Towards a predictive Skyrme pseudo-potential-based energy density functional

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Workshop on "New developments
in Nuclear Energy-Density-Functional Models"
Espace de Structure Nucléaire Théorique, Saclay, 25 November 2014

People involved

- Identifying problems with general functionals in MR EDF:
  M. B., T. Duguet, D. Lacroix, B. Avez, B. Bally, P.-H. Heenen
- construction of a Skyrme-type 3-body pseudo-potential:
  J. Sadoudi (thesis work in Saclay + postdoc in Bordeaux),
  T. Duguet, J. Meyer, M. Bender

Symmetry restoration

particle-number projector

$$\hat{P}_{N_0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi_N \; \underbrace{e^{-i\phi_N N_0}}_{\text{weight}} \quad \overbrace{e^{i\phi_N \hat{N}}}^{\text{rotation in gauge space}}$$

angular-momentum restoration operator

$$\hat{P}^J_{MK} = \frac{2J+1}{16\pi^2} \int_0^{4\pi} d\alpha \int_0^{\pi} d\beta \; \sin(\beta) \int_0^{2\pi} d\gamma \; \underbrace{\mathcal{D}^{*J}_{MK}(\alpha,\beta,\gamma)}_{\text{Wigner function}} \quad \overbrace{\hat{R}(\alpha,\beta,\gamma)}^{\text{rotation in real space}}$$

K is the z component of angular momentum in the body-fixed frame.
Projected states are given by

$$|JMq\rangle = \sum_{K=-J}^{+J} f_J(K) \; \hat{P}^J_{MK} \; \hat{P}^Z \; \hat{P}^N \; |\mathrm{MF}(q)\rangle = \sum_{K=-J}^{+J} f_J(K) \; |JM(qK)\rangle$$

$f_J(K)$ is the weight of the component K and determined variationally

Axial symmetry (with the z axis as symmetry axis) allows to perform the α and γ integrations analytically, while the sum over K collapses, $f_J(K) \sim \delta_{K0}$

Configuration mixing by the symmetry-restored Generator Coordinate Method

Superposition of projected self-consistent mean-field states $|\mathrm{MF}(\mathbf{q})\rangle$ differing in a set of collective and single-particle coordinates $\mathbf{q}$

$$|NZJM\nu\rangle = \sum_{\mathbf{q}} \sum_{K=-J}^{+J} f^{NZ}_{J,\kappa}(\mathbf{q},K)\, \hat{P}^J_{MK}\, \hat{P}^Z\, \hat{P}^N\, |\mathrm{MF}(\mathbf{q})\rangle = \sum_{\mathbf{q}} \sum_{K=-J}^{+J} f^{NZ}_{J\nu}(\mathbf{q},K)\, |NZ\, JM(\mathbf{q}K)\rangle$$

with weights $f^{NZ}_{J\nu}(\mathbf{q},K)$.

$$\frac{\delta}{\delta f^*_{J\nu}(\mathbf{q},K)} \frac{\langle NZ\, JM\nu|\hat{H}|NZ\, JM\nu\rangle}{\langle NZ\, JM\nu|NZ\, JM\nu\rangle} = 0 \quad \Rightarrow \quad \text{Hill-Wheeler-Griffin equation}$$

$$\sum_{\mathbf{q}'} \sum_{K'=-J}^{+J} \left[\mathcal{H}^{NZ}_J(\mathbf{q}K,\mathbf{q}'K') - E^{NZ}_{J,\nu}\, \mathcal{I}^{NZ}_J(\mathbf{q}K,\mathbf{q}'K')\right] f^{NZ}_{J,\nu}(\mathbf{q}'K') = 0$$

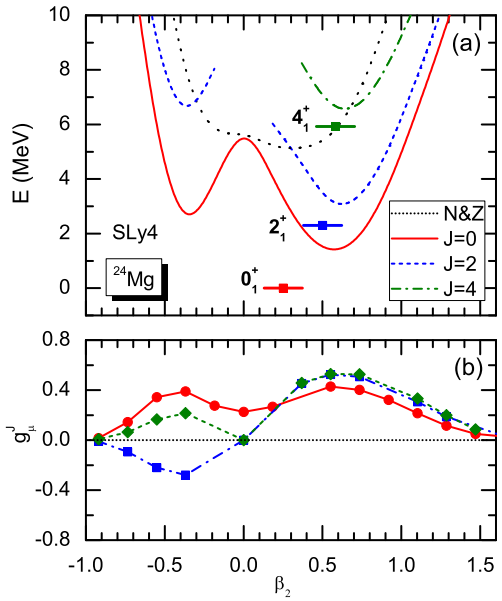
with

$$\mathcal{H}_J(\mathbf{q}K,\mathbf{q}'K') = \langle NZ\, JM\, \mathbf{q}K|\hat{H}|NZ\, JM\, \mathbf{q}'K'\rangle \quad \text{energy kernel}$$
$$\mathcal{I}_J(\mathbf{q}K,\mathbf{q}'K') = \langle NZ\, JM\, \mathbf{q}K|NZ\, JM\, \mathbf{q}'K'\rangle \quad \text{norm kernel}$$

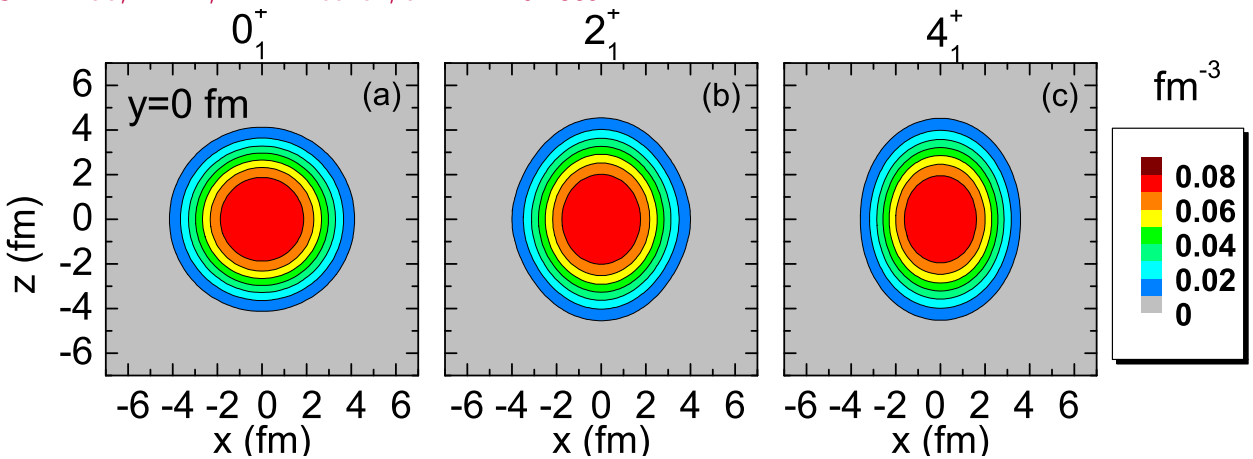
Angular-momentum projected GCM gives the

- correlated ground state for each value of J
- spectrum of excited states for each J

A comment on laboratory densities



J. M. Yao, M. B., P.-H. Heenen, arXiv:1410.2389



Contour plots of the 3D proton densities (in fm^{-3}) in the $y = 0$ plane for the 0_1^+ (a), 2_1^+ (b), 4_1^+ (c) states (with $M = 0$) in ^{24}Mg.

Transition density in the laboratory between GCM states $|J_i M_i \mu_i\rangle$ and $|J_f M_f \mu_f\rangle$ assuming axial HFB states

$$\rho^{J_f M_f \mu_f}_{J_i M_i \mu_i}(\mathbf{r}) = \sum_{q_f, q_i} f^{J_f *}_{\mu_f, q'} \langle q' | \hat{P}^{J_f}_{0M_f} \hat{\rho}(\mathbf{r}) \hat{P}^{J_i \dagger}_{0M_i} \hat{P}^N \hat{P}^Z | q\rangle f^{J_i 0}_{\mu_i, q}$$

with

$$\langle q' | \hat{P}^{J_f}_{0M_f} \hat{\rho}(\mathbf{r}) \hat{P}^{J_i \dagger}_{0M_i} \hat{P}^N \hat{P}^Z | q\rangle$$

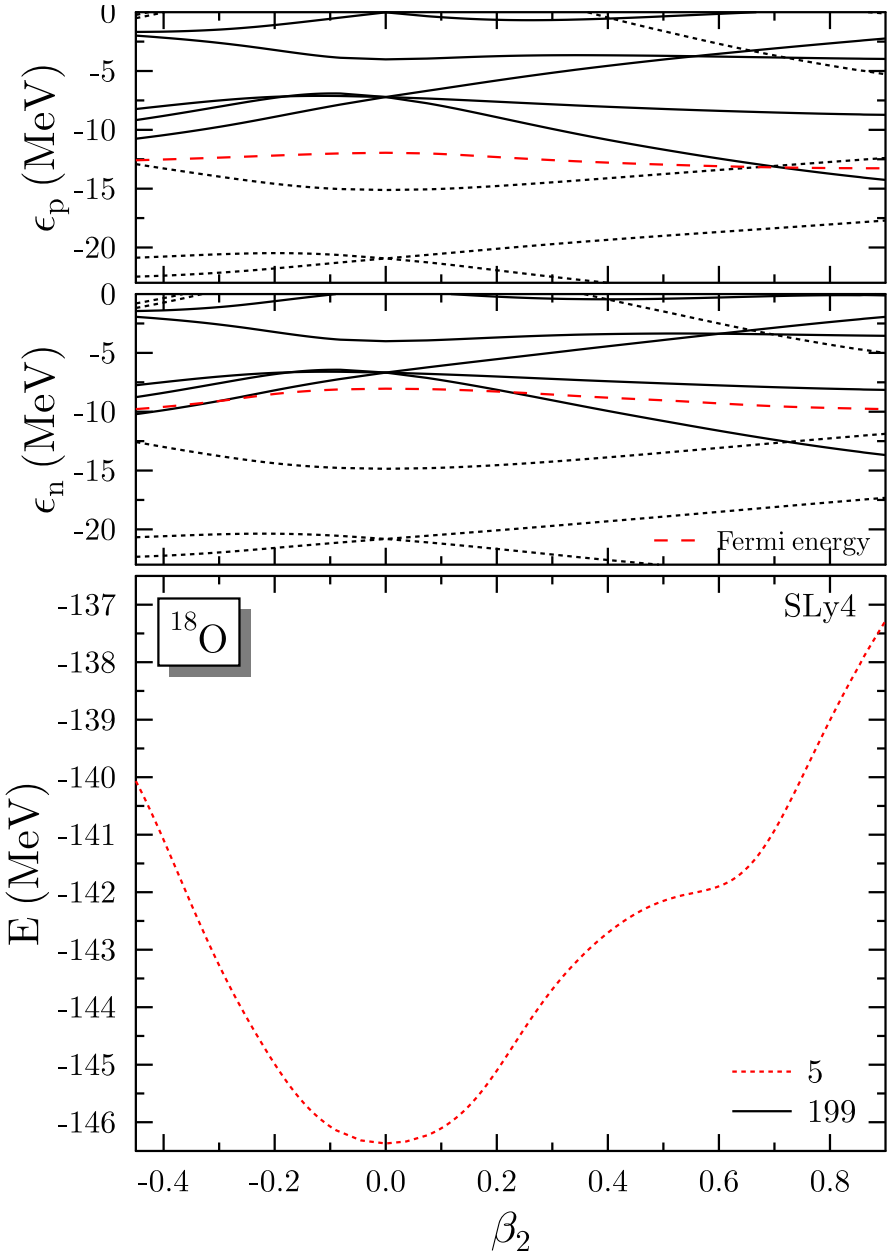
$$= \frac{\hat{J}_i^2 \hat{J}_f^2}{(8\pi^2)^2} \int d\Omega' \; D^{J_f *}_{0M_f}(\Omega') \sum_K D^{J_i}_{K0}(\Omega') \int d\Omega'' \; D^{J_i}_{0K}(\Omega'') \langle q' | \hat{\rho}(\tilde{\mathbf{r}}_{\Omega'}) \hat{P}^N \hat{P}^Z \hat{R}^\dagger(\Omega'') | q\rangle$$

$$\equiv \frac{\hat{J}_f^2}{8\pi^2} \int d\Omega' \; D^{J_f *}_{0M_f}(\Omega') \sum_K D^{J_i}_{KM_i}(\Omega') \hat{R}^\dagger(\Omega') \rho^{J_f J_i K 0}_{q' q}(\mathbf{r})$$

For the density of the GCM state $|JM\mu\rangle$ one obtains

$$\rho^{JM\mu}_{JM\mu}(\mathbf{r}) = \sum_{q_f, q_i} f^{J*}_{\mu, q'} f^{J0}_{\mu, q} \sum_\lambda Y_{\lambda 0}(\hat{\mathbf{r}}) \langle JM\lambda 0 | JM\rangle \sum_K \langle J0\lambda K | JK\rangle \int d\hat{\mathbf{r}}' \; \rho^{JJK0}_{q' q}(r, \hat{\mathbf{r}}') Y^*_{\lambda K}(\hat{\mathbf{r}}')$$

What kind of functionals to use? Problems with existing ones



- pure particle-number projection

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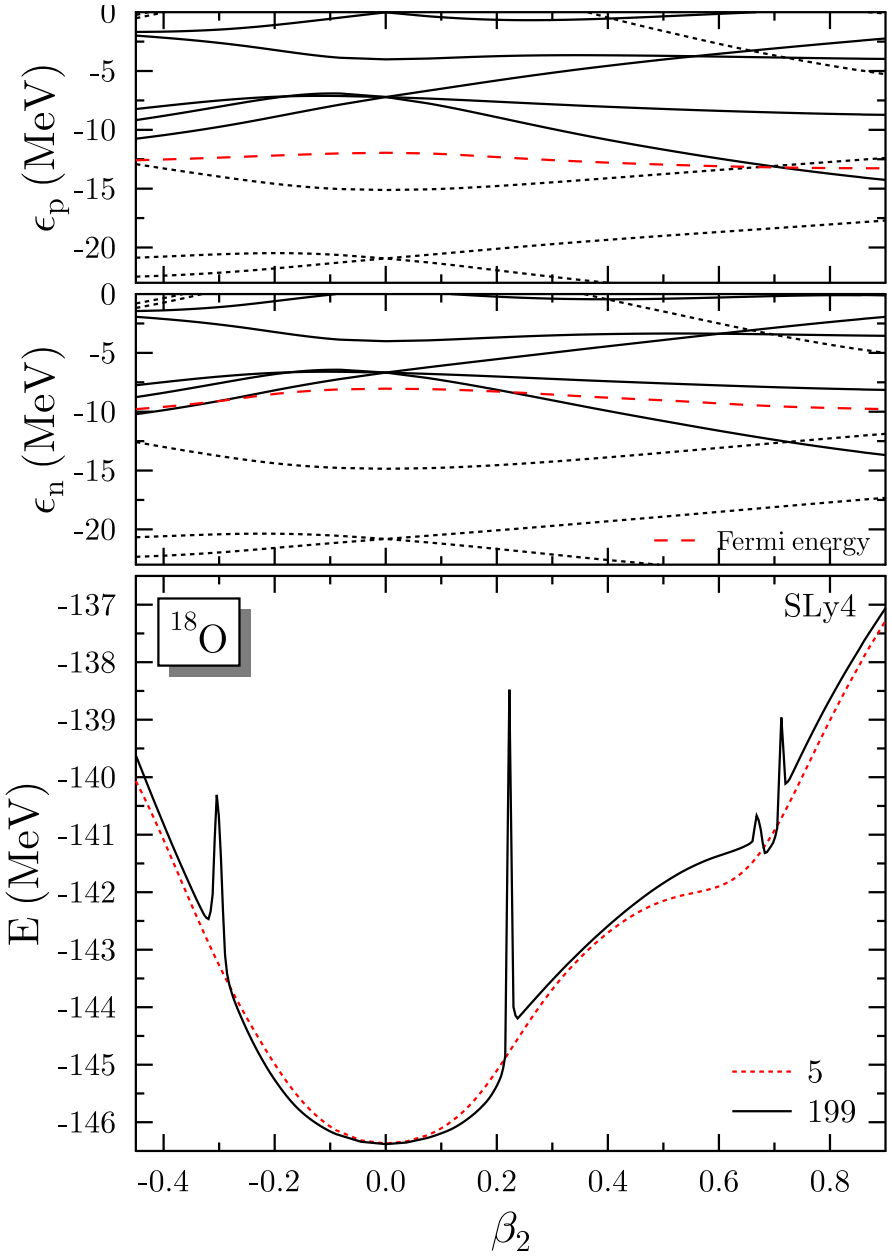

