Emergence, Effective Field Theory, Gravitation and Nuclei

1) "Philosophical" comments on QM, Emergence etc.

2) Effective field theory

3) General Relativity as an Effective Field Theory

4) Nuclear potential in the massless limit

John Donoghue – Saclay – Jan 17, 2017

<u>Physics is an experimental science</u>

Dreams of a final theory are nice, but …

We can only learn about fundamental theory layer by layer
- even our **theory techniques** (i.e. QM, QFT) come from expt.

Present layer is the Standard Model, but more layers needed

The structure of the ultimate high energy theory is always unknown

Do not know the particles nor their interactions, nor…..

So, how do we live with the unknown?

Why do quantum calculations work?

The problem: QM says to sum over **all** intermediate states

$$\sum_I \frac{<f|V|I><I|V|i>}{E_i - E_I}$$

So, how can you sum over all states if you don't know what they are or how they interact??

Some possible solutions:

1) The energy denominator suppresses high energy states

$$\sum_I \frac{1}{E_i - E_I} \to \int d^3 p_I \frac{1}{E_i - \frac{p_I^2}{2m}}$$

2) Perhaps matrix elements are small to high energy states

NO! All observables sensitive to high energy at some order in PT

Even worse: our fields and particles are tentative: Emergent fields

Take a series of masses interacting with neighbors:

$$S = \int dt L[y_i, \dot{y}_i] = \int dt \sum_i [\frac{1}{2} m \dot{y}_i^2 - V(y_i - y_{i-1})] \approx \int dt \sum_i [\frac{1}{2} m \dot{y}_i^2 - \frac{1}{2} k (y_i - y_{i-1})^2]$$

Go to the continuum limit:

$$y_j(t) \equiv \sqrt{\frac{1}{ka}} \phi(x,t) \qquad x = aj$$

Get a field satisfying the wave eq. (= massless 1D Klein Gordon equation

$$S = \int dx dt \frac{1}{2} [\frac{1}{v^2} (\frac{\partial \phi}{\partial t})^2 - (\frac{\partial \phi}{\partial x})^2] = \int dx dt \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

with

$$\partial_\mu = (\frac{1}{v_s} \frac{\partial}{\partial t}, -\frac{\partial}{\partial x}) \qquad v = \sqrt{\frac{ka^2}{m}}$$

Phonons

Atoms bumping into each other
- described as acoustic waves

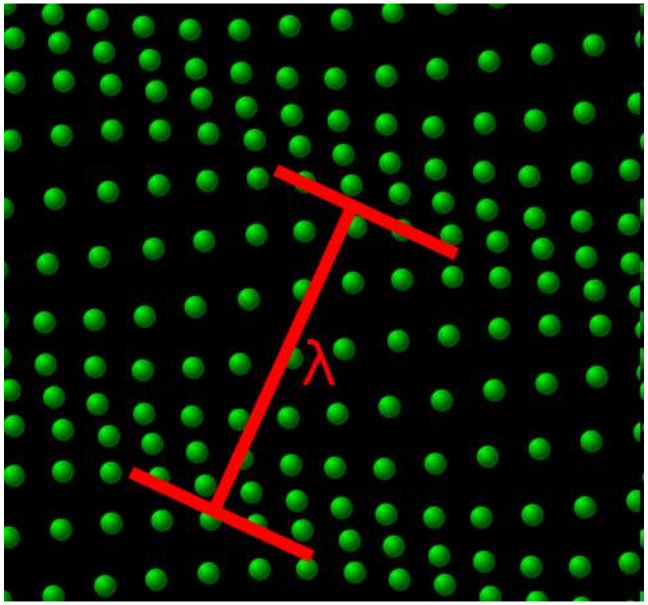
$$L = \int d^3x [\frac{1}{2}\partial_\mu \phi \partial^\mu \phi]$$

$$\partial_\mu = (\frac{1}{v_s}\frac{\partial}{\partial t}, -\nabla)$$

Generates massless wave equation:

$$[\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}]\phi = 0$$

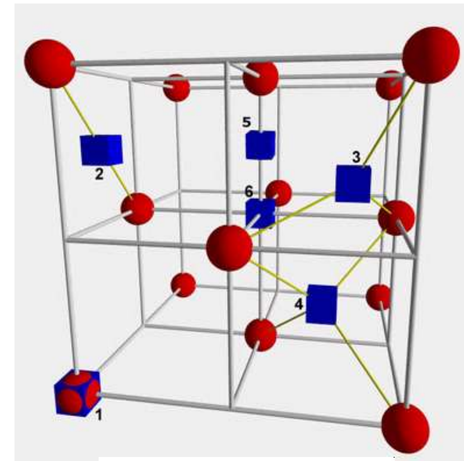
$$E = pv_s$$



Different status for light waves and sound waves
- FOR NOW!

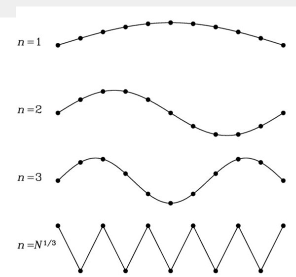
Even our symmetries may not be sacred: Emergent symmetry

Three emergent symmetries in phonon/string examples:



