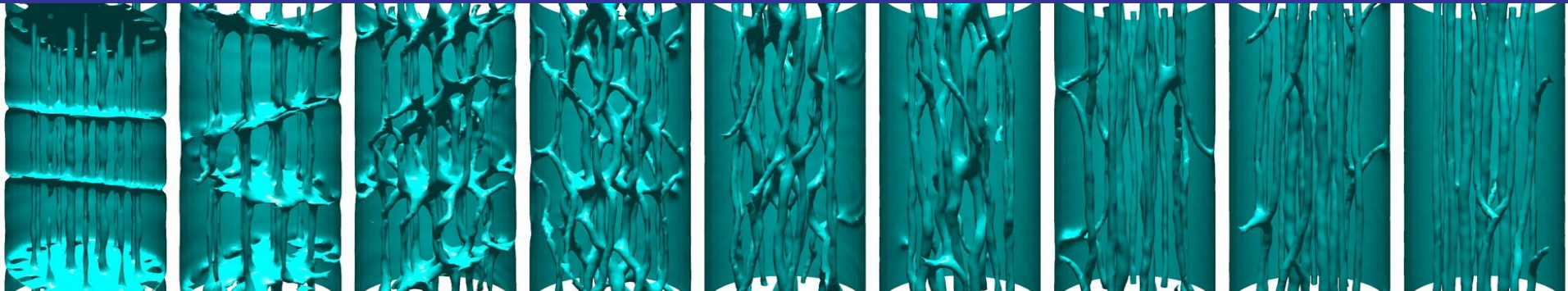


Stability and Dissipative Dynamics of Collective Modes in Ultracold Atomic Gases

Piotr Magierski
Warsaw University of Technology (WUT)



DE LA RECHERCHE À L'INDUSTRIE

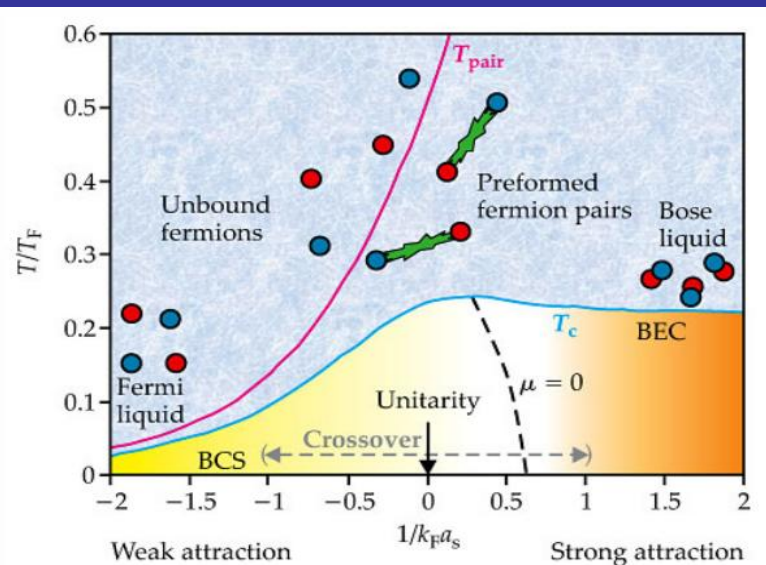
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ESNT

Espace de Structure Nucléaire Théorique
DSM - DAM

Many-body correlations, phase transitions and clustering

7-11 September

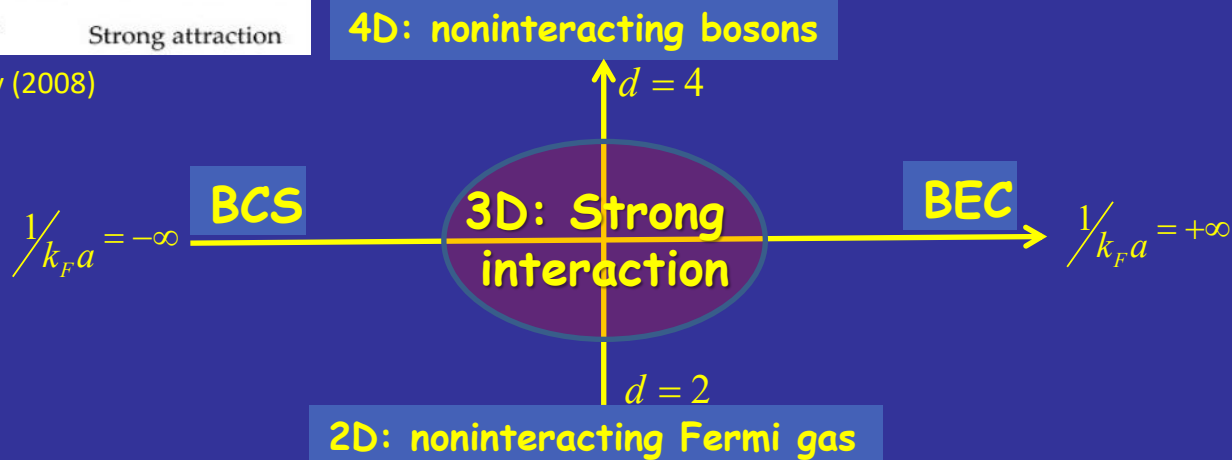


From Sa de Melo, Physics Today (2008)

Unitary Fermi gas.

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$$a \rightarrow \infty : R(r) \propto \frac{1}{r^{d-2}} + O(r^{4-d}), \text{ Two body wave function for } r \rightarrow 0.$$



Nussinov, Nussinov, Phys.Rev. A74, 053622(2006)

- For $d=2$ the singularity of the wave function disappears
- For $d=4$ $\int R(r)^2 d^d r$ diverges at the origin

$$\frac{\Delta}{\mathcal{E}_F} \approx 0.5$$

GOAL:

Unified description of superfluid dynamics of fermionic systems far from equilibrium based on microscopic theoretical framework.

Microscopic framework = explicit treatment of fermionic degrees of freedom.

Why Time Dependent Density Functional Theory (TDDFT)?

We need to describe the time evolution of (externally perturbed) spatially inhomogeneous, superfluid Fermi system.

Within current computational capabilities TDDFT allows to describe real time dynamics of strongly interacting, superfluid systems of hundred of thousands fermions.

Time dependent Kohn-Sham scheme for superfluids:

O.-J. Wacker, R. Kümmel, E.K.U. Gross, Phys. Rev. Lett. 73, 2915 (1994).

$$\sum_{\sigma'=\uparrow\downarrow} \left(\hat{h}_{\sigma\sigma'}(\mathbf{r}, t) u_{\sigma'}(\mathbf{r}, t) + \int d^3\mathbf{r}' \Delta_{0\sigma\sigma'}(\mathbf{r}, \mathbf{r}', t) v_{n\sigma'}(\mathbf{r}, t) \right) = i\hbar \frac{\partial}{\partial t} u_{\sigma}(\mathbf{r}, t),$$

$$\sum_{\sigma'=\uparrow\downarrow} \left(-\hat{h}_{\sigma\sigma'}^*(\mathbf{r}, t) v_{\sigma'}(\mathbf{r}, t) + \int d^3\mathbf{r}' \Delta_{0\sigma\sigma'}^*(\mathbf{r}, \mathbf{r}', t) u_{n\sigma'}(\mathbf{r}, t) \right) = i\hbar \frac{\partial}{\partial t} v_{\sigma}(\mathbf{r}, t)$$

Densities:

$$\rho_{\sigma}(\mathbf{r}, t) = \sum_n |v_{n,\sigma}(\mathbf{r}, t)|^2,$$

$$\chi_{\sigma,\sigma',t}(\mathbf{r}, \mathbf{r}', t) = \sum_n v_{n,\sigma}^*(\mathbf{r}, t) u_{n,\sigma'}(\mathbf{r}', t),$$

Potentials:

$$V_{0\sigma}(\mathbf{r}, t) = V_{\sigma}^{ext}(\mathbf{r}, t) + \sum_{\sigma'} \int d^3\mathbf{r}' V_{\sigma,\sigma'}^{int}(\mathbf{r}, \mathbf{r}') \rho_{\sigma'}(\mathbf{r}', t) + \frac{\delta V^{corr}[\rho, \chi, t]}{\delta \rho},$$

$$\Delta_{0\sigma\sigma'}(\mathbf{r}, \mathbf{r}', t) = \Delta_{\sigma\sigma'}^{ext}(\mathbf{r}, \mathbf{r}', t) + V_{\sigma,\sigma'}^{int}(\mathbf{r}, \mathbf{r}') \chi_{\sigma,\sigma',t}(\mathbf{r}, \mathbf{r}', t) + \frac{\delta \Delta^{corr}[\rho, \chi, t]}{\delta \chi}$$

The main difference between static and time-dependent functionals is that the latter contains memory term *i.e.* dependence on the past densities.

J.F. Dobson, M.J. Brunner, E.K.U. Gross, Phys. Rev. Lett. 79 1905 (1997).

G. Vignale, C. A. Ullrich, S. Conti, Phys. Rev. Lett. 79 4878 (1997).

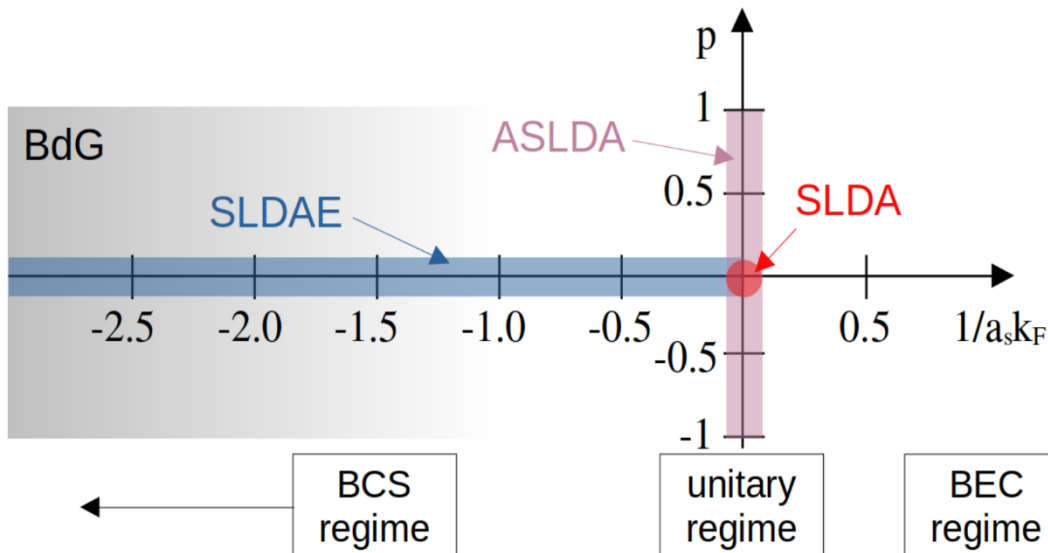
Adiabatic approximation (no memory effects)

$$\mathcal{H}(t) \begin{pmatrix} \tilde{u}_{n,\uparrow}(\mathbf{r}, t) \\ \tilde{u}_{n,\downarrow}(\mathbf{r}, t) \\ \tilde{v}_{n,\uparrow}(\mathbf{r}, t) \\ \tilde{v}_{n,\downarrow}(\mathbf{r}, t) \end{pmatrix} = i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \tilde{u}_{n,\uparrow}(\mathbf{r}, t) \\ \tilde{u}_{n,\downarrow}(\mathbf{r}, t) \\ \tilde{v}_{n,\uparrow}(\mathbf{r}, t) \\ \tilde{v}_{n,\downarrow}(\mathbf{r}, t) \end{pmatrix},$$

$$\mathcal{H}(t) = \begin{pmatrix} h_{\uparrow\uparrow}(\mathbf{r}, t) & h_{\uparrow\downarrow}(\mathbf{r}, t) & 0 & \tilde{\Delta}_0(\mathbf{r}, t) \\ h_{\downarrow\uparrow}(\mathbf{r}, t) & h_{\downarrow\downarrow}(\mathbf{r}, t) & -\tilde{\Delta}_0(\mathbf{r}, t) & 0 \\ 0 & -\tilde{\Delta}_0^*(\mathbf{r}) & -h_{\uparrow\uparrow}^*(\mathbf{r}, t) & -h_{\uparrow\downarrow}^*(\mathbf{r}, t) \\ \tilde{\Delta}_0^*(\mathbf{r}) & 0 & -h_{\downarrow\uparrow}^*(\mathbf{r}, t) & -h_{\downarrow\downarrow}^*(\mathbf{r}, t) \end{pmatrix},$$

Adiabatic approximation, although simplifies the equations, affects dissipative effects.

C. A. Ullrich, *Time-dependent density-functional theory: concepts and applications*, Oxford University Press, (2012).

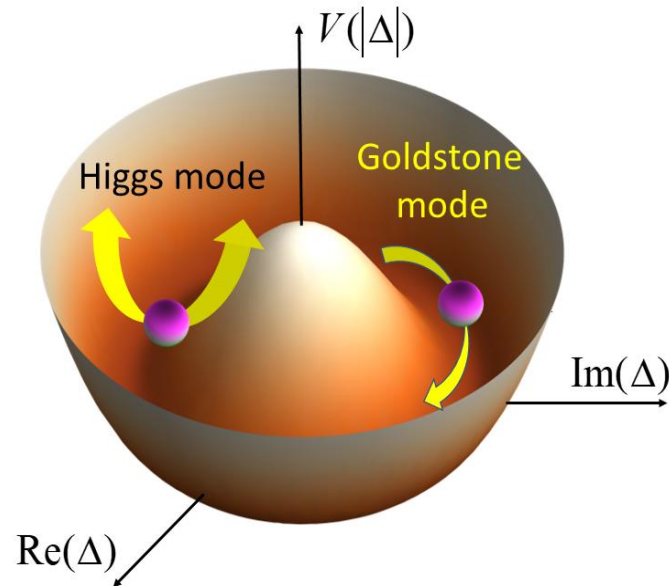


Energy Density Functionals:

- spin-symmetric unitary regime: **SLDA**
A.Bulgac, PRA 76 (2007) 040502
- spin-asymmetric unitary regime: **ASLDA**
A.Bulgac, M.M.Forbes, P.M., Lecture Notes in Physics (2012) 305
- **SLDAE** matching between unitary regime and deep BCS limit
A.Boulet, G.Wlazlowski, P. M., PRA 106 (2022) 013306

$$\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$$

Appearance of pairing field in Fermi systems is associated with U(1) symmetry breaking.



