Effective Field Theory For Halo Nuclei



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- separation of scales:
 - an observable's length scale is much larger than the interaction range $a \gg \ell$
 - physics at scale a is insensitive to physics at scale ℓ
- systematic expansion:
 - construct effective Lagrangian $\mathcal{L} = \sum c_{\nu} (\ell/a)^{\nu}$
 - calculate observables at scale a in the ℓ/a expansion
- universality:
 - separation of scales \rightarrow universality 2body universality: $B_2 = 1/ma^2$
 - a limited number of LECs enter at a given EFT order
 - observables are correlated through a limited number of parameters



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Effective Theory for Halo Nuclei

cluster configuration:

core + valence nucleons d.o.f.

separation of scales:

 $M_{\rm halo} \sim \sqrt{m_n S_{2n}} \\ M_{\rm core} \sim \sqrt{m_n E_c^*} \\ M_{\rm halo} \ll M_{\rm core}$



- o effects from underlying theory:
 - a theory in NN/NNN interactions (pionful, pionless, realistic, ...)
 - anti-symmetrization of core neutrons is not explicit in halo EFT
 - short-range effects are embedded in LECs controlled by systematic expansions in $M_{\rm halo}/M_{\rm core}$
- EFT unveils universality in halo nuclei

Examples of Halo Nuclei

● ²H

- simplest neutron halo
- pionless EFT for few-nucleon systems is a specific case of halo EFT



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- neutron halos with compound nuclear core
 - 6 He, 11 Be, ...



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- proton halos
 - ¹⁷F* (s-wave halo)
 - ⁸B (p-wave halo): $E_c^* = 1.59$ MeV; $S_{1p} = 0.14$ MeV $M_{\rm halo}/M_{\rm core} \approx 20\%$



FMD calculation (T. Neff, GSI)

• ²H

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- α -clustering
 - ⁹Be: α + α + n
 ⁸Be, ¹²C*, ¹⁶O*



- Halos in s-wave interactions
- Halos in p-wave interactions
- Halos with Coulomb (proton halos and α-clusters)
- Electromagnetic reactions on halo nuclei
- Discussions:
 - EFT construction and power counting
 - Universality in Halo EFT (mainly LO results)
 - Connection with underlying theory /experiments

"Someone" said:

There are a thousand Hamlets in a thousand people's eyes, so is the word "naturalness".

Naturalness in observables

naturalness of observables can be connected with naturalness in LECs

a two-body system in s-wave zero-range interactions

$$\mathscr{L} = \psi^{\dagger} \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - \frac{C_0}{2} (\psi^{\dagger}\psi)^2 \quad ; \text{where} \ C_0 = \frac{4\pi}{m} (-\Lambda + 1/a)^{-1}$$



Kaplan, Savage, Wise, '98

• scattering amplitude at $k \sim M_{halo}$: $t_0(k) = rac{2\pi}{\mu} \left(1/a + ik
ight)^{-1}$

• natural case: $a \sim 1/M_{\text{core}}$ $\rightarrow t_0 \approx C_0(\Lambda \rightarrow 0);$

 C_0 is natural and perturbative

- unnatural case: $a \sim 1/M_{halo}$ $\rightarrow t_0$ unnaturally enhanced by a bound/virtual pole ; $C_0(\Lambda \rightarrow 0)$ is unnatural, nonpertubative loop even for $\Lambda \sim p$
- In an ideal case, one can keep unnaturalness at LO and maintain naturalness in a perturbative expansion of $M_{\rm halo}/M_{\rm core}$ at higher orders

We adopt EFT with contact interactions to describe clustering in halo nuclei
introduce auxiliary dimer fields for bound/resonance states

$$\begin{aligned} \mathscr{L} &= \mathscr{L}_{1} + \mathscr{L}_{2} + \mathscr{L}_{3} \\ \mathscr{L}_{1} &= n^{\dagger} \left(i\partial_{0} + \frac{\nabla^{2}}{2m_{n}} \right) n + c^{\dagger} \left(i\partial_{0} + \frac{\nabla^{2}}{2m_{c}} \right) c \\ \mathscr{L}_{2} &= s^{\dagger} \left[\eta_{0} \left(i\partial_{0} + \frac{\nabla^{2}}{4m_{n}} \right) + \Delta_{0} \right] s + \sigma^{\dagger} \left[\eta_{1} \left(i\partial_{0} + \frac{\nabla^{2}}{2(m_{n} + m_{c})} \right) + \Delta_{1} \right] \sigma \\ &+ g_{0} \left[s^{\dagger} (nn) + \text{h.c.} \right] + g_{1} \left[\sigma^{\dagger} (nc) + \text{h.c.} \right], \\ \mathscr{L}_{3} &= h \left(\sigma n \right)^{\dagger} \left(\sigma n \right) \end{aligned}$$