1) **Translation symmetry**

$$x \to x + c$$



2) **Lorentz-like symmetry**

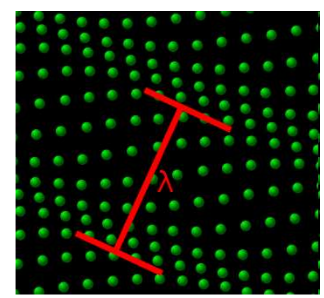
$$L = \int d^3x[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi]$$

leads to extra invariance

$$x'^\mu = \Lambda^\mu_\nu x^\nu$$

3) **Shift symmetry:**

- why the massless wave equation?
  shift symmetry $\varphi \to \varphi + c$

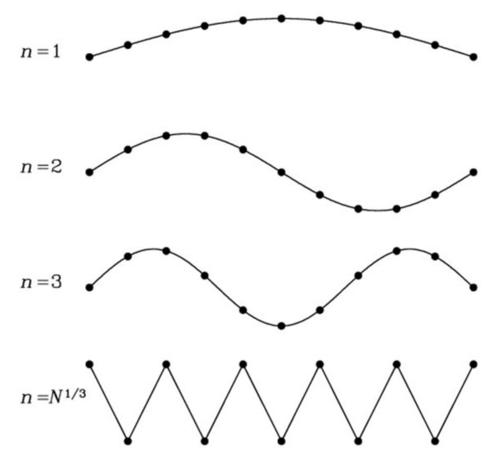


corresponds to translating the overall system
-no cost in energy

These are not symmetries of the original system but emerge

Examples of violation of emergent symmetry

1) **Translation invariance violated at small scales**



2) **Waves do not exist at small wavelength**
Emergent DOF no longer exist

3) **Next order in L is not Lorentz invariant:**

$$V(y_i - y_{i-1}) = \frac{1}{2}k(y_i - y_{i-1})^2 + \frac{1}{4}\lambda(y_i - y_{i-1})^4 + \ldots.$$

Then there is a **new term in the action without Lorentz-like symmetry**

$$S = \int dx dt \frac{1}{2}[\frac{1}{v^2}(\frac{\partial\phi}{\partial t})^2 - (\frac{\partial\phi}{\partial x})^2 + \bar{\lambda}(\frac{\partial\phi}{\partial x})^4]$$

These are generic features of an emergent symmetry
– eventually sensitive to lack of symmetry

Need to continue experimental tests

Tests for violation of gauge symmetries

- in case gauge symmetries are emergent

JFD
Anber
Aydemir
El-Menoufi

1) General relativity

$$\mathcal{L} = \mathcal{L}_{EH} + \sum_{i=1}^{7} a_i \mathcal{L}_i + \mathcal{L}_m \,.$$

with

$$\begin{aligned}
\mathcal{L}_1 &= -\, g^{\mu\nu}\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\alpha}\,, \quad \mathcal{L}_2 = -\, g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\lambda\alpha} \\
\mathcal{L}_3 &= -\, g^{\alpha\gamma}g^{\beta\rho}g_{\mu\nu}\Gamma^{\mu}_{\alpha\beta}\Gamma^{\nu}_{\gamma\rho}\,, \quad \mathcal{L}_4 = -\, g^{\alpha\gamma}g_{\beta\lambda}g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}\Gamma^{\beta}_{\gamma\alpha} \\
\mathcal{L}_5 &= -g^{\alpha\beta}\Gamma^{\lambda}_{\lambda\alpha}\Gamma^{\mu}_{\mu\beta}\,, \quad \mathcal{L}_6 = -g^{\mu\nu}\partial_\nu\Gamma^{\lambda}_{\mu\lambda} \\
\mathcal{L}_7 &= -g^{\mu\nu}\partial_\lambda\Gamma^{\lambda}_{\mu\nu}\,, \qquad (3)
\end{aligned}$$

2) QED

$$\mathcal{L}_{gv} = -\frac{1}{8}\kappa A_\mu A^\mu A_\nu A^\nu$$

Can get significant constraints

So, how do we proceed?

Experimental hint: Perfectly fine to quantize phonons and other quasiparticles even though they do not exist as fundamental particles at all scales

- specific heat of solids, transport properties…

We need to live with the particles and interactions that exist at our energy scale.

Effective field theory to the rescue

What is Effective Field Theory?

- clear thinking about energy scales in usual field theory

Effective:

1) Low energy limit of a more complete theory
    - no very massive particles in the effective theory
2) Useful – able to have effect

Assumptions:

QM (QFT) as we know it

Set of D.O.F seen at a given energy scale

Observed symmetries

In practice:

- include only low energy DOF in theory
- allow all terms in Lagrangian consistent with symmetry
- order predictions in powers of (low E scale)/(high E scale)

Why do quantum calculations work?

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So, how can you sum over all states if you don't know what they are or how they interact??

<u>Some possible solutions:</u>

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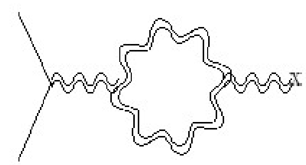
<u>NO!</u> All observables sensitive to high energy at some order in PT

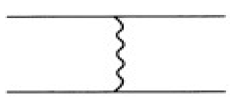
The key to the solution: the uncertainty principle

High energy effects look **local** - very short range
- Looks like some term in a local Lagrangian

Mass term or charge coupling
Shift in mass or coupling
We measure **total** mass and coupling



<u>Applequist Carrazone theorem</u>:
-effects of high energy either absorbed in "coupling constants" or suppressed by powers of the heavy scale
- uncertainty principle is more general

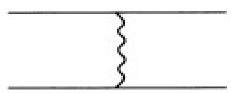


Note: Even U.P. and locality is experimentally based
-brane example

Key Steps

1) High energy effects are local (when viewed at low E)

Example = W exchange



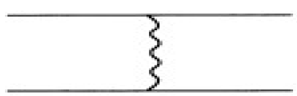
=> local 4 Fermi interaction

Even loops

=> local mass counterterm

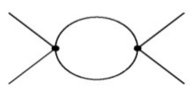
Low energy particle propagate long distances:

Photor



Not local

$$V \sim \frac{1}{q^2} \sim \frac{1}{r}$$



Even in loops – cuts, imag. parts….