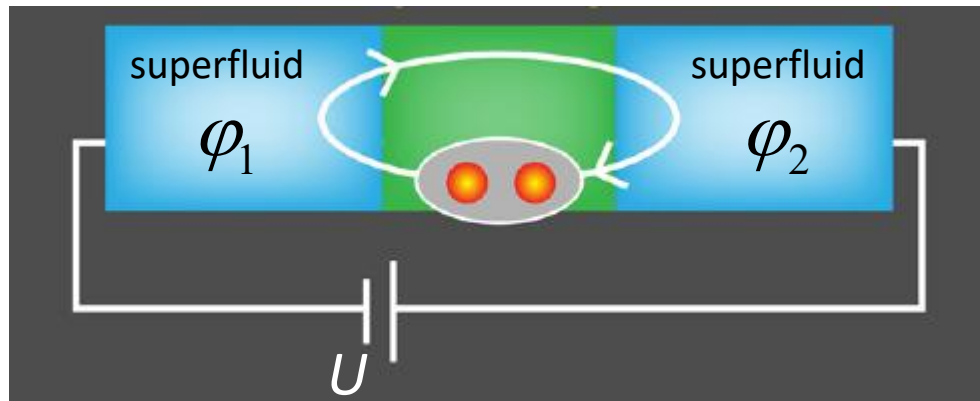
There are two characteristic modes associated with $\Delta(\vec{r}, t)$

- 1) **Nambu-Goldstone mode** explores the degree of freedom associated with the phase: $\phi(\vec{r}, t)$
- 2) **Higgs mode** explores the degree of freedom associated with the magnitude: $|\Delta(\vec{r}, t)|$

Probing phase degree of freedom of pairing field

The well known example is Josephson junction:

- DC Josephson junction: $U = 0$
- AC Josephson junction: $U \neq 0$



$$\Delta\varphi = \varphi_1 - \varphi_2$$

$$J(t) = J_c \sin(\Delta\varphi(t))$$

$$\frac{d(\Delta\varphi)}{dt} = \frac{2eU}{\hbar}$$

Relation between Josephson current and phase differences Between pairing fields.

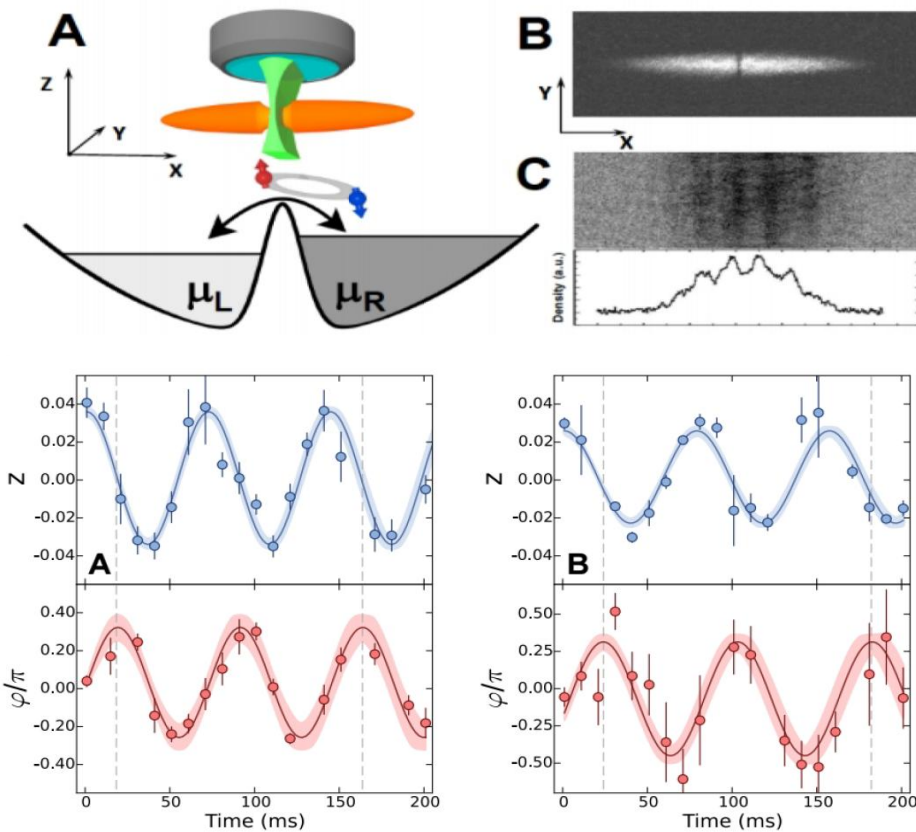
Important: Josephson junction means usually so-called **weak link**.

Pairing condensates on both sides are assumed to remain unperturbed by the Josephson current.

Ultracold atomic gases: two regimes for realization of the Josephson junction

Weak coupling (weak link)

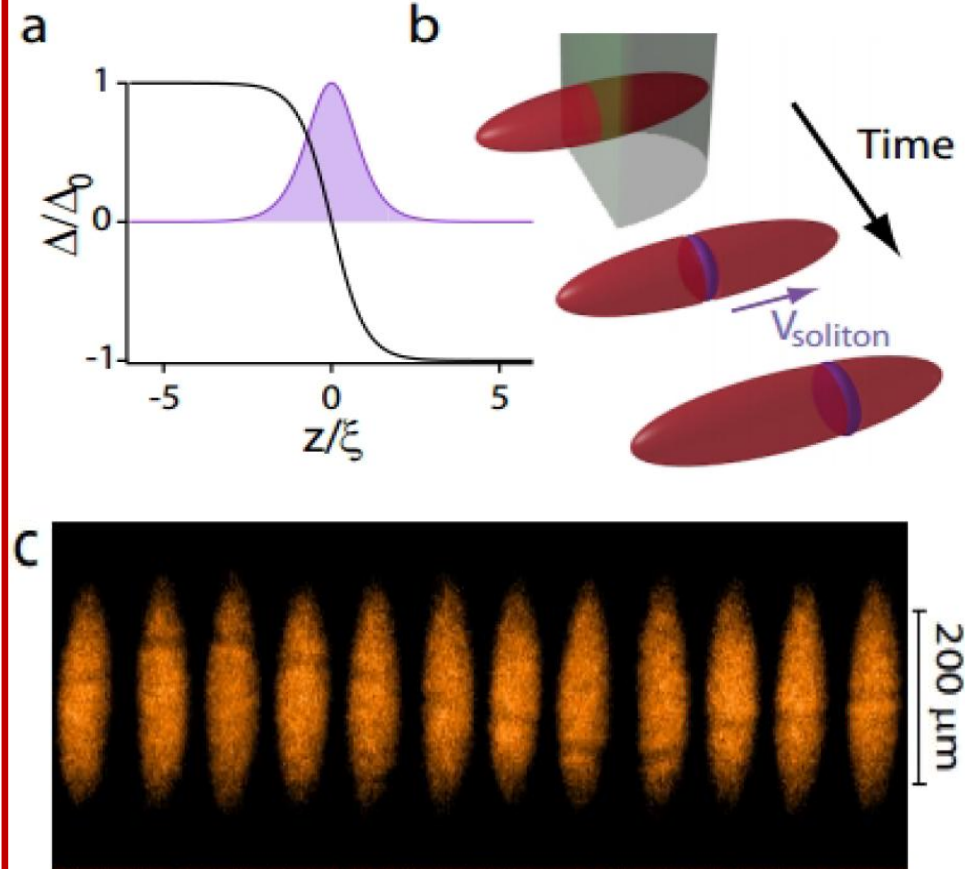
Strong coupling



Observation of **AC Josephson effect** between two ^6Li atomic clouds.

It need not to be accompanied by creation of a topological excitation.

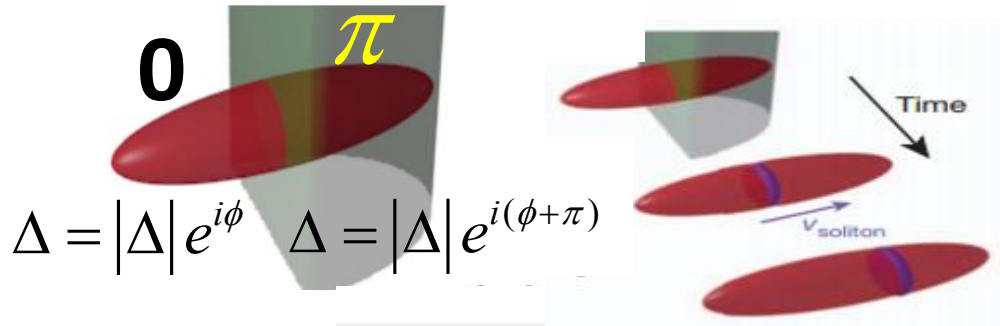
G. Valtolina et al., Science 350, 1505 (2015).



Creation of a „heavy soliton“ after merging two superfluid atomic clouds.

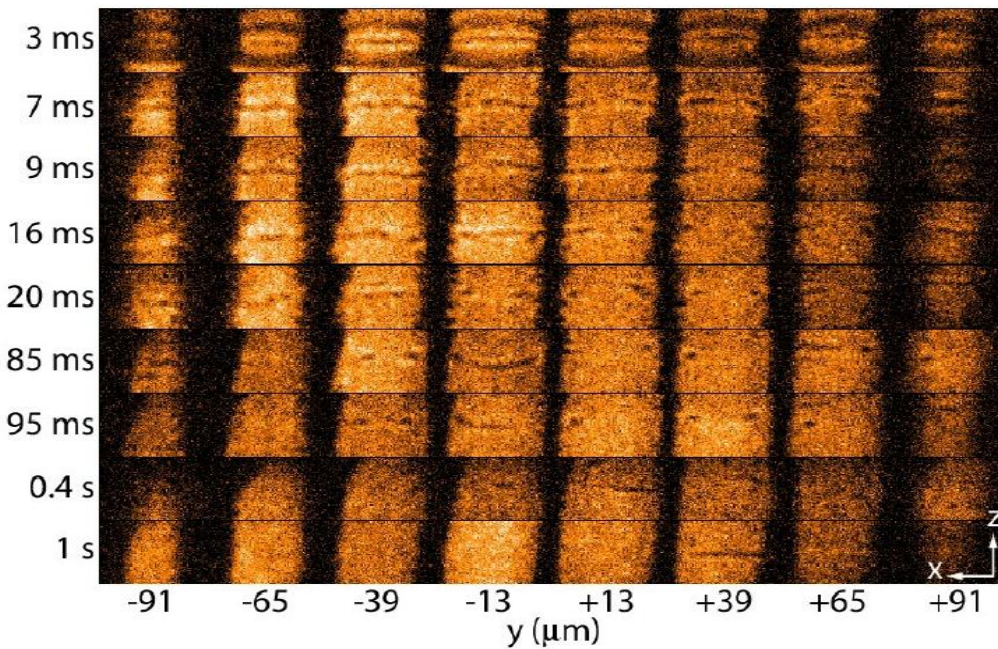
T. Yefsah et al., Nature 499, 426 (2013).

atomic cloud collisions

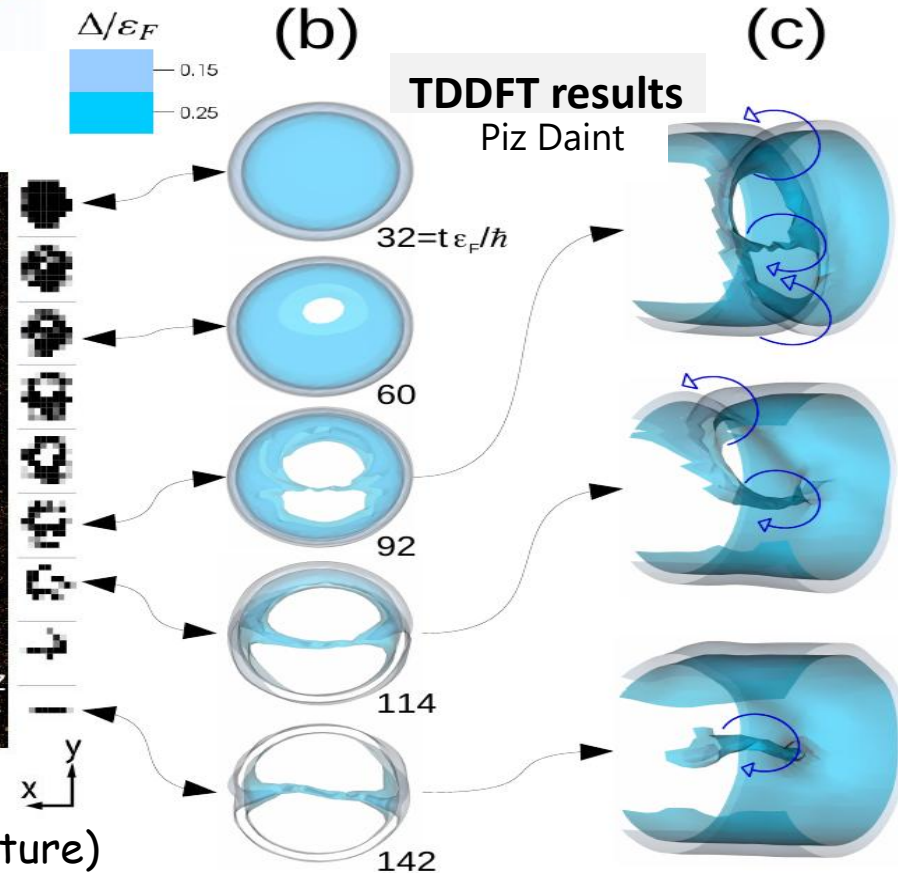


MIT experiment

Phys. Rev. Lett. 116, 045304 (2016)



Decay of solitonic excitation (pairing nodal structure) generates a sequence of topological excitations involving: "Phi"-soliton and vortex line.



