

Coherence and Collectivity in nuclear excitations

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Many-body correlations, phase transitions, and clustering: how to identify and quantify them?

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Outline

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- Collective nuclear excitations
- The Giant Resonances' paradigm
- Low-lying excitations in neutron-rich systems
- Are they collective?
- Coherence as building mechanism leading to Collectivity
- One- (and Two-) body operators/responses

Collective nuclear excitations

Collective nuclear excitations

- Collective nuclear excitations are well established phenomena
- Typical examples are Giant Resonances (GRs)
- GRs exhaust a large fraction of the Sum Rule
- GRs are excitations where many nucleons participate coherently

Coherence in collective nuclear excitations

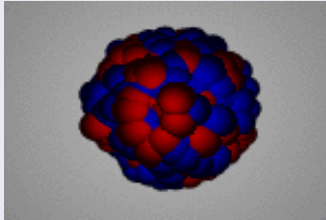
- The coherence is driven by the residual interaction (restoring force)
- Coherence is a fingerprint of correlations which can be inspected/interpreted in different ways (many-body, spatial, quantal, ...)

Collective nuclear excitations

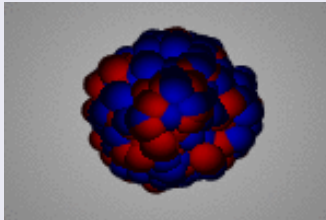
- Historically the first GRs experimentally discovered was the IVGDR
- GRs are identified in terms of the L, S, T quantum numbers
- Their study is still important to test many-body methods, constrain the nuclear interaction, infer information on the EoS

Giant Resonances, a macroscopic picture

IVGDR: neutrons (n) and protons (p) in opposite phase (isovector)

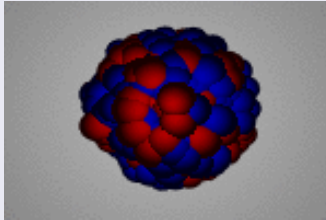


ISGMR: n and p in phase (isoscalar), compression (breathing) mode

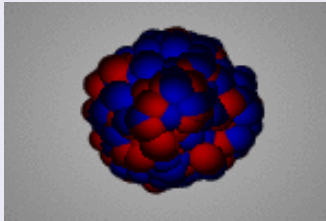


Giant Resonances, a macroscopic picture

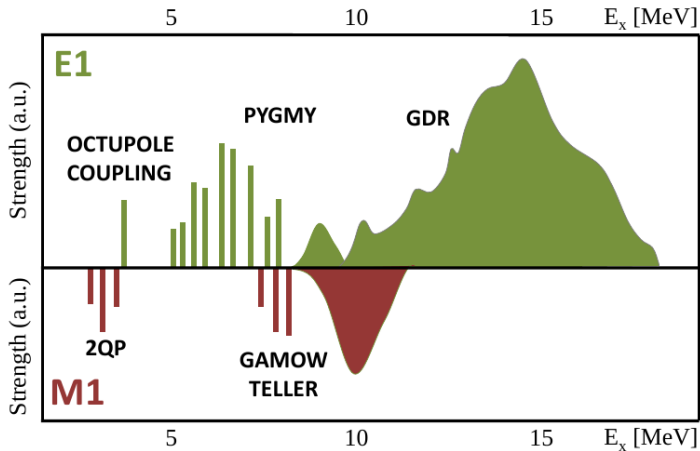
IVGDR: neutrons (n) and protons (p) in opposite phase (isovector)



ISGMR: n and p in phase (isoscalar), compression (breathing) mode



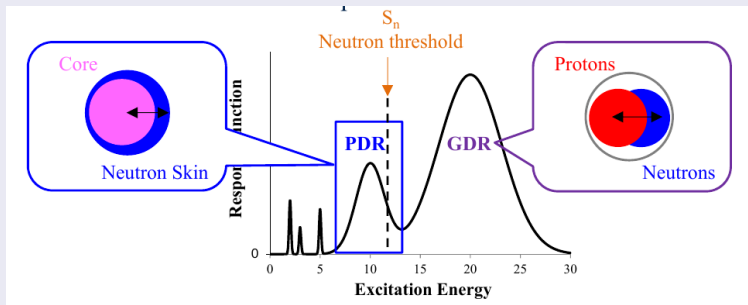
Nuclear Dipole Response: a qualitative picture



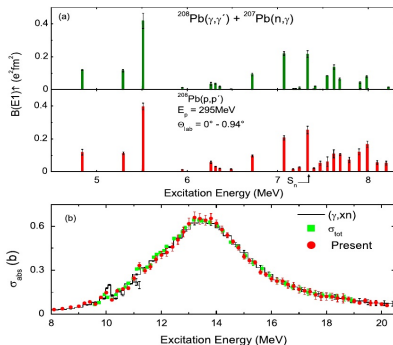
What is the Pygmy Dipole Resonance ?

The Pygmy Dipole Resonance (PDR)

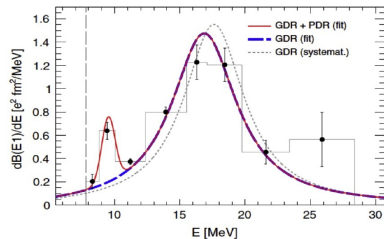
- low-energy E1 strength observed in neutron-rich nuclei $N > Z$
- exhausting few percents of the total strength (i.e., pygmy)
- located around the neutron threshold
- dipole oscillation of the neutron skin against the n-p core



PDR experimental results

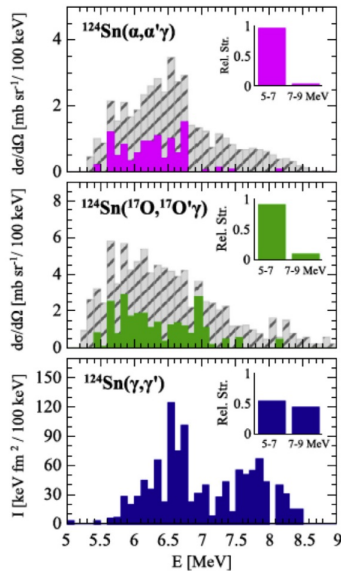
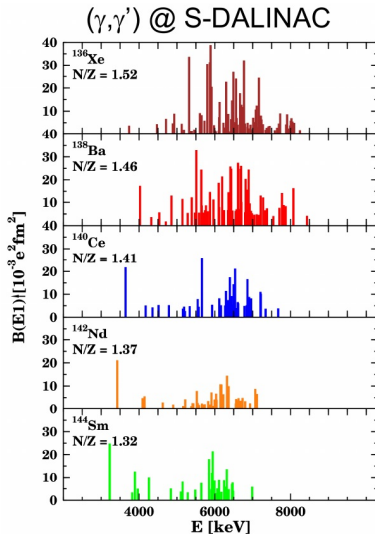


A. Tamii, et al., Phys. Rev. Lett. 107 (2011) 062502.



Electric dipole strength distribution in ^{68}Ni measured by Coulomb excitation
 D.M. Rossi, et al., Phys. Rev. Lett. 111 (2013) 242503.

PDR experimental results



L. Pellegrini, et al., Phys. Lett. B 738 (2014) 519.

Structure and reaction studies on PDR

PHYSICAL REVIEW C **79**, 054615 (2009)

Multiphonon excitations and pygmy resonances in tin isotopes

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CEA, Irfu/Dir, Centre de Saclay, F-91191 Gif-sur-Yvette, France

(Received 19 January 2009; published 29 May 2009)

PHYSICAL REVIEW C **84**, 064602 (2011)

Excitations of pygmy dipole resonances in exotic and stable nuclei via Coulomb and nuclear fields

E. G. Lanza,^{1,2} A. Vitturi,^{3,4} M. V. Andrés,⁵ F. Catara,^{2,1} and D. Gambacurta^{2,1}

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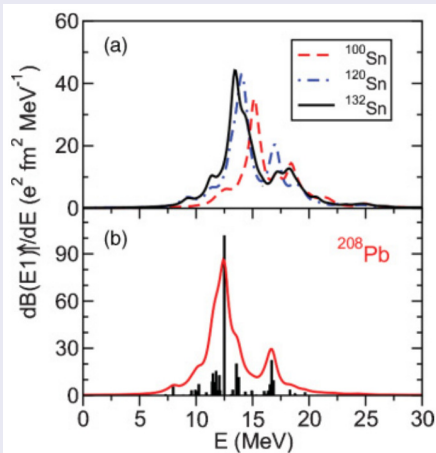
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(Received 25 July 2011; published 2 December 2011)

- Nuclear excitations described in RPA
- RPA transitions provide Form Factors (double-folding)
- Reaction calculations in Couple-Channel formalism

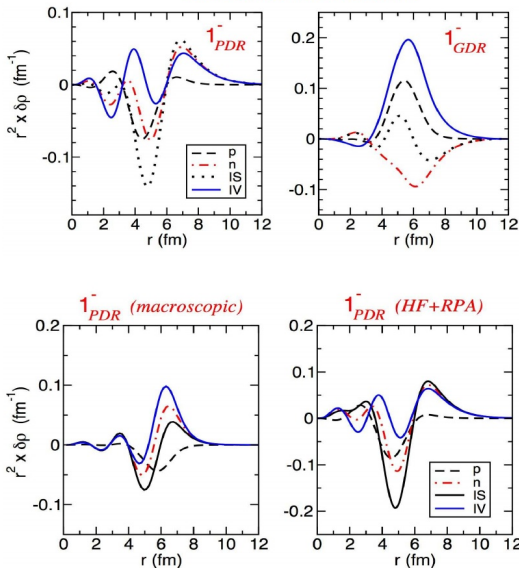
RPA Microscopic description of the dipole response

