Particle–Vibration Coupling with Non-Relativistic EDFs

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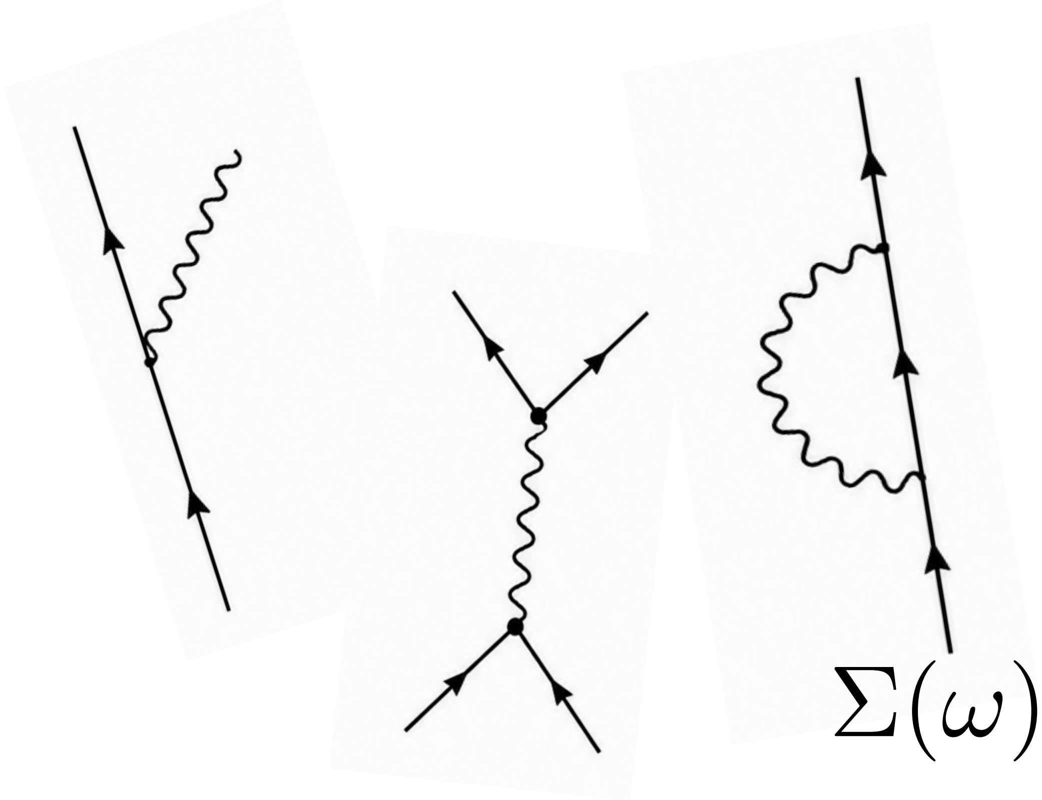


General idea / outline of the talk

There have been in the past models based on the idea that the low-energy nuclear spectra arise from an **interweaving of single-particle and collective degrees of freedom**. However, many are based on purely empirical input and *ad hoc* perturbative parameters.

- QPPM (quasiparticle-phonon model): Woods-Saxon and separable QRPA with fitted parameters.
- NFT (nuclear field theory): similar, with perturbative parameter Ω.

In the QPVC the input are consistently taken from e.g. Skyrme EDFs. The equations are derived via the Equation Of Motion (EOM) method.

We sketch this derivation and compare with other approaches. Applications will be also shown.

- The QRPA+QPVC framework

We start from a Hartree-Fock-Bogoliubov (HFB) set of states obtained with a Skyrme EDF.

First step: **diagonalize *H* on a two quasi-particle basis (QRPA).**

This produces a set of “vibrational quanta” or “phonons” associated with the operators Γ_n and Γ_n^+.

$$\Gamma_n^\dagger = \sum_{\alpha\beta} X_{\alpha\beta}^{(n)} \alpha_\alpha^\dagger \alpha_\beta^\dagger - Y_{\alpha\beta}^{(n)} \alpha_\beta \alpha_\alpha$$

$$|n\rangle = \Gamma_n^\dagger |\tilde{0}\rangle = \left(\sum_{\alpha\beta} X_{\alpha\beta}^{(n)} \alpha_\alpha^\dagger \alpha_\beta^\dagger - Y_{\alpha\beta}^{(n)} \alpha_\beta \alpha_\alpha \right) |\tilde{0}\rangle$$

Connecting with CC

$$|\Phi\rangle = \exp[\chi^\dagger] |\Phi_0\rangle$$

$$\chi^\dagger = \frac{1}{2} \sum_{\alpha\beta} c_{\alpha\beta} \alpha_\alpha^\dagger \alpha_\beta^\dagger$$

QRPA can be obtained by looking at variations of the total energy at second order in the coefficients *c* [M. Waroquier *et al.*, Phys. Rep. 148, 249 (1987)]

We wish to improve the structure of the excited states. We can go to the next order in the quasiparticle number.

$$|N\rangle = \left(\sum_{\alpha\beta} X^{(1)}_{\alpha\beta}\alpha^\dagger_\alpha\alpha^\dagger_\beta - Y^{(1)}_{\alpha\beta}\alpha_\beta\alpha_\alpha + \sum_{\alpha\beta\gamma\delta} X^{(2)}_{\alpha\beta\gamma\delta}\alpha^\dagger_\alpha\alpha^\dagger_\beta\alpha^\dagger_\gamma\alpha^\dagger_\delta - Y^{(2)}_{\alpha\beta\gamma\delta}\alpha_\delta\alpha_\gamma\alpha_\beta\alpha_\alpha \right) |\tilde{0}\rangle$$

This is **second QRPA**. Cf. the talk by Danilo Gambacurta for the case without pairing (second RPA).

Although the second QRPA or second RPA give already a rich structure in the wave function and often quite satisfactory results, not many evidences that one converges at 2p-2h, or 3p-3h... exist.

$$|\Phi\rangle = \exp[\chi^\dagger]|\Phi_0\rangle \qquad \chi^\dagger = \frac{1}{2}\sum_{\alpha\beta} c_{\alpha\beta}\alpha^\dagger_\alpha\alpha^\dagger_\beta + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} c_{\alpha\beta\gamma\delta}\alpha^\dagger_\alpha\alpha^\dagger_\beta\alpha_\delta\alpha_\gamma$$

J. Da Providência, Nucl. Phys. 61, 87 (1965)
D. Gambacurta *et al*., J. Phys. G 38, 035103 (2011)

We can go to the next order in the quasiparticle number in different ways, e.g. we can group two quasiparticle in in a QRPA phonon.

$$|N\rangle = \left(\sum_{\alpha\beta} X^{(1)}_{\alpha\beta} \alpha^\dagger_\alpha \alpha^\dagger_\beta - Y^{(1)}_{\alpha\beta} \alpha_\beta \alpha_\alpha + \sum_{\alpha\beta\gamma\delta} X^{(2)}_{\alpha\beta\gamma\delta} \alpha^\dagger_\alpha \alpha^\dagger_\beta \alpha^\dagger_\gamma \alpha^\dagger_\delta - Y^{(2)}_{\alpha\beta\gamma\delta} \alpha_\delta \alpha_\gamma \alpha_\beta \alpha_\alpha \right) |\tilde{0}\rangle$$

Second QRPA

QPVC

$$|N\rangle = \left(\sum_{\alpha\beta} X^{(1)}_{\alpha\beta} \alpha^\dagger_\alpha \alpha^\dagger_\beta - Y^{(1)}_{\alpha\beta} \alpha_\beta \alpha_\alpha + \sum_{\alpha\beta n} X^{(2)}_{\alpha\beta n} \alpha^\dagger_\alpha \alpha^\dagger_\beta \Gamma^\dagger_n - Y^{(2)}_{\alpha\beta n} \alpha_\beta \alpha_\alpha \Gamma_n \right) |\tilde{0}\rangle$$

QPVC means **quasiparticle-vibration coupling** and we can derive the equations from EoM method, given this ansatz for the wave function.

