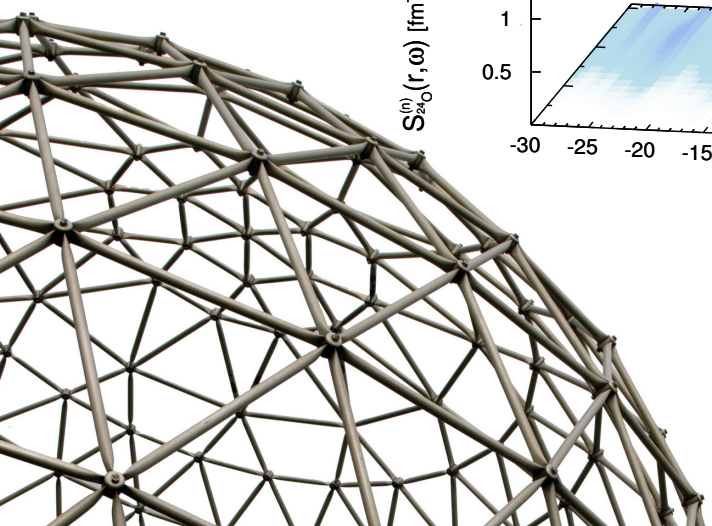
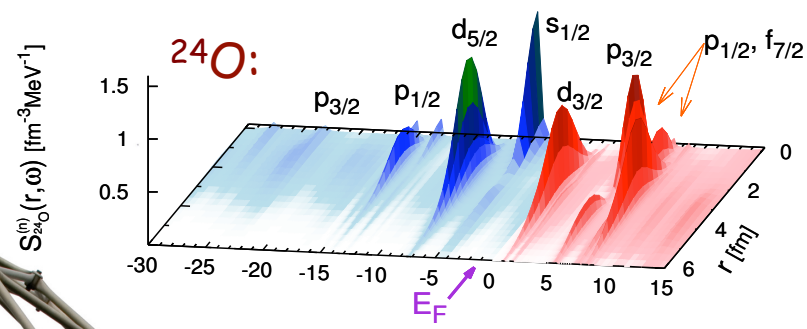
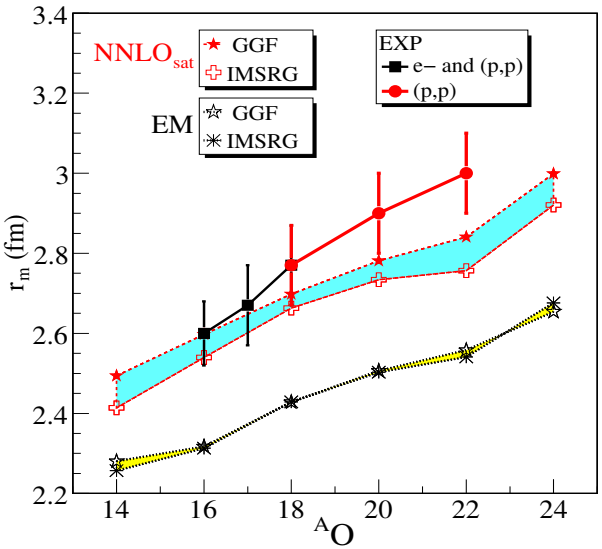
Origin of shell-structure in atomic nuclei

Carlo Barbieri — University of Surrey

17 January 2016







Current Status of low-energy nuclear physics

Composite system of interacting fermions

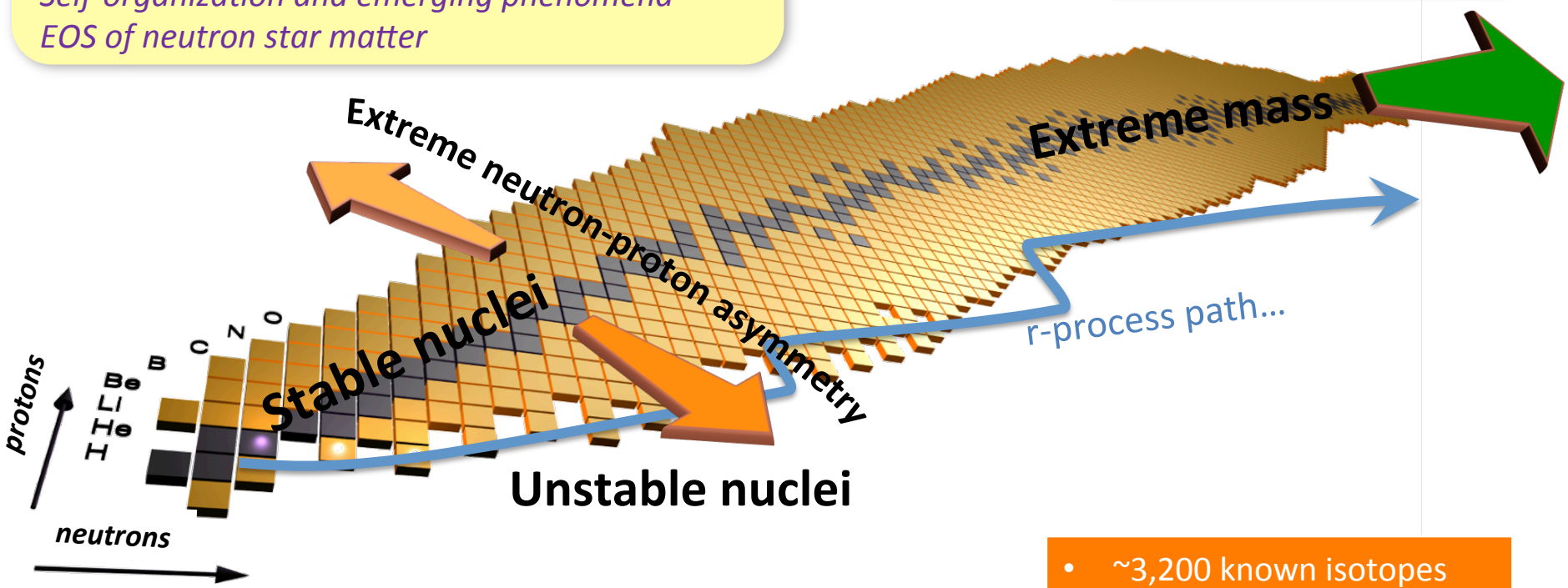
Binding and limits of stability

Coexistence of individual and collective behaviors

Self-organization and emerging phenomena

EOS of neutron star matter

Experimental programs
RIKEN, FAIR, FRIB

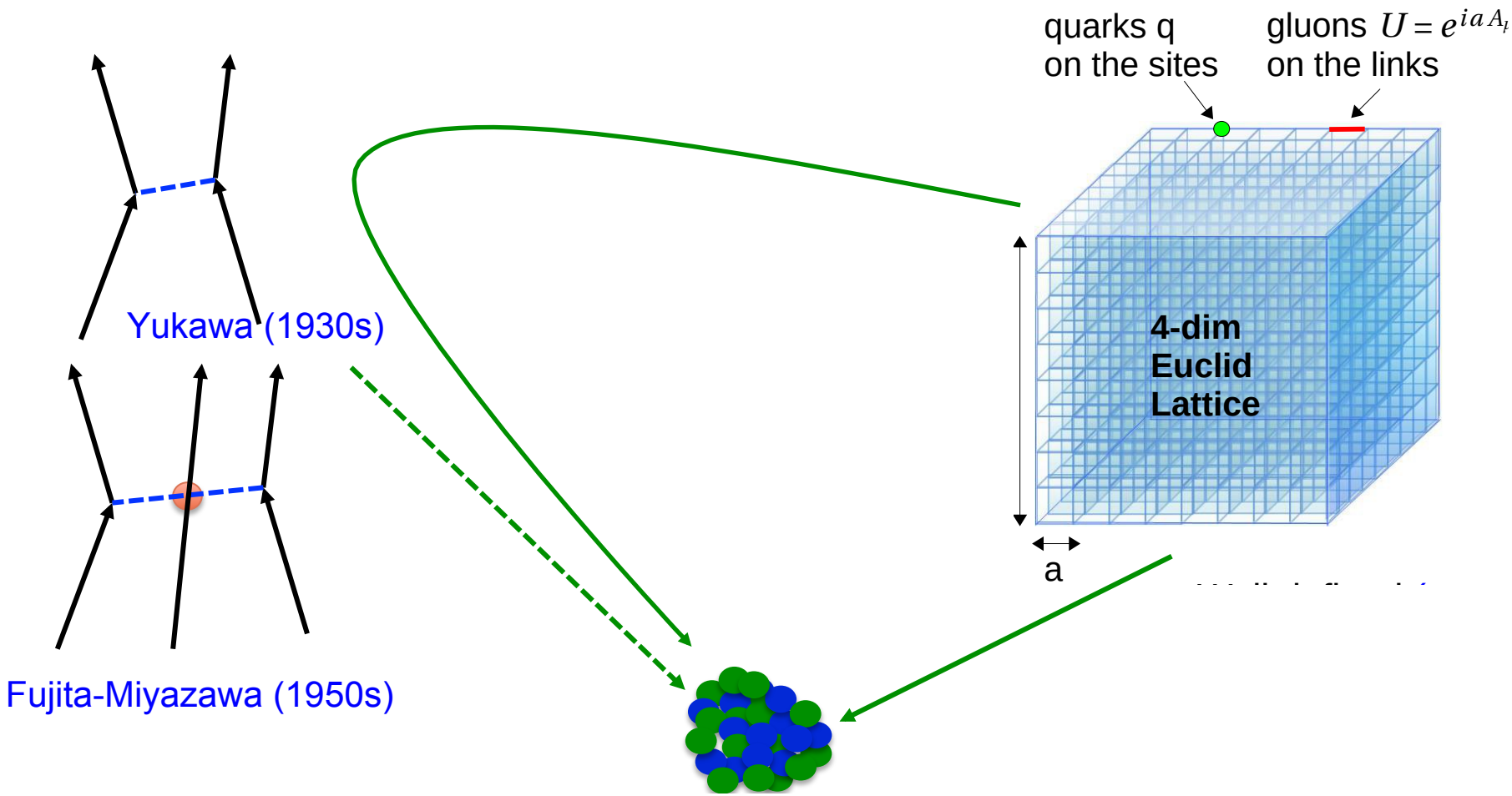


- ~3,200 known isotopes
- ~7,000 predicted to exist
- Correlation characterised in full for ~283 stable

Nature **473**, 25 (2011); **486**, 509 (2012)

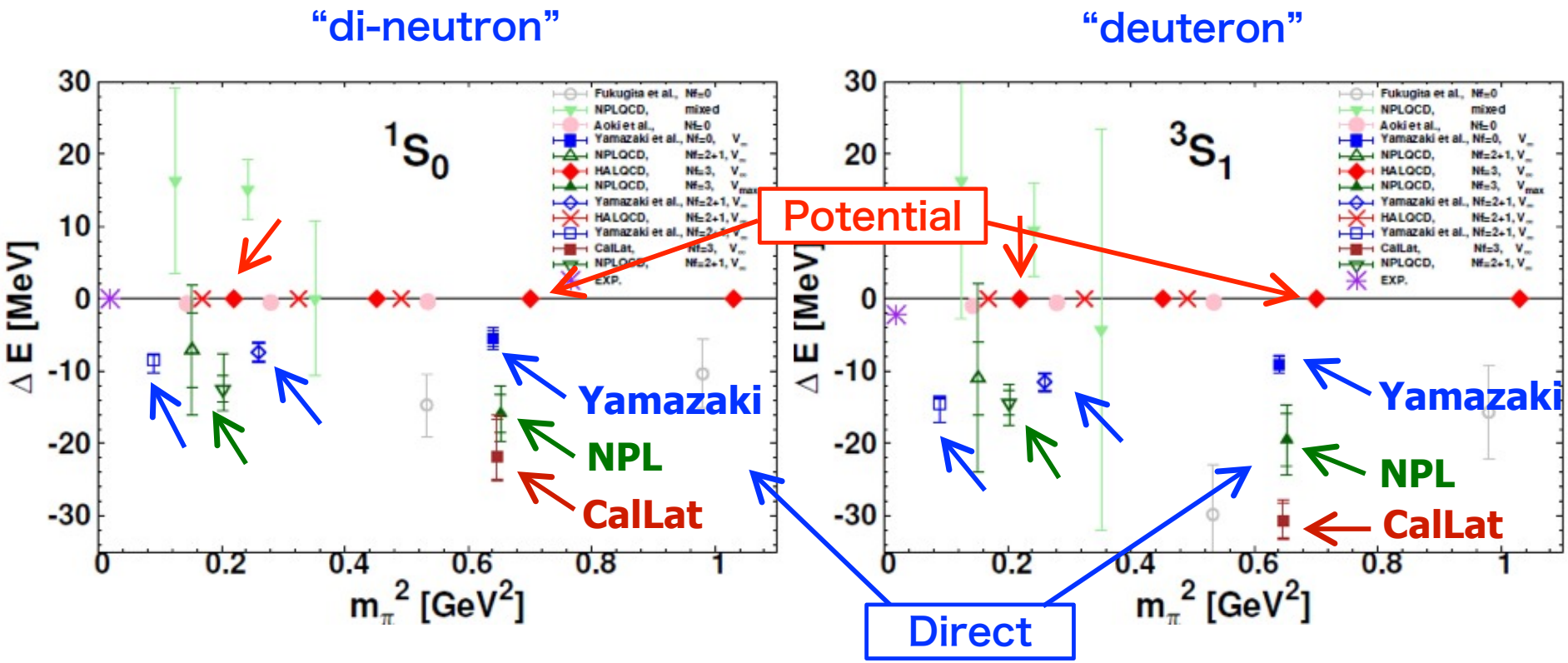
Approaches to nuclei from LQCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m\bar{q}q$$



Approaches to nuclei from LQCD

HALQCD coll. -- Talk of ***S. Aoki*** at Kavli institute, Oct. 2016



Potential method (HALQCD) :

Direct method (Yamazaki et al./NPL/CalLat):

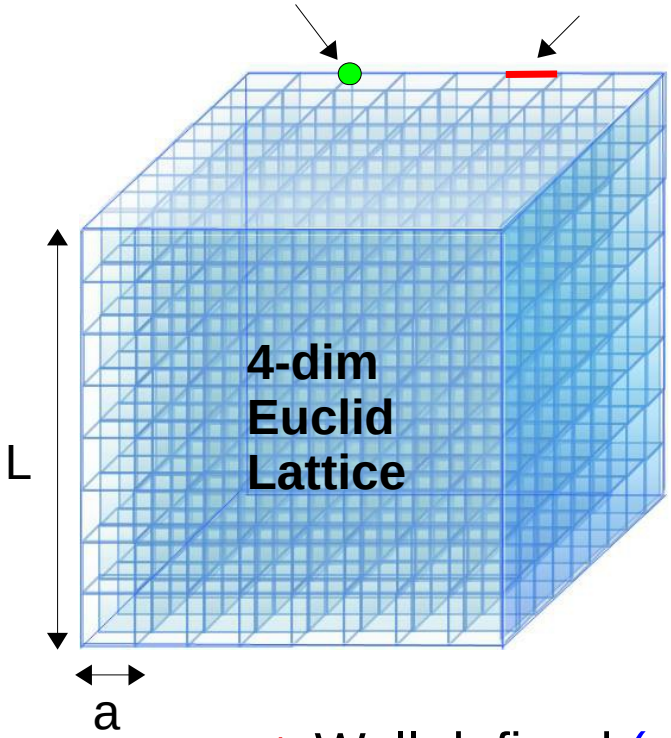
See Z. Davoudi talk on Wednesday!!

Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m\bar{q}q$$

quarks q on the sites

gluons $U = e^{iaA_\mu}$ on the links



Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U) e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ U_i } : ensemble of gauge conf. U generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- Well defined (reguralized)
- Manifest gauge invariance
- Fully non-perturvative
- Highly predictive

Slide, courtesy of T. Inoue (YITP talk, Oct. 8th 2015)

The HAL-QCD Method

Define a general potential U(***r***,***r'***) which is and non-local but energy independent up to inelastic threshold, such that:

$$\frac{-\nabla^2}{2\mu}\varphi_{\vec{k}}(\vec{r}) \; + \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\varphi_{\vec{k}}(\vec{r}\,') \; = \; E_{\vec{k}}\varphi_{\vec{k}}(\vec{r})$$

for the Nambu-Bethe-Salpeter (NBS) wave function,

$$\varphi_{\vec{k}}(\vec{r}) = \sum \langle 0|B_i(\vec{x}+\vec{r},t)B_j(\vec{x},t)|B=2,\vec{k}\rangle$$

Operationally, measure the 4-pt function on the QCD Lattice

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0|B_i(\vec{x}+\vec{r},t)B_j(\vec{x},t)\,J(t_0)|0\rangle = \sum_{\vec{k}} A_{\vec{k}}\varphi_{\vec{k}}(\vec{r})e^{-W_{\vec{k}}(t-t_0)} + \ldots$$

and extract U(***r***,***r'***) from:

$$\left\{2M_B - \frac{\nabla^2}{2\mu}\right\}\psi(\vec{r},t) \; + \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) \; = -\frac{\partial}{\partial t}\psi(\vec{r},t)$$

A *local potential* V(***r***) is then obtained through a derivative expansion of U(***r***,***r'***), which *must give the same observables* of the LQCD simulation:

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')\left\{V(\vec{r}) + \mathcal{O}(\nabla) + \mathcal{O}(\nabla^2) + \ldots\right\}$$

$$\rightarrow \; V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

Tensor/Yukawa force in S-D

Spin-orbit force, P waves

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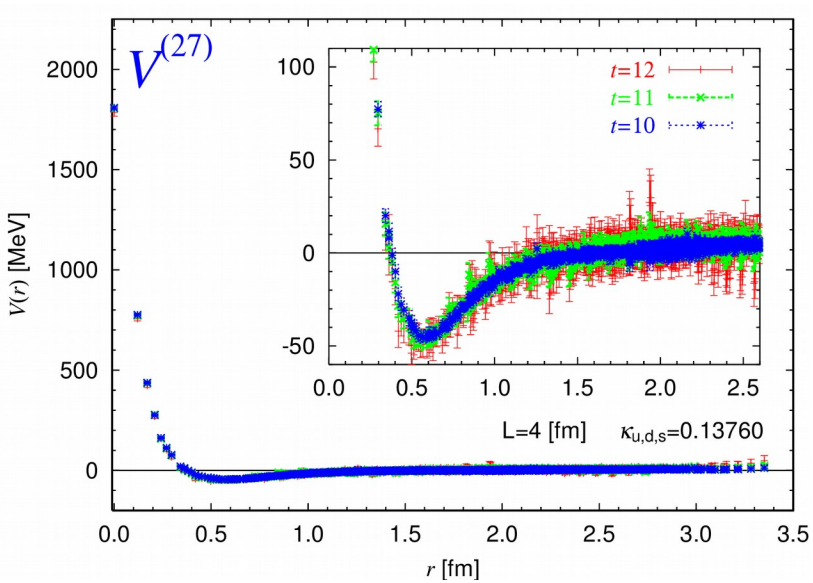
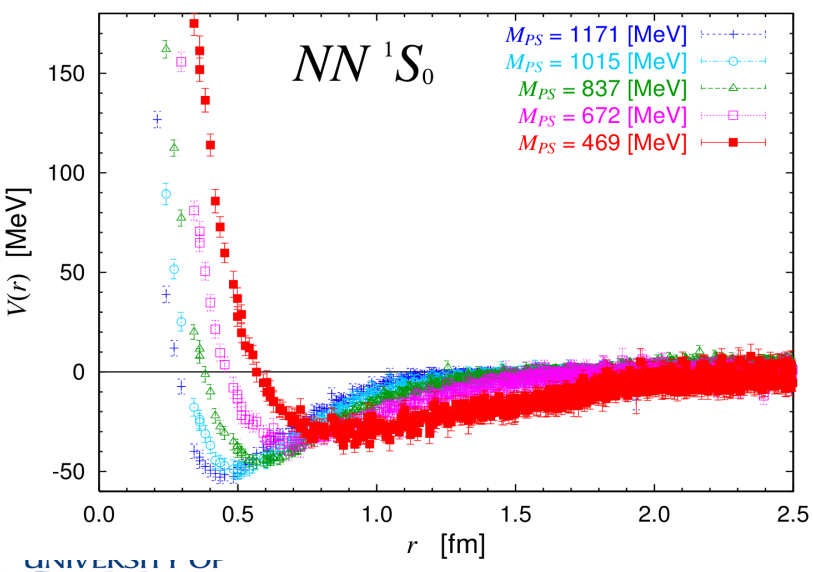
Prog. Theor. Phys. 123 89 (2010); Phys. Lett. B712 , 437 (2012); Prog. Theor. Exp. Phys. 01A105 (2012)

