Assigned Title: Is Something Wrong With Chiral EFT?

H. W. Grießhammer

Institute for Nuclear Studies
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1. The EFT Promise: Serious Theorists Have Error Bars
2. The Promise of Being Systematic
3. The Promise of Reliable Error Bars
4. Concluding Questions

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Providing reliable theoretical uncertainties,
testing non-perturbative EFTs.

hg: *Nucl. Phys.* **A744** (2004) 192;
hg: NNPSS 2008, Saclay workshop 04.03.2013, Benasque workshop 24.07.2014;
hg: Chiral Dynamics proceedings [arXiv:1511.00490 [nucl-th]]

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Answer: No, but with its Practitioners...

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1. The EFT Promise: Serious Theorists Have Error Bars

(a) Physical Models vs. Physical Theories – Sliding Scale

Model: Capture *some* aspects with lots of data – no “fail” but “tuned”. Cargo Cult mode.

The Trouble With Nuclear Physics

In fact the trouble in the recent past has been a surfeit of different *models* [of the nucleus], each of them successful in explaining the behavior of nuclei *in some situations*, and each in *apparent contradiction with other successful models* or with our ideas about nuclear forces.

Rudolph E. Peierls: “The Atomic Nucleus”, *Scientific American* **200 (1959)**, no. 1, p. 75; emphasis added

Theory: Comprehensive, prescriptive, predictive, may fail. Explain-All-To-Some-Degree mode.

Gelman’s Totalitarian Principle/Swiss Basic Law/ Weinberg’s “Folk Theorem”: Throw In the Kitchen Sink

As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you’re simply writing down the most general theory you could possibly write down.

Original: Weinberg: Physica 96A (1979) 327 – here 1997 version

“EFT $=$ Symmetries $+$ Parametrisation of Ignorance"?? WHAT CAN POSSIBLY GO WRONG???

(b) EFTs Can Go Wrong: Check & Follow Assumptions

Expand observables as $\mathcal{O} = c_0 + c_1 Q^1 + c_2 Q^2 + \dots$

with $Q = \dfrac{\text{typ. momentum } p_{\text{typ.}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} < 1.$

– No separation/jungle of scales? e.g. N^* at 2 GeV

– Incorrect usage: $p_{\text{typ.}} \nearrow \overline{\Lambda}_{\text{EFT}} \implies Q \not\ll 1$?

"EFTs carry seed of own destruction." D. R. Phillips

Check EFT's Fundamental Building Blocks

– **Which constituents?** – **The Elephant in the Room:**

Results at $k \gtrsim 200$ MeV **without** $\Delta(1232)$ **inconsistent.**

Breakdown of χEFT without it: $M_\Delta - M_N \approx 300$ MeV.

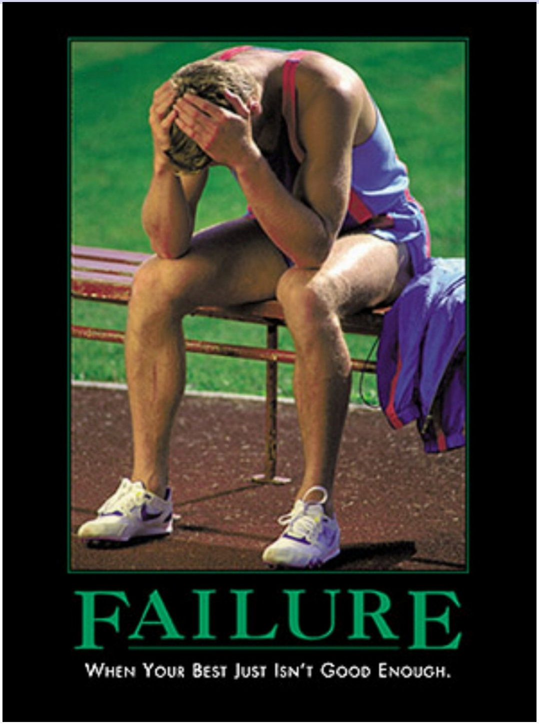
Often not considered (phase shift fits), although available.

UvK 1993, Krebs/...2007/8, Piarulli/Navarro Pérez/Amaro/Ruiz Arriola/...2016,..

– **Which symmetries?** e.g. impose Parity in weak processes

– **Check Quantitatively Predicted Convergence Pattern:**

Order by order smaller corrections & cut-off dependence.



– **EFT may converge, but not to Nature:** Wrong ordering scheme (e.g. perturbative in NN) – or any of the above.

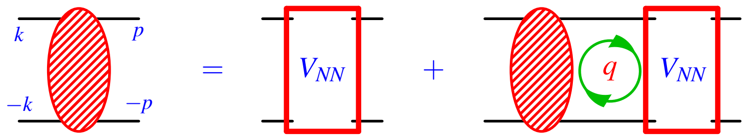
Convergence to Nature **tests assumptions. –** ***After*** **theoretical consistency & uncertainties determined.**

Humans abhor failure, but if an EFT fails, *"you have learned a lot"* UvK last Tuesday.

Weinberg 1991, van Kolck 1992-;
cf. hg 1511.00490 [nucl-th]

Phenomenology: Non-relativistic system with shallow (real/virtual) bound-state $\Longrightarrow$ LO *non-perturbative.*

$$T_{NN}(E \sim \frac{p^2,k^2}{M}) \sim Q^{-1}$$

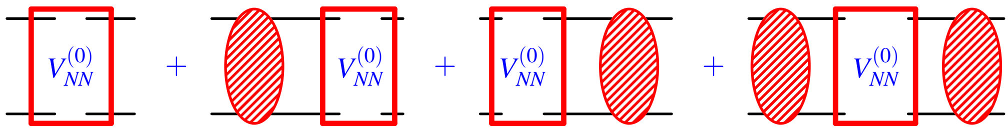


$$T_{\text{LO}} = V_{\text{LO}} + V_{\text{LO}}\, G_{NN}^{\text{nonrel.}}\, T_{\text{LO}}$$

Power-Counting: $Q^m \qquad Q^m \qquad Q^{2m+3-2} \overset{!}{=} Q^m \Longrightarrow m=-1$

All corrections in "strict perturbation" about LO (Distorted-Wave Born):
$\Longrightarrow$ Power-counting of amplitudes (observables); simple, no resummation artefacts.

Example NLO:



$$T_{\text{NLO}} = (\mathbb{1} + T_{\text{LO}}^\dagger)\, V_{\text{NLO}}\, (\mathbb{1} + T_{\text{LO}})$$

Alternative Weinberg 1990: Power-count & iterate $V_{NN} \Longrightarrow T = \dfrac{V_{\text{LO}} + V_{\text{NLO}} + \dots}{1 - (V_{\text{LO}} + V_{\text{NLO}} + \dots)\, G_{NN}}$.

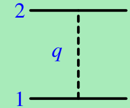
$\Longrightarrow$ Obscures PC of amplitudes/observables, unphysical poles just around $\overline{\Lambda}_{\text{EFT}}$,
small cutoff variation range $\Lambda \approx \overline{\Lambda}_{\text{EFT}} \pm 20\%$, implementation & numerics more difficult.