Dipole response in neutron rich systems



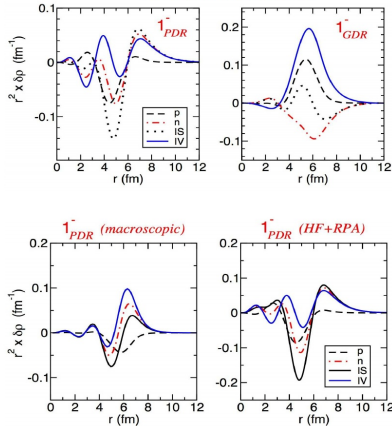
Transition densities in ^{132}Sn

Microscopic RPA: PDR vs GDR



Transition densities in ^{132}Sn

Microscopic RPA: PDR vs GDR



$$\delta\rho_N(r) = \beta \left[\frac{N_V}{A} \frac{d\rho_N^C(r)}{dr} - \frac{N_C + Z}{A} \frac{d\rho_N^V(r)}{dr} \right]$$

$$\delta\rho_P(r) = \beta \left[\frac{N_V}{A} \frac{d\rho_P(r)}{dr} \right], \quad \text{where C (V) stand for Core (Valence)}$$

Coherence and Collectivity

Coherence of the elementary $1p1h$ configurations in building the $B(E1)$

$$B^\nu(E\lambda) = \left| \sum_{ph} (X_{ph}^\nu - Y_{ph}^\nu) F_{ph}^\lambda \right|^2 = \left| \sum_{ph} b_{ph}^\nu(E\lambda) \right|^2$$

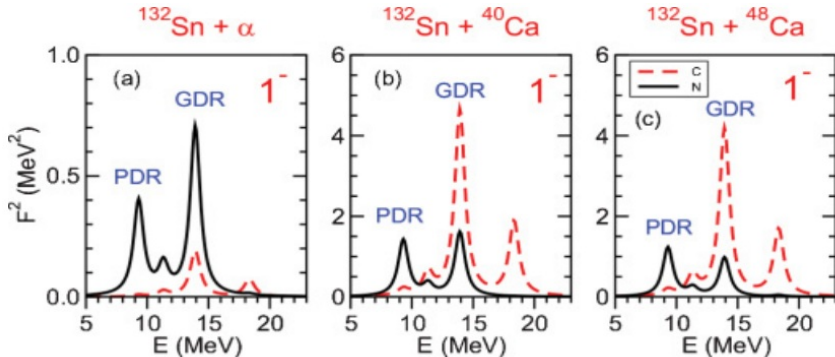
where F_{ph}^λ are the dipole transition amplitudes of the operator

$$F = e \frac{N}{A} \sum_{i=1}^Z r_i Y_{10}(\Omega_i) - e \frac{Z}{A} \sum_{i=1}^N r_i Y_{10}(\Omega_i)$$

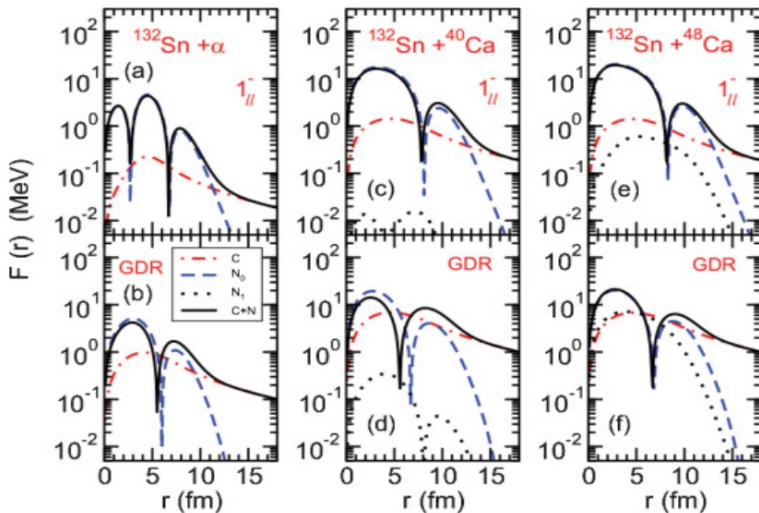
$b_{ph}^\nu(E\lambda) = (X_{ph}^\nu - Y_{ph}^\nu) F_{ph}^\lambda$ produce (or not) coherence $\mapsto$ Collectivity



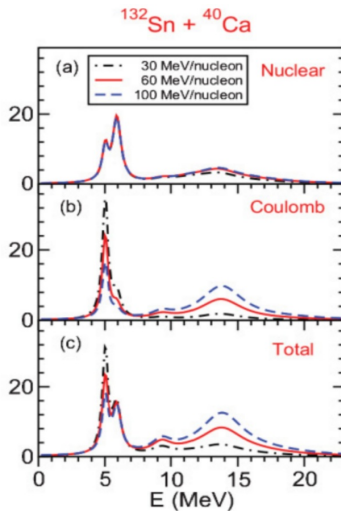
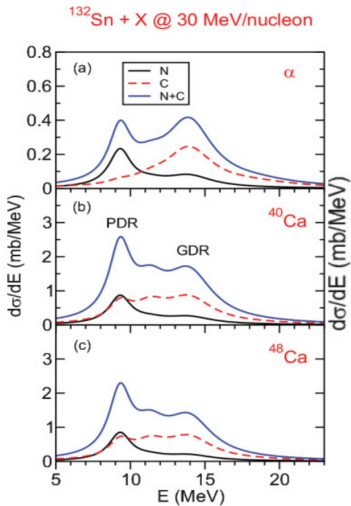
Reaction calculations in CC formalism



Reaction calculations in CC formalism



Reaction calculations in CC formalism



The Method of the Equations of Motion

The Method of the Equations of Motion

Set of exact eigenstates of the Hamiltonian H

$$H|\nu\rangle = E_\nu |\nu\rangle$$

where $|0\rangle$ is the ground state with energy E_0

Phonon operators

Let us introduce the operators Q' 's:

$$Q_\nu^\dagger |0\rangle = |\nu\rangle$$

$$Q_\nu |0\rangle = 0.$$

Equations of Motion:

$$\langle 0 | [\delta Q, [H, Q_\nu^\dagger]] | 0 \rangle = \omega_\nu \langle 0 | [\delta Q, Q_\nu^\dagger] | 0 \rangle$$

where

$$\omega_\nu = E_\nu - E_0.$$

The Random Phase Approximation

The Random Phase Approximation (RPA)

- The RPA is a widely used tool for the description of collective excitations
- Very successful especially within the Energy Density Functional framework (interactions á la Skyrme or Gogny, covariant versions)
- It provides global properties: centroid energies and total strength

However, extensions of the RPA are required for:

- Strength Fragmentation
- Fine Structure
- Spreading Width
- Low Lying excitations in closed shell nuclei
- Double excitations and Anharmonicities, ...

RPA vs SRPA

Phonon Operators: RPA vs SRPA

Random Phase Approximation (RPA)

$$Q_{\nu}^{\dagger} = \underbrace{\sum_{ph} X_{ph}^{(\nu)} \underbrace{a_p^{\dagger} a_h}_{1p-1h} - \sum_{ph} Y_{ph}^{(\nu)} \underbrace{a_h^{\dagger} a_p}_{1h-1p}}_{\text{Only Landau Damping, Centroid Energy and Total Strength of GRs}}$$

Only Landau Damping, Centroid Energy and Total Strength of GRs

Second Random Phase Approximation (SRPA)

$$Q_{\nu}^{\dagger} = \sum_{ph} (X_{ph}^{(\nu)} a_p^{\dagger} a_h - Y_{ph}^{(\nu)} a_h^{\dagger} a_p) + \underbrace{\sum_{p_1 < p_2, h_1 < h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{p_1}^{\dagger} a_{h_1} a_{p_2}^{\dagger} a_{h_2}}_{2p-2h} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{h_1}^{\dagger} a_{p_1} a_{h_2}^{\dagger} a_{p_2}}_{2h-2p})}_{\text{Spreading Width, Fragmentation, Double GRs and Anharmonicities, Low-Lying States}}$$

Spreading Width, Fragmentation, Double GRs and Anharmonicities, Low-Lying States

RPA and SRPA Equations

RPA Equations of Motion ($1 \mapsto 1p-1h$)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}_{11}^* & -\mathcal{A}_{11}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix}$$

RPA and SRPA Equations

RPA Equations of Motion ($1 \mapsto 1p-1h$)

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SRPA Equations of Motion ($1 \mapsto 1p-1h, 2 \mapsto 2p-2h$)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}_{11}^* & -\mathcal{B}_{12}^* & -\mathcal{A}_{11}^* & -\mathcal{A}_{12}^* \\ -\mathcal{B}_{21}^* & -\mathcal{B}_{22}^* & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix}$$

Transition Probabilities

Transition Probabilities in RPA and SRPA

- Let us consider a one-body operator $F = \sum_{\alpha,\beta} F_{\alpha\beta} a_{\alpha}^{\dagger} a_{\beta}$
- The RPA and SRPA transition amplitudes are

$$\langle \nu | F | 0 \rangle = \langle 0 | [Q_{\nu}, F] | 0 \rangle \approx \langle HF | [Q_{\nu}, F] | HF \rangle = \sum_{ph} \{ X_{ph}^{\nu*} F_{ph} + Y_{ph}^{\nu*} F_{hp} \}$$

SRPA with effective EDFs

- When the SRPA is applied within the EDF framework (e.g. with effective interaction) double-counting issues might arise
- The Subtraction method ^a is used $\mapsto$ Subtracted SRPA (SSRPA)
- Designed for beyond RPA approaches
- Regularization procedure removing double countings ^b

^aI. Tselyaev Phys. Rev. C 75, 024306 (2007)

^bD. Gambacurta *et al.* Phys. Rev. C 92, 034303 (2015), Phys. Rev. Lett. 125, 212501 (2020)

Some SRPA and SSRPA applications

Some SSRPA applications

- Dipole response in ^{48}Ca and ^{68}Ni
- Impact of BMF correlations on EoS
- Soft monopole modes in neutron-rich nuclei
- Two-Body response and Collectivity

Dipole response in ^{48}Ca : Low-lying states

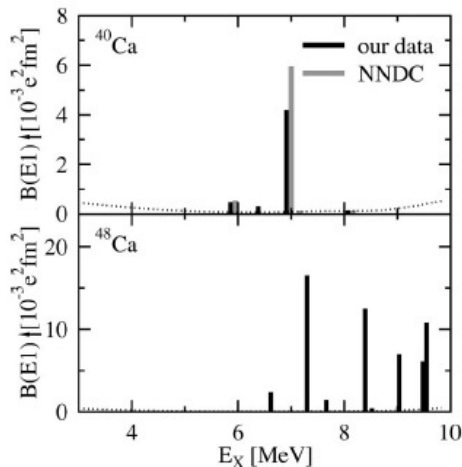
Low-lying dipole response in ^{48}Ca : Motivation

- Experimental low-lying dipole (from 5 to 10 MeV) response in ^{48}Ca stronger than in ^{40}Ca
- Pygmy Dipole Resonance (PDR) type?
- Not described in relativistic and non-relativistic RPA models
- What happens in SRPA ^a ?
- and in the SSRPA ^b ?

