“Ab-initio” nuclear response theory and emergent quasiparticle-vibration coupling

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The scope:

- Nuclear structure theory has strived for decades to achieve accurate computation of nuclear spectra, needed for many applications, but this is still difficult

- We address this problem by developing the relativistic nuclear field theory (RNFT) in the model-independent Equation of Motion (EOM) framework using the universal QFT language, quantifying fermionic correlation functions (FCFs). ~All models follow from it

- The theory (i) is transferable across the energy scales, (ii) capable of identifying the bottlenecks of the existing nuclear structure approaches, and (iii) opens the door to spectroscopic accuracy

- In this formulation, dynamical interaction kernels of the FCF EOMs are the source of the richness of the nuclear wave functions in terms of their configuration complexity and the major ingredient for an accurate description with quantified uncertainties

- Electromagnetic and weak probes are the most informative ones for benchmarking the theory and constitute the majority of applications: selected highlights for nuclear excited states

- Goals: (i) reaching shell-model quality in large model spaces and energy intervals; (ii) understanding the genesis of fundamental interactions

- Open problems

Exact “ab initio” equation of motion (EOM) for the two-fermion correlation function with an unspecified NN interaction

$$H=\sum_{12}\bar\psi_1(-i\gamma\cdot\nabla+M)_{12}\psi_2+\frac{1}{4}\sum_{1234}\bar\psi_1\bar\psi_2\bar v_{1234}\psi_4\psi_3\;+\;\ldots\;=T+V^{(2)}+\ldots$$

Bare (Relativistic) **Hamiltonian**

Particle-hole correlator or two-time propagator (response function):

$$R_{12,1'2'}(t-t')=-i\langle T(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle$$

In-medium scattering amplitude spectra of excitations, decays, …

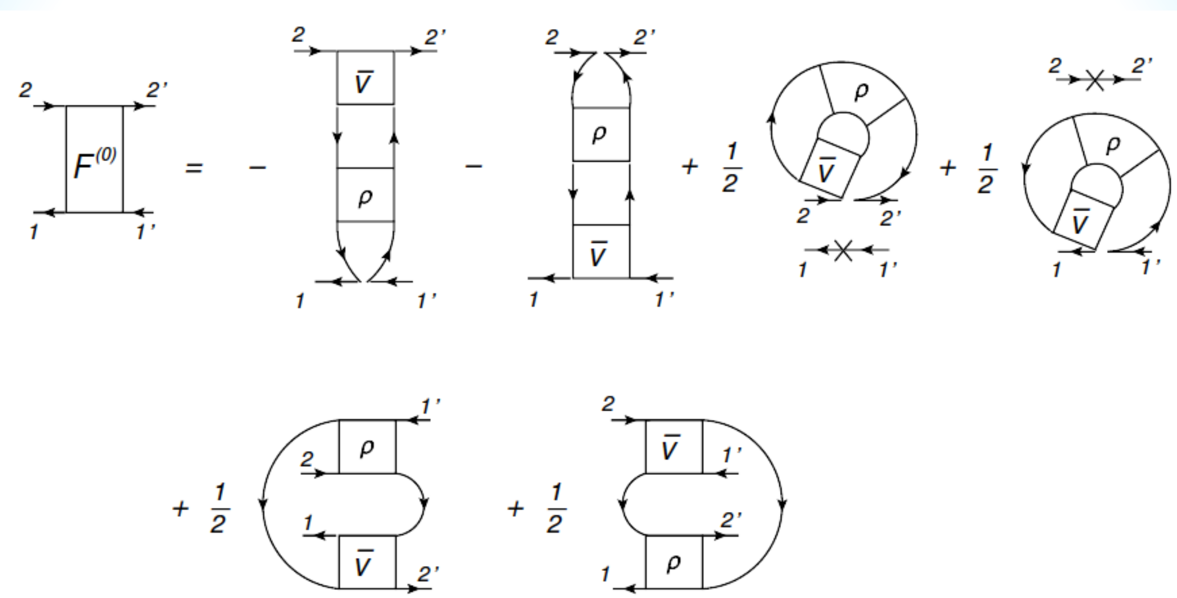
***EOM**: Bethe-Salpeter-(Dyson) Equation (**)*

$$R(\omega)=R^{(0)}(\omega)+R^{(0)}(\omega)F(\omega)R(\omega)$$

Irreducible kernel (exact): in-medium “interaction”

$$F(t-t')=F^{(0)}\delta(t-t')+F^{(r)}(t-t')$$

Instantaneous term (“bosonic” mean field):
Short-range correlations

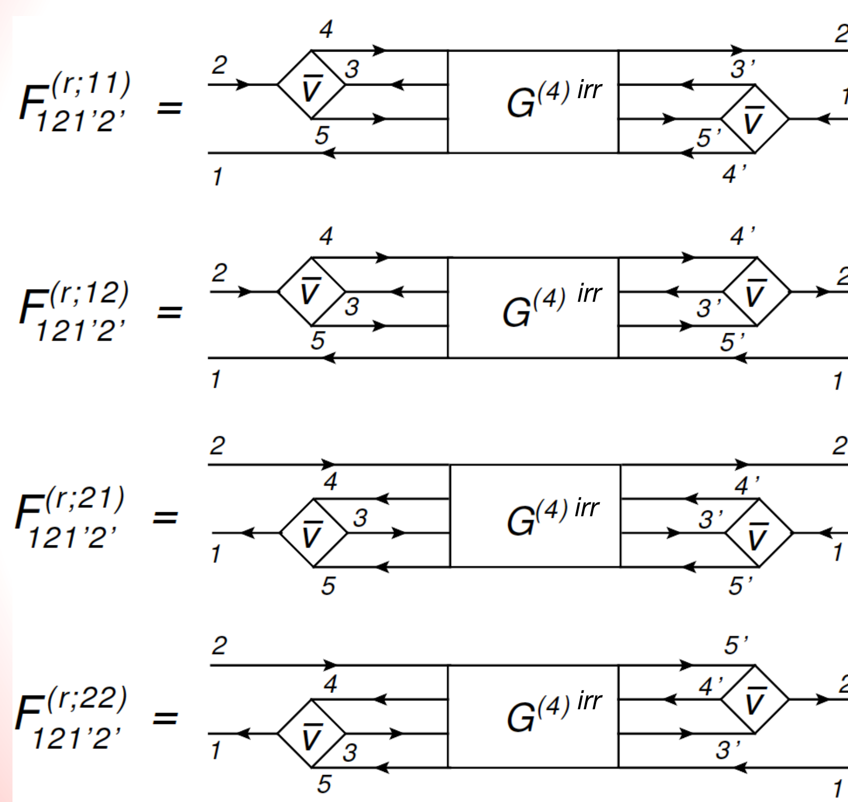


Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'}=\delta_{22'}\rho_{11'}-i\lim_{t'\to t+0}R_{2'1,21'}(t-t')$$

contains the full solution of (**) ***including the dynamical term!***

t-dependent (dynamical) term:
Long-range correlations:
Couples to higher-rank FCFs



E.L. & P. Schuck, PRC 100, 064320 (2019)

Peter Schuck (1940-2022)

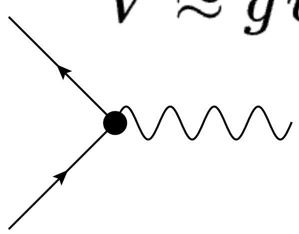
Symmetric form:
P. Schuck, J. Dukelsky, S. Adachi, et al.

Non-Symmetric form: see
J. Schwinger, F. Dyson, et al.

Language remarks: "ab initio"

- *Ab initio = "from the beginning" (lat.), in science and engineering "from first principles"*
- ***First-principle theory** does not (yet) exist:*
  Even the Standard Model (SM) is an effective theory:

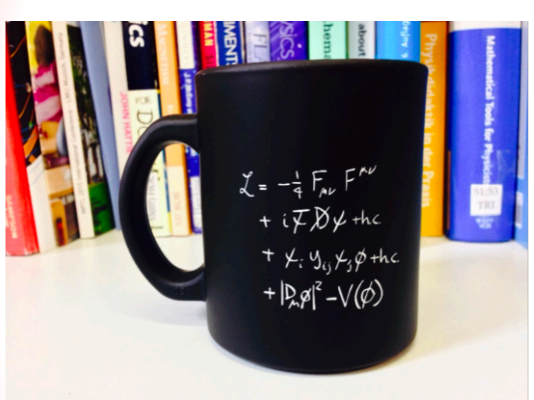
$$V \approx g \bar{\psi} \phi \psi$$



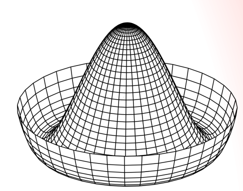
~20 parameters:
masses, coupling constants, etc.

Open questions:

- *Can the SM be UV-completed and include gravity?*
- *What are the SM interactions?*
- *What is the origin of the Higgs potential?*
- *What is dark matter & energy?*
- *Neutrino mass?*
- *…*



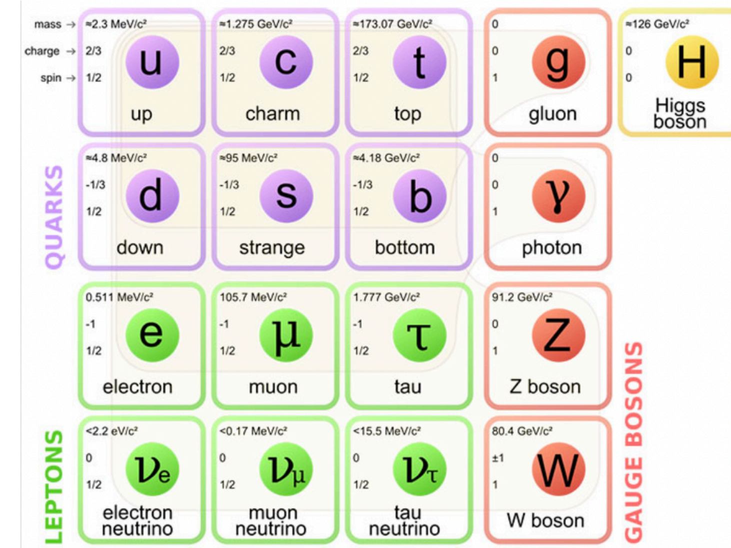
Higgs mechanism of mass generation (~ mean field)



$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi),$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.$$

The Standard Model

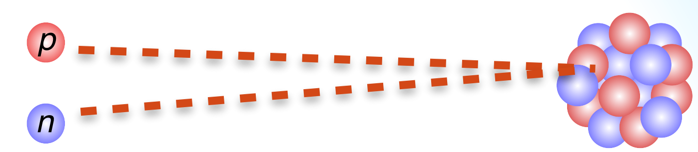


The Standard Model of physics describes the known particles and forces that operate at the tiny quantum scale. *(Wikimedia Commons: Miss J)*

"Bare" ***Interaction*** *(aka forces, potentials):*

$$V(\mathbf{r}_1 - \mathbf{r}_2)$$

Ab initio…



- *In **Theoretical Physics**: knowing the bare (vacuum) interaction between the two particles, quantitatively describing the many-particle system (yet impossible with desired accuracy for strong coupling)*
- *"Ab initio" (UV-complete) is the opposite of "effective"*

Language remarks [for medium-mass and heavy systems]

To the narratives:

(i) There is an ab initio nuclear theory as opposed to "traditional" nuclear theory

(ii) There is "the theory" (χPT potential) and there are "models" (the many-body approaches)

(iii) Only ab initio χPT can provide quantified uncertainties

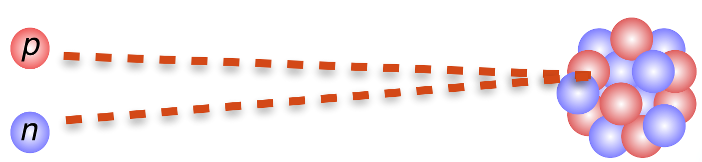
(iv) There are equally good many-body "solvers" which are unimportant attachments to χPT

Which are wrong

"Bare" ***Interaction*** *(aka forces, potentials):*

$V(\mathbf{r}_1 - \mathbf{r}_2)$

Ab initio…



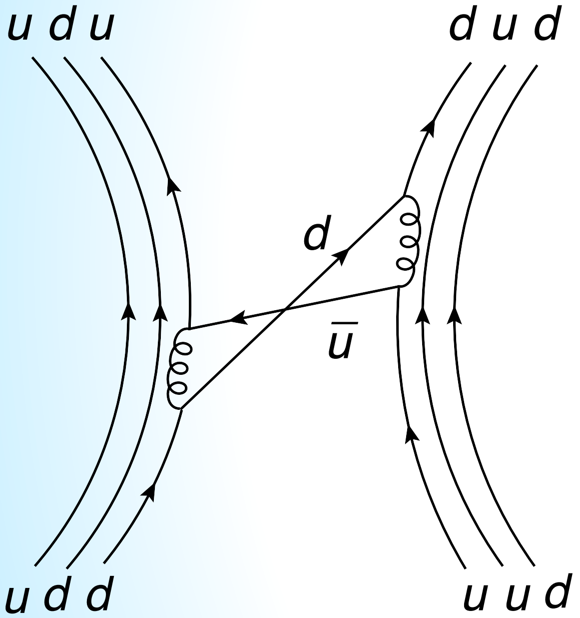
- *In **Theoretical Physics**: knowing the bare (vacuum) interaction between two particles, quantitatively describe the many-particle system*
- *"Ab initio = UV-complete" as opposed to "effective"*
- *Interactions adjusted to many-particle systems (e.g., finite nuclei) are not ab initio but effective*

Formulation of the theory and its implementation

- *We can formulate an ab initio theory, but it has not been implemented accurately (yet). Working with effective interactions, we still need an ab initio theory in the theoretical physics sense to constrain concrete implementations, e.g., for maintaining their algebraic structure.*
- *All "traditional" microscopic many-body models employing effective interactions can be derived ab initio and represent various approximations to the static and dynamical kernels of the same formally exact EOMs.*
- *Paraphrasing Steven Weinberg, saying that "all non-renormalizable theories are as renormalizable as renormalizable theories", we find that, in the theoretical physics sense,* ***"traditional" theories are as ab initio as ab initio theories :)***

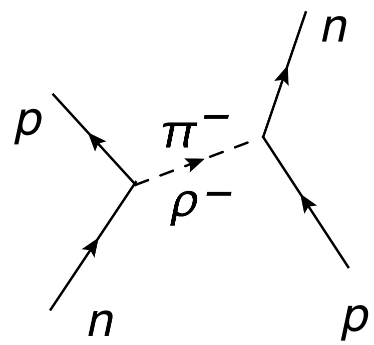
NN-interaction: a relay of EFTs

Quantum Chromodynamics (QCD, high energy)



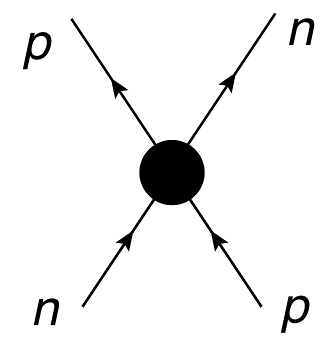
"Almost" calculable

Quantum Hadrodynamics (QHD, intermediate energy)



Meson exchange

"Almost" calculable

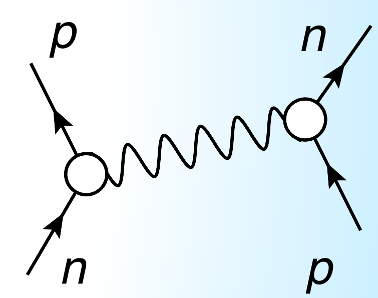


DFT: effective coupling

Nuclear Structure (NS, low energy)

"Almost" calculable

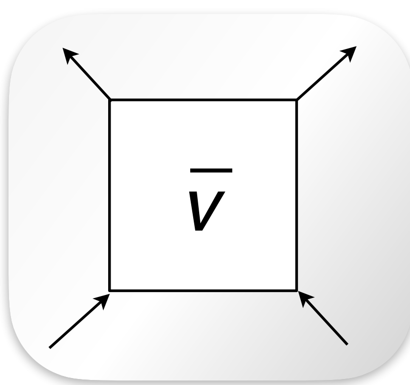
+



Induced in-medium phonon exchange

"Ab initio" many-body theory

- *Assumes a generic, unspecified bare "interaction": model-independent, all channels included on the tree level*
- *Higher orders are treated via* ***in-medium propagators***
- *No perturbation theory (strong coupling)*



Static, extracted from the NN-scattering Generally, it can contain retardation)

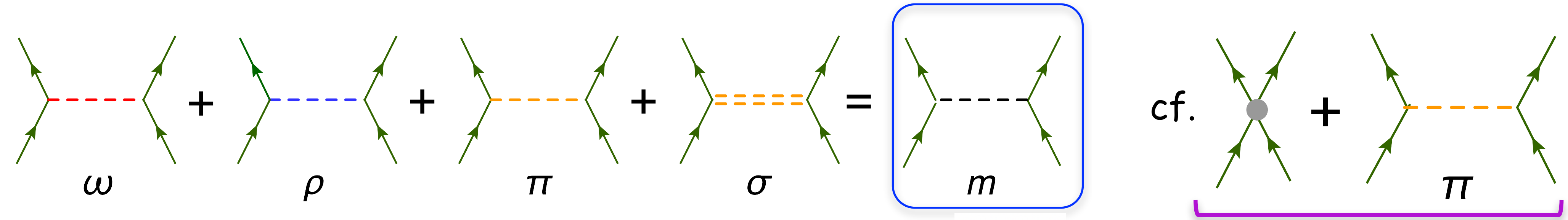
Implementations:

- *Meson-exchange (ME) at leading order*
- *Effective coupling constants/masses (adjusted at the mean-field (MF) level, NL3(*)) + subtraction of qPVC (P. Ring et al., V. Tselyaev)*
- *Bare ME + subtraction of MF artifacts (ongoing; first results are good)*
- ***Goal: accuracy + predictive power***

Linking the nucleon-nucleon NN(…N) interaction and the many-body theory

Quantum Hadrodynamics (QHD)

Tree-level:

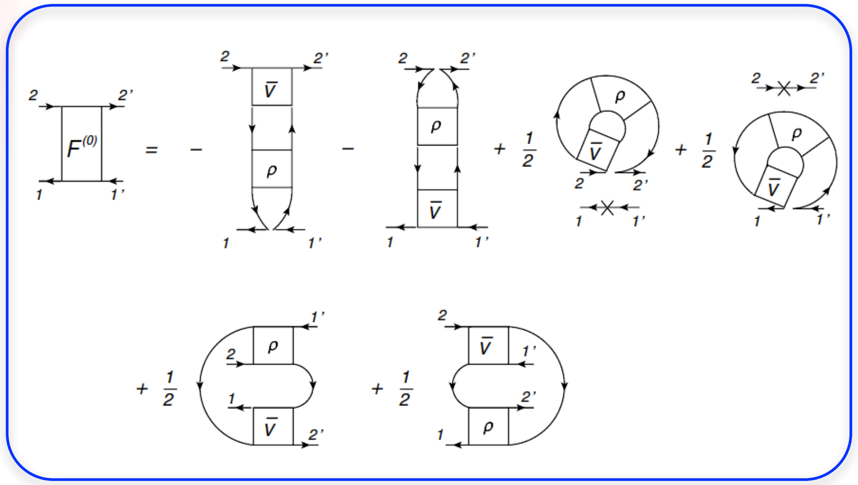


RNFT (this work): non-PT, in-medium beyond the tree-level

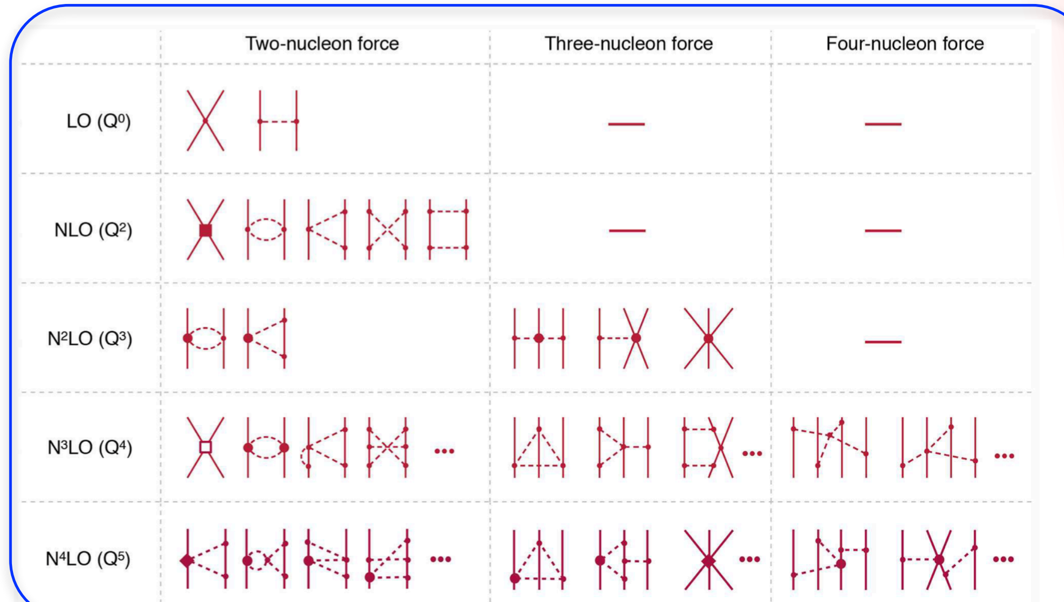
Beyond the leading order:

cf. χEFT: PT for hadrons in the vacuum

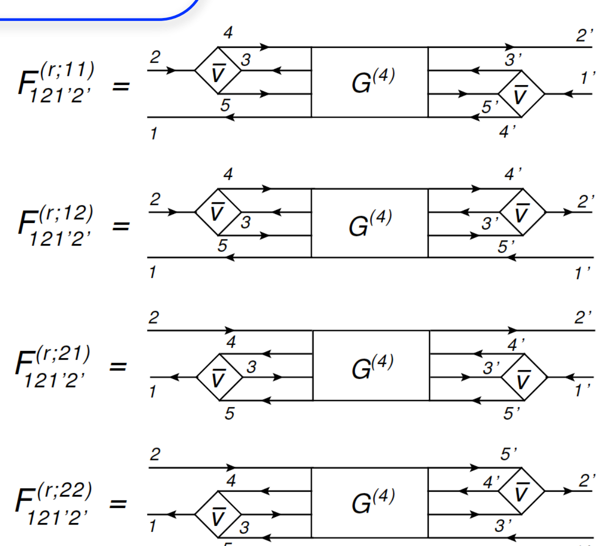
E. Epelbaum et al., Front. Phys. 8, 98 (2020)



Static



+ Dynamical



+ Dynamical

Many-body… "solver" (?)

Beyond light nuclei incorrect:

- *Standard "solvers" are too simplistic*
- *Bare NN and many-body theory should be linked consistently*
- ***In the correlated nuclear medium, the χEFT power counting becomes irrelevant: new Nambu-Goldstone modes emerge***

New spontaneously broken symmetries:

The in-medium power counting is associated with emergent scales of low-energy collective modes in medium-heavy nuclei

Dynamical kernels: bridging scales and emergent EFT

Quantum many-body problem: Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); $N_{Equations} \sim N_{Particles}$ & Coupled 🙈 !!!

Non-perturbative solutions:

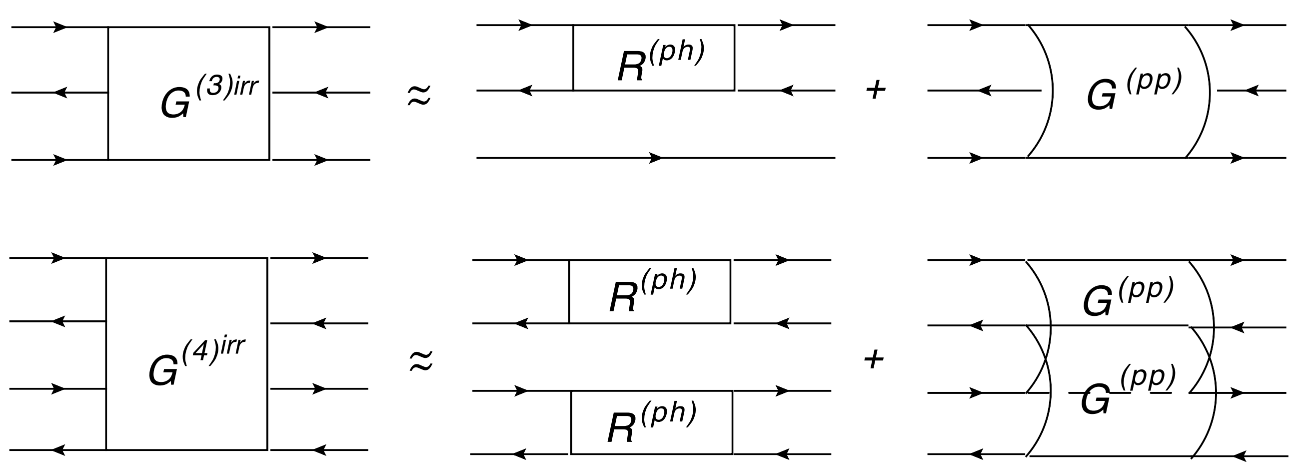
Cluster decomposition [QFT, S. Weinberg]

$$\bigstar\ G^{(3)} = \boxed{G^{(1)} G^{(1)} G^{(1)}} + \boxed{G^{(2)} G^{(1)}} + \boxed{\Xi^{(3)}}$$

$$\bigstar\ G^{(4)} = \boxed{G^{(1)} G^{(1)} G^{(1)} G^{(1)}} + \boxed{G^{(2)} G^{(2)}} + \boxed{G^{(3)} G^{(1)} + \Xi^{(4)}}$$

Leading at:	**weak**	**intermediate**	**strong**	coupling
	self-consistent GFs second RPA etc.	phonon coupling **this work**	**Faddeev + future work**	

Beyond weak coupling:

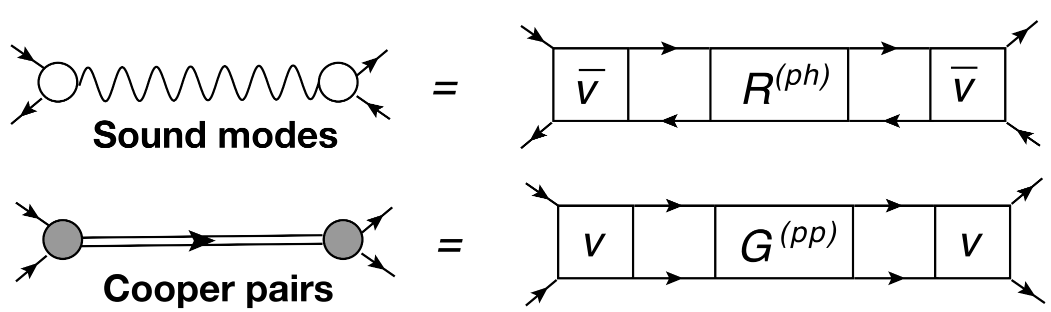


- P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).
- N. Vinh Mau, Trieste Lectures 1069, 931 (1970)
- P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)
- …

Exact mapping: particle-hole (2q) quasibound states

Emergence of low-energy Nambu-Goldstone bosons, phonons [SO(3) symmetry breaking]:

Emergence of superfluidity [U(1) symmetry breaking]:



Quasiparticle-vibration coupling (qPVC) in nuclei. Cf. NFT

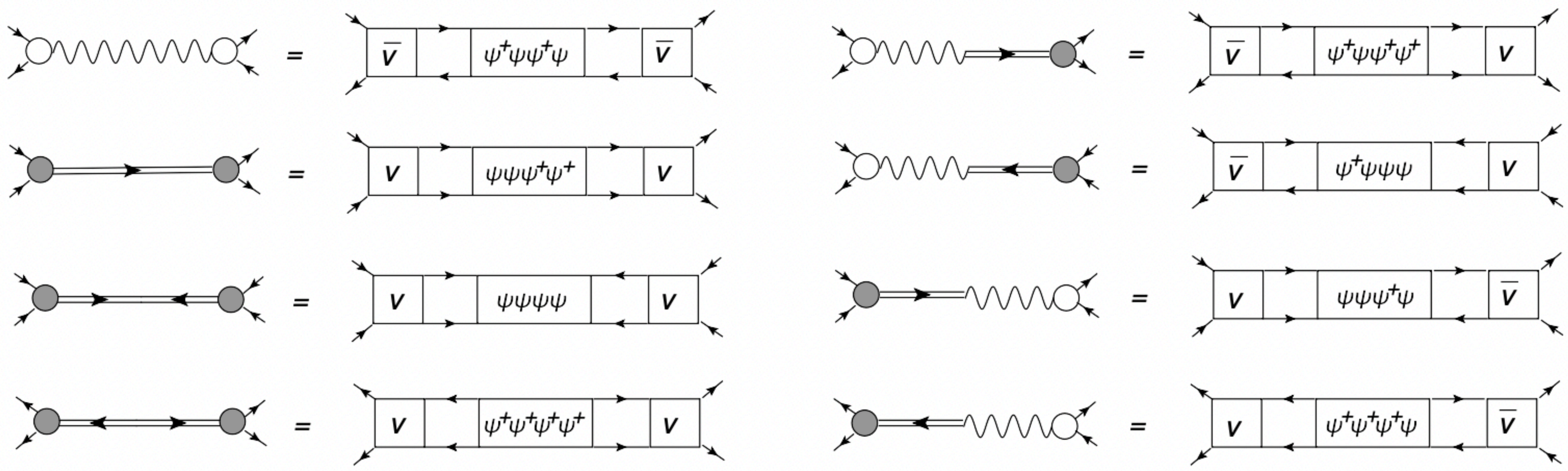
Diquarks in hadrons

Cf. C. Popovici, P. Watson, and H. Reinhardt [PRD83, 025013 (2011)]

"Ab-initio" qPVC in superfluid systems

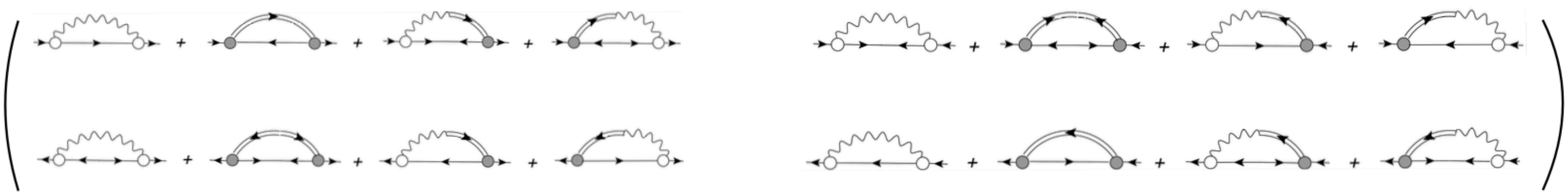
Superfluid dynamical kernel: adding particle-number violating contributions

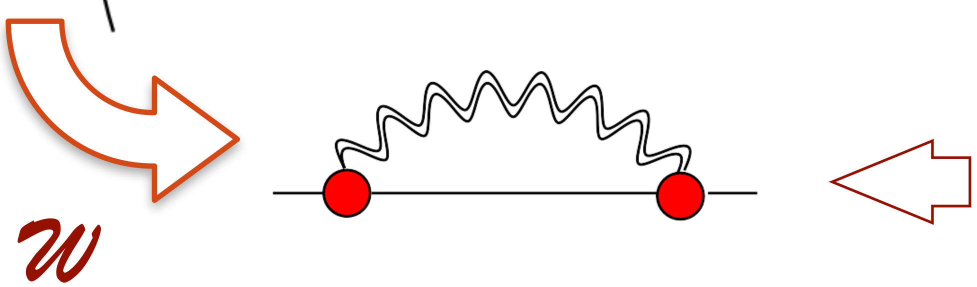
Mapping on the qPVC in the canonical basis



Quasiparticle dynamical self-energy (matrix): normal and pairing phonons are unified

$\hat{\Sigma}^{r} =$





Bogoliubov transformation

Compact form, (almost) as simple as non-superfluid

Cf.: Quasiparticle static self-energy (matrix) in HFB

$$\hat{\Sigma}^{0} = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^{*}_{11'} & -\tilde{\Sigma}^{T}_{11'} \end{pmatrix}$$

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

Finite-temperature superfluid formalism beyond QRPA

Theory is formulated in the HFB basis keeping 4x4 block matrix structure

superfluid

Correlated 2q-propagator with Matsubara frequencies

S. Bhattacharjee, E.L., arXiv:2412.20751, to appear in PRC (2026)

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}(\omega_n) = \hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) + \\ + \frac{1}{4}\sum_{\gamma\gamma'\delta\delta'} \hat{\mathcal{R}}^0_{\mu\mu'\gamma\gamma'}(\omega_n)\hat{\mathcal{K}}_{\gamma\gamma'\delta\delta'}(\omega_n)\hat{\mathcal{R}}_{\delta\delta'\nu\nu'}(\omega_n)$$

$$\mathcal{R}^{[lk]}_{\mu\mu'\nu\nu'}(\omega) = \sum_{fi}\sum_{\sigma=\pm} \sigma p_i \frac{\mathcal{Z}^{if[l\sigma]}_{\mu\mu'}\mathcal{Z}^{if[k\sigma]*}_{\nu\nu'}}{\omega - \sigma\omega_{fi}}$$

