Round Table T2:
“Comparison between beyond-QRPA approaches”

Elena Litvinova

Western Michigan University

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A strongly-correlated many body system: single-fermion propagator, particle-hole propagator and related observables

$$H = \sum_{12} \bar{\psi}_1 (-i\gamma \cdot \nabla + M)_{12} \psi_2 + \frac{1}{4} \sum_{1234} \bar{\psi}_1 \bar{\psi}_2 \bar{v}_{1234} \psi_4 \psi_3 \quad = T + V^{(2)}$$

Hamiltonian, *extendable to 3B forces (3BFs are minimized in covariant theories)*

$$G_{11'}(t-t') = -i\langle T \psi(1) \bar{\psi}(1') \rangle$$

$$1 = \{\xi_1, t\}$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$$\rangle = |0\rangle$$

Single-particle propagator

$$G_{11'}(\varepsilon) = \sum_n \frac{\eta_1^n \bar{\eta}_{1'}^{n*}}{\varepsilon - \varepsilon_n^+ + i\delta} + \sum_m \frac{\chi_1^m \bar{\chi}_{1'}^{m*}}{\varepsilon + \varepsilon_m^- - i\delta}$$

Fourier transform: Spectral expansion

$$\eta_1^n = \langle 0^{(N)} | \psi_1 | n^{(N+1)} \rangle \qquad \chi_1^m = \langle m^{(N-1)} | \psi_1 | 0^{(N)} \rangle$$

Ground state of N particles

(Excited) state of (N+1) particles

Residues *- spectroscopic (occupation) factors*

Poles *- single-particle energies*

$$R_{12,1'2'}(t-t') = -i\langle T(\bar{\psi}_1 \psi_2)(t)(\bar{\psi}_{2'} \psi_{1'})(t') \rangle$$

Particle-hole response function

Fourier transform: Spectral expansion

$$R_{12,1'2'}(\omega) = \sum_{\nu>0} \left[\frac{\rho_{21}^\nu \bar{\rho}_{2'1'}^{\nu*}}{\omega - \omega_\nu + i\delta} - \frac{\bar{\rho}_{12}^{\nu*} \rho_{1'2'}^\nu}{\omega + \omega_\nu - i\delta} \right]$$

Excitation energies

$$\rho_{12}^\nu = \langle 0 | \bar{\psi}_2 \psi_1 | \nu \rangle$$

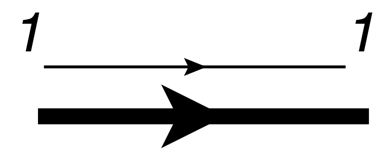
Residues *- transition densities*

Poles *- excitation energies*

Diagrammatic conventions

One-fermion propagator:

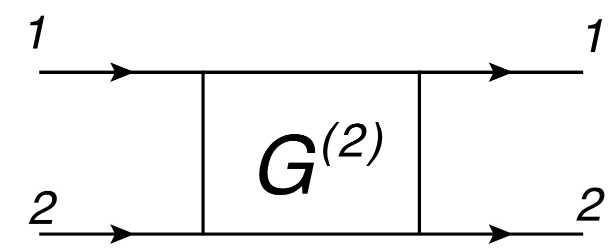
$$G(1,1') = -i\langle T\psi(1)\bar{\psi}(1')\rangle \quad = $$



uncorrelated
correlated

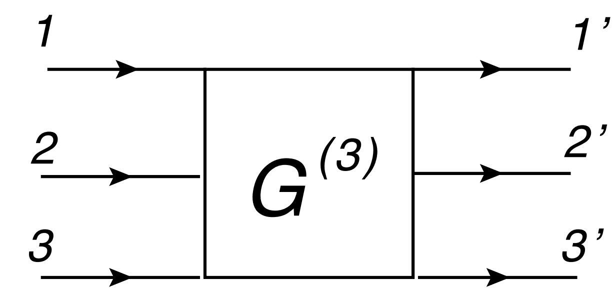
Two-fermion propagator:

$$G(12,1'2') = (-i)^2\langle T\psi(1)\psi(2)\bar{\psi}(2')\bar{\psi}(1')\rangle \quad = $$



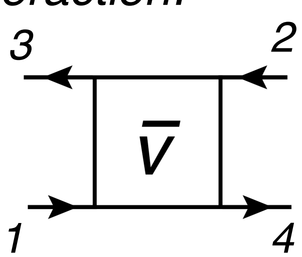
Three-fermion propagator:

$$G(123,1'2'3') = (-i)^3\langle T\psi(1)\psi(2)\psi(3)\bar{\psi}(3')\bar{\psi}(2')\bar{\psi}(1')\rangle \quad = $$



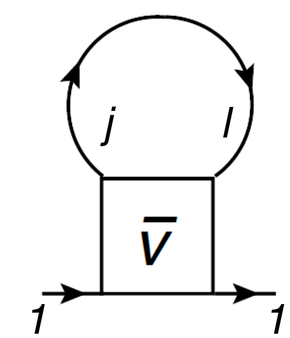
Two-fermion interaction:

$$\bar{v}_{1234} \quad = $$



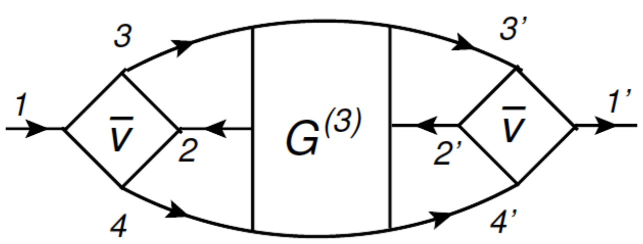
Static self-energy:

$$\sum_{jl} \bar{v}_{1j1'l}\,\rho_{lj} \quad = $$



Dynamical self-energy:

$$\sum_{234}\sum_{2'3'4'} \bar{v}_{1234}\,G(432',23'4')\,\bar{v}_{4'3'2'1'} \quad = $$



Exact equations of motion (EOM) with two-nucleon interactions: one-body problem

One-fermion propagator

$$G_{11'}(t-t') = -i\langle T\psi(1)\bar{\psi}(1')\rangle$$

***EOM**: Dyson Eq.*

$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \qquad (*)$$

$$\Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

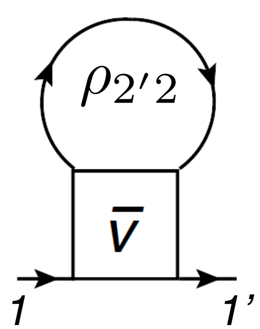
Irreducible kernel (aka Self-energy, exact)

Instantaneous term (Hartree-Fock incl. "tadpole")

Short-range correlations

$$\Sigma^{(0)}_{11'} = -\langle \gamma^0 \left\{ [V, \psi_1], \bar{\psi}_{1'} \right\} \gamma^0 \rangle$$

$$= \sum_{22'} \bar{v}_{121'2'} \langle \bar{\psi}_2 \psi_{2'} \rangle =$$



t-dependent (dynamical) term (symmetric version):

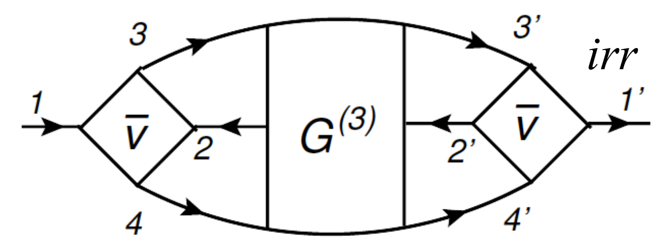
Long-range correlations: connecting scales

Symmetric form

$$\Sigma^{(r)}_{11'} = i\langle T\gamma^0[V,\psi_1](t)[V,\bar{\psi}_{1'}\ ](t')\gamma^0\rangle^{irr}$$

$$= -\frac{1}{4}\sum_{234}\sum_{2'3'4'} \bar{v}_{1234} G^{(3)irr}_{432',23'4'}(t-t')\bar{v}_{4'3'2'1'}$$

$$= -\frac{1}{4}$$



$$\rho_{11'} = -i \lim_{t=t'-0} G_{11'}(t-t')$$

is the full solution of (*):
includes the dynamical term!

Koltun-Migdal-Galitsky sum rule: ***the binding energy***

"Ab-initio DFT":

$$E_0 = \frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F^-} d\varepsilon \sum_{12}(T_{12} + \varepsilon\delta_{12})\mathrm{Im}G_{21}(\varepsilon)$$

Exact equation of motion (EOM) for the particle-hole response

Particle-hole propagator (response function):

$$R_{12,1'2'}(t-t') = -i\langle T(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t')\rangle$$

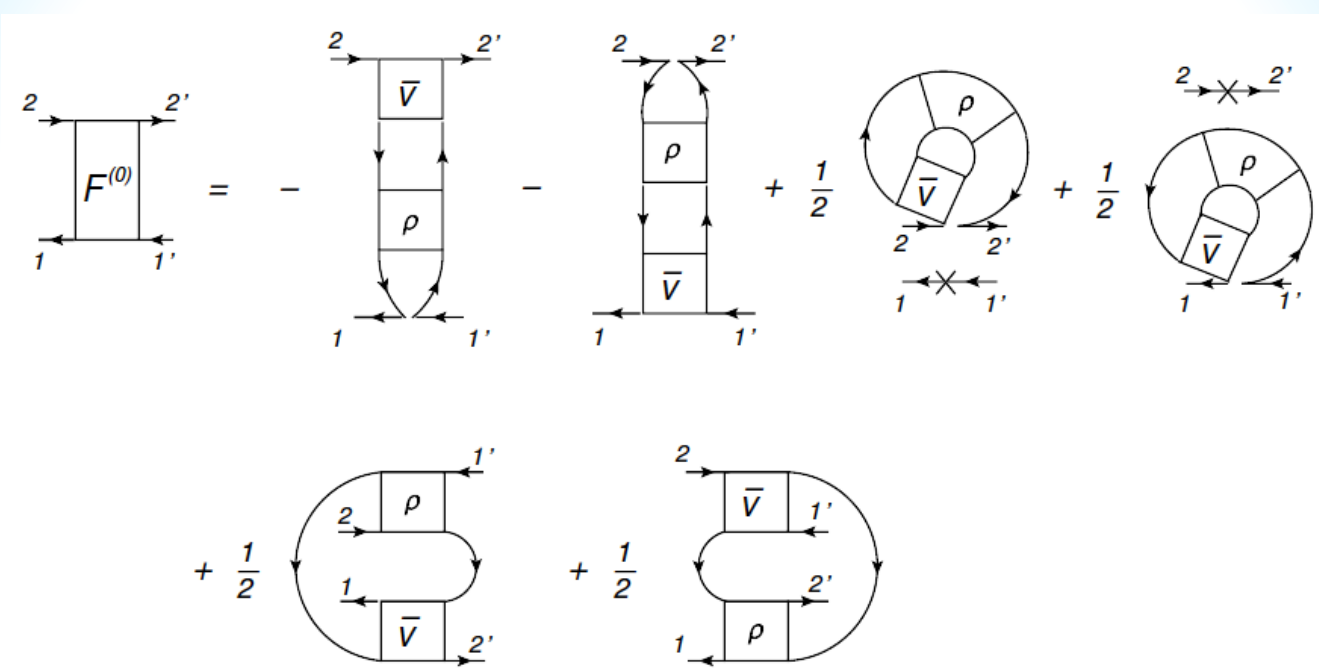
spectra of excitations, masses, decays, …