Derivation of the QPVC equations

$$\mathcal{O}_N^\dagger = \sum_{\alpha\beta} X^{(1)}_{\alpha\beta}\alpha^\dagger_\alpha\alpha^\dagger_\beta - Y^{(1)}_{\alpha\beta}\alpha_\beta\alpha_\alpha + \sum_{\alpha\beta n} X^{(2)}_{\alpha\beta n}\alpha^\dagger_\alpha\alpha^\dagger_\beta\Gamma^\dagger_n - Y^{(2)}_{\alpha\beta n}\alpha_\beta\alpha_\alpha\Gamma_n$$

We can use the Equation of Motion (EoM) in the form:

$$\langle\tilde{0}|\left[\delta\mathcal{O}_N,\left[H,\mathcal{O}_N^\dagger\right]\right]|\tilde{0}\rangle = E_\nu\langle\tilde{0}|\left[\delta\mathcal{O}_N,\mathcal{O}_N^\dagger\right]|\tilde{0}\rangle$$

green: usual QRPA matrix

yellow: interaction between 2qp and 2qp+phonon

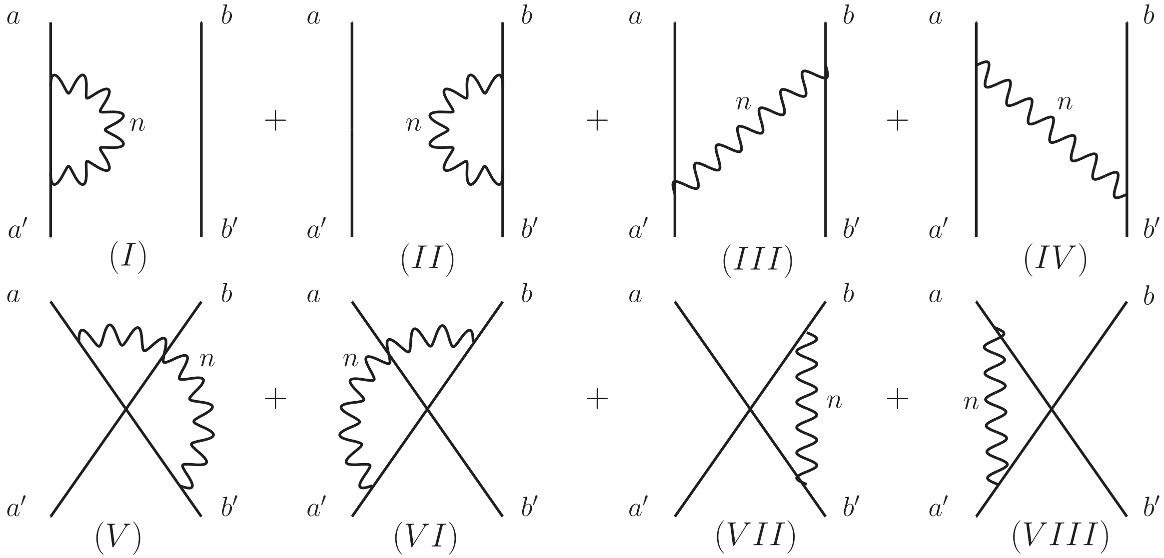
This leads to

$$\begin{pmatrix} A_{\alpha\beta,\alpha'\beta'} & B_{\alpha\beta,\alpha'\beta'} & A_{\alpha\beta,\alpha'\beta'n'} & 0 \\ -B^*_{\alpha\beta,\alpha'\beta'} & -A^*_{\alpha\beta,\alpha'\beta'} & 0 & -A^*_{\alpha\beta,\alpha'\beta'n'} \\ A_{\alpha\beta,\alpha'\beta'n'} & 0 & A_{\alpha\beta n,\alpha'\beta'n'} & 0 \\ 0 & -A^*_{\alpha\beta n,\alpha'\beta'} & 0 & -A^*_{\alpha\beta n,\alpha'\beta'n'} \end{pmatrix} \begin{pmatrix} X^{(1)}_{\alpha'\beta'} \\ Y^{(1)}_{\alpha'\beta'} \\ X^{(2)}_{\alpha'\beta'n'} \\ Y^{(2)}_{\alpha'\beta'n'} \end{pmatrix} = E \begin{pmatrix} X^{(1)}_{\alpha\beta} \\ Y^{(1)}_{\alpha\beta} \\ X^{(2)}_{\alpha\beta n} \\ Y^{(2)}_{\alpha\beta n} \end{pmatrix}$$

(Q)RPA + (Q)PVC

The scheme is PROJECTED on the space of two quasiparticle states.

$$\begin{pmatrix} A + \Sigma(E) & B \\ -B & -A - \Sigma^*(-E) \end{pmatrix} \qquad \Sigma_{ab,a'b'n} = \sum_\alpha \frac{\langle ab|V|\alpha\rangle\langle\alpha|V|a'b'\rangle}{E - E_\alpha + i\eta}$$

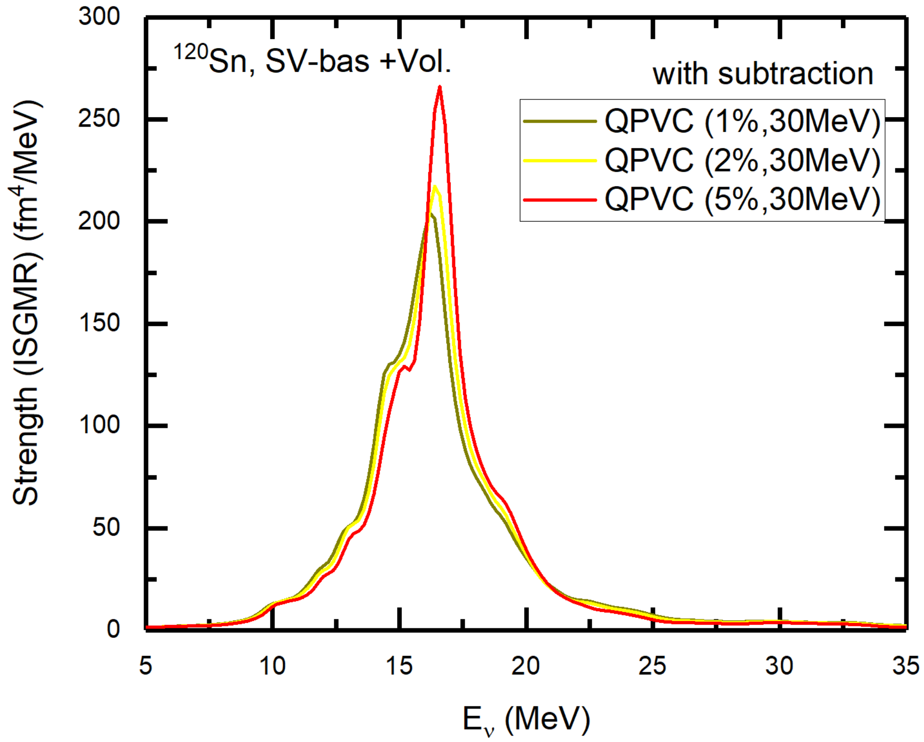


Some detail + the subtraction scheme

All QRPA calculations are performed in a model space which is large enough so that the EWSR is satisfied.

