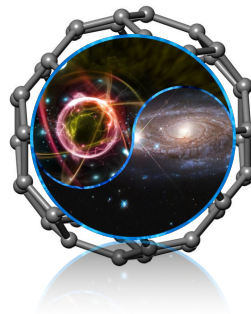




Statistical vs microscopic descriptions of transitions between excited states beyond QRPA



Elena Litvinova



Western Michigan University



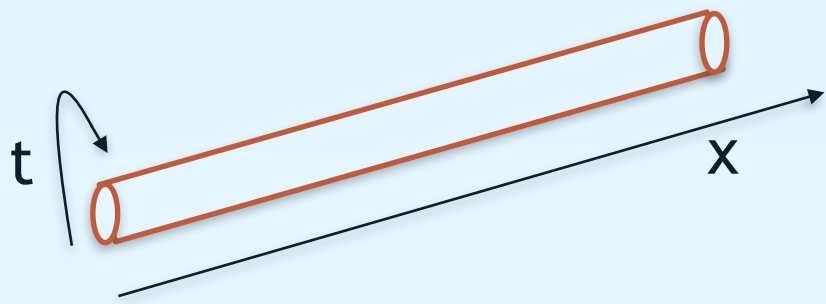
ESNT WS, CEA Saclay, July 7-9, 2026

Evolution of the Giant Dipole Resonance at $T > 0$

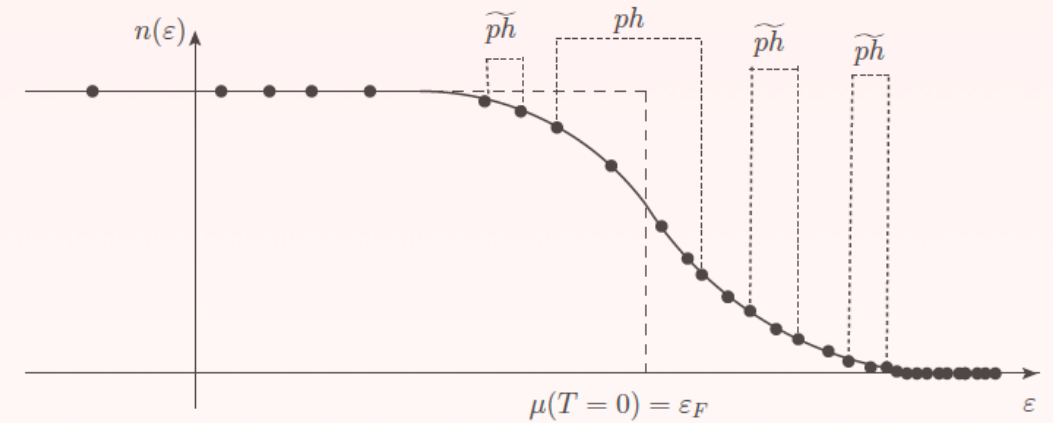
Finite temperature: *imaginary time technique*:

$$t \rightarrow i\tau, \quad -1/T < \tau < 1/T$$

$2/T$ periodic; time as a closed loop



Thermal unblocking:

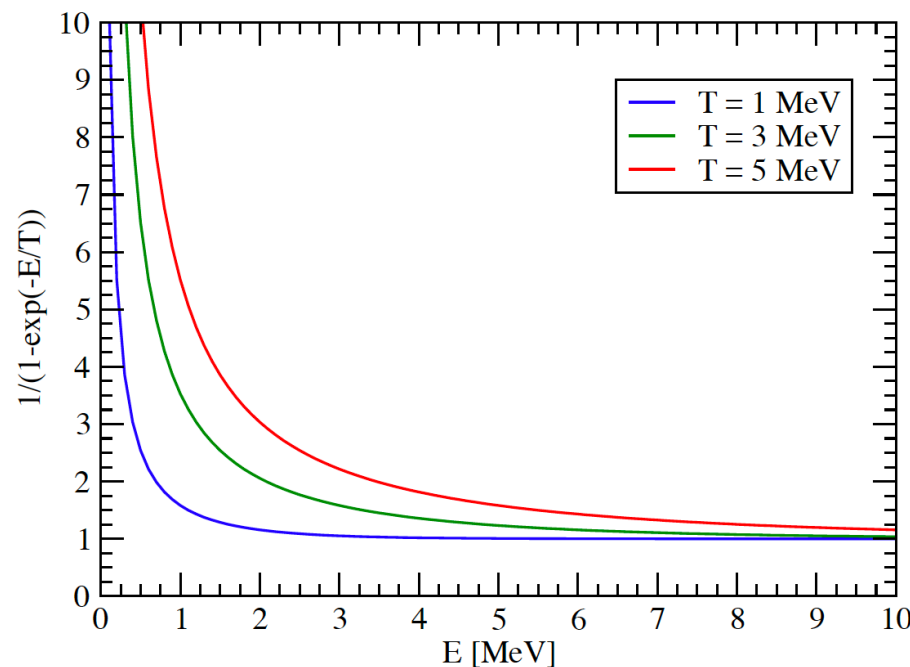


Uncorrelated propagator:

$$\tilde{R}_{14,23}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

Partial strength:

$$S(E, T) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im} \langle V^{0\dagger} \mathcal{R}(E + i\Delta, T) V^0 \rangle$$

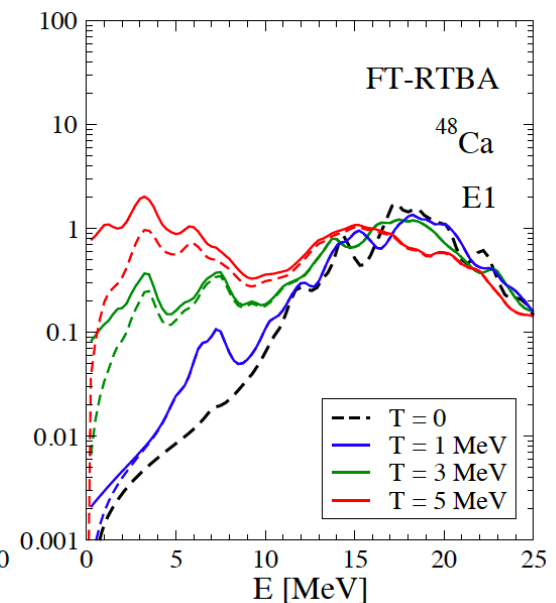
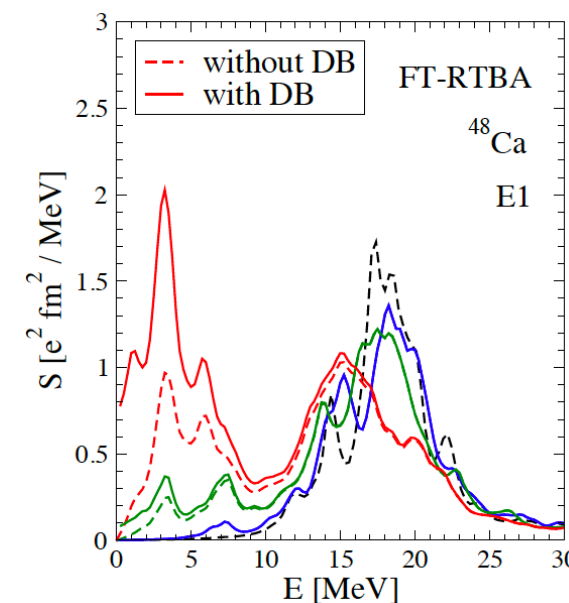


Total strength function:

$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} S(E)$$

The “Oslo upbend”

Giant Dipole resonance in ^{48}Ca



Finite-temperature response with the ph+phonon [REOM²] dynamical kernel

$$R_{12,1'2'}(t - t') = -i\langle \mathcal{T}(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t') \rangle \rightarrow -i\langle \mathcal{T}(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t') \rangle_T$$

**non-superfluid
limit**

$$\langle \dots \rangle \equiv \langle 0|\dots|0 \rangle \rightarrow \langle \dots \rangle_T \equiv \sum_n \exp\left(\frac{\Omega - E_n - \mu N}{T}\right) \langle n|\dots|n \rangle$$

averages

thermal averages

**Method: EOM
for Matsubara
Green's functions**

E.L., H. Wibowo,
PRL 121, 082501 (2018)
H. Wibowo, E.L.,
PRC 100, 024307 (2019)

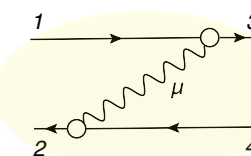
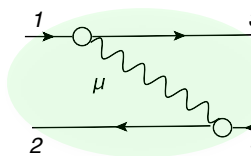
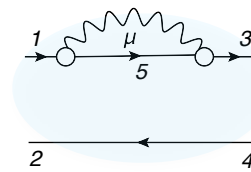
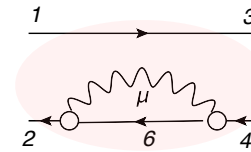
$$\begin{aligned} \mathcal{R}_{14,23}(\omega, T) &= \tilde{\mathcal{R}}_{14,23}^0(\omega, T) + \\ &+ \sum_{1'2'3'4'} \tilde{\mathcal{R}}_{12',21'}^0(\omega, T) [\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega, T)] \mathcal{R}_{3'4,4'3}(\omega, T) \\ \delta\Phi_{1'4',2'3'}(\omega, T) &= \Phi_{1'4',2'3'}(\omega, T) - \Phi_{1'4',2'3'}(0, T) \end{aligned}$$

$T > 0$:

Leading-order 1p1h+phonon dynamical kernel:

$T = 0$:

$$\begin{aligned} \Phi_{14,23}^{(ph)}(\omega, T) &= \frac{1}{n_{43}(T)} \sum_{\mu, \eta_\mu = \pm 1} \eta_\mu \left[\delta_{13} \sum_6 \gamma_{\mu;62}^{\eta_\mu} \gamma_{\mu;64}^{\eta_\mu*} \times \right. \\ &\times \frac{(N(\eta_\mu \Omega_\mu) + n_6(T))(n(\varepsilon_6 - \eta_\mu \Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_6 - \eta_\mu \Omega_\mu} + \\ &+ \delta_{24} \sum_5 \gamma_{\mu;15}^{\eta_\mu} \gamma_{\mu;35}^{\eta_\mu*} \times \\ &\times \frac{(N(\eta_\mu \Omega_\mu) + n_2(T))(n(\varepsilon_2 - \eta_\mu \Omega_\mu, T) - n_5(T))}{\omega - \varepsilon_5 + \varepsilon_2 - \eta_\mu \Omega_\mu} - \\ &- \gamma_{\mu;13}^{\eta_\mu} \gamma_{\mu;24}^{\eta_\mu*} \times \\ &\times \frac{(N(\eta_\mu \Omega_\mu) + n_2(T))(n(\varepsilon_2 - \eta_\mu \Omega_\mu, T) - n_3(T))}{\omega - \varepsilon_3 + \varepsilon_2 - \eta_\mu \Omega_\mu} - \\ &- \gamma_{\mu;31}^{\eta_\mu*} \gamma_{\mu;42}^{\eta_\mu} \times \\ &\times \left. \frac{(N(\eta_\mu \Omega_\mu) + n_4(T))(n(\varepsilon_4 - \eta_\mu \Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_4 - \eta_\mu \Omega_\mu} \right], \end{aligned}$$



$$\Phi_{14,23}^{(ph,ph)}(\omega) = \sum_{\mu} \times$$

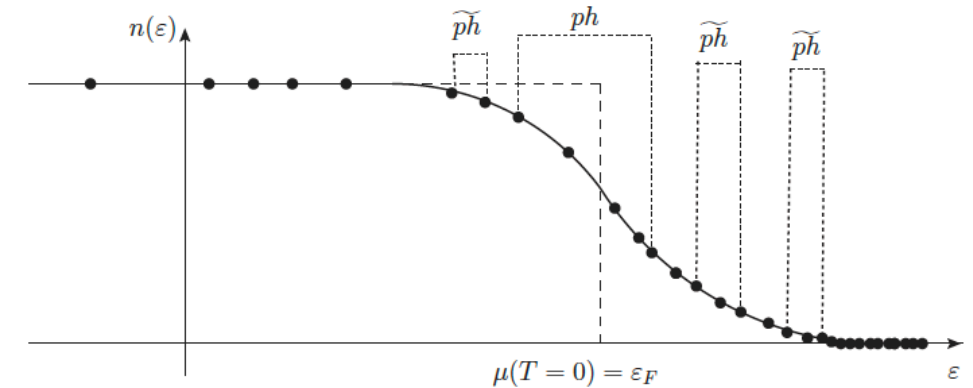
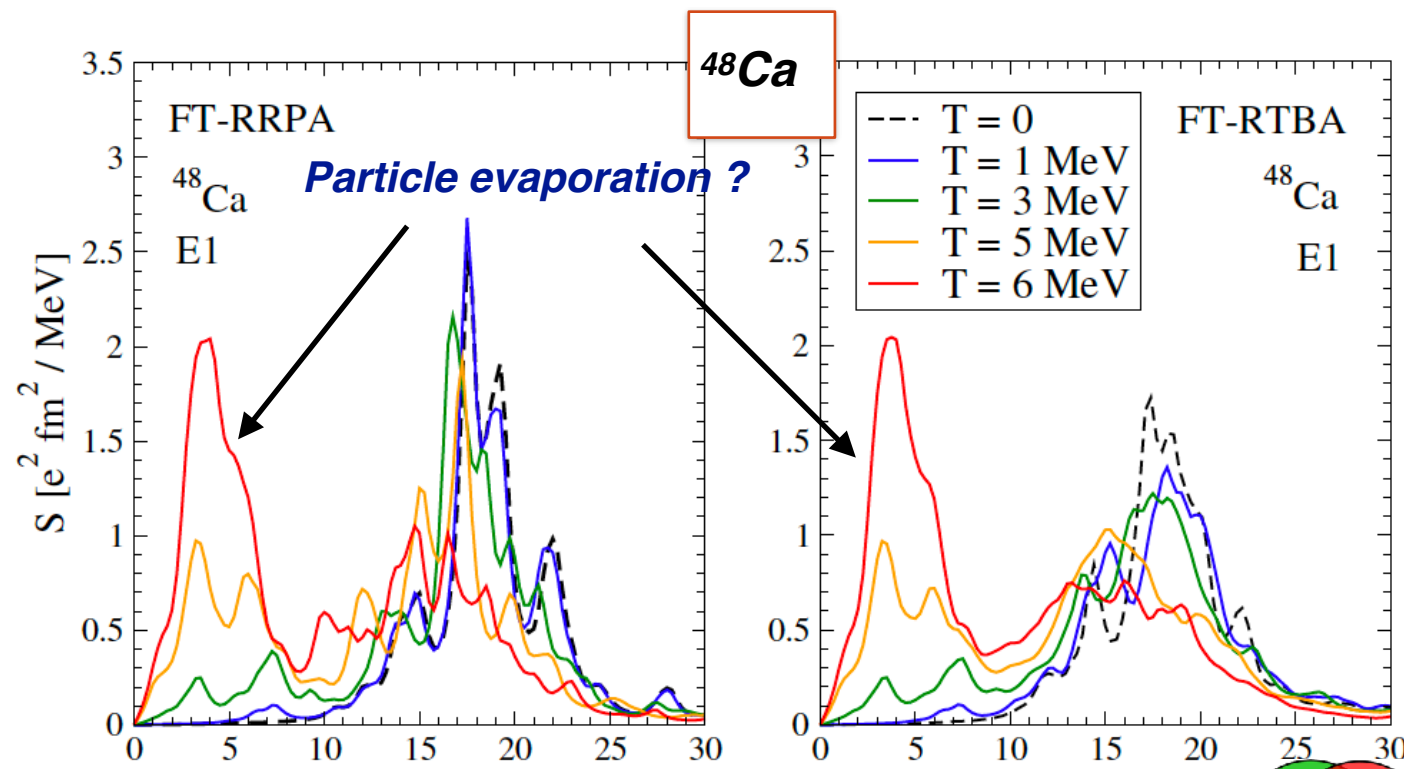
$$\begin{aligned} &\times \left[\delta_{13} \sum_6 \frac{\gamma_{62}^{\mu} \gamma_{64}^{\mu*}}{\omega - \varepsilon_1 + \varepsilon_6 - \Omega_\mu} + \right. \\ &+ \delta_{24} \sum_5 \frac{\gamma_{15}^{\mu} \gamma_{35}^{\mu*}}{\omega - \varepsilon_5 + \varepsilon_2 - \Omega_\mu} - \\ &- \frac{\gamma_{13}^{\mu} \gamma_{24}^{\mu*}}{\omega - \varepsilon_3 + \varepsilon_2 - \Omega_\mu} - \\ &- \left. \frac{\gamma_{31}^{\mu*} \gamma_{42}^{\mu}}{\omega - \varepsilon_1 + \varepsilon_4 - \Omega_\mu} \right] \end{aligned}$$

