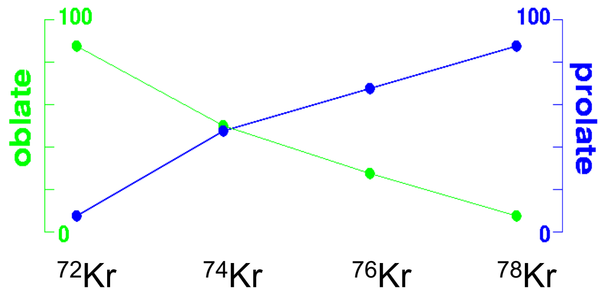
$$Q_{0,p} = \sqrt{\frac{16\pi}{5}}\langle 2_p^+|E2|0_p^+\rangle$$

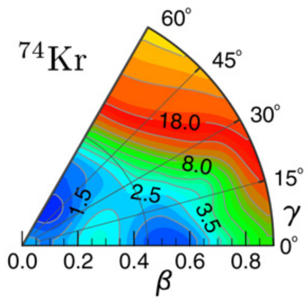
$$Q_{0,o} = \sqrt{\frac{16\pi}{5}}\langle 2_o^+|E2|0_o^+\rangle$$

using experimental coulex matrix elements for ^{74}Kr:

$$a_0^2 = 0.48(2)$$
$$a_2^2 = 0.82(20)$$
$$Q_{0,p} = 3.62(48)\ eb$$
$$Q_{0,o} = -0.66(86)\ eb$$

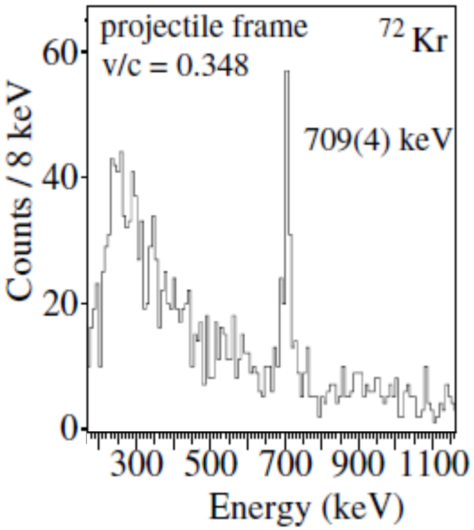
E. Clément et al., PRC 75, 054313 (2007)



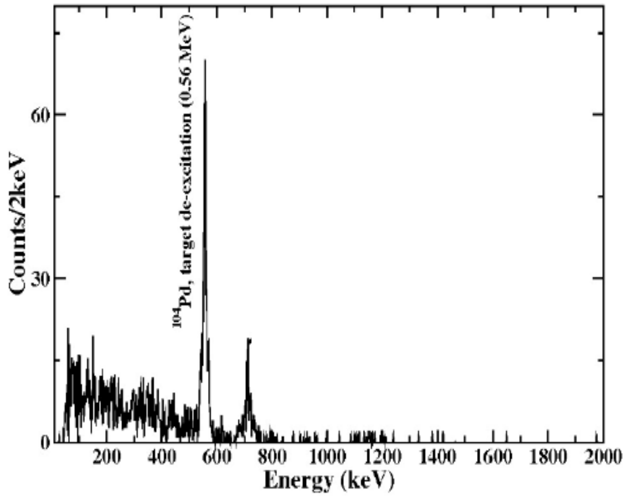


consistent with mixing obtained from E0 transitions

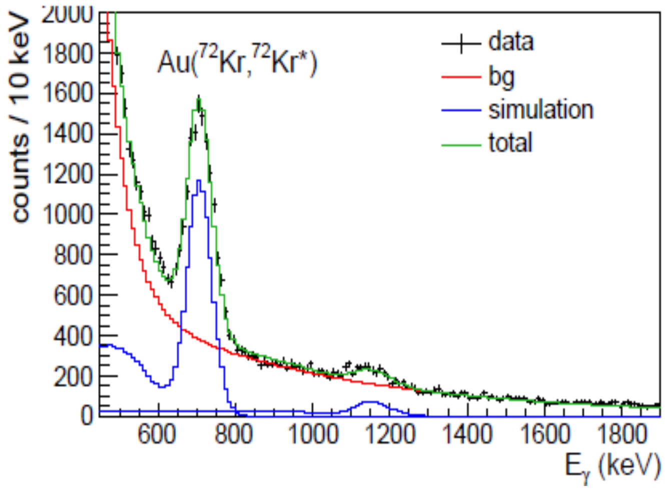
$B(E2;0^+ \rightarrow 2^+)$ measurements for ^{72}Kr



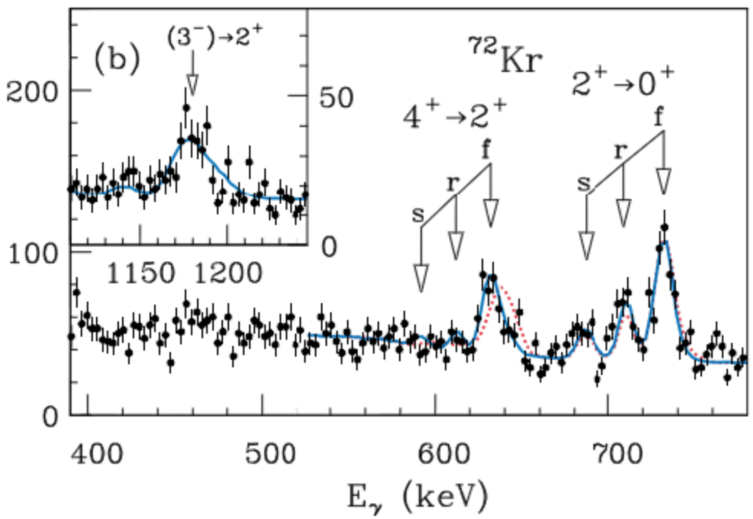
Au(^{72}Kr,^{72}Kr'), 70 MeV/u, MSU
A.Gade et al., PRL 95, 022502 (2005)



^{104}Pd(^{72}Kr,^{72}Kr'), 2.8 MeV/u
ISOLDE IS478 (2012)
B.S.Nara Singh et al. (unpublished)



