

Linear response theory in asymmetric nuclear matter for Skyrme functionals

(Part II)

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Saclay, November 2014

Thanks to all collaborators!

A very ambitious project: a lot of collaborations!

- Barcelona: X. Viñas
- Bordeaux: M. Bender, J. Sadoudi
- Belgium: S. Goriely, N. Chamel, P.-H. Heenen, V. Hellemans, M. Martini
- Jyväskylä: K. Bennaceur, M. Kortelainen, J. Dobaczewski
- Lyon: J. Meyer, R. Jodon, P. Becker, J. Margueron
- Orsay: P. Schuck, M. Urban
- Saclay: T. Duguet, T. Lesinski
- Seattle: J.W. Holt

Special thanks to all UNEDF members (N. Schunck, W. Nazarewicz, S.M. Wild, J. Sarich, P.G. Reinhard, J. Erler, E.H. Olsen, J.A.M. McDonnell,...)

Thanks also to other people not mentioned here!!

Outline

- 1 Introduction
- 2 Linear response theory with Skyrme functionals
- 3 Instabilities in Skyrme functionals
- 4 Neutrino mean free path
- 5 Conclusions

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Infinite nuclear matter (Ground State)

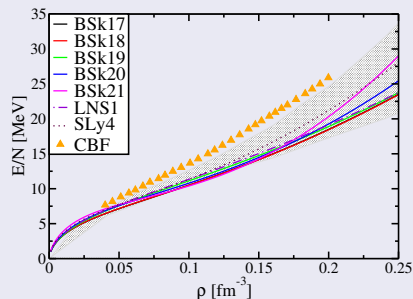
Main features

- Ideal system: neutron and protons
- No Coulomb
- plane-waves
- $Y = \frac{\rho_1}{\rho_0}$

Physics Report

A. Pastore, D. Davesne and J. Navarro (2014) [\[Accepted\]](#)

Pure Neutron Matter EoS

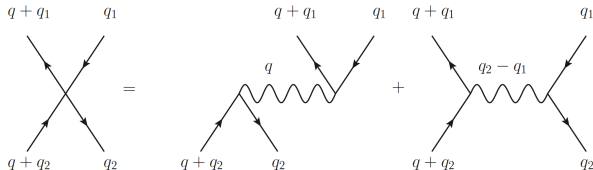


Just a simple model?

(Warm) Neutron stars can be described as infinite matter

Introduction (Excited States)

We use the Linear response theory to describe excited states in IM



The exchange term is the difficult one!

Different approximations

- 1 Landau (static) $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{k}_F$ and $q, \nu \approx \omega/q = 0$
- 2 Landau (dynamic) $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{k}_F$ and $q = 0$
- 3 Landau (extended) $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{k}_F$ and $q = 0$ (only for V_{ph})
- 4 RPA (complete) $\mathbf{q}_1, \mathbf{q}_2, q$

Landau theory of Fermi liquids (PNM)

The most general residual interaction reads [A. Schwenck and B. Friman, Phys. Rev. Lett. 92, 8 (2004)]

$$\begin{aligned} V_{ph} = & f(\theta_{12}) + g(\theta_{12}) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \tilde{h}(\theta_{12}) \frac{\mathbf{k}_{12}^2}{k_F^2} S_{12}(\hat{\mathbf{k}}_{12}) \\ & + \tilde{k}(\theta_{12}) \frac{\mathbf{P}_{12}^2}{k_F^2} S_{12}(\hat{\mathbf{P}}_{12}) + \tilde{l}(\theta_{12}) \frac{\mathbf{k}_{12} \cdot \mathbf{P}_{12}}{k_F^2} A_{12}(\hat{\mathbf{k}}_{12}, \hat{\mathbf{P}}_{12}). \end{aligned}$$

Tensor terms

$$S_{12}(\hat{\mathbf{k}}_{12}) = 3(\hat{\mathbf{k}}_{12} \cdot \boldsymbol{\sigma}_1)(\hat{\mathbf{k}}_{12} \cdot \boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2.$$

$$S_{12}(\hat{\mathbf{P}}_{12}) = 3(\hat{\mathbf{P}}_{12} \cdot \boldsymbol{\sigma}_1)(\hat{\mathbf{P}}_{12} \cdot \boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$

