

# Strategies for constructing energy density functionals beyond mean-field

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IPN Orsay

## Outline



EDF combined with configuration mixing have severe problems

- Strategy 1: keep the same functional and correct it

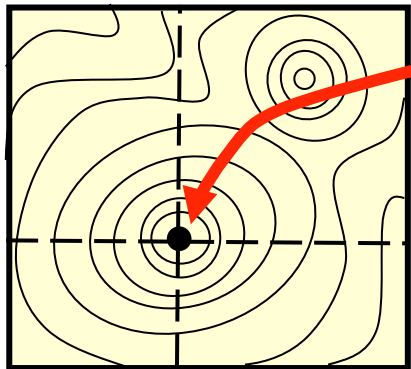


- Strategy 2: extend the EDF to incorporate complex correlations associated to symmetry breaking

- Strategy 3: back to basics , EDF from a true Hamiltonian

# Configuration Mixing within Energy Density Functional

## Mean-Field Theories



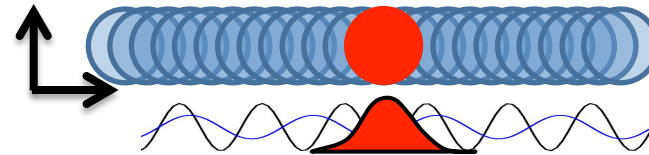
$$|\Phi\rangle = \Pi \alpha_k |0\rangle$$

(Skyrme, Gogny, ...)

$$\Phi \rightarrow \{\rho, \kappa\} \rightarrow \mathcal{E}(\rho, \kappa)$$

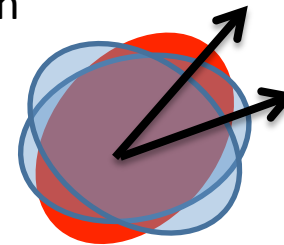
To make the functional predictive, we use and abuse of symmetry breaking

## Translation



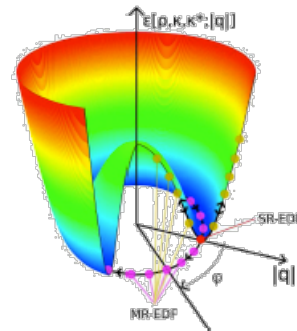
Surface  
Vibration

## Rotation

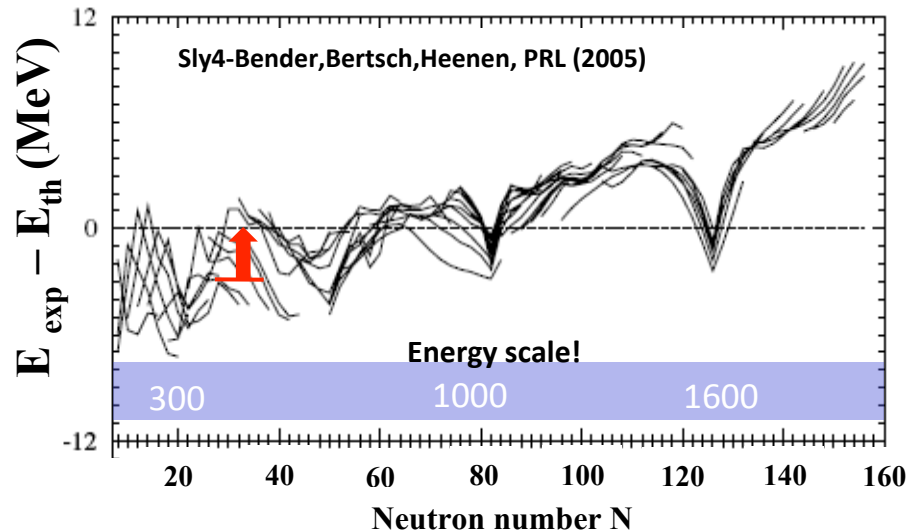


Quadrupole  
Correlation  
-  
Rotational  
Bands

## Particle number



Pairing  
Correlations  
Odd-even  
effects



## Dilemma

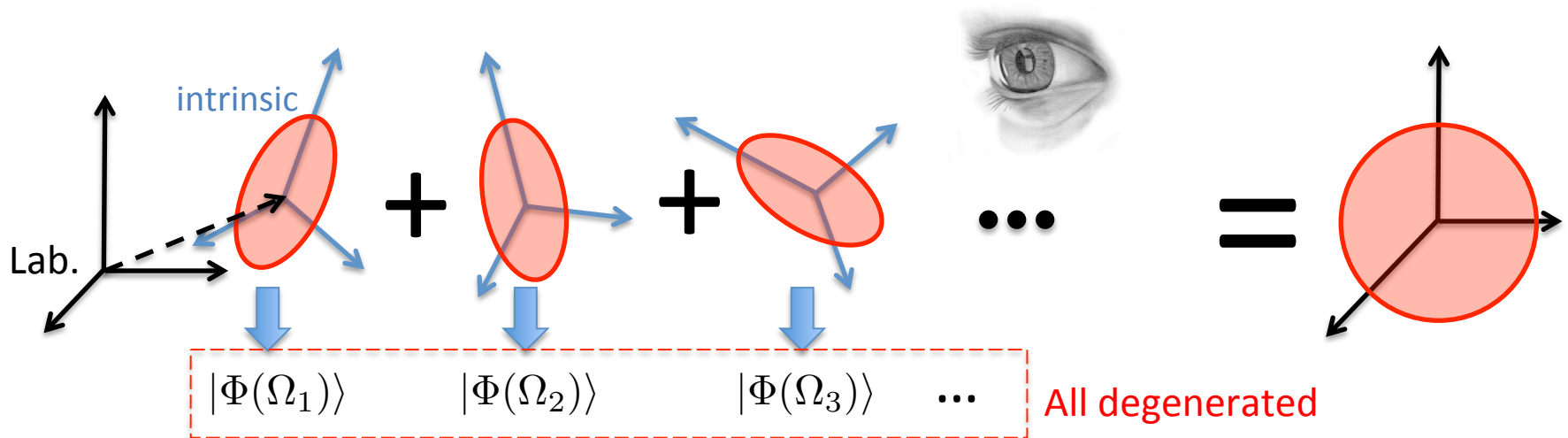
The many-body state  
respect all symmetries of  
H in the lab. frame

MB states should ultimately  
have good quantum numbers

EDF uses symmetry breaking to  
grasp correlation which are not  
easy to get without symmetry  
breaking

MB states do not have good  
quantum numbers

## Symmetry Restoration : the rotation case



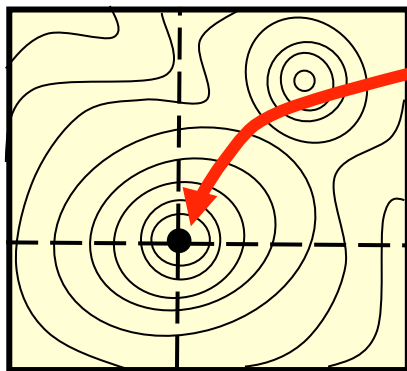
## Formulation with projectors

$$|\Psi^J\rangle = P_J |\Psi(0)\rangle \quad \text{with} \quad |\Psi^J\rangle = \int d\Omega f(\Omega) |\Phi(\Omega)\rangle$$

Generator Coordinate Method  
Configuration mixing  
Multi-Reference EDF

# Configuration Mixing within Energy Density Functional

Single Reference (SR)-  
Mean-Field

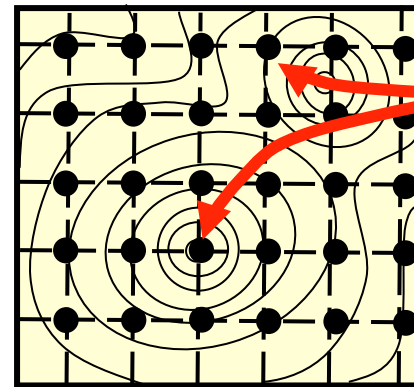


$$|\Phi\rangle = \Pi \alpha_k |0\rangle$$

(Skyrme, Gogny)

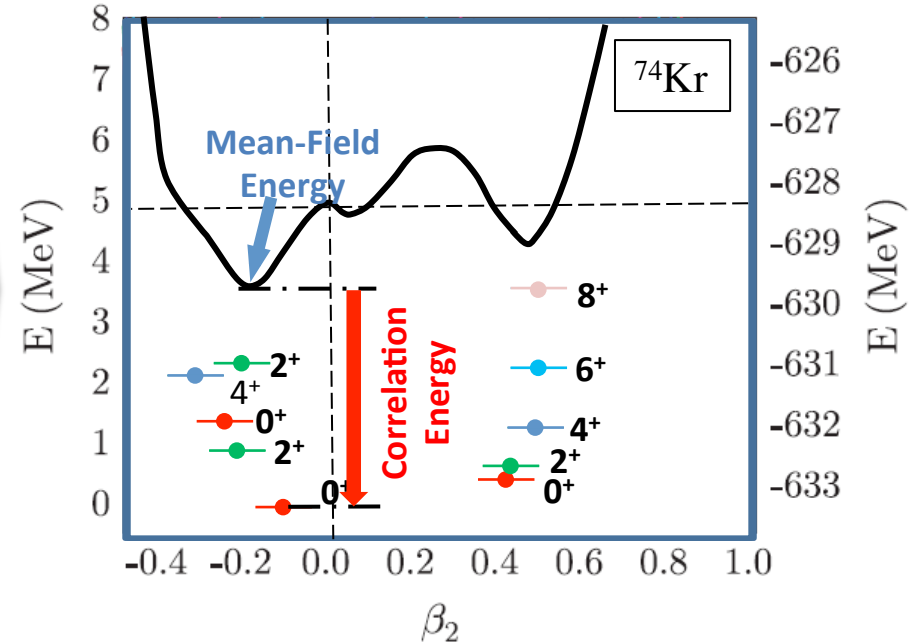
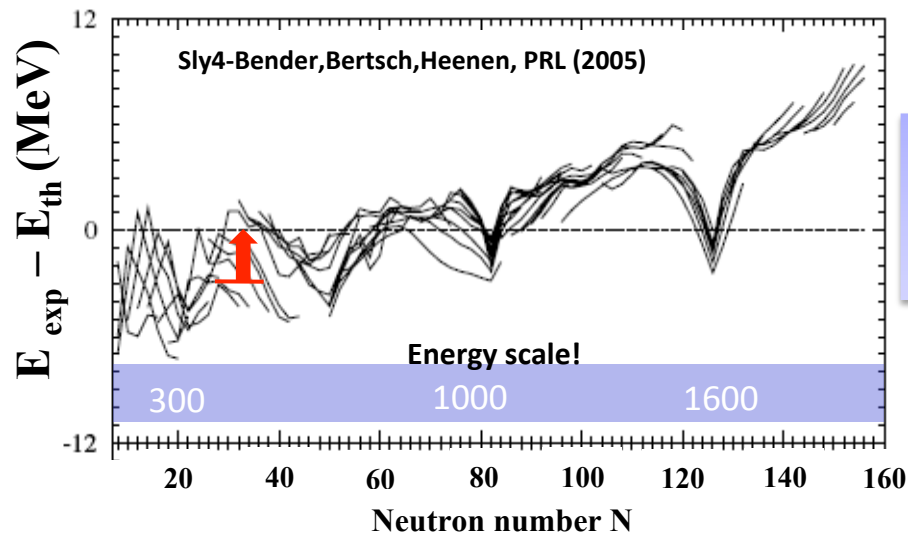
$$\Phi \rightarrow \{\rho, \kappa\} \rightarrow \mathcal{E}(\rho, \kappa)$$