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots.$$

Even if you don't know the true effect, you know that it is local
-use most general local Lagrangian

2) Energy Expansion

Order lagrangians by powers of (low scale/high scale)^N

Only a finite number needed to a given accuracy

Then:

Quantization: use lowest order Lagrangian

Renormalization:

-U.V. divergences are local

- can be absorbed into couplings of local Lagrangian

Remaining effects are predictions

Renormalization Program

Effects of high energy go into measured values of the parameters
-including unknown physics and potential divergences

Renormalizable Field Theory

- finite number of parameters sensitive to high energy
- terms (in Lagrangian) suppressed by powers of heavy scale are not allowed

Effective Field Theory

- allow terms suppressed by powers of heavy scale
- quantum effects from low energy D.O.F. only
- more general

General procedure from bottom up

1) Identify Lagrangian

-- include low energy particles
-- most general (given symmetries)
-- order by energy expansion

2) **Calculate and renormalize**

-- start with lowest order
-- renormalize parameters

3) **Phenomenology**

-- measure parameters
-- residual relations are predictions

Note: Two differences from textbook renormalizable field theory:
1) no restriction to renormalizable terms only
2) energy expansion

The "Hydrogen atom" of EFT:

Chiral Perturbation Theory

-QCD at very low energies –pions and photons

Non-linear lagrangian required by symmetry:

$$\mathcal{L} = F_\pi^2 Tr(D_\mu U D^\mu U^\dagger) + L_1 [Tr(D_\mu U D^\mu U^\dagger)]^2 + ...$$

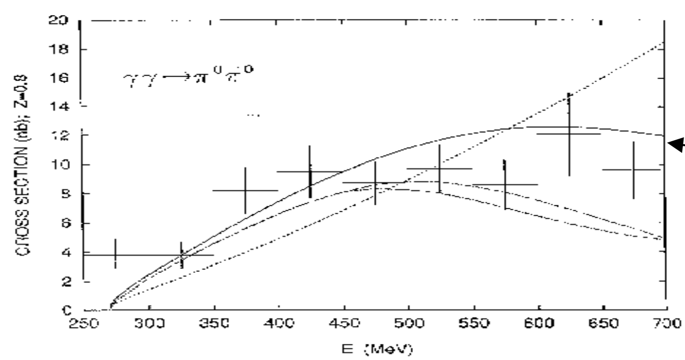
$$U = exp[i\frac{\tau \cdot \phi}{F_\pi}]$$

Very well studied: Theory and phenomenology
- energy expansion, loops, symmetry breaking, experimental constraints, connection to QCD.

Sample calculation:

-no direct couplings at low energy
– pure loops
-essentially parameter free at low energy

$\gamma\gamma \longrightarrow \pi^0\pi^0$



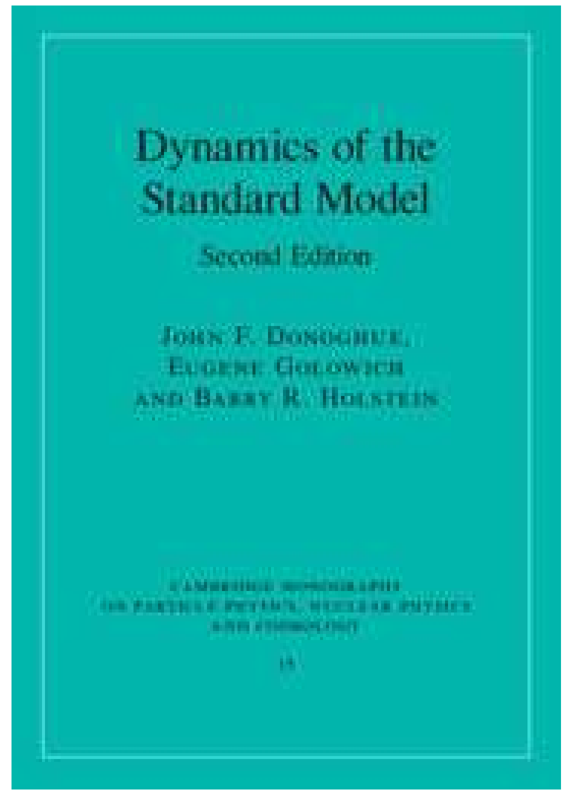
one loop

two loops

Want to learn more of practical EFT ?

Check out our book!!

Now in second edition



We have come to think of all of our theories as effective field theories

Theories are tested over some distance/energy scale
- -we know D.O.F. and interactions for that scale
- -can do calculations at that scale

But, there likely are new particles and new interactions at higher energy
- -these do not propagate at low energy
- -only give suppressed local interactions

All theories likely modified as we go to higher energy

Gravity as an Effective Field Theory

1) Calculable quantum correction to gravitational interactions

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

*

$$\theta \simeq \frac{4G_N M}{b} + \frac{15}{4}\frac{G_N^2 M^2 \pi}{b^2} + \left(8bu^S + 9 - 48\log\frac{b}{2b_0}\right)\frac{\hbar G_N^2 M}{\pi b^3} + \dots .$$

2) The quantum theory of general relativity <u>at ordinary energies</u> exists and is of the form called an "effective field theory"

What is the problem with quantum gravity?

"*Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct.*"

"*The existence of gravity clashes with our description of the rest of physics by quantum fields*"

"*Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences.*"

"The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory"

"*Quantum mechanics and general relativity are incompatible*"

These statements are old-fashioned and misleading

Rather: Quantum general relativity at $E<<M_P$ exists and is described by "effective field theory"

Effective field theory:

- calculate quantum effects at a given energy scale
- shifts focus from U.V. to I.R.
- handles main obstacle
  – quantum effects involve **all** scales

Completion of program of Feynman, De Witt,..Weinberg…
….. 't Hooft, Veltman ……

Previously: Quantization and divergence structure

E.F.T - Extraction of quantum predictions

Known vs unknown physics

Gravity as an effective theory

Weinberg
JFD

Both General Relativity and Quantum Mechanics known and tested over common range of scales

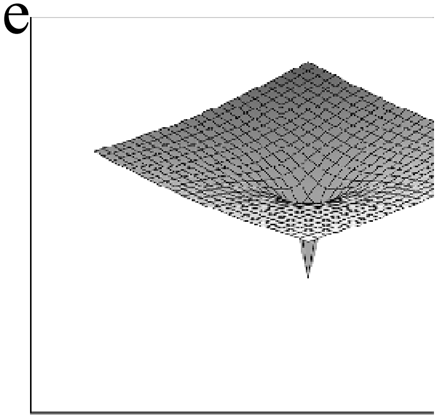
Is there an incompatibility **at those scales** ?

