

Quartet correlations in $N=Z$ nuclei

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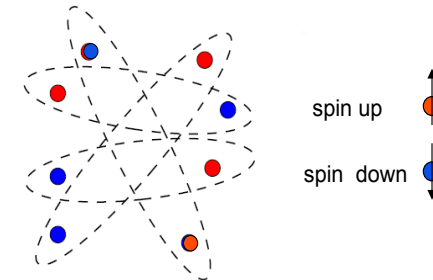
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Outline

- Quartet correlations for neutron pairing ?
- Quartet correlations for proton-neutron pairing
 - pn pairing in quark-meson coupling approach
- Quartet correlations for shell-model interactions
 - band structure in $N=Z$ nuclei in term of quartets
- How to probe quartet correlations ?

Like-particle pairing : PBCS versus exact solution

$$H = \sum_i^{\Omega} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i \geq j}^{\Omega} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$



Cooper pairs $\Gamma^+ = \sum_i x_i a_i^+ a_{\bar{i}}^+ \quad \text{two-body correlations}$

Condensate $|PBCS\rangle = (\Gamma^+)^N |0\rangle \rightarrow |2qp\rangle = \Gamma_{ex}^+ (\Gamma^+)^{N-1} |0\rangle$

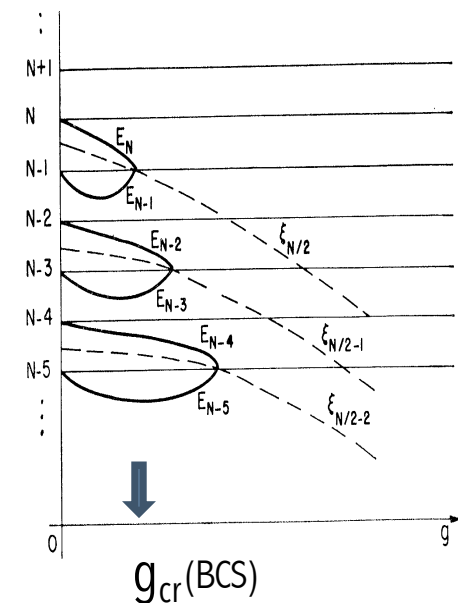
Exact solution (Richardson, 1964)

$$|\Psi_{gs}\rangle = B_1^+ B_2^+ B_3^+ \dots B_{N/2}^+ |0\rangle \quad B_i^+ = \sum_{k=1}^{\Omega} \frac{1}{2\varepsilon_k - E_i} a_k^+ a_{\bar{k}}^+$$

all pairs are different *pairs with real or **complex** E_i*

$$E_{2\lambda} = \xi_\lambda + i\eta_\lambda \quad E_{2\lambda-1} = \xi_\lambda - i\eta_\lambda$$

what is the physical meaning of the pairs with **complex** E_i ?



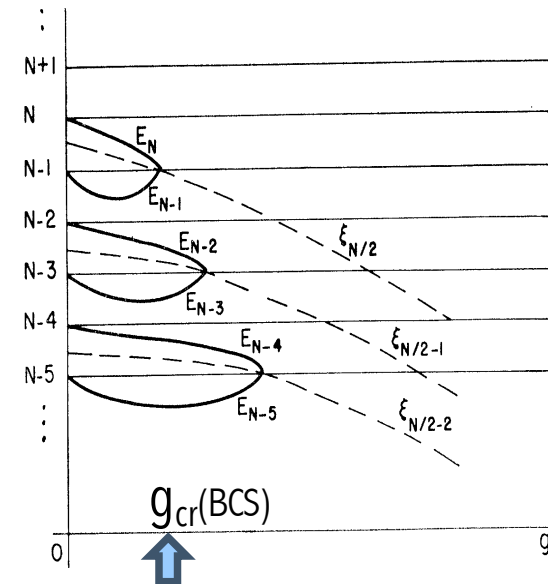
Like-particle pairing and 4-body correlations

complex conjugate pairs

$$E_{2\lambda-1} = \xi_\lambda - i\eta_\lambda, \quad E_{2\lambda} = \xi_\lambda + i\eta_\lambda$$

$$Q_\lambda^\dagger \equiv B_{2\lambda-1}^\dagger B_{2\lambda}^\dagger = \sum_{k,k'=1}^{\Omega} \gamma_{kk'}^{(\lambda)} P_k^\dagger P_{k'}^\dagger$$

4-body correlations !