Multi-Ref. (MR)-GCM



$$|\Psi\rangle = \int dQ f(Q) |\Phi(Q)\rangle$$

$$|\Phi(Q_i)\rangle$$

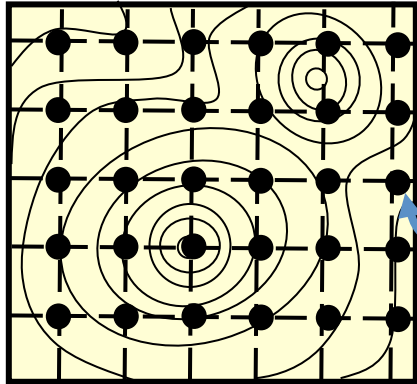


Bender et al, PRC74 (2006)

# Some recent discussions: specific aspects of EDF

➡ Application of conf. mixing in EDF  
Needs to be regularized

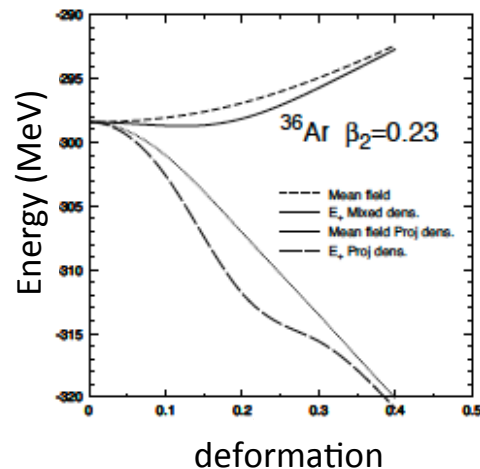
Multi- Ref. (MR)-GCM



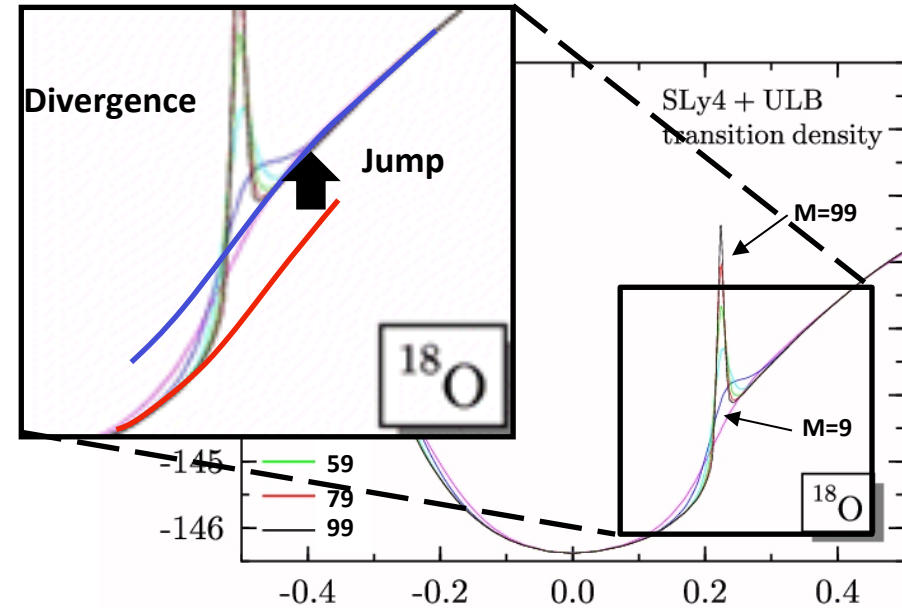
M: number of  
Mesh points

In LDA only functional of  $\rho$ ,  $\rho^2$ ,  $\rho^3$ ,  $\rho^4$ ,  $\rho^5$  could be used, not  $\rho^\alpha$  !!

➡ What should be the density to be used in the effective interaction ?



Taken from  
L. M. Robledo, J. Phys. G 37 (2010)



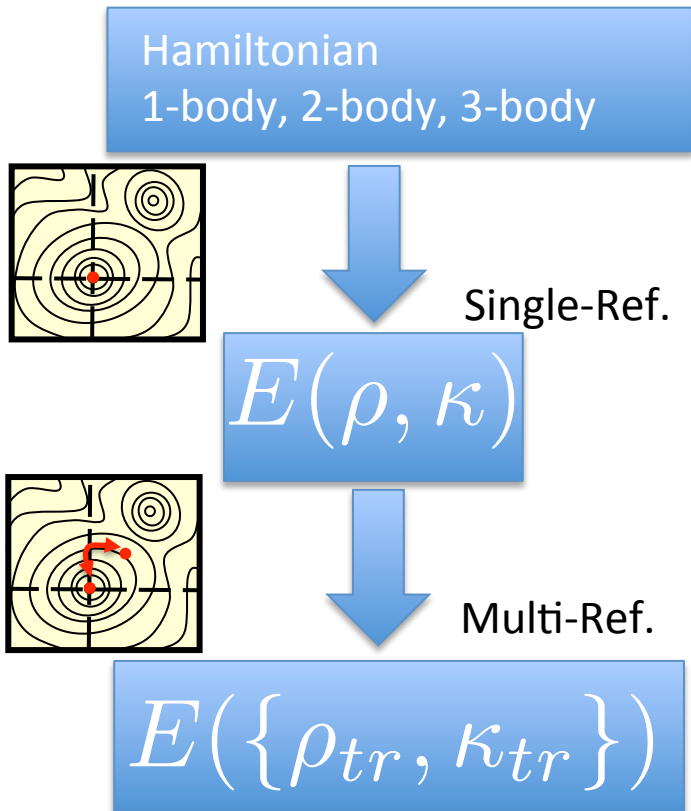
Lacroix et al, PRC79 (2009),  
Bender et al, PRC79 (2009),  
Duguet et al, PRC79 (2009)

➡ The very notion of symmetry restoration  
in EDF needs to be clarified

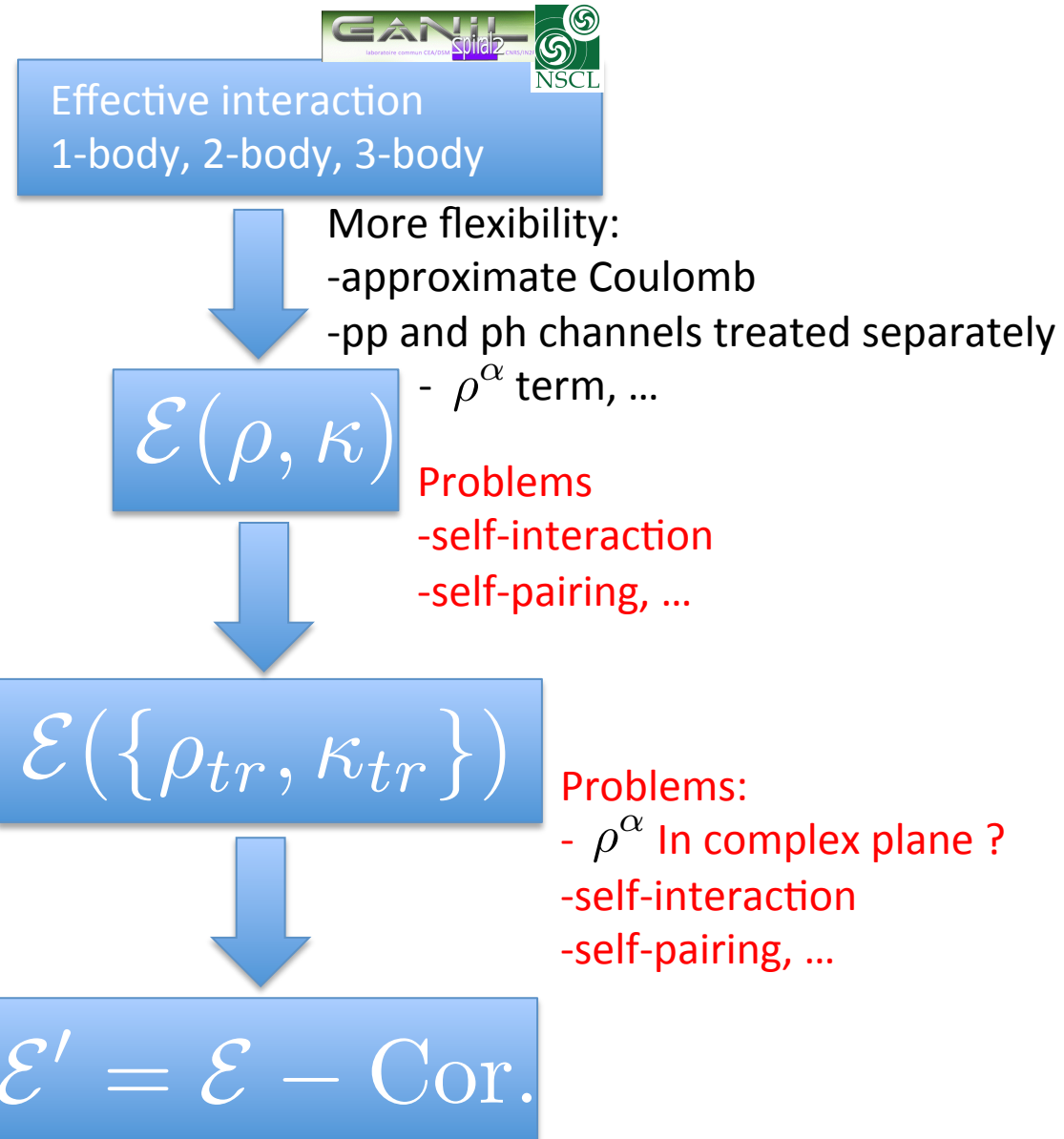
T. Duguet and J. Sadoudi, J. Phys. G 37 (2010)

Strategy 1: keep the same functional  
and try to correct it

## Many-body technology



## Nuclear EDF phenomenology



# Practical and conceptual difficulties in Configuration Mixing within EDF

(I) Lacroix, et al, PRC79 (2009), (II) Bender et al PRC79 (2009), (III) Duguet et al, PRC79 (2009).