Or are problems only at uncharted high energies?

Need to study GR with a careful consideration of scales

Aspects of GR that we will use:

1) The gravitational field –deviation from flat space



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

2) Symmetry – general coordinate invariance

- restricts Lagrangian to invariant terms

$$\mathcal{L} = \frac{1}{G} R \sim \frac{1}{G}\left[\partial h \partial h + h \partial h \partial h + ...\right]$$

3) Gravity couples to energy and to itself – Einstein's equation

$$\nabla^2 h = G\left[H_{\rm matter} + \partial h \partial h\right]$$

The general Lagrangian

The Einstein action:

$$S_{grav} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R \right]$$

$\kappa^2 = 32\pi G, g = det g_{\mu\nu}, g_{\mu\nu}$ is the metric tensor and $R = g^{\mu\nu} R_{\mu\nu}$

$$\begin{aligned} R_{\mu\nu} &= \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\mu\nu} \Gamma^\lambda_{\lambda\sigma} \\ \Gamma^\lambda_{\alpha\beta} &= \frac{g^{\lambda\sigma}}{2} \left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta} \right) \end{aligned}$$

But this is not the most general lagrangian consistent with general covariance.

Key: R depends on <u>two derivatives</u> of the metric
- Energy expansion – expansion in number of derivatives

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

Parameters

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right\}$$

***1)* Λ = cosmological constant**

$$\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$$

$$M_P = 1.22 \times 10^{19} \text{ GeV}$$

-this is observable only on cosmological scales
-neglect for rest of talk
-interesting aspects

Note: Naturalness **not** part of this process
- EFT takes what experiment measures
- these parameters need not be natural
- range of utility may depend on size, but EFT exists independent of this

2) Newton's constant

$$\kappa^2 = 32\pi G$$

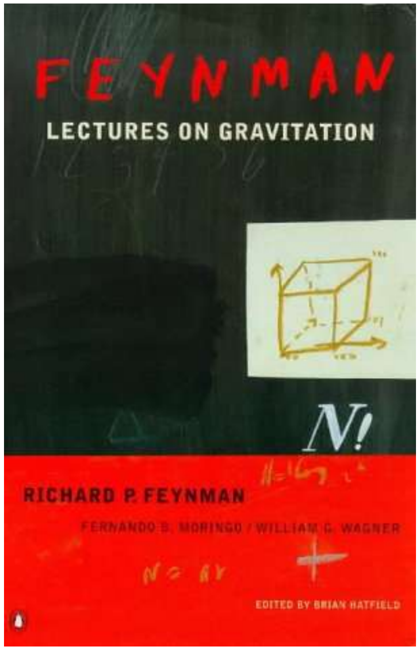
Note: G does not run

3) Curvature –squared terms c_1, c_2
- studied by Stelle
- modify gravity at very small scales
-essentially unconstrained by experiment

$$c_1(\mu), c_2(\mu) < 10^{65}$$

If new dimensions at M_* , then $c_i \sim (M_P/M_*)^2$

Quantizing general relativity



Feynman quantized gravity in the 1960's

Quanta = gravitons (massless, spin 2)

Rules for Feynman diagrams given

Subtle features:

$h_{\mu\nu}$ has 4x4 components – only 2 are physical DOF!

-need to remove effects of unphysical ones

Gauge invariance (general coordinate invariance)

- calculations done in some gauge
- need to maintain symmetry

In the end, the techniques used are very similar to other gauge theories

Quantization

“**Easy” to quantize gravity**:

-Covariant quantization   Feynman deWitt

-gauge fixing

-ghosts fields

-Background field method   ‘t Hooft Veltman

-retains symmetries of GR

-path integral

Background field:

$$
\begin{aligned}
g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\
g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \ldots
\end{aligned}
$$

Expand around this background:

$$
S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \ldots \right]
$$

$$
\begin{aligned}
\mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[\bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\
\mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\
&\quad + \bar{R} \left(\frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left(2h^{\lambda}_{\mu} h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu}
\end{aligned}
$$

Linear term vanishes by Einstein Eq. $\bar{R}^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$

Gauge fixing:
-harmonic gauge

$$\mathcal{L}_{gf} = \sqrt{-\bar{g}}\left\{\left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu}\right)\left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu}\right)\right\}$$

$$h \equiv h_{\lambda}^{\lambda}$$

Ghost fields:

$$\mathcal{L}_{ghost} = \sqrt{-\bar{g}}\eta^{*\mu}\left\{\eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu}\eta^{\nu}\right\}$$

vector fields
anticommuting,
in loops only

Interesting note:
Feynman introduced
ghost fields in GR
before F-P in YM

Quantum lagrangian:

$$S_{eff} = \int d_x^4 \sqrt{\bar{g}} \left\{ \bar{\mathcal{L}}(\bar{g}) - \frac{1}{2} h_{\alpha\beta} D^{\alpha\beta\gamma\delta} h_{\gamma\delta} + \dots \right\}$$

with

$$\begin{aligned} D^{\alpha\beta\gamma\delta} \;=\; & I^{\alpha\beta,\mu\nu} d_\lambda d^\lambda I_{\mu\nu,}{}^{\gamma\delta} - \frac{1}{2}\bar{g}^{\alpha\beta} d_\lambda d^\lambda \bar{g}^{\gamma\delta} + \bar{g}^{\alpha\beta} d^\gamma d^\delta + d^\alpha d^\beta \bar{g}^{\gamma\delta} \\ & -2I^{\alpha\beta,\mu\nu} d_\sigma d_\lambda I_\mu{}^{\sigma,\gamma\delta} + \bar{R}\left(I^{\alpha\beta,\gamma\delta} - \frac{1}{2} g^{\alpha\beta} g^{\gamma\delta} \right) \\ & + \left(\bar{g}^{\alpha\beta} \bar{R}^{\gamma\delta} + \bar{R}^{\alpha\beta} \bar{g}^{\gamma\delta} \right) - 4I^{\alpha\beta,\lambda\mu} \bar{R}_{\mu\nu} I_\lambda{}^{\nu,\gamma\delta} \end{aligned}$$

and

$$I^{\alpha\beta,\gamma\delta} = \frac{1}{2}\left(\bar{g}^{\alpha\gamma}\bar{g}^{\beta\delta} + \bar{g}^{\alpha\delta}\bar{g}^{\beta\gamma} \right)$$

