RPA as a Microscopic Theory of Nuclear Response: Foundations and Open Challenges

Nils Paar

Department of Physics, Faculty of Science, University of Zagreb, Croatia

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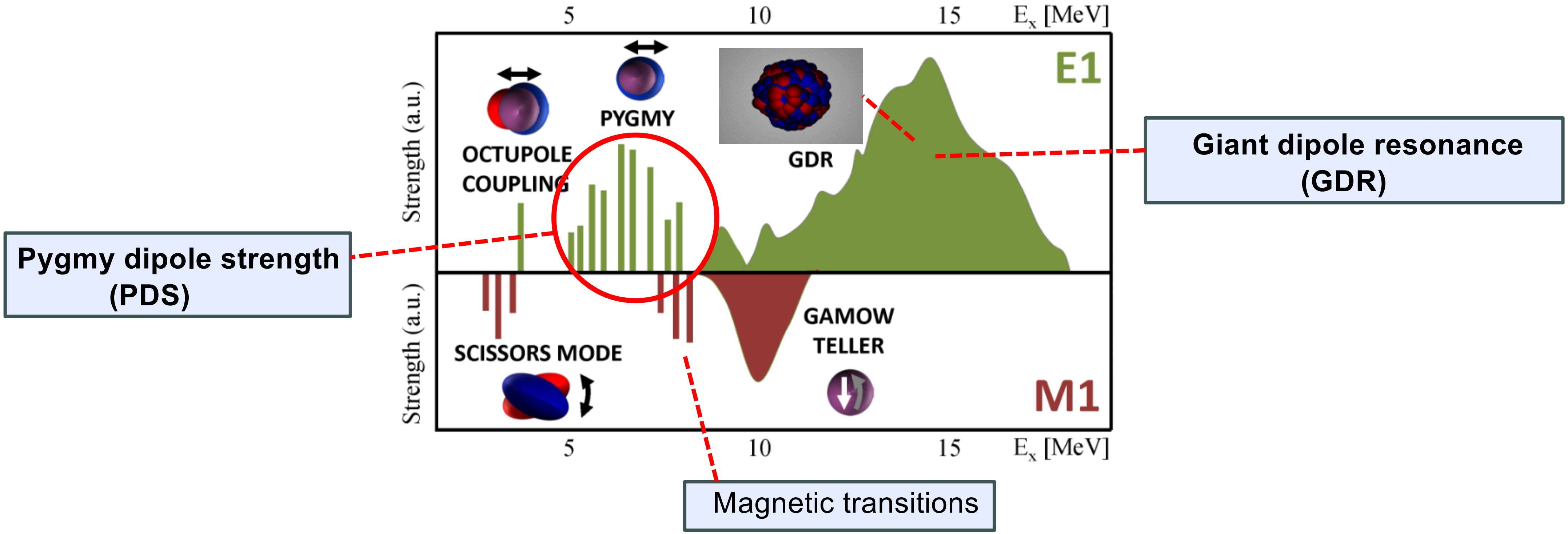
ELECTRIC AND MAGNETIC TRANSITIONS IN ATOMIC NUCLEI

Electric (E1) and magnetic (M1) transitions in nuclei Figure by A. Zilges.



- How various nuclear transition strengths evolve with isospin, deformation and temperature changes?
- Applications in nuclear processes and reactions of astrophysical relevance (beta decays, electron capture, neutron induced reactions, neutrino-nucleus interaction, etc.)

Random phase approximation (RPA) – baseline for more complex approaches

Mean field HF/HFB/RMF → Small amplitude TDHF/TDHFB → RPA / QRPA linear response → Observables Trensition strenghts → Beyond RPA complexity

- Microscopic theory – relies on particle-hole transitions in quantum mechanical description of single particle spectra and wave functions
- What QRPA adds: pairing, superfluid open-shell nuclei (2qp configuration space)
- What it misses: fragmentation, spreading widths, configuration mixing, and consistent uncertainty estimates.
  → Beyond RPA approaches are required.

Microscopic starting point: an energy density functional

- Hohenberg-Kohn theorem – the **exact energy of a quantum many body system** is a functional $E(\rho)$ of the local density $\rho(\vec{r})$
- Ground state density and other ground state observables are obtained by **minimizing a suitable energy functional** $E(\rho)$
- In nuclear physics, the total energy of a nucleus is expressed as a **functional of densities and pairing fields**

EDF variational input

$$E[\rho, \kappa, \kappa^*] \quad \Rightarrow \quad h = \frac{\delta E}{\delta \rho}, \qquad \Delta = \frac{\delta E}{\delta \kappa^*}$$

Hartree-Fock / Relativistic Hartree-Bogoliubov model

$$\begin{pmatrix} h - \lambda & \Delta \\ -\Delta^* & -h^* + \lambda \end{pmatrix} \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix} = E_\mu \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix}$$

RPA from small-amplitude time-dependent mean-field theory

Response of density matrix $\rho(t)$ to external field

$$\hat{F}(t) = \hat{F} e^{-i\omega t} + \mathrm{H.c.}$$

Equation of motion for density operator

$$i\partial_t \hat{\rho} = [\hat{h}(\hat{\rho}) + \hat{f}(t), \hat{\rho}]$$

linear expansion around static ground state

$$\hat{\rho}(t) = \hat{\rho}^{(0)} + \delta\hat{\rho}(t)$$

small-amplitude equation

$$i\partial_t \delta\hat{\rho} = [\hat{h}^{(0)}, \delta\hat{\rho}] + \left[\frac{\partial \hat{h}}{\partial \rho}\delta\rho, \hat{\rho}^{(0)}\right] + [\hat{f}, \hat{\rho}^{(0)}]$$

$$\frac{\partial \hat{h}}{\partial \rho}\delta\rho = \sum_{ph} \frac{\partial \hat{h}}{\partial \rho_{ph}}\delta\rho_{ph} + \frac{\partial \hat{h}}{\partial \rho_{hp}}\delta\rho_{hp} + \sum_{\alpha h} \frac{\partial \hat{h}}{\partial \rho_{\alpha h}}\delta\rho_{\alpha h} + \frac{\partial \hat{h}}{\partial \rho_{h\alpha}}\delta\rho_{h\alpha}$$

- Keep superpositions of 1p–1h configurations (plus 1α-1h in the relativistic case)
- Let the residual interaction mix them to produce collective eigenmodes.

