Second RPA for Nuclear Excitations

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Recent Developments and Challenges for (Q)RPA-Based Methods
7-9 July 2026, Saclay, France

Outline

- RPA and Second RPA (SRPA)
- A bit of history: First derivations (1960s) and applications (1970s-2000s), Large scale calculations (2010s)
- The SRPA within the EDF approach: $\mapsto$ the Subtracted SRPA (SSRPA)
- Some applications of the SSRPA
- Conclusions: Main difficulties and challenges, Perspective and Outlook

Recent Review Paper on SSRPA

Progress in Particle and Nuclear Physics 150 (2026) 104251

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Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

Recent applications of the subtracted second RPA method

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The Random Phase Approximation (RPA)

- The RPA is a widely used tool for the description of collective excitations
- Very successful especially within the Energy Density Functional framework (interactions á la Skyrme or Gogny, covariant versions)
- It provides global properties: centroid energies and total strength

However, extensions of the RPA are required for:

- Strength Fragmentation
- Fine Structure
- Spreading Width
- Low Lying excitations in closed shell nuclei
- Double exctiations and Anharmonicities, ...

Phonon Operators: RPA vs SRPA

Random Phase Approximation (RPA)

$$Q_\nu^\dagger = \underbrace{\sum_{ph} X_{ph}^{(\nu)} \underbrace{a_p^\dagger a_h}_{\text{1p-1h}} - \sum_{ph} Y_{ph}^{(\nu)} \underbrace{a_h^\dagger a_p}_{\text{1h-1p}}}_{\text{Only Landau Damping, Centroid Energy and Total Strength of GRs}}$$

Second Random Phase Approximation (SRPA)

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \underbrace{\sum_{p_1<p_2,h_1<h_2} (X^{(\nu)}_{p_1 h_1 p_2 h_2} \underbrace{a^\dagger_{p_1} a_{h_1} a^\dagger_{p_2} a_{h_2}}_{\text{2p-2h}} - Y^{(\nu)}_{p_1 h_1 p_2 h_2} \underbrace{a^\dagger_{h_1} a_{p_1} a^\dagger_{h_2} a_{p_2}}_{\text{2h-2p}})}_{\textbf{Spreading Width, Fragmentation}\text{, Double GRs and Anharmonicites, Low-Lying States}}$$

Phonon Operators: RPA vs SRPA

Random Phase Approximation (RPA)

$$Q_\nu^\dagger = \underbrace{\sum_{ph} X_{ph}^{(\nu)} \underbrace{a_p^\dagger a_h}_{\text{1p-1h}} - \sum_{ph} Y_{ph}^{(\nu)} \underbrace{a_h^\dagger a_p}_{\text{1h-1p}}}_{\text{Only Landau Damping, Centroid Energy and Total Strength of GRs}}$$

Second Random Phase Approximation (SRPA)

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \underbrace{\sum_{p_1<p_2, h_1<h_2} (X_{p_1h_1p_2h_2}^{(\nu)} \underbrace{a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2}}_{\text{2p-2h}} - Y_{p_1h_1p_2h_2}^{(\nu)} \underbrace{a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2}}_{\text{2h-2p}})}_{\textbf{Spreading Width, Fragmentation}\text{, Double GRs and Anharmonicites, Low-Lying States}}$$

Quasi Boson Approximation (QBA)

The phonon vacumm $\mid 0\rangle$ is not known: it is replaced by the HF one

$$\mid 0\rangle \mapsto \mid HF\rangle$$

RPA Equations of Motion (1$\mapsto$ 1p-1h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}^*_{11} & -\mathcal{A}^*_{11} \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu_1 \\ \mathcal{Y}^\nu_1 \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu_1 \\ \mathcal{Y}^\nu_1 \end{pmatrix}$$

RPA and SRPA Equations

RPA Equations of Motion (1$\mapsto$ 1p-1h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}^*_{11} & -\mathcal{A}^*_{11} \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu_1 \\ \mathcal{Y}^\nu_1 \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu_1 \\ \mathcal{Y}^\nu_1 \end{pmatrix}$$

SRPA Equations of Motion (1$\mapsto$ 1p-1h, 2$\mapsto$ 2p-2h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}^*_{11} & -\mathcal{B}^*_{12} & -\mathcal{A}^*_{11} & -\mathcal{A}^*_{12} \\ -\mathcal{B}^*_{21} & -\mathcal{B}^*_{22} & -\mathcal{A}^*_{21} & -\mathcal{A}^*_{22} \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu_1 \\ \mathcal{X}^\nu_2 \\ \mathcal{Y}^\nu_1 \\ \mathcal{Y}^\nu_2 \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu_1 \\ \mathcal{X}^\nu_2 \\ \mathcal{Y}^\nu_1 \\ \mathcal{Y}^\nu_2 \end{pmatrix}$$

Transition Probabilities in RPA and SRPA

- Let us consider a one-body operator $F = \sum_{\alpha,\beta} F_{\alpha\beta} a^\dagger_\alpha a_\beta$
- The RPA and SRPA transition amplitudes are

$$\langle \nu | F | 0 \rangle = \langle 0 | [Q_\nu, F] | 0 \rangle \approx \langle HF | [Q_\nu, F] | HF \rangle = \sum_{ph} \{ X^{\nu *}_{ph} F_{ph} + Y^{\nu *}_{ph} F_{hp} \}$$

Thouless Theorem

The first moment

$$m_1 = \sum_\nu \omega_\nu |\langle \nu \mid F \mid 0 \rangle|^2$$

is the same in RPA and SRPA[a] and Thouless theorem holds [b]

$$\sum_\nu \omega_\nu |\langle \nu \mid F \mid 0 \rangle|^2 = \frac{1}{2} \langle HF \mid [F, [H, F]] \mid HF \rangle,$$

Energy Weighted Sum Rules are preserved.