Uncorrelated 2q-propagator

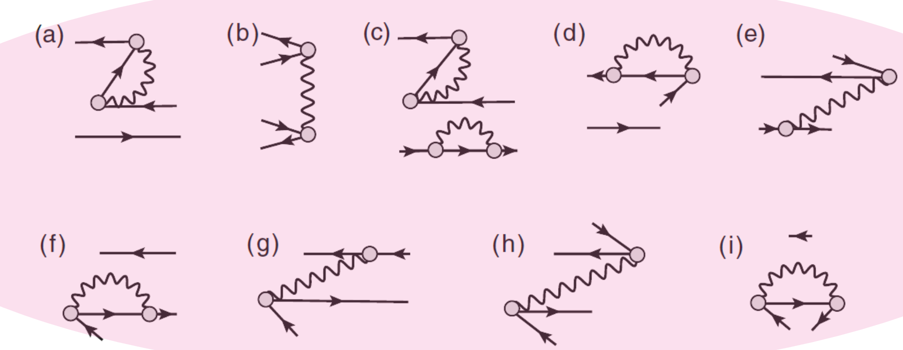
$$\hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) = [\omega_n - \hat{\Sigma}_3\hat{E}_{\mu\mu'}]^{-1}\hat{\mathcal{N}}_{\mu\mu'\nu\nu'}$$

The norm matrix

$$\hat{\mathcal{N}}_{\mu\mu'\nu\nu'} = \left\langle \begin{pmatrix} [A_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [A_{\mu\mu'}, A_{\nu\nu'}] & [A_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [A_{\mu\mu'}, C_{\nu\nu'}] \\ [A^\dagger_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [A^\dagger_{\mu\mu'}, A_{\nu\nu'}] & [A^\dagger_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [A^\dagger_{\mu\mu'}, C_{\nu\nu'}] \\ [C_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [C_{\mu\mu'}, A_{\nu\nu'}] & [C_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [C_{\mu\mu'}, C_{\nu\nu'}] \\ [C^\dagger_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [C^\dagger_{\mu\mu'}, A_{\nu\nu'}] & [C^\dagger_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [C^\dagger_{\mu\mu'}, C_{\nu\nu'}] \end{pmatrix} \right\rangle$$

$$\langle O \rangle = \sum_i p_i \langle i|O|i\rangle \qquad p_i = \frac{e^{-E_i/T}}{\sum_j e^{-E_j/T}}$$

Complex GSC: a source of "natural" quenching at both T=0 and T>0



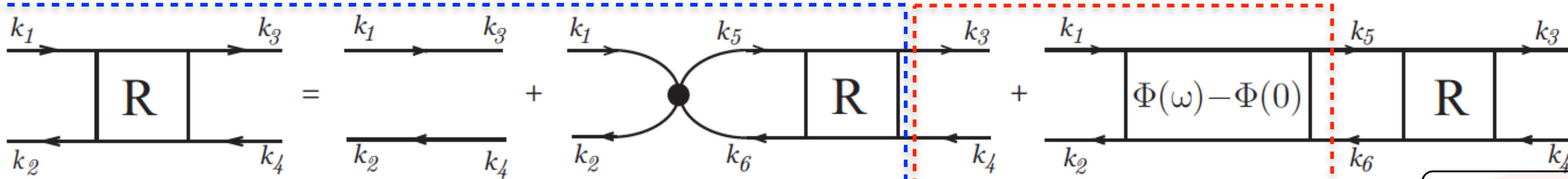
The complete superfluid dynamical qPVC kernel

$$\mathcal{K}^{r[11]cc(a)}_{\mu\mu'\nu\nu'}(\omega_k) = \sum_{\gamma\delta nm} p_{i_n} p_{i_m}\left(1 - e^{-(\omega_n+\omega_m)/T}\right) \\ \times \Big[\frac{(\Gamma^{(11)n}_{\mu\gamma}\mathcal{X}^m_{\mu'\gamma} + \Gamma^{(02)\bar{n}*}_{\mu\gamma}\mathcal{U}^m_{\mu'\gamma})(\mathcal{X}^{m*}_{\nu'\delta}\Gamma^{(11)n*}_{\nu\delta} + \mathcal{U}^{\bar{m}*}_{\nu'\delta}\Gamma^{(02)\bar{n}}_{\nu\delta})}{\omega_k - \omega_n - \omega_m} \\ - \frac{(\Gamma^{(11)n*}_{\gamma\mu}\mathcal{Y}^{m*}_{\mu'\gamma} + \Gamma^{(02)\bar{n}}_{\gamma\mu}\mathcal{V}^{\bar{m}*}_{\mu'\gamma})(\mathcal{Y}^m_{\nu'\delta}\Gamma^{(11)n}_{\delta\nu} + \mathcal{V}^m_{\nu'\delta}\Gamma^{(02)\bar{n}*}_{\delta\nu})}{\omega_k + \omega_n + \omega_m}\Big]$$

QRPA limit: $\mathcal{K}^r$ = 0; talk by E. Yuksel

Nuclear response: toward a complete theory

toward ab initio <= static kernel V *dynamic kernel => toward more complex configurations*



Dyson-Bethe-Salpeter Equation:

$$R(\omega) = R^0(\omega) + R^0(\omega)\ [V+\Phi(\omega)-\Phi(0)\ ]\ R(\omega)$$

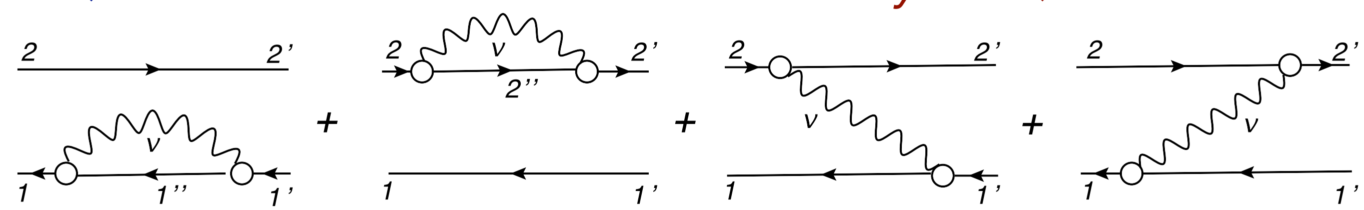
Subtraction [V.Tselyaev 2013]

for effective interactions V [P. Ring, A.Afanasjev, et al: CEDF]

NFT is reproduced as the leading approximation ***[talk by G. Còlo]***

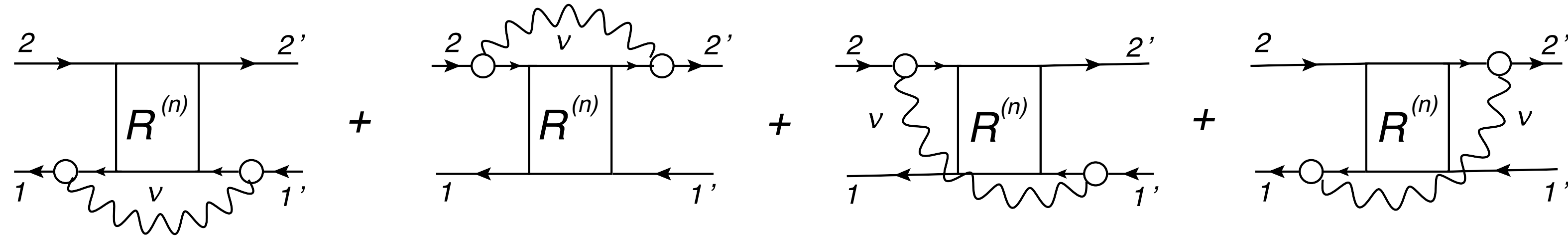
QRPA/REOM[1] *Beyond QRPA: REOM[n]*

$\Phi_{21,2'1'} =$



Extended NFT: closed cycle

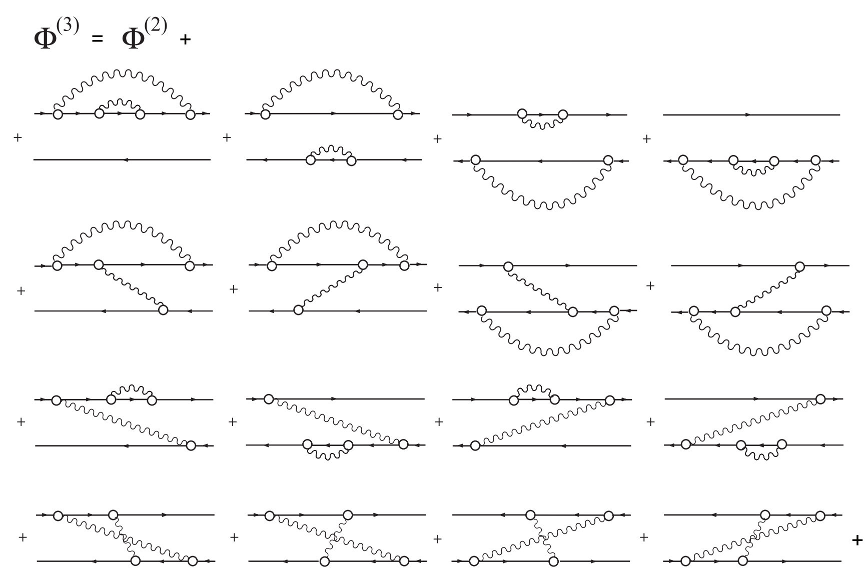
$\Phi^{(n+1)}_{212'1'} =$



Generalized approach for the correlated propagators

n-th order: E.L. PRC 91, 034332 (2015)

Ab-initio formulation, $\Phi^{(3)}$ implementation; 2q+2phonon correlations: E.L., P. Schuck, PRC 100, 064320 (2019)

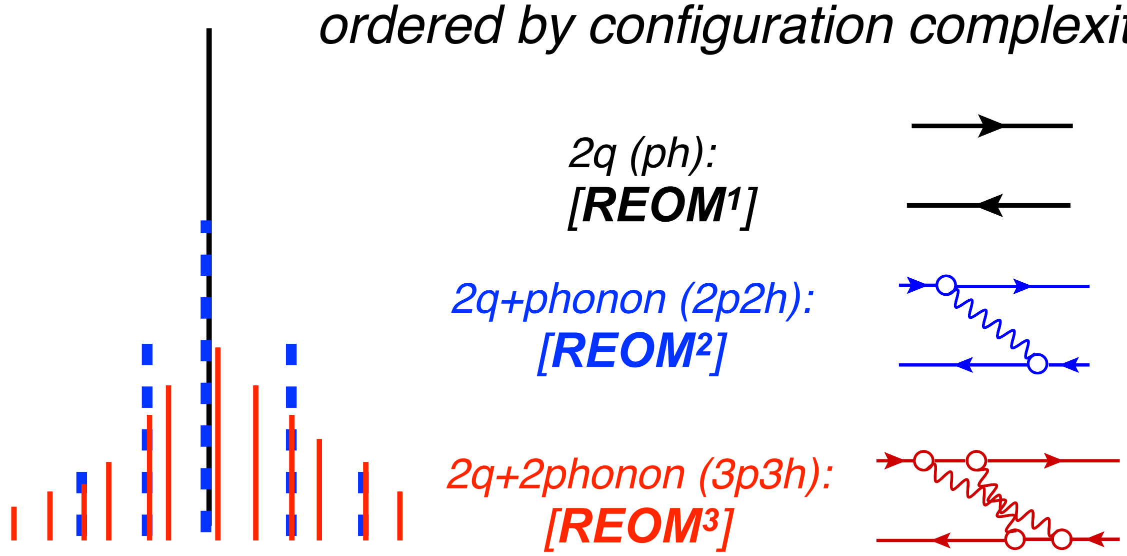


2q+2phonon configurations in the qPVC expansion: ***after two iterations***

Included in all orders (non-perturbatively)

Toward spectroscopic accuracy: correlated 3p3h beyond the leading qPVC

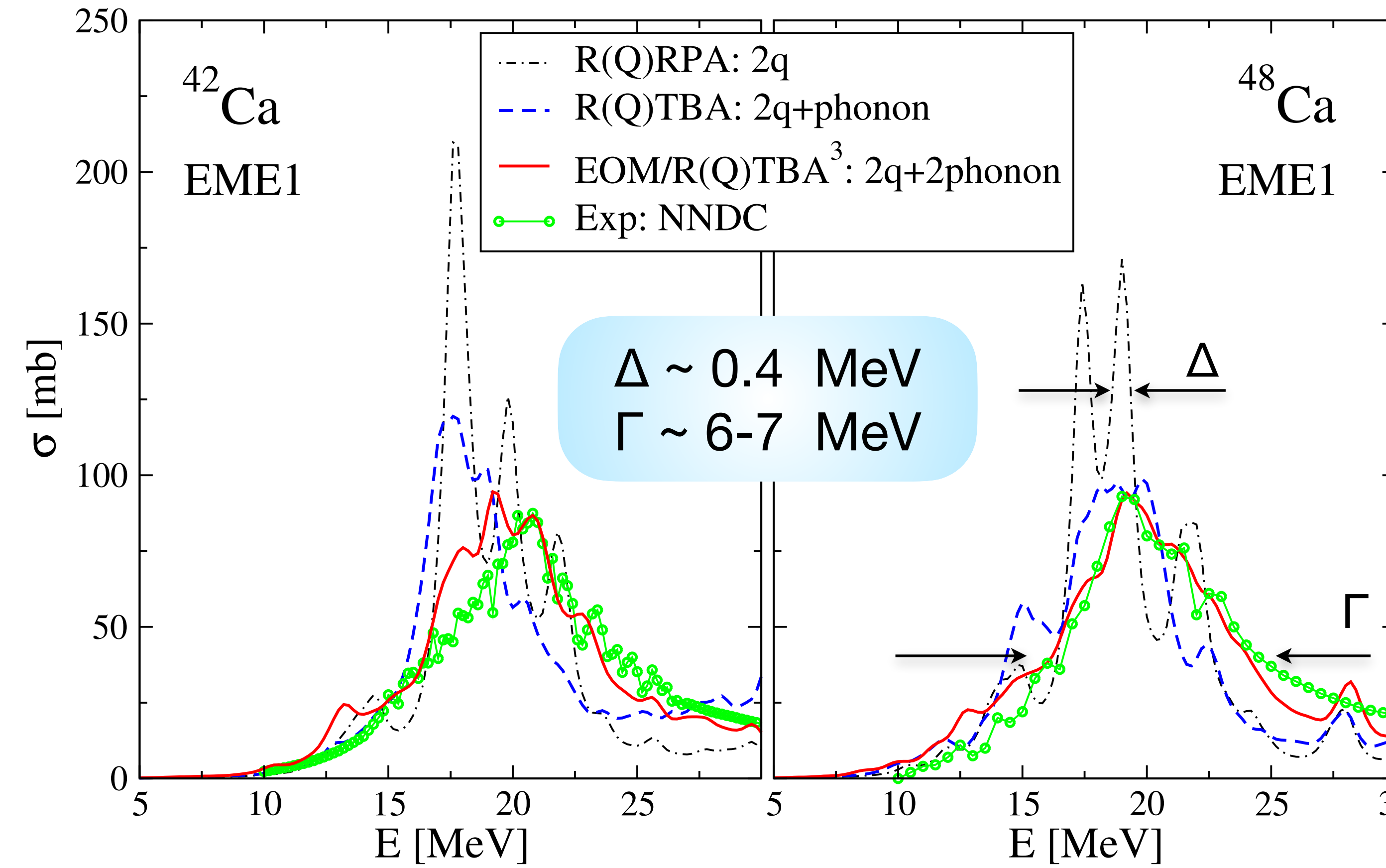
Fragmentation mechanism: ordered by configuration complexity



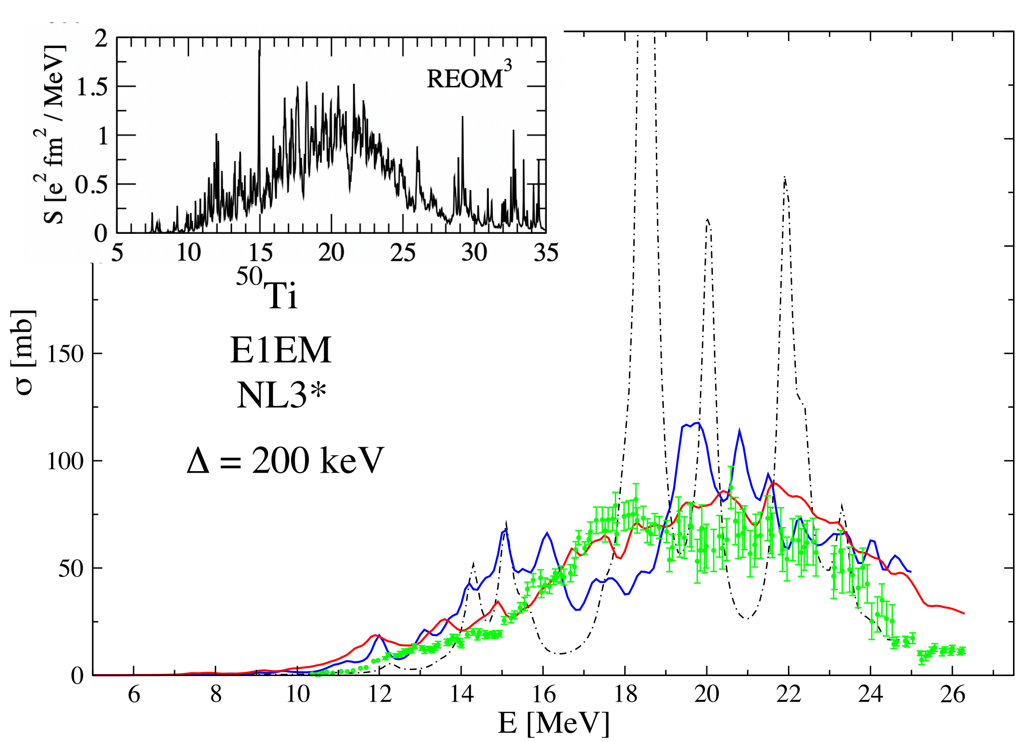
In principle exact spectral expansion

$$R_{12,1'2'}(\omega) = \sum_{\nu>0}\left[\frac{\rho^{\nu}_{21}\bar{\rho}^{\nu*}_{2'1'}}{\omega - \omega_\nu + i\delta} - \frac{\bar{\rho}^{\nu*}_{12}\rho^{\nu}_{1'2'}}{\omega + \omega_\nu - i\delta}\right]$$