(d) Long-Range Interaction: One Pion Exchange

One Pion Exchange Potential (OPE)

like mag. dipole-dipole int., **parameters fixed by πN.**

$$V_{OPE} = -\frac{g_A^2}{4f_\pi^2}\,\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^2+m_\pi^2}\,\tau_1^a\tau_{2a}$$



Central part Yukawa 1935:

$$V_C(r) = -\frac{g_A^2\,m_\pi^2}{16\pi f_\pi^2}\,\frac{e^{-m_\pi r}}{r} < 0 \quad \xrightarrow{\text{chiral limit}} \quad 0 + \text{CT}$$

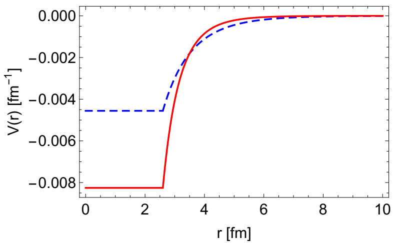
Tensor part cf. mag. dipoles:

$$V_T(r) = -\frac{g_A^2\,m_\pi^2}{16\pi f_\pi^2}\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r} < 0 \quad \xrightarrow{\text{chiral limit}} \quad -\frac{3g_A^2}{16\pi f_\pi^2}\,\frac{1}{r^3}$$

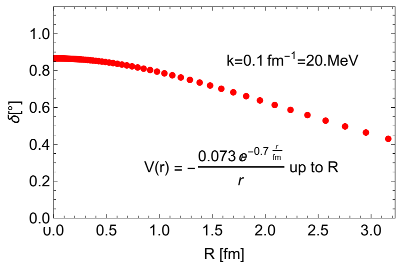
$$V_{OPE}[S=0] = V_C(r)\times\begin{cases} -3 & \text{: repulsive for } I=0\text{, i.e. } L \text{ odd} \\ +1 & \text{: attractive for } I=1\text{, i.e. } L \text{ even}\end{cases}$$

$$V_{OPE}[S=1] = \frac{1}{3}\left[V_C(r) + \left[6(\vec{S}\cdot\vec{e}_r)^2-4\right]\,V_T(r)\right]\times\begin{cases} +3 & \text{: attractive for } I=0\text{, i.e. } L \text{ even} \\ -1 & \text{: repulsive for } I=1\text{, i.e. } L \text{ odd}\end{cases}$$

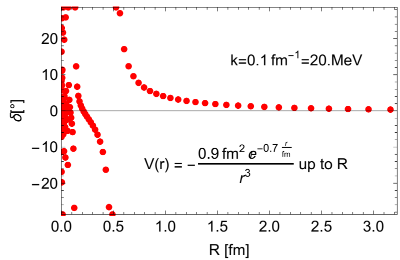
Regularise attractive $V_{C/T}$, study $R=\frac{1}{\Lambda}\to 0$:



$$V_{C/T} = \begin{cases} V_{C/T}(r) & \text{unchanged} \\ V_{C/T}(R) & \text{cut off at } R\end{cases}$$



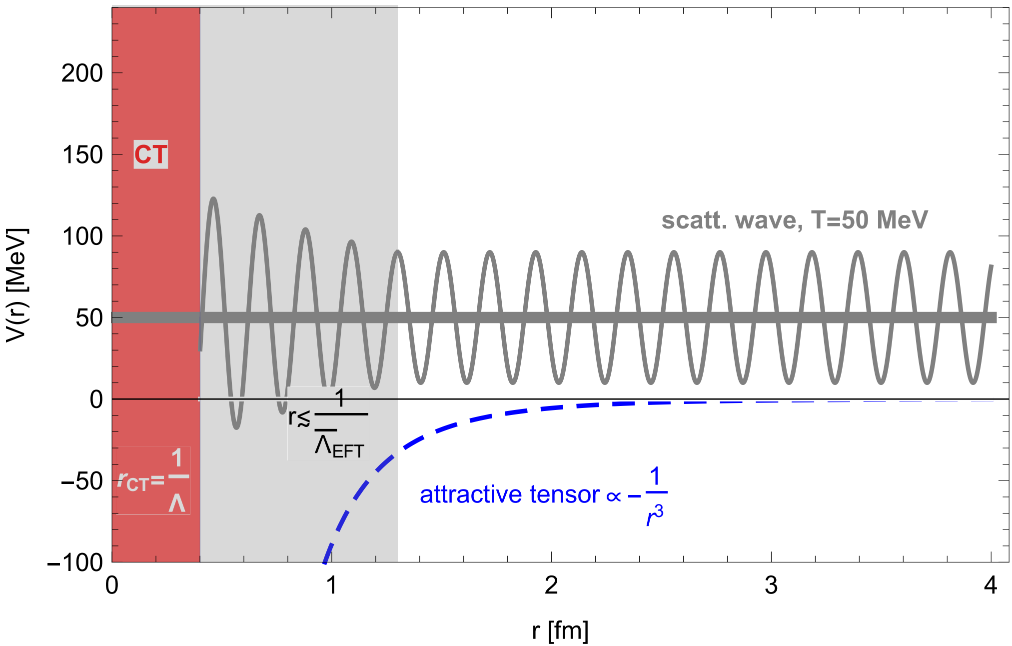
Coulombic "stable" as expected.



Thomas Collapse to $r=0$.

Unstable, RG mandates CT.

(e) Intuitive Argument for Attractive Triplet Partial Waves



RGE: Adjust CT strength $c(R = \frac{1}{\Lambda})$ with $R = \frac{1}{\Lambda \lesssim \overline{\Lambda}_\chi}$ such that observables cutoff-independent.

Initial condition set by one datum, e.g. scattering length.

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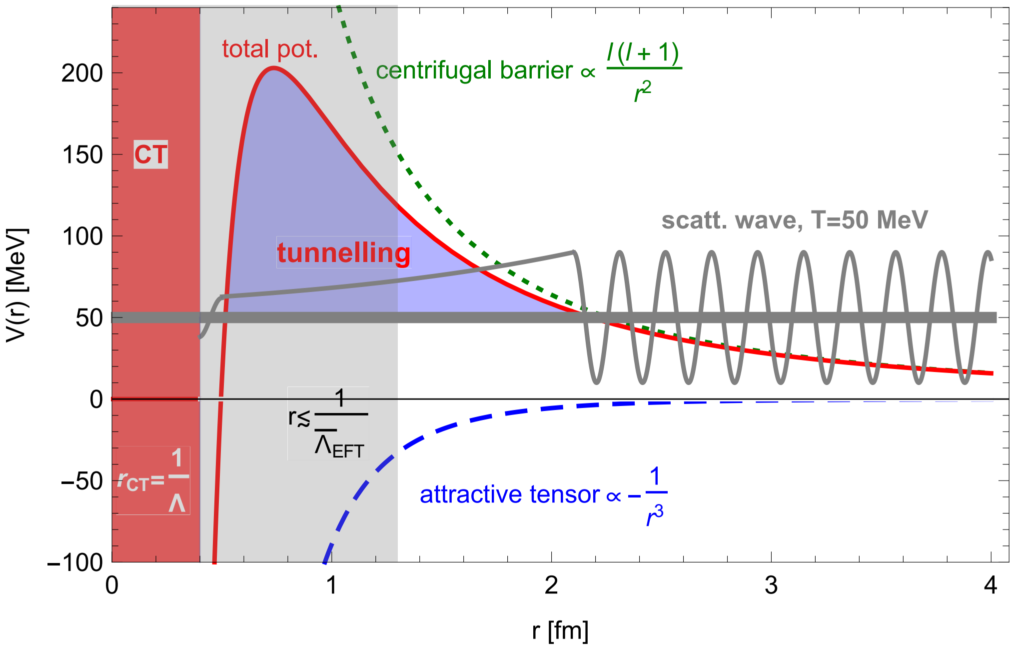

Higher PWs: Tunnelling through centrifugal barrier reduces sensitivity to details of short-distance Physics.

