Three-body forces in ab initio calculations of nuclei

Vittorio Somà (CEA Saclay)

ESNT Program

Radioactive Ion Beam experiments and three-nucleon forces

SPhN, 31 March 2014

---

1) Introduction

2) Nuclear Hamiltonian

3) RG techniques

4) Treatment of 3N forces

5) Error estimates

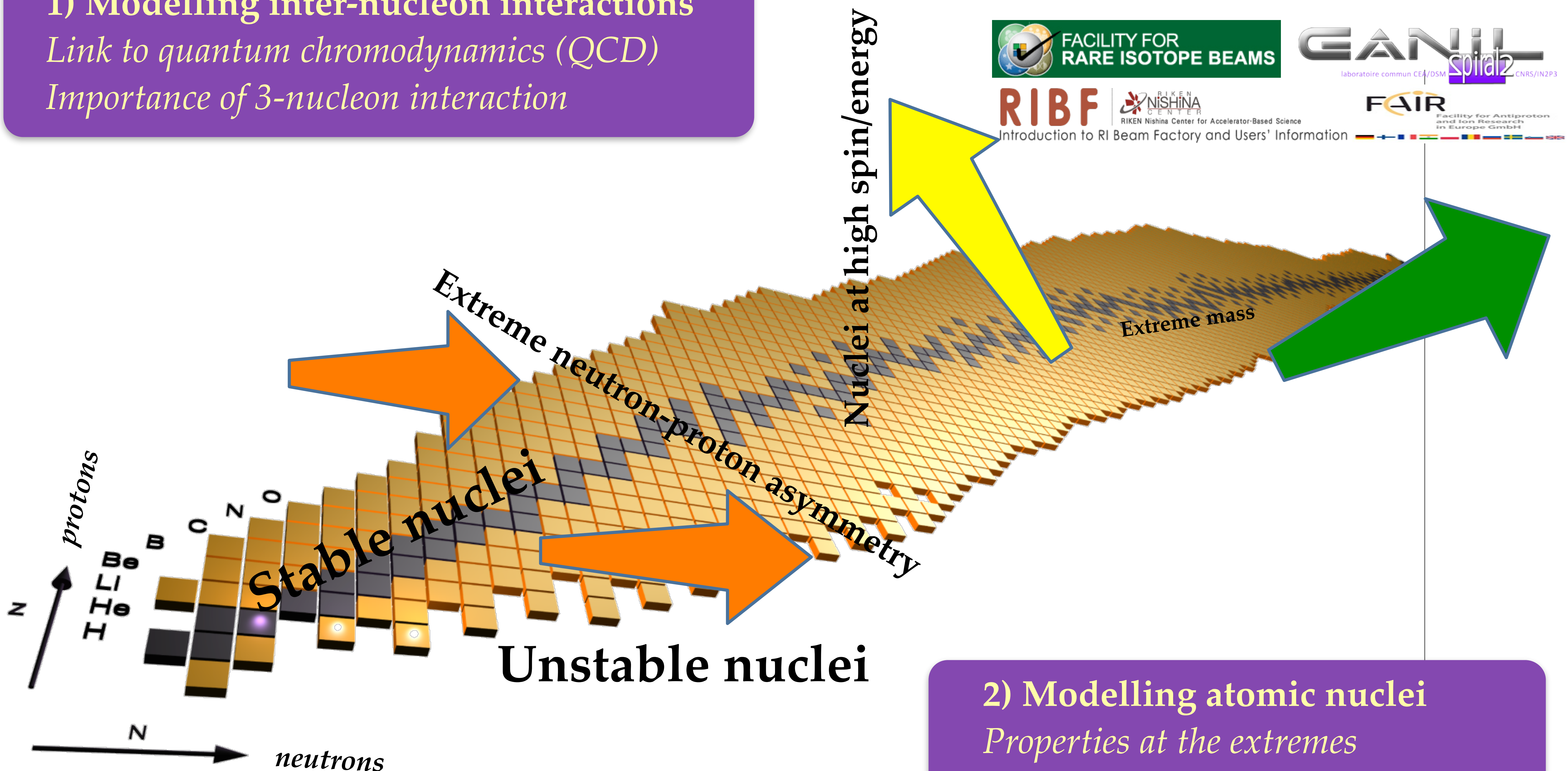
6) Results

Introduction

Low-energy nuclear physics: state of the art

Large-scale experimental facilities

1) Modelling inter-nucleon interactions
Link to quantum chromodynamics (QCD)
Importance of 3-nucleon interaction



2) Modelling atomic nuclei
Properties at the extremes
Reliable and consistent systematics

Ab initio vs effective many-body theories

Ab initio many-body theories

- ➠ Inter-nucleon interactions as input
- ➠ Solve A-body Schrödinger eq.
- ➠ Thorough assessment of errors

Limited applicability

Controlled extrapolations

Test fundamental interactions

Effective many-body theories

- ➠ Based on effective interactions
- ➠ Solve *simpler* many-body problem
- ➠ Partial assessment of errors

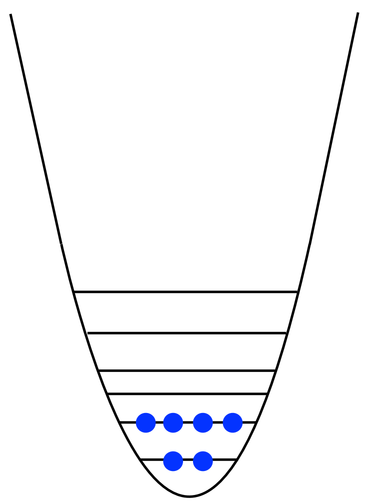
Extended reach

Uncontrolled extrapolations

Aim at reproduction of data

Different ab initio philosophies

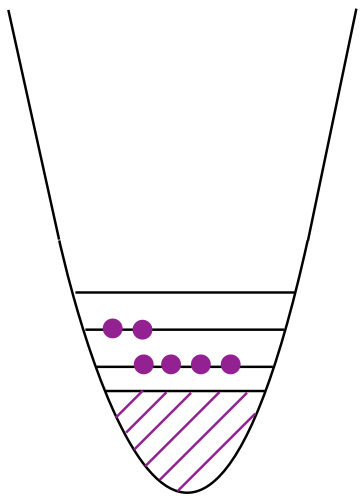
Light nuclei



"Exact"

NCSM, GFMC, ...

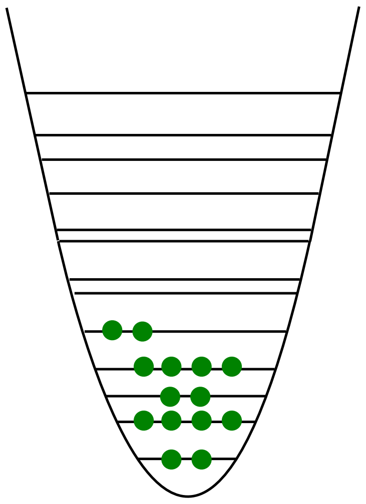
Medium-mass nuclei



Valence space

Miscroscopic SM

Medium-mass nuclei



Based on expansion

GF, CC, IM-SRG, ...

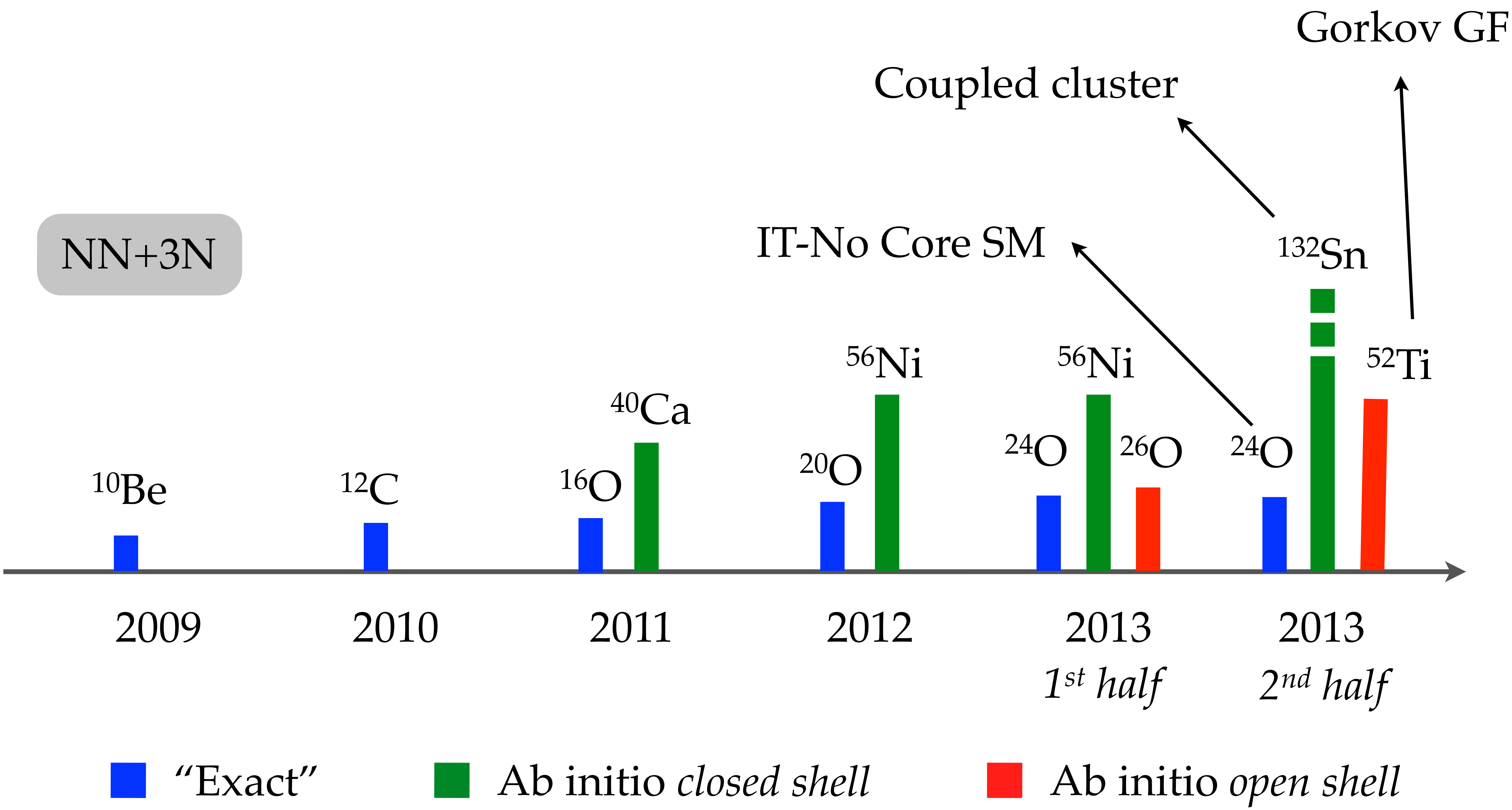
Closed-shell

Open-shell

All methods (should be able to) take the same input NN+3N interactions

Current limits / reach of ab initio calculations

➟ Heavier system computed in the different types of ab initio



Nuclear Hamiltonian

Traditional nuclear interactions

✪ Based on one-boson exchange models

➟ Based on one-boson exchange models

➟ Feature a *hard core*

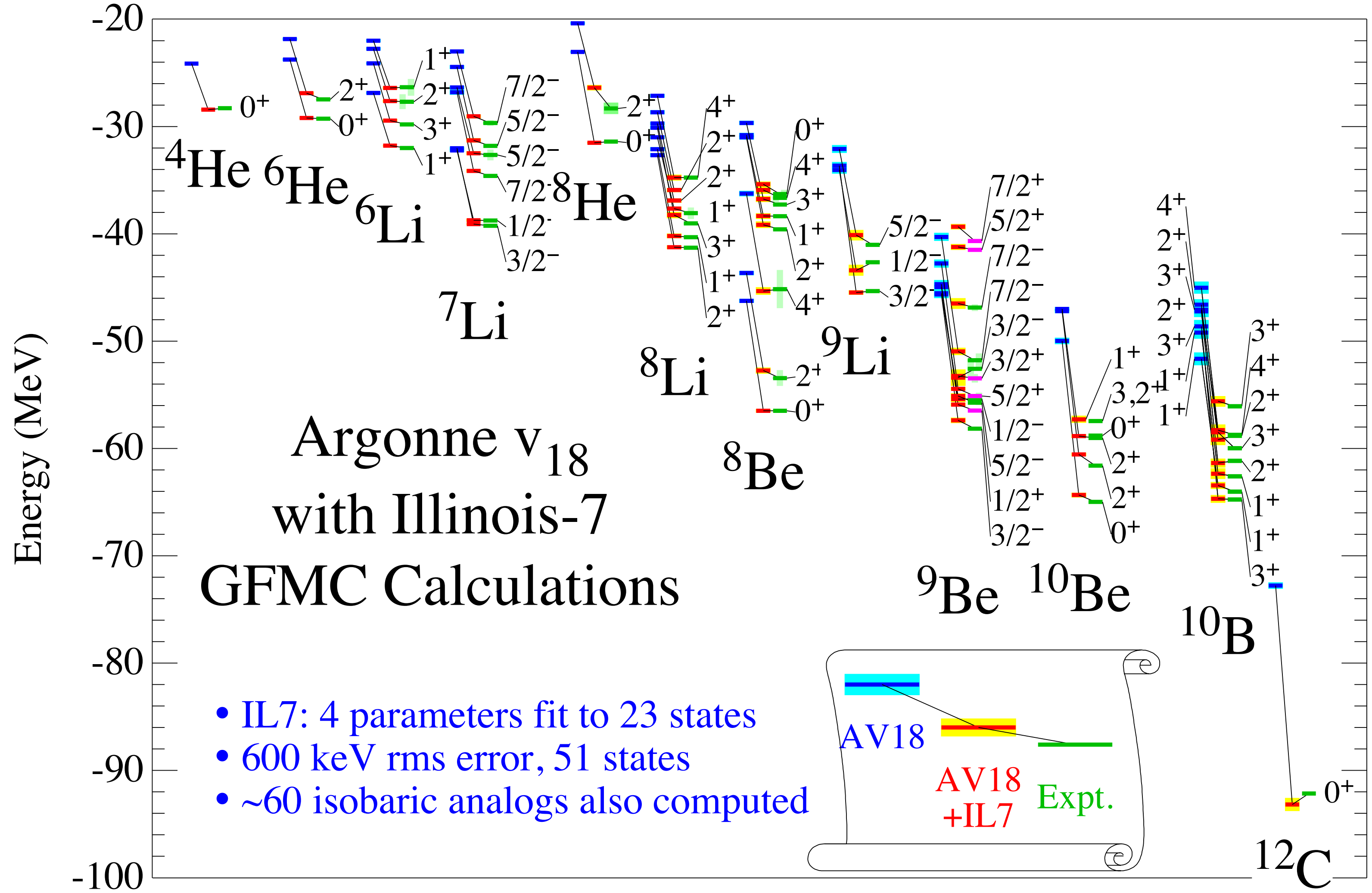
➟ Three-body forces mainly phenomenological

Not systematically improvable

Not consistent with NN

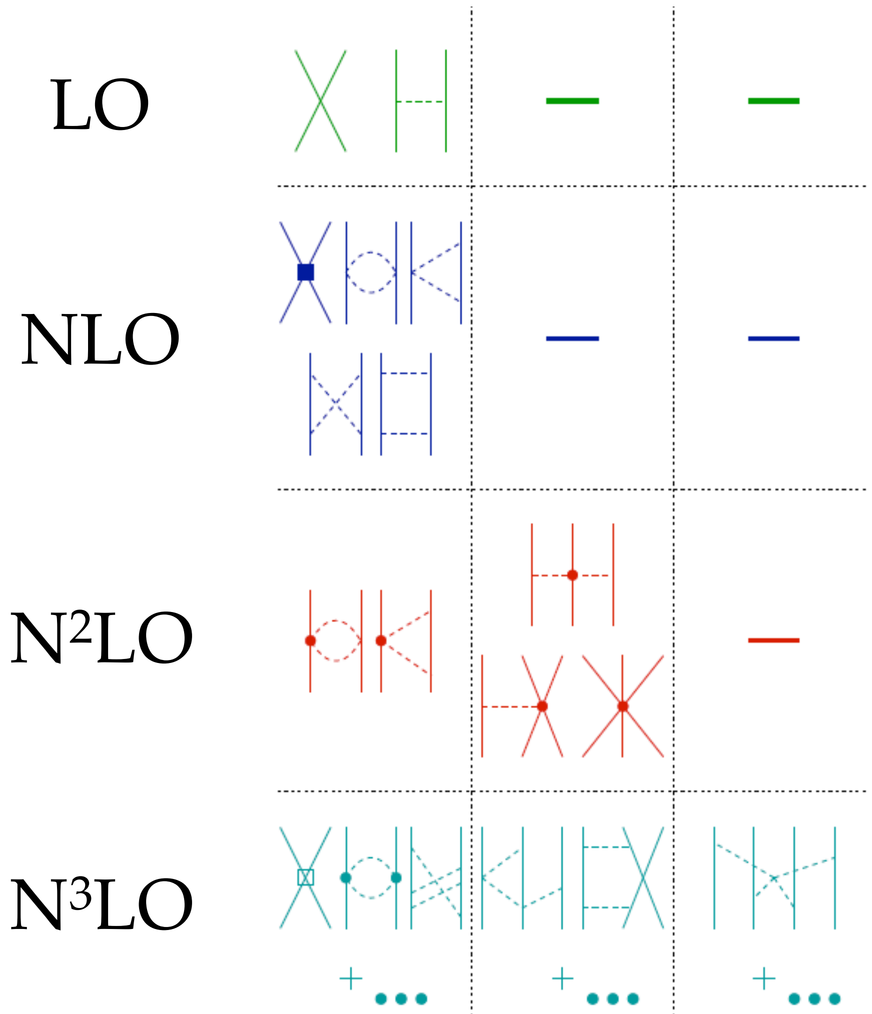
Severe constraint for the many-body approach

CD-Bonn
Av18
Reid
Nijmegen
…



[Pieper & Wiringa 2001]

Modern approach: interactions from chiral EFT

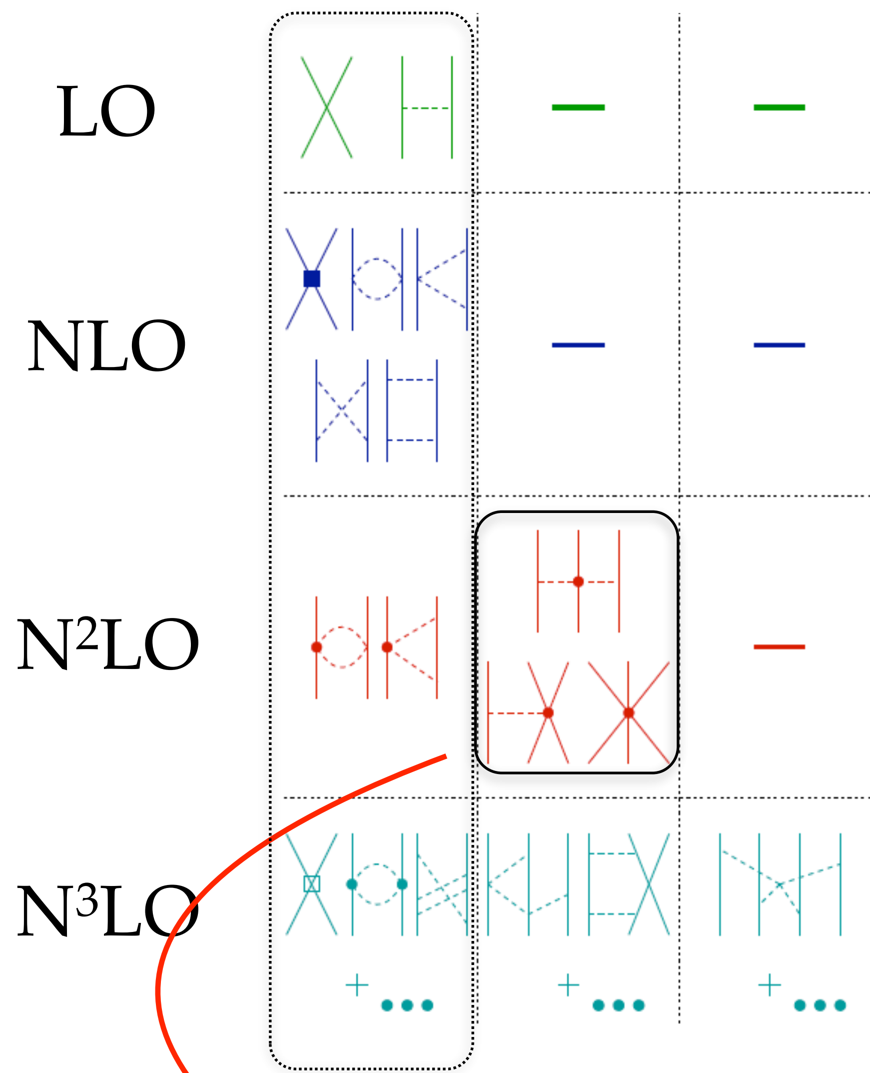


- ✪ Separation of scales
- ✪ Expansion in powers of momenta
- ✪ Long-range physics explicit
  +
  Short-range couplings
- ✪ Consistent many-body forces
- ✪ Systematic, provides error estimates

✪ Very promising, but yet not completely satisfactory

- ➠ Different orders in EFT
- ➠ Different cutoffs
- ➠ Nuclear matter indicates that N^3LO might not be enough
- ➠ More fundamental problem: EFT power counting

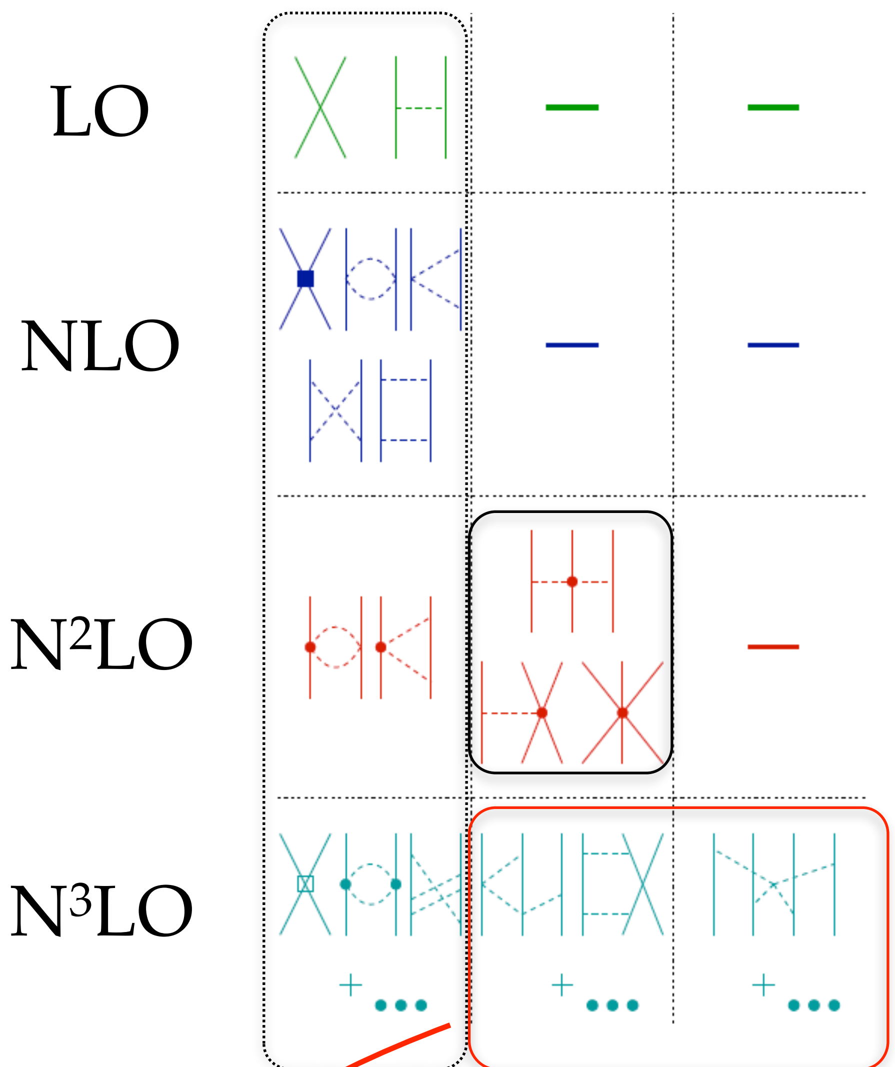
Modern approach: interactions from chiral EFT



- ✪ Separation of scales
- ✪ Expansion in powers of momenta
- ✪ Long-range physics explicit
  +
  Short-range couplings
- ✪ Consistent many-body forces
- ✪ Systematic, provides error estimates

Routinely included in nuclear structure calculations

Modern approach: interactions from chiral EFT

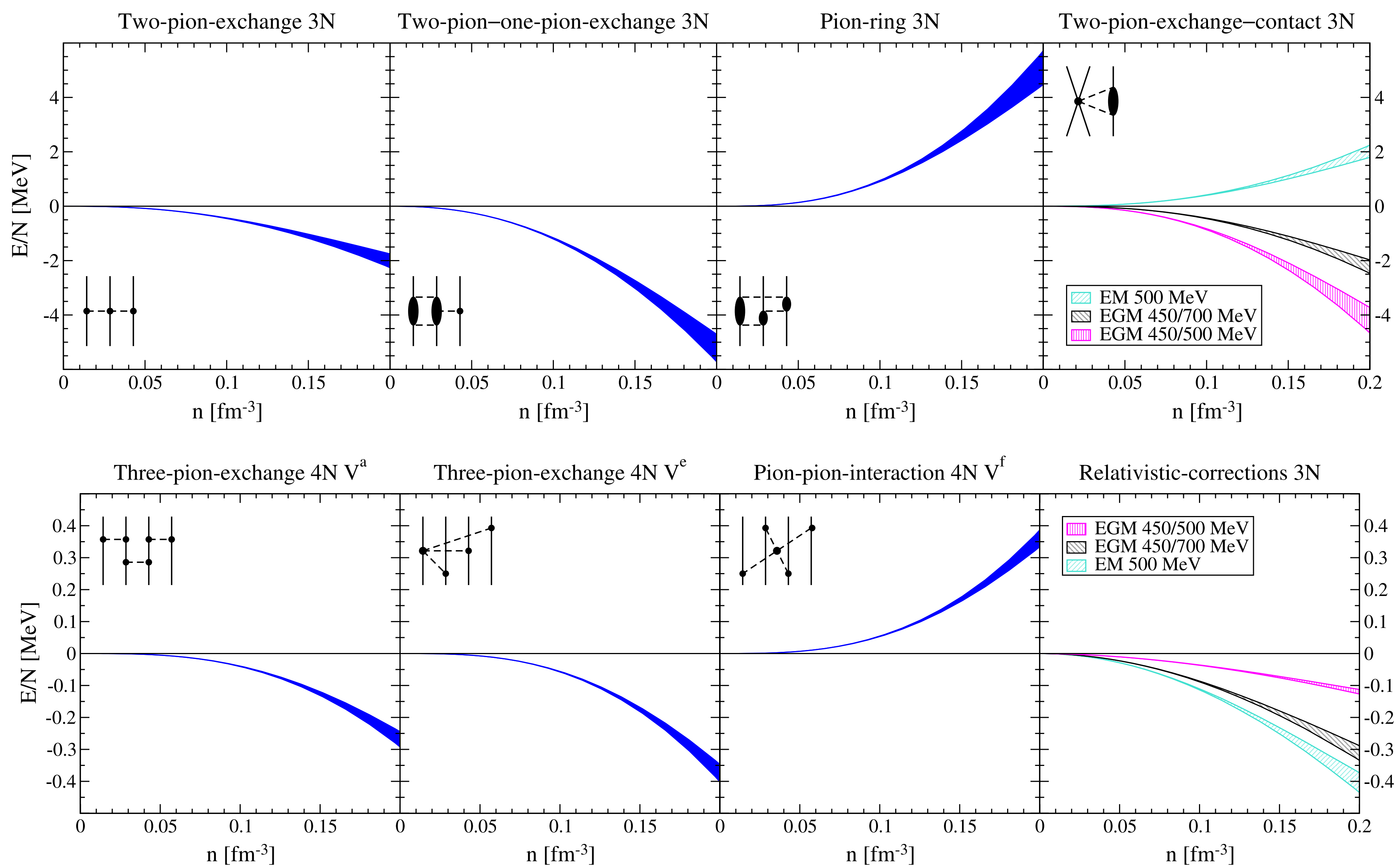


and this?