Propagator around flat space:

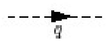
$$iD_{\mu\nu\alpha\beta}(q) = \frac{i}{q^2 - i\epsilon} P_{\mu\nu,\alpha\beta}$$

$$P_{\mu\nu,\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta} \right]$$

Feynman rules:

A.1 Scalar propagator

The massive scalar propagator is:



$$= \frac{i}{q^2 - m^2 + i\epsilon}$$

A.2 Graviton propagator

The graviton propagator in harmonic gauge can be written in the form:

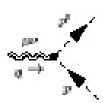
$$= \frac{i\mathcal{P}^{\alpha\beta\gamma\delta}}{q^2 + i\epsilon}$$

where

$$\mathcal{P}^{\alpha\beta\gamma\delta} = \frac{1}{2}\left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\beta\gamma}\eta^{\alpha\delta} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right]$$

A.3 2-scalar-1-graviton vertex

The 2-scalar-1-graviton vertex is discussed in the literature. We write it as:



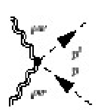
$$= \tau_1^{\mu\nu}(p, p', m)$$

where

$$\tau_1^{\mu\nu}(p, p', m) = -\frac{i\kappa}{2}\left[p^\mu p'^\nu + p^\nu p'^\mu - \eta^{\mu\nu}\left((p \cdot p') - m^2\right)\right]$$

A.4 2-scalar-2-graviton vertex

The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:



$$= \tau_2^{\eta\lambda\rho\sigma}(p, p', m)$$

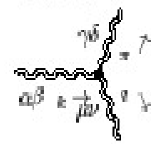
$$\tau_2^{\eta\lambda\rho\sigma}(p, p') = i\kappa^2\Bigg[\left\{I^{\eta\lambda\alpha\delta}I^{\rho\sigma\beta}{}_{\delta} - \frac{1}{4}\left\{\eta^{\eta\lambda}I^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma}I^{\eta\lambda\alpha\beta}\right\}\right\}(p_\alpha p'_\beta + p'_\alpha p_\beta)$$
$$- \frac{1}{2}\left\{I^{\eta\lambda\rho\sigma} - \frac{1}{2}\eta^{\eta\lambda}\eta^{\rho\sigma}\right\}\left[(p \cdot p') - m^2\right]\Bigg] \tag{61}$$

with

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}).$$

A.5 3-graviton vertex

The 3-graviton vertex can be derived via the background field method and has the form[9],[10]



$$= \tau_3{}^{\mu\nu}_{\alpha\beta\gamma\delta}(k, q)$$

where

$$\tau_3{}^{\mu\nu}_{\alpha\beta\gamma\delta}(k, q) = -\frac{i\kappa}{2} \times \Bigg(\mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right]$$
$$+ 2q_\lambda q_\sigma\left[I_{\alpha\beta}{}^{\sigma\lambda}I_{\gamma\delta}{}^{\mu\nu} + I_{\gamma\delta}{}^{\sigma\lambda}I_{\alpha\beta}{}^{\mu\nu} - I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta}{}^{\nu\lambda} - I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta}{}^{\nu\lambda}\right]$$
$$+ \Big[q_\lambda q^\mu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\nu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\nu\lambda}\right) + q_\lambda q^\nu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\lambda}\right)$$
$$- q^2\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\nu}\right) - \eta^{\mu\nu}q_\sigma q_\lambda\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\sigma\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\sigma\lambda}\right)\Big]$$
$$+ \Big[2q_\lambda\big(I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\nu}(k-q)^\mu + I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\mu}(k-q)^\nu - I_{\gamma\delta}{}^{\lambda\sigma}I_{\alpha\beta\sigma}{}^{\nu}k^\mu - I_{\gamma\delta}{}^{\lambda\sigma}$$
$$+ q^2\left(I_{\alpha\beta\sigma}{}^{\mu}I_{\gamma\delta}{}^{\nu\sigma} + I_{\alpha\beta}{}^{\nu\sigma}I_{\gamma\delta\sigma}{}^{\mu}\right) + \eta^{\mu\nu}q_\sigma q_\lambda\left(I_{\alpha\beta}{}^{\lambda\rho}I_{\gamma\delta\rho}{}^{\sigma} + I_{\gamma\delta}{}^{\lambda\rho}I_{\alpha\beta\rho}{}^{\sigma}\right)\Big]$$
$$+ \Big\{(k^2 + (k-q)^2)\left[I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta\sigma}{}^{\nu} + I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta\sigma}{}^{\nu} - \frac{1}{2}\eta^{\mu\nu}\mathcal{P}_{\alpha\beta\gamma\delta}\right]$$
$$- \left(I_{\gamma\delta}{}^{\mu\nu}\eta_{\alpha\beta}k^2 + I_{\alpha\beta}{}^{\mu\nu}\eta_{\gamma\delta}(k-q)^2\right)\Big\}\Bigg) \tag{62}$$

Performing quantum calculations

Quantization was straightforward, but what do you do next?
- calculations are not as simple

<u>Next step: Renormalization</u>
-divergences arise at high energies
-not of the form of the basic lagraingian

Solution: Effective field theory and renormalization
- renormalize divergences into parameters of the most general lagrangian ($c_1, c_2 \ldots$)

Power counting theorem: (pure gravity, $\Lambda=0$)
-each graviton loop = 2 more powers in energy expansion
-1 loop = Order $(E)^4$
-2 loop = Order $(E)^6$

Power counting theorem

Weinberg 1979

Simplified form – just count powers of $1/M^2_P$

1) Tree from leading Lagrangian is leading power of M_P
2) Effect of second Lagrangian is down by E^2/M_P^2
3) Each loop from lowest L contributes extra E^2/M^2_P
4) Loops from higher L's yet higher in energy

Effects ordered in **Energy Expansion**

Renormalization

One loop calculation: ‘t Hooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

dim. reg. preserves symmetry

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity "one loop finite" since $R_{\mu\nu}=0$

Note: Two loop calculation known in pure gravity — Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

What are the quantum predictions?