- pure particle-number projection
- first hints from Hamiltonian-based approaches: Dönau, PRC 58 (1998) 872; Almehed, Frauendorf, Dönau, PRC 63 (2001) 044311; Anguiano, Egido, Robledo NPA696 (2001) 467
- First analysis in a strict energy density functional (EDF) framework and of EDF-specific consequences by Dobaczewski, Stoitsov, Nazarewicz, Reinhard, PRC 76 (2007) 054315
- Further analysis of the EDF case by Lacroix, Duguet, Bender, PRC 79 (2009) 044318; Bender, Duguet, Lacroix, PRC 79 (2009) 044319; Duguet, Bender, Bennaceur, Lacroix, Lesinski, PRC 79 (2009) 044320; Bender, Avez, Duguet, Heenen, Lacroix, *in preparation*

The origin of the problem in a nutshell

- All standard energy density functionals (EDF) used for mean-field models and beyond do not correspond to the expectation value of a Hamiltonian for at least one of the following reasons:
  - density dependences
  - the use of different effective interactions in the particle-hole and pairing parts of the energy functional
  - the omission, approximation or modification of specific exchange terms

  that are all introduced for phenomenological reasons and/or the sake of numerical efficiency.
- consequence: breaking of the exchange symmetry ("Pauli principle") under particle exchange when calculating the energy, leading to non-physical interactions of a given nucleon or pair of nucleons with itself, or of three nucleons among themselves etc.
- the resulting self-interactions and self-pairing-interactions remain (usually) hidden in the mean field
- in the extension to symmetry-restored GCM, these terms cause
  - discontinuities and divergences in symmetry-restored energy surfaces
  - breaking of sum rules in symmetry restoration
  - potentially multi-valued EDF in case of standard density-dependences

Functionals corresponding to "true Hamiltonians" vs. "general" functionals

True contact pseudo-potential $t_0\,(1 + x_0 \hat{P}_\sigma)\,\delta(\mathbf{r} - \mathbf{r}')$

$$\begin{aligned}\mathcal{E} &= \int d^3r \left\{ \tfrac{3}{8}\, t_0\, \rho_0^2(\mathbf{r}) - \tfrac{1}{8}\, t_0\, (1 + 2x_0)\, \rho_1^2(\mathbf{r}) - \tfrac{1}{8}\, t_0\, (1 - 2x_0)\, \mathbf{s}_0^2(\mathbf{r}) \right. \\ &\quad \left. - \tfrac{1}{8}\, t_0\, \mathbf{s}_1^2(\mathbf{r}) + \tfrac{1}{8}\, t_0\, (1 + x_0)\, \check{\mathbf{s}}_0(\mathbf{r}) \cdot \check{\mathbf{s}}_0^*(\mathbf{r}) + \tfrac{1}{8}\, t_0\, (1 - x_0)\, \check{\rho}_1(\mathbf{r})\, \check{\rho}_1^*(\mathbf{r}) \right\}\end{aligned}$$

(see Perlinska *et al.* PRC 69 (2004) 014316 for definition of $\check{\mathbf{s}}_0(\mathbf{r})$ and $\check{\rho}_1(\mathbf{r})$)

Functional with contact vertices:

$$\begin{aligned}\mathcal{E} &= \int d^3r \left\{ C_0^\rho[\rho_0, \ldots]\, \rho_0^2(\mathbf{r}) + C_1^\rho[\rho_0, \ldots]\, \rho_1^2(\mathbf{r}) + C_0^s[\rho_0, \ldots]\, \mathbf{s}_0^2(\mathbf{r}) \right. \\ &\quad \left. + C_1^s[\rho_0, \ldots]\, \mathbf{s}_1^2(\mathbf{r}) + C_0^{\check{s}}[\rho_0, \ldots]\, \check{\mathbf{s}}_0(\mathbf{r}) \cdot \check{\mathbf{s}}_0^*(\mathbf{r}) + C_1^{\check{\rho}}[\rho_0, \ldots]\, \check{\rho}_1(\mathbf{r})\, \check{\rho}_1^*(\mathbf{r}) \right\}\end{aligned}$$

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Coulomb interaction $\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}$

$$\mathcal{E} = \frac{1}{2}\iint d^3r\,d^3r'\,\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}\Big[\rho_p(\mathbf{r})\rho_p(\mathbf{r}') - \rho_p(\mathbf{r},\mathbf{r}')\rho_p(\mathbf{r}',\mathbf{r}) + \kappa_p^*(\mathbf{r},\mathbf{r}')\kappa_p(\mathbf{r},\mathbf{r}')\Big]$$

Approximate Coulomb functionals

$$\mathcal{E} = \frac{e^2}{2}\iint d^3r\,d^3r'\,\frac{\rho_p(\mathbf{r})\rho_p(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} - \frac{3e^2}{4}\left(\frac{3}{\pi}\right)^{1/3}\int d^3r\,\rho_p^{4/3}(\mathbf{r})$$

Forgotten constraints in general functionals

Hamiltonian

$$\hat{H} = \sum_{ij} a_i^\dagger \, t_{ij}^{(1)} \, a_j + \frac{1}{2!} \sum_{ijkl} a_i^\dagger \, a_j^\dagger \, v_{ijkl}^{(2)} \, a_l \, a_k + \frac{1}{3!} \sum_{ijklmn} a_i^\dagger \, a_j^\dagger \, a_k^\dagger \, v_{ijklmn}^{(3)} \, a_n \, a_m \, a_l + \cdots$$

Energy

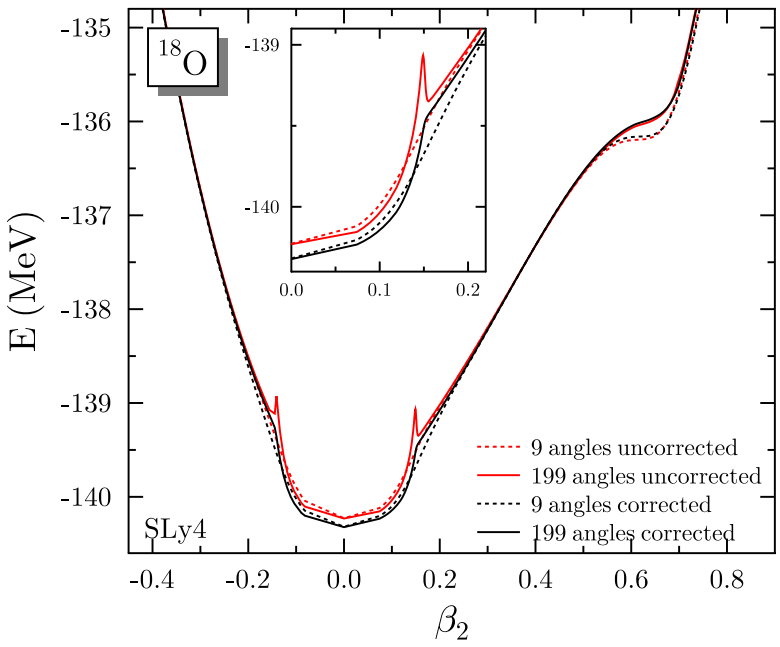
$$\begin{aligned} \langle \hat{H} \rangle &= \sum_{ij} t_{ij}^{(1)} \, \langle a_i^\dagger \, a_j \rangle + \frac{1}{2!} \sum_{ijkl} v_{ijkl}^{(2)} \, \langle a_i^\dagger \, a_j^\dagger \, a_l \, a_k \rangle + \frac{1}{3!} \sum_{ijklmn} v_{ijklmn}^{(3)} \, \langle a_i^\dagger \, a_j^\dagger \, a_k^\dagger \, a_n \, a_m \, a_l \rangle \\ &= \sum_{ij} t_{ij}^{(1)} \, \rho_{ji}^{(1)} + \frac{1}{2!} \sum_{ijkl} v_{ijkl}^{(2)} \, \rho_{lkji}^{(2)} + \frac{1}{3!} \sum_{ijklmn} v_{ijklmn}^{(3)} \, \rho_{nmlkji}^{(3)} + \cdots \end{aligned}$$

For BCS/Bogoliubov-type product states

$$\begin{aligned} \rho_{lkji}^{(2)} &= \rho_{lj}^{(1)} \, \rho_{ki}^{(1)} - \rho_{li}^{(1)} \, \rho_{kj}^{(1)} + \kappa_{ij}^* \, \kappa_{kl} \\ \rho_{nmlkji}^{(3)} &= \cdots \end{aligned}$$

Second problem: non-analytical density dependences

Non-viability of non-integer density dependences



Duguet, Lacroix, M. B., Bennaceur, Lesinski, PRC 79 (2009) 044320

- in symmetry restored GCM, the local densities $\rho^{qq'}(\mathbf{r})$ are in general complex
- $\left[\rho^{qq'}(\mathbf{r})\right]^{\alpha}$ is a multi-valued non-analytical function
- spurious contribution from branch cuts (see Dobaczewski*et al.* PRC 76 (2007) 054315, and Duguet *et al.* PRC 79 (2009) 044320 for complex plane analysis)
- (partial) workaround when conserving specific symmetries: use particle-number projected densities for density dependence instead (stategy currently used by L. Egido and collaborators).
  - Difficult to justify formally.
  - Does not bypass the problem anymore when using time-reversal-invariance breaking reference states.

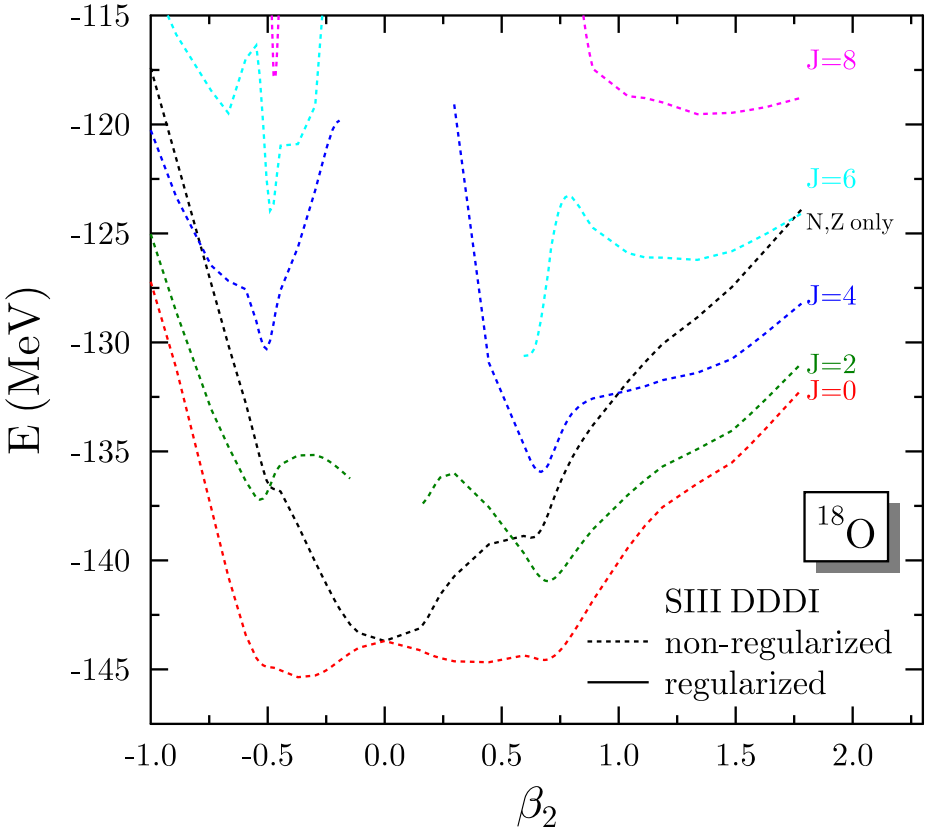
What to do?