***EOM**: Bethe-Salpeter-Dyson Eq.*

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (**) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Irreducible kernel (exact):

Instantaneous term (“bosonic” mean field):
Short-range correlations

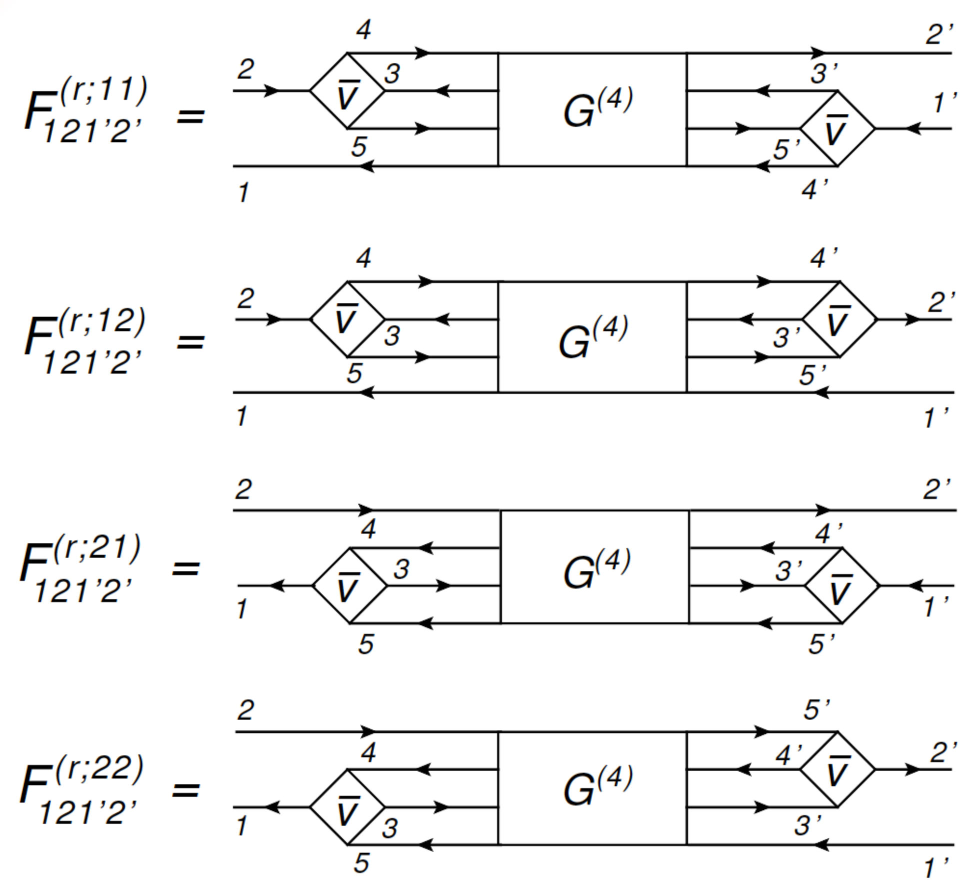


Self-consistent mean field F(0), where

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i \lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

*contains the full solution of (**) **including the dynamical term!***

t-dependent (dynamical) term:
Long-range correlations



$$F^{(r)}_{12,1'2'}(t-t') = \sum_{ij} F^{(r;ij)}_{12,1'2'}(t-t')$$

Dynamical kernels: bridging the scales

- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy or Schwinger-Dyson equations. $N_{Equations} = N_{Particles}$ & Coupled 🙈 !!!*
- ***Non-perturbative solutions:***

Cluster decomposition

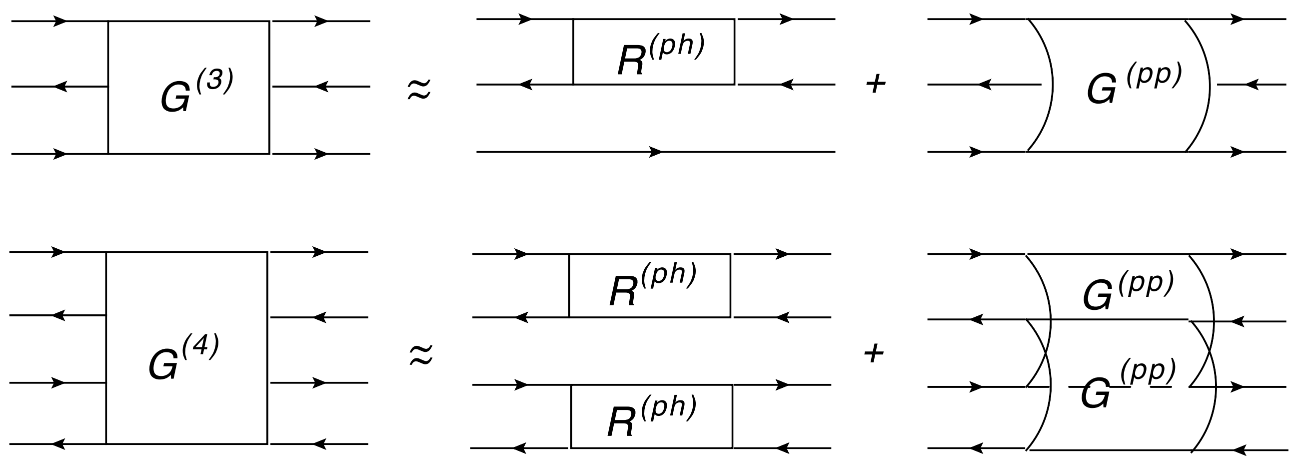
$$\blacklozenge\ G^{(3)} = \boxed{G^{(1)} G^{(1)} G^{(1)}} + \boxed{G^{(2)} G^{(1)}} + \boxed{\Xi^{(3)}}$$

$$\blacklozenge\ G^{(4)} = \boxed{G^{(1)} G^{(1)} G^{(1)} G^{(1)}} + \boxed{G^{(2)} G^{(2)}} + \boxed{G^{(3)} G^{(1)} + \Xi^{(4)}}$$

Leading at: **weak** / **intermediate** / **strong** *coupling*

weak	intermediate	strong
self-consistent GFs second RPA etc.	*phonon coupling* ***this work***	***Faddeev + future work***

Beyond weak coupling:

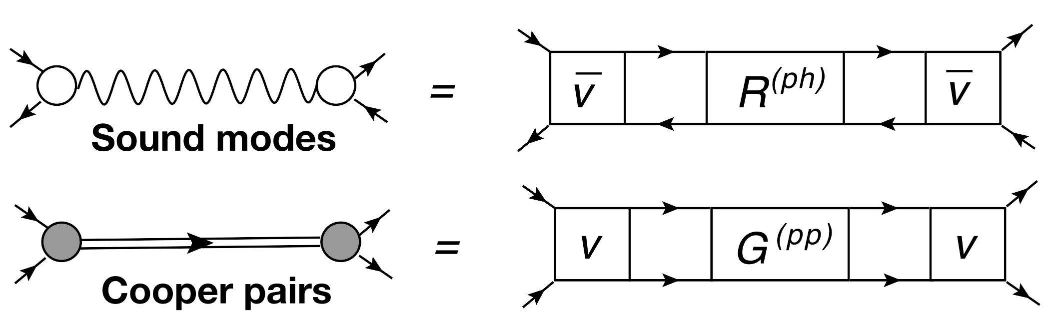


- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- *…*

Exact mapping: particle-hole (2q) quasibound states

Emergence of effective "bosons" (phonons, vibrations):

Emergence of superfluidity:

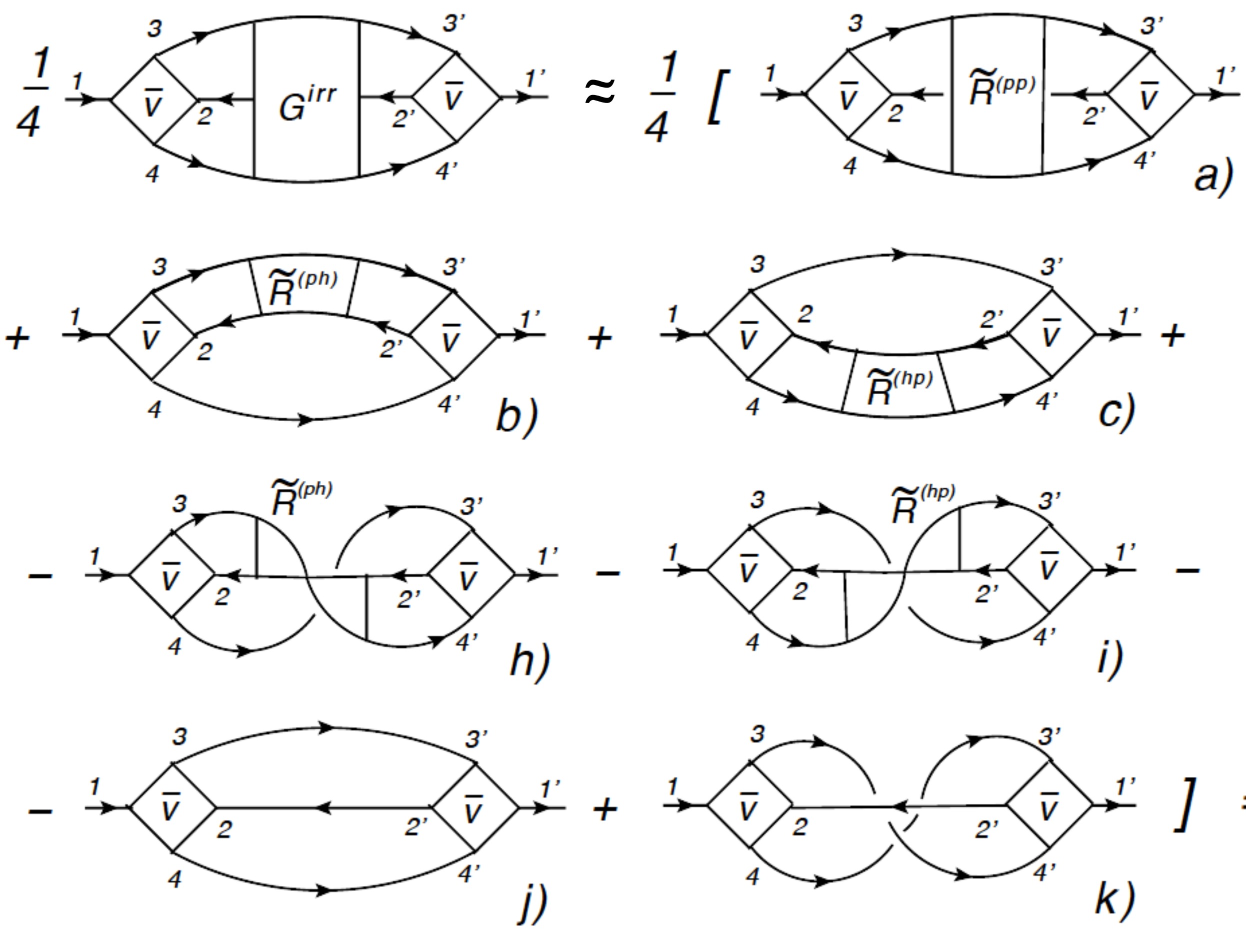


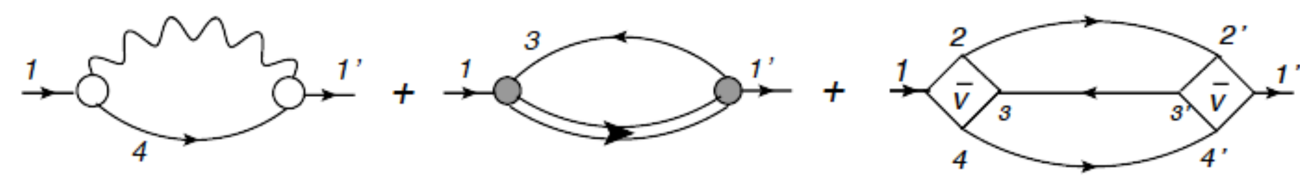
Quasiparticle-vibration coupling (qPVC) *in nuclei. Cf. NFT*

Diquarks in hadrons

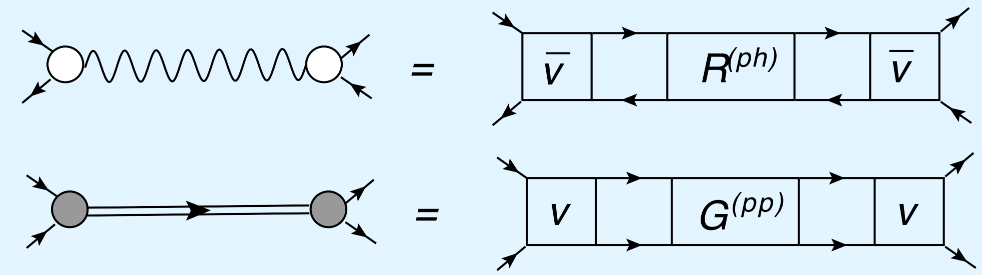
Cf. C. Popovici, P. Watson, and H. Reinhardt [PRD83, 025013 (2011)]

Exact mapping to the particle-vibration coupling (PVC)





ph correlator: coupling to normal phonons



pp correlator: coupling to pairing phonons

Dropping the uncorrelated term: “factor 1/2” problem” (Ring & Schuck, Book)
“sign problem” (Danielewicz & Schuck, NPA567 (1994) 78)
But: The lowest order is not the leading order in the strong coupling regime!
=> the uncorrelated term can be safely neglected

Model-independent mapping to the PVC:

$$\sum_{242'4'} \bar{v}_{1234} R^{ph}_{24,2'4'}(\omega) \bar{v}_{4'3'2'1'} = \sum_{\nu,\sigma=\pm1} g^{\nu(\sigma)}_{13} D^{(\sigma)}_{\nu}(\omega) g^{\nu(\sigma)*}_{1'3'}$$

“phonon” vertex:

$$g^{\nu}_{13} = \sum_{24} \bar{v}_{1234}\rho^{\nu}_{42}$$

“phonon” propagator:

$$R_{12,1'2'}(\omega) = \sum_{\nu>0}\Big[\frac{\rho^{\nu}_{21}\rho^{\nu*}_{2'1'}}{\omega-\omega_\nu+i\delta} - \frac{\rho^{\nu*}_{12}\rho^{\nu}_{1'2'}}{\omega+\omega_\nu-i\delta}\Big]$$

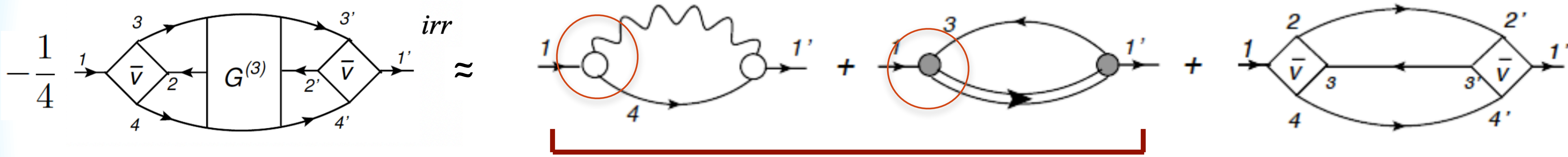
$$D^{(\sigma)}_{\nu}(\omega) = \frac{\sigma}{\omega-\sigma(\omega_\nu-i\delta)}$$

Emergence of effective degrees of freedom at intermediate coupling

Dynamical self-energy $\Sigma^{(r)}$:

"Radiative-correction"

"Second-order"



Can be treated in a unified way in the superfluid theory:
qPVC in nuclei, correlations in hadrons / matter

Emergent phonon vertices and propagators: calculable from the underlying H, which does not contain phonon degrees of freedom

"Ab-initio"

$$H = \sum_{12} h_{12}\psi_1^\dagger\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi_1^\dagger\psi_2^\dagger\psi_4\psi_3$$

"Effective" qPVC: NFT, QPM

$$H = \sum_{12}\tilde{h}_{12}\psi_1^\dagger\psi_2 + \sum_{\lambda\lambda'}\mathcal{W}_{\lambda\lambda'}Q_\lambda^\dagger Q_{\lambda'} + \sum_{12\lambda}\left[\Theta_{12}^{\lambda}\psi_1^\dagger Q_\lambda^\dagger\psi_2 + h.c.\right]$$

Cf. Nuclear Field Theory (NFT)

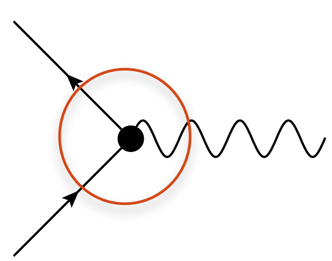
R.A. Broglia, D. Bes, P.-F. Bortignon, R. Liotta, G. Colò, E. Vigezzi, F. Barranco, G. Potel, et al.

&

Quasiparticle-Phonon Model (QPM)

V.G. Soloviev et al.

Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the input:



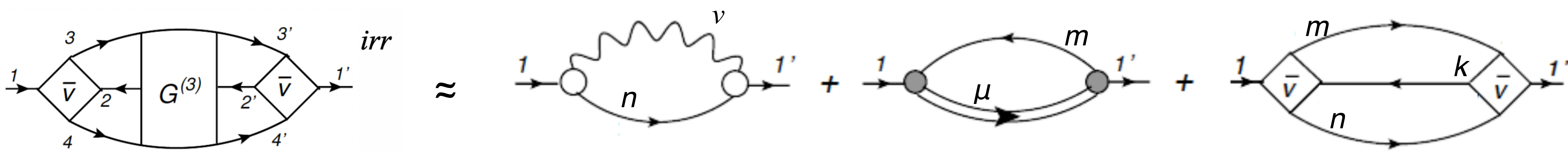
$$\gamma, g, W^{\pm}, Z^0$$

E.L., P. Schuck, PRC 100, 064320 (2019)
E.L., Y. Zhang, PRC 104, 044303 (2021)

Problems with approximate treatments: poles "mismatch", (non)-positivity and optical theorem

Dynamical self-energy:

"Radiative-correction" "Second-order"



Approximate:

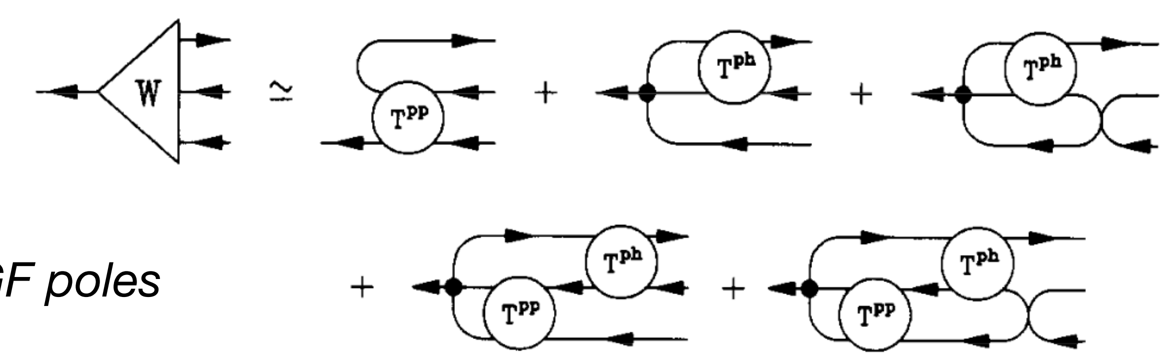
$$\Sigma^{(r)+}_{11'}(\omega) = \sum_{33'}\sum_{\nu n} \frac{\eta^{n}_{3} g^{\nu}_{13} g^{\nu *}_{1'3'} \eta^{n*}_{3'}}{\omega-\omega_{\nu}-\varepsilon^{(+)}_{n}+i\delta} + \sum_{22'}\sum_{\mu m} \frac{\chi^{m*}_{2}\gamma^{\mu(+)}_{12}\gamma^{\mu(+)*}_{1'2'}\chi^{m}_{2'}}{\omega-\omega^{(++)}_{\mu}-\varepsilon^{(-)}_{m}+i\delta} - \sum_{mnk}\frac{w^{mnk}_{1} w^{mnk*}_{1'}}{\omega-\varepsilon^{+}_{m}-\varepsilon^{+}_{n}-\varepsilon^{-}_{k}+i\delta}$$

Exact:

$$\Sigma^{(r)+}_{11'}(\omega) \sim \langle v G^{(3)+}(\omega) v \rangle^{irr}_{11'}$$

$$G^{(3)+}_{432',23'4'}(\omega) = \sum_{\varkappa} \frac{\langle 0|\bar\psi_2\psi_4\psi_3|\varkappa\rangle\langle\varkappa|\bar\psi_{3'}\bar\psi_{4'}\psi_{2'}|0\rangle}{\omega-\omega_{\varkappa}+i\delta}^{irr}$$

- Watson-Faddeev series: P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994):



