Bridging Effective Field Theories and Self-Consistent Gorkov-Green's Function method

- Stories from the EFT side

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What have we done?

❖ The existing chiral EFT potentials (WPC) are based on naive dimensional analysis (NDA) are wrong because the NN scattering amplitudes don't satisfy RG invariance (UV cutoff independence): WPC has to be modified by the guidance of RG invariance.

❖ At LO, attractive triplet channels (at lease 3P0 and 3P2) need a counterterm that are, however, considered Q^2 corrections in WPC.

❖ Modifications to WPC in subleading orders are also needed, but are different in (3P0, 3P2) vs. 1S0. Renormalization is most easily demonstrated when subleading interactions are treated as perturbations on top of LO.

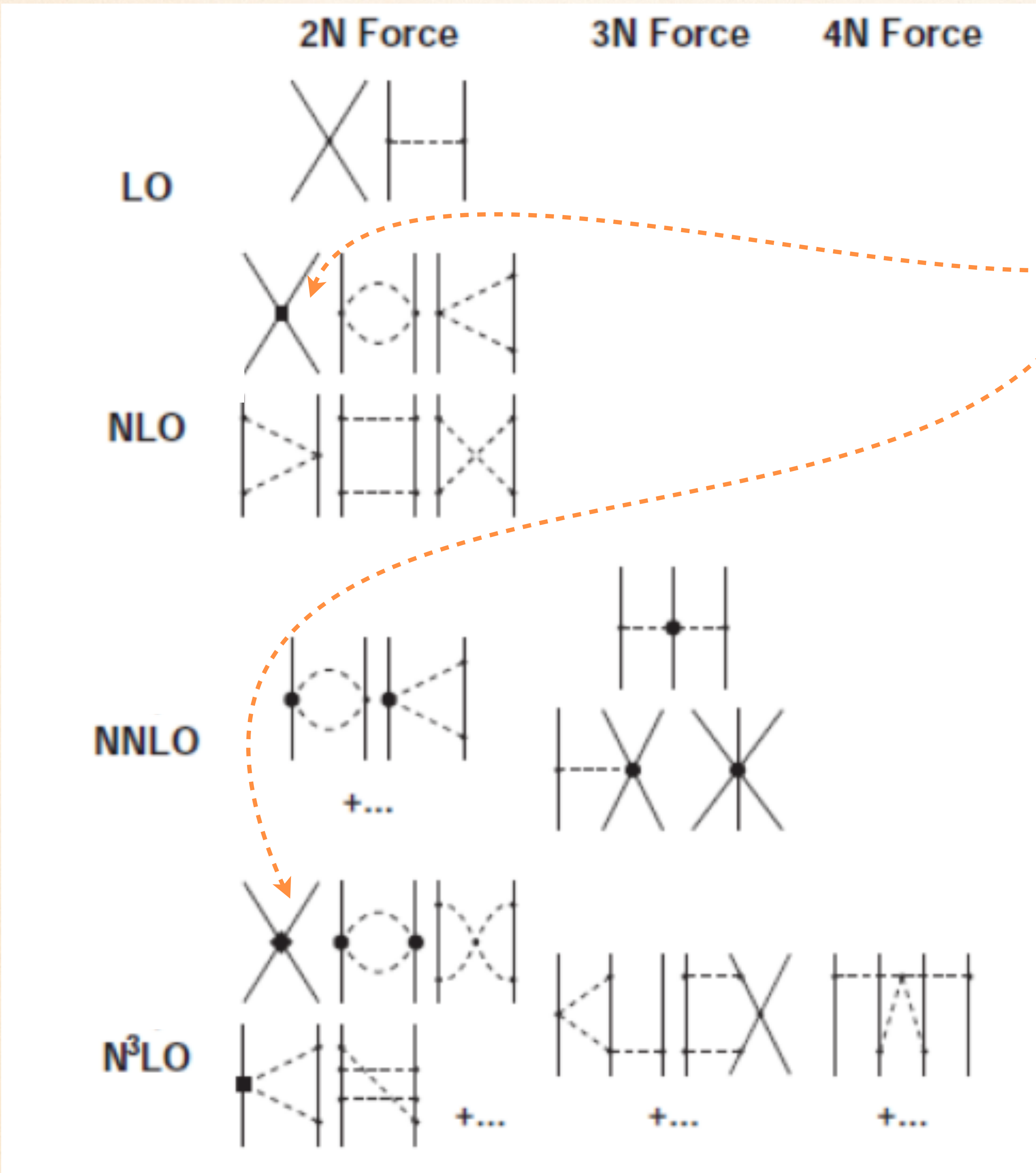
What to do now?