The GDR collectivity puzzle: Dipole Strength at $T > 0$: ^{48}Ca and ^{132}Sn

Static only (FT-RRPA)

Static + dynamic (FT-RTBA)

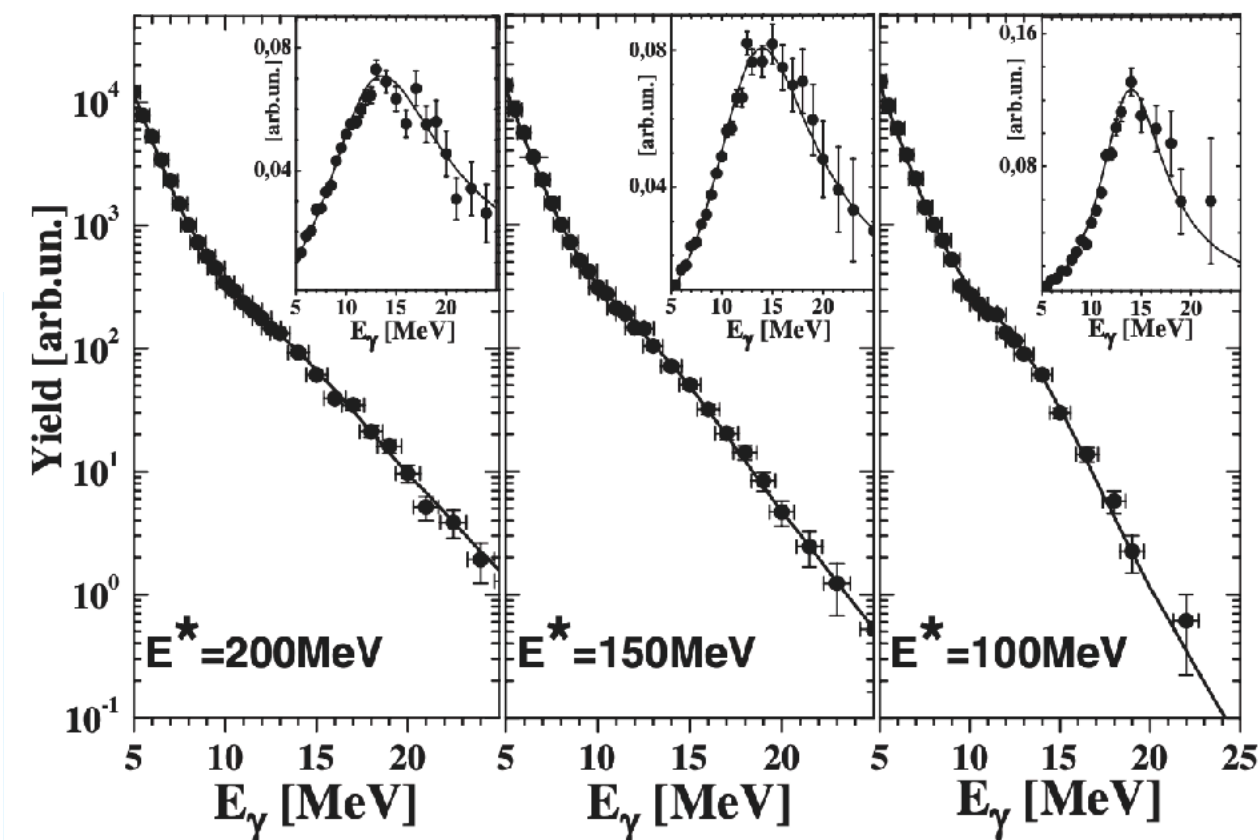
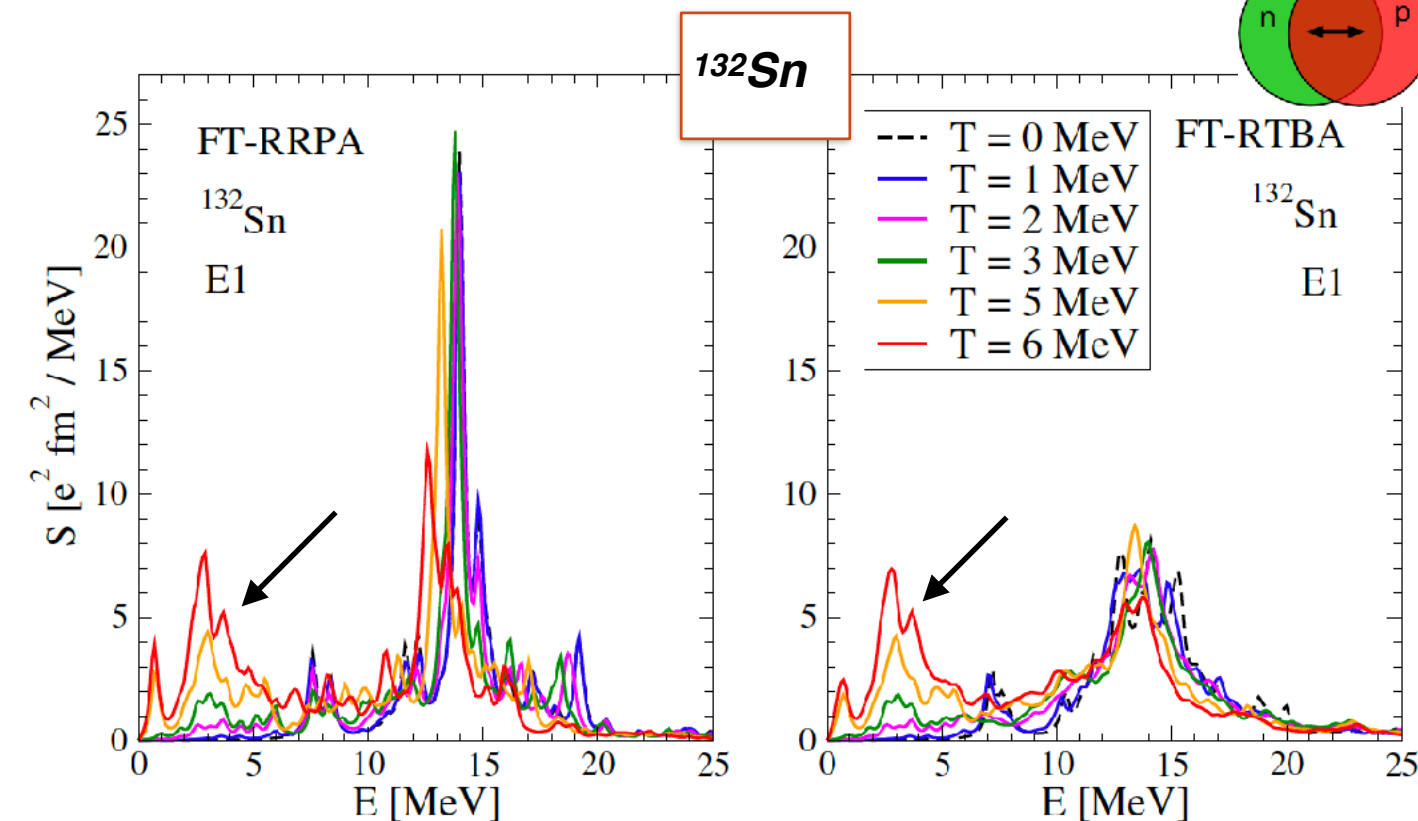
Thermal unblocking:



0th approximation:
Uncorrelated propagator

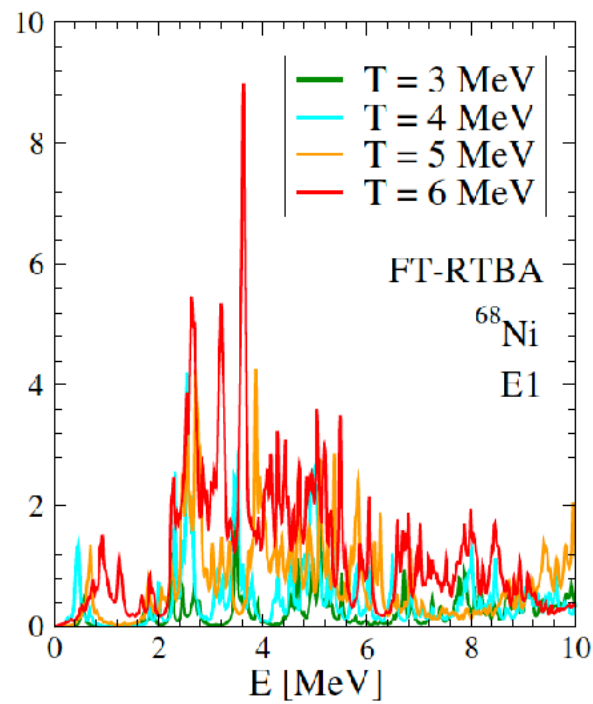
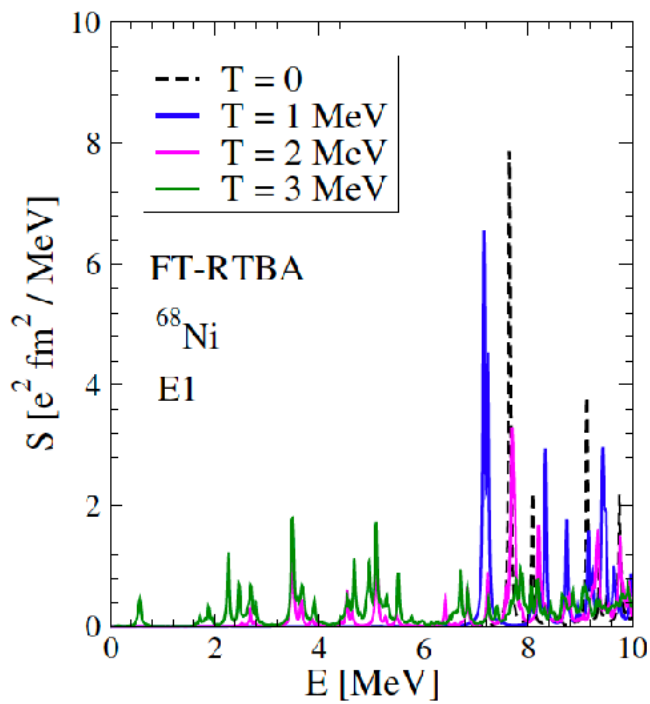
$$\tilde{R}_{14,23}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

O. Wieland et al., PRL 97, 012501 (2006):
GDR in ^{132}Ce

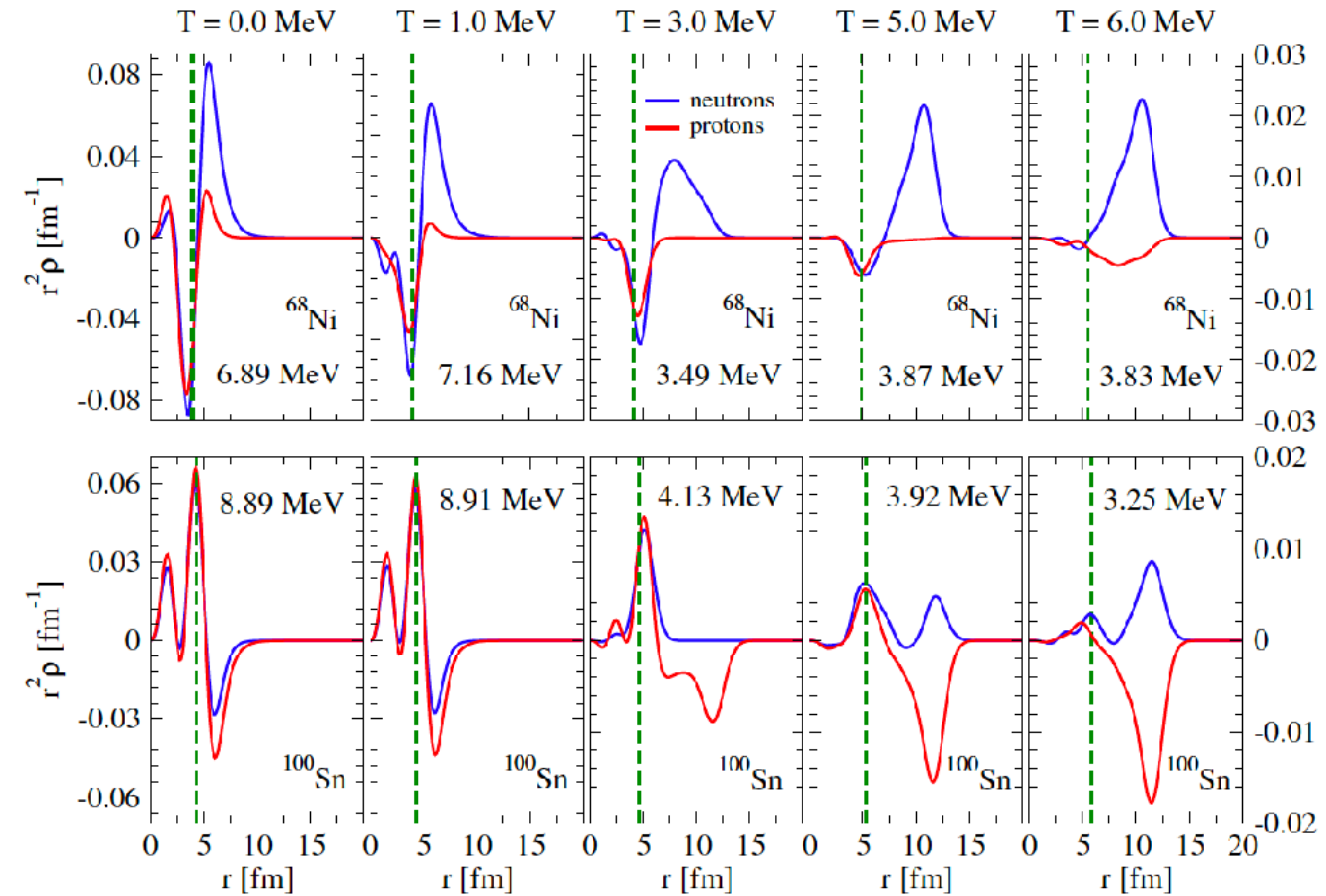


Evolution of the pygmy dipole resonance (PDR) at $T > 0$ and the GDR width

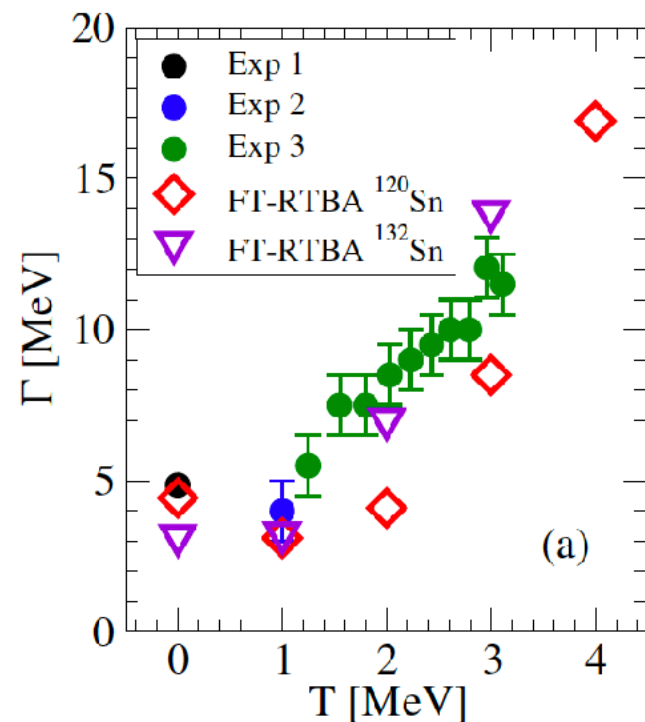
Low-energy strength distribution in ^{68}Ni



