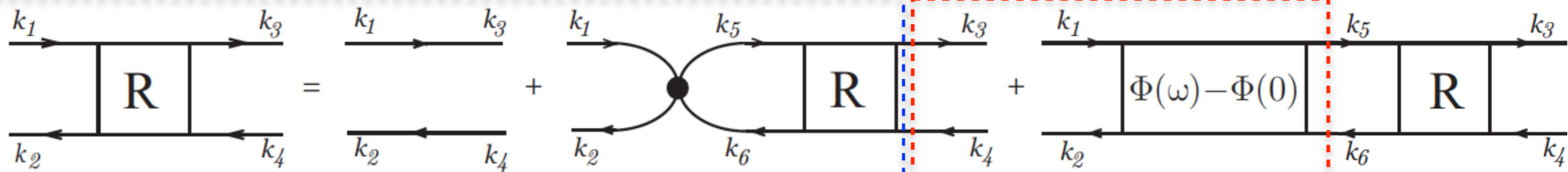


Nuclear response: toward a complete theory

toward ab initio $\Leftarrow$ static kernel V

dynamic kernel $\Rightarrow$ toward more complex configurations



Dyson-Bethe-Salpeter Equation:

$$R(\omega) = R^0(\omega) + R^0(\omega) [V + \Phi(\omega) - \Phi(0)] R(\omega)$$

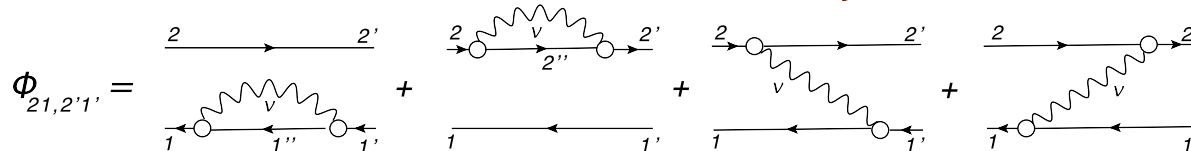
Subtraction
[V. Tselyaev 2013]

for effective interactions V
[P. Ring, A. Afanasjev, et al: CEDF]

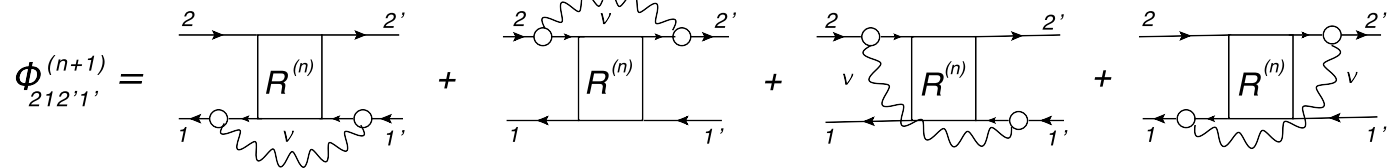
NFT is reproduced as the leading approximation
G. Còlo et al.]

QRPA/REOM¹

Beyond QRPA: REOMⁿ



Extended NFT: closed cycle



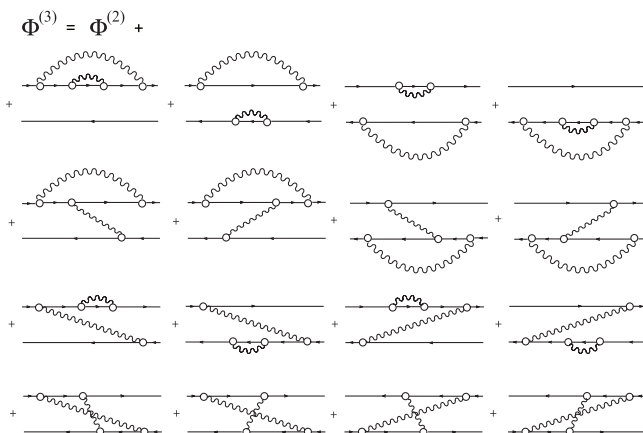
Generalized approach for the correlated propagators

n -th order: E.L. PRC 91, 034332 (2015)

Ab-initio formulation,

$\Phi^{(3)}$ implementation; $2q+2$ phonon correlations:

E.L., P. Schuck, PRC 100, 064320 (2019)

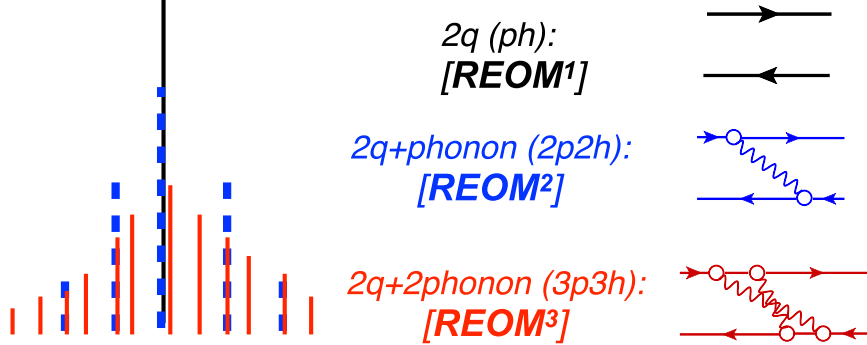


$2q+2$ phonon configurations in the qPVC expansion: **after two iterations**

Included in all orders (non-perturbatively)

Toward spectroscopic accuracy: correlated 3p3h beyond the leading qPVC

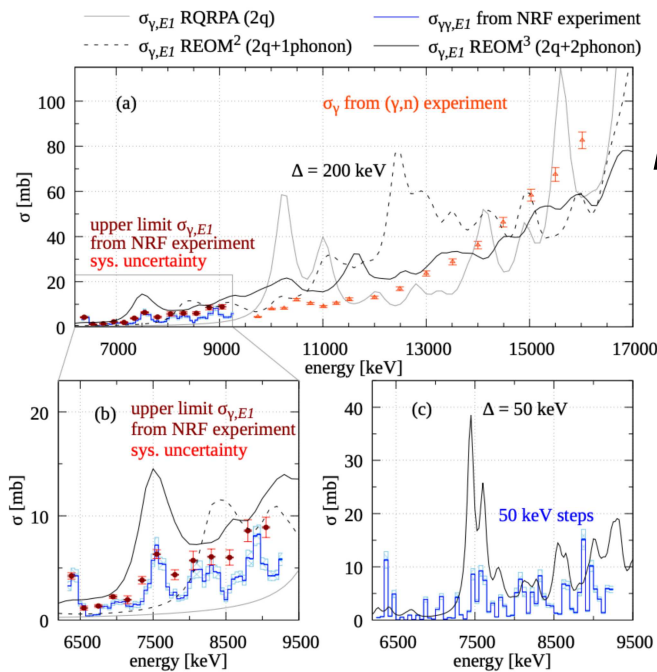
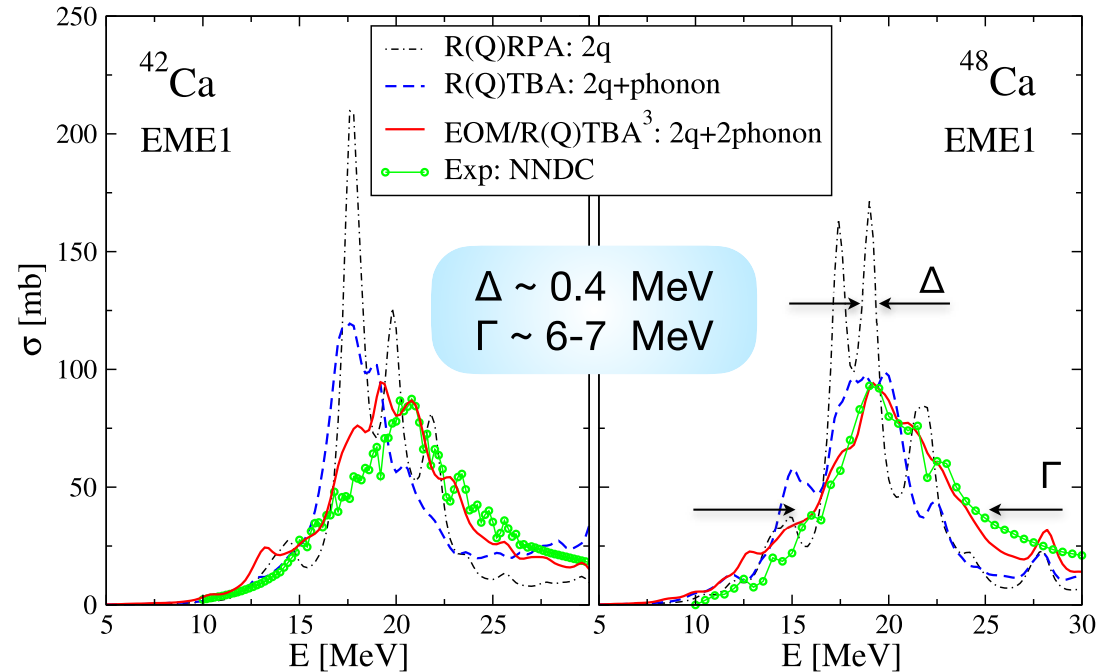
Fragmentation mechanism:
ordered by configuration complexity



In principle exact spectral expansion

$$R_{12,1'2'}(\omega) = \sum_{\nu>0} \left[\frac{\rho_{21}^{\nu} \bar{\rho}_{2'1'}^{\nu*}}{\omega - \omega_{\nu} + i\delta} - \frac{\bar{\rho}_{12}^{\nu*} \rho_{1'2'}^{\nu}}{\omega + \omega_{\nu} - i\delta} \right]$$

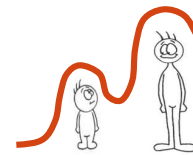
Giant dipole resonance in Ca: overlapping resonances



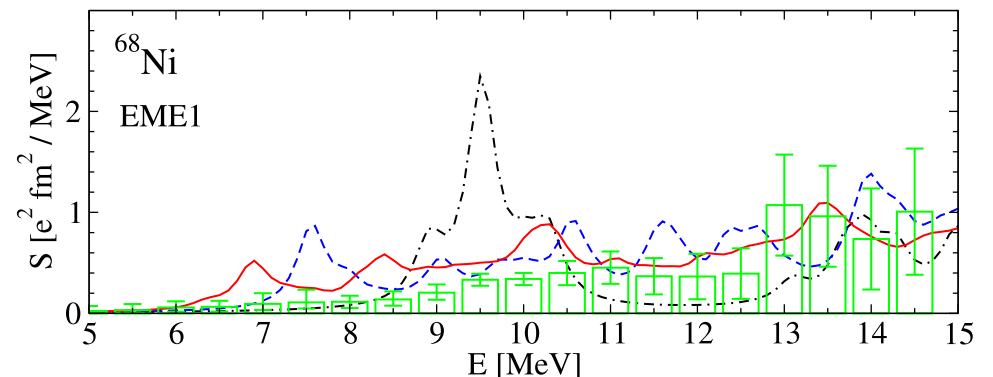
⁶⁴Ni: M. Müscher et al.,
PRC109, 044318 (2024)

$\beta_2 \sim 0.16$

New capability:
Description of
deformed systems
in spherical bases;
correlations “generate”
non-trivial geometry