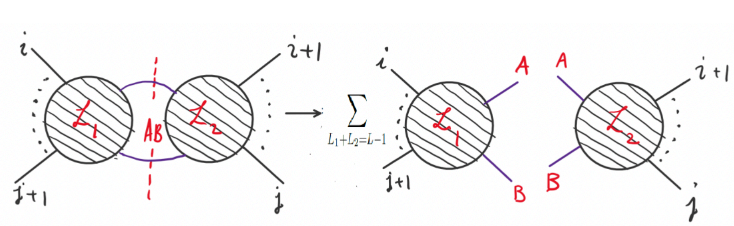
- 2p1h-RPA
P. Schuck, F. Villars and P. Ring
Nucl. Phys. A208, 302 (1973)

Reducible terms: unperturbed GF poles

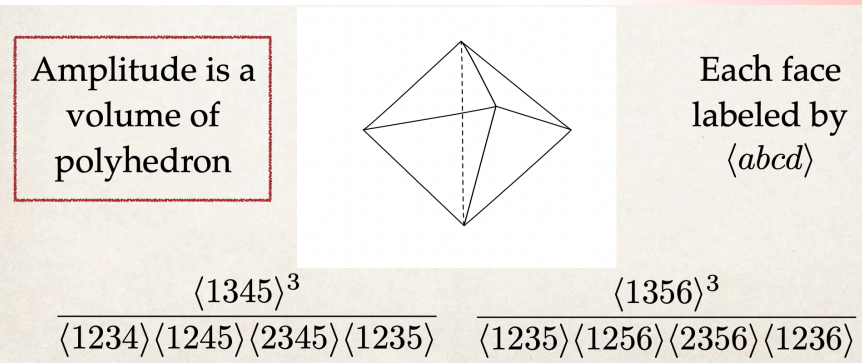
Scattering amplitudes *in particle physics (Yang-Mills theories etc.):*

- Positivity preserved when eliminating all the virtual particles
- Amplitudes <=> Polyhedron "living" in the kinematic space
- Emergent unitarity

Idea: eliminating virtual particles



N. Arkani-Hamed, J. Trnka, A. Hodges et al.



Toward concrete implementations: the elefant in the room

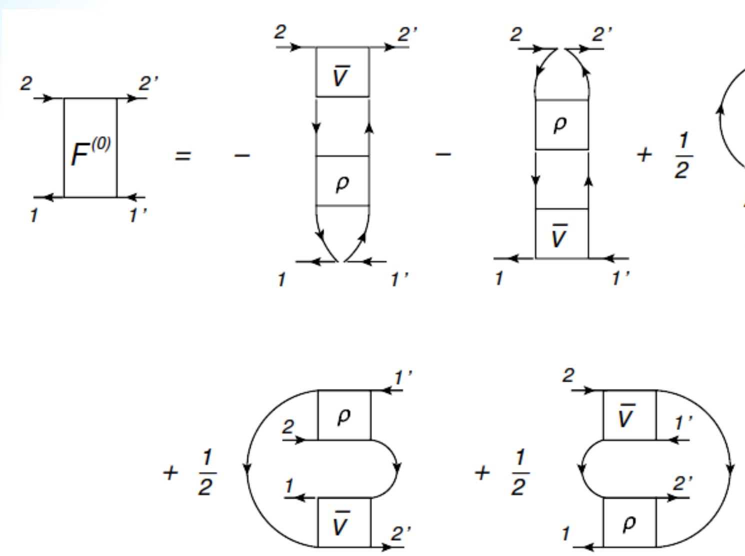
Particle-hole propagator (response function):

$$R_{12,1'2'}(t-t') = -i\langle T(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle$$

spectra of excitations, masses, decays, …

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (**) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

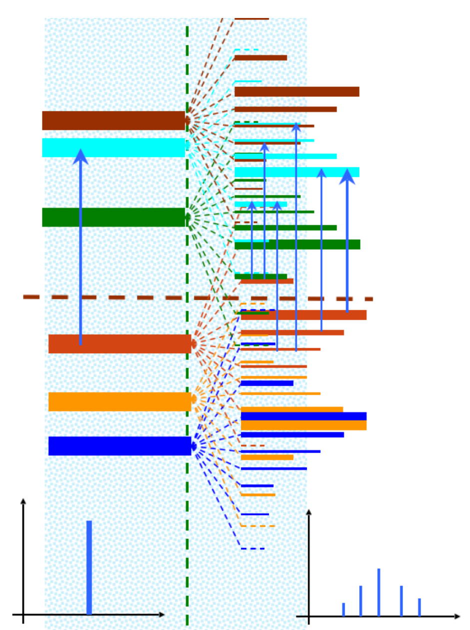
Static kernel:



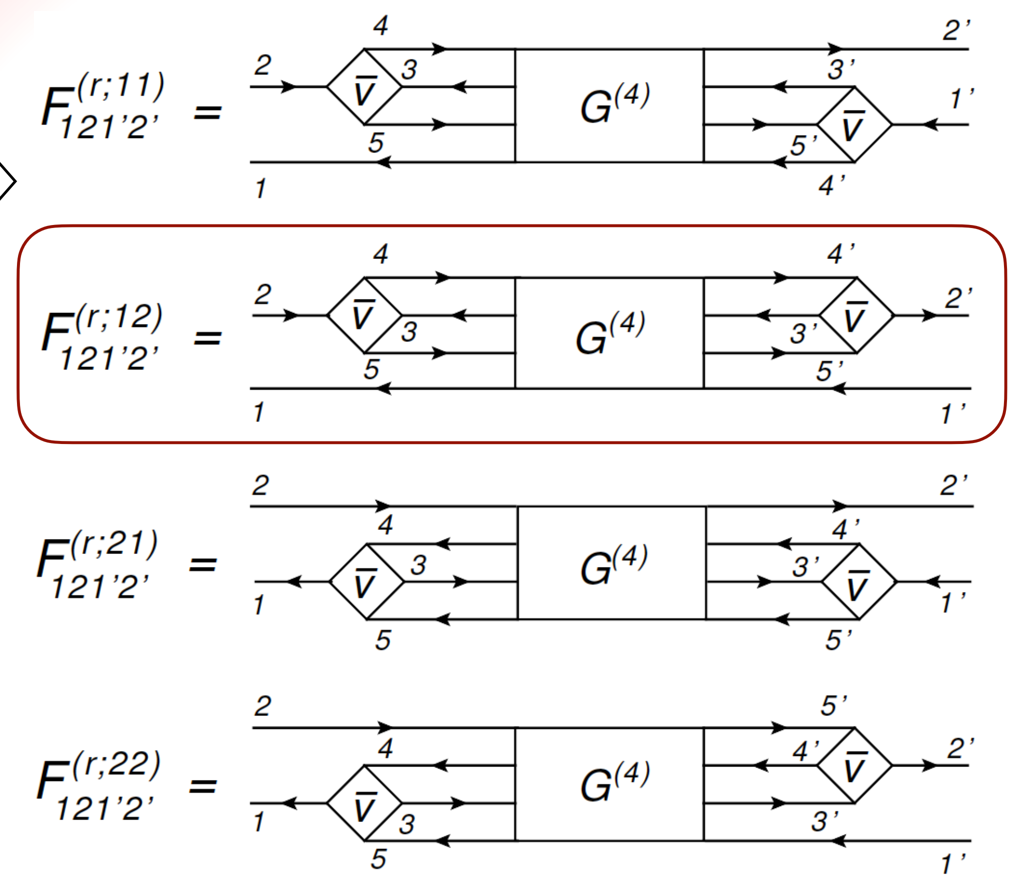
$$\sim \sum_{jkl} \bar v_{2jkl}\rho_{kl,2'j}$$

Static & regular:

- **Static kernel:** roughly defines the positions of the elementary excitation modes (1p1h)
- **Dynamical kernel**: generates the fragmentation of the 1p1h modes into the multitude of realistic states with complex structure (complex configurations)



Dynamical kernel:



$$\sim \sum_{345}\sum_{3'4'5'} \bar v_{4513} G(325'4',3'2'45)\bar v_{3'1'4'5'}$$

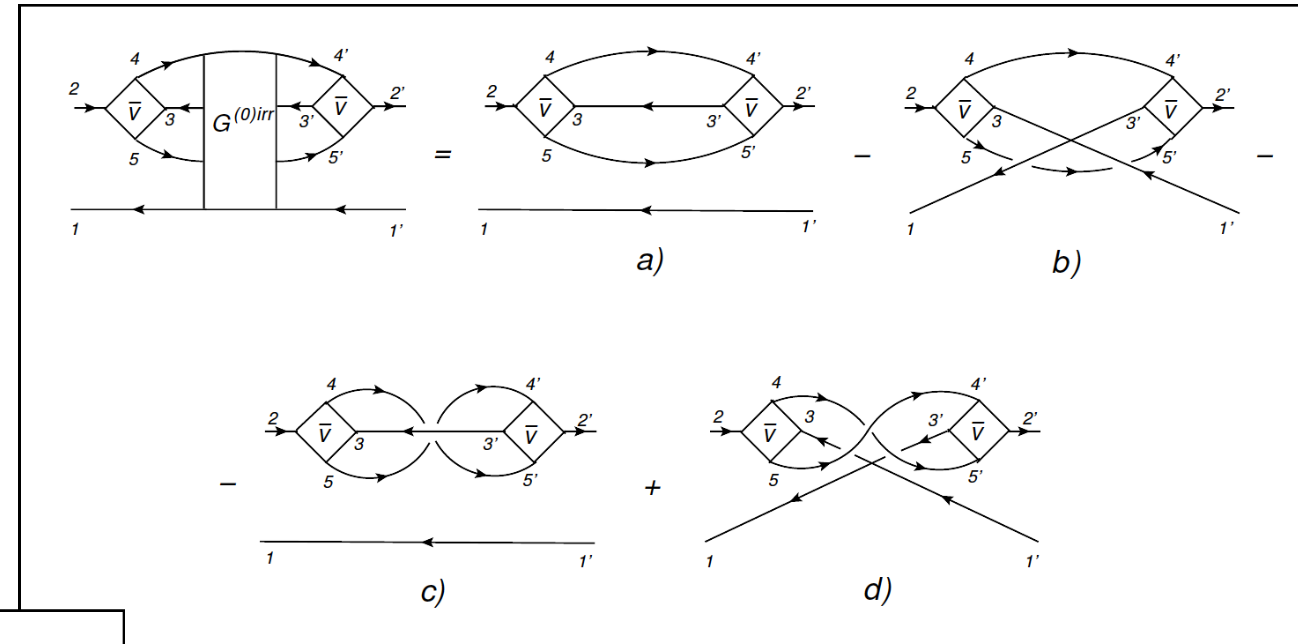
t(ω)-dependent & singular

Expansion of the dynamical kernel $F^{(r;12)irr}$: truncation at the 2-body level