We calculate natural-parity phonons with 0^+, 1^-, 2^+ ... 5^- and select those having energy less than 30 MeV and strength larger than 2% of the total strength.

The convergence of the results with respect to the choice of the model space has been carefully assessed.



Subtraction: $$\Sigma(E) \rightarrow \Sigma(E) - \Sigma(E=0)$$

THIS PRESCRIPTION KEEPS THE VALUE OF THE m_{-1} SUM RULE AS IN QRPA

More on subtraction

The subtraction procedure has been introduced by V. Tselyaev.

At the same time, it has been shown that it emerges naturally if one:

- assumes the universality of the EDF, according to the Hohenberg-Kohn theorem
- applies this concept to $E - \lambda\hat{Q}$

This leads to the conservation of the inverse energy-weighted sum rule.

Subtraction method in the second random-phase approximation: First applications with a Skyrme energy functional

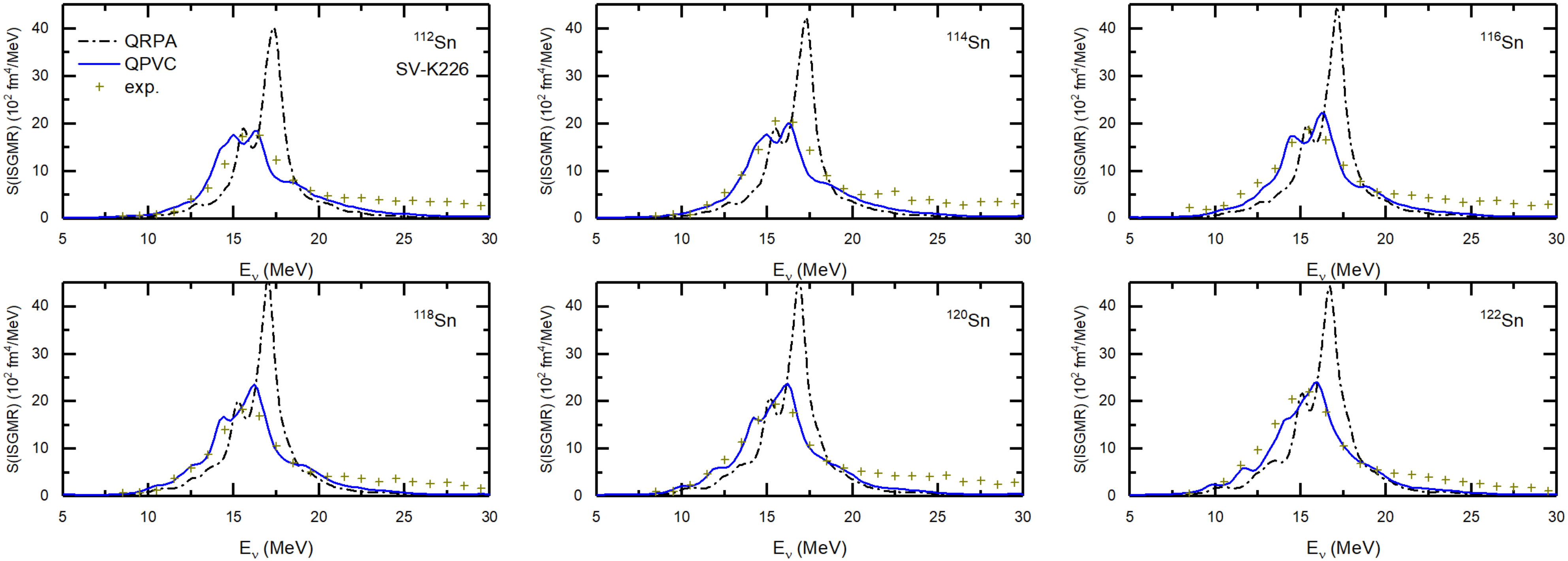
D. Gambacurta[1], M. Grasso[2], and J. Engel[3]

Phys. Rev. C **92**, 034303

The subtraction does not guarantee the conservation of the EWSR.

- Applications

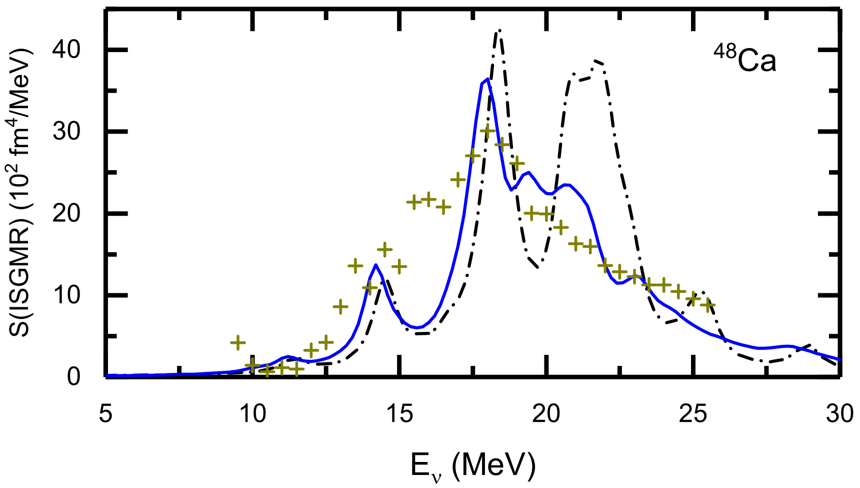
ISGMR in Sn isotopes

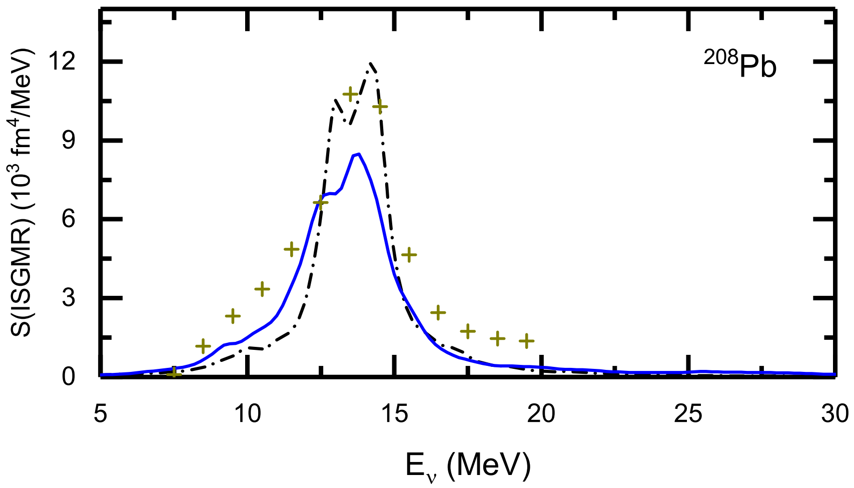


- Exp. data from D. Patel *et al*., Phys. Lett. B726, 178 (2013)
- QPVC reproduces the experimental data quite well.
- The best description is obtained with the Skyrme EDF SV-K226.