## Correction is possible

$$\mathcal{E} = \mathcal{E}^\rho + \mathcal{E}^{\rho\rho} + \mathcal{E}^{\kappa\kappa} + \dots$$

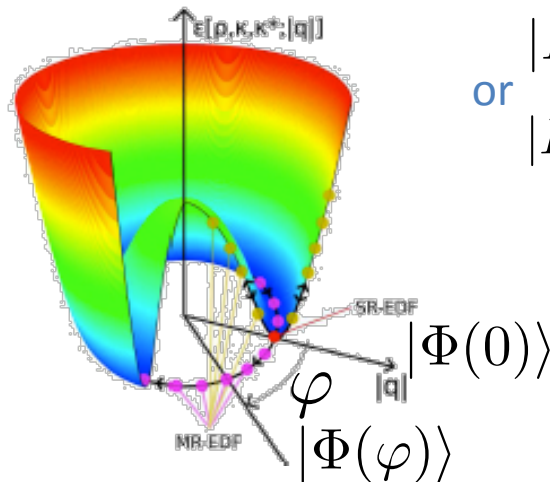
with

$$\mathcal{E}^{\rho\rho} \rightarrow \mathcal{E}^{\rho\rho} - \mathcal{E}^{\rho\rho}_{\text{Corr.}}$$

$$\mathcal{E}^{\kappa\kappa} \rightarrow \mathcal{E}^{\kappa\kappa} - \mathcal{E}^{\kappa\kappa}_{\text{Corr.}}$$

...

## Example: particle number projection



or

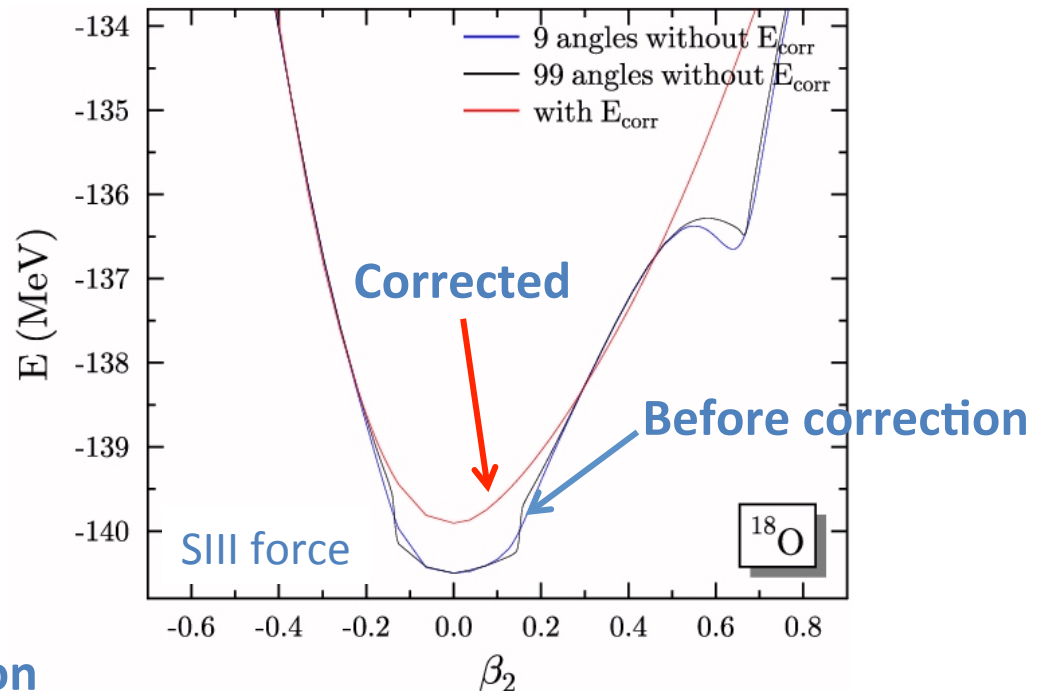
$$\begin{aligned} |BCS\rangle \\ |HFB\rangle \end{aligned} = \prod_{p>0} (u_p + v_p a_p^\dagger a_{\bar{p}}^\dagger) |-\rangle$$

Projection on  
Good particle number



$$|N\rangle = \int_0^{2\pi} d\varphi \frac{e^{-i\varphi N}}{2\pi c_N} |\Phi[\varphi]\rangle$$

with  $|\Phi[\varphi]\rangle = e^{i\varphi \hat{N}} |\Phi_0\rangle$



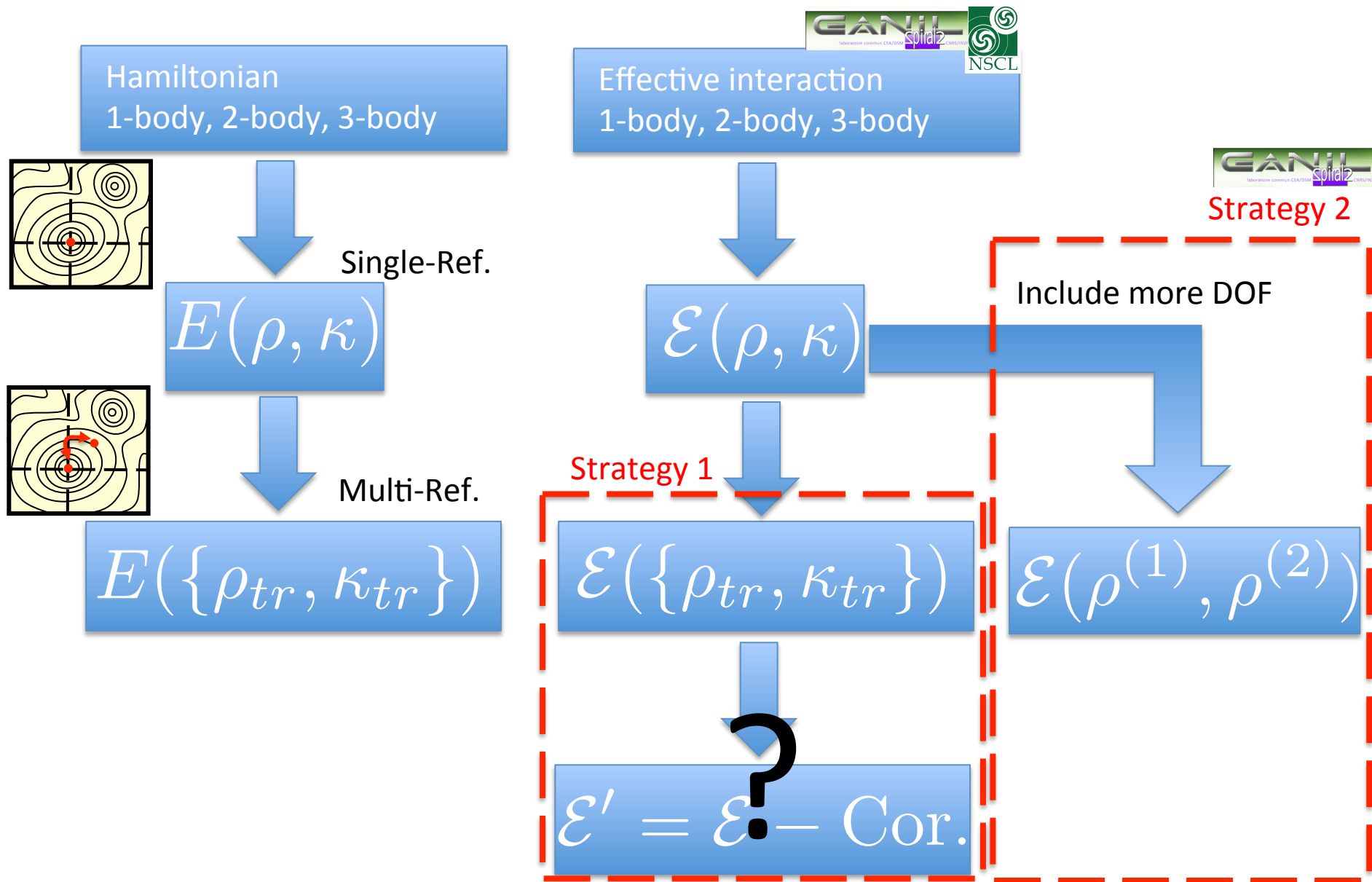
Some uncontrolled  
and not understood spurious  
contributions persists

M. Bender (private unless published)



## Many-body technology

## Nuclear EDF phenomenology



# Configuration mixing as a functional theory

The two-body Hamiltonian case: what is a functional of what?

$$H = \sum_{ij} t_{ij} a_i^\dagger a_j + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} a_i^\dagger a_j^\dagger a_l a_k$$

Mean-Field (with Pairing)

$$\begin{aligned} |\Phi_0\rangle &= \Pi \beta_\alpha^\dagger |-\rangle \\ \langle H \rangle &= \sum_i t_{ii} \rho_{ii} + \frac{1}{2} \sum_{i,j} \bar{v}_{ijij} \rho_{ii} \rho_{jj} + \frac{1}{4} \sum_{i,j} \bar{v}_{i\bar{i}j\bar{j}} \kappa_{i\bar{i}}^* \kappa_{j\bar{j}} \\ &= E_{SR}[\rho, \kappa, \kappa^*] \end{aligned}$$

$$\Phi_0 \rightarrow \{\rho, \kappa\} \rightarrow E_{SR}$$

Projection

$$\begin{aligned} |\Psi_\Omega\rangle &= P^\Omega |\Phi_0\rangle \\ &= \int dQ f(Q) |\Phi(Q)\rangle \\ E_{MR} &= \iint dQ dQ' \mathcal{N}(Q, Q') E_{SR}(\rho^{QQ'}, \kappa^{QQ'}, \kappa^{*QQ'}) \end{aligned}$$

$$\Phi_0 \rightarrow \{\rho^{QQ'}, \kappa^{QQ'}, \kappa^{*QQ'}\} \rightarrow E_{MR}$$

Alternative formulation

$$|\Psi_\Omega\rangle = P^\Omega |\Phi_0\rangle$$



$$\begin{aligned} E_{MR} &= \sum_{ij} t_{ij} \langle a_i^\dagger a_j \rangle_\Omega \\ &+ \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} \langle a_i^\dagger a_j^\dagger a_l a_k \rangle_\Omega \end{aligned}$$



$$E_{MR} = \sum_{ij} t_{ij} \rho_{ij}^\Omega + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} R_{kl,ij}^\Omega$$

$$\Phi_0 \rightarrow \Psi_\Omega \rightarrow \{\rho^\Omega, R^\Omega\} \rightarrow E_{MR}$$



# What about the EDF theory?

The particle number restoration case

Alternative formulation ?