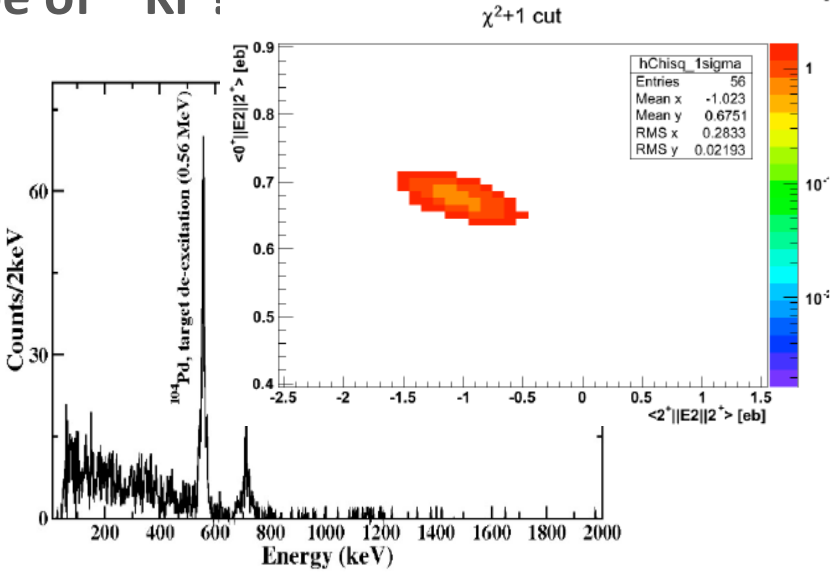
Au(^{72}Kr,^{72}Kr'), 150 MeV/u, RIKEN
T.Arici, PhD Univ. Giessen (2017)



^{9}Be(^{74}Kr,^{72}Kr), RDDS lifetime, MSU
H.Iwasaki et al., PRL 112, 142502 (2014)

$B(E2;0^+ \rightarrow 2^+)$ [e^2fm^4]	
4997(647)	Gade
4600(600)	Nara Singh
4050(750)	Iwasaki
4910(500)	Arici

Shape of ^{72}Kr ?

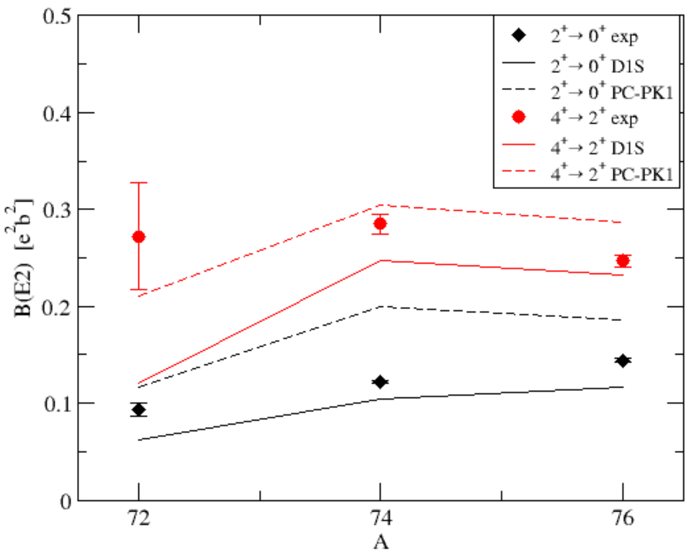


B.S.Nara Singh, HIE-ISOLDE Workshop 2016

high-energy Coulomb excitation (MSU, RIKEN)
- not sensitive to Q_s

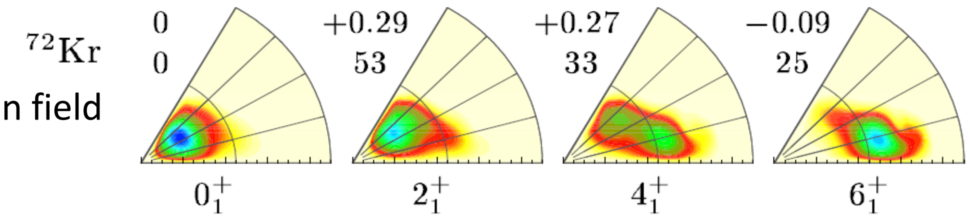
low-energy Coulomb excitation (ISOLDE)
- sensitive to $Q_s(2^+)$
- very low statistics
- favors prolate shape for 2^+ state

lifetime measurement (MSU)
- not sensitive to $Q_s(2^+)$
- large $B(E2; 4^+ \rightarrow\ 2^+)/B(E2; 2^+ \rightarrow\ 0^+)$



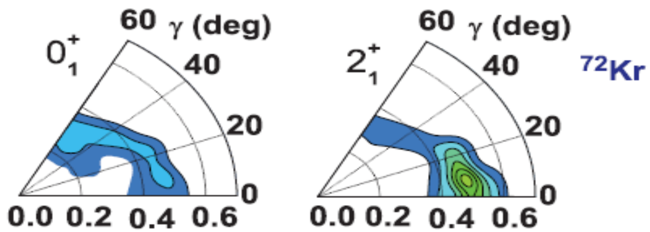
[74,76]Kr: E.Clément et al., PRC 75, 054313 (2007)
[72]Kr: H.Iwasaki et al., PRL 112, 142502 (2014)

beyond mean field
Gogny D1S
5DCH



M.Girod et al., PLB 676, 39 (2009)

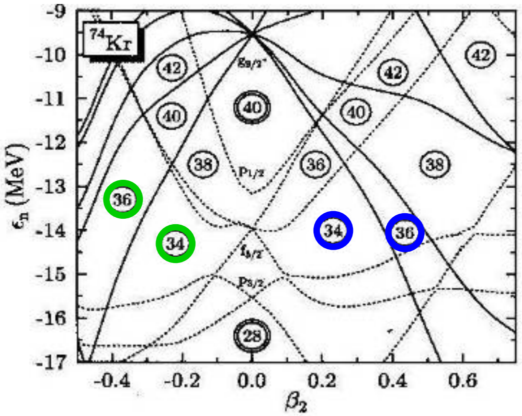
beyond mean field
RMF PC-PK1
5DCH

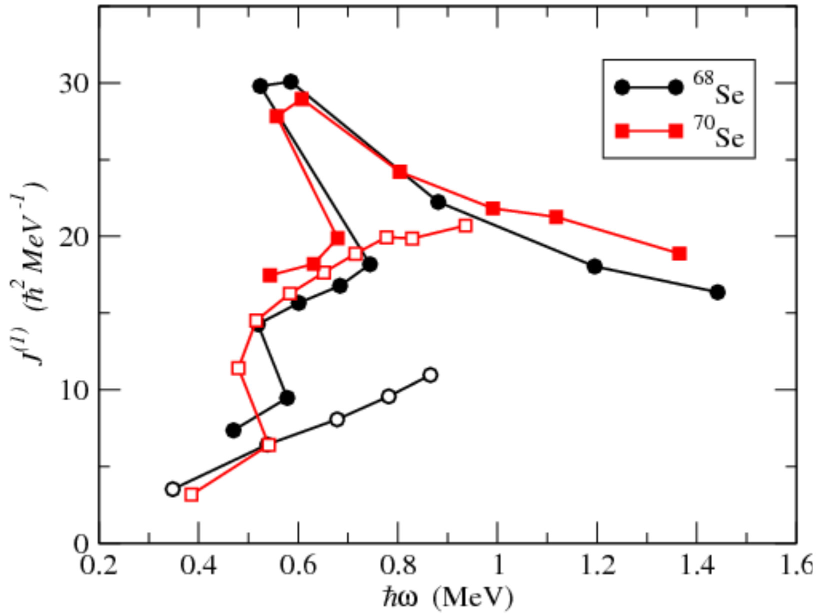


Y.Fu et al., PRC 87, 054305 (2013)

- rapid transition from oblate ground state to prolate 2^+ in ^{72}Kr ?