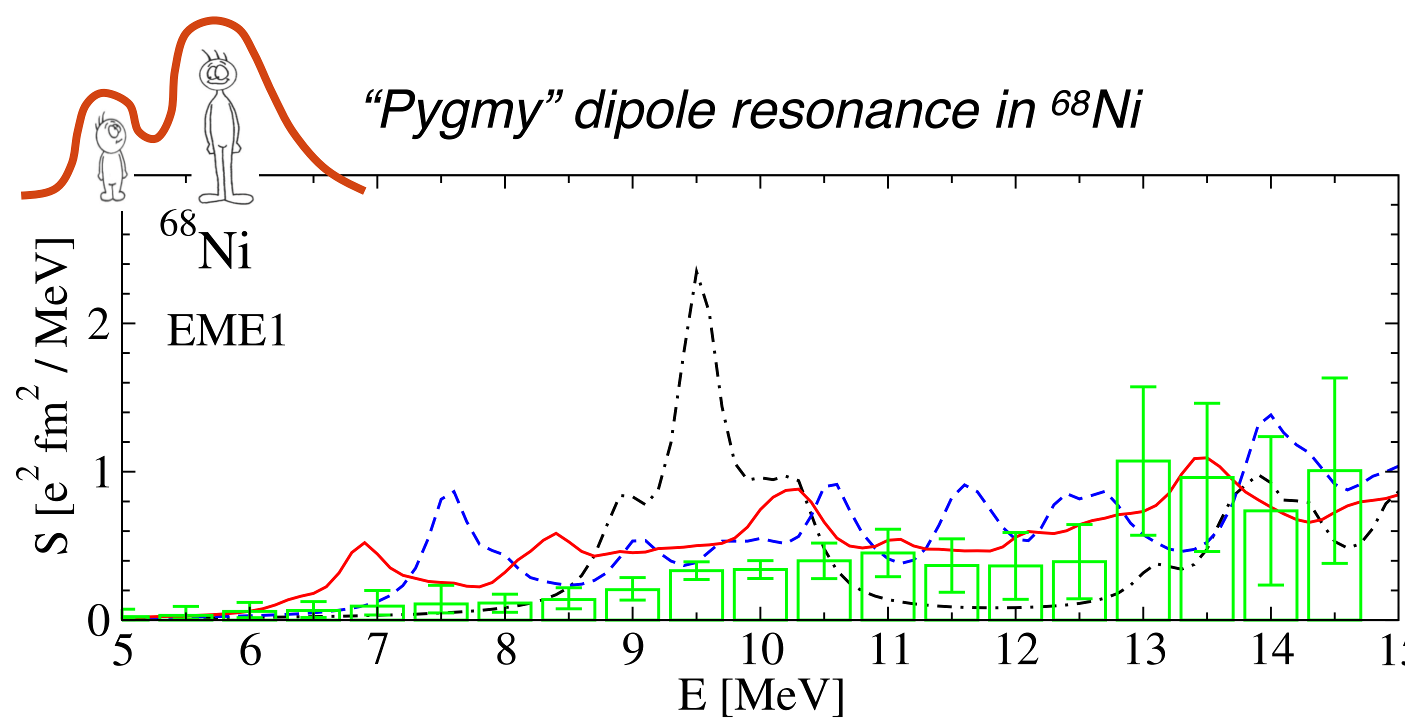
Giant dipole resonance in Ca: overlapping resonances



(Moderately) deformed: ^{50}Ti [$\beta_2 \sim 0.17$]:



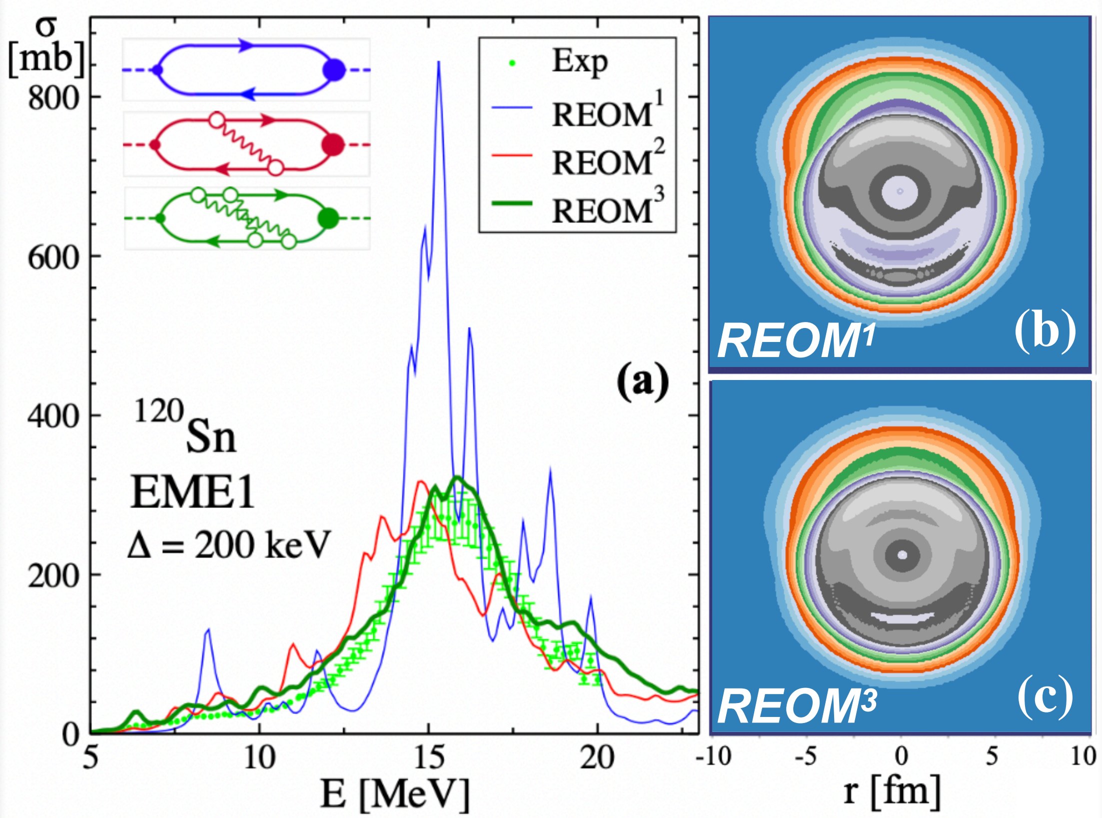
"Pygmy" dipole resonance in ^{68}Ni



Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)

Resolving long-term nuclear structure puzzles

Giant and soft resonances in heavy nuclei



- *Accurate computation of nuclear excited states in a wide energy range, including giant resonances, is pushed to heavy systems.*
- *The recently developed ab initio relativistic equation of motion method (REOM^n) with non-perturbative dynamical kernels enables a hierarchy of growing-complexity (n) approximations to the nuclear response organized by the qPVC power counting based on emergent collectivity.*
- *We demonstrated that this power counting remains valid in heavy systems and provides a firm uncertainty quantification of the many-body theory.*
- *A significantly improved description is achieved for the Sn-Sm mass region for giant resonances and soft modes.*

J. Novak, M. Hlatshwayo, E.L., arXiv:2405.02255
Exp. Data: S. Bassauer, PRC102, 034327 (2020)

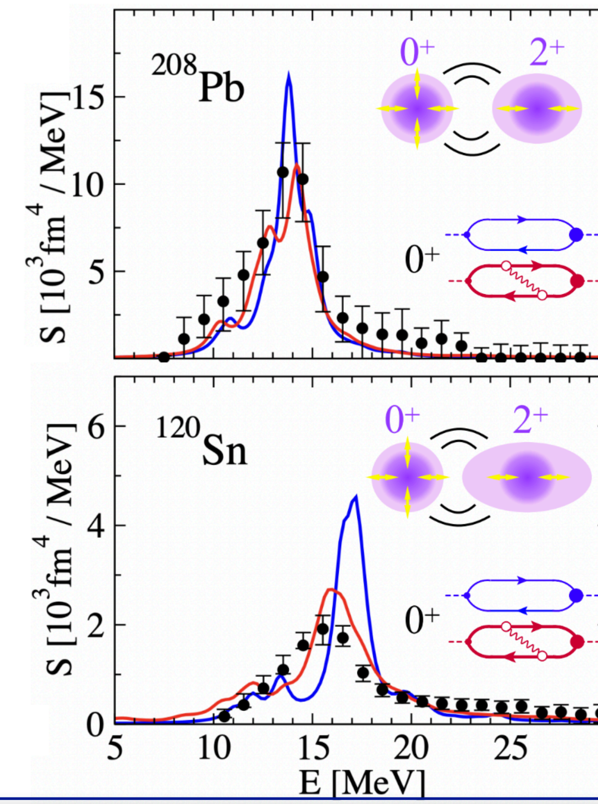
Giant monopole resonance and nuclear compressibility

A universal description of the monopole response across the nuclear chart is achieved.
It is found that nuclear "softness" increases:

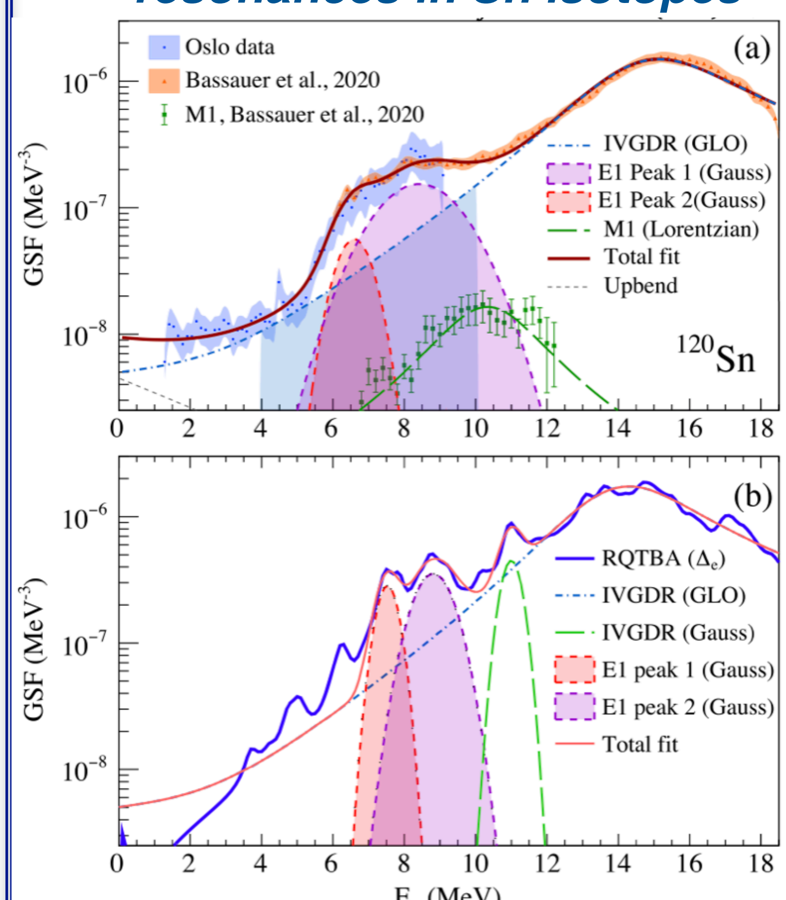
- *with the neutron number*
- *with superfluidity manifested via large quadrupole collectivity*
- *with correlations beyond QRPA (q)PVC*

Stiffer EOS can be used for nuclear matter computation

*E. L., PRC **107**, L041302 (2023)*

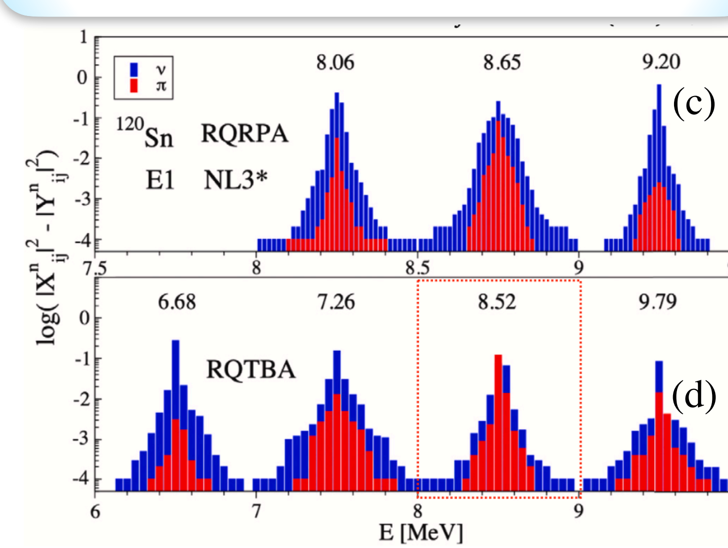


Giant and pygmy dipole resonances in Sn isotopes



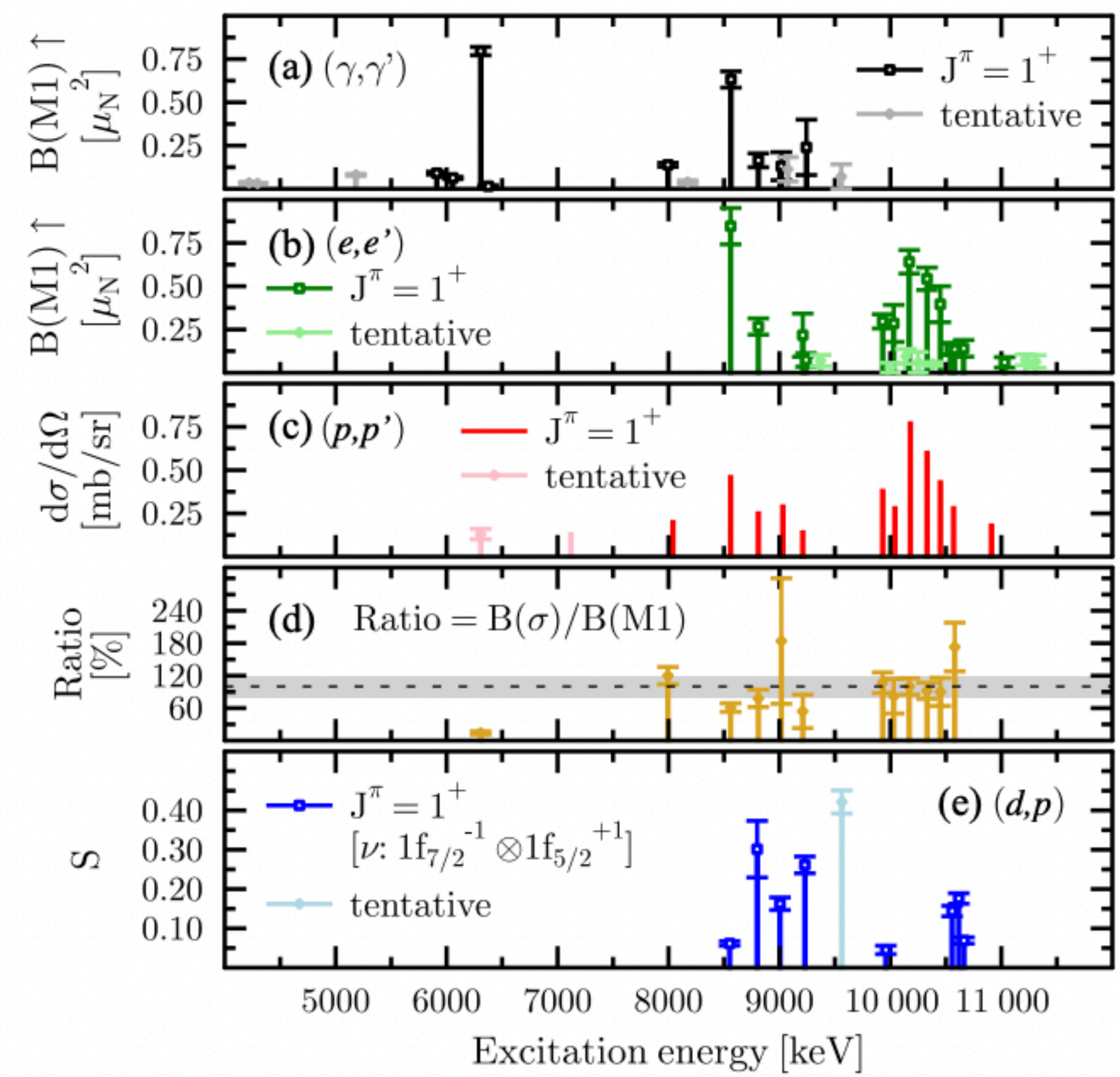
- *qPVC correlations of RQTBA (REOM²) are necessary to reproduce the two-component structure of the low-energy dipole strength: (a), (b).*
- *Only the lower component is associated with the pygmy dipole resonance (PDR) and neutron skin.*
- *PDR is linked to the transitions from (to) the ground state; the upper structure is formed by transitions between excited states: (d) vs (c).*