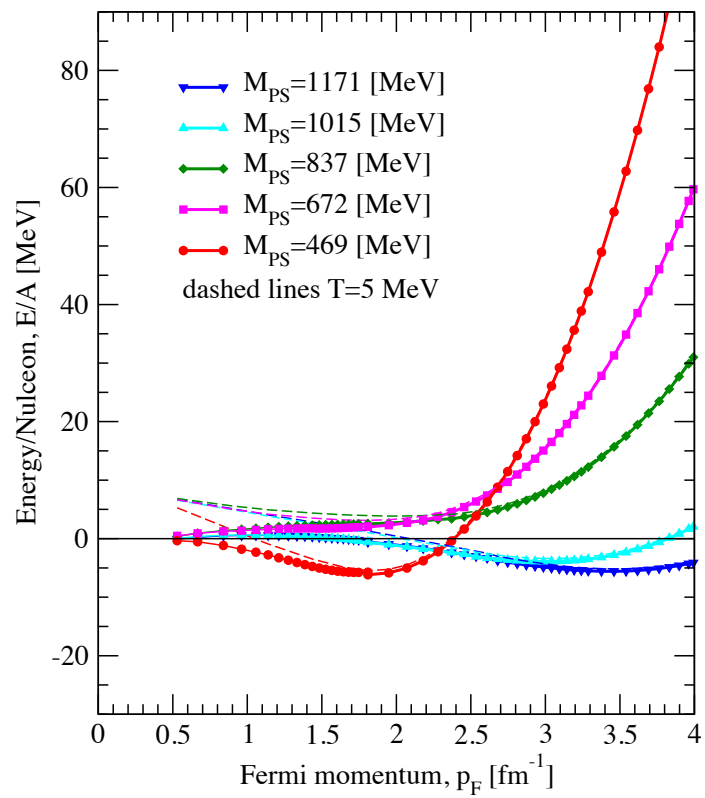
Two-Nucleon HAL potentials





Quark mass dependence of *V(r)* for NN partial wave ($^1S_0$, $^3S_1$, $^3S_1$-$^3D_1$)

➔ Potentials become stronger m_π as decreases.



T. Inoue *et al.*,
Phys. Rev. Lett. **111** 112503 (2013).

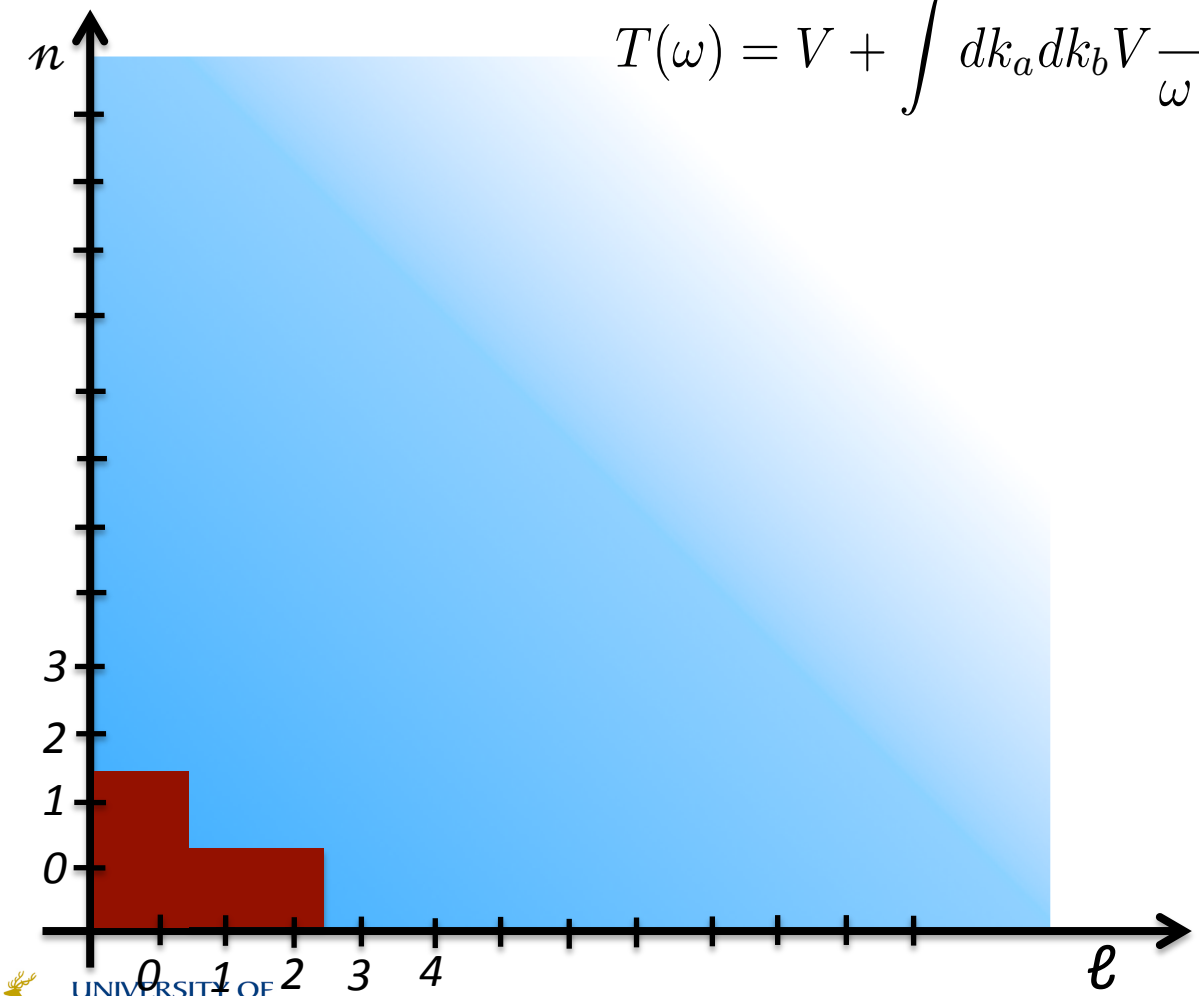
Prog. Theor. Exp. Phys. 01A105 (2012)

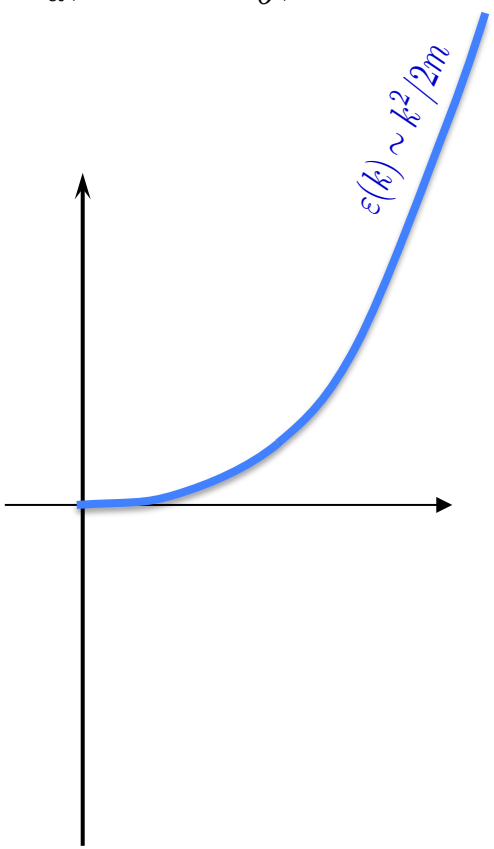
(Finite-T results by A. Carbone, priv. comm.)

Analysis of Brueckner HF

Scattering of two nucleon in free space:

$$T(\omega) = V + \int dk_a dk_b V \frac{1}{\omega - k_a^2/2m - k_b^2/2m + i\eta} T(\omega)$$

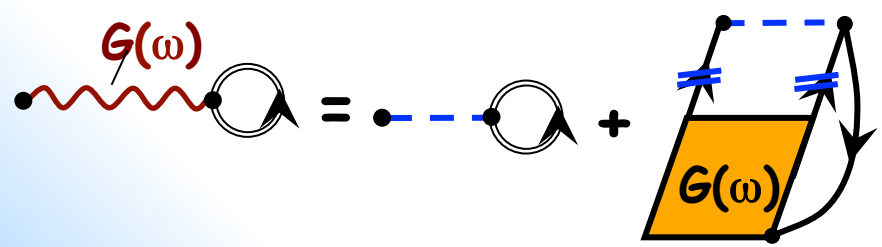


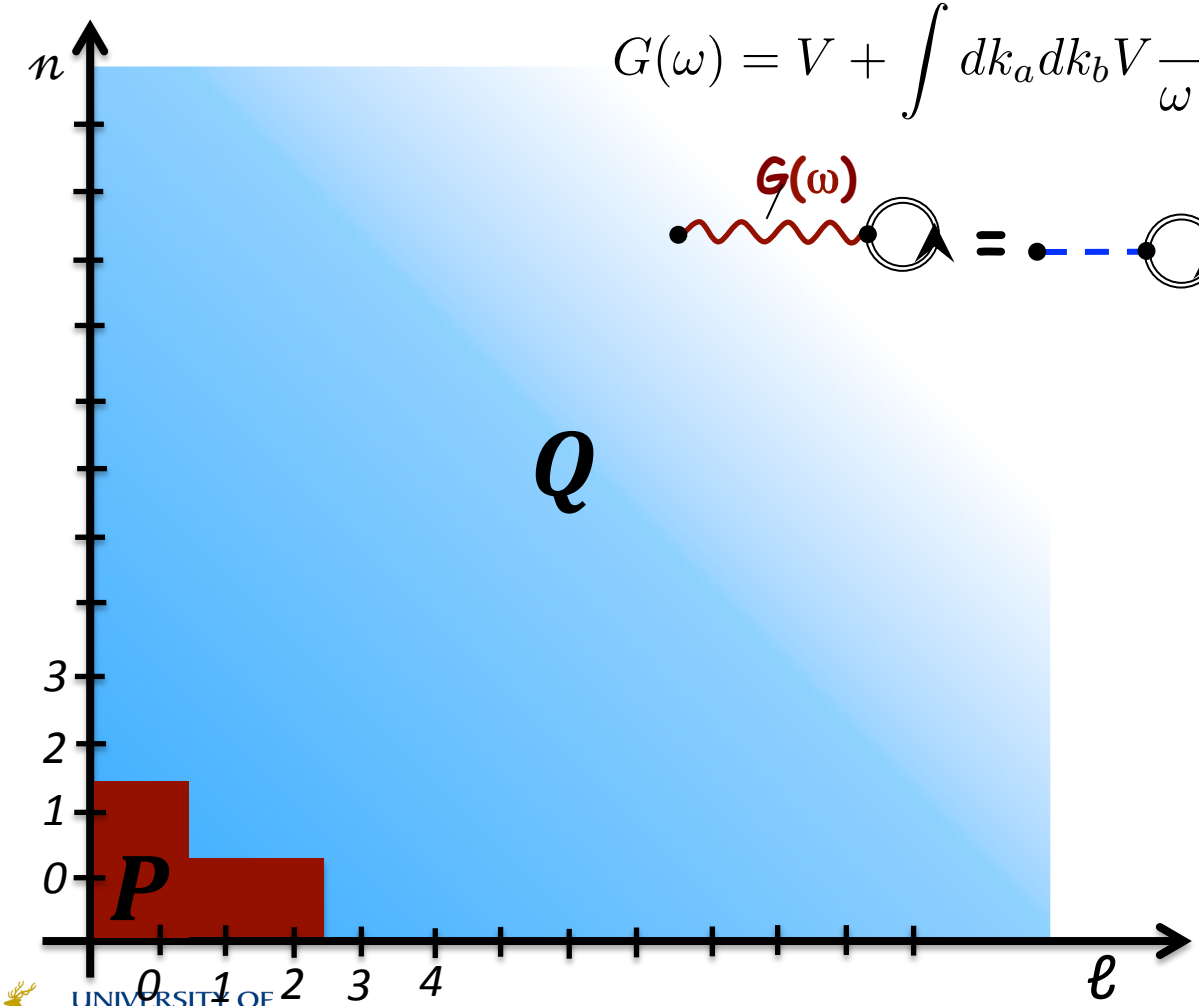


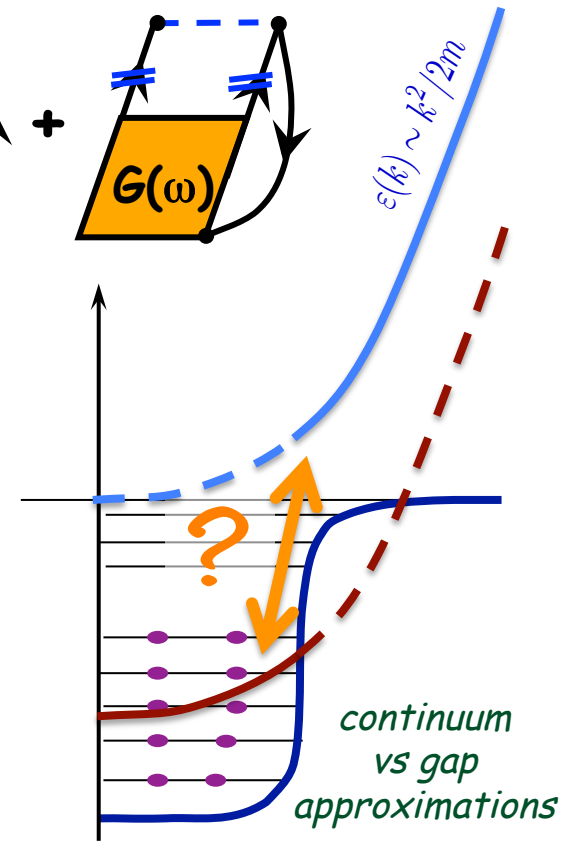
Analysis of Brueckner HF

Scattering of two nucleons outside the Fermi sea (➔BHF):

$$G(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G(\omega)$$



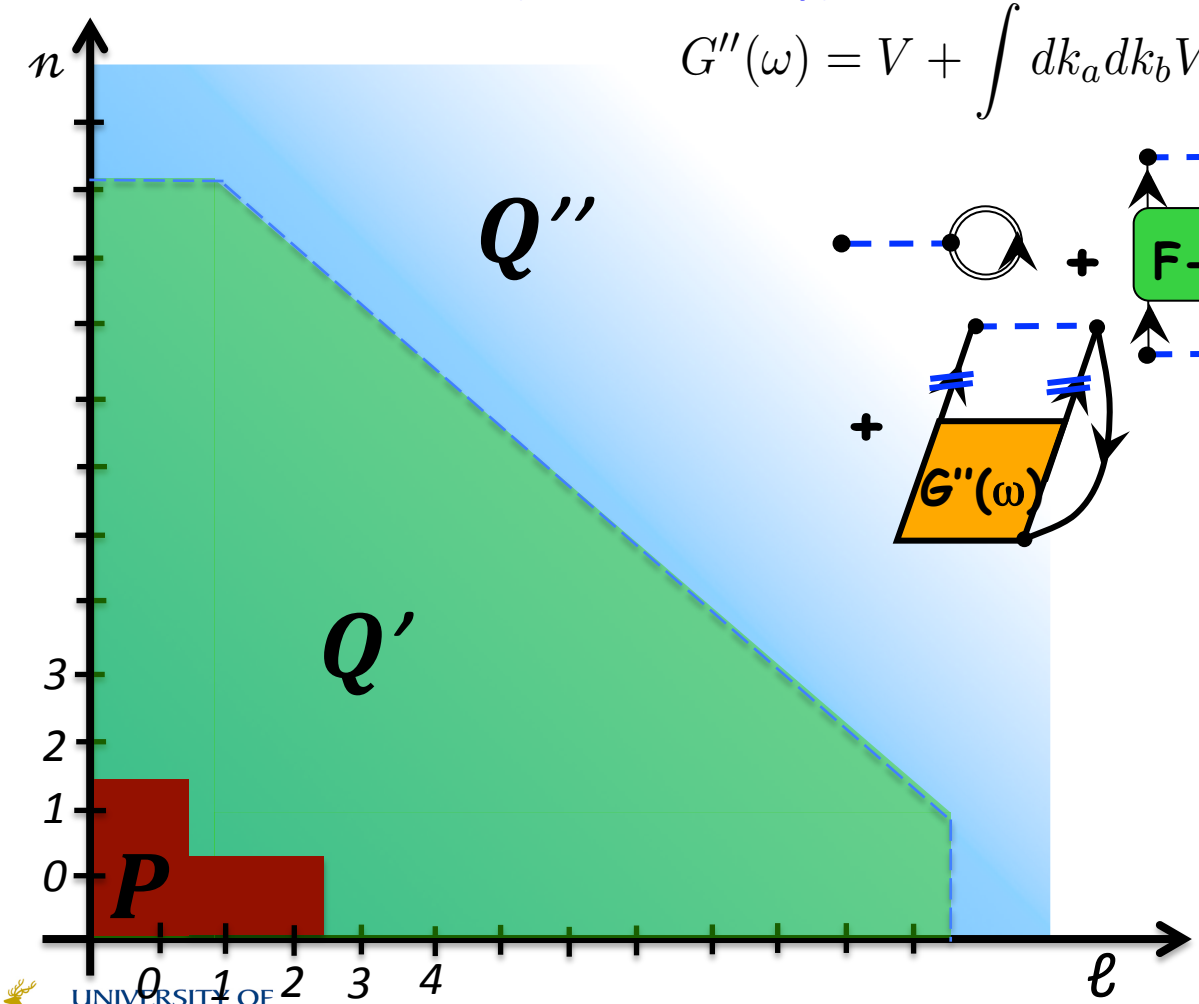


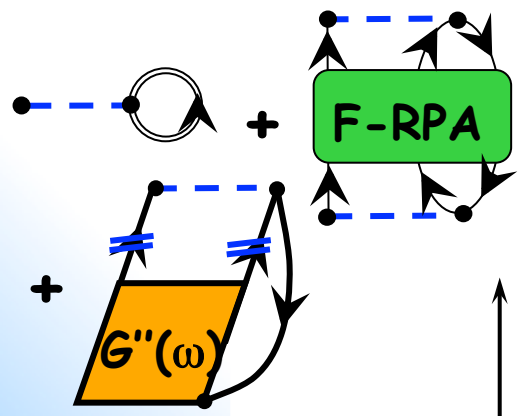


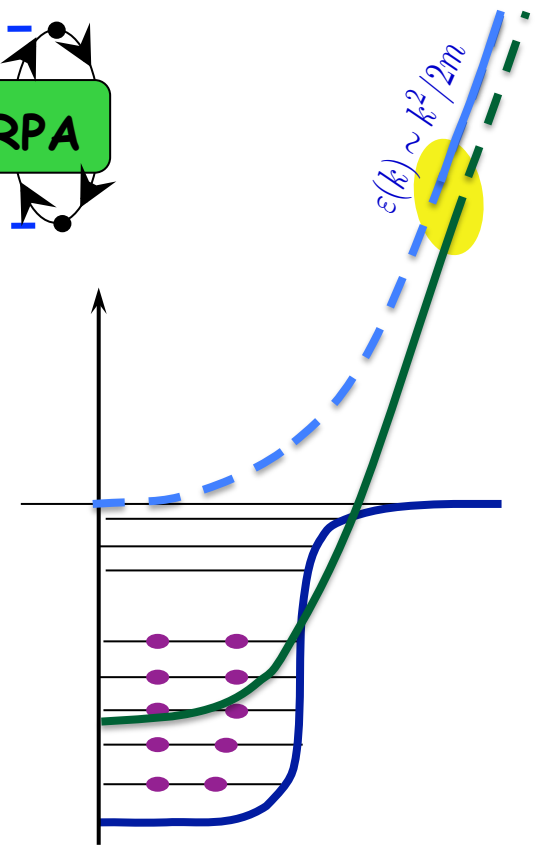
Mixed SCGF-Brueckner approach

Solve full many-body dynamics in model space (P+Q') and the Goldstone's ladders outside it (i.e. in Q'' only):