Growing centrifugal barrier $l \nearrow$ shields CT. $\Longrightarrow$ Higher partial waves perturbative Kaiser/Brockmann/Weise 1997.

Disputes: Cutoff $\Lambda \sim \overline{\Lambda}_{EFT}$ breakdown scale, or all $\Lambda \gtrsim \overline{\Lambda}_{EFT}$ equally acceptable, including $\Lambda \to \infty$?

Effect of higher orders (Distorted-Wave Born or resumming into Schrödinger eq.)?

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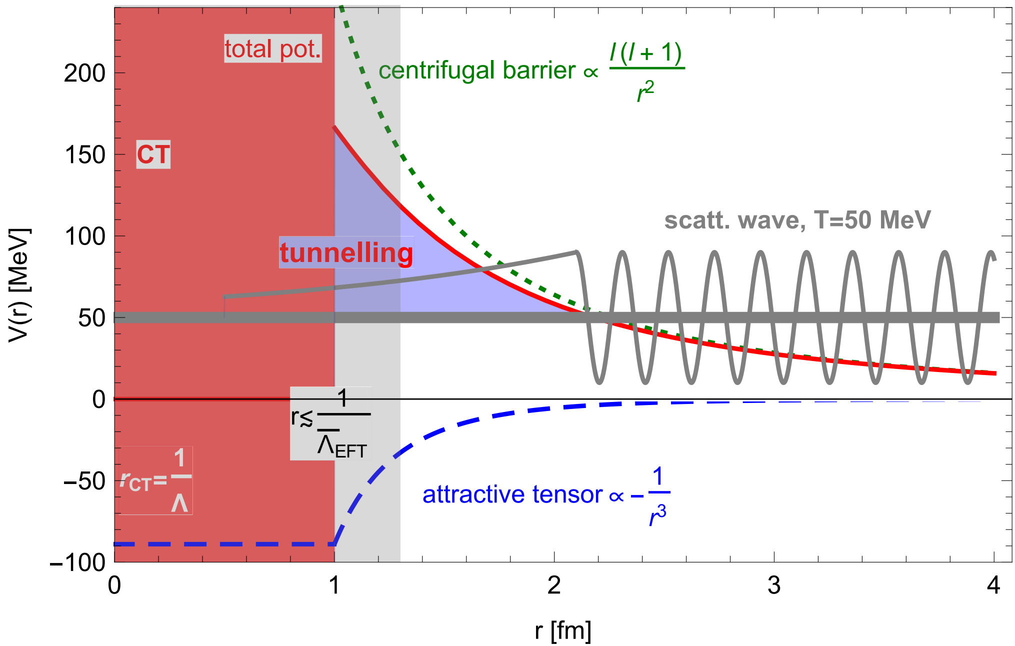

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Effect of higher orders (Distorted-Wave Born or resumming into Schrödinger eq.)?

Beane/...2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07;
NLO: Song/Lazauskas/van Kolck 1612.09090

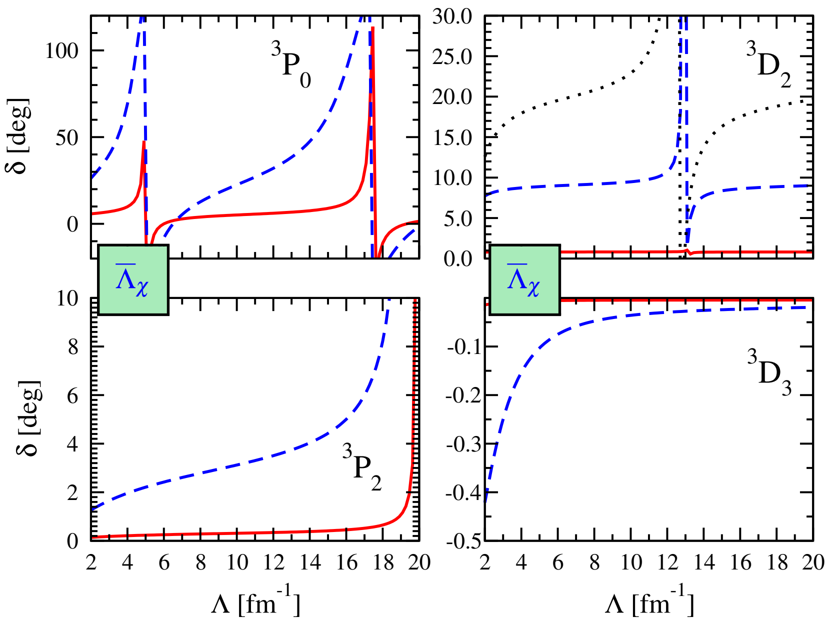
Check consistency of Weinberg's proposal: Observables cut-off dependent at LO?

Low attractive P/D-wave triplets: Weinberg predicts zero LECs at LO (momentum-independence).

phase-shift δ(cut-off Λ):

- $E_{\text{lab}} = 10\,\text{MeV}$ (solid)
- 50 MeV (dashed)
- 100 MeV (dotted)
- 190 MeV (dash-dotted)

$$V(r) \propto -\frac{\#}{r^3} + \frac{l(l+1)}{r^2}$$



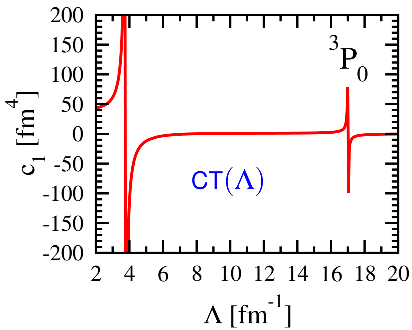
Cutoff-dependent, even for $\Lambda \approx \bar{\Lambda}_\chi$ $\Longrightarrow$ Short-distance missing!
$\Longrightarrow$ *Not renormalised!*

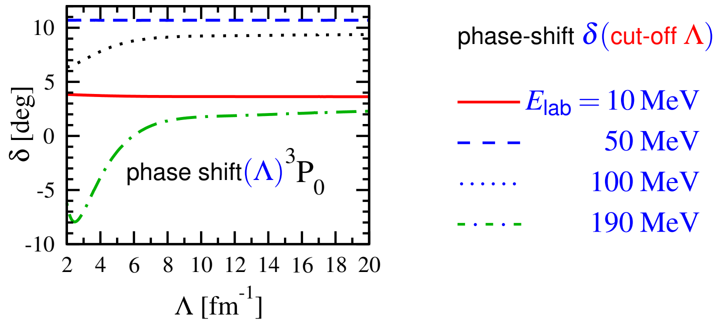
(f) Even Weinberg Can Be Wrong

Beane/. . . 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07;
NLO: Song/Lazauskas/van Kolck 1612.09090