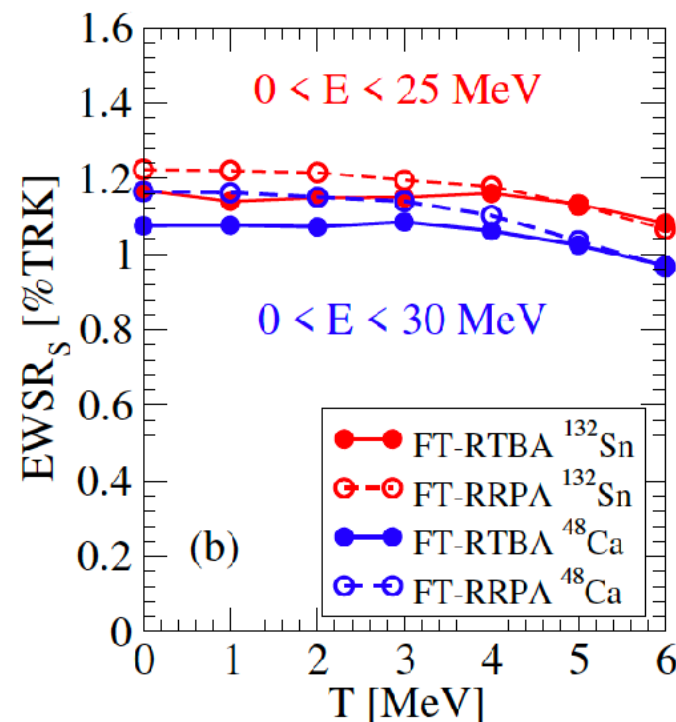
Transition density for the low-energy peak in ^{68}Ni , ^{100}Sn



GDR's width



Energy-weighted sum rule



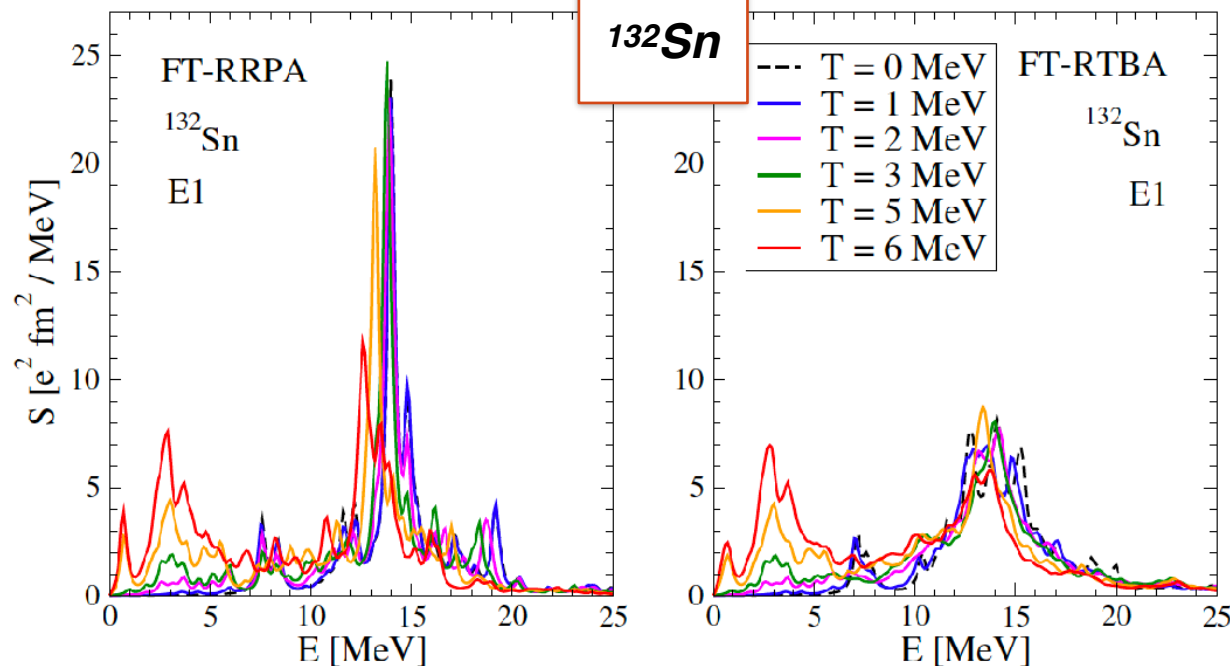
- The low-energy peak (PDR) gains the strength from the GDR with the temperature growth: $\text{EWSR} \sim \text{const}$
- The total width $\Gamma \sim T^2$ (as in the Landau theory); shape fluctuations are missing for $T \sim 2-3$ MeV
- The PDR develops a new type of collectivity originated from the thermal unblocking
- The same happens with other low-lying modes (2^+ , 3^- , ...) => strong PVC => "destruction" of the GDR at high temperatures

E.L., H. Wibowo, *Phys. Rev. Lett.* 121, 082501 (2018).
H. Wibowo, E.L., *Phys. Rev. C* 100, 024307 (2019).

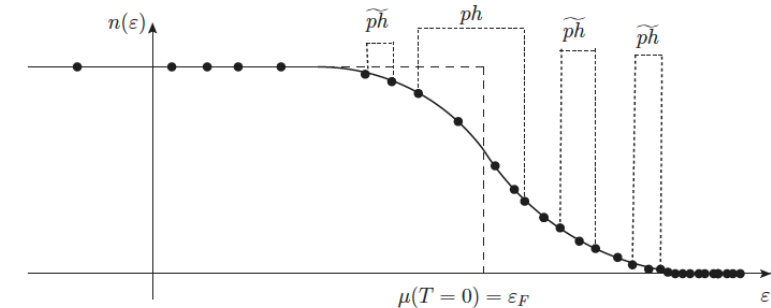
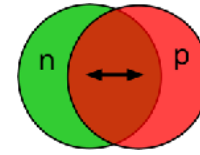
Finite temperature theory ($T > 0$): REOM² ^{48}Ca and ^{132}Sn

**Static only
(FT-REOM1/RRPA)**

**Static + dynamic
(FT-REOM2/RTBA)**



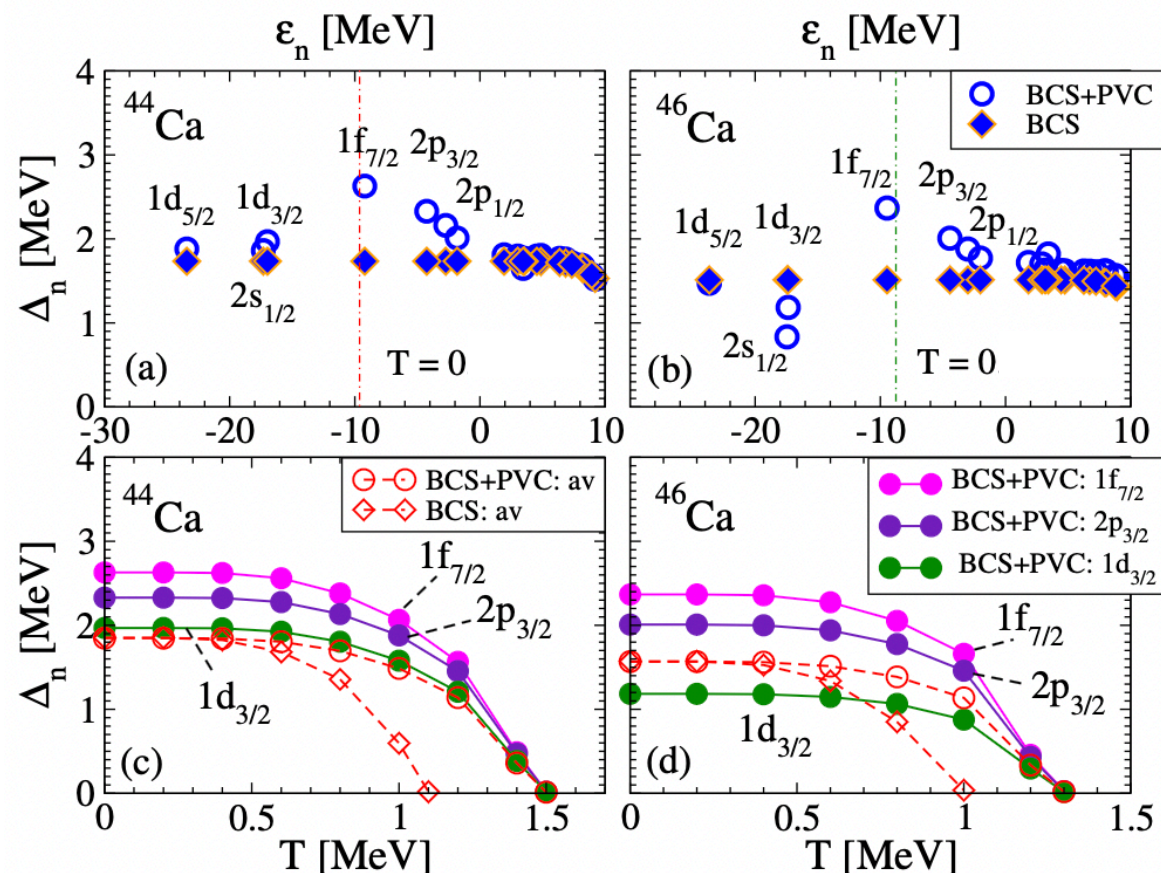
Thermal unblocking mechanism (simplified):



0th approximation:
Uncorrelated propagator

$$\tilde{R}_{14,23}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

Critical temperature: modified by qPVC



- New transitions due to the thermal unblocking effects
- More collective and non-collective modes contribute to the PVC self-energy (~400 modes at $T=5-6$ MeV)
- Broadening of the resulting GDR spectrum
- Development of the low-energy part => a feedback to GDR

- The spurious translation mode is properly decoupled as the mean field is modified consistently
- The role of the new terms in the Φ amplitude increases with temperature
- The role of dynamical correlations and fragmentation remain significant in the high-energy part

E.L., H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)

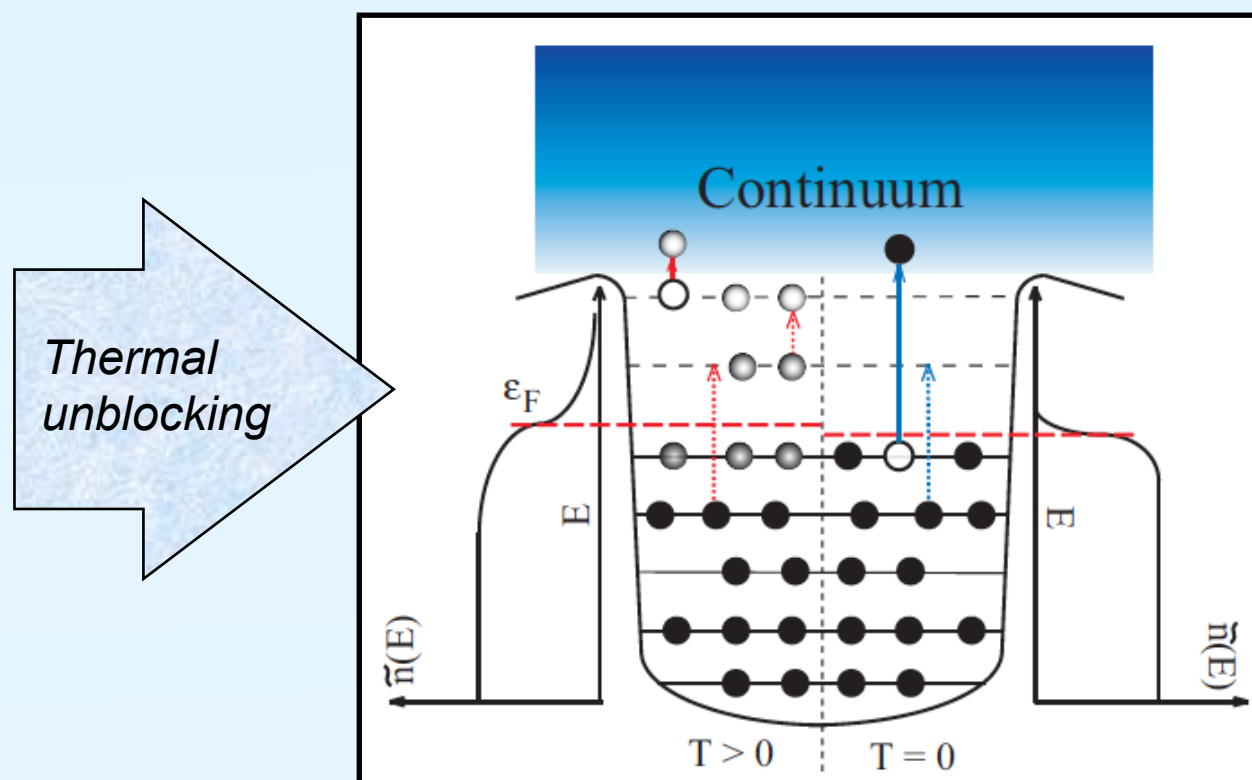
E.L., P. Schuck, Phys. Rev. C 104, 044330 (2021)

Continuum effects

- Theory including the continuum, QVC and superfluid pairing at $T=0$:
E.L., V.I. Tselyaev, PRC 75, 054318 (2007):