(as I understand it)

- Implement the correct LO potential in SCGGF. (Use RG invariance to further constrain the approximations in many-body calculations?)

- Explore the difference between pert. and non-pert. treatments of subleading potentials, in terms of RG invariance and describing phenomenology

Back to NN



Modify power counting of NN contact interactions, so as

(1) to satisfy renormalization group invariance;

(2) to better understand how much of nuclear physics is decided by short-range interactions as opposed to chiral symmetry.

Modified Game Plan

BwL & Yang (2012)

TABLE I. Power counting for pion exchanges and S- and P-wave counterterms up to $\mathcal{O}(Q^3)$. p (p') is the magnitude of the center-of-mass incoming (outgoing) momentum. The two-by-two matrices are for the coupled channels.

$\mathcal{O}(1)$	OPE, $C_{^1S_0}$, $\begin{pmatrix} C_{^3S_1} & 0 \\ 0 & 0 \end{pmatrix}$, $C_{^3P_0}\, p'p$, $\begin{pmatrix} C_{^3P_2}\, p'p & 0 \\ 0 & 0 \end{pmatrix}$
$\mathcal{O}(Q)$	$D_{^1S_0}(p'^2 + p^2)$
$\mathcal{O}(Q^2)$	TPE0, $E_{^1S_0}\, p'^2 p^2$, $\begin{pmatrix} D_{^3S_1}(p'^2 + p^2) & E_{SD}\, p^2 \\ E_{SD}\, p'^2 & 0 \end{pmatrix}$,
	$D_{^3P_0}\, p'p(p'^2 + p^2)$, $p'p\begin{pmatrix} D_{^3P_2}(p'^2 + p^2) & E_{PF}\, p^2 \\ E_{PF}\, p'^2 & 0 \end{pmatrix}$,
	$C_{^1P_1}\, p'p$, $C_{^3P_1}\, p'p$
$\mathcal{O}(Q^3)$	TPE1, $F_{^1S_0}\, p'^2 p^2(p'^2 + p^2)$

Effective field theory

EFT = Effective Lagrangian + Power Counting

. Low-energy Dofs
. Symmetries

organization principle:
a priori estimation of diagrams

Goal: expansion of amplitudes

$$\mathcal{M} = \sum_n \left(\frac{Q}{M_{hi}}\right)^n \mathcal{F}_n\left(\frac{Q}{M_{lo}}\right)$$