- ✪ Separation of scales
- ✪ Expansion in powers of momenta
- ✪ Long-range physics explicit
  +
  Short-range couplings
- ✪ Consistent many-body forces
- ✪ Systematic, provides error estimates

Modern approach: interactions from chiral EFT

✪ Estimated (HF level) in (infinite) neutron matter



➟ N^{3}LO 3N contributions significant

➟ N^{3}LO 4N contributions small

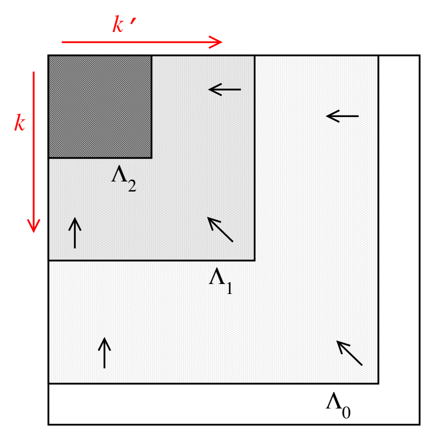
[Tews *et al.* 2013]

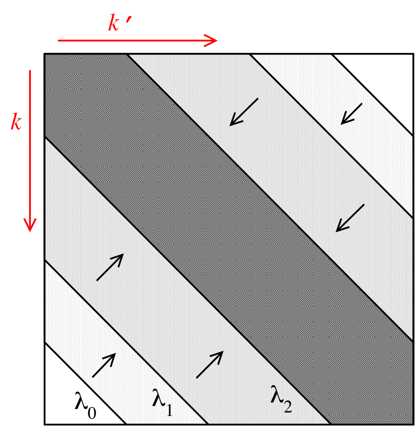
RG techniques

RG techniques for NN & 3N forces

- ✪ Renormalization group techniques for NN and 3N forces
  - ➟ Lower the *resolution scale* of the original Hamiltonian

$V_{\text{low-k}}$





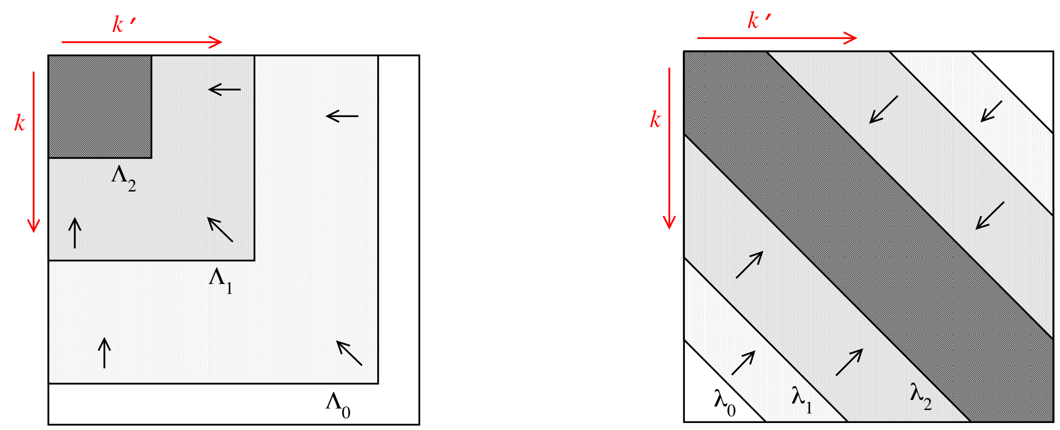
V_{SRG}

RG techniques for NN & 3N forces

✪ Renormalization group techniques for NN and 3N forces

➟ Lower the *resolution scale* of the original Hamiltonian

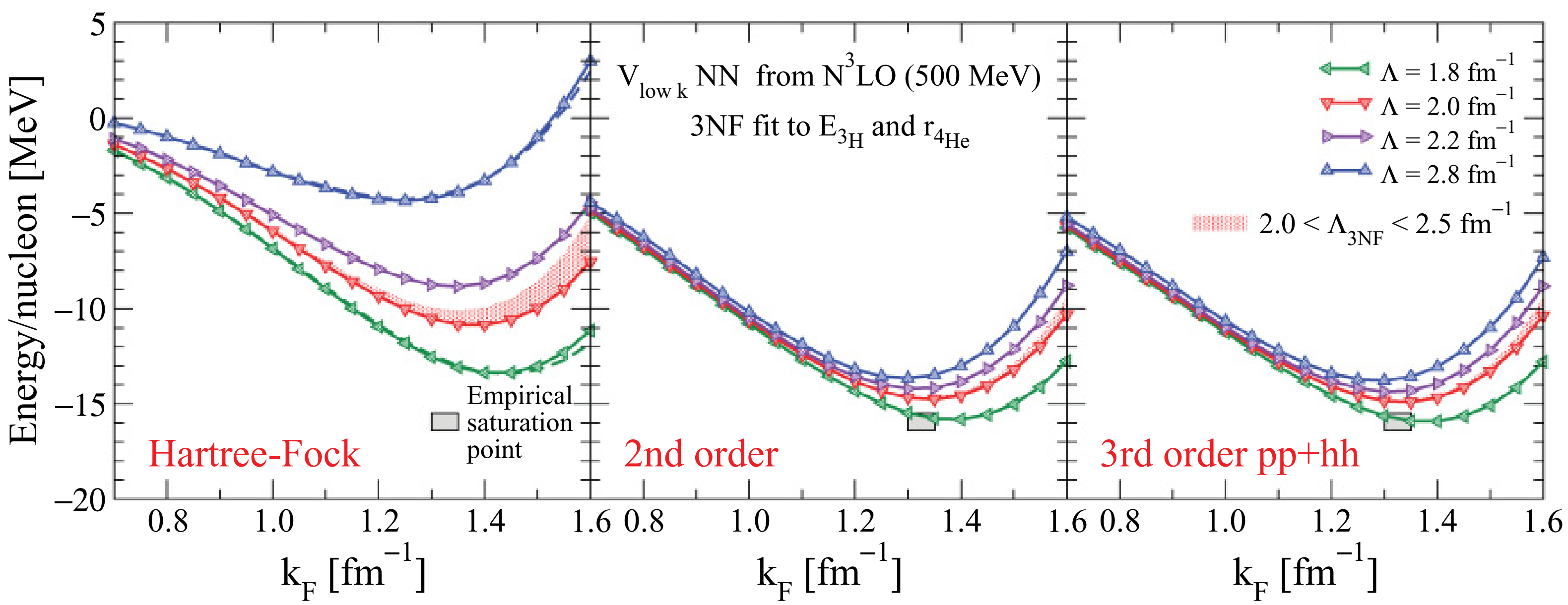
$V_{\text{low-k}}$



V_{SRG}

✪ Improved convergence of many-body calculations

➟ Many-body problem more perturbative

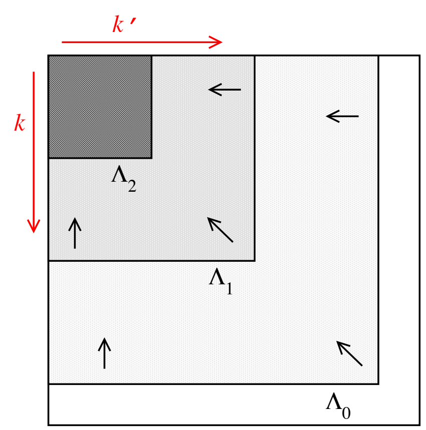


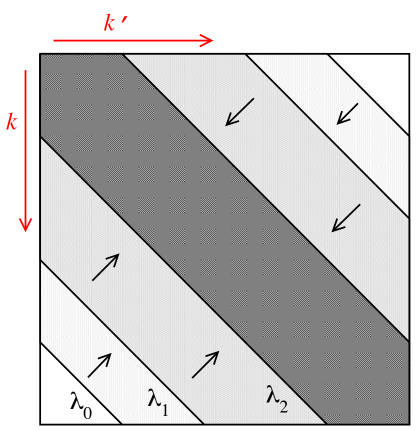
[Hebeler *et al.* 2011]

RG techniques for NN & 3N forces

- ✪ Renormalization group techniques for NN and 3N forces
  - ➟ Lower the *resolution scale* of the original Hamiltonian

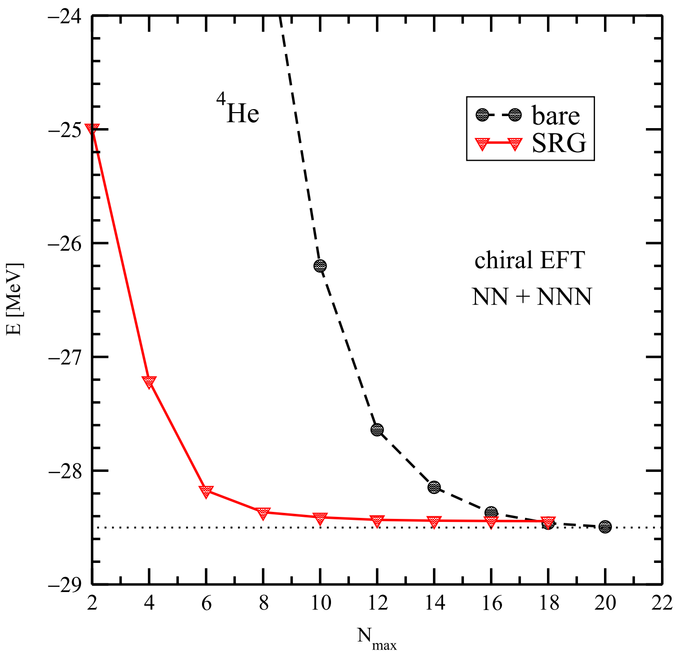
$V_{\text{low-k}}$





V_{SRG}

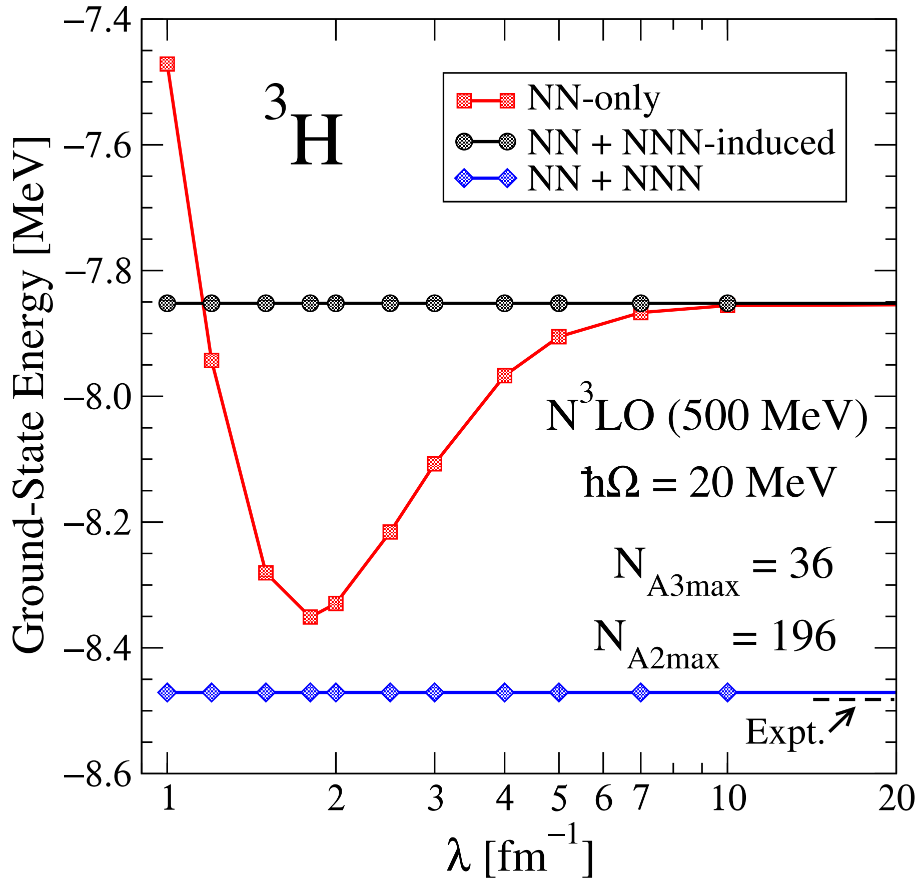
- ✪ Improved convergence of many-body calculations
  - ➟ Smaller model spaces needed

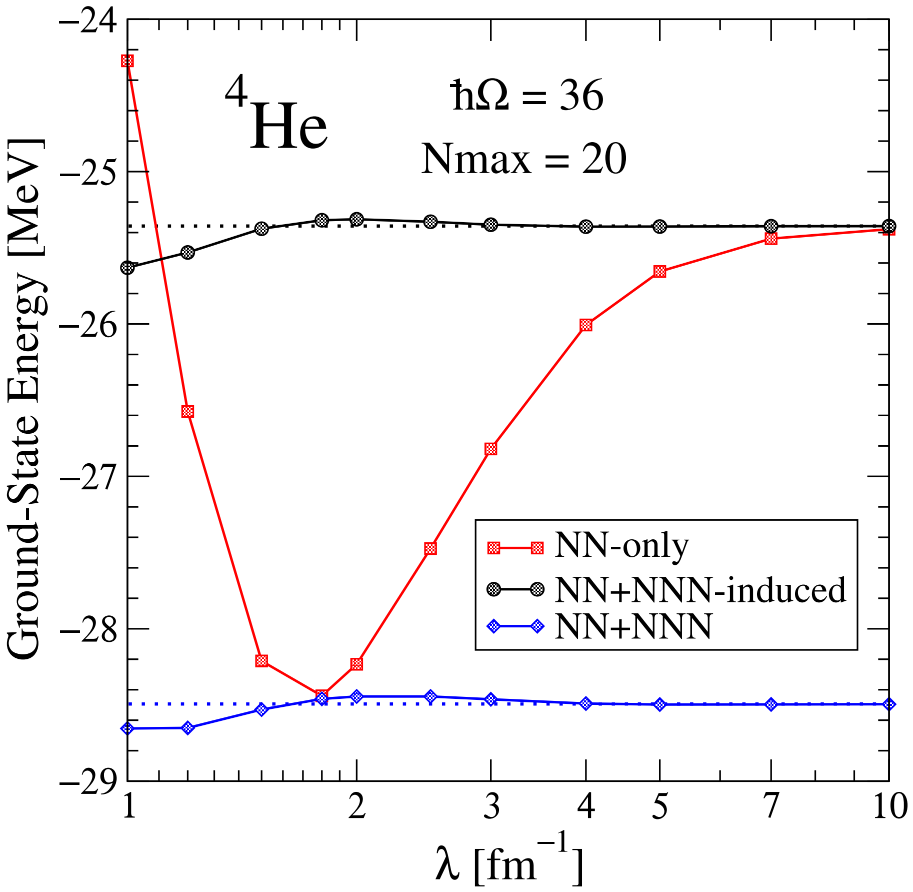


[Jurgenson, Navratil & Furnstahl 2013]

RG techniques for NN & 3N forces

➟ But… (additional) many-body forces are generated

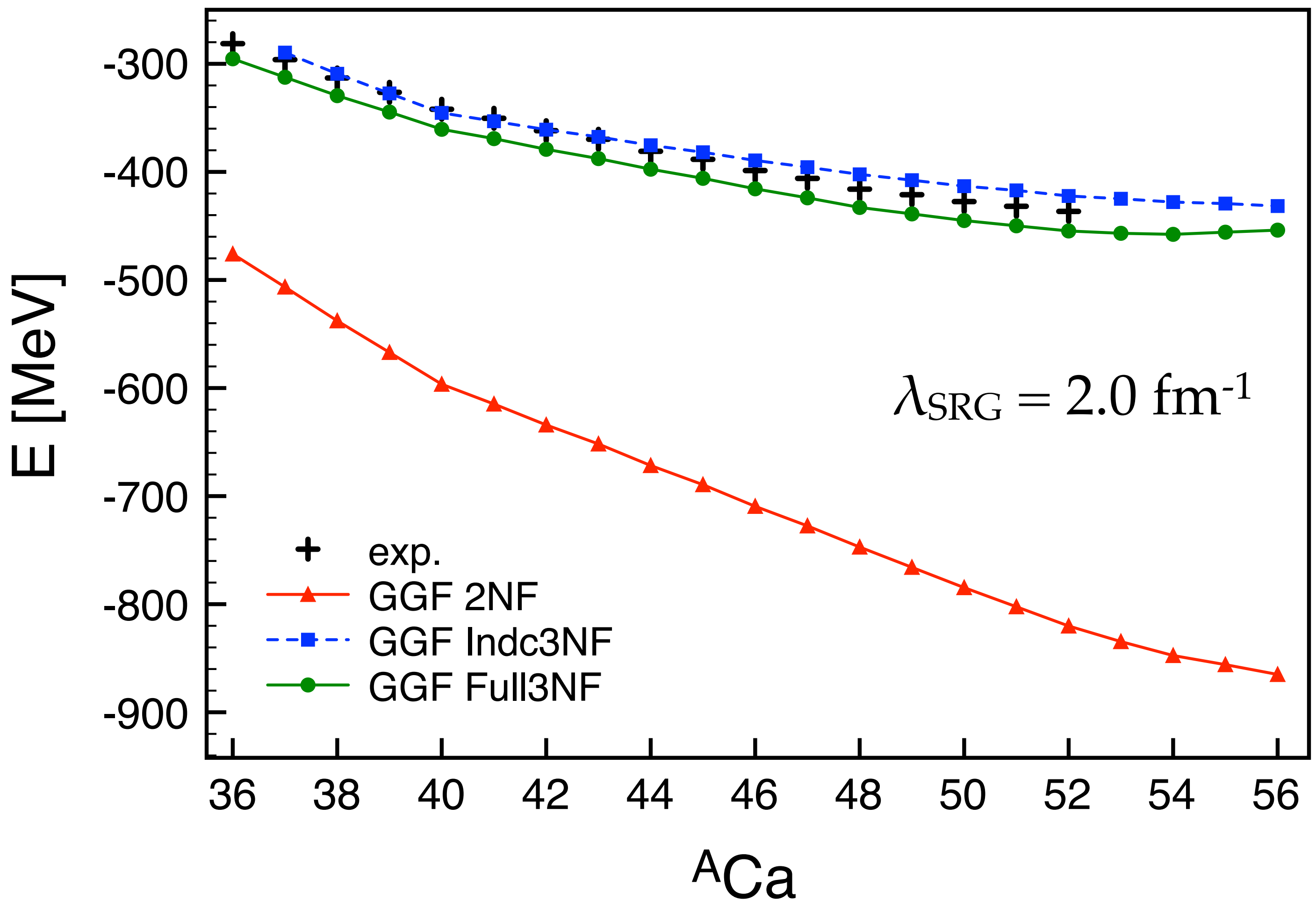




[Jurgenson *et al.* 2011]

RG techniques for NN & 3N forces

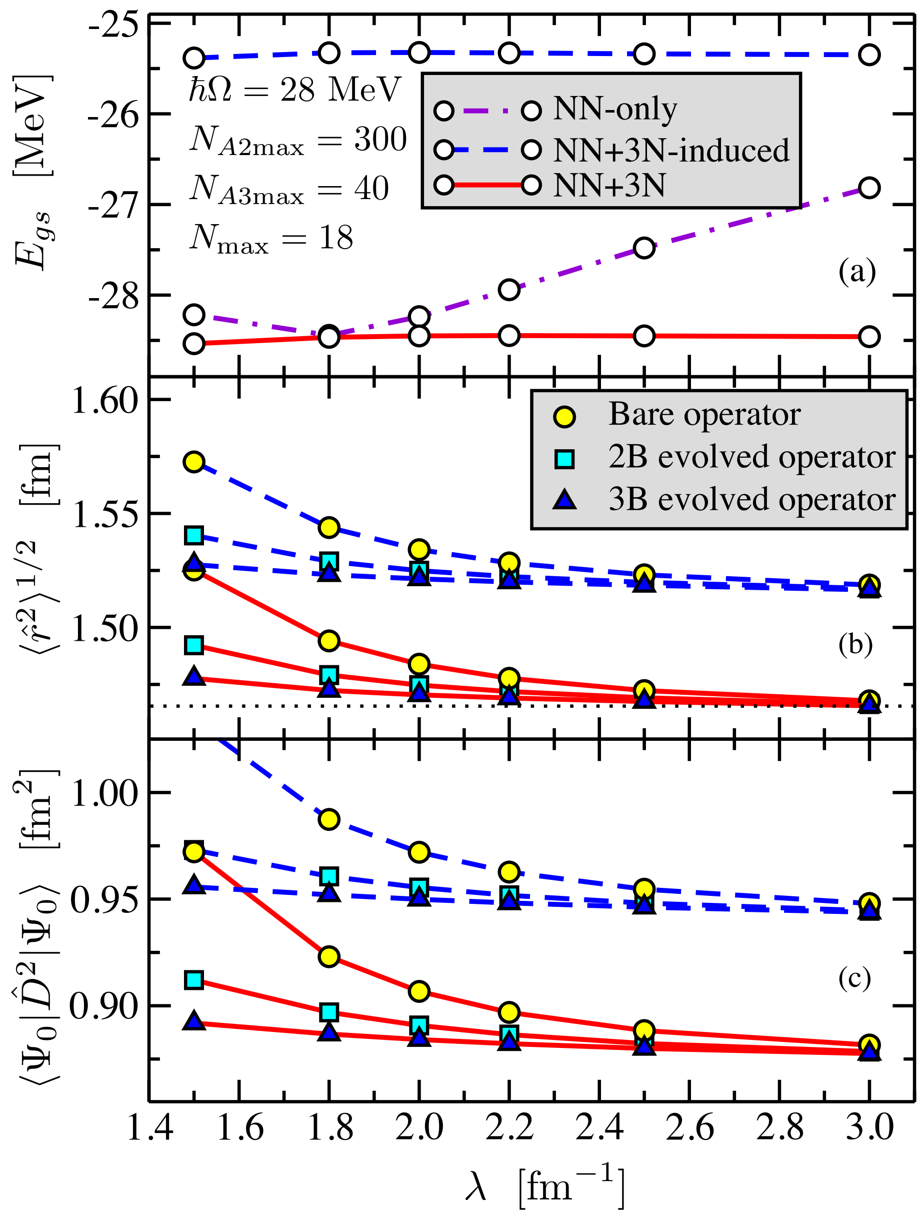
➟ One has to be careful about *which* 3NF is talking about



RG techniques for NN & 3N forces

✪ Evolution of operators

➟ Different operators should in principle be evolved consistently



^{4}He

[Schuster *et al.* 2014]

Treatments of three-body forces

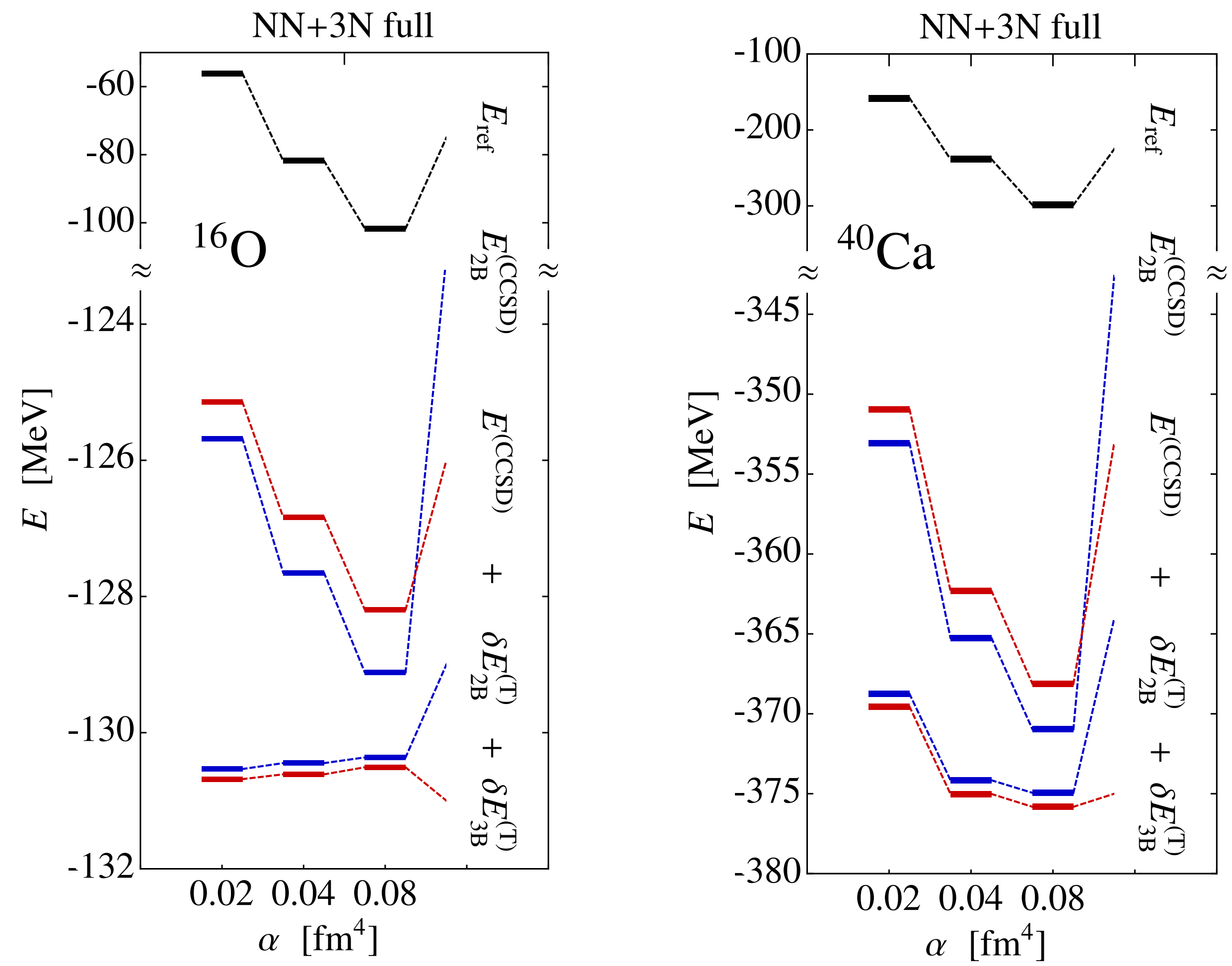
Treatment of three-body forces

✪ Full treatment of 3NF matrix elements is challenging

✪ Convenient solution: normal-ordered form of 3-body Hamiltonian

➟ Normal order with respect to an A-body Slater determinant

➟ Discard (residual/genuine) 3-body interaction



[Binder *et al.* 2013]