$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$



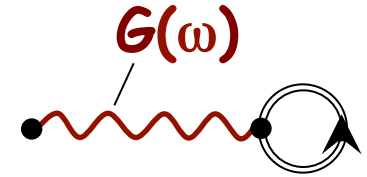


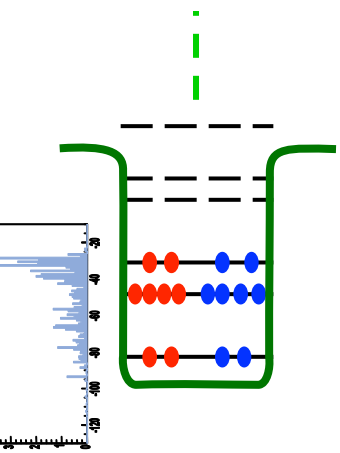


Treating short-range corr. with a G-matrix

- The short-range core can be treated by summing ladders outside the model space:

$$\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega) = i\sum_{\gamma\delta}\int\frac{d\omega'}{2\pi}G_{\alpha\gamma,\delta\beta}(\omega+\omega')g_{\delta\gamma}(\omega') \quad =$$





$$\Sigma^{\star}(\mathbf{r},\mathbf{r}';\omega) = \Sigma^{MF}(\mathbf{r},\mathbf{r}';\omega) + \tilde{\Sigma}(\mathbf{r},\mathbf{r}';\omega)\ .$$

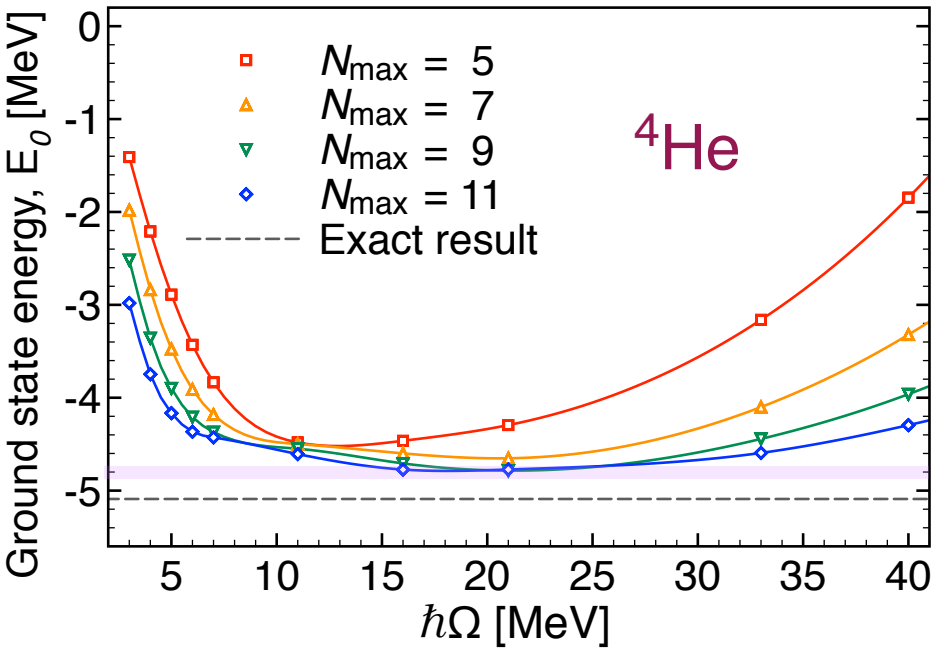
$$Z_\alpha = \int d\mathbf{r}\,|\psi^{A\pm1}_\alpha(\mathbf{r})|^2 = \left.\frac{1}{1-\dfrac{\partial\Sigma^{\star}_{\hat\alpha\hat\alpha}(\omega)}{\partial\omega}}\right|_{\omega=\pm(E^{A\pm1}_\alpha-E^A_0)}$$

Two contributions to the derivative:

- $\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega)$ is due to scattering to (high-k) states in the Q space
- $\Sigma(\mathbf{r},\mathbf{r}';\omega)$ accounts for low-energy (long range) correlations

Benchmark on ^{4}He

C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]

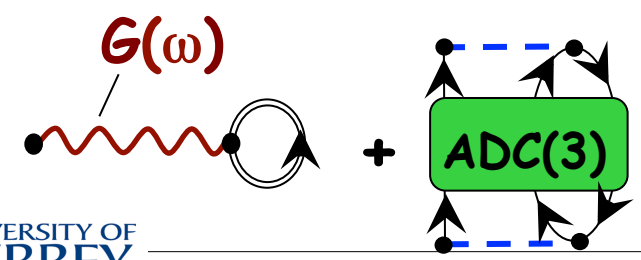


Can benchmark the Gmtx+ADC(3) method on light ^{4}He, where exact solutions are possible:

	G(ω) + ADC(3)	Exact
HALQCD @ $m_\pi \approx 470$MeV	4.79(3) MeV	5.09 MeV[1]

[1]H. Nemura *et al.*, Int. J. Mod. Phys. E **23**, 1461006 (2014)

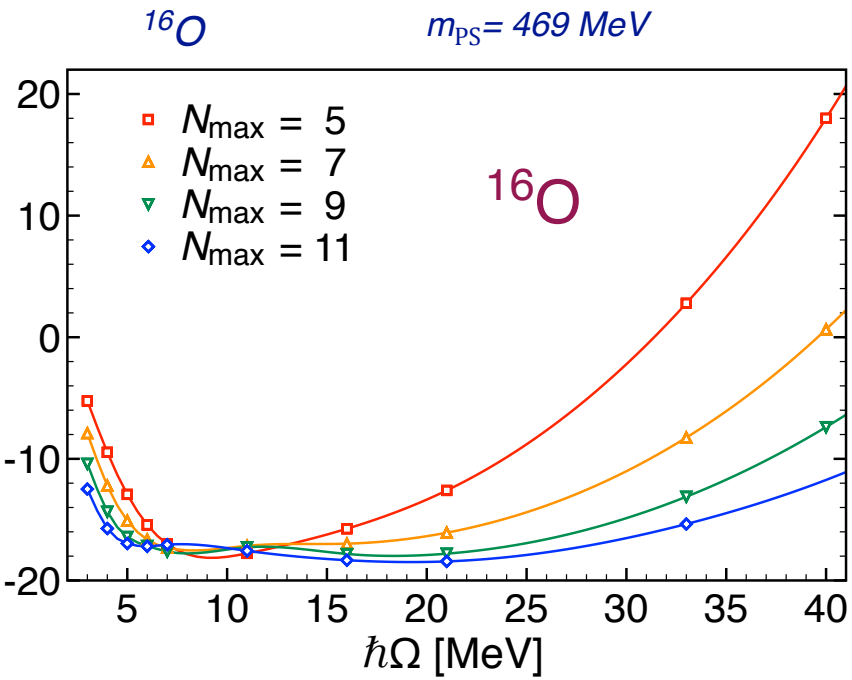
➔ *Can expect accuracy on binding energies at about 10%*

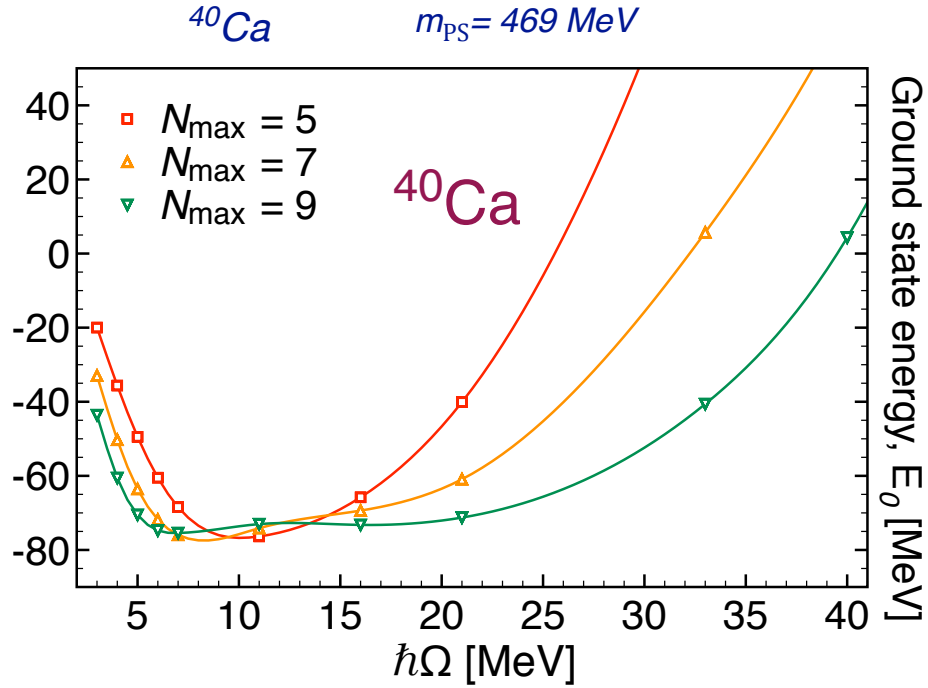


$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

Binding of ^{16}O and ^{40}Ca:

C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]





Binding energies are ~17 MeV ^{16}O and 70-75MeV for ^{40}Ca. Possibly being underestimated by 10%

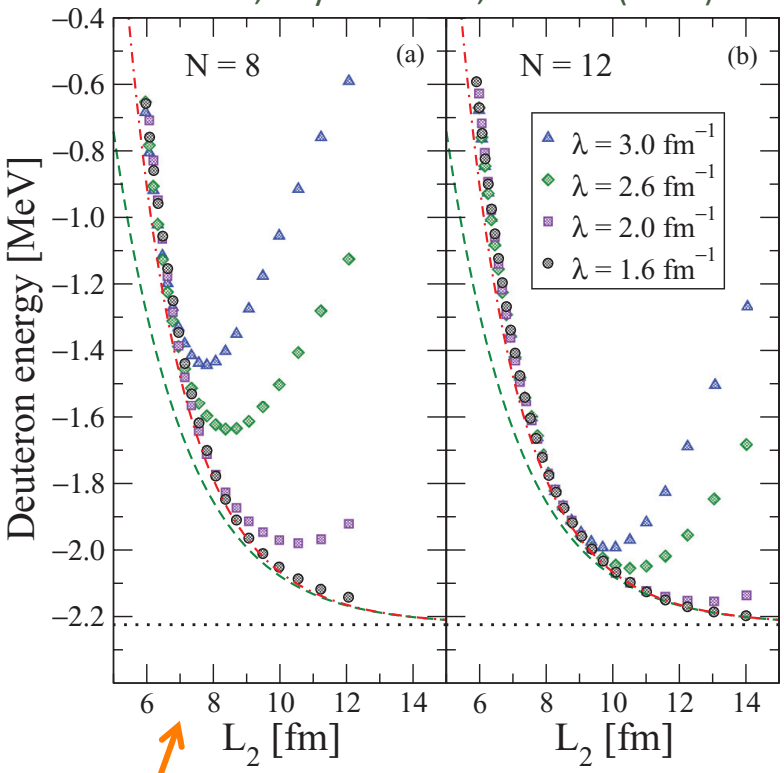
➔ ^{16}O at $m_\pi \approx$ 470 MeV is unstable toward 4-α breakup!

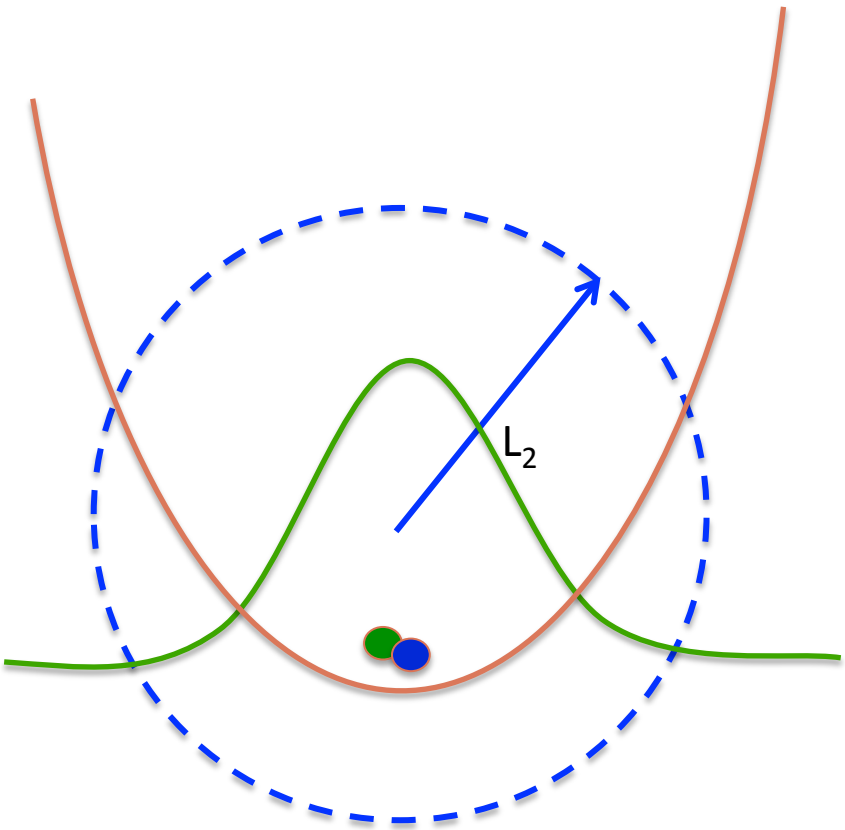
E_0^A [MeV]	^{4}He	^{16}O	^{40}Ca
BHF [22]	-8.1	-34.7	-112.7
G(ω) + ADC(3)	-4.80(0.03)	-17.9 (0.3) (1.8)	-75.4 (6.7) (7.5)
Exact Result [51]	-5.09	–	–
Separation into ^{4}He clusters:		-2.46 (0.3) (1.8)	24.5 (6.7) (7.5)

[C.S.McIlroy, CB, HAL coll., in prep]

Infrared convergence

Moore et al., Phy. Rev. C**87**, 044326 (2013)



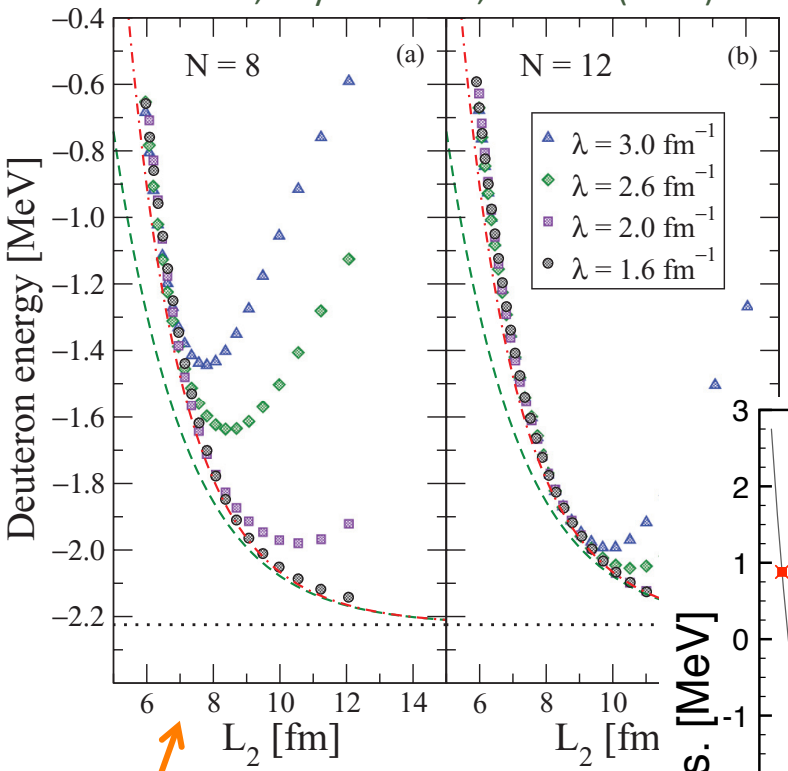


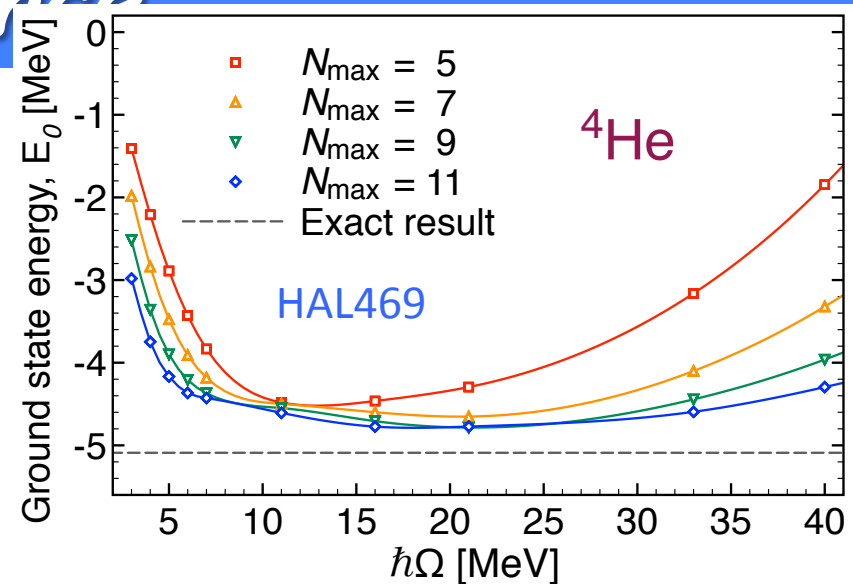
Deuteron g.s. Energy
EM(500) – N3LO two-nucleon force