G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. M.,
Phys. Rev. Lett. **120**, 253002 (2018)

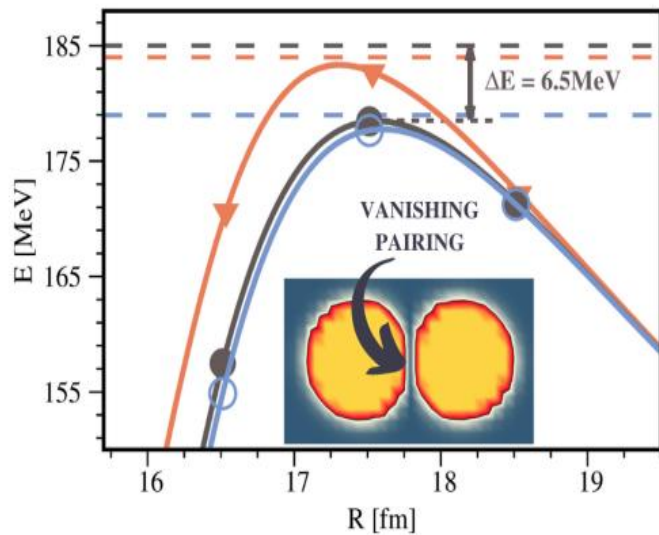


TABLE I: The minimum energies needed for capture in $^{90}\text{Zr}+^{90}\text{Zr}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ for the case of $\Delta\phi = 0$ [$E_{\text{thresh}}(0)$] and $\Delta\phi = \pi$ [$E_{\text{thresh}}(\pi)$]. The energy difference between the two cases is shown in the last column. The average pairing gap $\bar{\Delta}_i$ is defined by Eq. (4).

	$\bar{\Delta}_q$ (MeV)	$E_{\text{thresh}}(0)$ (MeV)	$E_{\text{thresh}}(\pi)$ (MeV)	ΔE_s
^{90}Zr	$\bar{\Delta}_n = 0.00$	184	184	0
	$\bar{\Delta}_p = 0.09$			
^{96}Zr	$\bar{\Delta}_n = 1.98$	179	185	6
	$\bar{\Delta}_p = 0.32$			
	$\bar{\Delta}_n = 2.44$	178	187	9
	$\bar{\Delta}_p = 0.33$			
	$\bar{\Delta}_n = 2.94$	178	187	9
	$\bar{\Delta}_p = 0.34$			

Dynamic nature of the effect:

Solid lines: static barrier between two nuclei (with pairing included):

$^{90}\text{Zr}+^{90}\text{Zr}$ - brown

$^{96}\text{Zr}+^{96}\text{Zr}$ - black (0-phase diff.) and
blue (Pi-phase diff.)

Static barriers are practically insensitive to the phase difference of pairing fields.

Dashed lines: Actual threshold for capture obtained in dynamic calculations.

Hence ΔE measures the additional energy which has to be added to the system to merge nuclei.

$$\Delta E = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\varphi}{2}$$

Dependence of the additional energy on pairing gap in colliding nuclei

P. M., K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

P. M., A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski, Phys. Rev. C 105, 064602, (2022)

G. Scamps, Phys. Rev. C 97, 044611 (2018): **barrier fluctuations extracted from experimental data provide evidence that the effect exists.**

Dissipation mechanisms in Fermionic Josephson junction

Characteristic velocities:

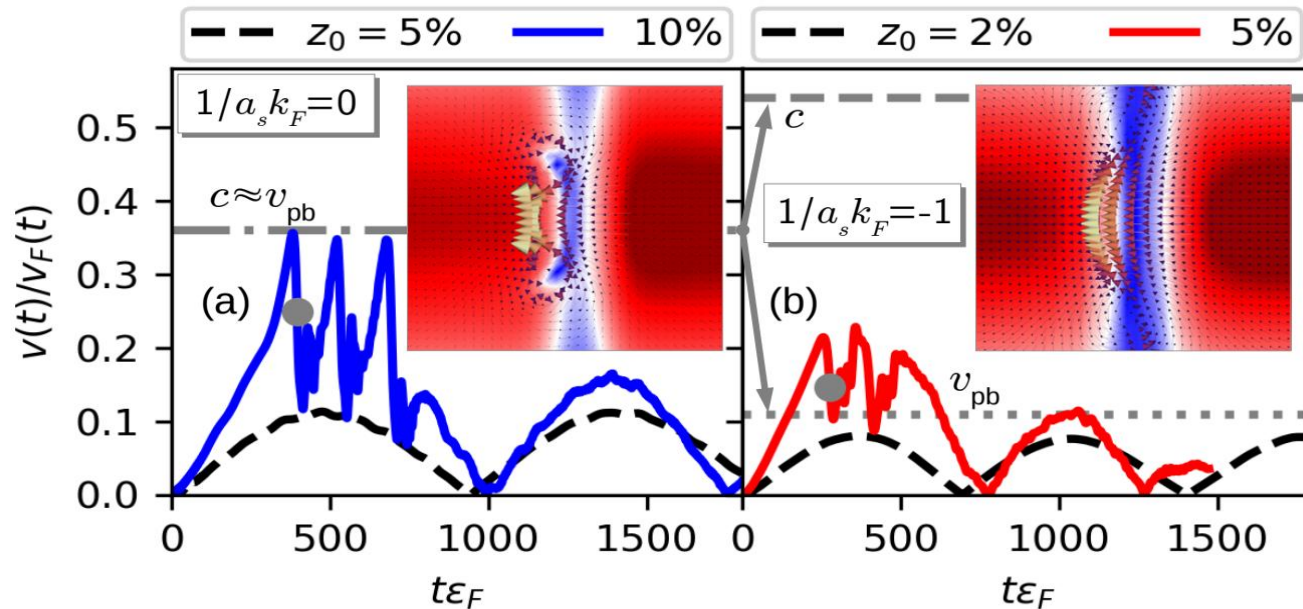
Sound velocity:
$$c_{\text{BCS}} = \sqrt{\frac{1}{3} - \frac{12}{e^4} e^{\pi/a_s k_F} \left[\left(\frac{\pi}{3a_s k_F} \right)^2 - \frac{2\pi}{3a_s k_F} + \frac{10}{9} \right]} v_F \quad c_{\text{UFG}} = \sqrt{\frac{\xi_0}{3}} v_F$$

Pair breaking velocity:
$$v_{\text{pb}} = \sqrt{\sqrt{\mu^2 + \Delta^2} - \mu}$$

unitary limit

BCS limit

$$v_{\text{pb}} \approx c_{\text{UFG}} \approx 0.36 v_F \quad v_{\text{pb}} \approx \frac{\Delta}{2\varepsilon_F} v_F \ll c_{\text{BCS}}$$

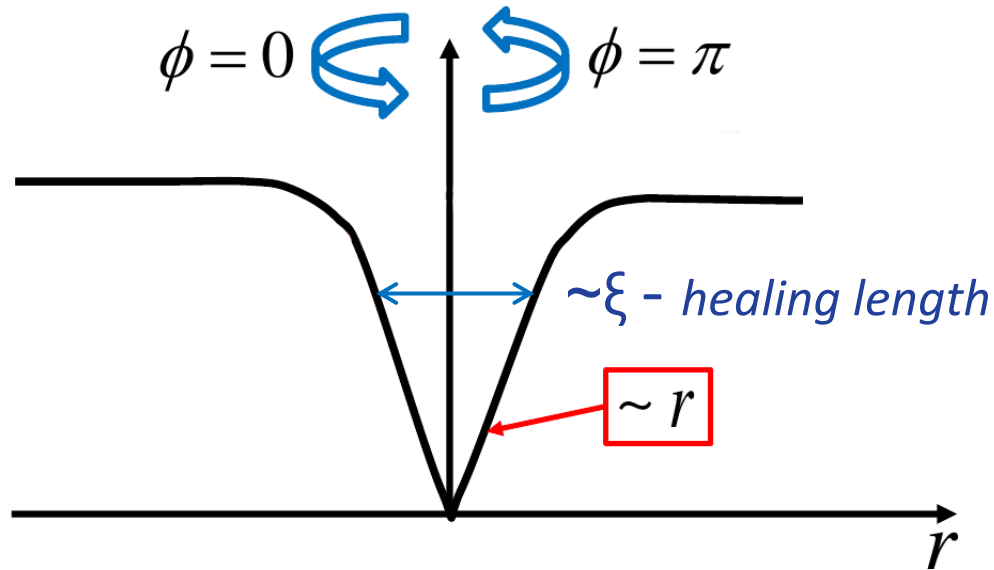


Anatomy of the vortex core

Bosonic vortex structure:

weakly interacting Bose gas at $T=0 \rightarrow$ Gross-Pitaevskii eq. (GPE)

$$\left[-\frac{1}{2m} \nabla^2 + g|\psi(\vec{r})|^2 + V_{ext}(\vec{r}) \right] \psi(\vec{r}) = \mu \psi(\vec{r})$$



Order parameter:

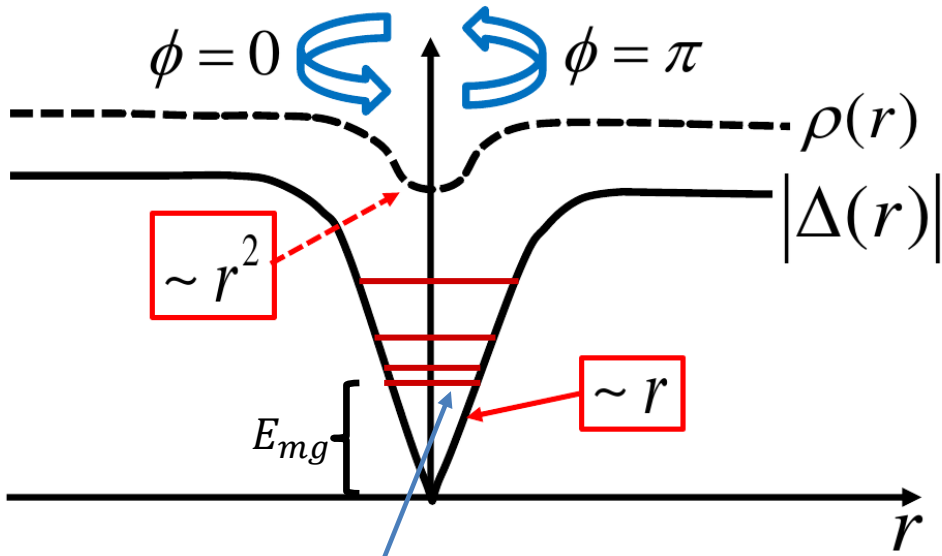
$$\psi(\vec{r}) = \sqrt{n(\vec{r})} e^{i\phi}$$

$$\vec{v}_s = \frac{\hbar}{m} \nabla \phi$$

$$\kappa = \oint d\vec{l} \cdot \vec{v}_s = \frac{\hbar}{m}$$

Fermionic vortex structure:
 Weakly interacting Fermi gas $\rightarrow$ Bogoliubov de Gennes (BdG) eqs.

$$\begin{pmatrix} h_{\uparrow} & \Delta \\ \Delta^* & -h_{\downarrow}^* \end{pmatrix} \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix}$$



Form of the vortex-like solutions:

$$u_{\eta}(\mathbf{r}) = u_{nmk_z}(\rho) e^{im\varphi} e^{ik_z z}$$

$$v_{\eta}(\mathbf{r}) = v_{nmk_z}(\rho) e^{i(m+1)\varphi} e^{ik_z z}$$

CdGM (Andreev) states
 C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

Minigap: $E_{mg} \sim \frac{|\Delta_{\infty}|^2}{\varepsilon_F}$ - energy scale for vortex core excitations.

Density of states: $g(\varepsilon) \sim \frac{\varepsilon_F}{|\Delta_{\infty}|^2} ; \varepsilon \ll |\Delta_{\infty}|$

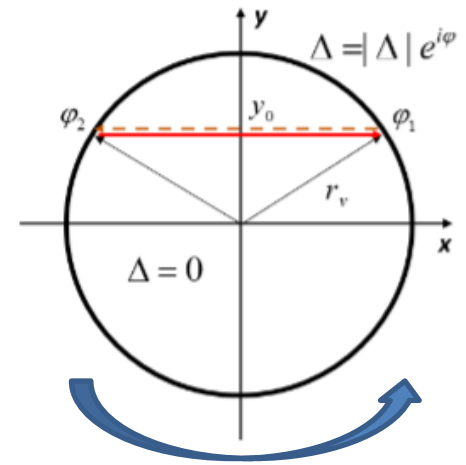
Vortex core structure in Andreev approximation:

$$\frac{E(0, L_z)}{\varepsilon_F} k_F r_V \sqrt{1 - \left(\frac{L_z}{k_F r_V} \right)^2} + \arccos \left(\frac{-L_z}{k_F r_V} \right) - \arccos \left(\frac{E(0, L_z)}{|\Delta_\infty|} \right) = 0$$