Not the divergences

-they come from the Planck scale

-unreliable part of theory

Not the parameters

-local terms in L

-we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

-not the same as terms in the Lagrangian

- most always non-analytic dependence in momentum space

-can't be Taylor expanded – can't be part of a local Lagrangian

-long distance in coordinate space

Example- Corrections to Newtonian Potential

Here discuss scattering potential of two heavy masses.

JFD 1994
JFD, Holstein, Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}$$

Potential found using from

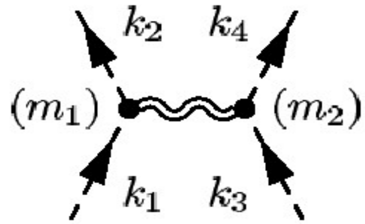
$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

Lowest order:

one graviton exchange



$$iM_{1(a)}(\vec{q}) = \tau_1^{\mu\nu}(k_1, k_2, m_1)\left[\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2}\right]\tau_1^{\alpha\beta}(k_3, k_4, m_2)$$

Non-relativistic reduction:

$$M_{1(a)}(\vec{q}) = -\frac{4\pi G m_1 m_2}{\vec{q}^2}$$

Potential:

$$V_{1(a)}(r) = -\frac{G m_1 m_2}{r}$$

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

short range

Non-analytic

analytic

Parameter free and divergence free

Recall: divergences like local Lagrangian ~R^2

Also unknown parameters in local Lagrangian ~c_1,c_2

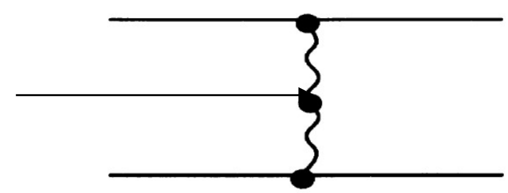
But this generates only "short distance term"

Note: R^2 has 4 derivatives $R^2 \sim q^4$

<u>Then:</u>

Treating R^2 as perturbation

R^2

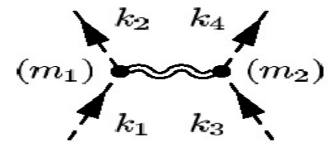


$$V_{R^2} \sim G^2 Mm \;\; \frac{1}{q^2} \; q^4 \; \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 Mm \delta^3(x)$$

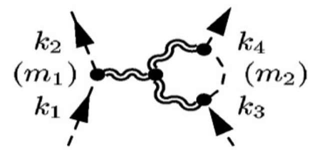
Local lagrangian gives only short range terms

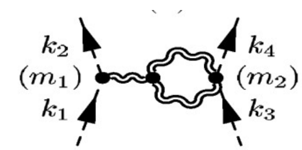
The calculation:

Lowest order:

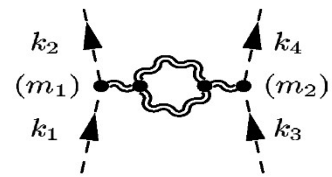


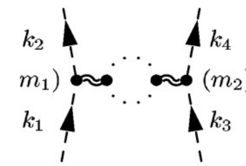
Vertex corrections:



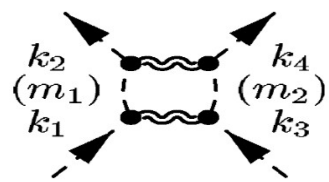


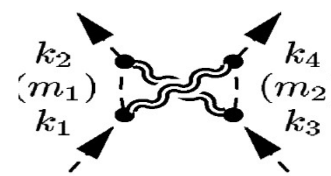
Vacuum polarization:
(Duff 1974)



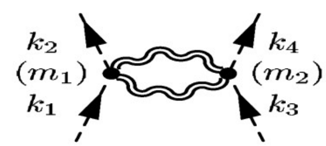


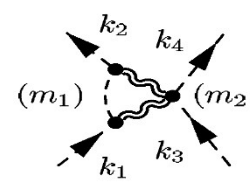
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right)$$

$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Gives precession of Mercury, etc (Iwasaki ; Gupta + Radford)

Quantum correction

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

5) Why not done 30 years ago?
   - power of effective field theory reasoning

Aside: Classical Physics from Quantum Loops:

JFD, Holstein
2004 PRL

Field theory folk lore:
Loop expansion is an expansion in ħ
"Proofs" in field theory books

This is not really true.
- numerous counter examples – such as the gravitational potential

- can remove a power of ħ via kinematic dependence

$$\sqrt{\frac{m^2}{-q^2}} = \frac{m}{\hbar\sqrt{-k^2}}$$

- classical behavior seen when massless particles are involved

On-shell techniques and loops from unitarity

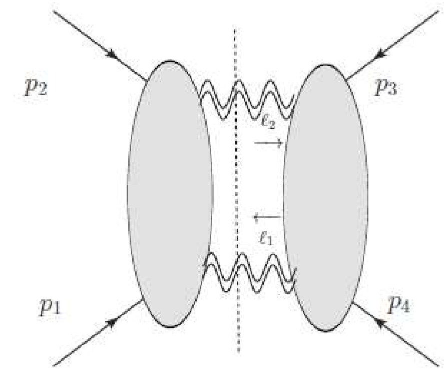
JFD, Bjerrum-Bohr
Vanhove

- On-shell amplitudes only
- No ghosts needed – axial gauge
- Both unitarity cuts and dispersion relation methods
- Gravity is square of gauge theory

$$iM^{1-\text{loop}}\Big|_{disc} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\sum_{\lambda_1,\lambda_2} M^{\text{tree}}_{\lambda_1\lambda_2}(p_1,p_2,-\ell_2^{\lambda_2},\ell_1^{\lambda_1})(M^{\text{tree}}_{\lambda_1\lambda_2}(p_3,p_4,\ell_2^{\lambda_2},-\ell_1^{\lambda_1}))^*}{\ell_1^2\ell_2^2}\Big|_{cut},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)},$$



Confirm results (and gauge invariance)

<u>One loop universality/soft theorem</u>

Low
Gell-Mann
Goldberger
Weinberg
Gross
Jackiw

<u>Tree level soft theorems</u>

- Compton amplitudes and gravitational Compton amplitudes are universal at leading order
- Conservation of charge/energy and ang. mom.