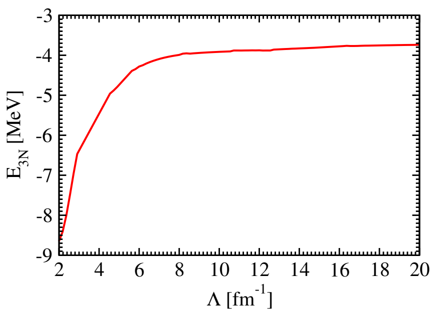
Check consistency of Weinberg's proposal: Observables cut-off dependent at LO?
Need 4 *new*, momentum-dependent LECs for low attractive triplets: $^3P_{0,2}$, $^3D_{2,3}$.

Low attractive P/D-wave triplets: Weinberg predicts zero LECs at LO (momentum-independence).





Extension to higher orders in progress. NLO: Song/Lazauskas/van Kolck 1612.09090



Triton binding-energy cutoff-independent.

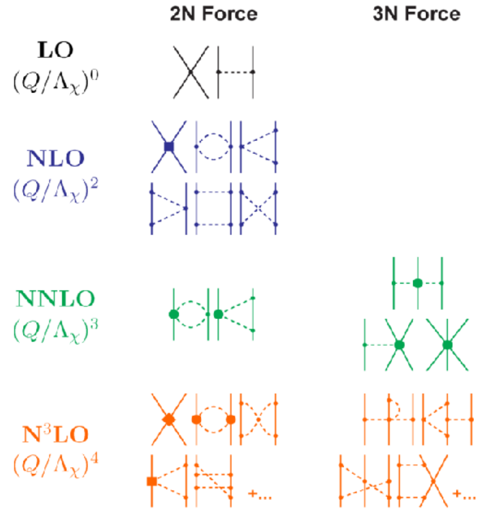
(g) The Demise of Weinberg's Pragmatic Proposal

Weinberg's Pragmatic Proposal (WPP) 1990:

(1) Order V_{NN} *including CTs* by explicit powers of $Q \sim p_{\text{typ}}$ ["simplistic NDA" hg NPA 760 (2005)]

LO: 2 Q-indep. CTs, ${}^{3,1}\mathrm{S}$ only: $\left(\frac{g_A^2}{4f_\pi^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_\pi^2} \tau_1^a \tau_{2a} \sim \frac{Q^2}{Q^2} \sim Q^0 \right) + C_S (N^\dagger N)^2 + C_T (N^\dagger \tau^a N)^2$

(2) For shallow bound states, **iterate** $T = \dfrac{V}{1 - V G_{NN}}$.



Pragmatic, widely used ("Everybody Does It").

But *conclusively show to be conceptually inconsistent*:

– $V_{\text{W}} \sim Q^0$ needs fine-tuning to justify iteration. $V_{\text{NN}} \overset{!!}{\sim} Q^{-1}$ slide 3

How to justify including LO CTs in ${}^3\mathrm{P}_{0,2},\ldots$? slide 7

– Not renormalised in low partial waves with attractive tensor.
Nogga/Timmermans/van Kolck 2005

– ${}^1\mathrm{S}_0$: m_π^2-dependence of CT and divergence do not match.
Beane/Bedaque/Savage/van Kolck 2002

Not just LO problem: LO Reg/Ren ricochets through all orders.

$\Longrightarrow$ WPP underestimates number of CTs per order.

$\Longrightarrow$ WPP at alleged order Q^n not as accurate as thought:

Accurate only to lower order $Q^{n-1,2,3,\ldots}$.

Fitting may obscure the problem. . . ; but final PC will contain many of WPP's features (we're already close).

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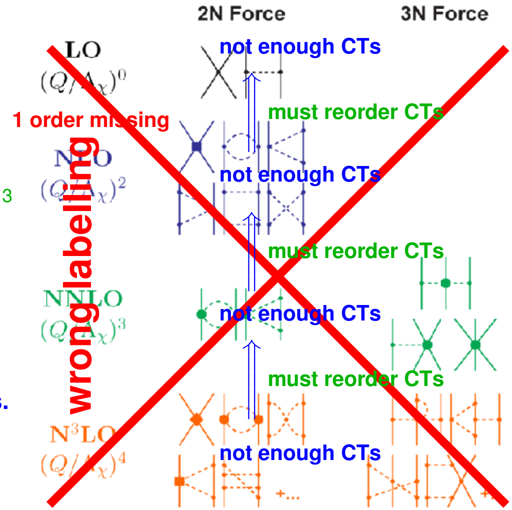


underlying plot by Epelbaum, butchered by hg

Fitting may obscure the problem. . . ; but final PC will contain many of WPP's features (we're already close).

We may be unable to say whose PC is right, but we know whose is wrong. WPP is; it's *In-Effective*.

Still, use it pragmatically to develop numerics & first glimpses at final theory – with *caveat on systematics*!

(h) $NN\ \chi$EFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.

All but WPP: RGE as construction principle, but different approximations at short-range lead to variant interpretations.

Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

order	**Weinberg** (modified) PL**B251** (1990) 288 etc.	**Birse** PR**C74** (2006) 014003 etc.	**Pavon Valderrama et al.** PR**C74** (2006) 054001 etc.	**Long/Yang** PR**C86**(2012) 024001 etc.
Q^{-1}	LO of $^1S_0$, $^3S_1$, OPE	LO of $^1S_0$, $^3S_1$, OPE; plus $^3D_1$, $^3SD_1$	LO of $^1S_0$, $^3S_1$, OPE; plus $^3P_{0,2}$, $^3D_2$	LO of $^1S_0$, $^3S_1$, OPE; plus $^3P_{0,2}$
$Q^{-\frac{1}{2}}$	none	LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$	LO of $^3SD_1$, $^3D_1$, $^3PF_2$, $^3F_2$	none
Q^0	none	NLO of $^1S_0$	NLO of $^1S_0$	NLO of $^1S_0$
$Q^{\frac{1}{2}}$	none	NLO of $^3S_1$, $^3D_1$, $^3SD_1$	none	none
Q^1	LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$	none	none	LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; N^{2}LO of $^1S_0$
# at Q^{-1}	2	4	5	4
# at Q^0	$+0$	$+7$	$+5$	$+1$
# at Q^1	$+7$	$+3$	$+0$	$+8$
total at Q^1	9	14	10	13

With same $\chi^2/$d.o.f., proposal with least parameters *wins*: minimum information bias.