• 2-body contact (LO) $= -i\sqrt{2}g$ $g \leftarrow 2$ -body observable 3-body contact (LO)



 $h \leftarrow$ 3-body observable

• Iterative summation



• scattering amplitude:
$$t_0(k) = \frac{2\pi}{\mu} \left(\frac{1}{a_0} - \frac{r_0}{2}k^2 + ik\right)^{-1}$$

- $a_0 \sim 1/M_{\rm halo}$; $r_0 \sim 1/M_{\rm core}$
- calculation in expansion of r_0/a_0
- tune coupling

• LO:
$$a_0 = \left(\frac{2\pi\Delta}{\mu g^2} + \Lambda\right)^{-1}$$

• NLO: $r_0 = -\eta \frac{2\pi}{\mu^2 g^2}$

• pole expansion: $t_0(k) = \frac{2\pi}{\mu} \frac{C_{\sigma}^2/C_{\sigma,LO}^2}{\gamma_0 + ik} + \text{regular}$

• LO:
$$C_{\sigma,LO} = \sqrt{2\gamma_0}$$

• NLO: $C_{\sigma}/C_{\sigma,LO} = 1/\sqrt{1-\gamma_0 r_0}$

One-neutron s-wave halos

| | ^{2}H | ^{11}Be | 15 C | 19 C |
|--|--------------|-------------|-----------|-----------|
| EXP | | | | |
| S_{1n} [MeV] | 2.224573(2) | 0.50164(25) | 1.2181(8) | 0.58(9) |
| E_c^* [MeV] | 140 | 3.36803(3) | 6.0938(2) | 1.62(2) |
| $\langle r_{nc}^2 angle^{1/2}$ [fm] | 3.936(12) | 6.05(23) | 4.15(50) | 6.6(5) |
| | 3.95014(156) | 5.7(4) | 7.2±4.0 | 6.8(7) |
| _ | | 5.77(16) | 4.5(5) | 5.8(3) |
| EFT | | | | |
| $M_{\rm halo}/M_{\rm core}$ | 0.33 | 0.39 | 0.45 | 0.6 |
| r_0/a_0 | 0.32 | 0.32 | 0.43 | 0.33 |
| $\mathcal{C}_{\sigma}/\mathcal{C}_{\sigma,LO}$ | 1.295 | 1.3 | 1.63 | 1.3 |
| $\langle r_{nc}^2 angle^{1/2}$ [fm] | 3.954 | 6.16 | 4.93 | 5.72 |

A three-body problem in Faddeev formalism



• three-body wave functions

$$\Psi_{n}(\boldsymbol{p},\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{p} \\ \boldsymbol{q} \end{bmatrix}^{\boldsymbol{p}} \cdot \boldsymbol{q} \\ \Psi_{c}(\boldsymbol{p},\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{bmatrix}^{\boldsymbol{p}} \cdot \boldsymbol{q} \\ \mathcal{A}_{c} \\$$

Three-body renormalization

• running of three-body coupling

- tune $H(\Lambda) = \Lambda^2 h/2mg^2$: reproduce one observable in a 2n-halo
- Iimit cycle:

 $H(\Lambda)$ periodic for $\Lambda \to \Lambda \exp(n\pi/s_0)$

A = 1: Bedaque *et al.* '00

discrete scale invariance \rightarrow Efimov physics



Efimov physics

• a universal spectrum of three-body bound states

$$B_3 = -\frac{1}{ma_0^2} + \left[e^{-2\pi n} f(\xi)\right]^{1/s_0} \frac{\kappa_*^2}{m}$$

Braaten, Hammer, Phys. Rept. '06

- ullet atomic physics: vary a_0 through Feshbach resonance
- nuclear physics: fixed a₀



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- unitary limit $(a \to \infty)$: $B_3 = e^{-2\pi n/s_0} \frac{\kappa_*^2}{m}$
- discrete scale invariance: $\kappa_* \to \kappa_*, \quad a_0 \to e^{\pi n/s_0} a_0$
- exploring Efimov physics in halo nuclei is an important subject



n-core in s-wave virtual/real bound state:
 ¹¹Li, ¹²Be, ²⁰C [Canham, Hammer, EPJA '08, NPA '10]
 ²²C Acharya, C.J., Phillips, PLB '13