⇒RPA eigenvalue problem

RPA matrix equation

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \Omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + \langle ph'|V_{\text{res}}|hp'\rangle$$

$$B_{ph,p'h'} = \langle pp'|V_{\text{res}}|hh'\rangle$$

strength distribution

$$S_F(E) = \sum_\nu B_\nu(F)\,\delta(E - E_\nu), \qquad B_\nu(F) = |\langle\nu|\hat{F}|0\rangle|^2$$

Self-consistency – the same EDF in the ground state and V_{res}

residual interaction

$$V_{res} = \frac{\delta\hat{h}}{\delta\rho} = \frac{\delta^2 E[\rho]}{\delta\rho\delta\rho}$$

Self-consistency matters::

- spurious center-of-mass state near zero
- energy-weighted sum rules

VARIOUS IMPLEMENTATIONS OF (Q)RPA

Nonrelativistic

- Hartree-Fock + Random Phase Approximation (RPA)
  - *Skyrme/Gogny functionals, etc.*
- Hartree-Fock-Bogoliubov model + quasiparticle RPA
  - *Skyrme/Gogny functionals (QRPA)*
- Ab initio – based on realistic nucleon-nuceon interactions
- …

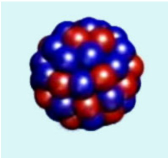
Relativistic

- Dirac-Hartree + Relativistic RPA (RRPA)
- Relativistic Hartree-Bogoliubov (RHB) model + relativistic QRPA (RQRPA)

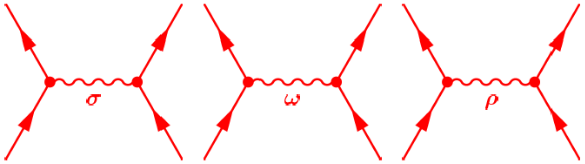
- Further extensions: finite temperature, spherical → deformed, …

  - (Q)RPA serves as basis and reference for beyond (Q)RPA approaches

RELATIVISTIC NUCLEAR ENERGY DENSITY FUNCTIONAL



i) Nucleons are Dirac particles coupled by the exchange mesons and the photon field



ii) Four-fermion contact interaction (Point-coupling model)

- The basis is effective Lagrangian density with relativistic symmetries:

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$

$$\mathcal{L}_N = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi$$

$$\begin{aligned}\mathcal{L}_m \;&=\; \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &\quad - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

$$\mathcal{L}_{int} = -\bar{\psi}\Gamma_\sigma\sigma\psi - \bar{\psi}\Gamma_\omega^\mu\omega_\mu\psi - \bar{\psi}\vec{\Gamma}_\rho^\mu\vec{\rho}_\mu\psi - \bar{\psi}\Gamma_e^\mu A_\mu\psi.$$

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma\cdot\partial - m)\psi \\ &- \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ &- \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ &- \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma\cdot A\frac{(1-\tau_3)}{2}\psi\end{aligned}$$

- Many-body correlations encoded in density-dependent coupling functions **(problem for beyond mean field approaches!)**

In the case of density-dependent interactions, for example in the case of meson-exchange,

$$g_i(\rho) = g_i(\rho_{\text{sat}}) f_i(x) \quad \text{for} \quad i = \sigma, \omega$$

$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$$

$$g_\rho(\rho) = g_\rho(\rho_{\text{sat}}) \exp[-a_\rho(x - 1)]$$

⇒ rearrangement terms appear at the (Q)RPA level, i.e. terms including derivatives of the meson-nucleon couplings

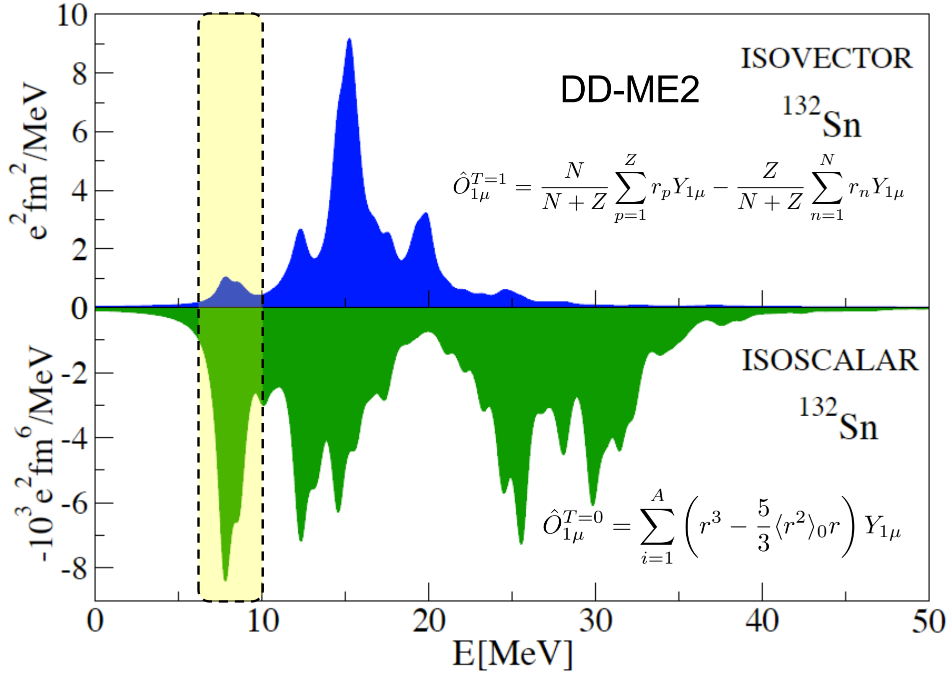
Expansion of couplings and their derivatives around ground state density:

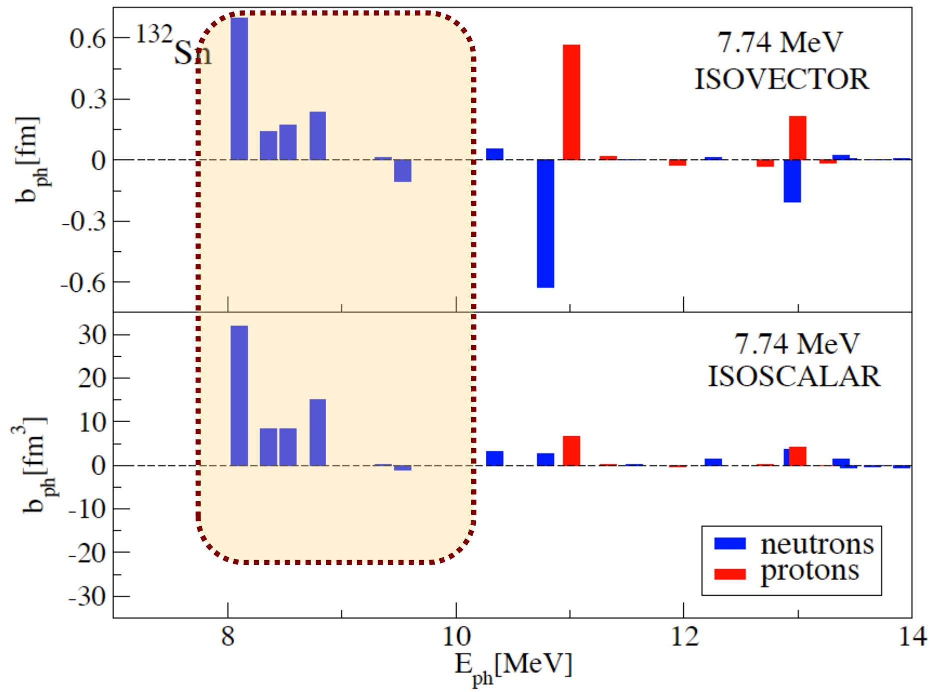
$$g_i(\rho_v) = g_i(\rho_v^0) + \left.\frac{\partial g_i}{\partial \rho_v}\right|_0 \delta\rho_v$$