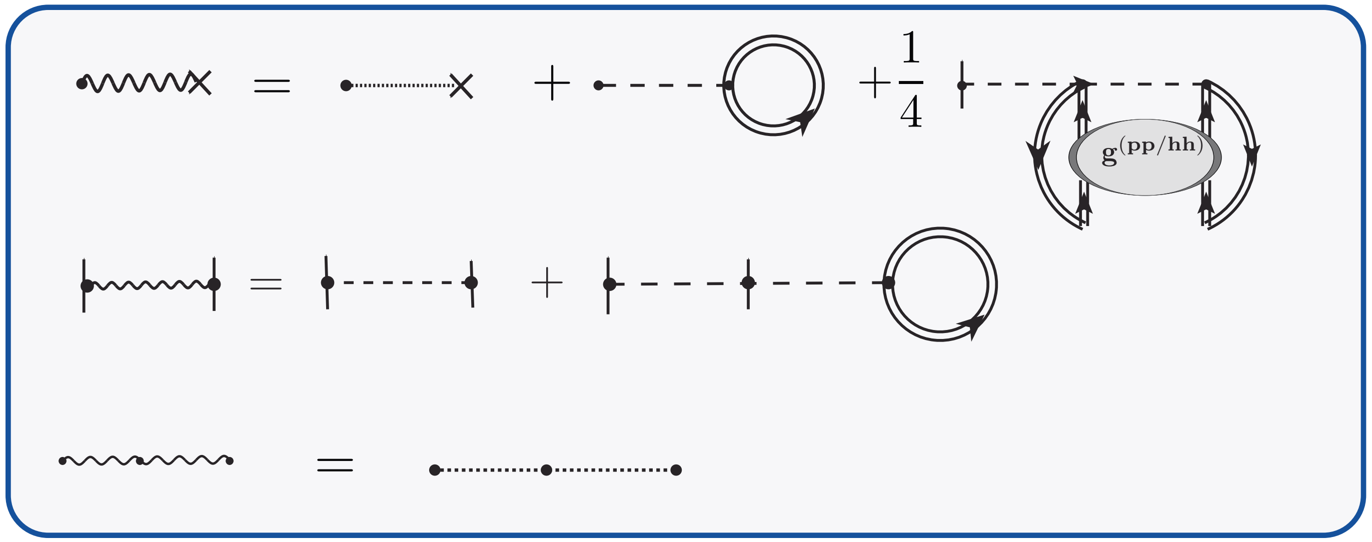
Treatment of three-body forces

✪ One- and two-body forces derived from the 3N part of the Hamiltonian

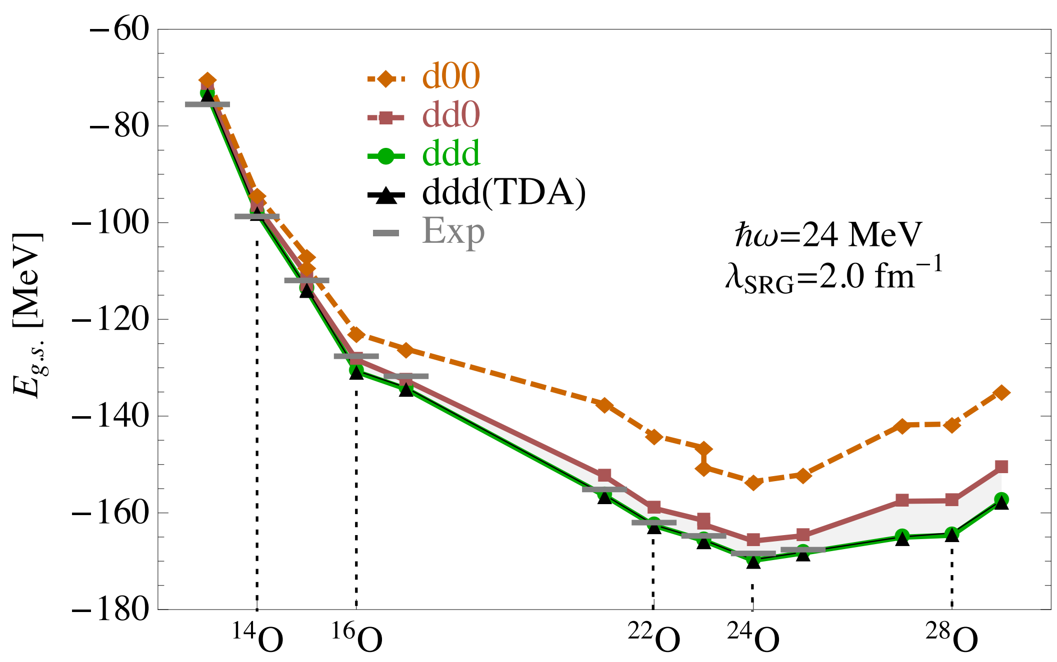
➟ Contractions with fully correlated density matrix

➟ Generalization of normal ordering

✪ Galitskii-Koltun sum rule modified to account for 3N piece



[Carbone, Cipollone *et al.* 2013]



➟ Use of dressed propagators provides significant extra correlations

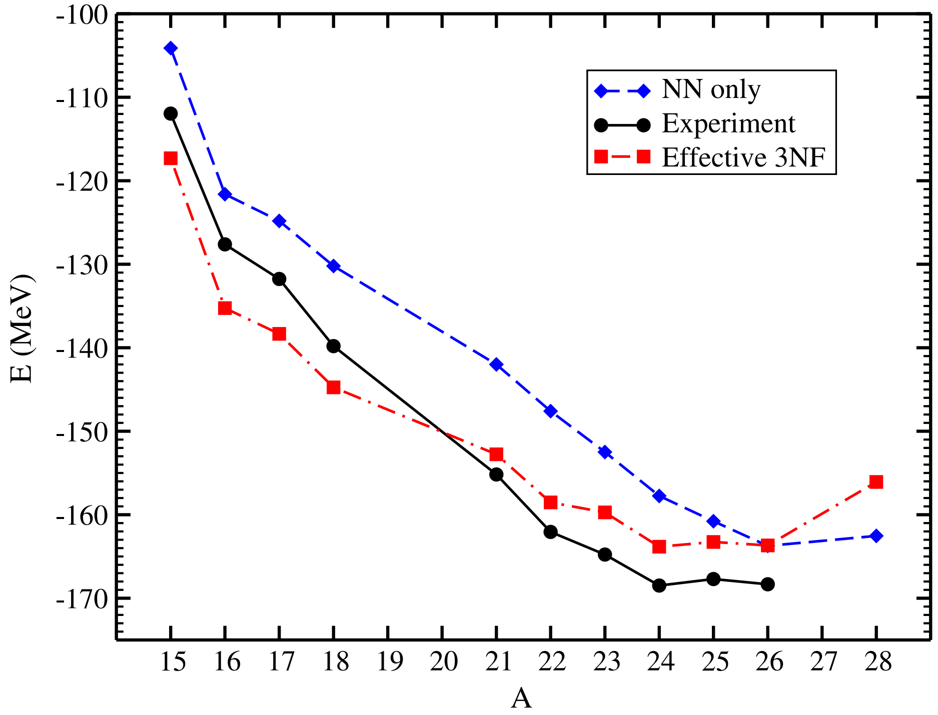
Treatment of three-body forces

✪ More phenomenological inclusions of 3N forces

➟ Derive effective NN by averaging in infinite matter

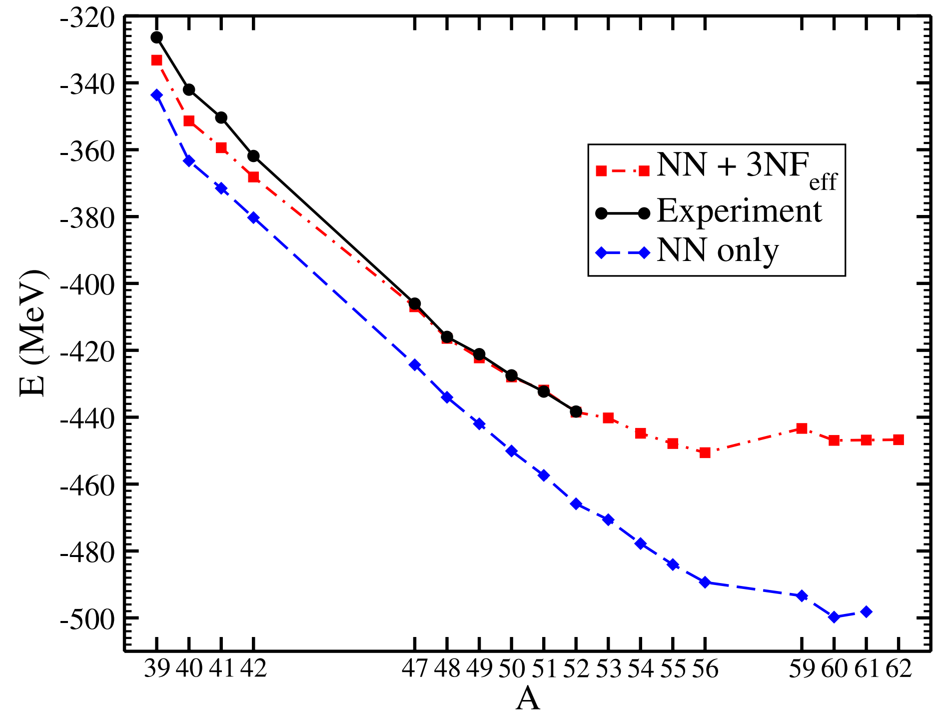
➟ Adjust Fermi momentum and c_E to reproduce some of the isotopes

^{16}O, ^{22}O



[Hagen *et al.* 2012]

^{48}Ca, ^{52}Ca



[Hagen *et al.* 2012]

Error estimates

Error estimates in ab initio calculations

- ✪ Long-term goal: predictive calculations with quantified theoretical errors
- ✪ Possible sources of error:

  1) Numerical algorithms
     - ➠ Usually the smallest source of error
     - ➠ Comparison within one method

  2) Model space truncation
     - ➠ Enters at different stages of the calculation
     - ➠ Comparison within one method + results from other methods

  3) Many-body expansion
     - ➠ Roughly under control, but difficult to assess precisely
     - ➠ Comparison within one method & different methods

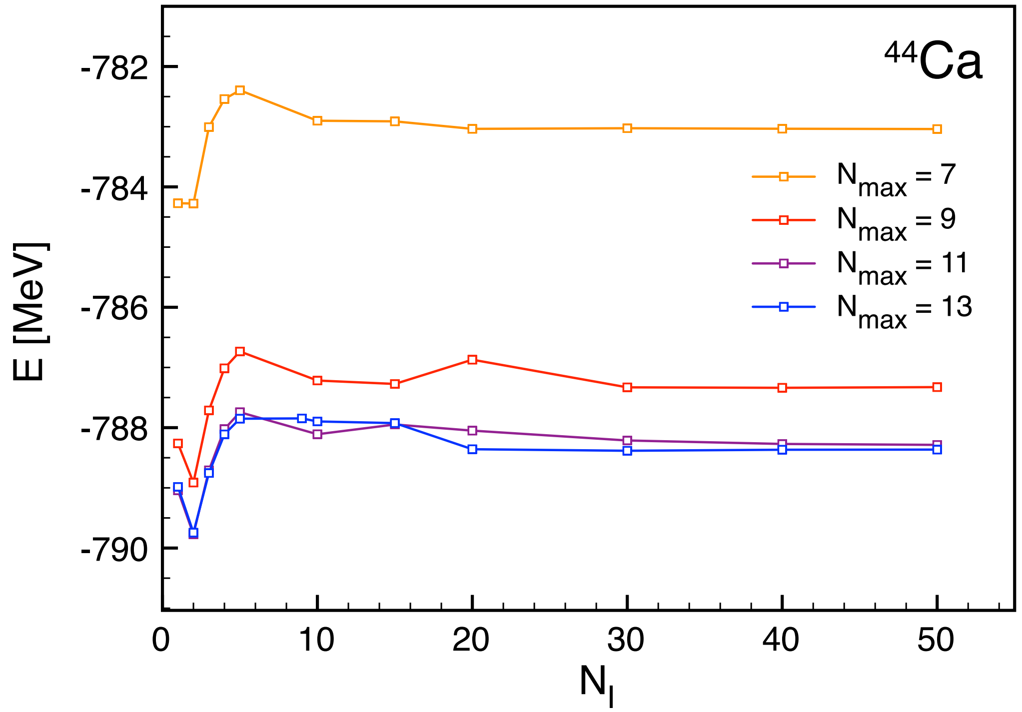
  4) Hamiltonian
     - ➠ Currently the hardest to assess in a thorough way
     - ➠ Comparison within one method, different methods & data

Error estimates 1: numerical algorithms

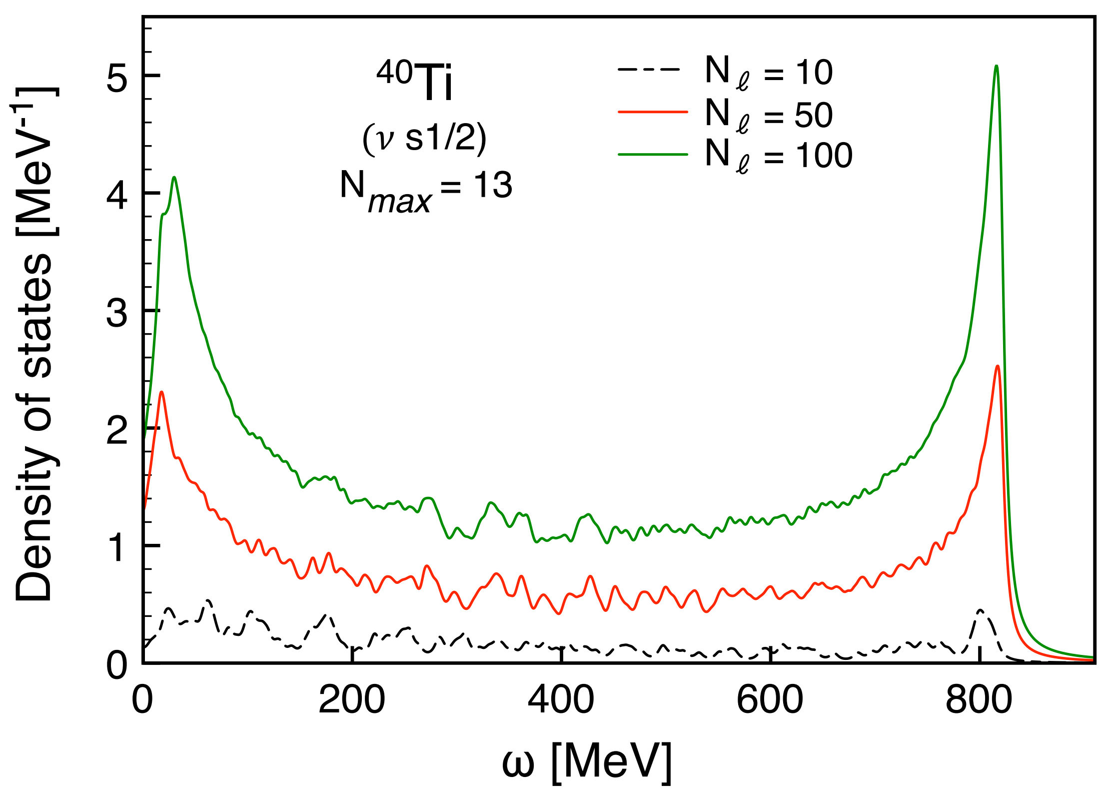
✪ Krylov projection in GGF

➟ Energy & spectral distribution independent of the projection

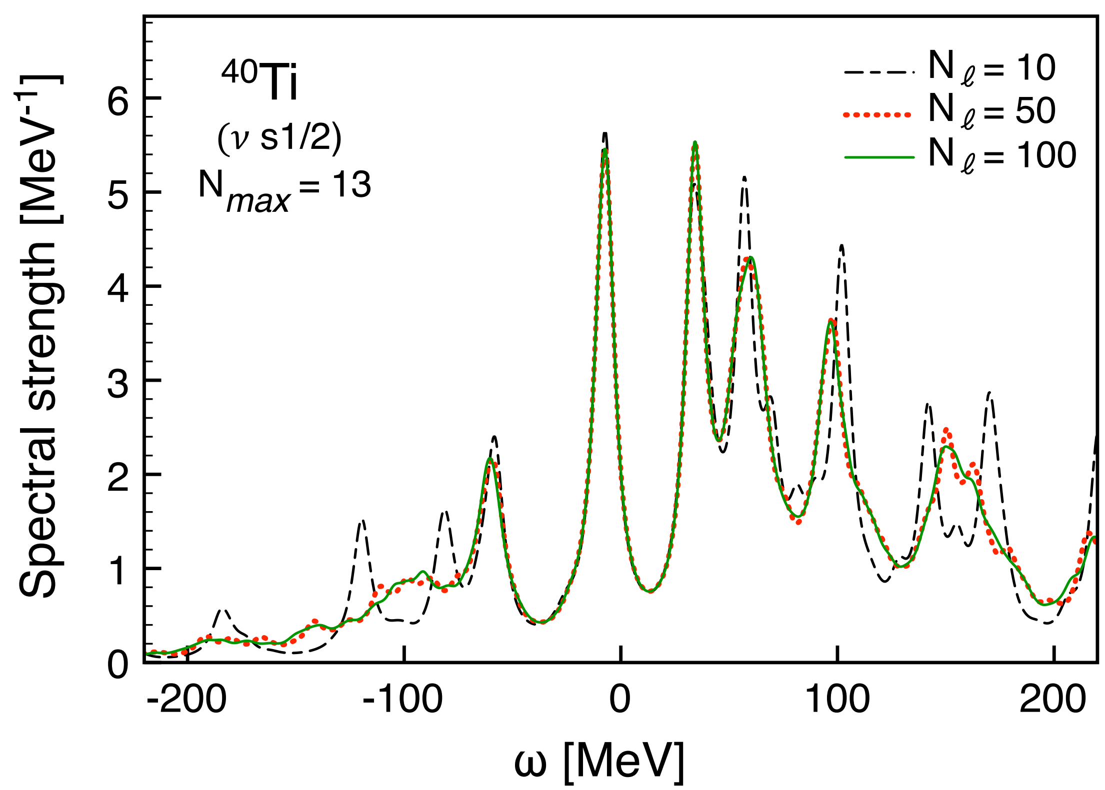
[Somà, Barbieri & Duguet 2014]



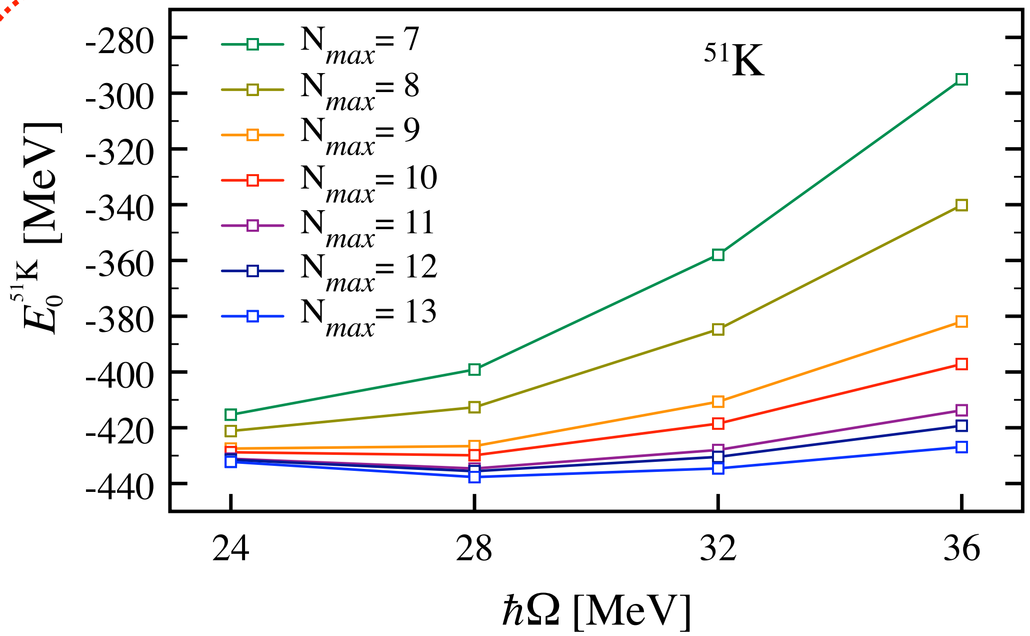
Density of states



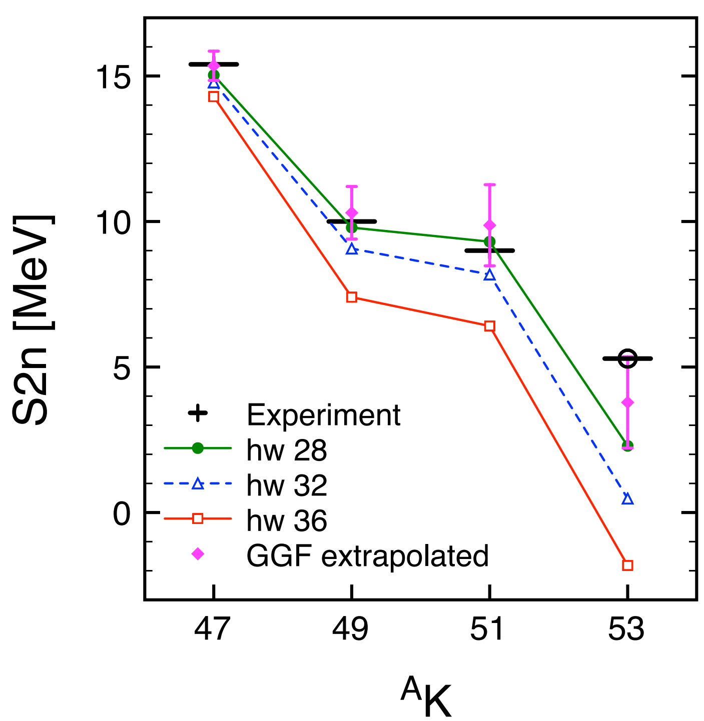
Spectral strength



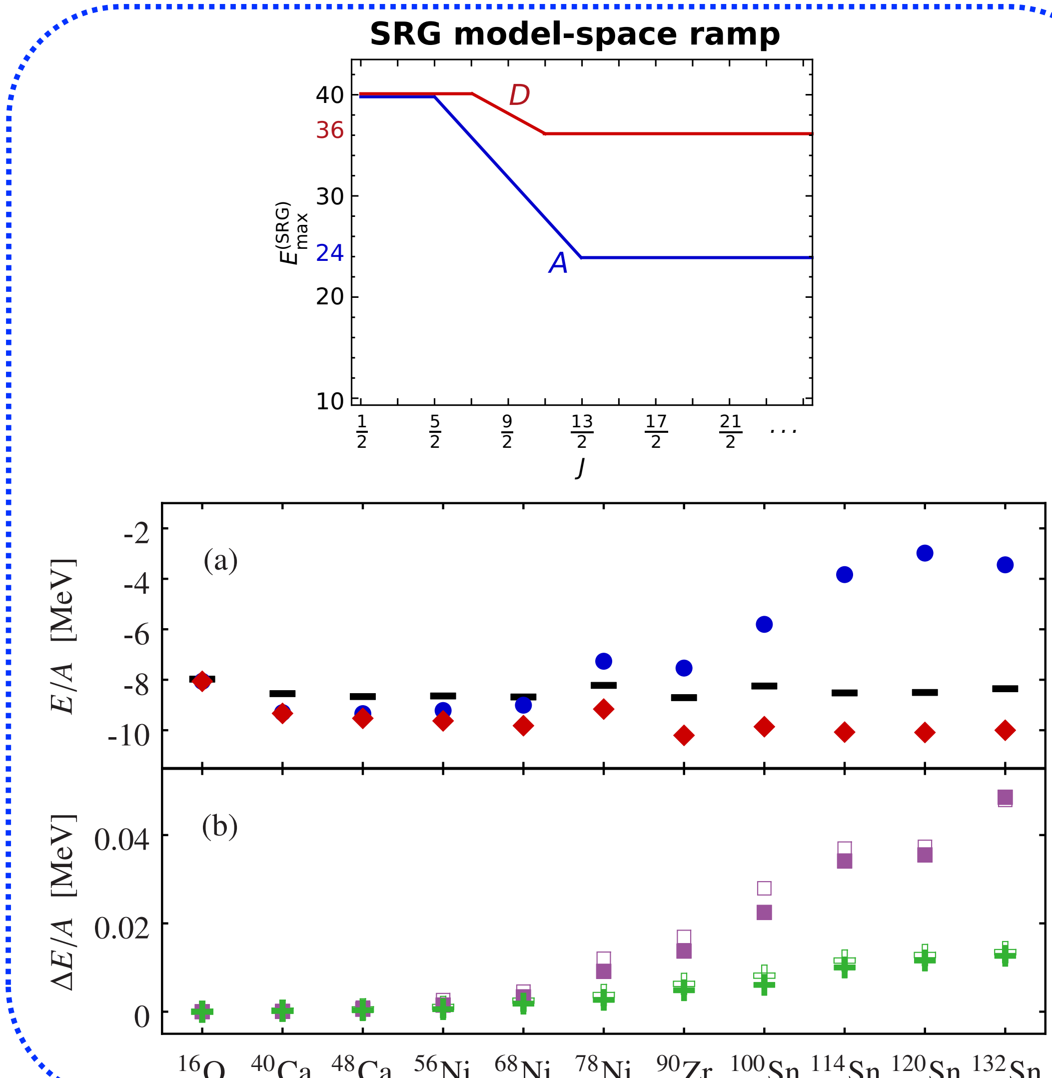
Error estimates 2: model space truncation



Extrapolation to ∞ model space



- ✪ Enters at various stages
  - ➟ Building of NN+3N matrix elements
  - ➟ SRG procedure
  - ➟ Many-body calculation

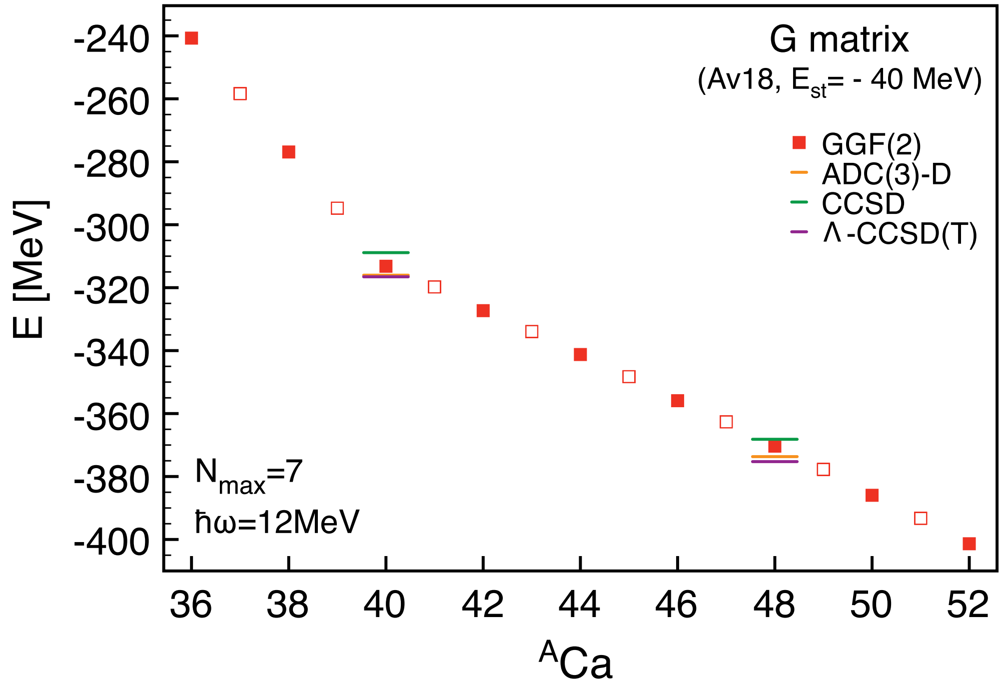


[Binder *et al.* 2013]

