



FINE-TUNING and the EMERGENCE of STRUCTURE in NUCLEAR PHYSICS

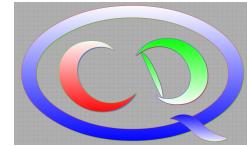
Ulf-G. Meißner, Univ. Bonn & FZ Jülich

Supported by BMBF 05P15PCFN1

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by CAS, PIFI

by Volkswagen Stiftung



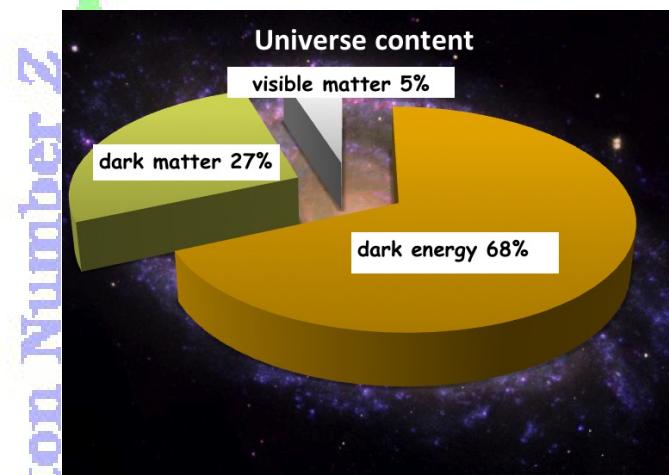
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- Tools I: Continuum physics
- Tools 2: Lattice physics
- Fine-tuning and the emergence of life
- Fine-tuning and the emergence of clustering
- Summary & outlook

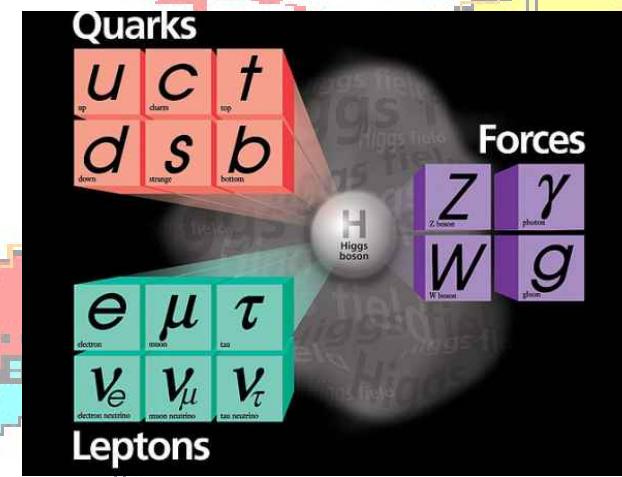
Introduction

WHY NUCLEAR PHYSICS?

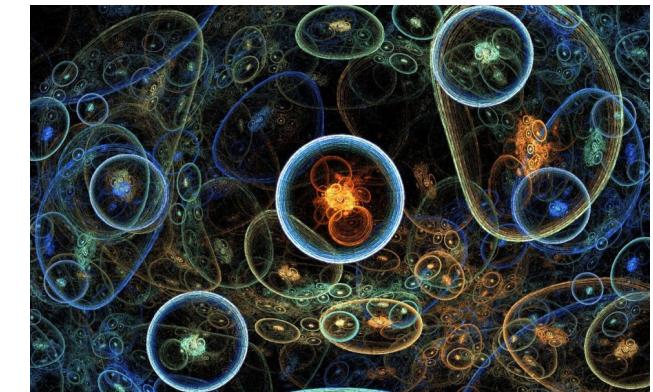
- The matter we are made off



- The last frontier of the SM



- Access to the Multiverse



Neutron Number N

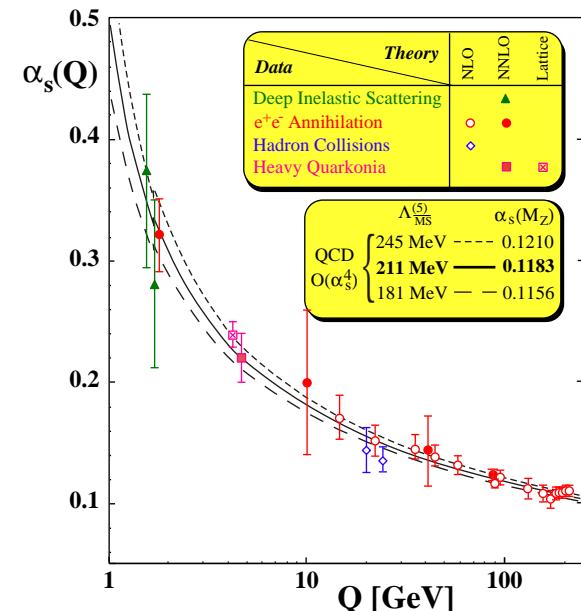
EMERGENCE of STRUCTURE in QCD

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- The strong interactions are described by **QCD**:

$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{q}_f (i\gamma_\mu D^\mu - m_f) q_f + \dots$$

- up** and **down** quarks are very light, a few MeV
 - Quarks and gluons are confined within **hadrons**
→ playground of lattice QCD
 - Protons and neutrons form **atomic nuclei**
- ⇒ This requires the inclusion of electromagnetism
described by QED with $\alpha_{\text{EM}} \simeq 1/137$ [+ weak int.]



- How can one describe nuclei *ab initio*? → chiral EFT
- How sensitive are these strongly interacting composites to variations of the fundamental parameters of QCD+QED?

CHIRAL EFT of QCD

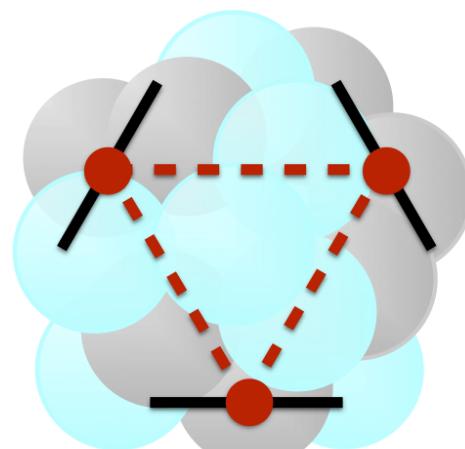
Gasser, Leutwyler, **Weinberg**, Bijnens, Ecker, Bernard, Kaiser, UGM, van Kolck, Kaplan, Savage, Wise, . . .

- At low energies: CHIRAL LAGRANGIAN (two flavors)

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{EFF}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

- dofs: quark & gluon fields \rightarrow pions, nucleons, external sources
- Spontaneous chiral symmetry breaking of QCD \rightarrow pions are Goldstone bosons
- Systematic expansion in powers of Q/Λ_χ & M_π/Λ_χ , with $\Lambda_\chi \simeq 1 \text{ GeV}$
- Pion and pion-nucleon sectors are perturbative in $Q \rightarrow$ chiral perturbation theory
- Parameters in $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\pi N}$ known from CHPT studies \rightarrow low-energy constants
- \mathcal{L}_{NN} collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation
 \rightarrow chirally expand V_{NN} , use in regularized LS equation

Continuum chiral EFT physics



LENPIC

www.lenpic.org

The NUCLEAR HAMILTONIAN

- Nucleons in nuclei are slow-moving particles, binding momenta $\sim M_\pi$

$$\left(- \sum_{i=1}^A \frac{\nabla_i^2}{2m_N} + V \right) |\Psi\rangle = E|\Psi\rangle , \quad V = V_{NN} + V_{NNN} + \dots$$

- The nuclear potential V classically build from meson exchange models
- Weinberg's idea: use chiral perturbation theory to construct V

Weinberg (1990,1991)

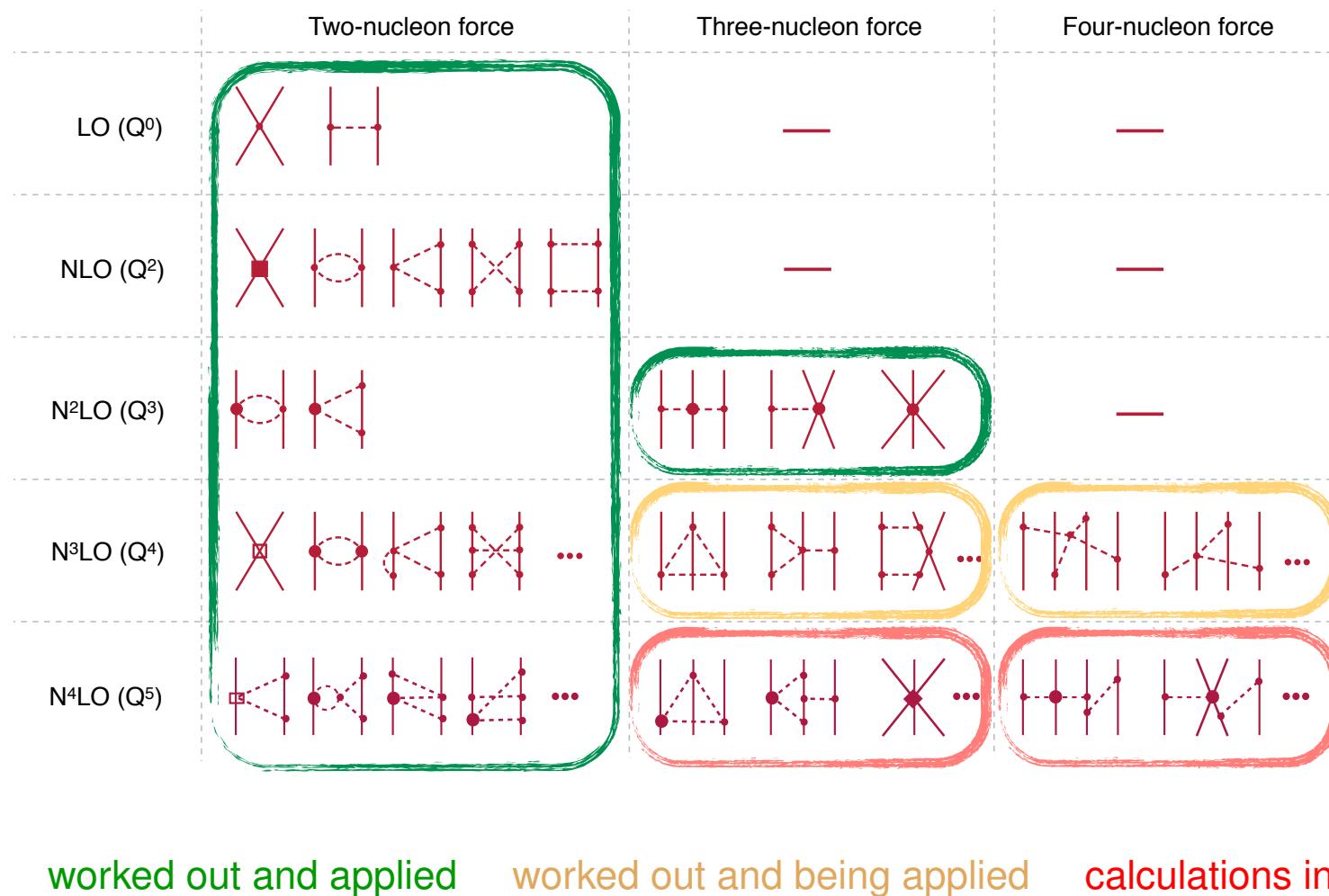
- ↪ direct link to QCD via symmetries and their breakings
- ↪ systematic approach that can be improved order-by-order
- ↪ allows for a consistent calculation of two-, three- and four-body forces
- ↪ allows for a consistent calculation of forces and currents
- ↪ systematic approach that allows for uncertainty quantifications
- ↪ gives access to the multiverse

NUCLEAR FORCES in CHIRAL NUCLEAR EFT

- expansion of the potential in powers of Q [small parameter]
- explains observed hierarchy of the nuclear forces

Weinberg, van Kolck

for a review, see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773



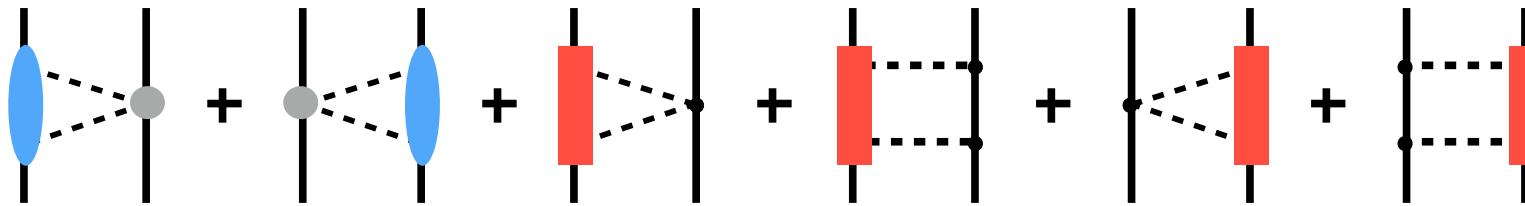
NN FORCES to FIFTH ORDER

Epelbaum, Krebs, UGM, Phys. Rev. Lett. **115** (2015) 122301

Entem, Kaiser, Machleidt, Nosyk, Phys.Rev. C **91** (2015) 014002

- No contact interactions at this order - odd in Q
- New contributions fixed from πN scattering, LECs c_i, d_i, e_i :

Büttiker, Fettes, UGM, Steininger (1998-2000); Krebs, Gasparian, Epelbaum (2012), Hoferichter et al. (2015)



$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}(c_i) + \mathcal{L}_{\pi N}^{(3)}(d_i) + \mathcal{L}_{\pi N}^{(4)}(e_i)$$

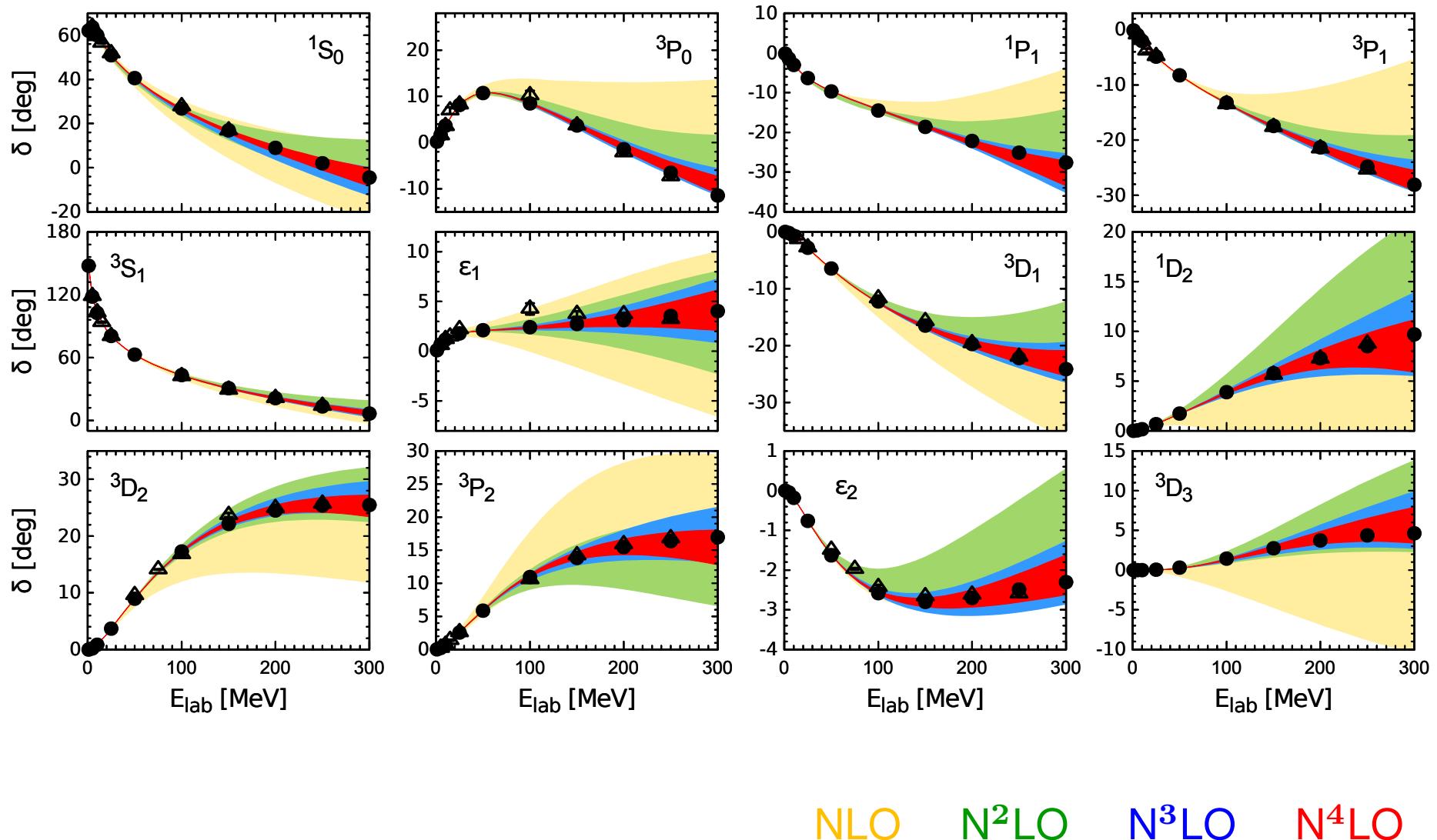
- Three-pion exchange can be neglected
 - explicit calculation of the dominant NLO contribution
 - no influence on phase shifts or deuteron properties