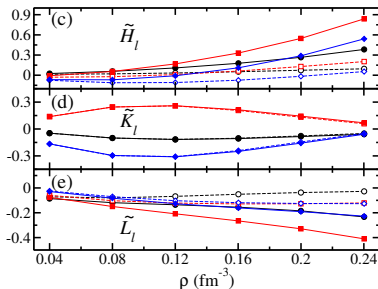
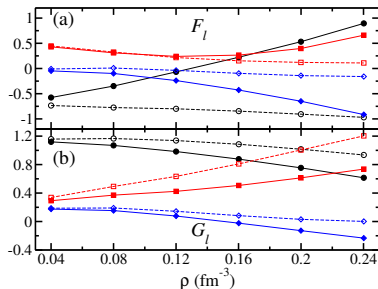
$$A_{12}(\hat{\mathbf{k}}_{12}, \hat{\mathbf{P}}_{12}) = (\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{P}}_{12})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{k}}_{12}) - (\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{k}}_{12})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{P}}_{12})$$

Question: how to interpret K_l, L_l ?

- H_l, K_l, L_l have the same order of magnitude
- They give important effects on the response function
- Not considered in phenomenological interactions
- A_{12} mixes $S=0, S=1$ in ground state (spin orbit?)

Landau parameters in PNM: χ -EFT (J.W. Holt)

Landau parameters obtained with χ -EFT ($\Lambda = 450$ MeV).



Full line \rightarrow 2NF+3NF; Dashed line \rightarrow 2NF only

Remarks

- Landau parameters are cut-off dependent (Λ)
- 3 body terms play a crucial role

[D. Davesne, J.W. Holt, A.P. and J. Navarro (2014) arXiv:1411.3117]

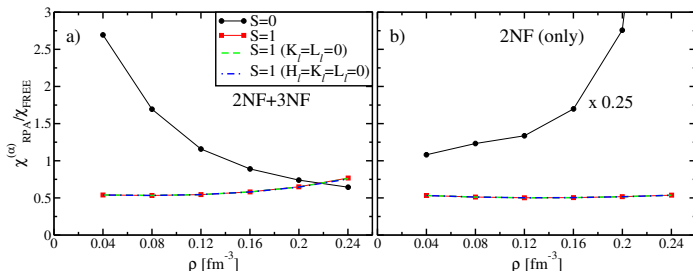
Static Landau with χ -EFT: effect of 2- and 3- body terms

Information on the static deformation of the Fermi surface

$$\frac{\chi_{HF}(0)}{\chi_{RPA}^{(S=0)}(0)} = 1 + F_0 \quad \frac{\chi_{HF}(0)}{\chi_{RPA}^{(S=1)}(0)} = 1 + G_0 + \frac{T_1}{T_2}$$

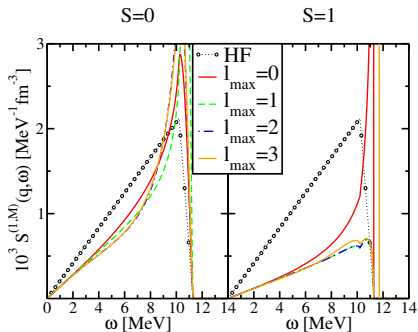
$$T_1 = -2 \left(\tilde{H}_0 - \frac{2}{3} \tilde{H}_1 + \frac{1}{5} \tilde{H}_2 + \tilde{K}_0 + \frac{2}{3} \tilde{K}_1 + \frac{1}{5} \tilde{K}_2 \right)^2,$$

$$T_2 = 1 + \frac{1}{5} G_2 - \frac{7}{15} \tilde{H}_1 + \frac{2}{5} \tilde{H}_2 - \frac{3}{35} \tilde{H}_3 + \frac{7}{15} \tilde{K}_1 + \frac{2}{5} \tilde{K}_2 + \frac{3}{35} \tilde{K}_3 + \frac{2}{5} \tilde{L}_1 - \frac{6}{35} \tilde{L}_3.$$



Extended Landau response function

PNM at $k_F = 1.68 \text{ fm}^{-1}$, $q/k_F = 0.1$ for χ -EFT ($H_l = K_l = L_l = 0$)



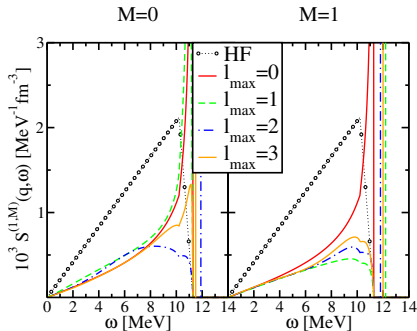
Convergence

We have an infinite sets of Landau parameters, at which l shall we stop?