Q: generic external momenta,

$M_{hi} = \Lambda_{SB}, m_\rho, \cdots \sim 1\text{GeV}$

$M_{lo} = m_\pi, f_\pi \sim 100\text{MeV}$

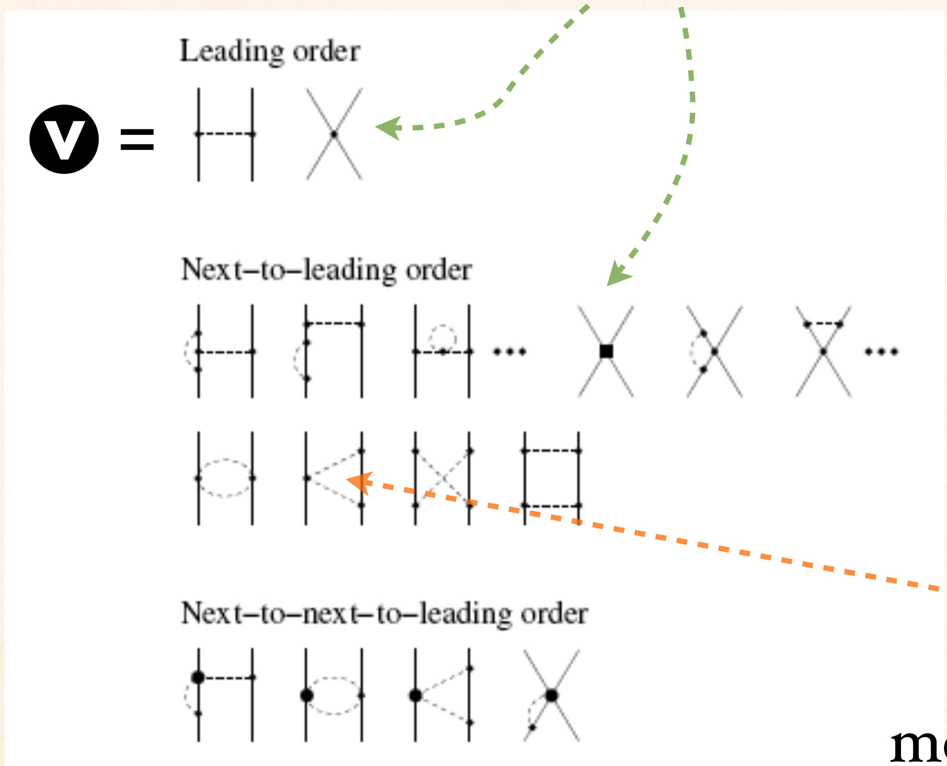
Main benefit: reliable estimates of theo. err.

Dr. W's prescriptions

1. Derivatives on couplings are always suppressed — naive dimensional analysis.

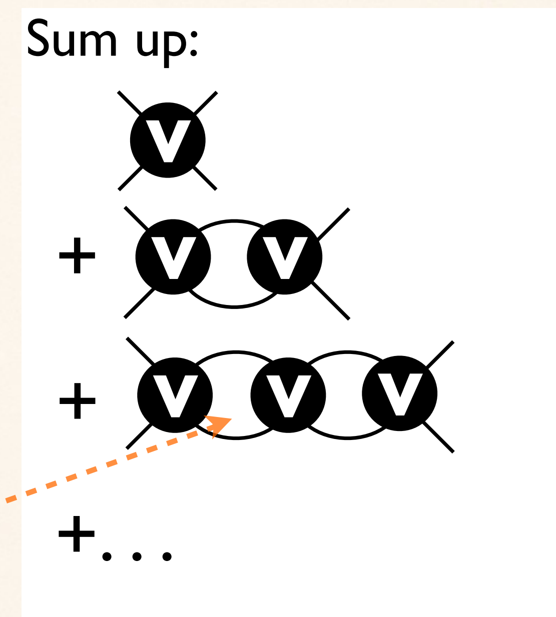
2. Nonperturbative iterations don't affect power counting.

Derive potentials from Chi. Lag.



Λ

momentum cutoff



Solve Sch. eqn. exactly

❖ State-of-art implemention by Epelbaum, et al (1999, 2003) and Entem et al (2002) with a small range of cutoffs

❖ However, cutoff dependence of resummed amplitudes was not addressed in W counting

Strength of OPE provides an infrared scale

$$V_{1\pi} = \frac{g_A^2}{4f_\pi^2}\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{m_\pi^2+q^2} \quad \to \frac{1}{m_N}\frac{\lambda}{M_{NN}r^3}e^{-m_\pi r}$$

$M_{NN} = 100 \sim 300\mathrm{MeV}$ varies for different partial waves

Naive dimensional analysis for estimating contact interactions is no longer reliable