^aD. G. , M. Grasso, and F. Catara, Phys. Rev. C 84, 034301 (2011)

^bD. G., M. Grasso and O. Vasseur, Physics Letters B 777 (2018) 163–168

Experimental low-lying dipole strength in $^{40,48}\text{Ca}$. (Photon Scattering)



$$\sum B(E1) = 5.1 \pm 0.8 (10^{-3} e^2 \text{fm}^2),$$

$$\sum B(E1) = 68.7 \pm 7.5 (10^{-3} e^2 \text{fm}^2),$$

From T. Hartmann *et al.*, PRC 65, 034301, (2002)

Relativistic RPA results

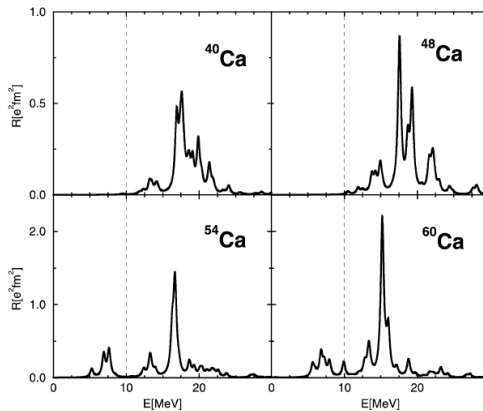


Fig. 5. RRPA isovector dipole strength distributions in Ca isotopes. The thin dashed line tentatively separates the region of giant resonances from the low-energy region below 10 MeV.

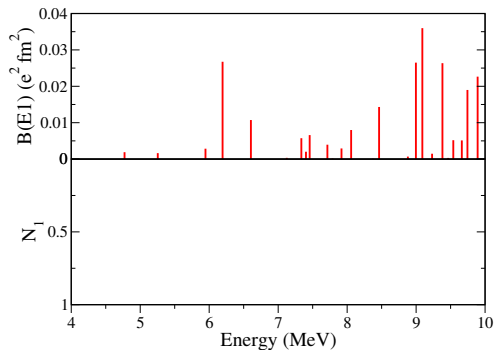
From D. Vretenar *et al.*, Nucl. Phys. A 692, 496 (2001)



1p1h-2p2h composition of the states

1p1h and 2p2h contents

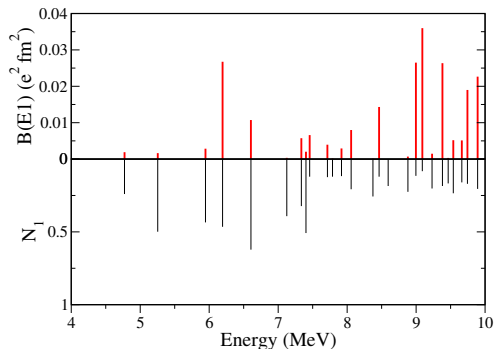
$$\begin{aligned}
 \langle \nu | \nu \rangle &= \sum_{ph} (|X_{ph}^\nu|^2 - |Y_{ph}^\nu|^2) + \sum_{p_1 < p_2, h_1 < h_2} (|X_{p_1 h_1 p_2 h_2}^\nu|^2 - |Y_{p_1 h_1 p_2 h_2}^\nu|^2) \\
 &= N_1 + N_2 = 1
 \end{aligned}$$



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Coherence and Collectivity

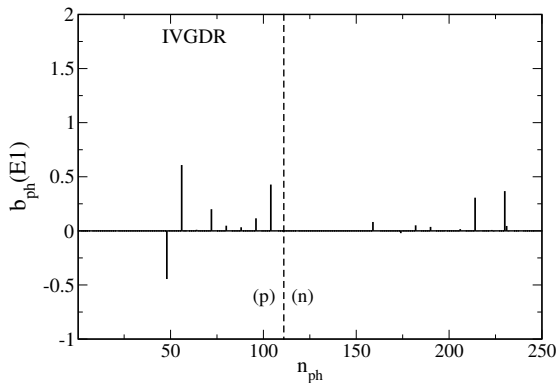
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where F_{ph}^λ are the dipole transition amplitudes of the operator

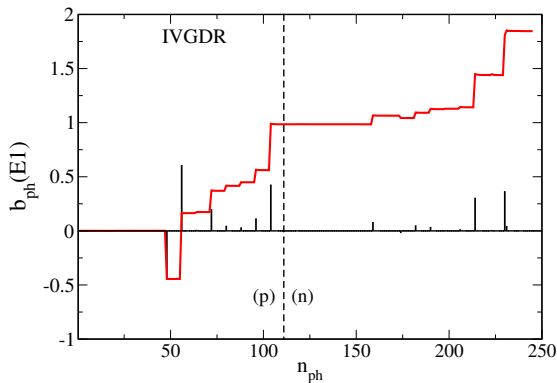
$$F = e \frac{N}{A} \sum_{i=1}^Z r_i Y_{10}(\Omega_i) - e \frac{Z}{A} \sum_{i=1}^N r_i Y_{10}(\Omega_i)$$

The IVGDR case



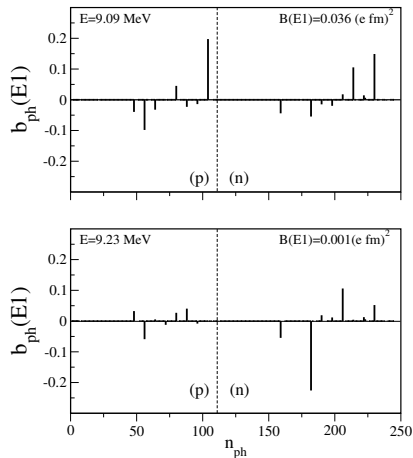
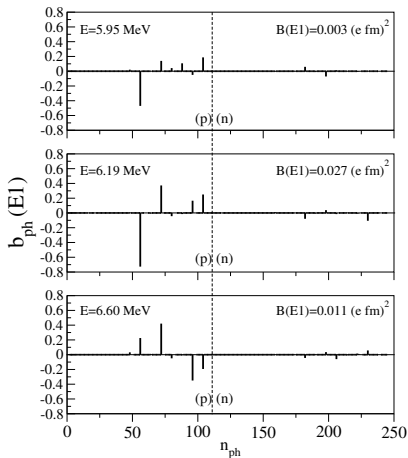
Bars are the single b_{ph} , the continuous line is the cumulative sum of the b_{ph}

The IVGDR case

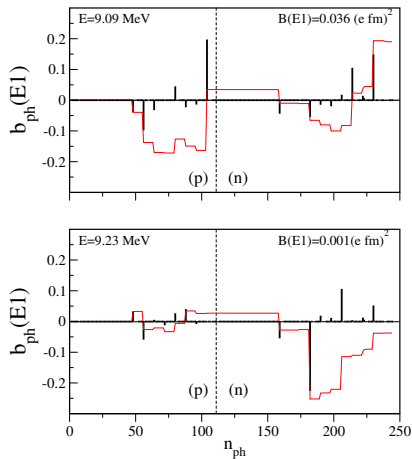
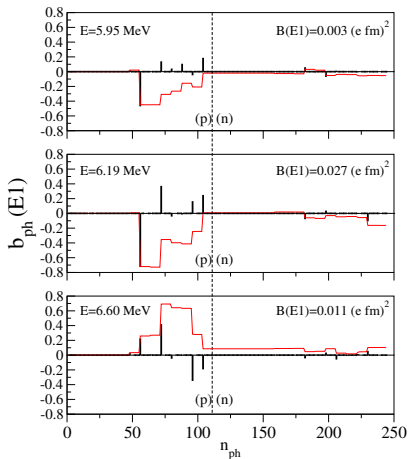


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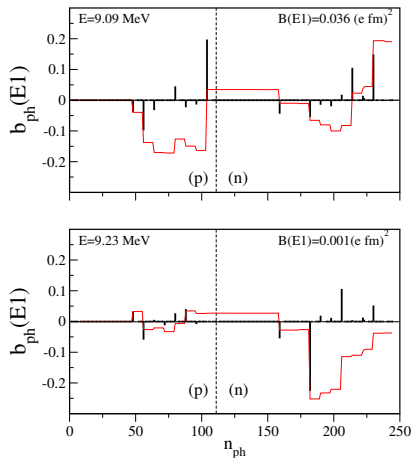
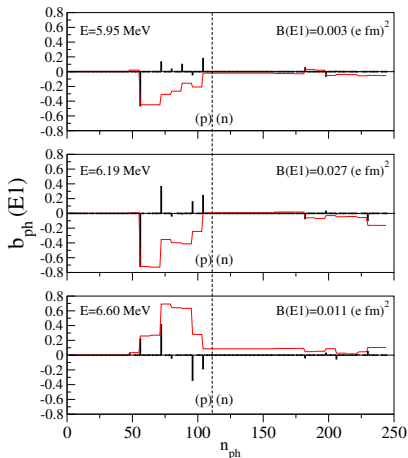
Collectivity: coherence of the elementary contributions