Exact solution

$$|\Psi_{gs}\rangle = Q_1^+ Q_2^+ \dots Q_{n_q}^+ B_{2n_q}^+ B_{2n_q-1}^+ \dots B_{N/2}^+ B_{N/2-1}^+ |-\rangle$$

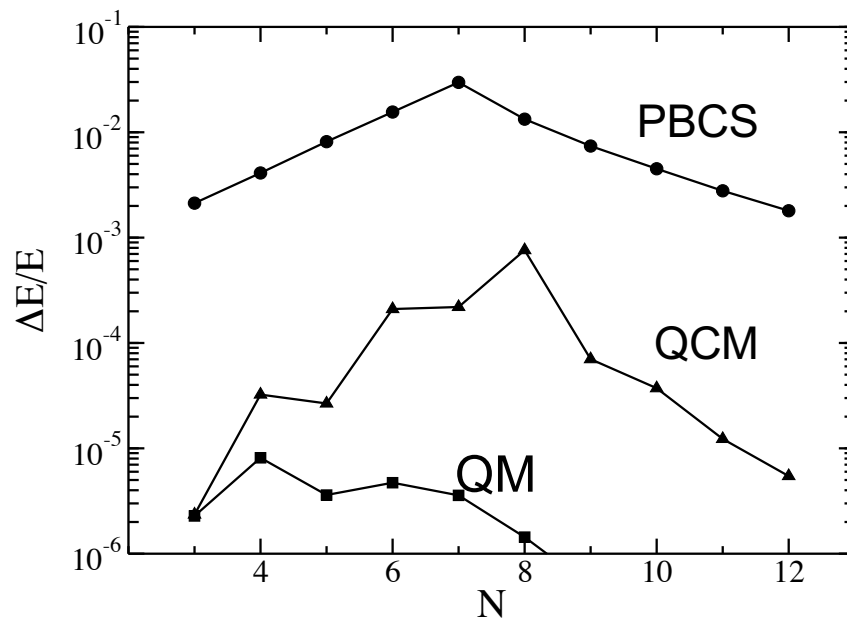
restriction around ε_F : only quartets $\longrightarrow$ quartet condensate ?

neutron skins/halos made of quartets ?

Like-particle pairing with quartets: Sn isotopes*

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}(i, j) P_i^+ P_j \quad Q_\lambda^\dagger \equiv \sum_{k, k'=1}^{\Omega} \gamma_{kk'}^{(\lambda)} P_k^\dagger P_{k'}^\dagger$$

$$|QM\rangle = Q_1^+ Q_2^+ \dots Q_{n_q}^+ |-\rangle \quad |QCM\rangle = Q^{+n_q} |-\rangle$$



QCM accurate for a general pairing force !

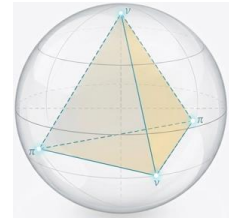
QCM calculations for neutron skin/halos ?

with continuum Skyrme-HF

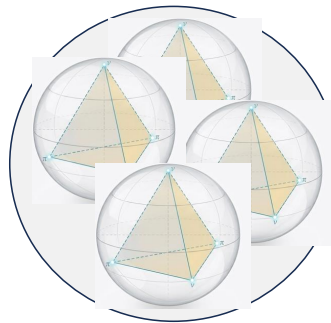
* M. Sambataro and N. S., J Phys G (2013)

quartets : 4-body structures of two neutrons and two protons
correlated in configuration space (e.g., isospin, spin)