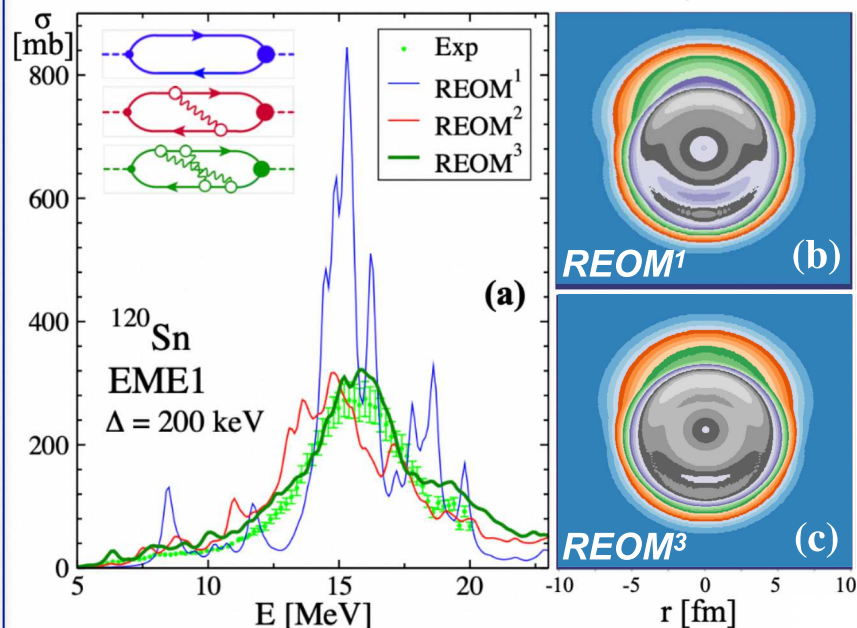
“Pygmy” dipole resonance in ⁶⁸Ni



Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)

Resolving long-term nuclear structure puzzles

Giant and soft resonances in heavy nuclei



- Accurate computation of nuclear excited states in a wide energy range, including giant resonances, is pushed to heavy systems.
- The recently developed *ab initio* relativistic equation of motion method (REOMⁿ) with non-perturbative dynamical kernels enables a *hierarchy of growing-complexity* (n) approximations to the nuclear response organized by the qPVC power counting based on emergent collectivity.
- We demonstrated that this *power counting* remains valid in heavy systems and provides a firm uncertainty quantification of the many-body theory.
- A significantly improved description is achieved for the Sn-Sm mass region for giant resonances and soft modes.

J. Novak, M. Hlatshwayo, E.L., arXiv:2405.02255
Exp. Data: S. Bassauer, PRC102, 034327 (2020)

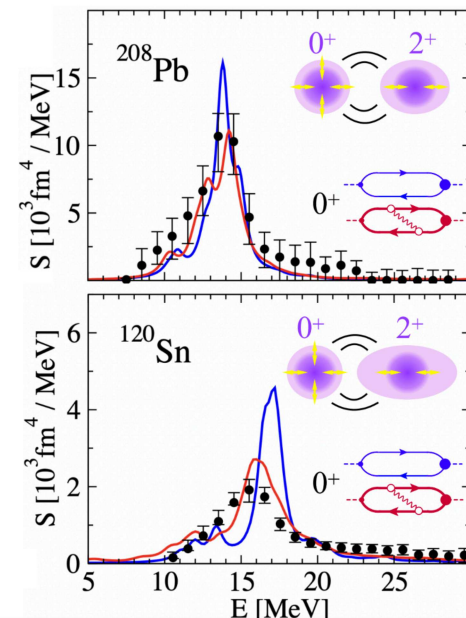
Giant monopole resonance and nuclear compressibility

A universal description of the monopole response across the nuclear chart is achieved. It is found that nuclear “softness” increases:

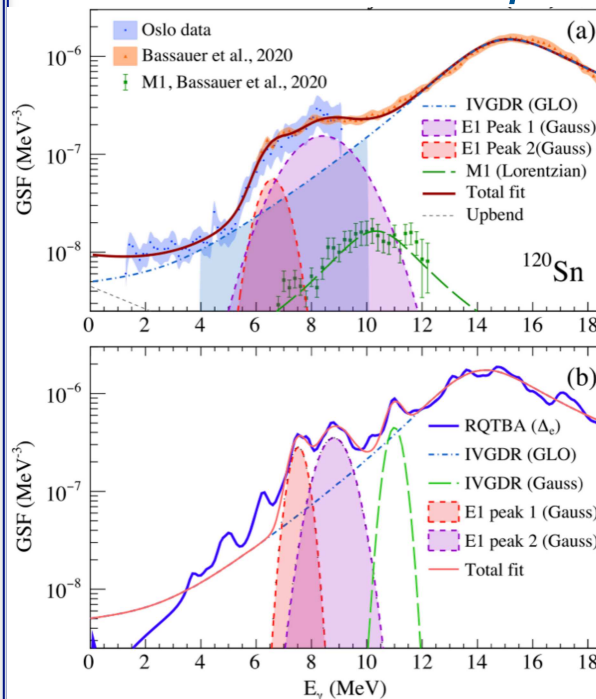
- with the neutron number
- with superfluidity manifested via large quadrupole collectivity
- with correlations beyond QRPA (q)PVC

Stiffer EOS can be used for nuclear matter computation

E. L., PRC **107**, L041302 (2023)

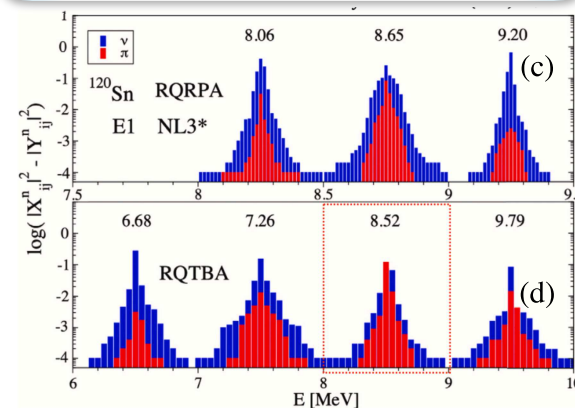


Giant and pygmy dipole resonances in Sn isotopes



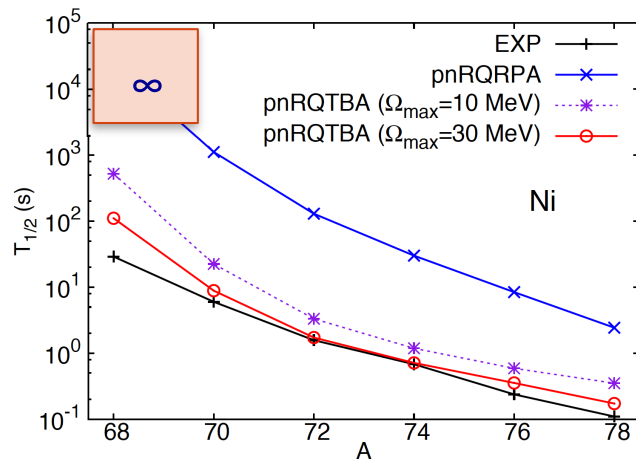
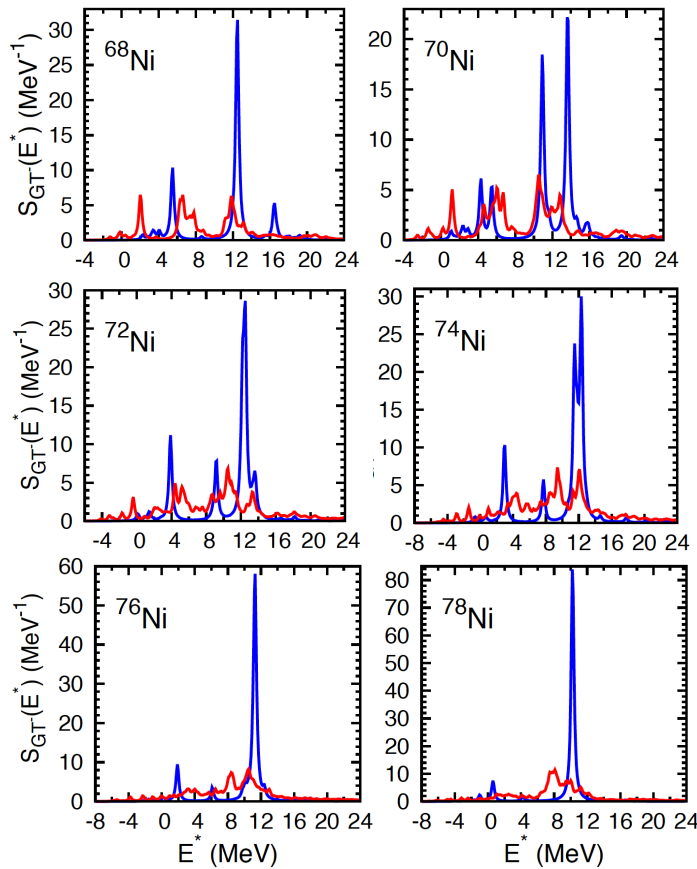
- qPVC correlations of RQTBA (REOM²) are necessary to reproduce the two-component structure of the low-energy dipole strength: (a), (b).
- Only the lower component is associated with the pygmy dipole resonance (PDR) and neutron skin.
- PDR is linked to the transitions from (to) the ground state; the upper structure is formed by transitions between excited states: (d) vs (c).

M. Markova, P. Von Neumann-Cosel, E. Litvinova, Phys. Lett. B860, 139216 (2025)

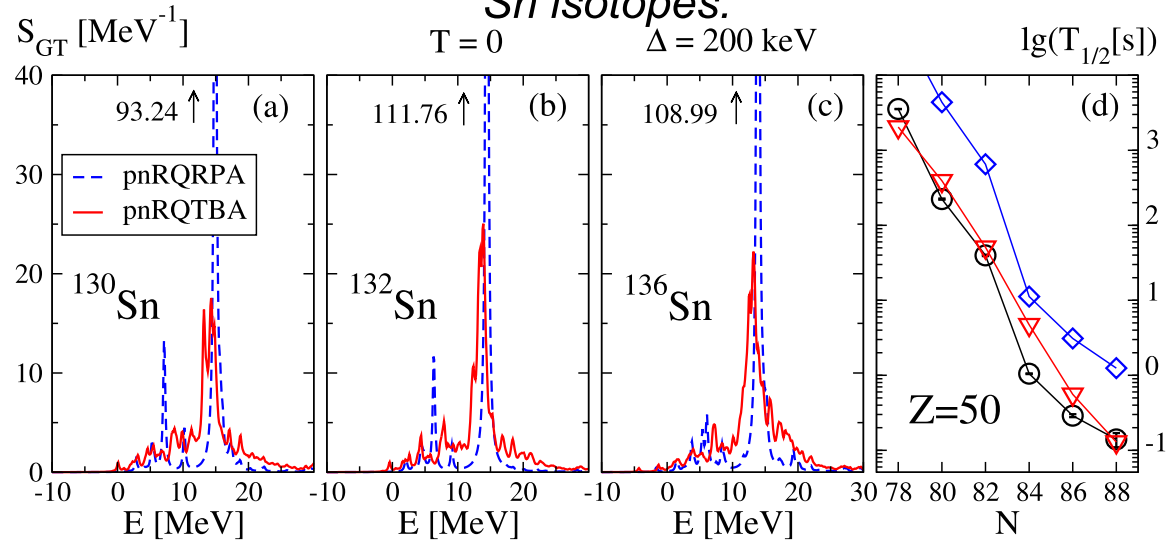


REOM²: charge-exchange resonances and beta decays

GTR in ⁶⁸⁻⁷⁸Ni isotopes:



Sn isotopes:



— *pn-RQRPA*
[REOM¹]
— *pn-RQTBA*
[REOM²]

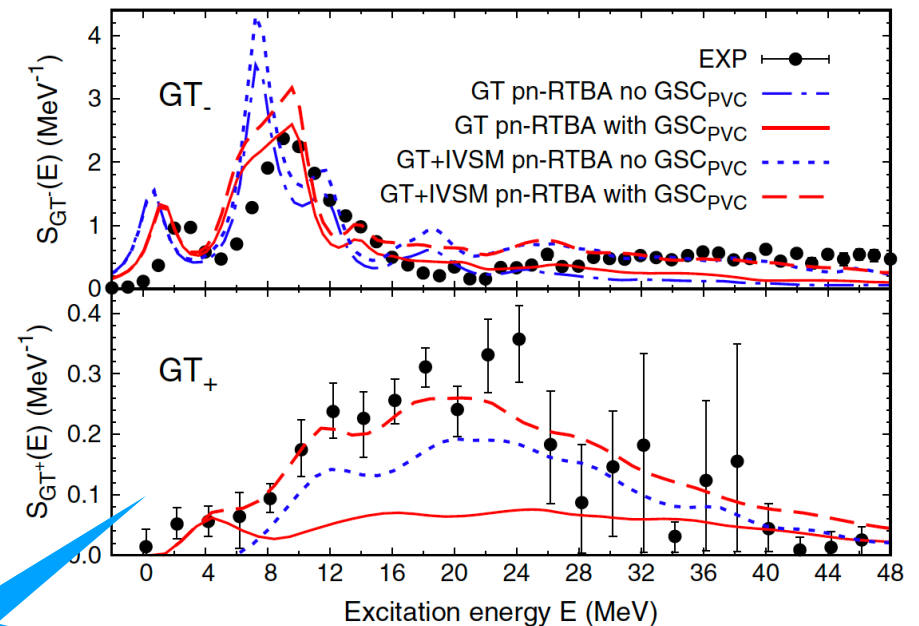
C. Robin, E.L., *EPJA*
52, 205 (2016)

E. L., C. Robin,
H. Wibowo, *PLB* 800,
135134 (2020)

C. Robin, E.L., *Phys.*
Rev. Lett. 123, 202501
(2019)

Complex GSC
added

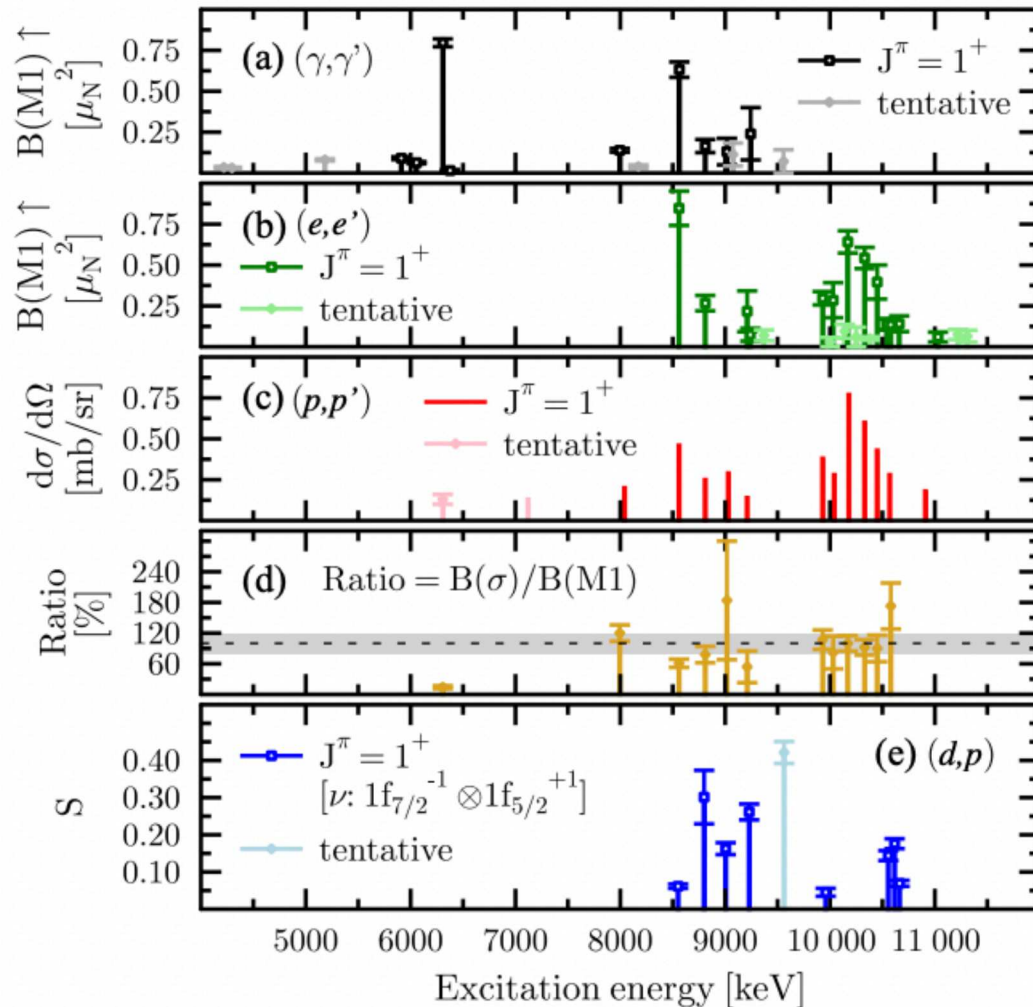
Gamow-Teller +IVSM strength in 90-Zr:



No fits, no artificial quenching,
no adjustable proton-neutron pairing

M1 transitions in ^{50}Ti : $^{49}\text{Ti}(d,p)^{50}\text{Ti}$ probe at FSU

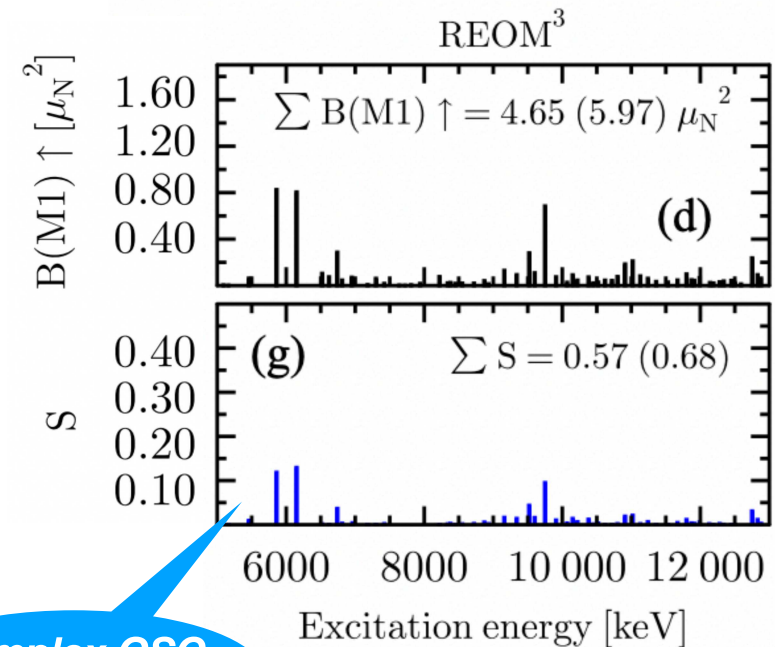
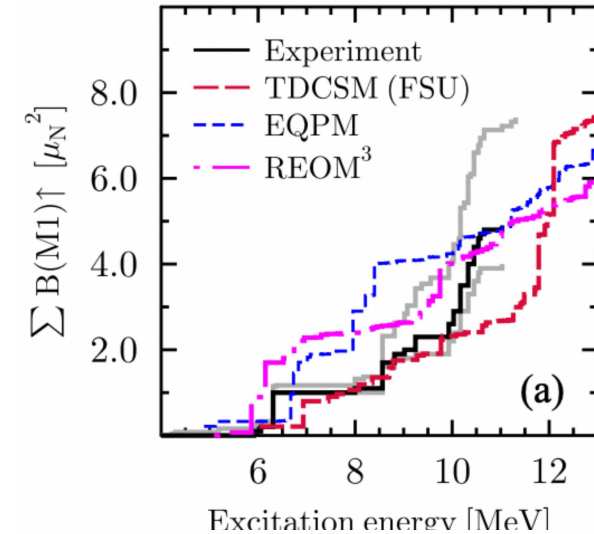
Combined M1 data



$^{49}\text{Ti}(d,p)^{50}\text{Ti}$ experiment: that 1^+ states with larger neutron $(1f_{7/2})^{-1}-(1f_{5/2})^{+1}$ spectroscopic factors S do not correspond to the ones with the largest $B(M1)$ strengths. Theory is in tension

B. Kelly, M. Spieker et al., PRL 136, 082502 (2026)

Total $B(M1)$



Complex GSC
to be included

Finite-temperature response with the $ph+phonon$ [REOM²] dynamical kernel

$$R_{12,1'2'}(t-t') = -i\langle \mathcal{T}(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t') \rangle \rightarrow -i\langle \mathcal{T}(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t') \rangle_T$$

non-superfluid

$$\langle \dots \rangle \equiv \langle 0|\dots|0 \rangle \rightarrow \langle \dots \rangle_T \equiv \sum_n \exp\left(\frac{\Omega - E_n - \mu N}{T}\right) \langle n|\dots|n \rangle$$

averages

thermal averages

**Method: EOM
for Matsubara
Green's functions**

E.L., H. Wibowo,
PRL 121, 082501 (2018)
H. Wibowo, E.L.,
PRC 100, 024307 (2019)

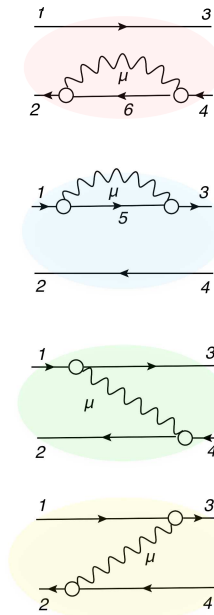
$$\begin{aligned} \mathcal{R}_{14,23}(\omega, T) &= \tilde{\mathcal{R}}_{14,23}^0(\omega, T) + \\ &+ \sum_{1'2'3'4'} \tilde{\mathcal{R}}_{12',21'}^0(\omega, T) [\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega, T)] \mathcal{R}_{3'4,4'3}(\omega, T) \\ \delta\Phi_{1'4',2'3'}(\omega, T) &= \Phi_{1'4',2'3'}(\omega, T) - \Phi_{1'4',2'3'}(0, T) \end{aligned}$$

$T > 0$:

Leading-order $1p1h+phonon$ dynamical kernel:

$T = 0$:

$$\begin{aligned} \Phi_{14,23}^{(ph)}(\omega, T) &= \frac{1}{n_{43}(T)} \sum_{\mu, \eta_\mu=\pm 1} \eta_\mu \left[\delta_{13} \sum_6 \gamma_{\mu;62}^{\eta_\mu} \gamma_{\mu;64}^{\eta_\mu*} \times \right. \\ &\times \frac{(N(\eta_\mu\Omega_\mu) + n_6(T))(n(\varepsilon_6 - \eta_\mu\Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_6 - \eta_\mu\Omega_\mu} + \\ &+ \delta_{24} \sum_5 \gamma_{\mu;15}^{\eta_\mu} \gamma_{\mu;35}^{\eta_\mu*} \times \\ &\times \frac{(N(\eta_\mu\Omega_\mu) + n_2(T))(n(\varepsilon_2 - \eta_\mu\Omega_\mu, T) - n_5(T))}{\omega - \varepsilon_5 + \varepsilon_2 - \eta_\mu\Omega_\mu} - \\ &- \gamma_{\mu;13}^{\eta_\mu} \gamma_{\mu;24}^{\eta_\mu*} \times \\ &\times \frac{(N(\eta_\mu\Omega_\mu) + n_2(T))(n(\varepsilon_2 - \eta_\mu\Omega_\mu, T) - n_3(T))}{\omega - \varepsilon_3 + \varepsilon_2 - \eta_\mu\Omega_\mu} - \\ &- \gamma_{\mu;31}^{\eta_\mu*} \gamma_{\mu;42}^{\eta_\mu} \times \\ &\times \left. \frac{(N(\eta_\mu\Omega_\mu) + n_4(T))(n(\varepsilon_4 - \eta_\mu\Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_4 - \eta_\mu\Omega_\mu} \right], \end{aligned}$$



$$\begin{aligned} \Phi_{14,23}^{(ph,ph)}(\omega) &= \sum_\mu \times \\ &\times \left[\delta_{13} \sum_6 \frac{\gamma_{62}^\mu \gamma_{64}^{\mu*}}{\omega - \varepsilon_1 + \varepsilon_6 - \Omega_\mu} + \right. \\ &+ \delta_{24} \sum_5 \frac{\gamma_{15}^\mu \gamma_{35}^{\mu*}}{\omega - \varepsilon_5 + \varepsilon_2 - \Omega_\mu} - \\ &- \frac{\gamma_{13}^\mu \gamma_{24}^{\mu*}}{\omega - \varepsilon_3 + \varepsilon_2 - \Omega_\mu} - \\ &\left. - \frac{\gamma_{31}^{\mu*} \gamma_{42}^\mu}{\omega - \varepsilon_1 + \varepsilon_4 - \Omega_\mu} \right] \end{aligned}$$

Finite-temperature superfluid formalism beyond QRPA

Theory is formulated in the HFB basis keeping 4x4 block matrix structure

superfluid

Correlated 2q-propagator
with Matsubara frequencies

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}(\omega_n) = \hat{\mathcal{R}}_{\mu\mu'\nu\nu'}^0(\omega_n) + \frac{1}{4} \sum_{\gamma\gamma'\delta\delta'} \hat{\mathcal{R}}_{\mu\mu'\gamma\gamma'}^0(\omega_n) \hat{\mathcal{K}}_{\gamma\gamma'\delta\delta'}(\omega_n) \hat{\mathcal{R}}_{\delta\delta'\nu\nu'}(\omega_n)$$

S. Bhattacharjee, E.L., arXiv:2412.20751,
to appear in PRC (2026)

$$\mathcal{R}_{\mu\mu'\nu\nu'}^{[lk]}(\omega) = \sum_{fi} \sum_{\sigma=\pm} \sigma p_i \frac{\mathcal{Z}_{\mu\mu'}^{if[l\sigma]} \mathcal{Z}_{\nu\nu'}^{if[k\sigma]*}}{\omega - \sigma\omega_{fi}}$$

Uncorrelated 2q-propagator

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}^0(\omega_n) = [\omega_n - \hat{\Sigma}_3 \hat{E}_{\mu\mu'}]^{-1} \hat{\mathcal{N}}_{\mu\mu'\nu\nu'}$$

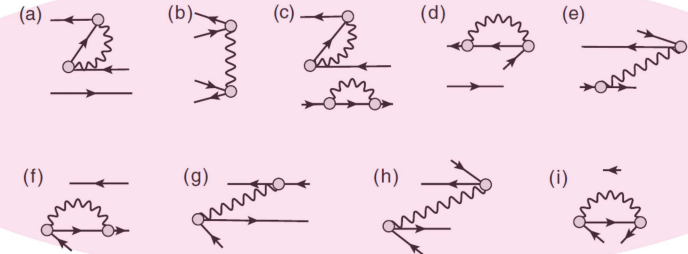
The norm matrix

$$\hat{\mathcal{N}}_{\mu\mu'\nu\nu'} = \left\langle \begin{pmatrix} [A_{\mu\mu'}, A_{\nu\nu'}^\dagger] & [A_{\mu\mu'}, A_{\nu\nu'}] & [A_{\mu\mu'}, C_{\nu\nu'}^\dagger] & [A_{\mu\mu'}, C_{\nu\nu'}] \\ [A_{\mu\mu'}^\dagger, A_{\nu\nu'}^\dagger] & [A_{\mu\mu'}^\dagger, A_{\nu\nu'}] & [A_{\mu\mu'}^\dagger, C_{\nu\nu'}^\dagger] & [A_{\mu\mu'}^\dagger, C_{\nu\nu'}] \\ [C_{\mu\mu'}, A_{\nu\nu'}^\dagger] & [C_{\mu\mu'}, A_{\nu\nu'}] & [C_{\mu\mu'}, C_{\nu\nu'}^\dagger] & [C_{\mu\mu'}, C_{\nu\nu'}] \\ [C_{\mu\mu'}^\dagger, A_{\nu\nu'}^\dagger] & [C_{\mu\mu'}^\dagger, A_{\nu\nu'}] & [C_{\mu\mu'}^\dagger, C_{\nu\nu'}^\dagger] & [C_{\mu\mu'}^\dagger, C_{\nu\nu'}] \end{pmatrix} \right\rangle$$

$$\langle O \rangle = \sum_i p_i \langle i | O | i \rangle \quad p_i = \frac{e^{-E_i/T}}{\sum_j e^{-E_j/T}}$$

The complete superfluid dynamical
qPVC kernel

Complex GSC: a source of “natural” quenching
at both $T=0$ and $T>0$

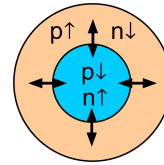
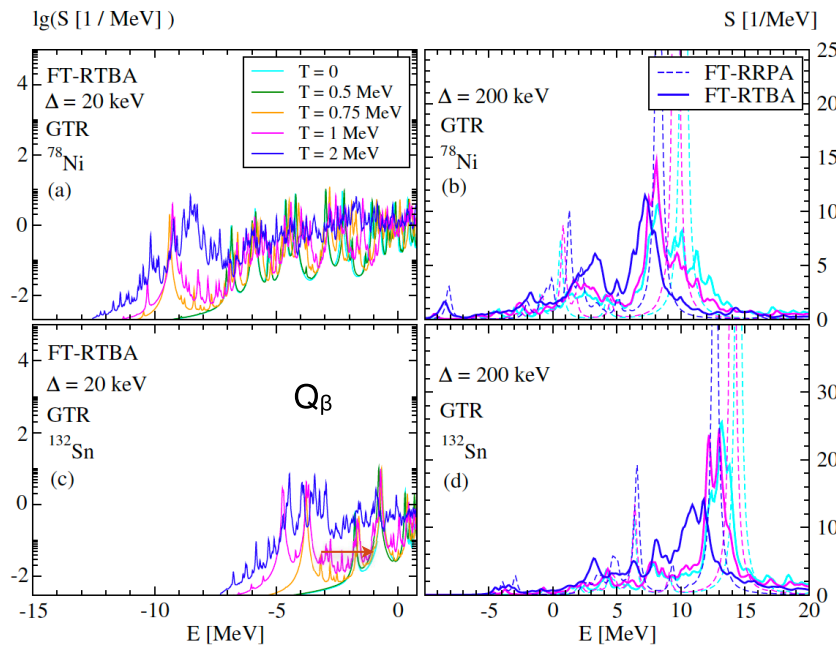


$$\mathcal{K}_{\mu\mu'\nu\nu'}^{r[11]cc(a)}(\omega_k) = \sum_{\gamma\delta n m} p_{i_n} p_{i_m} (1 - e^{-(\omega_n + \omega_m)/T}) \times \left[\frac{(\Gamma_{\mu\gamma}^{(11)n} \mathcal{X}_{\mu'\gamma}^m + \Gamma_{\mu\gamma}^{(02)\bar{n}*} \mathcal{U}_{\mu'\gamma}^m)(\mathcal{X}_{\nu'\delta}^{m*} \Gamma_{\nu\delta}^{(11)n*} + \mathcal{U}_{\nu'\delta}^{\bar{m}*} \Gamma_{\nu\delta}^{(02)\bar{n}})}{\omega_k - \omega_n - \omega_m} - \frac{(\Gamma_{\gamma\mu}^{(11)n*} \mathcal{Y}_{\mu'\gamma}^{m*} + \Gamma_{\gamma\mu}^{(02)\bar{n}} \mathcal{V}_{\mu'\gamma}^{\bar{m}*})(\mathcal{Y}_{\nu'\delta}^m \Gamma_{\delta\nu}^{(11)n} + \mathcal{V}_{\nu'\delta}^m \Gamma_{\delta\nu}^{(02)\bar{n}*})}{\omega_k + \omega_n + \omega_m} \right]$$



Spin-Isospin response and beta decay in stellar environments ($T > 0$)