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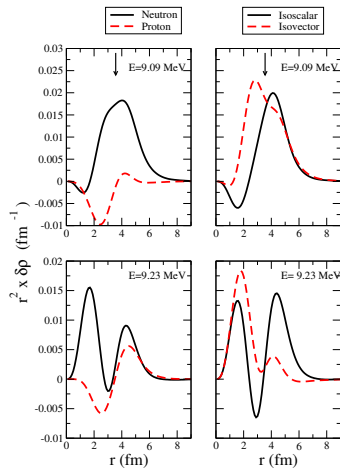
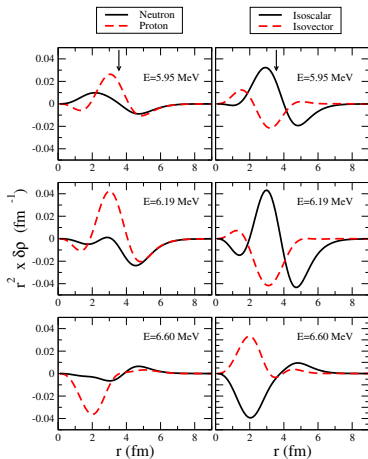


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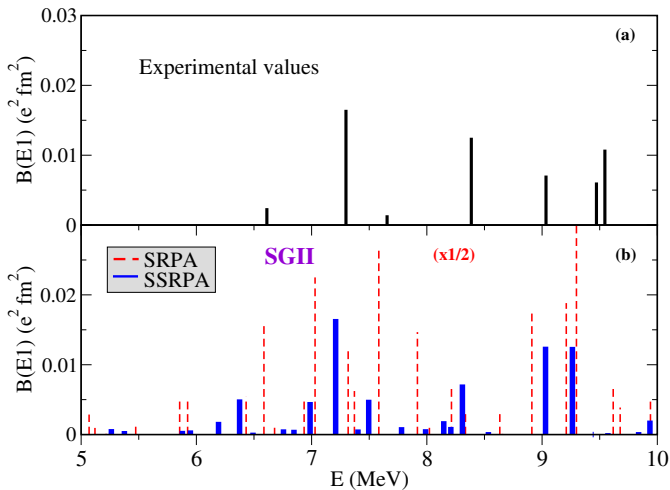


Several 1p-1h configurations participate but not coherently

Transition densities



SRPA, SSRPA vs Data



D. G. , M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163–168

Total $B(E1)$ and EWSRs (From 5 to 10 MeV)

	Exp	SRPA SGII	SSRPA SGII	SRPA SLy4	SSRPA SLy4
$\sum B(E1)$	0.068 ± 0.008	0.563	0.078	1.012	0.126
$\sum_i E_i B_i(E1)$	0.570 ± 0.062	4.618	0.621	8.795	1.062

Experimental and theoretical $\sum B(E1)$ in ($e^2 \text{ fm}^2$) and $\sum_i E_i B_i(E1)$ in ($\text{MeV } e^2 \text{ fm}^2$) summed between 5 and 10 MeV.

From D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163–168

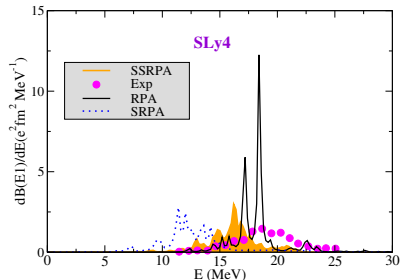
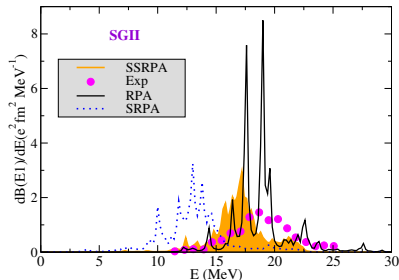
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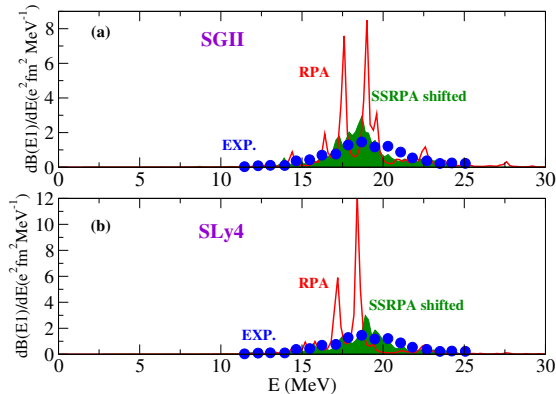
From D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163–168

SSRPA vs Data, GDR case



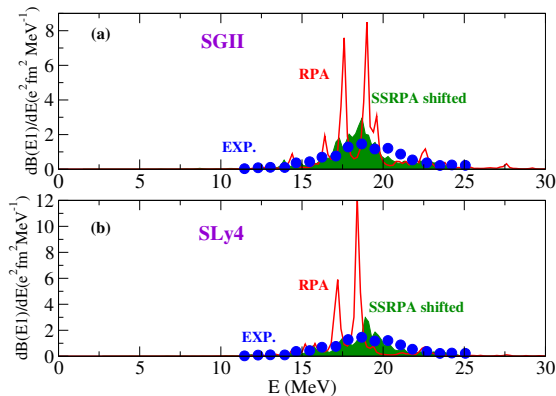
Data From J. Birkhan *et al.*, Phys. Rev. Lett. 118, 252501 (2017);
 Theoretical results folded with a Lorentzian having a width of 0.25 MeV
 D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163–168

SSRPA vs Data, GDR case



Shift ~ 1 MeV (SGII), ~ 1.5 MeV (SLy4);

SSRPA vs Data, GDR case



Shift ~ 1 MeV (SGII), ~ 1.5 MeV (SLy4);
 Room to improve on that (full subtraction)

PDR results in SM calculations

PHYSICAL REVIEW C **111**, 064308 (2025)

Electric dipole response of sd -shell nuclei within the configuration-interaction shell-model approach

O. Le Noan and K. Sieja

Université de Strasbourg, IPHC, 23 rue du Loess 67037 Strasbourg, France
and CNRS, UMR7178, 67037 Strasbourg, France

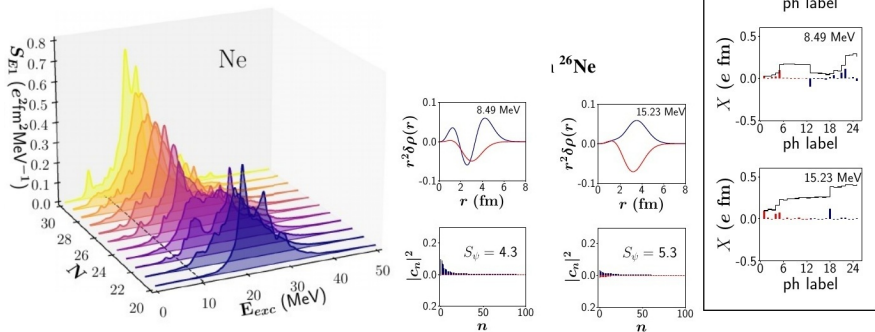


FIG. 6. Electric dipole strength functions obtained in the $1h\omega$ shell-model calculations Eq. (7) for even-even and even-odd neon isotopes with $N \leq 20$.

$$|\psi\rangle = \sum_n c_n^\psi |n\rangle.$$

Low-energy dipole states in ^{68}Ni

Experimental measurements: relativistic Coulomb excitations at GSI

- (I) O. Wieland et al., Phys. Rev. Lett. 102, 092502 (2009): strength centered at around 11 MeV with a contribution of 5% to the EWSR
- (II) D.M. Rossi et al., Phys. Rev. Lett. 111, 242503 (2013): strength centered at around 9.55 MeV with a contribution of 2.8% to the EWSR
- The discrepancy in the value of the centroid was explained as a possible 'energy-dependent branching ratio'.

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- (II) D.M. Rossi et al., Phys. Rev. Lett. 111, 242503 (2013): strength centered at around 9.55 MeV with a contribution of 2.8% to the EWSR
- The discrepancy in the value of the centroid was explained as a possible 'energy-dependent branching ratio'.
- Isoscalar excitation on ^{12}C target @ LNS: peak located around 10 MeV with a contribution of 10% to the EWSR (N.S. Martorana et al., Phys. Lett. B 782, 112 (2018))

Low-energy dipole states in ^{68}Ni

- We study low-energy dipole excitations in ^{68}Ni within the SSRPA ^a
- Strength distributions are compared with available experimental data
- Transition densities of some selected (low-lying) peaks are analyzed.
- We estimate beyond-mean-field (BMF) on the symmetry energy (J) and on its slope (L)

^aM. Grasso and DG Phys. Rev. 101, 064314 (2020)

Low-energy dipole states in ^{68}Ni

Low-energy dipole states in ^{68}Ni

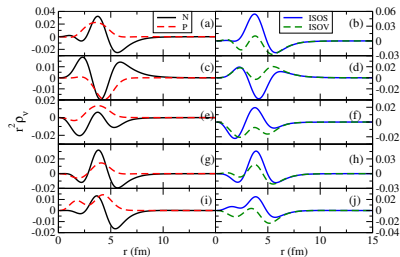
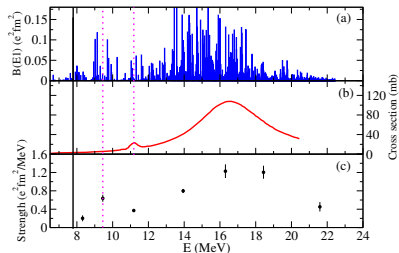
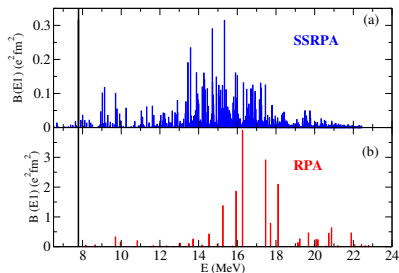
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^aM. Grasso and DG Phys. Rev. 101, 064314 (2020)