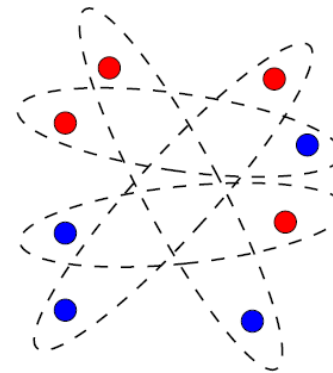
$$Q^\dagger = \sum x_{ijkl} (\nu_i^\dagger \nu_j^\dagger \pi_k^\dagger \pi_l^\dagger)^{T=0}$$



quartets



Cooper pairs



overlaps in coordinate space

EFFECT OF QUADRUPLE CORRELATIONS IN LIGHT NUCLEI

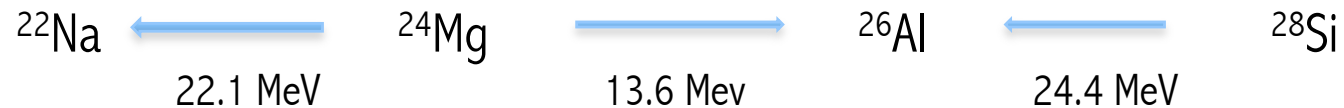
V G SOLOVIEV

Joint Institute of Nuclear Research, Dubna, USSR

Received 25 December 1959

Fingerprints of alpha-like (quadruple) correlations

- 1) Extracting a pn pair from a even-even N=Z nucleus costs more energy than adding to it a pn pair



- 2) Extracting one neutron from a even-even N=Z nucleus costs more energy than from neighbouring nuclei

$$B(^{24}\text{Mg}) - B(^{23}\text{Mg}) = 16.6 \text{ MeV}$$

$$B(^{25}\text{Mg}) - B(^{24}\text{Mg}) = 7.3 \text{ MeV}$$

$$B(^{26}\text{Mg}) - B(^{25}\text{Mg}) = 11.3 \text{ MeV}$$

Theoretical studies on pn pairing & alpha correlations

B. H. Flowers and M. Vijić, NPA49(1963)

$$\Phi = \prod_{ll'mm'} \prod_{pqrs}' (U_l + V_{ll'} \epsilon_{pqrs} a_{lp}^\dagger a_{l-mq}^\dagger a_{l'm'}^\dagger a_{l'-m's}^\dagger) |0\rangle$$

not actually solved !

B. Bremond and J. G. Valatin NP41(1963)

$$\prod_{\alpha} (S_{\alpha} + V_{\alpha p} a_{\alpha p}^\dagger a_{\bar{\alpha} p}^\dagger + V_{\alpha n} a_{\alpha n}^\dagger a_{\bar{\alpha} n}^\dagger + T_{\alpha} a_{\alpha p}^\dagger a_{\bar{\alpha} p}^\dagger a_{\alpha n}^\dagger a_{\bar{\alpha} n}^\dagger) |0\rangle$$

not included all relevant configurations !

J. Eichler and M. Yamamura, NPA182(1972)

non-collective quartets

R. Chasman, PLB577(2003)

J. Dobes and S. Pittel PRC57(1998)

quartet condensation for a degenerate state

R. A. Senkov and V. Zelevinski (2011)

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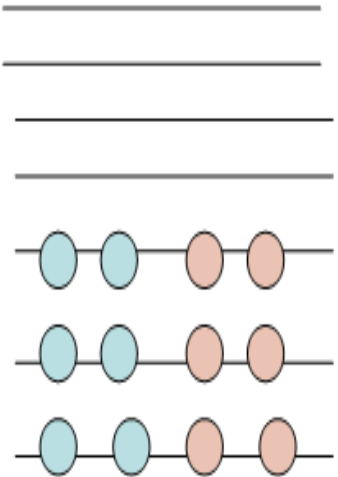
almost all studies on pn pairing: in BCS/HFB approximation

Goodman , ... , Bertsch & Gezerlis

drawbacks: particle-number, spin and isospin are not conserved !