Gamow-Teller GT response of ^{78}Ni and ^{132}Sn

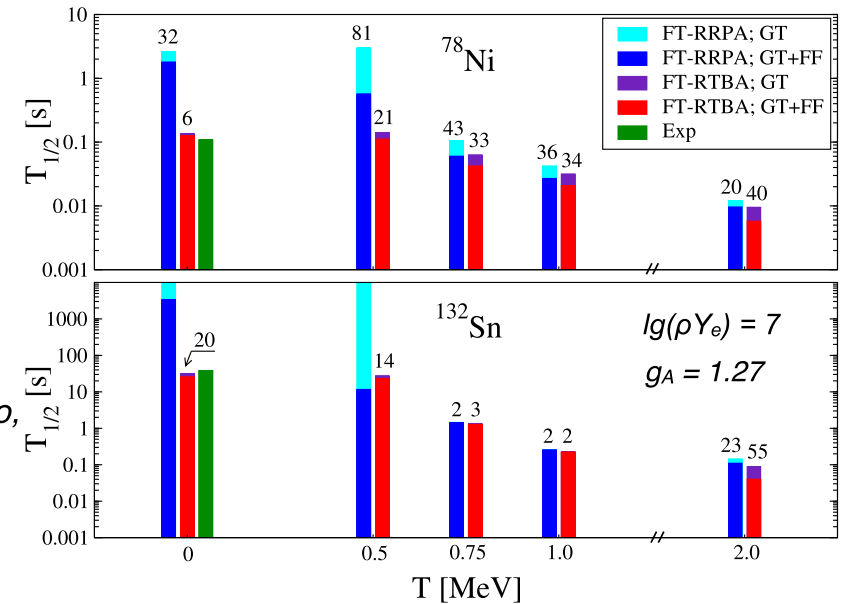


Thermal unblocking mechanism:
similar but with
proton-neutron pairs

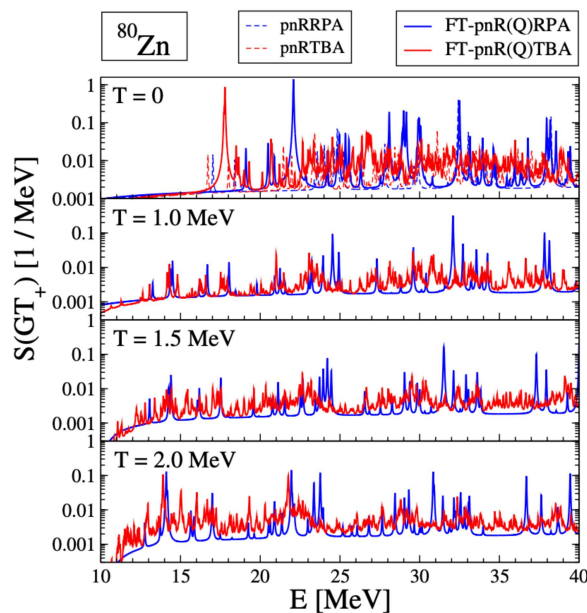
**Beta decay in r-process
at $T > 0$**

E. L., C. Robin, H. Wibowo,
PLB 800, 135134 (2020)

Beta decay half-lives in a stellar environment



GT+ response around ^{78}Ni



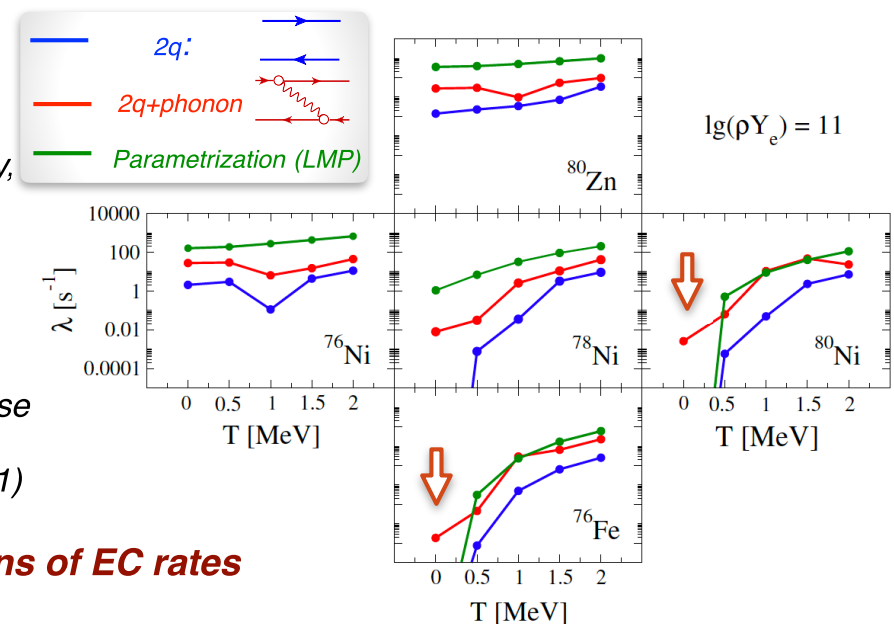
Interplay of superfluidity
and collective effects
in core-collapse supernovae:

- Amplifies the EC rates and, consequently,
- Reduces the electron-to-baryon ratio,
leading to lower pressure
- Promotes the gravitational collapse
- Increases the neutrino flux and effective
cooling
- Allows heavy nuclei to survive the collapse

E.L., C. Robin, PRC 103, 024326 (2021)

Ongoing: systematic calculations of EC rates

Electron capture rates around ^{78}Ni



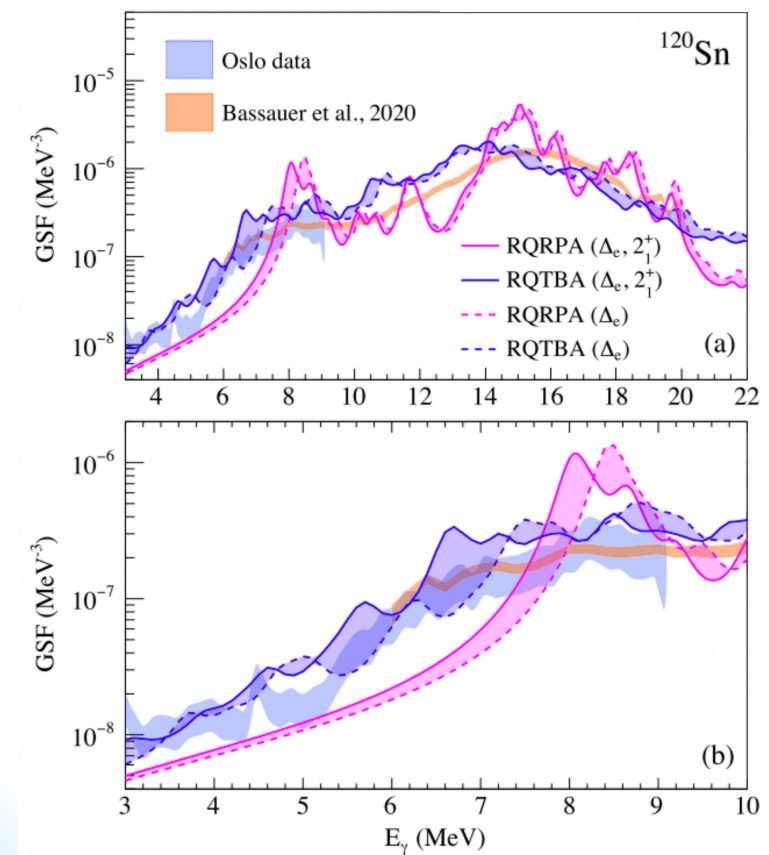
Problems with pairing correlations

- In the DFTs: pairing densities are the superfluid part of the mean field, and both are ill defined
- Ab initio: no quantitatively reasonable calculations

- ◆ The spectra are extremely sensitive to the pairing interaction, particularly at low energies
- ◆ Different fitting protocols lead to different spectral strength distributions

M. Markova, P. Von Neumann-Cosel,
E. L., Phys. Lett. B860, 139216 (2025)

Dipole strength in Sn isotopes



Exact “ab initio” theory for the two-fermion correlation function with an unspecified NN interaction: pp-channel

$$H = \sum_{12} \bar{\psi}_1 (-i\gamma \cdot \nabla + M)_{12} \psi_2 + \frac{1}{4} \sum_{1234} \bar{\psi}_1 \bar{\psi}_2 \tilde{v}_{1234} \psi_4 \psi_3 + \dots = T + V^{(2)} + \dots$$

Bare
(Relativistic)
Hamiltonian

Particle-particle correlator
two-time pair in-medium
propagator

$$G_{12,1'2'}(t - t') = -i \langle \mathcal{T}(\psi_1 \psi_2)(t) (\bar{\psi}_{2'} \bar{\psi}_{1'})(t') \rangle$$

**Superfluidity
and pair transfer**

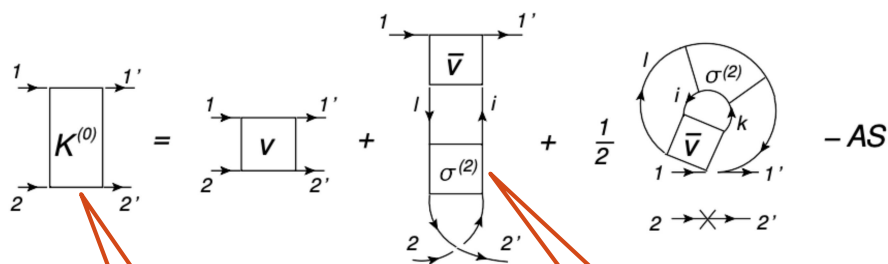
EOM: Bethe-Salpeter-(Dyson) Eq. ()**

$$G(\omega) = \tilde{G}^0(\omega) + \tilde{G}^0(\omega) K(\omega) G(\omega)$$

Irreducible kernel (exact): in-medium “pairing interaction”

$$K(t - t') = K^{(0)} \delta(t - t') + K^{(r)}(t - t')$$

Instantaneous term (“bosonic” mean field):
Short-range correlations



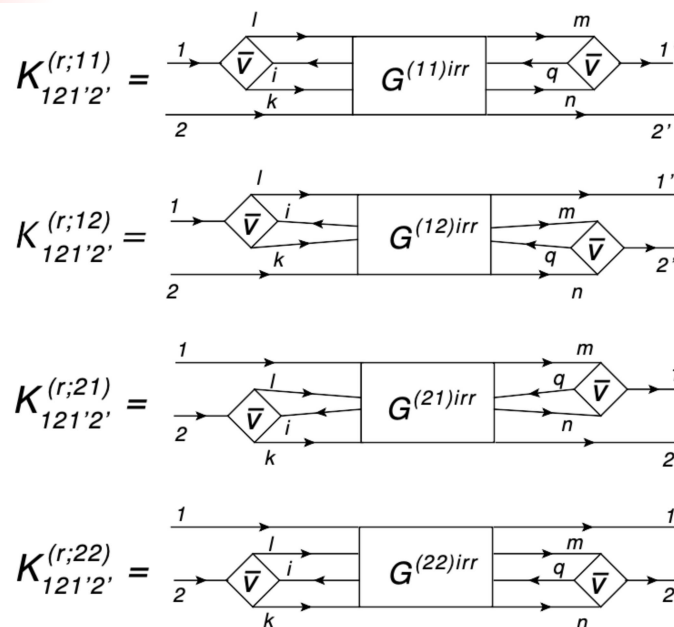
~ “Effective
pairing
interaction”

Correlated part
of the two-body
density

$$\rho_{ijkl} = \rho_{ik}\rho_{jl} - \rho_{il}\rho_{jk} + \sigma_{ijkl}^{(2)}$$

contains the full solution of (**) **including the dynamical term!**

t -dependent (dynamical) term: **Long-range correlations**: Couples to higher-rank FCFs



**Symmetric
form:**

P. Schuck,
J. Dukelsky,
S. Adachi, et al.

**Non-Symmetric
form: see**

J. Schwinger,
F. Dyson,
et al.

E.L. & P. Schuck, PRC 102, 034310 (2020)
E.L. & P. Schuck, PRC 104, 044330 (2021)

Factorization, qPVC mapping and the gap equation

Fourier transform
(spectral image)

$$iG_{12,1'2'}(\omega) = \sum_{\mu} \frac{\alpha_{21}^{\mu} \alpha_{2'1'}^{\mu*}}{\omega - \omega_{\mu}^{(++)} + i\delta} - \sum_{\pi} \frac{\beta_{12}^{\pi*} \beta_{1'2'}^{\pi}}{\omega + \omega_{\pi}^{(--)} - i\delta}$$

$$K_{121'2'}^{(r;11)} = \begin{array}{c} \text{Diagram 1: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 2: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 3: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 4: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array} + \begin{array}{c} \text{Diagram 5: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 6: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array}$$

$$= \begin{array}{c} \text{Diagram 7: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 8: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array} + \begin{array}{c} \text{Diagram 9: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 10: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array}$$