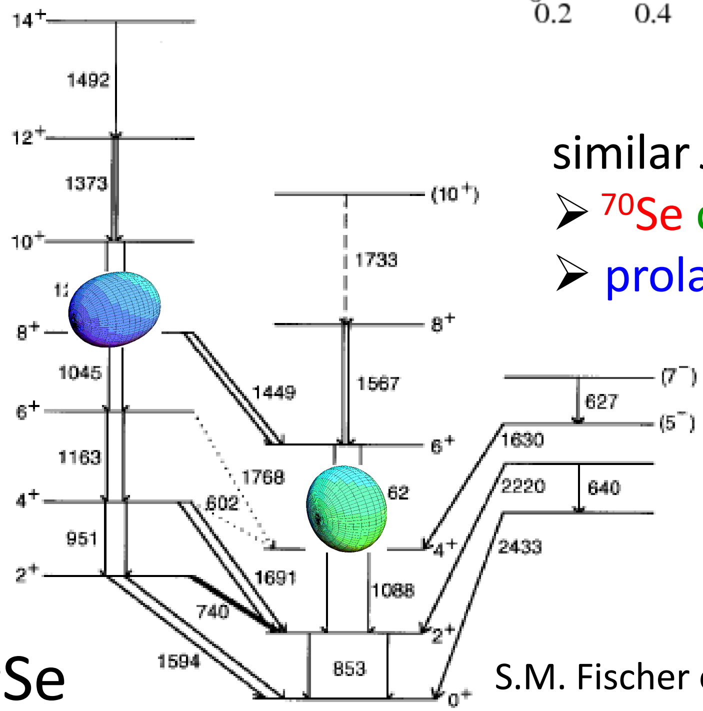
Shape coexistence in light Selenium isotopes





similar $J^{(1)}$ in ^{68}Se and ^{70}Se:

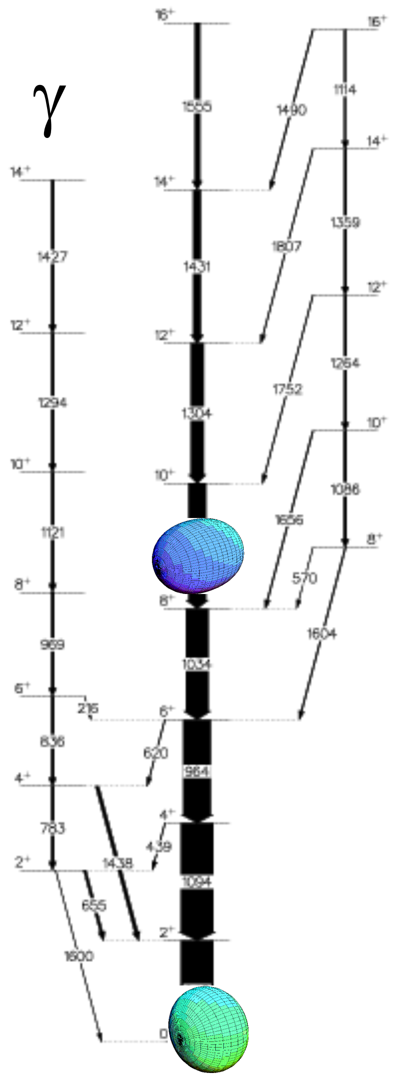
- ^{70}Se oblate near ground state
- prolate at higher spin



^{68}Se

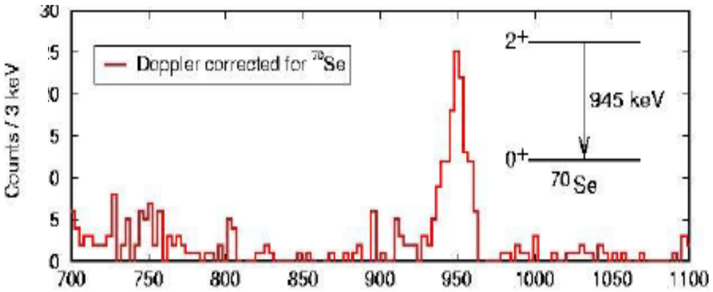
S.M. Fischer et al.,
PRC 67, 064318 (2003)