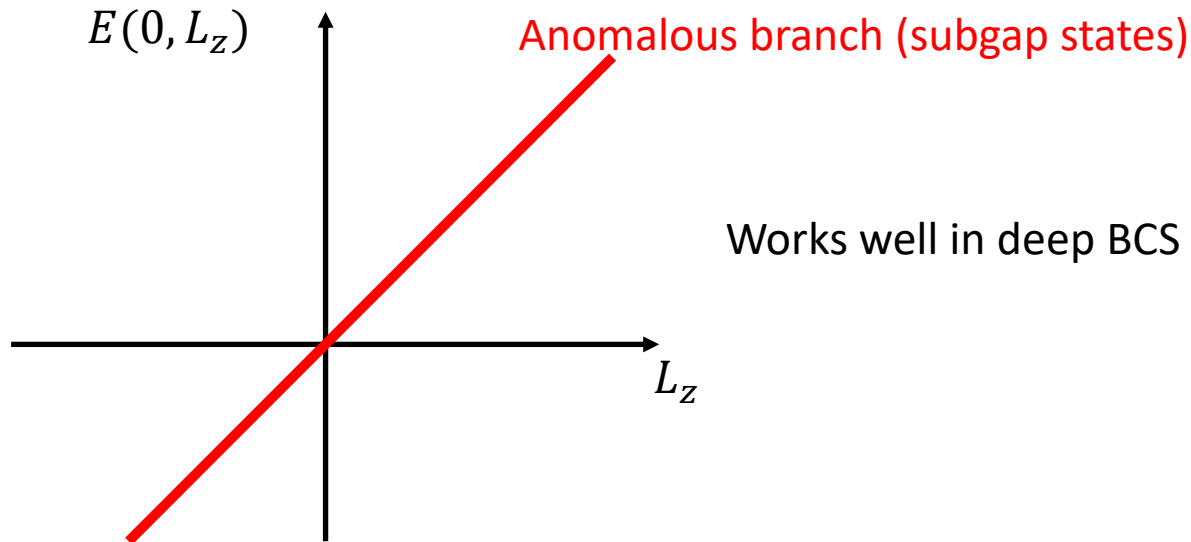
$$E(0, L_z) = E(0) L_z, \quad E \ll |\Delta_\infty|$$

$$E(0, L_z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \frac{r_V}{\xi} \left(\frac{r_V}{\xi} + 1 \right)} \frac{L_z}{\hbar}, \quad \xi = \frac{\varepsilon_F}{k_F |\Delta_\infty|}$$

Schematic section of the core

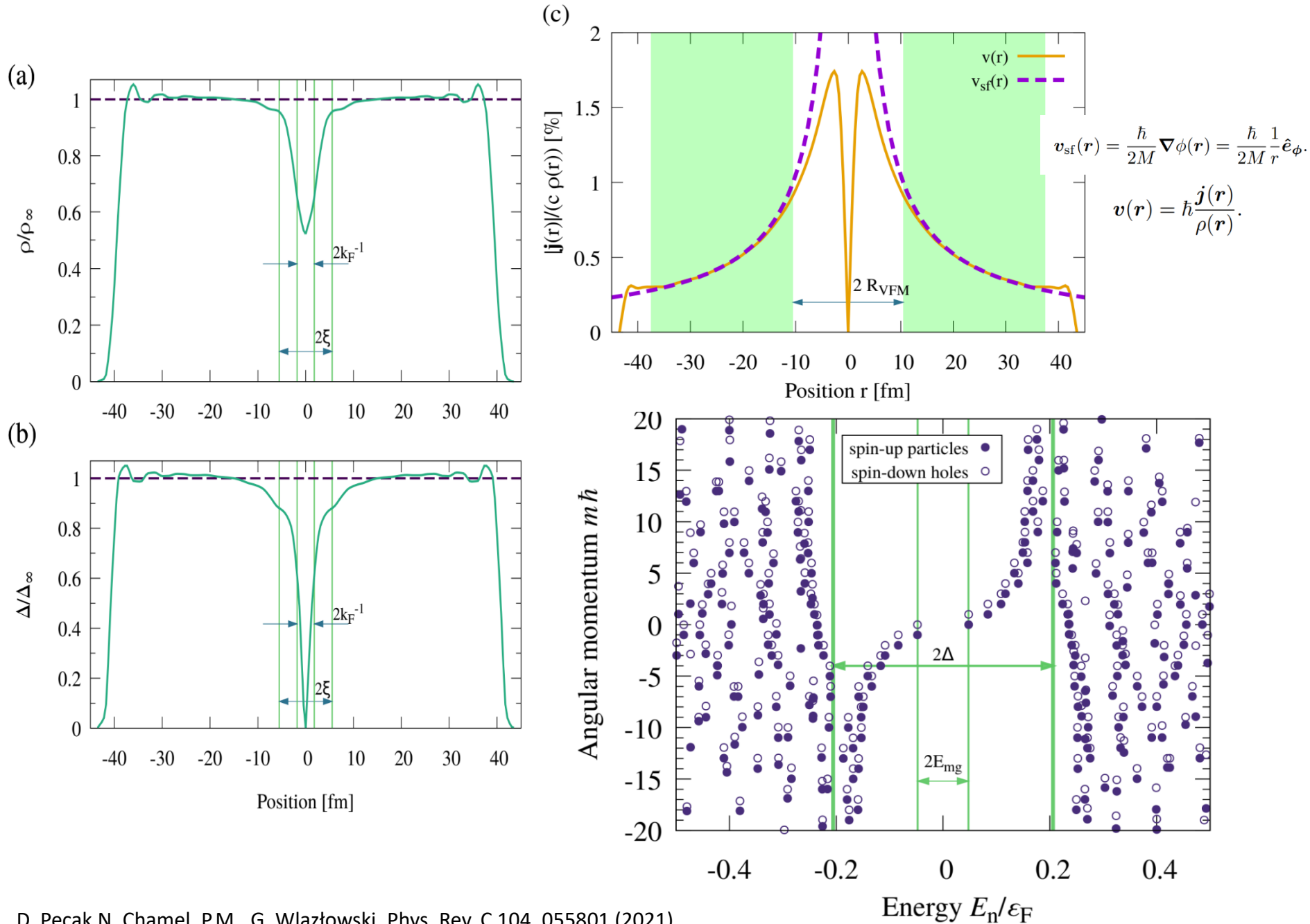


Spectrum of in-gap states



Works well in deep BCS limit: $\frac{1}{k_F a_s} \ll 0$

Example of HFB (BdG) calculations of the vortex structure (inner crust of neutron star).



ρ_∞ (fm ⁻³)	0.00036	0.0059	0.0112	0.0189	0.0231	0.0333
k_F^{-1} (fm)	4.52	1.79	1.45	1.21	1.14	1.01
ξ (fm)	8.44	5.53	5.97	7.00	7.78	10.28
R_{VFM} (fm)	15.0	10.5	10.5	12.0	13.5	16.5
Δ_∞ (MeV)	0.35	1.33	1.53	1.55	1.50	1.28
T_{crit} (MeV)	0.20	0.76	0.87	0.88	0.85	0.73
ε_F (MeV)	1.01	6.48	9.93	14.09	16.10	20.53
μ (MeV)	0.80	4.21	5.80	7.30	7.91	9.09
E_{mg} (MeV)	0.090	0.308	0.261	0.199	0.152	0.009
B_{crit} (10 ¹⁵ G)	7.76	26.5	22.5	17.2	13.1	0.82

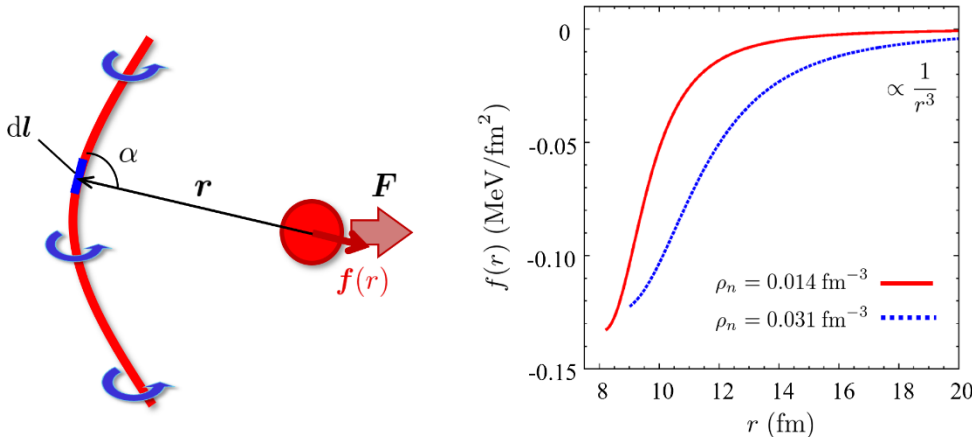
Properties of a vortex across the neutron star crust

Minigap values

Magnetic field needed to polarize the core

D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C **104**, 055801 (2021)

Vortex – impurity interaction (pinning force)



G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, M.M. Forbes, Phys. Rev. Lett. **117**, 232701(2016)

Is neutron star a turbulent system?

- What are differences and similarities of turbulence and its decay in Fermi and Bose superfluids?

A. Bulgac, A. Luo, P. Magierski, K. Roche, Y. Yu, Science **332**, 1288 (2011).

M. Tylutki, G. Wlazłowski, Phys. Rev. A **103**, 051302 (2021).

K. Hossain, K. Kobuszewski, M.M. Forbes, P. Magierski, K. Sekizawa, G. Wlazłowski Phys. Rev. A **105**, 013304 (2022).

G. Wlazłowski, M.M. Forbes, S.R. Sarkar, A. Marek, M. Szpindler, PNAS Nexus **3**, 160 (2024).

Quasiparticle mobility along the vortex line

$$E(k_z) = \frac{E(0)}{\sqrt{1 - \left(\frac{k_z}{k_F}\right)^2}} ; k_z < k_F$$

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

In Andreev approximation:

$$\sqrt{\varepsilon_F + E} \sin \alpha = \sqrt{\varepsilon_F - E} \sin \beta$$

$$k_h = \sqrt{2(\varepsilon_F - E)}$$

$$k_p = \sqrt{2(\varepsilon_F + E)}$$

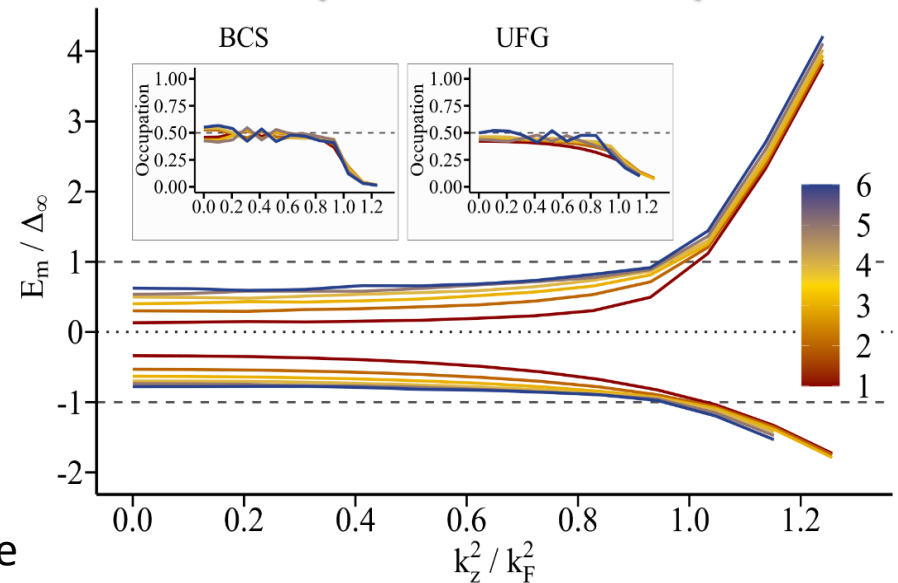
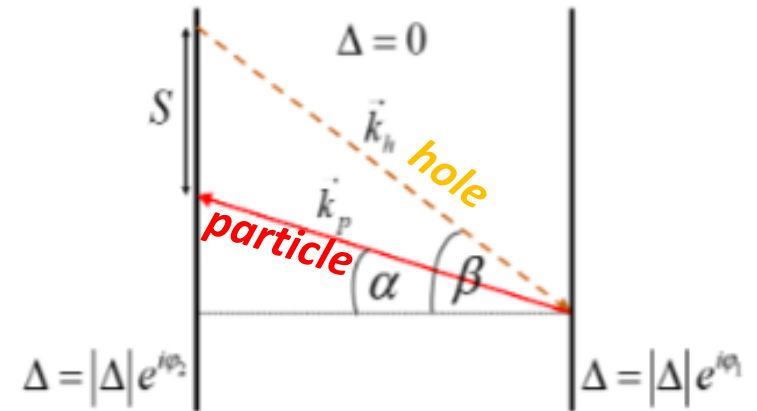
$$v_z = k_z \frac{\sqrt{k_p^2 - k_z^2} - \sqrt{k_h^2 - k_z^2}}{\sqrt{k_p^2 - k_z^2} + \sqrt{k_h^2 - k_z^2}} \quad \text{Velocity component along the vortex line}$$

It gives the same dispersion relations as above up to the second order.

$$M_{eff}^{-1}(L_z) \approx \frac{2}{3} \left(\frac{|\Delta_\infty|}{\varepsilon_F} \right)^2 \frac{L_z}{\hbar}$$

Effective mass of quasiparticle in the core carrying ang. mom. L_z

Schematic picture of Andreev reflection of particle-hole moving along the vortex line



P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Note that large value of effective mass along the vortex line originate from the fact that the occupations of hole and particle states below the gap are approximately equal.

Forces acting on a 2D vortex in an electrically neutral superfluid

Magnus Force

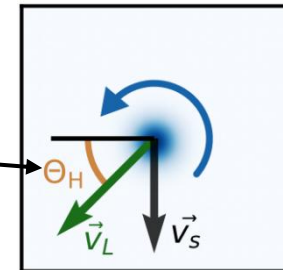
Dissipative Forces

$$\mathbf{F} = A\mathbf{e}_z \times \left(\frac{d\mathbf{R}_V}{dt} - \mathbf{v}_s \right) + B\mathbf{e}_z \times \left(\frac{d\mathbf{R}_V}{dt} - \mathbf{v}_n \right) + C \left(\frac{d\mathbf{R}_V}{dt} - \mathbf{v}_n \right) +$$

$$+ \mathbf{F}_{\text{pinning}} \left(\mathbf{R}_V, \frac{d\mathbf{R}_V}{dt}, \mathbf{e}_z \times \frac{d\mathbf{R}_V}{dt} \right),$$

Pinning Force (in inhomogeneous superfluid)

Hall angle



Dissipative processes involving vortex dynamics:

- Scattering of localized and delocalized quasiparticles on the vortex core giving rise to both longitudinal and transversal forces.
- Various particular mechanisms were considered:
lordanskii force, Kopnin-Kravtsov force, Spectral flow-force, Emission of quasiparticles from the core.