Isovector (T=1) and isoscalar (T=0) pairing in N=Z nuclei

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i, j) \sum_{\tau} P_{i\tau}^+ P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i, j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}$$

$P_{i,T_z}^+ = [a_i^+ a_i^+]_{T_z}^{T=1, J=0}$	N=Z 	$D_{ij,J_z}^+ = [a_i^+ a_j^+]_{J_z}^{J=1, T=0}$
$\Gamma_{\pi\nu}^+ = \sum_i x_i (\nu_i^+ \pi_i^+ + \pi_i^+ \nu_i^+)$		$\Delta_0^+ = \sum_i x_i (\nu_i^+ \pi_i^+ - \pi_i^+ \nu_i^+)$
$(\Gamma_{\nu\pi}^+)^{N_{\pi\nu}/2}$		$(\Delta_0^+)^{N_{\pi\nu}/2}$

not well-defined isospin

not well-defined spin/J

Isospin and spin projections

$$P_{T=0}(\Gamma_{\nu\pi}^\dagger)^{\frac{N+Z}{2}} \rightarrow (2\Gamma_{\nu\nu}^\dagger \Gamma_{\pi\pi}^\dagger - \Gamma_{\nu\pi}^\dagger \Gamma_{\nu\pi}^\dagger)^{\frac{N+Z}{4}} \equiv (Q_{T=1}^\dagger)^{n_q}.$$

$$P_{J=0}(\Delta_0^\dagger)^{\frac{N+Z}{2}} \rightarrow (2\Delta_1^\dagger \Delta_{-1}^\dagger - \Delta_0^\dagger \Delta_0^\dagger)^{\frac{N+Z}{4}} \equiv (Q_{T=0}^\dagger)^{n_q}$$

quartet condensates

a particular case of the [Quartet Condensation Model \(QCM\)](#)

Isovector (T=1,J=0) and isoscalar (J=1,T=0) pairing in QCM

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i, j) \sum_{\tau} P_{i\tau}^+ P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i, j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}$$

isovector

$$P_{i,T_z}^+ = [a_i^+ a_i^+]_{T_z}^{T=1, J=0}$$

collective quartets

$$Q_{\nu}^{+(iv)} = \sum_{i,j} x_{ij}^{(\nu)} [P_i^+ P_j^+]^{T=0}$$

generalised quartet

$$Q_{\nu}^+ = Q_{\nu}^{+(iv)} + Q_{\nu}^{+(is)}$$

ground state

$$|QCM\rangle = Q^{+n_q} | - \rangle$$

conserves exactly all the symmetries

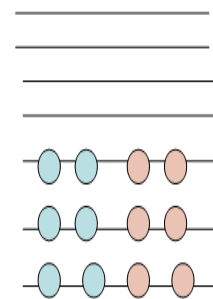
exact solution for degenerate levels (Dobes & Pittel PRC (1998))

isoscalar

$$D_{ij,J_z}^+ = [a_i^+ a_j^+]_{J_z}^{J=1, T=0}$$

$$Q_{\nu}^{+(is)} = \sum_{ij,kl} y_{ij,kl}^{(\nu)} [D_{ij}^+ D_{kl}^+]^{J=0}$$

N=Z



Quartet condensation versus pair condensation

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i, j) \sum_{\tau} P_{i\tau}^+ P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i, j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}$$

pairing forces: (T=1,J=0) and (T=0,J=1) components of shell model interactions

	$(Q^+)^{n_q} - \rangle$	$(\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+)^{n_q} - \rangle$	$(\Gamma_{\nu\pi}^+)^{2n_q} - \rangle$	$(\Delta_0^+)^{2n_q} 0 \rangle$
²⁰ Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
²⁴ Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
²⁸ Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
⁴⁴ Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
⁴⁸ Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
⁵² Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
¹⁰⁴ Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
¹⁰⁸ Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
¹¹² Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

correlations energies and errors relative to exact results

- isoscalar pairing condensation is not a good approximation !
- T=1 and T=0 pairing correlations always coexist in quartets

Excited states of T=1 & T=0 pairing in QCM

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i, j) \sum_{\tau} P_{i\tau}^+ P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i, j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}$$