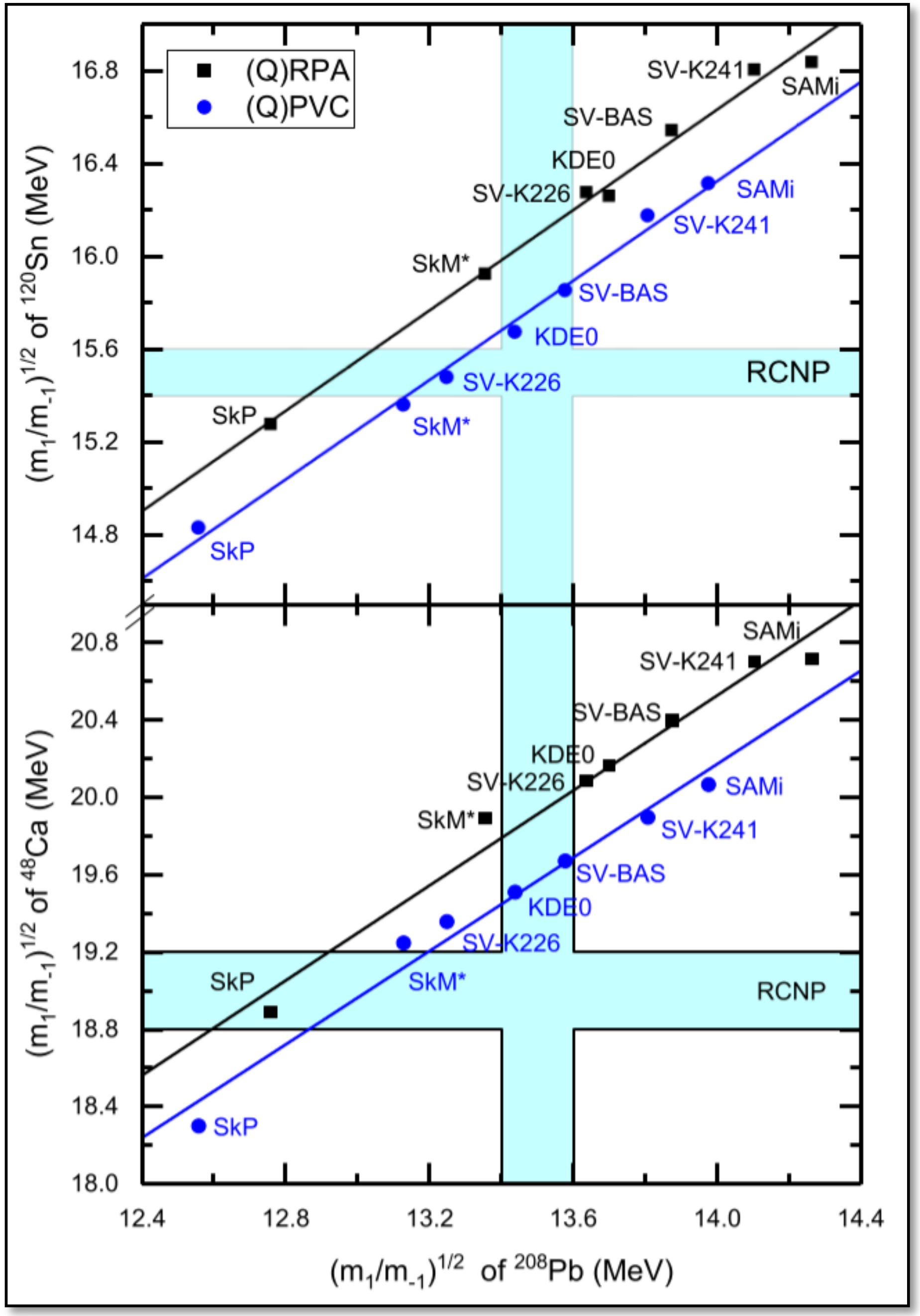
Klüpfel, Reinhard, *et al.*, PRC 79, 034310 (2009)

ISGMR in ^{48}Ca and ^{208}Pb





- Exp. data from T. Li *et al*., Phys. Rev. Lett. 99, 162503 (2007) and S.D. Olorunfunmi, Phys. Rev. C 105, 054319 (2022).
- In these two cases there is no pairing.



In our work, we have been able to analyse **in a systematic manner** the consistency between ISGMR energies in different nuclei.

We have used many Skyrme EDFs.

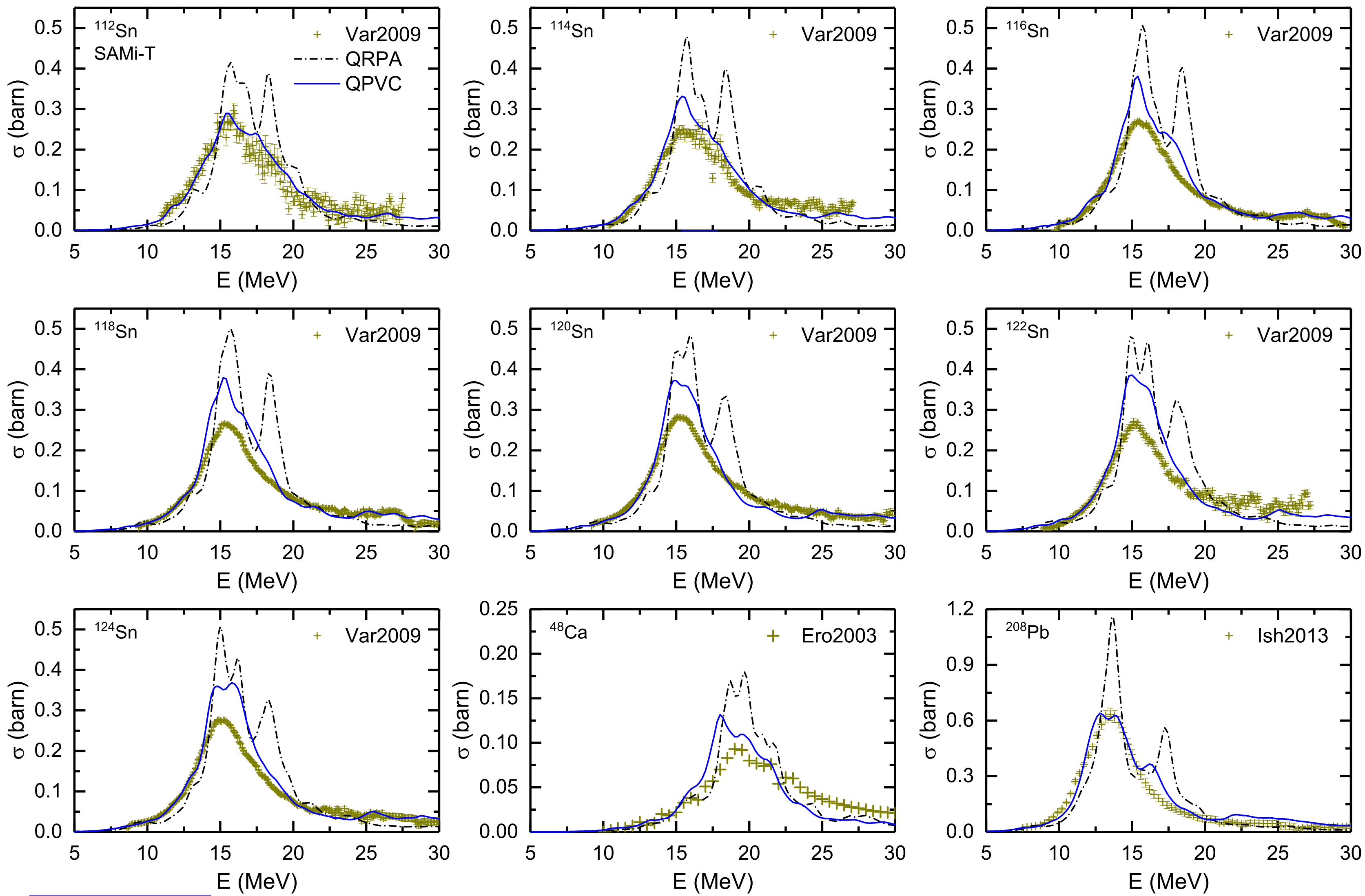
With the inclusion of QPVC effects, a big improvement is achieved.

We have, thus, a unique value for incompressibility.