^{70}Se $\pi(g_{9/2})^2 \otimes \nu(g_{9/2})^2$

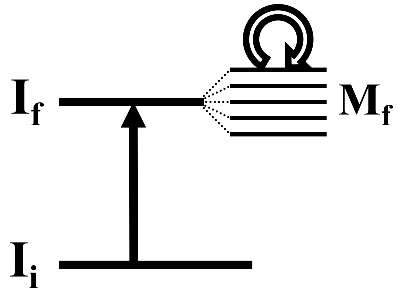


G. Rainovski et al.,
J.Phys.G 28, 2617 (2002)

The shape of ^{70}Se

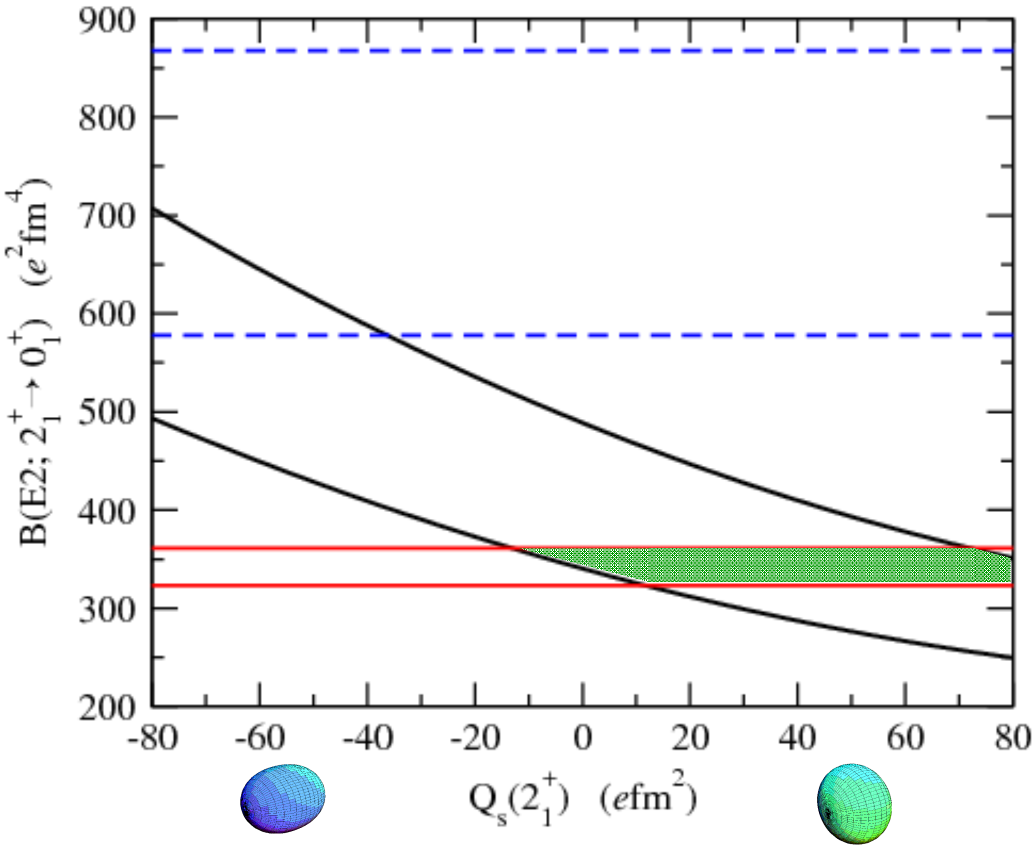


- Coulomb excitation at ISOLDE
- ^{70}Se on ^{104}Pd at 2.94 MeV/u
- integral measurement
- depends on **B(E2)** and **Q_s**
- one measurement, two unknowns !



A.M. Hurst et al.,
PRL 98, 072501 (2007)

- new lifetime measurement

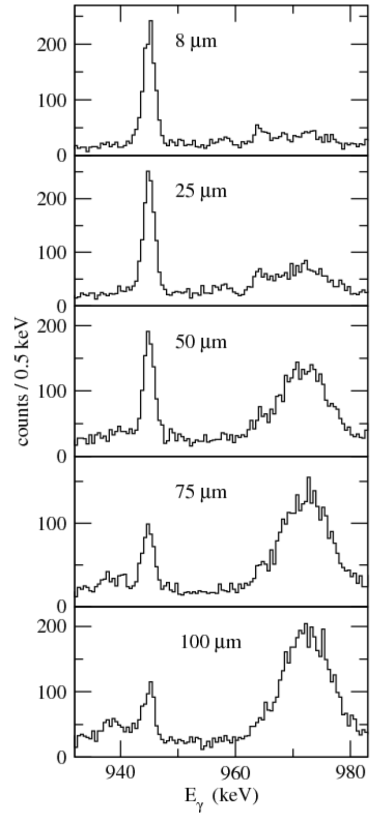


τ = 1.5(3) ps

J. Heese et al.,
Z. Phys. A 325, 45 (1986)

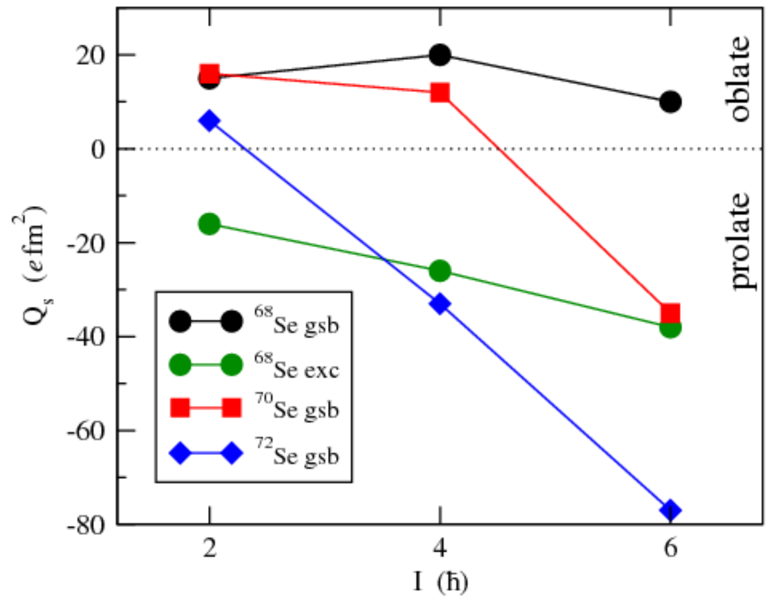
$\tau = 3.2(2)$ ps

J. Ljungvall et al.,
PRL 100, 102502 (2008)



Shape evolution in the light Selenium isotopes

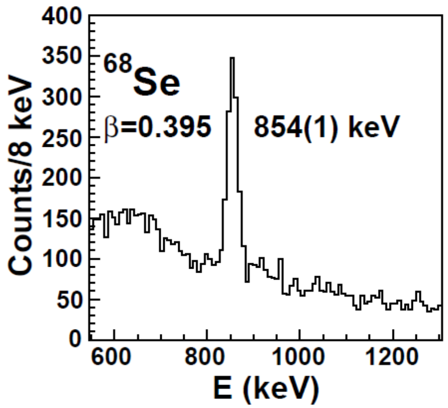
- oblate rotation prevails only in ^{68}Se
  $\Rightarrow$ best example for shape coexistence in A=70 region?
- need experimental quadrupole moments
- **where is the 0_2^+ state?**