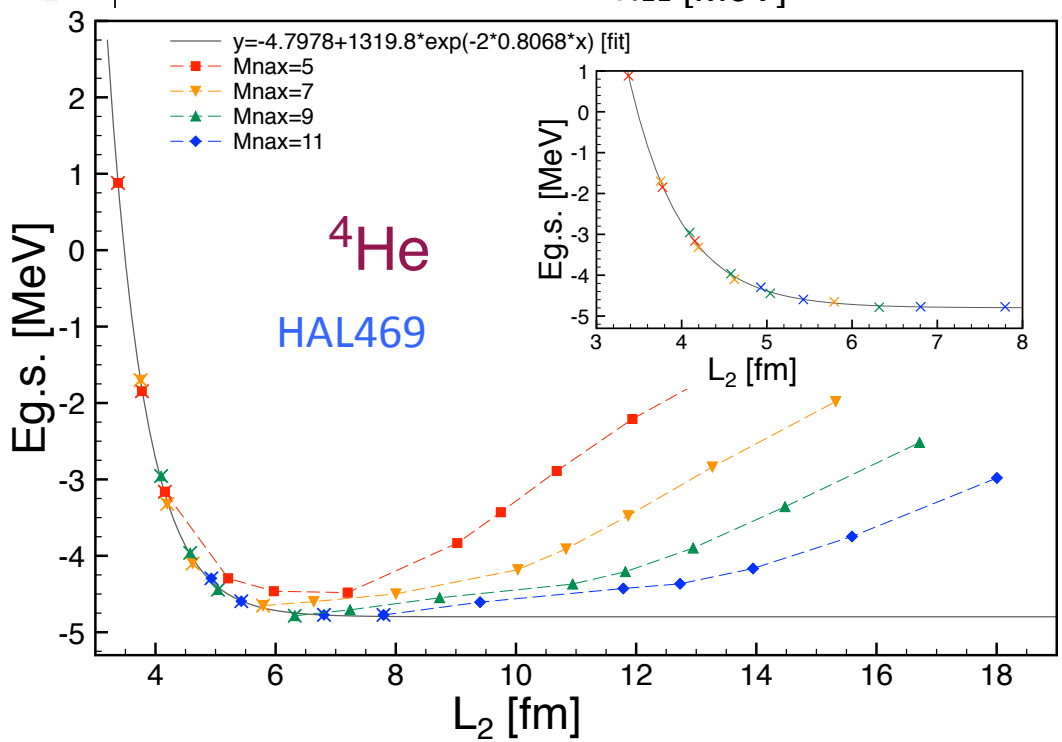
$$L_2 = \sqrt{2(N + 3/2 + 2)}b$$

Infrared convergence

Moore et al., Phy. Rev. C**87**, 044326 (2013)

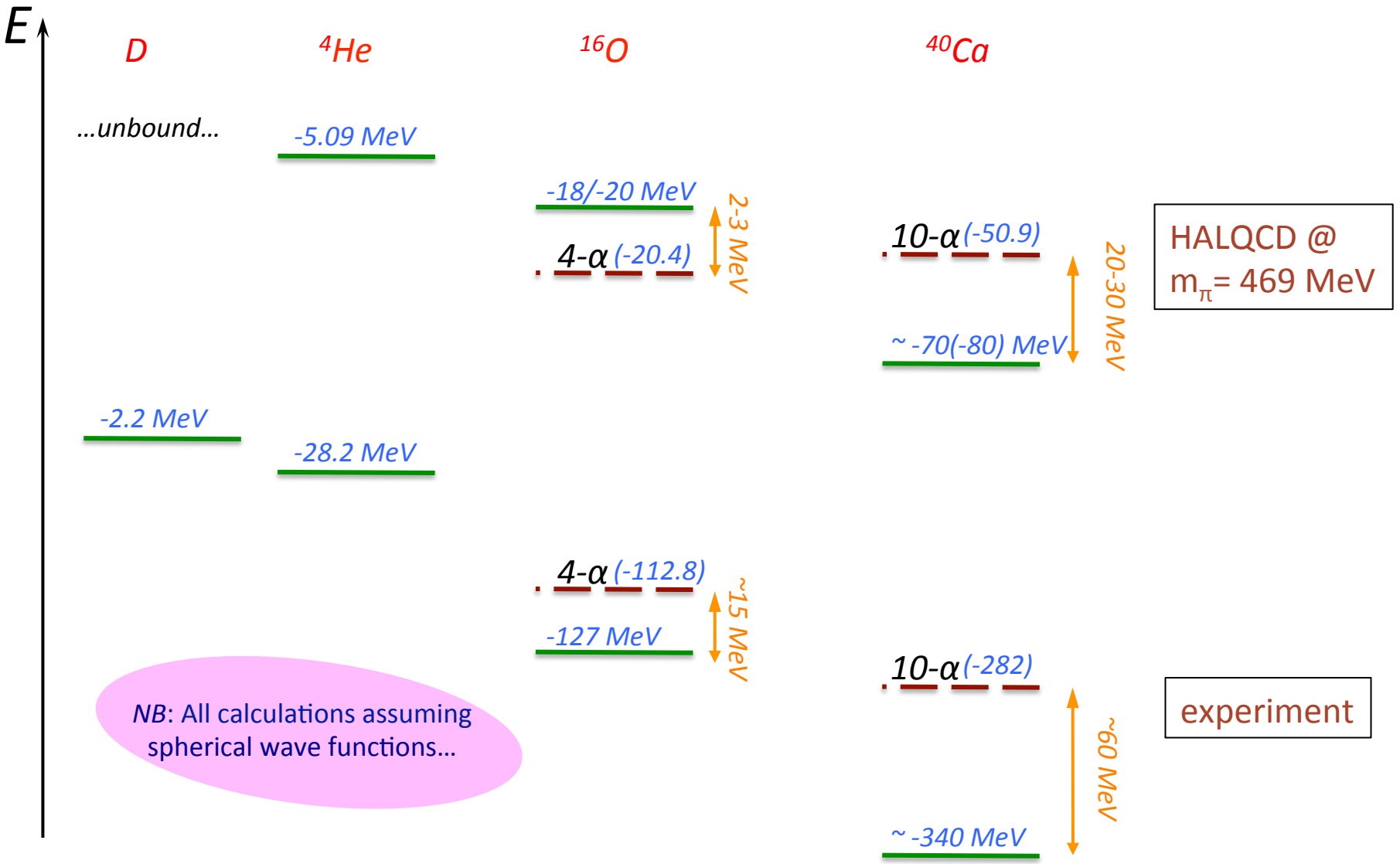






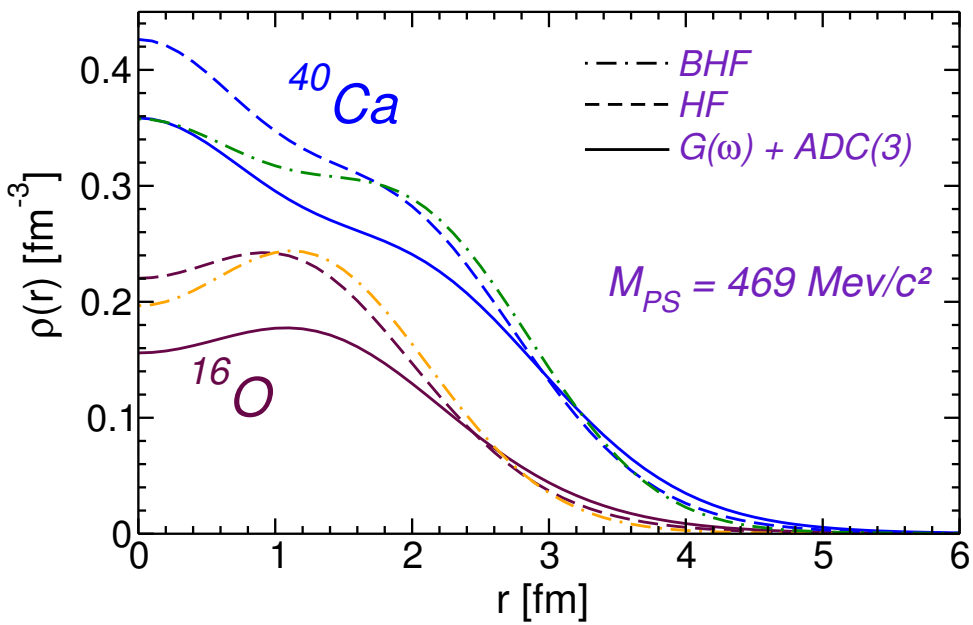
Deuteron g.s. Energy
EM(500) – N3LO two-nuc

Results for binding



C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]

Matter distribution of ^{16}O and ^{40}Ca:



Calculated matter radii at $m_\pi \approx 470$ MeV:

		^{16}O	^{40}Ca
$r_{pt-matter}$:	BHF [22]	2.35 fm	2.78 fm
	HF	2.39 fm	2.78 fm
	G(ω) + ADC(3)	2.64 fm	2.97 fm
r_{charge}:	G(ω) + ADC(3)	2.77 fm	3.08 fm
	Experiment [54, 55]	2.73 fm	3.48 fm

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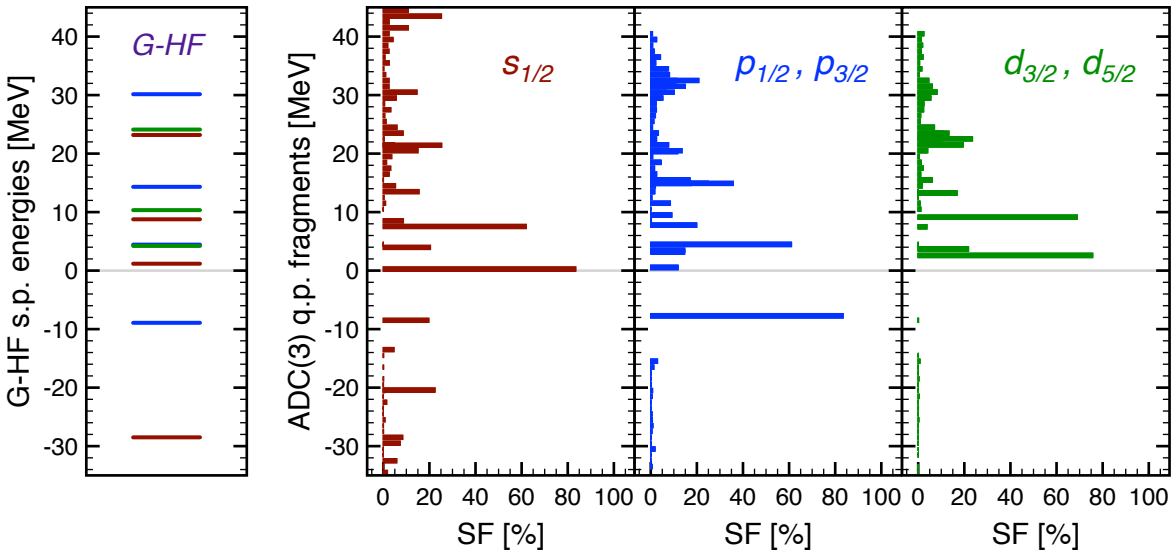
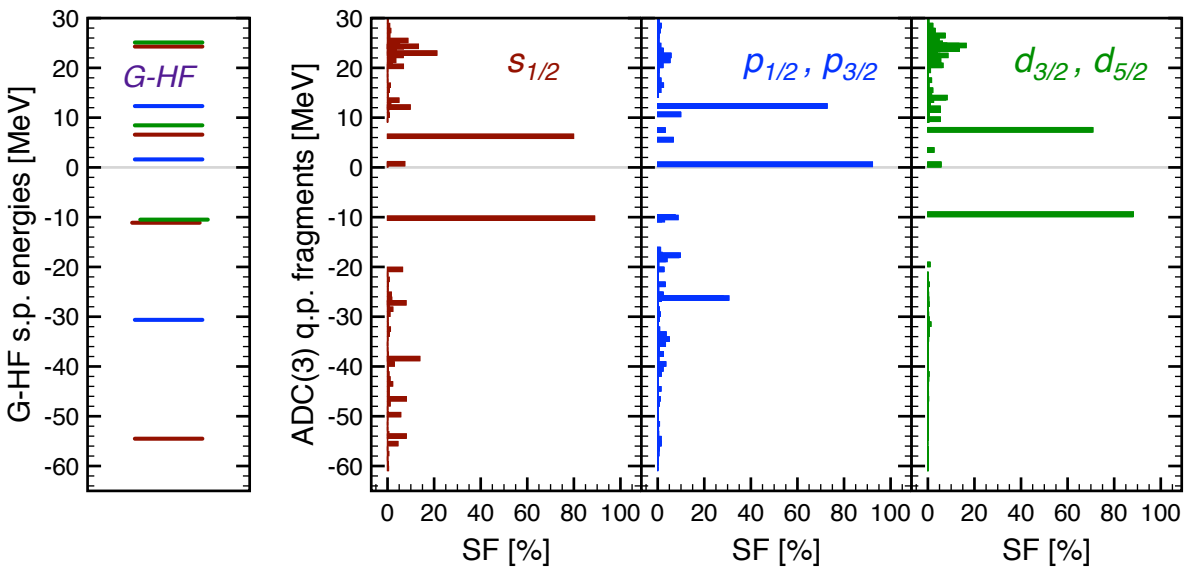
C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]

Spectral strength in ^{16}O and ^{40}Ca:





Particle-hole gaps:

^{16}O

m_π= 469 MeV: ~8 MeV

Expt (phys m_π): 11.5 MeV

^{40}Ca

m_π= 469 MeV: ~10 MeV

Expt (phys m_π): 7.5 MeV

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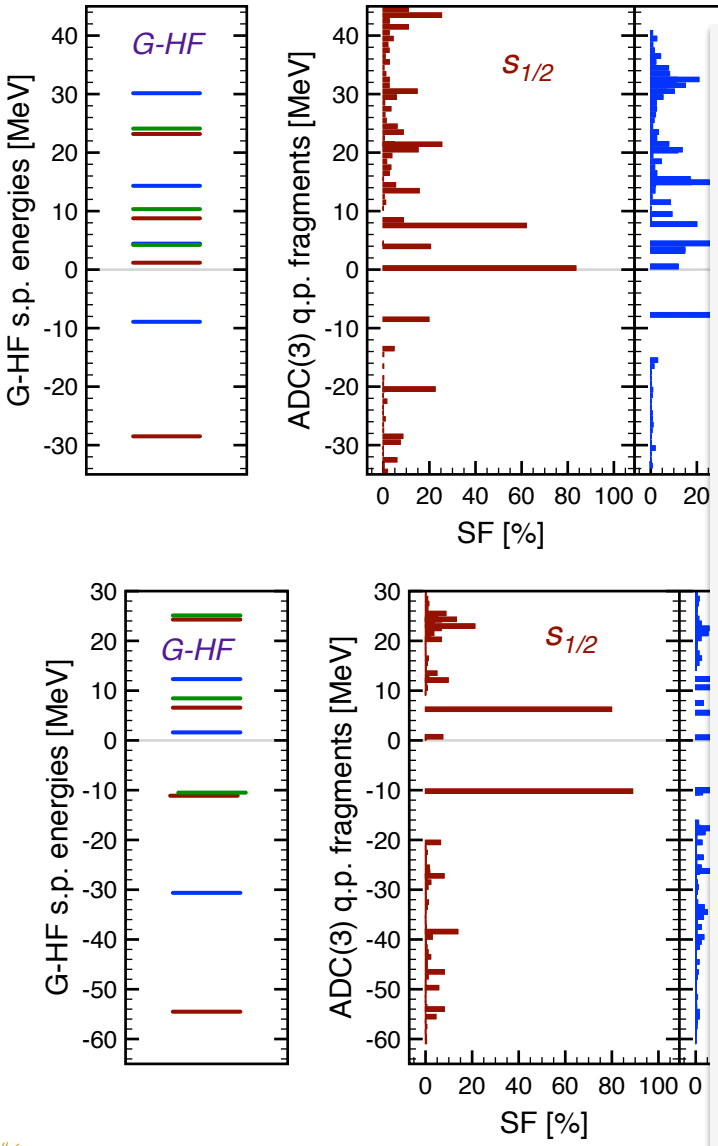
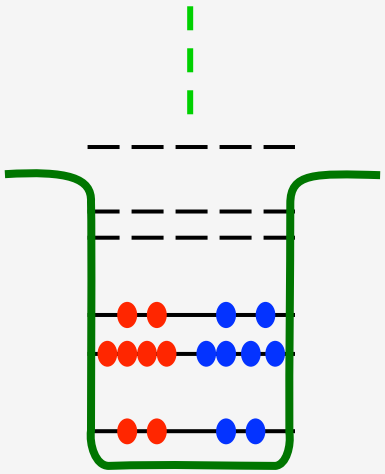
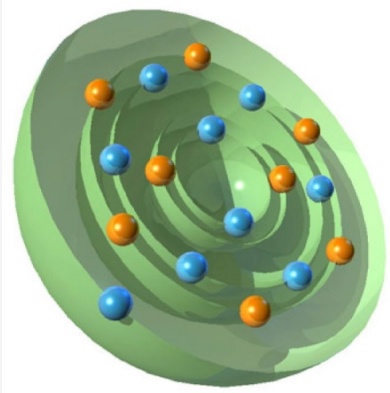
C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]

Spectral strength in ^{16}O and ^{40}Ca:





independent particle picture?

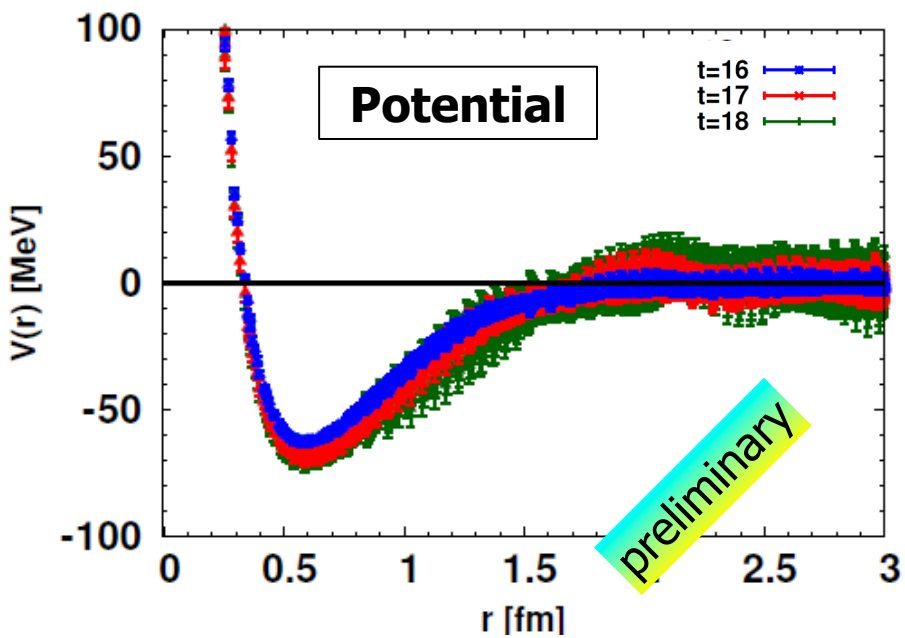


C. McIlroy, CB, et al., arXiv:1701.02607 [nucl-th]

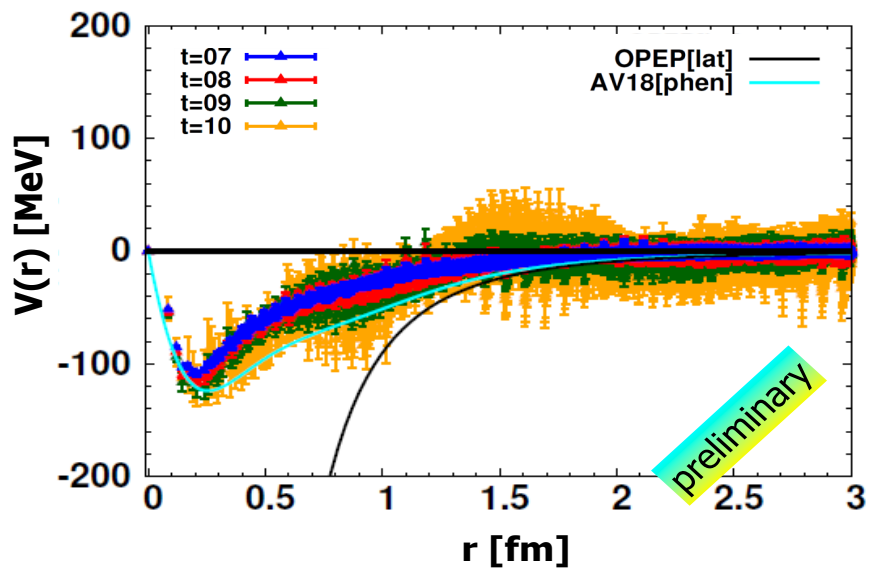
Future application for Ys in nuclei now possible