Silaev, Phys. Rev. Lett. 108, 045303 (2012)

Kopnin, Rep. Prog. Phys. 65, 1633 (2002)

Stone, Phys. Rev. B54, 13222 (1996)

Kopnin, Volovik, Phys. Rev. B57, 8526 (1998)

Kopnin, N.B., Kravtsov, V.E. JETP Lett. 23, 578 (1976)

lordanskii, S.V.:Sov. Phys. JETP, 22, 160 (1966)

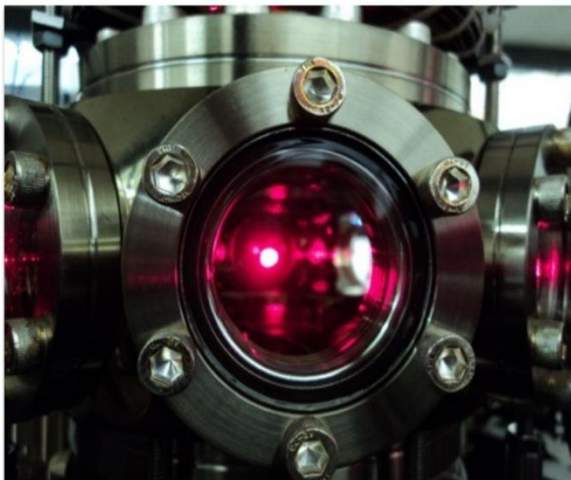
and many more (including those questioning e.g., the existence of lordanskii force)

Semiclassical approximation:

Classical treatment of states in the core + Boltzmann eq. In the relaxation time approx.
More applicable in deep BCS limit.

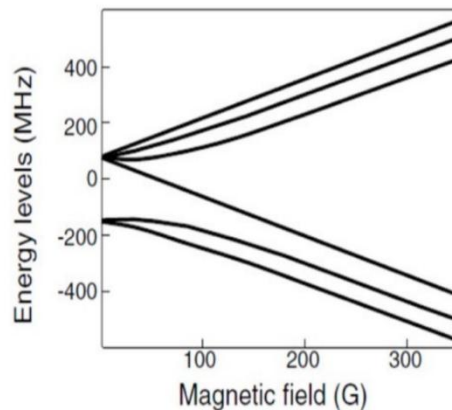
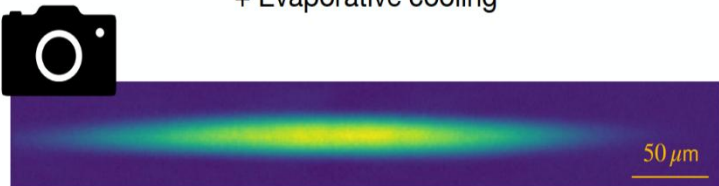
How can we measure the influence of core states in ultracold gases?

LITHIUM-6 FERMI SUPERFLUIDS



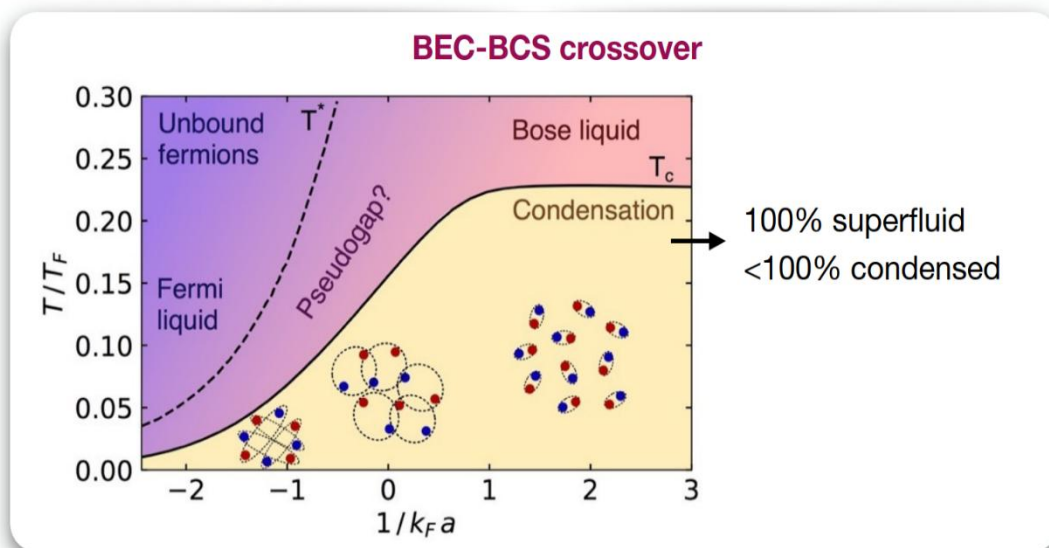
$N \sim 10^5$ at $T \sim 30$ nK

Magneto optical trap + D_1 molasses
+ Evaporative cooling



$$\sim \frac{4\pi\hbar^2}{m} a \delta(r)$$

$a \rightarrow$ Scattering length

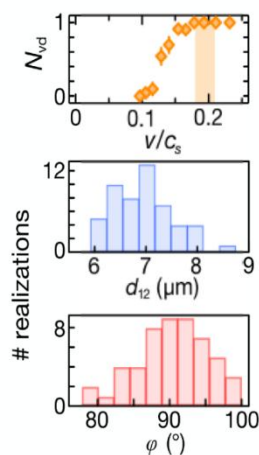
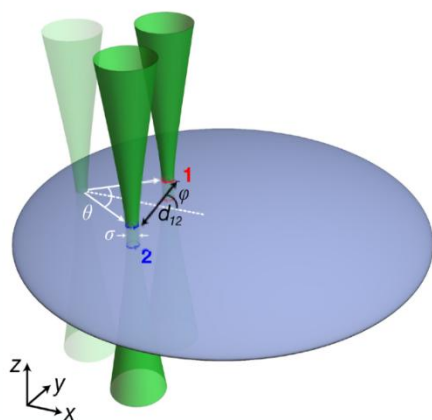


VORTICES IN ATOMIC FERMI SUPERFLUIDS

W. J. Kwon, et al. *Nature* **600**, 64–69 (2021).

Excitation of vortices

Chopstick method



With Bose superfluids:

E. C. Samson, *Phys. Rev. A* **93**, 023603 (2016).

T. Neely et al., arXiv:2402.09920v2

Detection of vortices

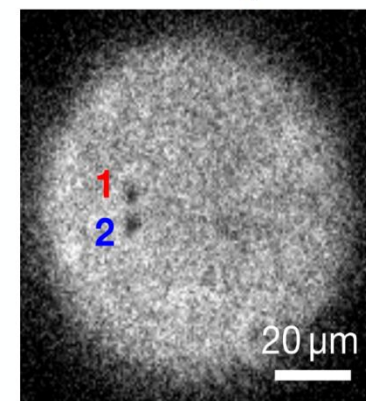
Chopstick ON

In situ



Chopstick OFF

Time-of-flight



- Create arbitrary configuration of **vortex dipole**
- Precisely track the vortex position

Velocities of vortex-antivortex pair (vortex dipole)

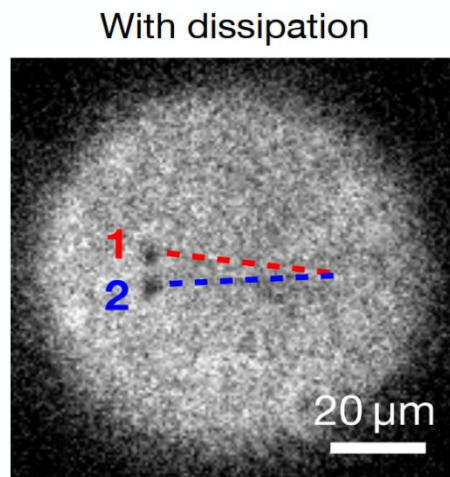
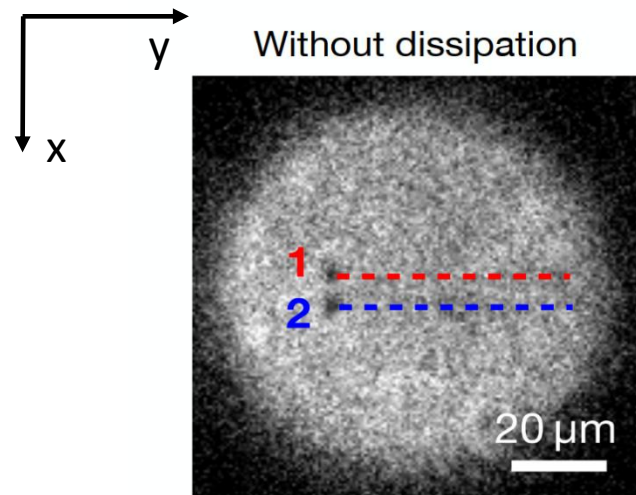
Dissipative Force

$$\mathbf{v}_1(t) = \frac{|\kappa|}{d(t)} [(1 - \alpha')\hat{\mathbf{y}} - |\alpha|\hat{\mathbf{x}}],$$

$$\mathbf{F}_N = D(\mathbf{v}_i - \mathbf{v}_n) + D'\hat{\mathbf{z}} \times (\mathbf{v}_i - \mathbf{v}_n)$$

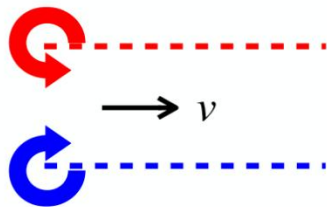
$$\mathbf{v}_2(t) = \frac{|\kappa|}{d(t)} [(1 - \alpha')\hat{\mathbf{y}} + |\alpha|\hat{\mathbf{x}}],$$

where $\alpha = \frac{\tilde{D}}{\tilde{D}^2 + (1 - \tilde{D}')^2}$, $1 - \alpha' = \frac{1 - \tilde{D}'}{\tilde{D}^2 + (1 - \tilde{D}')^2}$, $\tilde{D} = \frac{D}{\kappa\rho_s}$, $\tilde{D}' = \frac{D'}{\kappa\rho_s}$.

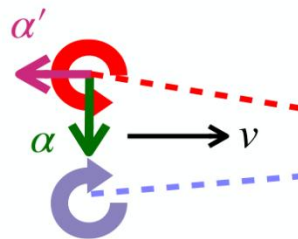


$$E \propto \log d$$

$$v \propto 1/d$$

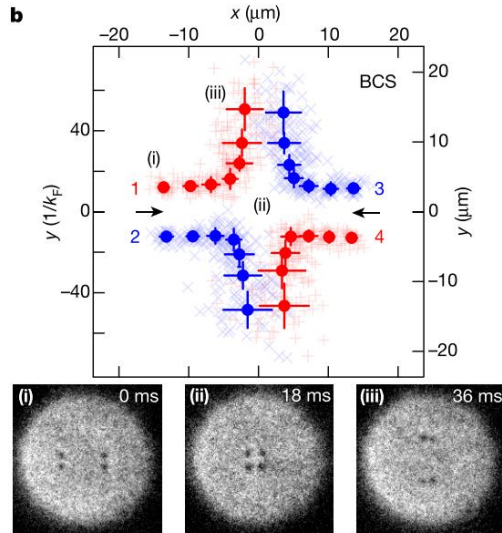


$$E_i = E_f$$



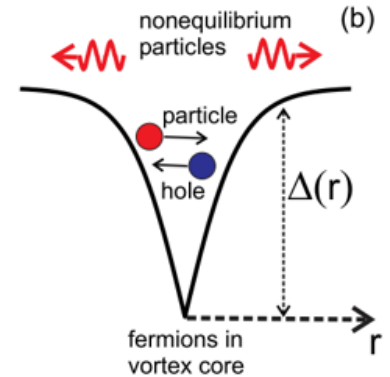
$$E_i > E_f$$

Vortex-antivortex scattering in 2D



„Further, our few-vortex experiments extending across different superfluid regimes reveal non-universal dissipative dynamics, suggesting that fermionic quasiparticles localized inside the vortex core contribute significantly to dissipation, thereby opening the route to exploring new pathways for quantum turbulence decay, vortex by vortex.“

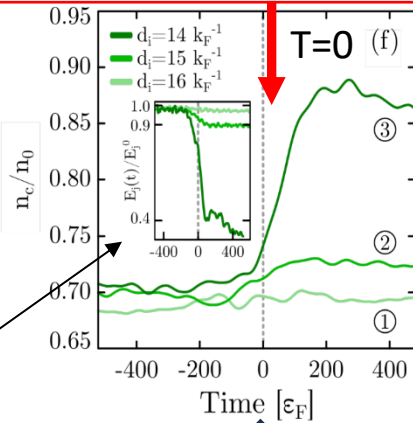
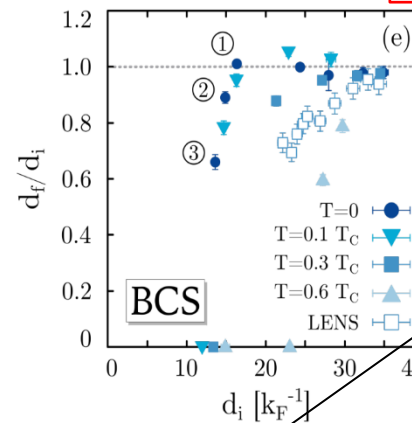
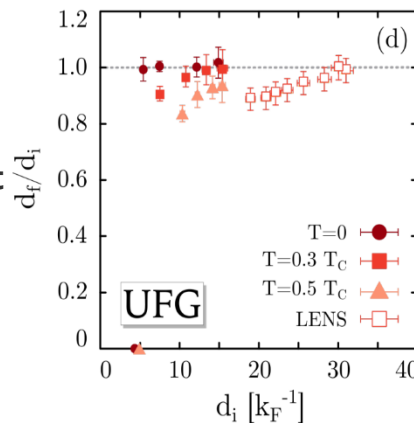
W.J. Kwon et al. Nature **600**, 64 (2021)



Exciting quasiparticles in the vortex core

Indeed quasiparticles in the core are excited due to vortex acceleration but the effect is too weak to account for the total dissipation rate.