M. Markova, P. Von Neumann-Cosel,
E. Litvinova, Phys. Lett. B860, 139216 (2025)

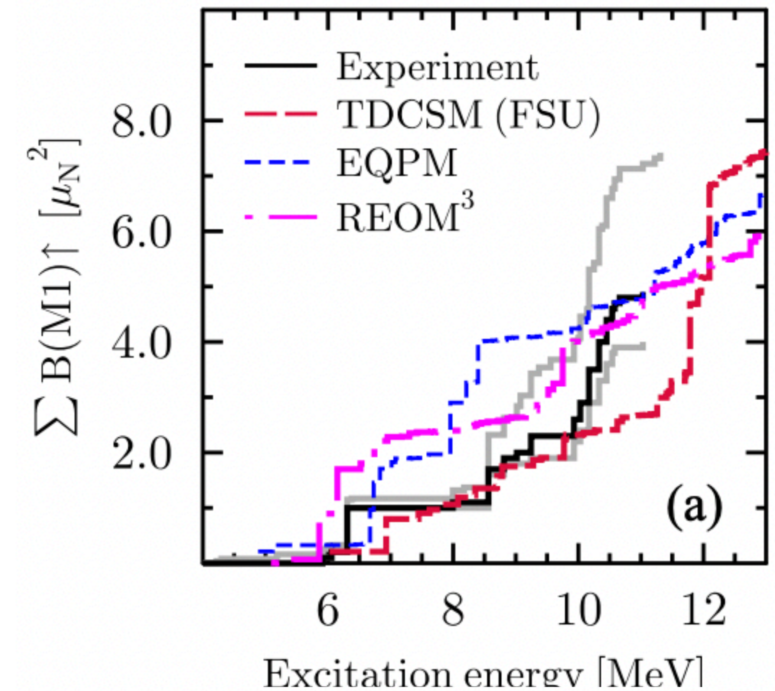


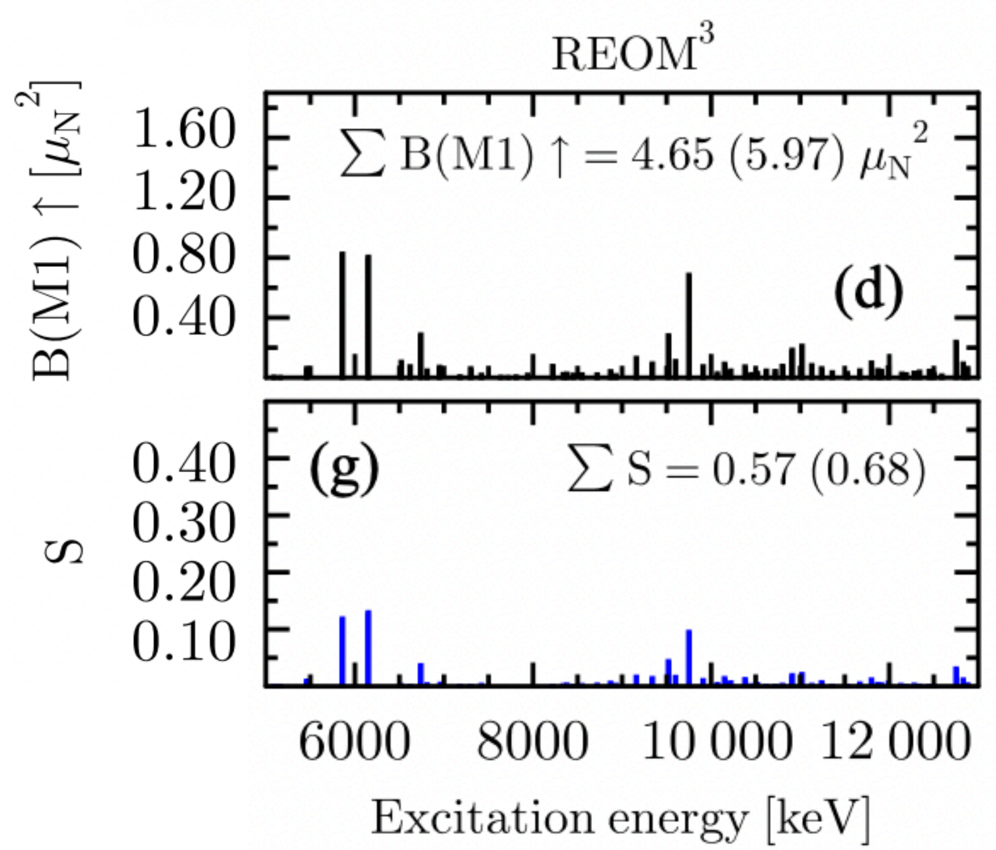
M1 transitions in ^{50}Ti: ^{49}Ti(d,p)^{50}Ti probe at FSU

Combined M1 data



Total B(M1)



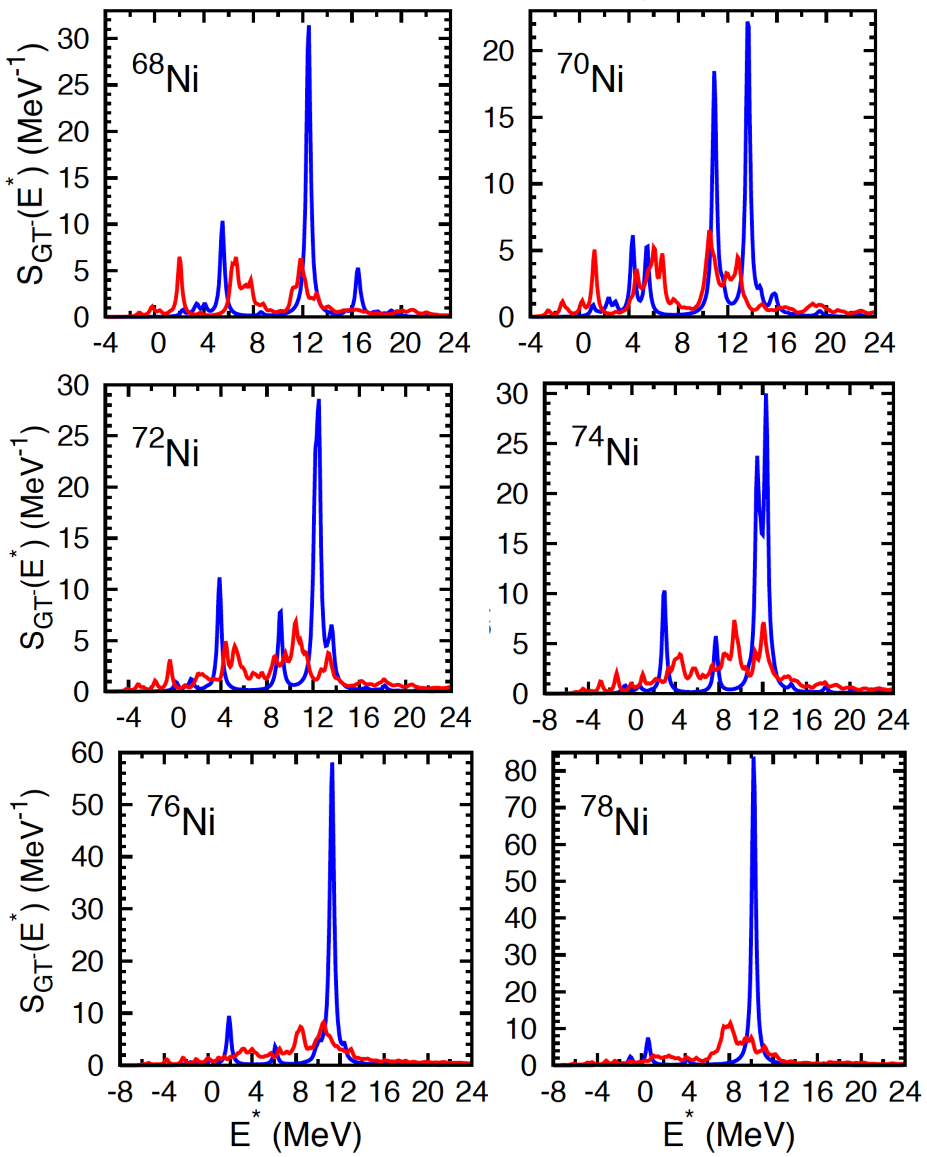


^{49}Ti(d,p)^{50}Ti experiment: that 1+ states with larger neutron $(1f7/2)^{-1}$-$(1f5/2)^{+1}$ spectroscopic factors S do not correspond to the ones with the largest B(M1) strengths. Theory is in tension

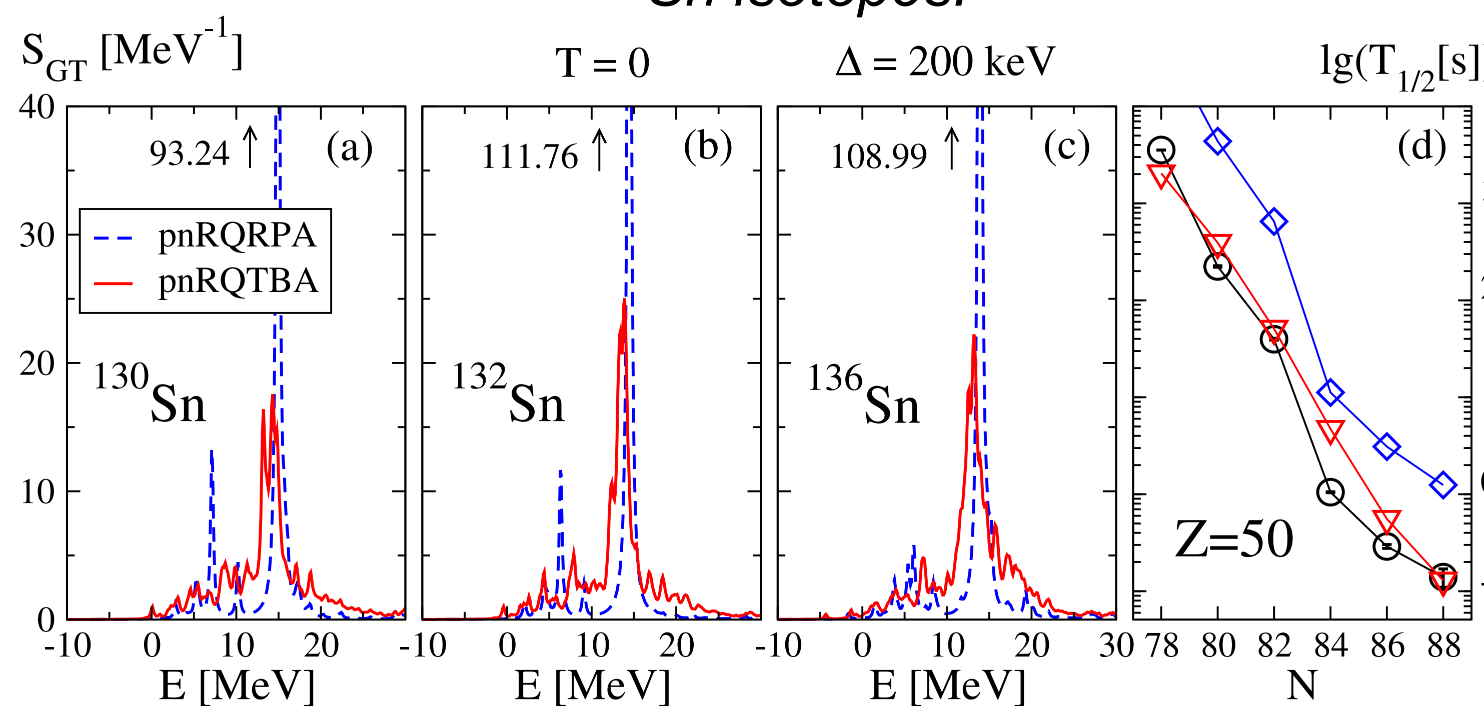
B. Kelly, M. Spieker, …, E.L.,… et al., PRL 136, 082502 (2026)

REOM²: charge-exchange resonances and beta decays

GTR in 68-78Ni isotopes:



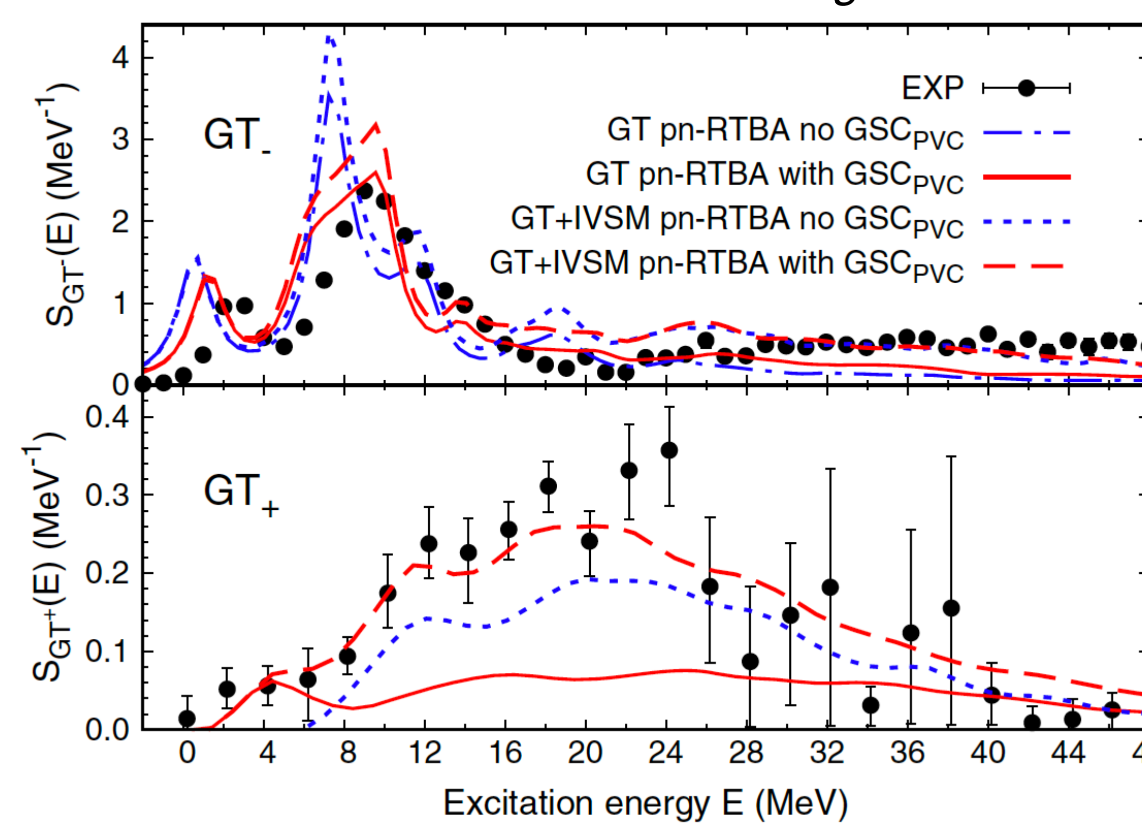
Sn isotopes:

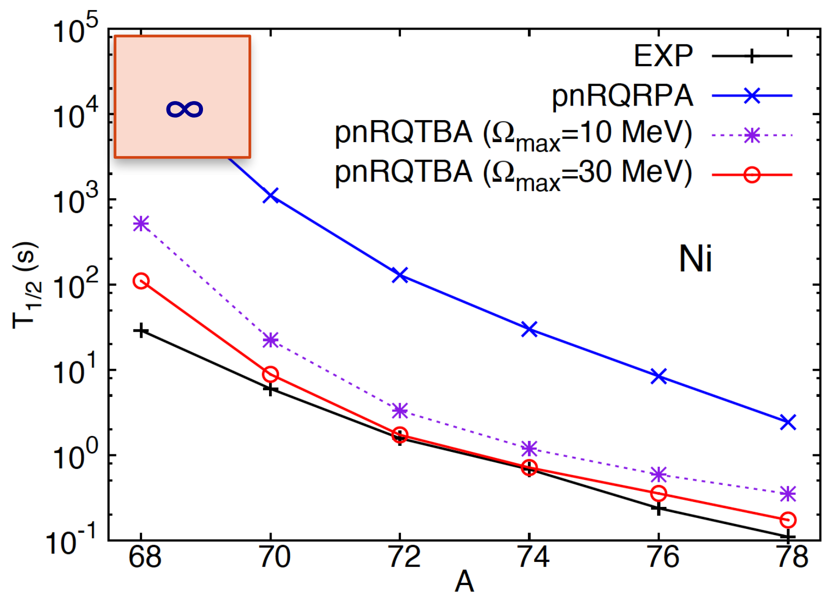


pn-RQRPA [REOM¹]

pn-RQTBA [REOM²]