(i) There's a War Going On

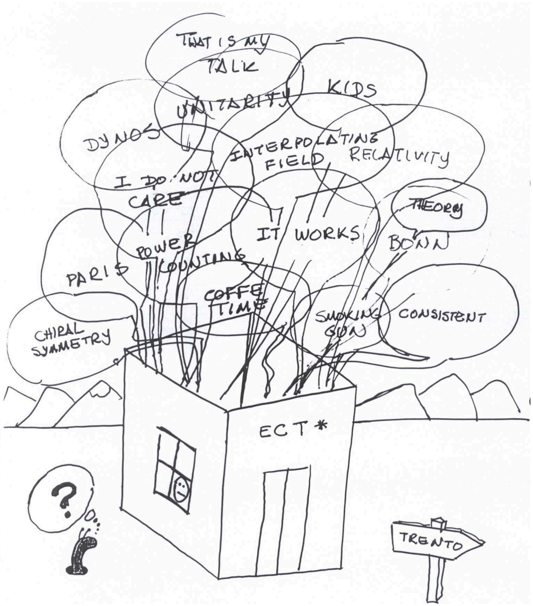
Reminder: Power-counting for non-perturbative EFTs is *not* straightforward.

Contentious is the *short-range* part, (mostly) *not* the long-range one.

Issue would not arrive if we could *derive* PC from underlying theory.

For the sake of this talk, I will be agnostic about who is right – if anyone.

But I want to test self-consistency of the proposals.



M. Robilotta: Impression of the Workshop on Nuclear Forces at the ECT*, Trento 1999

The Three Big Lies of Nuclear Physics

Nuclear Power is Safe.

They have Weapons of Mass Destruction.

The Three Big Lies of Nuclear Physics

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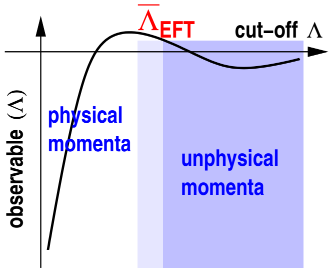
My Power-Counting is Systematic.

(a) Quantitative Predictions of Your PC: Advantage of Cut-Offs



Observable $\mathcal{O}(k)$ at momentum k, order Q^n in EFT, breakdown $\overline{\Lambda}_{\text{EFT}} \lesssim$ cut-off Λ:

$$\mathcal{O}_n(k;\Lambda) = \underbrace{\sum_i^n \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^i \mathcal{O}_i(k, p_{\text{typ.}})}_{\text{renormalised, } \Lambda\text{-indep.}} + \underbrace{\mathcal{C}(\Lambda; k, p_{\text{typ}}, \overline{\Lambda}_{\text{EFT}}) \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1}}_{\text{residual } \Lambda\text{-dependence}}$$

parametrically small
$\mathcal{C}$ "of natural size"

$\Longrightarrow$ **Difference between *any two* cut-offs:** $\dfrac{\mathcal{O}_n(k;\Lambda_1) - \mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\dfrac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \dfrac{\mathcal{C}(\Lambda_1) - \mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$

Isolate breakdown scale $\overline{\Lambda}_{\text{EFT}}$, order n by double-ln plot of "derivative of observable w. r. t. cut-off".

Ideally, *no resort to Data!* – Test consistency: Does numerics match predicted convergence pattern?

***After that*, quantitative test of EFT assumptions *against data*.**

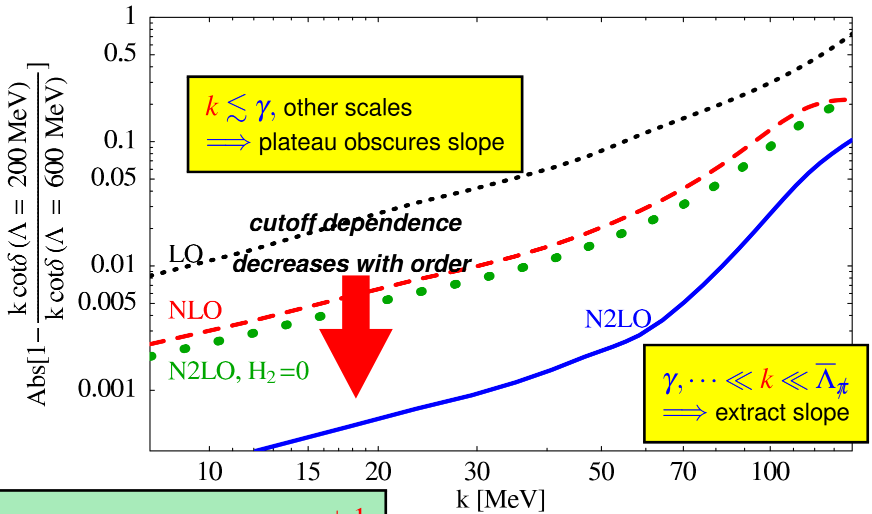
Renormalisation Group Evolution: $\Lambda_1 \to \Lambda_2 \Longrightarrow \dfrac{\Lambda}{\mathcal{O}} \dfrac{d\mathcal{O}}{d\Lambda} = \left(\dfrac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \dfrac{d \ln \mathcal{C}(\Lambda)}{d \ln \Lambda} \to 0$ if exact RGE.

Residual Λ-dependence decreases parametrically order-by-order.

Complication: Several intrinsic low-energy scales in few-N EFT:
scattering momentum k, m_π, inverse NN scatt. lengths $\gamma(^3S_1) \approx 45$ MeV, $\gamma(^1S_0) \approx 8$ MeV,...

(b) "Toy Model": nd Doublet-S Wave in EFT($\not\pi$)

Does momentum-dependent 3NI H_2 enter at N^{2}LO hg/...2002-4 – or higher Platter/Phillips 2006?



$$\left|1 - \frac{k\cot\delta(\Lambda = 200\ \text{MeV})}{k\cot\delta(\Lambda = \infty)}\right| \sim \underbrace{\left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\not\pi}}\right)^{n+1}}_{Q^{n+1}}$$

	LO	NLO	N^{2}LO	N^{2}LO without H_2
$n+1$ fitted	~ 1.9	2.9	4.8	3.1
$n+1$ predicted	2	3	4	**not renormalised**

$\Longrightarrow$ Fit to $k \in [70; 100 \dots 130]\ \text{MeV} \gg \gamma, \dots$: H_2 **is N^{2}LO**; re-confirmed by Ji/Phillips 2013.

Slope confirms Power Counting; estimates $\overline{\Lambda}_{\not\pi} \approx 140\ \text{MeV}$.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

– Estimate k-dependence of expansion parameter $Q(k) = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)$

$\Longrightarrow$ Lower limit of residual theoretical uncertainties.

– ***"Window of Opportunity"***: Fit is most transparent for $p_{\text{typ}} \ll k \ll \overline{\Lambda}_{\text{EFT}}$.

– Any two cutoffs Λ_1, Λ_2 – Numerical leverage?!

– Order n, $\overline{\Lambda}_{\text{EFT}}$ regulator independent. – But not $\mathcal{C}$: flexible regulator...

$\Longrightarrow$ Test robustness: cutoff range & schemes, fit window,...

– Non-integer powers, non-analyticities: $n+1 \to n+\text{Re}[\alpha]$ with $n \notin \mathbb{Z}$, $\text{Re}[\alpha] > 0$.