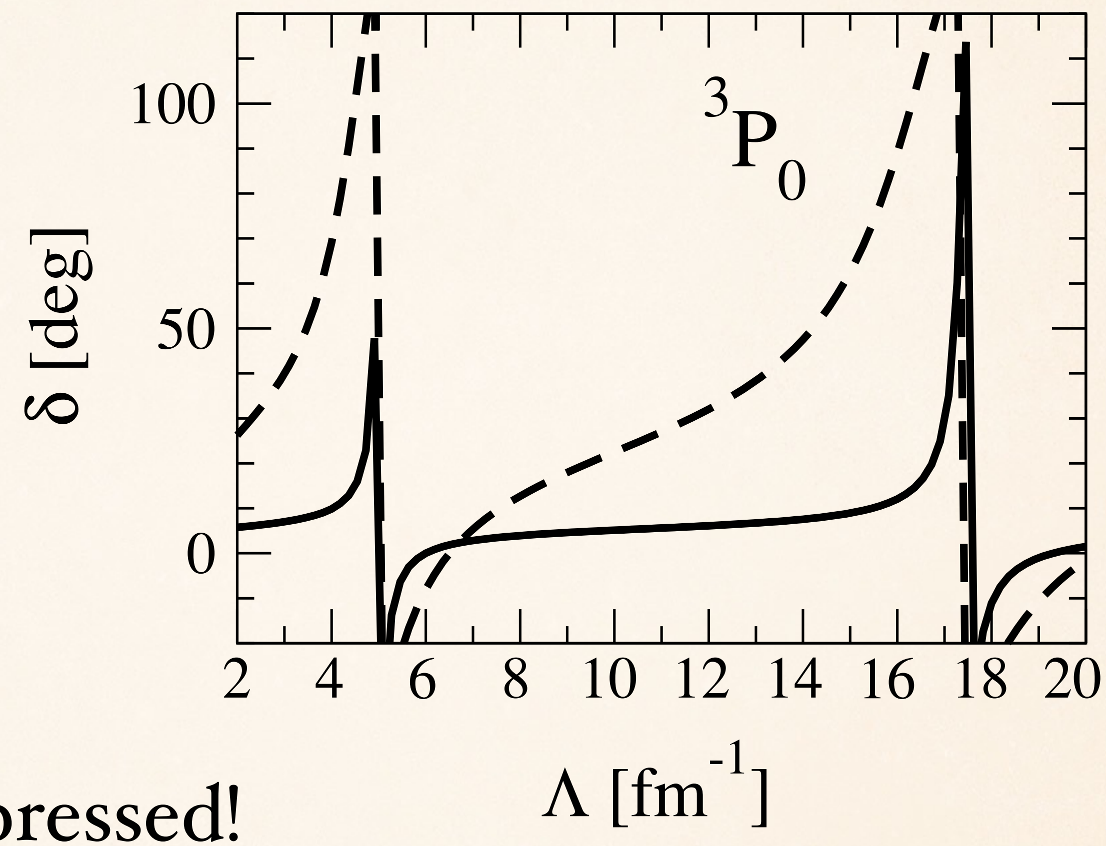
$$\frac{4\pi}{m_N}\left(\frac{\tilde{C}_0}{M_0}\delta^{(3)}(\vec{x}) + \frac{\tilde{C}_2}{M_2^2}\nabla^2\delta^{(3)}(\vec{x}) + \cdots\right)$$

Cutoff dependence of W counting

Nogga, Timmerman & van Kolck (2005)

E.g., 3Po

A singular attractive potential needs a counterterm — 4-nucleon operator with the same QM number as 3Po



→ A derivative coupling not suppressed!

Solid: Tlab = 10 MeV, dashed: 50 MeV

Very large cutoffs were used to illustrate the cutoff dependence, but we don't insist on using them in practical calculations once power counting is established.

Subleading orders in triplet channels

Renormalization of one insertion of two-pion exchange.