Kaiser (2001)

PHASE SHIFTS at N⁴LO

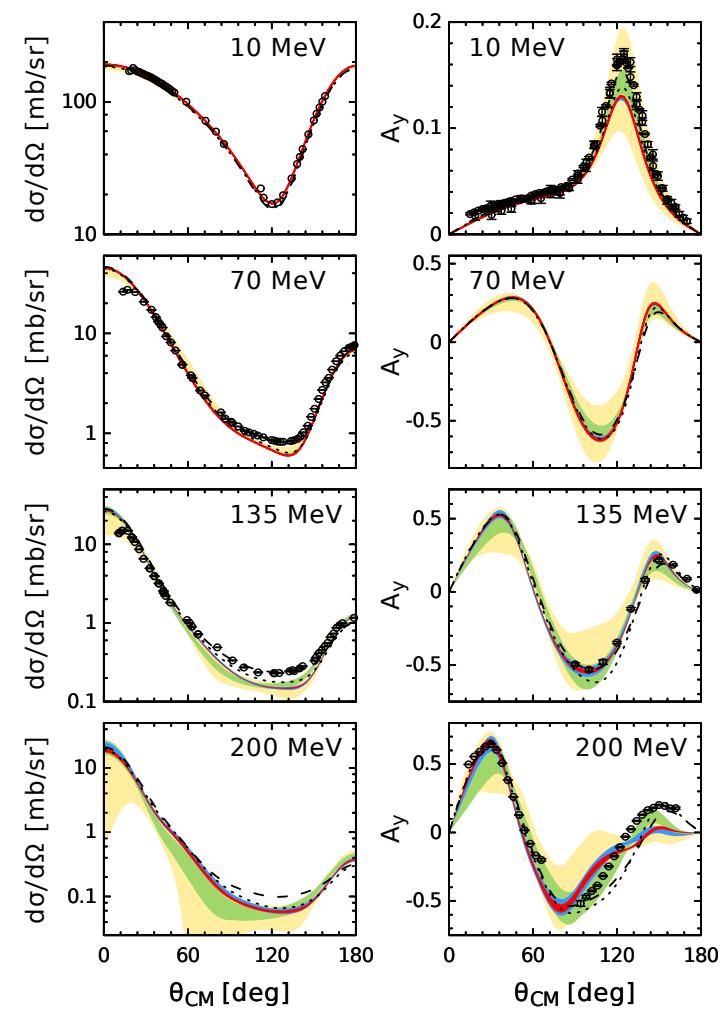
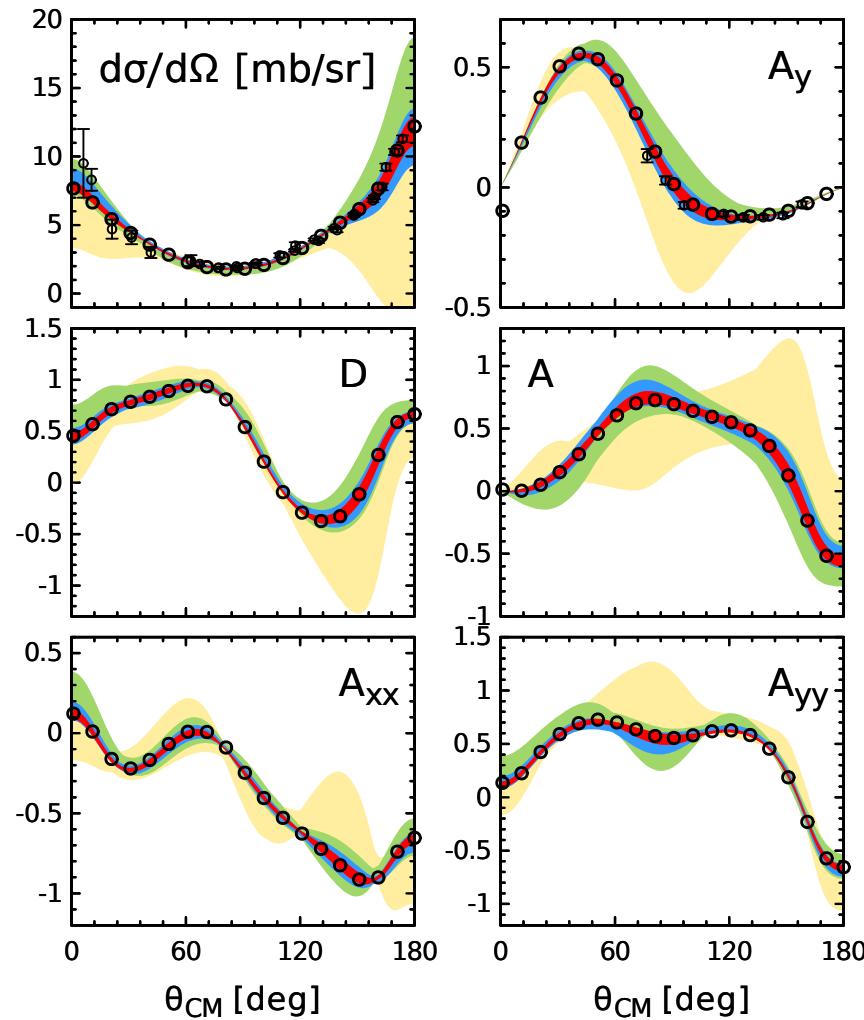
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⇒ Precision phase shifts with small uncertainties up to $E_{\text{lab}} = 300 \text{ MeV}$



EVIDENCE for THREE-NUCLEON FORCES

- Two-nucleon system under control, nuclei with $A \geq 3$ require 3NFs!
 - being implemented [LENPIC collaboration] Phys.Rev. C93 (2016) 044002
 - np scattering at 200 MeV
 - nd scattering [2NFs only]



NLO
N²LO
N³LO
N⁴LO

Quark mass dependence of the nuclear forces and impact on BBN

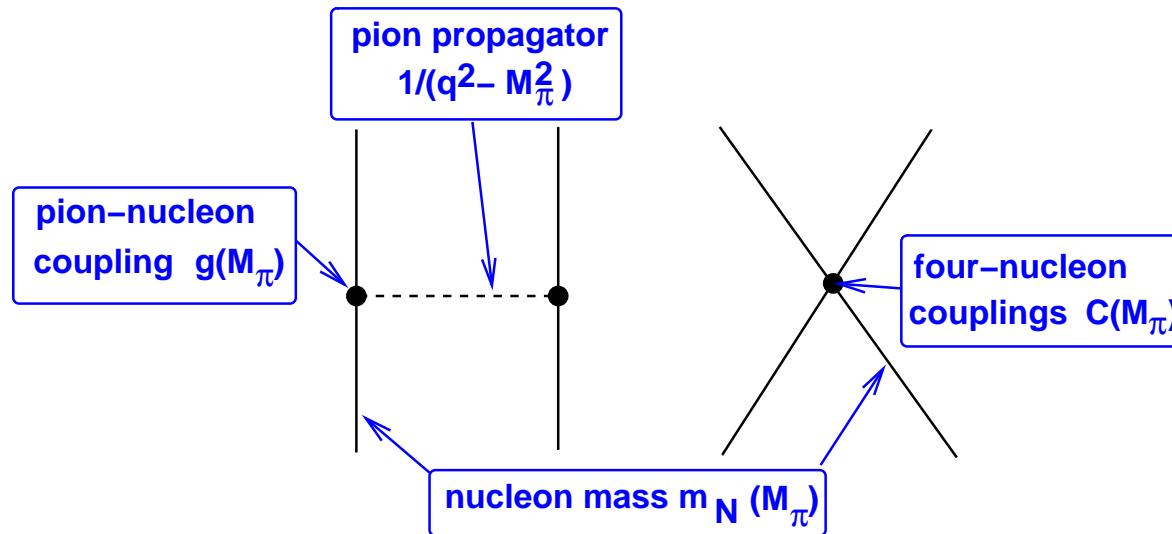
“Fine-tuning in the early Universe”

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,
Phys. Rev. D **87** (2013) 085018

INGREDIENTS

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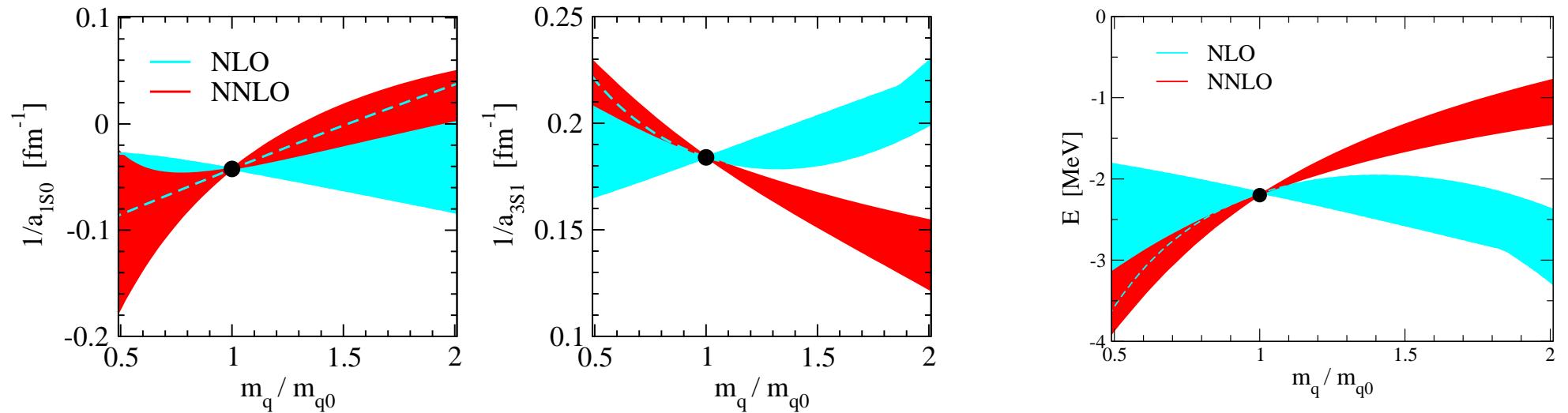
- Nuclear forces: Pion-exchange contributions & short-distance multi-N operators
- graphical representation of the quark mass dependence of the LO potential



- always use the Gell-Mann–Oakes–Renner relation:
$$M_{\pi^\pm}^2 \sim (m_u + m_d)$$
- fulfilled in QCD to better than 94% Colangelo, Gasser, Leutwyler 2001
- ⇒ Quark mass dependence of hadron properties from lattice QCD,
contact interaction require modelling

RESULTS for the NN SYSTEM

- Putting pieces together for the two-nucleon system:



- uncertainties mostly due to modelling the contact interactions

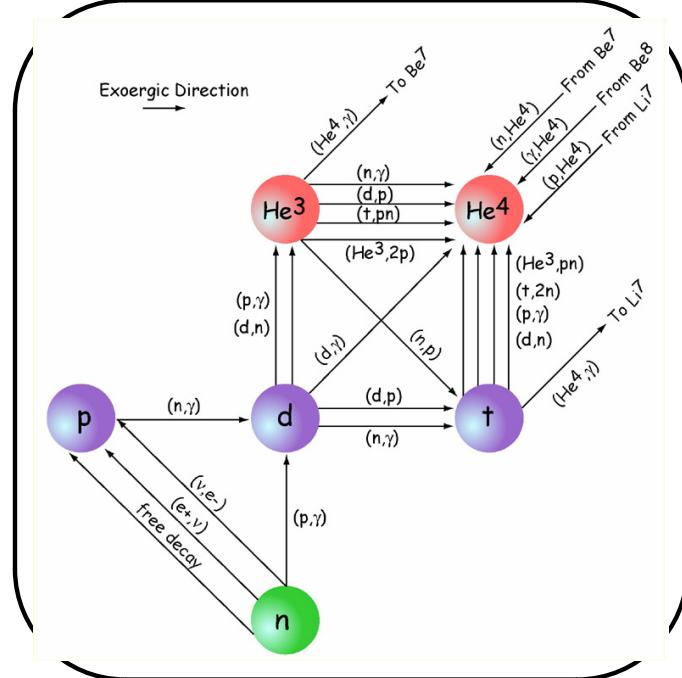
→ contact to lattice QCD required

Baru et al., Phys. Rev. C92 (2015) 014001

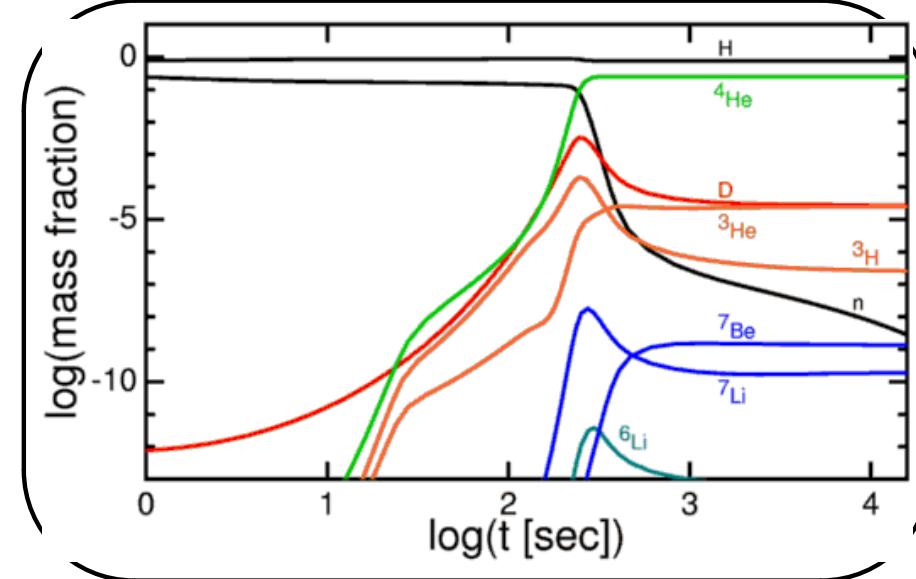
- extends and improves earlier work based on EFTs and models

Beane, Savage (2003), Epelbaum, UGM, Glöckle (2003), Mondejar, Soto (2007), Flambaum, Wiringa (2007), Bedaque, Luu, Platter (2011), ...

BBN NETWORK & ELEMENT ADUNDANCES



from Cococubed.com



from Burles, Nollett & Turner

- consider element generation in the Big Bang up to ^7Li , ^7Be
- how does this network / the abundances of the elements change under variations of the quark masses?
 ⇒ use results just shown, extended also to $A = 3, 4, \dots$

LIMITS for the QUARK MASS VARIATION

- Work first in the isospin limit: Average of [deut/H] and ${}^4\text{He}(Y_p)$:

$$\frac{\delta m_q}{m_q} = 0.02 \pm 0.04$$

- in contrast to earlier studies, we provide reliable error estimates (EFT)
- Isospin breaking: stronger constraint due to the neutron life time (affects $Y({}^4\text{He})$)

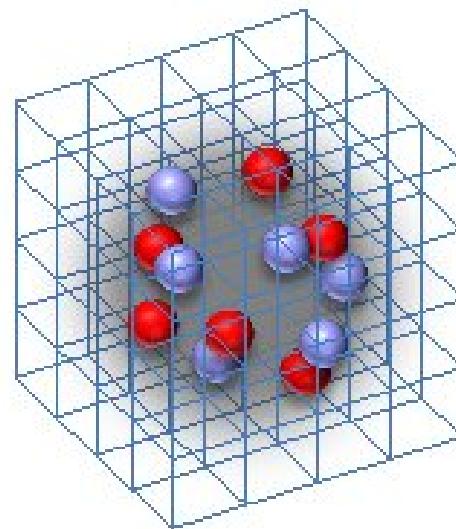
Bedaque, Luu, Platter, Phys. Rev. C **83** (2011) 045803

- re-evaluate this under the model-independent assumption that
all quark & lepton masses vary with the Higgs VEV v

⇒ results are dominated by the ${}^4\text{He}$ abundance:

$$\left| \frac{\delta v}{v} \right| = \left| \frac{\delta m_q}{m_q} \right| \leq 0.9\%$$

Lattice chiral EFT physics



NLEFT

THE TOOL: NUCLEAR LATTICE SIMULATIONS

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Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