- ground state $|\Psi_{gs}\rangle = (Q_{ivs}^+)^{n_q} |0\rangle$. $Q_{ivs}^+ = Q_{iv}^+ + Q_{is}^+$,
- excited states $|\Phi_{v, JJ_z}\rangle = \tilde{Q}_{v, JJ_z} (Q_{ivs}^+)^{n_q-1} |-\rangle$,

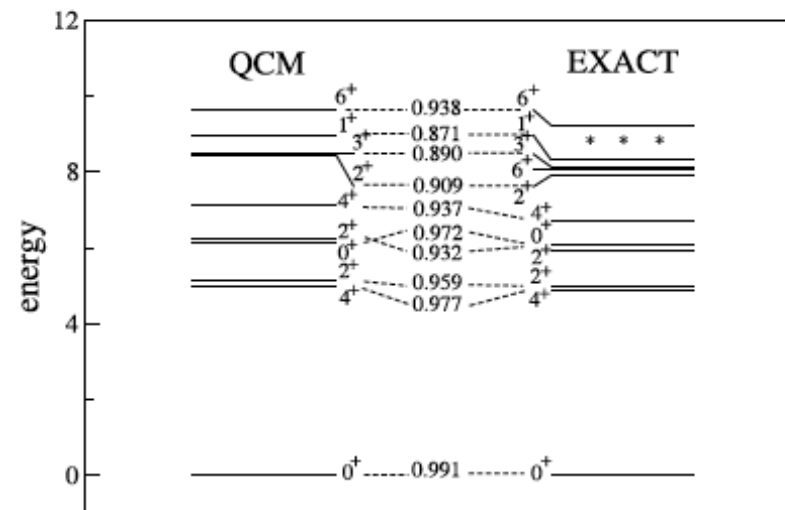


Fig. 4. The low-lying spectrum provided by the QCM approximation (24) for the valence nucleons of ^{28}Si interacting by an isovector-isoscalar pairing force extracted from the USDB interaction. The numbers are the overlaps between the QCM and the exact wave functions. Energies are in MeV.

Isoscalar – isovector pairing in axially deformed mean-fields

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P_{i,t}^+ P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D_{i,0}^+ D_{j,0}$$

pairing in time-reversed deformed states

$$Q_{T=1}^+ = \sum_{ij} x_i x_j [P_{i\tau}^+ P_{j\tau'}^+]^{T=0} \quad \Delta_0^+ = \sum y_i D_{i,0}^+$$

- Ground state $|\Psi\rangle = (Q_{T=1}^+ + \Delta_0^{+2})^{n_q} |-\rangle$

exact solution for degenerate states

non-degenerate levels: errors under 1%

(N.S, D.Negrea, D. Gambacurta, Phys. Lett. B 2015)



- Applications : Skyrme-HF+ QCM (D. Negrea, N.S , D. Gambacurta, PRC105 (2022))

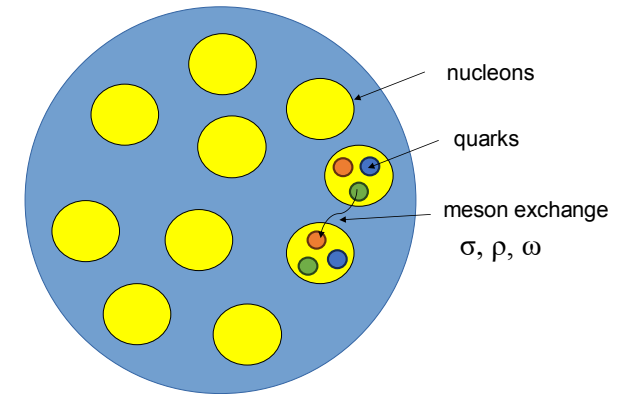
Quark-Meson-Coupling+ QCM (T. Popa, N.S, D. Gambacurta, J.Phys.G, in press)

Quark Meson Coupling approach

- Quark-Meson Coupling (QCM) model : basic assumptions

P. Guichon, PLB(1988); P. Guichon, J. Stone, A. W. Thomas, Progr. Part. and N.P. (2018)

*nucleons described as clusters of 3 quarks (MIT bag model);
the quarks interact by the exchange of σ , ρ and ω mesons*



- QMC energy density functional (EDF): Skyrme- like structure

P. Guichon, H. Metosovian, N. Sandulescu, A. W. Thomas,, NPA (2006)

spin-orbit, tensor and pairing interactions derived within QMC

depends only on 5 parameters

- Systematic QMC calculations of nuclei :

K. L. Martinez, A. W. Thomas, J. R. Stone, P. Guichon PRC102 (2020)

accuracy comparable with the best Skyrme calculations !

no proton-neutron pairing

predictions for a T=0 phase ?