[A. P., D. Davesne, J. Navarro J.Phys. G41 (2014) 055103]

Extended Landau response function

PNM at $k_F = 1.68 \text{ fm}^{-1}$, $q/k_F = 0.1$ for χ -EFT

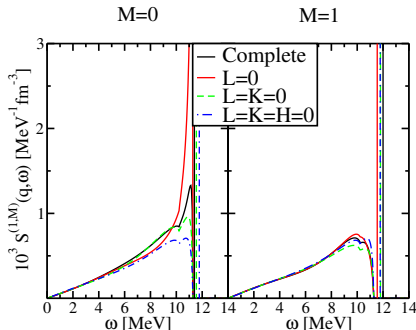


Convergence

Less clear convergence.... better results with $K_t = 0$

Extended Landau response function

PNM at $k_F = 1.68 \text{ fm}^{-1}$, $q/k_F = 0.1$ for χ -EFT

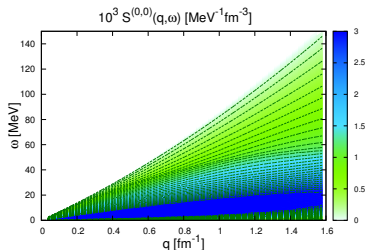
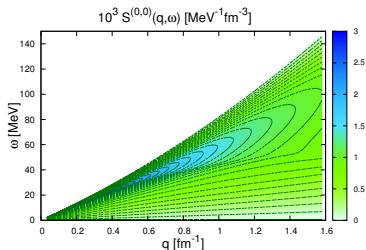


Remarks

- The H_l, K_l, L_l have a *similar* importance
- Landau parameters do depend on cut off, interaction and model!!

3NF in S=0 channel

Using Landau parameters of χ EFT we can study the impact of the 3 body terms!



Left: complete 2-3- body. Right 2-body. In S=1 channel we see no (great) differences.

From infinite matter to finite nuclei

- What about the giant monopole resonance (S=0 channel)?
- Possible relation IM \leftrightarrow finite nuclei (Thomas Fermi?)
- 3NF have a strong density dependence
- Is $t_3(1 + x_3 P_\sigma)\rho^\alpha \delta_{r_{12}}$ sufficient to describe this physics?

[D. Davesne, J.W. Holt, A.P. and J. Navarro (2014) arXiv:1411.3117]

Tensor term with phenomenological functionals

Simple tensor term

$$V_{ph}^{tensor} = \sum_l (h_l + h'_l \tau_1 \tau_2) \frac{k_{12}^2}{k_F^2} S_{12} P_l(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)$$

where $\mathbf{k}_{12} = \mathbf{k}_1 - \mathbf{k}_2$ and $S_{12} = 3(\hat{k}_{12} \cdot \sigma_1)(\hat{k}_{12} \cdot \sigma_2) - \sigma_1 \sigma_2$

Possible origins

- ① Gogny family (D1MT,D1ST,...) (perturbative)

[M. Anguiano *et al.* Phys. Rev. C83, 064306 (2011).]

- ② M3Y (fully self consistent) [H. Nakada, Phys. Rev. C68, 014316 (2003).]

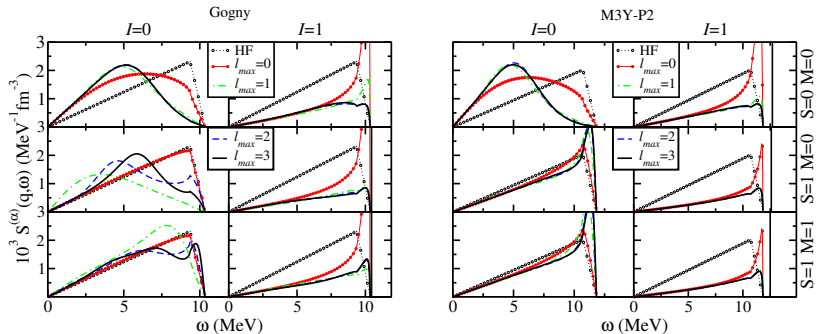
- ③ Skyrme (fully self consistent) [T Lesinski *et al.* Phys. Rev C76, 014312, (2007)]

Major problem

How to fix the tensor parameters? Which observables?