Continuum is mostly important for light nuclei

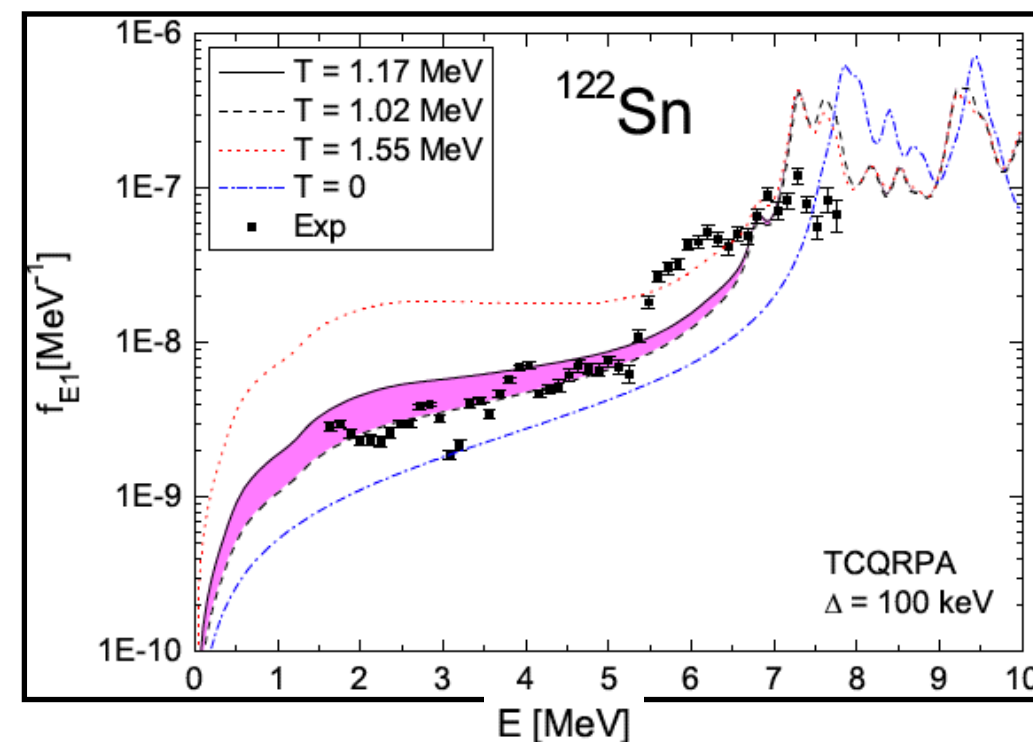
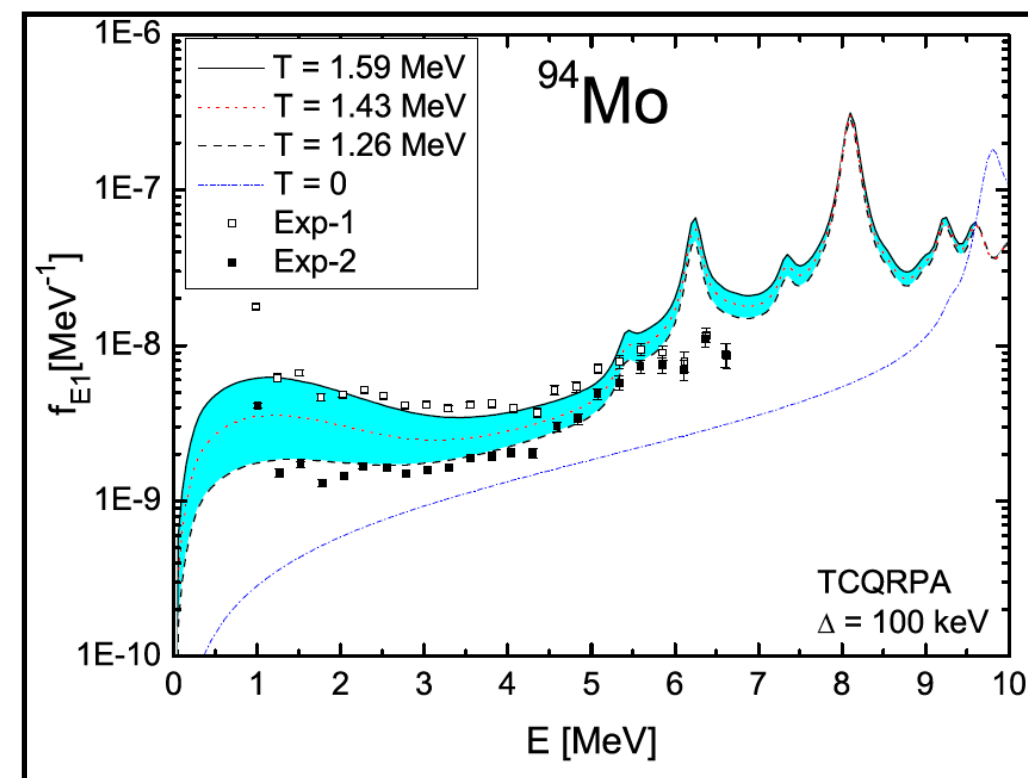
- However, the continuum effects become important also in heavy systems *at finite temperatures*:
excited compound nuclei (CN), for instance, after the thermal neutron capture



Theory: E. Litvinova, N. Belov, PRC 88, 031302(R)(2013)

Exp.: Oslo data M. Guttormsen et al., PRC 71, 044307 (2005),
S. Goriely et al., PRC 78, 064307 (2008)

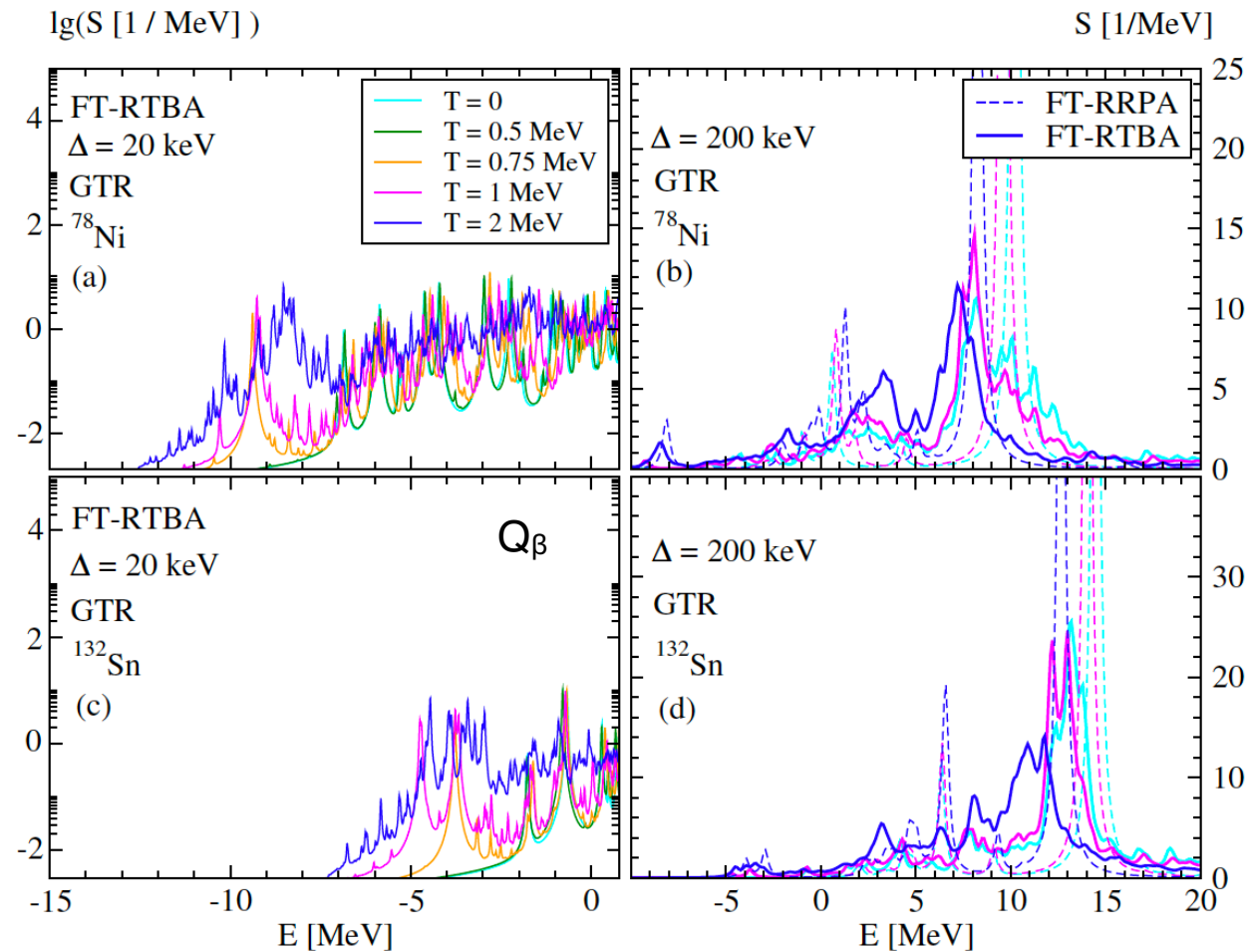
*Low-energy limit of the
radiative dipole strength functions*



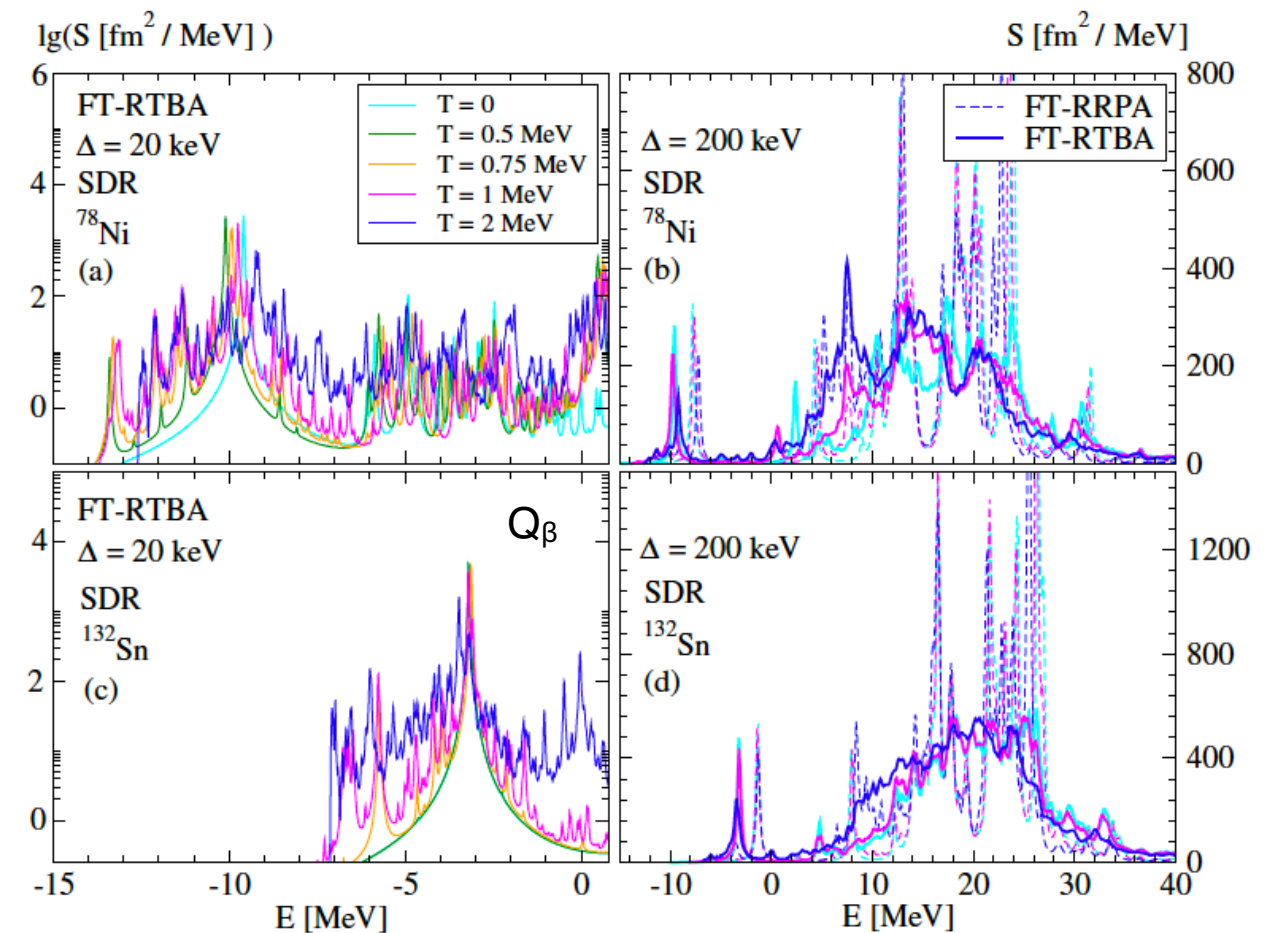
*Exponential factor not included;
QVC not included; to be done!*

The “Astro” puzzles: Spin-Isospin response and beta decay in hot stellar environments

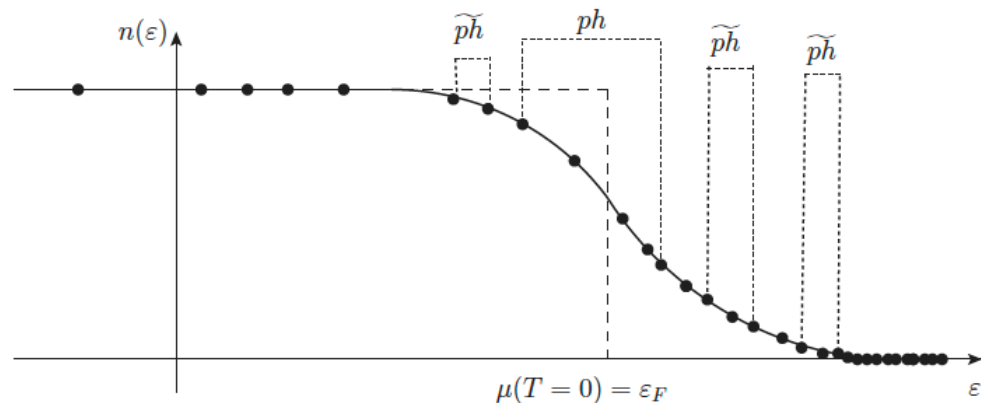
Gamow-Teller GT_{-} response of ^{78}Ni and ^{132}Sn



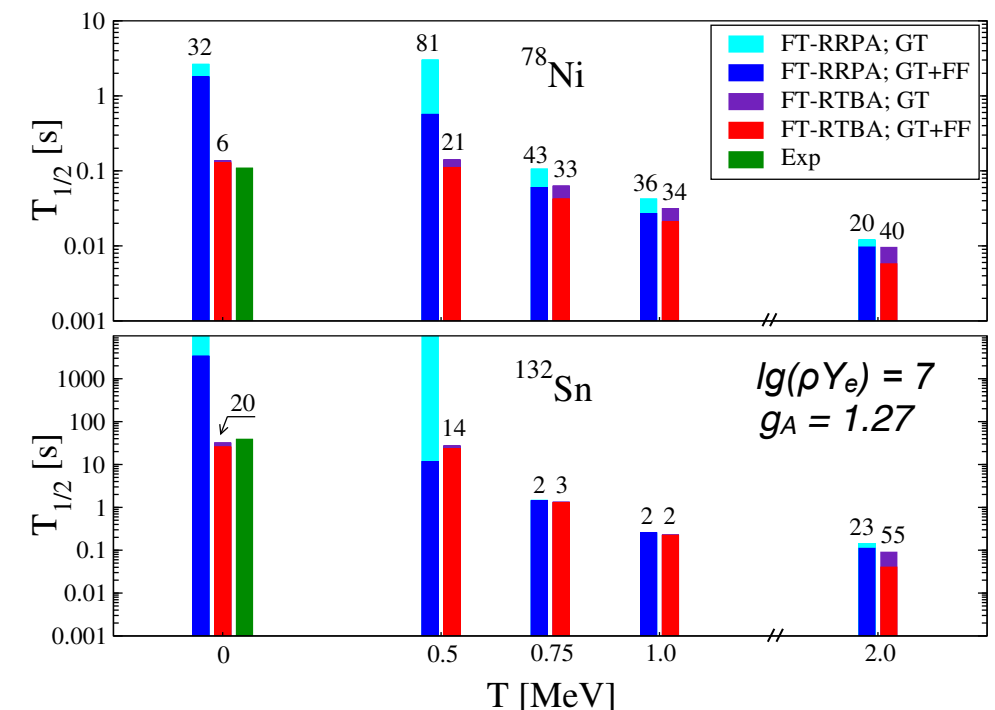
Spin Dipole response of ^{78}Ni and ^{132}Sn