$$\frac{\partial g_i}{\partial \rho_v} = \left.\frac{\partial g_i}{\partial \rho_v}\right|_0 + \left.\frac{\partial^2 g_i}{\partial \rho_v^2}\right|_0 \delta\rho_v$$

ISOVECTOR AND ISOSCALAR DIPOLE EXCITATIONS

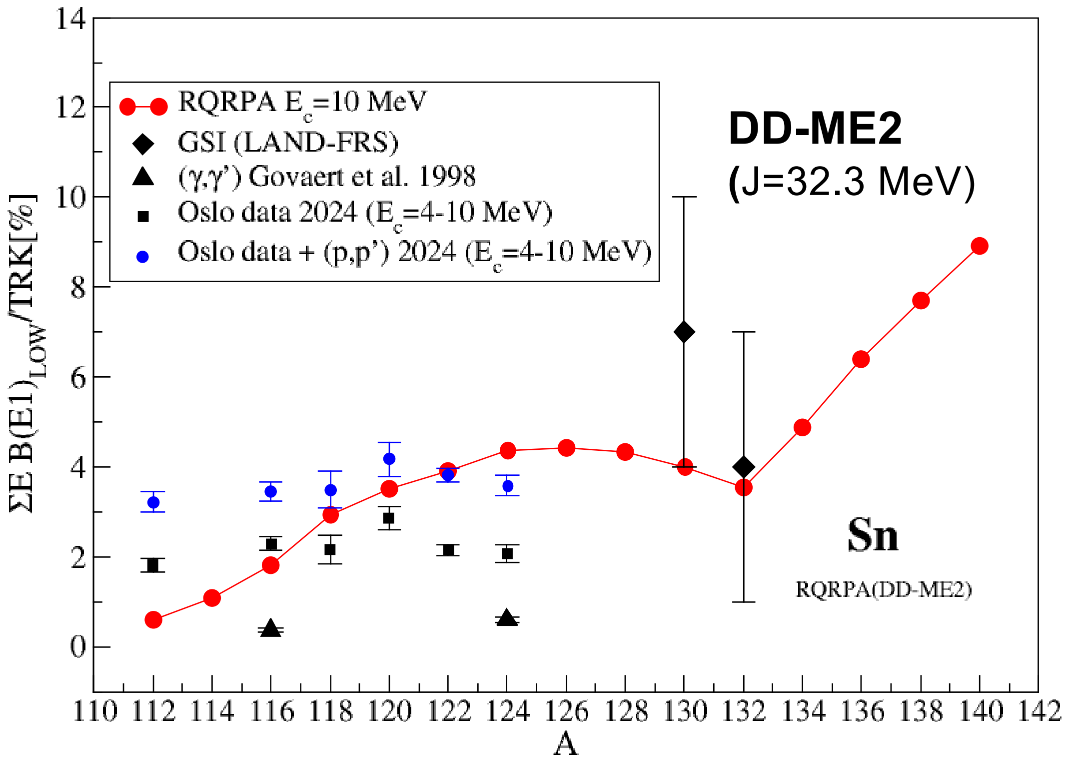
(Q)RPA allows in-depth analysis of the underlying 2qp structure of excited states





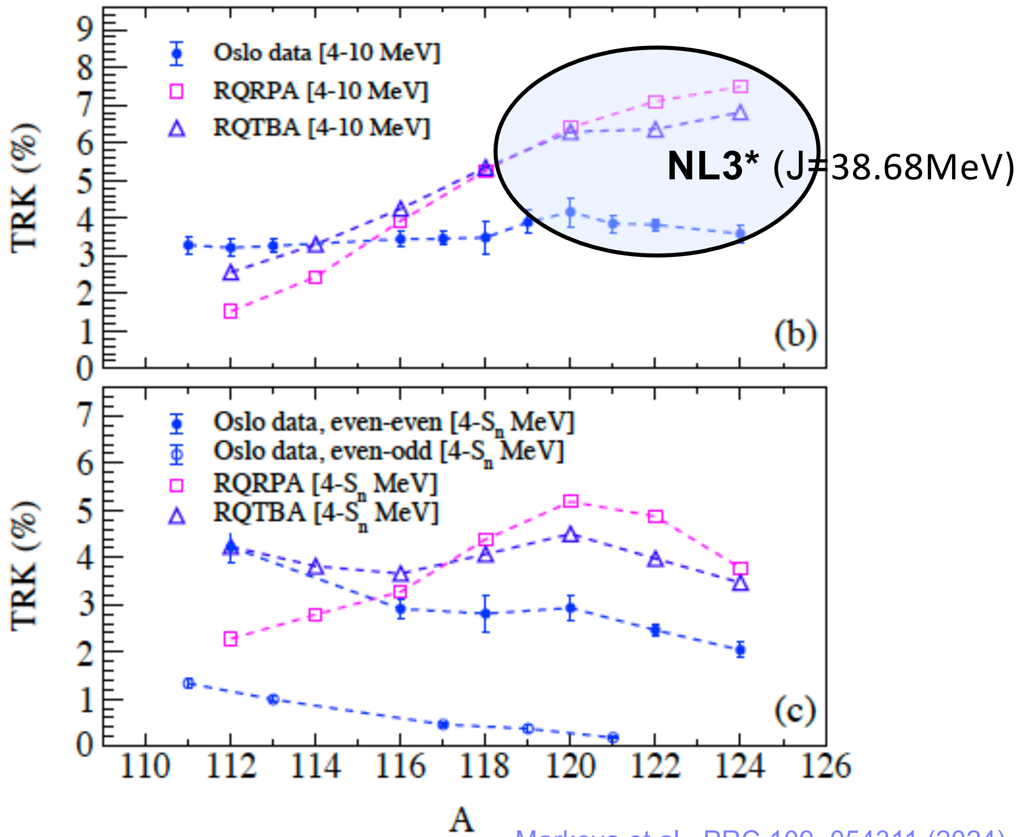
- ➢ Isoscalar and isovector strengths at 7.74 MeV include identical neutron transitions with similar relative contributions (apart from the decoherence in the isovector channel).
- ➢ Decoherence in the isovector response is mechanism that makes the strength small
- ➢ Has the underlying structure been investigated in beyond QRPA approaches?