Mean-Field (with Pairing)

$$|\Phi_0\rangle = \Pi \beta_\alpha^\dagger |-\rangle$$

$$\mathcal{E}_{SR}[\rho, \kappa, \kappa^*] = \sum t_{ii} \rho_{ii} + \frac{1}{2} \sum \bar{v}_{ijij}^{\rho\rho} \rho_{ii} \rho_{jj} + \frac{1}{4} \sum \bar{v}_{i\bar{i}j\bar{j}}^{\kappa\kappa} \kappa_{i\bar{i}}^* \kappa_{j\bar{j}}$$

$$\Phi_0 \rightarrow \{\rho, \kappa\} \rightarrow \mathcal{E}_{SR}$$

Projection

$$\mathcal{E}_N[\Psi_N] \equiv \int_0^{2\pi} d\varphi \mathcal{E}_{SR}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}] \mathcal{N}_N(0, \varphi)$$

$$\Phi_0 \rightarrow \{\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{*\varphi 0}\} \rightarrow \mathcal{E}_N$$

$$\begin{aligned} \mathcal{E}_N[\Psi_N] = & \sum_i t_{ii} n_i^N \\ & + \frac{1}{2} \sum_{i,j,j \neq \bar{i}} \bar{v}_{ijij}^{\rho\rho} R_{ijij}^N \\ & + \frac{1}{4} \sum_{i \neq j, i \neq \bar{j}} \bar{v}_{i\bar{i}j\bar{j}}^{\kappa\kappa} R_{j\bar{j}i\bar{i}}^N \\ & + \frac{1}{2} \sum_i \bar{v}_{i\bar{i}i\bar{i}}^{\rho\rho} \int_0^{2\pi} d\varphi n_i^{0\varphi} n_i^{0\varphi} \mathcal{N}_N(0, \varphi) \\ & + \frac{1}{2} \sum_i \bar{v}_{i\bar{i}i\bar{i}}^{\kappa\kappa} \int_0^{2\pi} d\varphi \kappa_{i\bar{i}}^{\varphi 0*} \kappa_{i\bar{i}}^{0\varphi} \mathcal{N}_N(0, \varphi) \end{aligned}$$

OK

# What about the EDF theory

The particle number restoration case

Alternative formulation ?

Mean-Field (with Pairing)

$$|\Phi_0\rangle = \Pi \beta_\alpha^\dagger |-\rangle$$

$$\mathcal{E}_{SR}[\rho, \kappa, \kappa^*] = \sum t_{ii} \rho_{ii} + \frac{1}{2} \sum \bar{v}_{ijij}^{\rho\rho} \rho_{ii} \rho_{jj} + \frac{1}{4} \sum \bar{v}_{i\bar{i}j\bar{j}}^{\kappa\kappa} \kappa_{i\bar{i}}^* \kappa_{j\bar{j}}$$

$$\Phi_0 \rightarrow \{\rho, \kappa\} \rightarrow \mathcal{E}_{SR}$$

Projection

$$\mathcal{E}_N[\Psi_N] \equiv \int_0^{2\pi} d\varphi \mathcal{E}_{SR}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}] \mathcal{N}_N(0, \varphi)$$

$$\Phi_0 \rightarrow \{\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{*\varphi 0}\} \rightarrow \mathcal{E}_N$$

For non-density dependent effective int.  
After regularization proposed in

Lacroix, Duguet, Bender, PRC79 (2009)

$$\begin{aligned} \mathcal{E}_N[\Psi_N] = & \sum_i t_{ii} n_i^N \\ & + \frac{1}{2} \sum_{i,j,j\neq\bar{i}} \bar{v}_{ijij}^{\rho\rho} R_{ijij}^N \\ & + \frac{1}{4} \sum_{i\neq j,j\neq\bar{i}} \bar{v}_{i\bar{i}j\bar{j}}^{\kappa\kappa} R_{j\bar{j}i\bar{i}}^N \\ & + \frac{1}{2} \sum_i \bar{v}_{i\bar{i}i\bar{i}}^{\rho\rho} (n_i^N n_i^N - \delta n_i \delta n_i) \\ & + \frac{1}{2} \sum_i \bar{v}_{i\bar{i}i\bar{i}}^{\kappa\kappa} [n_i^N (1 - n_i^N) + \delta n_i \delta n_i] \end{aligned}$$

OK

$\delta n_i = n_i^N - n_i^0$

$\Psi_N$

$\Phi_0$

# What about the EDF theory

The particle number restoration case

$$|\Psi_N\rangle = P^N |\Phi_0\rangle$$

$$P^N = \frac{1}{2\pi} \int_0^{2\pi} d\varphi e^{i\varphi(\hat{N}-N)}$$

Mean-Field (with Pairing)

$$|\Phi_0\rangle = \Pi \beta_\alpha^\dagger |-\rangle$$

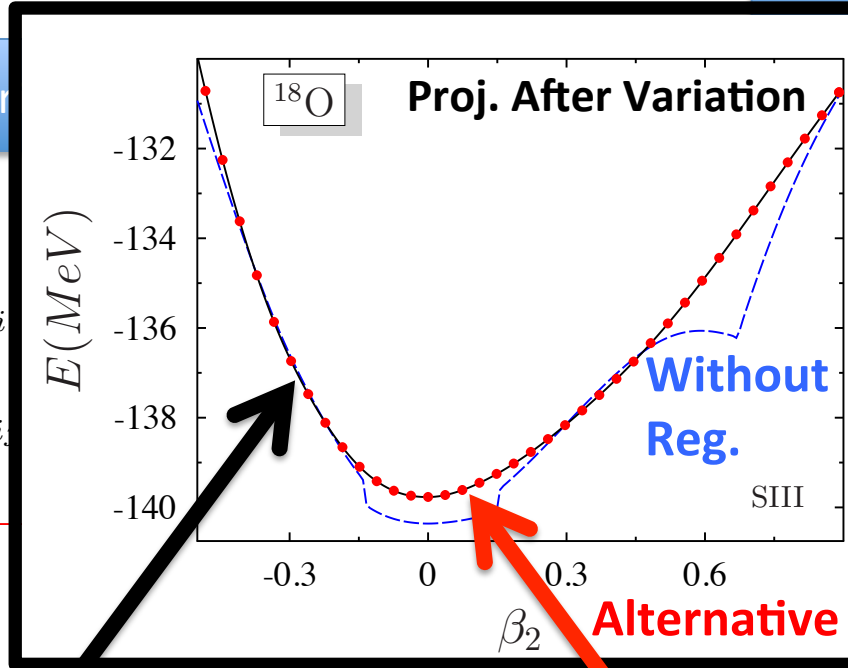
$$\mathcal{E}_{SR}[\rho, \kappa, \kappa^*] = \sum t_{ii} \rho_{ii} + \frac{1}{2} \sum \bar{v}_{ijij}^{\rho\rho}$$

$$\Phi_0 \rightarrow \{\rho, \kappa\}$$

Projection

$$\mathcal{E}_N[\Psi_N] \equiv \int_0^{2\pi} d\varphi \mathcal{E}_{SR}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{0\varphi*}] \mathcal{N}_N(0, \varphi)$$

$$\Phi_0 \rightarrow \{\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{*0\varphi}\} \rightarrow \mathcal{E}_N$$



Alternative formulation ?

density dependent effective int.  
restoration proposed in  
Lacroix, Duguet, Bender, PRC7 (2009)

$$\begin{aligned} & t_{ii} n_i^N \\ & \sum_{j, j \neq \bar{i}} \bar{v}_{ijij}^{\rho\rho} R_{ijij}^N \\ & \sum_{j, j \neq \bar{i}} \bar{v}_{i\bar{i}j\bar{j}}^{\kappa\kappa} R_{j\bar{j}i\bar{i}}^N \\ & + \frac{1}{2} \sum_i \bar{v}_{i\bar{i}i\bar{i}}^{\rho\rho} (n_i^N n_{\bar{i}}^N - \delta n_i \delta n_{\bar{i}}) \\ & + \frac{1}{2} \sum_i \bar{v}_{i\bar{i}i\bar{i}}^{\kappa\kappa} [n_i^N (1 - n_{\bar{i}}^N) + \delta n_i \delta n_{\bar{i}}] \end{aligned}$$

**OK**

$\delta n_i = n_i^N - n_i^0$

$\Psi_N$

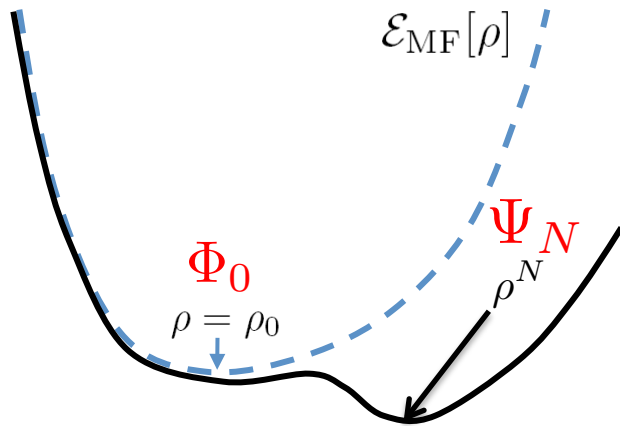
$\Phi_0$

Direct formulation:

$$\Psi_N \rightarrow \{\rho^N, R^N\} \rightarrow \mathcal{E}_{SC}(\rho^N, R^N)$$

Advantages

- the functional is automatically symmetry conserving.
- It is equivalent to MR-EDF for non density dependent term
- It is a natural extension of SR-EDF