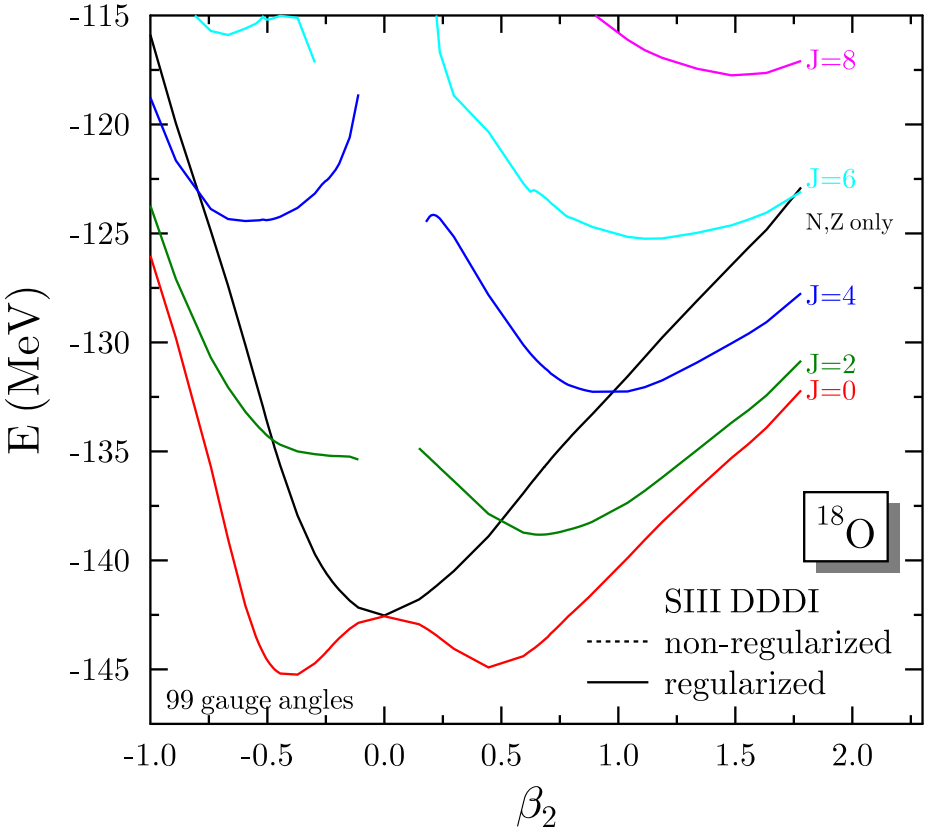
1. constructing the EDF as expectation value of a strict Hamiltonian. New problem: numerically very costly due to Coulomb exchange & pairing; no available parameterizations of high quality (the difficulties to construct such parametrizations was the main motivation to use EDFs in the 1970s).
2. construct the EDF from a density-dependent Hamiltonians with special treatment of the density entering density dependent terms for which numerically efficient high-quality parameterizations can be easily constructed. Problem: numerically very costly due to Coulomb exchange & pairing; cannot be defined for all possible confifuration mixing [Robledo, J. Phys. G 37 (2010) 064020].
3. introducing a regularization scheme of the EDF that allows for the use of (almost) standard functionals [Lacroix, Duguet, & Bender, PRC 79 (2009) 044318] for which numerically efficient high-quality parameterizations can be easily constructed [Washiyama, Bennaceur, Avez, Bender, Heenen, & Hellemans, PRC 86 (2012) 054309]. Problem: complicated formalism.

We tried the last option first, unleashing a monster.

The regularisation: it *almost* works

Usual number of Euler and gauge angles:

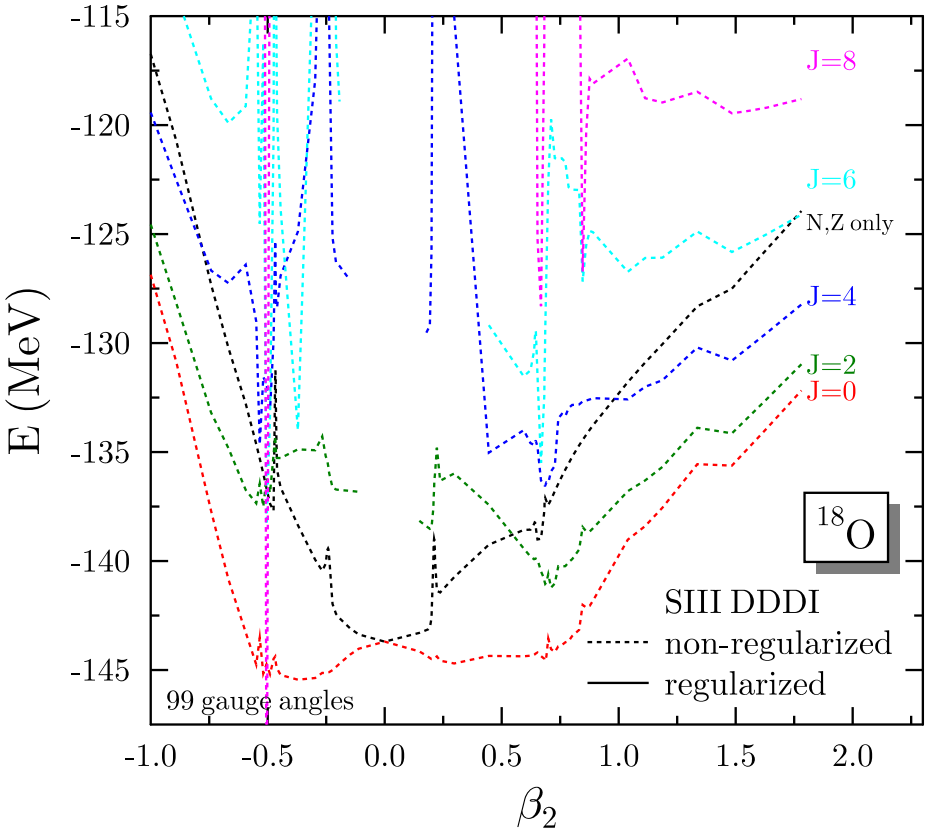


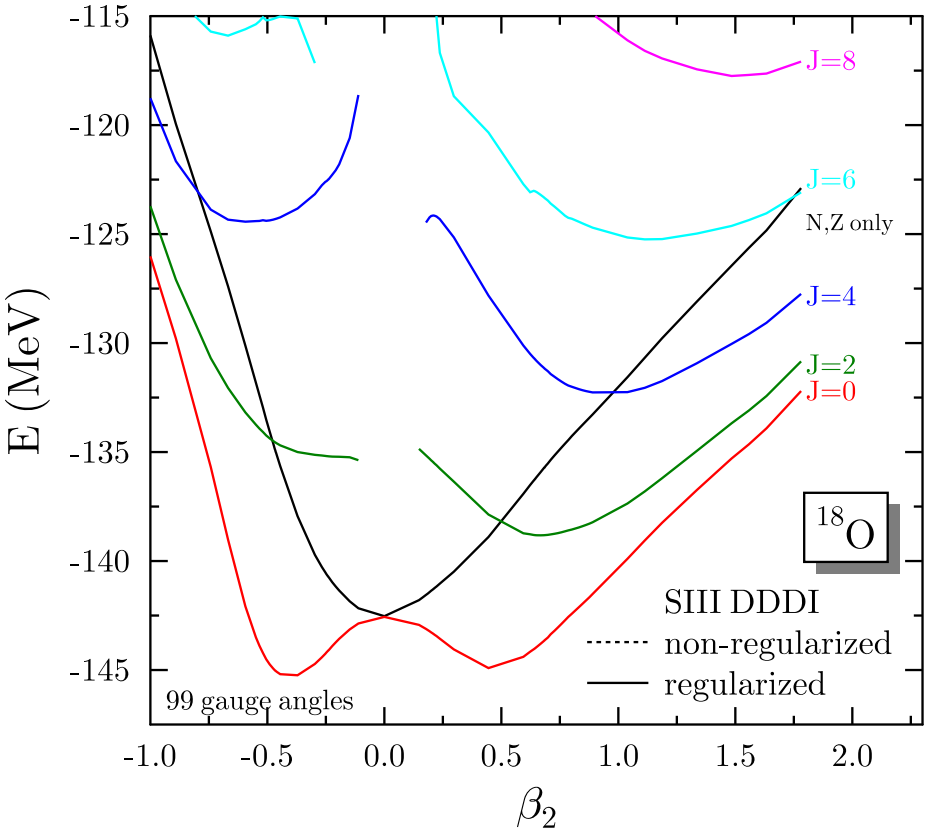


M. B., B. Avez, T. Duguet, P.-H. Heenen, D. Lacroix, unpublished

The regularisation: it *almost* works

Usual number of Euler and ridiculously large number of gauge angles:

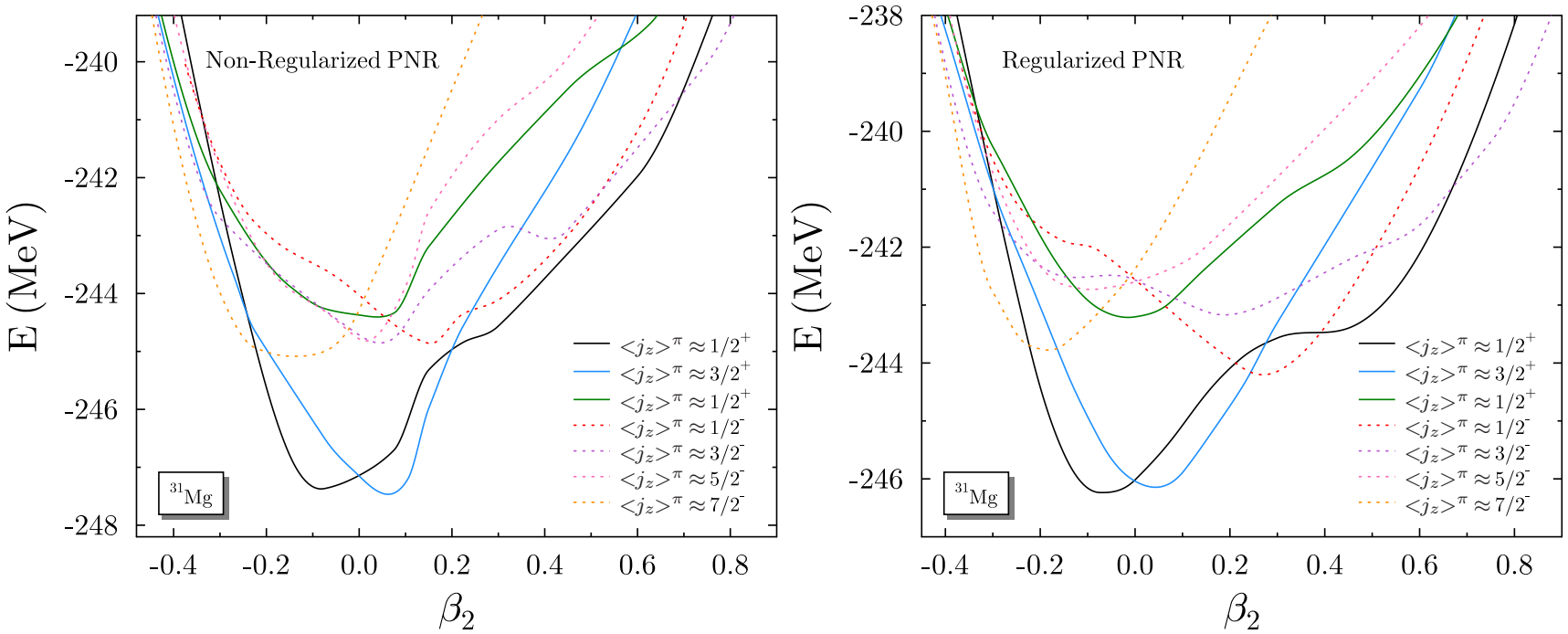




M. B., B. Avez, T. Duguet, P.-H. Heenen, D. Lacroix, unpublished

$\Rightarrow$ dependence on the discretization that becomes visible only when using unreasonably fine discretizations

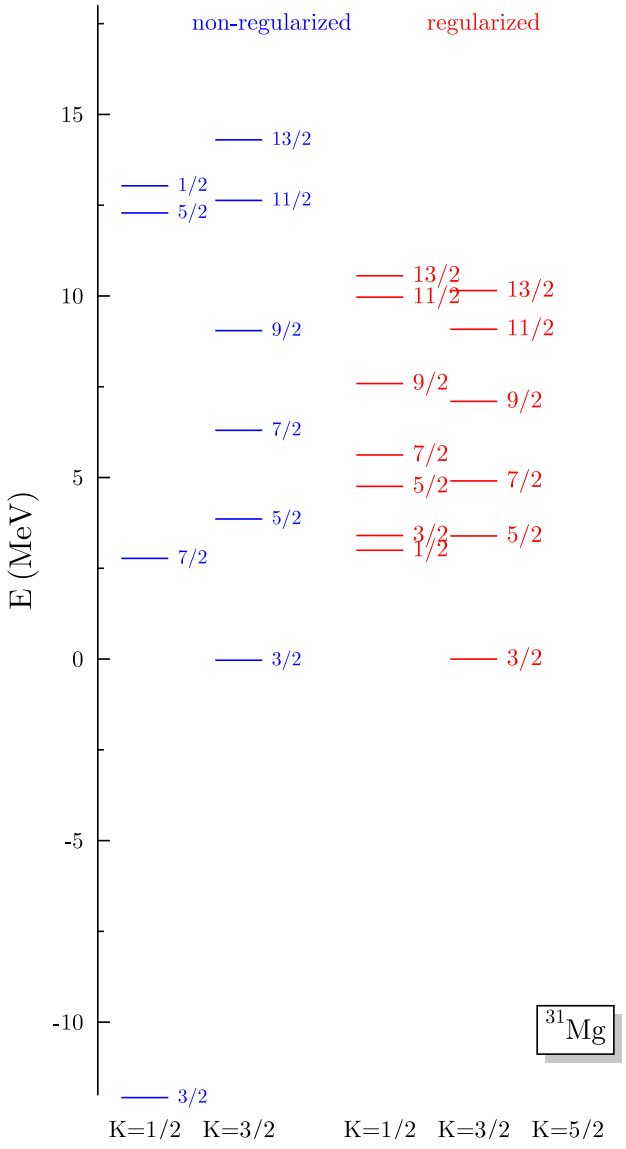
Particle-number restoration of ^{31}Mg



B. Bally, B. Avez, M. B., P.-H. Heenen (unpublished)

All states are constructed as blocked one-quasiparticle HFB states

Non-regularized MR EDF with general functionals can give unphysical spectra



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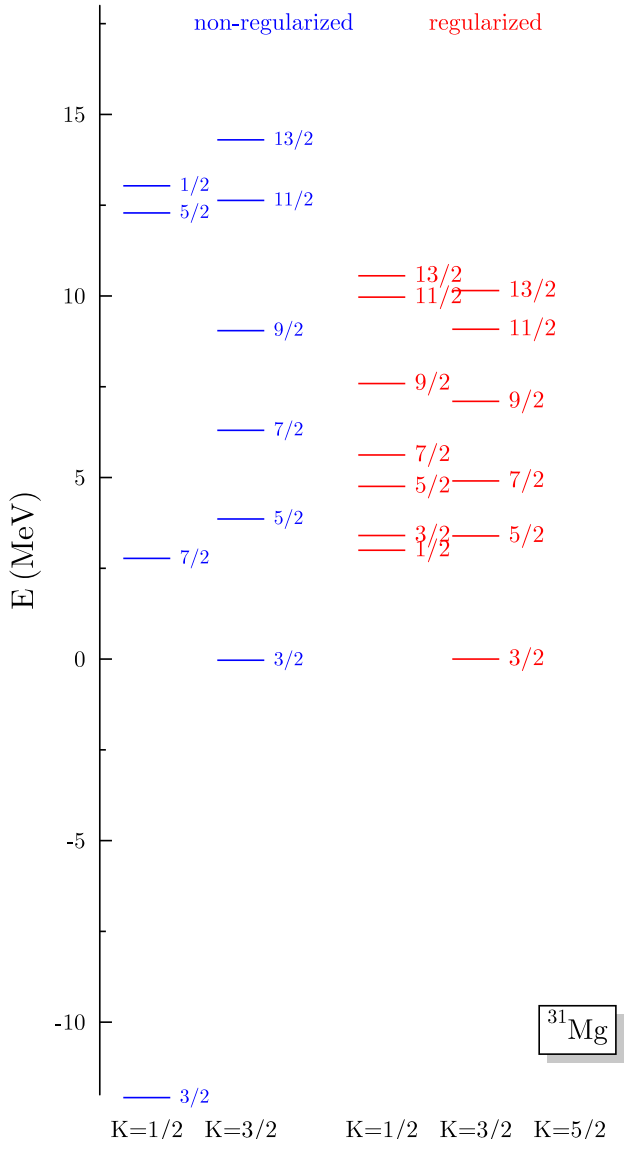
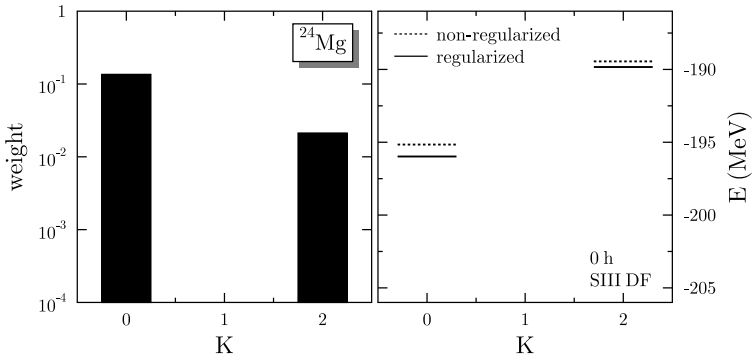


