Emergent geometry and duality in ^{24}Mg and other self-conjugate nuclei

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I. LOGO

For the present workshop, the adoption of the logo in Fig. 1 is proposed.

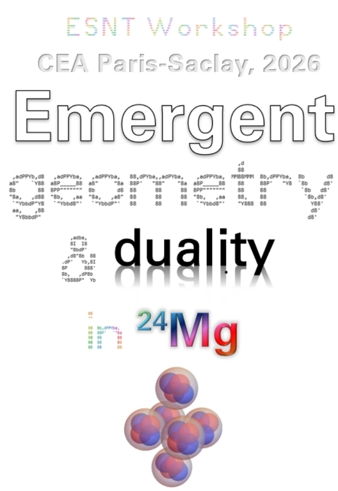

Figure 1. Logo for the ESNT workshop *Emergent geometry and duality in ^{24}Mg and other self-conjugate nuclei.*

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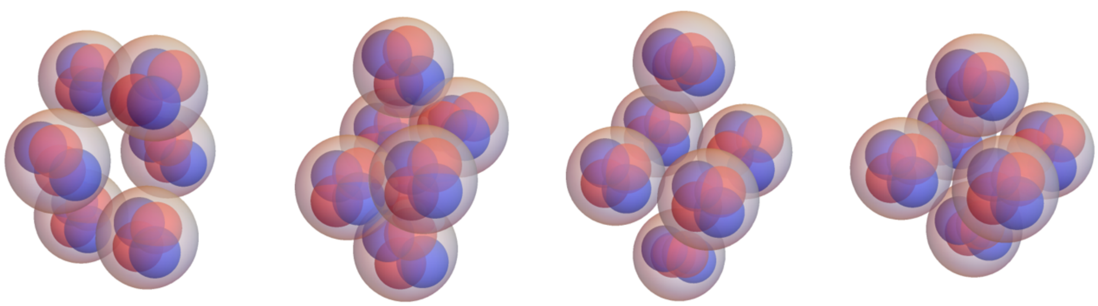
II. SCIENTIFIC ISSUE

Since the discovery of the α particle, the study of the structure of α-conjugate nuclei is a subject of major interest for nuclear theorists, as α particles are favoured configurations within certain nuclei.

More in general, the observed nuclear spectra are the result of the interplay between independent-particle effects in a mean field and spatial few-body correlations between the constituents [1)]. While the former give rise to shell-structure features, the latter are responsible of clustering phenomena. Namely in even-even self-conjugate nuclei, such correlations lead to the partial or complete partitioning of nuclear matter into α-particle groupings. In recent literature, this twofold nature of the nuclear phenomenology is referred to as *duality* [2–7)].

The appearance of clusters affects the intrinsic structure of the nucleus, which acquires a *molecular* shape [8)]. As a result, molecular or *exotic* discrete symmetries [9)] add on top of continuous symmetries. Probably, the most striking example is represented by ^{12}C. Its ground state of is a mean-field, *i.e.* shell-model-like, state dominated by the $(p_{3/2})^4$ closed-subshell configuration with significant overlap with 3α-cluster wavefunctions [1)] with $\mathcal{D}_{3h}$ symmetry [10,11)], indicating that α-clusters are partially formed even in the lowest-energy 0^+ state.

The most abundant isotope of magnesium, ^{24}Mg, makes no exception in this sense and has been recently investigated by means of a macroscopic α-cluster model [12,13)]. There, the relevance of the point-symmetry group $\mathcal{D}_{4h}$ for the prediction of spectrum and electromagnetic properties of the ^{24}Mg nucleus has been subject of recent investigations in the framework of the *geometric α-cluster model* (GαCM) at leading order [12,13)]. The latter represents a macroscopic approach [14,15)] wherein nuclear excitations are described in terms of rotations and vibrations of ^{4}He clusters about their equilibrium positions, at the vertices of a square bipyramid [16,17)]. The finite group associated with the latter regulates the composition of the rotational bands as well as the γ-transitions between the energy levels, by means of additional selection rules, of molecular nature [12)].