Gamow-Teller +IVSM strength in 90-Zr:





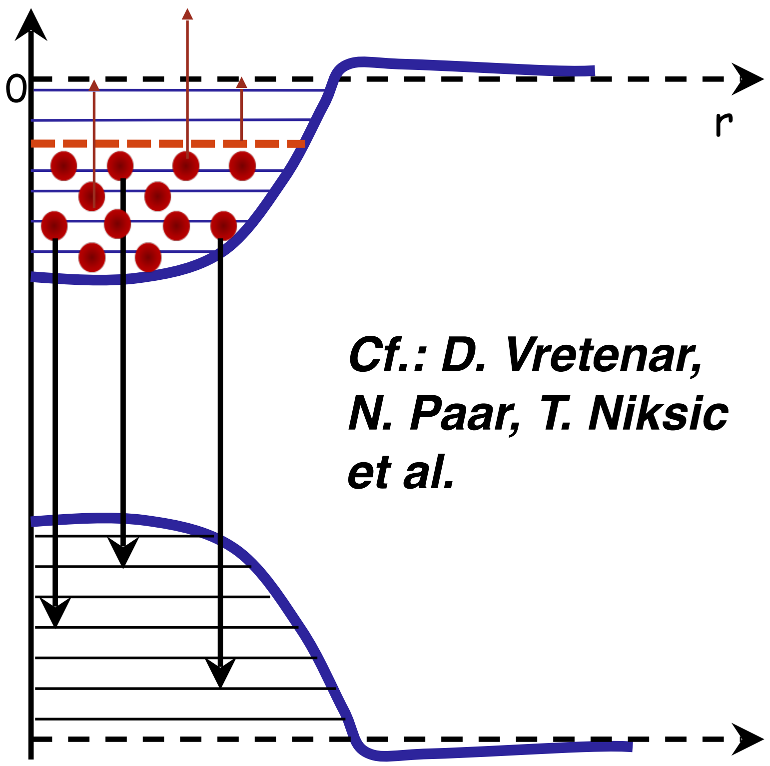
C. Robin, E.L., EPJA 52, 205 (2016)

E. L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)

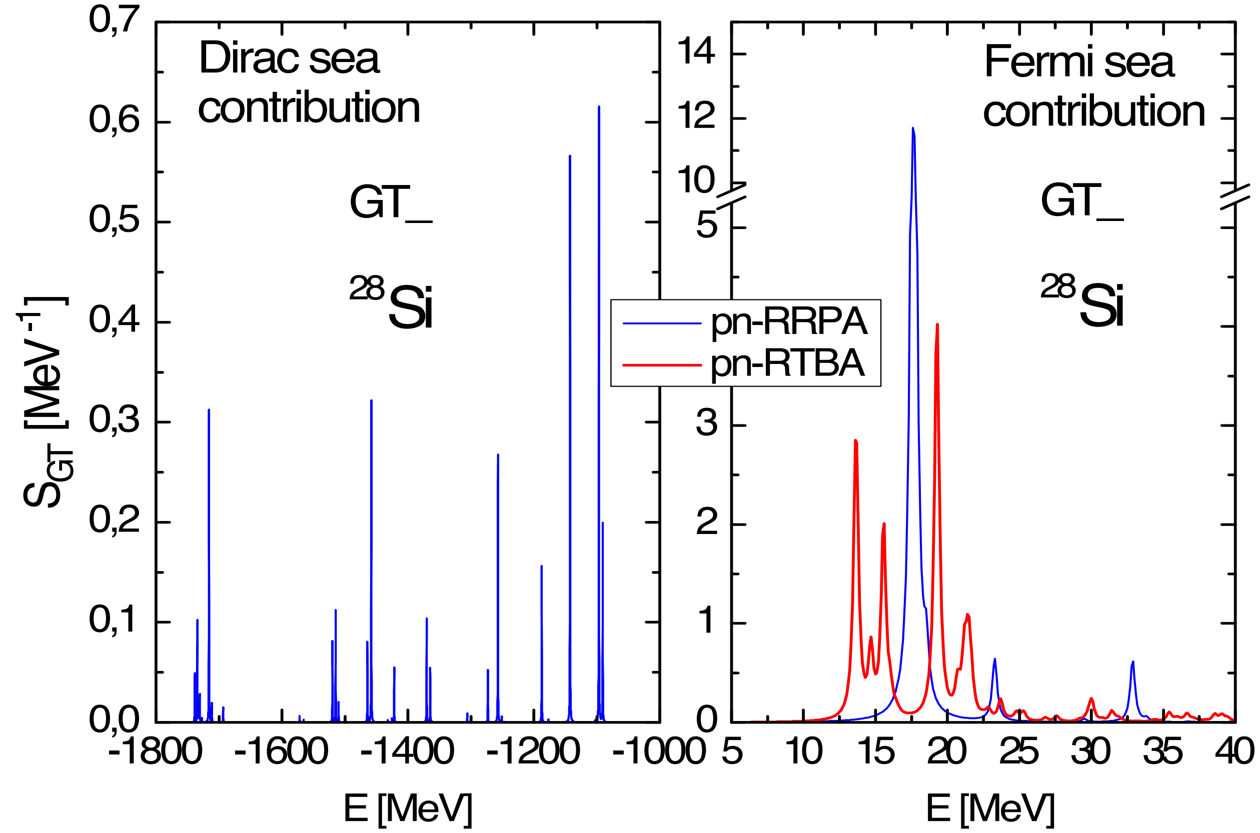
C. Robin, E.L., Phys. Rev. Lett. 123, 202501 (2019)

No fits, no artificial quenching, no adjustable proton-neutron pairing

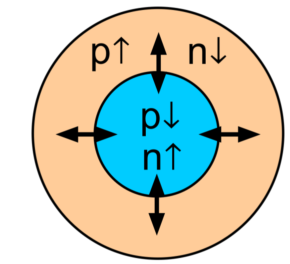
The Gamow-Teller resonance quenching puzzle: 28-Si



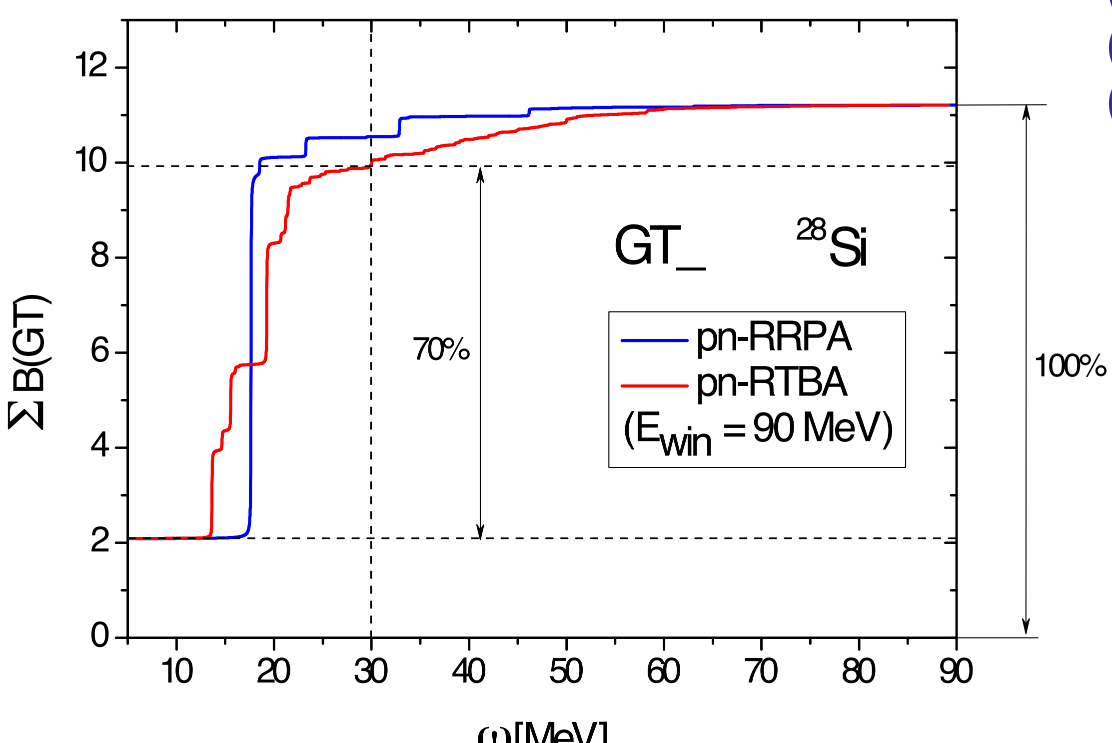
Cf.: D. Vretenar, N. Paar, T. Niksic et al.



$$P = \sum_i \sigma^{(i)} \tau_{\pm}^{(i)}$$



ΔL = 0
ΔT = 1
ΔS = 1

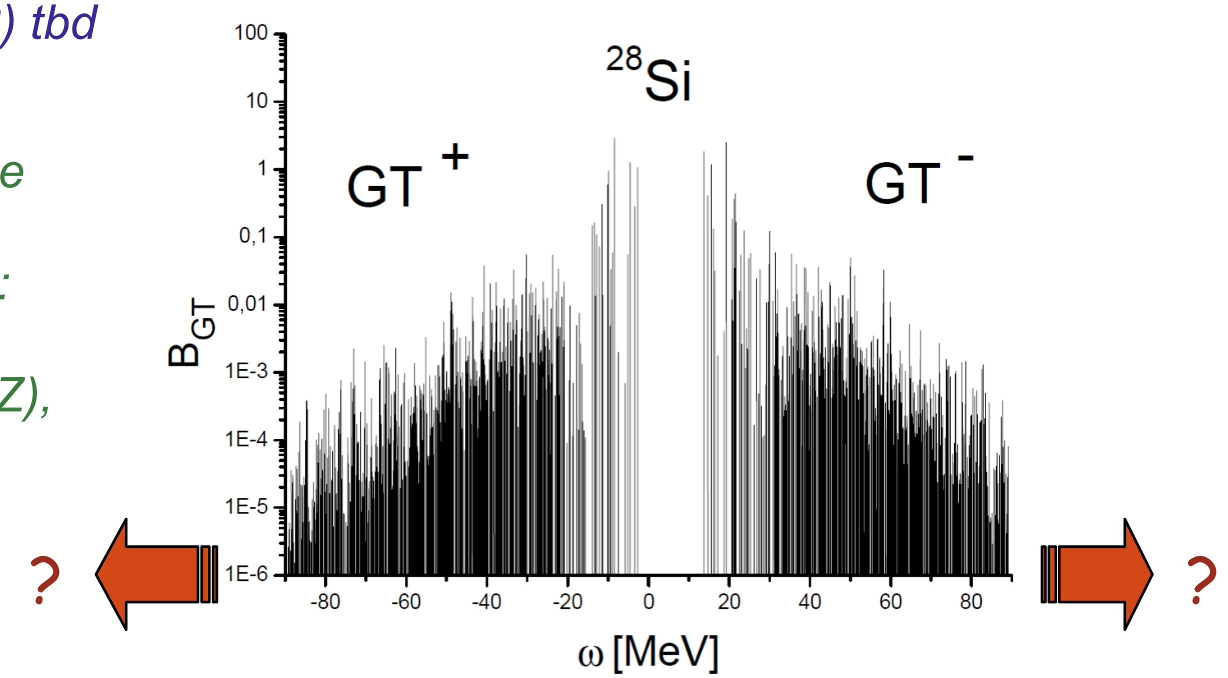


„Microscopic" quenching of B(GT):
(i) relativistic effects,
(ii) ph+phonon and higher configurations
(iii) 2b currents (?) tbd

Cf.: H. Sagawa, Y. Niu, D. Gambacurta

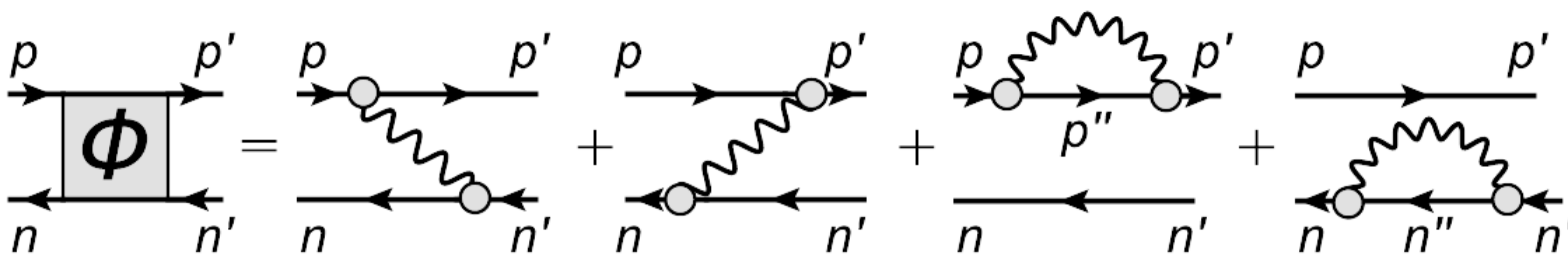
Ikeda Sum rule (model independent):