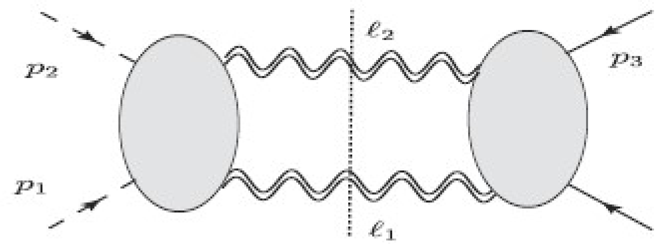
<u>One loop soft theorem</u>

- E&M and gravitational potentials
- formed by square of Compton amplitudes
- quantum term down from classical by $\sqrt{-q^2}$
- first found in direct calculations by Holstein and Ross

Light bending at one loop

Bjerrum-Bohr, JFD, Holstein
Plante, Vanhove
Bai and Huang

- Again using unitarity methods
- Gravity Compton as square of EM Compton
- Compare massless spin 0 and photon



$$\mathcal{M}_\eta = \kappa^2 \frac{M^2\omega^2}{\mathbf{q}^2} + \kappa^4 \frac{15 M^3\omega^2}{512|\mathbf{q}|} + \kappa^4 \frac{15 M^2\omega^2}{512\pi^2} \log\left(\frac{\mathbf{q}^2}{M^2}\right) - \kappa^4 \frac{M^2\omega^2}{32\pi^2} \log^2\left(\frac{\mathbf{q}^2}{\mu^2}\right)$$

$$- \kappa^4 \frac{\mathrm{bu}^\eta}{(8\pi)^2} M^2\omega^2 \log\left(\frac{\mathbf{q}^2}{\mu^2}\right) - \kappa^4 \frac{M^3\omega^3}{8\pi} \frac{i}{\mathbf{q}^2} \log\left(\frac{\mathbf{q}^2}{M^2}\right),$$

$$\mathrm{bu}^\varphi = \frac{371}{120}, \qquad \mathrm{bu}^\gamma = \frac{113}{120}.$$

bu^i is different coefficient for spin 0 and 1

Bending angle calculatable using eikonal amplitude
- saddle point approximation

Using
Akhoury
Ryo
Sterman

$$\mathcal{M}^{0+1}\left(\Delta^{\perp}\right) = 2(s-M_{\sigma}^{2}) \int d^{2}\mathrm{b}^{\perp} e^{-i\Delta^{\perp}\cdot\mathrm{b}^{\perp}} \left[e^{i(\chi_0 - i\ln[1+i\chi_2])} - 1\right]$$

r_0 is IR cutoff can turn into energy dependence depending on resolution of IR singularity

Result:

$$\theta_{\eta} = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^{\eta} - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}.$$

Massless particles no longer move on null geodesics!
- irreducible tidal effects from loops
- also non-universal – violation of some forms of EP
- perhaps energy dependence

Limitations of the effective field theory

Corrections grow like $Amp \sim A_0\left[1 + Gq^2 + Gq^2 \ln q^2\right]$

Overwhelm lowest order at $q^2 \sim M_P^2$

Also sicknesses of $R+R^2$ theories beyond M_P
(J. Simon)

Effective theory predicts its own breakdown at M_P
-**could in principle be earlier**

Needs to be replaced by more complete theory
at that scale
(String theory??)

Treating quantum GR beyond the Planck scale
is likely not sensible

Reformulate problem of quantum gravity

Old view: GR and Quantum Mechanics incompatible

Unacceptable

New view: We need to find the right “high energy” theory which includes gravity

Less shocking:
- not a conflict of GR and QM
- just incomplete knowledge

THIS IS PROGRESS!

A Modern Viewpoint:

A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.

Frank Wilczek
Physics Today
August 2002

Another thoughtful quote:

"I also question the assertion that we presently have no quantum field theory of gravitation. It is true that there is no *closed*, internally consistent theory of quantum gravity valid at all distance scales. But such theories are hard to come by, and in any case, are not very relevant in practice. But as an *open* theory, quantum gravity is arguably our *best* quantum field theory, not the worst. ….

{Here he describes the effective field theory treatment}

From this viewpoint, quantum gravity, when treated –as described above- as an effective field theory, has the largest bandwidth; it is credible over 60 orders of magnitude, from the cosmological to the Planck scale of distances."

J.D. Bjorken

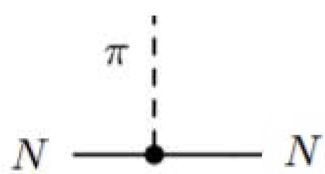
Nuclear Physics

Imagine that pions were strictly massless – chiral symmetry

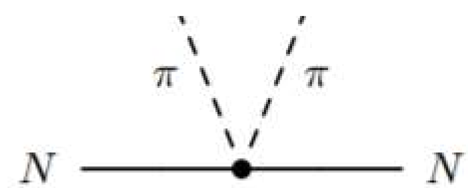
They would be almost as ubiquitous as light