(a) (b) (c) (d)

Figure 2. Intrinsic structures of ^{24}Mg in terms of α-particles, constituted by two protons (blue) and two neutrons (red) with polygonal symmetry, adopted for the construction of NLEFT trial states. Among these, we mention the bitetrahedron (a), corresponding to the discrete group $\mathcal{D}_{2h}$; the square bipyramid with rotated basis (b), with the same symmetry; the square bipyramid (c), with group $\mathcal{D}_{4h}$ and the octahedron (d), with group $\mathcal{O}_h$.

A sample of electric multipole transition strengths of intraband nature has been calculated [13)]. It is found that reduced intraband $E2$ transition probabilities are better predicted by the square-bipyramidal rather than the bitetrahedral α-cluster equilibrium configuration with $\mathcal{D}_{2h}$ symmetry [18)]. Moreover, the identification of the candidates for the 9 expected rotational bands corresponding to normal modes of vibration with a single quantum of excitation [11,12)] corroborates the $\mathcal{D}_{4h}$-symmetric ansatz.

Besides, the scarcity of $J^\pi = 0^-$ states in the observed spectrum as well as the difficulty in finding the members of the predicted 12 rotational bands with a quantum of vibration predicted by the $\mathcal{D}_{2h}$ model, seem to favour the bipyramidal structure over the bitetrahedral one. Conversely, comparison between the experimental squared transition form factors and the LO GαCM counterparts seem to favour $\mathcal{D}_{2h}$ symmetry, corresponding to either a bitetrahedron or a deformed square bipyramid with the planar α-clusters rotated by $\approx 30^\circ$ about an axis orthogonal to the principal symmetry axis [13)] (cf. Fig. 2).

On the wave of the recent investigation on the α-cluster structure of ^{12}C [3)] as well as earlier works [19,20)], the application of *nuclear lattice effective field theory* (NLEFT) [21–24)] on ^{24}Mg has become the subject of a new collaborative project involving the organizers of the proposed workshop. Among the objectives, there is the one of obtaining a model-independent density map of the geometry of the $J^\pi = 2^+_1$, 2^+_2, 4^+_1 and 3^+_1 excited states of the nucleus. Other levels of interest are represented by the Hoyle-state analogues lying in the vicinity of decay thresholds such as $\alpha+^{20}$Ne, ^{12}C+^{12}C and 2α + ^{16}O at 9.316, 13.934 and 14.047 MeV respectively.

The observed abundance of ^{24}Mg, in fact, stems partly from the ^{12}C+^{12}C channel in carbon-burning stars, whose initial mass is greater than ≈ 4 solar masses ($M_{\odot}$). Significant is also the production by means of the $\alpha+^{20}$Ne reaction channel in neon-burning stars, occurring in more massive stars ($\gtrsim 8M_{\odot}$).

An essential element of this study is the *pinhole* algorithm ^{12}C [3], which permits the reconstruction of the nuclear density distribution associated with a given A-body state in the *auxiliary field Monte Carlo* method adopted by NLEFT [24,25]. Observables such as charge radii, form factors, $M1$ and $E2$ moments as well as reduced electric and magnetic multipole transition probabilities are sampled stochastically as the energy eigenvalues.

The designated tool is provided by an SU(4)-symmetric Hamiltonian with Coulomb interaction [3,25], whose parameters have been adjusted in order to reproduce the experimental ground-state ($g.s.$) energy of ^{24}Mg as well as the experimental *Tjon ratio* [26] between the binding energies of ^{3}H and ^{4}He. The fit is accompanied by the calculation of the $g.s.$ energy on the mirror nuclei ranging from ^{2}H to ^{40}Ca. The latter shows that the new parameters of the Hamiltonian are capable of capturing the experimental trend of the binding energy per nucleon as well as reproducing simultaneously within 2 % deviation the observed $g.s.$ energies of ^{10}B, ^{18}F, ^{22}Na, ^{26}Al, ^{28}Si, ^{30}P and ^{32}S (cf. Fig. 3).