- Physical mass now under reach ($m_\pi \approx$ 145 MeV) for hyperons
- Need to improve on statistic for the NN sector

$\Omega\Omega$ potential



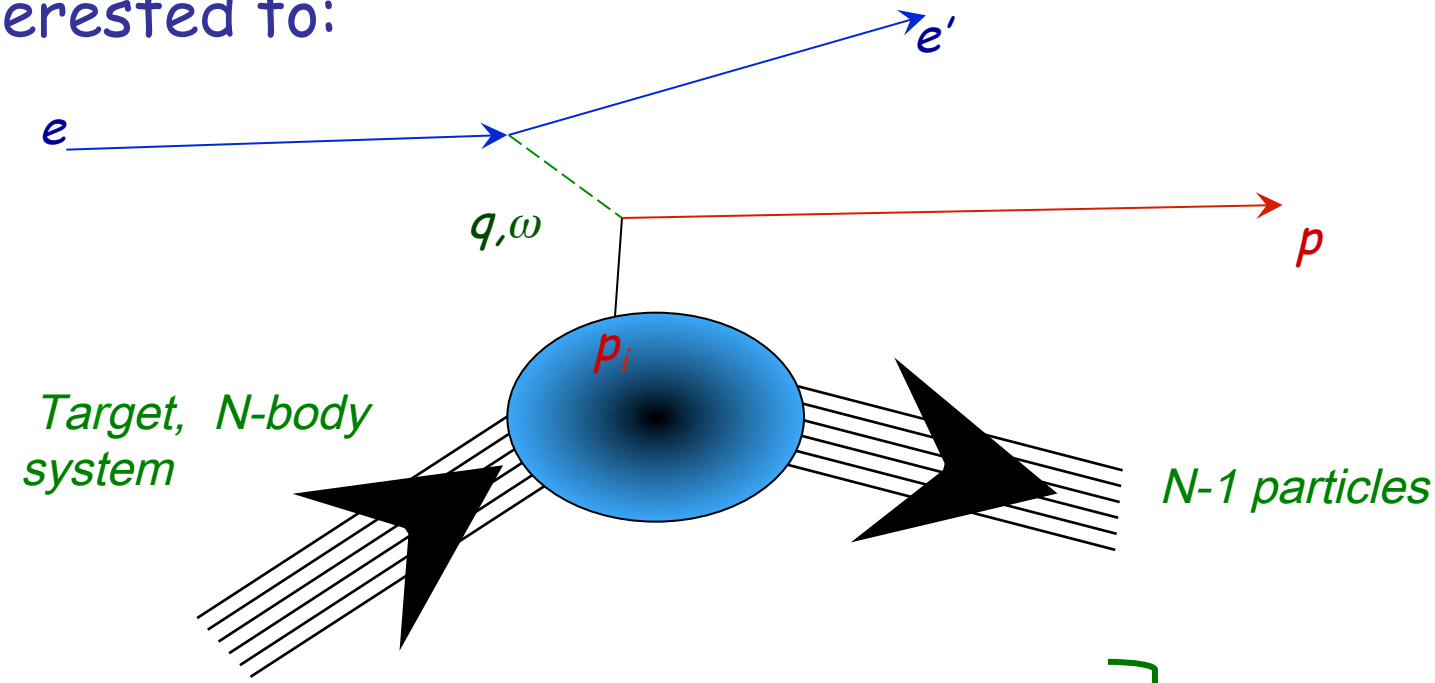
$NN(^3S_1)$ tensor potential



HALQCD coll. -- Talk of ***S. Aoki*** at Kavli institute, Oct. 2016

Spectroscopy via knock out reactions-basic idea

Use a probe (ANY probe) to eject the particle we are interested to:



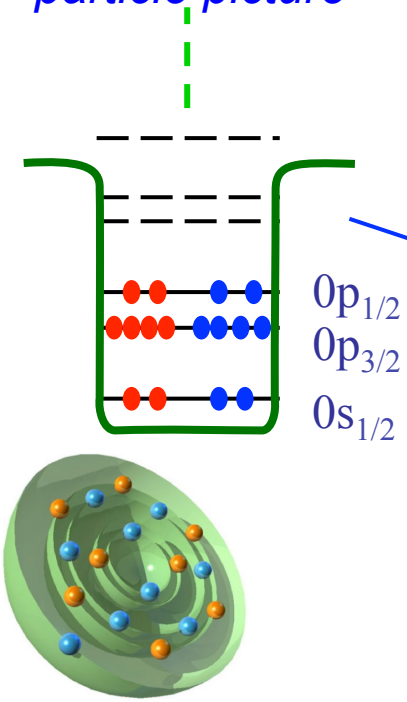
Basic idea:

- we know, e, e' and p
- "get" *energy and momentum* of p_i: $p_i = k_e' + k_p - k_e$
  $E_i = E_e' + E_p - E_e$

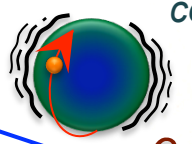
Better to choose large transferred momentum and weak probes!!!

Concept of correlations

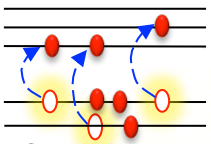
independent particle picture



Particle-vibration coupling (PV)



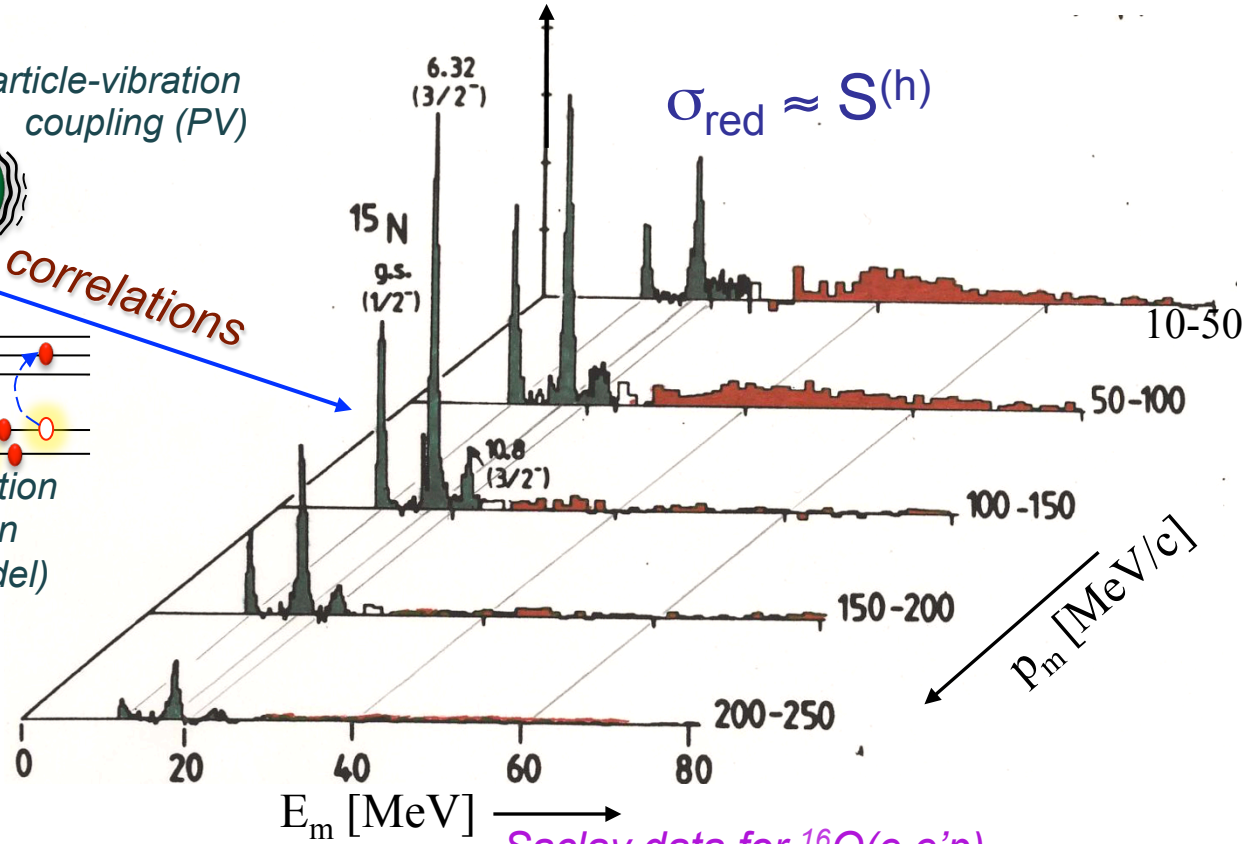
correlations



Configuration interaction (shell model)

Spectral function: distribution of momentum (p_m) and energies (E_m)

$\sigma_{red} \approx S^{(h)}$



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

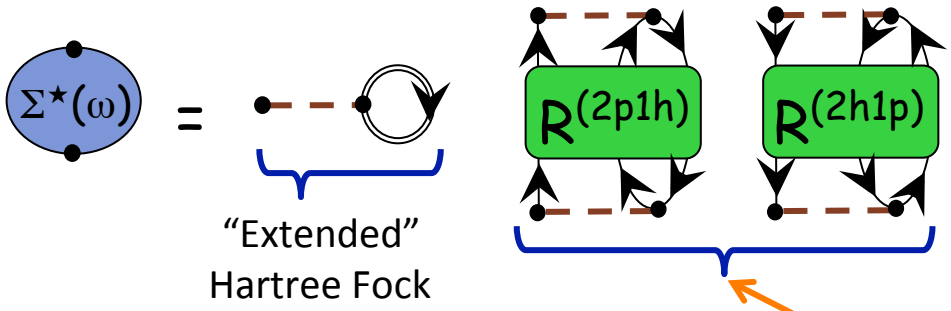
Understood for a few stable closed shells:

*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface…

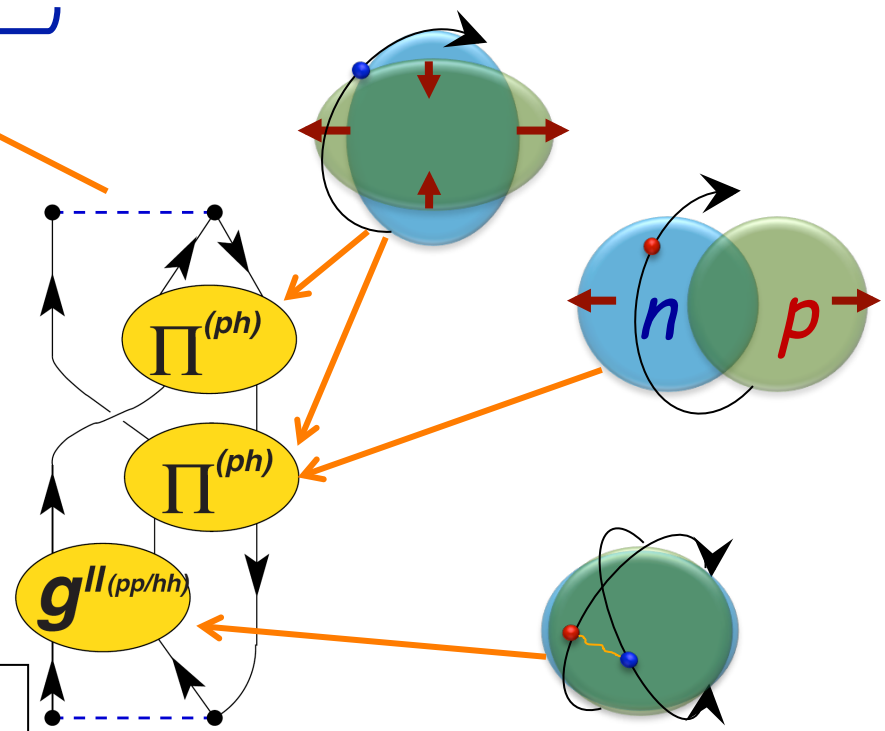
CB et al.,
*Phys. Rev. C**63**, 034313 (2001)*
*Phys. Rev. A**76**, 052503 (2007)*
*Phys. Rev. C**79**, 064313 (2009)*



- A complete expansion requires *all types* of particle-vibration coupling

  …these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

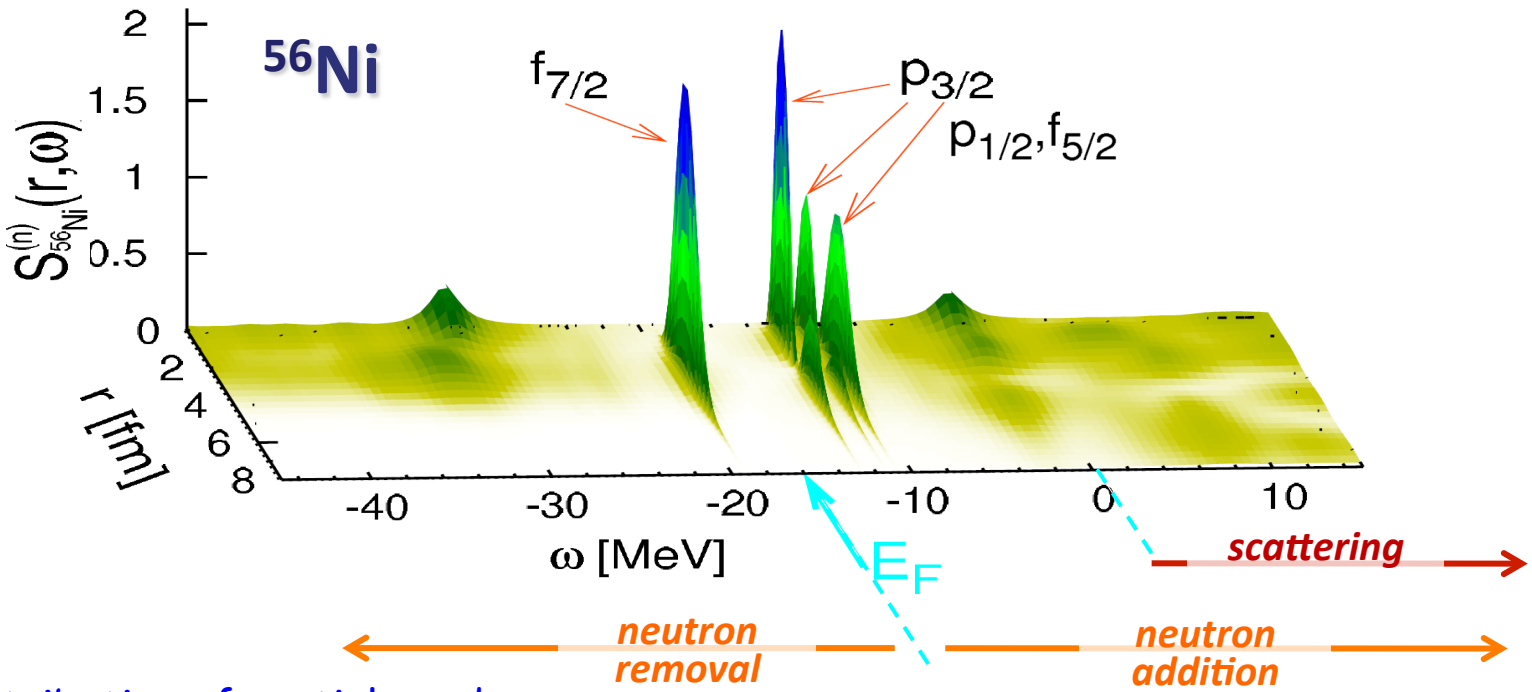
- The Self-energy $\Sigma^{\star}(\omega)$ yields *both* single-particle states and scattering



≡ particle ≡ hole

One-nucleon spectral function

$$S^{p,h}(r,\omega) = \mp\frac{1}{\pi}\,\mathrm{Im}\,g(r=r';\omega)$$

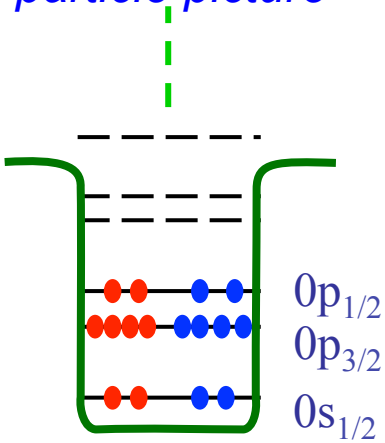


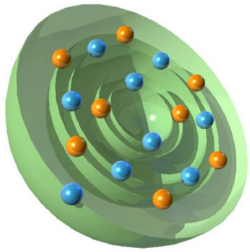
Distribution of particle and hole neutron states in ^{56}Ni

W. Dickhoff, CB, Prog. Part. Nucl. Phys. 53, 377 (2004)
CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009)

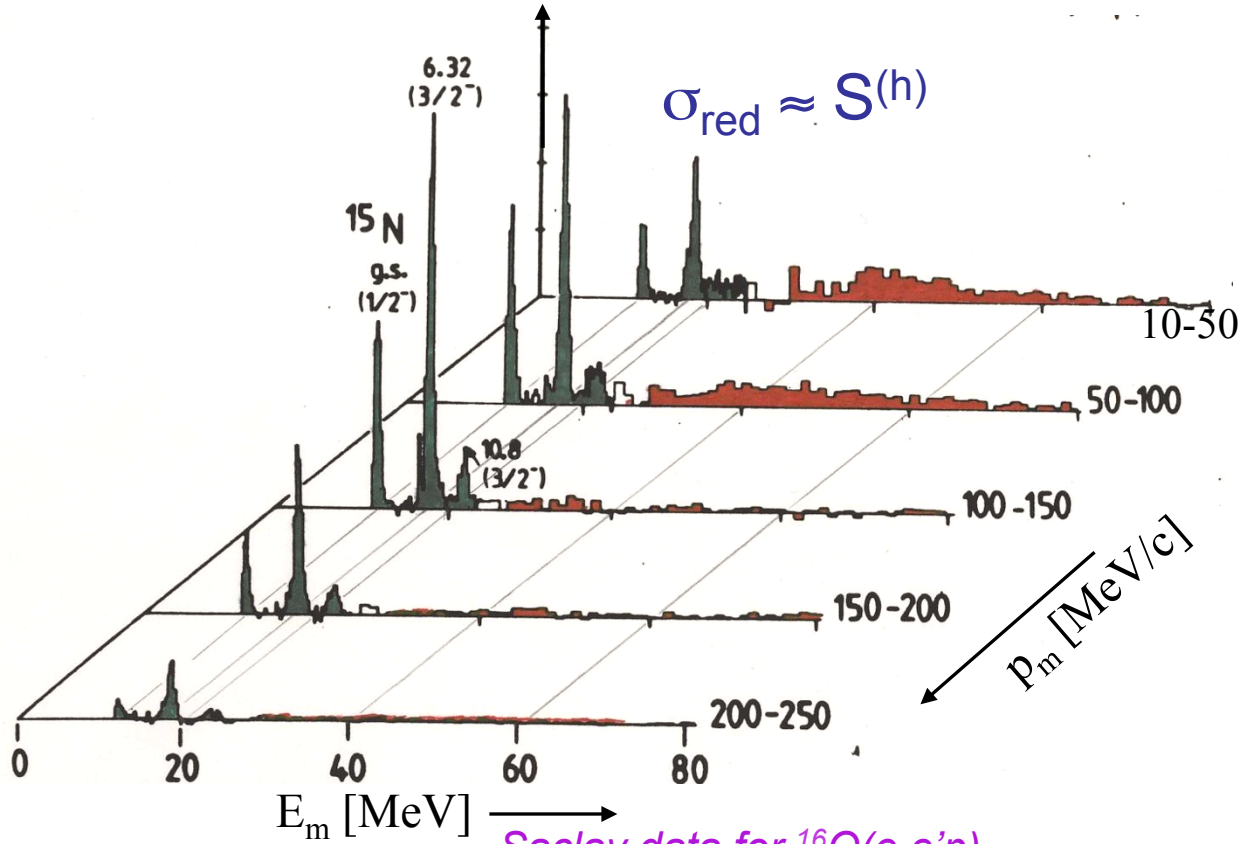
Concept of correlations

independent particle picture





Spectral function: distribution of momentum (p_m) and energies (E_m)