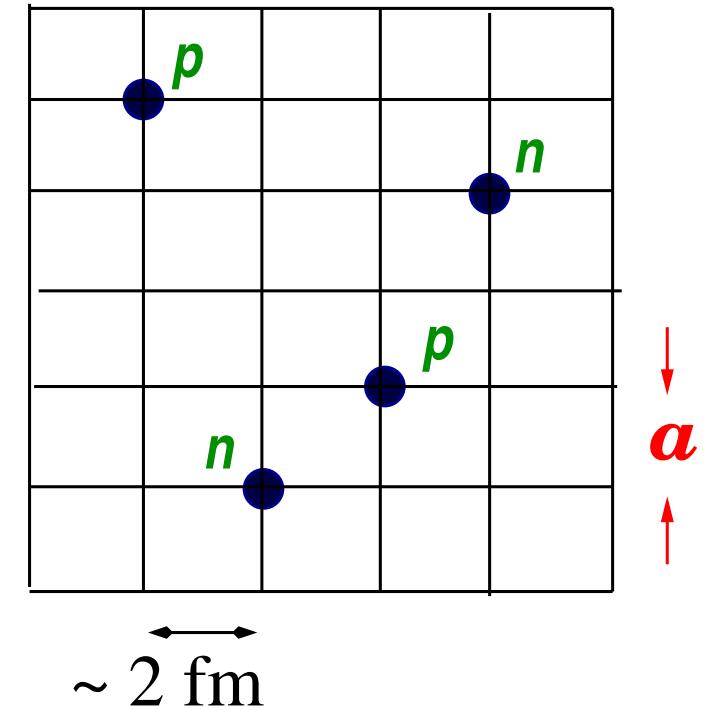
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
nucleons are point-like fields on the sites

- discretized chiral potential w/ pion exchanges
and contact interactions + Coulomb

- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300 \text{ MeV} \text{ [UV cutoff]}$$

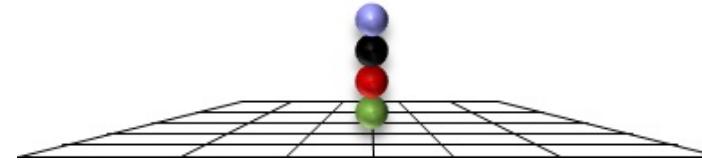
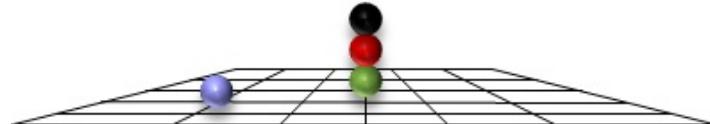
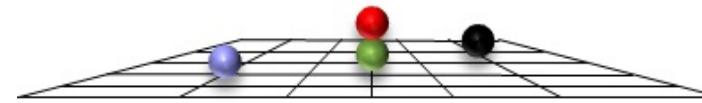
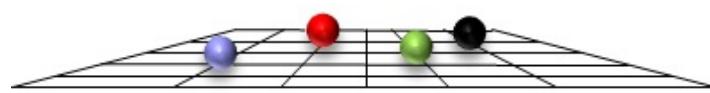


- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302, T. Lähde et al., Eur. Phys. J. **A 52** (2015) 92

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

CONFIGURATIONS



⇒ all possible configurations are sampled
⇒ clustering emerges naturally

COMPUTATIONAL EQUIPMENT

- Present = JUQUEEN (BlueGene/Q)

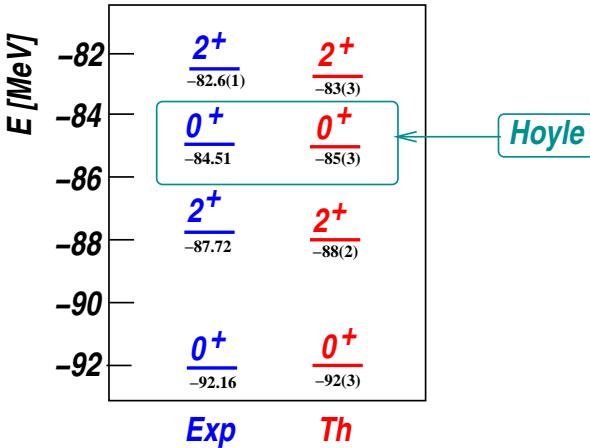


6 Pflops

RESULTS from LATTICE NUCLEAR EFT

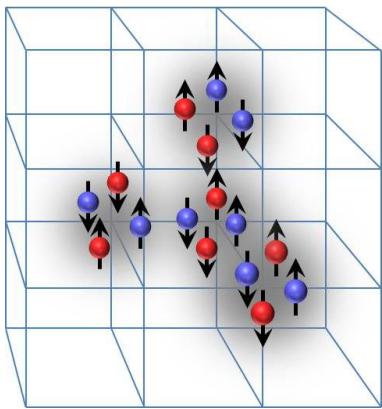
- Hoyle state in ^{12}C

PRL 106 (2011)



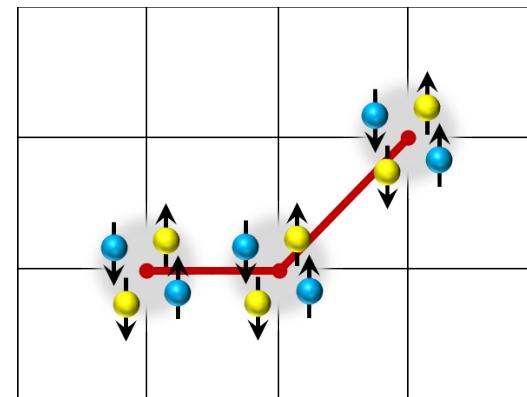
- Spectrum of ^{16}O

PRL 112 (2014)



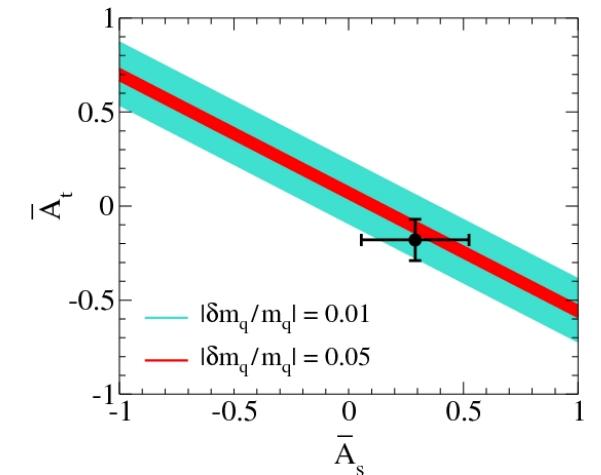
- Structure of the Hoyle state

PRL 109 (2012)



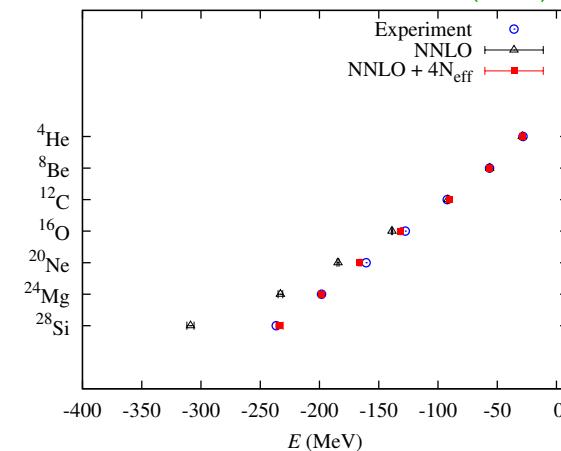
- Fate of carbon-based life

PRL 110 (2013), EPJ A49 (2013)



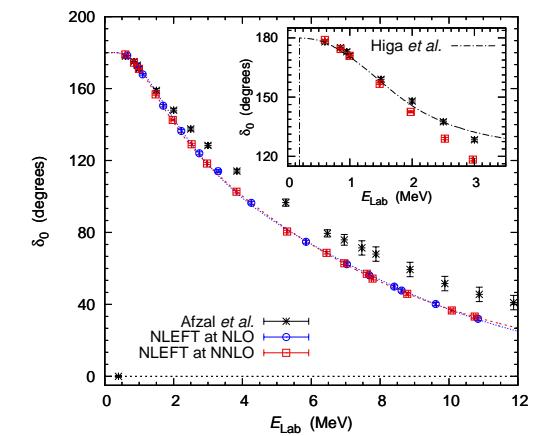
- Going up the α -chain

PLB 732 (2014)



- Ab initio α - α scattering

Nature 528 (2015)

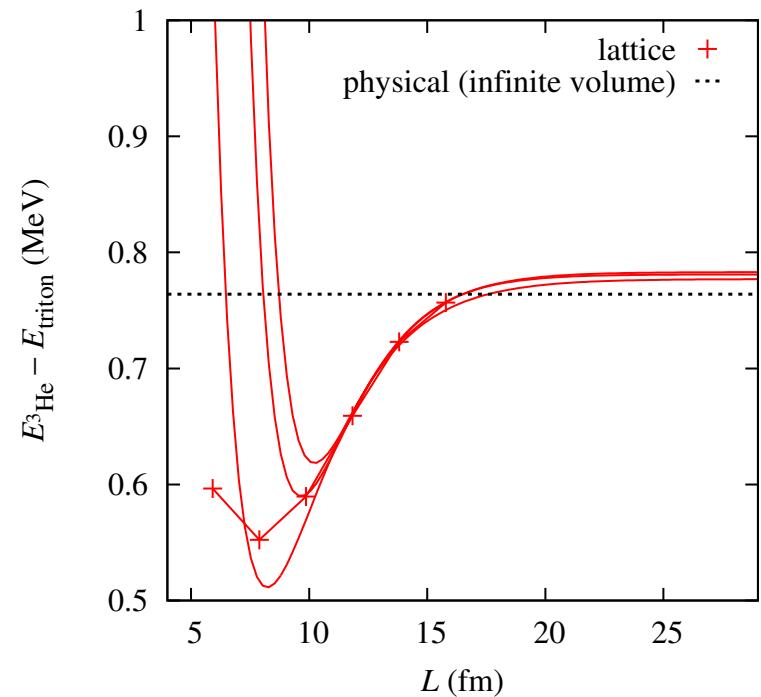


RESULTS

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A**45** (2010) 335

- some groundstate energies and differences [NNLO, 11+2 LECs]

	E [MeV]	NLEFT	Exp.
old algorithm			
new algorithm			
$^3\text{He} - ^3\text{H}$	0.78(5)	0.76	
^4He	-28.3(6)	-28.3	
^8Be	-55(2)	-56.5	
^{12}C	-92(3)	-92.2	
^{16}O	-131(1)	-127.6	
^{20}Ne	-166(1)	-160.6	
^{24}Mg	-198(2)	-198.3	
^{28}Si	-234(3)	-236.5	



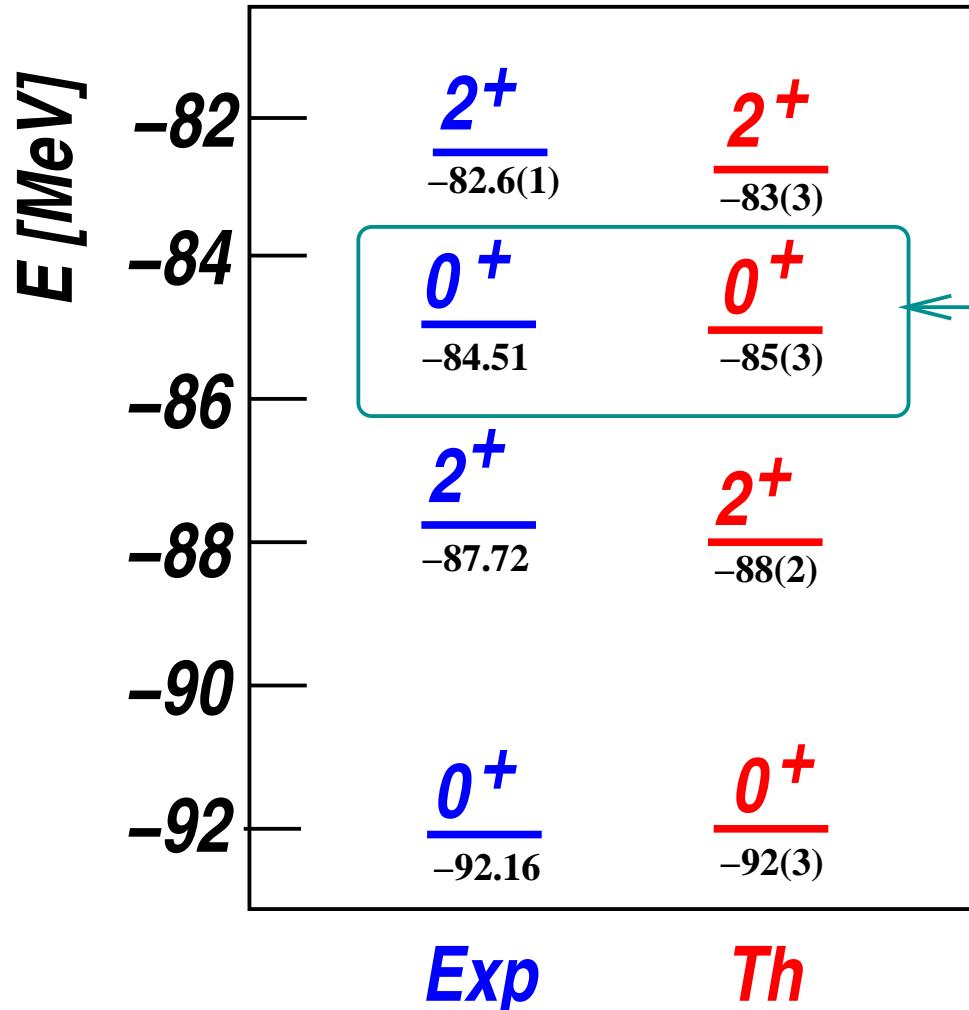
- promising results \Rightarrow uncertainties down to the 1% level
- excited states more difficult \Rightarrow projection MC method + triangulation

The SPECTRUM of CARBON-12

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

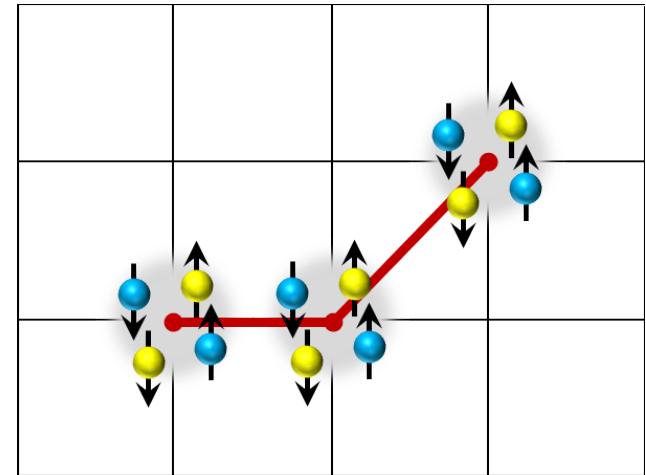
Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **109** (2012) 252501

- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and “some” human work)



⇒ First ab initio calculation
of the Hoyle state ✓

Structure of the Hoyle state:



Fine-tuning and the emergence of life

The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600

From: Steven Weinberg <weinberg@zippy.ph.utexas.edu>

To: Ulf-G. Meissner <meissner@hiskp.uni-bonn.de>

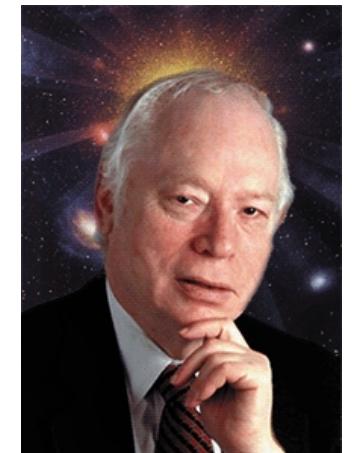
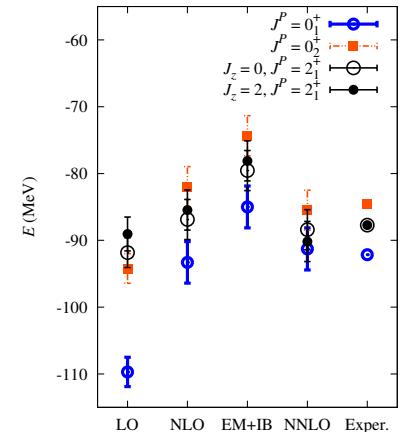
Subject: Re: Hoyle state in 12C