B. Bally, B. Avez, M. B., P.-H. Heenen, unpublished

J	K	weight	E_{nonreg}	regul	E_{reg}
1	1	0.001006	−234.071	10.037	−244.108
3	1	0.001809	−259.183	−15.481	−243.702
5	1	0.001820	−234.818	7.531	−242.349
7	1	0.001797	−244.332	−2.848	−241.484
9	1	0.001271	−267.849	−28.332	−239.517
11	1	0.000902	−201.965	35.172	−237.137
13	1	0.000544	−336.901	−100.352	−236.549
3	3	0.039376	−247.137	−0.032	−247.105
5	3	0.030730	−243.247	0.467	−243.714
7	3	0.023390	−240.805	1.395	−242.199
9	3	0.013372	−238.060	1.948	−240.007
11	3	0.007914	−234.473	3.548	−238.021
13	3	0.004087	−232.805	4.150	−236.956
5	5	0.000015	−582.874	−371.932	−210.942
7	5	0.000014	−103.953	94.559	−198.512
9	5	0.000010	−127.945	95.658	−223.603
11	5	0.000007	860.956	1075.711	−214.755
13	5	0.000004	−530.816	−334.758	−196.058
7	7	0.000005	790.818	977.088	−186.270
9	7	0.000004	−2215.259	−1916.331	−298.928
11	7	0.000003	−3657.395	−3321.042	−336.353
13	7	0.000002	−4077.760	−3715.879	−361.881

Regularized MR EDF using general functionals also can give unphysical results

- small components (still) take unphysical values when regularising which can be demonstrated using a cranking constraint to vary the size of $K = 1$ components
- example shown in plots: K decomposition in projection on $J = 2$ of cranked HFB states cranked to different rotational frequencies



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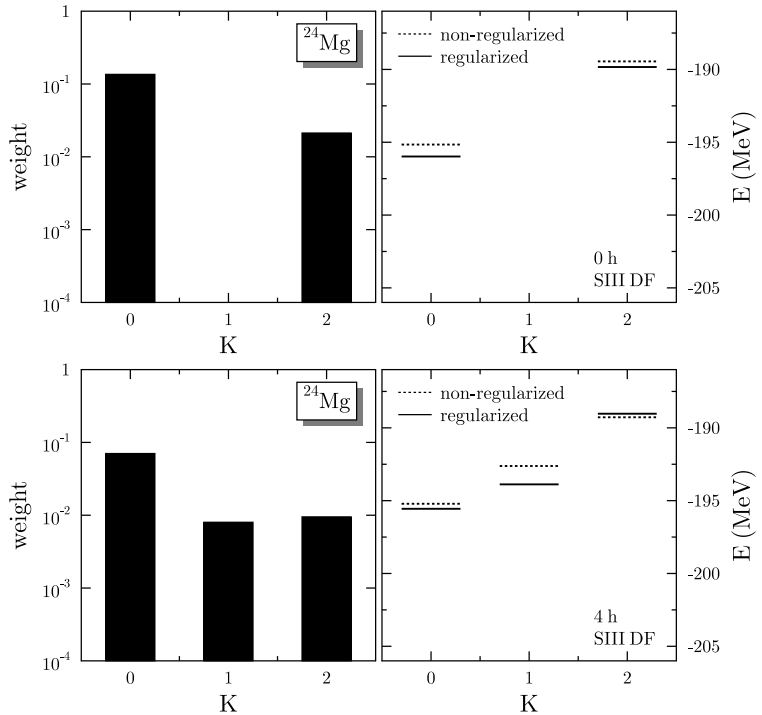

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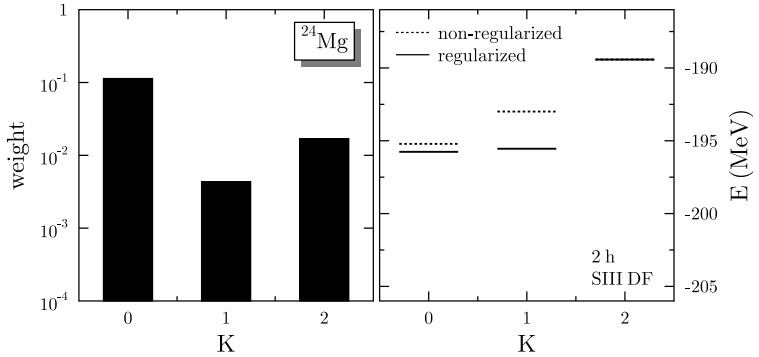

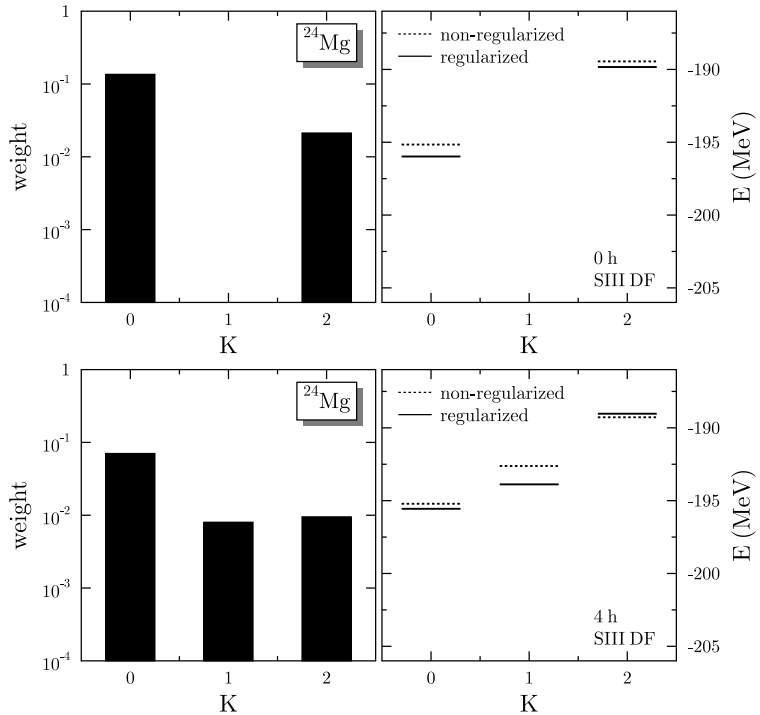

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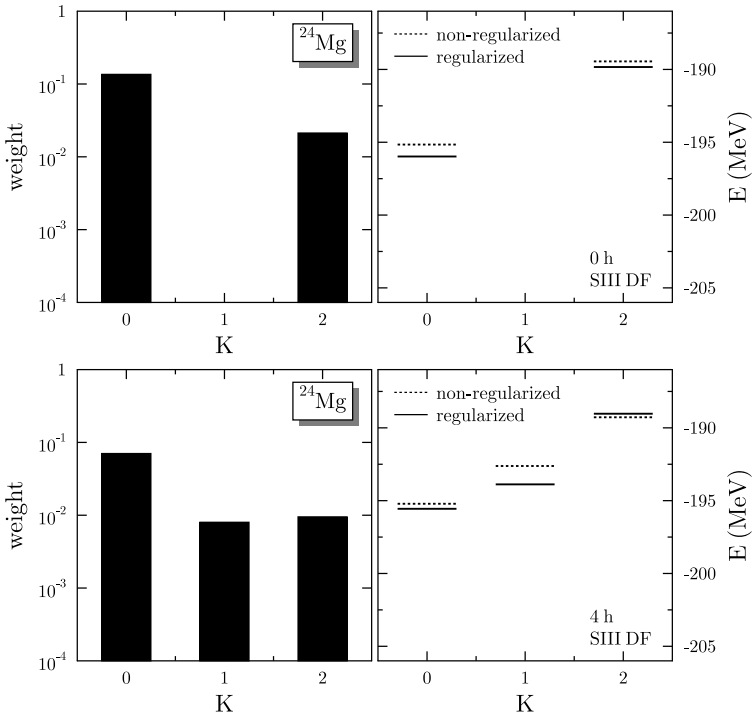
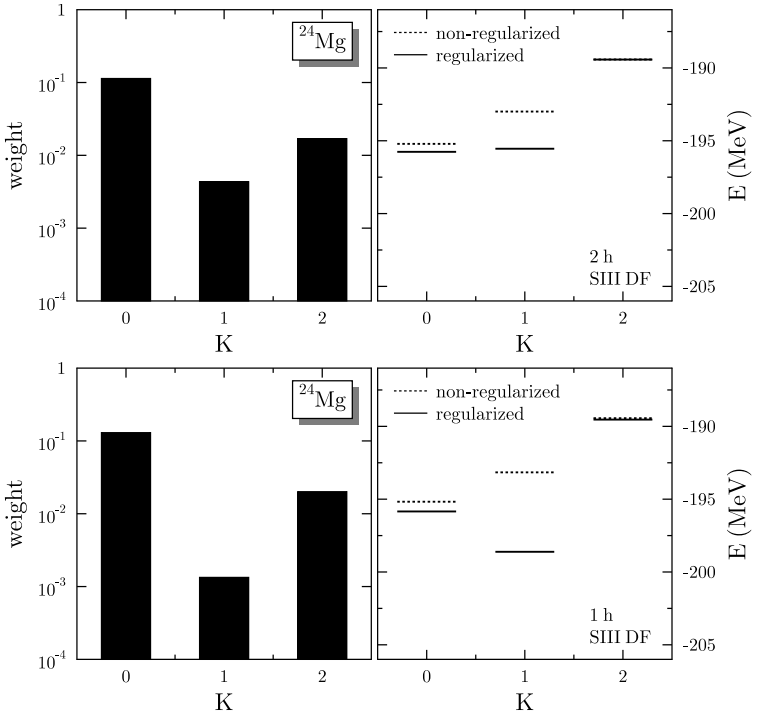
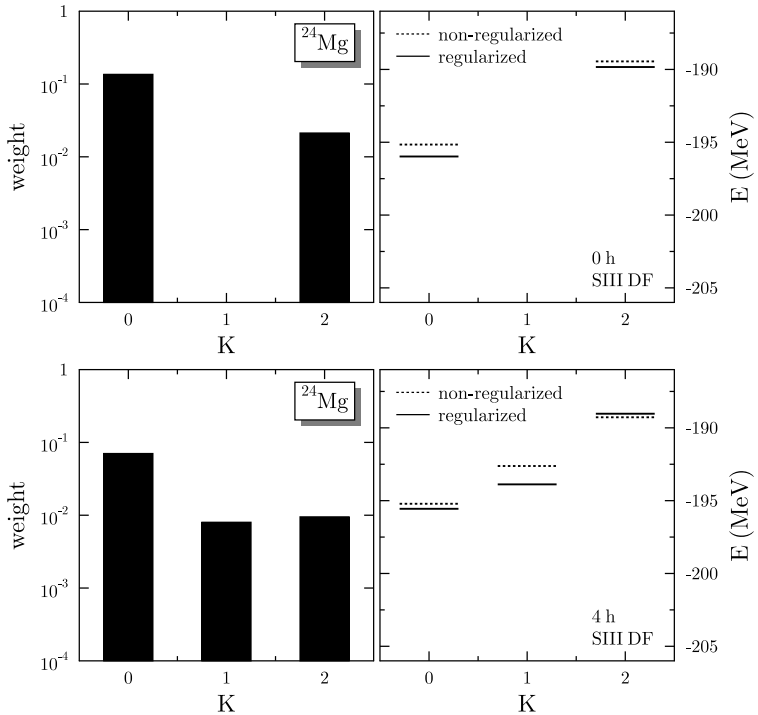
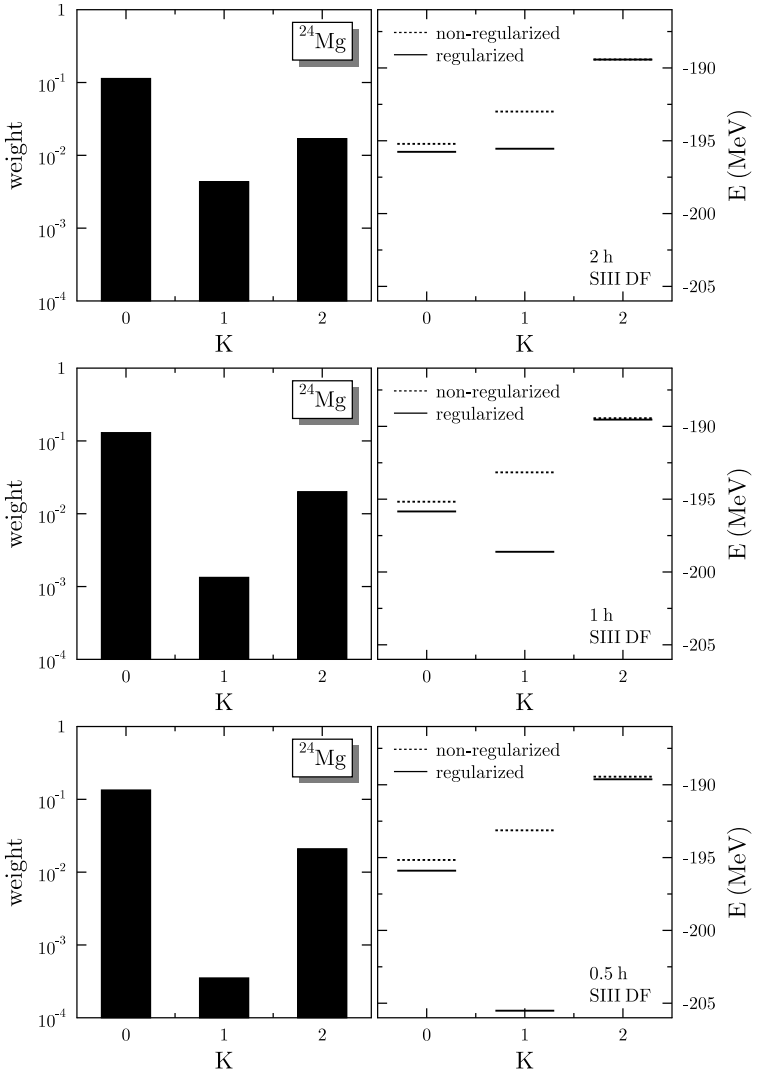