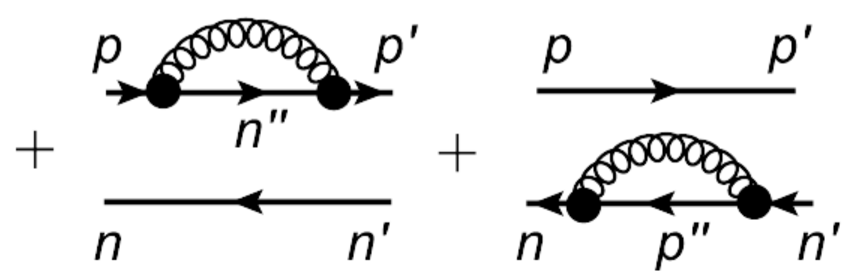
$S_- - S_+ = 3(N - Z),$



Problem: finite basis

The role of coupling to charge-exchange (CE) vibrations

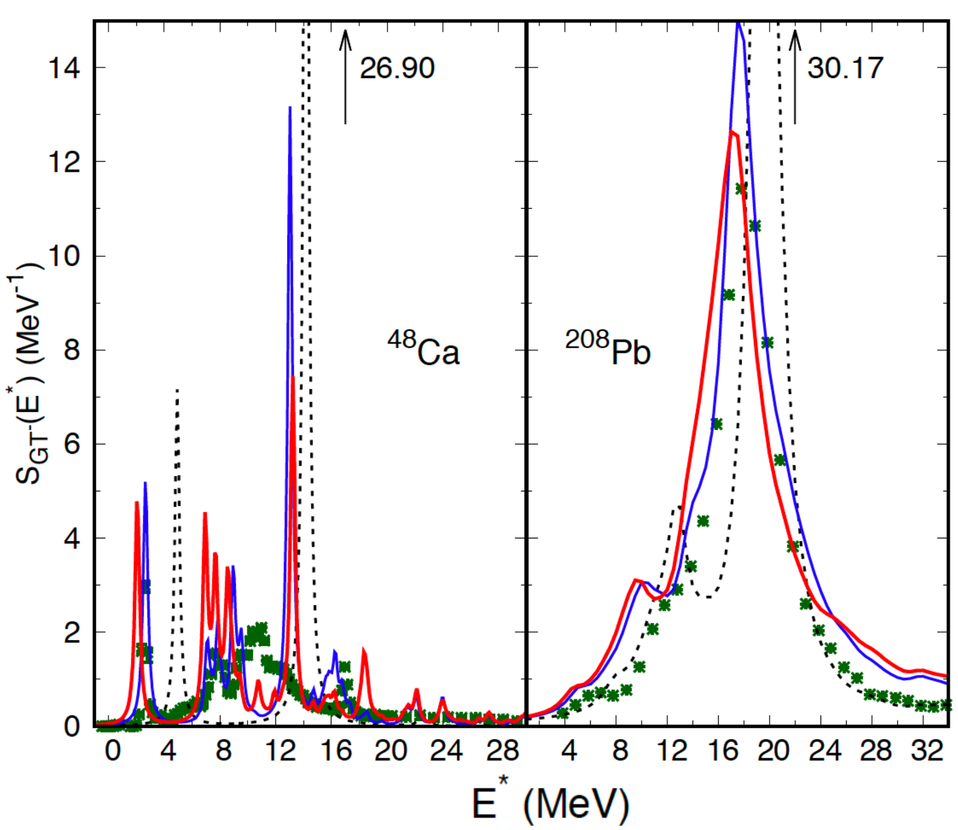


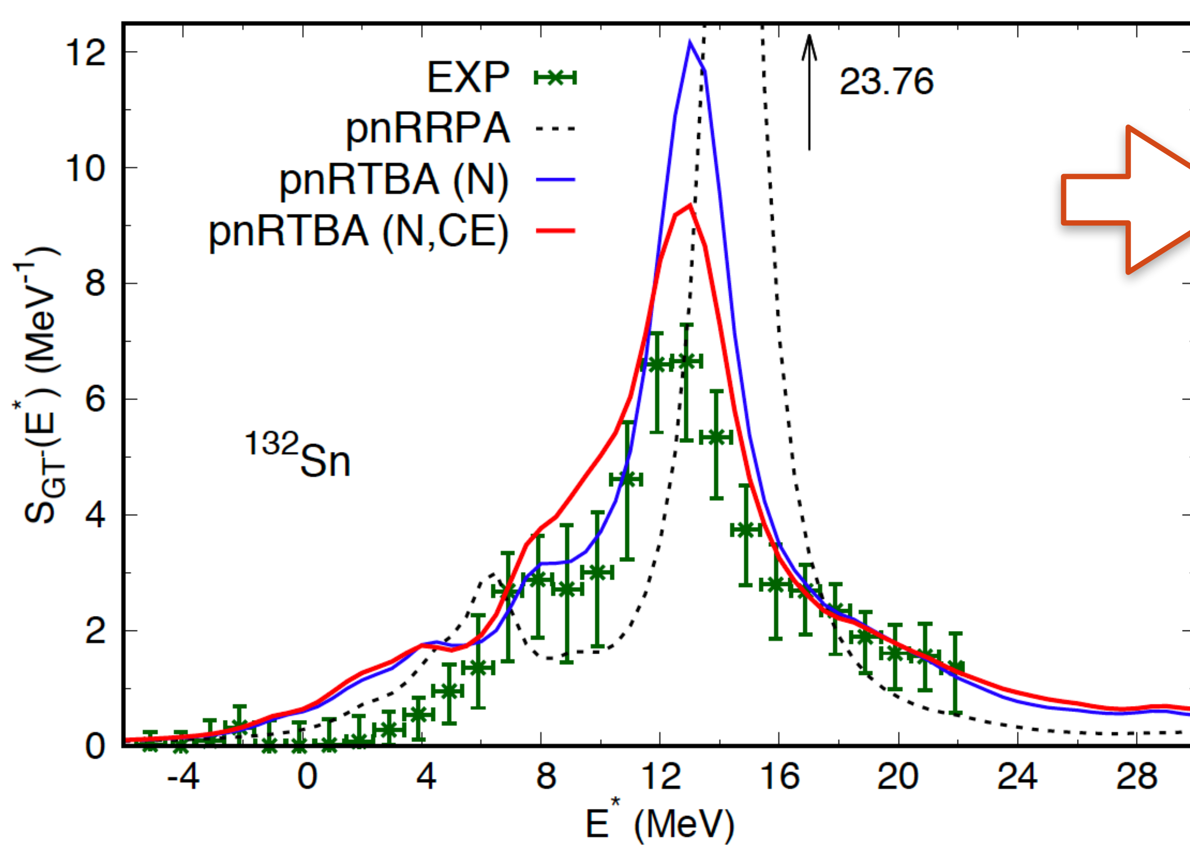


Neutral phonons:
Both phonon-exchange and self-energy contributions

Charge-exchange phonons:
no phonon-exchange counterparts
=> larger than expected contribution!

Gamow-Teller response





3p3h-configurations are needed for a consistent description without applying quenching factors (in progress)

C. Robin, E.L. PRC 98, 051301(R) (2018)

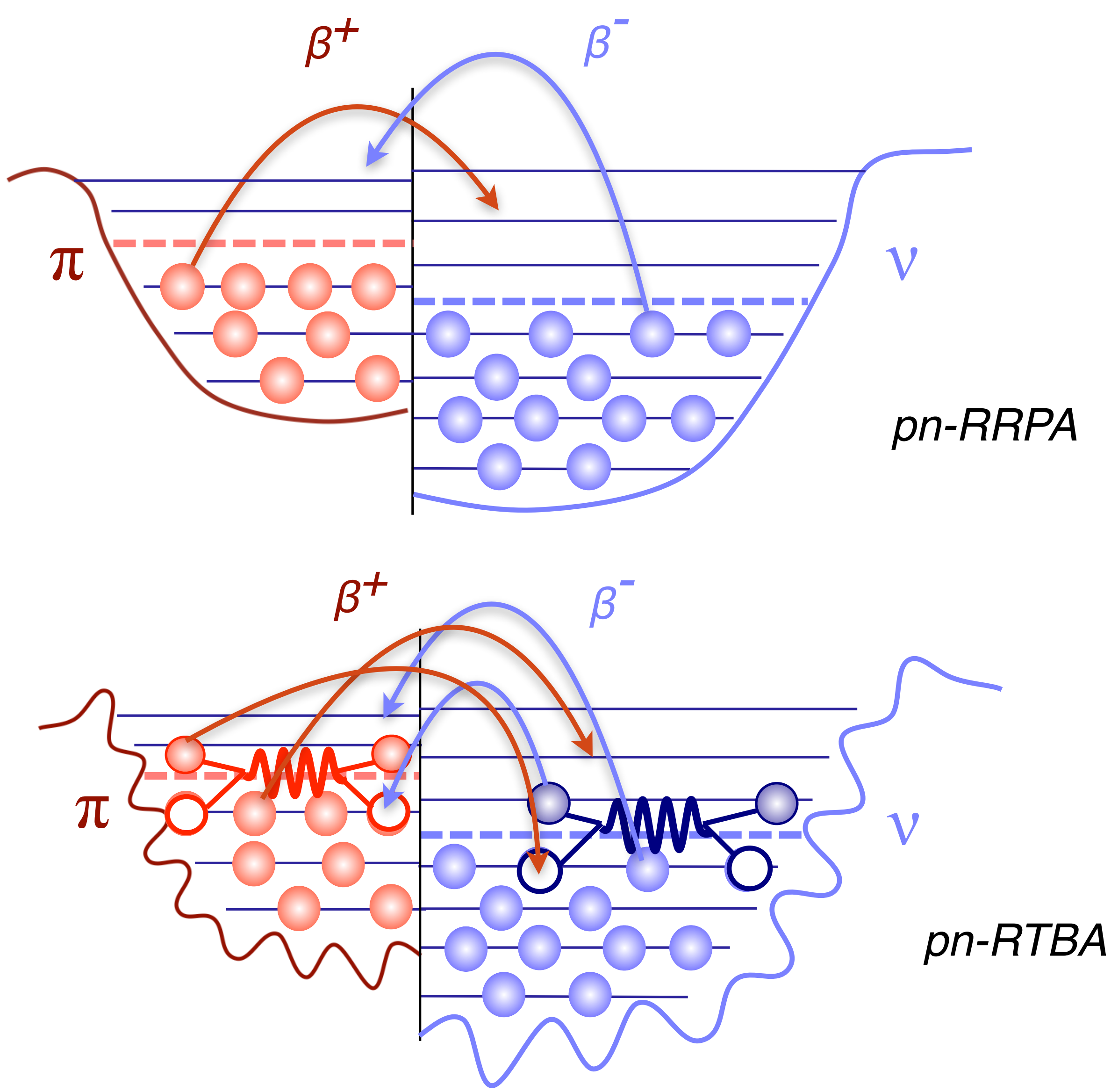
Data: J. Yasuda et al., Phys. Rev. Lett. 121, 132501 (2018)

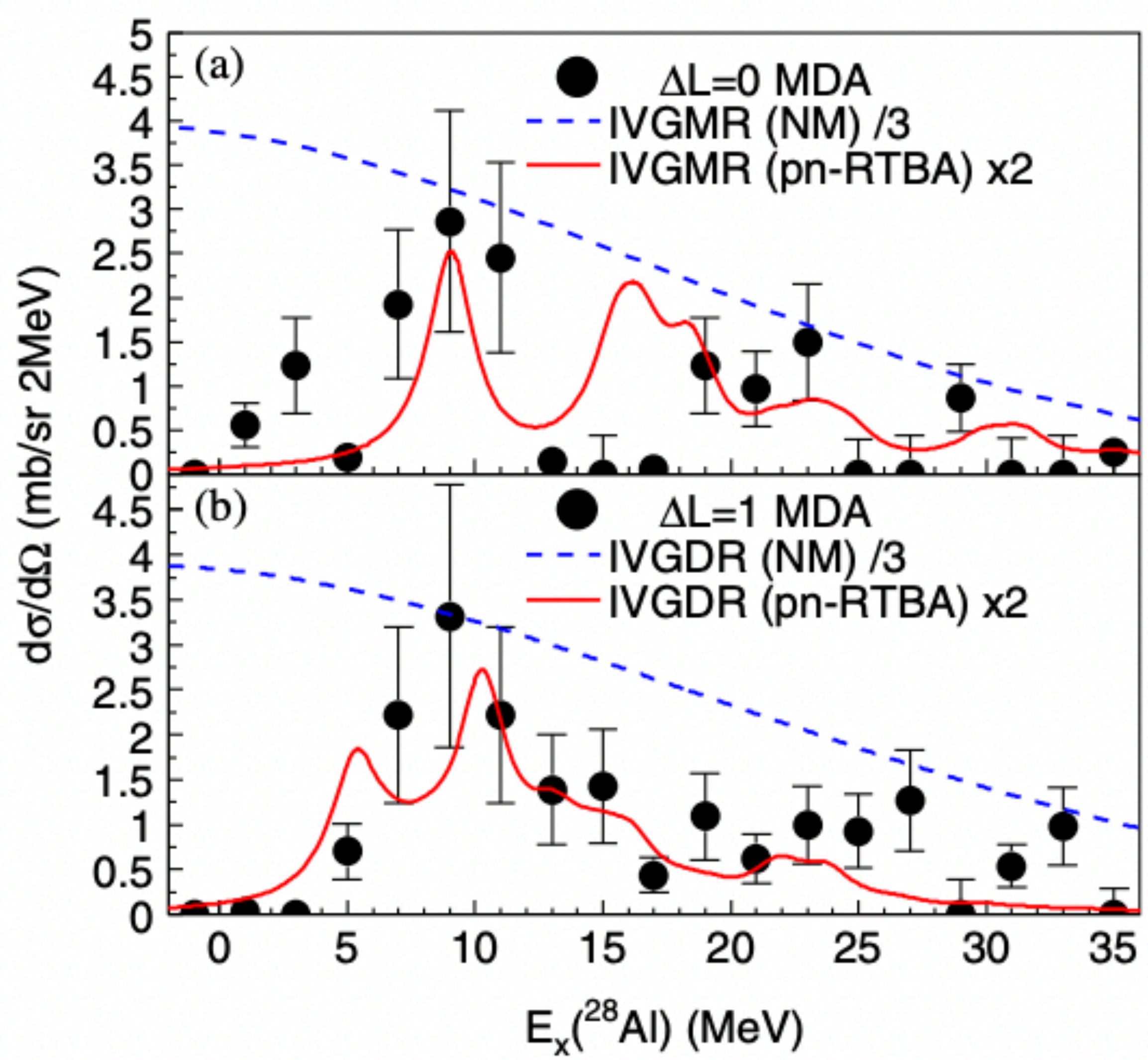
Exotic modes: the Isovector Giant Monopole Resonance

PRL 118, 172501 (2017) PHYSICAL REVIEW LETTERS week ending 28 APRIL 2017

Observation of the Isovector Giant Monopole Resonance via the $^{28}Si(^{10}Be, ^{10}B^*[1.74\ MeV])$ Reaction at 100 AMeV

M. Scott,[1,2,3] R. G. T. Zegers,[1,2,3,*] R. Almus,[4] Sam M. Austin,[1,2,3] D. Bazin,[1] B. A. Brown,[1,2,3] C. Campbell,[5] A. Gade,[1,3] M. Bowry,[1] S. Galès,[6,7] U. Garg,[8] M. N. Harakeh,[9] E. Kwan,[1] C. Langer,[1,2] C. Loelius,[1,2,3] S. Lipschutz,[1,2,3] E. Litvinova,[10,1,2] E. Lunderberg,[1,3] C. Morse,[1,3] S. Noji,[1,2] G. Perdikakis,[4,1,2] T. Redpath,[4] C. Robin,[10,2] H. Sakai,[11] Y. Sasamoto,[12,11] M. Sasano,[11] C. Sullivan,[1,2,3] J. A. Tostevin,[13,1] T. Uesaka,[11] and D. Weisshaar[1]





A path to axially deformed systems: transformation to quasiparticle basis + FAM

Bogolyubov transformation:

$$\psi_1 = \sum_\nu \left(U_{1\nu}\alpha_\nu + V^*_{1\nu}\alpha^\dagger_\nu\right), \qquad \psi^\dagger_1 = \sum_\nu \left(V_{1\nu}\alpha_\nu + U^*_{1\nu}\alpha^\dagger_\nu\right)$$

$$G^{(+)}_{\nu\nu'}(\varepsilon) = \sum_{12}\begin{pmatrix} U^\dagger_{\nu 1} & V^\dagger_{\nu 1}\end{pmatrix}\hat{G}_{12}(\varepsilon)\begin{pmatrix} U_{2\nu'} \\ V_{2\nu'}\end{pmatrix}$$

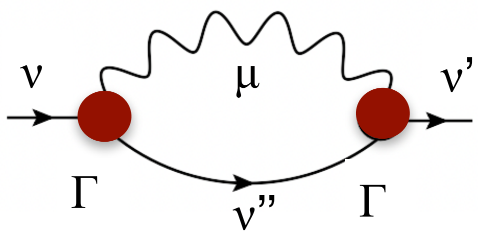
$$G^{(-)}_{\nu\nu'}(\varepsilon) = \sum_{12}\begin{pmatrix} V^T_{\nu 1} & U^T_{\nu 1}\end{pmatrix}\hat{G}_{12}(\varepsilon)\begin{pmatrix} V^*_{2\nu'} \\ U^*_{2\nu'}\end{pmatrix}$$

Propagator becomes diagonal

Dyson Eqs. decouple for η=1 and η=-1: Eq. for η=-1 is redundant

$$G^{(\eta)}_{\nu\nu'}(\varepsilon) = \tilde{G}^{(\eta)}_{\nu\nu'}(\varepsilon) + \sum_{\mu\mu'}\tilde{G}^{(\eta)}_{\nu\mu}(\varepsilon)\Sigma^{r(\eta)}_{\mu\mu'}(\varepsilon)G^{(\eta)}_{\mu'\nu'}(\varepsilon)$$

$$\Sigma^{r(+)}_{\nu\nu'}(\varepsilon) = \sum_{\nu''\mu}\left[\frac{\Gamma^{(11)\mu}_{\nu\nu''}\Gamma^{(11)\mu *}_{\nu'\nu''}}{\varepsilon - E_{\nu''} - \omega_\mu + i\delta} + \frac{\Gamma^{(02)\mu *}_{\nu\nu''}\Gamma^{(02)\mu}_{\nu'\nu''}}{\varepsilon + E_{\nu''} + \omega_\mu - i\delta}\right]$$



Dynamical self-energy: acquires the same form as the non-superfluid one!

Superfluid quasiparticle-vibration coupling (QVC) vertices:

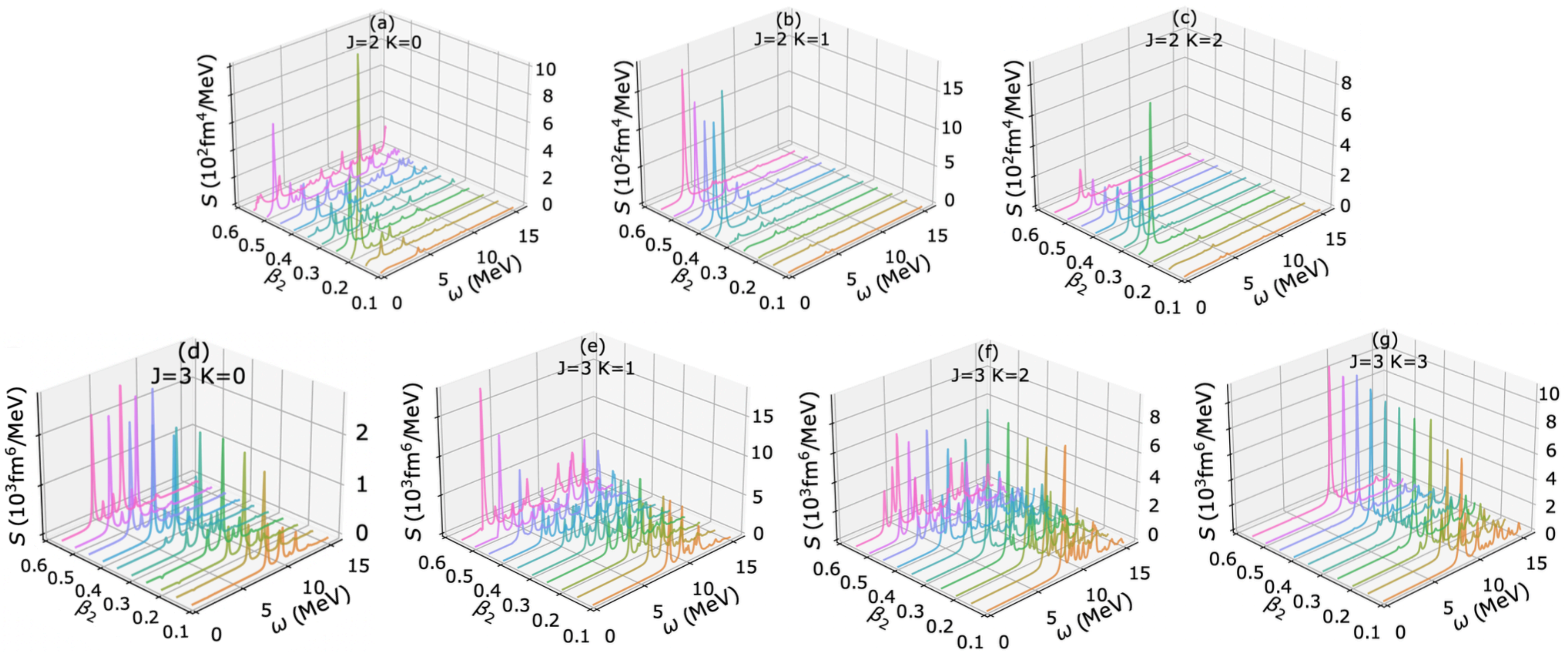
$$\Gamma^{(11)\mu}_{\nu\nu'} = \sum_{12}\left[U^\dagger_{\nu 1}g^\mu_{12}U_{2\nu'} + U^\dagger_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - V^\dagger_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - V^\dagger_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\right]$$

$$\Gamma^{(02)\mu}_{\nu\nu'} = -\sum_{12}\left[V^T_{\nu 1}g^\mu_{12}U_{2\nu'} + V^T_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - U^T_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - U^T_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\right]$$

E.L., Y. Zhang, PRC 104, 044303 (2021)

The phonon spectrum in ^{38}Si and QVC

(i) Relativistic meson-nucleon Lagrangian + (ii) Relativistic Hartree-Bogoliubov (RHB) + (iii) Quasiparticle random phase approximation (QRPA): $J = 2^+ - 5^-$, $K = [0,J]$. Finite amplitude method (FAM): A. Bjelčić et al., CPC 253, 107184 (2020). Relativistic DD-PC1 interaction.