[a] C. Yannouleas, Phys. Rev. C35, 1159 (1987)
[b] For self-consistent calculations

THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893
SECOND SERIES, VOL. 122, No. 2 APRIL 15, 1961

Higher Random-Phase Approximations in the Many-Body Problem

H. SUHL AND N. R. WERTHAMER*

The usual random-phase approximation combined with an equations-of-motion technique for the many-electron problem is extended, yielding many of the known results of series summation methods in a straightforward manner. The method should apply to other types of many-body problems as well.

PHYSICAL REVIEW VOLUME 126, NUMBER 6 JUNE 15, 1962

Higher Random Phase Approximation and Energy Spectra of Spherical Nuclei

J. SAWICKI
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(Received January 8, 1962)

A method which is an extension of the usual random phase approximation (RPA) is proposed in order to study the spectra of spherical nuclei. In contrast to the usual RPA method based on the linearized equations of motion of the one-nucleon excitation operators, in the higher random phase approximation (HRPA) the many-nucleon excitations are also included. Nuclear states are superpositions of the one- and many-nucleon excitations (i.e., particle-hole pairs). We have especially studied the modes consisting of one- and two-nucleon excitations. In this second RPA, one solves the secular problem obtained from the closed system of linearized equations of motion for the one particle-hole pair operators ("doubles") and the two-pair operators ("quadruples"). States of definite nuclear spin, parity, and isotopic spin are considered. The method of the second RPA is illustrated in the example of the 6.06-Mev, $J^{\pi}=0^{+}$, $T=0$ state of O^{16}. Satisfactory semiquantitative agreement with the experiment is obtained for this case. The second RPA is also discussed in connection with the "aligned coupling scheme." The importance of the method for the study of the vibrational 4^{+} states is indicated.

Computationally very demanding

- Number of 2ph ($\sim 10^{6-8}$ for a medium-mass nucleus) is considerable higher than 1ph ($\sim$ a few hundreds) in J-coupled scheme
- First applications used strong approximations
  - Restricted 2ph spaces
  - Unperturbed 2ph configurations, e.g. no residual interaction, diagonal approximation (see later)
  - Real part of the self-energy neglected

Some references (not exhaustive)

- B. Schwesinger and J. Wambach, Nuclear Physics A426 (1984) 253-275
- S.Drozdz, S. Nishizaki, J. Speth and J. Wambach, Physics Reports 197,1 (1990) and references therein
- S. Ait-Tahar and D.M. Brink, Nuclear Physics A560 (1993) 765-796
- S. Nishizaki, J. Wambach, Physics Letters B 349 (1995) 7-10
- S. Ayik, D. Lacroix and P. Chomaz, PRC, 61, 014608 (1999)
- D. Lacroix *et al.*, Physics Letters B 479 (2000) 15

Computationally very demanding

- Number of 2ph ($\sim 10^{6,7}$ for a medium-mass nucleus) is considerable higher than 1ph ($\sim$ a few hundreds) in J-coupled scheme
- First applications used strong approximations
  - Restricted 2ph spaces
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From SRPA to an Energy dependent RPA-like problem

- The SRPA problem can be reduced to an RPA eigenvalue problem but with $\mathcal{A}_{1,1}$ **depending on the Energy** ω

$$A_{1,1'} \mapsto \tilde{A}_{1,1'}(\omega) = A_{1,1'} + \sum_{2,2'} A_{1,2}(\omega + i\eta - A_{2,2'})^{-1} A_{2',1'}$$

- Exact projection onto the 1ph space
- The matrix inversion is very **cumbersome**
- If we neglect the **residual interaction** among the $2p-2h$ states

$$A_{2,2} \simeq \delta_{h_1 h_1'} \delta_{p_1 p_1'} \delta_{h_2 h_2'} \delta_{p_2 p_2'} (\epsilon_{p_1} + \epsilon_{p_2} - \epsilon_{h_1} - \epsilon_{h_2})$$

the inversion is algebraic, (**diagonal approximation**)

SRPA as an energy-dependent RPA problem

$$\begin{pmatrix} \mathcal{A}_{11}(\omega) & \mathcal{B}_{11} \\ -\mathcal{B}_{11}^* & -\mathcal{A}_{11}^*(-\omega) \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu(\omega) \\ \mathcal{Y}_1^\nu(\omega) \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu(\omega) \\ \mathcal{Y}_1^\nu(\omega) \end{pmatrix}$$

PHYSICAL REVIEW C, VOLUME 61, 014608

Fragmentation and damping of the collective response in extended random-phase approximati

Sakir Ayik,[1] Denis Lacroix,[2] and Philippe Chomaz[2]

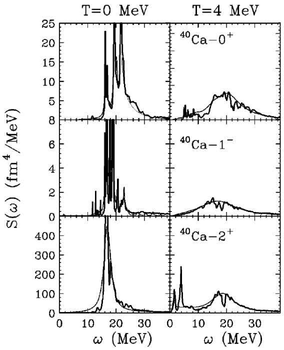

FIG. 2. The extended RPA strength distributions of the monopole, dipole, and quadrupole excitations at $T=0$ MeV (left) and $T=4$ MeV (right). Thin lines represent the strength obtained in the pole approximation whereas thick lines are obtained by including the frequency dependence of the imaginary part of the self-energy. In both cases, the real part of the self-energy is neglected.