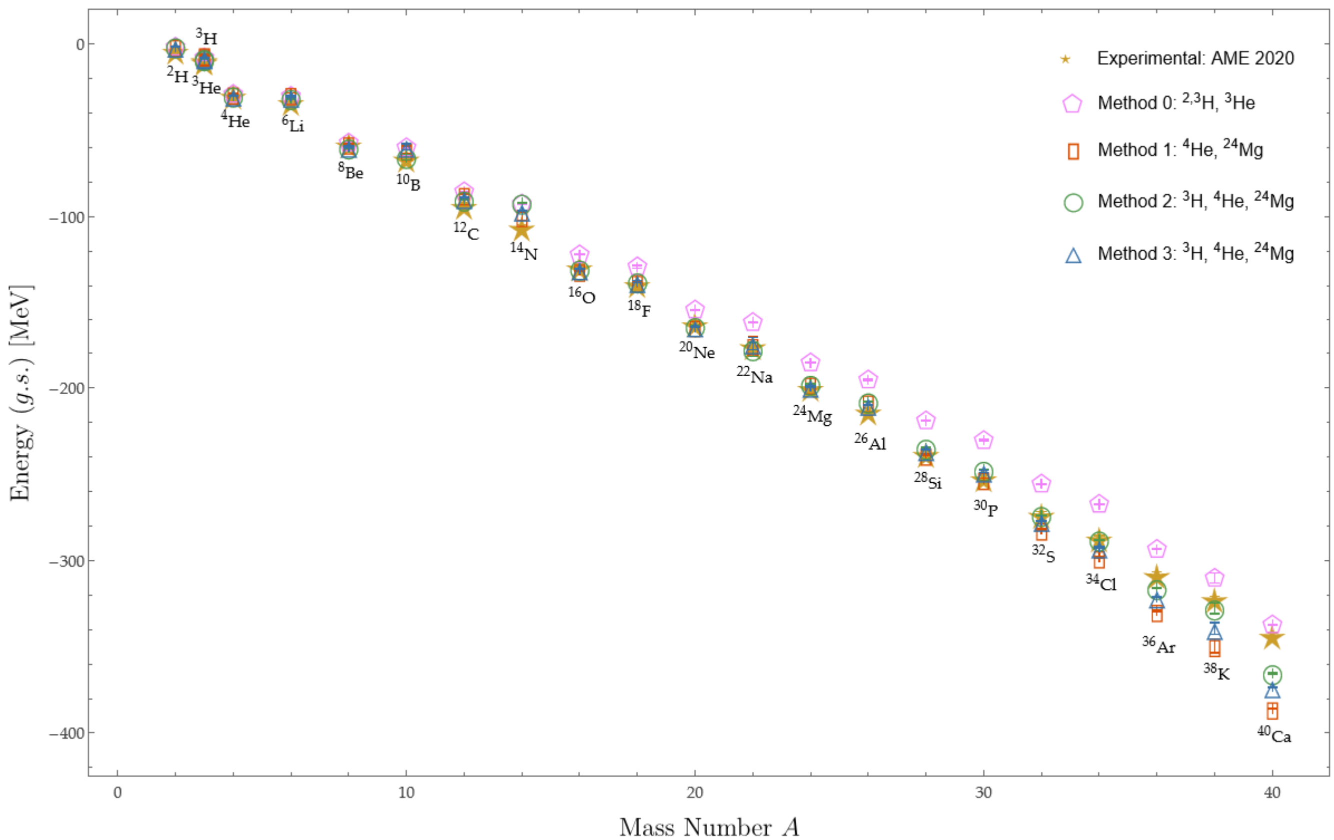


Figure 3. Ground-state energy of s, p and sd-shell mirror nuclei with the addition of ^{3}H and ^{3}He as a function of the mass number, A. The experimental values (stars) [25] are compared with the counterparts obtained from NLEFT with isospin-symmetric interactions and parameters C_2 and C_3 fitted (0) on the $g.s.$ energies of ^{2,3}H and ^{3}He (pentagons) [27]; (1) on the $g.s.$ energies of ^{4}He and ^{24}Mg (rectangles); (2) on the $g.s.$ energy of ^{24}Mg with constrained *Tjon ratio* (circles); (3) on the $g.s.$ energy of ^{24}Mg and minimizing the deviation between the binding energies of ^{3}H and ^{4}He and the respective experimental values (triangles).

In comparison with *mean-field* states consisting of single Slater determinants of harmonic-oscillator eigenfunctions, preliminary calculations of the energy eigenvalues of the 0^+ ground state of ^{24}Mg based on Euclidean-time extrapolations [22,24] show that trial wavefunctions of α-cluster type allow for a faster convergence towards the expectation value of energy. Among the cluster structures tested for the same state, the square bipyramidal, and the bitetrahedral configurations are favoured with respect to the octahedral configuration ($\mathcal{O}_h$ symmetry). Nevertheless, only the reconstruction of the nuclear density profile will permit to determine the possible α-cluster structure of the ground state. Analogous NLEFT simulations for the energy eigenvalue of the lowest 2^+, 3^+ and 4^+ states, in combination with the calculation of the elastic form factor of the ground state are presently ongoing.

Finally, the recent advances in the project on ^{24}Mg, along with the ones planned to come and the new developments in lattice field theory methods [27–32)], indicate that it is timely to bring together the members of the present collaboration as well as some practitioners of NLEFT to discuss in person the status the project, bridge between the different adopted techniques and delineate future perspectives.

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III. GOALS OF THE PROJECT