Condensed Matter and Atomic physics would involve pions

New couplings would be known:



$$\frac{g_A}{2F_\pi}\sigma \cdot q \; \tau^i$$



$$\frac{c_{(2,3,4)}}{F_\pi^2}q^2$$

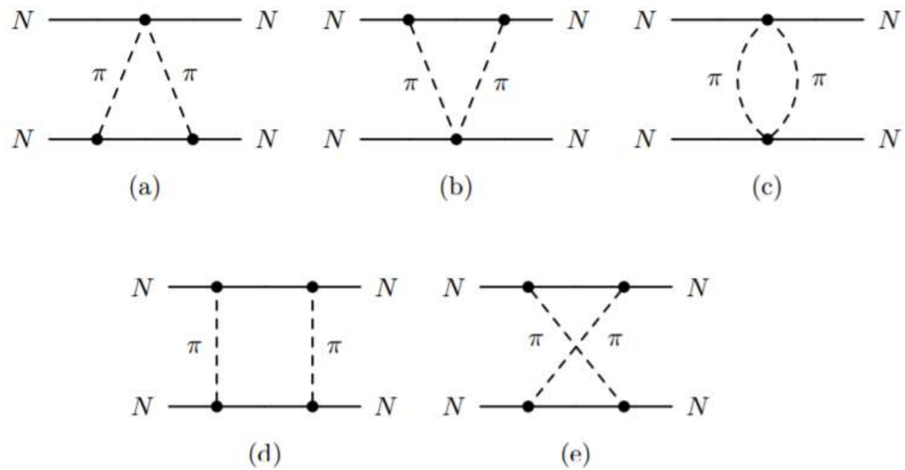
The pions would interact with themselves also
- weakly

Perhaps someone clever (Albert Pionstein?) would propose a theory with an effective Lagrangian

$$\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \qquad \text{with} \qquad U = exp\left[\frac{i\tau\cdot\pi}{F}\right]$$

But this we would need EFT to interpret this

EFT techniques could be applied unambiguously at low energy



In this case, many components to the potential:

$$V(r) = V_S(r) + V_T(r)\ \tau_1 \cdot \tau_2 (\sigma_1 \cdot \hat{r} \sigma_1 \cdot \hat{r} - \frac{1}{3} \sigma_1 \cdot \sigma_2) + ...$$

Van Kolck, Kaiser, Epelbaum,
Meißner …

Results:

$$V_T(r) = \frac{1}{16\pi^4}\frac{g_A^2}{F_\pi^2}\frac{1}{r^3}\left[1 - \frac{c_4}{\pi F_\pi^2}\frac{1}{r^3}\right]$$

$$V_S(r) = \frac{9}{16\pi^2 F_\pi^2}\left[\frac{g_A^2 c_3}{r^6} - \frac{2\bar{c}^2}{\pi r^7}\right] \qquad \bar{c}^2 = [c_3 + \frac{c_2}{6}]^2 + \frac{c_2^2}{45}$$

Plus local contact interactions:

- need to cut off singular potentials and adjust LECs

Low energy folks would see new scale

- having all the features of the Planck scale for us

$$V_T(r) = -29\ \mathrm{MeV}\left[\left(\frac{r_0}{r}\right)^3 - 0.66\left(\frac{r_0}{r}\right)^6\right] \qquad r_0 = 1\ \mathrm{fm} = (200\mathrm{MeV})^{-1})$$

$$V_S(r) = -300\ \mathrm{MeV}\left[\left(\frac{r_0}{r}\right)^6 + 0.24\left(\frac{r_0}{r}\right)^7\right]$$

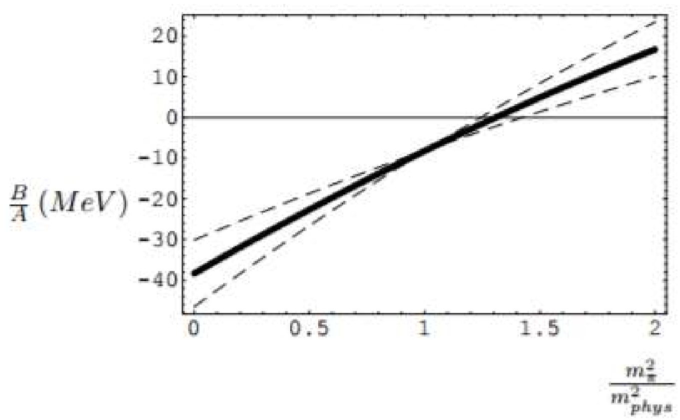
What is the chance that they could come up with QCD?

<u>This could be a good starting point for Nuclear Physics</u>

Start with pion massless and nucleon heavy
- unambiguous EFT
- power-law potentials

Then back off from these limits towards the real world

I have some papers that (sort of) do the reverse (2006,8)
- calculate these with present masses, then vary mass
- some deviations from low order ChPTh (unitarization)



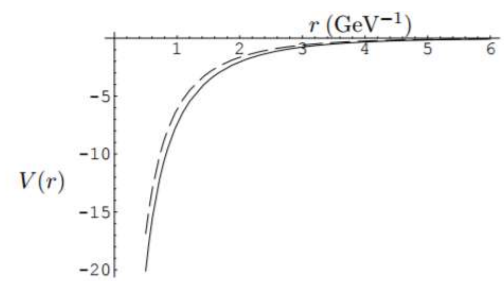


Figure 9: The attractive scalar-isoscalar potential in the physical case (dashed line) and in the chiral limit (solid line).

Summary:

EFT helps us make sense of the world

Quantum Theory of General Relativity exists
- it is an EFT

Rigorous predictions can be made

Still work to be done – in EFT and for UV completion

Our Core Theory

quantum mechanics — spacetime — gravity

$$W = \int_{k<\Lambda} [Dg][DA][D\psi][D\Phi] \exp\left\{ i \int d^4x \sqrt{-g} \left[\frac{m_p^2}{2} R - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi}^i \gamma^\mu D_\mu \psi^i + \left(\bar{\psi}^i_L V_{ij} \Phi \psi^j_R + \text{h.c.} \right) - |D_\mu \Phi|^2 - V(\Phi) \right] \right\}$$

other forces — matter — Higgs

Slide due to
Sean Carroll