Dear Professor Meissner,

Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,

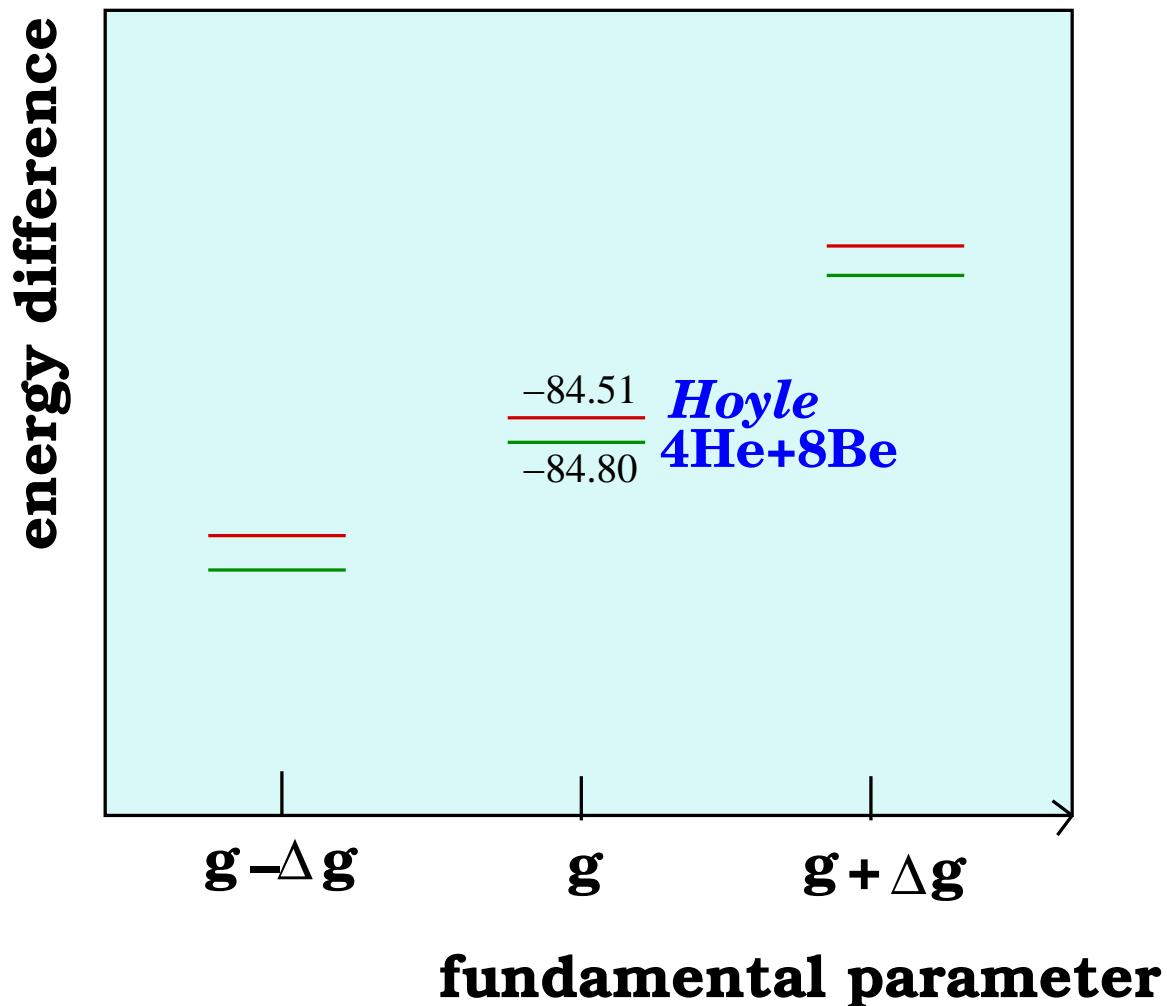
Steve Weinberg



- How does the Hoyle state move relative to the ${}^4\text{He} + {}^8\text{Be}$ threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

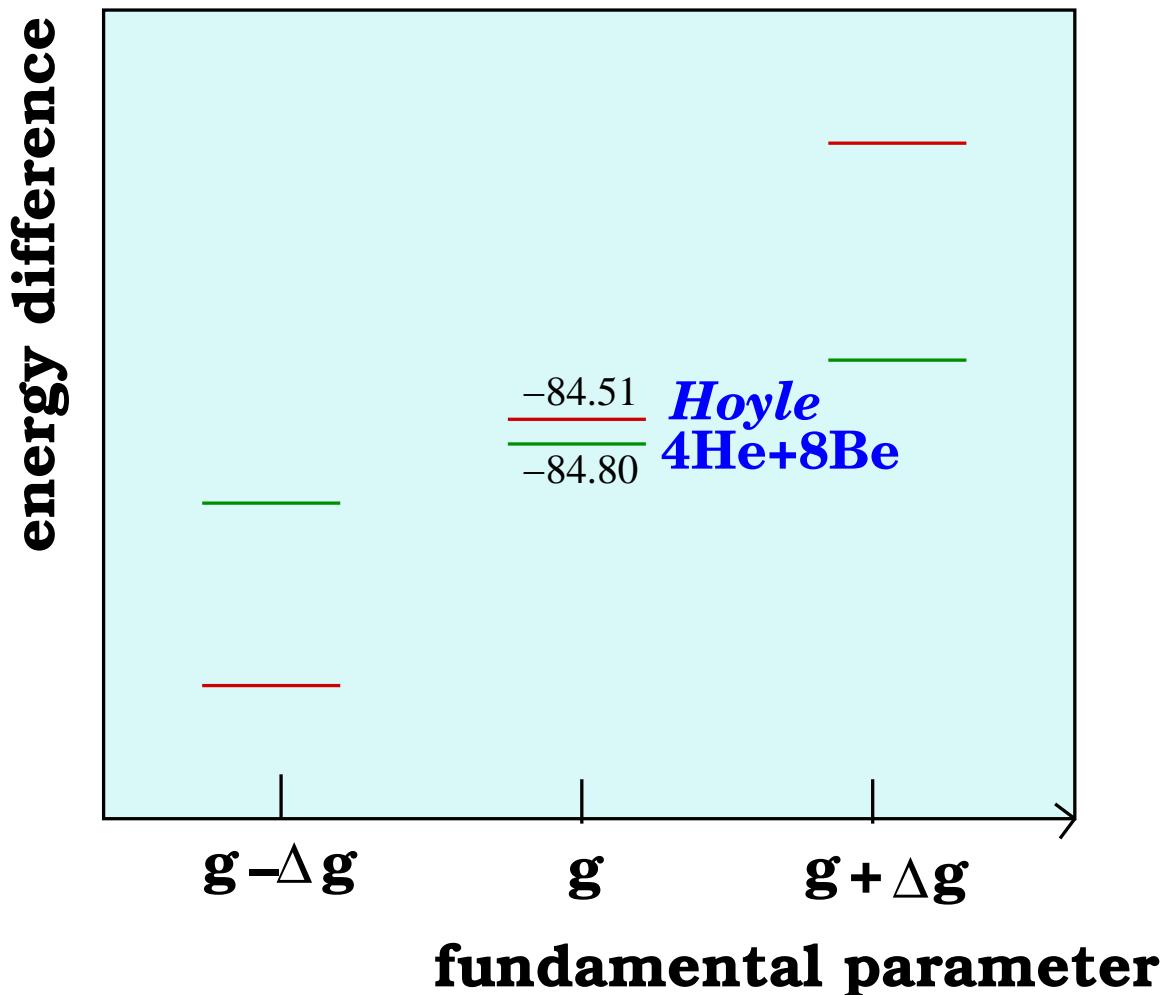
The NON-ANTHROPIC SCENARIO

- Weinberg's assumption: The Hoyle state stays close to the $4\text{He}+8\text{Be}$ threshold



The ANTHROPIC SCENARIO

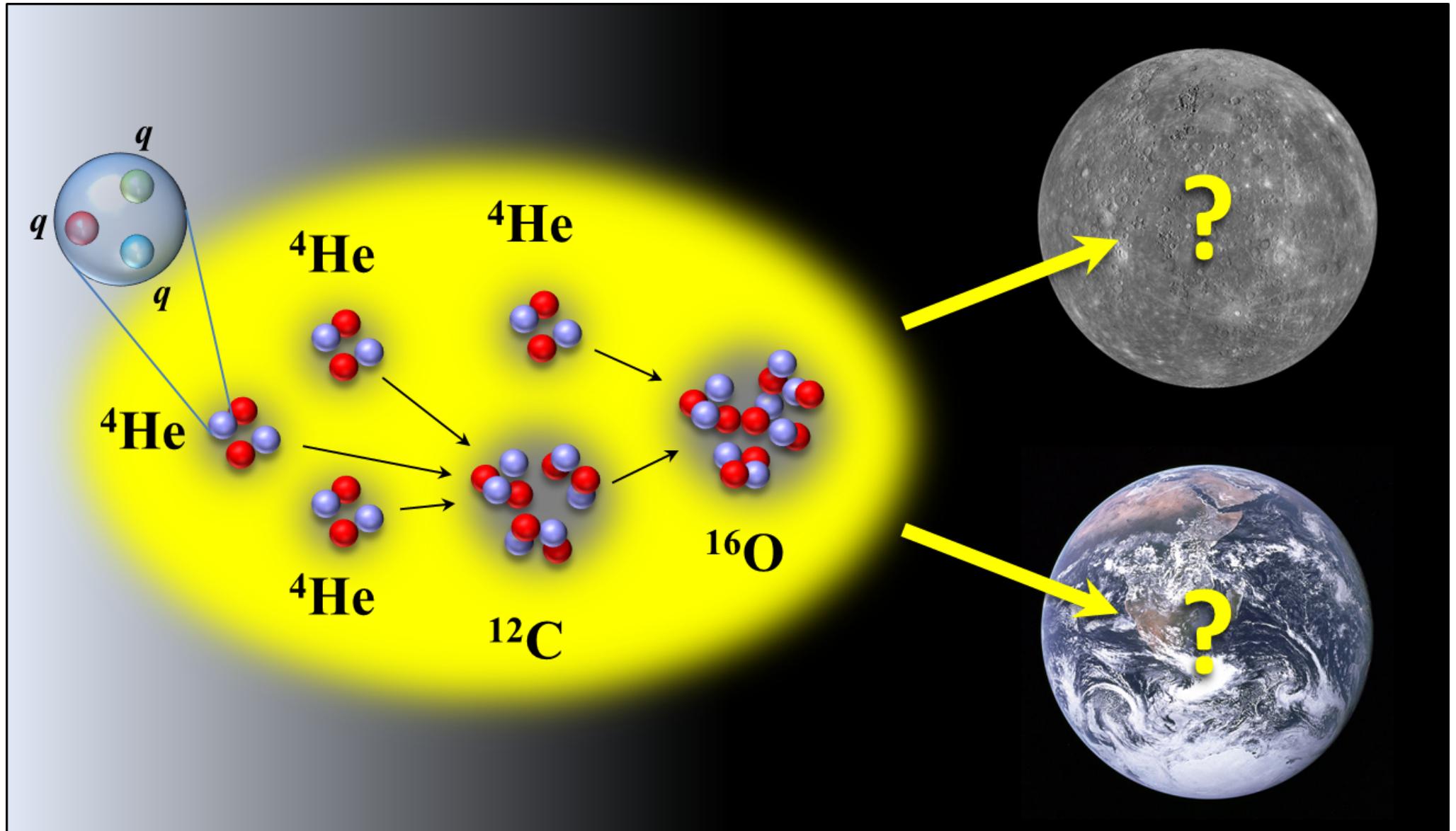
- The AP strikes back: The Hoyle state moves away from the $4\text{He}+8\text{Be}$ threshold



FINE-TUNING of FUNDAMENTAL PARAMETERS

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Fig. courtesy Dean Lee



EARLIER STUDIES of the ANTHROPIC PRINCIPLE

- rate of the 3α -process: $r_{3\alpha} \sim \Gamma_\gamma \exp\left(-\frac{\Delta E_{h+b}}{kT}\right)$
- $$\Delta E_{h+b} = E_{12}^\star - 3E_\alpha = 379.47(18) \text{ keV}$$

- how much can ΔE_{h+b} be changed so that there is still enough ^{12}C and ^{16}O ?

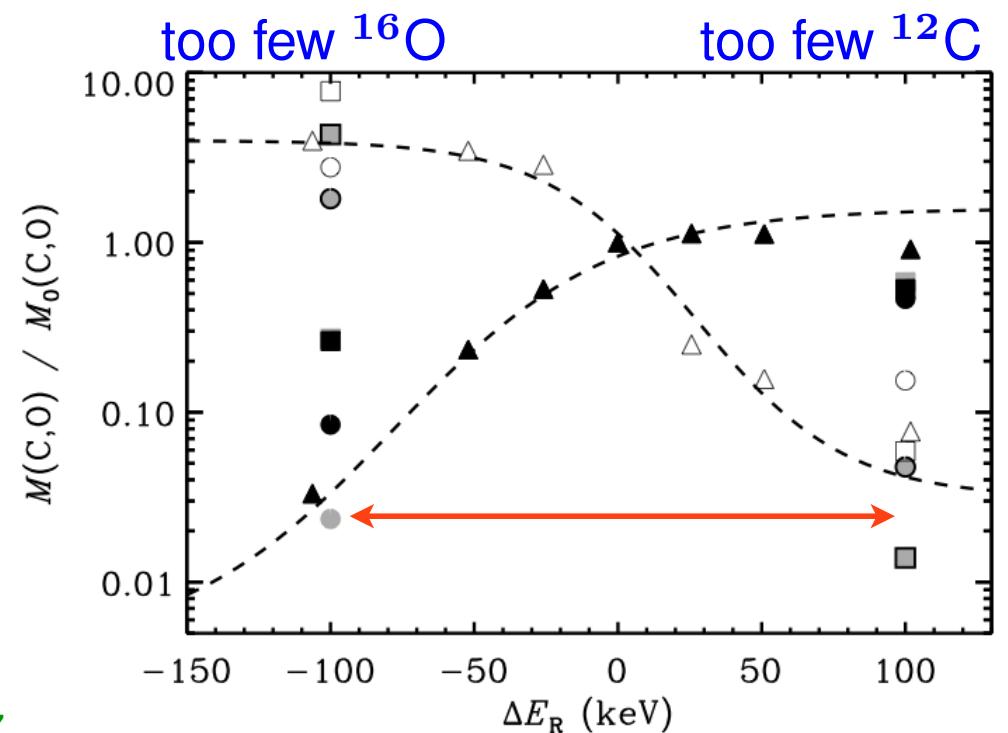
$$\Rightarrow \boxed{\delta|\Delta E_{h+b}| \lesssim 100 \text{ keV}}$$

Oberhummer et al., Science **289** (2000) 88

Csoto et al., Nucl. Phys. A **688** (2001) 560

Schlattl et al., Astrophys. Space Sci. **291** (2004) 27

[Livio et al., Nature **340** (1989) 281]



FINE-TUNING: MONTE-CARLO ANALYSIS

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Epelbaum, Krebs, Lähde, Lee, UGM, PRL 110 (2013) 112502

- consider first QCD only → calculate $\partial\Delta E/\partial M_\pi$
- relevant quantities (energy *differences*)

$$\Delta E_h \equiv E_{12}^* - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4$$

- energy differences depend on parameters of QCD (LO analysis)

$$E_i = E_i\left(M_\pi^{\text{OPE}}, m_N(M_\pi), g_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\right)$$

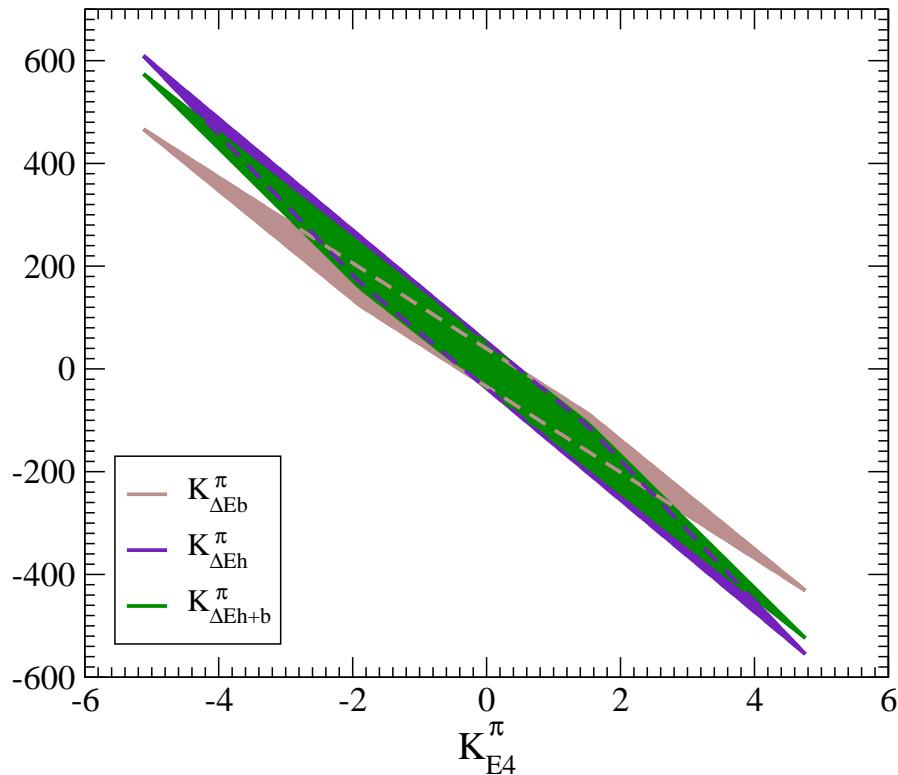
$$g_{\pi N} \equiv g_A/(2F_\pi)$$

- remember: $M_{\pi^\pm}^2 \sim (m_u + m_d)$ Gell-Mann, Oakes, Renner (1968)