Employed linear correlations

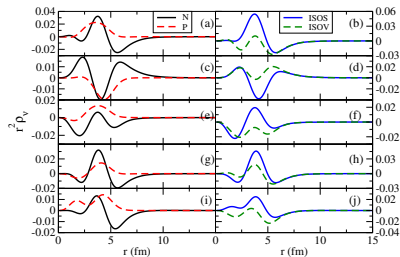
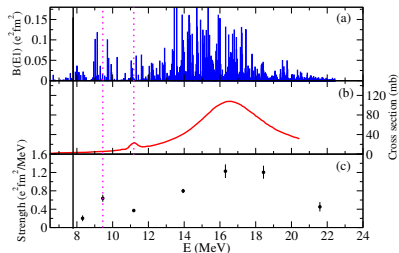
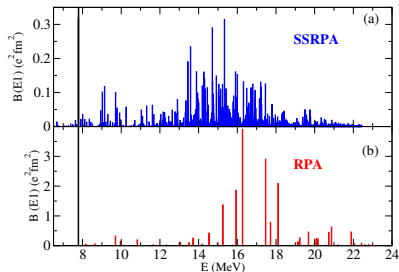
- Percentage of EWSR and L:
A. Klimkiewicz et al., Phys. Rev. C 76, 051603 (R) (2007); A. Carbone et al., Phys. Rev. C 81, 041301 (R) (2010)
- L and neutron-skin thickness:
M. Centelles et al., Phys. Rev. Lett. 102, 122502 (2009); M. Wardaet al., Phys. Rev. C 80, 024316 (2009)
- Neutron-skin thickness and electric dipole polarizability times the J coefficient: X. Roca-Maza et al. Phys. Rev. C 92, 064304 (2015).

Low-energy dipole states in ^{68}Ni (SGII case)



Peaks located at 9.14 ((a) and (b)), 9.70 ((c) and (d)), 10.25 ((e) and (f)), 11.10 ((g) and (h)), and 11.31 ((i) and (j))

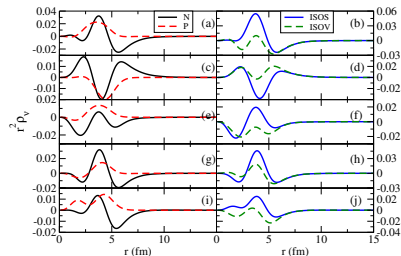
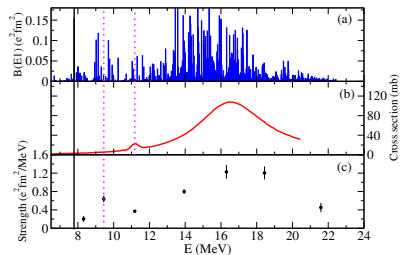
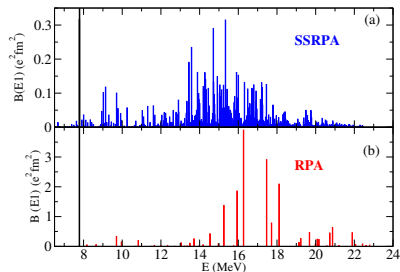
Low-energy dipole states in ^{68}Ni (SGII case)



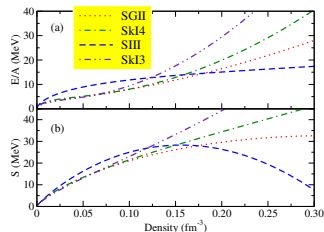
Peaks located at 9.14 ((a) and (b)), 9.70 ((c) and (d)), 10.25 ((e) and (f)), 11.10 ((g) and (h)), and 11.31 ((i) and (j))

Skymre	J (MeV)	L (MeV)
SIII	28.16	9.90
SGII	26.83	37.70
SkI4	29.50	60.00
SkI3	34.27	100.49

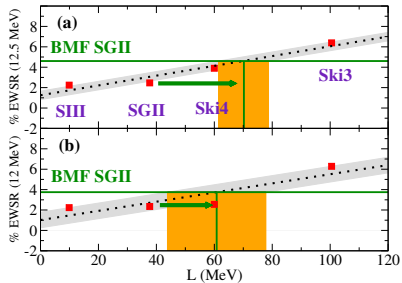
Beyond-mean-field effects on the symmetry energy and its slope



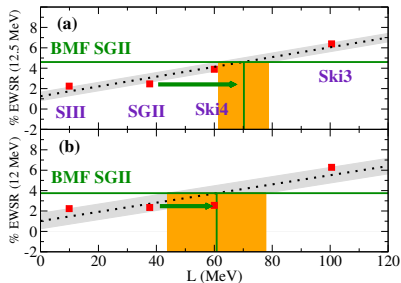
Peaks located at 9.14 ((a) and (b)), 9.70 ((c) and (d)), 10.25 ((e) and (f)), 11.10 ((g) and (h)), and 11.31 ((i) and (j))



Beyond-mean-field effects on the symmetry energy and its slope



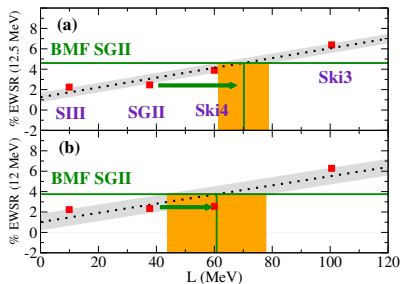
Beyond-mean-field effects on the symmetry energy and its slope



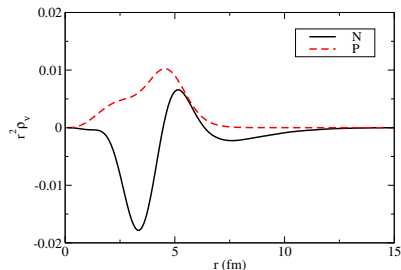
$SGII(RPA) = 2.35\%$ (up to 12 MeV)

$SGII(SSRPA) = 3.75\%$ (up to 12 MeV)

Beyond-mean-field effects on the symmetry energy and its slope

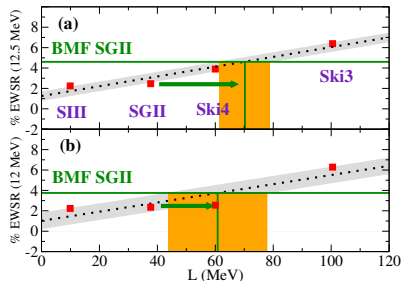


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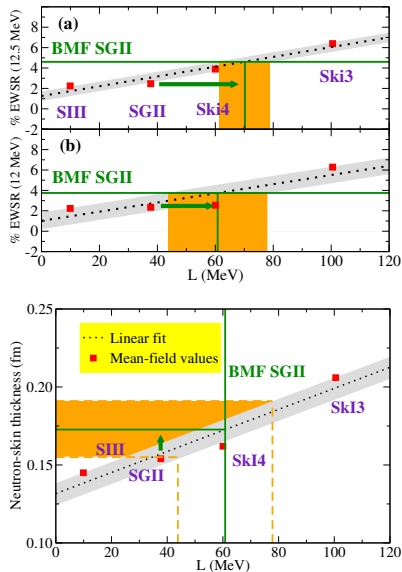


Neutron and proton transition densities
 for the state located at 12.17 MeV.

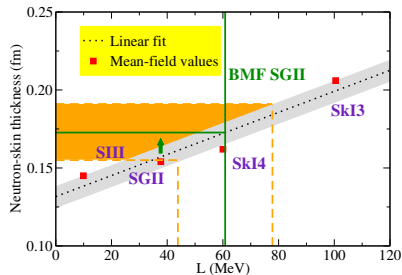
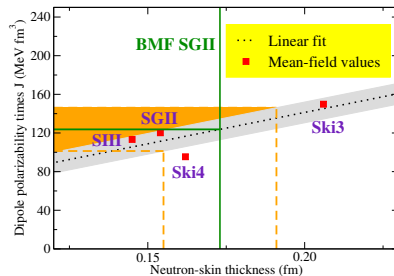
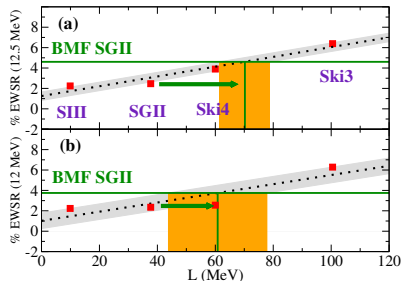
Beyond-mean-field effects on the symmetry energy and its slope