Some Limitations:

– Cannot see LECs which do *not absorb cutoff-dependence*.

– Can be numerically indecisive (e.g. small coefficients).

Test is necessary but not sufficient for consistency.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

What observable to choose?: Avoid Accidental Zeroes $\mathcal{O}(\Lambda_1)-\mathcal{O}(\Lambda_2)=0$ & Infinities $\mathcal{O}(\Lambda)=0$.

Best if unconstrained: Isolate dynamics!

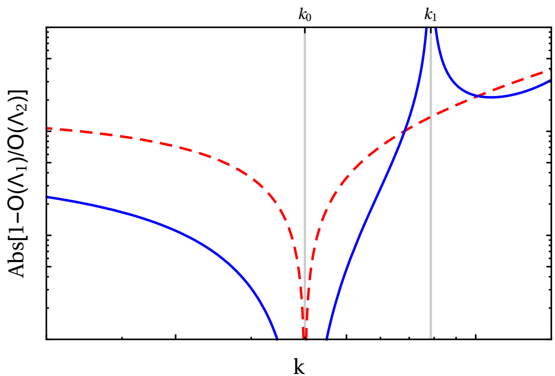
e.g. $k^{2l+1}\cot\delta_l(k)$ for lth scattering wave.

Not $\delta_l(k)$: $\delta_l(k\to 0)\propto k^{2l+1}$: constrained.

Best if same sign for all $k \lesssim \overline{\Lambda}_{\text{EFT}} \implies$ Peruse Λ_1, Λ_2.

If LECs need fitting, do for $k \lesssim p_{\text{typ}}$.

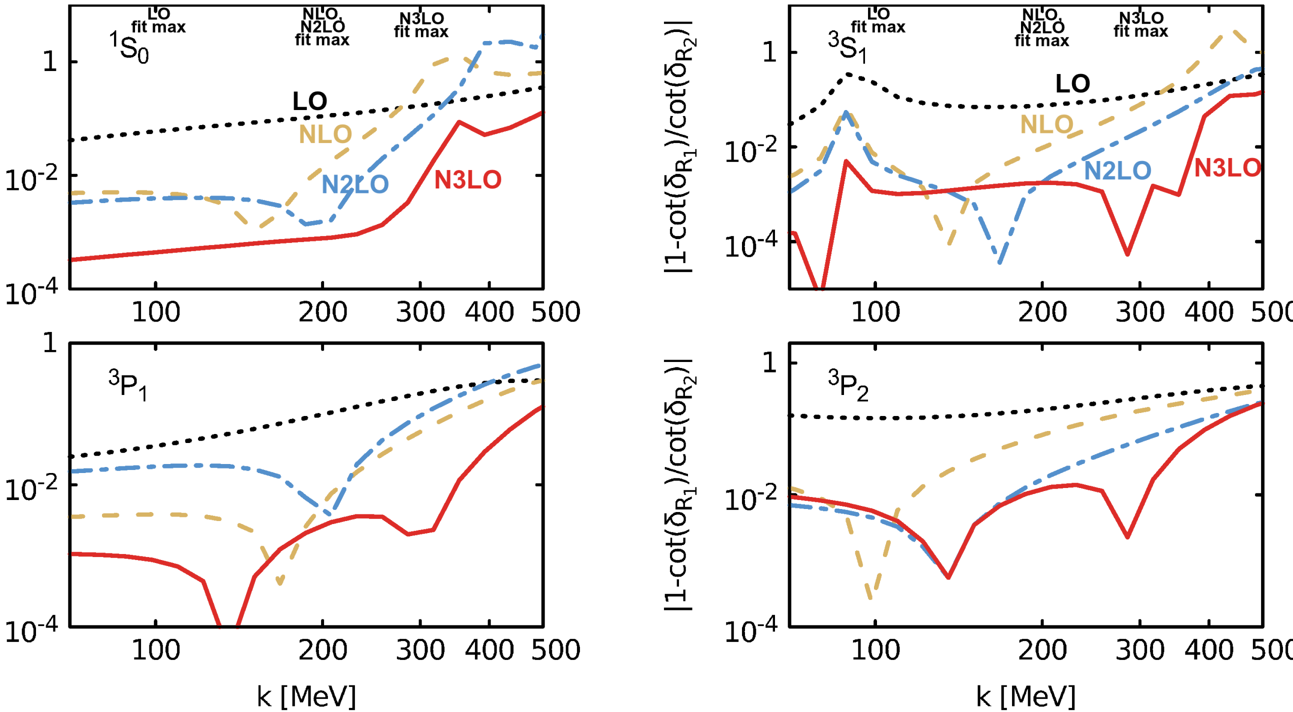
Slope may still emerge for $k \nearrow \overline{\Lambda}_{\text{EFT}}$; larger LEC fit error.



Goal: Test of Self-Consistency, not of Convergence to Data. $\implies$ Minimal resort to experiment.

(d) Case of Interest: NN in χEFT: Fitting Parameters Obscures Slopes

Weinberg's Hunch is wrong, but nobody else published: Plot stolen from **Epelbaum/Krebs/Meißner EPJA51 (2015) 5, 53.**



Inconclusive: Breakdown $400 - 500$ MeV, fit- & slope-regions not clearly separated.

$k \gtrsim 200$ MeV, but no $\Delta(1232)$ degree of freedom.

Coupled channels; NLO & N^{2}LO parallel? Slopes?

3. The Promise of Reliable Error Bars

(a) (Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83 (2011) 040001

PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication

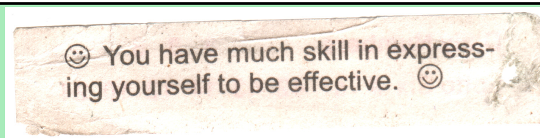
The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers

physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

Non-Theory Errors: Numerical $\Longrightarrow$ better computers. Statistical/parameter $\Longrightarrow$ better data.

Theoretical uncertainty: Truncation of Physics

EFT claim: systematic in $Q = \dfrac{\textit{typ. low scale } p_{\textit{typ}}}{\textit{typ. high scale } \overline{\Lambda}_{\textit{EFT}}}$



Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

Need procedure which is established, economical, reproducible: room to argue about "error on the error".

"Double-Blind" Theory Errors: Assess with pretense of no/very limited data.

(b) Fit Discussion: What Does "Conservative" Error Mean?

hg/JMcG/DRP
1511.01952

Proton polarisability $[10^{-4}\text{ fm}^3]$ χEFT: $[12.5_{\text{LO}} - 2.3_{\text{NLO}} + 0.4_{\text{N}^2\text{LO}} = 10.6 \pm 0.4_{\text{stat}} \pm ???_{\text{th}}] \Longleftrightarrow$ PDG: $[12.0 \pm 0.6]$

Observable as series $\mathcal{O} = Q^n \left(c_0 + c_1 Q^1 + c_2 Q^2 + \textbf{unknown} \times Q^3\right) \implies$

Estimate next term *"most conservatively"* as $|\textbf{unknown } c_3| \lesssim \max\{|c_0|;|c_1|;|c_2|\}$.