(iv) QVC vertex extraction:

Alt: contour integration; see talk by T. Nikšić

$$\Gamma^{(ij)\varkappa}_{\nu\nu'} = \lim_{\delta\to 0}\sqrt{\frac{\delta}{\pi S(\omega_\varkappa)}}\,\mathrm{Im}\left(\delta\mathcal{H}^{(ij)}_{\nu\nu'}(\omega_\varkappa + i\delta)\right)$$

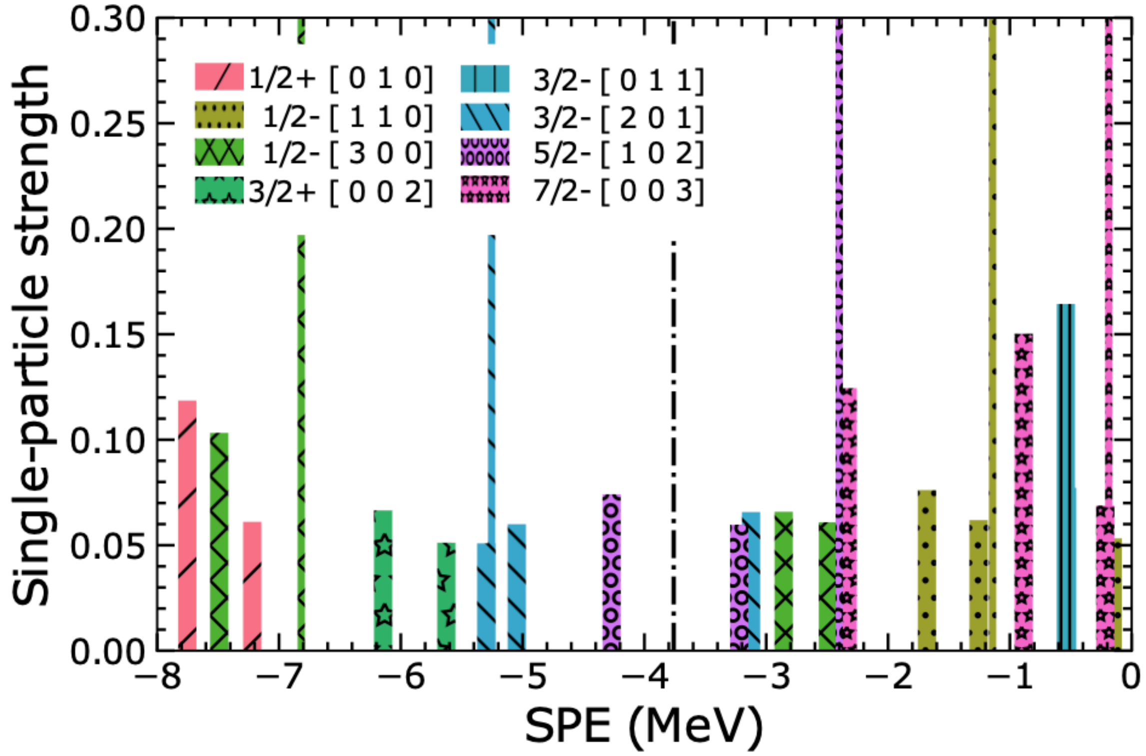
Variation of the HFB Hamiltonian at the QRPA pole

(v) Dyson Eq. solution

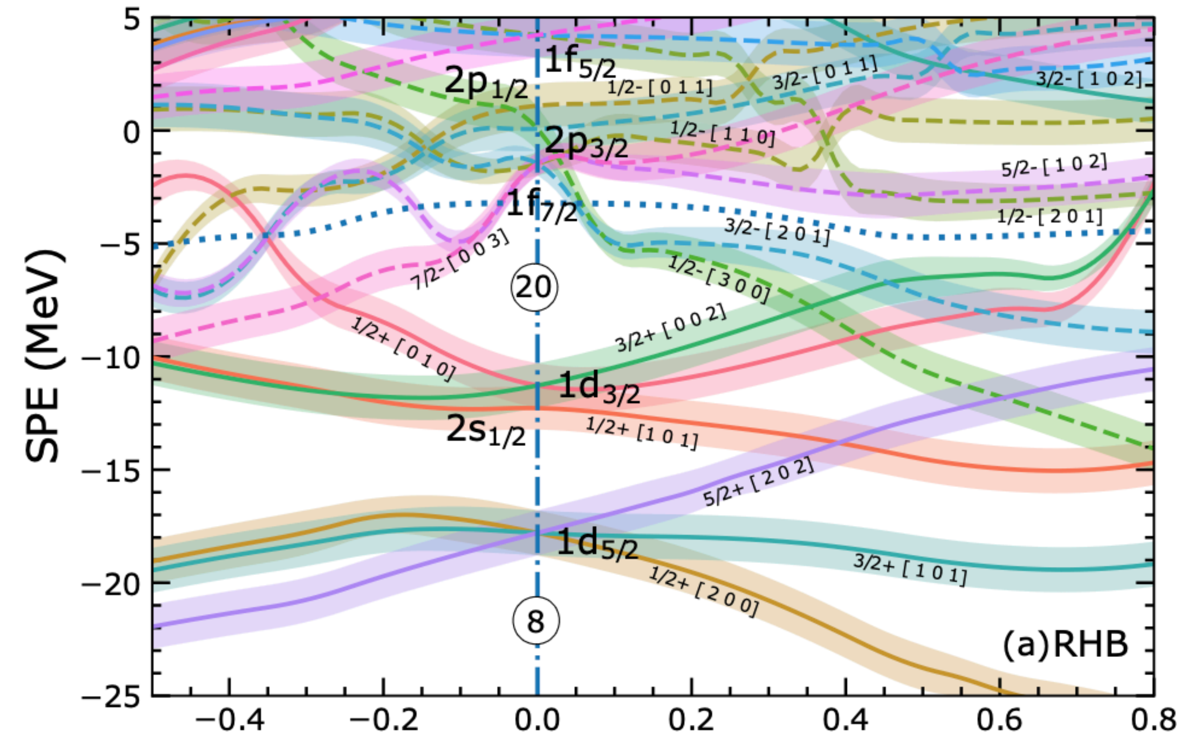
[E.L., Y. Zhang, PRC 104, 044303 (2021)]

Single-(quasi)particle states in ^{38}Si

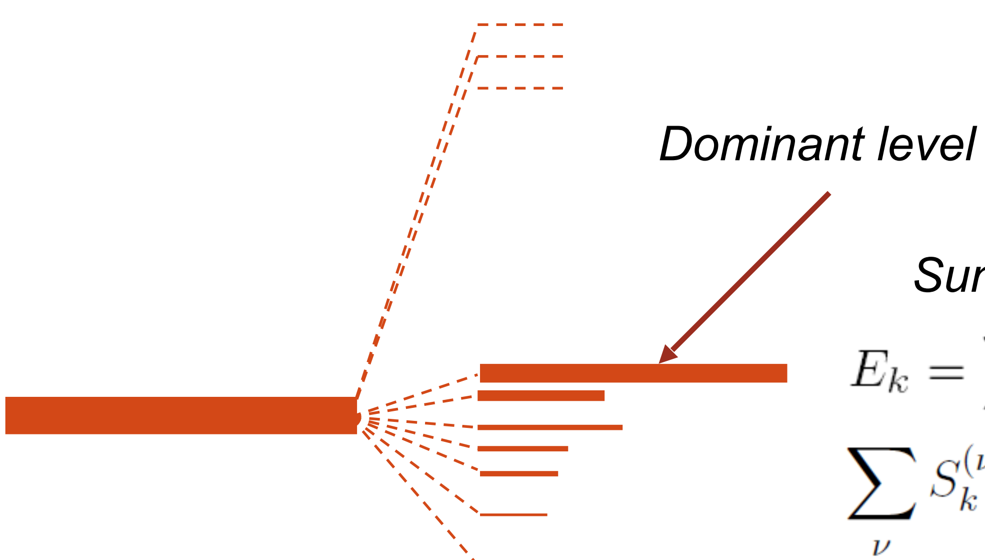
Fragmentation of quasiparticle states: RHB vs RHB+QVC



Nilsson diagram: RHB



Fragmentation mechanism: schematic

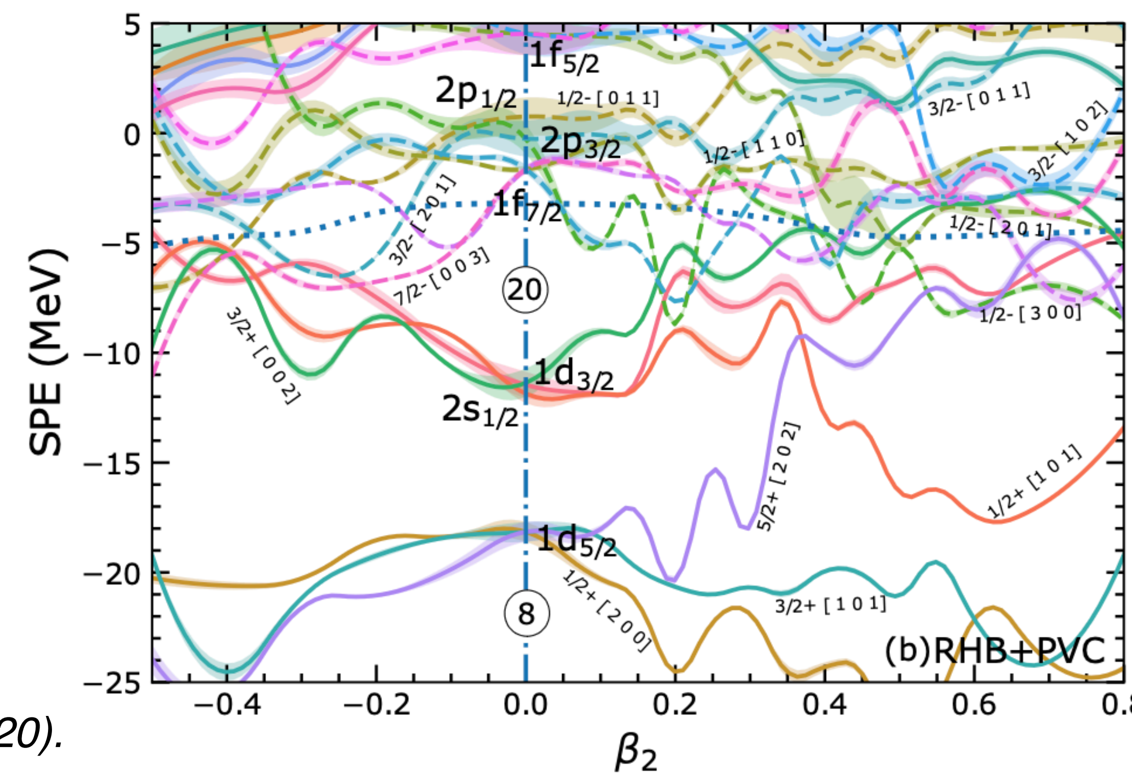


Sum rules:

$$E_k = \sum E_k^{(\nu)} S_k^{(\nu)}$$

$$\sum_\nu S_k^{(\nu)} = 1$$

Nilsson diagram: RHB+QVC (dominant only)



Y. Zhang et al., PRC 105, 044326 (2022)
Based on the code by A. Bjelčić , T. Nikšić et al., CPC 253, 107184 (2020).

Finite amplitude method extended beyond QRPA (frozen):

Generalized FAM (FAM-QVC)

QVC

$$\delta\mathcal{R}^{(20)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{20}_{\mu\nu}(\omega) + \sum\limits_{\mu'\nu'} \Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(20)}_{\mu'\nu'}(\omega) + F^{20}_{\mu\nu}}{\omega - E_\mu - E_\nu}$$

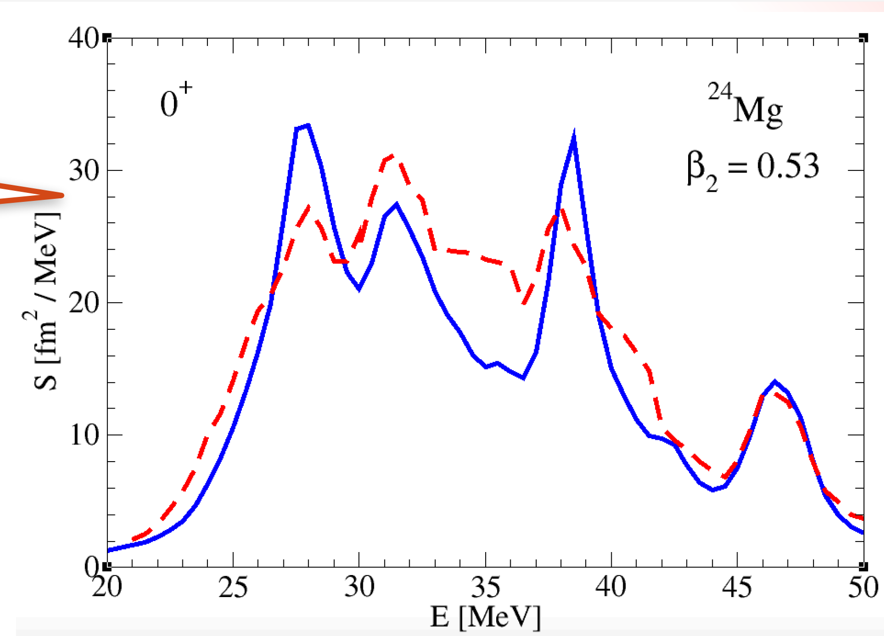
$$\delta\mathcal{R}^{(02)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{02}_{\mu\nu}(\omega) + \sum\limits_{\mu'\nu'} \Phi^{(-)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(02)}_{\mu'\nu'}(\omega) + F^{02}_{\mu\nu}}{-\omega - E_\mu - E_\nu}.$$

QVC amplitude (leading approximation):

$$\Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega) = \sum_n \Big[\delta_{\mu\mu'} \sum_{\nu''} \frac{\bar{\Gamma}^{(11)n}_{\nu''\nu}\bar{\Gamma}^{(11)n*}_{\nu''\nu'}}{\omega - E_\mu - E_{\nu''} - \omega_n} + \delta_{\nu\nu'} \sum_{\mu''} \frac{\Gamma^{(11)n}_{\mu\mu''}\Gamma^{(11)n*}_{\mu'\mu''}}{\omega - E_{\mu''} - E_\nu - \omega_n} - \frac{\Gamma^{(11)n}_{\mu\mu'}\bar{\Gamma}^{(11)n*}_{\nu\nu'}}{\omega - E_{\mu'} - E_\nu - \omega_n} - \frac{\Gamma^{(11)n*}_{\mu'\mu}\bar{\Gamma}^{(11)n}_{\nu'\nu}}{\omega - E_\mu - E_{\nu'} - \omega_n} \Big],$$

E.L., Y. Zhang, PRC (2022) [formalism]

Proof of principle:
GMR in 24-Mg
in a restricted model space



Ongoing:
- *Convergence improvement*
- *Optimization*
- *Cross-check routines*