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

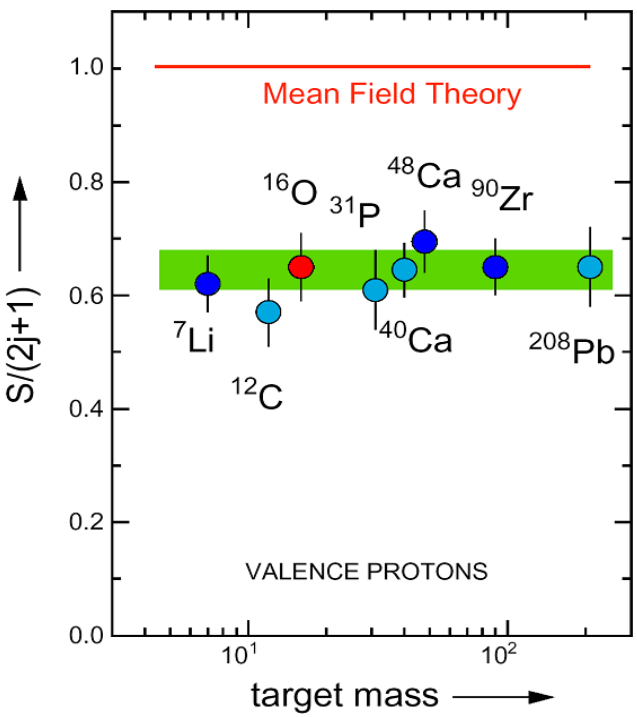
Understood for a few stable closed shells:

*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

Quenching of SF in stable nuclei

Nucl. Phys. A553 (1993) 297c

NIKHEF:



A common misconception about SRC:

"The quenching is constant over all stable nuclei, so it must be a short-range effect"

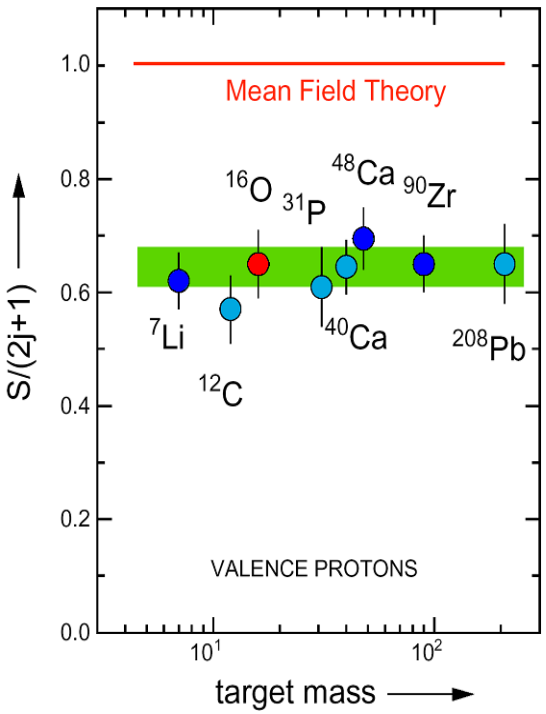
Actually, NO!

All calculations show that SRC have just a small effect at the Fermi surface. And the correlation to the experimental p-h gap is much more important.

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 *(2004)*]

Quenching of SF in stable nuclei

NIKHEF:
Nucl. Phys. A553 (1993) 297c



	$S_{p1/2}$	$S_{p3/2}$
• Short-range correlations oriented methods:		
– VMC [Argonne, '94]	0.90	
– GF(SRC) [St.Louis-Tübingen '95]	0.91	0.89
– FHNC/SOC [Pisa '00]	0.90	
• Including particle-phonon couplings:		
– GF(FRPA) [St.Louis '01]	0.77	0.72
[CB et al., Phys. Rev. C**65**, (02)]		
• Experiment:	0.63	0.67 ±0.07 (estimated uncertainty)

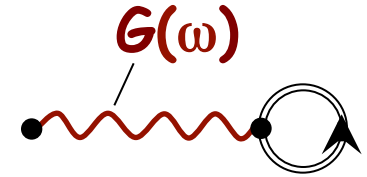
SRC are present and verified experimentally

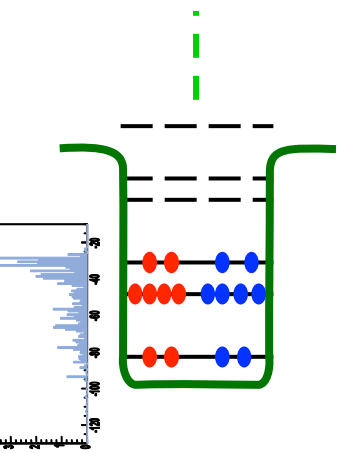
BUT the are NOT the dominant mechanism for quenching SF!!!

Treating short-range corr. with a G-matrix

- The short-range core can be treated by summing ladders outside the model space:

$$\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega) = i\sum_{\gamma\delta}\int\frac{d\omega'}{2\pi}G_{\alpha\gamma,\delta\beta}(\omega+\omega')g_{\delta\gamma}(\omega') \quad =$$





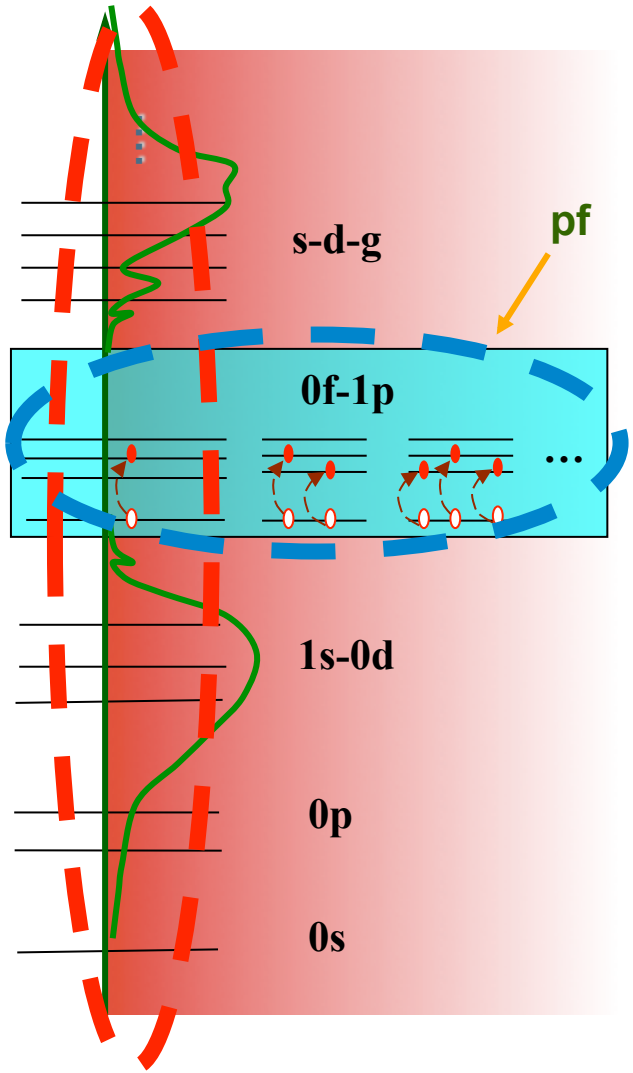
$$\Sigma^{\star}(\mathbf{r},\mathbf{r}';\omega) = \Sigma^{MF}(\mathbf{r},\mathbf{r}';\omega) + \tilde{\Sigma}(\mathbf{r},\mathbf{r}';\omega)\ .$$

$$Z_\alpha = \int d\mathbf{r}\,|\psi^{A\pm1}_\alpha(\mathbf{r})|^2 = \left.\frac{1}{1-\dfrac{\partial\Sigma^{\star}_{\hat\alpha\hat\alpha}(\omega)}{\partial\omega}}\right|_{\omega=\pm(E^{A\pm1}_\alpha-E^A_0)}$$

Two contributions to the derivative:

- $\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega)$ is due to scattering to (high-k) states in the Q space
- $\Sigma(\mathbf{r},\mathbf{r}';\omega)$ accounts for low-energy (long range) correlations

Quenching of absolute spectroscopic factors



Particle-vibration coupling dominates the quenching of spectroscopic factors

Relative strength among fragments requires shell-model approach

Quenching of absolute spectroscopic factors

[CB, *Phys. Rev. Lett.* **103**, 202520 (2009)]

Overall quenching of *spectroscopic factors* is driven by:

SRC → *~10%*

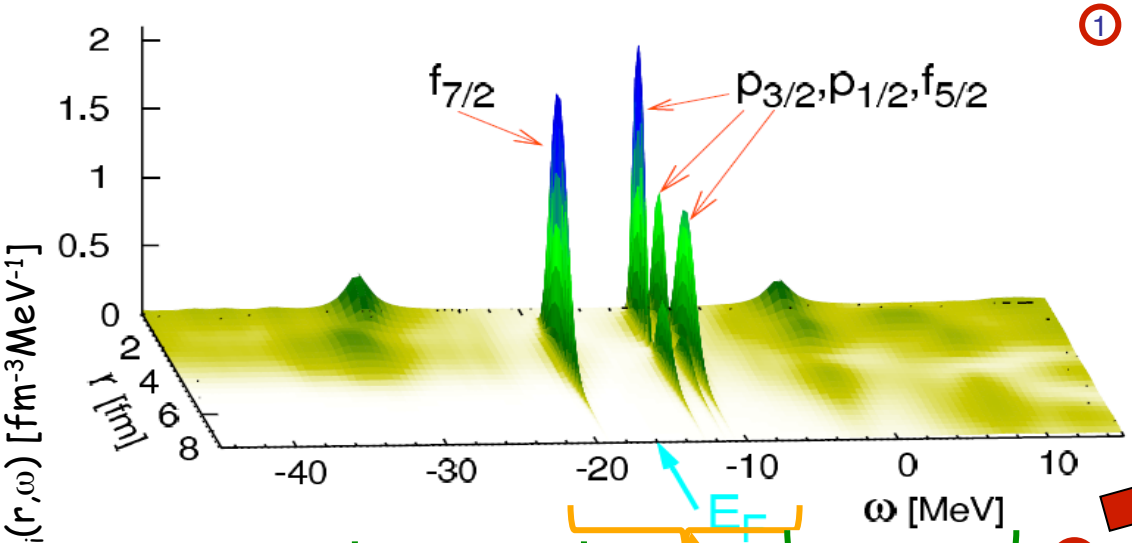
part-vibr. coupling → *dominant*

"shell-model" → *in open shell*

...with analogous conclusions for ^{48}Ca

		10 osc. shells			Exp. [30]	$1p0f$ space		
		FRPA (SRC)	full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
	^{57}Ni:							
^{57}Ni	$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
	$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
	$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
	^{55}Ni:							
^{55}Ni	$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
		1	2 + 3			3		

$$Z_\alpha = \int d^3r\, |\psi_\alpha^{overlap}(\mathbf{r})|^2 = \left. \frac{1}{1 - \frac{\partial \Sigma_{\hat{\alpha}\hat{\alpha}}(\omega)}{\partial \omega}} \right|_{\omega=\varepsilon_\alpha}$$



1 SHORT RANGE CORRELATIONS

3 SHELL MODEL

2 PARTICLE-VIBRATION COUPLING

Dependence of Spect. Fact. from p-h gap

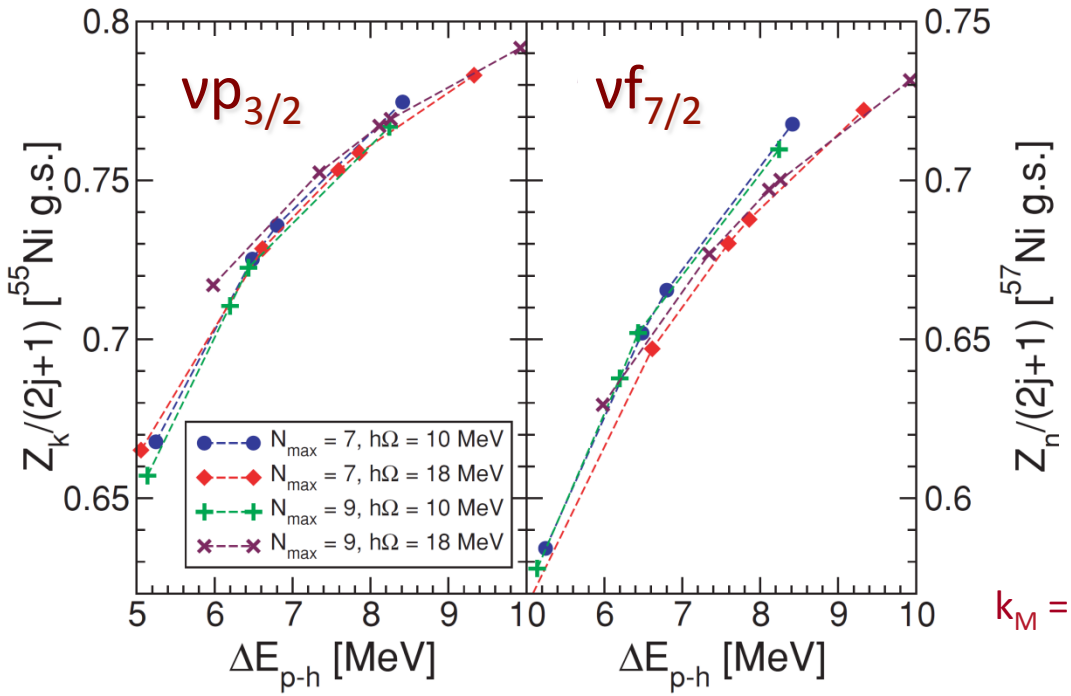
N3LO needs a monopole correction to fix the p-h gap:

$$\begin{cases} \Delta V^T_{fr} \rightarrow \Delta V^T_{fr} - (-1)^T \kappa_M, \\ \Delta V^T_{ff} \rightarrow \Delta V^T_{ff} - 1.5(1-T)\kappa_M, \end{cases}$$

$r \equiv p_{3/2}, p_{1/2}, f_{5/2}$

$f \equiv f_{7/2}$

Experimental Eph is found for k_M =0,57



k_M = 0.4-0.7 MeV

small k_M ← → large k_M

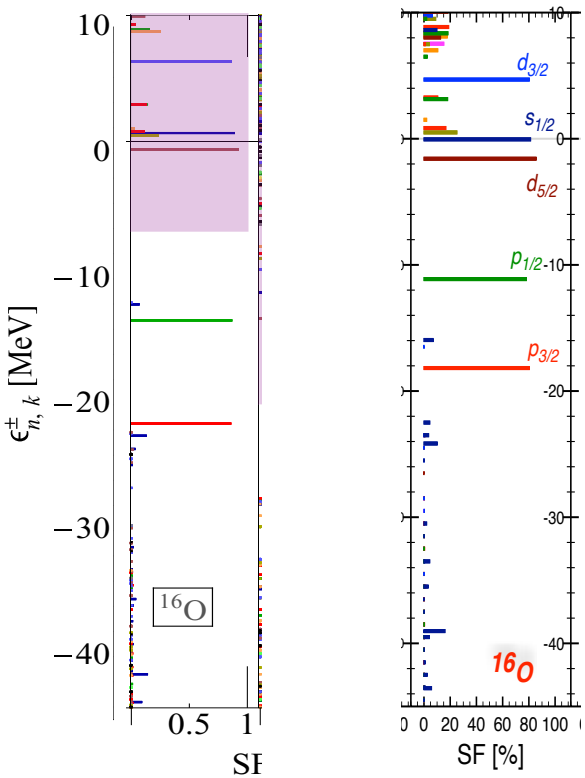
N3LO interaction + monopole corr.

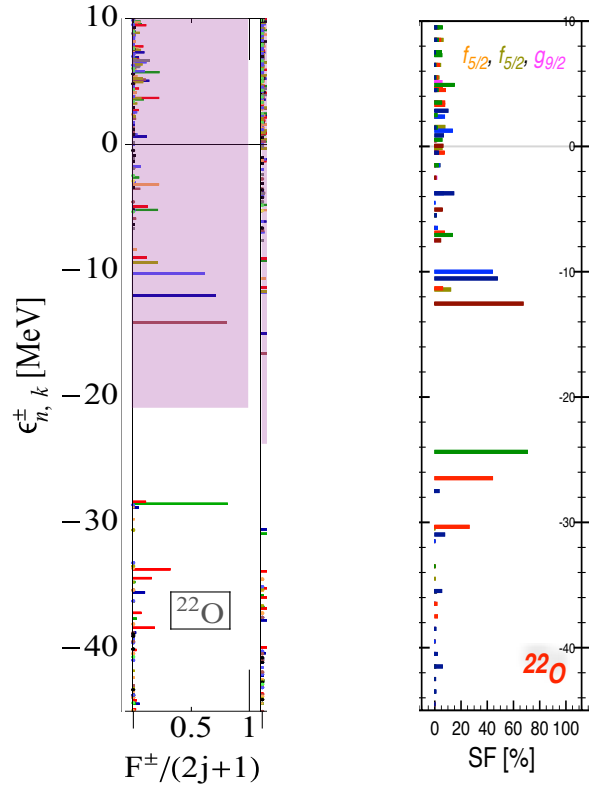
[CB, M.Hjorth-Jensen, Pys.Rev.C79, 064313 (2009)]

Proton spectral strength in Oxygen

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)
and ***in preparation***

More in detail:

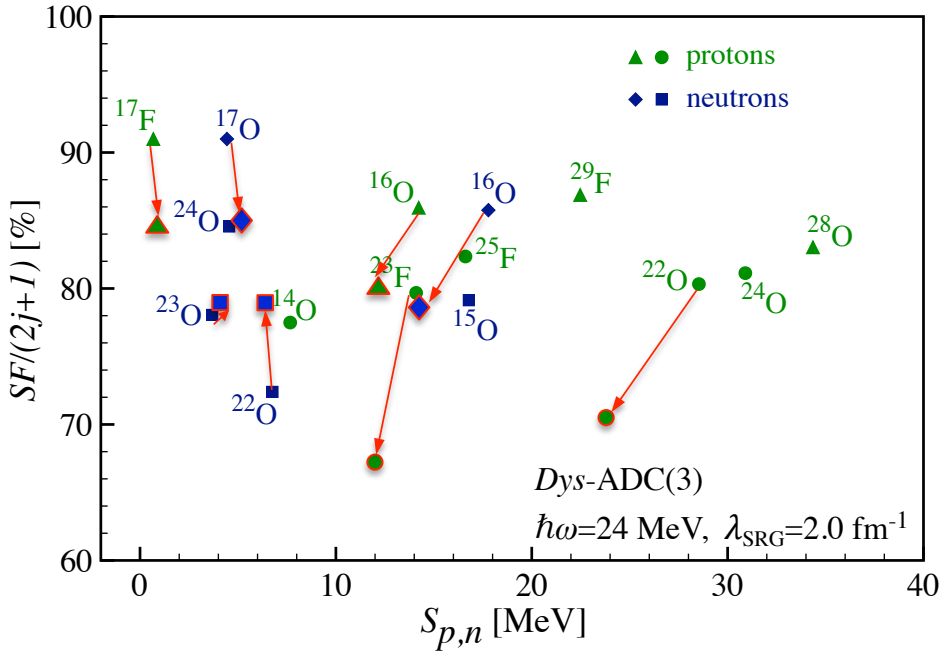




Z/N asymmetry dependence of SFs - Theory

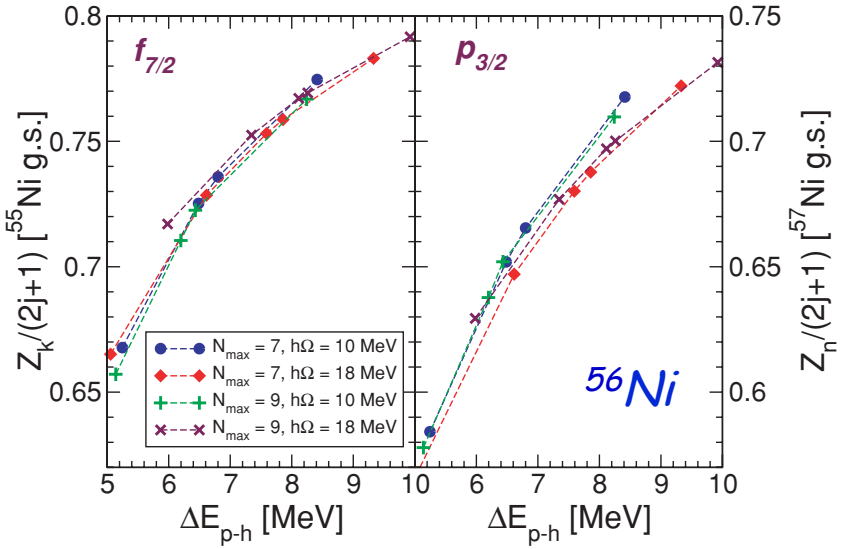
Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenching is high correlated to the gap at the Femi surface.