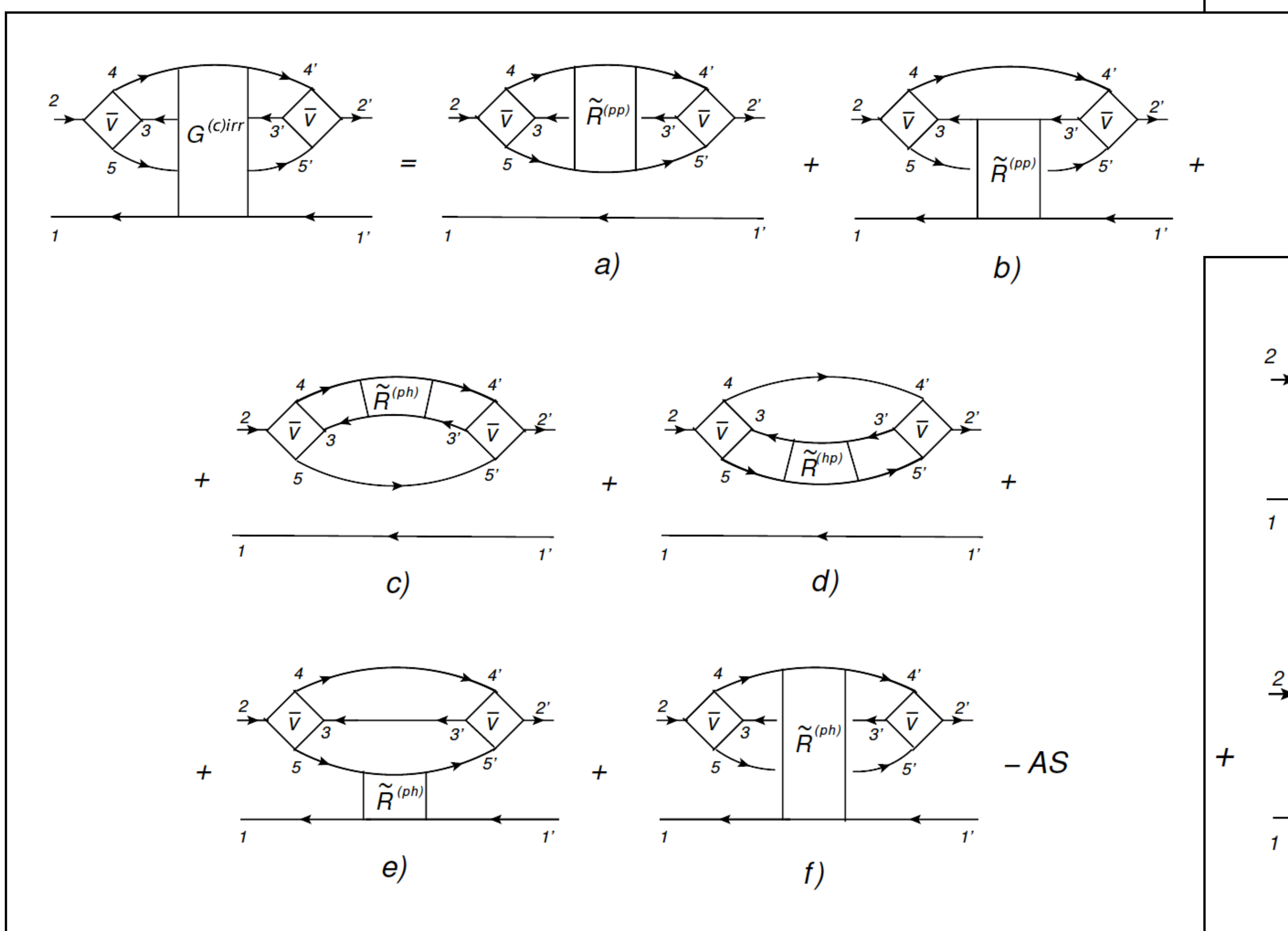
P. Schuck, V. Olevano et al.:
Irreducible part of $G^{(4)}$ is decomposed (approximately) into uncorrelated, singly-correlated and doubly-correlated terms (cluster expansion):

$$\boxed{\begin{aligned} G^{irr}(543'1',5'4'31) &= G^{(0)irr}(543'1',5'4'31) + \\ &+ G^{(c)irr}(543'1',5'4'31) + G^{(cc)irr}(543'1',5'4'31) \end{aligned}}$$

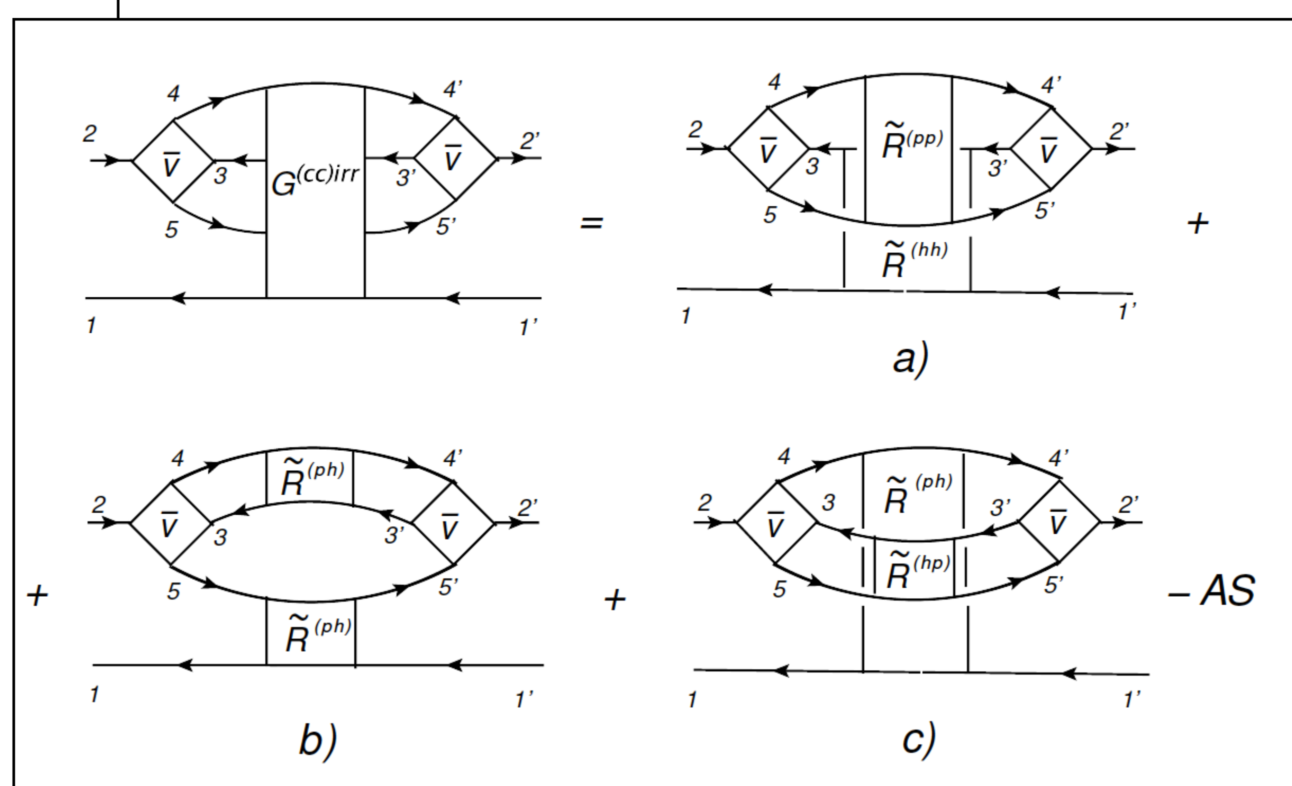
(i) Uncorrelated terms ("Second RPA"):



(ii) Singly-correlated terms (PVC-NFT):



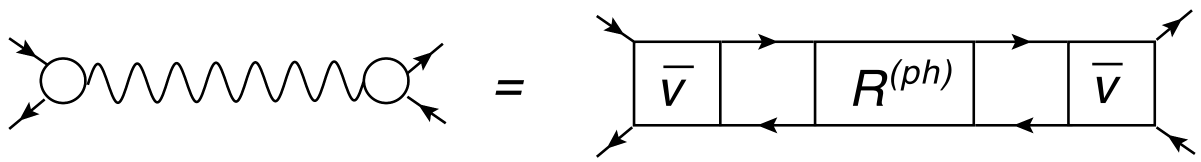
(iii) Doubly-correlated terms: beyond NFT



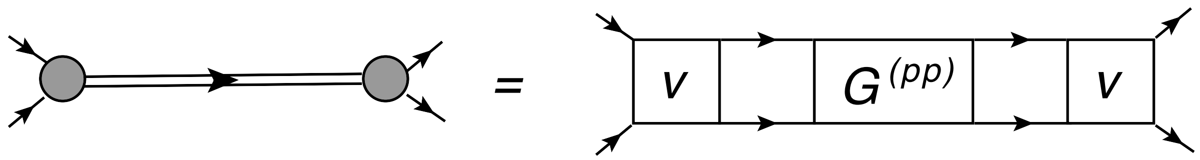
Mapping to the (quasi)particle-vibration coupling (qPVC)

Model-independent (exact) mapping to the qPVC (Effective Hamiltonian derived!):

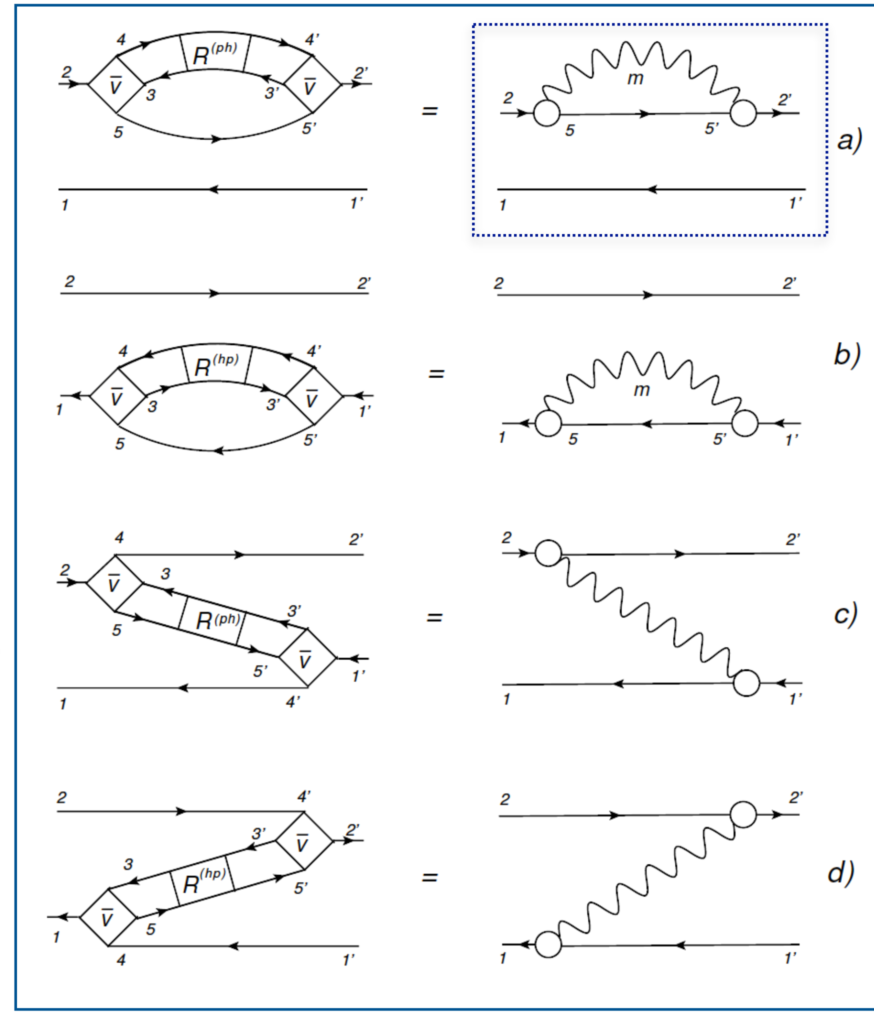
normal vibration (phonon)