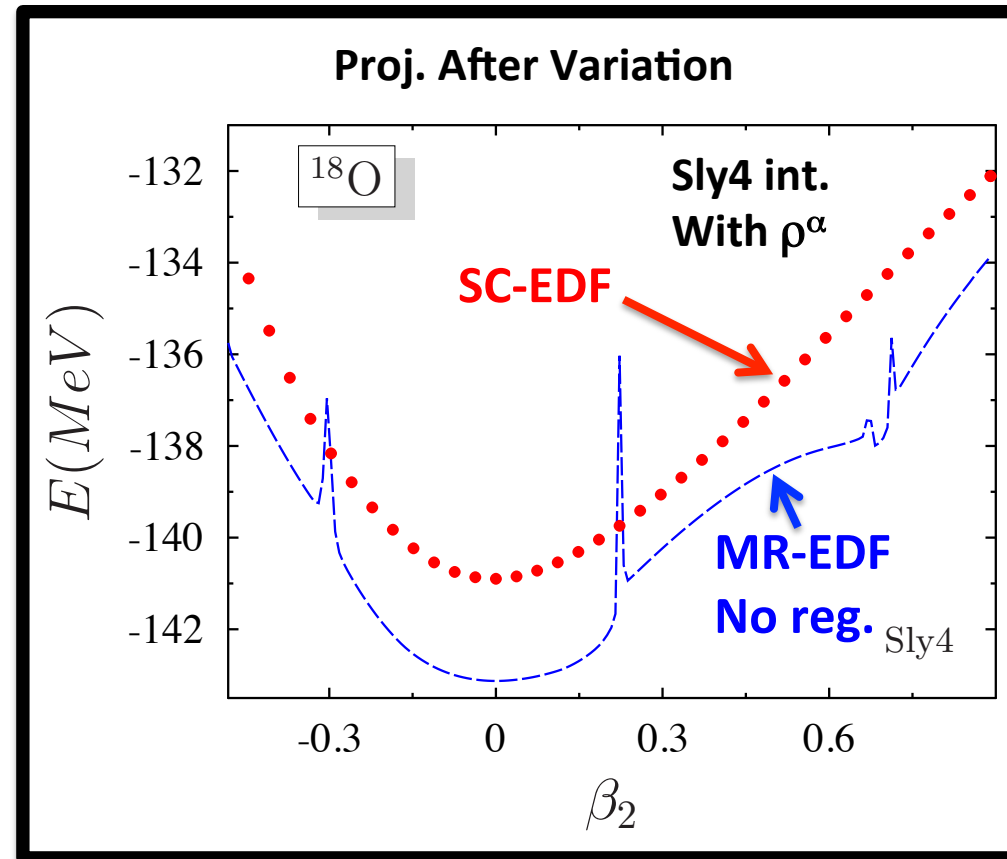


- It is free of jumps/divergence

## The Symmetry Conserving EDF

PAV :Hupin, Lacroix, Bender, Phys. Rev. C84 (2011)

VAP: Hupin, Lacroix, PRC86 (2012)

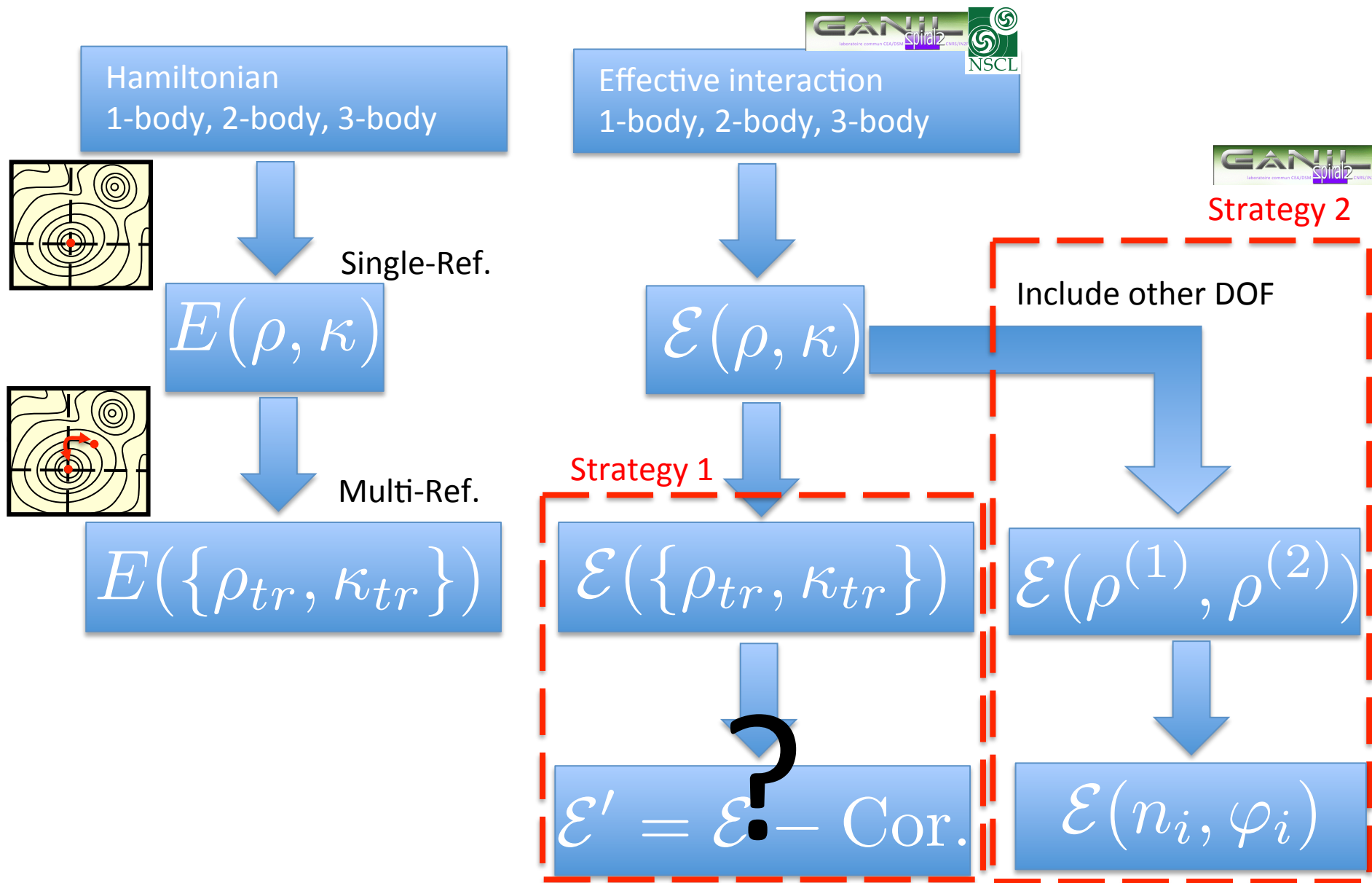


- It could be extended to density dependent interaction

$$\bar{v}^{\rho\rho}[\rho] \Rightarrow \bar{v}^{\rho\rho}[\rho^N], \quad \bar{v}^{\kappa\kappa}[\rho] \Rightarrow \bar{v}^{\kappa\kappa}[\rho^N]$$

## Many-body technology

## Nuclear EDF phenomenology



## Occupation-number based energy functionals

Lieb (1983), Papenbrock, Bhattacharyya, PRC75 (2007) ; Bertolli, Papenbrock, PRC78 (2008) ; Lacroix, PRC79 (2009)

$$F(\{n_j\}) = \boxed{E} - \sum_{k=1}^{\Omega} n_k \varepsilon_k \quad \text{with} \quad n_k \equiv \frac{\partial E}{\partial \varepsilon_k}$$

↓

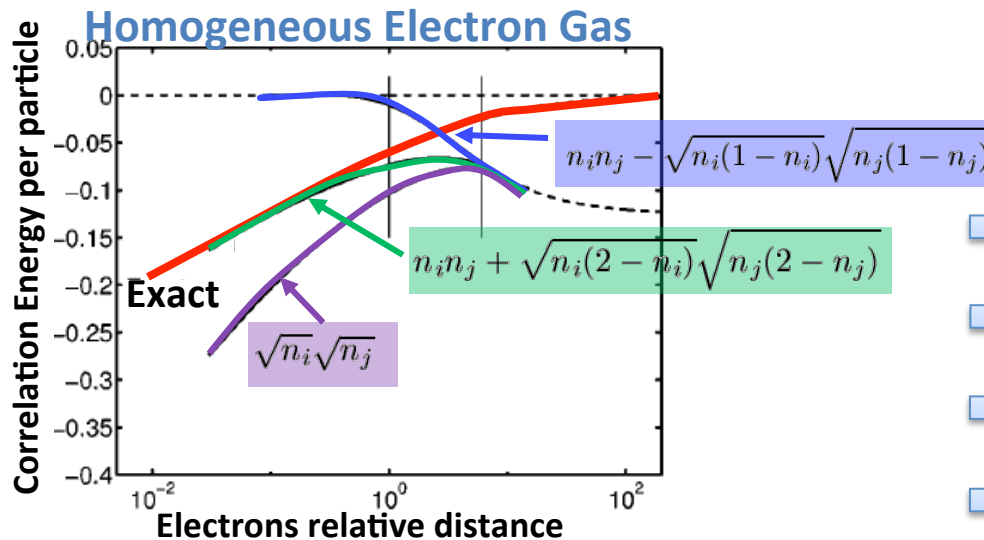
BCS, RPA, NLO...

- Richardson model
- 2- and 3-level Lipkin
- Some motivation for the Duflo-Zuker mass formula

## Density Matrix Functional Theory (DMFT) for electronic systems

Gilbert (1975). Klooster, <http://theochem.chem.rug.nl/publications/Abstracts.html#587>.

$$\mathcal{F}[\{\varphi_i\}, \{n_i\}] = \mathcal{E}[\{\varphi_i\}, \{n_i\}] - \mu \{Tr(\rho) - N\} - \sum_{ij} \lambda_{ij} (\langle \varphi_i | \varphi_j \rangle - \delta_{ij})$$



From Csanyi et al, PRA65 (2002)

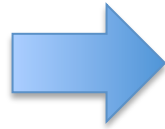
- ➡ DMFT for self-bound systems ?
- ➡ DMFT with symmetry breaking?
- ➡ DMFT for excited states?
- ➡ More than ground state : dynamics ?