- small components (still) take unphysical values when regularising which can be demonstrated using a cranking constraint to vary the size of $K = 1$ components
- example shown in plots: K decomposition in projection on $J = 2$ of cranked HFB states cranked to different rotational frequencies





Besides the low-lying $J = 2$, $K = 1$ state there is also a low-lying $J = 1$, $K = 1$ state that becomes the ground state for small $\hbar\omega$

Further problems encountered in regularized MR EDF calculations

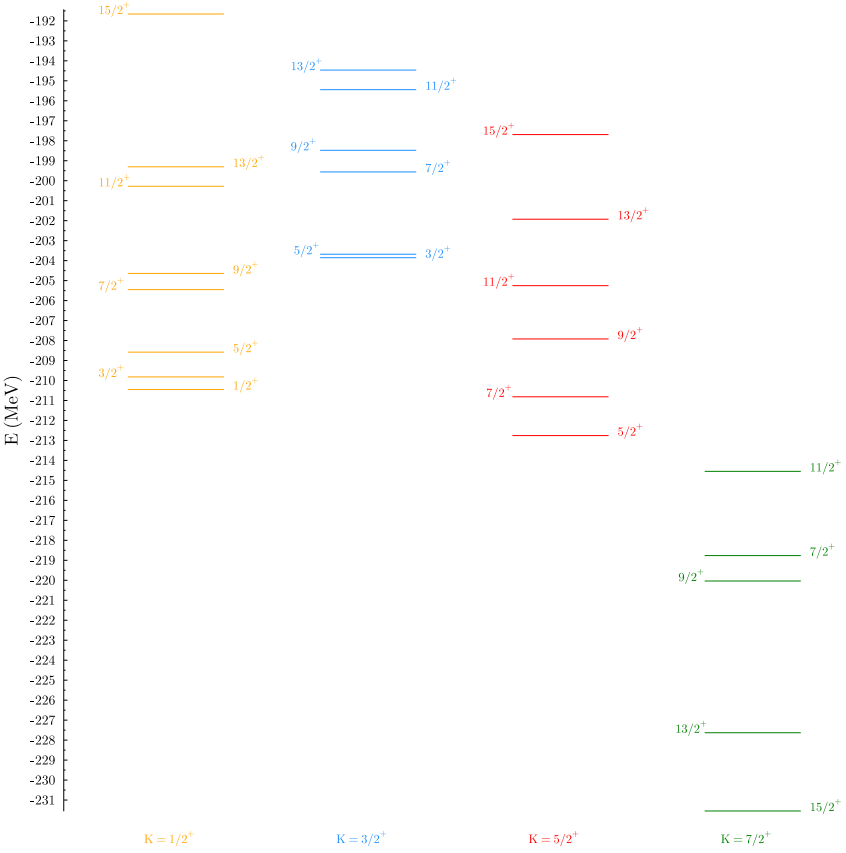
- regularizing to the left and to the right does not give the same energy ($\Rightarrow$ averaging necessary)
- non-convergence of combined N and J projection when increasing the number of gauge and/or Euler angles (on a very small scale that would be hardly visible on the previous plots, though)
- non-diagonal regularized particle-number projected MR EDF kernels can be decomposed on unphysical particle numbers (i.e. components that have strictly zero overlap), including *negative* particle numbers
- violation of physical sum rules in particle-number projection of non-diagonal regularized MR EDF kernels
- counter-intuitive behavior of moments of inertia in angular-momentum projection in MR EDF with genuine functionals (no obvious scaling with pairing strength)
- ...

publication in preparation ...

Again: What to do?

1. constructing the EDF as expectation value of a strict Hamiltonian. New problem: numerically very costly due to Coulomb exchange & pairing; no available parameterizations of high quality (the difficulties to construct such parametrizations was the main motivation to use EDFs in the 1970s).
2. construct the EDF from a density-dependent Hamiltonians with special treatment of the density entering density dependent terms for which numerically efficient high-quality parameterizations can be easily constructed. Problem: numerically very costly due to Coulomb exchange & pairing; cannot be defined for all possible confifuration mixing [Robledo, J. Phys. G 37 (2010) 064020].
3. introducing a regularization scheme of the EDF that allows for the use of (almost) standard functionals [Lacroix, Duguet, & Bender, PRC 79 (2009) 044318] for which numerically efficient high-quality parameterizations can be easily constructed [Washiyama, Bennaceur, Avez, Bender, Heenen, & Hellemans, PRC 86 (2012) 054309]. Problem: complicated formalism.

Density dependence with particle-number projected density



^{25}Mg

B. Bally, B. Avez, M. Bender, unpublished (2012).

- effective interaction: standard density-dependent Skyrme taking all exchange and pairing terms into account, courtesy of K. Bennaceur (unpublished, 2012).
- exact Coulomb exchange and Coulomb pairing
- particle-number projected (mixed) density entering the linear density dependence $\rho^\alpha = \rho$
- No obvious problems when projecting and mixing time-reversal invariance conserving HFB states.
- On a very small level, projected energies depend on the number of discretization points and sumrules might not be fulfilled.
- unrealistic decomposition into J, K components when projecting *time-reversal-invariance breaking* HFB states (where the particle-number projected mixed densities are *complex*)

Once more: What to do?

1. constructing the EDF as expectation value of a strict Hamiltonian. New problem: numerically very costly due to Coulomb exchange & pairing; no available parameterizations of high quality (the difficulties to construct such parametrizations was the main motivation to use EDFs in the 1970s).
2. construct the EDF from a density-dependent Hamiltonians with special treatment of the density entering density dependent terms for which numerically efficient high-quality parameterizations can be easily constructed. Problem: numerically very costly due to Coulomb exchange & pairing; cannot be defined for all possible confifuration mixing [Robledo, J. Phys. G 37 (2010) 064020].
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At last, we try the first option.

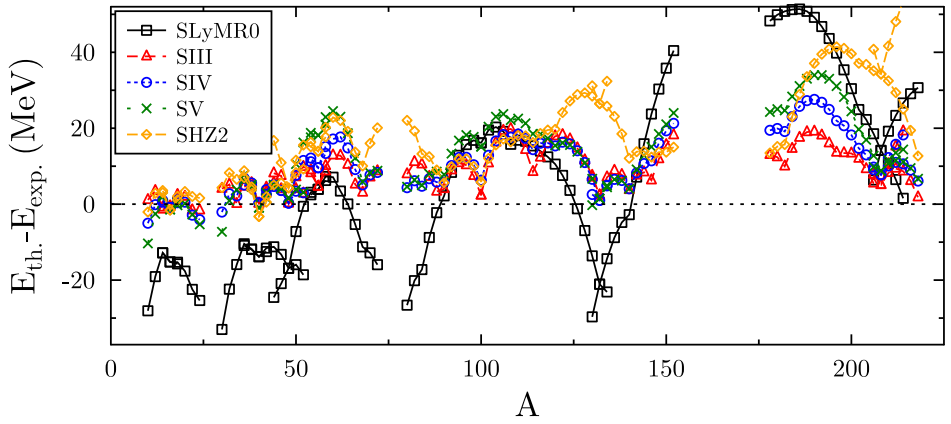
Proof-of-principle: Does using Hamiltonians remove all problems?

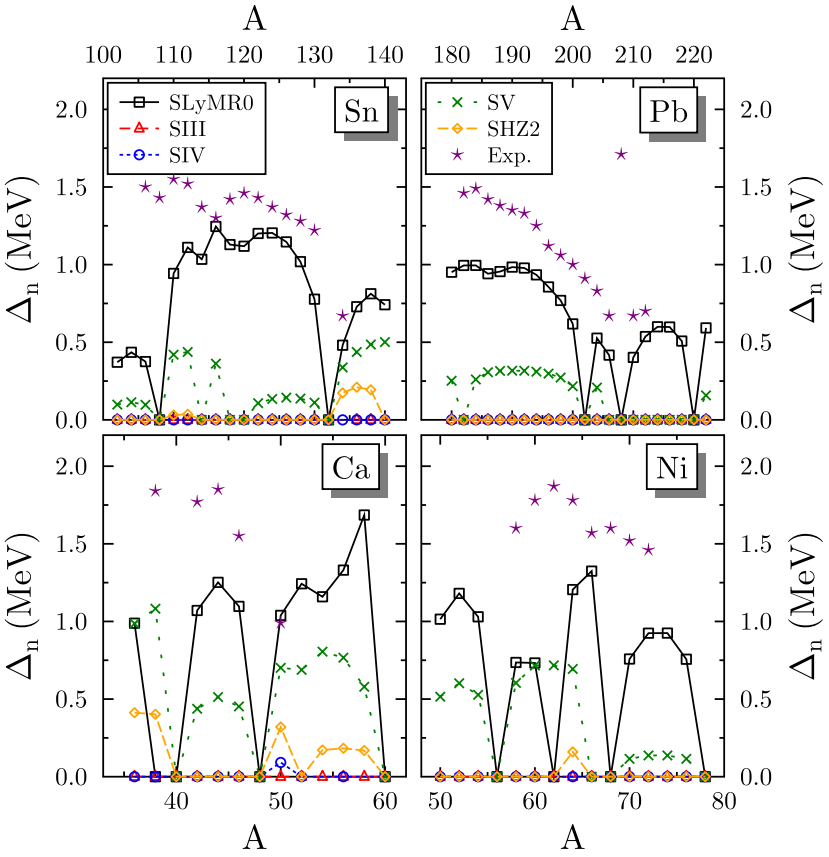
First try: SLyMR0

$$
\begin{aligned}
\hat{v} = {} & t_0 \left(1 + x_0 \hat{P}_\sigma\right) \hat{\delta}_{r_1 r_2} \\
& + \frac{t_1}{2} \left(1 + x_1 \hat{P}_\sigma\right) \left(\hat{\mathbf{k}}_{12}^{\prime\,2} \hat{\delta}_{r_1 r_2} + \hat{\delta}_{r_1 r_2} \hat{\mathbf{k}}_{12}^{2}\right) \\
& + t_2 \left(1 + x_2 \hat{P}_\sigma\right) \hat{\mathbf{k}}_{12}^{\prime} \cdot \hat{\delta}_{r_1 r_2} \hat{\mathbf{k}}_{12} \\
& + \mathrm{i}\, W_0 \left(\hat{\boldsymbol{\sigma}}_1 + \hat{\boldsymbol{\sigma}}_2\right) \cdot \hat{\mathbf{k}}_{12}^{\prime} \times \hat{\delta}_{r_1 r_2} \hat{\mathbf{k}}_{12} \\
& + u_0 \left(\hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} + \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_1 r_2} + \hat{\delta}_{r_2 r_1} \hat{\delta}_{r_3 r_1}\right) \\
& + v_0 \left(\hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} \hat{\delta}_{r_3 r_4} + \hat{\delta}_{r_1 r_2} \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_2 r_4} + \cdots\right)
\end{aligned}
$$

J. Sadoudi, M. Bender, K. Bennaceur, D. Davesne, R. Jodon, and T. Duguet, Physica Scripta T154 (2013) 014013

Proof-of-principle: Does using Hamiltonians remove all problems?



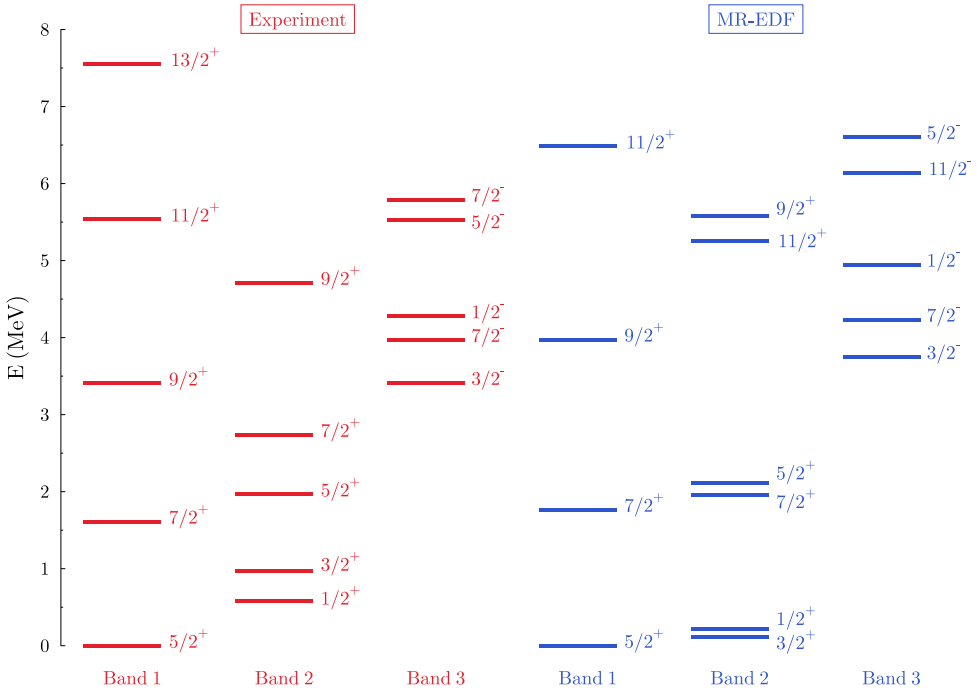


- it is impossible to fullfil the usual nuclear matter constraints , to have stable interactions and attractive pairing
- no "best fit" possible
- very bad performance compared to standard general functionals

J. Sadoudi, M. Bender, K. Bennaceur, D. Davesne, R. Jodon, and T. Duguet, Physica Scripta T154 (2013) 014013

Commercial break: Fun with SLyMR0 – ^{25}Mg

Angular-momentum and particle-number projected GCM of blocked triaxial one-quasiparticle states

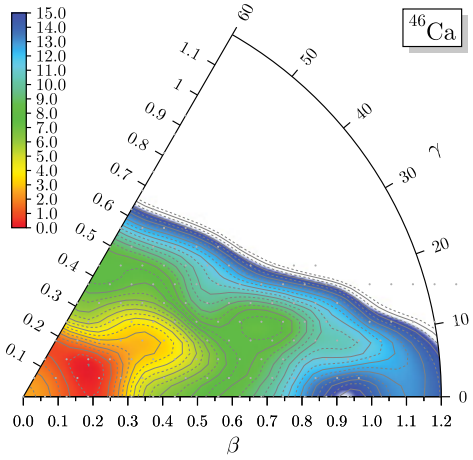


B. Bally, doctoral thesis, Université de Bordeaux (2014)

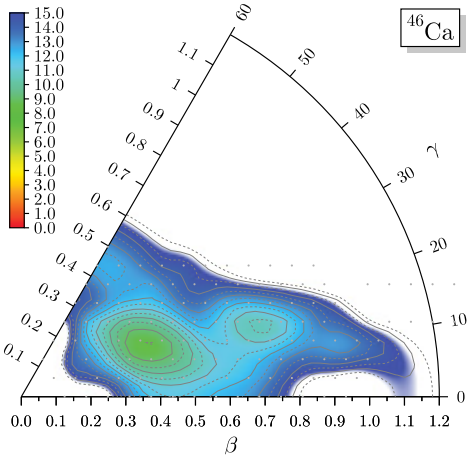
B. Bally, B. Avez, M. B., and P.-H. Heenen, PRL 113 (2014) 162501