Probing (Q)RPA and beyond QRPA approaches on pygmy dipole strength: Sn isotope chain



RQRPA calculation + new Oslo+(p,p') data

N.P., A.Kaur, APP 18, 2-A31 (2025)

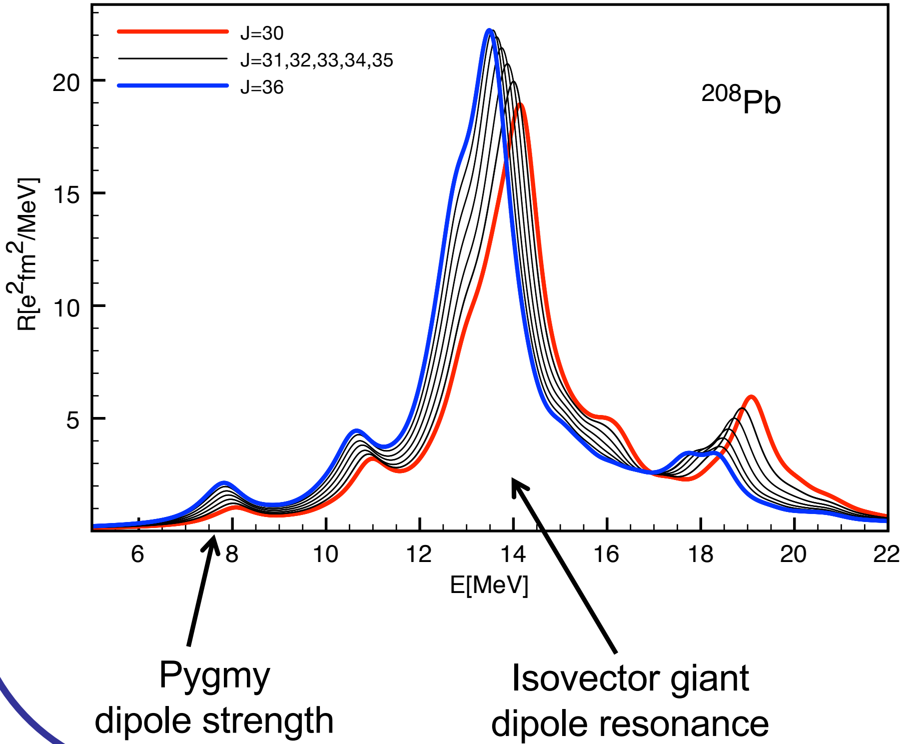


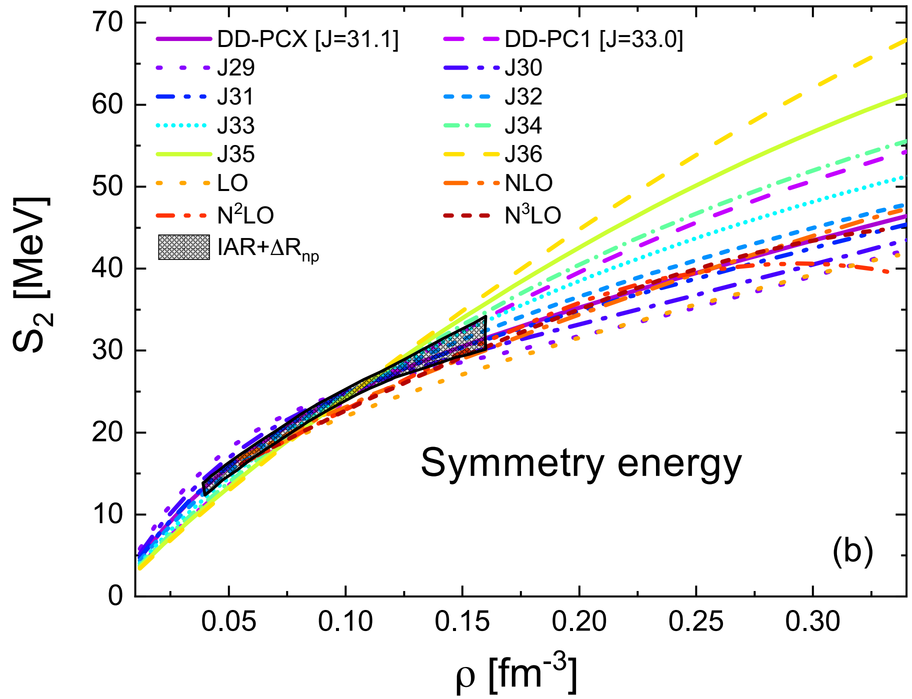
Markova et al., PRC 109, 054311 (2024)

→ effective interaction with realistic symmetry energy (e.g. DD-ME2) fits data better

→ in the integrated quantity (EWSR) **selection of EDF can be more important than beyond QRPA effects**

Constraining the symmetry energy with (Q)RPA calculations of dipole transitions in nuclei