Cluster decomposition:

$$\diamond G(4) = G(1) G(1) G(1) G(1) + G(2) G(2) + G(3) G(1) + \Xi(4)$$

E.L., P.Schuck, PRC 102, 034310 (2020)

qPVC mapping:

$$\begin{array}{c} \text{Diagram 1: } \text{Wavy line} \\ \text{Diagram 2: } \text{Double line} \end{array} = \begin{array}{c} \text{Diagram 3: } \bar{V} \xrightarrow{l} i \xrightarrow{k} R^{(ph)} \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 4: } V \xrightarrow{l} i \xrightarrow{k} G^{(pp)} \xrightarrow{m} V \xrightarrow{q} 1' \end{array}$$

$$K_{121'2'}^{(r;12)} = \begin{array}{c} \text{Diagram 1: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 2: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 3: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 4: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array} + \begin{array}{c} \text{Diagram 5: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 6: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array}$$

$$= \begin{array}{c} \text{Diagram 7: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 8: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array} + \begin{array}{c} \text{Diagram 9: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} R \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \\ \text{Diagram 10: } 1 \rightarrow \bar{V} \xrightarrow{l} i \xrightarrow{k} G \xrightarrow{m} \bar{V} \xrightarrow{q} 1' \end{array}$$

Pairing gap: $\Delta_1 = 2E_1 \alpha_{11}^s$

$$E_1 = \sqrt{(\tilde{\varepsilon}_1 - \lambda)^2 + \Delta_1^2}$$

The Gap equation:

$$\Delta_1 = - \sum_2 \mathcal{V}_{1\bar{1}2\bar{2}} \frac{\Delta_2}{2E_2}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2} \left(K_{121'2'}^{(0)} + K_{121'2'}^{(r)}(2\lambda) \right)$$

Microscopic formalism at $T>0$: the pairing channel

Averages redefined:

$$\mathcal{G}(12, 1'2') \equiv \mathcal{G}_{12,1'2'}(\tau - \tau') = -\langle T_\tau \psi(1)\psi(2)\bar{\psi}(2')\bar{\psi}(1') \rangle$$

Grand Canonical average:

$$\langle O \rangle = \sum_{\nu} w_{\nu} \langle \nu | O | \nu \rangle$$

Wick rotated:

$$\begin{aligned} \psi(1) &\equiv \psi_1(\tau_1) = e^{\mathcal{H}\tau_1} \psi_1 e^{-\mathcal{H}\tau_1}, \\ \bar{\psi}(1) &\equiv \bar{\psi}_1(\tau_1) = e^{\mathcal{H}\tau_1} \psi_1^\dagger e^{-\mathcal{H}\tau_1}. \end{aligned}$$

Matsubara imaginary-time formalism: temperature-dependent dynamical kernel: E.L. & P. Schuck, PRC 104, 044330 (2021)

Direct:

$$\begin{aligned} \mathcal{K}_{121'2'}^{(r;11)}(\omega_n) &= - \sum_{\nu'\nu''} w_{\nu'} w_{\nu''} \\ &\times \left[\sum_{\nu\mu} \frac{\Theta_{121'2'}^{\mu\nu;\nu'\nu''(+)} }{i\omega_n - \omega_{\nu\nu'} - \omega_{\mu\nu''}^{(++)}} (e^{-(\omega_{\nu\nu'} + \omega_{\mu\nu''}^{(++)})/T} - 1) \right. \\ &\left. - \sum_{\nu\kappa} \frac{\Theta_{121'2'}^{\kappa\nu;\nu'\nu''(-)} }{i\omega_n + \omega_{\nu\nu'} + \omega_{\kappa\nu''}^{(--)}} (e^{-(\omega_{\nu\nu'} + \omega_{\kappa\nu''}^{(--)})/T} - 1) \right] \end{aligned}$$

Exchange:

$$\begin{aligned} \mathcal{K}_{121'2'}^{(r;12)}(\omega_n) &= \sum_{\nu'\nu''} w_{\nu'} w_{\nu''} \\ &\times \left[\sum_{\nu\mu} \frac{\Sigma_{121'2'}^{\mu\nu;\nu'\nu''(+)} }{i\omega_n - \omega_{\nu\nu'} - \omega_{\mu\nu''}^{(++)}} (e^{-(\omega_{\nu\nu'} + \omega_{\mu\nu''}^{(++)})/T} - 1) \right. \\ &\left. - \sum_{\nu\kappa} \frac{\Sigma_{121'2'}^{\kappa\nu;\nu'\nu''(-)} }{i\omega_n + \omega_{\nu\nu'} + \omega_{\kappa\nu''}^{(--)}} (e^{-(\omega_{\nu\nu'} + \omega_{\kappa\nu''}^{(--)})/T} - 1) \right], \end{aligned}$$

E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

BCS-like gap Eq., but with non-trivial T -dependence in $K^{(r)}$:

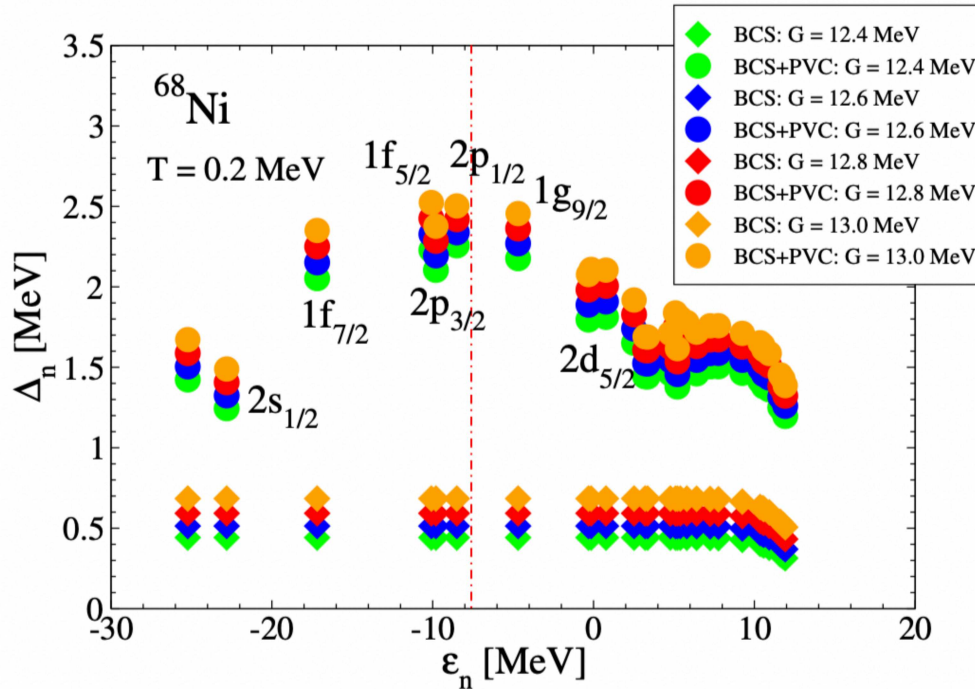
$$\Delta_1(T) = - \sum_2 \mathcal{V}_{1\bar{1}2\bar{2}} \frac{\Delta_2(T)(1 - 2f_2(T))}{2E_2}$$

$$f_1(T) = \frac{1}{\exp(E_1/T) + 1}$$

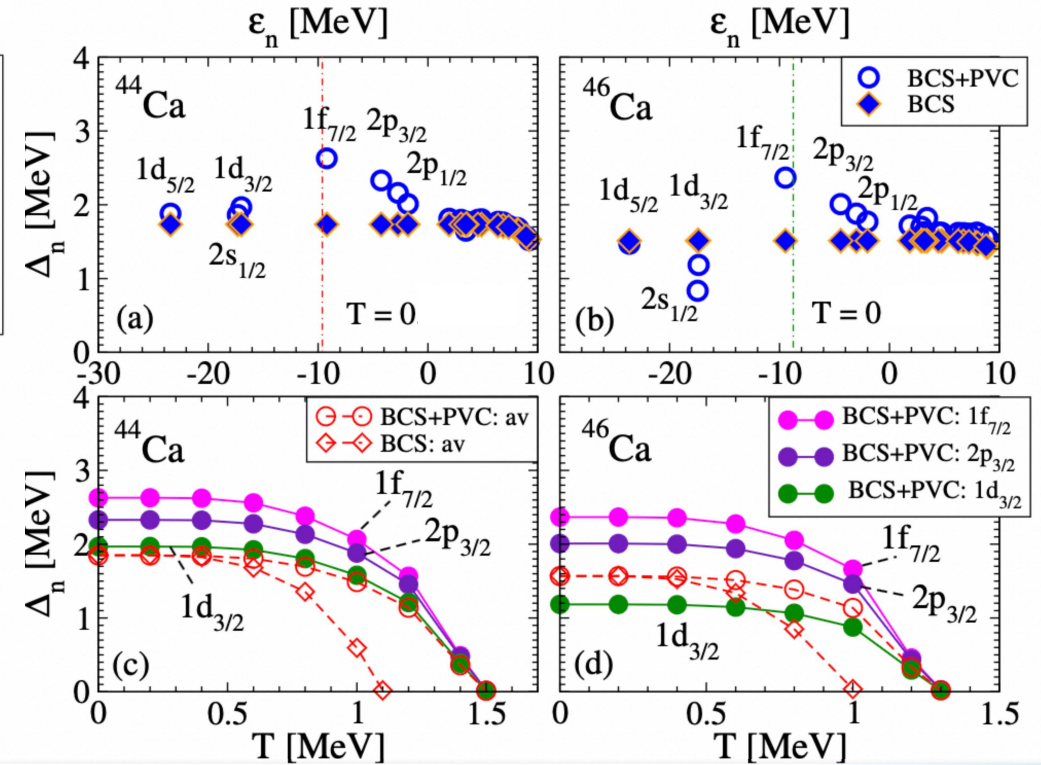
$$\mathcal{V}_{121'2'} = \frac{1}{2} \left(K_{121'2'}^{(0)} + K_{121'2'}^{(r)}(2\lambda) \right)$$

Dynamical qPVC in the pairing gap at $T = 0$, $T > 0$ and critical temperature

Dynamical pairing fraction: ^{68}Ni



Dynamical pairing fraction: $^{44,46}\text{Ca}$



Pairing gap:

$$\Delta_1(T) = \alpha_{11}^s \frac{2E_1}{1 - 2f_1(T)}$$

$$f_1(T) = \frac{1}{\exp(E_1/T) + 1}$$

BCS-like gap Eq., but with non-trivial T -dependence in $K^{(r)}$:

$$\Delta_1(T) = - \sum_2 \mathcal{V}_{1\bar{1}2\bar{2}} \frac{\Delta_2(T)(1 - 2f_2(T))}{2E_2}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2} \left(K_{121'2'}^{(0)} + K_{121'2'}^{(r)}(2\lambda) \right)$$

Proton-neutron "pairing"