The event foresees the in-person meeting between the members of the NLEFT collaboration which are presently involved in the investigation of the excited states of the ^{24}Mg nucleus. The idea underlying this project has arisen during the ESNT Workshop "*Light nuclei between single-particle and clustering features*"[1], in December 2024. The latter turned out to be an occasion for fruitful discussions between invited speakers and local experimentalists' groups, namely in the session "*Clustering as a phase transition and nuclear shape evolution from microscopic viewpoints*". In particular, an experiment targeting the search for Hoyle-state candidates in ^{24}Mg through the ^{26}Mg(p,t)^{24}Mg* reaction in the facilities of GANIL (Caen) has been proposed.

Unlike the aforementioned workshop, *Emergent geometry and duality in ^{24}Mg and other self-conjugate nuclei* possesses a markedly theoretical character. However, discussions on experimental aspects of the nuclear observables covered by the presentations, such as direct reactions suitable for populating excited states or techniques for improving the precision in the measurements of magnetic dipole moments, are encouraged.

Moreover, in the talks, the key results of the collaboration on ^{24}Mg are expected to be outlined, in combination with a selection of the most recent advances in nuclear lattice effective field theory. Thematically, the topics of the oral contributions have ties with the ESNT workshop "*Many-body correlations, phase transitions and clustering: how to identify and quantify them?*" scheduled for September 2026.

Exchanges and technical discussions on the prediction of structure observables in ^{24}Mg and other mirror nuclei are expected to take place namely during the roundtables of the afternoon sessions. On the whole, the workshop aims at (i) taking stock of the current state of the project; (ii) bridging the spectroscopic results of phenomenological and macroscopic approaches with ones achievable by NLEFT; (iii) weighting the role of α-clustering and assessing advantages and limitations of macroscopic approaches; (iv) providing a comprehensive understanding of the topic for all participants, potentially including the experimentalists involved in the spectroscopy of $N = Z$ mirror nuclei and (v) initiating possible new collaborations.

IV. SHORT-TERM VISITORS

Due to its symposium-like character, the project does not envisage the prolonged stay of participants from external institutions. However, the latter could eventually decide to extend their stay for research purpose at their own expense.