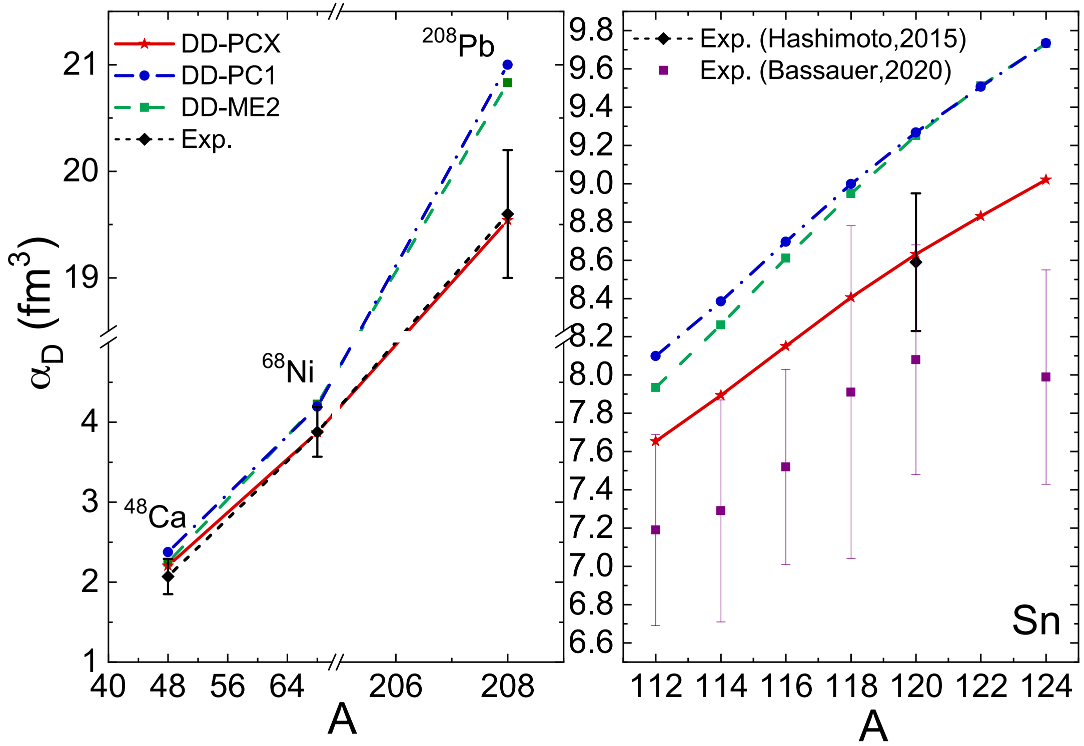




- **Strategy to constrain symmetry energy parameters (J,L):**
  - build the EDF which accurately reproduces experimental data on dipole transitions (e.g. DD-PCX)

DIPOLE POLARIZABILITY

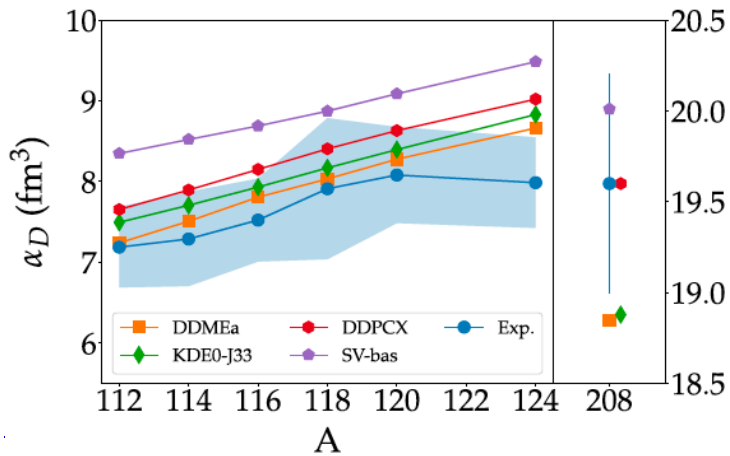
- Overview of dipole polarizability α_D for several nuclei: theory vs. exp. data



DD-PCX interaction improves description of overall exp. data for α_D

...but there are discrepancies with measurement for Sn isotope chain (low-energy E1 strength ?)

Could beyond QRPA approaches resolve this?



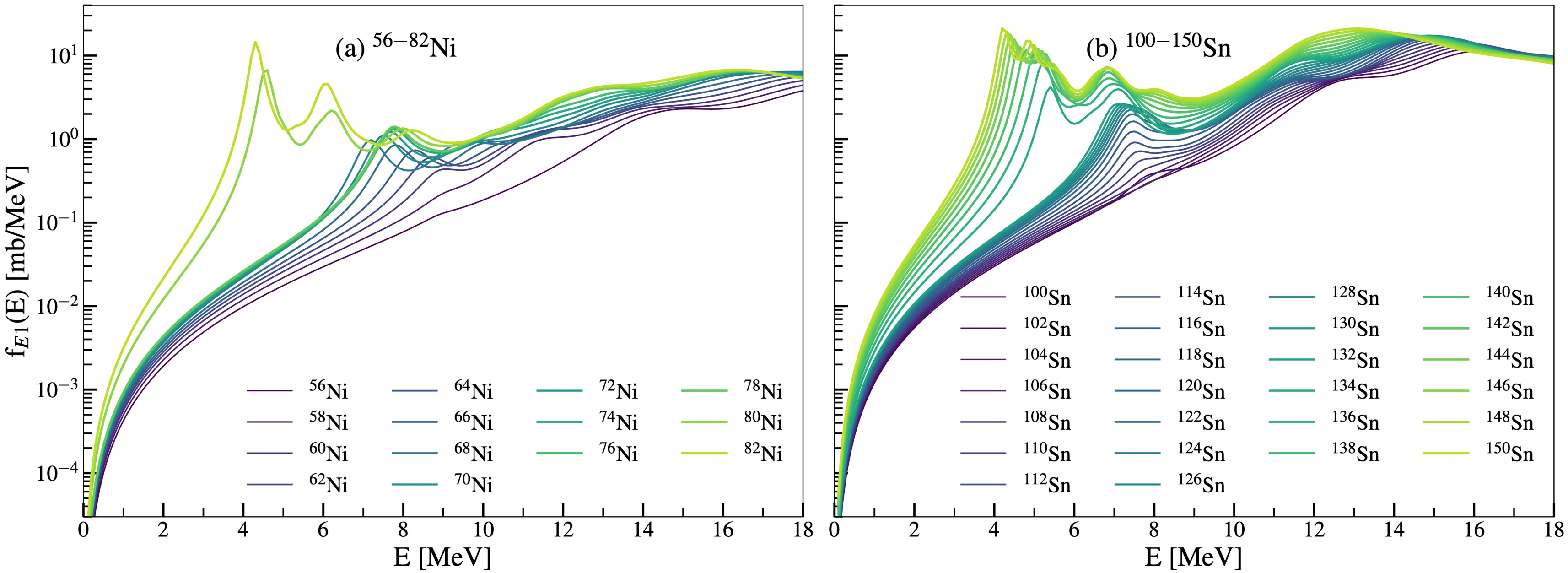
S. Bassauer et al., PLB 810, 135804 (2020).