Error estimates 3: many-body expansion

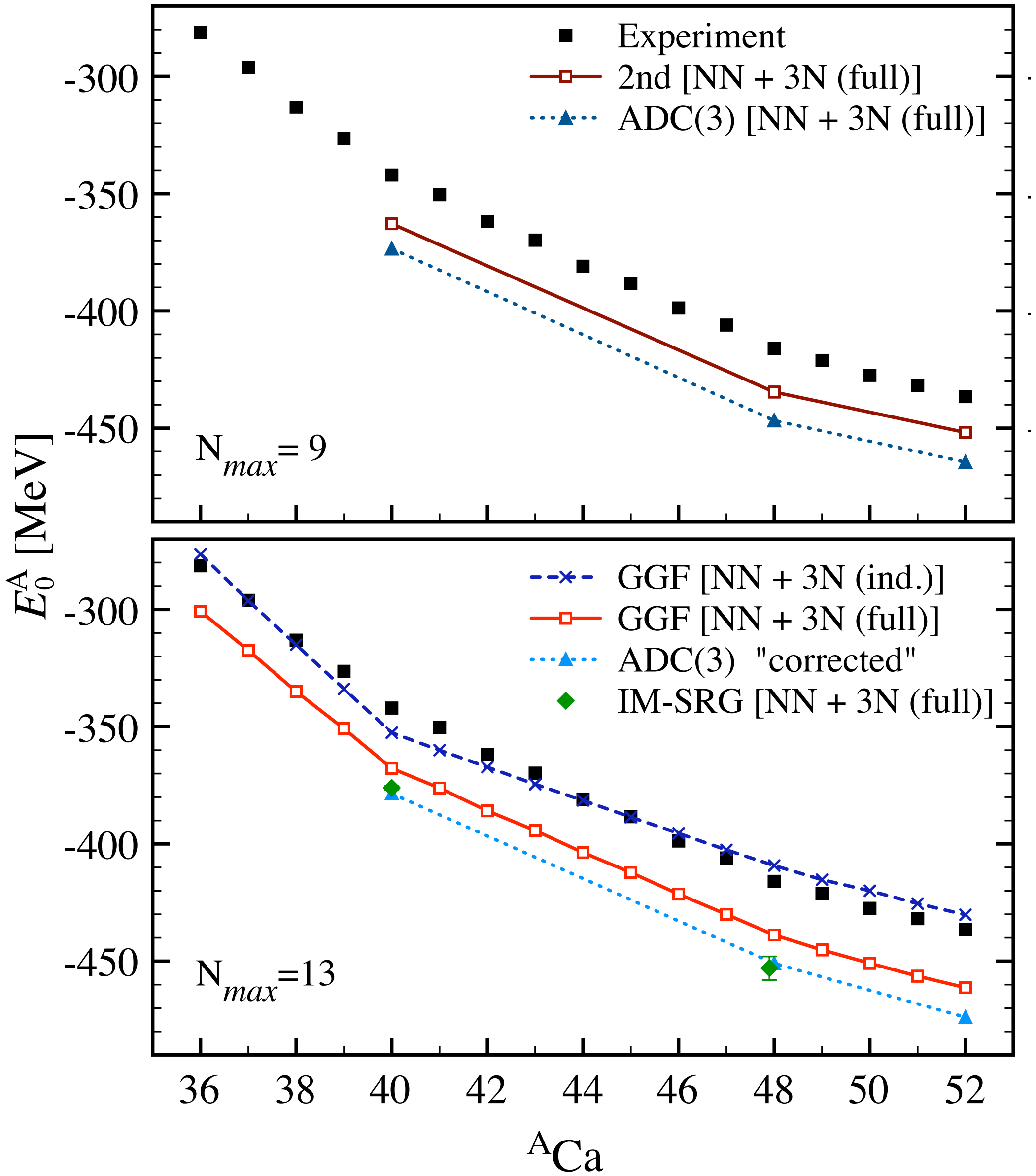
✪ Gorkov-Green's functions

vs CC



[Somà *et al.* 2013]

within SCGF

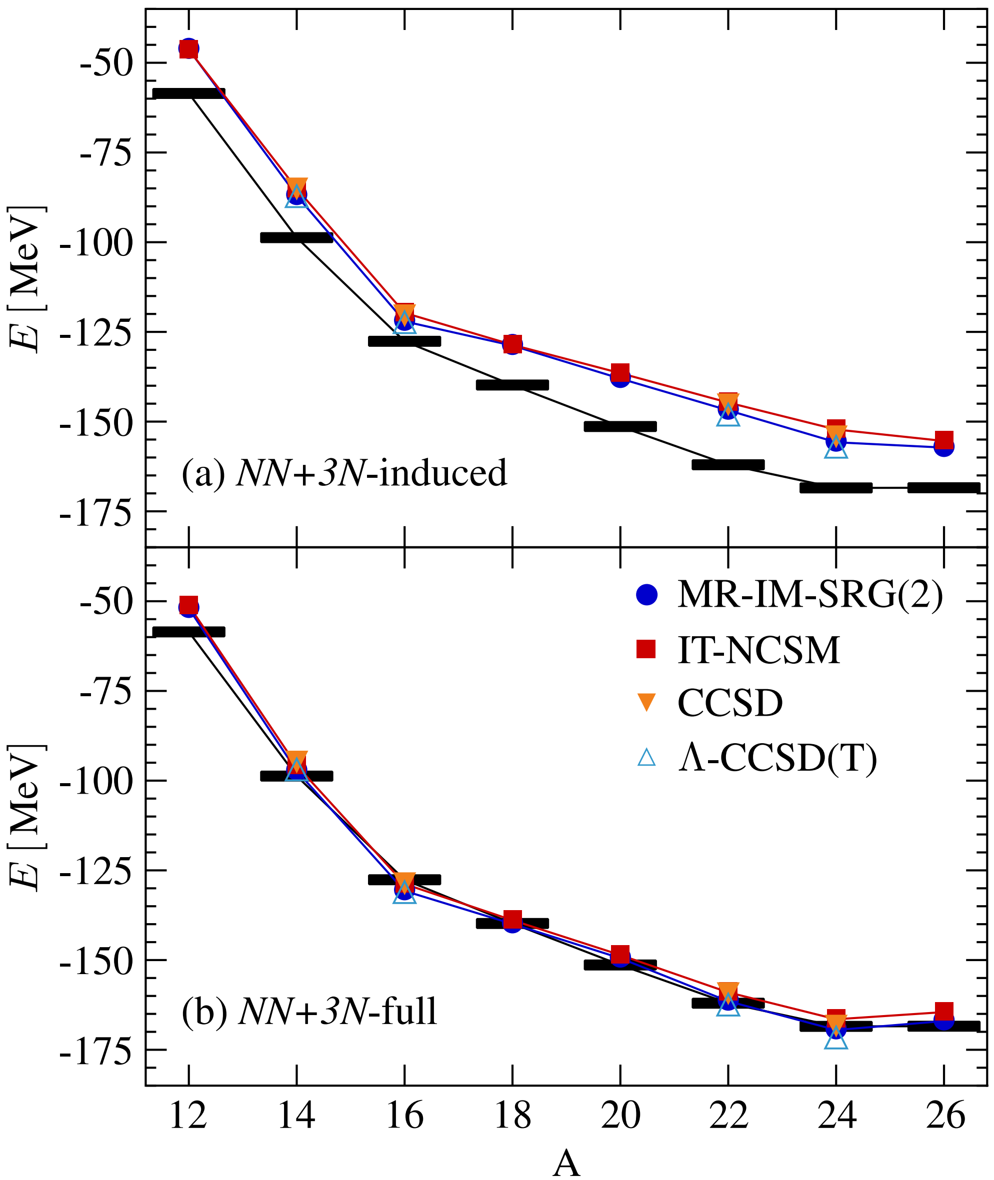


[Somà *et al.* 2013]

Error estimates 3: many-body expansion

✪ IM-SRG

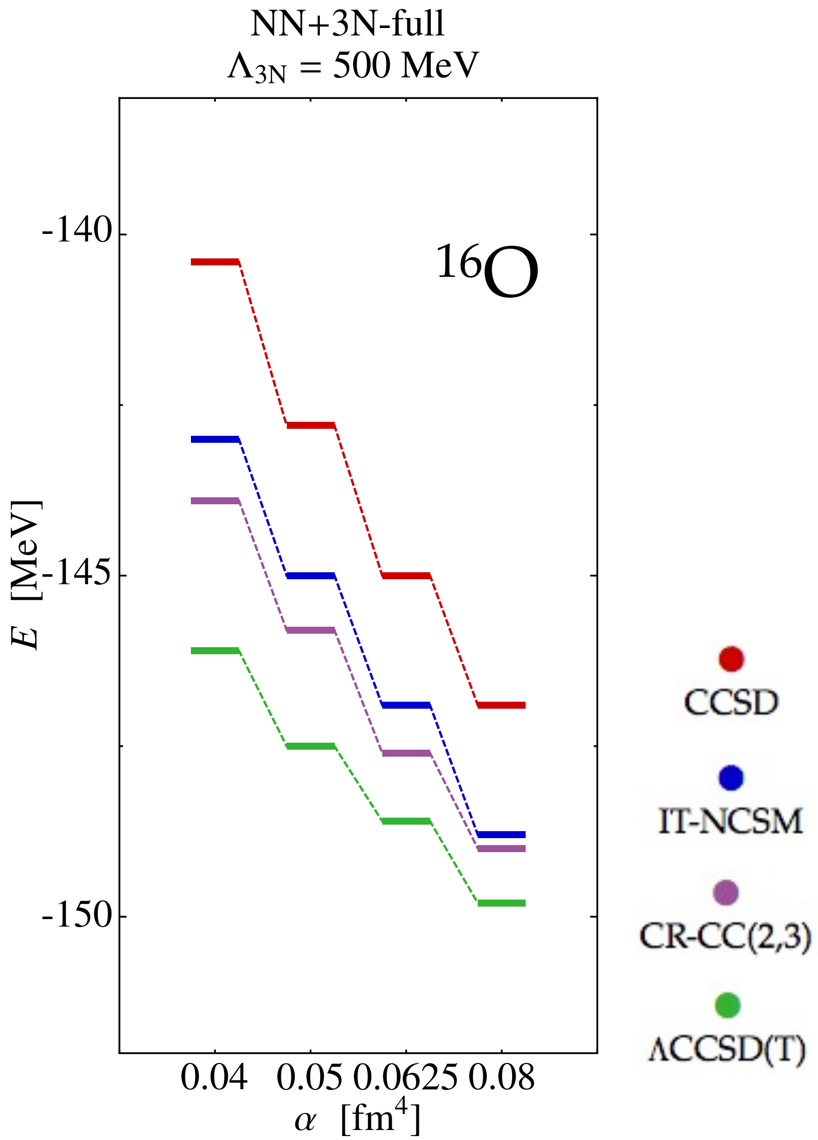
vs CC & NCSM



[Hergert *et al.* 2013]

✪ Coupled cluster

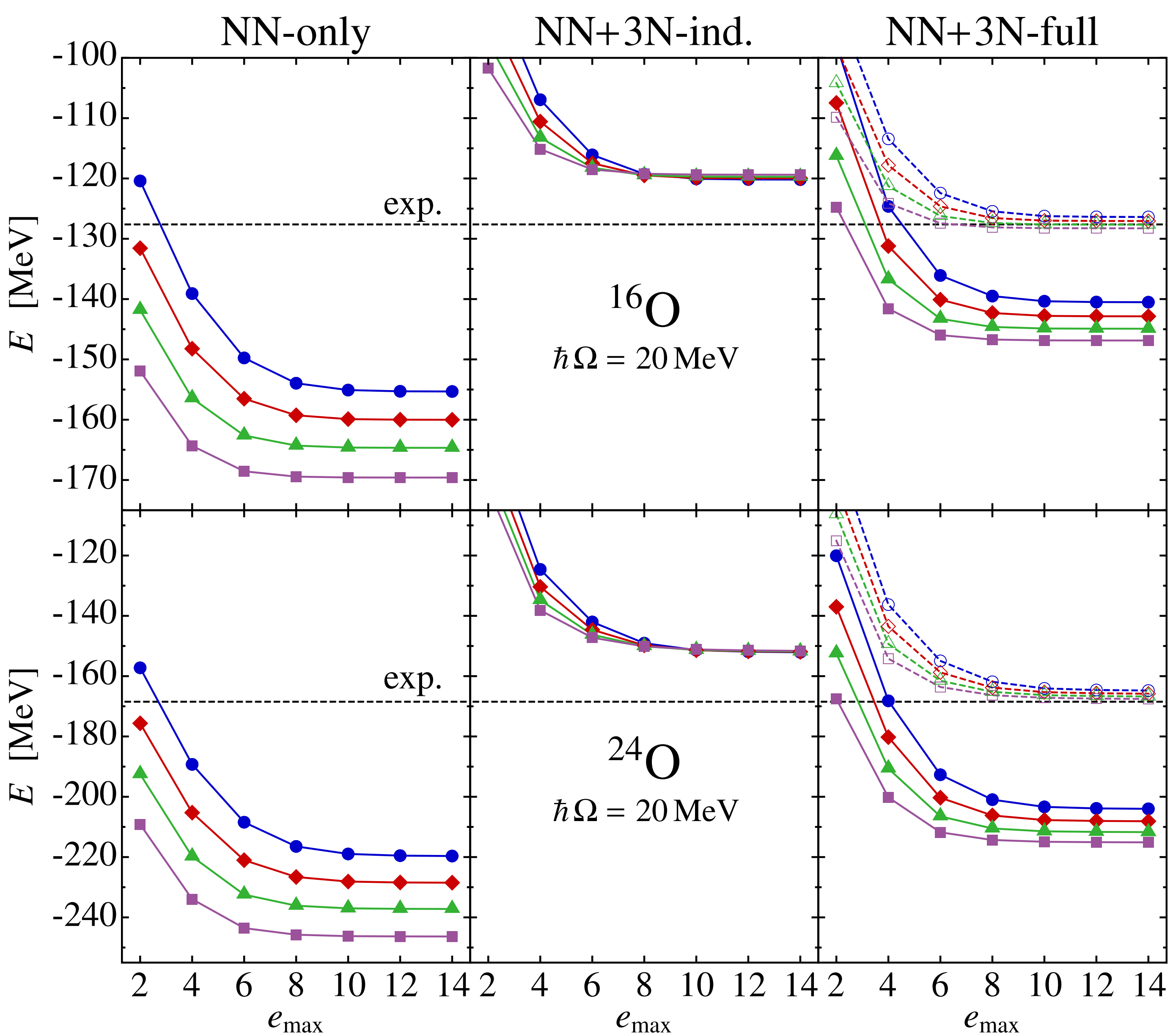
within CC



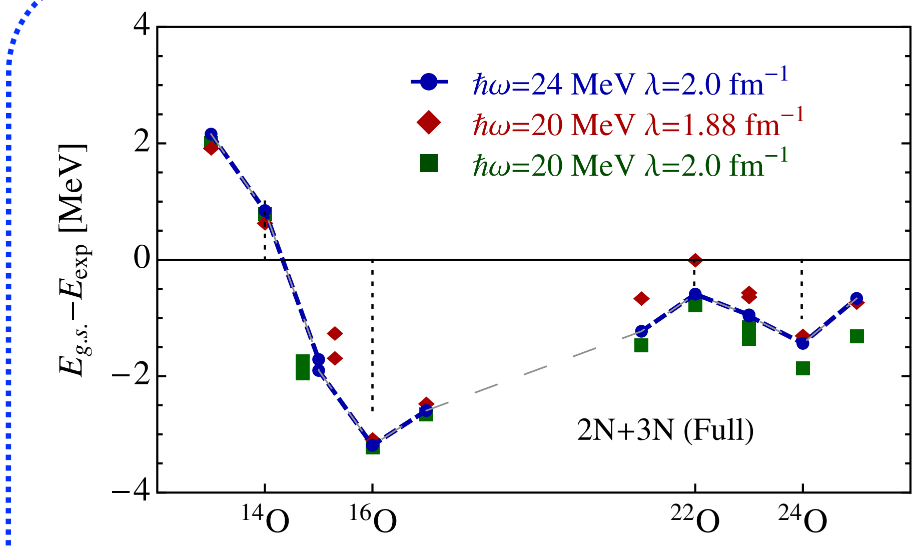
[Binder *et al.* unpublished]

Error estimates 4: input Hamiltonian

✪ Within one Hamiltonian by varying λ_{SRG}



[Roth *et al.* 2012]

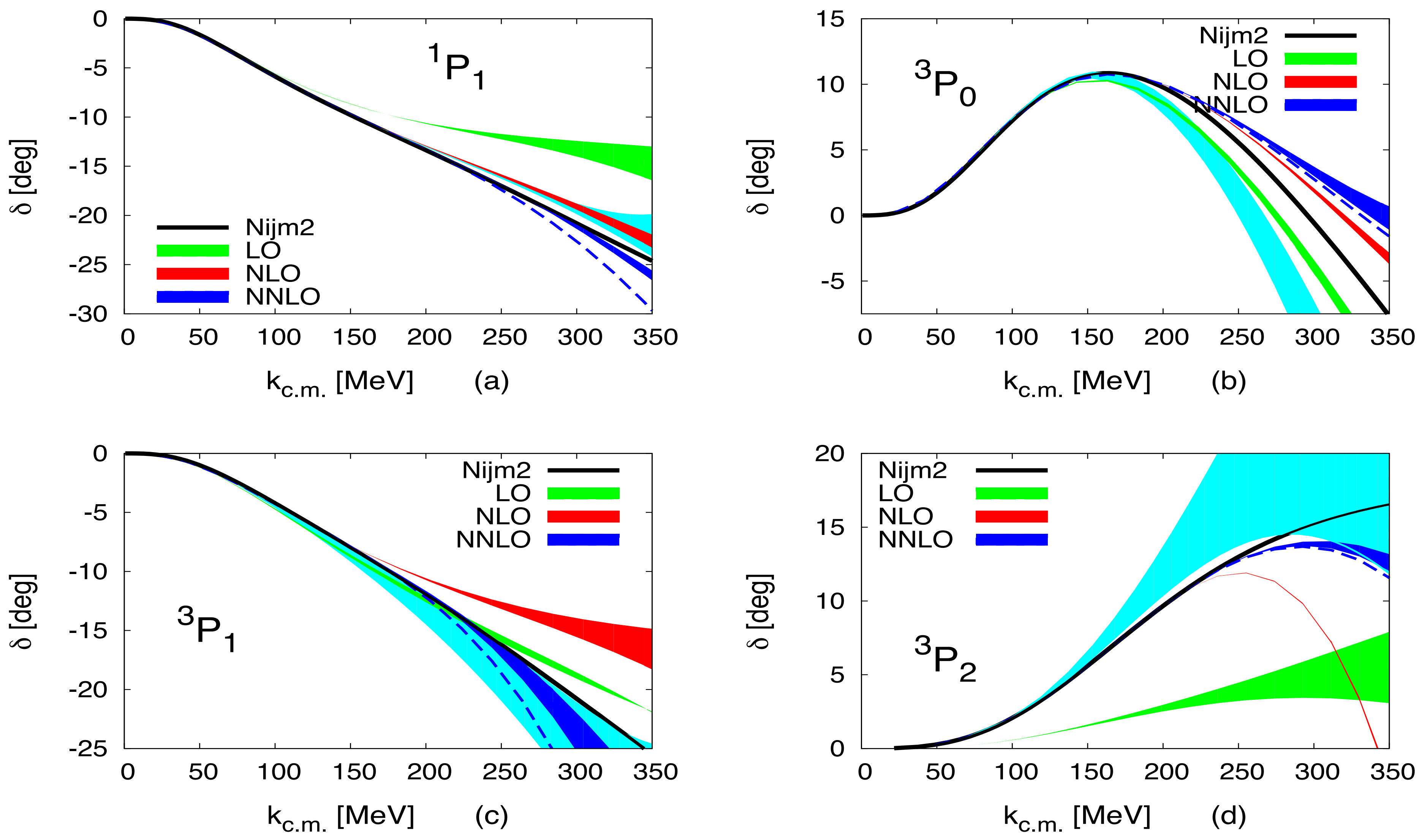


[Cipollone *et al.* 2013]

FIG. 4 (color online). CCSD ground-state energies for ^{16}O and ^{24}O as a function of $e_{\max}$ for the three types of Hamiltonians (see column headings) using the NO2B approximation for a range of flow parameters: α = 0.04 fm^4 (●), 0.05 fm^4 (◆), 0.0625 fm^4 (▲), and 0.08 fm^4 (■). The filled symbols for the $NN + 3N$-full Hamiltonian are for the standard chiral $3N$ interaction with cutoff 500 MeV, the open symbols for a modified $3N$ interaction with cutoff 400 MeV (see text).

Error estimates 4: input Hamiltonian

✪ One would like to test different chiral orders…

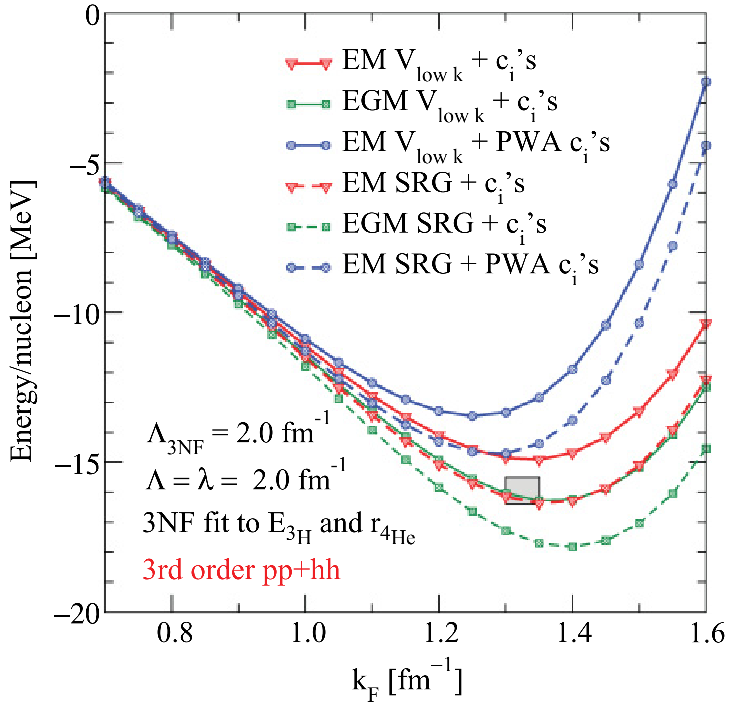


[Pavon Valderrama 2011]

Error estimates 4: input Hamiltonian

… and eventually different Hamiltonians

➠ Infinite (symmetric) matter



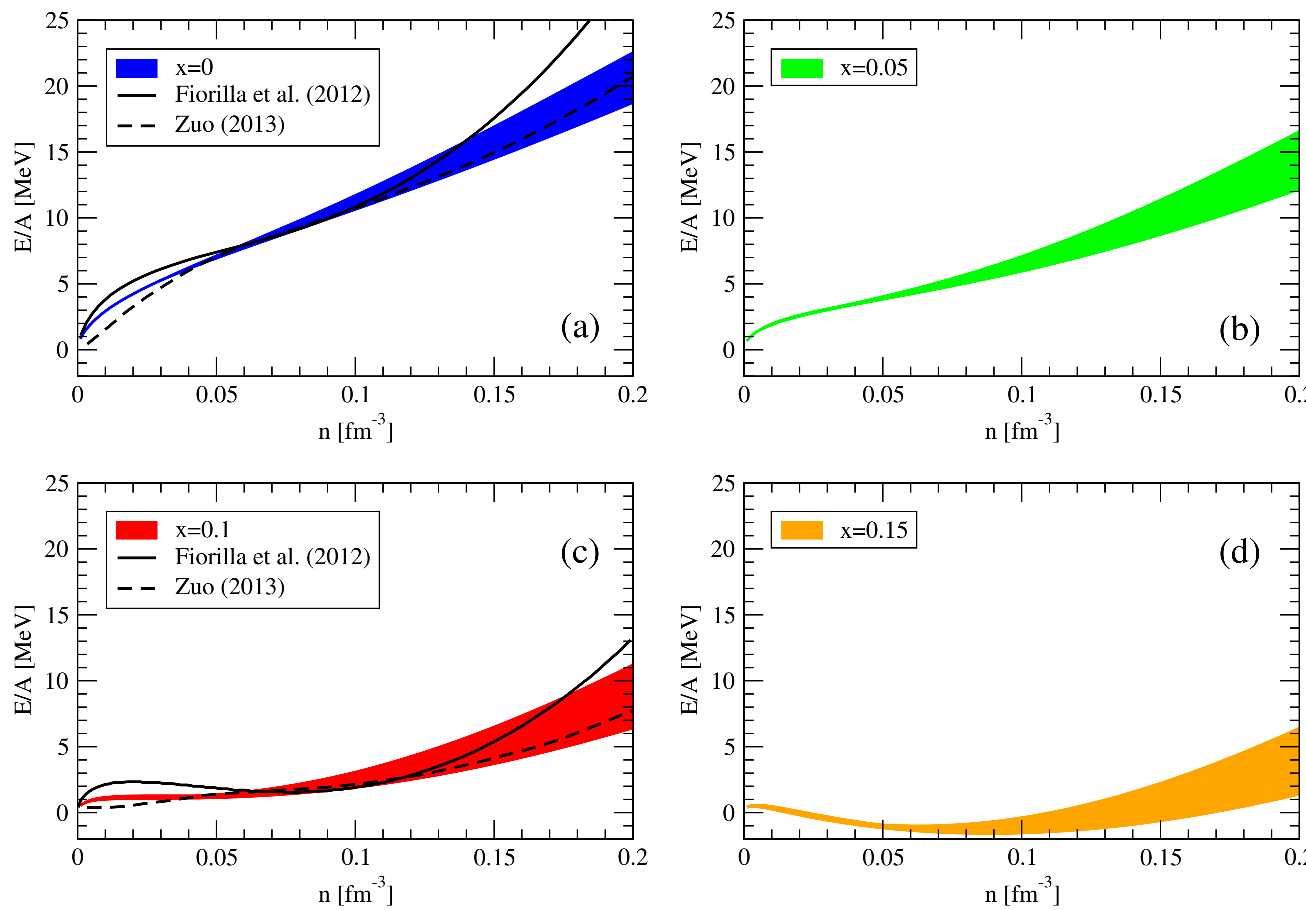
[Hebeler *et al.* 2011]

TABLE I. Results for the c_D and c_E couplings fit to $E_{^3\text{H}} = -8.482$ MeV and to the point charge radius $r_{^4\text{He}} = 1.464$ fm (based on Ref. [26]) for the NN/3N cutoffs and different EM/EGM/PWA c_i values used. For $V_{\text{low}\,k}$ (SRG) interactions, the 3NF fits lead to $E_{^4\text{He}} = -28.22 \ldots -28.45$ MeV ($-28.53 \ldots -28.71$ MeV).

	$V_{\text{low}\,k}$		SRG	
Λ or $\lambda/\Lambda_{3\text{NF}}$ (fm)	c_D	c_E	c_D	c_E
1.8/2.0 (EM c_i's)	+1.621	−0.143	+1.264	−0.120
2.0/2.0 (EM c_i's)	+1.705	−0.109	+1.271	−0.131
2.0/2.5 (EM c_i's)	+0.230	−0.538	−0.292	−0.592
2.2/2.0 (EM c_i's)	+1.575	−0.102	+1.214	−0.137
2.8/2.0 (EM c_i's)	+1.463	−0.029	+1.278	−0.078
2.0/2.0 (EGM c_i's)	−4.381	−1.126	−4.828	−1.152
2.0/2.0 (PWA c_i's)	−2.632	−0.677	−3.007	−0.686

➠ Infinite (asymmetric) matter

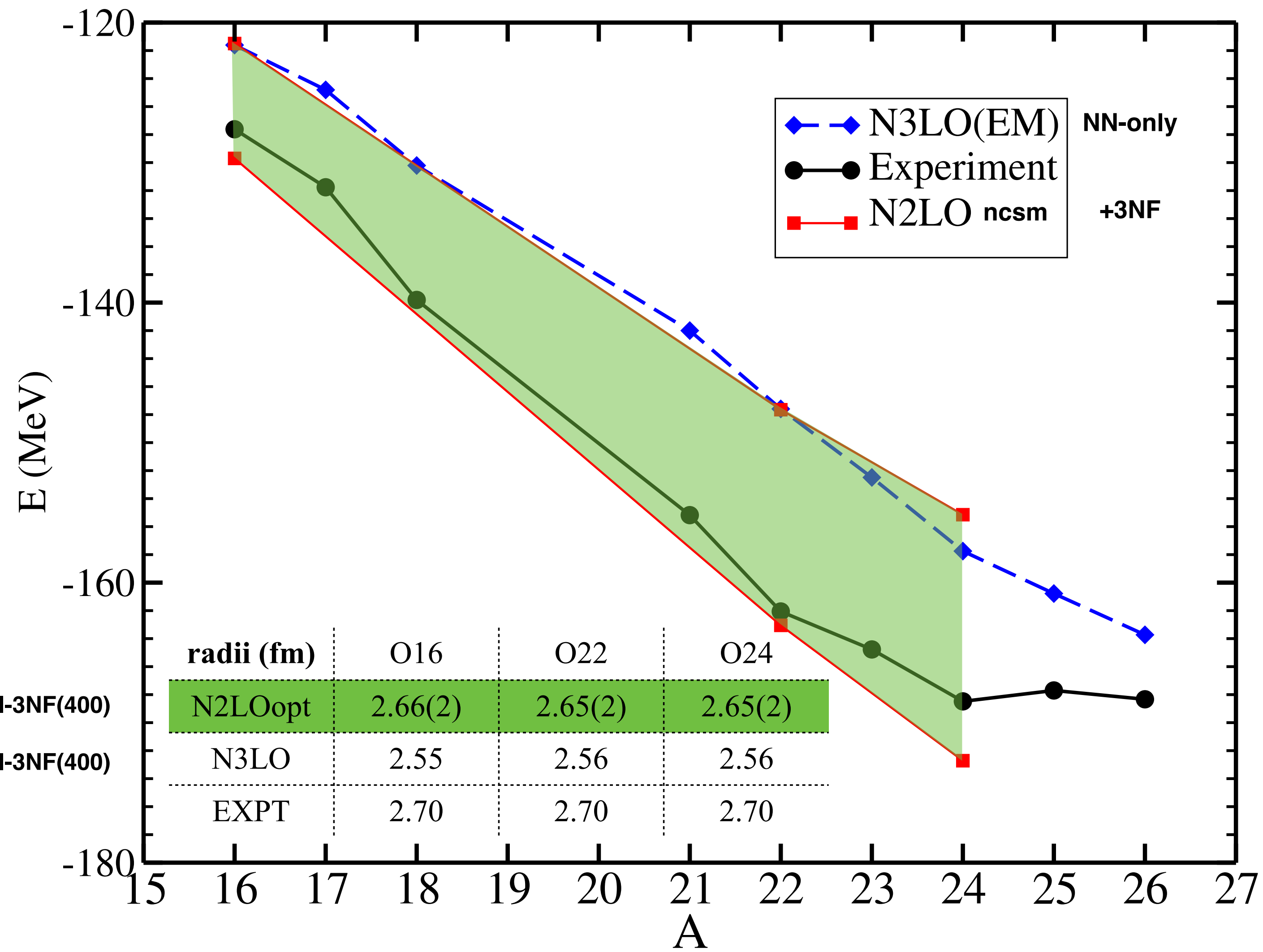
[Drischler *et al.* 2012]