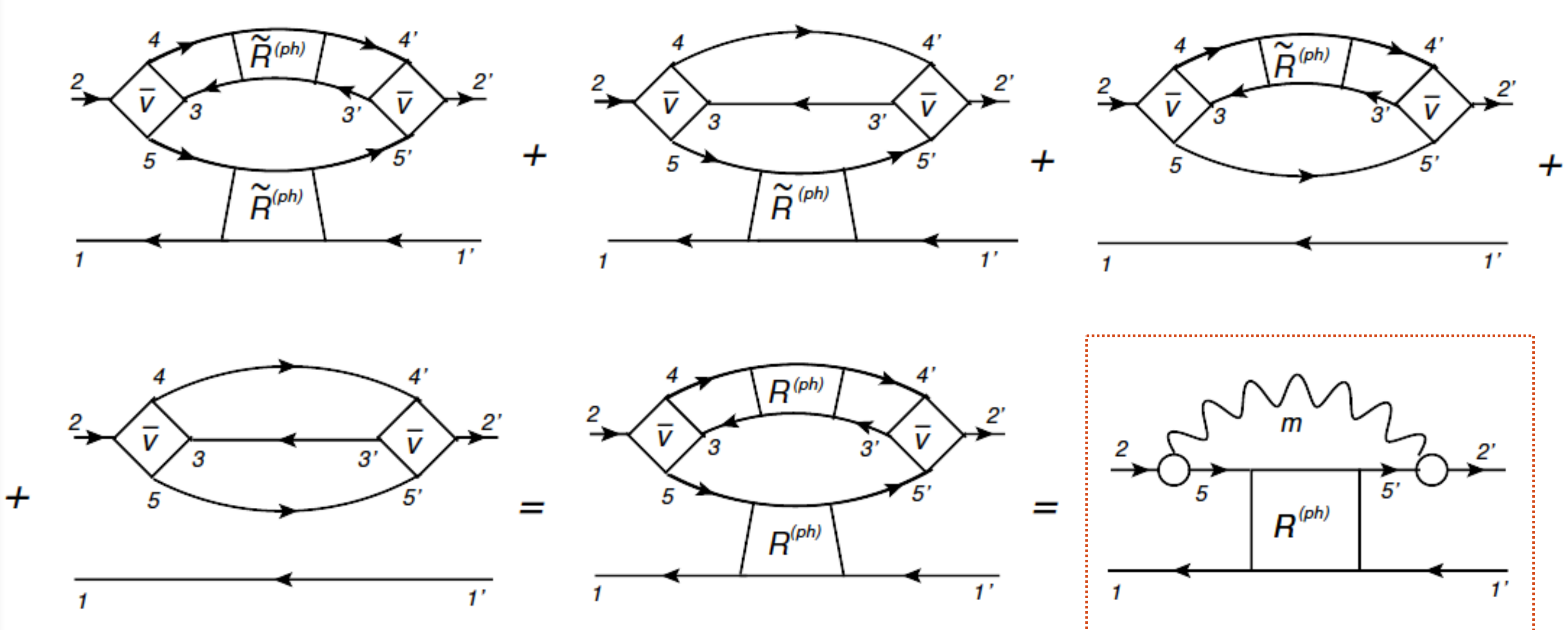
pairing vibration



Original qPVC, (Relativistic) **Nuclear Field Theory**, (Relativistic) Quasiparticle Time Blocking Approximation (RQTBA): only part of the singly-correlated terms



Higher-rank correlations:



Self-consistent closed system of equations:

$$\hat{G}(\omega) = \hat{G}^{(0)}(\omega) + \hat{G}^{(0)}(\omega)\mathcal{K}[\hat{G}(\omega)]\hat{G}(\omega)$$

All channels are coupled in $\mathcal{K}[\hat{G}(\omega)]$

$$\hat{G} = \left\{ R^{(ph)}, R^{(hp)}, G^{(pp)}, G^{(hh)} \right\}$$

E.L., P. Schuck, PRC 100, 064320 (2019).

(Quasi)particle-vibration coupling (qPVC) vs QPM

QPM, qPVC/NFT

$$H = \sum_{jm} E_j\, \alpha^\dagger_{jm}\alpha_{jm} + \sum_{\lambda\mu i} \omega_{\lambda i}\, Q^\dagger_{\lambda\mu i} Q_{\lambda\mu i} + \sum_{\substack{jj'\\ \lambda\mu i}} V^{(\lambda i)}_{jj'} \left[\alpha^\dagger_j \alpha_{j'} \left(Q^\dagger_{\lambda\mu i} + (-1)^{\lambda-\mu} Q_{\lambda,-\mu i}\right)\right]$$

If Q are the QRPA phonons:

$$Q^{n\dagger} = \frac{1}{2}\sum_{\mu\mu'} (X^n_{\mu\mu'}\alpha^\dagger_\mu \alpha^\dagger_{\mu'} - Y^n_{\mu\mu'}\alpha_{\mu'}\alpha_\mu),$$

$$Q^n = \frac{1}{2}\sum_{\mu\mu'} (X^{n*}_{\mu\mu'}\alpha_{\mu'}\alpha_\mu - Y^{n*}_{\mu\mu'}\alpha^\dagger_\mu \alpha^\dagger_{\mu'})$$

$$\sum_n (X^{n*}_{\mu\mu'} Q^{n\dagger} + Y^n_{\mu\mu'} Q^n) = \alpha^\dagger_\mu \alpha^\dagger_{\mu'},$$

$$\sum_n (Y^{n*}_{\mu\mu'} Q^{n\dagger} + X^n_{\mu\mu'} Q^n) = \alpha_{\mu'}\alpha_\mu.$$

it is always possible to express 2q via phonons and vice versa

At the 2p2h-level:

$$A^\dagger_{ab}(JM) = \left[\alpha^\dagger_a \alpha^\dagger_b\right]_{JM}$$

$$|\Psi_{JM}\rangle = \sum_i R_i\, Q^\dagger_i |0\rangle + \sum_{i_1 i_2} P_{i_1 i_2} Q^\dagger_{i_1} Q^\dagger_{i_2} |0\rangle + \cdots$$