Exp. data on α_D:

^{48}Ca - J. Birkhan et al., PRL 118, 252501 (2017).
^{68}Ni - D. M. Rossi et al., PRL 111, 242503 (2013).
^{120}Sn – T. Hashimoto et al., PRC 92, 031305(R) (20
$^{1112-124}$Sn – S. Bassauer et al., PLB 810, 135804 (2020).
^{208}Pb – A. Tamii et al., PRL 107, 062502 (2011)
X. Roca-Maza et al., PRC 92, 064304 (2015)

(n,γ) and (γ,n) Reaction Rates: QRPA calculated γ-Strength Function (γSF)

- In the r-process, **waiting point nuclei** are determined by equilibrium $(\gamma, n) \rightleftarrows (n, \gamma)$ reactions
- γSF controls **γ emission/absorption** probabilities
- → **key input to reaction rate** calculations in Hauser–Feshbach statistical model



→Pygmy Dipole Strength (PDS) enhances low-energy γSF and (n, γ) **reaction rates**

T. Ghosh, A. Kaur, N.P., arXiv:2603.12928 (2026)

E1 and M1 gamma strength functions

E1 and M1 gamma strength functions at T=0 MeV

$$f_{E1}^{RQRPA}(E_\gamma)(MeV^{-3}) = \frac{16\pi e^2}{27\hbar^3 c^3} \times S_{E1}(E_\gamma)$$

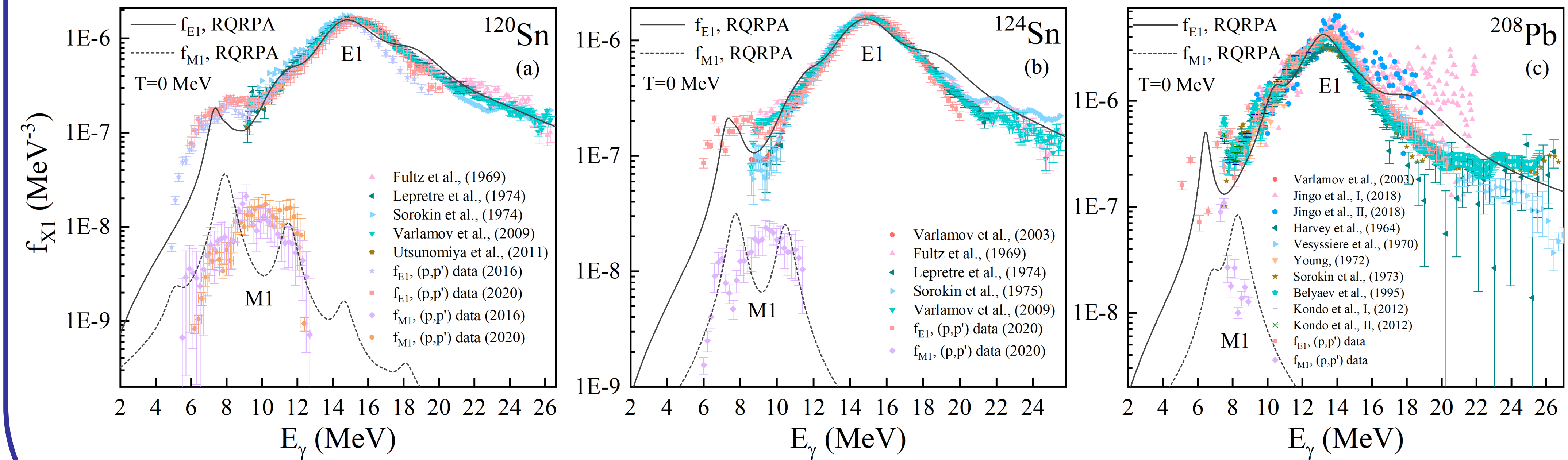
$$f_{M1}^{RQRPA}(E_\gamma)(MeV^{-3}) = \frac{16\pi}{27\hbar^3 c^3} \times S_{M1}(E_\gamma)$$

The RQRPA discrete spectrum is averaged with a Lorentzian function

$S_{E1}(E) = \sum B(E1, 0 \to \nu) L(E, E_\nu, \Gamma)$

$$L(E, E_\nu, \Gamma) = \frac{1}{2\pi} \frac{\Gamma(E, C_G)}{\left(E - E_\nu - \Delta(E, C_E)\right)^2 + \Gamma(E, C_G)^2/4}$$

S. Drozdz et al.; Phys. Rep.197, 1 (1990).



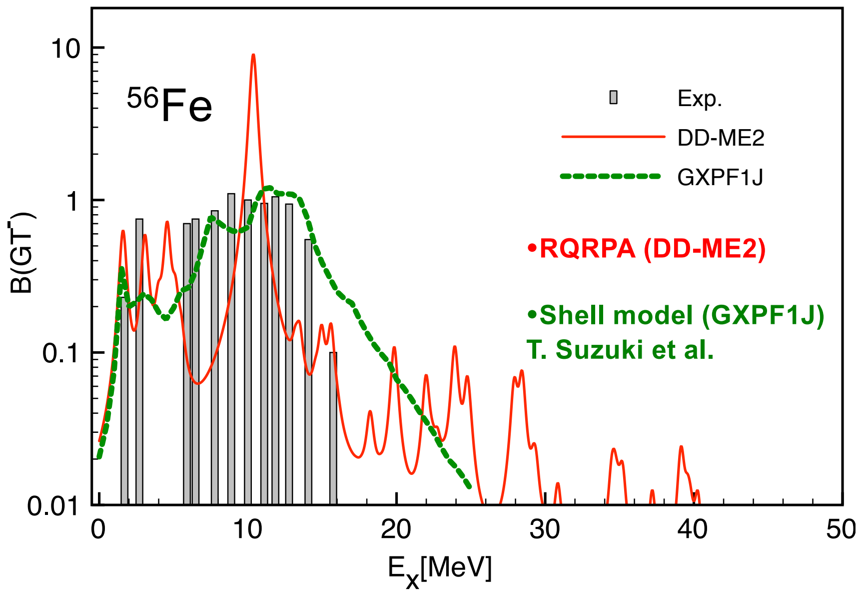
- The calculated E1 and M1 gamma strength functions are compared with the available experimental photoabsorption/photoneutron strength functions and (p,p') data.

A. Kaur, E. Yuksel, N.P., PRC 112, 014307 (2025)

Open limitations of the RPA/QRPA approach

- Restricted configuration space and harmonic approximation
- Strength is often too concentrated: missing fragmentation and spreading width.
- RPA cannot capture multi-phonon states or 2p–2h states.

Gamow-Teller (GT) transitions



- RQRPA reproduces total GT strength and global properties of transition strength. It allows systematic calculations of high multipole excitations (forbidden transitions), possible extrapolations toward nuclei away from the valley of stability.

- Shell model includes additional correlations among nuclei, accurately reproduces the experimental GT strength. However, already in medium mass nuclei the model spaces become large, many nuclei and forbidden transitions remain beyond reach.