Error estimates 4: input Hamiltonian

… and eventually different Hamiltonians

➟ First steps in this direction using N^{2}LO$_{opt}$



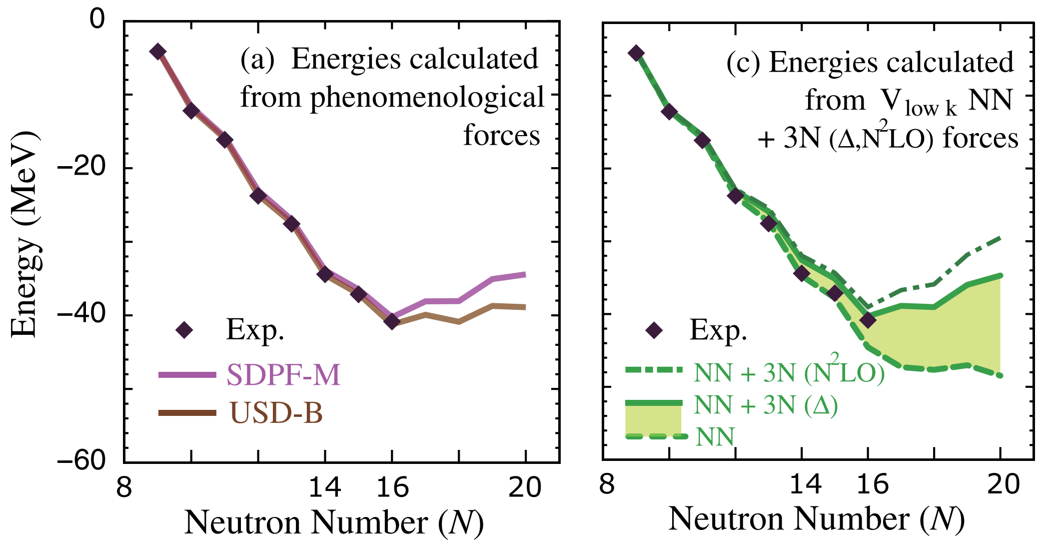
	radii (fm)	O16	O22	O24
+local-3NF(400)	N2LOopt	2.66(2)	2.65(2)	2.65(2)
+local-3NF(400)	N3LO	2.55	2.56	2.56
	EXPT	2.70	2.70	2.70

[Ekström *et al.* unpublished]

Results

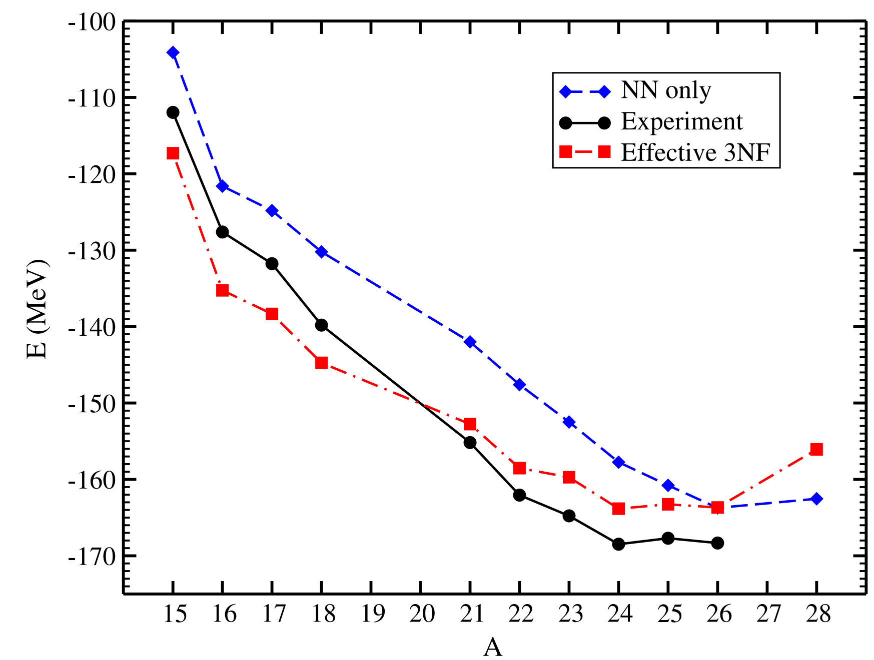
Results: g.s. energies in the O chain

✪ Shell Model (NN + 3N)



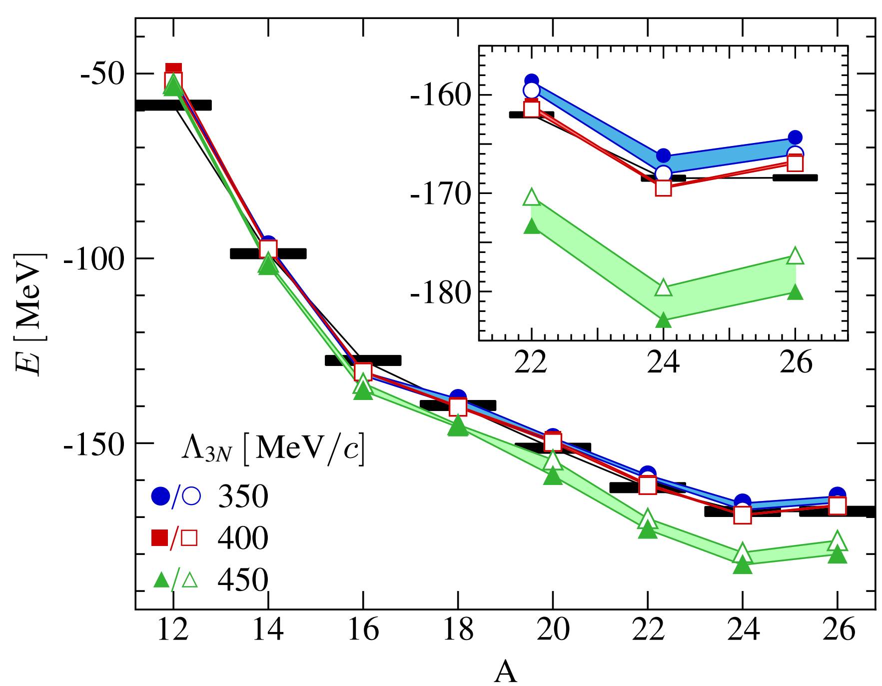
[Otsuka *et al.* 2010]

✪ CC (NN + effective 3N)



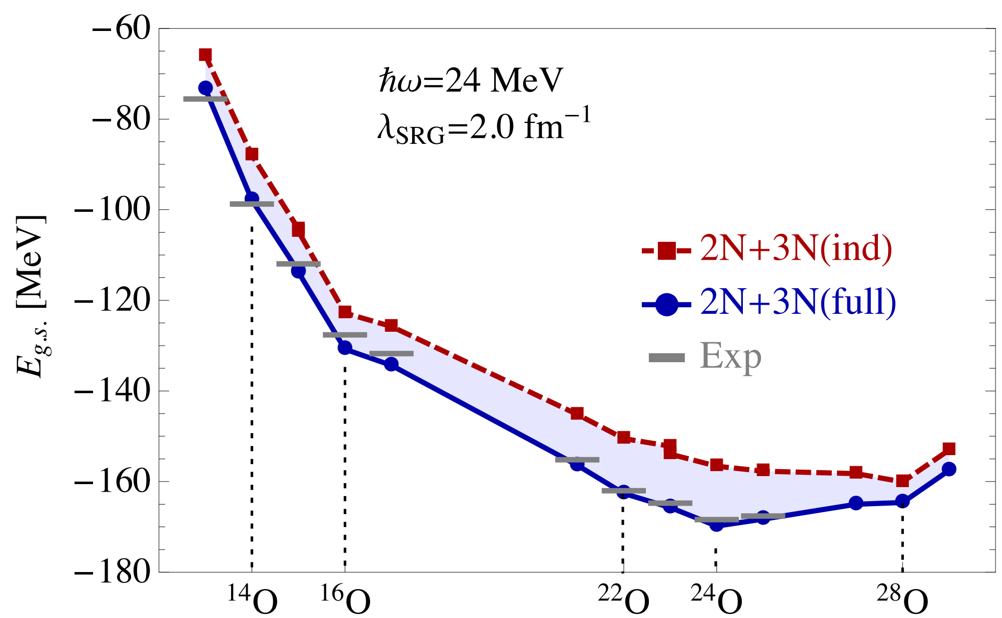
[Hagen *et al.* 2012]

✪ IM-SRG (NN + 3N)



[Hergert *et al.* 2013]

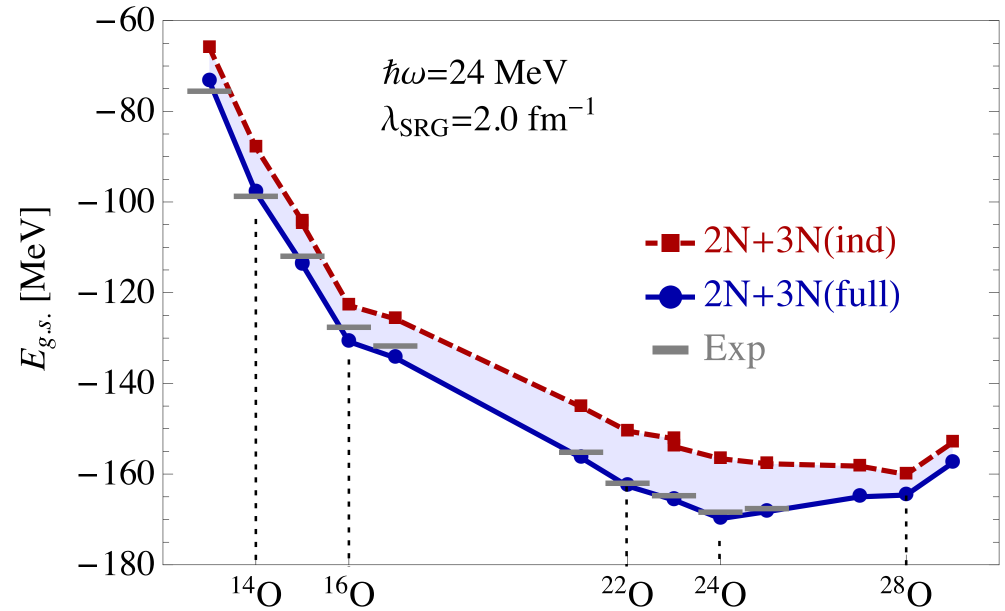
✪ SCGF (NN + 3N)

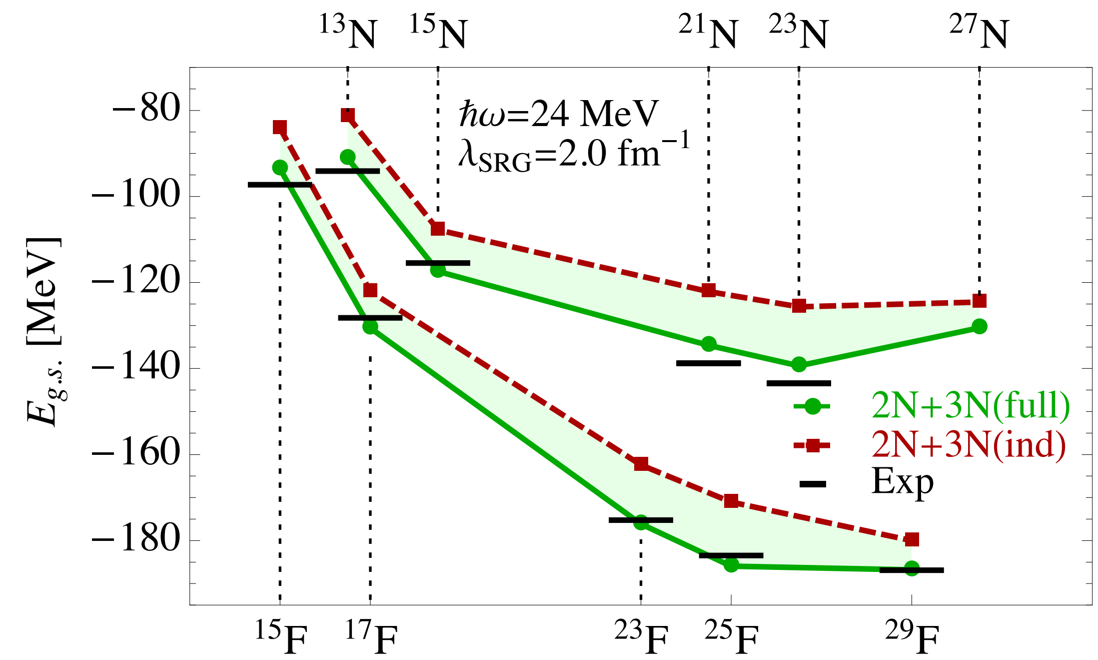


[Cipollone *et al.* 2013]

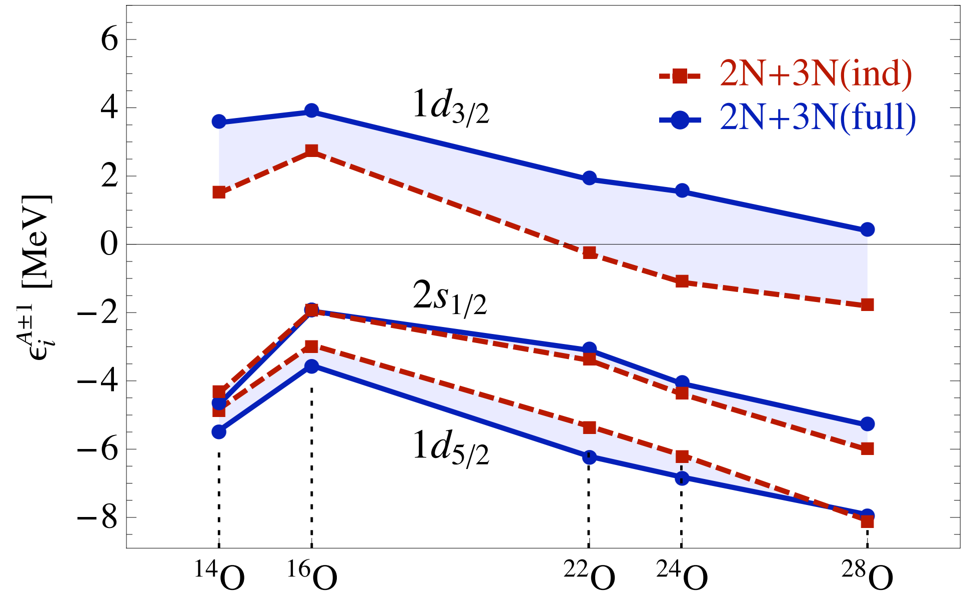
Not only oxygen...

✪ Consistent description of Z = 7, 8, 9 isotopic chains with GF method





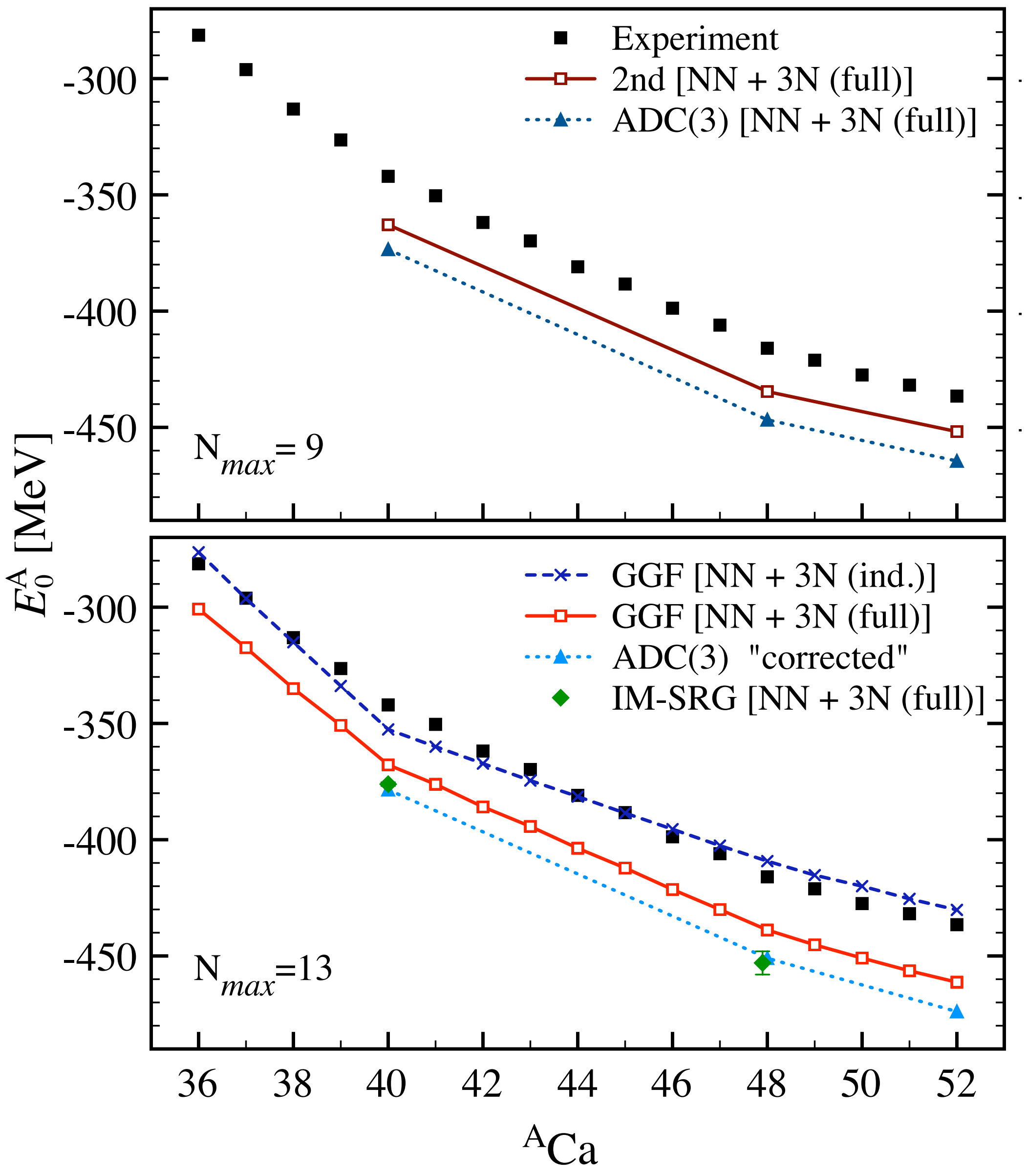
[Cipollone, Barbieri & Navrátil 2013]