⇒ quark mass dependence \equiv pion mass dependence

CORRELATIONS

- map $C_{0,I}(M_\pi)$ onto $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1} / \partial M_\pi \Big|_{M_\pi^{\text{phys}}}$ [singlet/triplet scatt. length]
- vary the derivatives $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1} / \partial M_\pi \Big|_{M_\pi^{\text{phys}}}$ within $-1, \dots, +1$:



$$\Delta E_b = E(^8\text{Be}) - 2E(^4\text{He})$$

$$\Delta E_h = E(^{12}\text{C}^*) - E(^8\text{Be}) - E(^4\text{He})$$

$$\Delta E_{h+b} = E(^{12}\text{C}^*) - 3E(^4\text{He})$$

$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \frac{O_H}{M_\pi}}$$

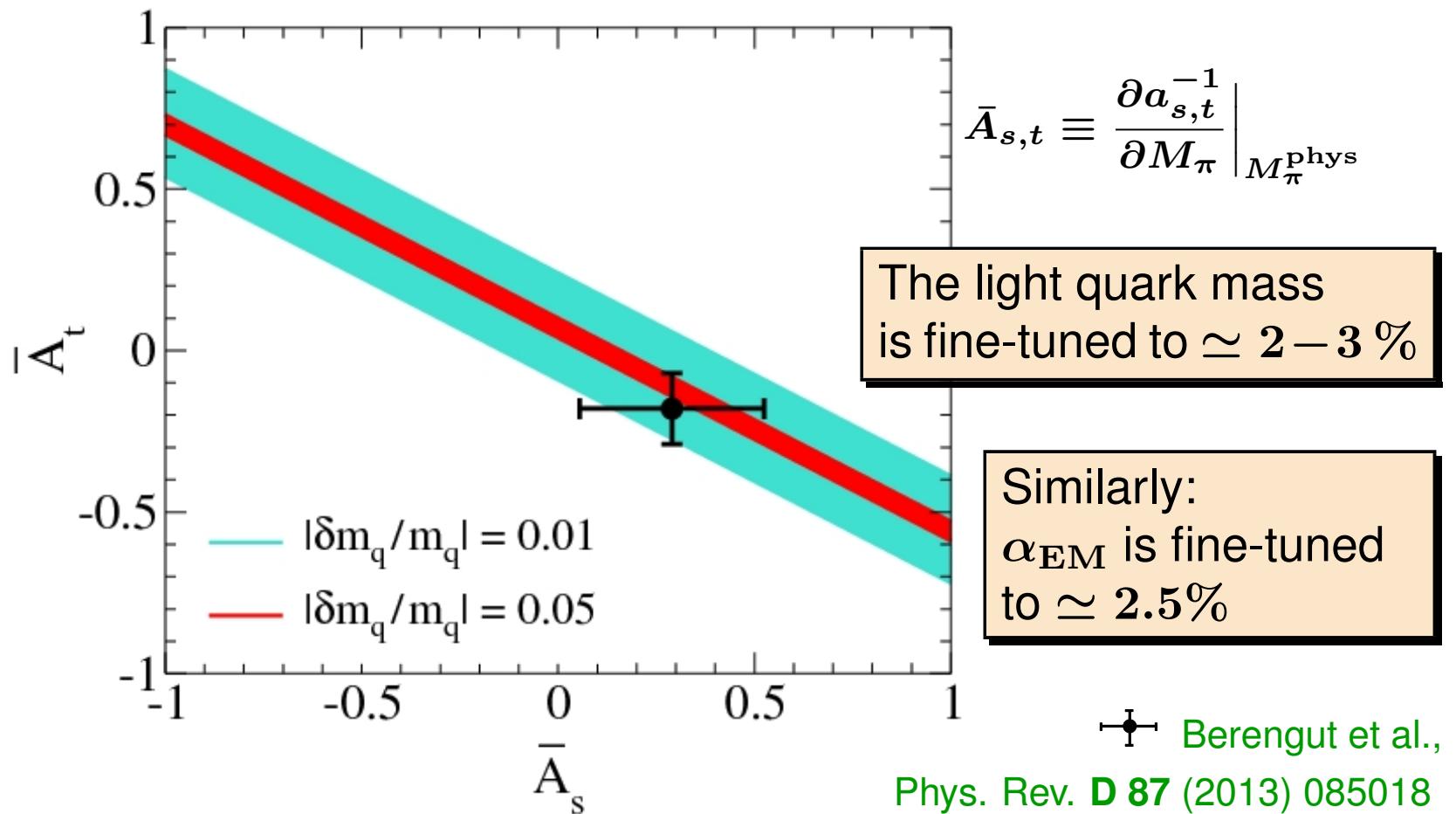
- clear correlations: speculated before but could not be calculated

Weinberg (2001)

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100 \text{ keV}$ [exp: 387 keV] Oberhummer et al., Science (2000)

$$\rightarrow \left| \left(0.571(14) \bar{A}_s + 0.934(11) \bar{A}_t - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$



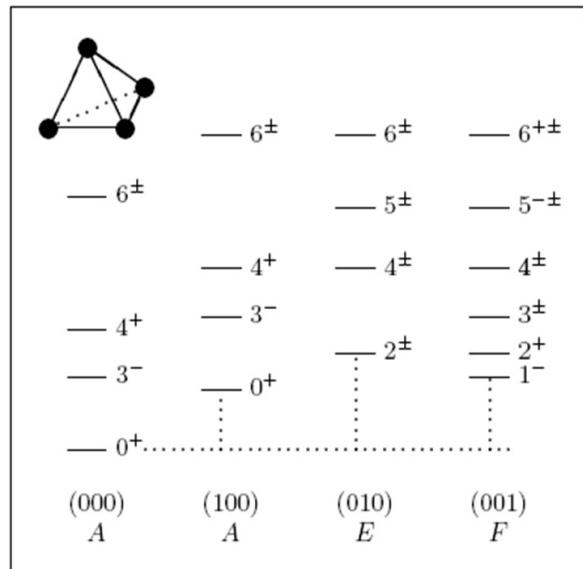
Nuclear binding near a quantum phase transition

Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, UGM, Epelbaum,
Krebs, Lähde, Lee, Rupak,
Phys. Rev. Lett. **117** (2016) 132501 [arXiv:1602.04539]

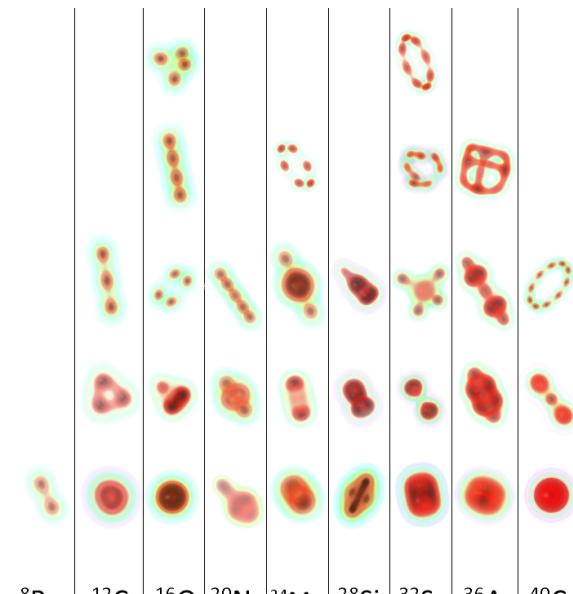
Editors' suggestion, featured in Physics viewpoint: D.J. Dean, Physics 9 (2016) 106

CLUSTERING in NUCLEI

- Introduced theoretically by Wheeler already in 1937 Phys. Rev. **52** (1937) 1083
- many works since then... Ikeda, Horiuchi, Freer, Schuck, Zhou, Khan, ...



Bijker, Iachello (2014)



Ebran, Khan, Niksic, Vretenar (2014)

- ⇒ can we understand this phenomenon from *ab initio* calculations?
- ⇒ yes, we can! NLEFT [and also DFT] soon to be published in RMP
- ⇒ new aspect: the emergence of α -clusters is also related to fine-tuning!

GENERAL CONSIDERATIONS

- *Ab initio* chiral EFT is an excellent theoretical framework
- not guaranteed to work well with increasing A
 - possible sources of problems:
higher-body forces, higher orders, cutoff dependence, . . .
- very many ways of formulating chiral EFT at any given order (smearing etc.)
 - use not only NN scattering and light nuclei BEs
but also light nucleus-nucleus scattering data
to pin down the pertinent interactions
 - troublesome corrections might be small
 - investigate these issues using two seemingly equivalent interactions
[not a precision study!]

LOCAL and NON-LOCAL INTERACTIONS

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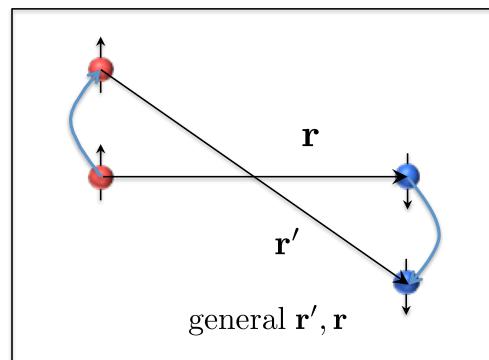
- General potential: $V(\vec{r}, \vec{r}')$

- Two types of interactions:

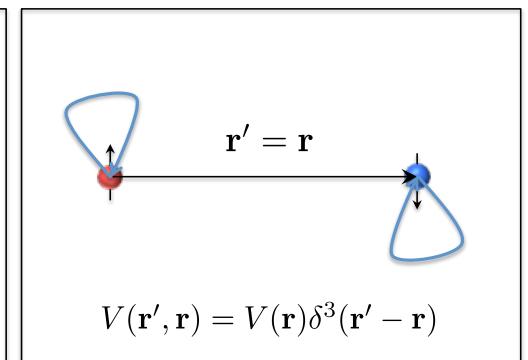
local: $\vec{r} = \vec{r}'$

non-local: $\vec{r} \neq \vec{r}'$

Nonlocal interaction



Local interaction



- Taylor two very different interactions:

Interaction A at LO (+ Coulomb)

Non-local short-range interactions
+ One-pion exchange interaction
(+ Coulomb interaction)

→ tuned to NN phase shifts

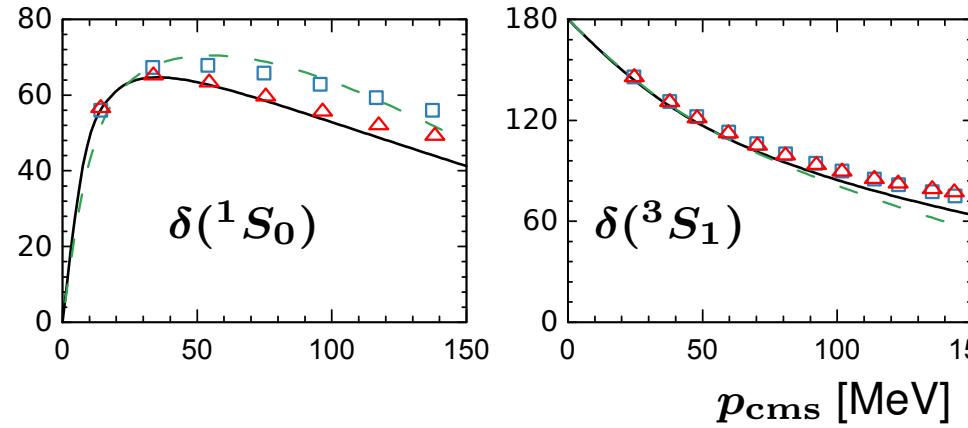
Interaction B at LO (+ Coulomb)

Non-local short-range interactions
+ Local short-range interactions
+ One-pion exchange interaction
(+ Coulomb interaction)

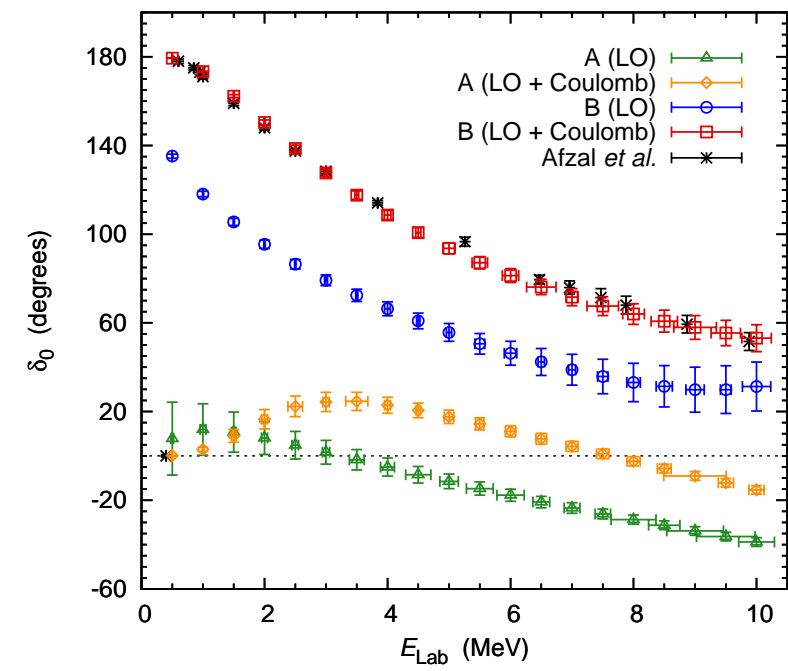
→ tuned to NN + α - α phase shifts

NN and ALPHA-ALPHA PHASE SHIFTS

- Both interactions very similar for NN but **not** for α - α phase shifts:



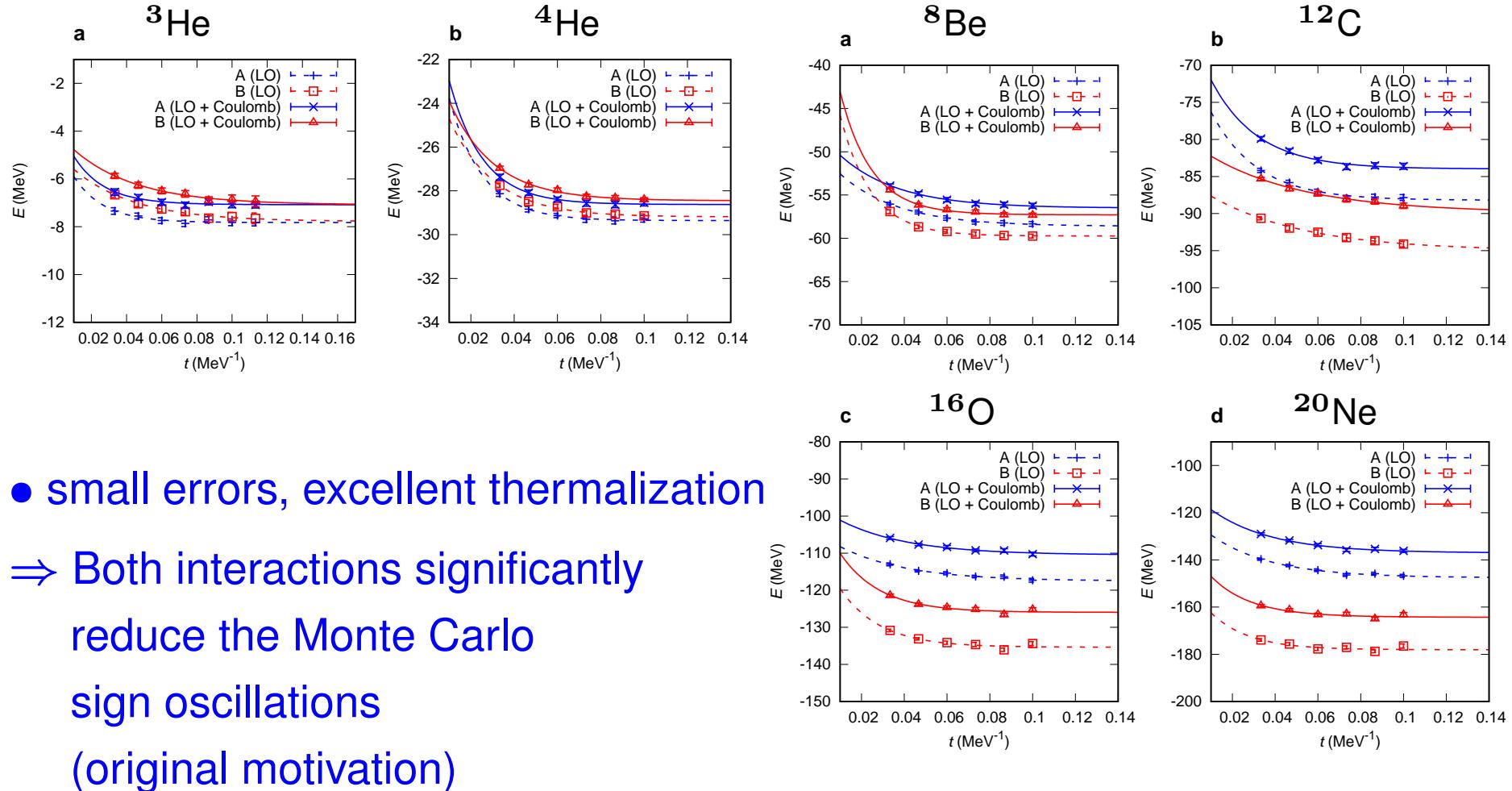
Nijmegen PWA —
 Continuum LO - -
 Lattice LO-A □
 Lattice LO-B △



- Interaction A fails, interaction B fitted
- ↪ consequences for nuclei?