# Density functional theory for projected BCS states

Starting points

$$|\Psi\rangle = P_N \prod_i (1 + x_i a_i^\dagger a_i) |-\rangle$$



$$n_i = N |x_i|^2 \frac{\sum_{(i_1, \dots, i_{N-1}) \neq (i)} |x_{i_1}|^2 \dots |x_{i_{N-1}}|^2}{\sum_{(i_1, \dots, i_N)} |x_{i_1}|^2 \dots |x_{i_N}|^2}$$

$$C_{ij} = N x_i^* x_j \frac{\sum_{(i_1, \dots, i_K) \neq (i,j)} |x_{i_1}|^2 \dots |x_{i_K}|^2}{\sum_{(i_1, \dots, i_N)} |x_{i_1}|^2 \dots |x_{i_N}|^2}$$

After some (6 months) efforts ...

$$x_i = \mathcal{F}(n_i)$$

Explicitly...

$$|x_i|^2 = \left( \frac{n_i}{1 - n_i} \right) [a_0 + a_1 n_i + \dots]$$

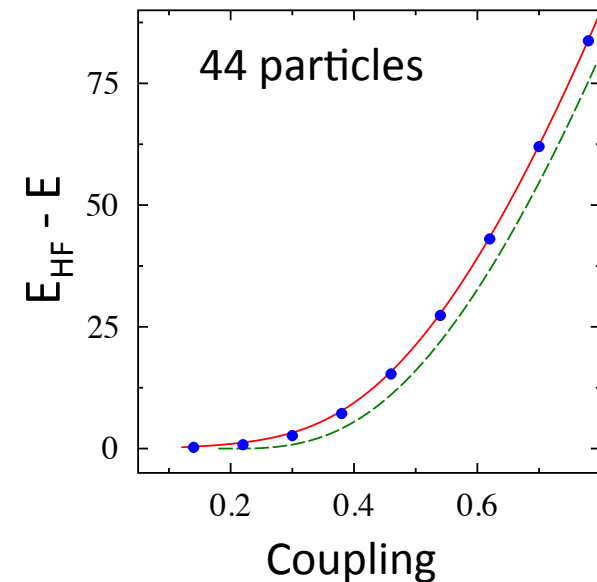
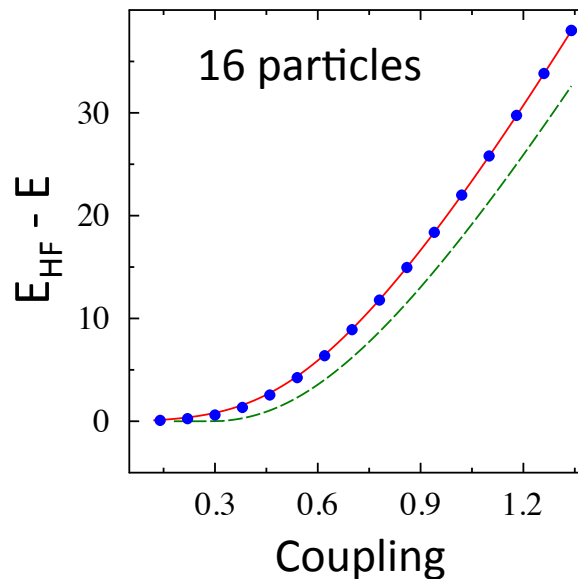
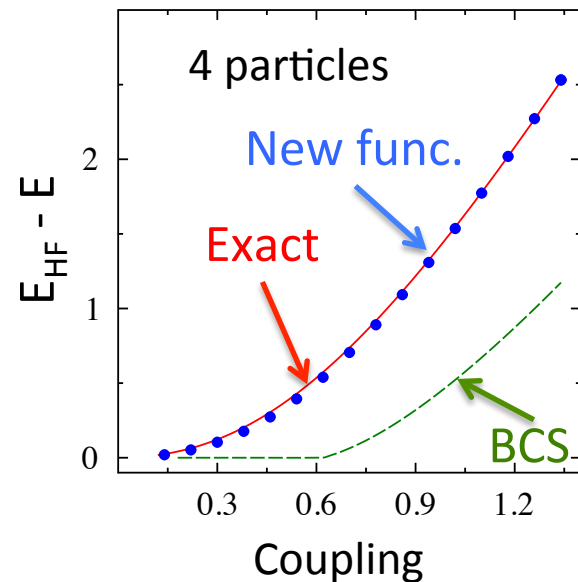
BCS

$$a_1 = -\frac{1}{N} \frac{1 - s_2^N}{1 - s_2}$$

$$a_0 = 1 - (s_2 - s_3) \frac{\partial a_1}{\partial s_2}$$

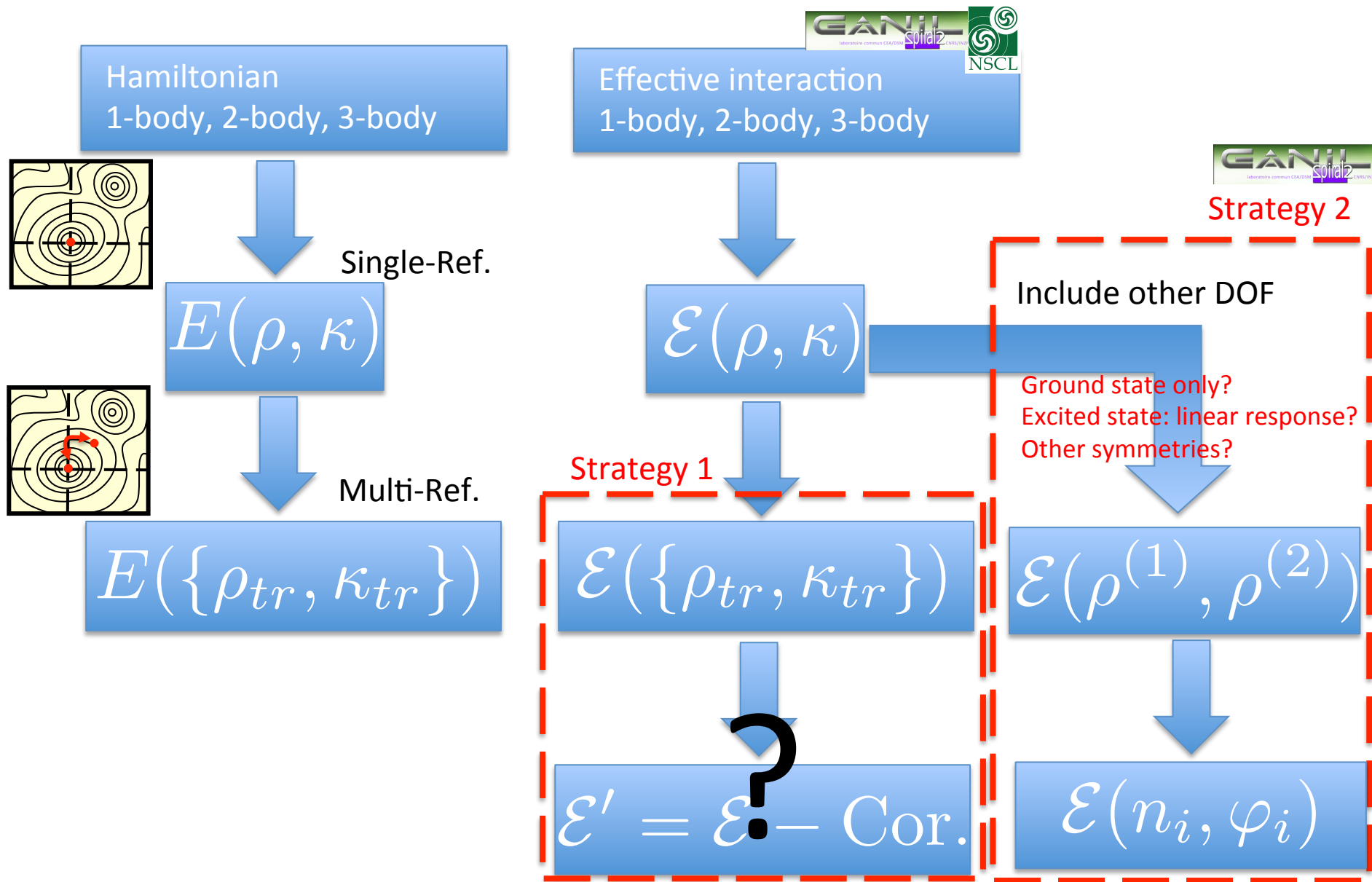
Application

Lacroix and Hupin, PRB82 (2010),  
Hupin, Lacroix, PRC83 (2011)



## Many-body technology

## Nuclear EDF phenomenology



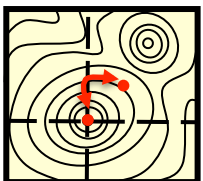
## Many-body technology Strategy 3

Hamiltonian  
1-body, 2-body, 3-body



Single-Ref.

$$E(\rho, \kappa)$$



Multi-Ref.

$$E(\{\rho_{tr}, \kappa_{tr}\})$$

Back to the original strategy

Constraints:

- start from a Hamiltonian
- no  $\rho^\alpha$

## Nuclear EDF phenomenology



Effective interaction  
1-body, 2-body, 3-body



$$\mathcal{E}(\rho, \kappa)$$



Strategy 1

$$\mathcal{E}(\{\rho_{tr}, \kappa_{tr}\})$$



$$\mathcal{E}' = \mathcal{E} - \text{Cor.}$$

Strategy 2

Include other DOF

Ground state only?  
Excited state: linear response?  
Other symmetries?



$$\mathcal{E}(\rho^{(1)}, \rho^{(2)})$$



$$\mathcal{E}(n_i, \varphi_i)$$

## Some random remarks on the DFT for fermions and non-integer powers of the density

Interacting fermions in different regimes of density

At low density

Galitskii formula

$$\left(\frac{m^*}{m}\right) = 1 + \frac{8}{15\pi^2}(7\ln 2 - 1)(k_F a_S)^2$$

$\downarrow$   
 $\rho^{2/3}$

Unitary gas

$$E[\rho] = E_0[\rho] \times \xi = \xi \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} \rho$$



$$\mathcal{E}[\rho, \nu] = \alpha \frac{\tau(\mathbf{r})}{2} + \beta c \rho^{5/3}(\mathbf{r}) + \gamma \frac{|\nu(\mathbf{r})|^2}{\rho(\mathbf{r})^{1/3}}$$

Lee-Yang formula

Bulgac, Forbes, ...