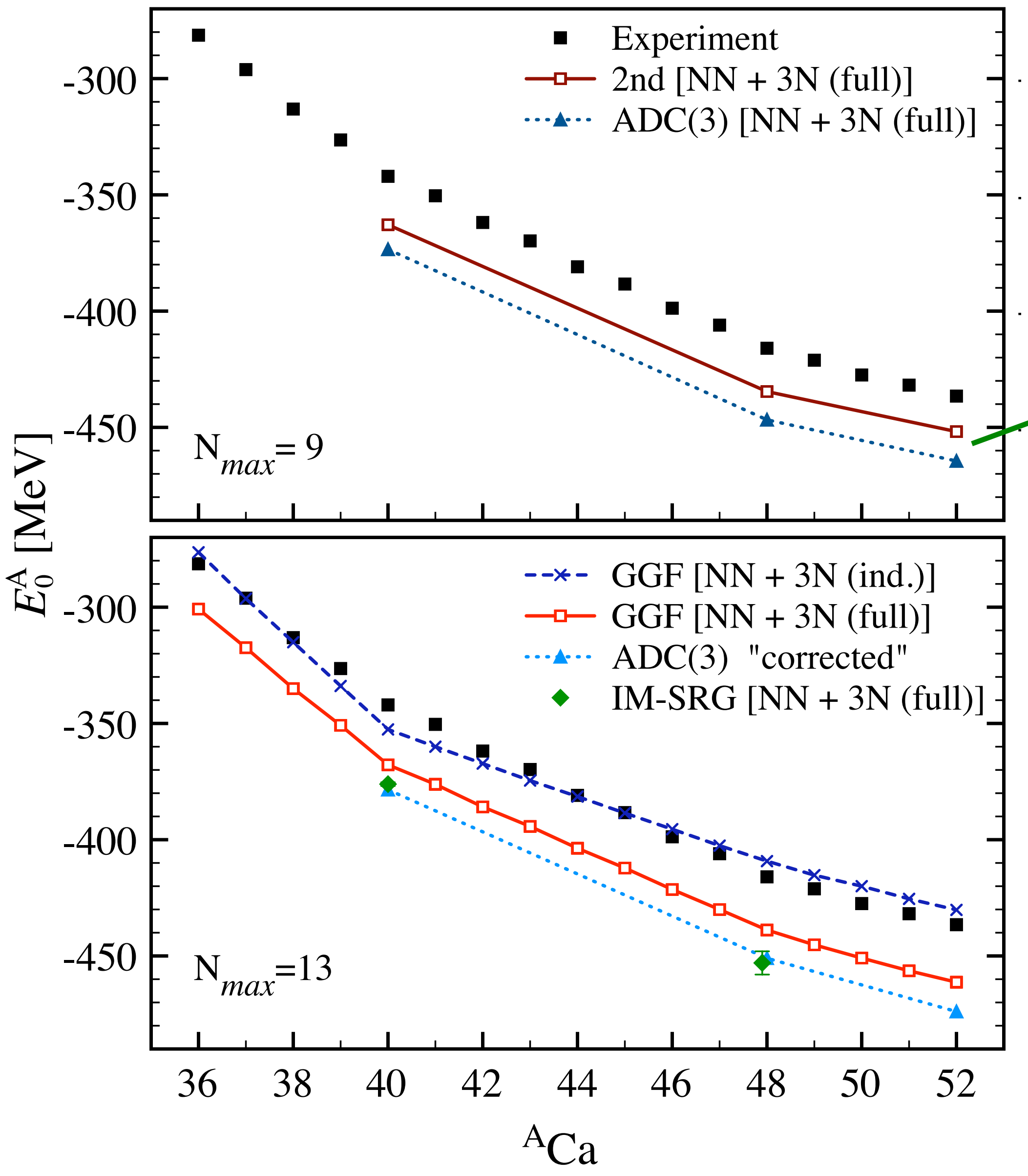
➟ 3NF crucial for reproducing driplines

➟ $d_{3/2}$ raised by genuine 3NF

Binding energies around Ca

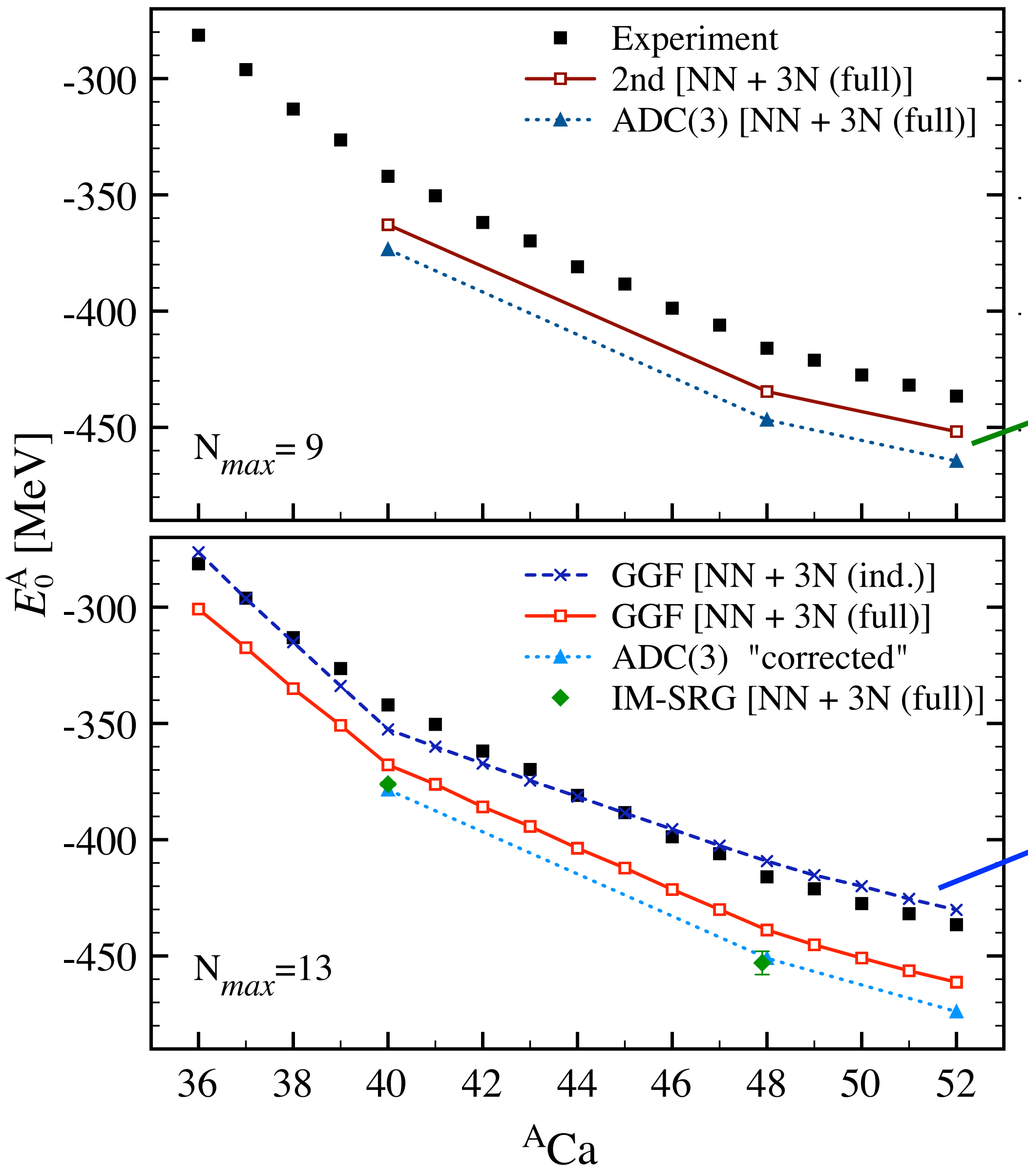


Binding energies around Ca



Estimate of the many-body truncation error

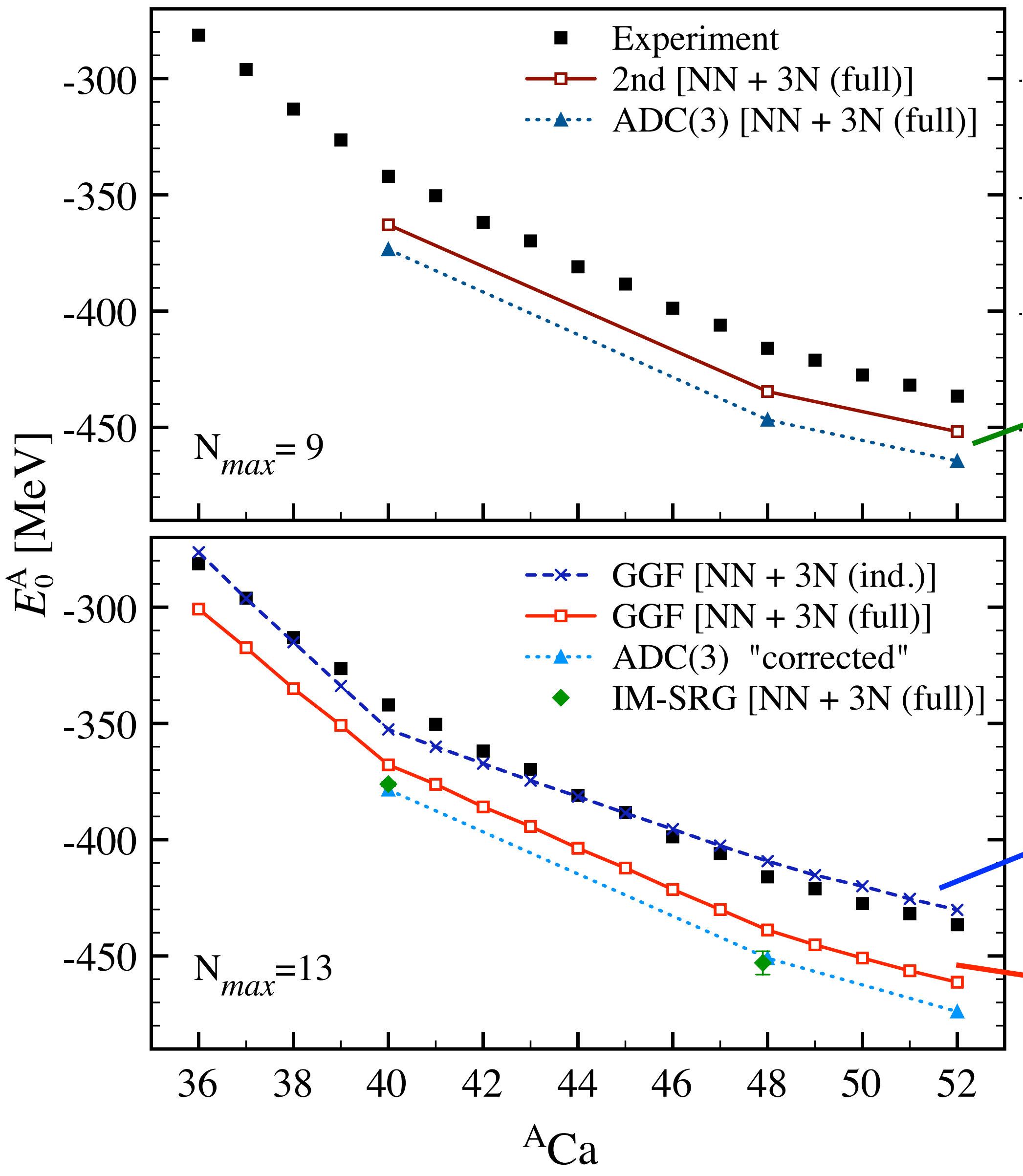
Binding energies around Ca



Estimate of the many-body truncation error

Original 3NF correct for trend

Binding energies around Ca

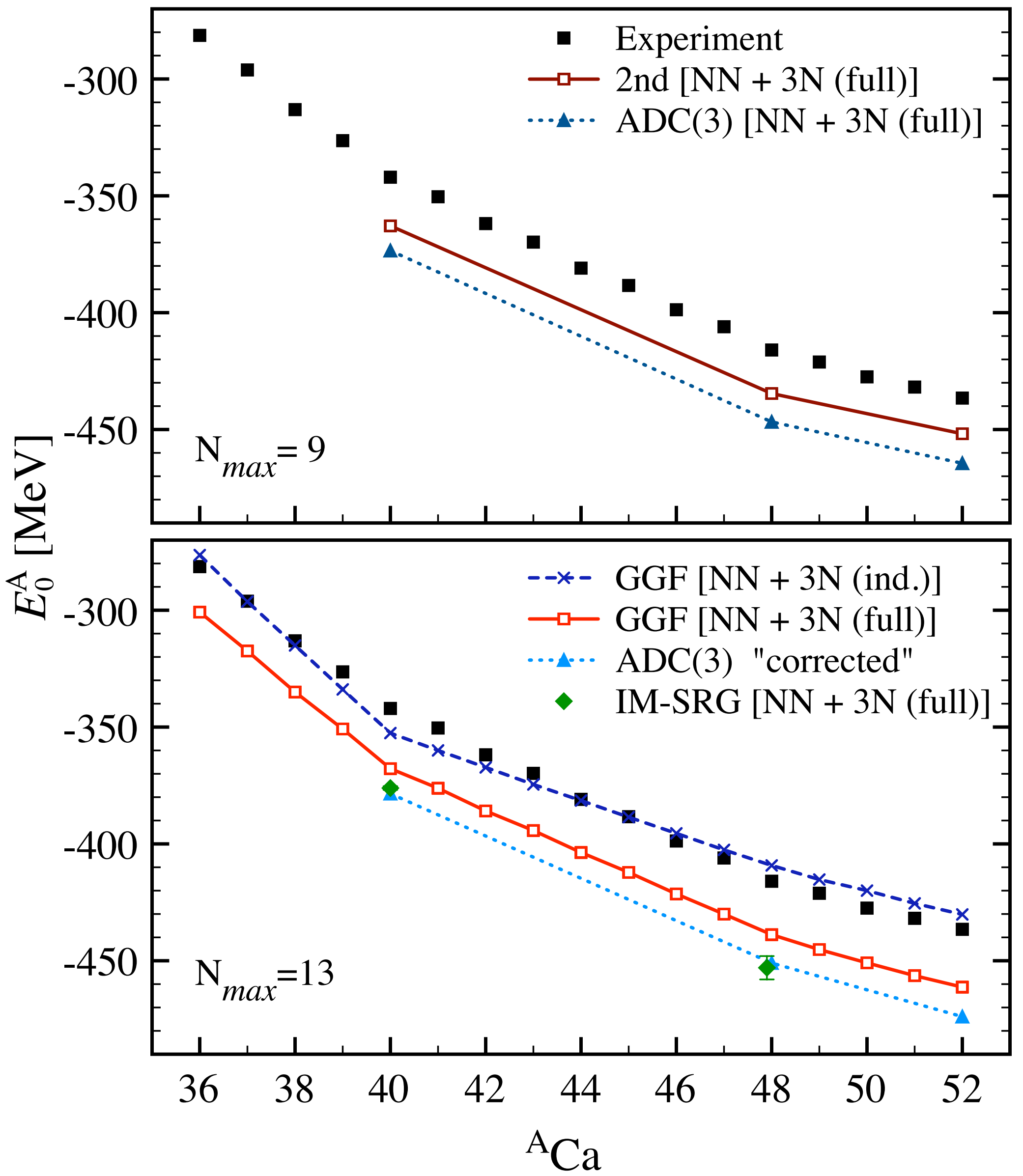


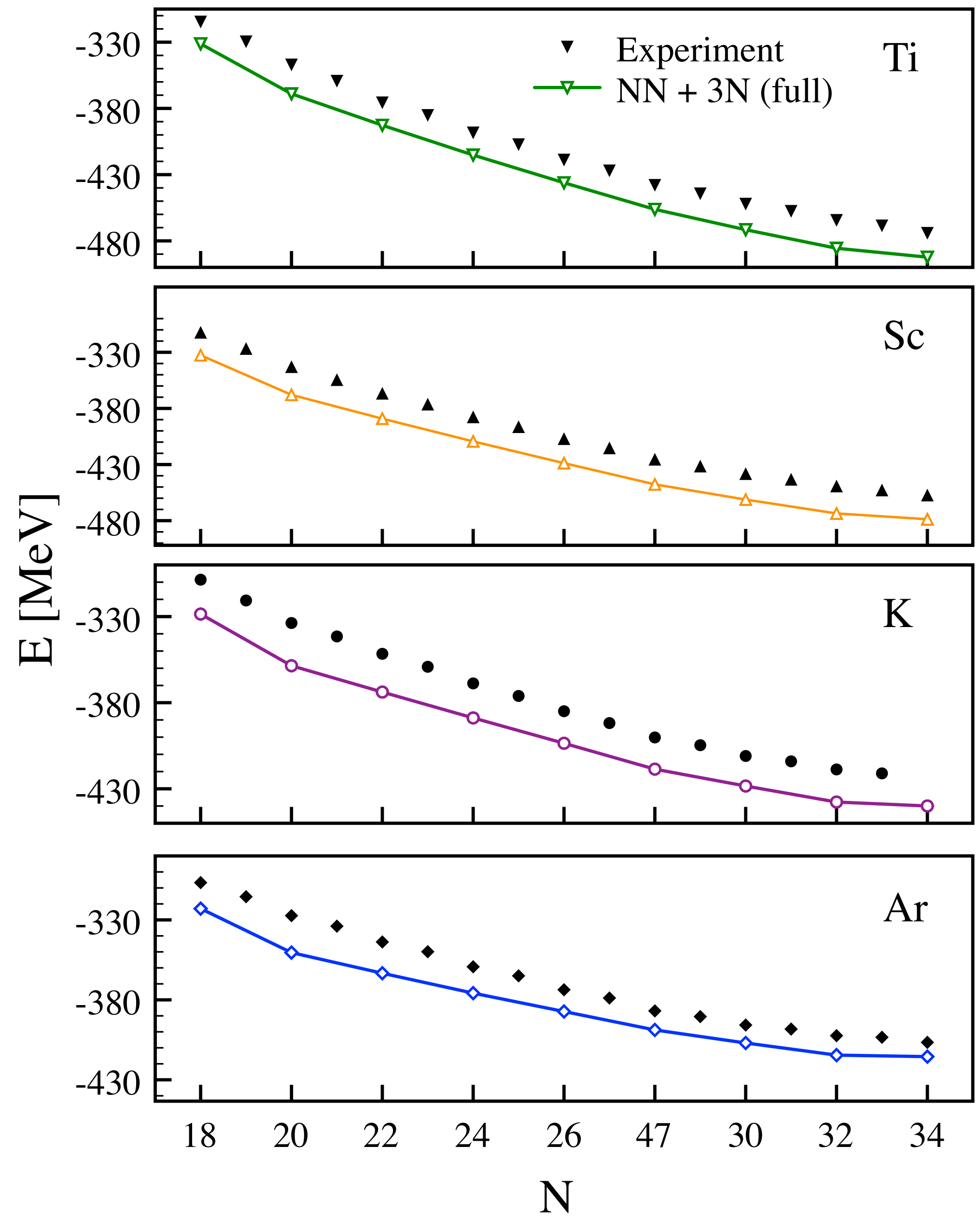
Estimate of the many-body truncation error

Original 3NF correct for trend

Systematic overbinding

Binding energies around Ca



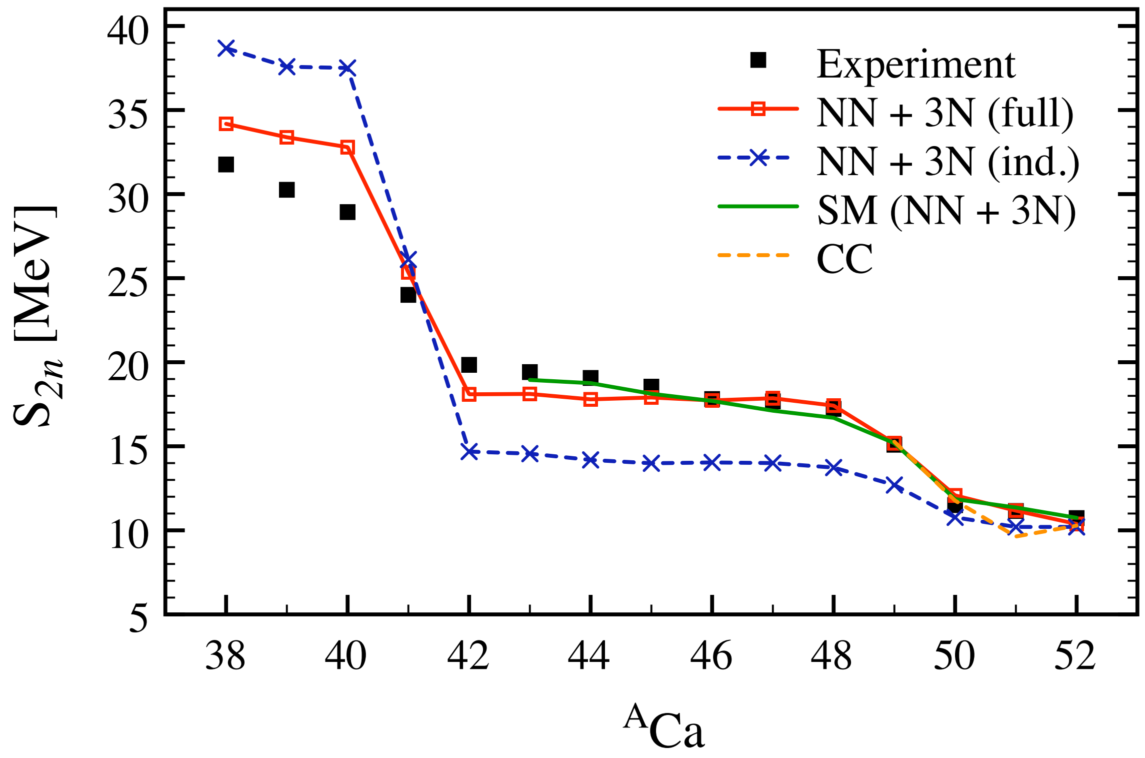


➠ Results confirmed within different many-body approaches

➠ NN + full 3N correct the trend of binding energies

➠ Systematic overbinding through all chains around Z=20

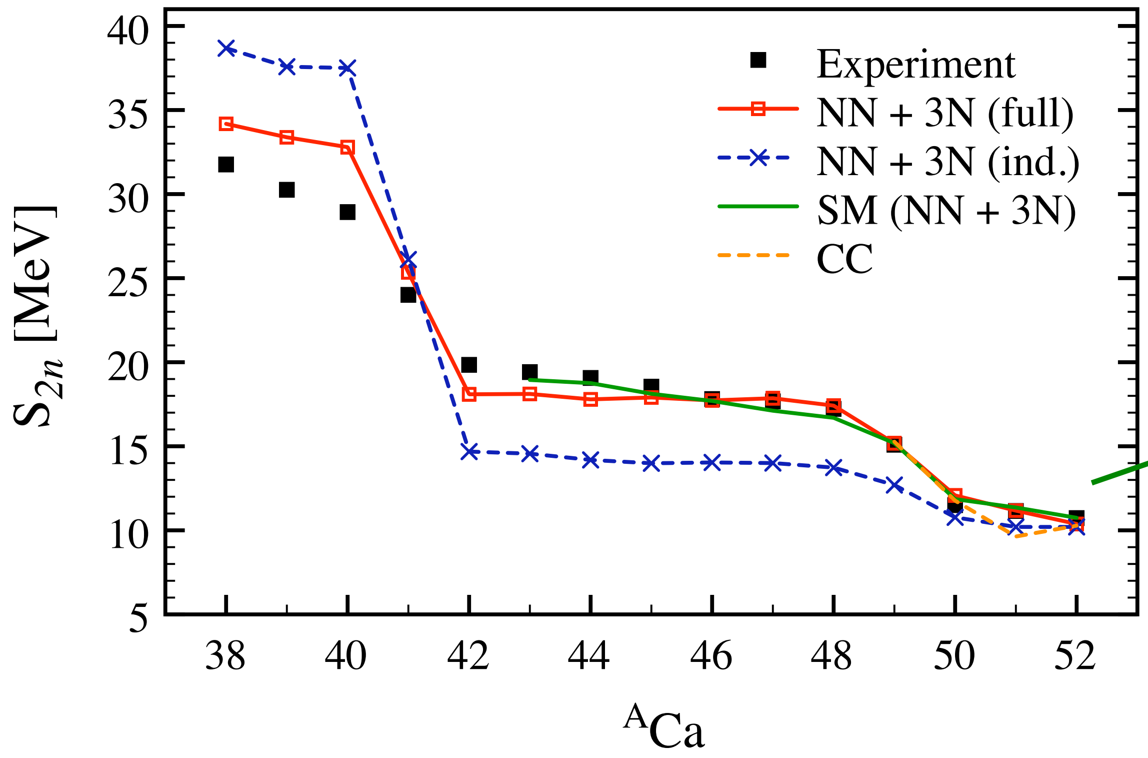
Two-neutron separation energies around Ca



➟ S_{2n} well reproduced with chiral NN + 3N interactions

➟ Microscopic calculations extended to the whole Ca chain

Two-neutron separation energies around Ca



Challenging new data

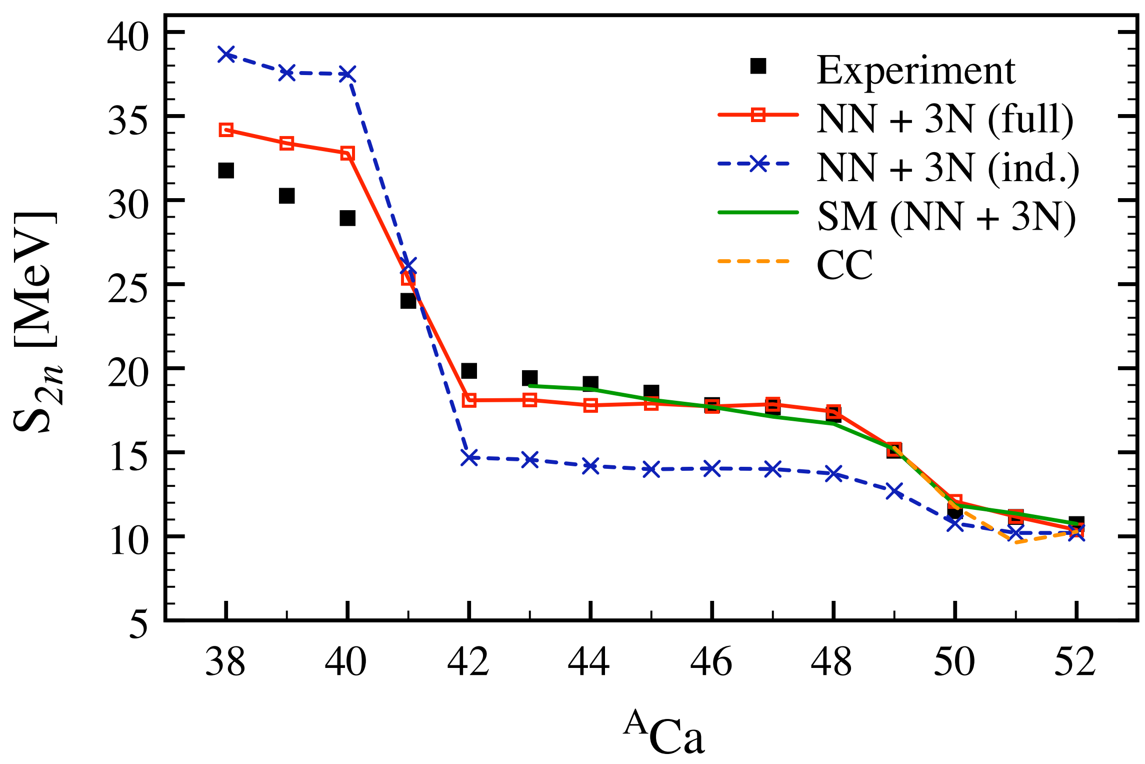
[Gallant *et al.* 2012]

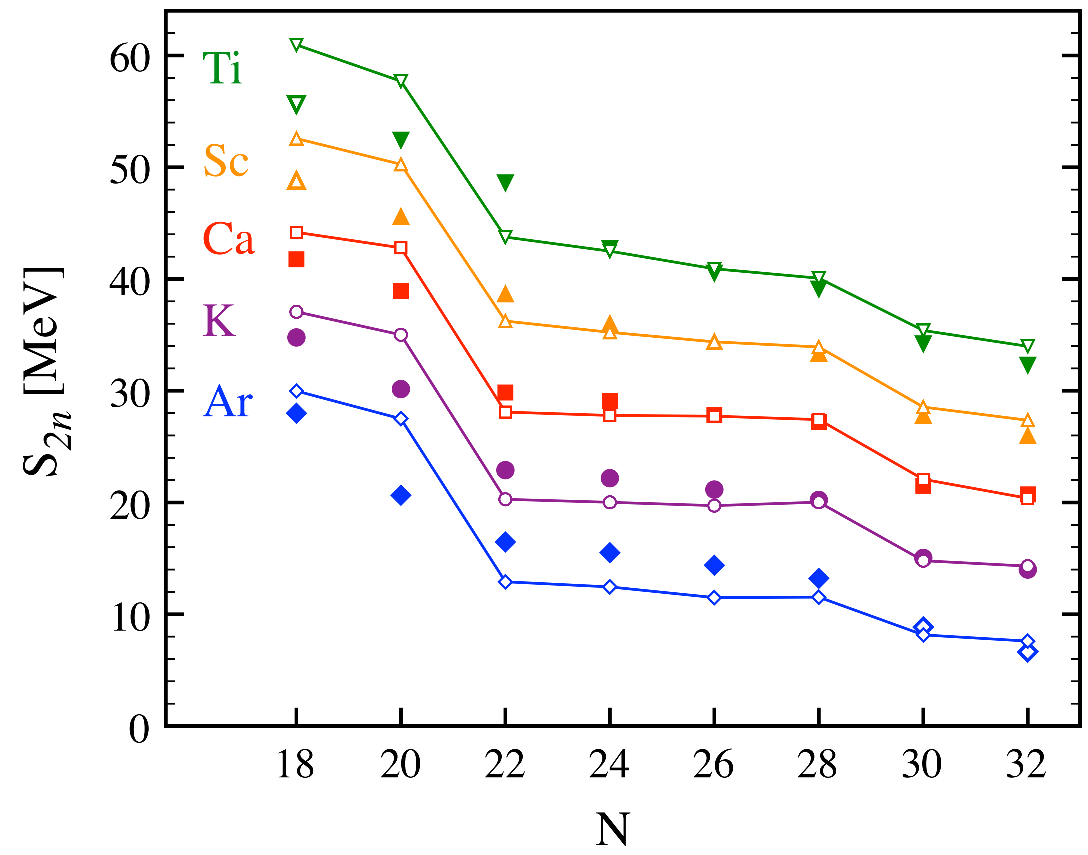
[Wienholtz *et al.* 2013]

➟ S_{2n} well reproduced with chiral NN + 3N interactions

➟ Microscopic calculations extended to the whole Ca chain

Two-neutron separation energies around Ca



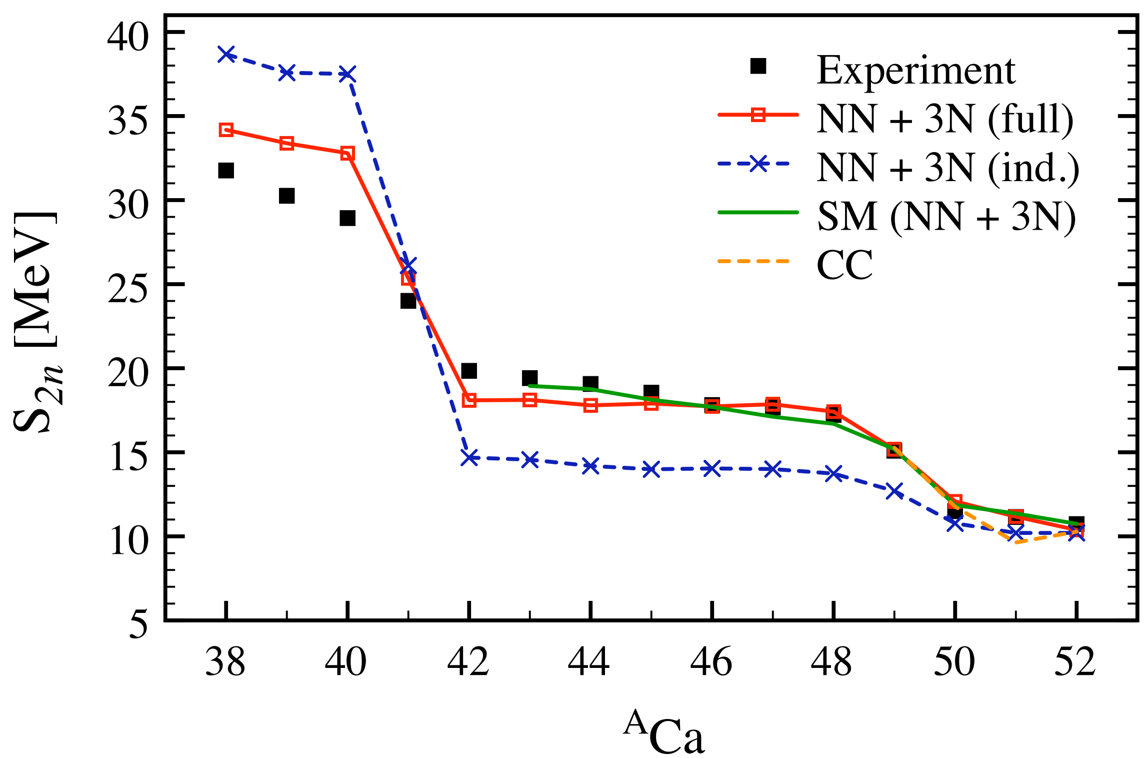


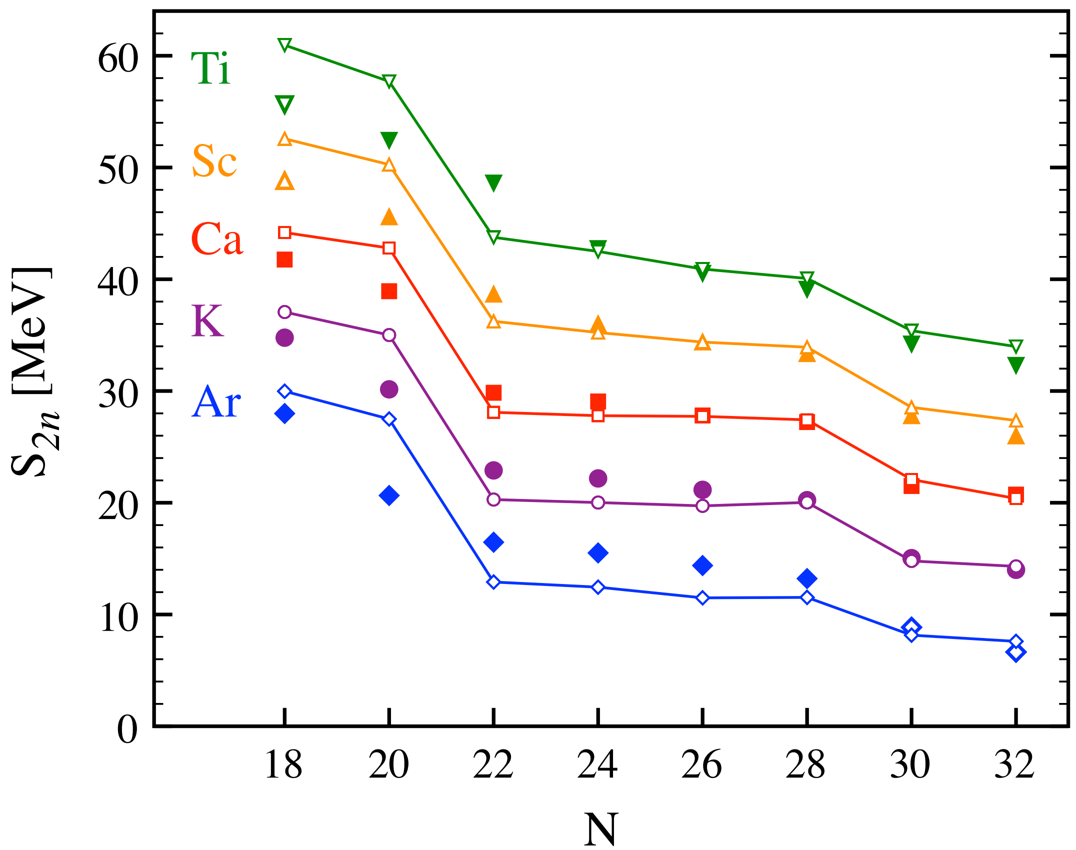
➠ S_{2n} well reproduced with chiral NN + 3N interactions

➠ Microscopic calculations extended to the whole Ca chain

➠ Neighbouring Z=18-22 chains computed within the same GGF framework

Two-neutron separation energies around Ca





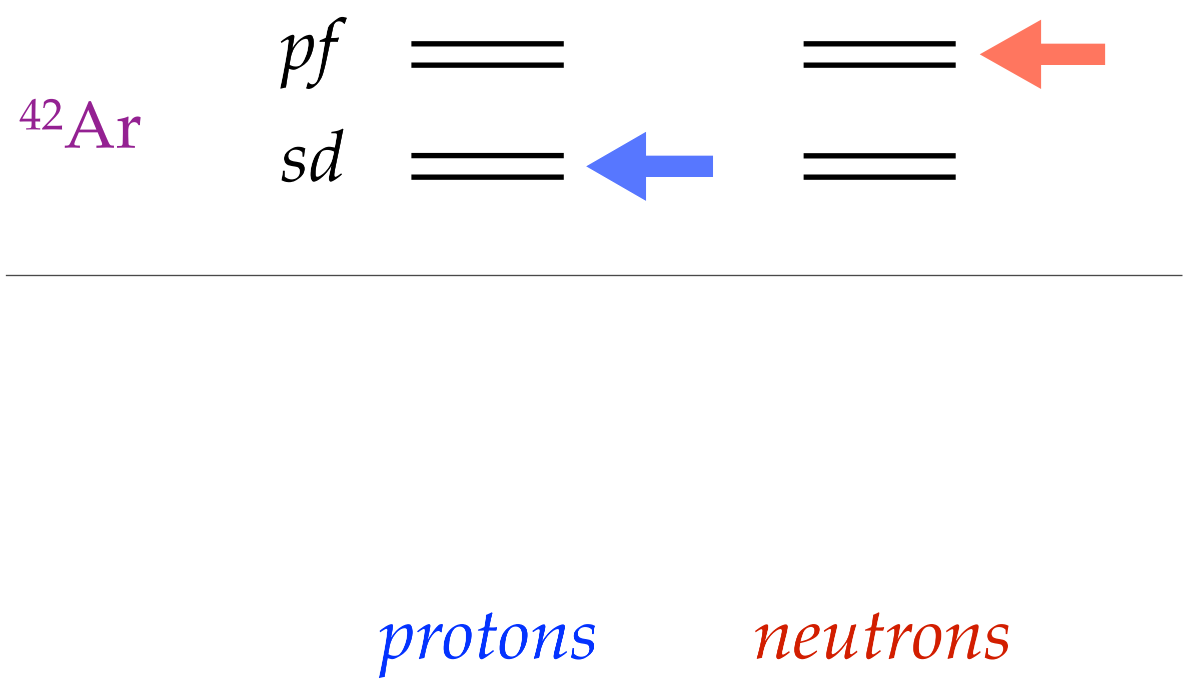
➟ S_{2n} well reproduced with chiral NN + 3N interactions