Commercial break: Fun with SLyMR0 – ^{46}Ca

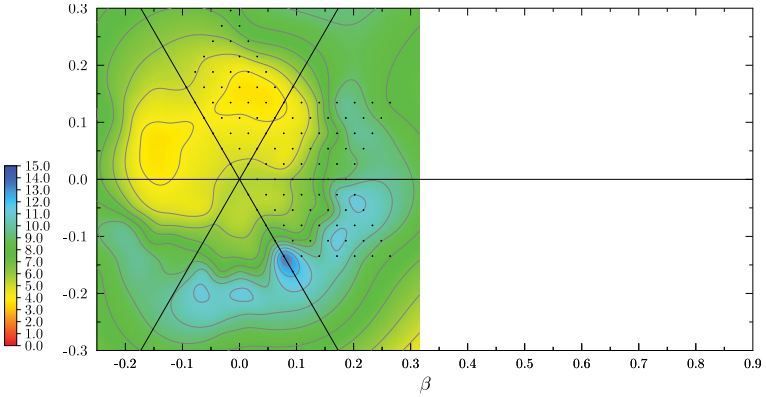
N, Z, $J = 0$ projected



N, Z, $J = 6$ projected

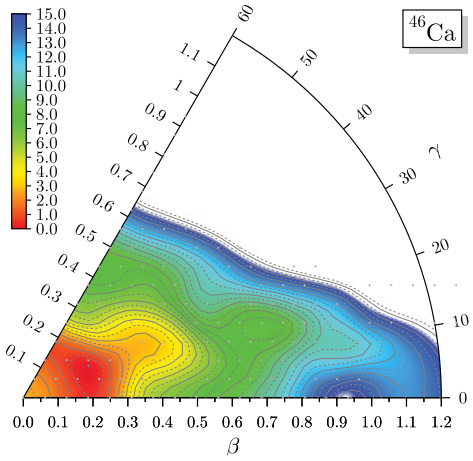


N, Z, $J = 6$ projected blocked/cranked states

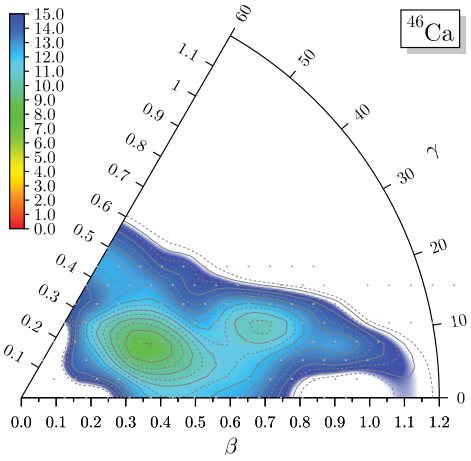


Commercial break: Fun with SLyMR0 – ^{46}Ca

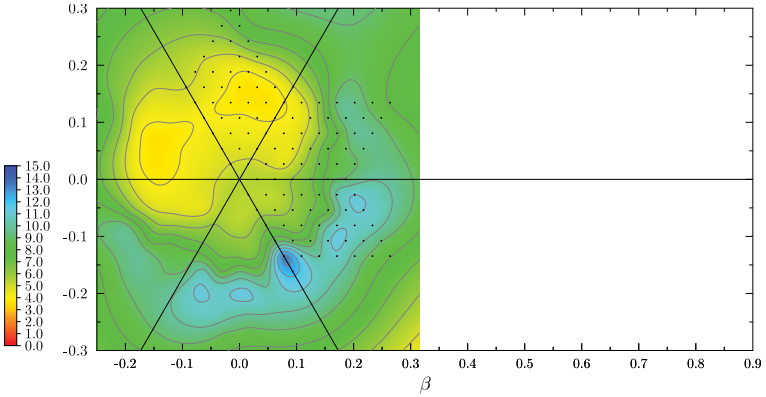
N, Z, $J = 0$ projected

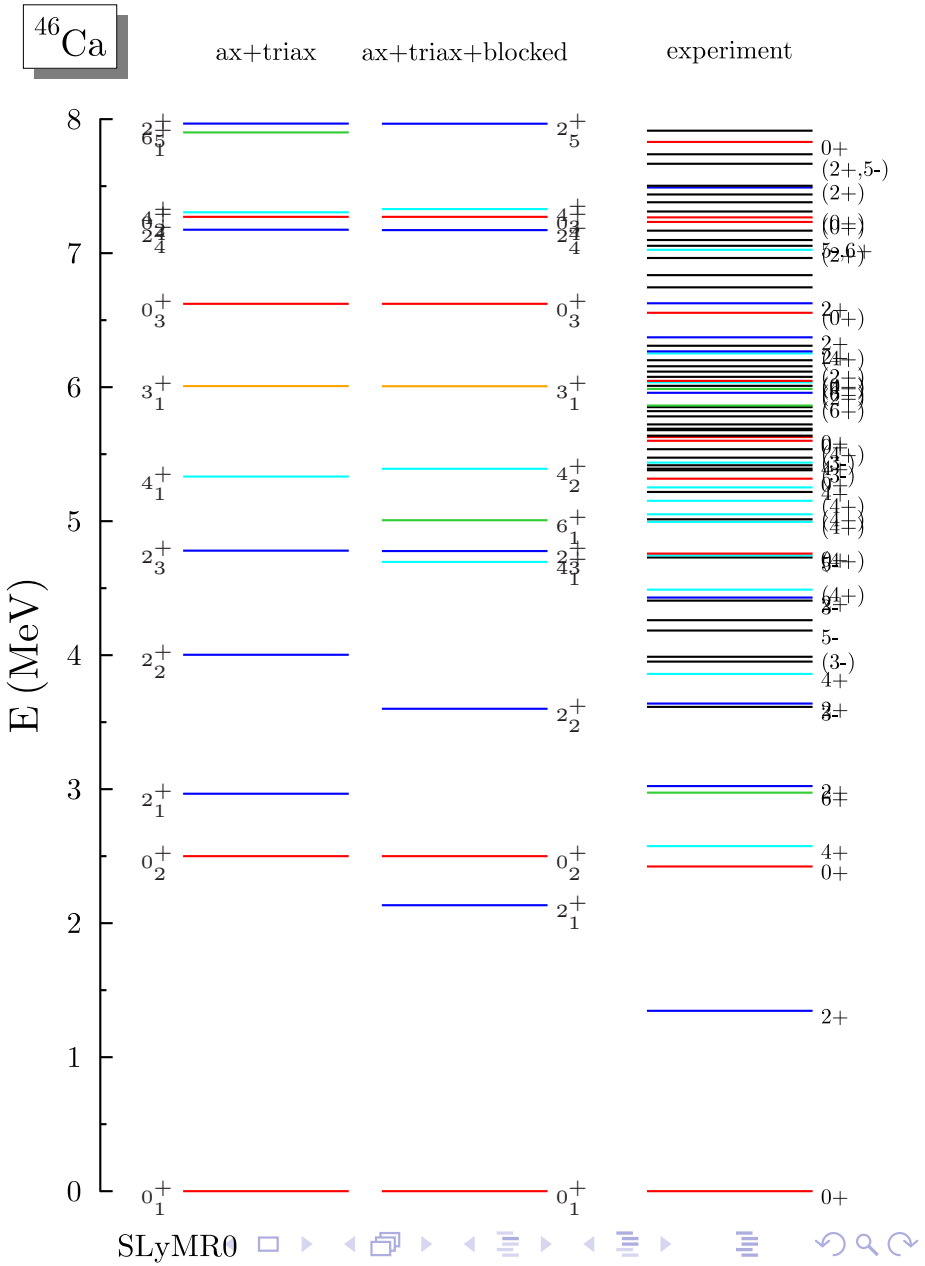


N, Z, $J = 6$ projected



N, Z, $J = 6$ projected blocked/cranked states





Proof-of-principle: Does using Hamiltonians remove all problems?

Some observations from first calculations:

- no divergences / steps
- impressive tolerance of the Hill-Wheeler-Griffin equation to handle large configuration spaces with many near-redundant states

3-body terms of 2nd order in gradients

The most general *central* Skyrme-type 3-body force up to 2nd order in gradients has been constructed by J. Sadoudi with a dedicated formal algebra code

1. write down all possible 3-body pseudo-potentials compatible with symmetry requirements (invariance under time reversal, space inversion, rotations, translations, isospin rotations, ...
2. derive complete functional in particle-hole and $T = 1$ pairing channels as HFB expectation value of the pseudo-potential with the help of a dedicated *in-house* formal algebra code.
3. separate the linearly independent terms in the pseudo-potential from the redundant ones by SVD of the matrix that relates the coupling constants of the *functional* with those of the *pseudo potential*.
4. derive mean fields and pairing fields
5. derive properties of infinite nuclear matter for symmetric, polarized symmetric, neutron and polarized neutron matter, including effective masses and Landau parameters.

J. Sadoudi, T. Duguet, J. Meyer, M. B., PRC 88 (2013) 064326

3-body terms of 2nd order in gradients

- structure

$$
\begin{aligned}
\hat{v}_{123} = & \, u_0 \left(\hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} + \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_1 r_2} + \hat{\delta}_{r_2 r_1} \hat{\delta}_{r_3 r_1} \right) \\
& + \frac{u_1}{2} \left[1 + y_1 P^{\sigma}_{12} \right] \left(\hat{\mathbf{k}}_{12} \cdot \hat{\mathbf{k}}_{12} + \hat{\mathbf{k}}'_{12} \cdot \hat{\mathbf{k}}'_{12} \right) \hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} \\
& + \frac{u_1}{2} \left[1 + y_1 P^{\sigma}_{31} \right] \left(\hat{\mathbf{k}}_{31} \cdot \hat{\mathbf{k}}_{31} + \hat{\mathbf{k}}'_{31} \cdot \hat{\mathbf{k}}'_{31} \right) \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_1 r_2} \\
& + \frac{u_1}{2} \left[1 + y_1 P^{\sigma}_{23} \right] \left(\hat{\mathbf{k}}_{23} \cdot \hat{\mathbf{k}}_{23} + \hat{\mathbf{k}}'_{23} \cdot \hat{\mathbf{k}}'_{23} \right) \hat{\delta}_{r_2 r_1} \hat{\delta}_{r_3 r_1} \\
& + u_2 \left[1 + y_{21} P^{\sigma}_{12} + y_{22} (P^{\sigma}_{13} + P^{\sigma}_{23}) \right] \left(\hat{\mathbf{k}}_{12} \cdot \hat{\mathbf{k}}'_{12} \right) \hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} \\
& + u_2 \left[1 + y_{21} P^{\sigma}_{31} + y_{22} (P^{\sigma}_{32} + P^{\sigma}_{12}) \right] \left(\hat{\mathbf{k}}_{31} \cdot \hat{\mathbf{k}}'_{31} \right) \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_1 r_2} \\
& + u_2 \left[1 + y_{21} P^{\sigma}_{23} + y_{22} (P^{\sigma}_{21} + P^{\sigma}_{31}) \right] \left(\hat{\mathbf{k}}_{23} \cdot \hat{\mathbf{k}}'_{23} \right) \hat{\delta}_{r_2 r_1} \hat{\delta}_{r_3 r_1}
\end{aligned}
$$

J. Sadoudi, T. Duguet, J. Meyer, M. B., PRC 88 (2013) 064326

- implementation in spherical HFB code
- implementation in 3d SR and MR EDF codes
- implemtation into semi-infinite nuclear matter code
- implementation into code calculating linear response in infinite homogeneous matter
- first preliminary fits have been achieved (⇒ Karim Bennaceur's talk)

Earlier work

Table: Three-body terms labeled by their parameters that have been considered in earlier (non-systematic) work as indicated by $+$.

Ref.	u_0	u_1	$u_1 y_1$	u_2	$u_2 y_{21}$	$u_2 y_{22}$
Blaizot et al. (1975)	$+$	$+$	$-$	$-$	$-$	$-$
Liu et al. (1975)	$+$	$+$	$-$	$-$	$-$	$-$
Onishi et al. (1978)	$+$	$-$	$-$	$+$	$+$	$-$
Waroquier et al. (1983)	$+$	$+$	$-$	$-$	$-$	$-$
Arima et al. (1986)	$+$	$+$	$-$	$+$	$+$	$-$
Zheng et al. (1990)	$+$	$+$	$-$	$-$	$-$	$-$
Liu et al. (1991)	$+$	$+$	$+$	$+$	$+$	$-$

(cf. Table III of J. Sadoudi, T. Duguet, J. Meyer, M. B., PRC 88 (2013) 064326 for complete references)

The normal part of the functional

$$
\begin{aligned}
\mathcal{E}^{\rho\rho\rho} \quad = \quad & \sum_{t=0,1} \Bigg\{ B_t^{\rho}\, \rho_t\, \rho_t\, \rho_0 + B_t^{s}\, \mathbf{s}_t \cdot \mathbf{s}_t\, \rho_0 + B_t^{\tau}\, \rho_t\, \tau_t\, \rho_0 + B_t^{\tau s}\, \tau_t\, \mathbf{s}_t \cdot \mathbf{s}_0 \\
& + B_t^{T}\, \mathbf{s}_t \cdot \mathbf{T}_t\, \rho_0 + B_{t\bar{t}}^{T}\, \mathbf{s}_t \cdot \mathbf{T}_{\bar{t}}\, \rho_1 + B_t^{\nabla\rho}\, (\boldsymbol{\nabla}\rho_t) \cdot (\boldsymbol{\nabla}\rho_t)\, \rho_0 + B_t^{j}\, \mathbf{j}_t \cdot \mathbf{j}_t\, \rho_0 \\
& + \sum_{\mu\nu} \Big[B_t^{\nabla s}\, (\nabla_\mu s_{t,\nu})(\nabla_\mu s_{t,\nu})\, \rho_0 + B_t^{J}\, J_{t,\mu\nu} J_{t,\mu\nu}\, \rho_0 \\
& + B_{t\bar{t}}^{\nabla\rho s}\, (\nabla_\mu \rho_t)(\nabla_\mu s_{\bar{t},\nu})\, s_{1,\nu} + B_t^{\nabla\rho s}\, (\nabla_\mu \rho_t)(\nabla_\mu s_{t,\nu})\, s_{0,\nu} \\
& + B_t^{Js}\, j_{t,\mu}\, J_{t,\mu\nu}\, s_{0,\nu} + B_{t\bar{t}}^{Js}\, j_{t,\mu}\, J_{\bar{t},\mu\nu}\, s_{1,\nu} \Big] \\
& + \sum_{\mu\nu\lambda\kappa} \epsilon_{\nu\lambda\kappa} \Big[B_t^{\nabla sJ}\, (\nabla_\mu s_{t,\nu})\, J_{t,\mu\lambda}\, s_{0,\kappa} + B_{t\bar{t}}^{\nabla sJ}\, (\nabla_\mu s_{t,\nu})\, J_{\bar{t},\mu\lambda}\, s_{1,\kappa} \Big] \Bigg\} \\
& + B_{10}^{s}\, \mathbf{s}_1 \cdot \mathbf{s}_0\, \rho_1 + B_{10}^{\tau}\, \rho_1\, \tau_0\, \rho_1 + B_{10}^{\tau s}\, \tau_0\, \mathbf{s}_1 \cdot \mathbf{s}_1 + B_{10}^{\nabla\rho}\, (\boldsymbol{\nabla}\rho_1) \cdot (\boldsymbol{\nabla}\rho_0)\, \rho_1 \\
& + \sum_{\mu\nu} B_{10}^{\nabla s}\, (\nabla_\mu s_{1,\nu})(\nabla_\mu s_{0,\nu})\, \rho_1 + B_{10}^{j}\, \mathbf{j}_1 \cdot \mathbf{j}_0\, \rho_1 + \sum_{\mu\nu} B_{10}^{J}\, J_{1,\mu\nu}\, J_{0,\mu\nu}\, \rho_1
\end{aligned}
$$

- 8 terms are the usual central two-body terms $\times \rho_0$, the other 15 have different vector and/or isospin coupling.