B. Energy shift with zero-range effective forces

A zero range Skyrme-type force provides a good description for the mean-field properties, but it is not very suitable for calculation of $2p$-$2h$ self-energies of the collective modes. Matrix elements of a zero range force with a quadratic momentum dependence does not involve a natural cutoff for large values of the momentum transfer. This behavior is not a serious problem for the damping width since the high frequency matrix elements are naturally cut off due to the Lorentzian factor in $\Gamma_\lambda(\omega)$ [Eq. (14)]. However, the situation is different for the energy shift. The frequency dependent factor in the expression of $\Delta_\lambda(\omega)$ does not provide such a cutoff. As a result, the shift of the collective frequency $\Delta_\lambda(\omega)$ is strongly affected by the unrealistic large matrix elements for high momentum transfers. As an example, Fig. 3 shows the strength distribution for quadrupole excitations at temperature $T=4$ MeV. Thin lines are obtained in the pole approximation and neglecting the shift of the resonance due to the real part of the self-energy, whereas, thick line is the result of full calculations including the real and imaginary parts of the self-energy. The strength is shifted down by an unreasonable amount of about 10 MeV from the RPA value. A similar result was reported in [23]. Moreover, the magnitude of the energy shift depends on the size of the $2p$-$2h$ space, which clearly indicates a serious weakness of the zero range forces in the calculation of $2p$-$2h$ self-energies.

PHYSICAL REVIEW C, VOLUME 61, 014608

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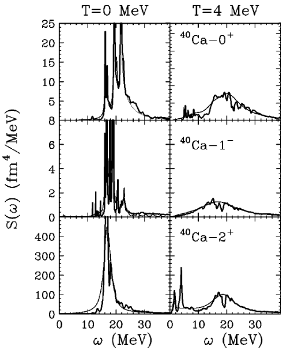

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Large scale SRPA calculations

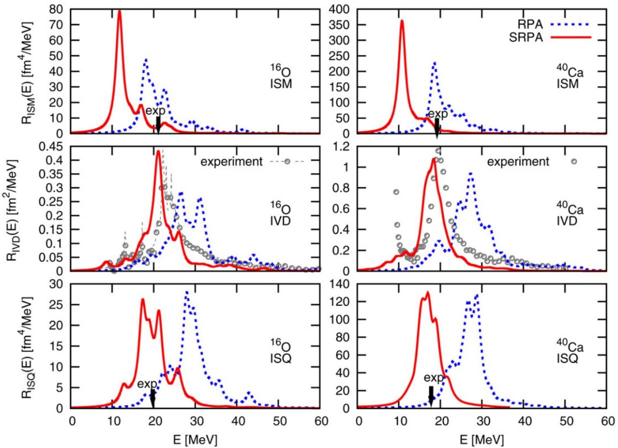
- Large scale SRPA calculations become feasible recently (2010s) [a]
- Model spaces large enough to preserve EWSRs and to study stability
- No approximation in the evaluation of the matrix elements
- Self-consistency (same interaction in mean-field and in linear response)

[a] P. Papakonstantinou and R. Roth, PLB 671, 356 (2009); D.G et al. PRC 81, 054312 (2010)

.

Some references

- P. Papakonstantinou and R. Roth, Physics Letters B 671, 356 (2009), Phys. Rev. C.81, 024317 (2010)
- P. Papakonstantinou, Phys. Rev. C 90, 024305 (2014)
- DG, M. Grasso and F. Catara, PRC 81, 054312 (2010), PRC 84, 034301 (2011)
- DG, M. Grasso and F. Catara, Journal of Physics G 38 035103 (2011)
- DG *et al.*, Phys. Rev. C 86 021304 (2012)



SRPA with UCOM method

SRPA for closed-shell nuclei using an interaction derived from the Argonne V18 potential with the Unitary Correlation Operator Method

PHYSICAL REVIEW C **81**, 054312 (2010)

Collective nuclear excitations with Skyrme-second random-phase approximation

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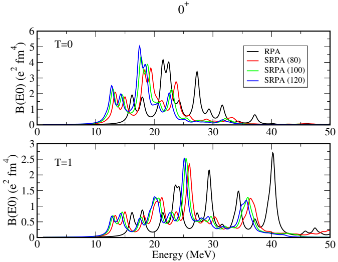
[2]*Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy*
[3]*Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, F-91406 Orsay Cedex, France*
(Received 22 February 2010; published 20 May 2010)

PHYSICAL REVIEW C **86**, 021304(R) (2012)

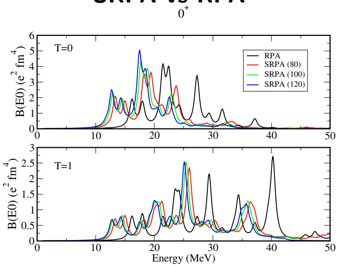
Second random-phase approximation with the Gogny force: First applications

D. Gambacurta,[1] M. Grasso,[2] V. De Donno,[3] G. Co',[3] and F. Catara[4]
[1]*Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DSM-CNRS/IN2P3, Boulevard Henri Becquerel, F-14076 Caen, France*
[2]*Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, F-91406 Orsay Cedex, France*
[3]*Dipartimento di Matematica e Fisica "E. De Giorgi," Università del Salento and INFN, Sezione di Lecce, I-73100 Lecce, Italy*
[4]*Dipartimento di Fisica e Astronomia and INFN, Via Santa Sofia 64, I-95123 Catania, Italy*
(Received 7 May 2012; published 30 August 2012)