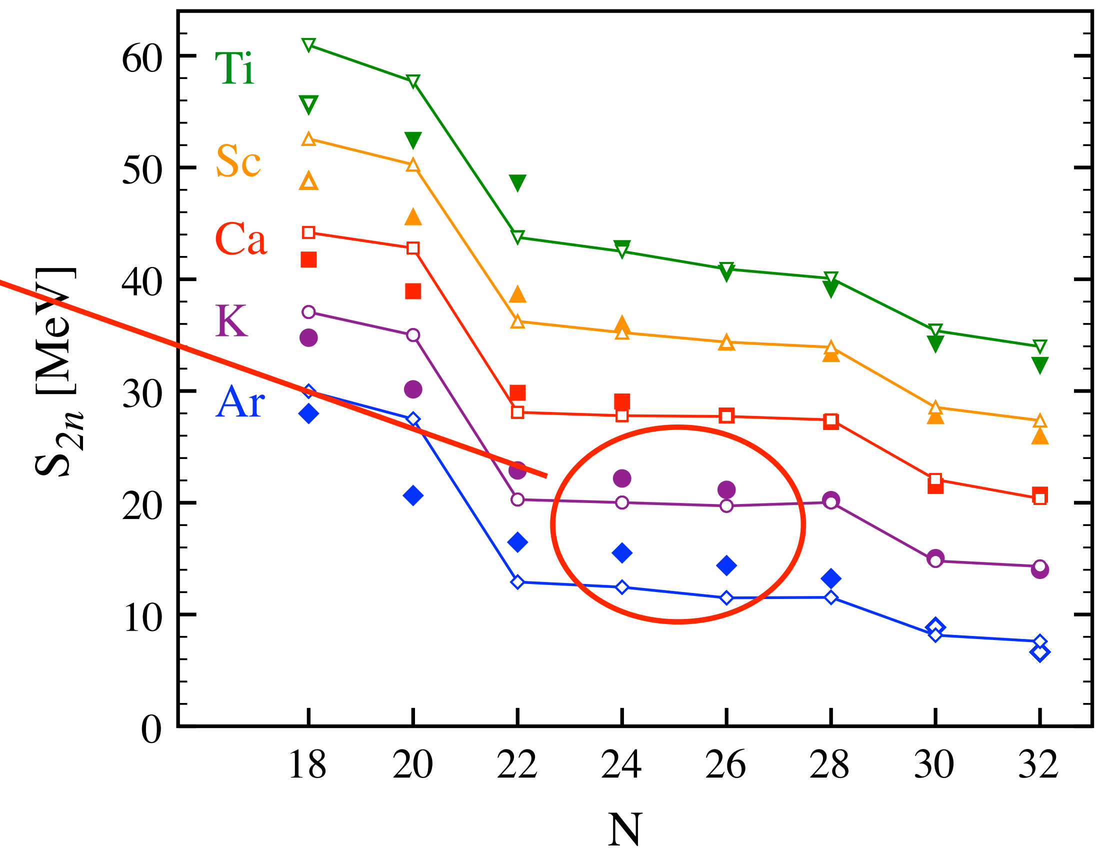
➟ Microscopic calculations extended to the whole Ca chain

➟ Neighbouring Z=18-22 chains computed within the same GGF framework

➟ Overestimation of N=20 gap traced back to spectrum too spread out

Two-neutron separation energies around Ca





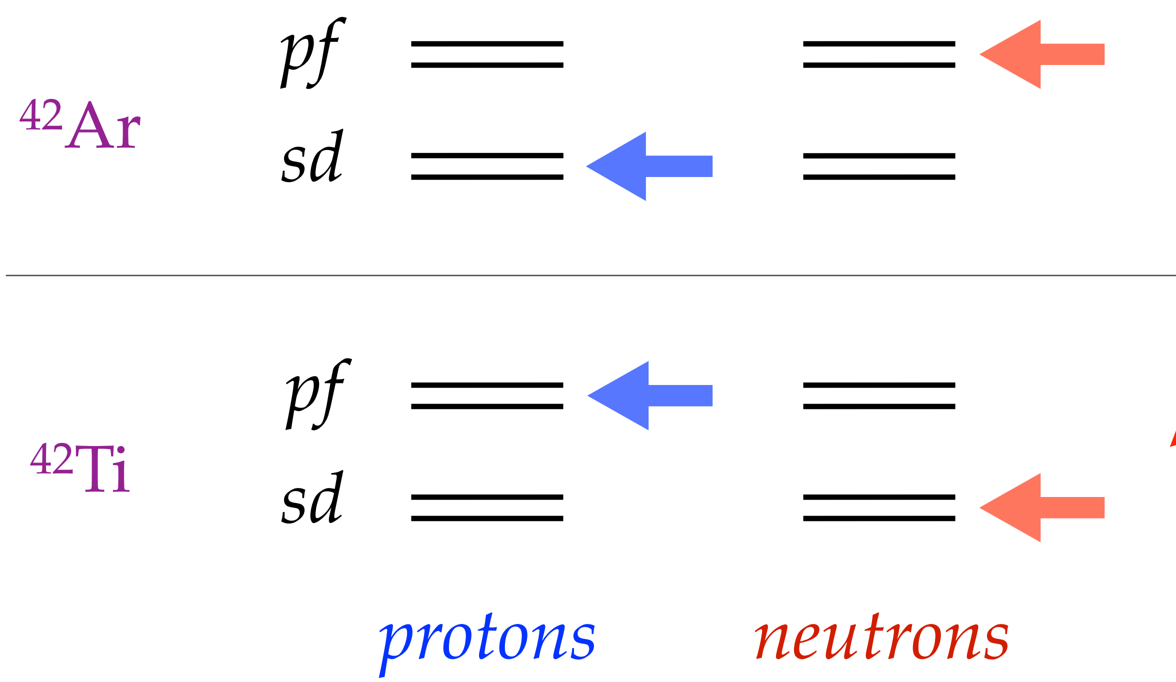
➟ S_{2n} well reproduced with chiral NN + 3N interactions

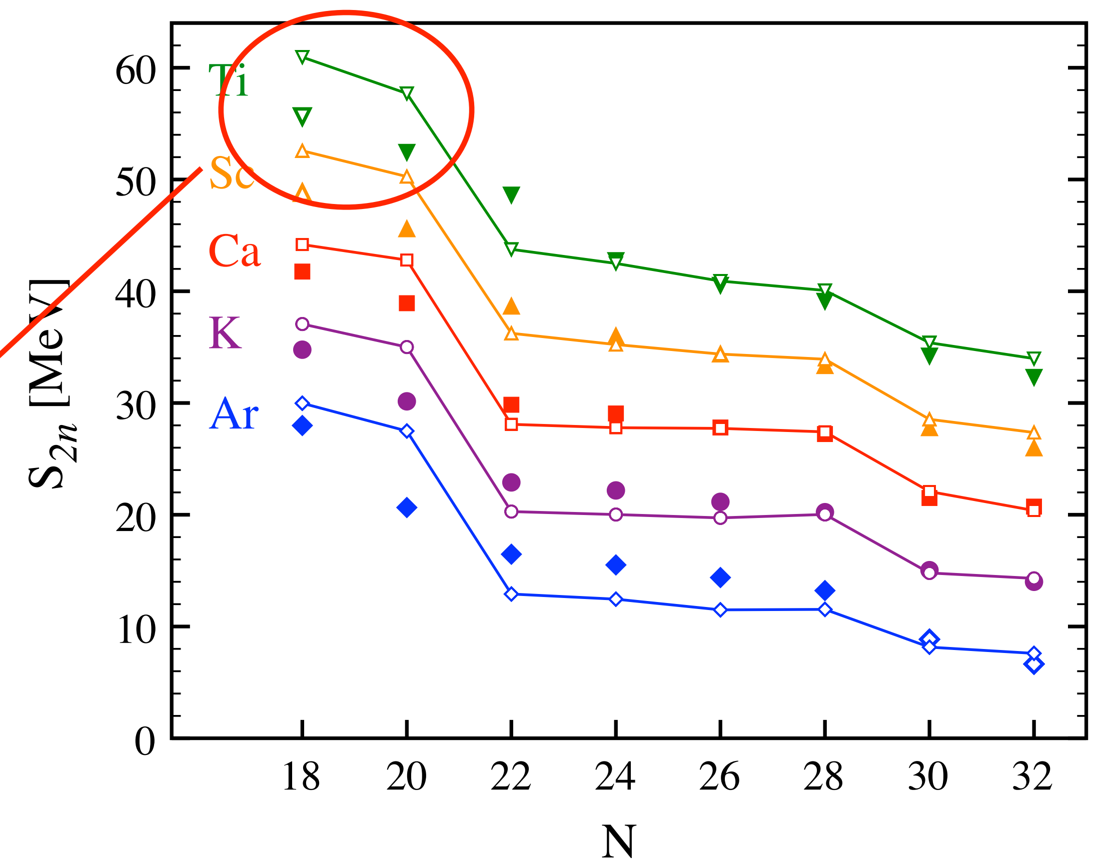
➟ Microscopic calculations extended to the whole Ca chain

➟ Neighbouring Z=18-22 chains computed within the same GGF framework

➟ Overestimation of N=20 gap traced back to spectrum too spread out

Two-neutron separation energies around Ca





➟ S_{2n} well reproduced with chiral NN + 3N interactions

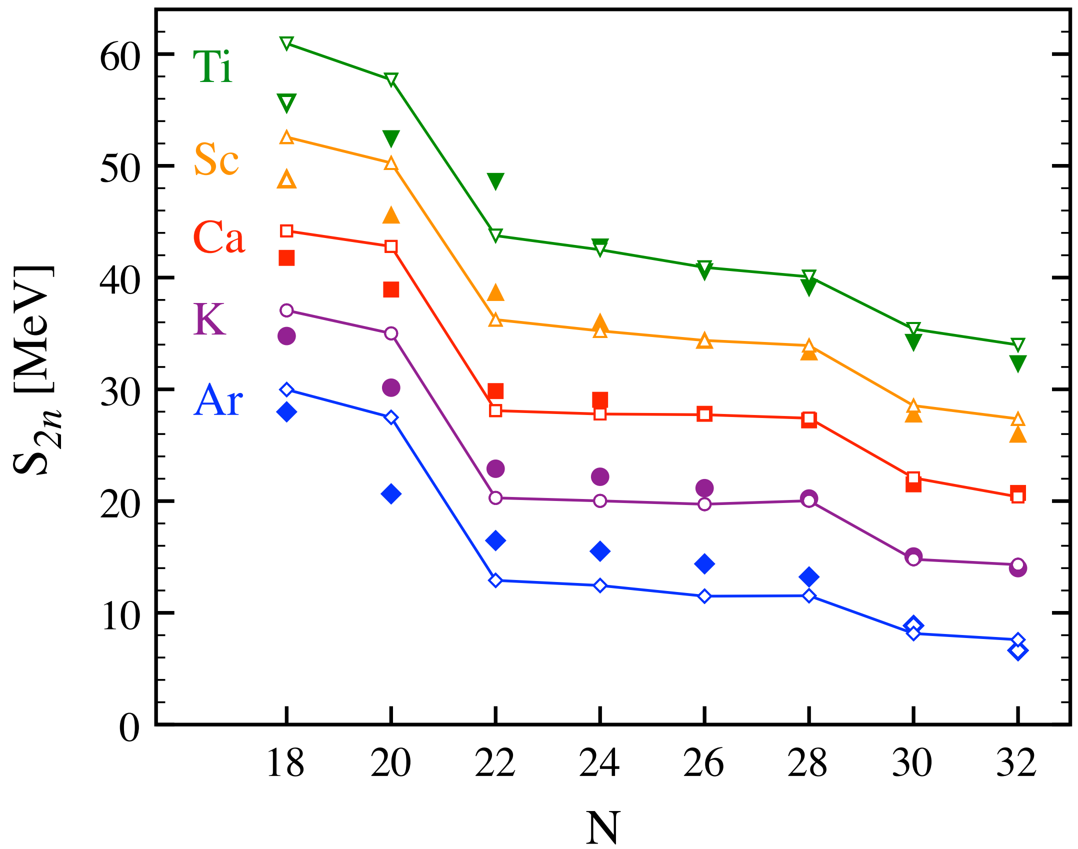
➟ Microscopic calculations extended to the whole Ca chain

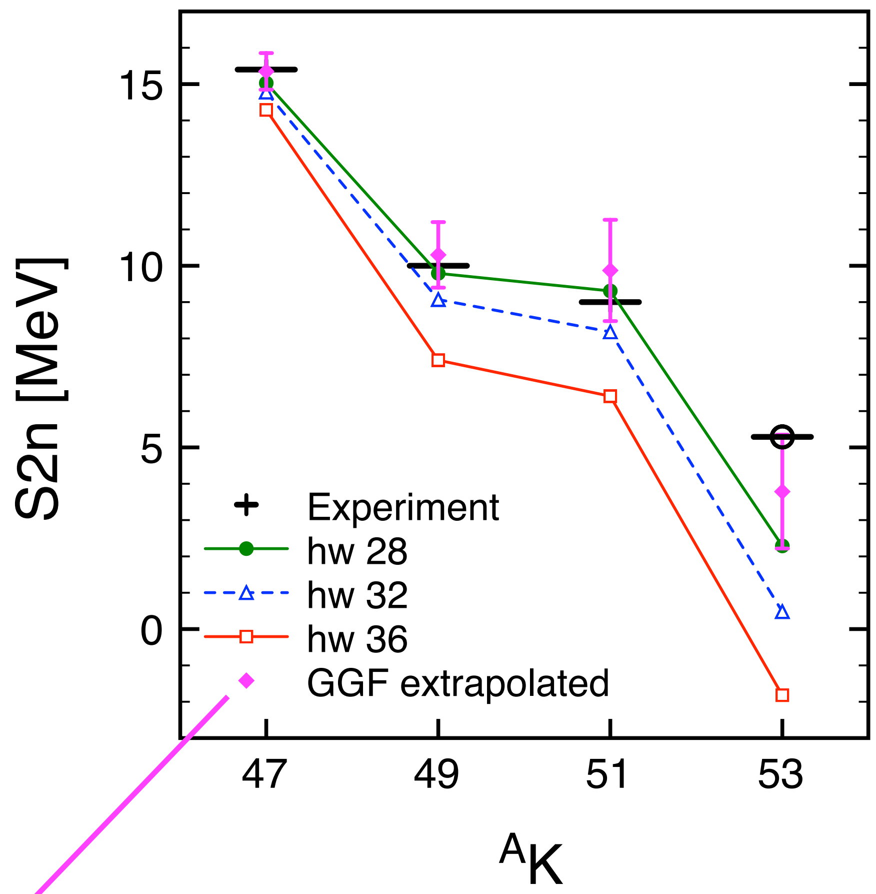
➟ Neighbouring Z=18-22 chains computed within the same GGF framework

➟ Overestimation of N=20 gap traced back to spectrum too spread out

Extrapolation of the neutron-rich end

✪ Convergence worsens after N=32





Extrapolation to infinite model space [Coon *et al.*, Furnstahl *et al.*]

Appendix

Going *open-shell:* Gorkov-Green's functions

✪ Self-consistent Green's functions

➟ Many-body truncation in the self-energy expansion (cf. CC, IM-SRG, ...)

➟ Access to *A±1* systems via spectral function

➟ Natural connection to scattering (e.g. optical potentials)

✪ Gorkov scheme

➟ Goes beyond standard expansion schemes limited to doubly closed-shell

- Formulate the expansion scheme around a Bogoliubov vacuum
- Single-reference method (cf. MR in quantum chemistry or IM-SRG)
- Exploit breaking (and restoration) of U(1) symmetry

➟ From few tens to hundreds of medium-mass open-shell nuclei

- *Formalism* VS, Duguet & Barbieri, PRC 84 064317 (2011)
- *Proof of principle* VS, Barbieri & Duguet, PRC 87 011303 (2013)
- *Technical aspects* VS, Barbieri & Duguet, PRC 89 024323 (2014)
- *NN+3N* VS, Cipollone, Barbieri, Navrátil & Duguet, arXiv:1312.2068 (2013)

Gorkov framework

✪ Expand around an auxiliary many-body state

$$|\Psi_0\rangle \equiv \sum_{A}^{\text{even}} c_A \, |\psi_0^A\rangle$$

Breaks particle-number symmetry

➟ Introduce a "grand-canonical" potential $\Omega = H - \mu A$

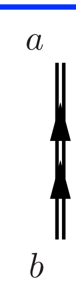
➟ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle \Psi_0 | \Omega | \Psi_0 \rangle$ under the constraint $A = \langle \Psi_0 | A | \Psi_0 \rangle$

➟ Observables of the A-body system $\Omega_0 = \sum_{A'} |c_{A'}|^2 \, \Omega_0^{A'} \approx E_0^A - \mu A$

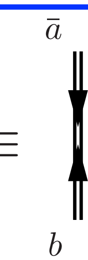
set of 4 Gorkov propagators

[Gorkov 1958]

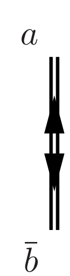
$$i\, G_{ab}^{11}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) a_b^\dagger(t') \right\} | \Psi_0 \rangle \quad \equiv$$



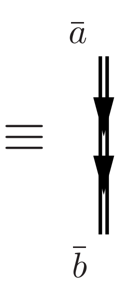
$$i\, G_{ab}^{21}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}_a^\dagger(t) a_b^\dagger(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\, G_{ab}^{12}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\, G_{ab}^{22}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}_a^\dagger(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



Inside the Green's function

✪ Separation energy spectrum

$$G^{11}_{ab}(\omega) = \sum_k \left\{ \frac{\mathcal{U}^k_a \mathcal{U}^{k*}_b}{\omega - \omega_k + i\eta} + \frac{\bar{\mathcal{V}}^{k*}_a \bar{\mathcal{V}}^k_b}{\omega + \omega_k - i\eta} \right\}$$

Lehmann representation

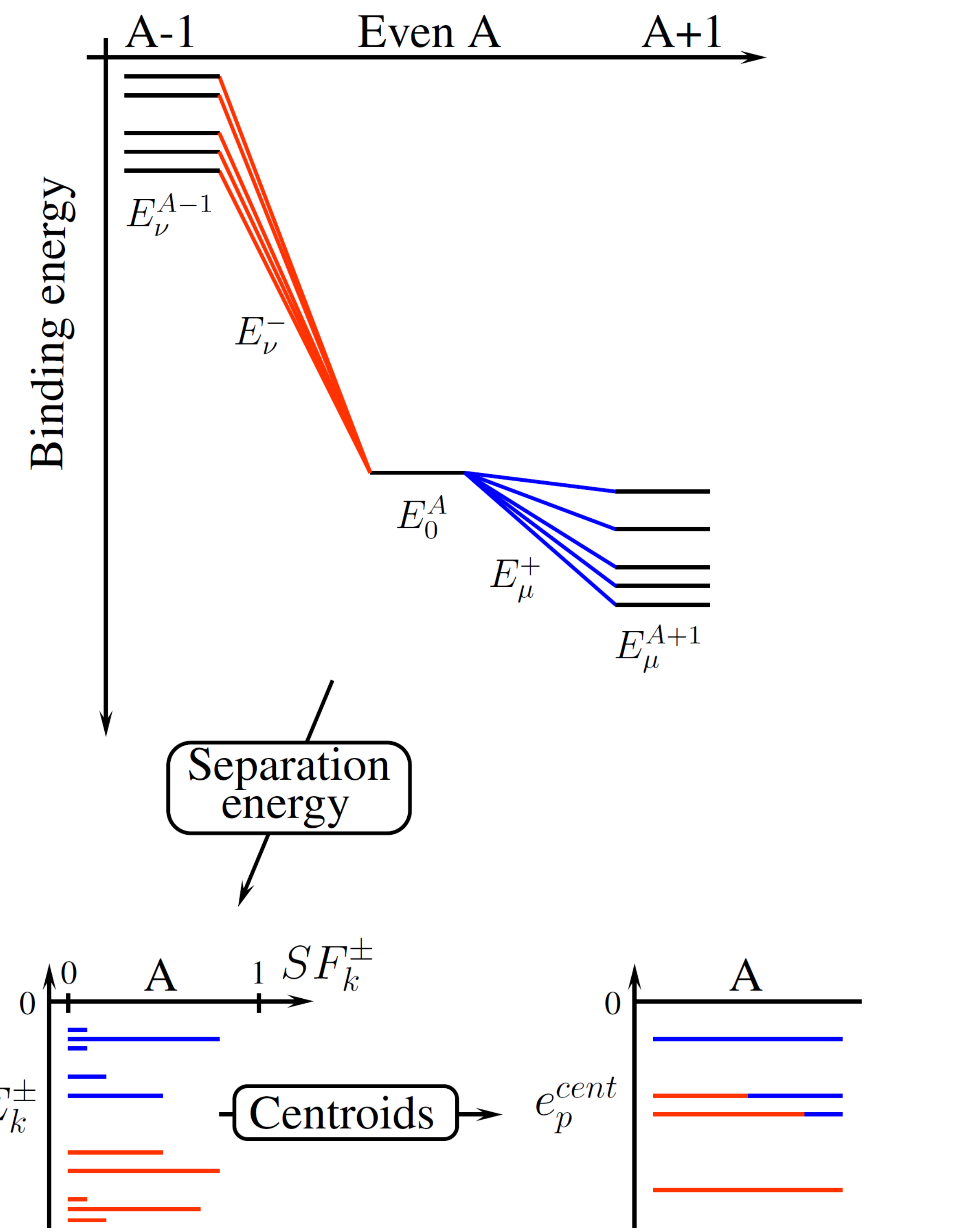
where $\begin{cases} \mathcal{U}^{k*}_a \equiv \langle \Psi_k | a^\dagger_a | \Psi_0 \rangle \\ \mathcal{V}^{k*}_a \equiv \langle \Psi_k | \bar{a}_a | \Psi_0 \rangle \end{cases}$

and $\begin{cases} E^{+\,(A)}_k \equiv E^{A+1}_k - E^A_0 \equiv \mu + \omega_k \\ E^{-\,(A)}_k \equiv E^A_0 - E^{A-1}_k \equiv \mu - \omega_k \end{cases}$

✪ Spectroscopic factors

$$SF^+_k \equiv \sum_{a \in \mathcal{H}_1} \left| \langle \psi_k | a^\dagger_a | \psi_0 \rangle \right|^2 = \sum_{a \in \mathcal{H}_1} \left| \mathcal{U}^k_a \right|^2$$

$$SF^-_k \equiv \sum_{a \in \mathcal{H}_1} \left| \langle \psi_k | a_a | \psi_0 \rangle \right|^2 = \sum_{a \in \mathcal{H}_1} \left| \mathcal{V}^k_a \right|^2$$



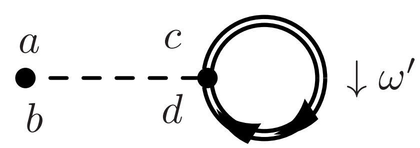
[figure from J. Sadoudi]

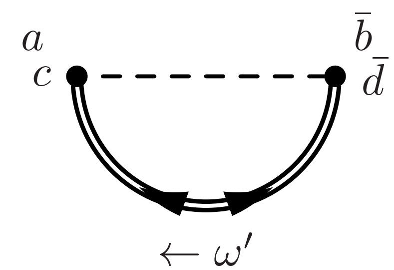
Gorkov equation

✪ Gorkov equation ⟶ energy *dependent* eigenvalue problem

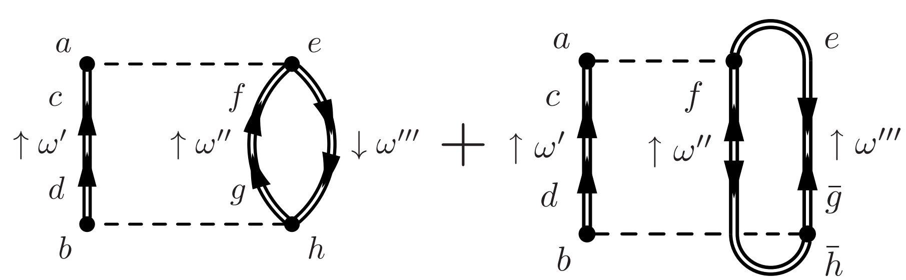
$$\boxed{\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}}$$

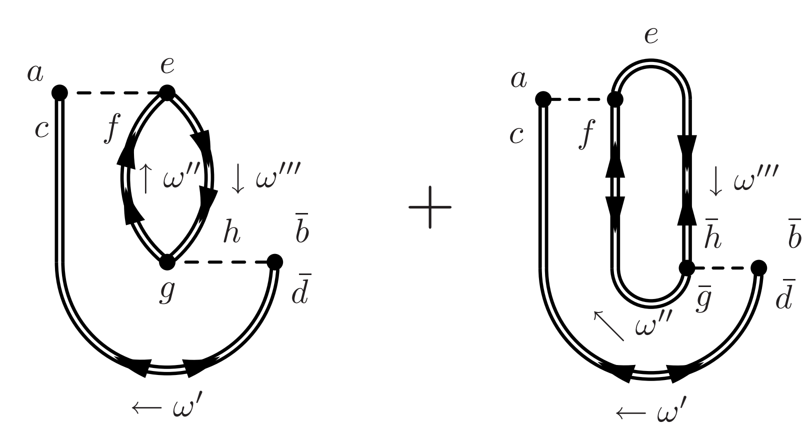
✪ 1^{st} order ➟ energy-independent self-energy

$\Sigma^{11\,(1)}_{ab} =$ 

$\Sigma^{12\,(1)}_{ab} =$ 

✪ 2^{nd} order ➟ energy-dependent self-energy

$\Sigma^{11\,(2)}_{ab}(\omega) =$ 

$\Sigma^{12\,(2)}_{ab}(\omega) =$ 

Gorkov equation

✪ Gorkov equation $\longrightarrow$ energy *dependent* eigenvalue problem

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}$$

[Schirmer & Angonoa 1989]

energy *independent* eigenvalue problem

$\propto N_b^3$

typically ~10^6-10^7

$$\begin{pmatrix} T - \mu + \Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T + \mu - \Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$

Krylov space eigenvalue problem

$\propto N_{\text{Lanczos}}$

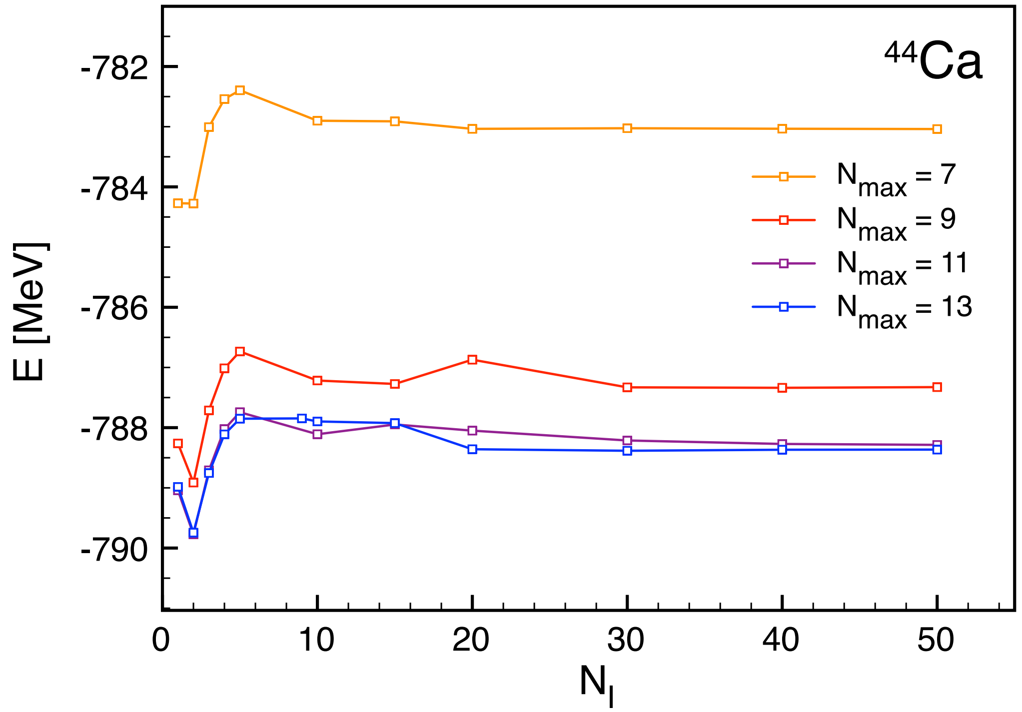
typically ~10^2-10^3

$$\begin{pmatrix} T - \mu + \Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T + \mu - \Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$