GROUND STATE ENERGIES I

- Ground state energies for alpha-type nuclei plus ${}^3\text{He}$:



- small errors, excellent thermalization
 ⇒ Both interactions significantly
 reduce the Monte Carlo
 sign oscillations
 (original motivation)

GROUND STATE ENERGIES I

- Ground state energies for alpha-type nuclei (in MeV):

	A (LO)	A (LO+C.)	B (LO)	B (LO+C.)	Exp.
⁴ He	-29.4(4)	-28.6(4)	-29.2(1)	-28.5(1)	-28.3
⁸ Be	-58.6(1)	-56.5(1)	-59.7(6)	-57.3(7)	-56.6
¹² C	-88.2(3)	-84.0(3)	-95.0(5)	-89.9(5)	-92.2
¹⁶ O	-117.5(6)	-110.5(6)	-135.4(7)	-126.0(7)	-127.6
²⁰ Ne	-148(1)	-137(1)	-178(1)	-164(1)	-160.6

- B (LO+Coulomb) quite close to experiment (within 2% or better)
- A (LO) describes a Bose condensate of particles:

$$E(^8\text{Be})/E(^4\text{He}) = 1.997(6) \quad E(^{12}\text{C})/E(^4\text{He}) = 3.00(1)$$

$$E(^{16}\text{O})/E(^4\text{He}) = 4.00(2) \quad E(^{20}\text{Ne})/E(^4\text{He}) = 5.03(3)$$

CONSEQUENCES for NUCLEI and NUCLEAR MATTER

- Define a one-parameter family of interactions that interpolates between the interactions A and B:

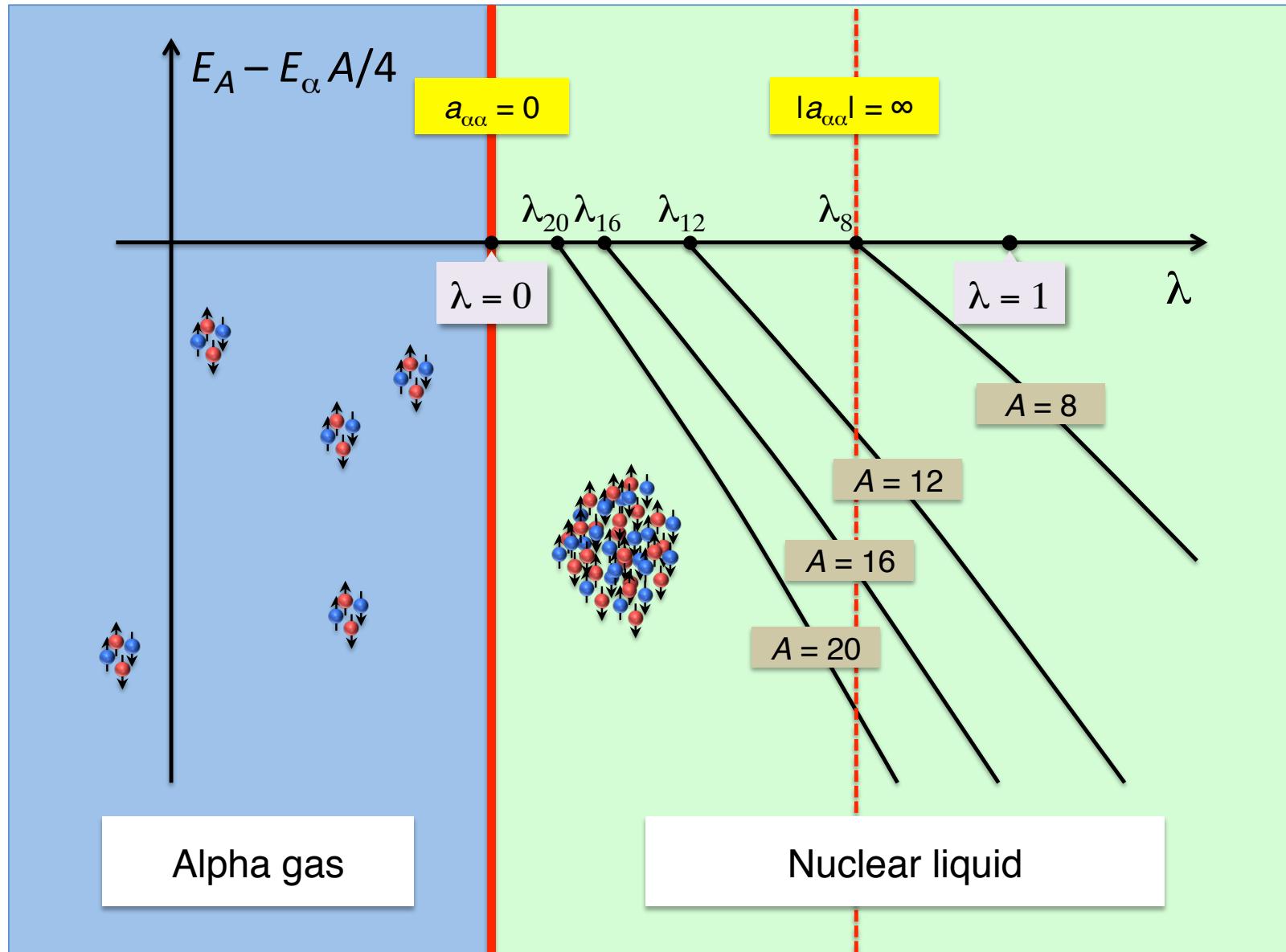
$$V_\lambda = (1 - \lambda) V_A + \lambda V_B$$

- To discuss the many-body limit, we turn off the Coulomb interaction and explore the zero-temperature phase diagram
- As a function of λ , there is a quantum phase transition at the point where the alpha-alpha scattering length vanishes

Stoff, Phys. Rev. A **49** (1994) 3824

- The transition is a first-order transition from a Bose-condensed gas of alpha particles to a nuclear liquid

ZERO-TEMPERATURE PHASE DIAGRAM

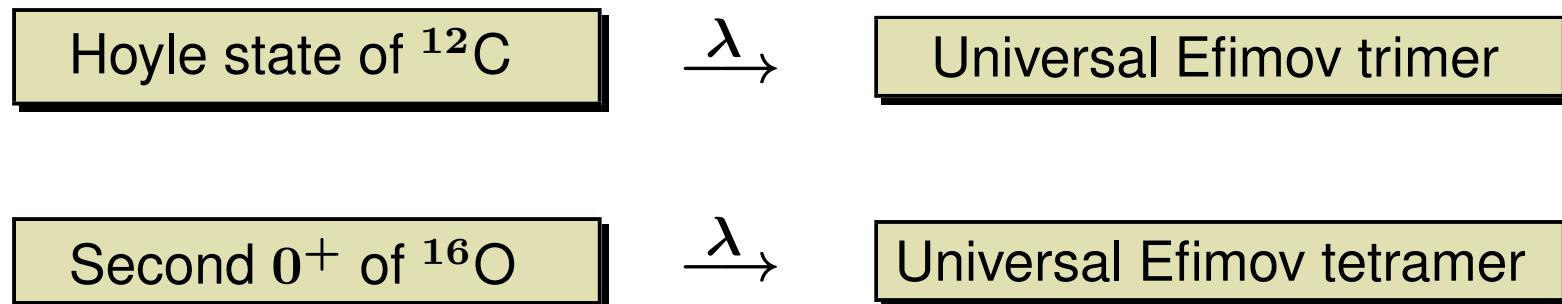


$$\begin{aligned}
 \lambda_8 &= 0.7(1) \\
 \lambda_{12} &= 0.3(1) \\
 \lambda_{16} &= 0.2(1) \\
 \lambda_{20} &= 0.2(1) \\
 \lambda_\infty &= 0.0(1)
 \end{aligned}$$

FURTHER CONSEQUENCES

- By adjusting the parameter λ in *ab initio* calculations, one can move the of any α -cluster state up and down to alpha separation thresholds.
→ This can be used as a new window to view the structure of these exotic nuclear states
- In particular, one can tune the α - α scattering length to infinity!
→ In the absence of Coulomb interactions, one can thus make contact to **universal Efimov physics**:

for a review, see Braaten, Hammer, Phys. Rept. **428** (2006) 259



SUMMARY & OUTLOOK

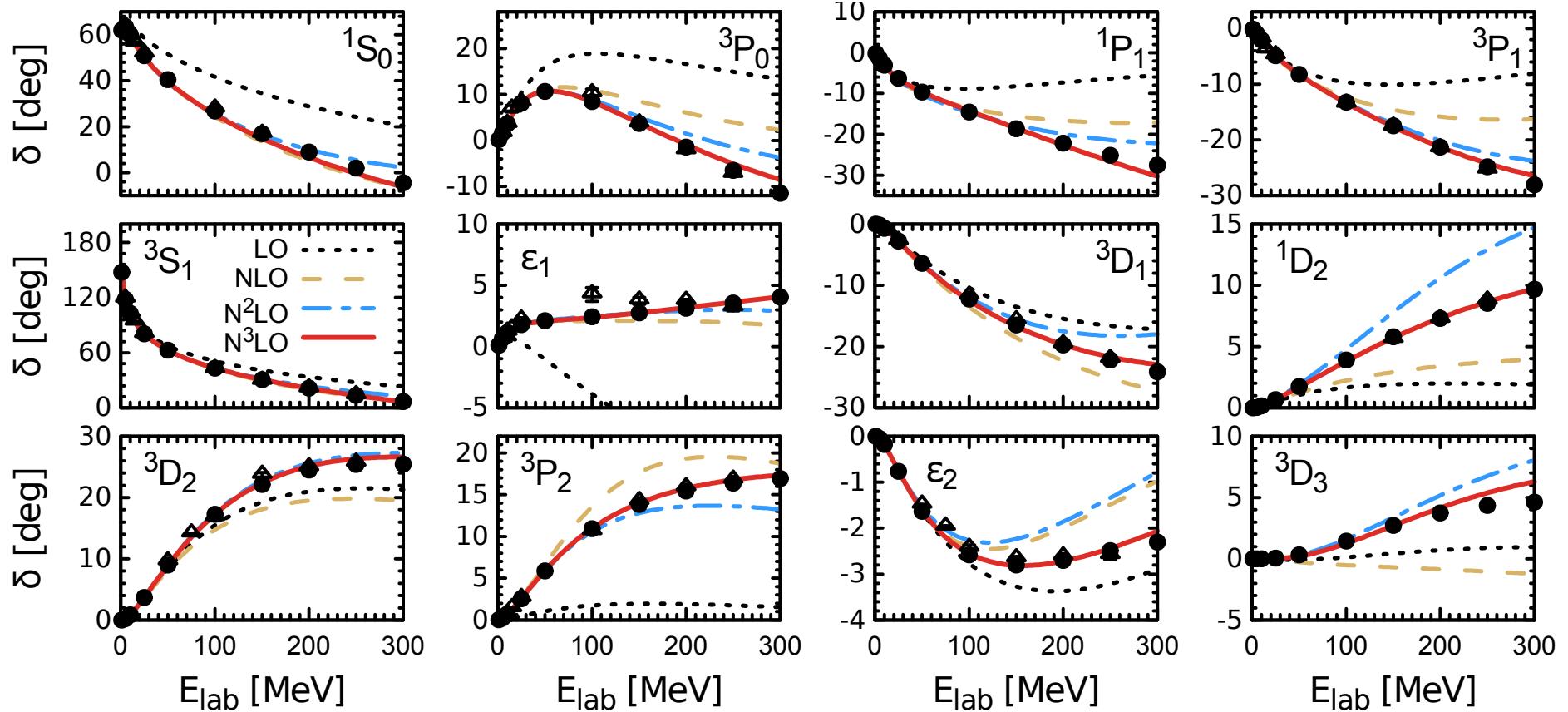
- Chiral nuclear EFT: continuum & lattice formulations
 - allows for precision calculations
 - solid method to calculate the theoretical uncertainties
 - allows for variations of the fundamental parameters
 - clustering emerges naturally, α -cluster nuclei
- Fine-tuning & the emergence of structure
 - BBN: the Higgs vev is fine-tuned to 1%
 - Hoyle state resonance condition: fine-tuned to a few %
 - ↪ need lattice QCD to reduce uncertainties
 - nuclear physics near a quantum phase transition
 - ↪ cluster formation fine-tuned to $a_{\alpha\alpha}$

SPARES

CONVERGENCE of the CHIRAL SERIES

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- phase shifts show expected convergence [large N²LO corrections understood]

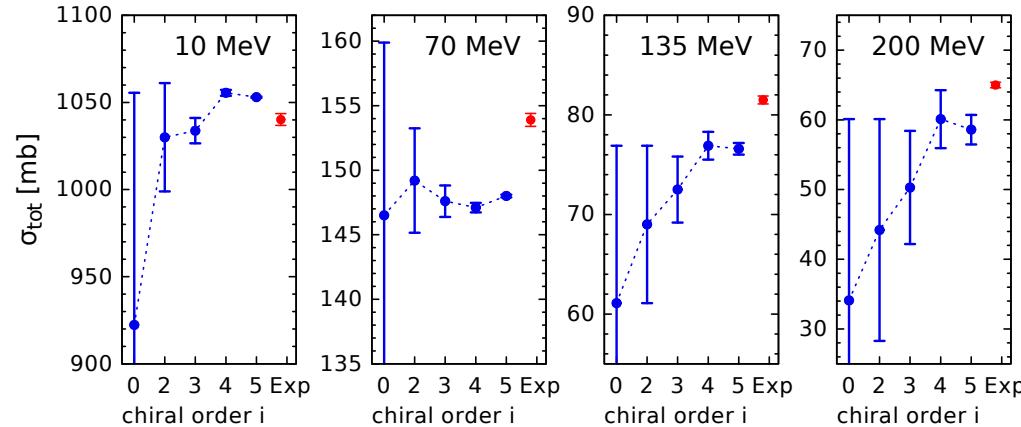


⇒ clear improvement comp. to earlier N³LO potentials [momentum space reg.]
Entem, Machleidt; Epelbaum, Glöckle, UGM

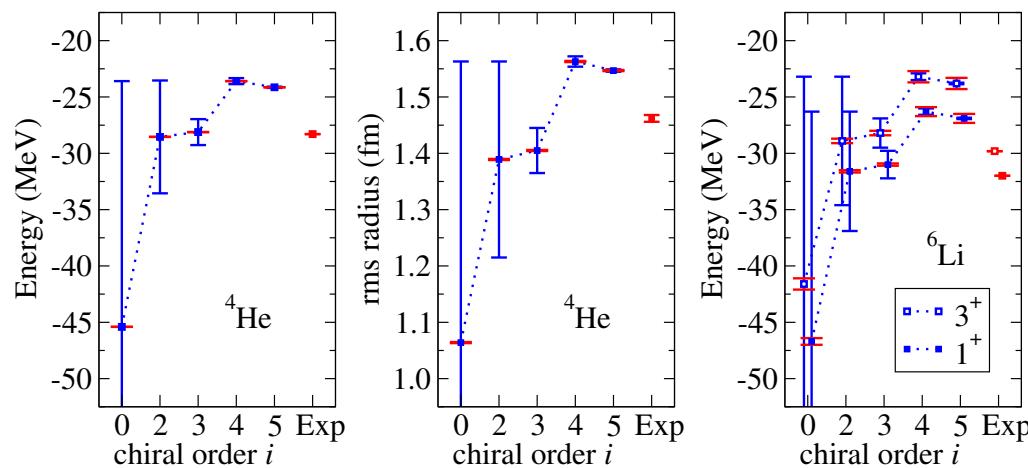
MORE EVIDENCE for THREE-NUCLEON FORCES

Binder et al. [LENPIC collaboration], arXiv:1505.07218

- Total cross section for Nd scattering [2NFs only]



- Binding energy and rms radius of ⁴He, lowest levels in ⁶Li [2NFs only]



QUARK MASS DEPENDENCE of HADRON MASSES etc⁴⁸

- Quark mass dependence of hadron properties:

$$\frac{\delta O_H}{\delta m_f} \equiv K_H^f \frac{O_H}{m_f}, \quad f = u, d, s$$

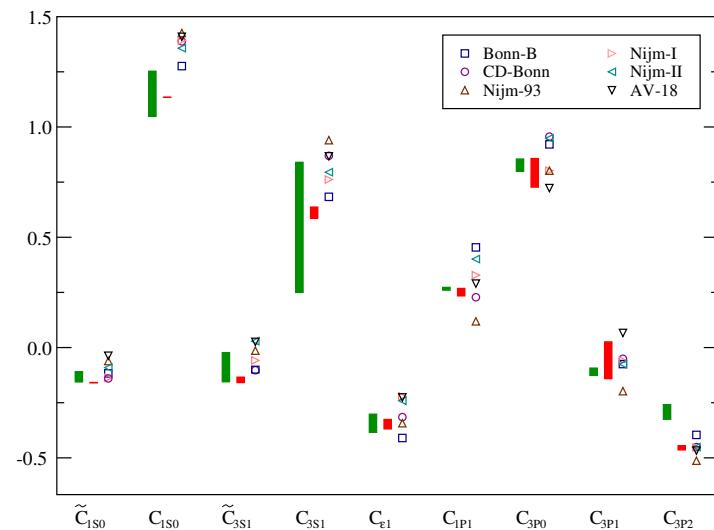
- Pion and nucleon properties from lattice QCD combined with CHPT
- Contact interactions modeled by heavy meson exchanges