Underlying scale: QHD

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \quad \text{A. Bouyssy et al., PRC 36, 380 (1989)}$$

$$\begin{aligned} \mathcal{L}_0 = & \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ & + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu \\ & - \frac{1}{4}\mathbf{G}_{\mu\nu}\cdot\mathbf{G}^{\mu\nu} + \frac{1}{2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}^2) - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_I = & -g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi + \frac{f_\omega}{2M}\bar{\psi}\sigma_{\mu\nu}\partial^\nu\omega^\mu\psi \\ & - g_\rho\bar{\psi}\gamma_\mu\boldsymbol{\rho}^\mu\cdot\boldsymbol{\tau}\psi + \frac{f_\rho}{2M}\bar{\psi}\sigma_{\mu\nu}\partial^\nu\boldsymbol{\rho}^\mu\cdot\boldsymbol{\tau}\psi \\ & - e\bar{\psi}\gamma_\mu\frac{1}{2}(1+\tau_3)A^\mu\psi + \mathcal{L}_{\pi NN}, \end{aligned}$$

Δ and η mesons are weak but can be included (present in Bonn interactions)

PS πN coupling

$$\mathcal{L}_{\pi NN}^{\text{PS}} = -ig_\pi\bar{\psi}\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau}\psi$$

PV πN coupling

$$\mathcal{L}_{\pi NN}^{\text{PV}} = -\frac{f_\pi}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau}\psi$$

Non-relativistic reduction of the t-components:

σ and $\omega \iff T=1$ pairing

$$\begin{aligned} (2E_\omega)^{1/2}\langle {}^\omega_N(p_f) | \mathcal{L}_I^\omega | N(p_i) \rangle \\ = g_\omega\chi_f^\dagger \left[1 - \frac{(\mathbf{p}_f + \mathbf{p}_i)^2}{8M^2} \pm i\boldsymbol{\sigma}\cdot\frac{\mathbf{p}_f \times \mathbf{p}_i}{4M^2} \right] \chi_i, \end{aligned}$$

$$\sim (1/M_N)^0$$

π and $\rho \iff T=0$ pairing

$$\begin{aligned} \sqrt{2E_\pi}\langle {}^\pi_N(p_f) | \mathcal{L}_I^\pi | N(p_i) \rangle &= ig_\pi\chi_f^\dagger \frac{\boldsymbol{\sigma}\cdot(\mathbf{p}_f - \mathbf{p}_i)}{2M}\chi_i, \\ \sqrt{2E_\rho}\langle {}^\rho_N(p_f) | \mathcal{L}_I^\rho | N(p_i) \rangle &= ig_\rho\chi_f^\dagger \boldsymbol{\epsilon}\cdot\frac{\boldsymbol{\sigma} \times (\mathbf{p}_f - \mathbf{p}_i)}{2M}\chi_i, \end{aligned}$$

$$\sim (1/M_N)^1 \text{ for both PS and PV}$$

**The static proton-neutron coupling is suppressed: no pn-condensate
Some dynamical pn-pairing emerges aligning with the qPVC scale**

Isospin transfer response function: proton-neutron particle-particle REOM² (pn-pp-RTBA)

Response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega) \bar{W}(\omega) R(\omega)$$

Interaction
kernel

$$\bar{W}(\omega) = V_\rho + V_\pi + V_{\delta\pi} + \underbrace{\Phi(\omega) - \Phi(0)}_{\text{Subtraction to avoid double counting of } p \text{ (if CDFT-based)}}$$

Static:
R(Q)RPA

$$\left\{ \begin{array}{l} V_\rho(1, 2) = g_\rho^2 \vec{\tau}_1 \vec{\tau}_2 (\beta \gamma^\mu)_1 (\beta \gamma_\mu)_2 D_\rho(\mathbf{r}_1, \mathbf{r}_2) \\ V_\pi(1, 2) = - \left(\frac{f_\pi}{m_\pi} \right)^2 \vec{\tau}_1 \vec{\tau}_2 (\boldsymbol{\Sigma}_1 \nabla_1) (\boldsymbol{\Sigma}_2 \nabla_2) D_\pi(\mathbf{r}_1, \mathbf{r}_2), \\ V_{\delta\pi}(1, 2) = g' \left(\frac{f_\pi}{m_\pi} \right)^2 \vec{\tau}_1 \vec{\tau}_2 \boldsymbol{\Sigma}_1 \boldsymbol{\Sigma}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) \end{array} \right.$$

Subtraction
to avoid double
counting of p
(if CDFT-based)

free-space
coupling

fixed strength:
ab initio
if the Fock term
is present

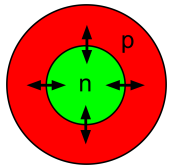
Dynamic
(retardation),
2-nd order:

quasiparticle-
vibration
coupling

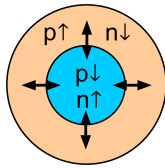
in time blocking
approximation

$$\begin{aligned} \Phi_{p_1 n_2, p_3 n_4}^\eta(\omega) = & \\ = \eta \sum_\mu & \left[\delta_{p_1 p_3} \sum_{n_6} \frac{\gamma_{\mu; n_2 n_6}^\eta \gamma_{\mu; n_4 n_6}^{\eta*}}{\omega - \varepsilon_{p_1} - \varepsilon_{n_6} - \eta \Omega_\mu} + \delta_{n_2 n_4} \sum_{p_5} \frac{\gamma_{\mu; p_1 p_5}^\eta \gamma_{\mu; p_3 p_5}^{\eta*}}{\omega - \varepsilon_{p_5} - \varepsilon_{n_2} - \eta \Omega_\mu} \right. \\ & \left. - \left(\frac{\gamma_{\mu; p_1 p_3}^\eta \gamma_{\mu; n_4 n_2}^{\eta*}}{\omega - \varepsilon_{p_3} - \varepsilon_{n_2} - \eta \Omega_\mu} + \frac{\gamma_{\mu; p_3 p_1}^{\eta*} \gamma_{\mu; n_2 n_4}^\eta}{\omega - \varepsilon_{p_1} - \varepsilon_{n_4} - \eta \Omega_\mu} \right) \right] \end{aligned}$$

Response in proton-neutron particle-particle (deuteron transfer) channel: quest for deuteron condensate and pn-pairing



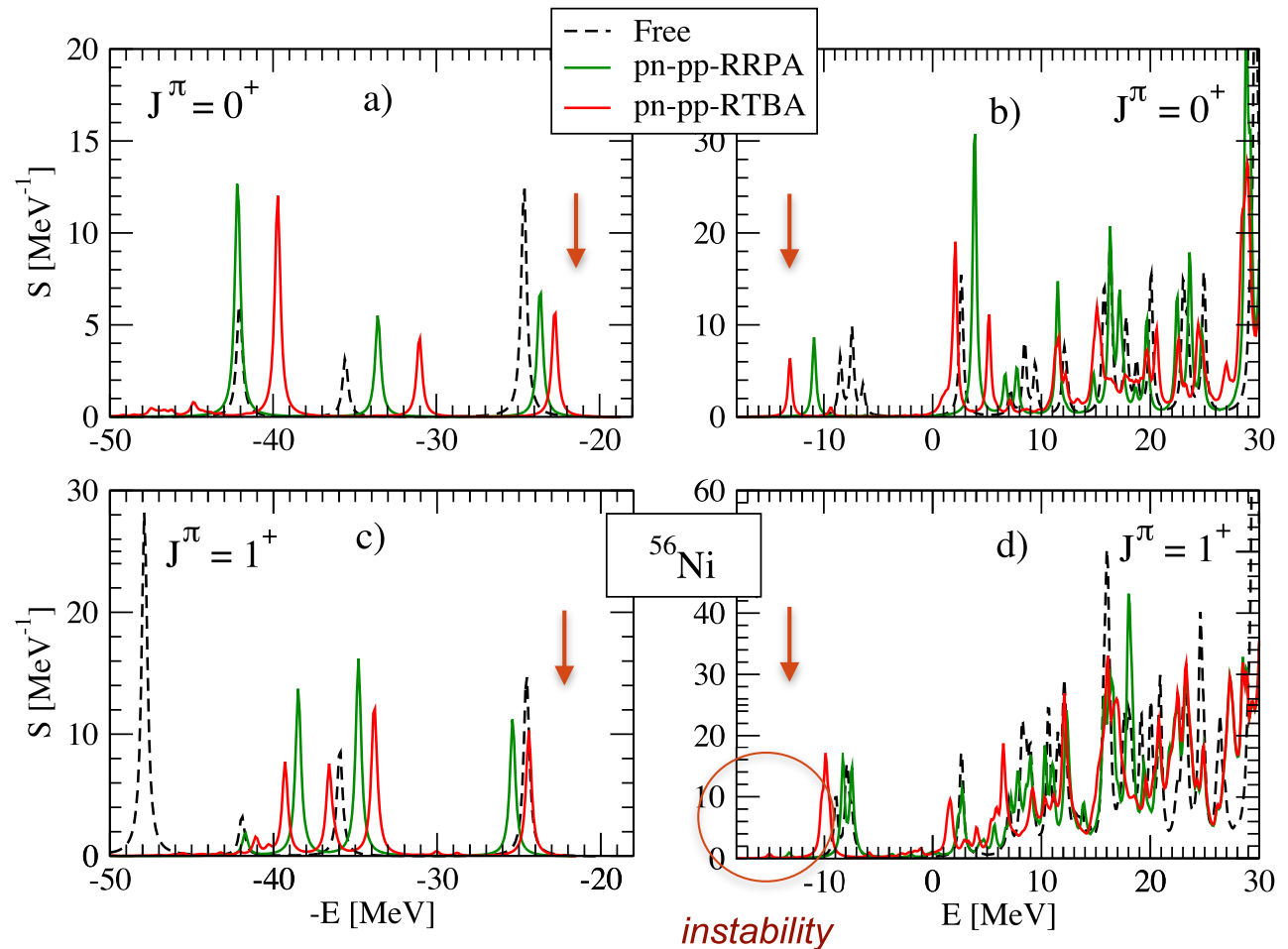
$$J^\pi = 0^+$$



$$J^\pi = 1^+$$

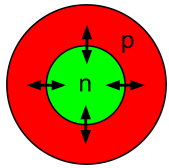


- The pairing interaction in the proton-neutron channel is a delicate interplay of the ρ -meson and π exchanges, and the exchange by core vibrations
- In the odd-odd $N=Z$ nuclei around closed shells the lowest 0^+ states are accurately described

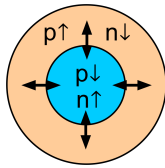


- The lowest 1^+ solutions in the addition channel become unstable, which may indicate the onset of the deuteron condensate.
- The particle-vibration coupling provides an overall attractive interaction and, thus, slightly reinforces the condensate formation

Response in proton-neutron particle-particle (deuteron transfer) channel: quest for deuteron condensate and pn-pairing