V. LIST OF POTENTIAL SPEAKERS

The workshop aims at involving primarily nuclear theoreticians, whose recent or ongoing works are focused on structure properties of mirror nuclei, with a preference for α-conjugate nuclei. Although subject to changes, the list of speakers accompanied by the contributions' titles is presented below:

- Serdar Elhatisari, "*Wavefunction matching for solving quantum many-body problems*", *Physics Department, King-Fahd university of Petroleum and Minerals*, Dhahran (Saudi Arabia) and *Faculty of Natural Sciences and Engineering*, *Gaziantep Islam Science and Technology University*, Gaziantep (Turkey), serdar.elhatisari@kfupm.edu.sa;

- Timo A. Lähde, "*Exploring the cluster character of Light Nuclei*", *Institute for Advanced Simulation* (IAS-4) *Forschungszentrum Jülich*, (Jülich, Germany) t.laehde@fz-juelich.de;

- Dean Lee, "*Charged-particle bound states in periodic boxes*", *Facility for Rare Isotope Beams and Department of Physics and Astronomy*, *Michigan State University*, East Lansing (USA) leed@frib.msu.edu;

- Fabian Hildenbrand, "*Lattice calculations near the proton dripline: the $^{99-102}$Sn isotopes* ", *Institute for Advanced Simulation* (IAS-4) *Forschungszentrum Jülich*, (Jülich, Germany) f.hildenbrand@fz-juelich.de;

- Bing-Nan Lu, "*Exploiting ^{20}Ne Isotopes for precision characterizations of collectivity in small Systems*", *Graduate School of China Academy of Engineering Physics*, Beijing (China), bnlv@gscaep.ac.cn;

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[1] cf. https://esnt.cea.fr/Phocea/Page/index.php?id=123 (website)

- Zhengxue Ren, ”*Quantifying α-clustering in the ground states of* ^{16,17,18,20}*O and* ^{20}*Ne isotopes*”, *School of Physics*, *Nankai University*, Tianjin (China), zxren@nankai.edu.cn;
- Shihang Shen, ”*Tomography of light nuclei through NLEFT: an introduction*”, *Peng Huanwu Collaborative Center for Research and Education* and *International Institute for Interdisciplinary and Frontiers* and *School of Physics*, *Beihang University*, Beijing (China), sshen@buaa.edu.cn;
- Gianluca Stellin, ”*Emergent geometry and duality in the* ^{24}*Mg nucleus*”, *IJCLab*, *CNRS*, *Université Paris-Saclay*, Orsay (France), gianluca.stellin@ijclab.in2p3.fr;
- Teng Wang, ”*Multi-reference trial states for lattice quantum Monte-Carlo simulations*”, *School of Physics*, *Peking University*, Beijing (China) tenggeer@pku.edu.cn;

VI. PRELIMINARY PROGRAMME

The temporary schedule of the workshop is displayed in the table below, where standard contributions consist of a 35′ *talk* followed by a 10′ *discussion* each. Besides, the welcome and concluding speeches entail presentations of 15′ each. Roundtables consist in working sessions and discussions taking place in the conference room. These are expected to involve the speakers and, possibly, the interested audience from the DPhN or the nearby IJCLab.

	Tuesday 01/12		Wednesday 02/12		Thursday 03/12
10h15	Welcome	10h30	**T. Wang**	10h30	**B.N. Lu**
10h30	**S. Shen**	11h15	Break	11h15	Break
11h15	Break	11h30	**T.A. Lähde**	11h30	**G. Stellin**
11h30	**S. Elhatisari**	12h15	Lunch	12h15	Lunch
12h15	Lunch	14h00	**D. Lee**	14h00	**X. Ren**
14h00	**F. Hildenbrand**	14h45	Break	14h45	Break
14h45	Break	15h00	Roundtable	15h00	Roundtable
15h00	Roundtable			18h00	Conclusion