A word on isospin pair densities I

- The particle-hole part of the energy density functional is usually represented in isospin representation
- The pairing part of the energy density functional is usually represented in proton-neutron representation

In the case with p-p and n-n pairing, $|\mathrm{HFB}_n\rangle \otimes \mathrm{HFB}_p\rangle$

$$\kappa(\mathbf{r}\sigma q, \mathbf{r}'\sigma' q') \equiv \langle\Phi|a_{\mathbf{r}'\sigma' q'} a_{\mathbf{r}\sigma q}|\Phi\rangle \quad \text{with} \quad \kappa(\mathbf{r}\sigma q, \mathbf{r}'\sigma' q') = -\kappa(\mathbf{r}'\sigma' q', \mathbf{r}\sigma q)$$

There is no local proton or neutron anomalous density $\kappa_q(\mathbf{r})$ analogous to $\rho_q(\mathbf{r}) = \sum_\sigma \rho(\mathbf{r}\sigma q, \mathbf{r}\sigma q)$

$$\kappa_q(\mathbf{r}) = \sum_\sigma \kappa(\mathbf{r}\sigma q, \mathbf{r}\sigma q) = -\sum_\sigma \kappa(\mathbf{r}\sigma q, \mathbf{r}\sigma q) = 0$$

Define pairing density instead

$$\tilde{\rho}_q(\mathbf{r}) \equiv \sum_\sigma (-\sigma)\kappa(\mathbf{r}\sigma q, \mathbf{r} - \sigma q) = -\sum_\sigma (-\sigma)\kappa(\mathbf{r} - \sigma q, \mathbf{r}\sigma q) \neq 0$$

But there is no isoscalar or isovector pairing density $\tilde{\rho}_t$ analogous to

$$\rho_0(\mathbf{r}) \equiv \sum_{\sigma q} \rho(\mathbf{r}\sigma q, \mathbf{r}\sigma q)$$

$$\rho_{1,\mathfrak{a}}(\mathbf{r}) \equiv \sum_{\sigma q' q} \rho(\mathbf{r}\sigma q, \mathbf{r}\sigma q')\, \tau_{\mathfrak{a},q'q} \quad \rightarrow \quad \rho_{1,1} = \rho_{1,2} = 0, \quad \rho_{1,3} = \rho_p(\mathbf{r}) - \rho_n(\mathbf{r})$$

A word on isospin pair densities II

Pairing densities

$$\breve{\rho}_0(\mathbf{r}) \equiv \sum_{\sigma q}(-\sigma)(-q)\,\kappa(\mathbf{r}\sigma q, \mathbf{r}-\sigma-q) = \sum_{\sigma q}\sigma\, q\,\kappa(\mathbf{r}-\sigma-q, \mathbf{r}\sigma q) = 0$$

$$\breve{\rho}_{1,\mathfrak{a}}(\mathbf{r}) \equiv \sum_{\sigma q' q}(-\sigma)(-q')\,\kappa(\mathbf{r}\sigma q, \mathbf{r}-\sigma-q')\,\tau_{\mathfrak{a},q'q}$$

- the first ($\mathfrak{a}=1$) and second ($\mathfrak{a}=2$) isovector components of all normal densities are zero,
- all isoscalar pairing densities are zero,
- the third component ($\mathfrak{a}=3$) of all isovector pairing densities is zero.

E. Perlinska, S. G. Rohoziński, J. Dobaczewski, W. Nazarewicz, PRC **69**, 014316 (2004)

S. G. Rohoziński, J. Dobaczewski, W. Nazarewicz, PRC 81, 014313 (2010)

$$\mathcal{P}_n \equiv \frac{1}{2}\left(\mathcal{P}_0 + \mathcal{P}_{1,3}\right) \qquad \mathcal{P}_p \equiv \frac{1}{2}\left(\mathcal{P}_0 - \mathcal{P}_{1,3}\right)$$

$$\tilde{\mathcal{P}}_n \equiv \frac{1}{2}\left(\breve{\mathcal{P}}_{1,1} + \mathrm{i}\breve{\mathcal{P}}_{1,2}\right) \qquad \tilde{\mathcal{P}}_p \equiv \frac{1}{2}\left(\breve{\mathcal{P}}_{1,1} - \mathrm{i}\breve{\mathcal{P}}_{1,2}\right)$$

The anomalous part of the functional

$$
\begin{aligned}
\mathcal{E}^{\kappa\kappa\rho} \quad = \quad & \sum_{\mathfrak{a}=1,2} \Big\{ B_0^{\breve\rho}\, \breve\rho^*_{1,\mathfrak{a}} \breve\rho_{1,\mathfrak{a}}\, \rho_0 + B_0^{\breve\tau\,*}\, \breve\tau^*_{1,\mathfrak{a}} \breve\rho_{1,\mathfrak{a}}\, \rho_0 + B_0^{\breve\tau}\, \breve\rho^*_{1,\mathfrak{a}} \breve\tau_{1,\mathfrak{a}}\, \rho_0 + B_0^{\breve\rho\tau}\, \breve\rho^*_{1,\mathfrak{a}} \breve\rho_{1,\mathfrak{a}}\, \tau_0 \\
& + B_0^{\nabla\breve\rho} (\boldsymbol{\nabla} \breve\rho^*_{1,\mathfrak{a}}) \cdot (\boldsymbol{\nabla} \breve\rho_{1,\mathfrak{a}}) \rho_0 + B_0^{\nabla\breve\rho^*\,\breve\rho} (\boldsymbol{\nabla} \breve\rho^*_{1,\mathfrak{a}}) \breve\rho_{1,\mathfrak{a}} \cdot (\boldsymbol{\nabla} \rho_0) + B_0^{\breve\rho^*\,\nabla\breve\rho} \breve\rho^*_{1,\mathfrak{a}} (\boldsymbol{\nabla} \breve\rho_{1,\mathfrak{a}}) \cdot (\boldsymbol{\nabla} \rho_0) \\
& + \mathrm{i} B_0^{\nabla\breve\rho^*\,j} (\boldsymbol{\nabla} \breve\rho^*_{1,\mathfrak{a}}) \breve\rho_{1,\mathfrak{a}} \cdot \mathbf{j}_0 + \mathrm{i} B_0^{\nabla\breve\rho j} \breve\rho^*_{1,\mathfrak{a}} (\boldsymbol{\nabla} \breve\rho_{1,\mathfrak{a}}) \cdot \mathbf{j}_0 \\
& + \sum_{\mu\nu} \big[B_0^{\breve J} \breve J^*_{1,\mathfrak{a},\mu\nu}\, \breve J_{1,\mathfrak{a},\mu\nu}\, \rho_0 + B_0^{\breve J^*\,\breve\rho} \breve J^*_{1,\mathfrak{a},\mu\nu} \breve\rho_{1,\mathfrak{a}} J_{0,\mu\nu} + B_0^{\breve\rho^*\,\breve J} \breve\rho^*_{1,\mathfrak{a}} \breve J_{1,\mathfrak{a},\mu\nu} J_{0,\mu\nu} \\
& + \mathrm{i} B_0^{\nabla\breve\rho^*\,\breve J} (\nabla_\mu \breve\rho^*_{1,\mathfrak{a}}) \breve J_{1,\mathfrak{a},\mu\nu} s_{0,\nu} + \mathrm{i} B_0^{\breve J^*\,\nabla\breve\rho} \breve J^*_{1,\mathfrak{a},\mu\nu} (\nabla_\mu \breve\rho_{1,\mathfrak{a}})\, s_{0,\nu} \\
& + \mathrm{i} B_0^{\breve J^*\,\nabla s} \breve J^*_{1,\mathfrak{a},\mu\nu} \breve\rho_{1,\mathfrak{a}} (\nabla_\mu s_{0,\nu}) + \mathrm{i} B_0^{\breve J \nabla s} \breve\rho^*_{1,\mathfrak{a}}\, \breve J_{1,\mathfrak{a},\mu\nu} (\nabla_\mu s_{0,\nu}) \big] \\
& + \sum_{\mu\nu\lambda\kappa} \epsilon_{\nu\lambda\kappa} \big[\mathrm{i} B_0^{\breve J 2 s}\, \breve J^*_{1,\mathfrak{a},\mu\nu}\, \breve J_{1,\mathfrak{a},\mu\lambda}\, s_{0,\kappa} \big] \Big\} \\
& + \sum_{\mathfrak{a},\mathfrak{b}=1,2} \sum_{\mathfrak{c}=3} \epsilon_{\mathfrak{a}\mathfrak{b}\mathfrak{c}} \Big\{ \mathrm{i} B_1^{\breve\rho} \breve\rho^*_{1,\mathfrak{a}}\, \breve\rho_{1,\mathfrak{b}}\, \rho_{1,\mathfrak{c}} + \mathrm{i} B_1^{\breve\tau\,*}\, \breve\tau^*_{1,\mathfrak{a}}\, \breve\rho_{1,\mathfrak{b}}\, \rho_{1,\mathfrak{c}} + \mathrm{i} B_1^{\breve\tau} \breve\rho^*_{1,\mathfrak{a}} \breve\tau_{1,\mathfrak{b}} \rho_{1,\mathfrak{c}} \\
& + \mathrm{i} B_1^{\breve\rho\tau} \breve\rho^*_{1,\mathfrak{a}}\, \breve\rho_{1,\mathfrak{b}}\, \tau_{1,\mathfrak{c}} + \mathrm{i} B_1^{\nabla\breve\rho} (\boldsymbol{\nabla} \breve\rho^*_{1,\mathfrak{a}}) \cdot (\boldsymbol{\nabla} \breve\rho_{1,\mathfrak{b}}) \rho_{1,\mathfrak{c}} + \mathrm{i} B_1^{\nabla\breve\rho^*\,\breve\rho} (\boldsymbol{\nabla} \breve\rho^*_{1,\mathfrak{a}}) \breve\rho_{1,\mathfrak{b}} \cdot (\boldsymbol{\nabla} \rho_{1,\mathfrak{c}}) \\
& + \mathrm{i} B_1^{\breve\rho^*\,\nabla\breve\rho} \breve\rho^*_{1,\mathfrak{a}} (\boldsymbol{\nabla} \breve\rho_{1,\mathfrak{b}}) \cdot (\boldsymbol{\nabla} \rho_{1,\mathfrak{c}}) + B_1^{\nabla\breve\rho^*\,j} (\boldsymbol{\nabla} \breve\rho^*_{1,\mathfrak{a}}) \breve\rho_{1,\mathfrak{b}} \cdot \mathbf{j}_{1,\mathfrak{c}} + B_1^{\nabla\breve\rho j} \breve\rho^*_{1,\mathfrak{a}} (\boldsymbol{\nabla} \breve\rho_{1,\mathfrak{b}}) \cdot \mathbf{j}_{1,\mathfrak{c}} \\
& + \sum_{\mu\nu} \Big[\mathrm{i} B_1^{\breve J} \breve J^*_{1,\mathfrak{a},\mu\nu} \breve J_{1,\mathfrak{b},\mu\nu} \rho_{1,\mathfrak{c}} + \mathrm{i} B_1^{\breve J^*\,\breve\rho} \breve J^*_{1,\mathfrak{a},\mu\nu} \breve\rho_{1,\mathfrak{b}} J_{1,\mathfrak{c},\mu\nu} + \mathrm{i} B_1^{\breve\rho^*\,\breve J} \breve\rho^*_{1,\mathfrak{a}} \breve J_{1,\mathfrak{b},\mu\nu} J_{1,\mathfrak{c},\mu\nu} \\
& + B_1^{\nabla\breve\rho^*\,\breve J} (\nabla_\mu \breve\rho^*_{1,\mathfrak{a}}) \breve J_{1,\mathfrak{b},\mu\nu} s_{1,\mathfrak{c},\nu} + B_1^{\breve J^*\,\nabla\breve\rho} \breve J^*_{1,\mathfrak{a},\mu\nu} (\nabla_\mu \breve\rho_{1,\mathfrak{b}}) s_{1,\mathfrak{c},\nu} + B_1^{\breve J^*\,\nabla s} \breve J^*_{1,\mathfrak{a},\mu\nu} \breve\rho_{1,\mathfrak{b}} (\nabla_\mu s_{1,\mathfrak{c},\nu}) \\
& + B_1^{\breve J \nabla s} \breve\rho^*_{1,\mathfrak{a}} \breve J_{1,\mathfrak{b},\mu\nu} (\nabla_\mu s_{1,\mathfrak{c},\nu}) \Big] + \sum_{\mu\nu\lambda\kappa} \epsilon_{\nu\lambda\kappa} \Big[B_1^{\breve J 2 s} \breve J^*_{1,\mathfrak{a},\mu\nu} \breve J_{1,\mathfrak{b},\mu\lambda} s_{1,\mathfrak{c},\kappa} \Big] \Big\}
\end{aligned}
$$

Generic isospin structure in case of *n*-*n* and *p*-*p* pairing

$\mathcal{P}_{1,\mathfrak{a}}, \mathcal{P}'_{1,\mathfrak{a}}$	generic normal isospin densities
$\breve{\mathcal{P}}_{1,\mathfrak{a}}, \breve{\mathcal{P}}'_{1,\mathfrak{a}}$	generic anomalous isospin densities