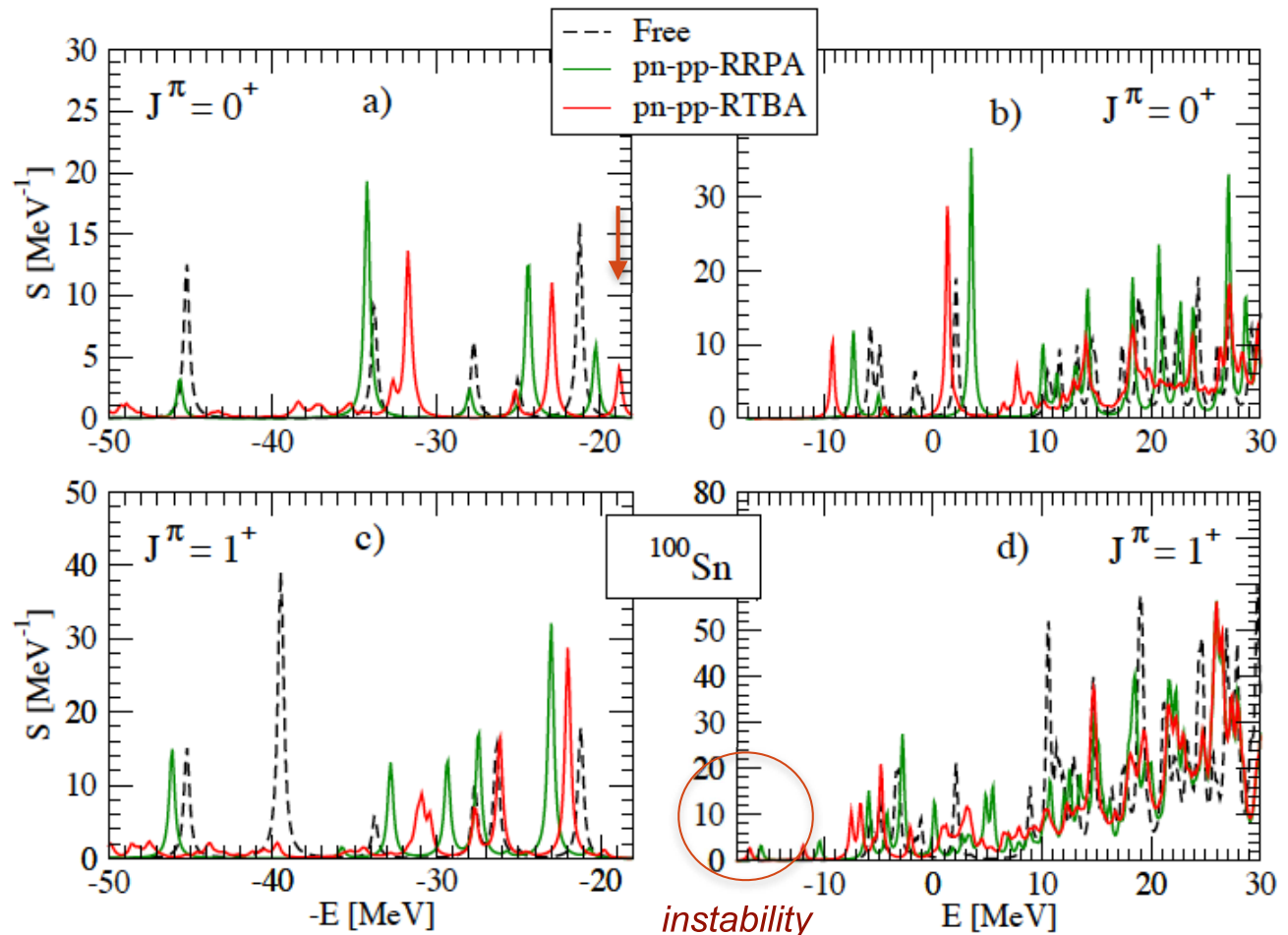


$$J^\pi = 0^+$$



$$J^\pi = 1^+$$

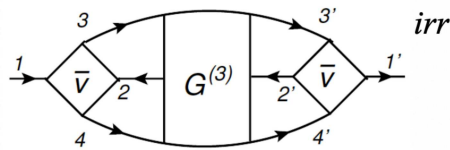
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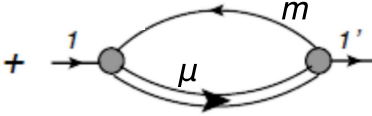
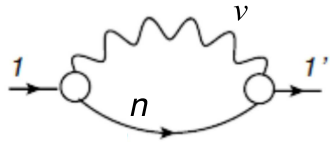
- The lowest 1^+ solutions in the addition channel become unstable, which may indicate the onset of the deuteron condensate.
- The particle-vibration coupling provides an overall attractive interaction and, thus, slightly reinforces the condensate formation

Problems with approximate treatments: poles “mismatch”, (non)-positivity and optical theorem

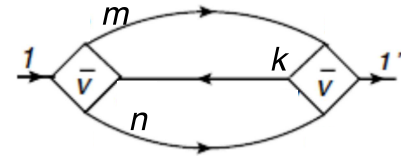
Dynamical self-energy:



“Radiative-correction”



“Second-order”



Approximate:

$$\Sigma_{11'}^{(r)+}(\omega) = \sum_{33'} \sum_{\nu n} \frac{\eta_3^n g_{13}^\nu g_{1'3'}^{\nu*} \eta_{3'}^{n*}}{\omega - \omega_\nu - \varepsilon_n^{(+)} + i\delta} + \sum_{22'} \sum_{\mu m} \frac{\chi_2^{m*} \gamma_{12}^{\mu(+)} \gamma_{1'2'}^{\mu(+)*} \chi_2^m}{\omega - \omega_\mu^{(++)} - \varepsilon_m^{(-)} + i\delta} - \sum_{mnk} \frac{w_1^{mnk} w_{1'}^{mnk*}}{\omega - \varepsilon_m^+ - \varepsilon_n^+ - \varepsilon_k^- + i\delta}$$

Exact:

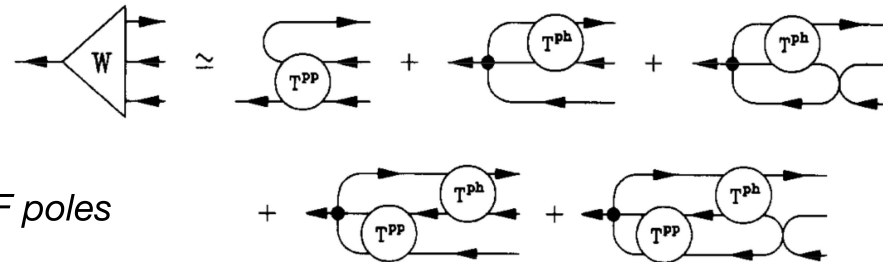
$$\Sigma_{11'}^{(r)+}(\omega) \sim \langle v G^{(3)+}(\omega) v \rangle_{11'}^{irr}$$

• Watson-Faddeev series:

P. Danielewicz and P. Schuck, Nucl.

Phys. A567, 78 (1994):

$$G_{432',23'4'}^{(3)+}(\omega) = \sum_{\kappa} \frac{\langle 0 | \bar{\psi}_2 \psi_4 \psi_3 | \kappa \rangle \langle \kappa | \bar{\psi}_3' \bar{\psi}_4' \psi_{2'} | 0 \rangle}{\omega - \omega_\kappa + i\delta}^{irr}$$



• 2p1h-RPA

P. Schuck, F. Villars and P. Ring

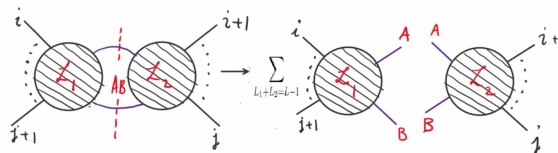
Nucl. Phys. A208, 302 (1973)

Reducible terms: unperturbed GF poles

Scattering amplitudes in particle physics
(Yang-Mills theories etc.):

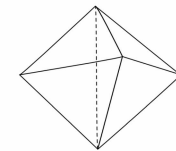
- Positivity preserved when eliminating all the virtual particles
- Amplitudes $\Leftrightarrow$ Polyhedron “living” in the kinematic space
- Emergent unitarity

Idea: eliminating virtual particles



N. Arkani-Hamed, J. Trnka,
A. Hodges et al.

Amplitude is a
volume of
polyhedron



Each face
labeled by
 $\langle abcd \rangle$

$$\frac{\langle 1345 \rangle^3}{\langle 1234 \rangle \langle 1245 \rangle \langle 2345 \rangle \langle 1235 \rangle} \quad \frac{\langle 1356 \rangle^3}{\langle 1235 \rangle \langle 1256 \rangle \langle 2356 \rangle \langle 1236 \rangle}$$



Outlook

Summary:

- The **emergent collective effects** associated with the dynamical kernels of the fermionic EOMs renormalize interactions in correlated media, underlie the spectral fragmentation mechanisms, and affect superfluidity and weak decay rates
- A hierarchy of converging growing-complexity approximations generates **solutions of growing accuracy and quantifies the uncertainties** of the many-body theory
- The recently enabled capabilities are **higher configuration complexity, finite temperature, and transitions between excited states**. The notion of a multicomponent PDR* has been introduced

Open theoretical problems:

- Correct separation of and delicate relationship between the static and dynamical kernels in the practical approximate solutions.
- Artificial poles and virtual “particles” in approximate solutions.
- Consistency between direct and pairing channels in the dynamical kernels in practical implementations.
- Ambiguities of numerical implementations on the 2p2h and 3p3h levels of complexity.
- An adequate assessment of the quantitative role of fully connected multi-fermion correlators (~ irreducible three- and many-body “forces” in the medium).

Open practical problem:

MORE QUALIFIED MANPOWER IS NEEDED!!!





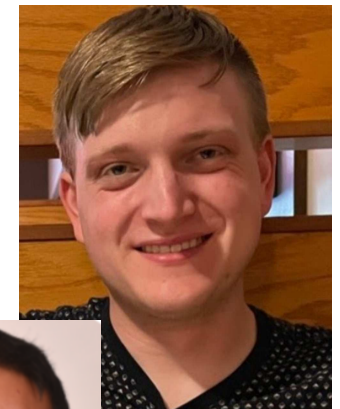
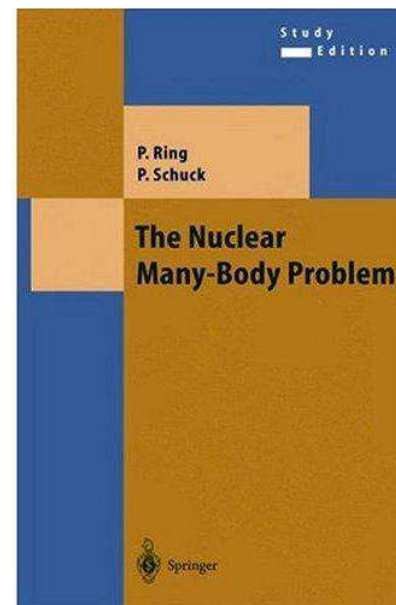
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Caroline Robin (U. Bielefeld & GSI)
Bryan Kelly (FSU)
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Edoardo Lanza (INFN)
Andreas Zilges (U. Cologne)
Peter Schuck^t (U. Grenoble)
R. Li (IJCLab)
M. Harakeh (KVI)
Anthea Fantina (GANIL)
Francesca Gulminelli (LPC Caen)

...

ALTO & PARIS
GANIL visitor program



PHY-2209376
PHY-2515056



Kyle
Morrisey



Sumit Bhattacharjee