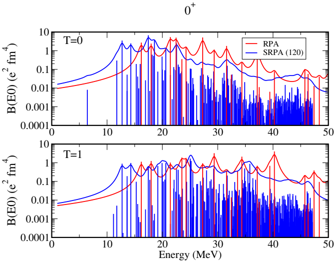
SRPA vs RPA



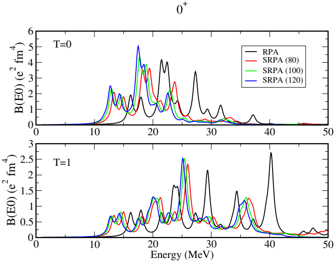
SRPA vs RPA



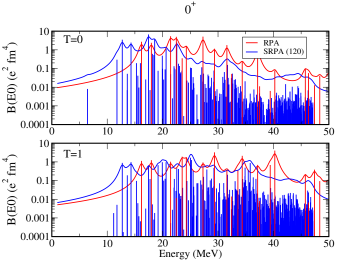
RPA vs SRPA Fragmentation



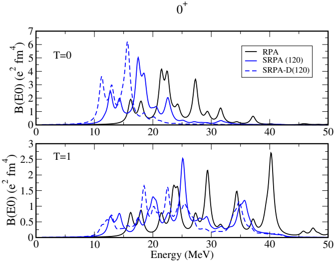
SRPA vs RPA



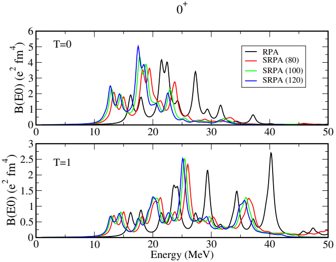
RPA vs SRPA Fragmentation



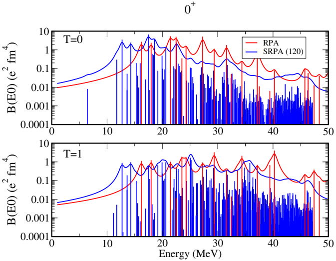
Full vs Diagonal



SRPA vs RPA



RPA vs SRPA Fragmentation



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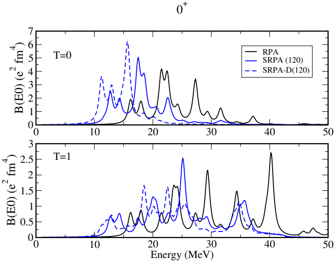
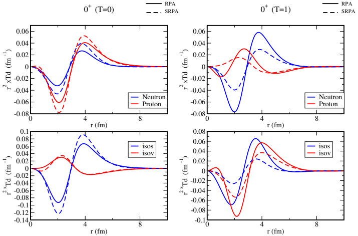


Transition Densities



EWSR Monopole Case in ^{16}O

First moment

$$m_1 = \sum_{\nu}^{\omega_{max}} \omega_\nu |\langle \nu \mid F \mid 0\rangle|^2; \qquad m_1^{RPA} = m_1^{SRPA}?$$

Isoscalar and Isovector EWSR

EWSR		
	$m_1(T=0)$	$m_1(T=1)$
RPA	671.6516	201.2494
ω_{max}	SRPA	SRPA
40	626.4381	115.4153
50	648.9699	147.8026
60	661.0194	182.7364
70	664.3803	193.7896
80	669.7185	197.6874
90	671.4575	200.6472
100	671.6515	201.2473
110	671.6515	201.2473

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100	671.6515	201.2473
110	**671.6515**	**201.2473**

Monopole State

Low Lying 0^+ state (MEV)			
Exp*	RPA	SRPA	SRPA-D
$\sim$6	16.19	6.43	11.23

Quadrupole State

Low Lying 2^+ state (MEV)			
Exp	RPA	SRPA	SRPA-D
$\sim$7	16.03	7.16	12.44

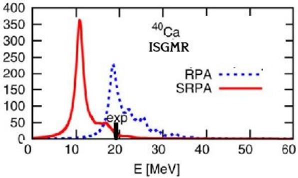
Transition Probabilities

However the $B(E0)$ and $B(E2)$ data values are underestimated (factor 5)

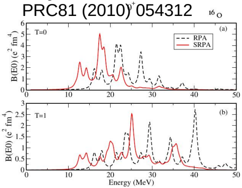
*Nucl. Phys. A375, 1 (1982);Progress in Part. and Nucl. Phys. 61 (2008)

SRPA results with different interactions

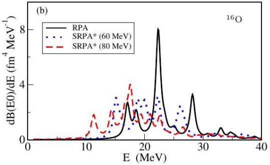
UCOM P. Papakonstantinou, R. Roth, PLB 671 (3) (2009) 356–360



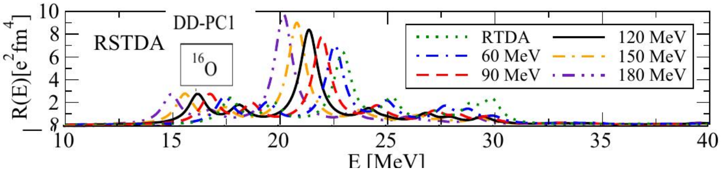
Skyrme EDF: DG et al., PRC81 (2010) 054312



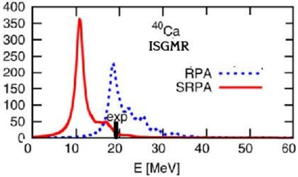
Gogny EDF: DG et al., PRC86 (2012) 021304.