Landau theory with Gogny (D1MT) and Nakada

Extended Landau in SNM at $\rho = \rho_{sat}$ and $q = 0.1k_F$.



Problem with perturbative tensor!

The response function does not converge in the isoscalar channel!! [A. P., D.

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From Landau theory to RPA (complete)

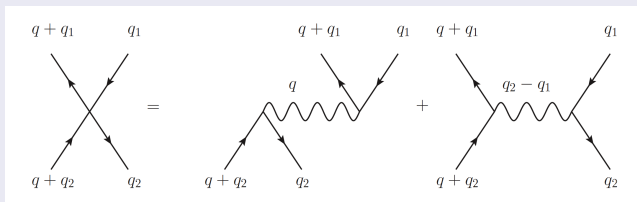
The particle-hole pair can be outside the Fermi sphere

Similarities

- Bethe-Salpeter equations
- Same formal method to obtain the solution (Green's functions)

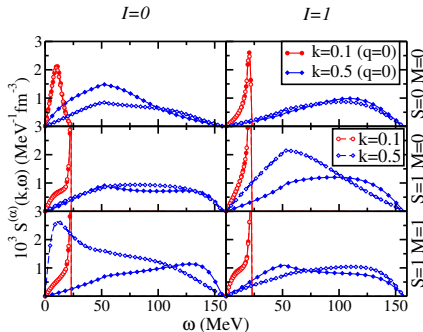
Differences

- The residual interactions has a dependence on q
- New coordinates $\mathbf{k}_1, \mathbf{k}_1, \mathbf{q}$



Landau vs RPA (complete)

PNM at $\rho = \rho_0$ using Skyrme-T44 (tensor).



Results

- The Landau approximation is very good for very low transferred momenta.
- Possible extension: LAFET [J. Margueron, J. Navarro, N Van Giai, P Schuck Phys. Rev. C77, 064306 (2008).]

Skyrme functionals

The Skyrme functional can be written as

$$\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}_{Skyrme} + \mathcal{E}_{pairing} + \mathcal{E}_{Coulomb} + \mathcal{E}_{corr.}$$

where at second order (N1LO) reads

Skyrme functional

$$\begin{aligned} \mathcal{E}_{Skyrme} = \sum_{t=0,1} \int d^3\mathbf{r} \left\{ C_t^\rho [\rho_0] \rho_t^2 + C_t^{\Delta\rho} \rho_t \Delta\rho_t + C_t^\tau \rho_t \tau_t + C_t^j \mathbf{j}_t^2 + C_t^s [\rho_0] s_t^2 \right. \\ + C_t^{\nabla s} (\nabla \cdot \mathbf{s}_t)^2 + C_t^{\Delta s} \mathbf{s}_t \cdot \Delta \mathbf{s}_t + C_t^T \mathbf{s}_t \cdot \mathbf{T}_t + C_t^F \mathbf{s}_t \cdot \mathbf{F}_t + C_t^{\nabla J} \rho_t \nabla \cdot \mathbf{J}_t \\ \left. + C_t^{\nabla j} \mathbf{s}_t \cdot (\nabla \times \mathbf{j}_t) + C_t^{J(0)} (J_t^{(0)})^2 + C_t^{J(1)} (\mathbf{J}_t^{(1)})^2 + C_t^{J(2)} \sum_{\mu\nu=x}^z J_{t\mu\nu}^{(2)} J_{t\mu\nu}^{(2)} \right\} \end{aligned}$$

[E. Perlinska et al. Phys. Rev C 69, 014316 (2004)]

The terms in red do not contribute in the Landau limit

Asymmetric Nuclear Matter (ANM) at finite temperature

Main features

- No pairing
- No Coulomb
- No charge-exchange process
- Asymmetry parameter $Y = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$

$$\begin{aligned} G_{RPA}^{\tau\tau', SM}(\mathbf{k}_1, q, \omega) &= \delta_{\tau\tau'} G_{HF}^{\tau\tau'}(\mathbf{k}_1, q, \omega) \\ &+ \sum_{S'M'\tau''\tau'''} \int d^3\mathbf{k}_2 \langle \tau\tau' SM | V_{ph} | \tau''\tau''' S'M' \rangle G_{RPA}^{\tau''\tau''', S'M'}(\mathbf{k}_2, q, \omega) \end{aligned}$$