Q_s from Gogny D1S calculation

intermediate-energy Coulomb excitation MSU

$B(E2; 2_1^+ \rightarrow 0_1^+) = 432(58)\ e^2\text{fm}^4$

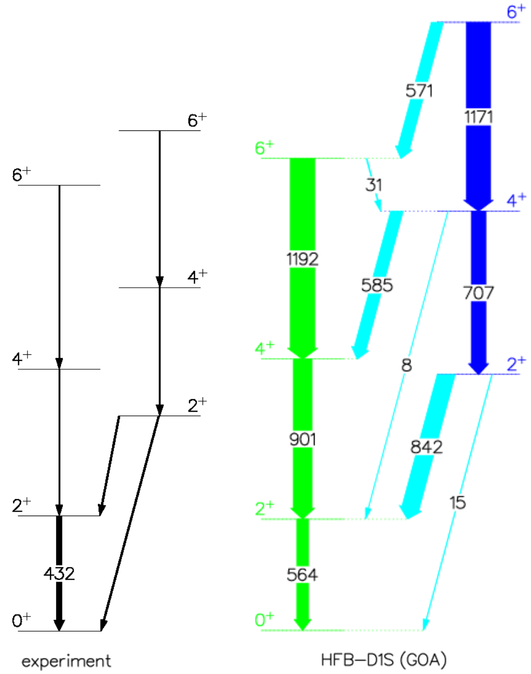


A. Obertelli et al.
PRC 80 (2009) 031304(R)

lifetime measurement at MSU

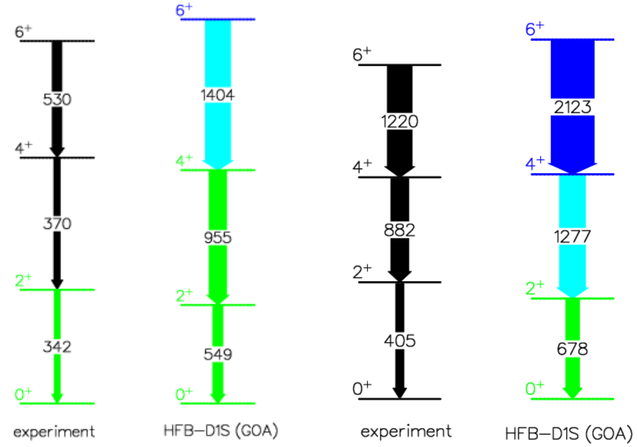
$B(E2;\downarrow) = 392(70)\ e^2\text{fm}^4$

A.J. Nichols et al., PLB 733, 52 (2014)



^{68}Se

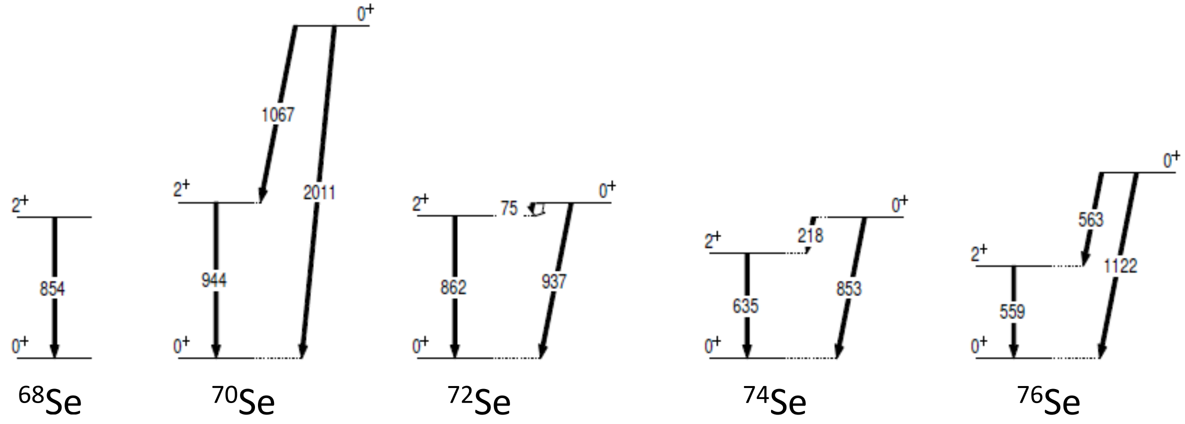
B(E2,↓) in e^2fm^4

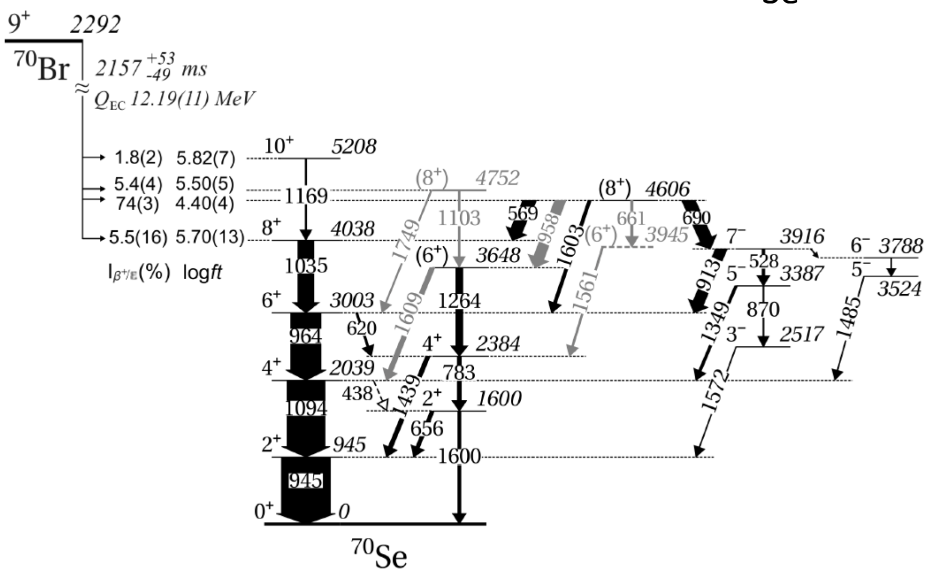


^{70}Se ^{72}Se

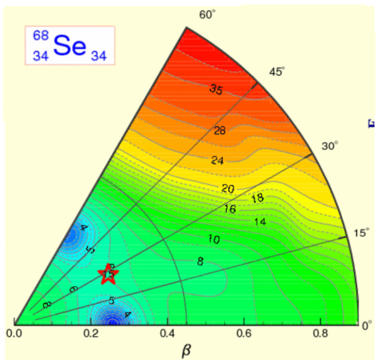
0_2^+ states in Se isotopes

- spectroscopy of ^{70}Se following β decay of ^{70}Br at RIKEN
- no sign of low-lying 0_2^+ state





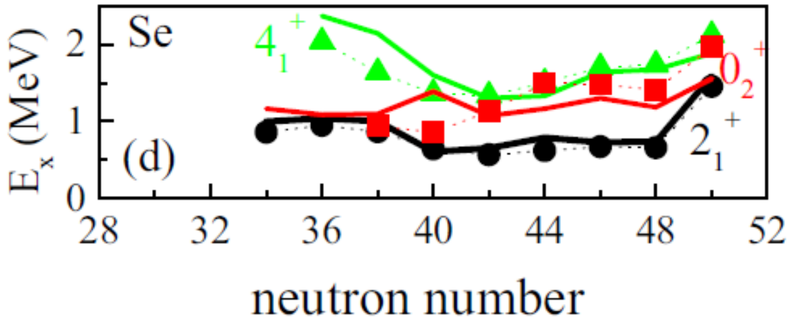
A.I. Morales et al.
Phys.Rev. C 95, 064327 (2017)