Going beyond RPA

1p–1h → 2p–2h → phonons

- including more complex configurations than 1p-1h, and their couplings
- **Second RPA (1p1h⊕2p2h)** → Relativistic variant of Second RPA: D. Vale, G. Saxena, N.P, PRC 113, 054313 (2026)

SRPA is especially useful:

- when the observable depends strongly on fragmentation
- spreading widths
- states dominated by multiparticle–multihole structure

Excited state:

$$
\begin{aligned}
|\nu\rangle = & \sum_{\mathrm{ph}} \left(X^{(1)\nu}_{\mathrm{ph}} a^\dagger_{\mathrm{p}} a_{\mathrm{h}} - Y^{(1)\nu}_{\mathrm{ph}} a^\dagger_{\mathrm{h}} a_{\mathrm{p}}\right)|\Phi_0\rangle \\
& + \sum_{\alpha\mathrm{h}} \left(X^{(1)\nu}_{\alpha\mathrm{h}} a^\dagger_{\alpha} a_{\mathrm{h}} - Y^{(1)\nu}_{\alpha\mathrm{h}} a^\dagger_{\mathrm{h}} a_{\alpha}\right)|\Phi_0\rangle \\
& + \frac{1}{4} \sum_{\mathrm{pp'hh'}} \left(X^{(2)\nu}_{\mathrm{pp'hh'}} a^\dagger_{\mathrm{p}} a^\dagger_{\mathrm{p'}} a_{\mathrm{h'}} a_{\mathrm{h}} - Y^{(2)\nu}_{\mathrm{pp'hh'}} a^\dagger_{\mathrm{h}} a^\dagger_{\mathrm{h'}} a_{\mathrm{p'}} a_{\mathrm{p}}\right)|\Phi_0\rangle
\end{aligned}
$$

SRPA Matrix Equations (couplings 1p1h ⊗2p2h, 2p2h⊗2p2h) :

$$
\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} X \\ Y \end{pmatrix} \qquad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \qquad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}
$$

Challenges for beyond RPA theory

- **SRPA based on EDF does not work well.**

  - Convergence issues, large energy shifts of excitation spectra
  - Double counting of static correlations

  SRPA response function does not reduce to RPA result in the zero frequency limit

- **Subtraction method** - removal of the static part in the energy dependent effective interaction

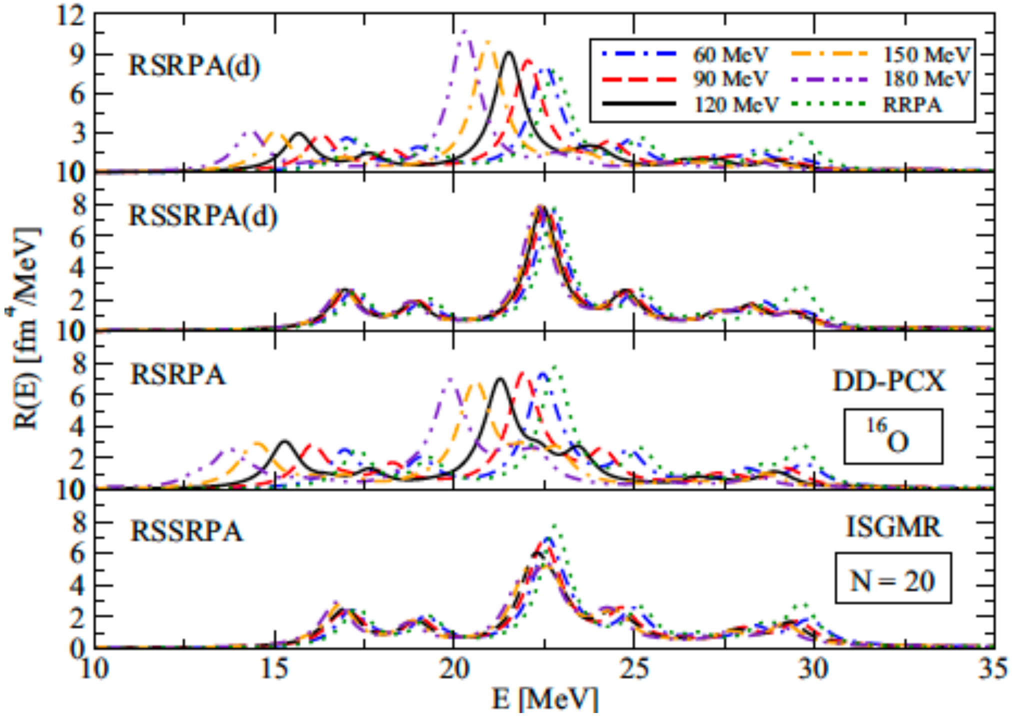
  V. I. Tselyaev, Phys. Rev. C 75, 024306 (2007).

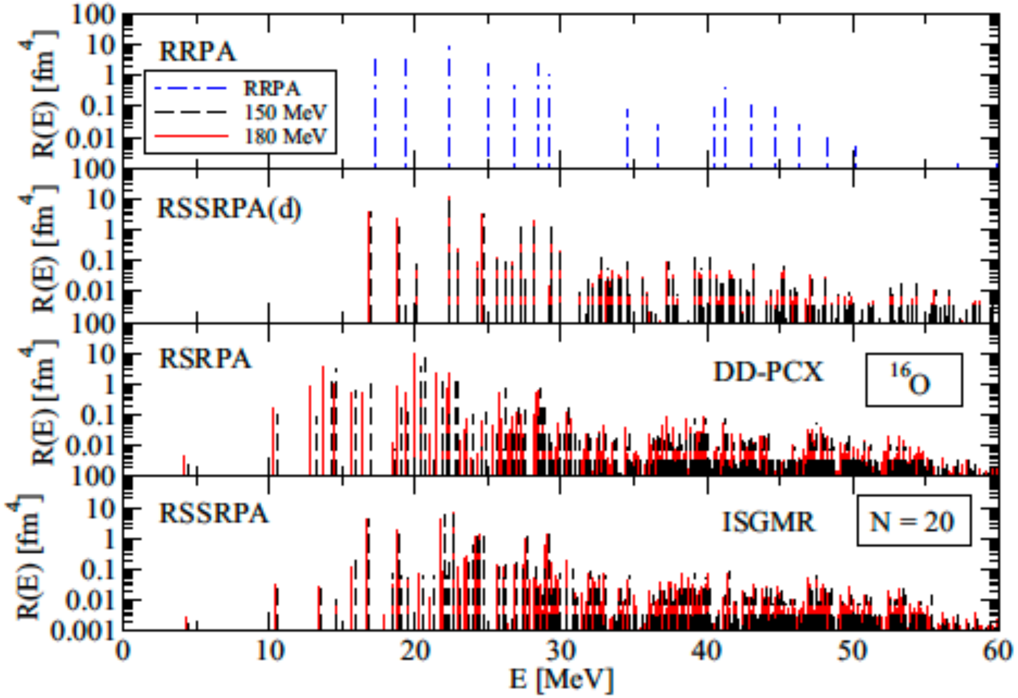
$$V^{\mathrm{SRPA}}(\omega) \rightarrow V^{\mathrm{SRPA}}(\omega) - [V^{\mathrm{SRPA}}(0) - V^{\mathrm{RPA}}]$$

Implementation in SRPA based on Skyrme EDF D. Gambacurta et al., Phys. Rev. C 92, 034303 (2015)

- preserves the static response already encoded in the fitted EDF
- reduces double counting
- weakens cutoff dependence
- removes much of the anomalously large downward energy shift found in ordinary SRPA calculations.