Within QPVC, the ISGMR energy in ^{208}Pb is consistent with ^{120}Sn.

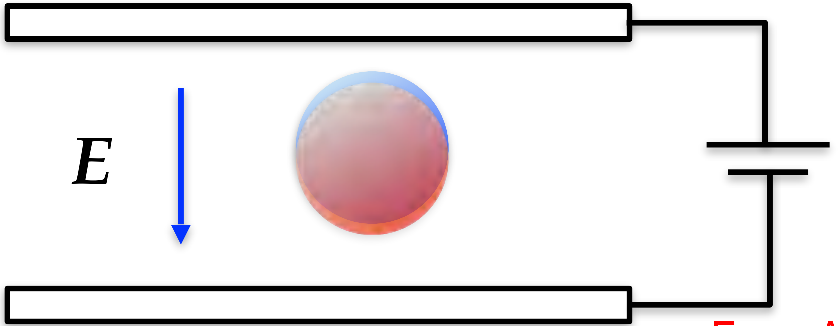
Z.Z. Li, Y.F. Niu, GC, Phys. Rev. Lett. 131, 082501 (2023)

IVGDR in different nuclei



Z.Z. Li *et al*., Phys. Rev. C110, 064317 (2024). SAMi-T.

The dipole polarizability



From A. Tamii

If the nucleus were under the action of a static electric field

$$p = \alpha_{\mathrm{D}} E$$

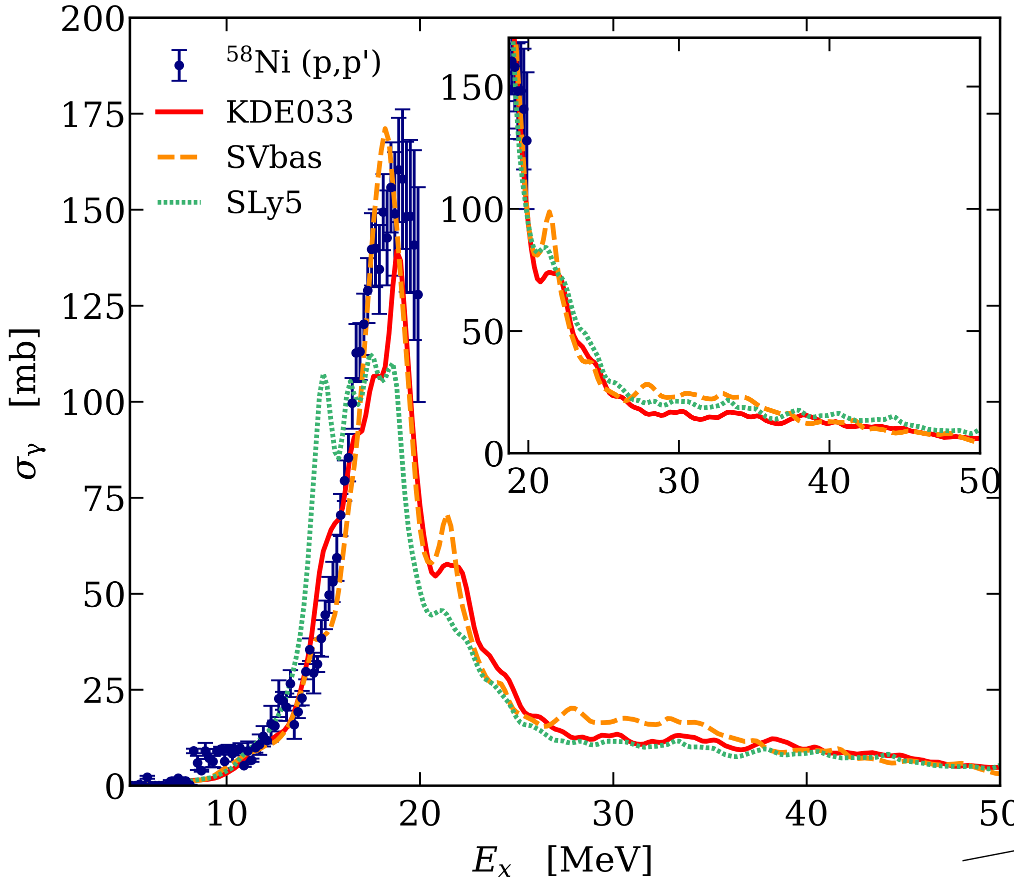
The **polarizability** can be deduced from the so-called **inverse energy-weighted sum rule** associated with dipole field

$$\alpha_{\mathrm{D}} = \frac{8\pi e^2}{9} \int dE \, \frac{dB(E1)}{dE} \equiv \frac{8\pi e^2}{9} \, m_{-1}$$

A. Migdal, Quadrupole and dipole γ-radiation of nuclei, J. Phys. Acad. Sci. USSR 8(1-6), 331-336 (1944)

$$\alpha_{\mathrm{D}} = \frac{\pi e^2}{54} \frac{A\langle r^2 \rangle}{J}$$

Photoabsorbtion of ^{58}Ni



Comparison with recent data from (p,p') measured at RCNP, Osaka. Exp. data from I. Brandherm *et al.*

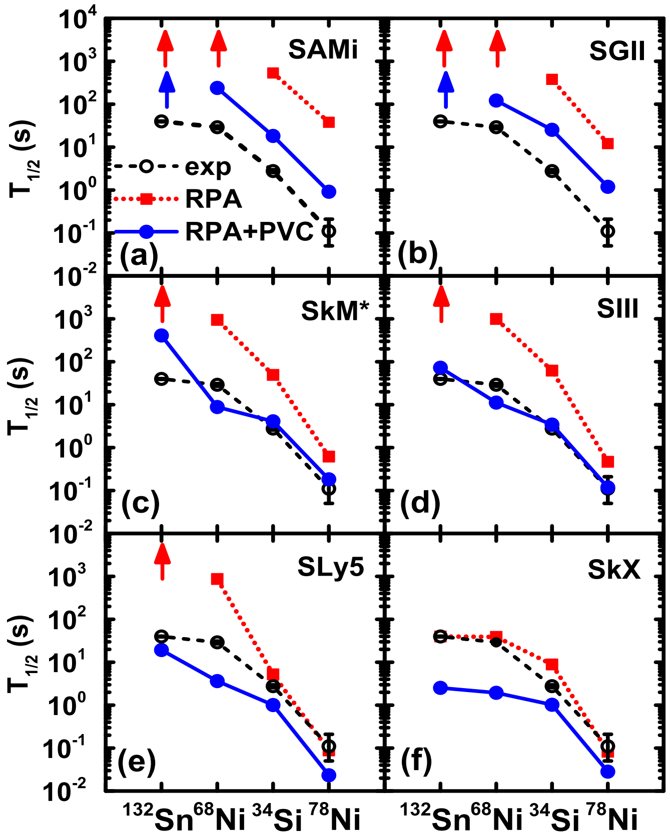
Theory reproduces very well the dipole polarizabilty and can be used to estimate the high-energy dipole tail.