- isomer spectroscopy at GANIL / LISE
- 0_2^+ state near or below 2_1^+ state excluded

E. Clément et al., NIM A 587, 292 (2008)

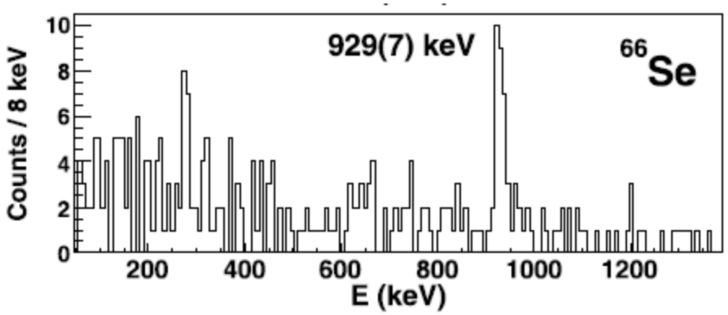
- shell model predicts 0_2^+ states near 2_1^+ for ^{68}Se and ^{70}Se



K. Kaneko et al., PRC 92, 044331 (2015)

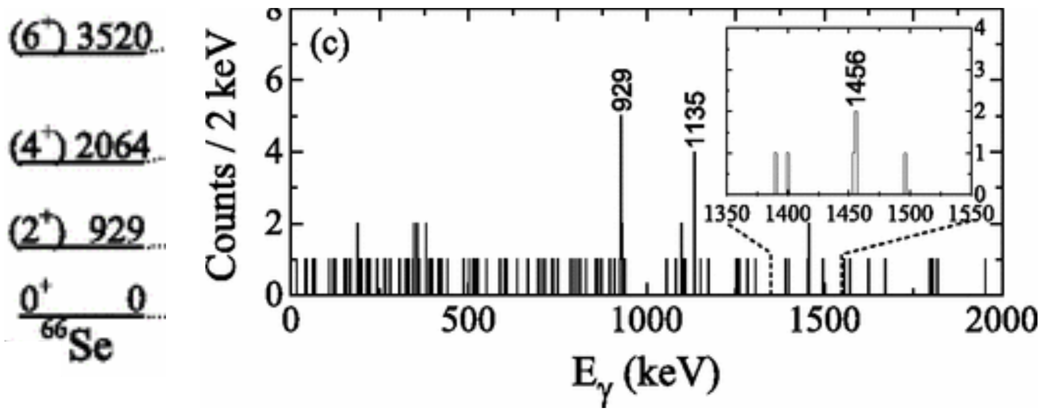
Beyond $N = Z$: Se

- ^{9}Be(^{68}Se,^{66}Se) @ 78 MeV
- two-nucleon knockout @ MSU
- 2_1^+ identified

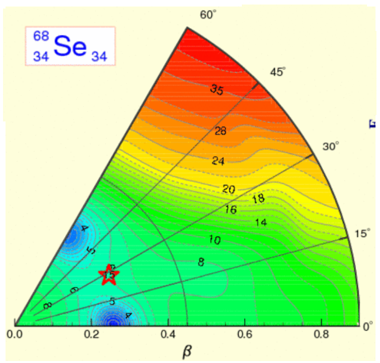


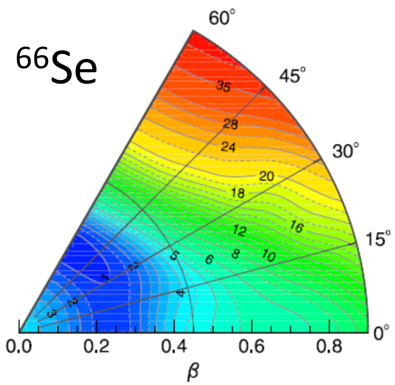
A. Obertelli et al.,
Phys. Lett. B 701, 417 (2011)

- β-decay tagging @ Jyväskylä
- candidates for ground-state band up to 6$^+$



P. Ruotsalainen et al.,
Phys.Rev. C 88, 041308 (2013)

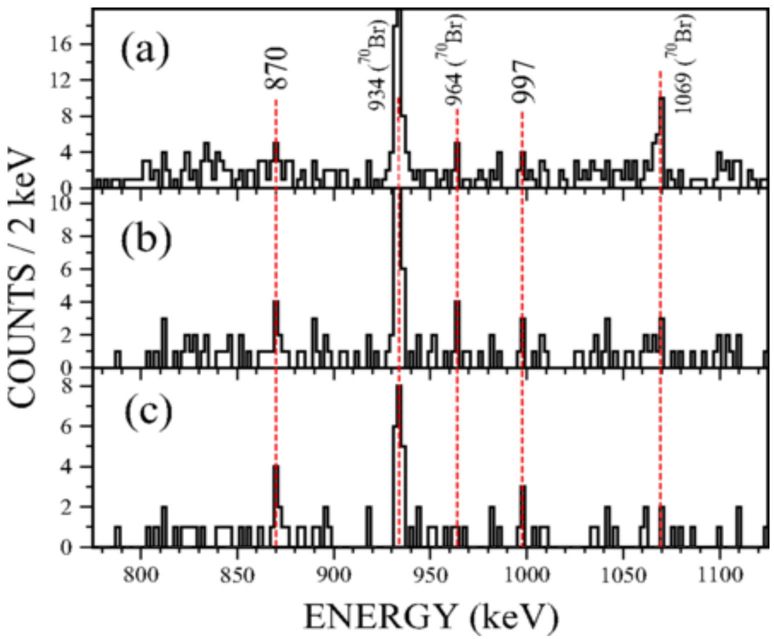




- PES (Gogny D1S) suggests vanishing barrier between oblate and prolate minima
- γ softness
- more experimental data needed