Def. *Naturalness (weak$_{\text{UvK}}$)*: Higher orders shall not spoil perturbation,

i.e. $|c_i|\,Q < |c_{i+1}|$ **"*in most cases*".** ($c = 10^{60}$ may be natural if $Q \sim 10^{-300}$.)

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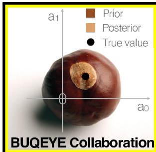
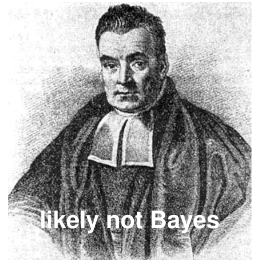


No infinite sampling pool; data fixed; more data changes confidence.

$\Longrightarrow$ **Call upon the Reverend Bayes!**

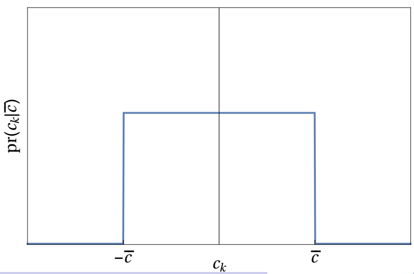
see e.g. **BUQEYE collaboration** **Furnstahl/Phillips/... 1506.01343**



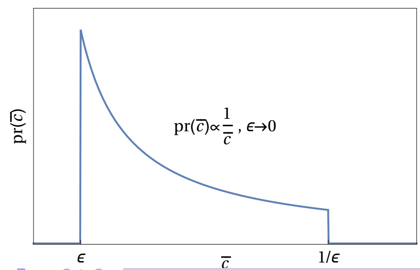
Bayes makes you specify your premises/assumptions about series.

Priors: leading-omitted term dominates ($Q \ll 1$); putative distributions of *all* c_k's and of largest value $\bar{c}$ in series.

"Least informed/informative": All values c_k equally likely, given upper bound $\bar{c}$ of series.



"Any upper bound": ln-uniform prior sets no bias on scale of $\bar{c}$.



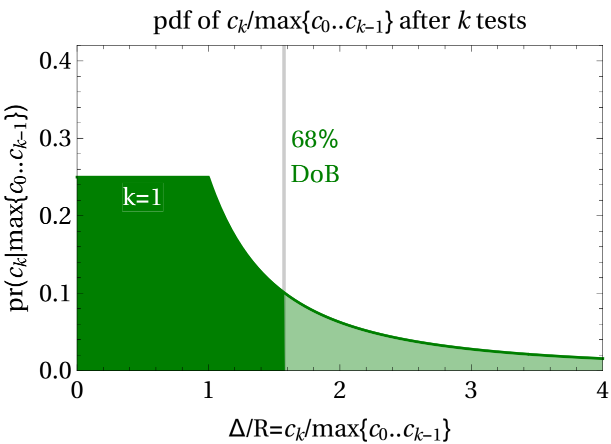
(c) Quantifying One's Beliefs in $\mathcal{O} = Q^n(c_0 + c_1 Q^1 + c_2 Q^2 + \dots)$

Information: Convergence LO→NLO→N^{2}LO gives

probable "largest number" $R = Q^k \max\{|c_0| \dots |c_{k-1}|\}$.

Result: **Posterior** $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k Q^k$ differs from order-k central value by Q.

$$\mathrm{pr}(\Delta|\max.\ R, \text{order } k) \propto \int\limits_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(c_k = \frac{\Delta}{Q^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\frac{1}{2R} \begin{cases} 1 & |\Delta| \le R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	50%	$1.6\,R$	$11R = 7\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

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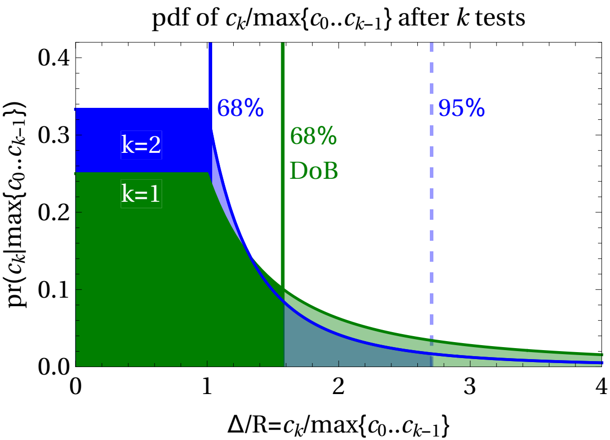
$$\mathrm{pr}(\Delta|\mathrm{max.}\ R, \mathrm{order}\ k) \propto \int_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(c_k = \frac{\Delta}{Q^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1} \frac{1}{2R} \begin{cases} 1 & |\Delta| \le R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	50%	$1.6\,R$	$11R = 7\sigma$
NLO	66.7%	$1.0\,R$	$2.7R = 2.6\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

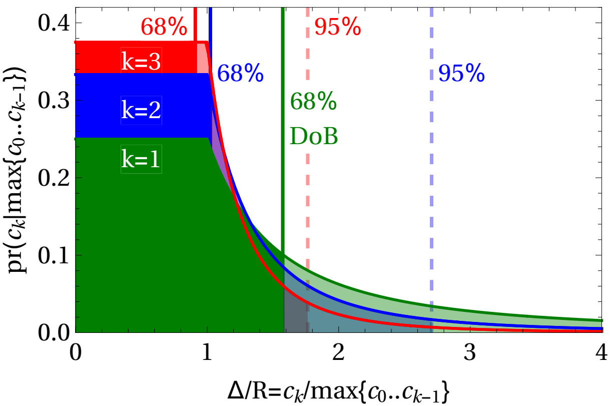
(c) Quantifying One's Beliefs in $\mathcal{O} = Q^n(c_0 + c_1 Q^1 + c_2 Q^2 + \dots)$

Information: Convergence LO→NLO→N²LO gives | probable "largest number" $R = Q^k \max\{|c_0| \dots |c_{k-1}|\}$.

Result: **Posterior** $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k Q^k$ differs from order-k central value by Q.

$$\mathrm{pr}(\Delta|\max. R, \mathrm{order}\ k) \propto \int_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(c_k = \frac{\Delta}{Q^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\frac{1}{2R}\begin{cases} 1 & |\Delta| \leq R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$

pdf of $c_k/\max\{c_0..c_{k-1}\}$ after k tests



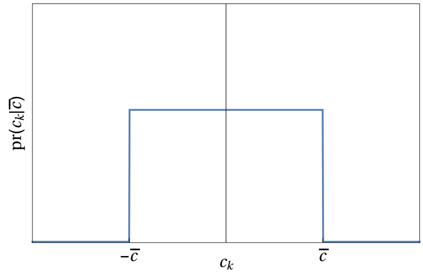
order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	50%	$1.6\,R$	$11R = 7\sigma$
NLO	66.7%	$1.0\,R$	$2.7R = 2.6\sigma$
N²LO	75%	$0.9\,R$	$1.8R = 1.9\sigma$
k	$\frac{k}{k+1}$		
Gauß	68.27%	$1.0\,R$	2.0σ

For "high enough" order, largest number R limits $\gtrsim 68\%$ **degree-of-belief interval.**

$\Longrightarrow$ ***Interpretation of all theory uncertainties, with these priors; "$A \pm \sigma$": 68% DoB interval*** $[A - \sigma; A + \sigma]$.