M

$g =$

$$\frac{g^2}{t-M^2} = -\frac{g^2}{M^2} - \frac{g^2 t}{M^4} + \dots$$

Epelbaum, UGM, Glöckle, Elster (2002)



QUARK MASS VARIATIONS of HEAVIER NUCLEI

- In BBN, we also need the variation of ${}^3\text{He}$ and ${}^4\text{He}$. All other BEs are kept fixed.
- use the method of BLP:

Bedaque, Luu, Platter, PRC 83 (2011) 045803

$$K_{A\text{He}}^q = K_{a, 1S0}^q K_{A\text{He}}^{a, 1S0} + K_{\text{deut}}^q K_{A\text{He}}^{\text{deut}}, \quad A = 3, 4$$

with

$$K_{{}^3\text{He}}^{a, 1S0} = 0.12 \pm 0.01, \quad K_{{}^3\text{He}}^{\text{deut}} = 1.41 \pm 0.01$$

$$K_{{}^4\text{He}}^{a, 1S0} = 0.037 \pm 0.011, \quad K_{{}^4\text{He}}^{\text{deut}} = 0.74 \pm 0.22$$

so that

$$\Rightarrow K_{{}^3\text{He}}^q = -0.94 \pm 0.75, \quad K_{{}^4\text{He}}^q = -0.55 \pm 0.42$$

- calculate BBN response matrix of primordial abundances Y_a ($a = {}^2\text{H}, {}^3\text{H}, {}^3\text{He}, {}^4\text{He}, {}^6\text{Li}, {}^7\text{Li}, {}^7\text{Be}$) at fixed baryon-to-photon ratio ($n_B/n_\gamma \simeq 6 \cdot 10^{-10}$)

SCATTERING CLUSTER WAVE FUNCTIONS

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- During Euclidean time interval τ_ϵ , each cluster undergoes spatial diffusion:

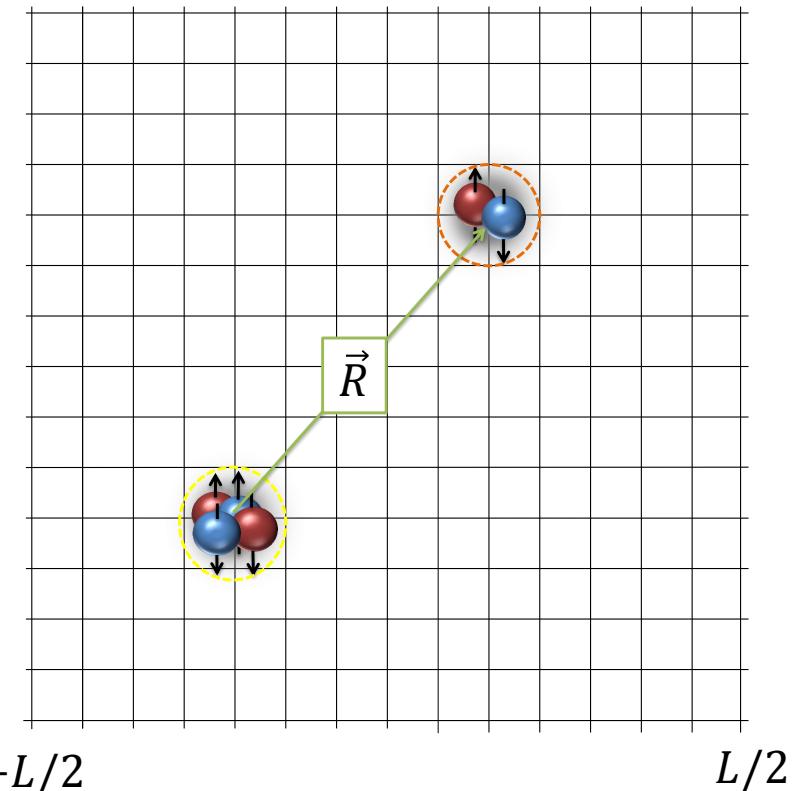
$$d_{\epsilon,i} = \sqrt{\tau_\epsilon/M_i}$$

- Only non-overlapping clusters if

$$|\vec{R}| \gg d_{\epsilon,i} \Rightarrow |\vec{R}\rangle_{\tau_\epsilon}$$

- Defines asymptotic region, where the amount of overlap between clusters is less than ϵ

$$|\vec{R}| > R_\epsilon$$

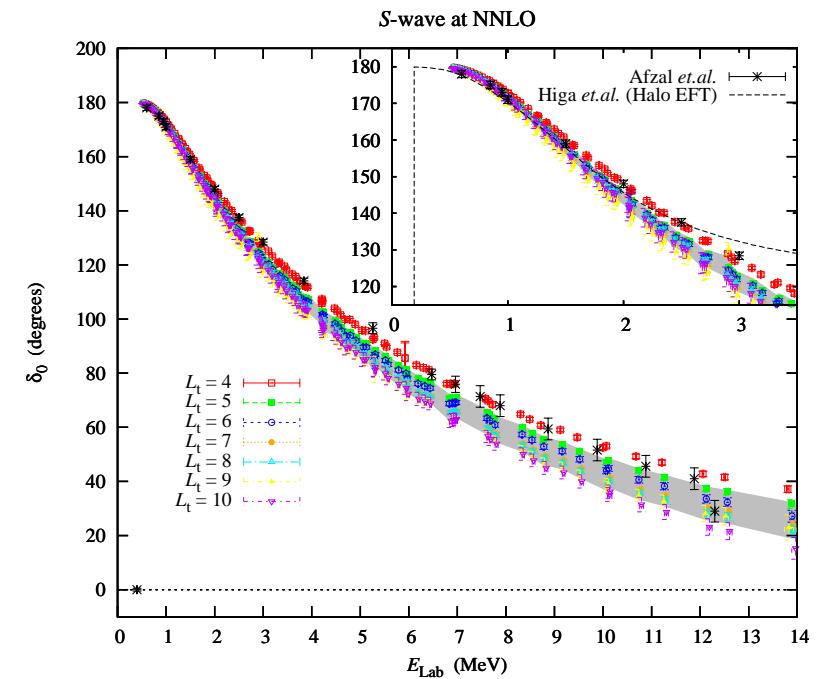
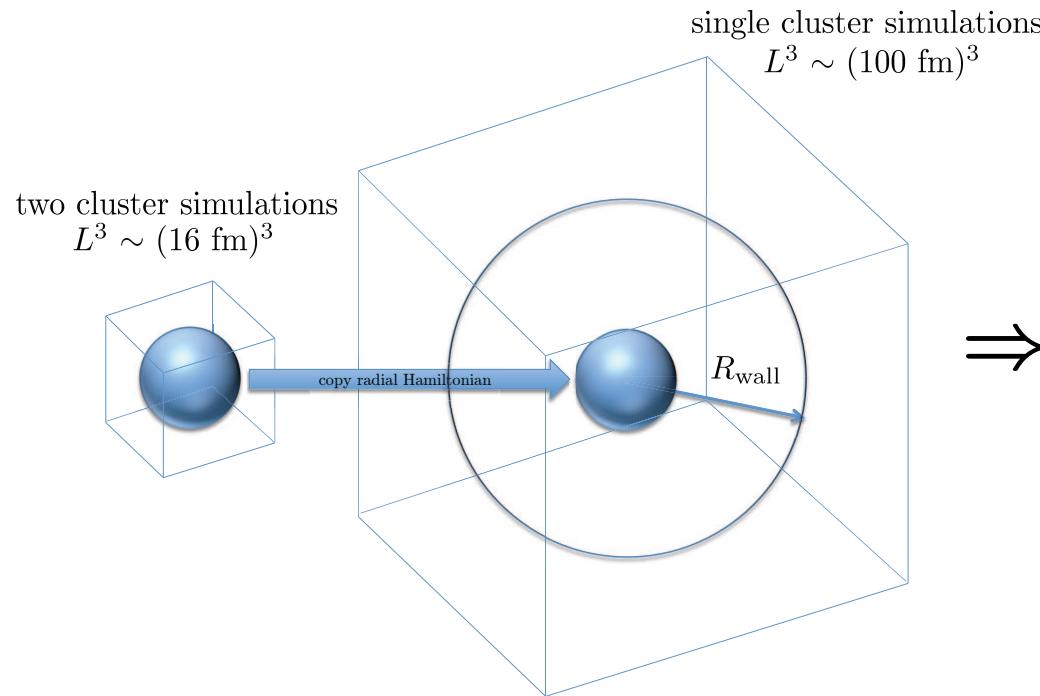


In the asymptotic region we can describe the system in terms of an effective cluster Hamiltonian (the free lattice Hamiltonian for two clusters) plus infinite-range interactions (like the Coulomb int.)

AB INITIO CALCULATION of α - α SCATTERING

- use lattice MC to construct an ab-initio cluster (adiabatic) Hamiltonian
- Use adiabatic Hamiltonian to compute scattering/reaction amplitudes

Elhatisari et al. 2015



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- D-wave equally well described

ADIABATIC HAMILTONIAN

- Construct the adiabatic Hamiltonian from the dressed cluster states:

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau \langle \vec{R} | H | \vec{R}' \rangle_\tau$$

- States are i.g. not normalized, require *norm matrix*:

$$[N_\tau]_{\vec{R}\vec{R}'} = {}_\tau \langle \vec{R} | \vec{R}' \rangle_\tau$$

- construct the full adiabatic Hamiltonian:

$$[H_\tau^a]_{\vec{R}\vec{R}'} = \sum_{\vec{R}_n \vec{R}_m} [N_\tau^{-1/2}]_{\vec{R}\vec{R}_n} [H_\tau]_{\vec{R}_n \vec{R}_m} [N_\tau^{-1/2}]_{\vec{R}_m \vec{R}'}$$

→ The structure of the adiabatic Hamiltonian is similar to the Hamiltonian matrix used in recent ab initio NCSM/RGM calculations

Navratil, Quaglioni, Phys. Rev. C 83 (2011) 044609
 Navratil, Roth, Quaglioni, Phys. Lett. B 704 (2011) 379
 Navratil, Quaglioni, Phys. Rev. Lett. 108 (2012) 042503

TESTING the ADIABATIC HAMILTONIAN

- Consider fermion-dimer scattering:

Microscopic Hamiltonian

$$L^{3(A-1)} \times L^{3(A-1)}$$



Two-cluster adiabatic Hamiltlonian

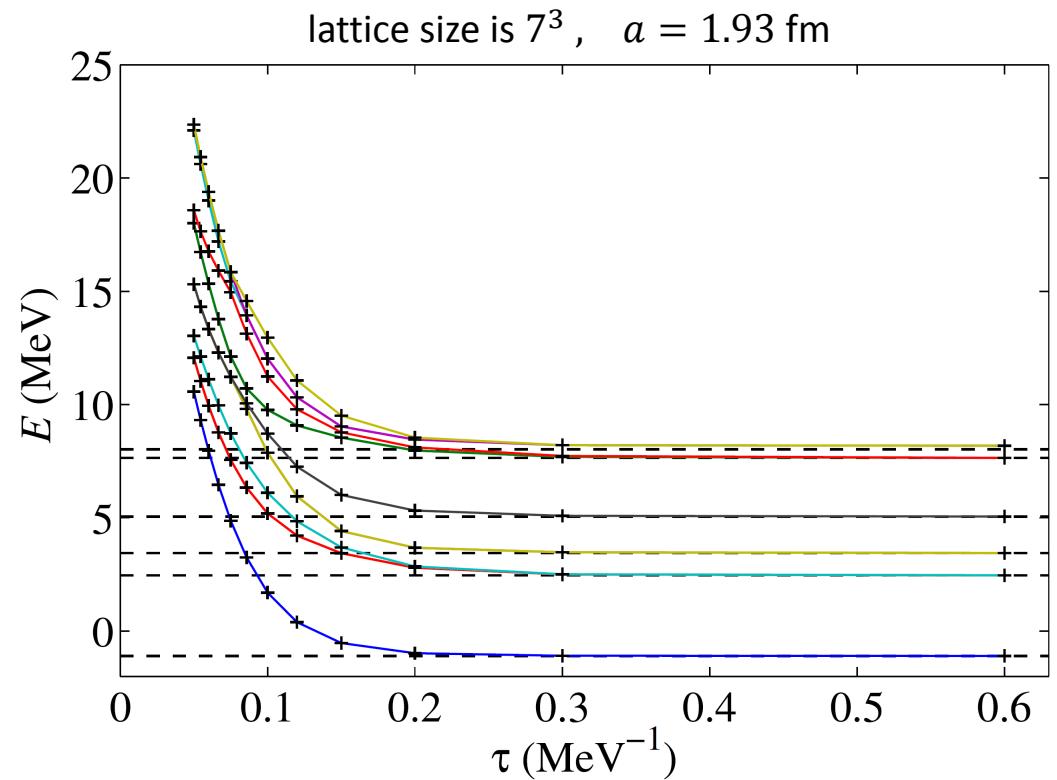
$$L^3 \times L^3$$

- calculation of a 7^3 lattice,
lattice spacing $a = 1.93$ fm

Pine, Lee, Rupak, EPJA 49 (2013) 151

exact Lanczos: black dashed lines

adiab. Ham.: solid colored lines



EXTRACTING PHASE SHIFTS on the LATTICE

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- Lüscher's method:

Two-body energy levels below the inelastic threshold on a periodic lattice are related to the phase shifts in the continuum

Lüscher, Comm. Math. Phys 105 (1986) 153

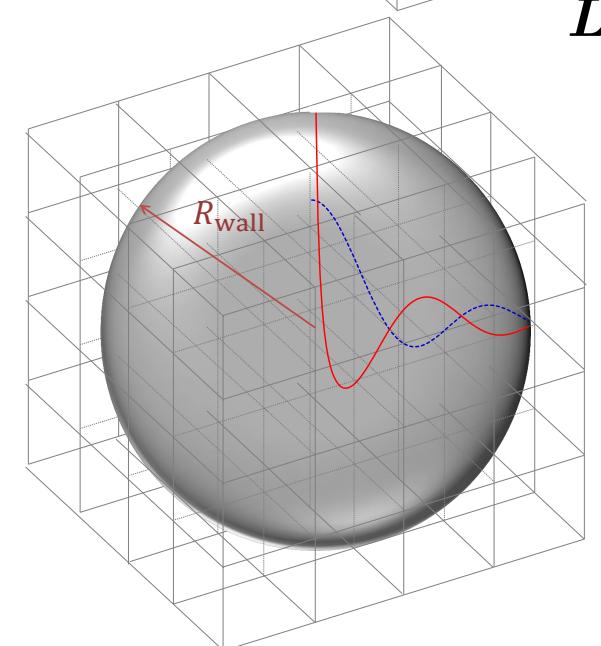
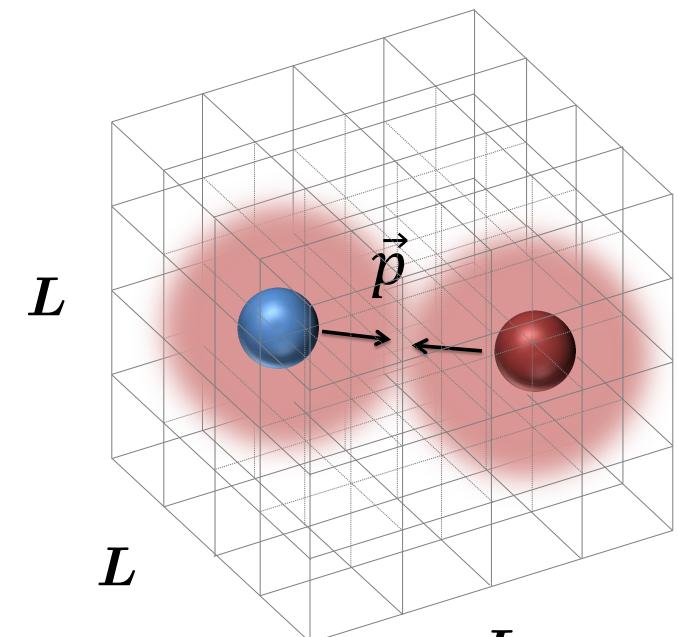
Lüscher, Nucl. Phys 354 (1991) 531

- Spherical wall method:

Impose a hard wall on the lattice and use the fact that the wave function vanishes for $r = R_{\text{wall}}$:

$$\psi_\ell(r) \sim [\cos \delta_\ell(p) F_\ell(pr) + \sin \delta_\ell(p) G_\ell(pr)]$$