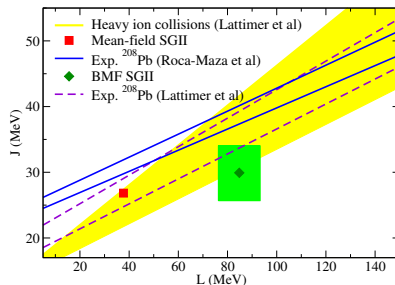
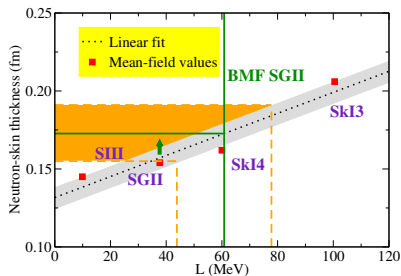
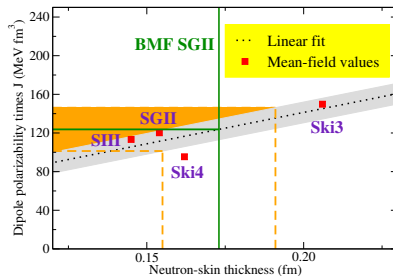
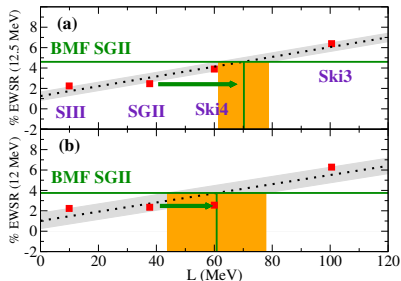
Beyond-mean-field effects on the symmetry energy and its slope



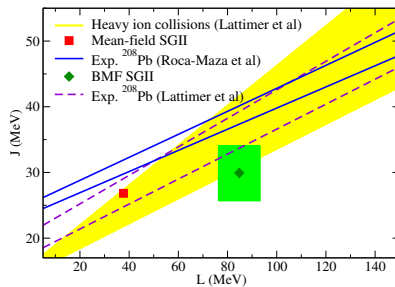
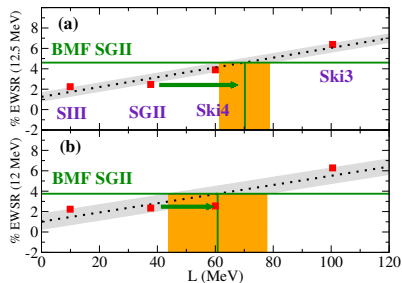
Beyond-mean-field effects on the symmetry energy and its slope



Beyond-mean-field effects on the symmetry energy and its slope



Beyond-mean-field effects on the symmetry energy and its slope



	L (MeV)	Δr_{np} (fm)	J (MeV)
Mean field	37.70	0.154	26.83
BMF	60.815 ± 16.982	0.173 ± 0.018	27.617 ± 5.004

Monopole breathing modes in neutron-rich nuclei

Monopole response in RPA and SSRPA

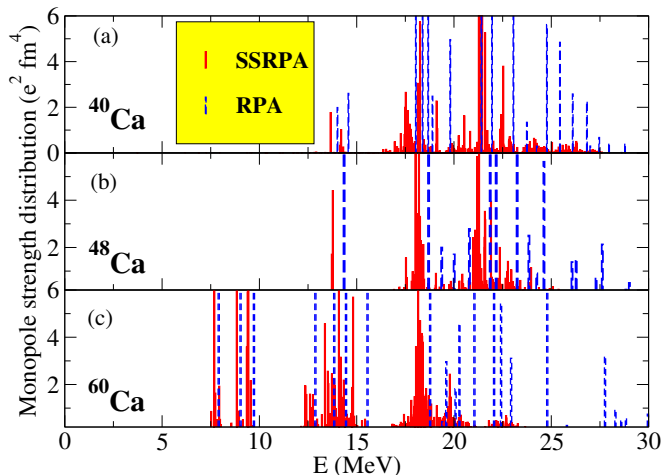
- Evolution of the response in Ca isotopes from ^{40}Ca to ^{60}Ca ^a
- $N = 20$ isotones : ^{40}Ca , ^{36}S and ^{34}Si
- The case of ^{68}Ni

^aPhys. Rev. Lett. 121, (2018) @ RIKEN

The key-points of our study

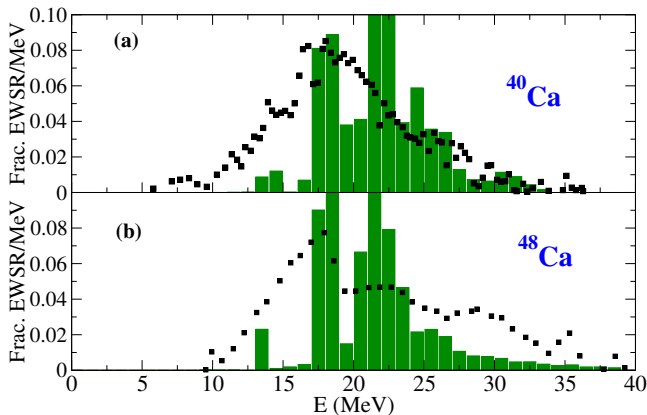
- Similarities and differences with respect PDR
- Microscopic properties
- Properties as a function of the isospin asymmetry
- More details: DG, M. Grasso and O. Sorlin, PRC 100, 014317 (2019)

Monopole strength distribution in Ca isotopes



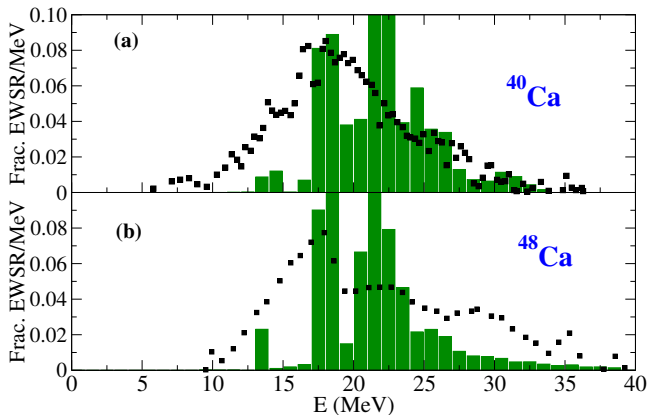
(a) Monopole strength distribution computed with RPA (dashed blue bars) and SSRPA (full red bars) for ^{40}Ca ; (b) Same as in (a) but for ^{48}Ca ; (c) Same as in (a) but for ^{60}Ca .

Comparison with data from Y.-W. Lui, et al., PRC 83, 044327 (2011)



(a) Black squares: experimental results ; green bars: SSRPA predictions for ^{40}Ca ; (b) Same as in (a) but for ^{48}Ca .

Comparison with data from Y.-W. Lui, et al., PRC 83, 044327 (2011)

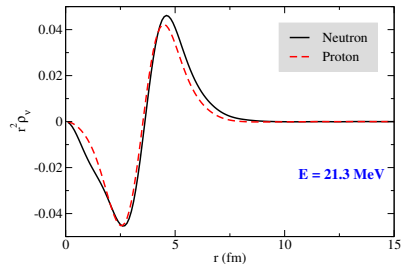
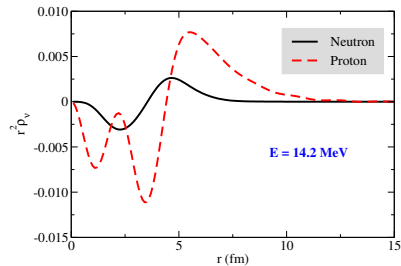
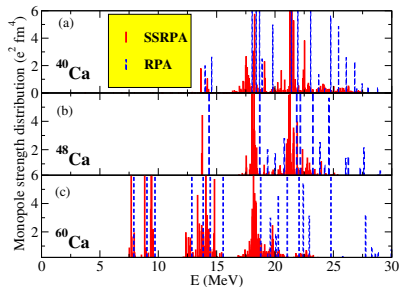


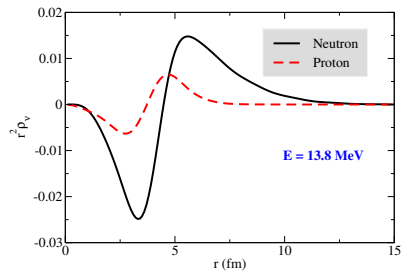
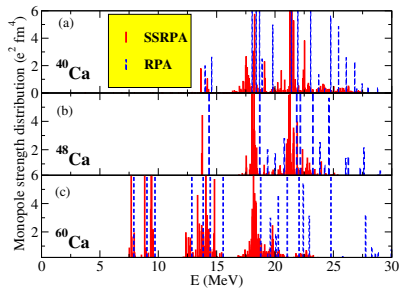
(a) Black squares: experimental results ; green bars: SSRPA predictions for ^{40}Ca ; (b) Same as in (a) but for ^{48}Ca .

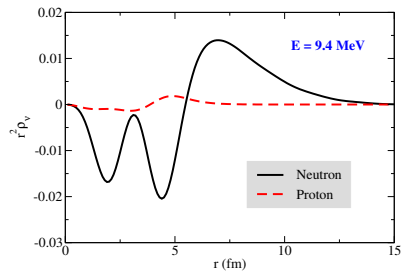
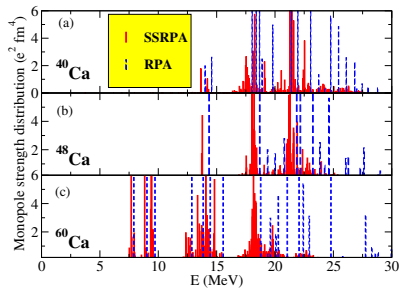
Centroids (MeV)

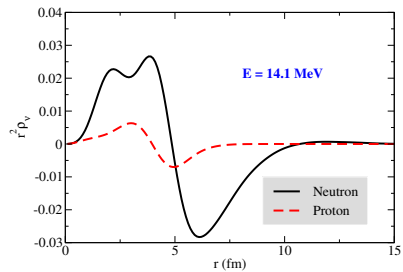
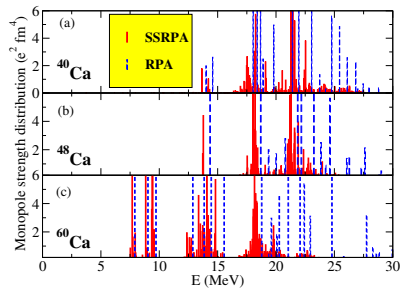
^{40}Ca : 21.3 (RPA), 20.7 (SSRPA), 18.3 (Exp)