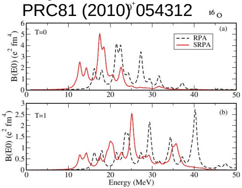
Relativistic EDF D.Vale and N. Paar PRC112, 034327 (2025)



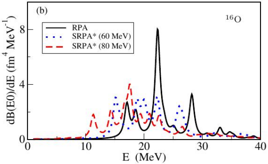
UCOM P. Papakonstantinou, R. Roth, PLB 671 (3) (2009) 356–360



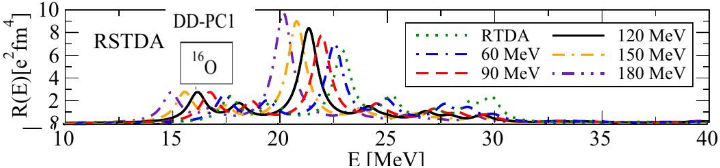
Skyrme EDF: DG et al., PRC81 (2010) 054312



Gogny EDF: DG et al., PRC86 (2012) 021304.



Relativistic EDF D.Vale and N. Paar PRC112, 034327 (2025)



Systematic downward-shift of SRPA with respect to RPA!

Large scale SRPA calculations have shown that:

- The SRPA strength distribution is sistematically shifted towards lower energies compared to the RPA one
- This shift is very strong ($\simeq$ 5-7 MeV), RPA description often spoiled
- That's why in "old" applications real part of the self-energy was neglected

Origins and Causes:

1. Quasi Boson Approximation more severe in SRPA [a] [b]
2. Use of effective interactions in beyond-mean field methods

[a] K. Takayanagi *et al.*, Nucl. Phys. A477 (1988); A. Mariano et al. PRC 49, 2824 (1994).
[b] P. Papakonstantinou, Phys. Rev. C 90, 024305 (2014)

The Subtraction procedure (I. Tselyaev Phys. Rev. C 75, 024306 (2007))

- Designed for beyond RPA approaches
- Static ($\omega = 0$) limit of the SRPA imposed to be equal to the RPA one
- Regularization procedure removing double countings, (Π^{RPA} restored)

PHYSICAL REVIEW C **92**, 034303 (2015)

Subtraction method in the second random-phase approximation: First applications with a Skyrme energy functional

D. Gambacurta,[1] M. Grasso,[2] and J. Engel[3]

[1] *Dipartimento di Fisica e Astronomia and INFN, Via Santa Sofia 64, I-95123 Catania, Italy*

[2] *Institut de Physique Nucléaire, IN2P3-CNRS, Université Paris-Sud, F-91406 Orsay Cedex, France*

[3] *Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27516-3255*

PHYSICAL REVIEW LETTERS **125**, 212501 (2020)

Gamow-Teller Strength in ^{48}Ca and ^{78}Ni with the Charge-Exchange Subtracted Second Random-Phase Approximation

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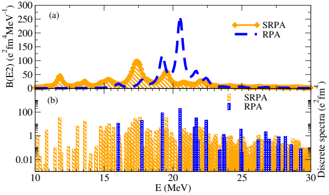
[1] *INFN-LNS, Laboratori Nazionali del Sud, 95123 Catania, Italy*

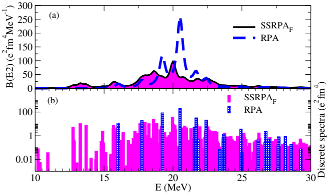
[2] *Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France*

[3] *Department of Physics and Astronomy, CB 3255, University of North Carolina, Chapel Hill, North Carolina 27599-3255*

Quadrupole Distributions



Quadrupole Distributions



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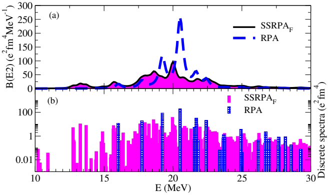
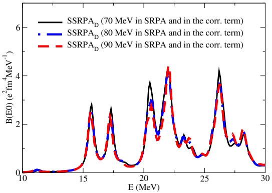
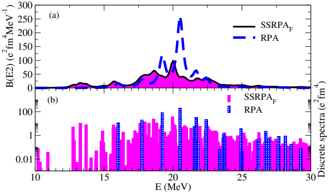


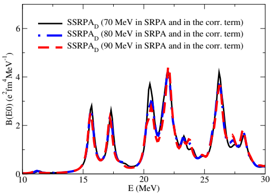
Cutoff dependence



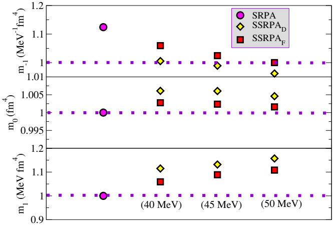
Quadrupole Distributions



Cutoff dependence

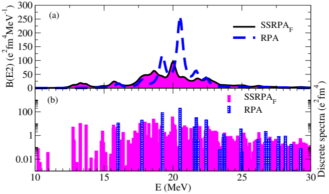


RPA, SRPA, SSRPA moments

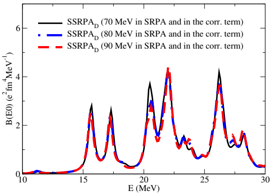


SSRPA Strength Distribution in ^{16}O

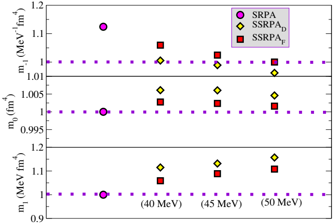
Quadrupole Distributions



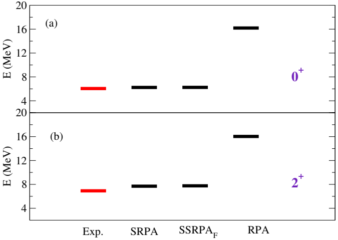
Cutoff dependence



RPA, SRPA, SSRPA moments



Low–lying 0^+ and 2^+ states



PHYSICAL REVIEW C **112**, 034327 (2025)

Subtracted second Tamm-Dancoff approximation in the relativistic point-coupling model

Deni Vale
Istrian University of Applied Sciences, Preradovićeva 9D, HR-52100 Pula, Croatia

Nils Paar
Department of Physics, Faculty of Science, University of Zagreb, Bijenička Cesta 32, HR-10000 Zagreb, Croatia

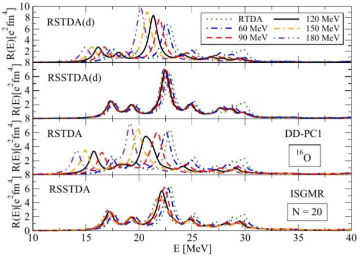


FIG. 1. ISGMR strength distributions for ^{16}O using TDA, RSTDA, and RSSTDA with the full A_{22} form and diagonal approximation [denoted with suffix (d)] as functions of excitation energy, for different 2p-2h energy cutoffs as given in the legend. For details see the text.