The generic isospin structure of the terms containing isovector densities is

$$\mathcal{P}_0\,\mathcal{P}'_0 \qquad ab$$

$$\sum_{\mathfrak{a}} \mathcal{P}_{1,\mathfrak{a}}\,\mathcal{P}'_{1,\mathfrak{a}} = \mathcal{P}_{1,3}\,\mathcal{P}'_{1,3} \qquad \mathbf{a}\cdot\mathbf{b}$$

$$\sum_{\mathfrak{a}} \breve{\mathcal{P}}^*_{1,\mathfrak{a}}\,\breve{\mathcal{P}}'_{1,\mathfrak{a}} = \breve{\mathcal{P}}^*_{1,1}\,\breve{\mathcal{P}}'_{1,1} + \breve{\mathcal{P}}^*_{1,2}\,\breve{\mathcal{P}}'_{1,2} \qquad \mathbf{A}^*\cdot\mathbf{B}$$

$$\sum_{\mathfrak{a}} \mathcal{P}_{1,\mathfrak{a}}\,\mathcal{P}'_{1,\mathfrak{a}}\,\mathcal{P}''_0 = \mathcal{P}_{1,3}\,\mathcal{P}'_{1,3}\,\mathcal{P}''_0 \qquad (\mathbf{a}\cdot\mathbf{b})\,c$$

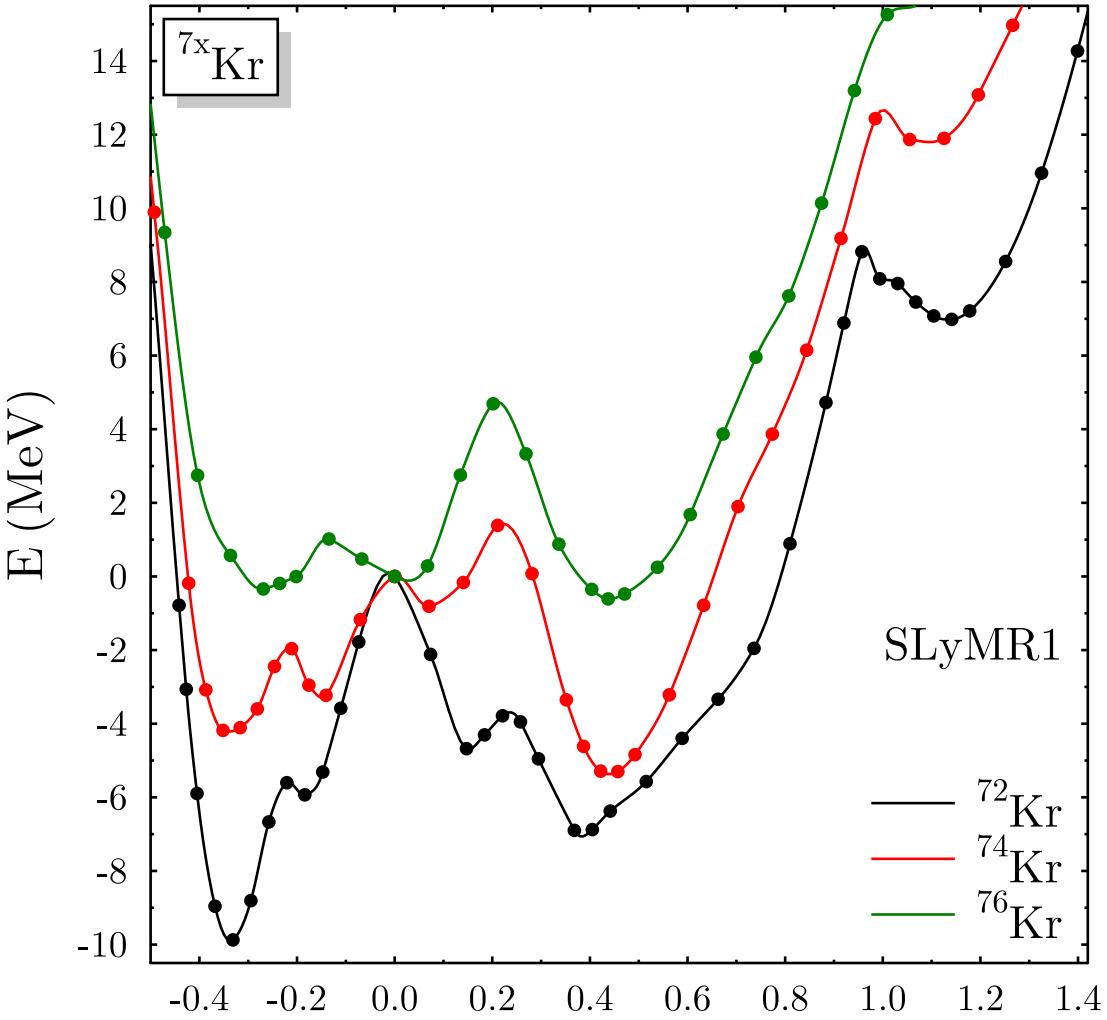
$$\sum_{\mathfrak{a}} \breve{\mathcal{P}}^*_{1,\mathfrak{a}}\,\breve{\mathcal{P}}'_{1,\mathfrak{a}}\,\mathcal{P}''_0 = \left(\breve{\mathcal{P}}^*_{1,1}\,\breve{\mathcal{P}}'_{1,1} + \breve{\mathcal{P}}^*_{1,2}\,\breve{\mathcal{P}}'_{1,2}\right)\mathcal{P}''_0 \qquad (\mathbf{A}^*\cdot\mathbf{B})\,c$$

$$\sum_{\mathfrak{abc}} \epsilon_{\mathfrak{abc}}\breve{\mathcal{P}}^*_{1,\mathfrak{a}}\,\breve{\mathcal{P}}'_{1,\mathfrak{b}}\,\mathcal{P}''_{1,\mathfrak{c}} = \left(\breve{\mathcal{P}}^*_{1,1}\,\breve{\mathcal{P}}'_{1,2} - \breve{\mathcal{P}}^*_{1,2}\,\breve{\mathcal{P}}'_{1,1}\right)\mathcal{P}''_{1,3} \qquad (\mathbf{A}^*\times\mathbf{B})\cdot\mathbf{c}$$

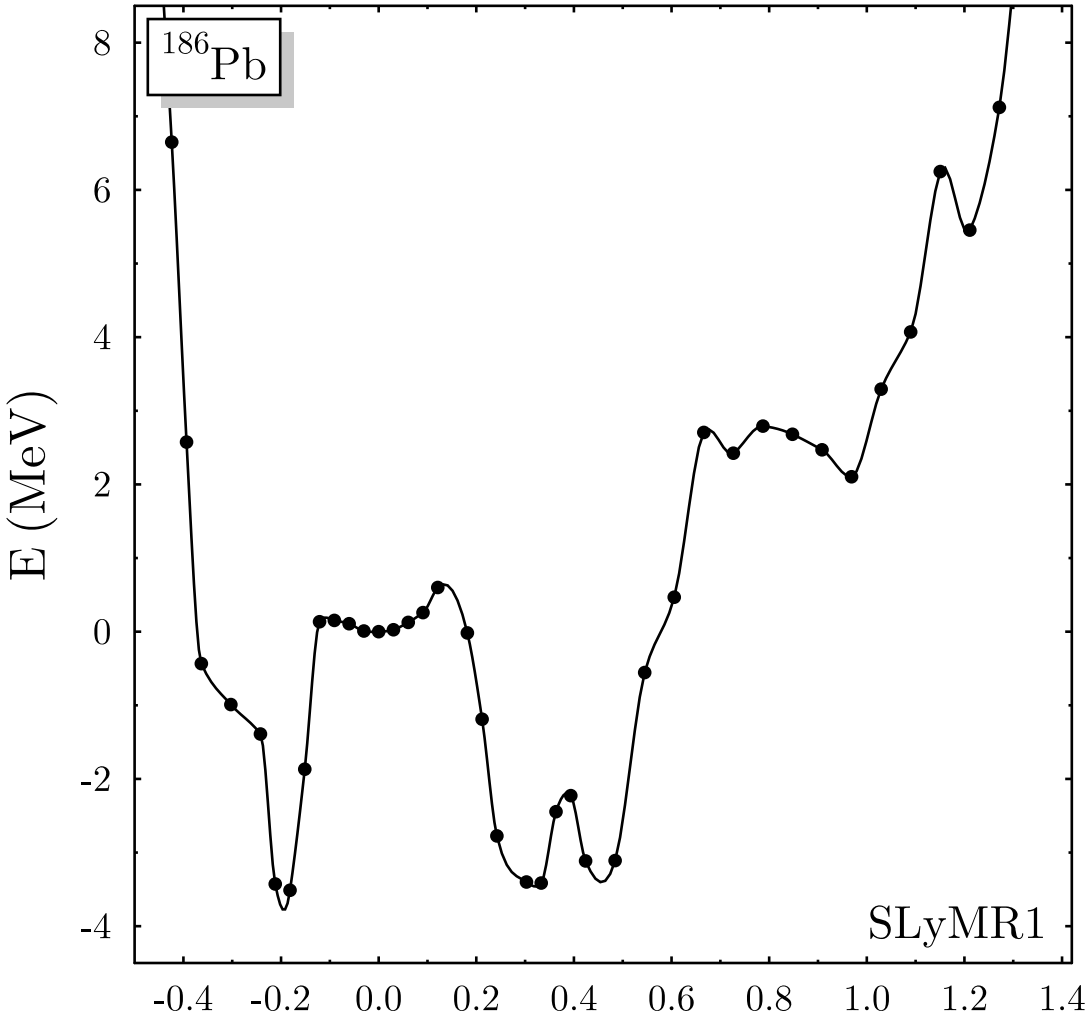
Some (very) preliminary results obtained with SLyMR1

- Implementation into various codes (1d spherical HFB code, 3d time-reversal-breaking HFB code, corresponding projected energy kernel code) by J. Sadoudi
- various tools (semi-infinite nuclear matter, linear response in homogeneous infinite matter) by the Lyon group
- First fit by Robin Jodon and Karim Bennaceur

Some (very) preliminary results obtained with SLyMR1



Some (very) preliminary results obtained with SLyMR1



Ongoing work

construction of most general spin-orbit + tensor 3-body force of 2nd order in gradients has been started by J. Sadoudi. This is how the time-even part looks like for spin-orbit (top) and tensor (bottom)

$$
\begin{aligned}
E^{\rho\rho\rho} = \int d\vec{r} \Bigg\{ \sum_{q_1} \Bigg[& + B^{J_{1\mu\mu} J_{1\nu\nu} \rho_1} J_{q_1,\mu\mu} J_{q_1,\nu\nu} \rho_{q_1} + B^{J_{1\mu\nu} J_{1\nu\mu} \rho_1} J_{q_1,\nu\mu} J_{q_1,\mu\nu} \rho_{q_1} \\
& + B^{\nabla_\mu \rho_1 J_{1\nu\kappa} \rho_1 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_1,\kappa\nu} \rho_{q_1} \epsilon_{\mu\nu\kappa} \Bigg] \\
& + \sum_{q_1 \neq q_2} \Bigg[+ B^{J_{1\mu\mu} J_{1\nu\nu} \rho_2} J_{q_1,\mu\mu} J_{q_1,\nu\nu} \rho_{q_2} + B^{J_{1\mu\nu} J_{1\nu\mu} \rho_2} J_{q_1,\nu\mu} J_{q_1,\mu\nu} \rho_{q_2} \\
& + B^{J_{1\mu\mu} J_{2\nu\nu} \rho_1} J_{q_1,\mu\mu} J_{q_2,\nu\nu} \rho_{q_1} + B^{J_{1\mu\nu} J_{2\nu\mu} \rho_1} J_{q_1,\nu\mu} J_{q_2,\mu\nu} \rho_{q_1} \\
& + B^{\nabla_\mu \rho_1 J_{1\nu\kappa} \rho_2 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_1,\kappa\nu} \rho_{q_2} \epsilon_{\mu\nu\kappa} + B^{\nabla_\mu \rho_1 J_{2\nu\kappa} \rho_1 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_2,\kappa\nu} \rho_{q_1} \epsilon_{\mu\nu\kappa} \\
& + B^{\nabla_\mu \rho_1 J_{2\nu\kappa} \rho_2 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_2,\kappa\nu} \rho_{q_2} \epsilon_{\mu\nu\kappa} \Bigg] \Bigg\} .
\end{aligned}
$$

$$
\begin{aligned}
E^{\rho\rho\rho} = \int d\vec{r} \Bigg\{ \sum_{q_1} \Bigg[& + B^{J_{1\mu\mu} J_{1\nu\nu} \rho_1} J_{q_1,\mu\mu} J_{q_1,\nu\nu} \rho_{q_1} + B^{J_{1\nu\mu} J_{1\mu\nu} \rho_1} J_{q_1,\nu\mu} J_{q_1,\mu\nu} \rho_{q_1} \\
& + B^{\nabla_\mu \rho_1 J_{1\kappa\nu} \rho_1 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_1,\kappa\nu} \rho_{q_1} \epsilon_{\mu\nu\kappa} \Bigg] \\
& + \sum_{q_1 \neq q_2} \Bigg[+ B^{J_{1\nu\mu} J_{1\nu\mu} \rho_2} J_{q_1,\nu\mu} J_{q_1,\nu\mu} \rho_{q_2} + B^{J_{1\mu\mu} J_{1\nu\nu} \rho_2} J_{q_1,\mu\mu} J_{q_1,\nu\nu} \rho_{q_2} \\
& + B^{J_{1\nu\mu} J_{1\mu\nu} \rho_2} J_{q_1,\nu\mu} J_{q_1,\mu\nu} \rho_{q_2} + B^{J_{1\mu\mu} J_{2\nu\nu} \rho_1} J_{q_1,\mu\mu} J_{q_2,\nu\nu} \rho_{q_1} \\
& + B^{J_{1\nu\mu} J_{2\mu\nu} \rho_1} J_{q_1,\nu\mu} J_{q_2,\mu\nu} \rho_{q_1} + B^{J_{1\nu\mu} J_{2\nu\mu} \rho_1} J_{q_1,\nu\mu} J_{q_2,\nu\mu} \rho_{q_1} \\
& + B^{\nabla_\mu \rho_1 J_{1\kappa\nu} \rho_2 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_1,\kappa\nu} \rho_{q_2} \epsilon_{\mu\nu\kappa} + B^{\nabla_\mu \rho_1 J_{2\kappa\nu} \rho_1 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_2,\kappa\nu} \rho_{q_1} \epsilon_{\mu\nu\kappa} \\
& + B^{\nabla_\mu \rho_1 J_{2\kappa\nu} \rho_2 \epsilon_{\mu\nu\kappa}} \nabla_\mu \rho_{q_1} J_{q_2,\kappa\nu} \rho_{q_2} \epsilon_{\mu\nu\kappa} \Bigg] \Bigg\} .
\end{aligned}
$$

J. Sadoudi, M. B., J. Meyer et al, in slow progress

Problems, problems . . .

- promising results
- clear signs of deficiencies that require further terms
- Before drawing further conclusions, let's wait until the end of the talks by Karim Bennaceur, Alessandro Pastore, and Dany Davesne.