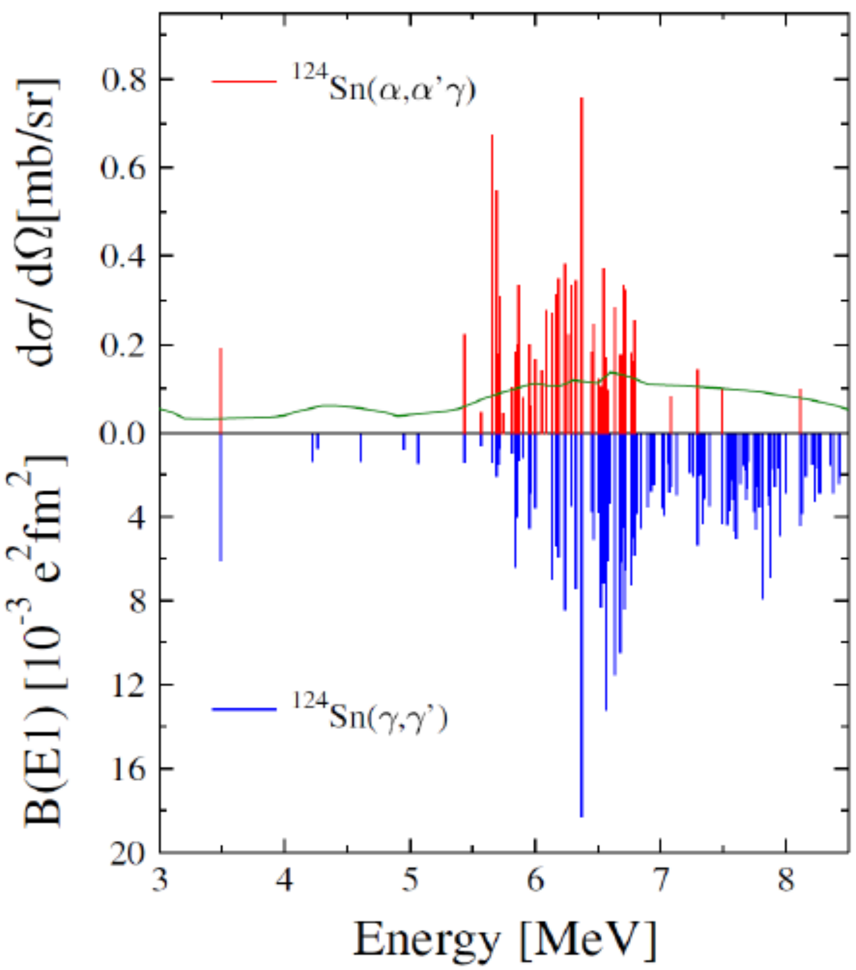
qPVC/NFT

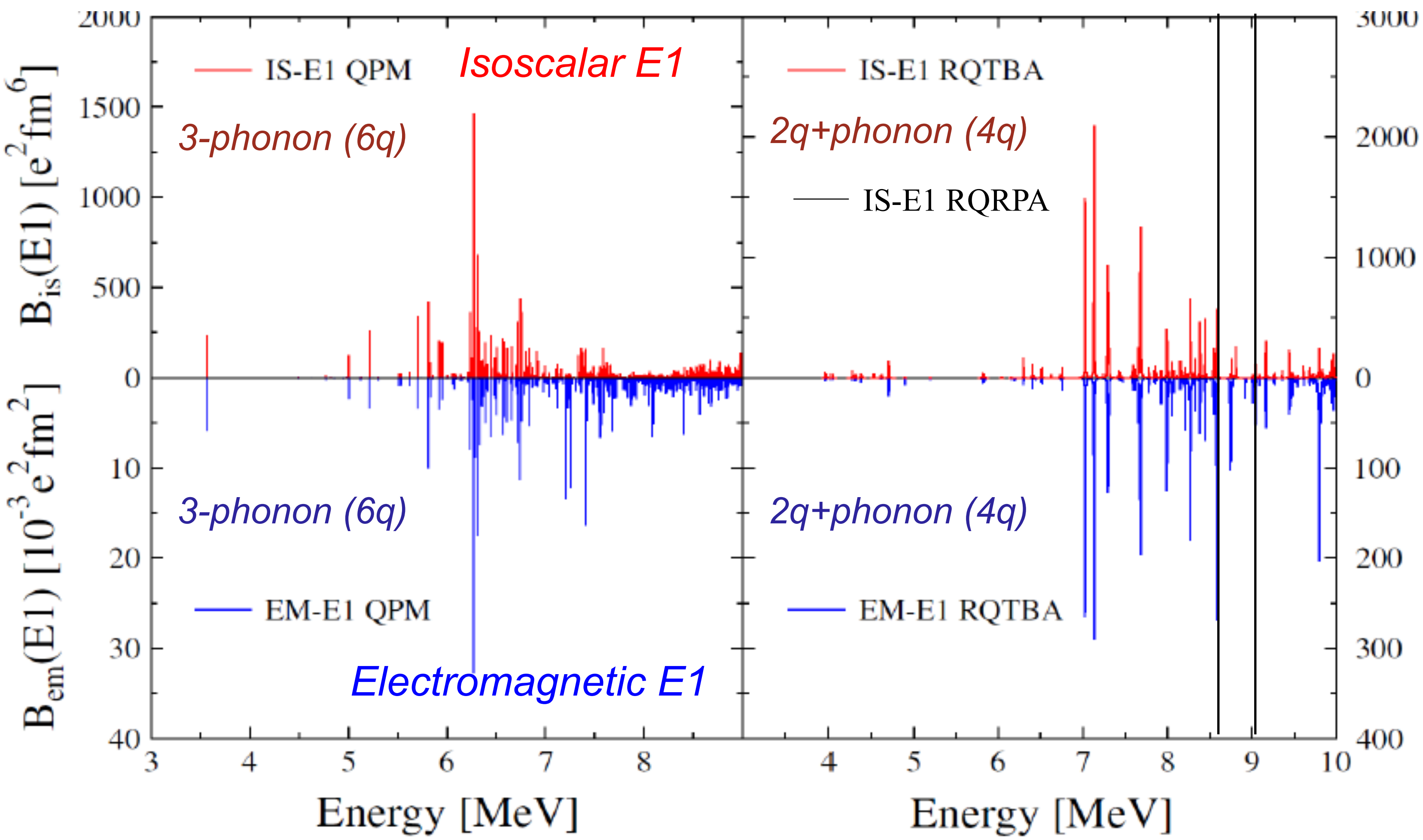
$$|\Psi_\nu(JM)\rangle = \sum_{ab} X^{(\nu)}_{ab} A^\dagger_{ab}(JM)|0\rangle + \sum_{ab,\lambda i} Y^{(\nu)}_{ab,\lambda i} \left[A^\dagger_{ab} Q^\dagger_{\lambda i}\right]_{JM} |0\rangle$$

QPM

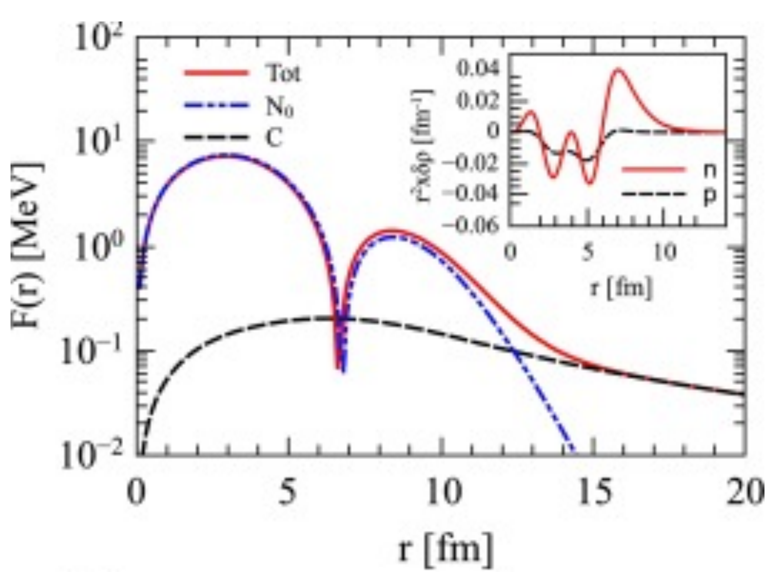
Quasiparticle-Phonon Model (QPM): the necessity of 3-phonon configurations

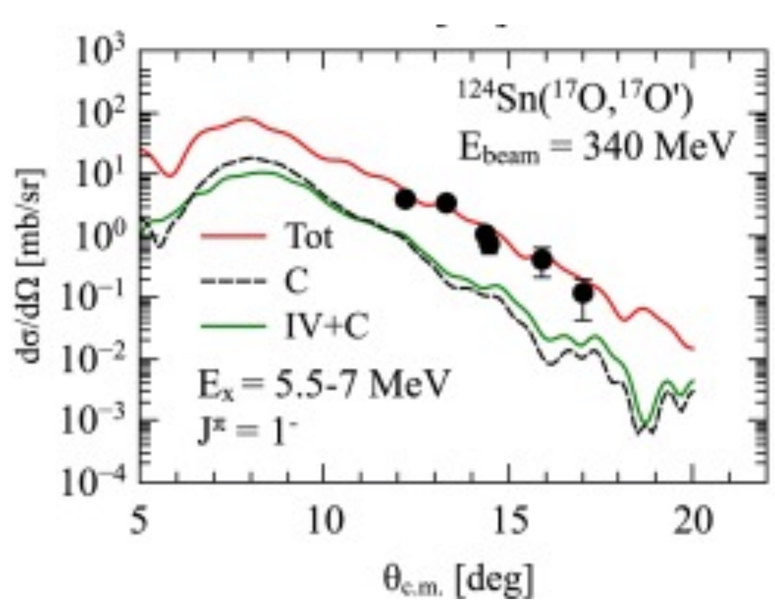
J. Endres, E.L., D. Savran et al., PRL 105, 212503 (2010) & *E. Lanza, A. Vitturi, E.L., D. Savran, PRC 89, 041601(R) (2014)*