Thermal unblocking mechanism:



Beta decay half-lives in a hot stellar environment



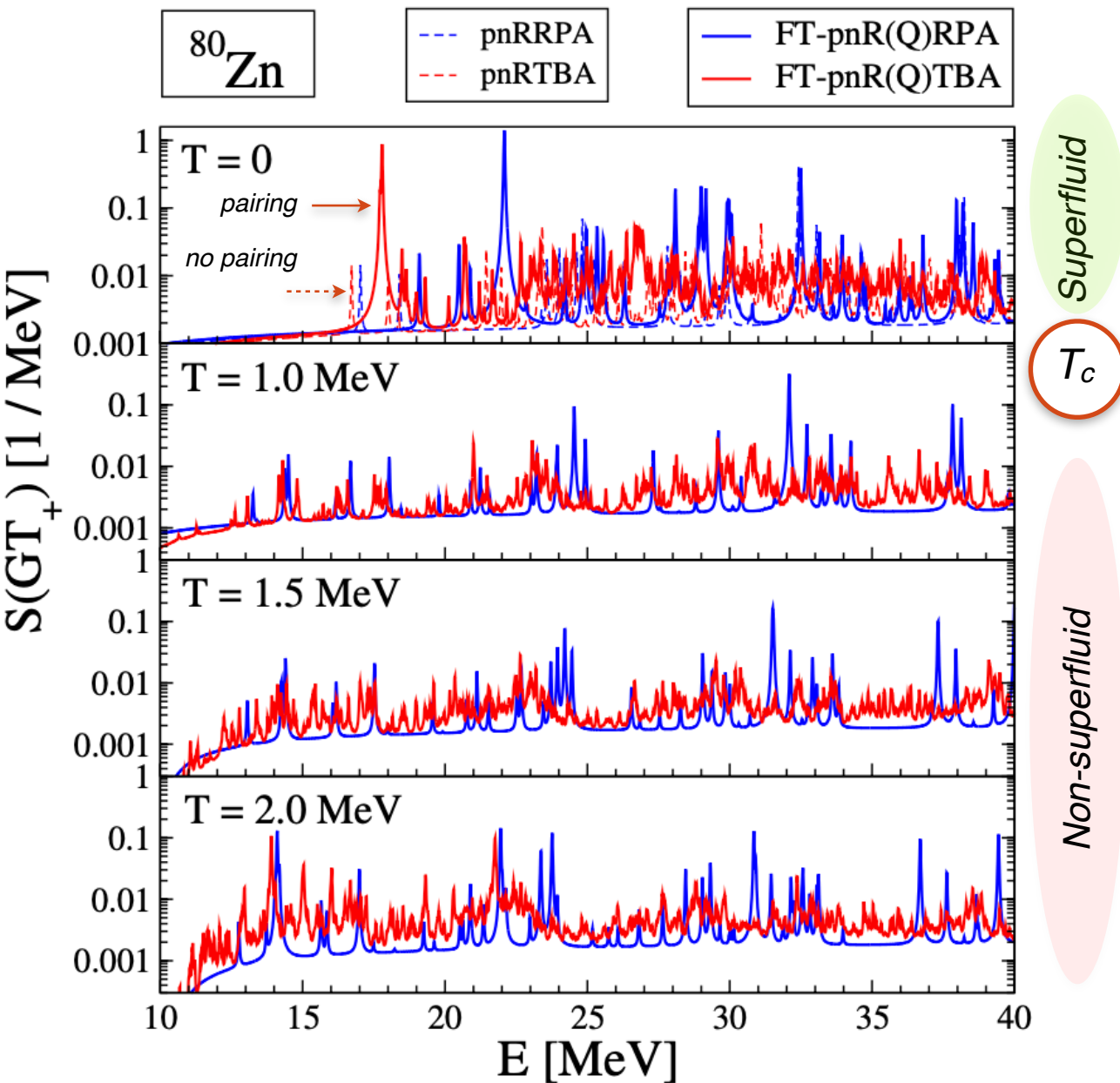
E. Litvinova, H. Wibowo, *Phys. Rev. Lett.* 121, 082501 (2018)

H. Wibowo, E. Litvinova, *Phys. Rev. C* 100, 024307 (2019)

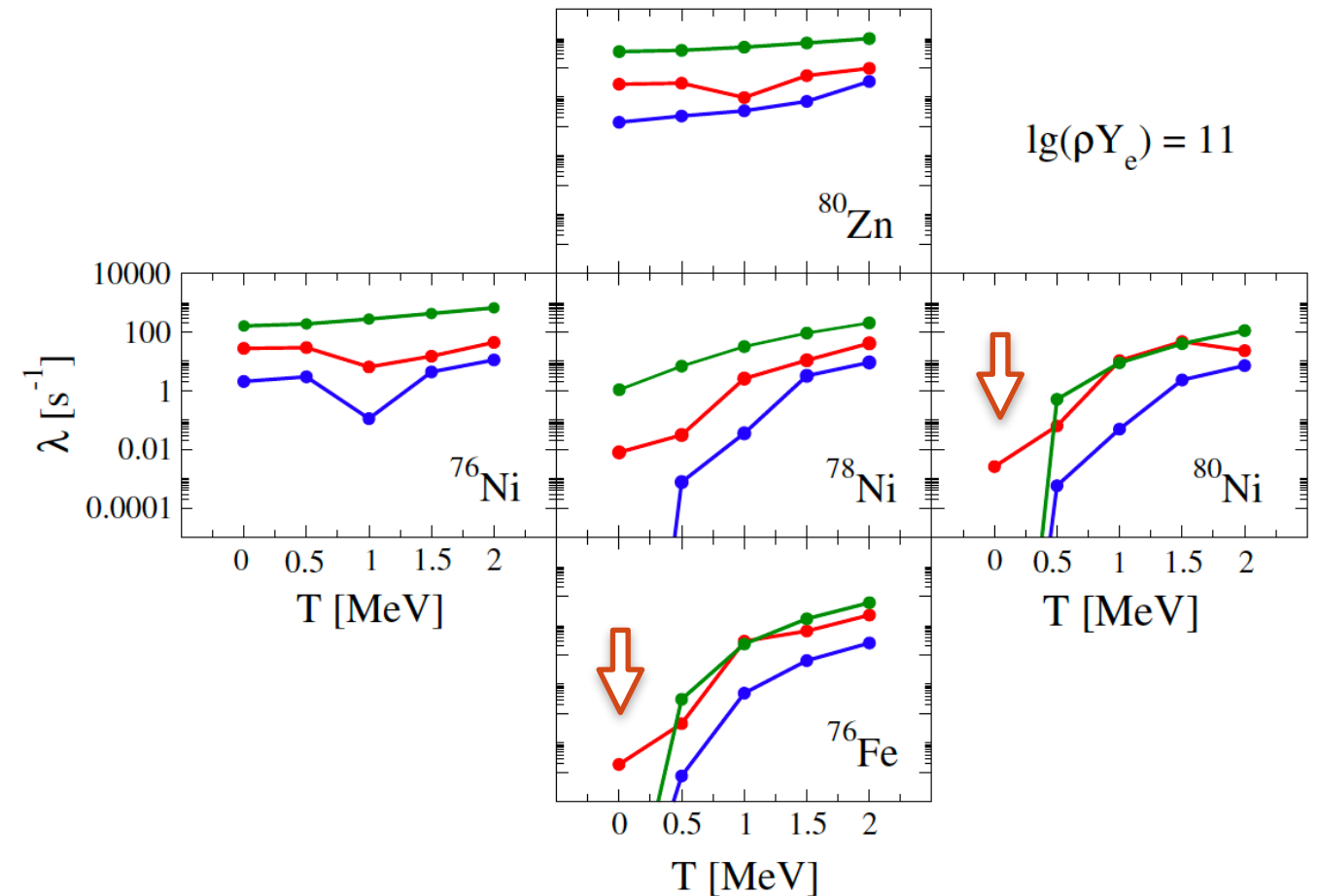
E. Litvinova, C. Robin, H. Wibowo, *Phys. Lett. B* 800, 135134 (2020)

GT+ response and electron capture (EC) rates at $T > 0$: the neighborhood of ^{78}Ni

GT+ response

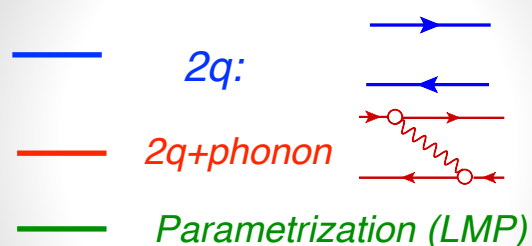


Electron capture rates around ^{78}Ni



Interplay of superfluidity and collective effects
in core-collapse supernovae:

- Amplifies the EC rates and, consequently,
- Reduces the electron-to-baryon ratio leading to lower pressure
- Promotes the gravitational collapse
- Increases the neutrino flux and effective cooling
- Allows heavy nuclei to survive the collapse



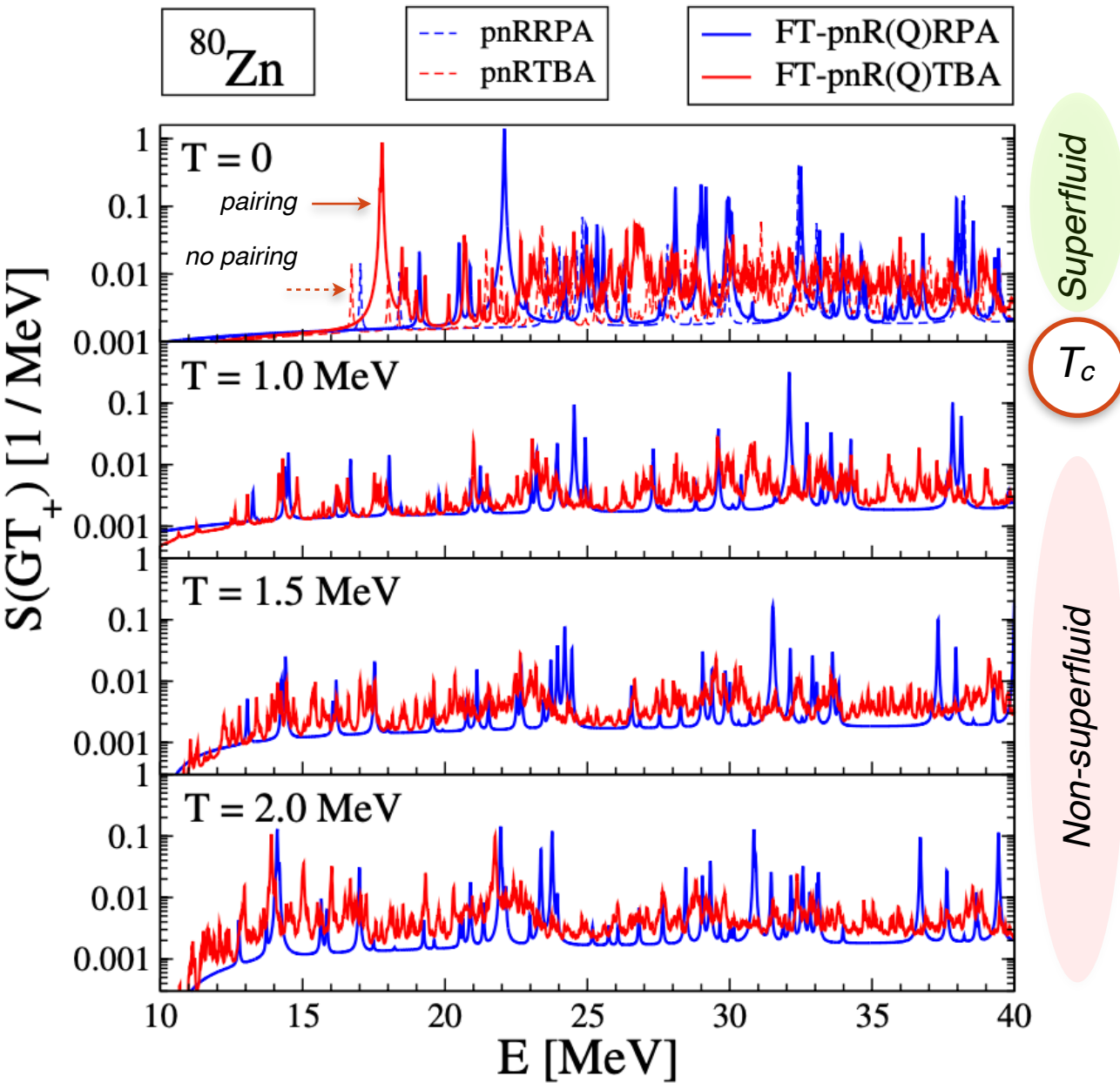
E.L., C. Robin, H. Wibowo, PLB 800,
135134 (2020)

E.L., P. Schuck, PRC 104, 044330 (2021)

E.L., C. Robin, PRC 103, 024326 (2021)

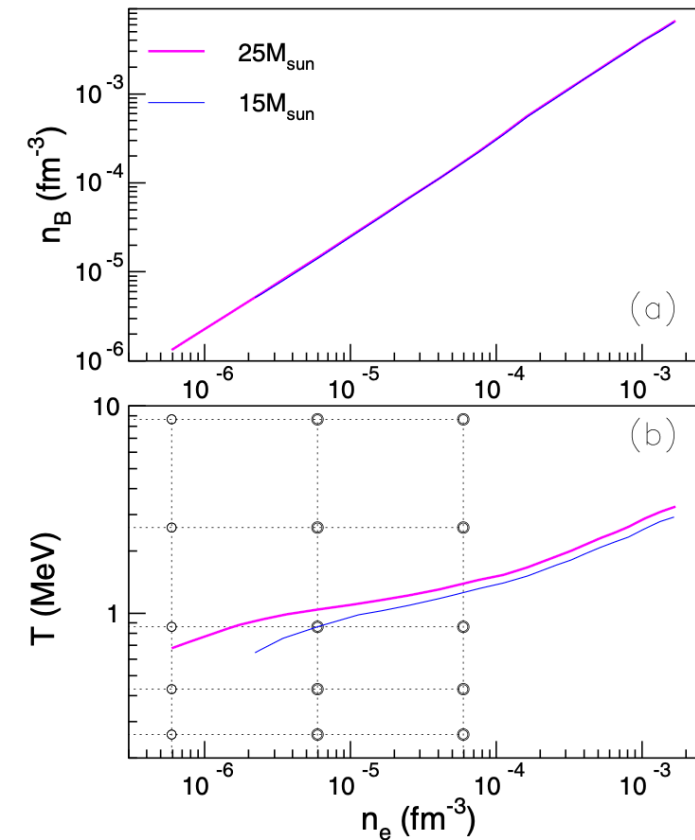
GT+ response and electron capture (EC) rates at $T > 0$: the neighborhood of ^{78}Ni