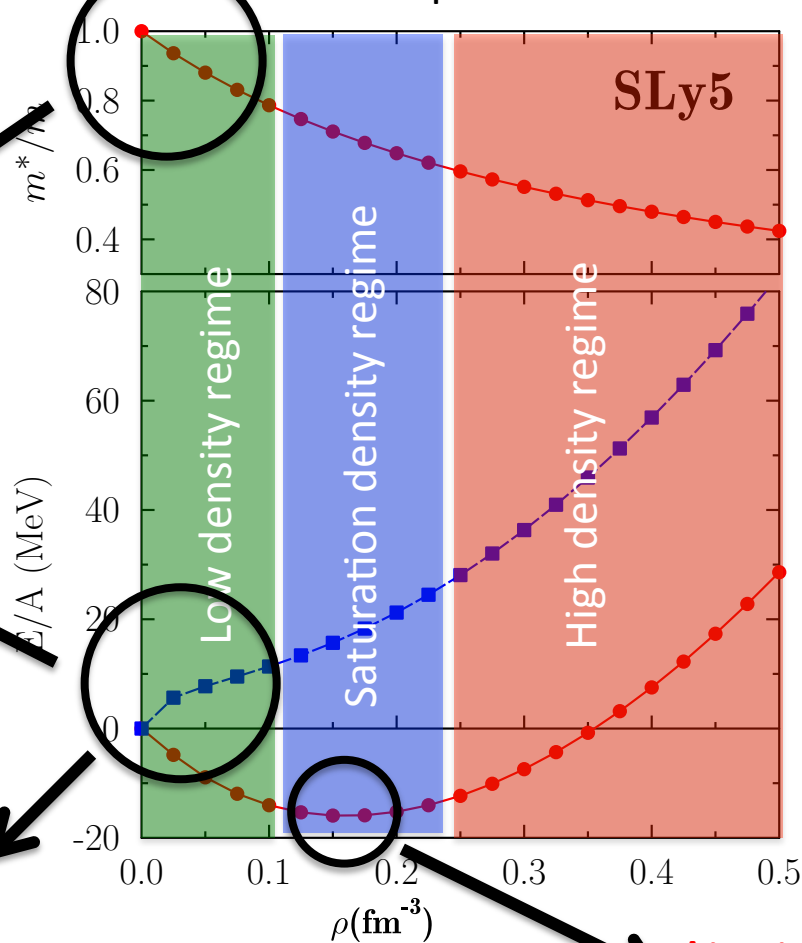
$$\frac{E}{A} = \frac{E_0}{A} \left( 1 + \frac{10}{9\pi}(a_S k_F) + \frac{4}{21\pi^2}(11 - 2\ln 2)(a k_F)^2 \right)$$



$$\int d^3r d^3r' \frac{C}{|\mathbf{r} - \mathbf{r}'|} \rho_i(\mathbf{r}, \mathbf{r}') \rho_j(\mathbf{r}, \mathbf{r}') \rho_k(\mathbf{r}, \mathbf{r}')$$

Gezerlis, Bertsch, ...

Fictitious equation of state



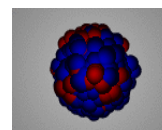
High density?

$$k_F \propto \rho^{1/3}$$

At saturation

$K_\infty, m^*$

$$E(\rho) = \dots + c \rho^{2+\alpha}$$



# Back to the construction of effective Hamiltonian

## With finite range interaction

For zero-range based interaction (see K. Bennaceur talk)

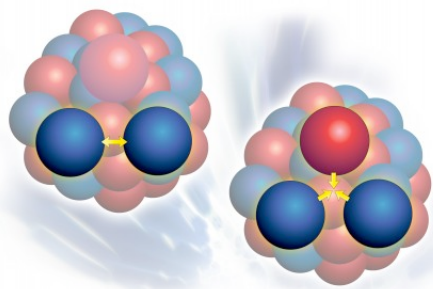
Goal: mimic the proper dependence of the energy *around saturation*

i.e. 
$$E(\rho) = \dots + c\rho^{2+\alpha}$$

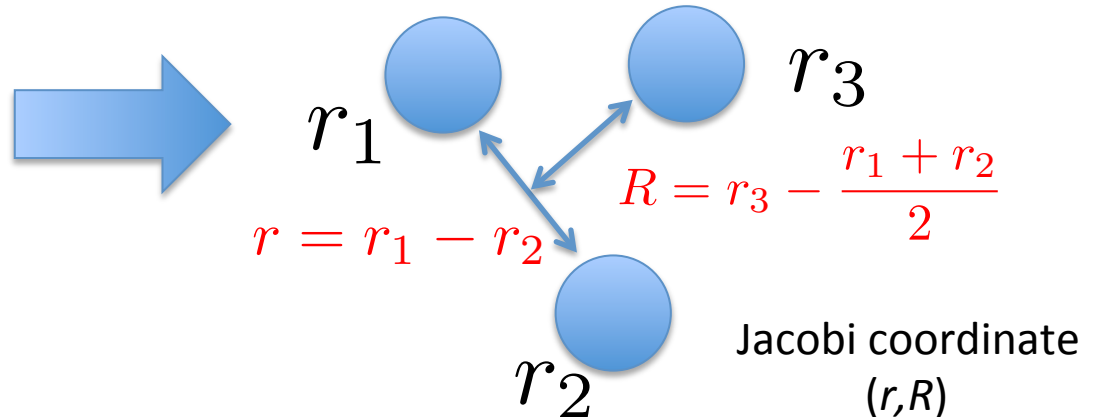
➡ Two-body finite-range interaction can mimic a dependence  $\rho^{1+\alpha}$

➡ Necessity to introduce at least three-body finite range interaction (see next)

Effective Hamiltonian  
with 2-body and 3-body



Semi-contact 3-body interaction



Idea: take a zero range in  $R$

Our starting point:

$$v_{ijk} = \left\{ V_0(r) + V_\sigma(r)P_\sigma + V_\tau(r)P_\tau + V_{\sigma\tau}(r)P_\sigma P_\tau \right\} \times \delta \left( \mathbf{r}_k - \left[ \frac{\mathbf{r}_i + \mathbf{r}_j}{2} \right] \right)$$

and (antisymmetrization)

$$\bar{v}_{ijk} = (v_{ijk} + v_{ikj} + v_{kij})/3$$



EDF = HF of the effective Hamiltonian

$$\begin{aligned} \frac{E^{\rho\rho\rho}}{A} &= c_1 \frac{\rho^2}{6} \int d^3r g(r) [1 - c_3 f(k_F r/2)^2] \\ &+ c_2 \frac{\rho^2}{6} \int d^3r g(r) [f(k_F r)^2 - c_3 f(k_F r) f(k_F r/2)^2], \end{aligned}$$

with  $f(x) = 3j_1(x)/x$

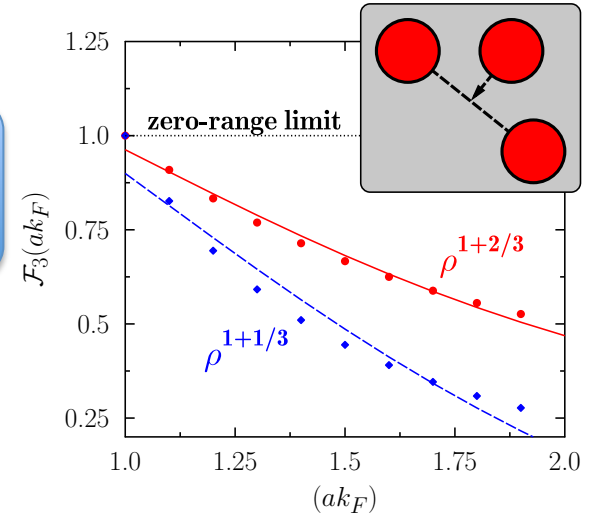
and  $g$ =Gaussian

	$c_1^{\text{SM}}$	$c_2^{\text{SM}}$	$c_1^{\text{NM}}$	$c_2^{\text{NM}}$	$c_1^{\text{PM}}$	$c_2^{\text{PM}}$	$c_1^{\text{PNM}}$	$c_2^{\text{PNM}}$
$v_0$	1	-1/4	1	-1/2	1	-1/2	1	-1
$v_\sigma$	1/2	-1/2	+1/2	-1	1	-1/2	1	-1
$v_\tau$	1/2	-1/2	1	-1/2	1/2	-1	1	-1
$v_{\sigma\tau}$	1/4	-1	+1/2	-1	1/2	-1	1	-1

Lacroix, Bennaceur, arXiv:1411.0360

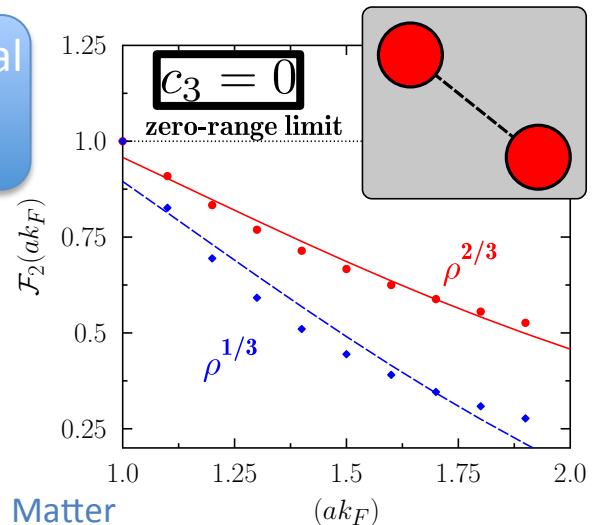
3-body functional

$$\frac{E^{\rho\rho\rho}}{A} = \frac{\rho^2}{6} \mathcal{F}_3(ak_F)$$



2-body functional

$$\frac{E^{\rho\rho\rho}}{A} = \frac{\rho}{2} \mathcal{F}_2(ak_F)$$



EDF = 2-body + 3-body (semi-contact)