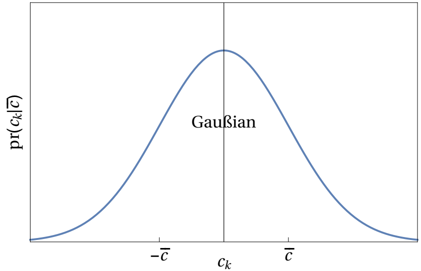
(d) Prior Choice: What is “Natural Size”? (SCOTUS: I Know It When I see It.)

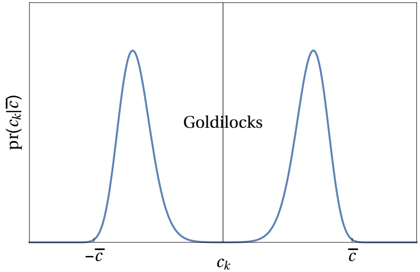
Observable/Series $\mathcal{O} = c_0 + c_1 Q^1 + c_2 Q^2 +$ **unknown** $\times Q^3$ with ***“naturally-sized coefficients”*** c_i.



“Least informative/informed”:

characterised by 1 number: $\bar{c}$.

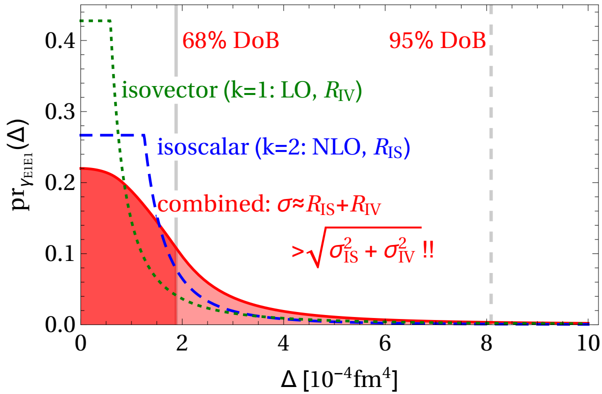




More informed choices: more complicated structures, more thought, more parameters: $\bar{c}$, typ. size, spread,. . .

BUQEYE (Wesolowski/Klco/. . .): When $k \geq 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$ for some “reasonable priors”.

(e) Final Bayes Comments



Posterior pdf *not Gauß'ian:*

Plateau & power-law tail.

$\Longrightarrow$ **Do not add in quadrature for convolution**

(more like linear).

Bayes provides well-defined *procedure*!

Example: χEFT predicts nucleon polarisabilities

hg/JMcG/DRP 1511.01952

e.g. $\gamma_{M1M1} = [2.2 \pm 0.5_{\text{stat/indir.}} \pm 0.6_{\text{th}}] \times 10^{-4}\ \text{fm}^4$

MAMI 2015: $[3.2 \pm 0.9_{\text{stat}}] \times 10^{-4}\ \text{fm}^4$

Bayes in EFTs also used to estimate:

- **Momentum-dependent expansion parameter $Q(k)$;**
- **breakdown scale $\overline{\Lambda}_{\text{EFT}}$;**
- **momentum-dependent data-weighting for LEC fitting/extraction;**
- **build LEC hierarchy into fit;**
- **"model quality"$\equiv$ correctness of EFT assumptions,...**

BUQEYE collaboration Furnstahl/Phillips/... 1506.01343, 1511.03618,...

$\Longrightarrow$ **Finally quantitative theoretical uncertainties which make the EFT falsifiable.**

Expansion parameter $Q = \frac{\text{typ. low scale } p_{\text{typ}}}{\text{typ. high scale } \overline{\Lambda}_{\text{EFT}}} \implies \mathcal{O} = Q^m \sum_{i=0}^{k-1} c_i(\Lambda)\, Q^i$ **complete at** $\mathcal{O}(Q^{k-1})$ **(N^{k}LO).**

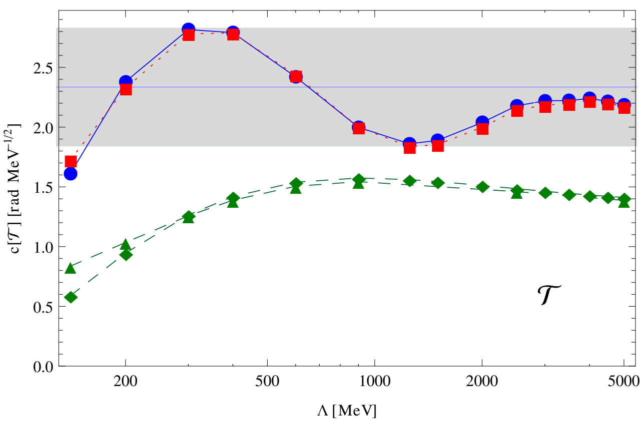
- A priori: Q^k of LO.
- Less dependence on particular low-E data taken for LECs. (e.g. Z-param. vs. ERE; fit H_0 to a_3 vs. B_3,...)
- Include selected higher-order RG- & gauge-invariant effects: *does not increase accuracy.*
- ***Any*** Λ **between** $\overline{\Lambda}_{\text{EFT}}$ **and** ∞ **is equally acceptable.**

$\Longrightarrow$ Corridor mapped by Λ *in wide range*.

Should decrease order-by-order.

Example: PV coefficient in nd at $k = 0$.

hg/Schindler/Springer 2012



Choose most conservative/worst-case error for final estimate! Clearly state your choice!

4. Concluding Questions

We have not quite followed through on EFT's promises.

– **Quantitative, falsifiable predictions test EFT's assumptions**: symmetries, constituents, naturalness,. . .

An EFT may be consistent and converge, but not with & to Nature.

– If **non-perturbative EFT** not derived from underlying theory, finding a consistent Power-counting is *non-trivial*.

$\Longrightarrow$ **Much debate, but agreement that Weinberg is wrong:** no RG-invariance,. . .

Consistency Test "Momentum-dependent Renormalisation Group flow of observable with cut-off":

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} \propto \left(\frac{k,p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1}$$ **for any two cut-offs $\Lambda_1,\Lambda_2 \gtrsim \overline{\Lambda}_{\text{EFT}}$.**

- For ***order $\mathcal{O}(Q^n)$ to which result is complete***: slope at $k \gg$low scales;
- For ***breakdown scale $\overline{\Lambda}_{\text{EFT}}$***: k at which different orders show same-size variations;
- For lower bound on ***expansion parameter Q***: vary Λ_1,Λ_2 over wide range.

Minimal resort to data, but may be inconclusive. – One of hopefully many arrows in the quiver.

– ***EFT results must have reproducible, defensible assessment of theoretical uncertainties!!*** *Bayes helps.*

Goal: World Domination by Uncertainty Quantification. – ***Error Bars for Nuclear Theory!*** **–**

The efficient person gets the job
done right. The effective person
gets the right job done.

There is always an easy solution

to every human problem ——

neat, plausible, and wrong.

H. L. Mencken