GT+ response



Density-temperature trajectory [Raduta et al., PRC2017]

PHYSICAL REVIEW C **95**, 025805 (2017)

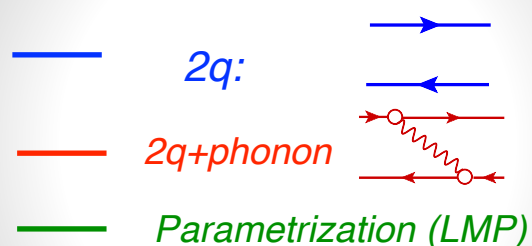


EC rate (GT+ only):

$$\lambda_{A,Z}(\mu_e, T) = \frac{g_A^2 \ln 2}{K} \int_1^\infty d\varepsilon_e \varepsilon_e p_e S_e(\varepsilon_e, \mu_e, T) F(A, Z, \varepsilon_e) \times \int_{-\infty}^{\varepsilon_e} dE (\varepsilon_e - E)^2 S_{\text{GT}+}(E, T), \quad (22)$$

Convergence :

at 2.59 MeV $E \sim 60 \text{ MeV}$



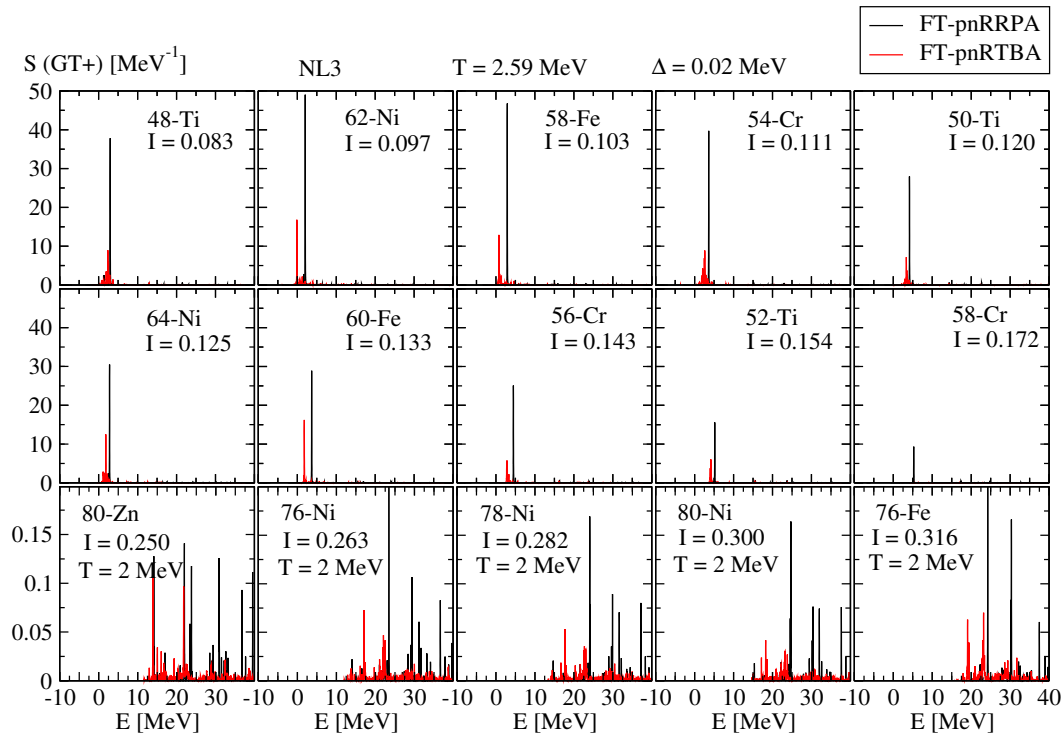
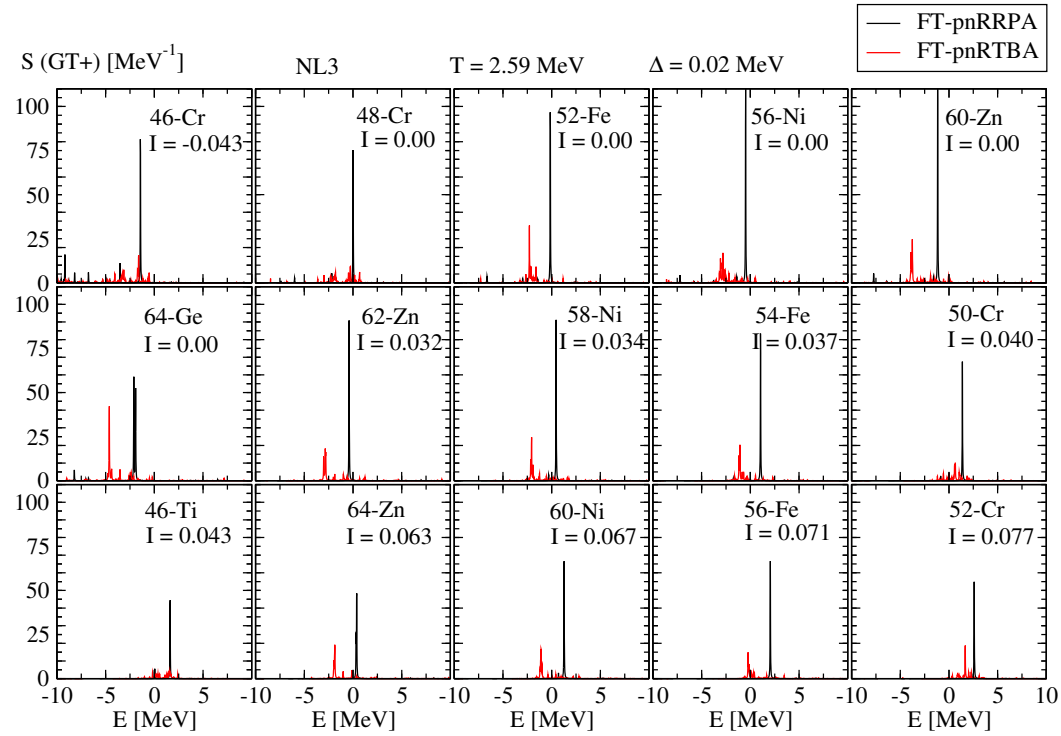
E.L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)

E.L., P. Schuck, PRC 104, 044330 (2021)

E.L., C. Robin, PRC 103, 024326 (2021)

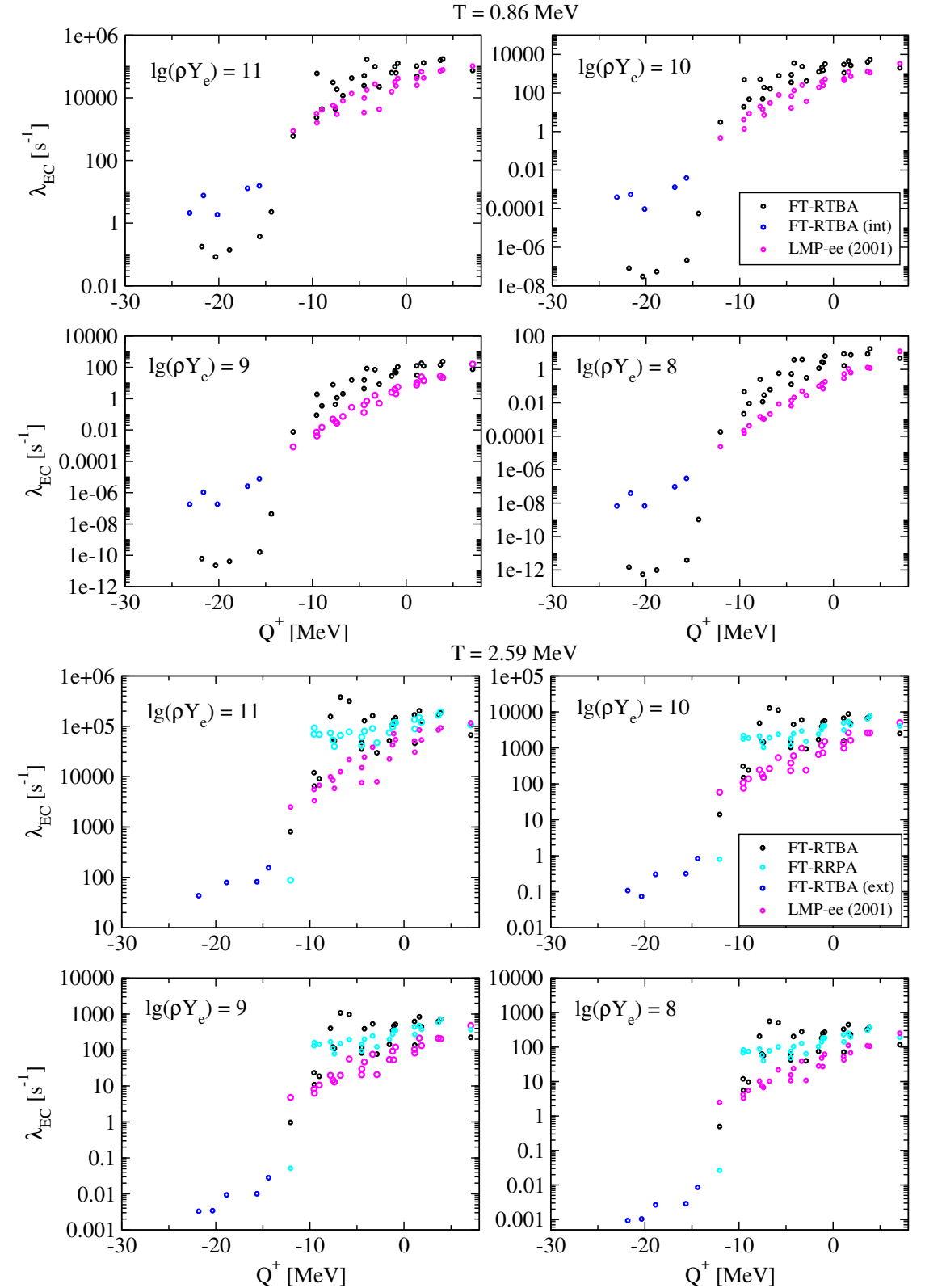
Large-scale calculations beyond (Q)RPA: relativistic PVC for GT^+

GT^+ strength:



In progress: analysis of model dependencies and comparison to available data on GT^+

EC rates for SSCN:



Temperature evolution of the “fluffiness” puzzle

$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} S(E)$$

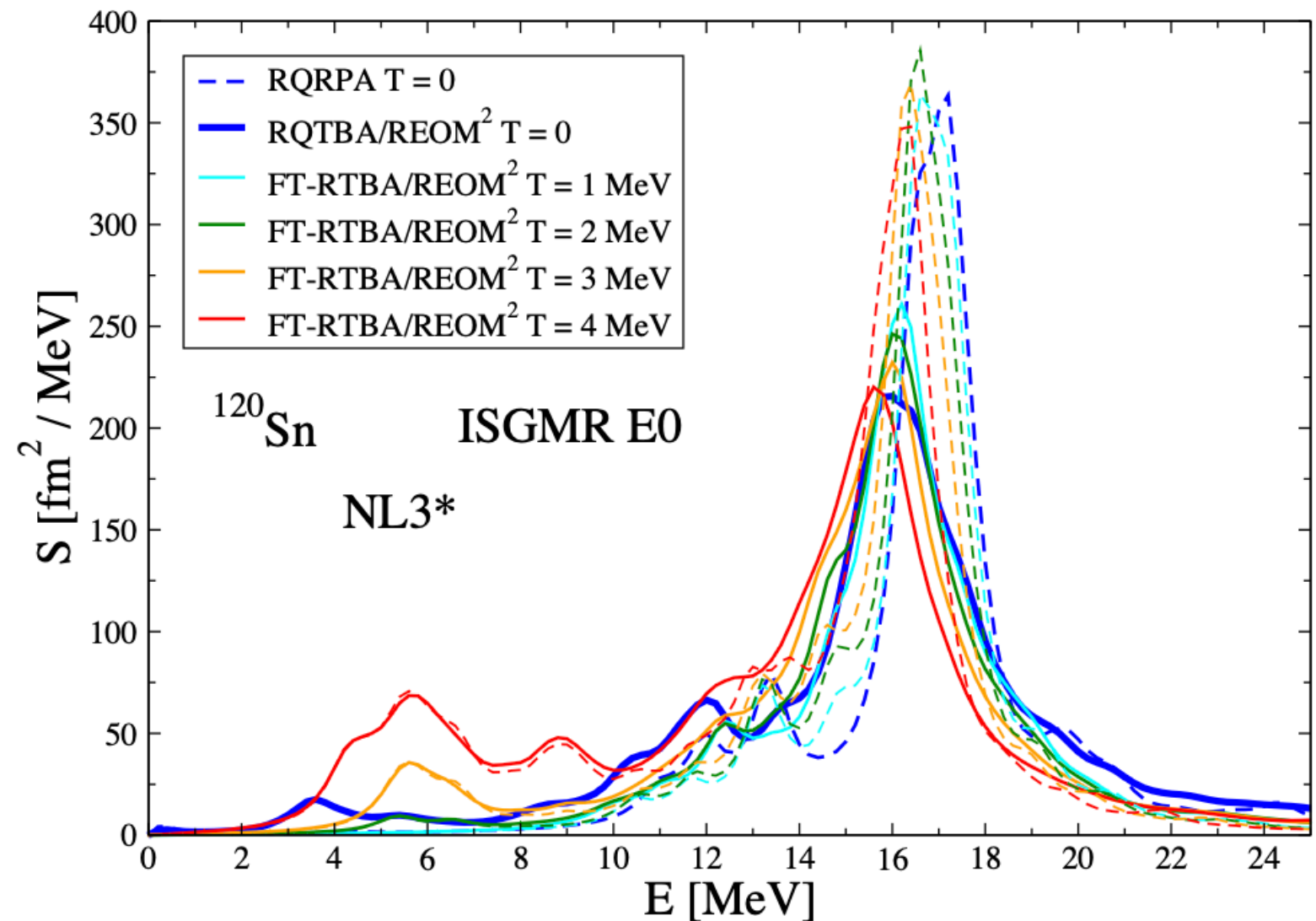
Not included here

Centroid

$$E_{\text{ISGMR}} = \sqrt{\frac{\hbar^2 A K_A}{m \langle r^2 \rangle_0}}$$

(In)compressibility:

$$K_A = K_\infty + K_{\text{surf}} A^{-1/3} + K_\tau \left(\frac{N - Z}{A} \right)^2 + K_{\text{Coul}} Z^2 A^{-1/3}$$



Centroid shift:

$$\Delta E = E(\text{RQRPA}) - E(\text{REOM}^2)$$

Decreases with temperature

Finite-temperature superfluid formalism beyond QRPA: Bridging the gap $0 < T < T_c$