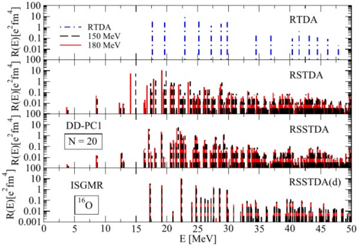
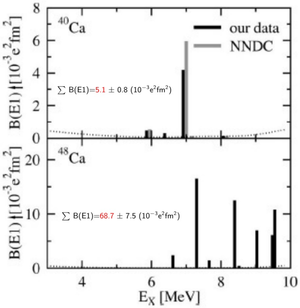


FIG. 2. Discrete ISGMR strength distribution in ^{16}O for RTDA, RSTDA, RSSTDA, and RSSTDA(d) as functions of ex for 2p-2h energy cutoffs 150 and 180 MeV.

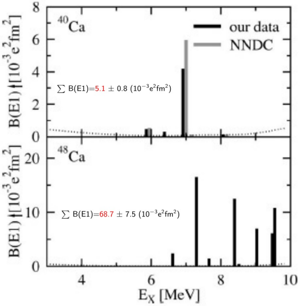
Some SSRPA applications

- Low-lying dipole response in ^{48}Ca
- Isoscalar quadrupole giant resonance
- Gamow-Teller excitations and beta-decay

Photon Scattering Data



Photon Scattering Data



Relativistic RPA results

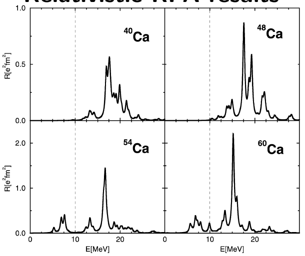
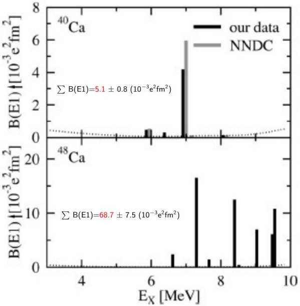


Fig. 5. RRPA isovector dipole strength distributions in Ca isotopes. The thin dashed line tentatively separates the region of giant resonances from the low-energy region below 10 MeV.

Photon Scattering Data



Relativistic RPA results

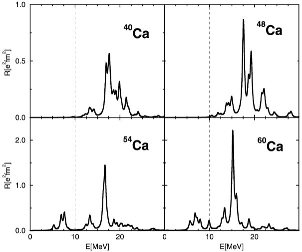
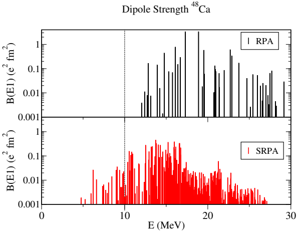


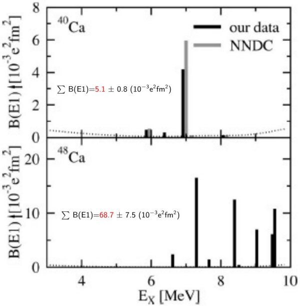
Fig. 5. RRPA isovector dipole strength distributions in Ca isotopes. The thin dashed line tentatively separates the region of giant resonances from the low-energy region below 10 MeV.

Skyrme RPA vs SRPA

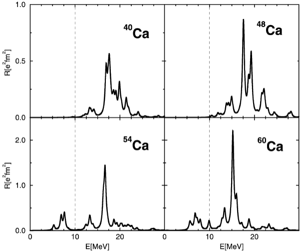


Low-lying dipole strength in ^{40,48}Ca

Photon Scattering Data



Relativistic RPA results



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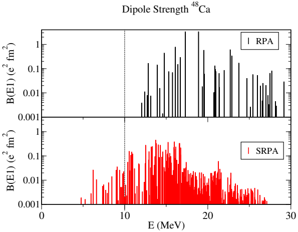
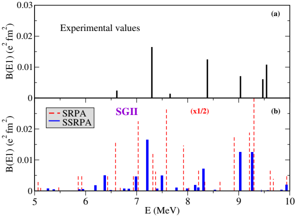
Skyrme RPA vs SRPA



SRPA, SSRPA vs Data



Total $B(E1)$ and EWSRs (From 5 to 10 MeV)

	Exp	SRPA SGII	SSRPA SGII	SRPA SLy4	SSRPA SLy4
$\sum B(E1)$	0.068 $\pm$ 0.008	0.563	0.078	1.012	0.126
$\sum_i E_i B_i(E1)$	0.570 $\pm$ 0.062	4.618	0.621	8.795	1.062

Experimental and theoretical $\sum B(E1)$ in (e^2 fm^2) and $\sum_i E_i B_i(E1)$ in (MeV e^2 fm^2) summed between 5 and 10 MeV.
From D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163–168