^{48}Ca : 20.7 (RPA), 20.4 (SSRPA), 19.0 (Exp)

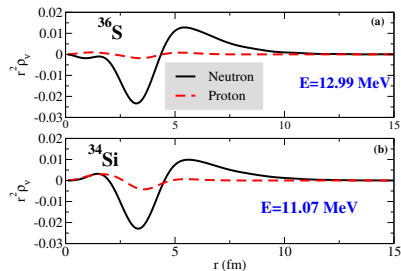
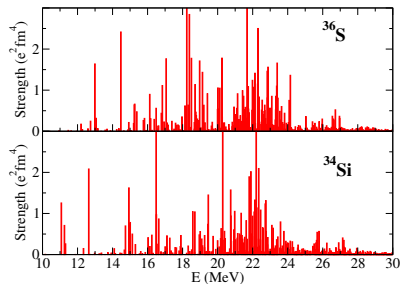
Transition densities in ^{40}Ca 

Transition densities in ^{48}Ca 

Transition densities in ^{60}Ca 

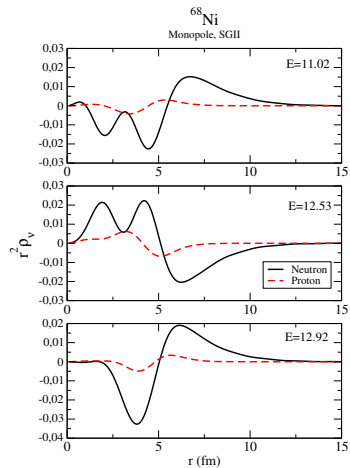
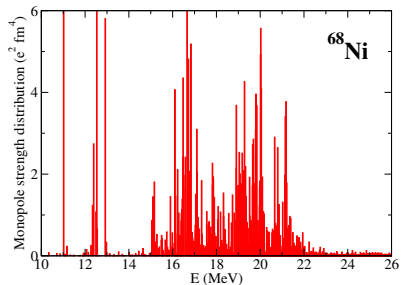
Transition densities in ^{60}Ca 

Monopole strength distributions and transition densities in ^{34}Si and ^{36}S

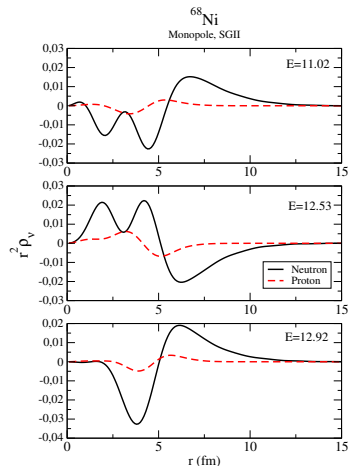
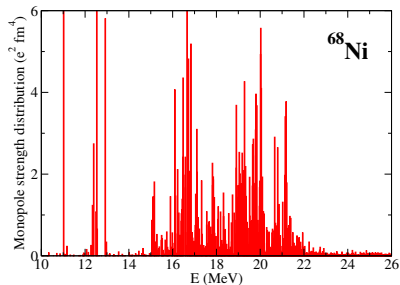


Composition of the peak located at 11.07 (12.99) MeV for ^{34}Si (^{36}S).

^{34}Si	1p1h 54 %	2p2h 46 %
	$[\nu 2d_{3/2}, \nu 1d_{3/2}]^{J=0}$	$[[\pi 3p_{1/2}, \nu 3f_{7/2}]^{J_p=3} [\pi 1d_{5/2}, \nu 2s_{1/2}]^{J_h=3}]^{J=0}$ $[[\pi 4p_{1/2}, \nu 1f_{5/2}]^{J_p=2} [\pi 1d_{5/2}, \nu 2s_{1/2}]^{J_h=2}]^{J=0}$ $[[\pi 4p_{1/2}, \nu 1f_{5/2}]^{J_p=3} [\pi 1d_{5/2}, \nu 2s_{1/2}]^{J_h=3}]^{J=0}$ $[[\pi 6s_{1/2}, \nu 2d_{3/2}]^{J_p=2} [\pi 1d_{5/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$ $[[\pi 6s_{1/2}, \nu 2d_{5/2}]^{J_p=2} [\pi 1d_{5/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$ $[[\pi 3d_{3/2}, \nu 3s_{1/2}]^{J_p=2} [\pi 1d_{5/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$ $[[\pi 3d_{3/2}, \nu 2d_{5/2}]^{J_p=1} [\pi 1d_{5/2}, \nu 1d_{3/2}]^{J_h=1}]^{J=0}$ $[[\pi 3d_{3/2}, \nu 2d_{5/2}]^{J_p=2} [\pi 1d_{5/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$
^{36}S	1p1h 52 %	2p2h 48 %
	$[\nu 2d_{3/2}, \nu 1d_{3/2}]^{J=0}$	$[[\pi 3d_{3/2}, \nu 4d_{3/2}]^{J_p=2} [\pi 1d_{5/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$ $[[\pi 4d_{3/2}, \nu 4s_{1/2}]^{J_p=2} [\pi 2s_{1/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$ $[[\pi 4d_{3/2}, \nu 5s_{1/2}]^{J_p=1} [\pi 2s_{1/2}, \nu 1d_{3/2}]^{J_h=1}]^{J=0}$ $[[\pi 4d_{3/2}, \nu 4d_{3/2}]^{J_p=2} [\pi 2s_{1/2}, \nu 1d_{3/2}]^{J_h=2}]^{J=0}$ $[[\pi 4d_{3/2}, \nu 2d_{5/2}]^{J_p=1} [\pi 2s_{1/2}, \nu 1d_{3/2}]^{J_h=1}]^{J=0}$

Transition densities in ^{68}Ni 

Transition densities in ^{68}Ni

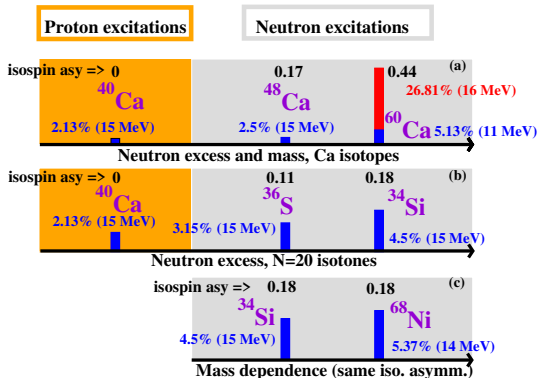


M. Vandebrouck et al. PRL 113, 032504 (2014)

$$E_{ISGMR}^{exp} = 21.1 \pm 1.9 \quad E_{ISGMR}^{SSRPA} \sim 19.8$$

$$E_{SOFT}^{exp} = 12.9 \pm 1.0 \quad E_{SOFT}^{SSRPA} \sim 12.92 \quad 70\%[\nu 2f_{5/2}, \nu 1f_{5/2}]^{J=0}$$

Low-energy states contribution to the EWSR



Percentages of the EWSR for several nuclei and corresponding isospin asymmetry $\delta = (N - Z)/A$.

- (a) Ca isotopes: evolution as a function of the neutron excess and the mass;
- (b) $N = 20$ isotones: evolution as a function of the neutron excess;
- (c) Evolution as a function of the mass for two nuclei with the same isospin asymmetry, ^{34}Si and ^{68}Ni .

Monopole breathing modes in neutron-rich nuclei

Main results

- Low-lying states appear also in the monopole channel
- Strength increases with neutron excess
- Soft monopole modes driven by neutron excitations
- Not only at the surface of the nucleus but over its entire volume
- However, they have mostly single 1ph nature
- More details: DG, M. Grasso and O. Sorlin, PRC 100, 014317 (2019)

Two-Body response and Collectivity

Nuclear Responses to Two-Body External Fields Studied with the Second Random-Phase-Approximation

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(Dated: August 27, 2026)

arXiv:2512.19961v2 [nucl-th] 25 Aug 2026

Two-Body External Fields

$$\mathcal{F}_{JM;L_1L_2} \equiv [F_{L_1} G_{L_2}]^{JM}$$

$$|\langle \nu, J || \mathcal{F}_{J;L_1L_2} || \text{SRPA} \rangle|^2 \approx \left| \sum_{\substack{p_1 < p_2 \\ h_1 < h_2}} \sum_{J_p J_h} C_{\nu; p_1 p_2 h_1 h_2}^{(J_p J_h; L_1 L_2) J} \right|_{12}^2 \quad C_{\nu; p_1 p_2 h_1 h_2}^{(J_p J_h; L_1 L_2) J} = \left(\mathcal{X}_{\nu; p_1 p_2 h_1 h_2}^{(J_p J_h) J} + (-)^{L_1 + L_2} \mathcal{Y}_{\nu; p_1 p_2 h_1 h_2}^{(J_p J_h) J} \right) \mathcal{F}_{p_1 p_2 h_1 h_2}^{(J_p J_h; L_1 L_2) J}$$

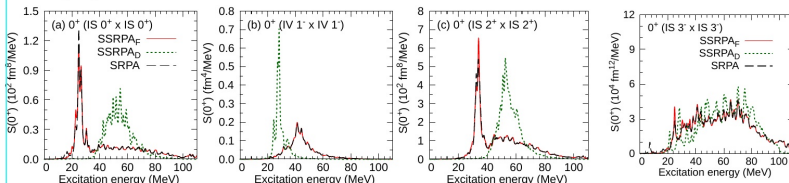
$$\mathcal{F}_{p_1 p_2 h_1 h_2}^{(J_p J_h) J} = 2 \sqrt{\frac{(2J_p + 1)(2J_h + 1)}{(1 + \delta_{p_1 p_2})(1 + \delta_{h_1 h_2})}} \left[\begin{pmatrix} j_{p_1} & j_{p_2} & J_p \\ j_{h_1} & j_{h_2} & J_h \\ L_1 & L_2 & J \end{pmatrix} \times F_{p_1 h_1}^{(L_1)} G_{p_2 h_2}^{(L_2)} - (-)^{j_{h_1} + j_{h_2} - J_h} \begin{pmatrix} j_{p_1} & j_{p_2} & J_p \\ j_{h_2} & j_{h_1} & J_h \\ L_1 & L_2 & J \end{pmatrix} F_{p_1 h_2}^{(L_1)} G_{p_2 h_1}^{(L_2)} \right]$$

$$F_{ph}^{(J)} = \langle p || F_J || h \rangle \equiv \langle j_p l_p || F_J || j_h l_h \rangle$$