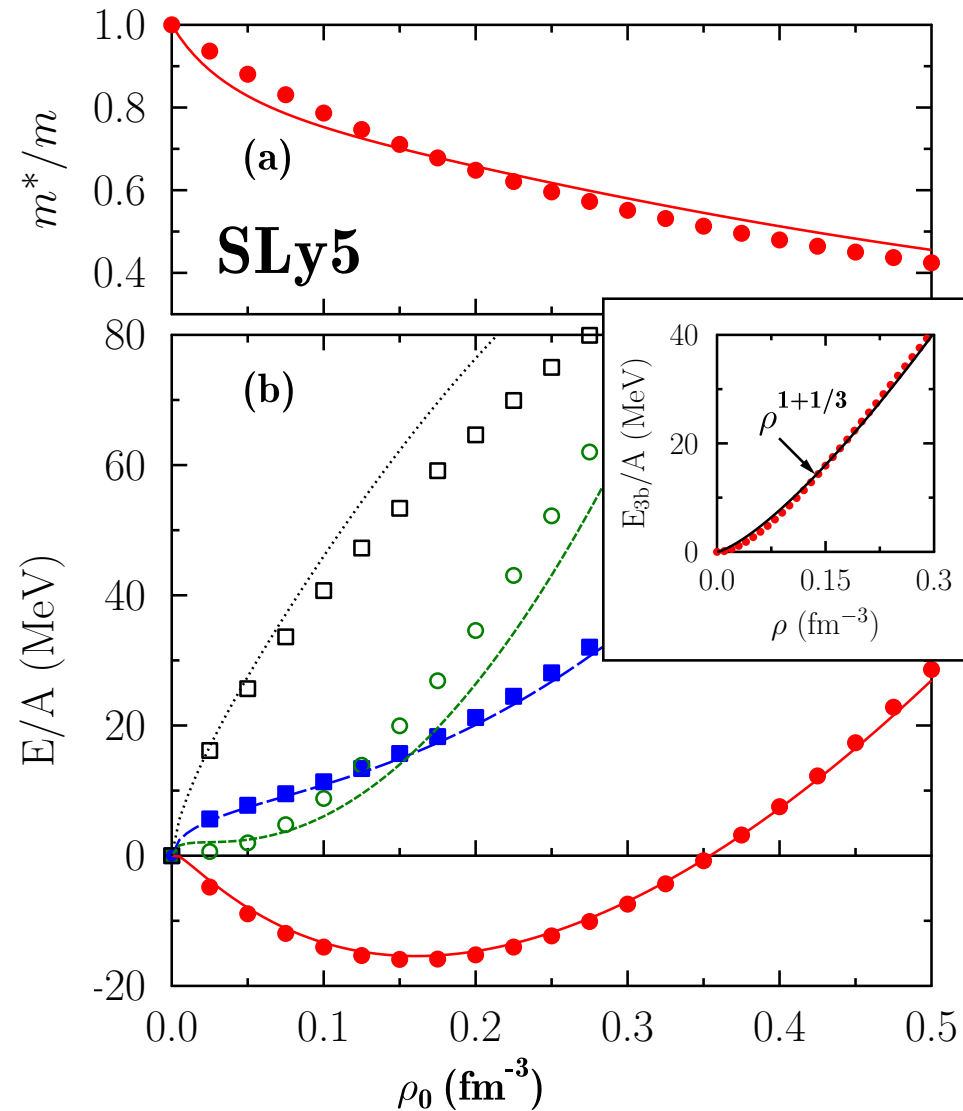
zero-range  
or  
finite-range

Illustration I : Skyrme (2-body)+3-body

Properties after a global fit:

$t_0$ (GeV fm <sup>3</sup> )	$t_1$ (GeV fm <sup>5</sup> )	$t_2$ (GeV fm <sup>5</sup> )	
-1.283 (0.05%)	0.874 (0.18%)	-0.808 (0.69%)	
$x_0$	$x_1$	$x_2$	
0.29 (1.8%)	0.51 (1.8%)	-1.08 (0.03%)	
$v_0$ (GeV)	$v_\sigma$ (GeV)	$v_\tau$ (GeV)	$v_{\sigma\tau}$ (GeV)
-15.03 (0.56%)	24.13 (0.82%)	24.99 (0.65%)	-42.12 (0.25%)

	SLy5	SLy5 <sup>3b</sup>
$\rho_{\text{sat}}$ [fm <sup>-3</sup> ]	0.160	0.161
$B/A$ [MeV]	-15.98	-15.42
$K_\infty$ [MeV]	229.92	236.59
$m^*/m$	0.697	0.691



EDF = 2-body + 3-body (semi-contact)



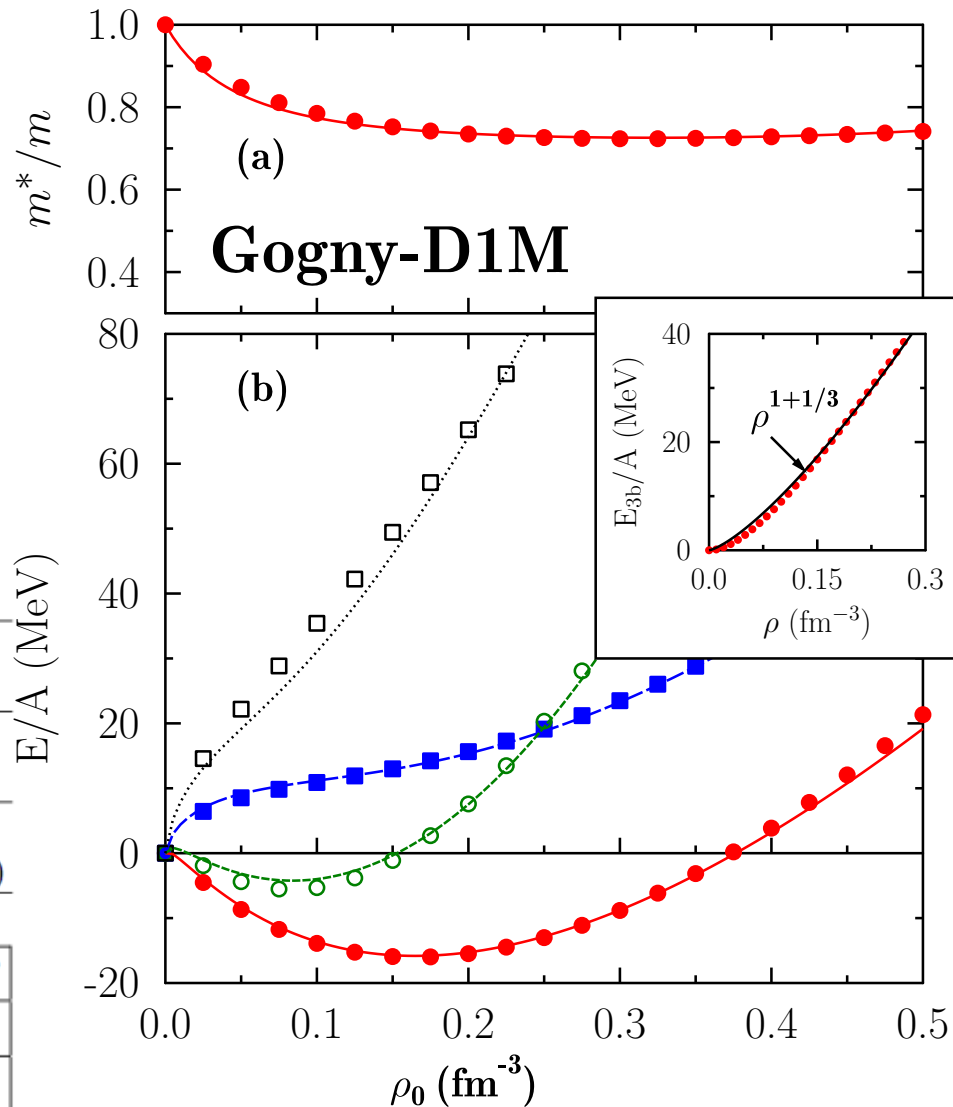
zero-range  
or  
finite-range

Illustration II : Gogny (2-body)+3-body

Properties after a global fit:

$W_1$ (GeV)	$B_1$ (GeV)	$H_1$ (GeV)	$M_1$ (GeV)
7.60 (1.3%)	2.54 (5.4%)	2.18 (5.8%)	-1.47 (8.0%)
$W_2$ (MeV)	$B_2$ (MeV)	$H_2$ (MeV)	$M_2$ (MeV)
-1047.97 (0.80%)	-86.54 (10%)	-681.06 (1.3%)	47.07 (10%)
$v_0$ (GeV)	$v_\sigma$ (GeV)	$v_\tau$ (GeV)	$v_{\sigma\tau}$ (GeV)
-14.65 (1.1%)	30.06 (0.82%)	19.02 (0.96%)	-42.71 (0.49%)

	SLy5	SLy5 <sup>3b</sup>	D1M	D1M <sup>3b</sup>
$\rho_{\text{sat}}$ [fm <sup>-3</sup> ]	0.160	0.161	0.165	0.165
$B/A$ [MeV]	-15.98	-15.42	-16.02	-15.82
$K_\infty$ [MeV]	229.92	236.59	224.98	228.58
$m^*/m$	0.697	0.691	0.746	0.744





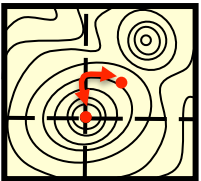
## Many-body technology Strategy 3

Hamiltonian  
1-body, 2-body, 3-body



Single-Ref.

$$E(\rho, \kappa)$$



Multi-Ref.

$$E(\{\rho_{tr}, \kappa_{tr}\})$$

Back to the original strategy

Constraints:

- start from an Hamiltonian
- no  $\rho^\alpha$

## Nuclear EDF phenomenology



Effective interaction  
1-body, 2-body, 3-body



$$\mathcal{E}(\rho, \kappa)$$



Strategy 1

$$\mathcal{E}(\{\rho_{tr}, \kappa_{tr}\})$$



$$\mathcal{E}' = \mathcal{E} - \text{Cor.}$$

Strategy 2

Include other DOF

Ground state only?  
Excited state: linear response?  
Other symmetries?



$$\mathcal{E}(\rho^{(1)}, \rho^{(2)})$$



$$\mathcal{E}(n_i, \varphi_i)$$



We should clarify what we are doing?

Certainly, but ...

It is also important to progress on the practical things ...

And to propose new physics ...

Should we really continue to say that what we are doing is unclear?

DFT, DMFT, RDM, 2-RDM, SR-EDF, MR-EDF ...

My feeling is that we exactly know what we are doing...

I also think we know where are the problems ...

My feeling is that we are doing too much noise...

If we know says that we have clarified many issues, and go ahead.

# More results of the correction

➡ **Correction strongly  
Constraint the functional**

only  $\rho$ ,  $\rho^2$ ,  $\rho^3$ ,  $\rho^4$ ,  $\rho^5$  could be used

No  $\rho^\alpha$  !!!

➡ **Correction leads to  
a new functional**

➡ **Application to other  
Symmetry breaking?**

(M. Bender et al, in prep?)

Application is complex but  
possible