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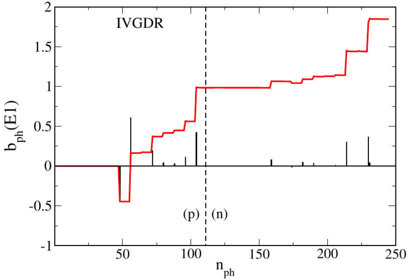
From D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163–168

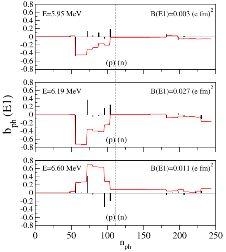
Coherence of the elementary $1p1h$ configurations in building the B(E1)

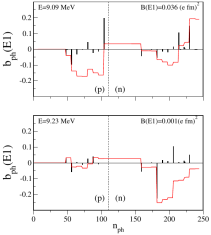
$$B^{\nu}(E\lambda) = |\sum_{ph}(X^{\nu}_{ph} - Y^{\nu}_{ph})F^{\lambda}_{ph}|^2 = |\sum_{ph} b^{\nu}_{ph}(E\lambda)|^2$$

where F^{λ}_{ph} are the dipole transition amplitudes of the operator

$$F = e\frac{N}{A}\sum_{i=1}^{Z} r_i Y_{10}(\Omega_i) - e\frac{Z}{A}\sum_{i=1}^{N} r_i Y_{10}(\Omega_i)$$







D. G. , M. Grasso, and F. Catara, Phys. Rev. C 84, 034301 (2011)

PHYSICAL REVIEW C **98**, 044313 (2018)

Systematic study of giant quadrupole resonances with the subtracted second random-phase approximation: Beyond-mean-field centroids and fragmentation

O. Vasseur,[1] D. Gambacurta,[2] and M. Grasso[1]

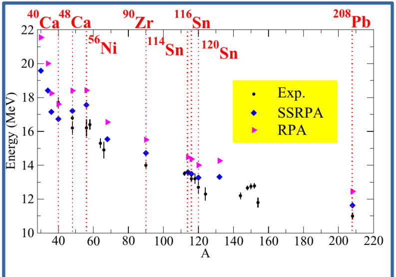


FIG. 1. Centroids of the IS GQR. The experimental data are

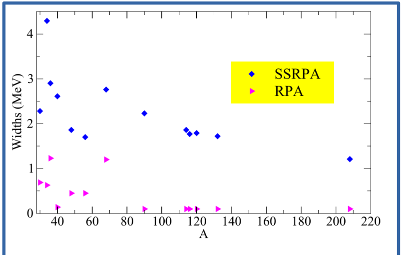
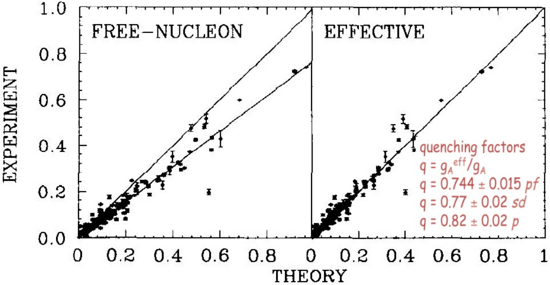


FIG. 2. Theoretical widths calculated with the fit of a Lorentzian distribution within RPA and SSRPA models.

The quenching problem

- Computed GT matrix elements **are larger** than the experimental ones.
- The problem is "cured" by **quenching** the strength by $q \sim 0.7$ or using effective axial constant g_A (~ 1) instead of the "bare" value ~ 1.27.

GT matrix elements deduced from the β decay



(from Brown & Wildenthal, Ann.Rev.Nucl. Part.Sci.**38**,(1988)29)

Shell Model calculations

The quenching problem

- Computed GT matrix elements **are larger** than the experimental ones.
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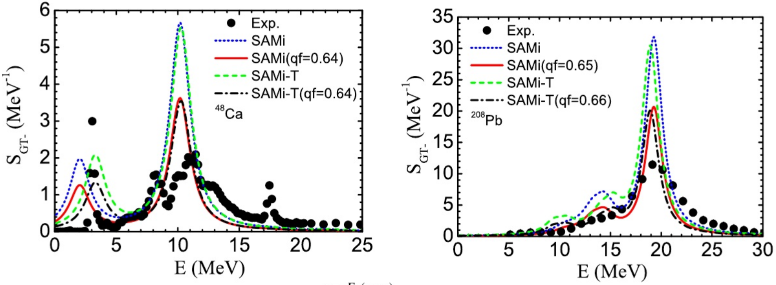


$$qf = \frac{\sum_{E_x=0}^{E_x(\max)} B(GT : E_x)_{\text{expt}}}{\sum_{E_x=0}^{E_x(\max)} B(GT)_{\text{calc}}}$$

Li-Gang Cao , Shi-Sheng Zhang, and H. Sagawa, PHYSICAL REVIEW C 100, 054324 (2019)

Skyrme-RPA calculations

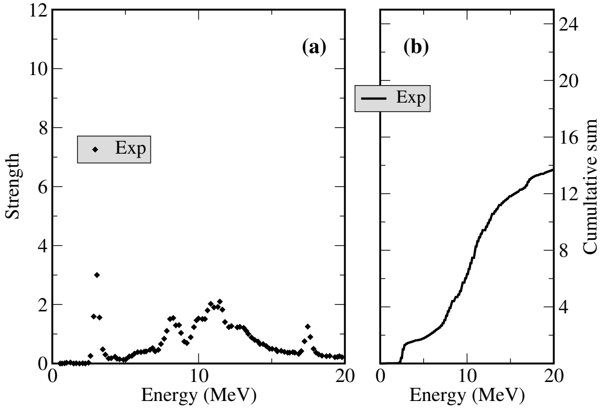
Theory vs Experiment: the quenching problem