PHYSICAL REVIEW C **111**, 024312 (2025)

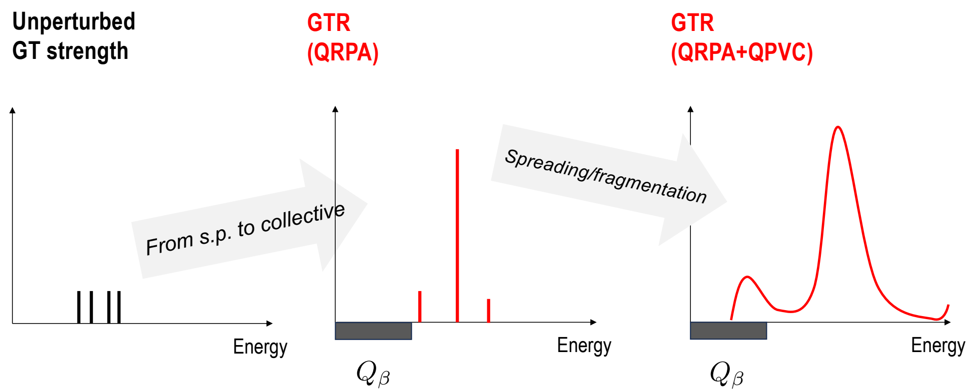
Electric dipole polarizability of ^{58}Ni

I. Brandherm,[1] F. Bonaiti,[2,3,4] P. von Neumann-Cosel,[1,*] S. Bacca,[2] G. Colò,[5,6]
G. R. Jansen,[7,4] Z. Z. Li (李征征),[8,9,10] H. Matsubara,[11,12] Y. F. Niu (牛一斐),[9,10]
P.-G. Reinhard,[13] A. Richter,[1] X. Roca-Maza,[14,15,5,6] and A. Tamii[11]

Applying QPVC to β-decay



Y. F. Niu *et al.*, Phys. Rev. Lett. 114, 142501



RPA may not leave strength in the Q_β window.

PVC redistributes the GT strength.

The IAR: a stringent test

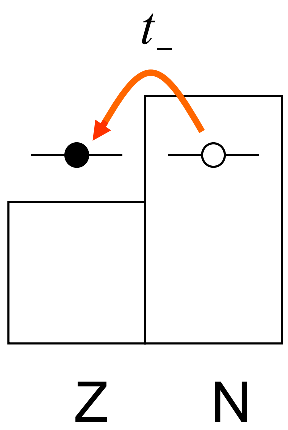


TABLE I. IAR results with the interaction SIII. Three different types of interactions based on SIII are used in the calculations: (I) Skyrme interaction without Coulomb force, (II) Skyrme interaction with Coulomb force, and (III) Skyrme interaction with Coulomb force and CSB-CIB forces. Three different microscopic models are also adopted: (a) TDA without the coupling to the continuum, (b) RPA with the coupling to the continuum, and (c) RPA with the couplings to both the continuum and the phonons. Energies are given in MeV and widths are in keV. The percentage of total strength $m_0=(N-Z)/2$ exhausted by the IAR is also shown.

	(a) Discrete TDA		(b) RPA + $W^{\uparrow}$			(c) RPA + $W^{\uparrow}+W^{\downarrow}$			
	E_{IAR}	% of m_0	E_{IAR}	$\Gamma^{\uparrow}$	% of m_0	E_{IAR}	Γ_{tot}	$\Gamma^{\downarrow}$	% of m_0
I	0.268	99.9	-	-	-	0.267	24	24	99.7
II	18.50	85	18.50	124	97	18.36	194	70	97
	18.28	16							
III	18.64	80	18.65	128	96	18.54	228	100	96
	18.39	11							

I : w/o Coulomb - a test of isospin symmetry

II : with Coulomb – narrow width

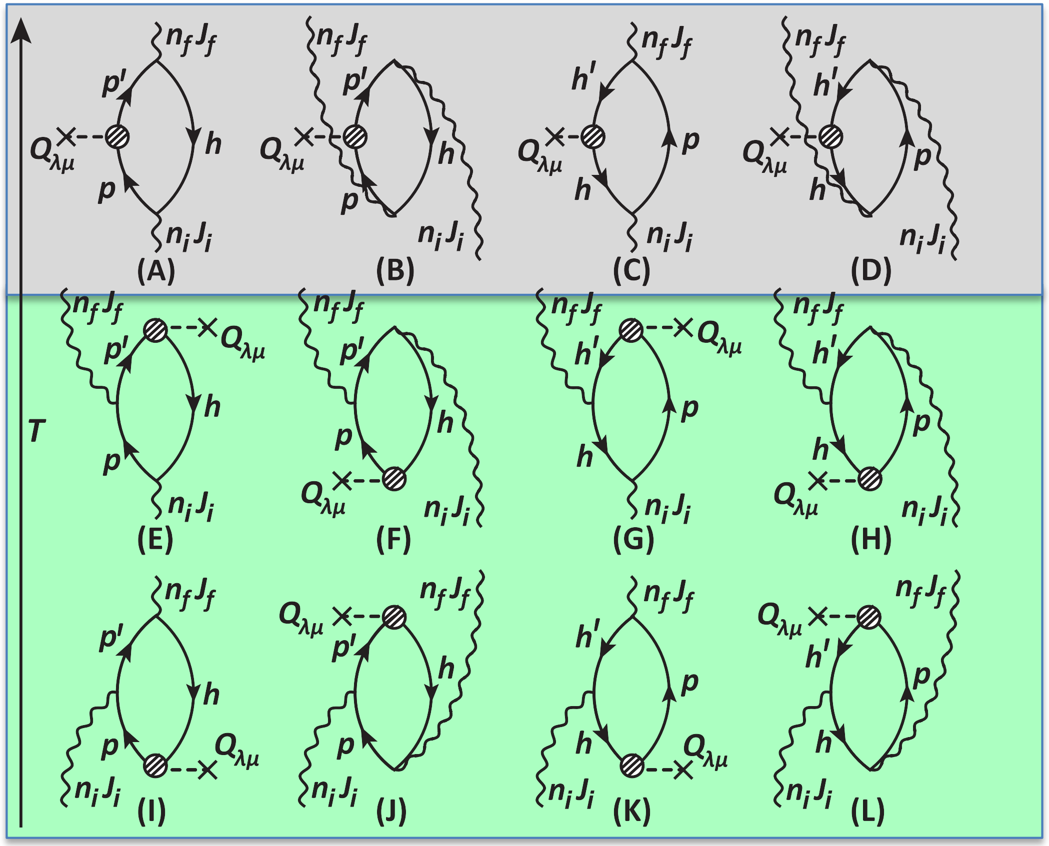
III : with Coulomb and isospin symmetry-breaking forces

- Gamma-decay

Formalism

$$\Gamma_\gamma(E\lambda; i \to f) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]}\left(\frac{E}{\hbar c}\right)^{2\lambda+1} B(E\lambda; i \to f)$$

$$B(E\lambda; i \to f) = \frac{1}{2J_i+1}|\langle J_f||Q_\lambda||J_i\rangle|^2$$



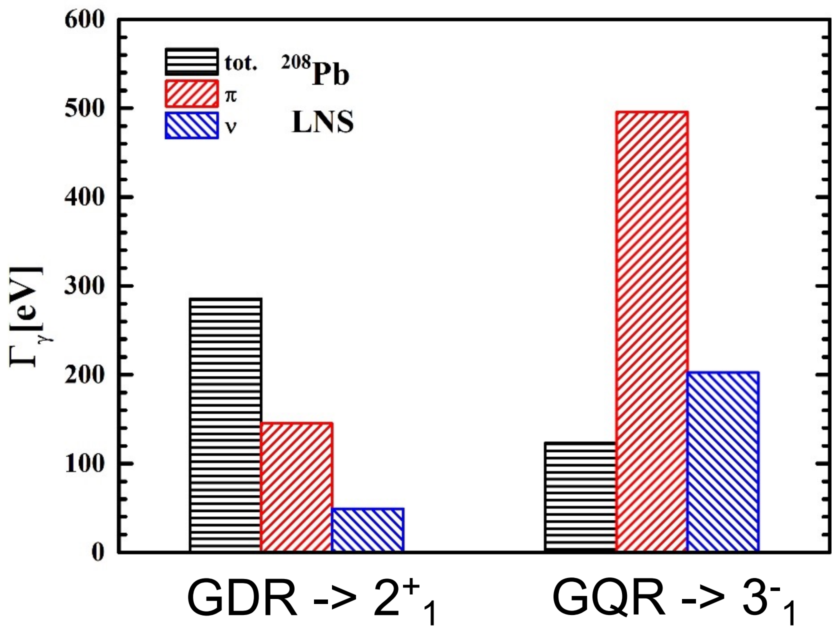
All the Feynman diagrams contribute to the matrix element:

First line: 1p-1h, RPA

Second and third line: PVC

This decay is a tiny (≈10^{-3}) part of the total width; there are cancellations between the amplitudes.