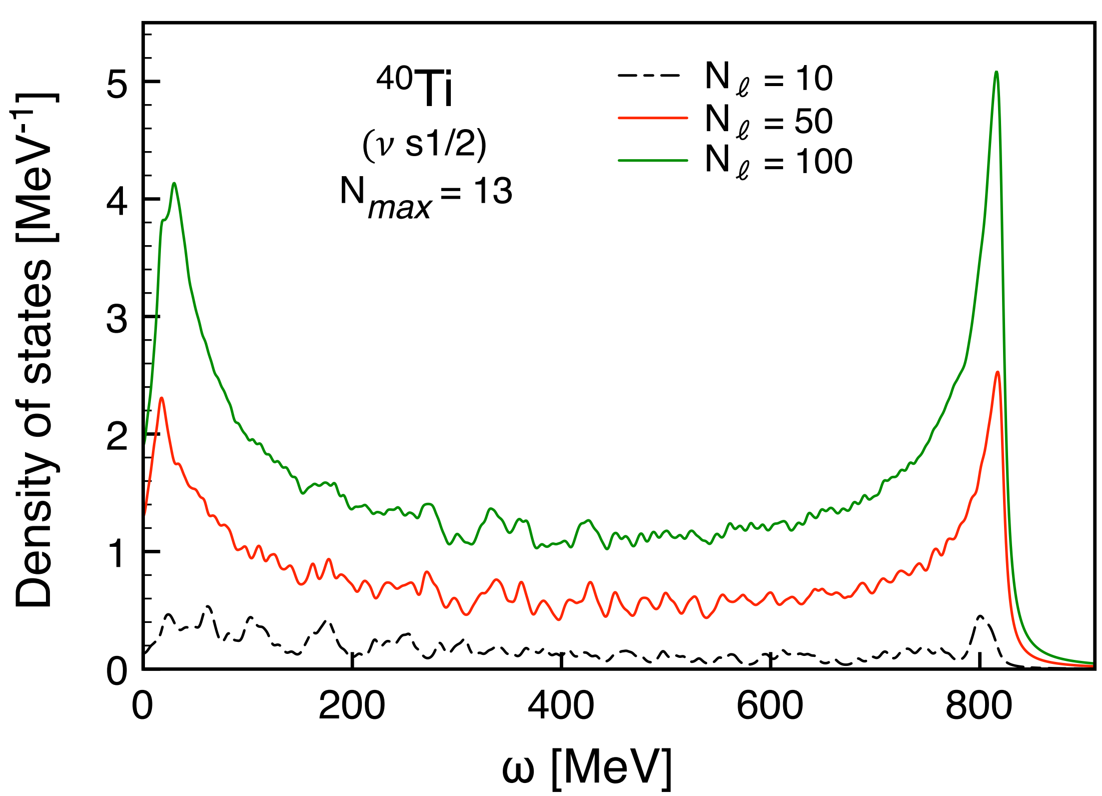
Testing Krylov projection

- ✪ Energy & spectral distribution independent of the projection
- ✪ Same behavior for all model spaces

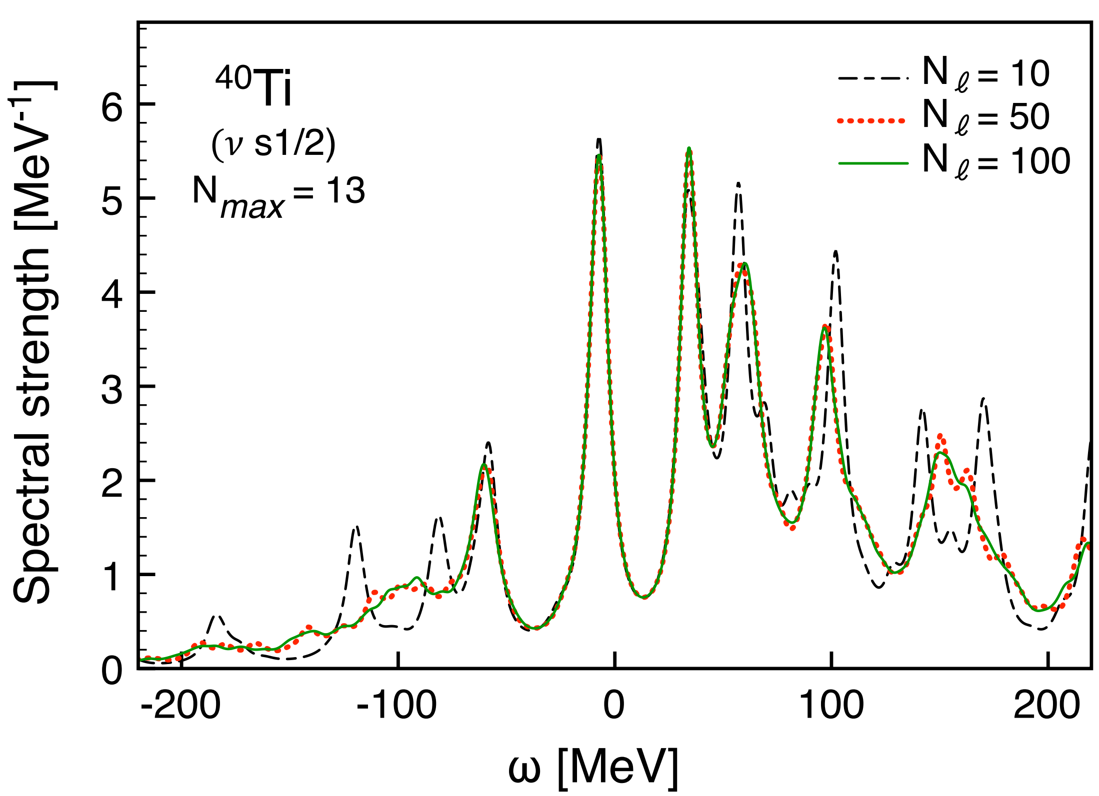
[VS, Barbieri & Duguet PRC 2014]



Density of states



Spectral strength



Odd-even systems

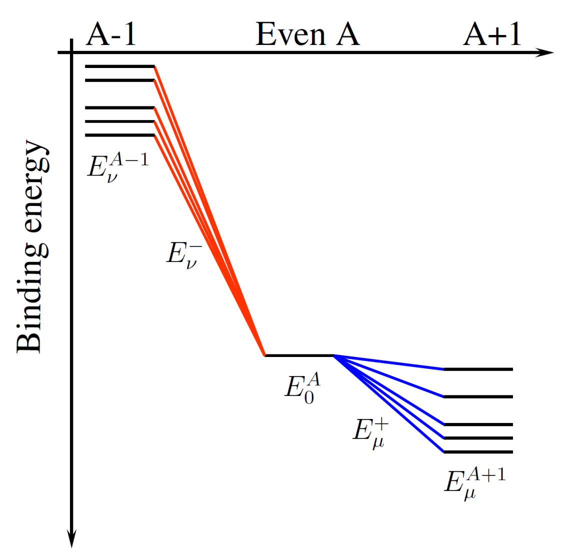
✪ Current implementation targets $J^{\pi} = 0^{+}$ states

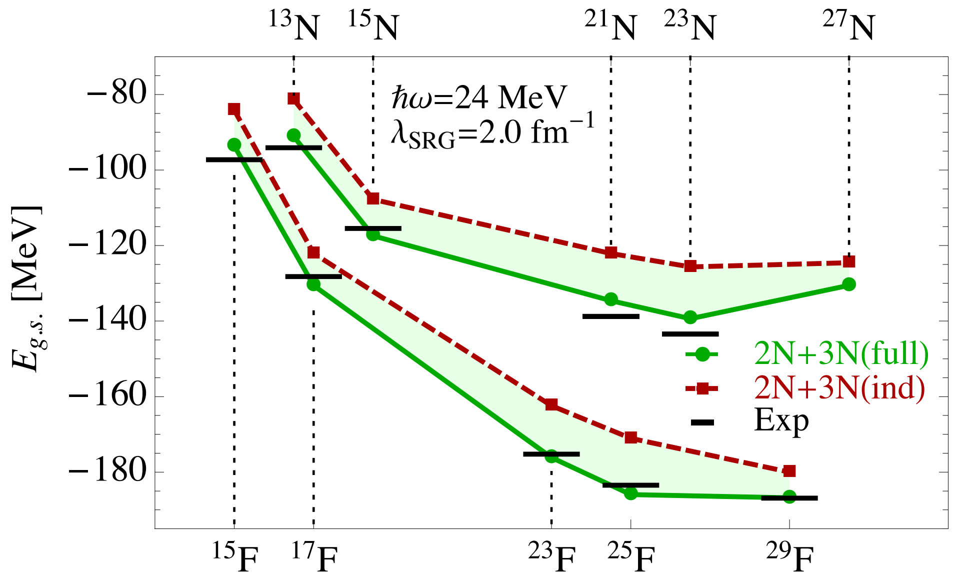
➟ Equations simplify: j-coupled scheme, block-diagonal structure, ...

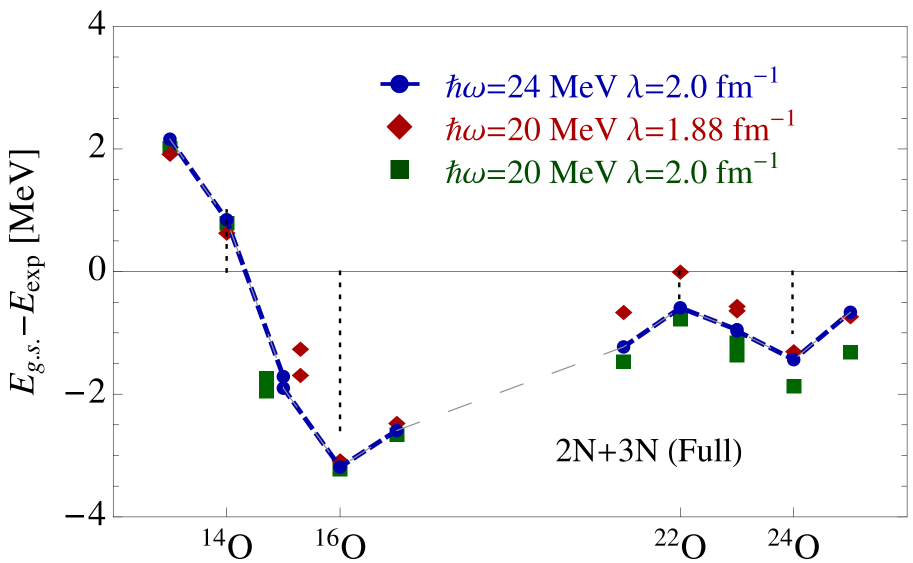
✪ Different possibilities to compute odd-even g.s. energies:

① From separation energies

➟ Either from A-1 or A+1







[Cipollone, Barbieri & Navrátil PRL 2013]

Odd-even systems

✪ Current implementation targets $J^{\pi} = 0^{+}$ states

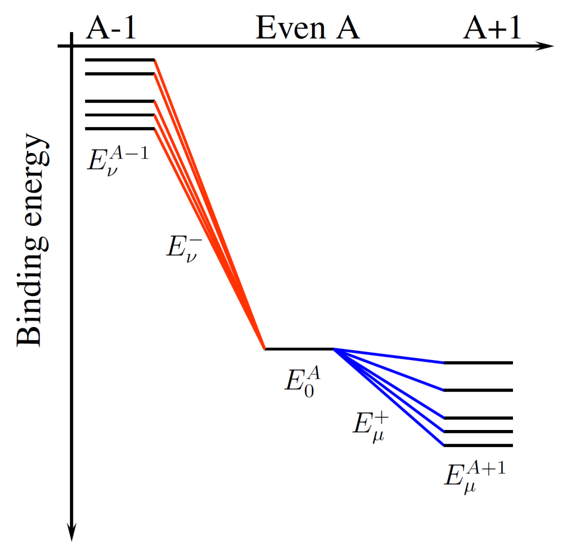
➟ Equations simplify: j-coupled scheme, block-diagonal structure, ...

✪ Different possibilities to compute odd-even g.s. energies:

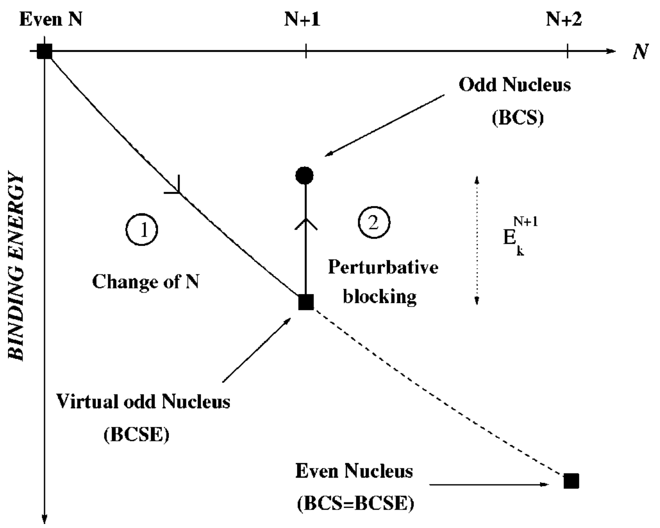
① From separation energies

② From fully-paired even number-parity state

➟ Either from A-1 or A+1



➟ "Fake" odd-A plus correction



[Duguet *et al.* 2001]

Two methods agree within 2-300 keV

The GGF input: NN & 3N interactions

- ✪ NN potential: chiral N^3LO (500 MeV) SRG-evolved to 2.0 fm^{-1}

  [Entem and Machleidt 2003]

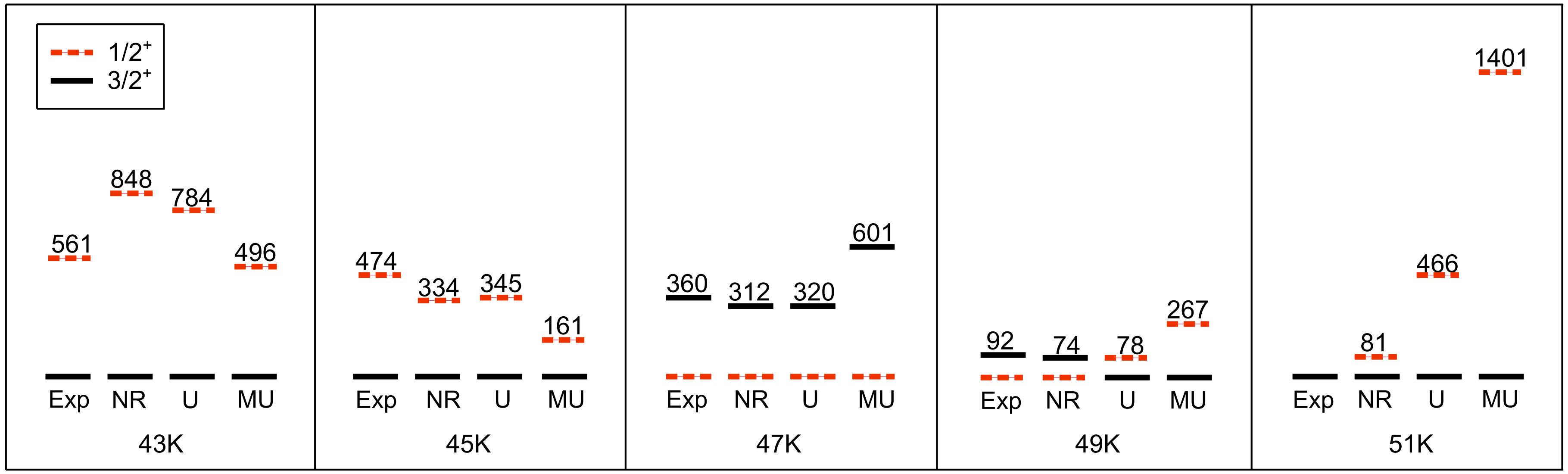
- ✪ 3N potential: chiral N^2LO (400 MeV) SRG-evolved to 2.0 fm^{-1} [Navrátil 2007]
  - ➟ Fit to three- and four-body systems only
  - ➟ Modified cutoff to reduce induced 4N contributions [Roth *et al*. 2012]

- ✪ In the future:
  - ➟ Chiral 3NF at N^3LO
  - ➟ Δ-full chiral interactions
  - ➟ NN & 3N consistently SRG-evolved in momentum space
  - ➟ ...
  - ➟ Chiral interactions with improved/correct power counting
  - ➟ Inputs from lattice QCD: couplings & YN interactions

Potassium ground states (re)inversion

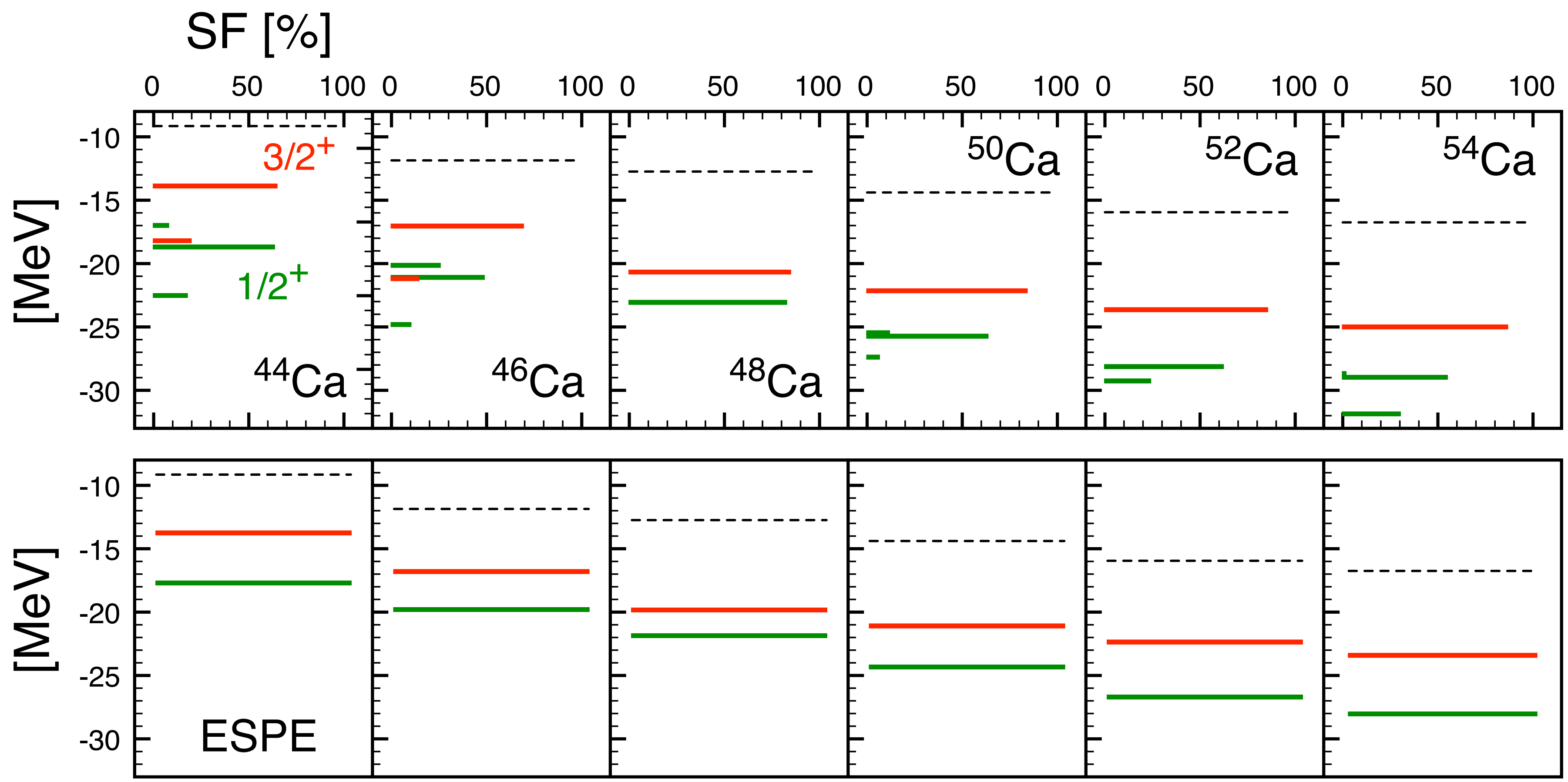
Laser spectroscopy (@ ISOLDE)

[Papuga *et al.* 2013]



Theory (GGF)

[VS *et al.* unpublished]



Knockout & transfer experiments

✪ Neutron removal from proton- and neutron-rich Ar isotopes @ NSCL

				(theo.)	(expt.)		(expt.)	
Isotopes	lj^{π}	Sn(MeV)	ΔS (MeV)	SF(LB-SM)	SF(JLM + HF)	Rs(JLM + HF)	SF(CH89)	Rs(CH89)
^{34}Ar	$s1/2^+$	17.07	12.41	1.31	0.85 ± 0.09	0.65 ± 0.07	1.10 ± 0.11	0.84 ± 0.08
^{36}Ar	$d3/2^+$	15.25	6.75	2.10	1.60 ± 0.16	0.76 ± 0.08	2.29 ± 0.23	1.09 ± 0.11
^{46}Ar	$f7/2^-$	8.07	-10.03	5.16	3.93 ± 0.39	0.76 ± 0.08	5.29 ± 0.53	1.02 ± 0.10

[Lee *et al.* 2010]

	Sn (MeV)	ΔS (MeV)	SF
^{34}Ar	33.0	18.6	1.46
^{36}Ar	27.7	7.5	1.46
^{46}Ar	16.0	-22.3	5.88

Gorkov GF NN

ΔS = Sn - Sp

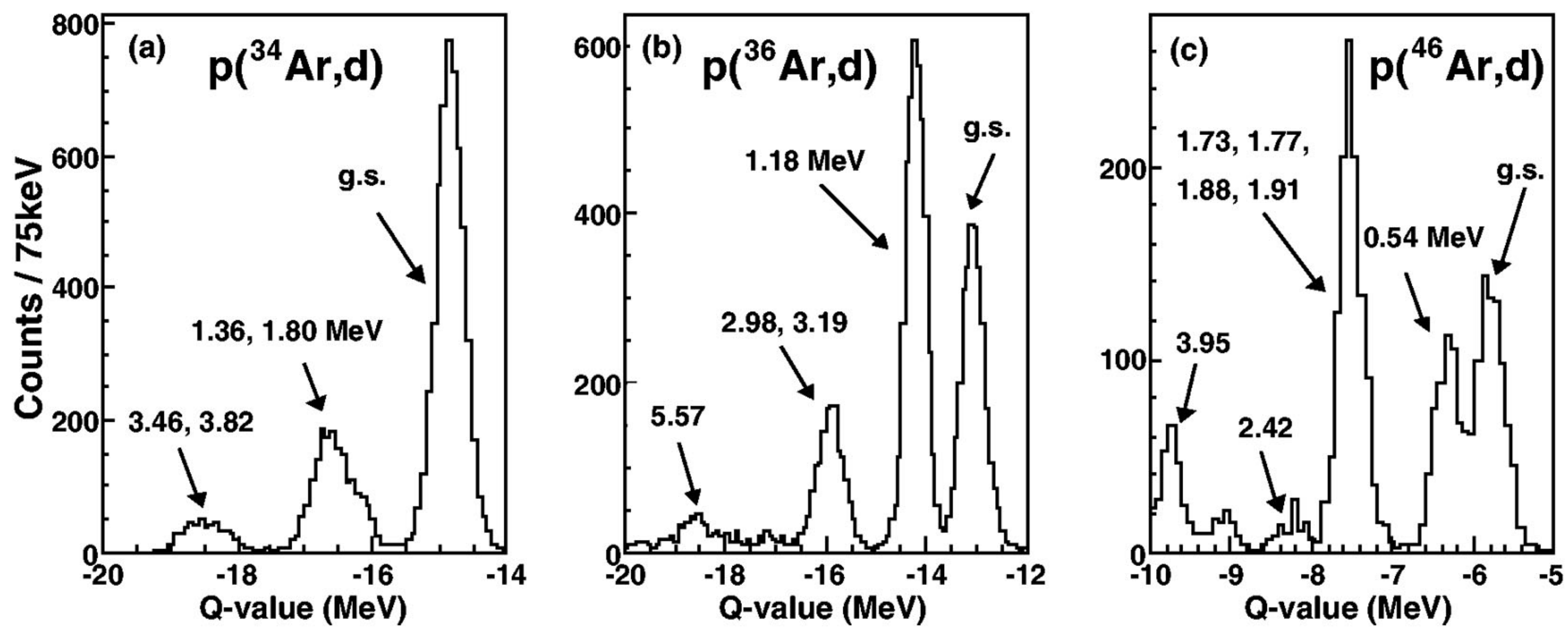
^{34}Ar	22.4	15.5	1.56
^{36}Ar	15.3	7.2	1.54
^{46}Ar	6.5	-15.7	6.64

Gorkov GF NN + 3N

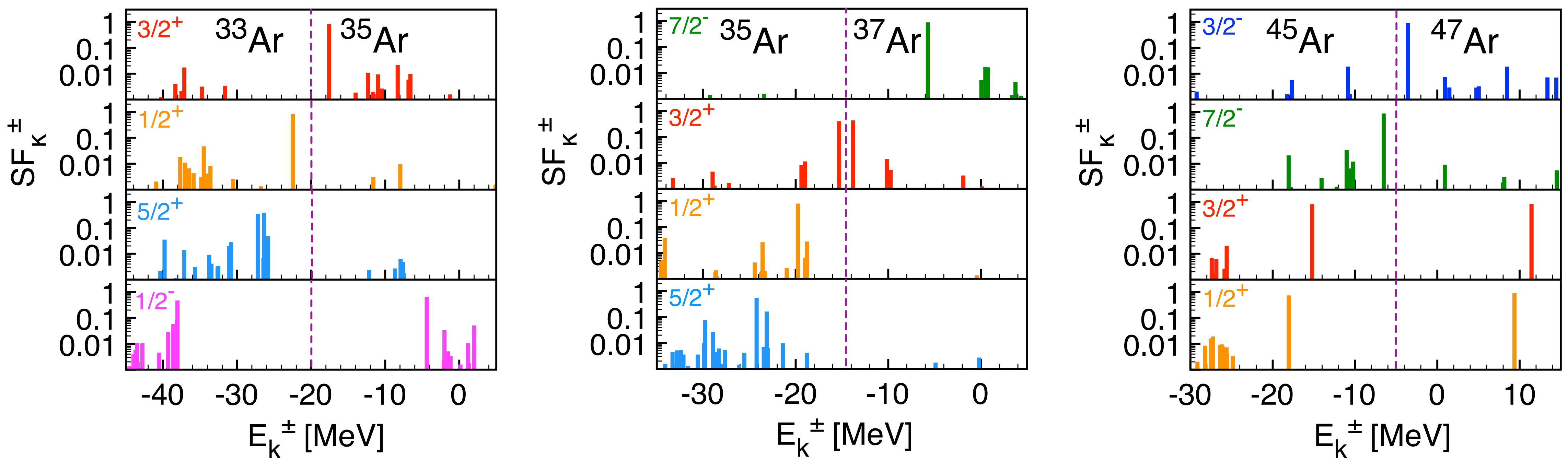
[VS *et al.* unpublished]

Knockout & transfer experiments

✪ Neutron removal from proton- and neutron-rich Ar isotopes @ NSCL



[Lee *et al.* 2010]



[VS *et al.* unpublished]

Summary & outlook

✪ Agreement between different many-body methods

➟ Model independent calculations challenge chiral interactions

✪ Gorkov-Green's functions

➟ Novel path to extend first-principle calculations to open-shells

➟ GGF(2) provides good reproduction of S2n around Ca

➟ Separation spectra at a qualitative level

➟ Work in progress: GGF(3)