- **Isoscalar monopole transitions** in ^{16}O calculated using RPA, RSRPA (full), RSRPA (diagonal approximation in A22), and their subtracted variants





E_{2p-2h} [MeV]	30	60	90	120	150	180	210	240	270
N_{2p-2h}	9	408	1890	4326	7390	10797	14691	18611	22700
N_{tot}	117	516	1998	4434	7498	10905	14799	18719	22808
N_{dim}	234	1032	3996	8868	14996	21810	29598	37438	45616

Large number of 2p-2h configurations required

➢ D. Vale, G. Saxena, N.P, PRC 113, 054313 (2026)

- Energy-weighted moments, excitation energies, widths, and energy-weighted sum rule (EWSR)
  (Numbers in brackets correspond to calculation in the experimental energy range)

	RRPA	RSSRPA(d)	RSSRPA	Expt. [24]
m_0 [fm^4]	29.794 (28.780)	29.789 (27.736)	29.838 (27.235)	
m_1 [MeVfm^4]	726.357 (680.288)	790.074 (638.375)	801.735 (624.961)	
m_2 [MeV^{2}fm^4]	18 725.64 (16 545.340)	28 768.79 (15 175.80)	29 548.40 (14897.78)	
m_{-1} [MeV^{-1}fm^4]	1.276 (1.254)	1.276 (1.243)	1.277 (1.230)	
m_1/m_0 [MeV]	24.380 (23.638)	26.522 (23.016)	26.870 (22.947)	21.13 ± 0.49
$\sqrt{m_1/m_{-1}}$ [MeV]	23.856 (23.296)	24.879 (22.661)	25.058 (22.539)	19.63 ± 0.38
$\sqrt{m_3/m_1}$ [MeV]	26.873 (24.659)	47.860 (24.178)	48.011 (24.329)	24.89 ± 0.59
Δ [MeV]	5.843 (4.020)	16.196 (4.172)	16.380 (4.522)	8.76 ± 1.82
EWSR[%]	99.67 (93.35)	108.41 (87.60)	110.01 (85.76)	48 ± 10

At RPA level sum rule is accurately reproduced

SRPA level:
Is there overcounting of correlations?

CONCLUDING REMARKS

RPA/QRPA is the linearized microscopic approach to collective excitations with many applications

- ➢ It serves as basis for extensions including couplings to complex configurations

APPROACHES BEYOND (Q)RPA OPEN MANY QUESTIONS

- Is double counting properly handled for EDFs constrained to experimental data?
- Which observables best discriminate beyond-(Q)RPA mechanisms against (Q)RPA?
- Is there systematics in beyond-(Q)RPA approaches, how different methods compare?
- In which way can theoretical uncertainties be estimated in beyond RPA calculations?
- How can one overcome computational difficulties of beyond RPA approaches?
- Can one account effectively beyond-(Q)RPA effects for implementation in large-scale calculations of astrophysically relevant processes (because these calculations of beyond (Q)RPA models are not feasible)?

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EMPIRICAL APPROACH TO SPREADING WIDTHS

- The RQRPA discrete spectrum is averaged with a Lorentzian function → $S_{E1}(E) = \sum B(E1,0 \rightarrow \nu) L(E, E_\nu, \Gamma)$

$$L(E, E_\nu, \Gamma) = \frac{1}{2\pi} \frac{\Gamma(E, C_G)}{\left(E - E_\nu - \Delta(E, C_E)\right)^2 + \Gamma(E, C_G)^2/4}$$

S. Drozdz et al.; Phys. Rep.197, 1 (1990).

Where, $\Gamma(E,C_G)$ is width at half maximum and $\Delta(E,C_E)$ is energy shift.

- The energy-dependent width $\Gamma(E,C_G)$ can be calculated from the measured decay width of particle and hole states:

$$\Gamma(E, C_G) = \frac{1}{E} \int_0^E (1 + C_G)\left[\gamma_p(\epsilon) + \gamma_h(\epsilon - E)\right] d\epsilon$$

- This empirical way of determining $\Gamma(E,C_G)$ has the advantage of including, contributions from the excitation beyond 2p-2h.

- $\Delta(E,C_E)$ is energy shift can be obtained from $\Gamma(E,C_G)$ by a dispersion relation.

- C_E and C_G are used to fit the experimental data of E1 photoabsorption and photoneutron strength functions.

Second RPA equations (relativistic)

- Include additional couplings 1p1h ⊗2p2h, 2p2h⊗2p2h

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega\begin{pmatrix} X \\ Y \end{pmatrix} \qquad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \qquad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$A_{11} = \begin{pmatrix} A_{\mathrm{ph;p'h'}} & A_{\mathrm{ph};\alpha'\mathrm{h'}} \\ A_{\alpha h;\alpha' h'} & A_{\alpha h;\alpha' h'} \end{pmatrix} \qquad A_{12} = \begin{pmatrix} A_{\mathrm{ph;p'_1p'_2h'_1h'_2}} \\ A_{\alpha h;\mathrm{p'_1p'_2h'_1h'_2}} \end{pmatrix}$$

$$A_{22} = A_{\mathrm{p_1p_2h_1h_2;p'_1p'_2h'_1h'_2}} \qquad B_{11} = \begin{pmatrix} B_{\mathrm{ph}\overline{\mathrm{p'h'}};0} & B_{\mathrm{ph}\overline{\alpha'\mathrm{h'}};0} \\ B_{\alpha h\overline{\alpha' h'};0} & B_{\alpha h\overline{\alpha' h'};0} \end{pmatrix}$$

$$B_{12} = \begin{pmatrix} B_{\mathrm{ph}\overline{\mathrm{p'_1p'_2h'_1h'_2}};0} \\ B_{\alpha h\overline{\mathrm{p'_1p'_2h'_1h'_2}};0} \end{pmatrix} \qquad B_{22} = 0$$