Sensitivity of γ-decay to isospin



The quenching of the decay of the ISGQR, with respect to the IVGDR, manifests itself in **the destructive interference of the neutron and proton amplitudes.**

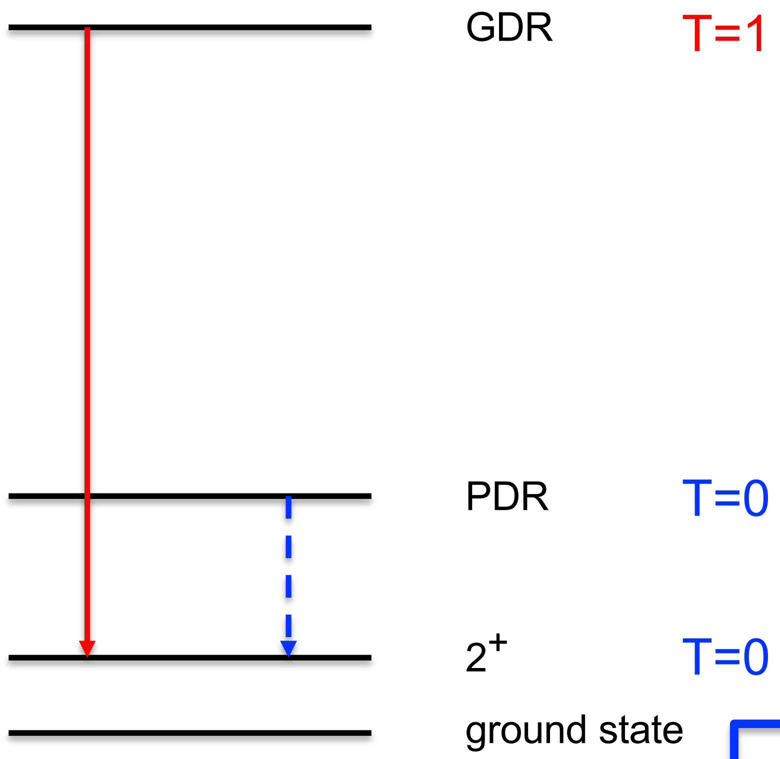
This would be complete in the case of perfect isospin symmetry.

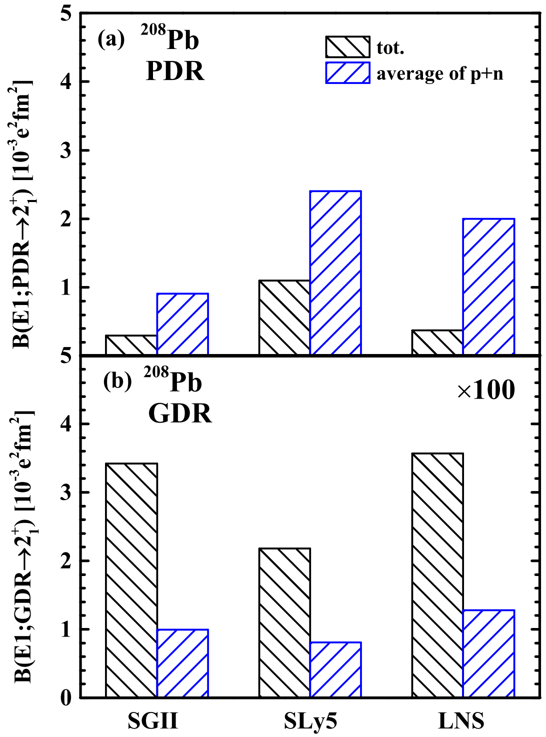
We have tested this property in the case of ^{56}Ni (N=Z) without Coulomb.

^{56}Ni	Γ_γ	Γ_γ^π	Γ_γ^ν
GDR $\rightarrow 2_1^+$	3877.72	969.43	969.43
GQR $\rightarrow 3_1^-$	0.00	84.95	84.95

Consequently, γ-decay data can be used to determine **unknown isospin properties of, e.g., pygmy dipole states.**

GDR and PDR decay to the low-lying 2+





W.L. Lv, Y.F. Niu, GC, arXiv:2602.19052v2

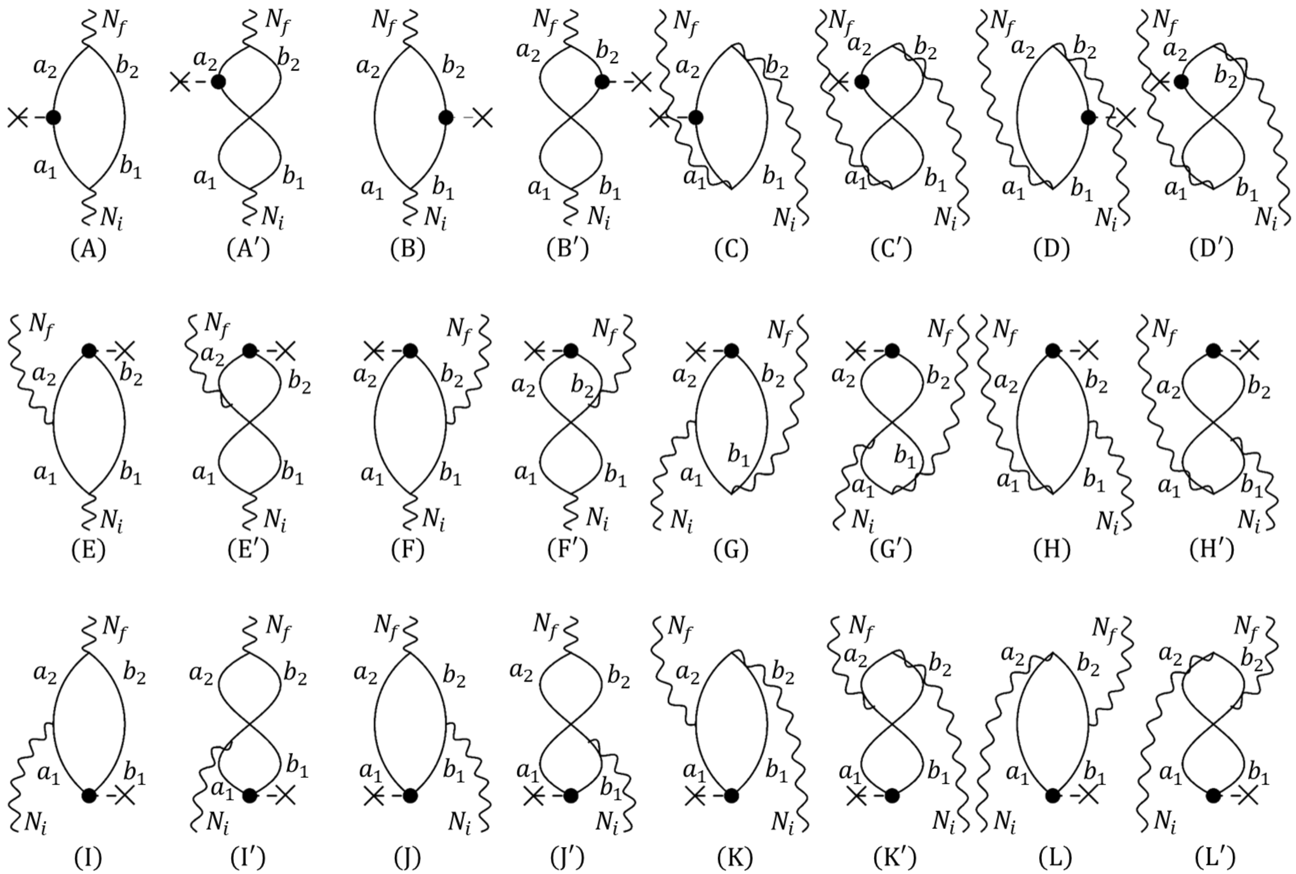
GDR: B is larger than the average $\frac{1}{2}(B_\pi + B_\nu)$

PDR: B is smaller than $\frac{1}{2}(B_\pi + B_\nu)$

The conclusion that the "pygmy" strength is mainly IS is consistent with other findings.