Theory is formulated in the HFB basis keeping 4x4 block matrix structure

superfluid

Correlated 2q-propagator
with Matsubara frequencies

S. Bhattacharjee, E.L., arXiv:2412.20751,
to appear in PRC (2026)

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}(\omega_n) = \hat{\mathcal{R}}_{\mu\mu'\nu\nu'}^0(\omega_n) + \frac{1}{4} \sum_{\gamma\gamma'\delta\delta'} \hat{\mathcal{R}}_{\mu\mu'\gamma\gamma'}^0(\omega_n) \hat{\mathcal{K}}_{\gamma\gamma'\delta\delta'}(\omega_n) \hat{\mathcal{R}}_{\delta\delta'\nu\nu'}(\omega_n)$$

$$\mathcal{R}_{\mu\mu'\nu\nu'}^{[lk]}(\omega) = \sum_{fi} \sum_{\sigma=\pm} \sigma p_i \frac{\mathcal{Z}_{\mu\mu'}^{if[l\sigma]} \mathcal{Z}_{\nu\nu'}^{if[k\sigma]*}}{\omega - \sigma\omega_{fi}}$$

Uncorrelated 2q-propagator

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}^0(\omega_n) = [\omega_n - \hat{\Sigma}_3 \hat{E}_{\mu\mu'}]^{-1} \hat{\mathcal{N}}_{\mu\mu'\nu\nu'}$$

The norm matrix

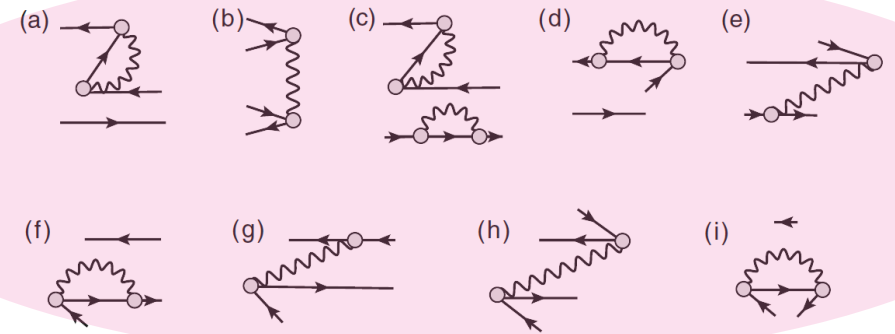
$$\hat{\mathcal{N}}_{\mu\mu'\nu\nu'} =$$

$$\left\langle \begin{pmatrix} [A_{\mu\mu'}, A_{\nu\nu'}^\dagger] & [A_{\mu\mu'}, A_{\nu\nu'}] & [A_{\mu\mu'}, C_{\nu\nu'}^\dagger] & [A_{\mu\mu'}, C_{\nu\nu'}] \\ [A_{\mu\mu'}^\dagger, A_{\nu\nu'}^\dagger] & [A_{\mu\mu'}^\dagger, A_{\nu\nu'}] & [A_{\mu\mu'}^\dagger, C_{\nu\nu'}^\dagger] & [A_{\mu\mu'}^\dagger, C_{\nu\nu'}] \\ [C_{\mu\mu'}, A_{\nu\nu'}^\dagger] & [C_{\mu\mu'}, A_{\nu\nu'}] & [C_{\mu\mu'}, C_{\nu\nu'}^\dagger] & [C_{\mu\mu'}, C_{\nu\nu'}] \\ [C_{\mu\mu'}^\dagger, A_{\nu\nu'}^\dagger] & [C_{\mu\mu'}^\dagger, A_{\nu\nu'}] & [C_{\mu\mu'}^\dagger, C_{\nu\nu'}^\dagger] & [C_{\mu\mu'}^\dagger, C_{\nu\nu'}] \end{pmatrix} \right\rangle$$

$$\langle O \rangle = \sum_i p_i \langle i | O | i \rangle \quad p_i = \frac{e^{-E_i/T}}{\sum_j e^{-E_j/T}}$$

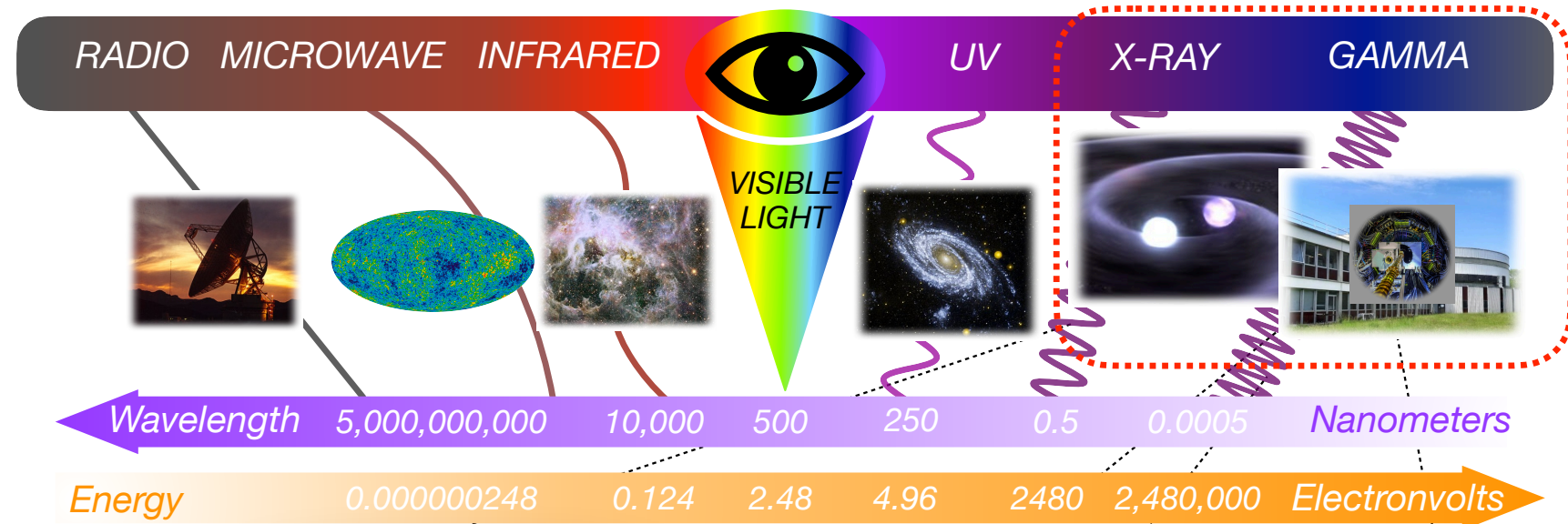
The complete superfluid dynamical
qPVC kernel

**Complex GSC: a source of “natural” quenching
at both $T=0$ and $T>0$**

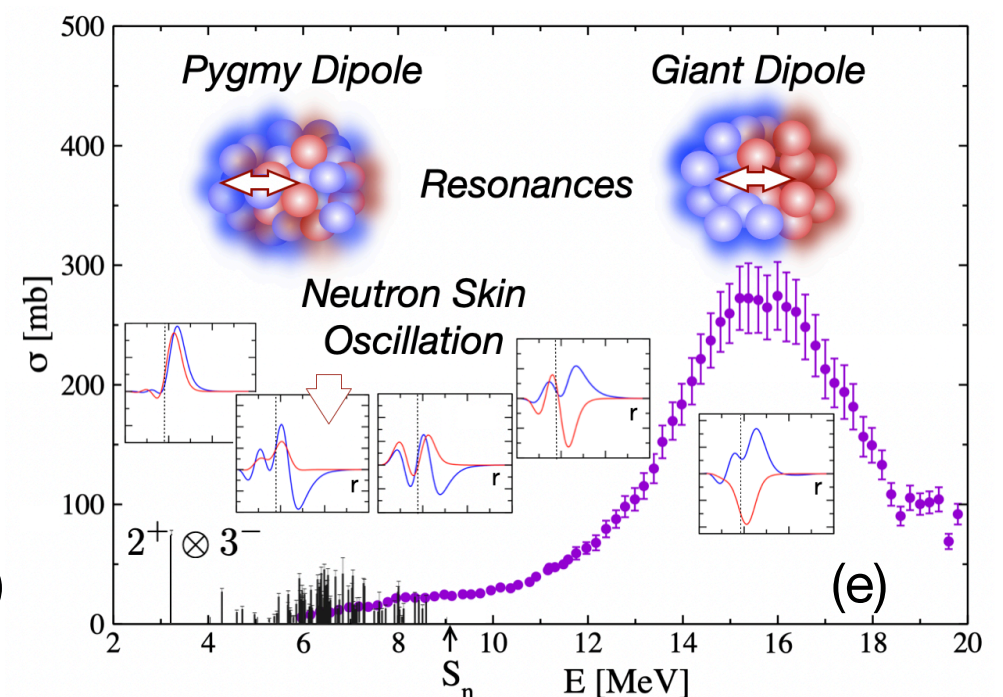
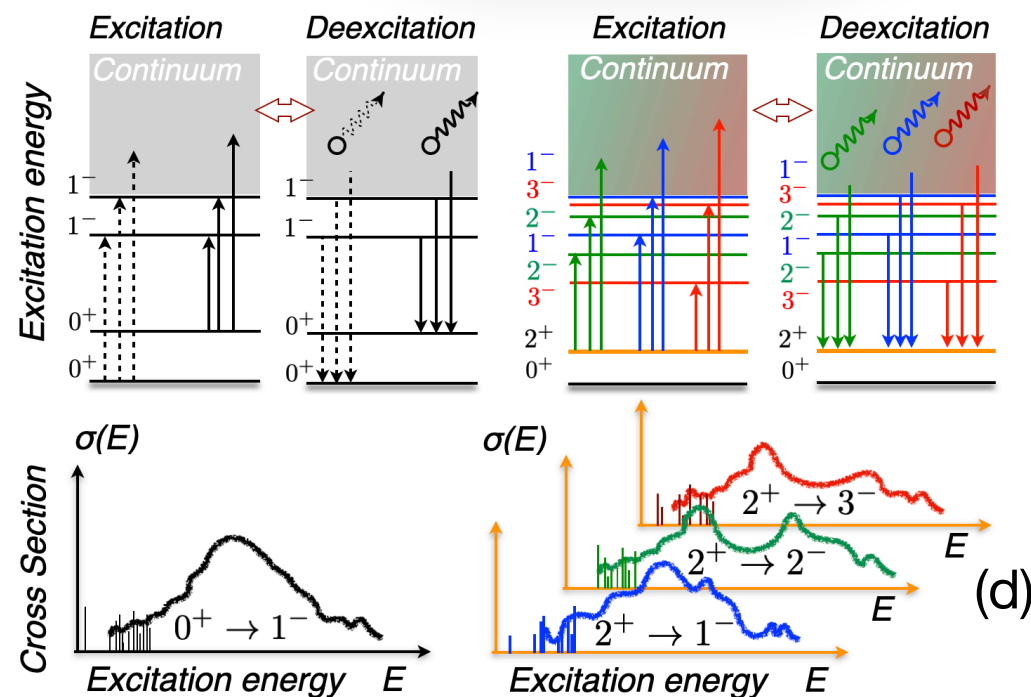
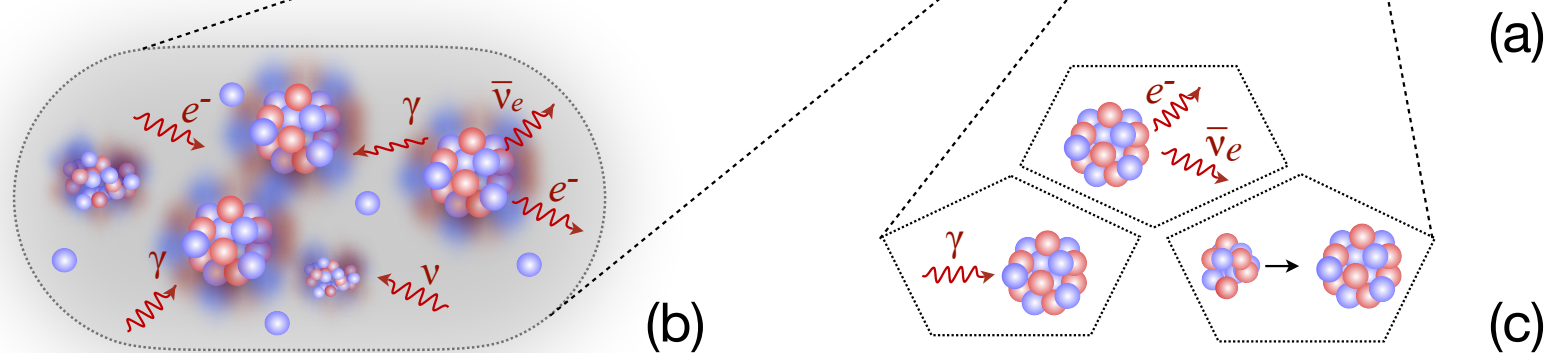


$$\mathcal{K}_{\mu\mu'\nu\nu'}^{r[11]cc(a)}(\omega_k) = \sum_{\gamma\delta nm} p_{i_n} p_{i_m} (1 - e^{-(\omega_n + \omega_m)/T}) \times \left[\frac{(\Gamma_{\mu\gamma}^{(11)n} \chi_{\mu'\gamma}^m + \Gamma_{\mu\gamma}^{(02)\bar{n}*} \mathcal{U}_{\mu'\gamma}^m)(\chi_{\nu'\delta}^{m*} \Gamma_{\nu\delta}^{(11)n*} + \mathcal{U}_{\nu'\delta}^{\bar{m}*} \Gamma_{\nu\delta}^{(02)\bar{n}})}{\omega_k - \omega_n - \omega_m} - \frac{(\Gamma_{\gamma\mu}^{(11)n*} \mathcal{Y}_{\mu'\gamma}^{m*} + \Gamma_{\gamma\mu}^{(02)\bar{n}} \mathcal{V}_{\mu'\gamma}^{\bar{m}*})(\mathcal{Y}_{\nu'\delta}^m \Gamma_{\delta\nu}^{(11)n} + \mathcal{V}_{\nu'\delta}^m \Gamma_{\delta\nu}^{(02)\bar{n}*})}{\omega_k + \omega_n + \omega_m} \right]$$