The response function is then obtained as

$$\chi^{\tau\tau'}(q, \omega) = n_g \int d^3\mathbf{k}_1 G_{RPA}^{\tau\tau', SM}(\mathbf{k}_1, q, \omega).$$

The role of Isospin quantum number

We can decompose the response function as

$$\begin{aligned}\langle G_{RPA}^{(S,M,I=0)} \rangle &= \langle G_{RPA}^{nn,S,M} \rangle + \langle G_{RPA}^{pn,S,M} \rangle \\ &+ \langle G_{RPA}^{np,S,M} \rangle + \langle G_{RPA}^{pp,S,M} \rangle, \\ \langle G_{RPA}^{(S,M,I=1)} \rangle &= \langle G_{RPA}^{nn,S,M} \rangle - \langle G_{RPA}^{pn,S,M} \rangle \\ &- \langle G_{RPA}^{np,S,M} \rangle + \langle G_{RPA}^{pp,S,M} \rangle.\end{aligned}$$

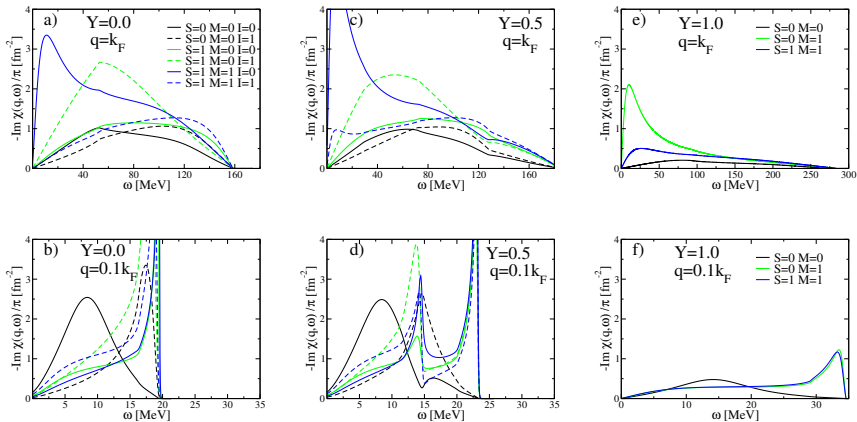
Pure Neutron Matter

There is no discontinuity going from SNM to PNM. In PNM we have

$$\langle G_{RPA}^{(S,M,I=0)} \rangle = \langle G_{RPA}^{(S,M,I=1)} \rangle$$

Response function for T44

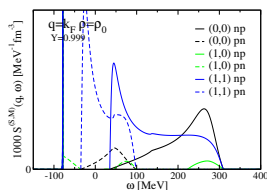
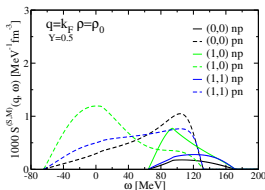
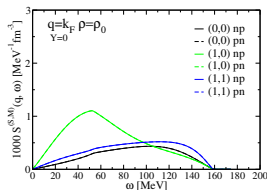
Response function at $\rho = \rho_0$ and different asymmetries



Asymmetric Nuclear Matter (ANM) II

Charge-exchange process

$$Q_{\pm}^S = \sum_j e^{i\mathbf{q} \cdot \mathbf{r}_j} \Theta_j^S \tau_j^{\pm}$$



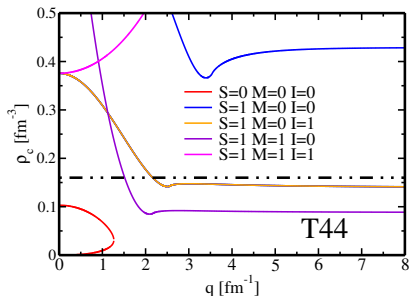
Preliminary results

Charge exchange process are already included in SNM. Numerical noise at $q \rightarrow 0$

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Instabilities in SNM

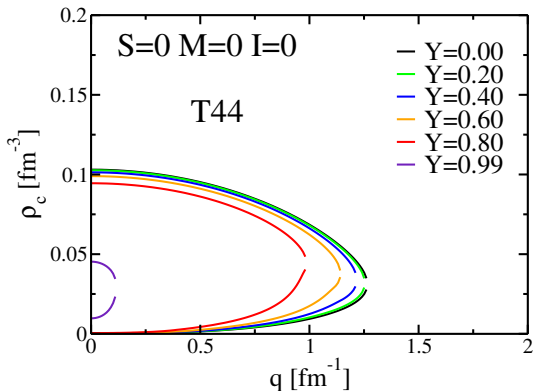


Features

- Instabilities appear in different (S,I) channels
- Instabilities can appear for finite values of the transfer momentum q
- ≈ 1 second CPU time; analytic formulas simple to code
- There is no natural cut-off for exchanged momentum!!!