Beyond $N = Z$: Kr

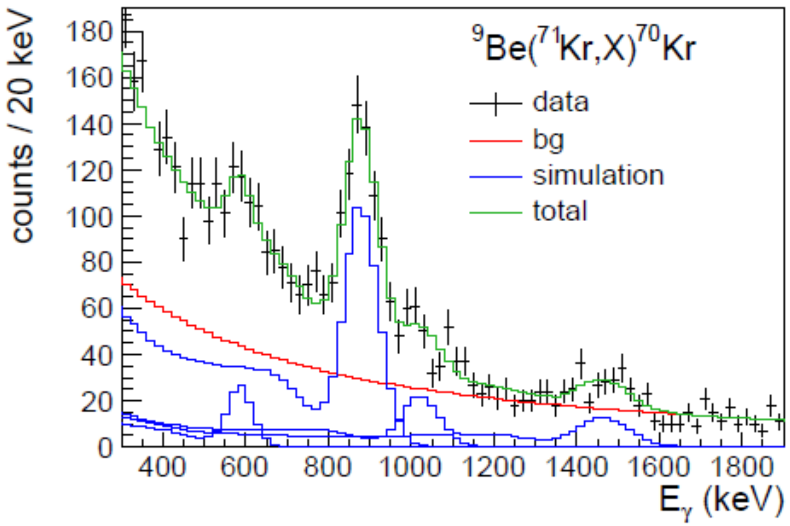
- $^{40}Ca(^{32}S,2n)^{70}Kr$
- β-decay tagging Jyväskylä
- candidates for 2^+ and 4^+ states

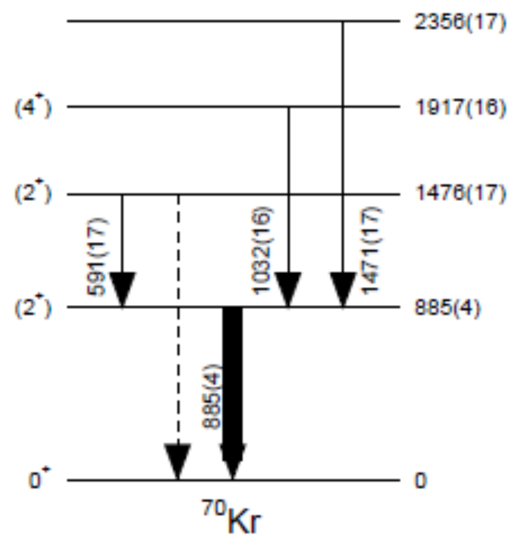


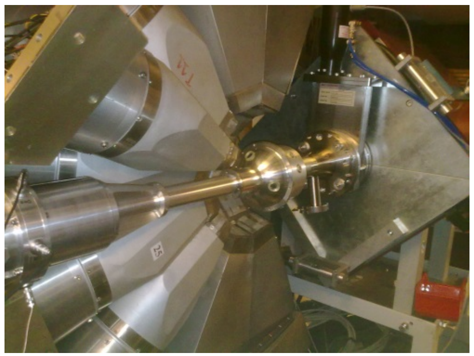
D.M. Debenham *et al.*, PRC 94, 054311 (2016)

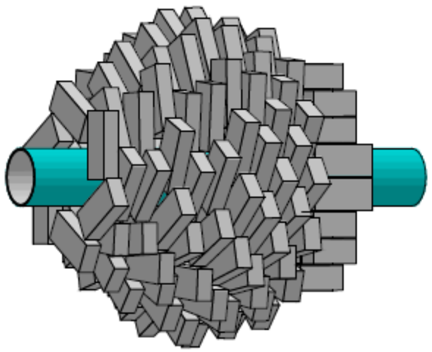
- spectroscopy following knockout @ RIKEN