- charge radius of 2n s-wave halos [Hagen, Hammer, Platter, EPJA '13]
 [Vanasse, arXiv '15, '16]
- heaviest 2n s-wave halo:

 $^{62}\mathsf{Ca}$ [Hagen, Hagen, Hammer, Platter, PRL '13] fit $n^{-60}\mathsf{Ca}$ scattering length from coupled-cluster calculations

Universality in 2n s-wave halo

• contour constraints on ground-state energy S_{2n} if the excited-state energy $B_3^* = \max\{0, E_{nn}, S_{1n}\}$



Canham, Hammer, EPJA '08; Frederico *et al.* PPNP '12 15 / 33

Correlations in ²²C

$$\langle r_m^2 \rangle_{2n-\text{halo}} = \frac{1}{m_n S_{2n}} f\left(\frac{E_{nn}}{S_{2n}}, \frac{S_{1n}}{S_{2n}}; A\right)$$

Acharya, C.J., Phillips, PLB '13

Experimental input: $\langle r_m^2 \rangle_{2n-{\rm halo}} - \langle r_m^2 \rangle_{{\rm core}} = 3.01^{+0.85}_{-0.72}$ fm Togano *et al.* PLB '16



bands: uncertainty from NLO EFT $\sim \max\left\{\frac{\sqrt{mE_{nn}}}{M_{core}}, \frac{\sqrt{mS_{1n}}}{M_{core}}, \frac{\sqrt{mS_{2n}}}{M_{core}}\right\}$

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Other experimental bound: • AME2012 $S_{2n} < 170 \text{ keV}$

• Gaudefroy et al., PRL '12 $S_{1n} < -2.9 \ {\rm MeV} \label{eq:sigma}$

P-wave neutron halos

• nc interaction in a p-wave bound/resonance state

$$n = \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

P-wave neutron halos

• *nc* interaction in a p-wave bound/resonance state

$$a = \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

- causality $r_1 < 0$ and $r_1 \not\rightarrow 0$ Nishida '12
- both a_1 and r_1 enter at leading order
- p-wave power countings
 - [Bertulani, Hammer, van Kolck NPA '02] $a_1 \sim M_{
 m halo}^{-3}$, $r_1 \sim M_{
 m halo}$

[Bedaque, Hammer, van Kolck PLB '03] a₁ ~ M⁻²_{halo}M⁻¹_{core}, r₁ ~ M_{core}
a₁ > 0: shallow bound state: ¹¹Be (1/2⁻)
a₁ < 0: shallow resonance: ⁵He (3/2⁻)

p-wave power counting (Bertulani)

• p-wave EFT power counting: Bertulani, Hammer, van Kolck NPA '02

- $a_1 \sim 1/M_{\text{halo}}^3$ $r_1 \sim M_{\text{halo}}$
- two fine tunings at LO



$n - \alpha$ p-wave power counting (Bedaque)

- $n\alpha$ EFT power counting: Bedaque, Hammer, van Kolck PLB '03
 - $a_1 \sim 1/(M_{\rm halo}^2 M_{\rm core})$ $r_1 \sim M_{\rm core}$
 - one fine tuning at LO

| $-k_P - i\Gamma$ | $X \gamma_1 \sim -\frac{r_1}{2}$ | |
|------------------|----------------------------------|--|
| kp | \times $k_R - i\Gamma$ \sim | \longrightarrow $\operatorname{Re}(k)$ |

T (1)

$n - \alpha$ p-wave power counting (Bedaque)

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$$a_1 \sim 1/(M_{\text{halo}}^2 M_{\text{core}})$$
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$n-\alpha$ scattering cross sections



- expansion without resumming ik^3 $t\propto rac{1}{r_1(k^2-k_R^2)}$ is only valid if $|k^2-k_R^2|$ is not small
- expansion with resumming ik^3 is required when $k \sim k_R$

cluster model

- separable potential Ghovanlou, Lehman '74
- variational method Funada et al. '94
- density-dependent nn contact interaction Esbensen et al. '97
- Wood Saxon $n \alpha$ + GPT nn Danilin, Thompson, Vaagen, Zhukov '98

ab intio calculation

- no-core shell model Navrátil et al. '01; Sääf, Forssén '14, Romero-Redondo
- NCSM-RGM/Continuum Romero-Redondo et al. '14 '16
- Green's function Monte Carlo Pieper et al. '01; '08
- hyperspherical harmonics (EIHH) Bacca et al. '12