Evolution of instabilities with Y (1)

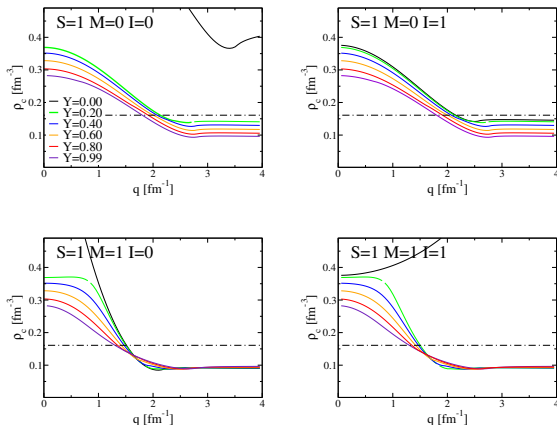
Evolution of spinodal instability with Y



From SNM to PNM

The spinodal has a physical meaning: instability against cluster formation

Evolution of instabilities with Y (2)

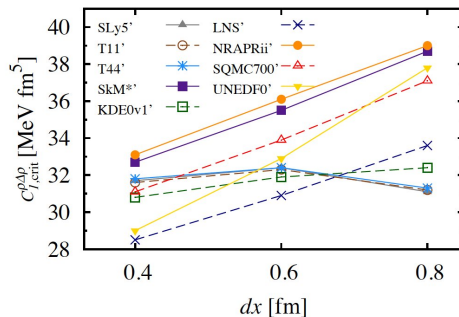


A simple result

We go smoothly from the poles of SNM to PNM. The lowest poles are either in SNM or PNM

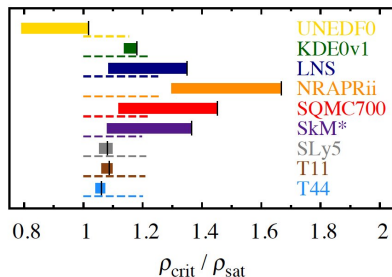
A first systematic study for finite nuclei I

- Extensive calculations using different codes
 - ① HOSPHE (spherical HO code)
 - ② Lenteur (spherical $r - space$ code)
 - ③ Ev8 (3D $r - space$ code)
- We study the instability related to $C_1^{\rho\Delta\rho}$ (SkP,LNS)



A first systematic study for finite nuclei II

For calculate the RPA response function for the values of $C_1^{\rho\Delta\rho}$ (lowest-maximum) and we deduce the corresponding value of the critical density in SNM.

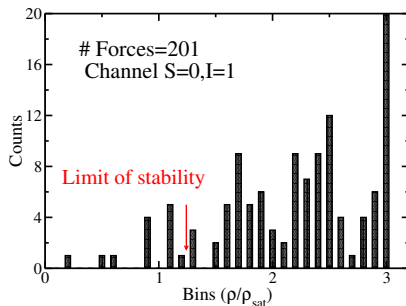


Param.	Ref.	$C_1^{\rho\Delta\rho}$	ρ_{min}/ρ_{sat}	$C_{1,crit}^{\rho\Delta\rho}$	ρ_{crit}/ρ_{sat}
KDE0v1	[26]	11.498	2.39	30.8(1)	1.18
LNS	[27]	33.750	1.25	28.5(1)	1.35
NRAPRii	[28]	16.599	4.21	33.1(1)	1.67
SQMC700	[29]	15.884	4.77	31.1(1)	1.45
SkM*	[30]	17.109	2.94	32.7(2)	1.36
SLy5	[31]	16.375	1.72	31.7(2)	1.08
T11	[32]	14.252	1.92	31.6(2)	1.08
T44	[32]	-4.300	6.63	31.8(2)	1.05
UNEDF0	[33]	-55.623	4.13	29.0(1)	1.02

... we need more work...