The quenching problem

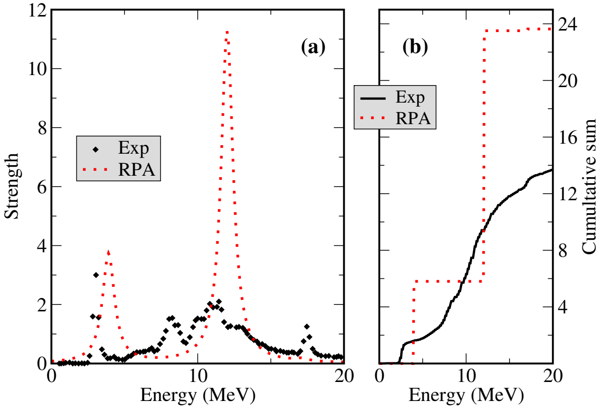
- Computed GT matrix elements **are larger** than the experimental ones.
- The problem is "cured" by **quenching** the strength by $q \sim 0.7$ or using effective axial constant g_A (~ 1) instead of the "bare" value ~ 1.27.

Possible causes fall in two main classes:

- **Nuclear many-body correlations not included in the calculations**: (truncation of the model space, short-range correlations, multi-phonon states, multi particle-hole excitations, ...)
- **Non-nucleonic degrees of freedom**: (Many-nucleon weak currents, Δ-isobar excitations, in-medium modification of pion physics, ...)



(a) GT$^-$ strength in RPA and SSRPA compared with (GT$^-$ plus IVSM) data.
(b) Cumulative strengths up to 20 MeV.
Data from: K. Yako *et al.*, Phys. Rev. Lett. 103, 012503 (2009)
From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)



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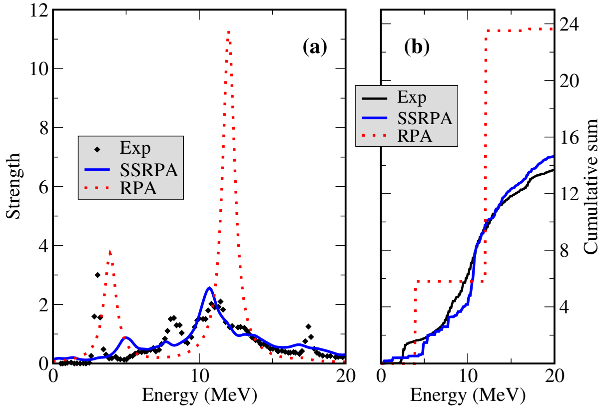

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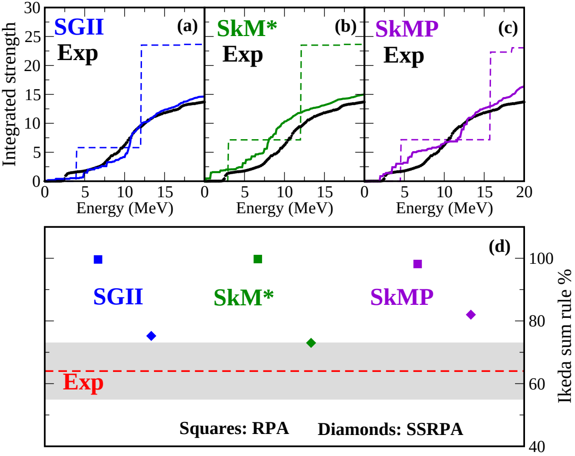
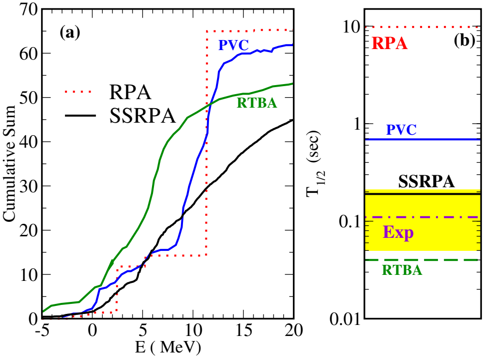
GT$^-$ Strength Distribution ^{48}Ca



(a), (b), (c) Strengths integrated up to 20 MeV with different parameterizations.

(d) RPA and SSRPA percentages of the Ikeda sum rule below 30 MeV compared with the experimental one.

From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)



(a) Cumulative sum for the nucleus ^{78}Ni within the SSRPA, PVC and RTBA models;
(b) β-decay half-life for ^{78}Ni. **No quenching, bare $g_a = 1.27$;**
Data from: P. T. Hosmer *et al.* Phys. Rev. Lett. 94, 112501 (2005)
PVC: Y. F. Niu, G. Coló and E. Vigezzi, Phys. Rev. C 90, 054328 (2014)
RTBA:C. Robin and E. Litvinova, Phys. Rev. C 98, 051301(R), 2018

From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

Conclusions and Outlook

SSRPA merits

- More general description than the RPA
- The EDF Subtracted SRPA is robust and reliable (double-counting free)
- Better description of Fragmentation, Spreading, Low-lying states

SSRPA limitations

- Numerically demanding (large matrices and number of eigenvalues)
- Extension to deformed case not (at present) viable (unless FAM + diagonal approximation)

SSRPA future work

- Applications: double-beta and -gamma decay
- Pairing, e.g. Second QRPA (SQRPA) and SSQRPA
- Comparison with PVC model (and possible merging)
- Continuum SRPA: escape and spreading width
- FT SRPA: hot nuclei (width $\sim E^*$), β-decay in astrophysical environments