A. Barresi, A. Boulet, P.M., G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)

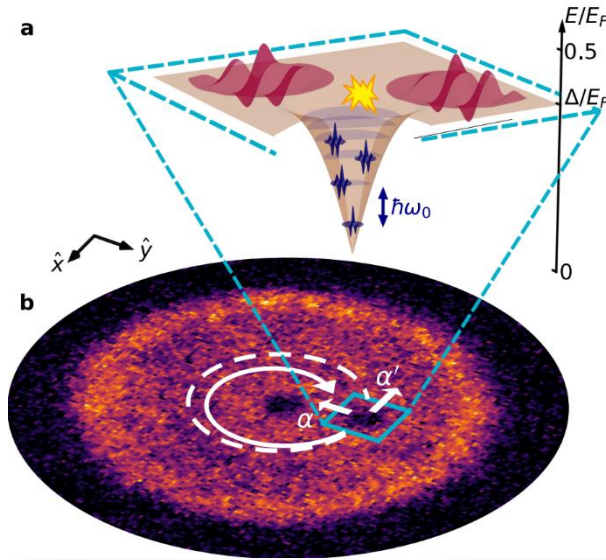


$$E_j = \int \frac{j^2}{2n} d^3r$$

$$n_A(\mathbf{r}) = 2 \sum_{0 < E_n < 0.85\Delta} (|u_n(\mathbf{r})|^2 f_n^+ + |v_n(\mathbf{r})|^2 f_n^-)$$

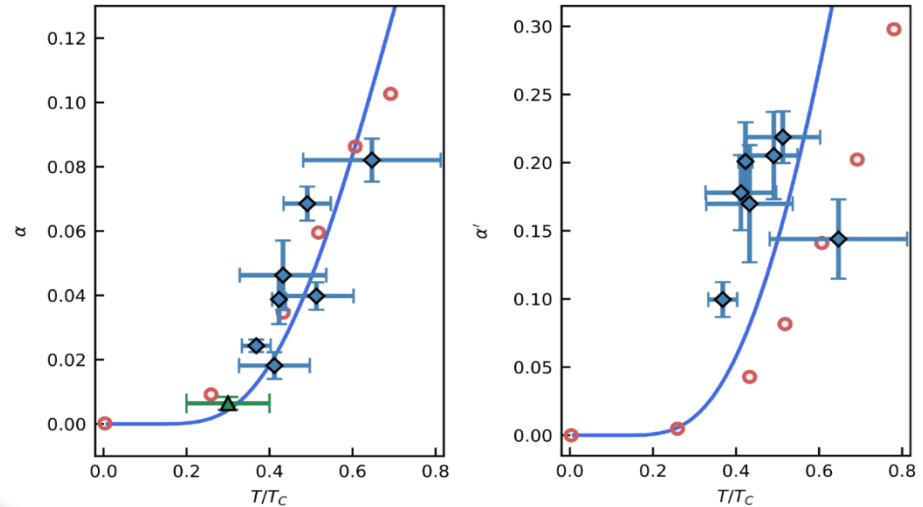
MUTUAL FRICTION IN A VORTEX DIPOLE

N. Grani, D. Hernández-Rajkov, C. Daix, P. Pieri, M. Pini, P.M., G. Wlazłowski, M.F. Fernández, F. Scazza, G. Del Pace, G. Roati, *Nature Comm.* 16 (2025) 10245



- First experimental observation of **nonzero** α' in ultra cold atoms
- Connection with **microscopic mechanism** of dissipation

Mutual friction coefficients



◦ SLDA simulation

— Analytical model including localized QP
Kopnin, Rep. Prog. Phys. 65, 1633 (2002)

$$\Theta_H = \tan^{-1} ((1 - \alpha')/\alpha)$$



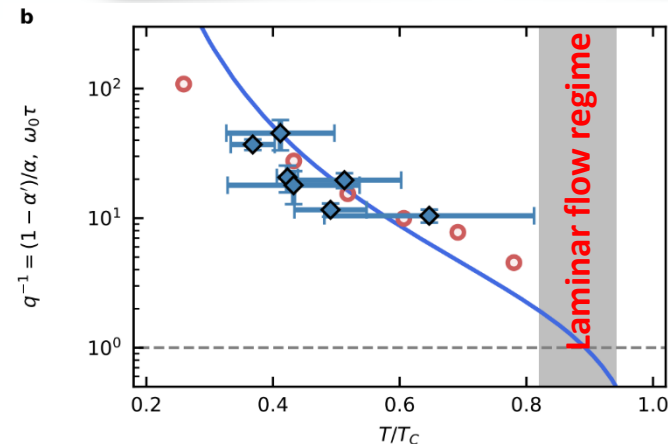
$$\left. \begin{aligned} \alpha &= \frac{\rho_s}{\rho} \left(\omega_0 \tau \tanh \frac{|\Delta|}{2k_B T} \right)^{-1} \\ \alpha' &= 1 - \frac{\rho_s}{\rho} \left(\tanh \frac{|\Delta|}{2k_B T} \right)^{-1} \end{aligned} \right\}$$

$$\omega_0 = E_{mg}/\hbar$$

$$\frac{1 - \alpha'}{\alpha} = \frac{1 - d_{\perp}}{d_{\parallel}} = \omega_0 \tau.$$

Determines two regimes:

- Free Kelvin wave propagation (>1)
- Overdamped Kelvin waves (<1)



Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

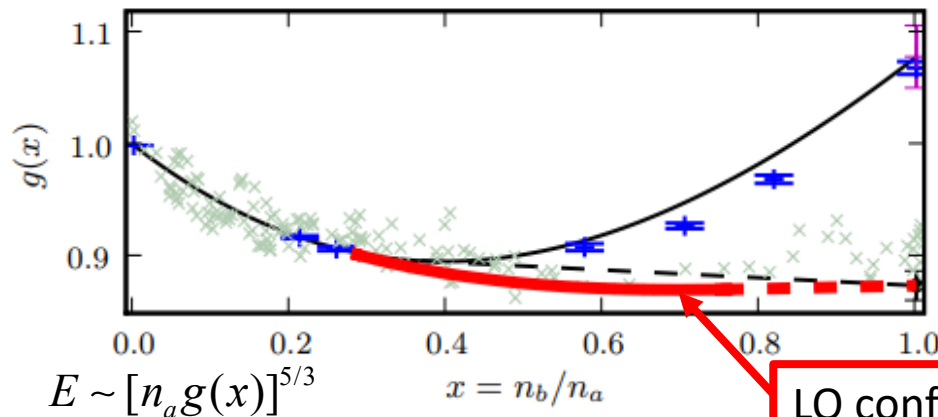
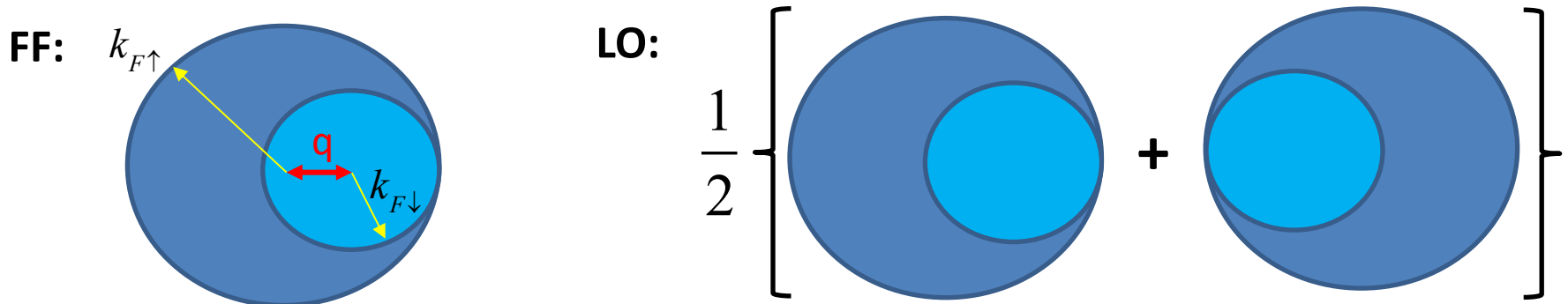
Larkin-Ovchinnikov (LO): $\Delta(r) \sim \cos(\vec{q} \cdot \vec{r})$

Fulde-Ferrell (FF): $\Delta(r) \sim \exp(i\vec{q} \cdot \vec{r})$

A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)

P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

Spatial modulation of the pairing field cost energy proportional to q^2 but may be compensated by an increased pairing energy due to the mutual shift of Fermi spheres:



Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG) (realized experimentally in ultracold atomic clouds)

LO configuration – supersolid state

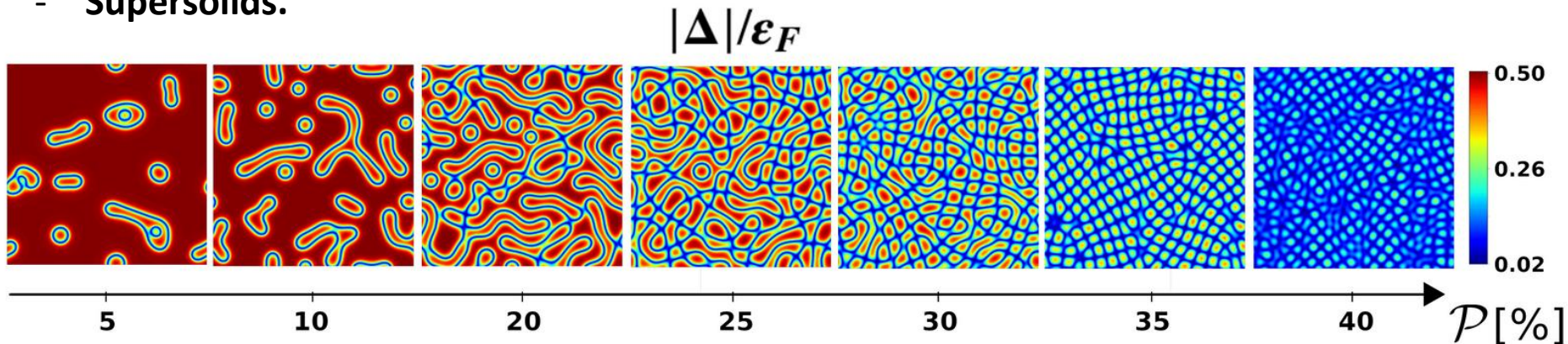
A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)

See also review of mean-field theories : Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

What is going to happen if we keep increasing spin imbalance?

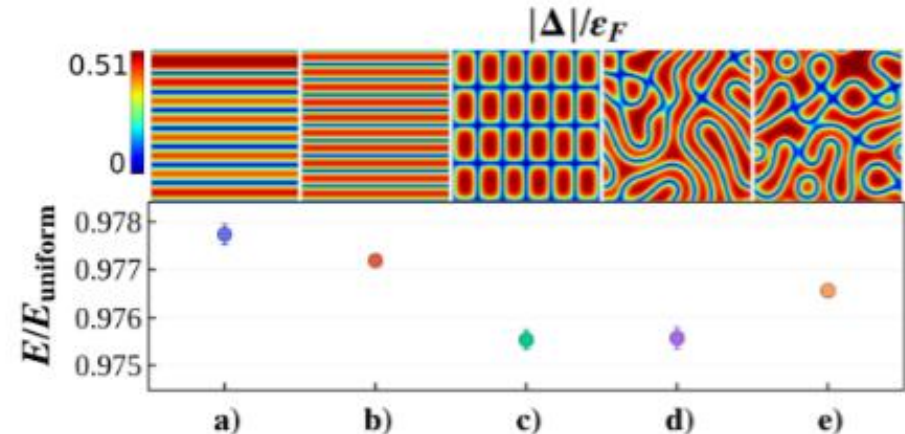
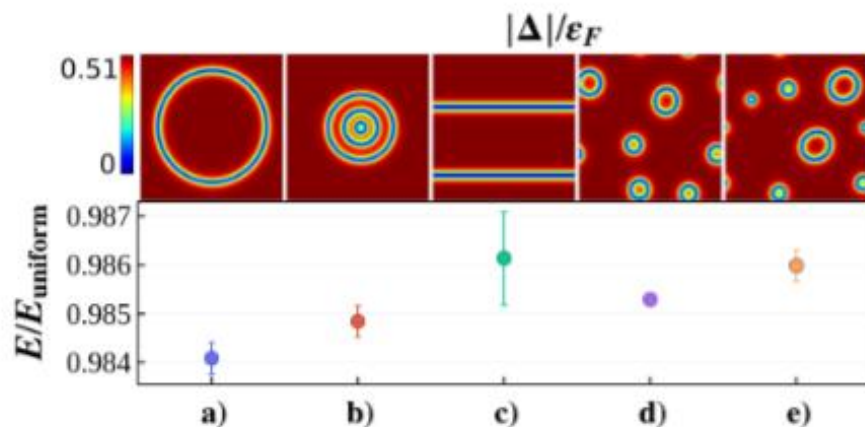
In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

- **Separate impurities (fermons),**
- **Liquid crystal-like structure,**
- **Supersolids.**

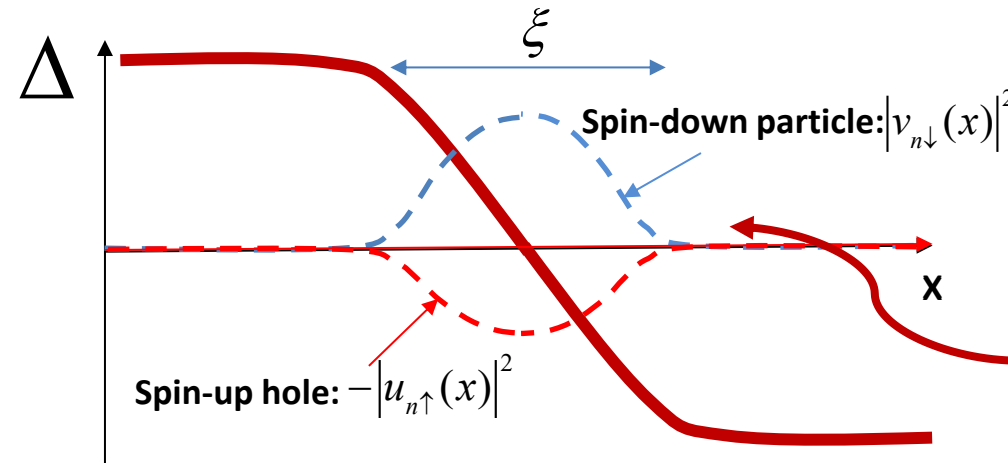


B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023)