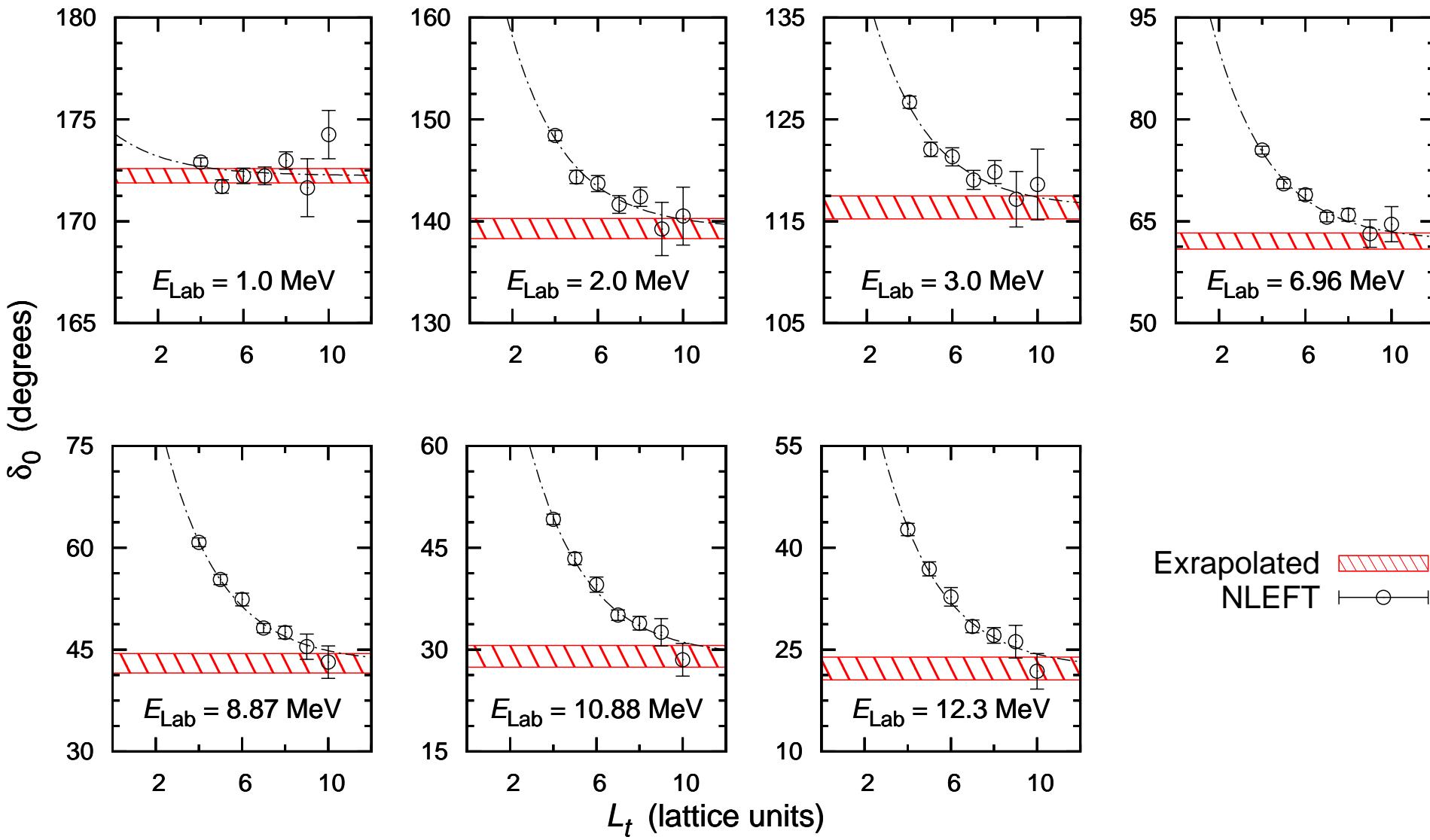
Borasoy, Epelbaum, Krebs, Lee, UGM,
EPJA 34 (2007) 185



LATTICE DATA I

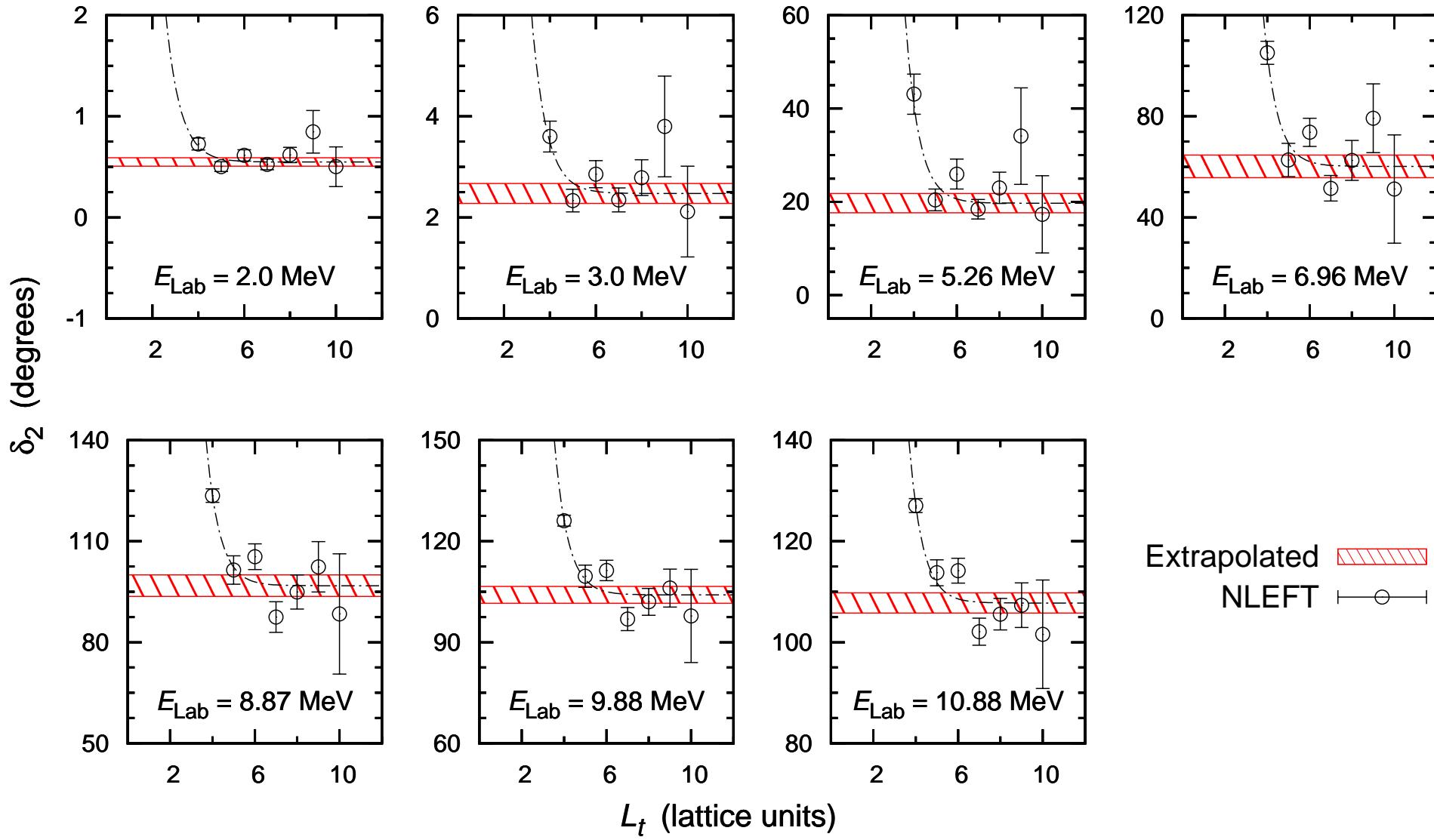
55

- Show data for the S-wave:



LATTICE DATA II

- Show data for the D-wave:



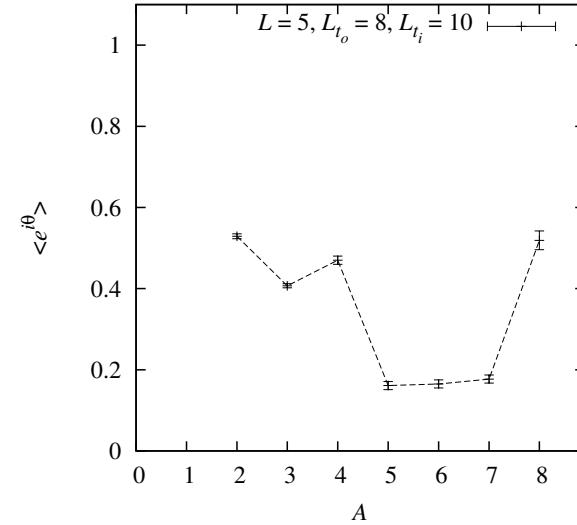
SYMMETRY-SIGN EXTRAPOLATION METHOD

Epelbaum, Krebs, Lähde, Lee, Luu, UGM, Rupak, arXiv:1502.06787

- so far: nuclei with $N = Z$, and $A = 4 \times \text{int}$
as these have the least sign problem
due to the approximate SU(4) symmetry

$$\langle \text{sign} \rangle = \langle \exp(i\theta) \rangle = \frac{\det M(t_o, t_i, \dots)}{|\det M(t_o, t_i, \dots)|}$$

$M(t_o, t_i, \dots)$ is the transition matrix



Borasoy et al. (2007)

- Symmetry-sign extrapolation (SSE) method: control the sign oscillations

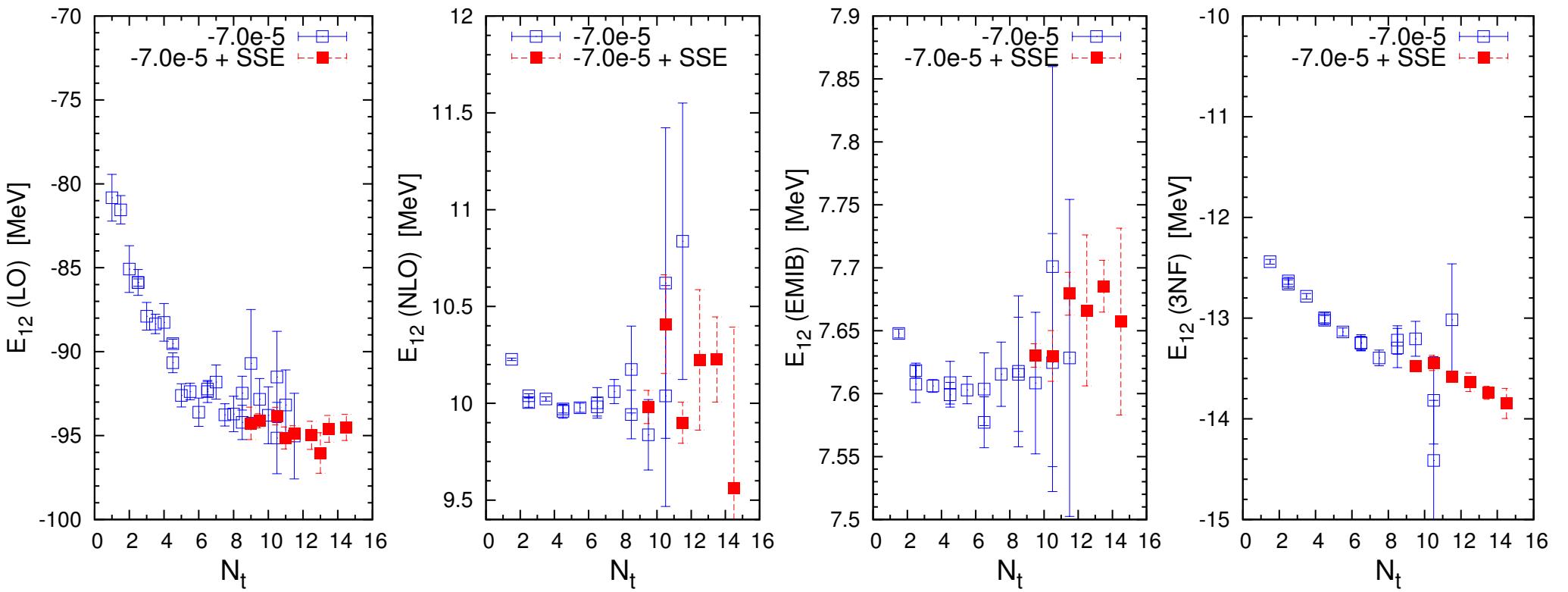
$$H_{d_h} = d_h \cdot H_{\text{phys}} + (1 - d_h) \cdot H_{\text{SU}(4)}$$

$$H_{\text{SU}(4)} = \frac{1}{2} C_{\text{SU}(4)} (N^\dagger N)^2$$

→ family of solutions for different SU(4) couplings $C_{\text{SU}(4)}$
that converge on the physical value for $d_h = 1$

RESULTS for ^{12}C

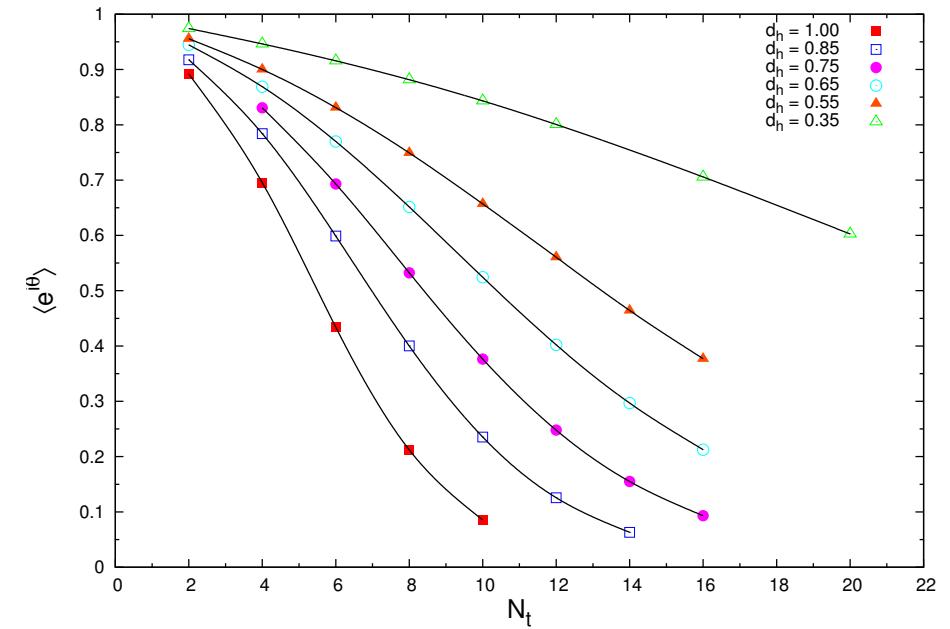
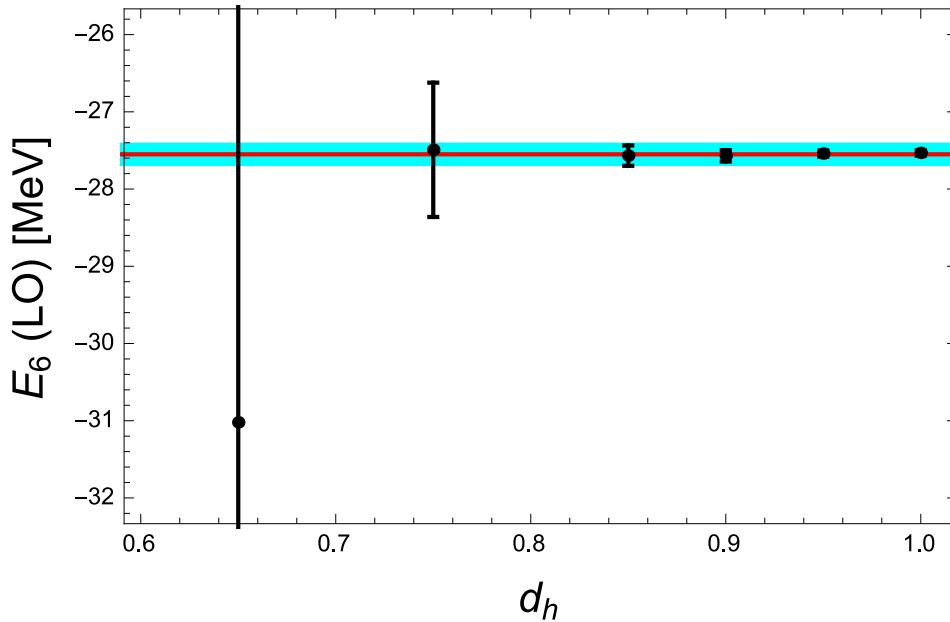
- generate a few more MC data at large N_t using SSE



- promising results → no more exponential deterioration of the MC data
- results w/ small uncertainties for $d_h \geq 0.8$

RESULTS for $A = 6$

- Simulations for ${}^6\text{He}$ and ${}^6\text{Be}$



⇒ methods works for nuclei with $A \neq Z$

⇒ neutron-rich nuclei can now be systematically explored (larger volumes)

FIRST INSIGHT

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- Interaction B was tuned to the nucleon-nucleon phase shifts, the deuteron binding energy, and the S-wave α - α phase shift
- Interaction A starts from interaction B, but *all* local short-distance interactions are switched off, then the LECs of the non-local terms are refitted to describe the nucleon-nucleon phase shifts and the deuteron binding energy
 - The alpha-alpha interaction is sensitive to the degree of locality of the NN int.
 - Qualitative understanding: tight-binding approximation (eff. α - α int.)