T. Arici, PhD Univ. Giessen (2017)





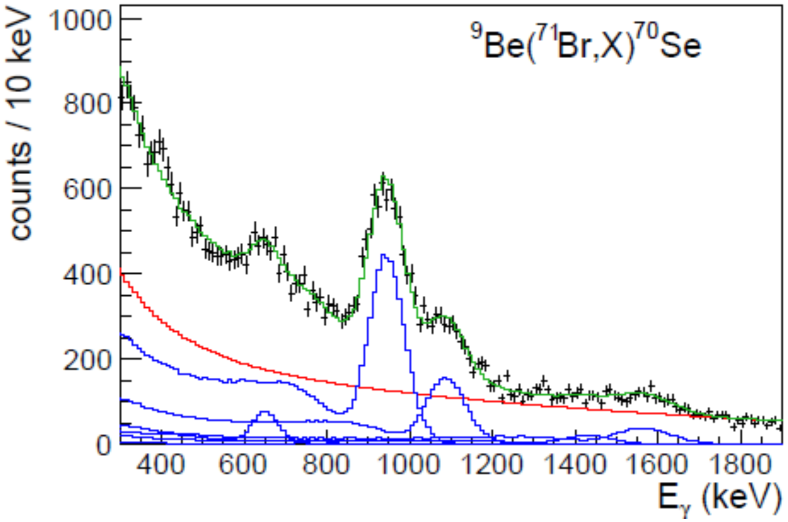


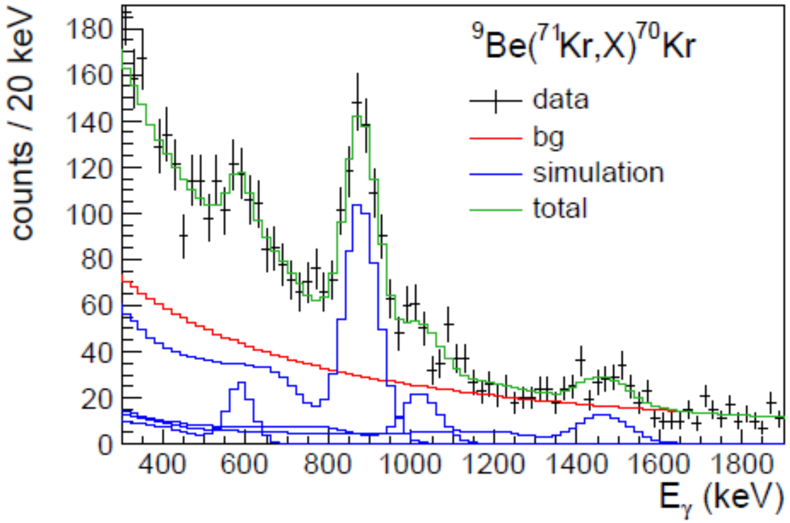


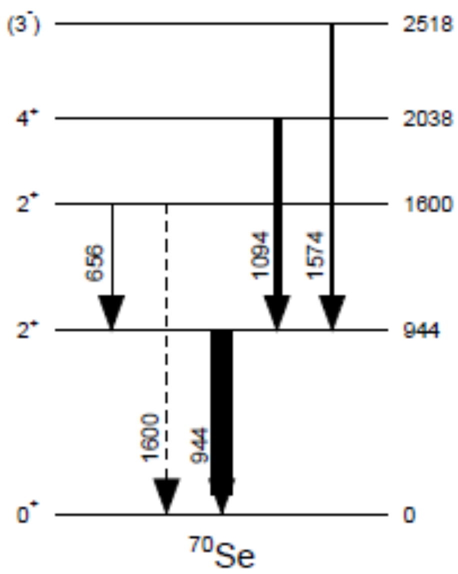
Mirror symmetry ^{70}Se - ^{70}Kr

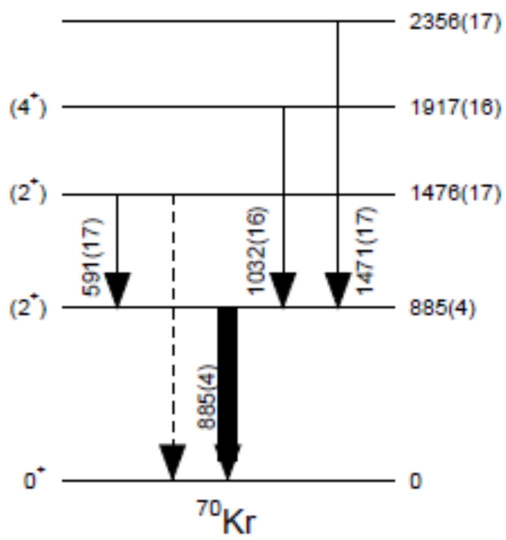
- spectroscopy following knockout @ RIKEN

T. Arici, PhD Univ. Giessen (2017)

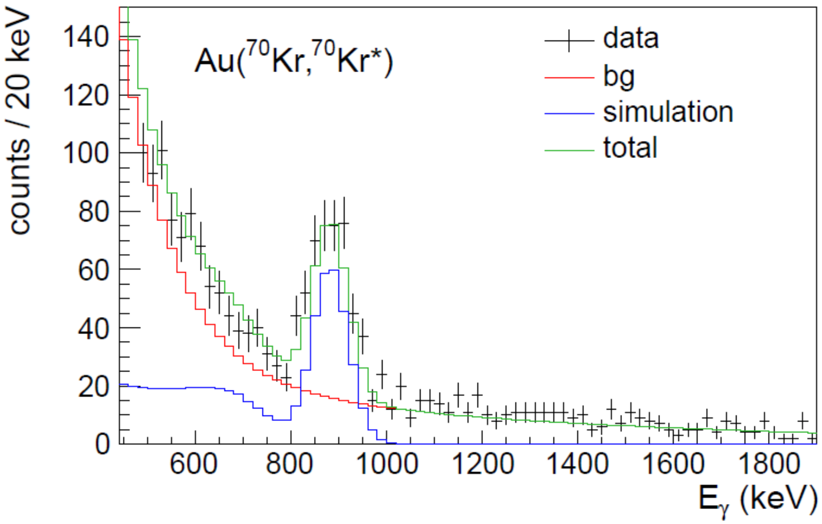








Coulomb excitation of ^{70}Kr



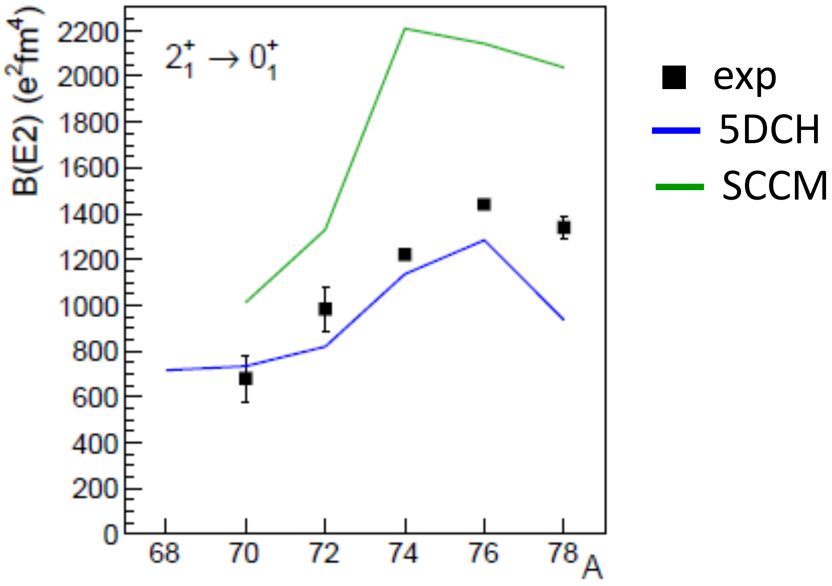
^{70}Kr: $B(E2;\downarrow) = 680(100)e^2\mathrm{fm}^4$

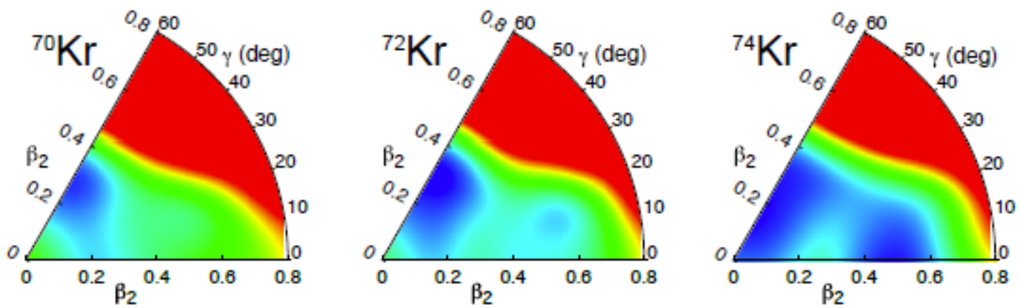
T. Arici, PhD Univ. Giessen (2017)

comparison with theory:

- 5DCH: Gogny D1S, $Q_{20} + Q_{22}$, GCM + GOA
- SCCM (symmetry-conserving configuration mixing)
  Gogny D1S, $Q_{20} + Q_{22}$, exact spin and
  particle-number restauration

T.R. Rodriguez, PRC 90, 034306 (2014)





Questions / Discussion

- Do we understand ^{72}Kr?
  - oblate ground state shape that turns prolate already for 2^+?
  - remarkable agreement for Gogny 5DCH for ^{76}Kr, ^{74}Kr, (^{70}Kr) but less for ^{72}Kr?

- Is ^{68}Se the best case for shape coexistence in this region (two rotational bands)?
- Why are there no low-lying 0_2^+ states in ^{68}Se and ^{70}Se?

- Do we understand the large B(E2) value in ^{70}Kr?
  (or the low B(E2) value in ^{70}Se)

- Which experimental measurements should we focus on?