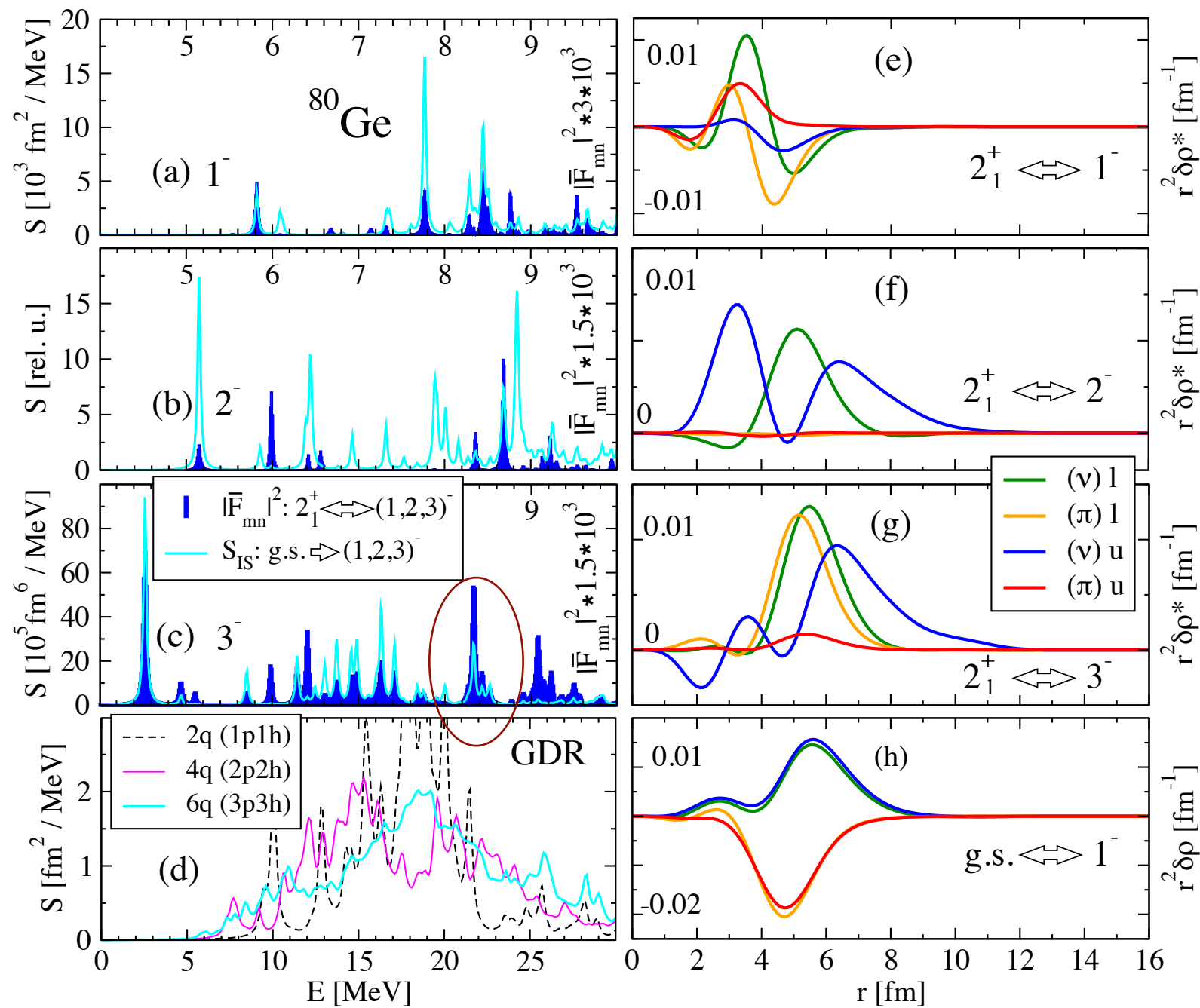
Nuclear Structure and Multimessenger Astrophysics



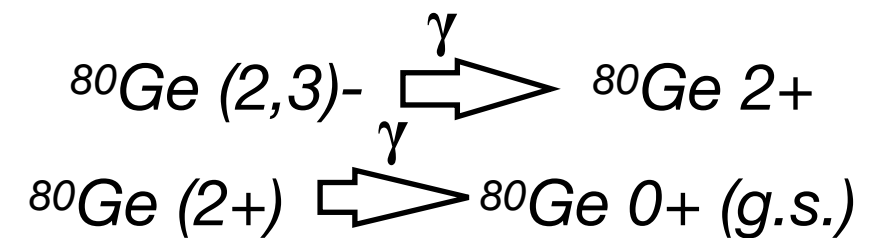
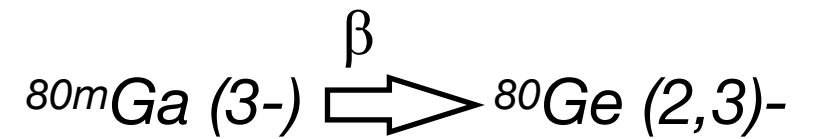
Statistical ($T > 0$)
vs
Transitions between
nuclear excited states



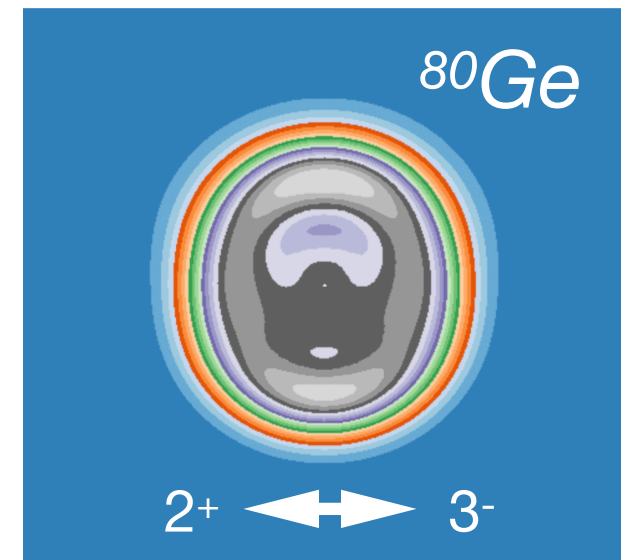
Explicit transitions between excited states in REOM³ : Dipole transitions $2^+ \leftrightarrow (1,2,3)^-$ in ^{80}Ge



Measured at ALTO as



$$Q_\beta = 10.3 \text{ MeV}$$

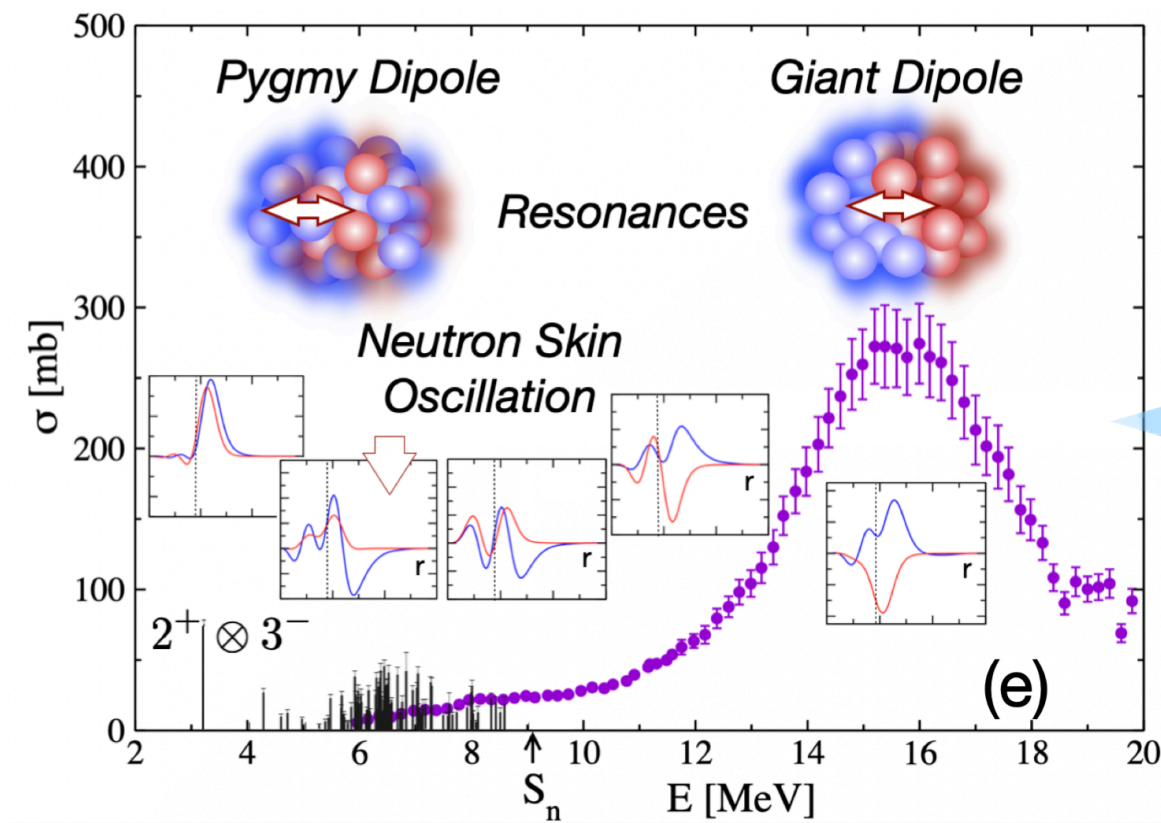


$$\mathcal{X}_{jk}^n = \langle 0 | \alpha_k \alpha_j | n \rangle \quad \mathcal{Y}_{jk}^n = \langle 0 | \alpha_j^\dagger \alpha_k^\dagger | n \rangle$$

$$\mathcal{F}_{mn} = \langle m | \mathcal{F} | n \rangle = f_{ij} (\mathcal{X}_{ik}^{m*} \mathcal{X}_{jk}^n + \mathcal{Y}_{jk}^{m*} \mathcal{Y}_{ik}^n)$$

Leading order; recoupling via 6j-symbols

Transition densities across PDR & PDR*(2₁⁺)

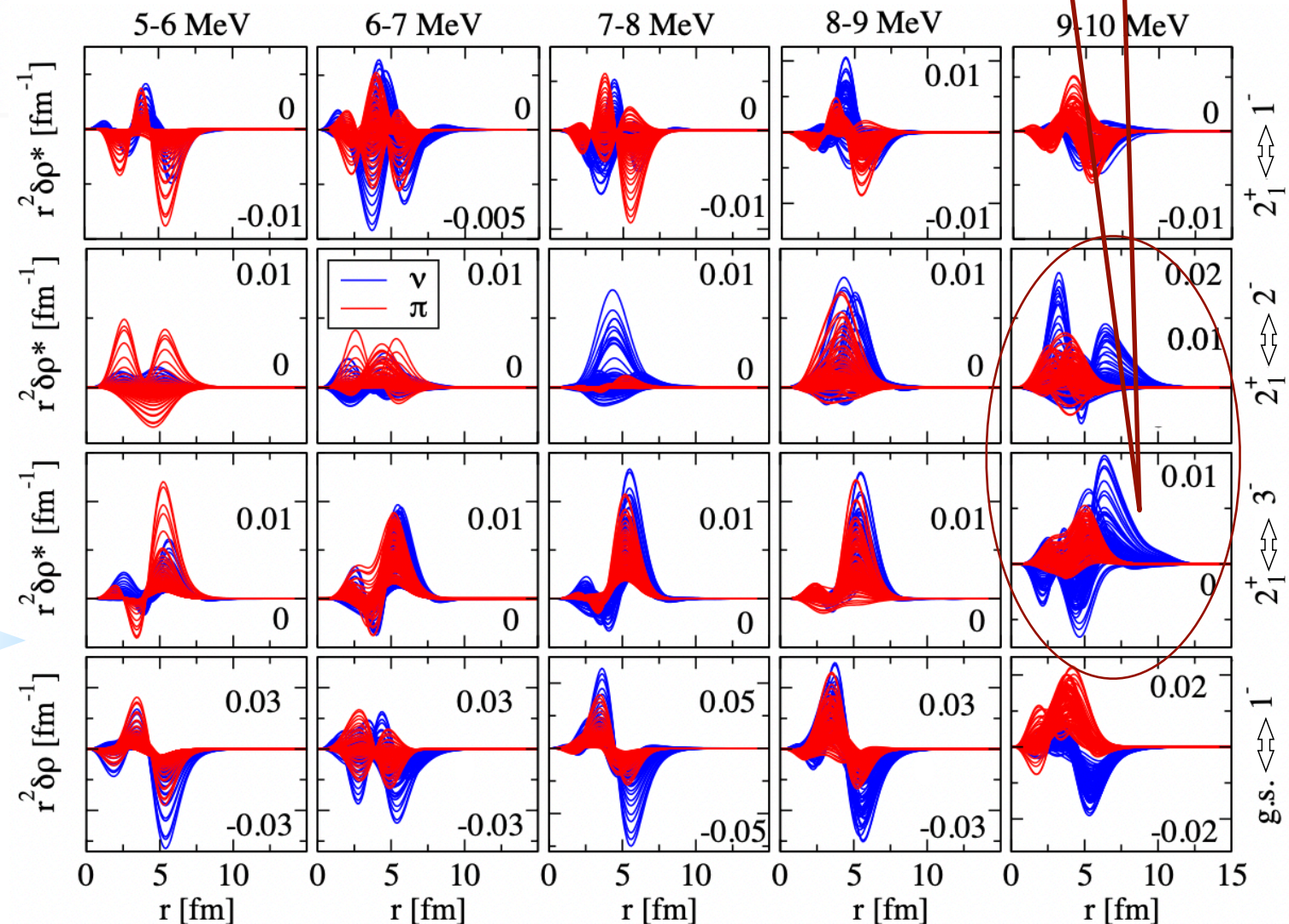


- The PDR in neutron-rich nuclei span a range of transition density patterns: (i) isoscalar, (ii) neutron skin-dominant, (iii) mixed with GSR [M. Markova, P. von Neumann-Cosel, E. L., PLB 860, 139216 (2025)]
- (i) [2⁺ x 3⁻] two-phonon state, (ii) transitions predominantly to the ground state, (iii) transitions predominantly to excited states

- The PDR in ⁸⁰Ge follows the pattern similar to ¹²⁰Sn
- The PDR*(2₁⁺) transition densities are defined as

$$\delta\rho^{nm}(1^-) = [\delta\rho^{0m}(J_m^{\pi_m}) \otimes \delta\rho^{0n}(J_n^{\pi_n})]_{1^-}$$

where $|m\rangle$ and $|n\rangle$ are the initial and final states



Outlook

Summary:

- The **emergent collective effects** associated with the dynamical kernels of the fermionic EOMs renormalize interactions in correlated media, underlie the spectral fragmentation mechanisms, and affect superfluidity and weak decay rates
- A hierarchy of converging, growing-complexity approximations generates **solutions of growing accuracy and quantifies the uncertainties** of the many-body theory
- The recently enabled capabilities are **higher configuration complexity, finite temperature, transitions between excited states, automated QRPA+qPVC computation, ab-initio implementation.**

Open theoretical problems:

- Correct separation of and delicate relationship between the static and dynamical kernels in the practical approximate solutions.
- Artificial poles and virtual “particles” in approximate solutions.
- Consistency between direct and pairing channels in the dynamical kernels in practical implementations.
- Ambiguities of numerical implementations on the 2p2h and 3p3h levels of complexity.
- An adequate assessment of the quantitative role of fully connected multi-fermion correlators (~irreducible three- and many-body “forces” in the medium).

Open practical problem:

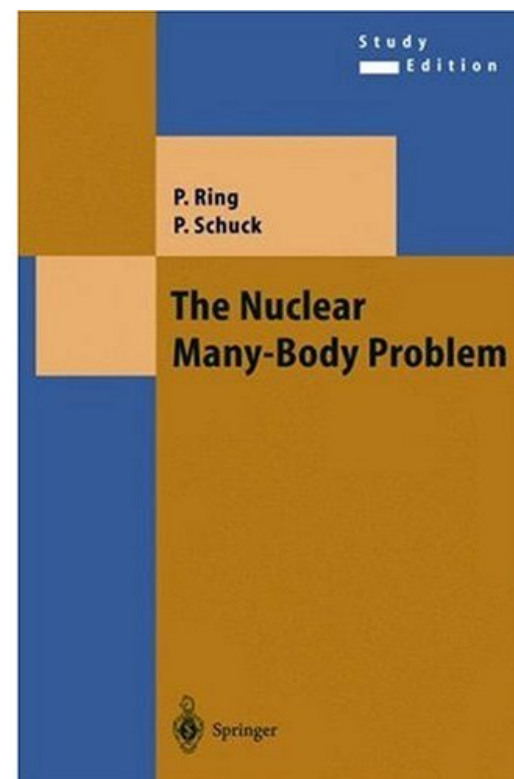
MORE MANPOWER IS NEEDED





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