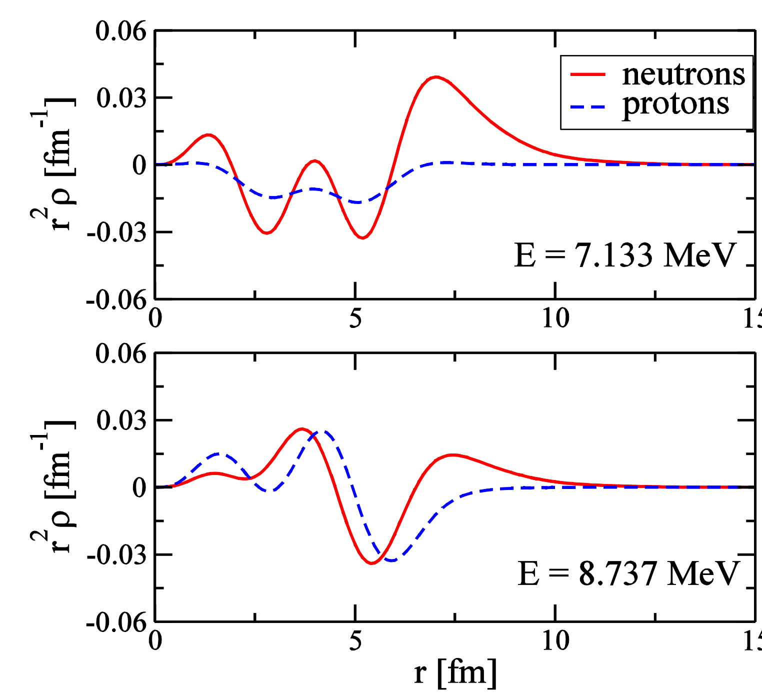




L. Pellegri et al., PLB 738, 519 (2014). Calculations: E. Lanza

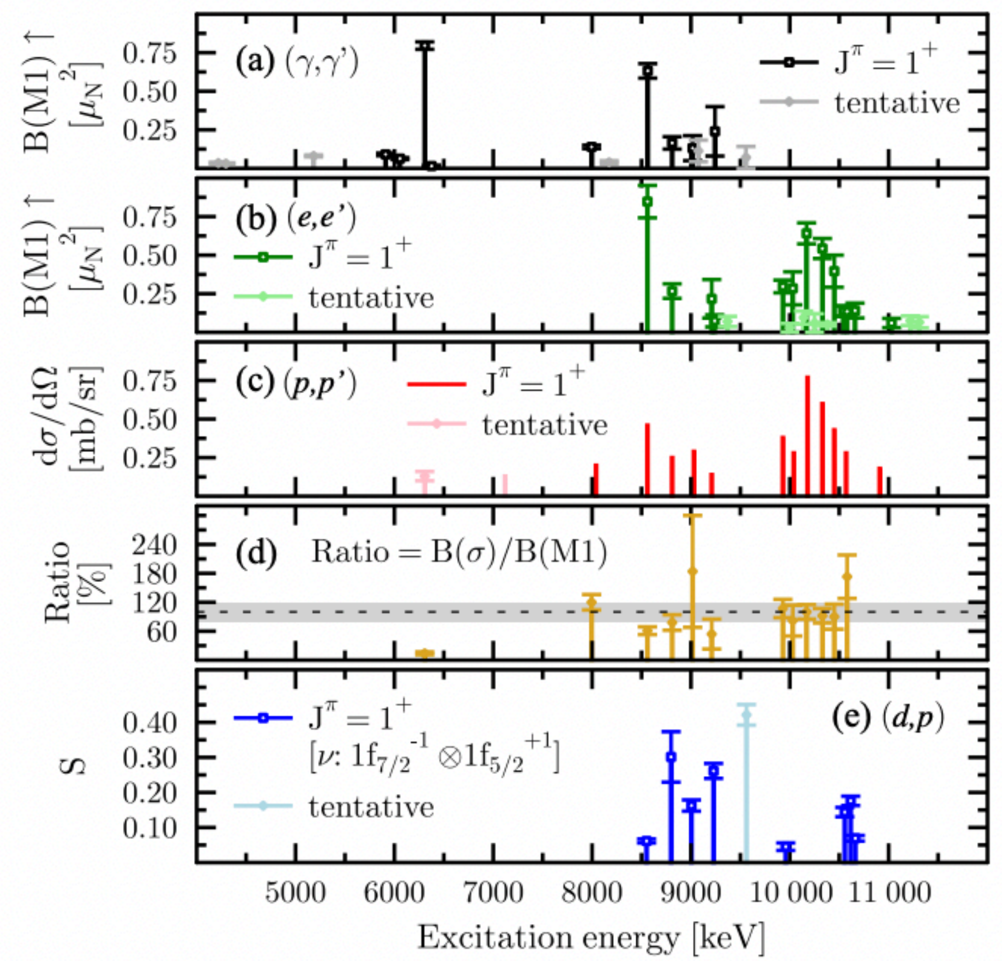






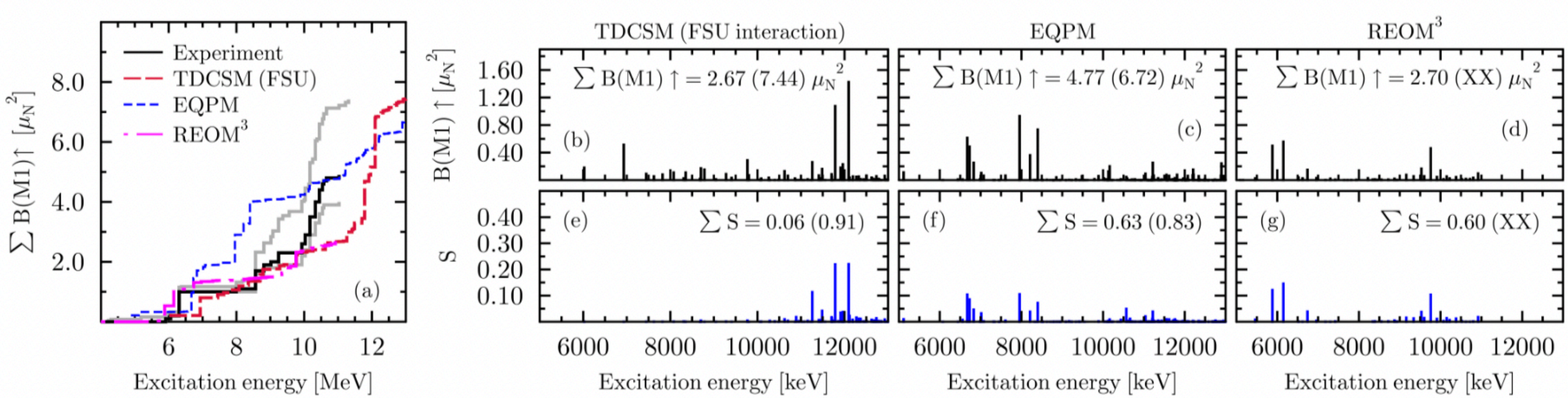
Comparison of REOM3 to Quasiparticle-Phonon Model (QPM) and shell-model (TDCSM)

M1 strength in 50-Ti

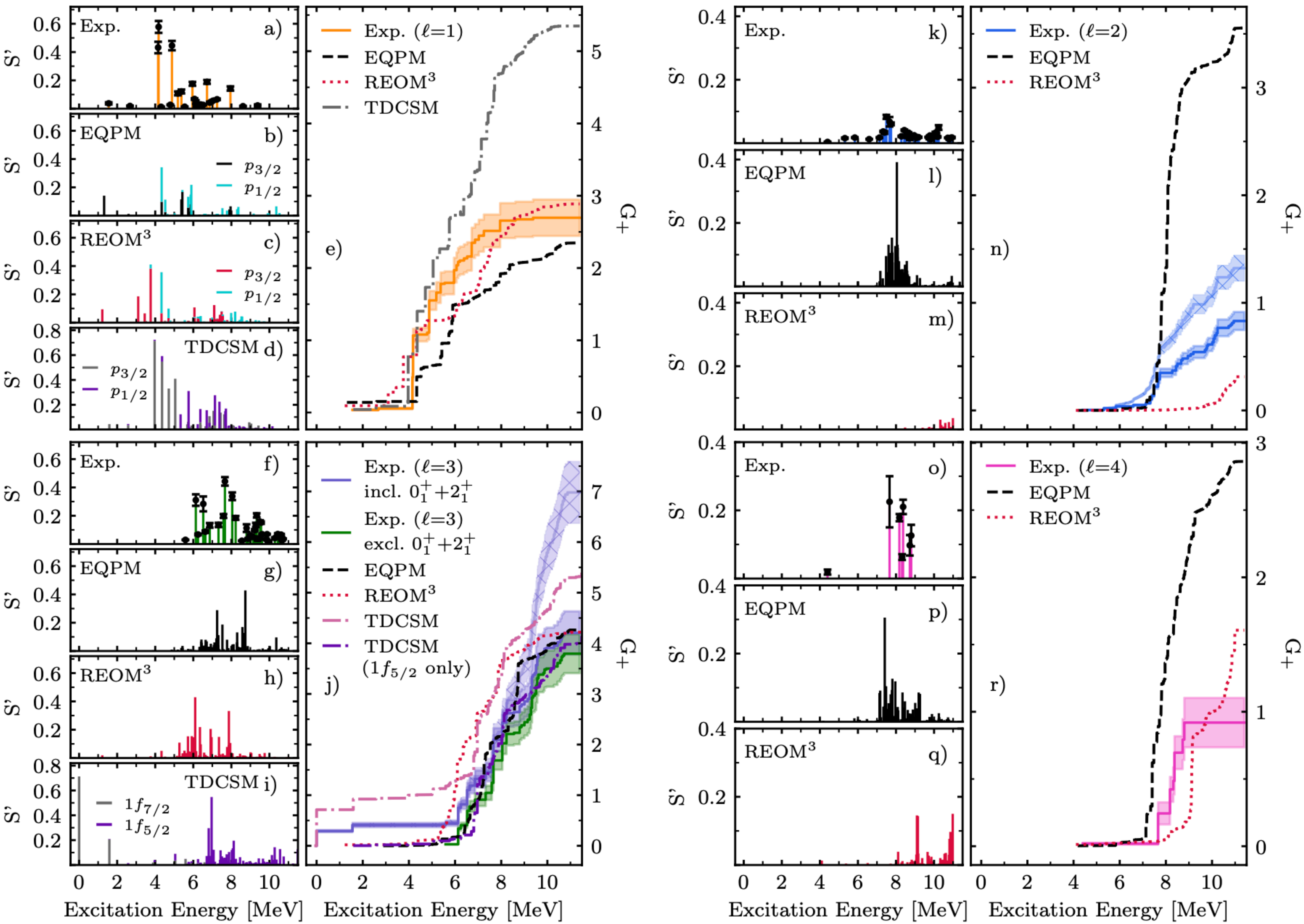


^{49}Ti(d,p)^{50}Ti experiment: 1+ states with larger neutron (1f7/2)$^{-1}$-(1f5/2)$^{+1}$ spectroscopic factors S do not correspond to the ones with the largest B(M1) strengths. Theory is in tension

B. Kelly, M. Spieker, …, E.L., N. Tsoneva, A Volya et al., PRL 136, 082502 (2026)



Other multipoles



B. Kelly, M. Spieker, …, E.L., N. Tsoneva, A Volya et al., PRC (2026)

SRPA vs STDA vs EMPM

F. Knapp et al., Phys. Rev. C 107, 014305 (2023)

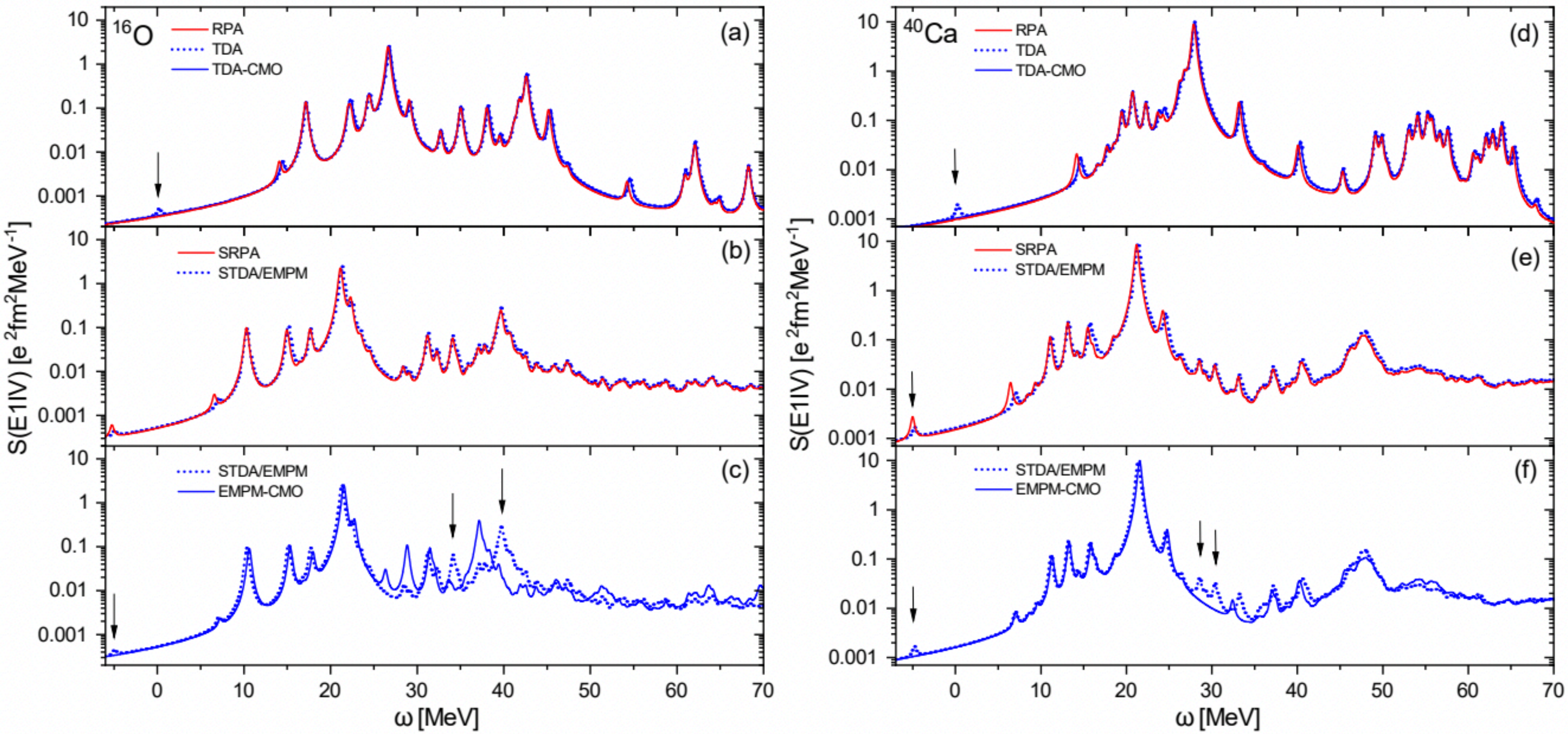


FIG. 2. (Color online) Isovector $E1$ strength functions in ^{16}O and ^{40}Ca in different approaches. The TDA strength is computed before (TDA) and after the application of Gram-Schmidt c.m. orthogonalization procedure (TDA-CMO). In this and the following figures, a single line is drawn for STDA and EMPM since they yield identical results. The arrows indicate states with significant spurious components which disappear if c.m. SVD orthogonalization procedure (EMPM-CMO) is applied.