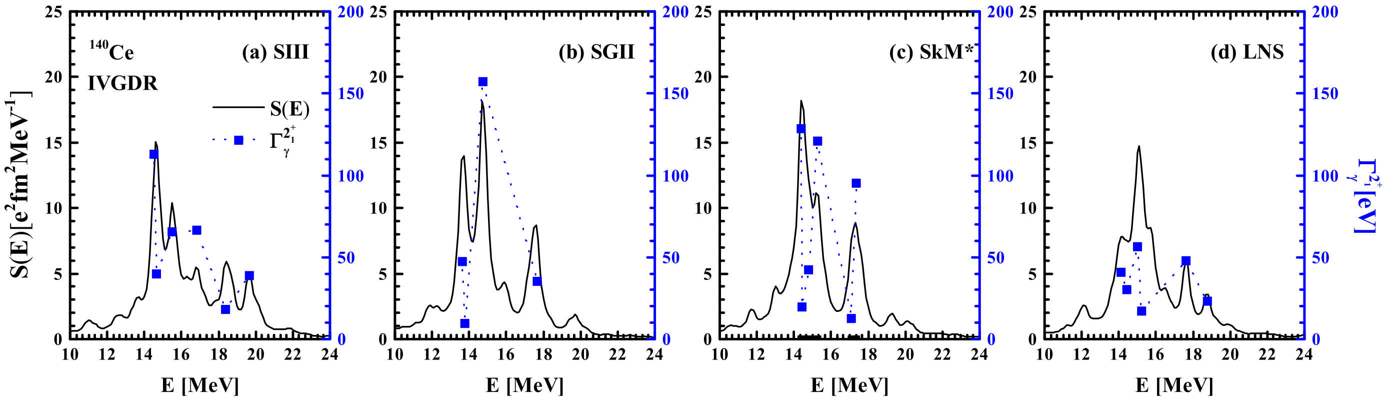
The decay from the IVGDR to 2^+ in ^{140}Ce (I)

The formalism has been extended to open-shell systems.



W.L. Lv, Y.F. Niu, GC, arXiv:2603.27243

The decay from the IVGDR to 2+ in ^{140}Ce (II)



SIII				SGII				SkM*				LNS			
E[MeV]	$\Gamma_\gamma^{2_1^+}$ [eV]	$\Gamma_\gamma^{g.s.}$[eV]	R[%]	E[MeV]	$\Gamma_\gamma^{2_1^+}$ [eV]	$\Gamma_\gamma^{g.s.}$[eV]	R[%]	E[MeV]	$\Gamma_\gamma^{2_1^+}$ [eV]	$\Gamma_\gamma^{g.s.}$[eV]	R[%]	E[MeV]	$\Gamma_\gamma^{2_1^+}$ [eV]	$\Gamma_\gamma^{g.s.}$[eV]	R[%]
14.53	112.9	2952.9	3.82	13.62	47.5	4029.6	1.18	14.38	128.3	8209.0	1.56	14.13	40.9	2790.4	1.47
14.65	39.8	9174.5	0.43	13.77	9.5	2624.9	0.36	14.45	19.6	3642.8	0.54	14.42	30.1	2282.5	1.32
15.50	65.4	7491.7	0.87	14.73	157.3	12744.4	1.23	14.78	42.4	4084.2	1.04	15.02	56.3	6966.4	0.81
16.84	66.4	4400.8	1.51	17.65	35.5	6937.5	0.51	15.26	120.8	7547.8	1.60	15.22	17.1	2682.0	0.64
18.37	18.0	6532.9	0.28					17.10	12.7	5141.9	0.25	17.63	48.0	8215.3	0.58
19.65	38.6	8592.5	0.45					17.34	95.2	6887.4	1.38	18.76	23.4	5183.9	0.45
Total	341.0	39145.2	0.87	Total	249.9	26336.5	0.95	Total	419.0	35513.1	1.18	Total	215.8	28120.5	0.77

The **partial widths** for the decay to the 2+ are or the order of **10^2 eV** but they fluctuate.

This corresponds to a ratio of 0.7-1.2% with respect to the ground-state decay.

W.L. Lv, Y.F. Niu, GC, arXiv:2603.27243

Courtesy: N. Pietralla

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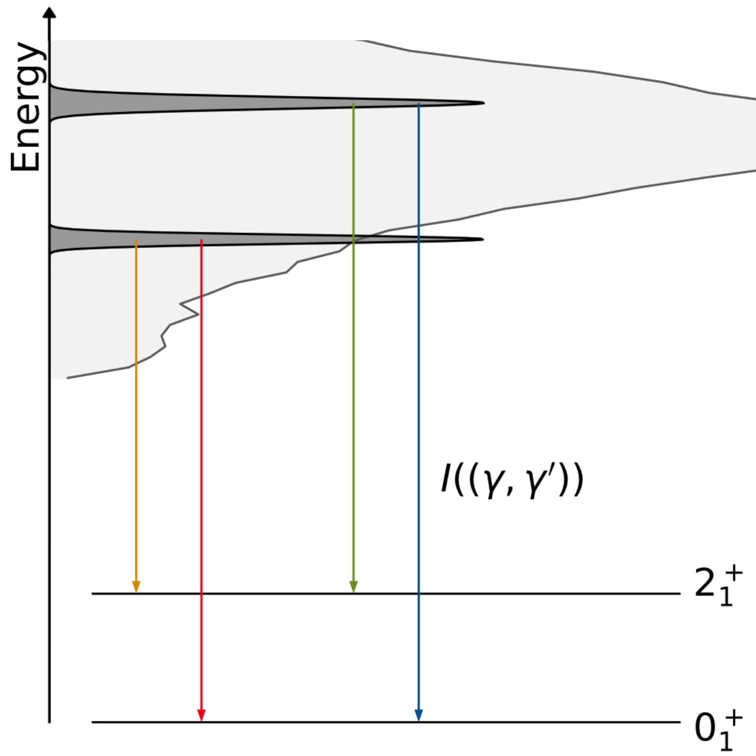

E_{beam} (MeV)	$I_{\gamma,2_1^+}/I_{\gamma,0_1^+}$
11.4	0.60(40)
12.6	0.0034(24)
14.3	0.0010(7)
15.4	0.0007(7)
16.2	0.0009(6)
17.8	0.0019(13)

Observation:

- Decays to 2^+_1 (or any other excited states) are VERY small. At all energies.

➢ The GDR decays by γ according to its collective part.

➢ The timescale for the γ–decay of the GDR must be the timescale of the collective state, i.e., 13 zs.