Pavon Valderrama(2005, 2011, 2012)
BwL, van Kolck (2008)
BwL, Yang (2011, 2012)

for LO potential $\sim -1/r^3$,

$$\psi_k^{(0)}(r) \sim \left(\frac{\lambda}{r}\right)^{\frac{1}{4}} \left[u_0(r/\lambda) + k^2 r^2 \sqrt{\frac{r}{\lambda}} u_1(r/\lambda) + \mathcal{O}(k^4)\right]$$

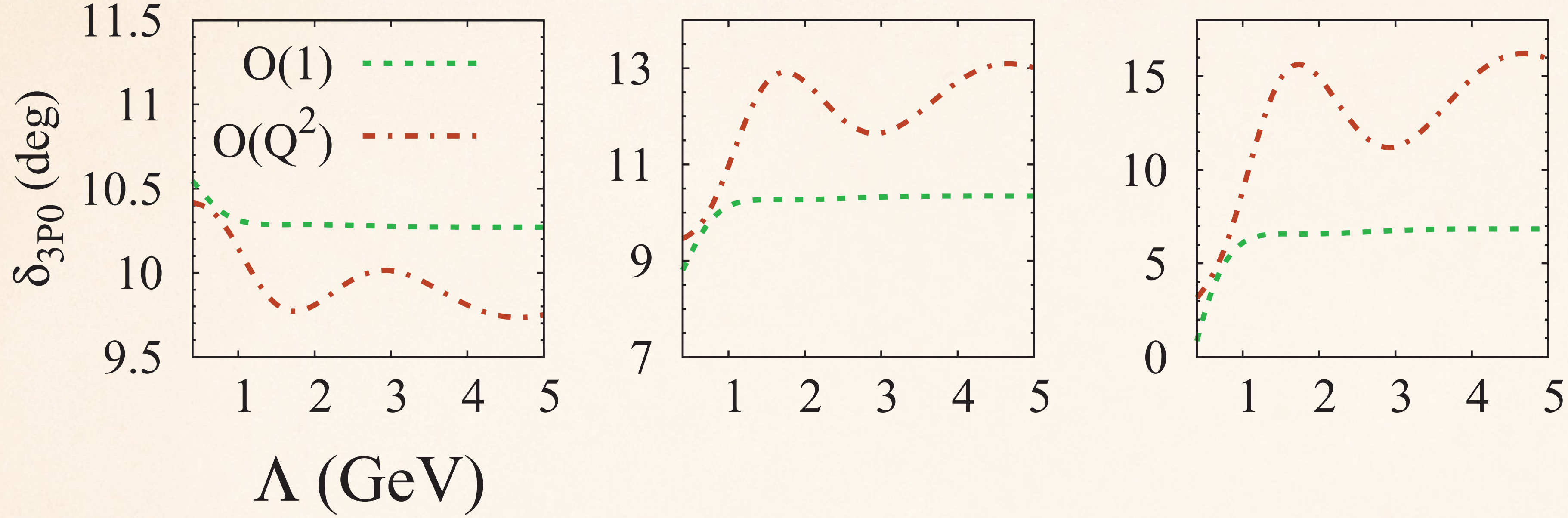
$$\lambda = \frac{3g_A^2 m_N}{8\pi f_\pi^2} \qquad u_{1,\,2}(x) \sim \mathcal{O}(1)$$