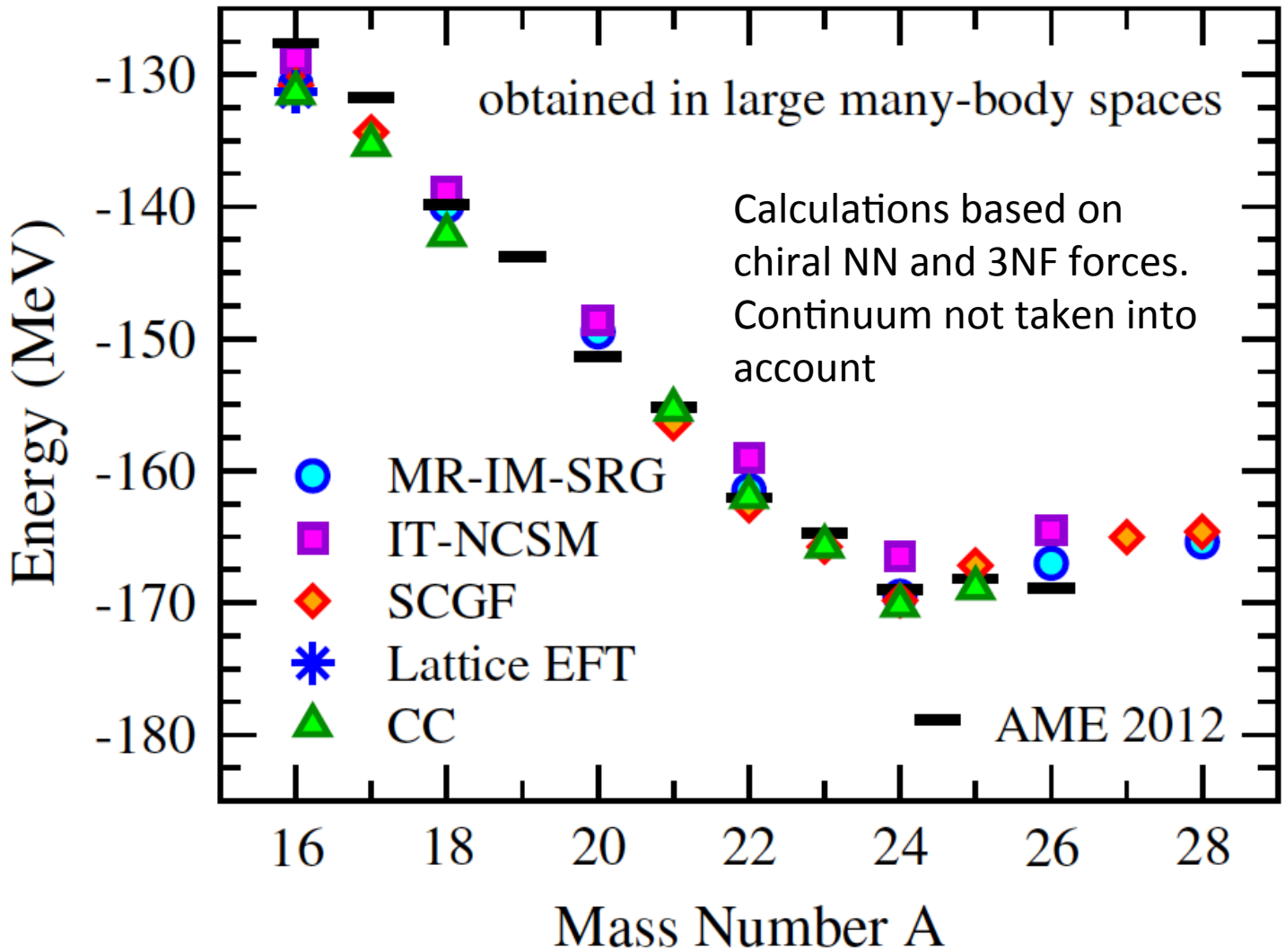
A. Cipollone, CB, P Navrátil
Phys. Rev. C**92**, 014306 (2015)

Spectroscopic factor are strongly correlated to p-h gaps:



CB, M. Hjorth-Jensen,
Phys. Rev. C **79**, 064313 (2009)

Benchmark of ab-initio methods in the oxygen isotopic chain



Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. in press (2015)

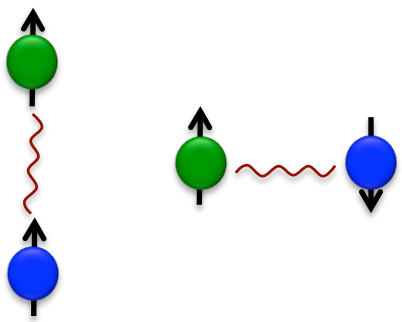
Nuclear forces in exotic nuclei

Nucleon interactions are very complex and difficult to handle…

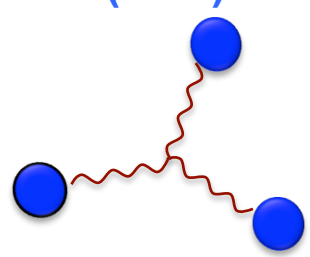
Symmetric matter:
N ≈ Z

Neutron-rich matter (N ≫ Z):
- **Neutron star matter EoS**
- **Symmetry energy**
- **new shell closures**

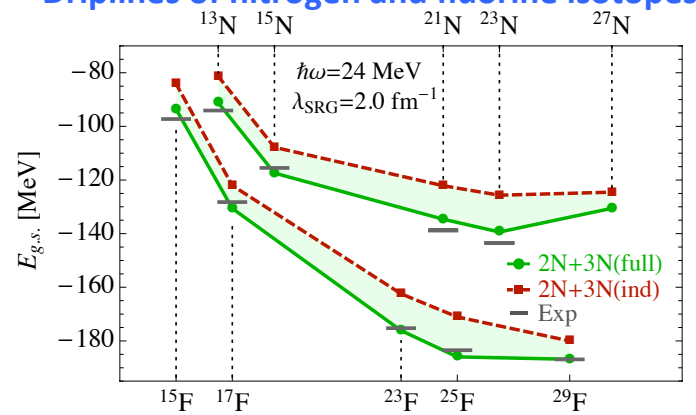
Tensor force (p-n)



Three-nucleon Force (3NF)



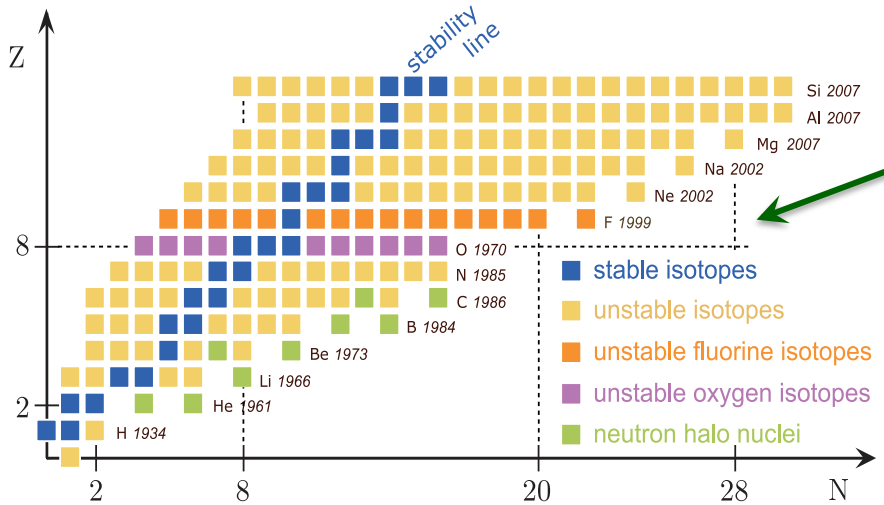
Driplines of nitrogen and fluorine isotopes



[A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)]

Change of regime from stable to dripline isotopes !

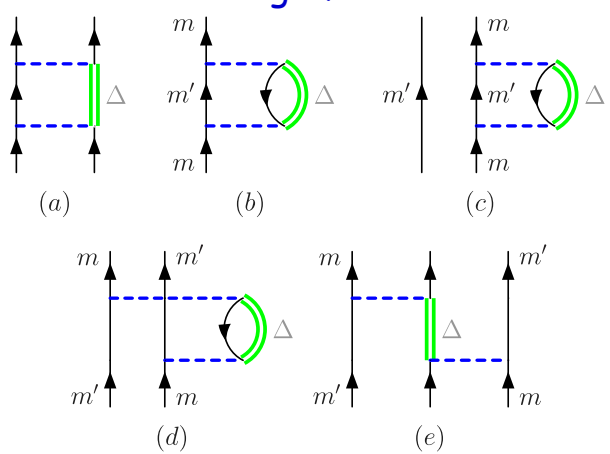
Oxygen puzzle…

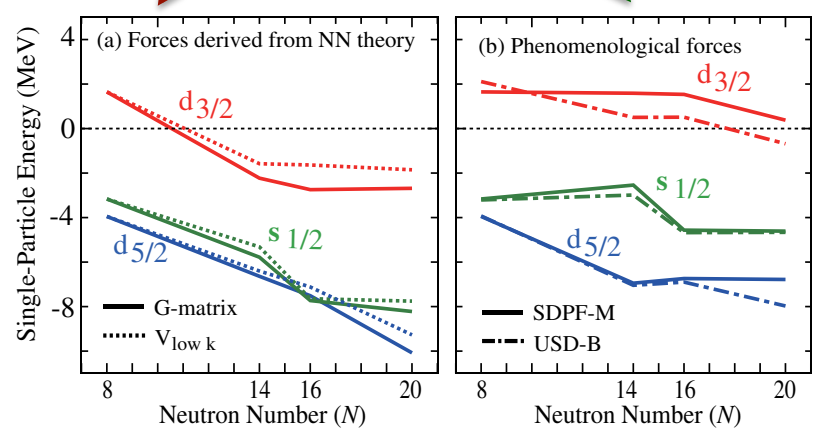


The oxygen dripline is at ^{24}O, at odds with other neighbor isotope chains.

Phenomenological shell model interaction reflect this in the s.p. energies but no realistic NN interaction alone is capable of reproducing this…

The fujita-Miyazawa 3NF provides repulsion through Pauli screening of other 2NF terms:

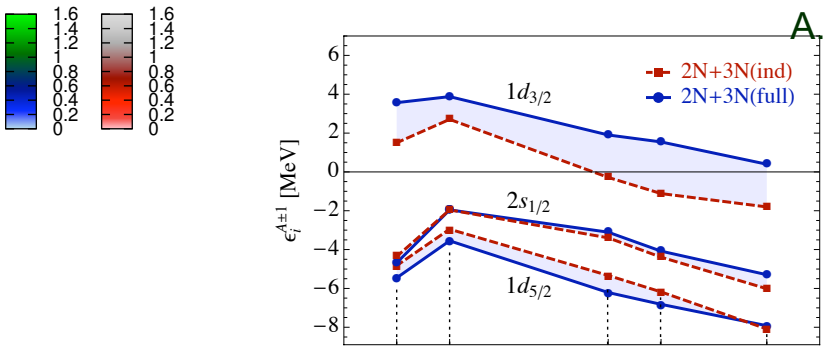




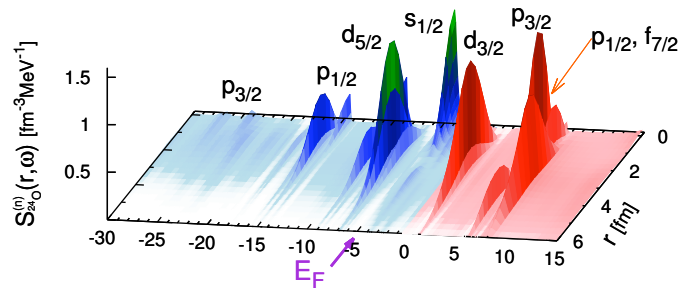
[T. Otsuka et al., Phys Rev. Lett **105**, 32501 (2010)]

Neutron spectral function of Oxygens

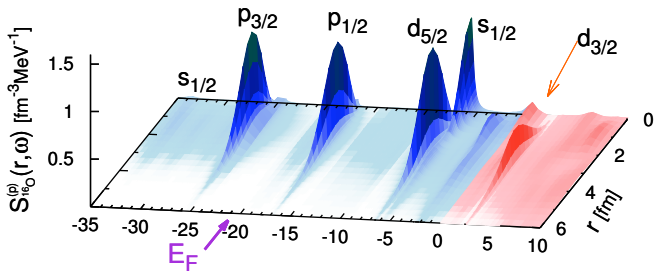
A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* **92**, 014306 (2015)



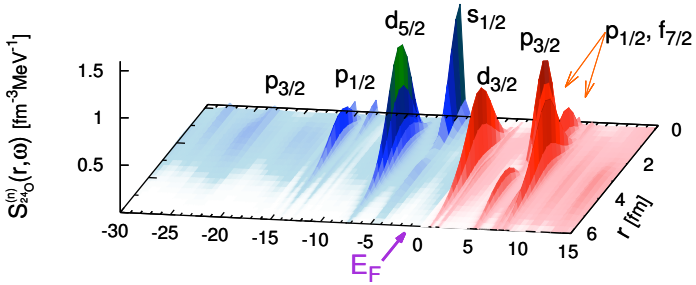
^{22}O:



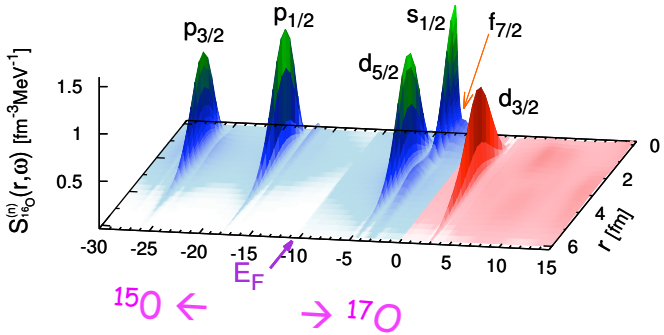
^{14}O:



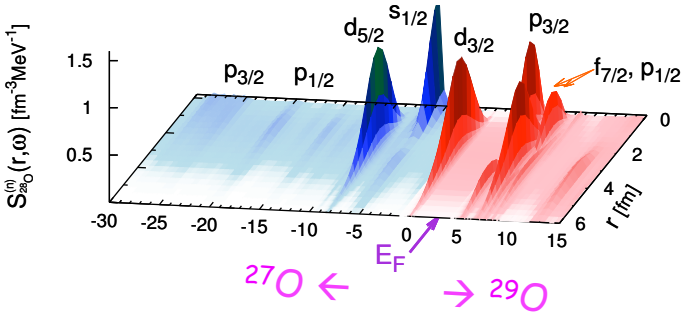
^{24}O:



^{16}O:

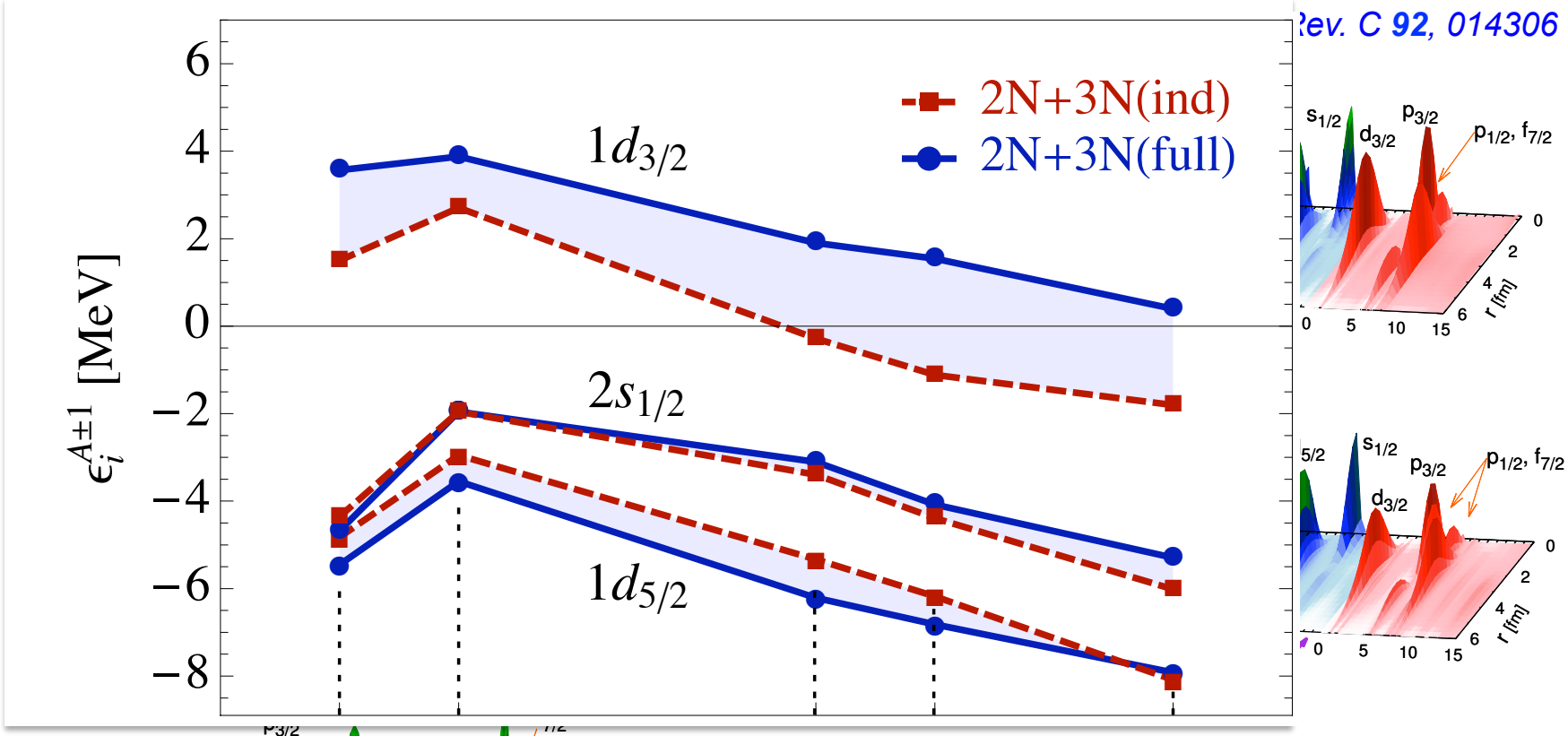


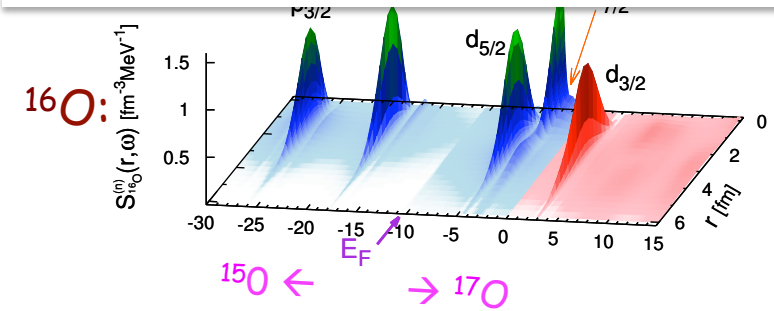
^{28}O:

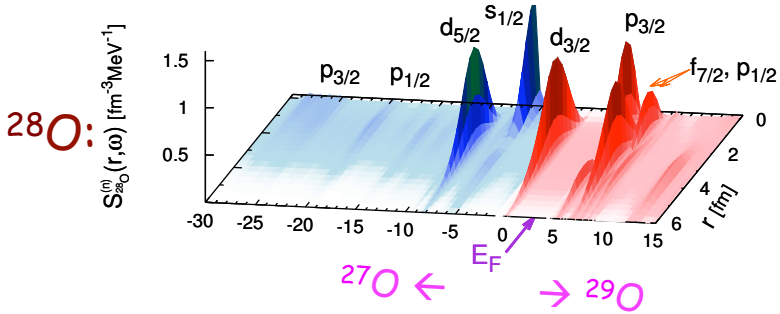


Neutron spectral function of Oxygens

Rev. C **92**, 014306 (2015)



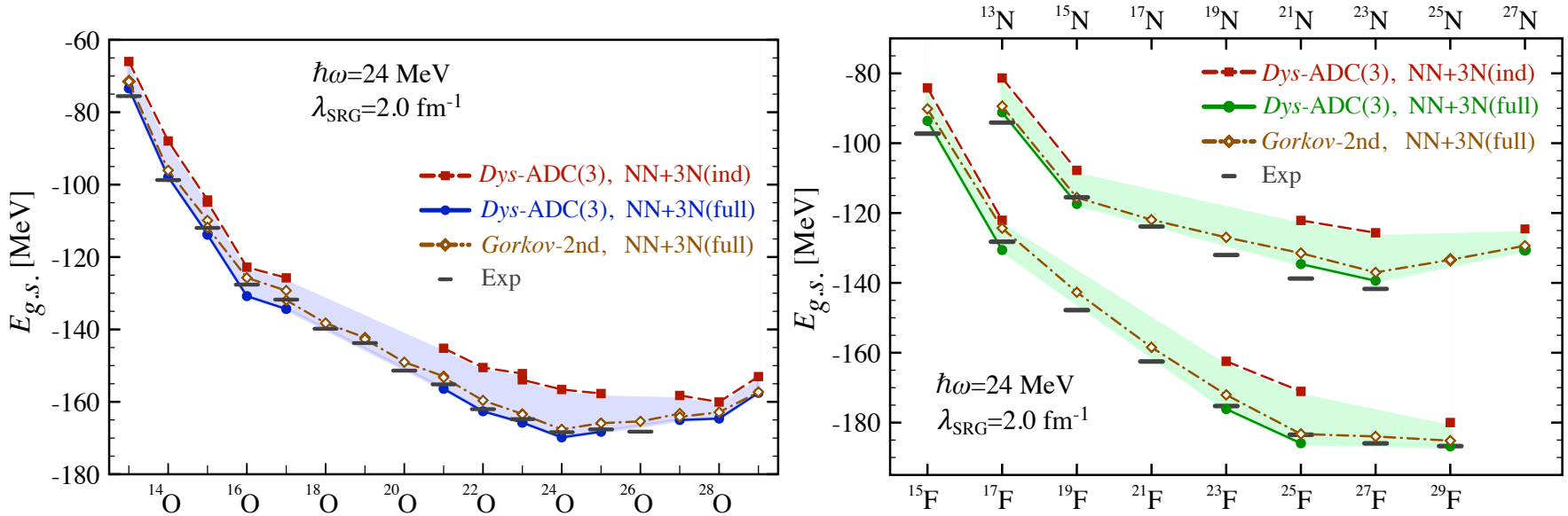




UNIVERSITY OF SURREY

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)



→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

PRL 117, 052501 (2016) PHYSICAL REVIEW LETTERS week ending 29 JULY 2016

Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,[1,*] V. Somà,[1] C. Barbieri,[2] H. Hergert,[3] J. D. Holt,[4] and S. R. Stroberg[4]

- New fits of chiral interactions (NNLOsat) highly improve comparison to data

- Deficiencies remain for neutron rich isotopes

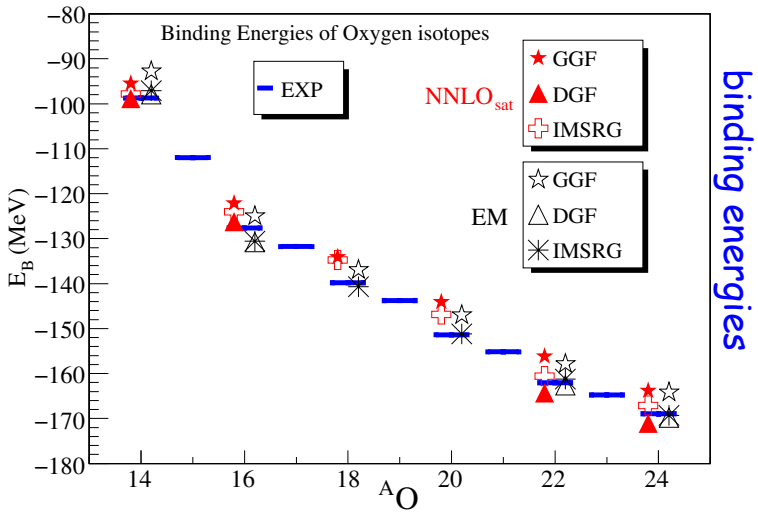
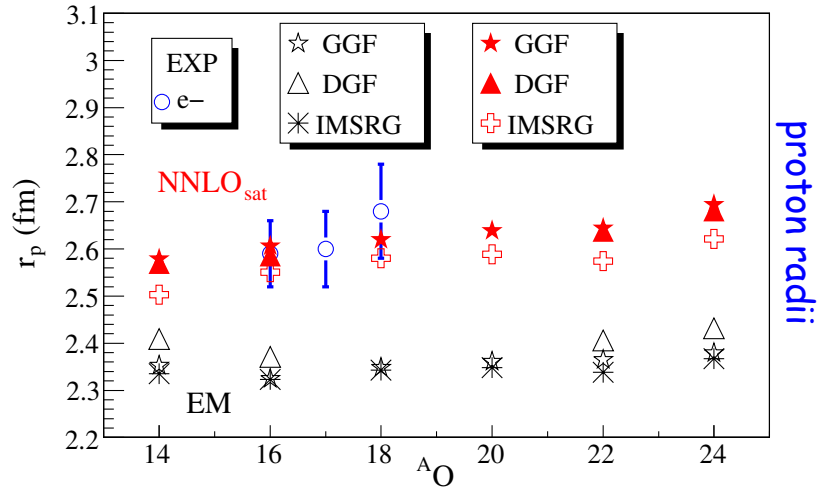
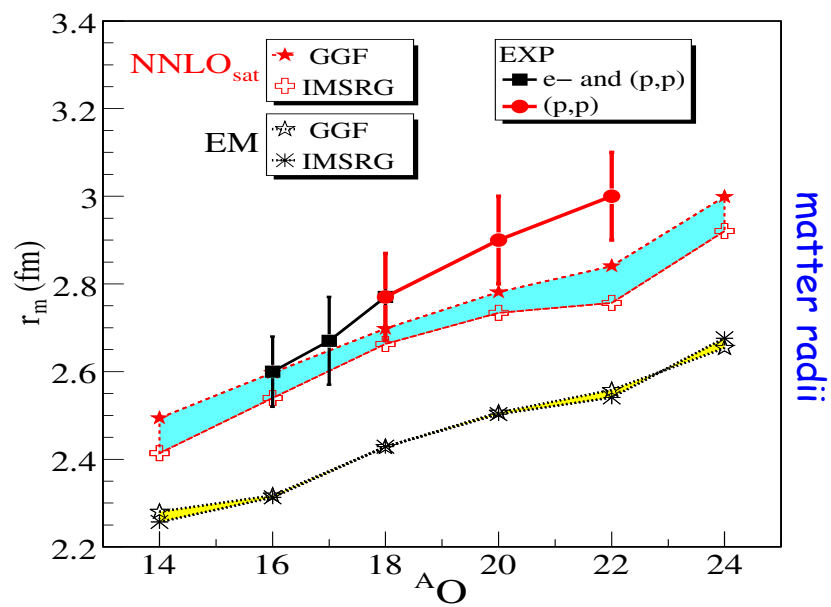


FIG. 1. Oxygen binding energies. Results from SCGF and IMSRG calculations performed with EM [20–22] and NNLO$_{sat}$ [26] interactions are displayed along with available experimental data.





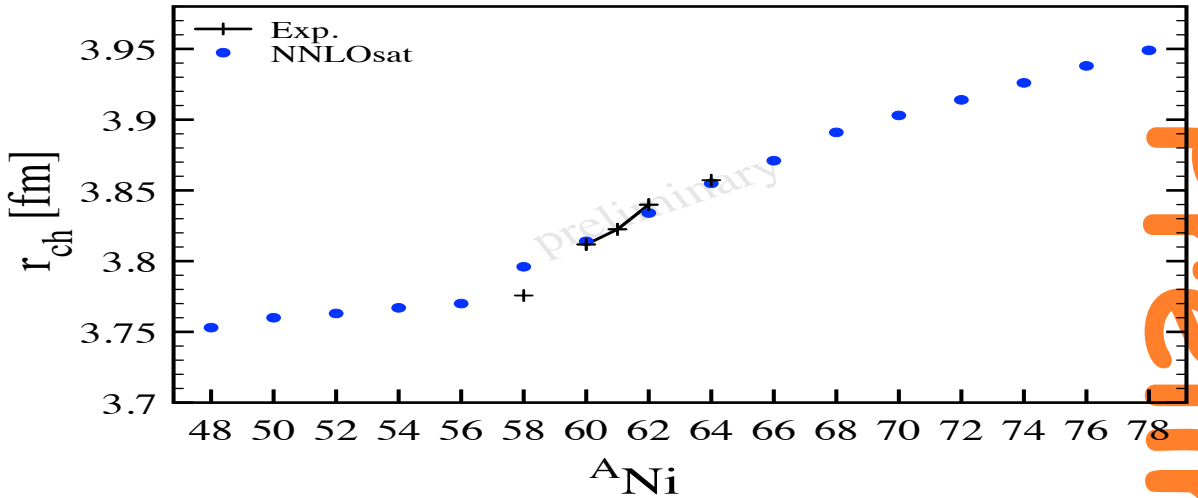
charge radii in the pf shell

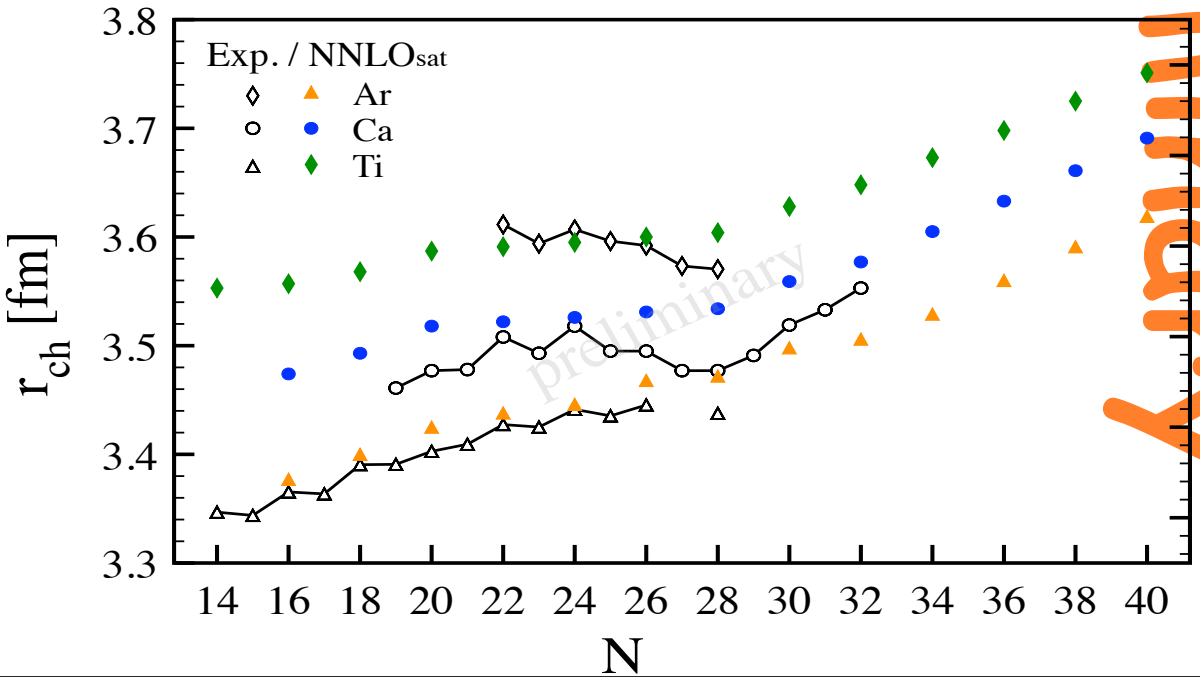
Size of radii not prefect but remains overall correct throughout the ***pf*** shell with NNLO-sat.

This suggests that saturation is indeed under control.

➔ Improvements of many-body truncations beyond 2nd order Gorkov will also be relevant.

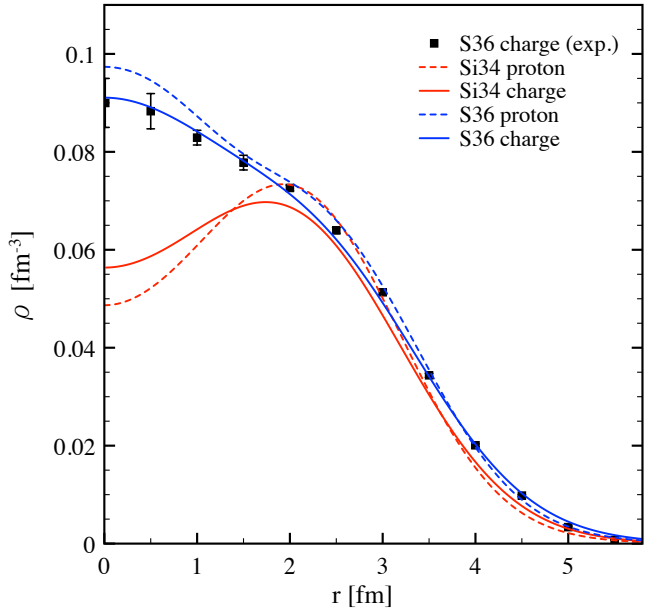
(work in progress!)





Preliminary

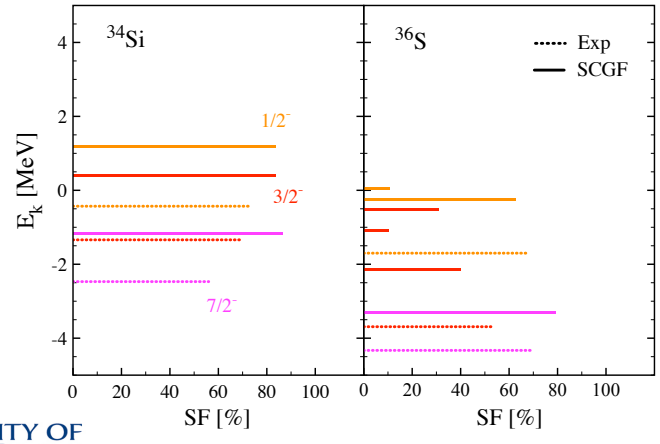
Bubble nuclei... ^{34}Si prediction

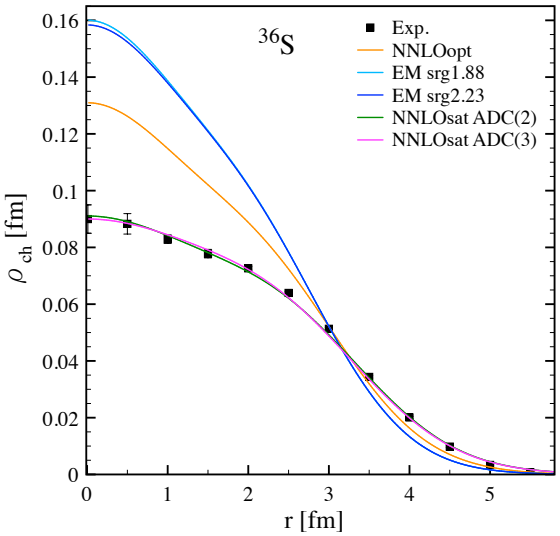


Duguet, Somà, CB, et al. arXiv:1611.08570 [nucl-th]

- ^{34}Si is unstable, charge distribution still unknown
- Suggested central depletion from mean-field simulations
- *Ab-initio* theory confirms predictions

Validated by charge distributions and neutron quasiparticle spectra:





Some considerations:

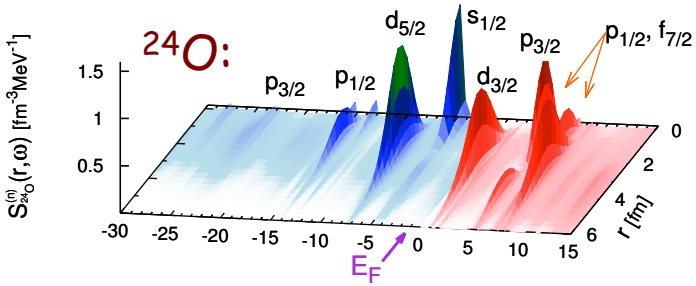
Thank you for your attention!!!

→ Emergence of shell effects in trivially linked to the size of a nuclear well
→ Nuclear radii are the most important constraint

→ However:
- actual "shell structure" is highly linked to details of the nuclear force
- 3NF and spin-orbit change shell ordering and limit of stability

→ Phenomenological understanding of atomic nuclei seem (so far) well explained by underlying theories

→ Deeper understanding of macroscopic dynamics better understood from larger scales??



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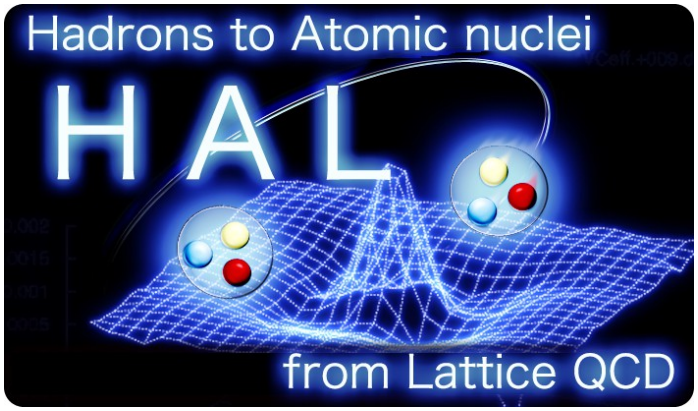

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