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$



Andreev states and stability of pairing nodal points



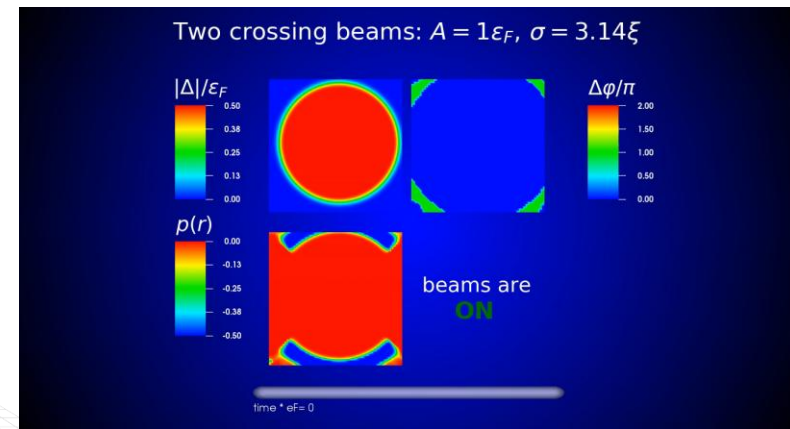
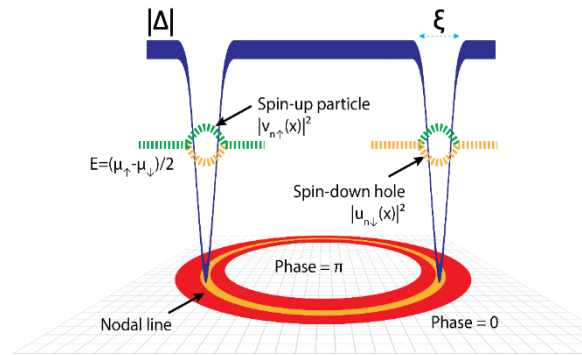
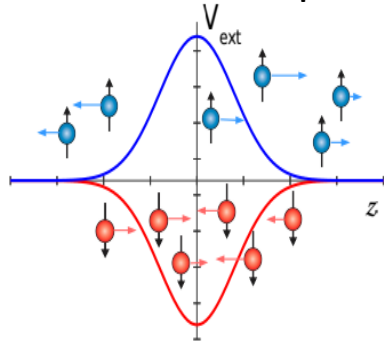
Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

BdG in the Andreev approx. ($\Delta \ll k_F^2$)

$$\begin{bmatrix} -2ik_F \frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \frac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

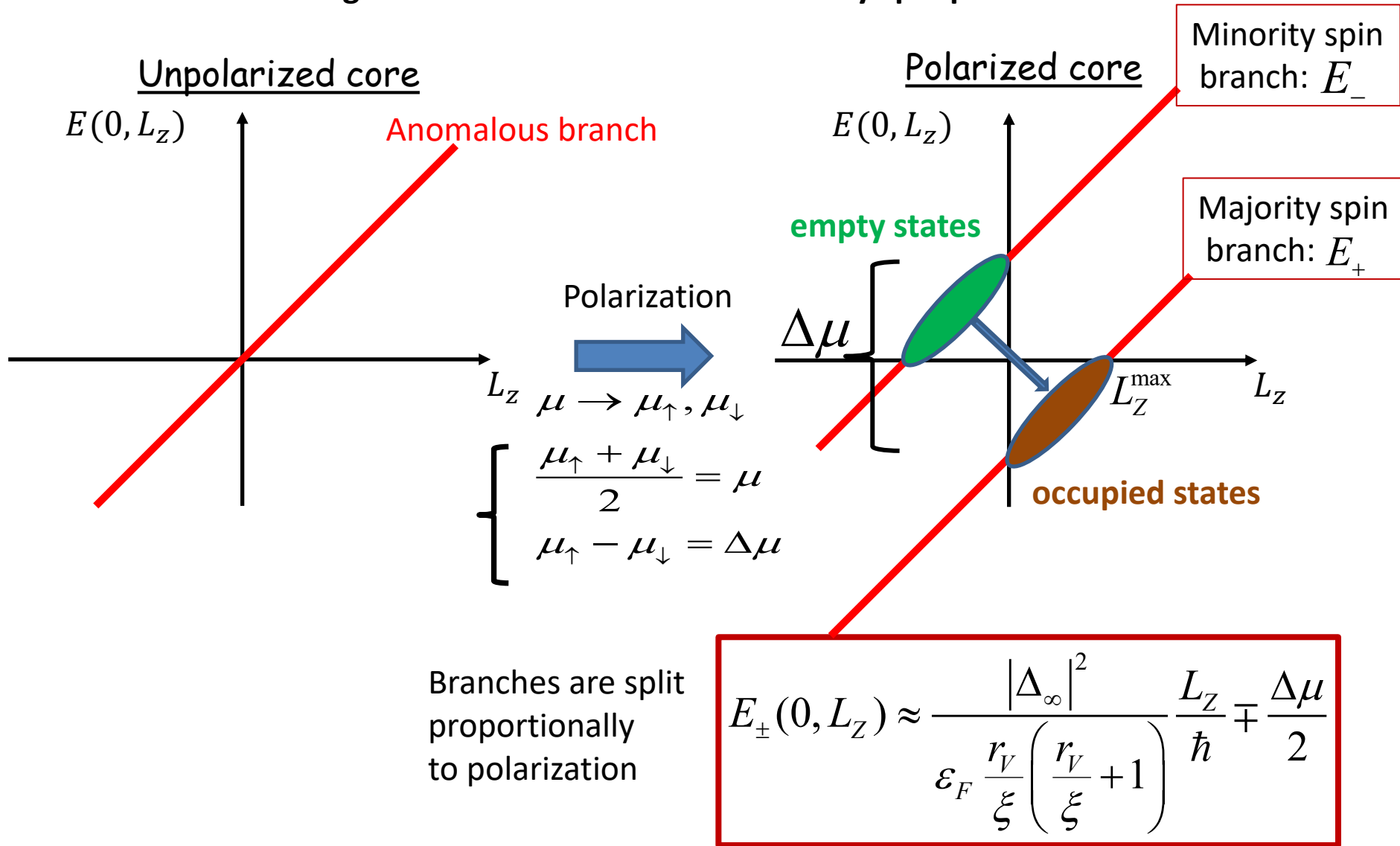
Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:



Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.

Changes of the core structure induced by spin polarization



Certain fraction of majority spin particles rotate in the opposite direction!

$$L_z^{\max} \approx \frac{1}{2} \frac{\varepsilon_F}{|\Delta_{\infty}|^2} \frac{r_V}{\xi} \left(\frac{r_V}{\xi} + 1 \right) \hbar \Delta\mu$$

Two consequences of vortex core polarization:

- 1) Minigap vanishes.
- 2) Direction of the current in the core reverses.

- 1) Since the polarization correspond to relative shift of anomalous branches therefore the quasiparticle spectrum of spin-up and spin-down components is asymmetric for $k_z = 0$.

However the symmetry of the spectrum has to be restored in the limit of $k_z \rightarrow \infty$.
Since for a straight vortex one can decouple the degree of freedom along the vortex line:

$$H = \begin{pmatrix} h_{2D}(\mathbf{r}) + \frac{1}{2}k_z^2 - \mu_{\uparrow} & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_{2D}^*(\mathbf{r}) - \frac{1}{2}k_z^2 + \mu_{\downarrow} \end{pmatrix}$$

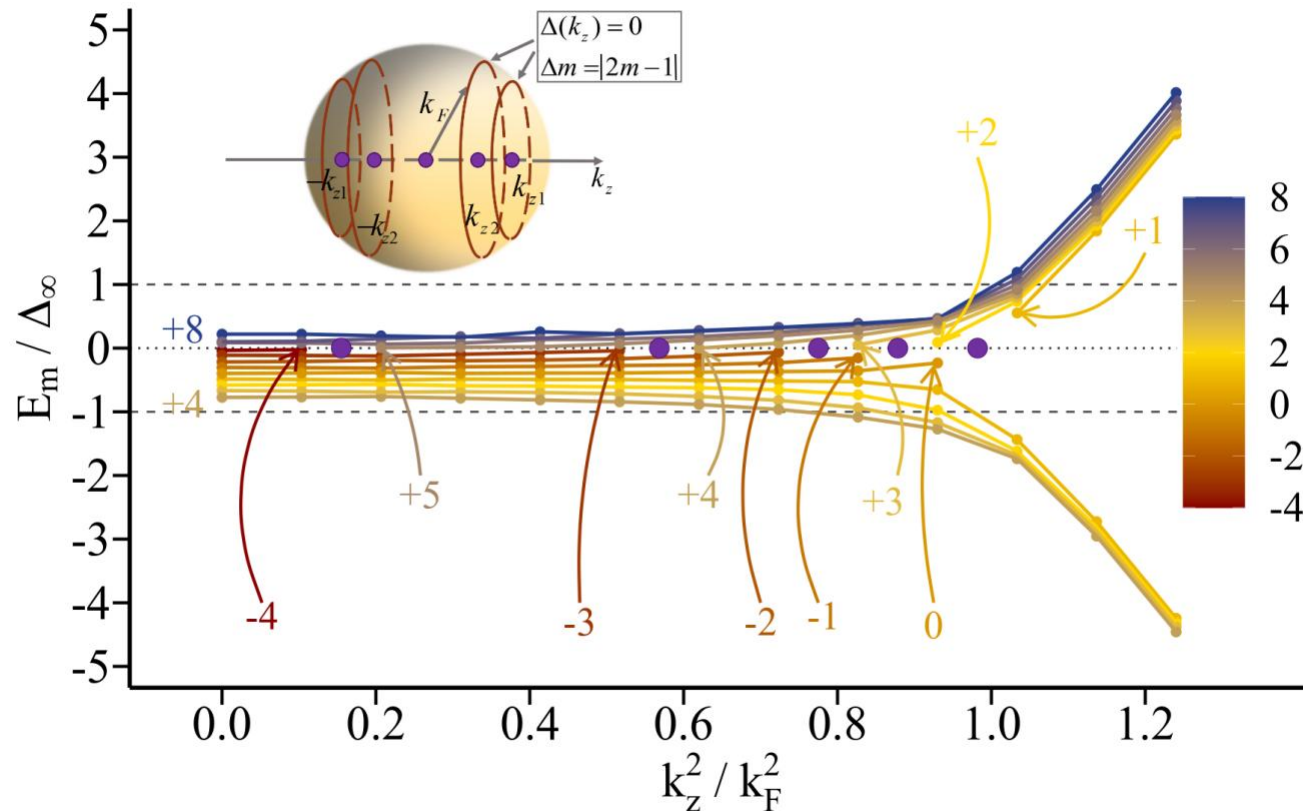
therefore $E(k_z) \propto \pm k_z^2$ when $k_z \rightarrow \infty$

As a result there must exist a sequence of values: $k_z = \pm k_{z1}, \pm k_{z2}, \dots$
for which:

$$E(\pm k_{zi}) = 0$$

Moreover the crossings occur between levels of particular projection of angular momentum on the vortex line.

Namely, the crossing occurs in such a way that the particle state: $v_{\uparrow}$ of ang. momentum m is converted into a hole $u_{\uparrow}$ of momentum $-m+1$
Hence the configuration changes by $\Delta m = |2m - 1|$