- The simplest criterion requires $\rho_c > 1.2\rho_{sat}$
- Some forces are not under control (NRAPR, SQMC700, SKM*,....). (Not fitted!!)

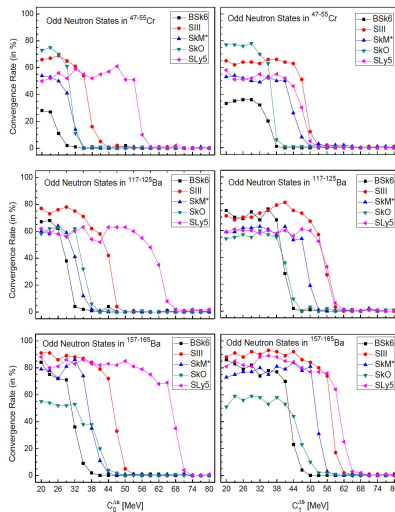
Systematic study in the $S=0$ channel



Results

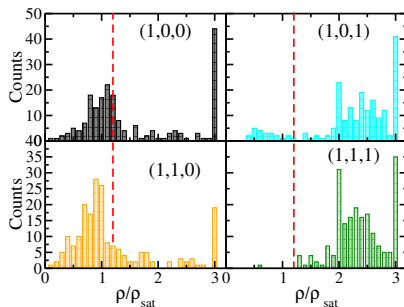
15 functionals of 201 are not stable (LNS, SkP ,SkS2 ,SKz0,...)

Similar problems in $S=1$ channel: the $s_1\Delta_{s_1}$ term



Results only based on HFODD (3d code HO basis), courtesy of N. Schunck

Systematic study in the $S=1$ channel



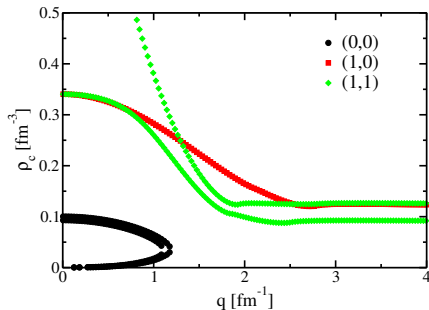
Results

Only 58 forces out of 201 pass the test.

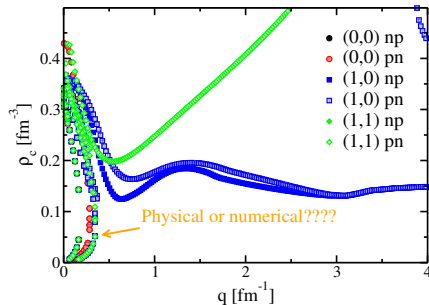
Charge-exchange instabilities (I=1 channel)

(Very Preliminary!!) T44 at Y=0.5 T=0

No Isospin-flip



Isospin-flip



Outline

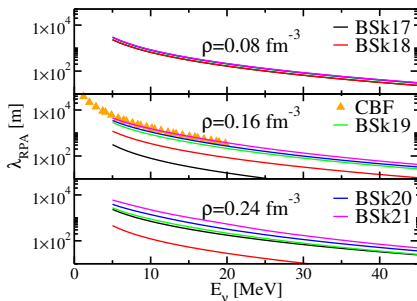
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Neutrino mean free path

$$n + \nu \longrightarrow n' + \nu'$$

The cross section of this process is

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{G_F^2 E_\nu^2}{16\rho\pi^2} \left\{ (1 + \cos\theta) S^{(0,0)} + g_A^2 F_{10}(\theta) S^{(1,0)} + 2g_A^2 F_{11}(\theta) S^{(1,1)} \right\}$$



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Conclusions and perspectives

- The LR theory can be used to fit functionals
 - ① We use it to detect instabilities ($S=1$ channel still to be fully investigated)
 - ② the LR method is now applied into the Saclay-Lyon protocol
 - ③ Finite size instabilities can also affect other functionals
- Extensions of the LR formalism to treat Landau
 - ① Limit of stability of Landau theory
 - ② Possible extension of Landau q^2 dependence
 - ③ Application to Neutron stars (NMFP, Specific heat...)
- Extension of the LR formalism to asymmetric nuclear matter (soon also charge exchange)
- Extension to finite range interactions
- Inclusion of pairing

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THANK YOU!!