$$V_{2\pi} \sim \frac{1}{r^5}, r \to 0$$

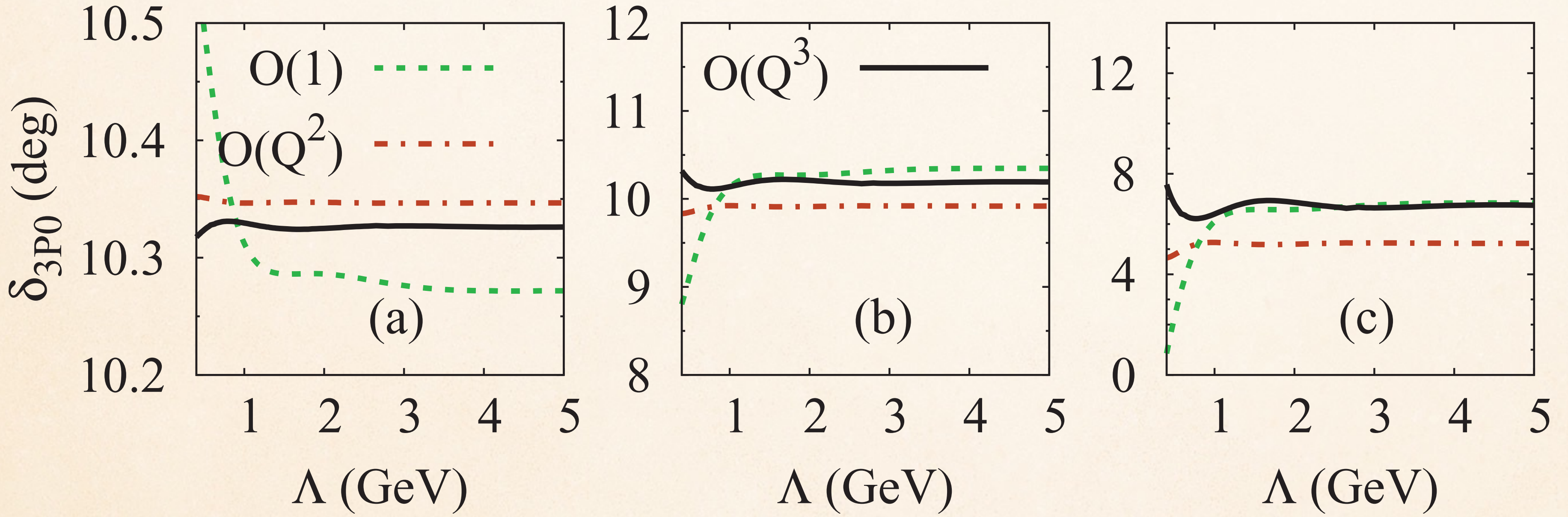
$$T^{(2)} = \langle \psi^{(0)} | V_{2\pi} | \psi^{(0)} \rangle$$

$$\sim \int_{\sim 1/\Lambda} dr r^2 |\psi^{(0)}(r)|^2 \frac{1}{r^5} \sim \alpha_0(\Lambda)\Lambda^{5/2} + \beta_0(\Lambda) k^2 + \mathcal{O}(k^4 \Lambda^{-5/2})$$

Numerics



Modify WPC at LO but not at O(Q^2)



Modify WPC at LO, O(Q^2) and O(Q^3)

WPC for 3P0

LO	OPE
$O(Q)$	-
$O(Q^2)$	TPE0 + $C_2\, p^2$
$O(Q^3)$	TPE1
$O(Q^4)$	other p.e. + $C_4\, p^4$

Modified PC for 3P0

LO	OPE
$O(Q)$	-
$O(Q^2)$	TPE_0 + $C_2\, p^2$
$O(Q^3)$	TPE_1
$O(Q^4)$	other p.e. + $C_4\, p^4$

Similar modifications to other attractive triplet channels (3P2, maybe 3D2)

The saga of 1S0

$$V^{(0)}_{1S0} = -\frac{g_A^2 m_\pi^2}{4 f_\pi^2}\frac{e^{-m_\pi r}}{r} + C_0\,\delta(\vec{r})$$

OPE becomes regular near the origin ~ 1/r → no singular attraction

Note: Co is really (Co + D2 mpi^2), for renormalization purpose. And D2 contributes to NN -> NN pi pi, through chiral symmetry.

O(k/\Lambda) cutoff error at LO suggests O(Q) should not be vanishing as WPC prescribes -> C2 is promoted to O(Q) from O(Q^2)

Modified Game Plan

BwL & Yang (2012)

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Other modified p.c.s exist !

Summary

1. Weinberg's scheme for chiral nuclear forces needs modifications

2. Some of the NN contact operators need promotions
   - In attractive triplet channels, due to renormalization
   - In 1S0, due to fine tuning of underlying theory