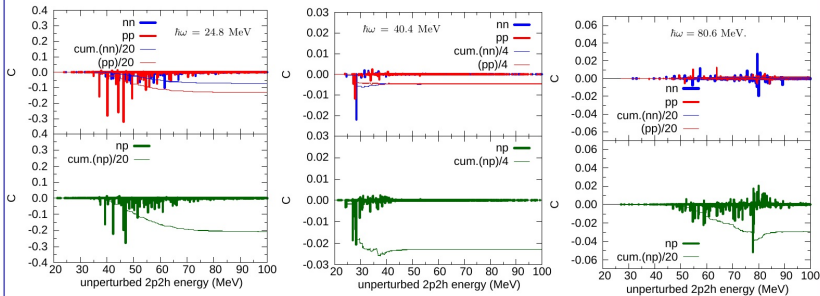
$$\begin{aligned} \mathcal{F}_{0;00} &= \left[[r^2 Y_0] [r^2 Y_0] \right]^0 \\ \mathcal{F}_{0;11} &= \left[[c_q r Y_1 \tau_z] [c_q r Y_1 \tau_z] \right]^0 \\ \mathcal{F}_{0;22} &= \left[[r^2 Y_2] [r^2 Y_2] \right]^0 \\ \mathcal{F}_{0;33} &= \left[[r^3 Y_3] [r^3 Y_3] \right]^0, \end{aligned}$$

Two-Body response and Collectivity

Two-body response in ^{16}O



Coherence and collectivity of two-body operator



Conclusions

Collectivity and coherence

- We have analyzed the coherence mechanism building up collectivity
- We studied soft modes in the dipole and monopole channel, compared to GRs's collectivity
- Dipole low-lying states build by many $1p - 1h$ configurations but not fully coherently
- Monopole soft modes are almost entirely single $1p - 1h$
- Two-Body operator collectivity (preliminary results... to be further investigated)

Extra Slides

Extra Slides

Two-Body response and Collectivity

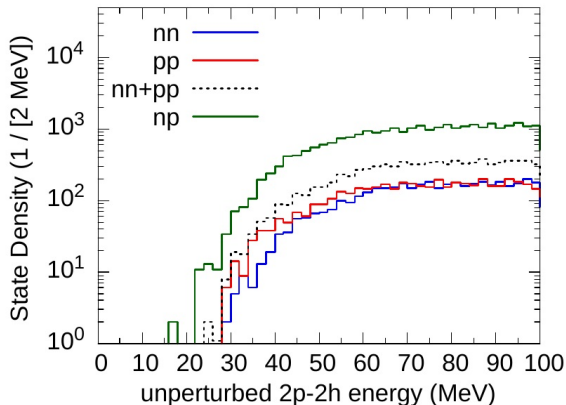


FIG. 12. 0^+ state densities of neutron-neutron (nn), proton-proton (pp), their sum ($nn + pp$) and neutron-proton (np) 2p-2h configurations as functions of the unperturbed 2p-2h energy in ^{16}O . The energy bin width is 2 MeV.

Backup Slides

Backup Slides

Matrices

RPA Matrices (1p-1h configurations)

$$A_{1,1'} = \langle HF | [a_h^\dagger a_p, [H, a_{p'}^\dagger a_{h'}]] | HF \rangle$$

$$B_{1,1'} = -\langle HF | [a_h^\dagger a_p, [H, a_{h'}^\dagger a_{p'}]] | HF \rangle.$$

SRPA Matrices (1p-1h and 2p-2h configurations)

$$A_{1,2} = A_{2,1}^* = \langle HF | [a_h^\dagger a_p, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]] | HF \rangle$$

$$A_{2',2} = \langle HF | [a_{h_2}^\dagger a_{h_1'}^\dagger a_{p_2'} a_{p_1'}, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]] | HF \rangle$$

$$B_{1,2} = B_{2,1}^* = -\langle HF | [a_p^\dagger a_h, [H, a_{p_2}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]] | HF \rangle = 0$$

$$B_{2',2} = -\langle HF | [a_{p_1'}^\dagger a_{p_2'}^\dagger a_{h_1'} a_{h_2'}, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]] | HF \rangle = 0$$

Matrices

RPA Matrices (1p-1h configurations)

$$A_{1,1'} \sim \epsilon_{1ph} + \langle ph|V|ph\rangle$$

$$B_{1,1'} \sim +\langle pp|V|hh\rangle$$

SRPA Matrices (1p-1h and 2p-2h configurations)

$$A_{1,2} \sim \langle ph|V|pp\rangle + \langle hh|V|hp\rangle$$

$$A_{2',2} \sim \epsilon_{2ph} + \langle ph|V|ph\rangle + \langle pp|V|pp\rangle + \langle hh|V|hh\rangle$$

The Subtraction procedure

Large scale SRPA calculations have shown that:

- The SRPA strength distribution is systematically shifted towards lower energies compared to the RPA one
- This shift is very strong ($\simeq 3\text{-}4\text{ MeV}$), RPA description often spoiled

Origins and Causes:

- 1 Quasi Boson Approximation and stability problems in SRPA
- 2 Use of effective interactions in beyond-mean field methods

The Subtraction procedure (I. Tselyaev Phys. Rev. C 75, 024306 (2007))

- Designed for beyond RPA approaches
- It restores the Thouless theorem, e.g. instabilities are removed
- Static ($\omega = 0$) limit of the SRPA imposed to be equal to the RPA one

The Subtraction procedure

From SRPA to an Energy dependent RPA-like problem

- The SRPA problem as an energy-dependent RPA problem

$$A_{1,1'} \mapsto \tilde{A}_{1,1'}(\omega) = A_{1,1'}^{RPA} + \sum_{2,2'} A_{1,2}(\omega + i\eta - A_{2,2'})^{-1} A_{2',1'} = A_{1,1'}^{RPA} + A_{1,1'}^{Cor}(\omega)$$

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The Subtraction procedure is SRPA (SSRPA)

- Subtraction of the zero-frequency limit of the SRPA correction

$$A_{1,1'}^{Cor} \mapsto \tilde{A}_{1,1'}^{Cor}(\omega) = A_{1,1'}(\omega) - A_{1,1'}(\omega = 0) \Rightarrow$$

$$\tilde{A}_{1,1'}(\omega = 0) = A_{1,1'}^{RPA}$$

$$\Rightarrow \Pi^{SSRPA}(\omega = 0) = \Pi^{RPA}$$

The Subtraction procedure

From SRPA to an Energy dependent RPA-like problem

- The SRPA problem as an energy-dependent RPA problem

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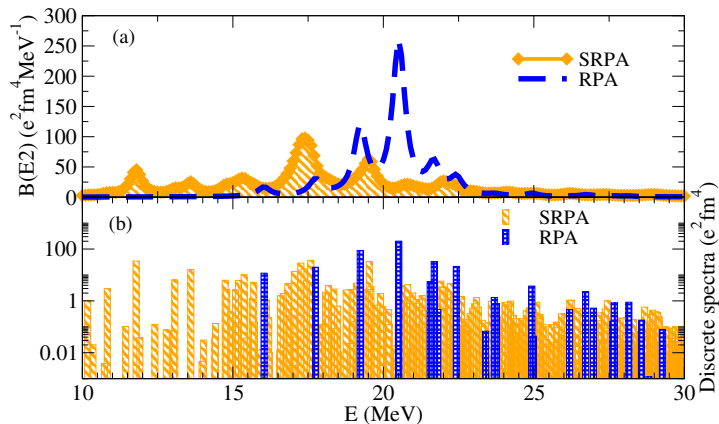
$$\Rightarrow \Pi^{SSRPA}(\omega = 0) = \Pi^{RPA}$$

Numerical implementation

- Subtraction performed in diagonal approximation, e.g. $A_{2,2'} \approx \delta_{2,2'} A_{2,2}$
- Full subtraction recently performed (GT strength) ^a

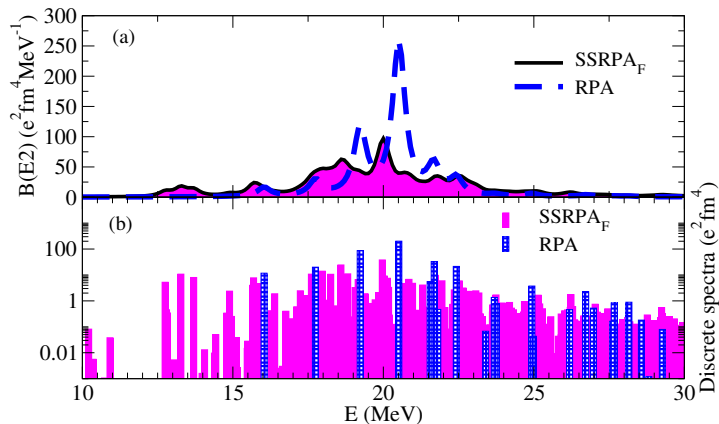
^aDG, M. Grasso, J. Engel, Physical Review Letters 125, 212501 (2020)

Quadrupole Strength Distribution in ^{16}O : RPA, SRPA and SSRPA



D. G., M. Grasso and J.Engel, Phys. Rev. C 92 , 034303 (2015)

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