• Halo EFT in ⁶He ground state

- EFT+Gamow shell model Rotureau, van Kolck Few Body Syst. '13
- EFT+Faddeev equation C.J., Elster, Phillips, PRC '14

Faddeev equations for ⁶He



Running of 3BF Coupling

 p-wave 3BF: reproduce S_{2n} = 0.973 MeV





Running of 3BF Coupling

• p-wave 3BF: reproduce $S_{2n} = 0.973$ MeV







Renormalized Faddeev Components

 \mathcal{A}_{α} and \mathcal{A}_{n} are cutoff independent



C.J., Elster, Phillips, PRC '14

Universal correlations btw ⁶He radii & S_{2n}



- In halo/clustering systems with Coulomb interactions, a new scale $k_c = Q_c \alpha_{em} \mu$ enters
 - $k_c \gtrsim M_{halo}$: Coulomb interaction is nonperturbative p-p scattering [Kong, Ravndal, PLB '99; NPA '10] $p-\alpha$ and $\alpha-\alpha$ scattering [Higa, Hammer, van Kolck, NPA '08; Higa, FBS '11]
 - ¹⁷F* [Ryberg, Forssén, Hammer, Platter, PRC '14; AnnPhys '16]
 - $k_c \ll M_{halo}$: Coulomb interaction is perturbative ³H and ³He [König, Grießhammer, Hammer, van Kolck, JPG '16]

Fine tuning in α clustering and proton-halos



Higa, Hammer, van Kolck, NPA '08

- $k_c \gg k_R$
- a highly fine tuned system
- LO $k_R^2 \approx 2/(a_0 r_0)$
- NLO a_0, r_0, P_0

universality in proton halos



Ryberg, Forssén, Hammer, Platter, AnnPhy '16

- $k_c \gg \gamma$
- fine tuning both a_0 and r_0
- large cancelation btw Coulomb repulsion and strong interaction

Electromagnetic reactions on halo nuclei

- Coulomb dissociation
 - breakup by colliding a halo nucleus with a high-Z nucleus
 - the halo dynamics dominates when $Q_\gamma \sim M_{
 m halo}$



EFT on Coulomb dissociation of 1n halos



EFT on Coulomb dissociation of 1n halos



universal E1 transition



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EFT on Coulomb dissociation of 1n halos



LO EFT: universal E1 transition

$$\frac{d\sigma}{dE} = \frac{16\pi^3}{9} N_{E1}(E, R) \frac{d\mathbf{B}(E1)}{dE}$$



Coulomb dissociation differential cross section in $^{11}\mathrm{Be}$ and $^{19}\mathrm{C}$





E1 S-factor for ${}^{7}\text{Be}(p,\gamma){}^{8}\text{B}$

Zhang, Nollett, Phillips, PRC '14
 — NSCM-GRM result
 [Navratil, Roth, Quaglioni, PLB '11]
 ---- LO EFT: fit to NSCM-GRM ANC
 LO EFT: fit to ANC from VMC
 VMC [Nollett, Wiringa, PRC '11]



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 Ryberg, Forssén, Hammer, Platter, EPJA '14
 correlation btw S(0) and r_C[⁸B]



NLO Halo EFT calculation of E1 S-factor for ${}^{7}\text{Be}(p,\gamma){}^{8}\text{B}$ combined with ab initio ANC and Bayesian data-error analysis Zhang, Nollett, Phillips, PLB '15 0.6 0.3 0. 01 0.3 0.4 0.5 0.224 22 22 20 20 18 18 16 16 S (eV b) vs. E (MeV) 14 14 0.6 0.3 0. 0.1 0.2 0.3 0.4 0.5

 $S(0) = (21.3 \pm 0.7 eVb)$

- \bullet Halo EFT describes structure/reaction in halo nuclei in a controlled expansion in $M_{\rm halo}/M_{\rm core}$
- Halo EFT rejuvenate cluster models with a systematic uncertainty estimates
- Halo EFT can be complimentary to ab initio calculations
 - adopt inputs from *ab initio* results
 - benchmark with ab initio calculations
 - explain universal correlations from observables in ab initio work

n-n s-wave power counting

• *nn* EFT power counting:

• EFT:
$$a_0 \sim 1/Q$$
 $r_0 \sim 1/\Lambda_{EFT}$

• $Q/\Lambda_{EFT} \sim 0.15$