Vortex dipole in spin imbalanced system

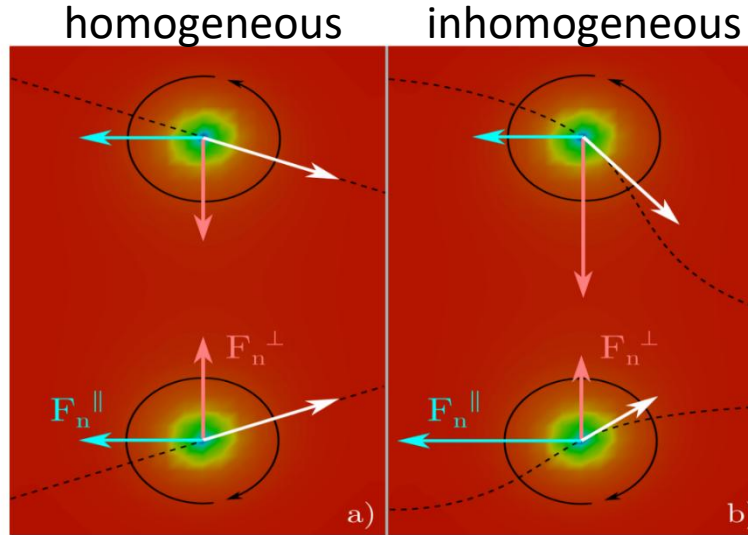


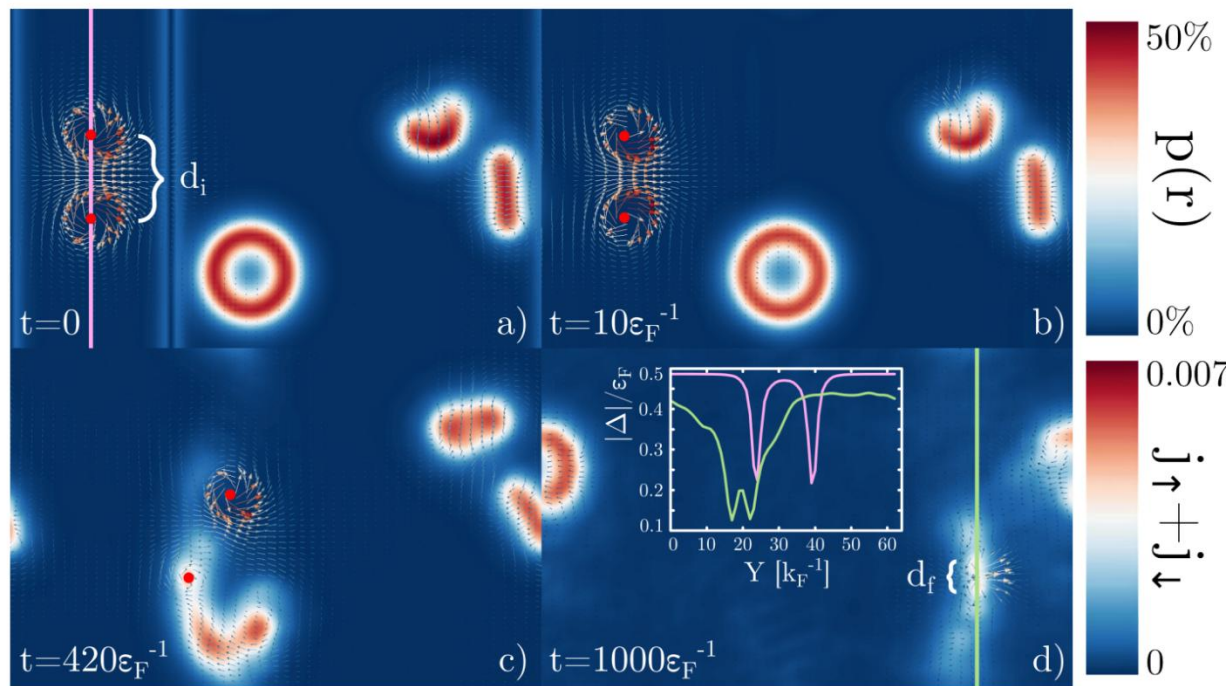
Figure 1. Comparison of forces acting on vortex cores under different density regimes. a) Both normal and superfluid component are isotropic ($\rho_s = \text{const}, \rho_n = \text{const}$); b) Both components are distributed inhomogeneously ($\rho_s = \rho_s(\mathbf{r}), \rho_n = \rho_n(\mathbf{r})$), but their sum is isotropic $\rho_s(\mathbf{r}) + \rho_n(\mathbf{r}) \approx \text{const}$.

Velocities of vortex-antivortex pair (vortex dipole)

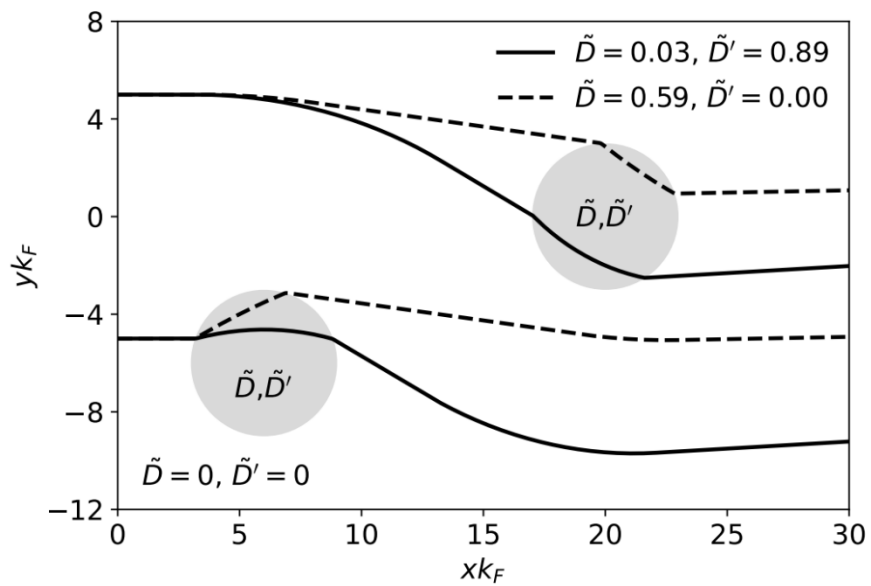
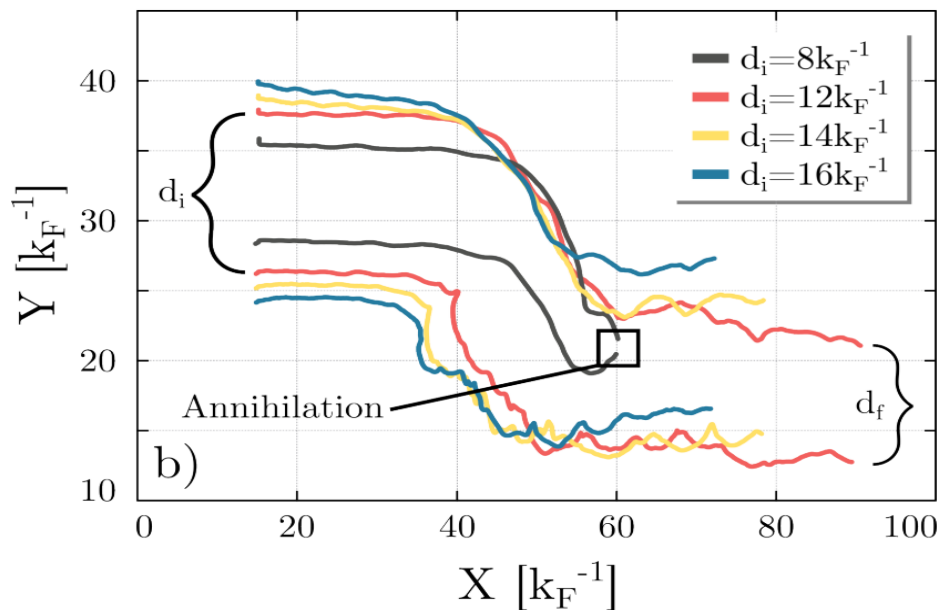
$$\mathbf{v}_1(t) = \frac{|\kappa|}{d(t)} [(1 - \alpha')\hat{\mathbf{y}} - |\alpha|\hat{\mathbf{x}}],$$

$$\mathbf{v}_2(t) = \frac{|\kappa|}{d(t)} [(1 - \alpha')\hat{\mathbf{y}} + |\alpha|\hat{\mathbf{x}}],$$

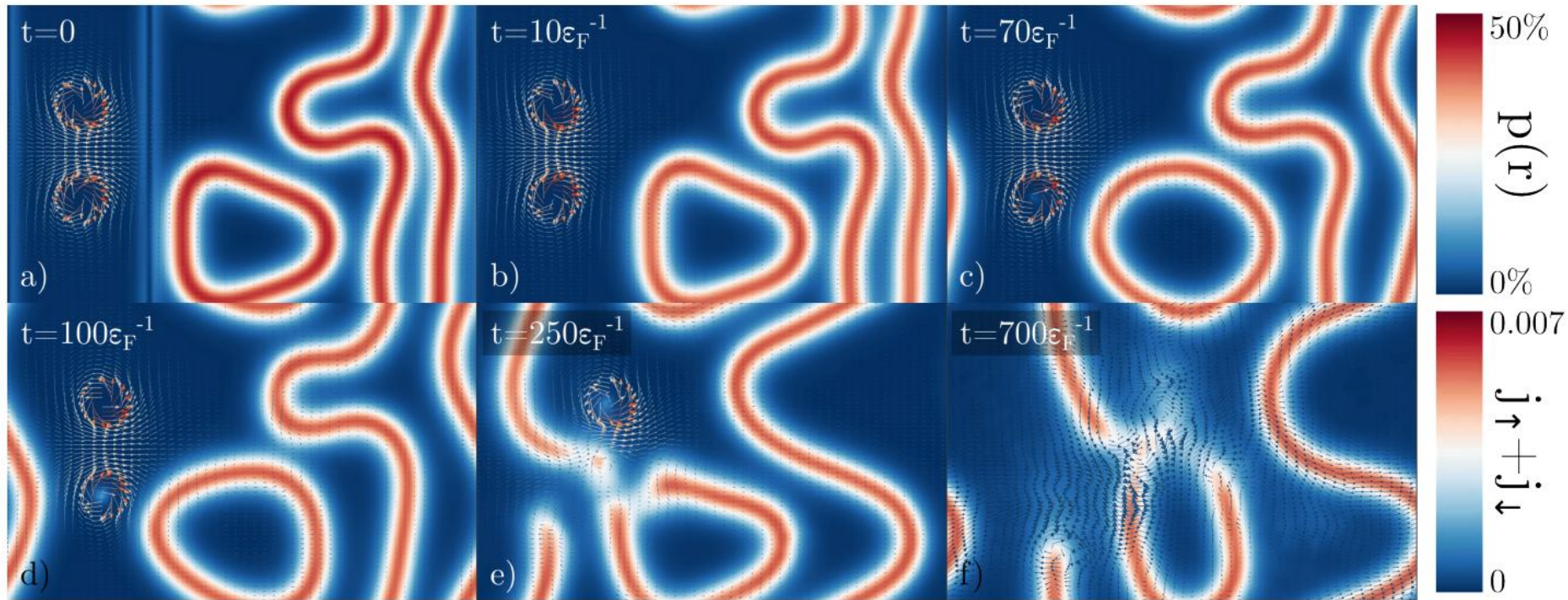
$$\text{where } \alpha = \frac{\tilde{D}}{\tilde{D}^2 + (1 - \tilde{D}')^2}, \quad 1 - \alpha' = \frac{1 - \tilde{D}'}{\tilde{D}^2 + (1 - \tilde{D}')^2}, \quad \tilde{D} = \frac{D}{\kappa\rho_s}, \quad \tilde{D}' = \frac{D'}{\kappa\rho_s}.$$



$$1.37\% < P < 1.84\%$$



P=10%

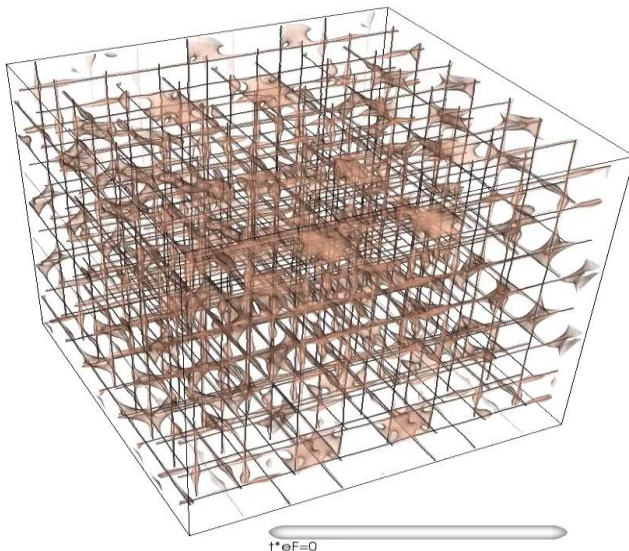


Vortex dipole is practically immobilized and annihilated!

Conclusions

- SLDA predicts surprisingly good dissipative parameters for vortex motion as a function of T (T must be rescaled to yield the correct T_c).
- Surprisingly, even the semiclassical approach of Kopnin et al. produces results of comparable quality to SLDA for dissipative effects.
- Enhanced dissipative effects are expected in spin-imbalanced systems due to the normal component and modifications of the vortex core structure.
- These effects lead to the trapping and annihilation of vortices when polarization P exceeds 10%.

Towards understanding of quantum turbulence in Fermi superfluids



Time- dependent DFT simulation of compressible turbulence in a unitary Fermi gas on a 100^3 cubic lattice.

G. Wlazłowski et al. PNAS Nexus 3, p. 160 (2024)

W-SLDA Toolkit

*Self-consistent solver
of mathematical problems
which have structure
formally equivalent to
Bogoliubov-de Gennes equations.*

static problems: st-wslda

$$\begin{pmatrix} h_a(\mathbf{r}) - \mu_a & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_b^*(\mathbf{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

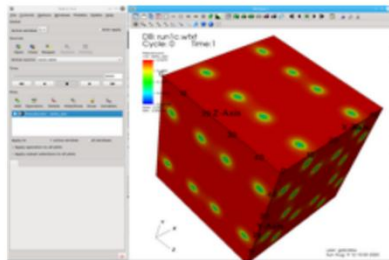
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r}, t) - \mu_a & \Delta(\mathbf{r}, t) \\ \Delta^*(\mathbf{r}, t) & -h_b^*(\mathbf{r}, t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix}$$

Extension to nuclear matter
in neutron stars

Unified solvers for static and
time-dependent problems

Dimensionalities of
problems: 3D, 2D and 1D

Integration with VisIt: visualization, animation and analysis tool



W-SLDA is integrated with the open-source VisIt tool. It allows for:
visualizing 3D, 2D and 1D results,
data processing,
creating animations for time-dependent simulations.

Integration with VisIt:
visualization, animation and
analysis tool

Speed-up calculations by
exploiting High Performance
Computing

Functionals for studies of
BCS and unitary regimes

ALL FUNCTIONALITIES →

Getting the code



DOWNLOAD

The W-SLDA & W-BSk Toolkits are free to download. It is published as open source under GNU GPL License. In order to get W-SLDA or W-BSk Toolkit click "Read more" and follow instructions.

Info

» Contributors

W-SLDA Toolkit: A simulation platform for ultracold Fermi gases

Gabriel Wlazlowski, Piotr Magierski, Michael McNeil Forbes, Aurel Bulgac

arxiv:2602.08982

Time-Dependent Nuclear Energy-Density Functional Theory Toolkit for Neutron Star Crust: Dynamics of a Nucleus in a Neutron Superfluid

Daniel Pęcak^{1,2,*}, Agata Zdanowicz¹, Nicolas Chamel^{3,†}, Piotr Magierski^{1,4,‡}, and Gabriel Wlazlowski^{1,4,§}

Phys. Rev. X **14**, 041054 – Published 3 December, 2024