Proton-neutron pairing interaction in QMC

- Isovector pairing force: **derived from QMC**

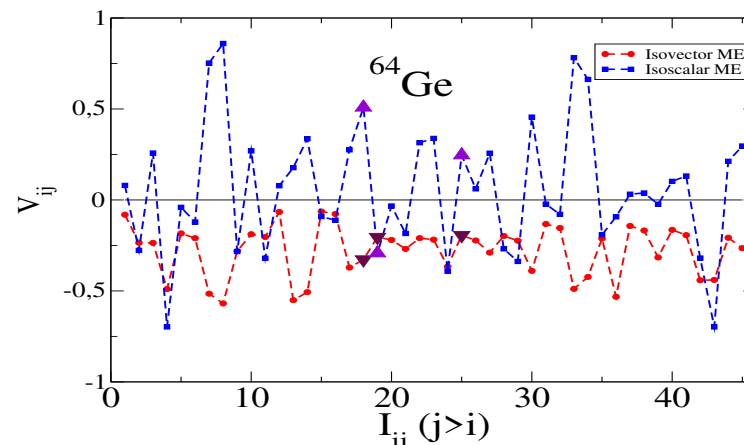
$$V^{T=1} = -s \left(\frac{G_\sigma}{1 + d' G_\sigma \rho(\vec{r})} - G_\omega - \frac{G_\rho}{4} \right) \delta(\vec{r} - \vec{r}')$$

depends of quark-meson couplings $G_i = \frac{g_i^2}{m_i^2}$

s : scaling fixed from OEMD of neutron-rich nuclei

- Isoscalar pairing force: $V^{T=0} = w V^{T=1}$ $w=1.6$

Example: off diagonal matrix elements for ^{64}Ge

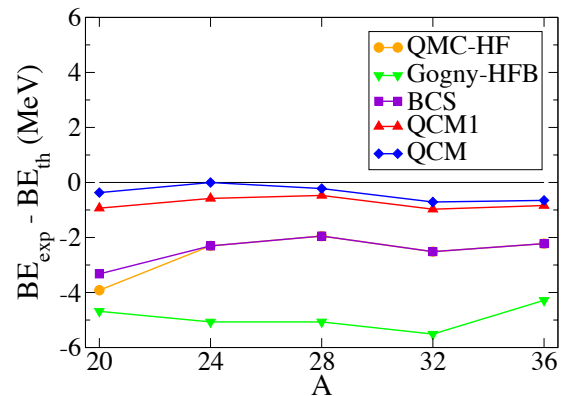


isocalar force has repulsive m.e $\Rightarrow$ isoscalar pairing suppression !

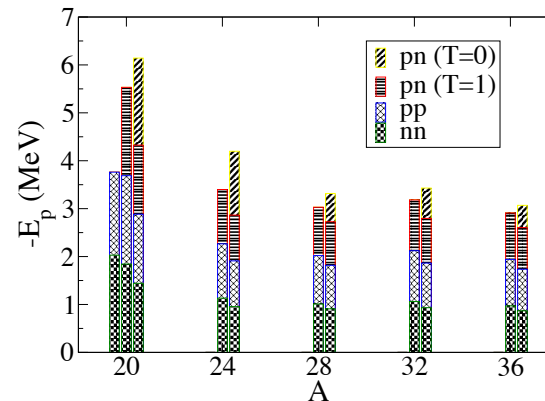
QMC+QCM calculations for N=Z nuclei: sd-shell

A. Popa, N.S, D. Gambacurta, J. Phys. G, in press

Binding energy residuals



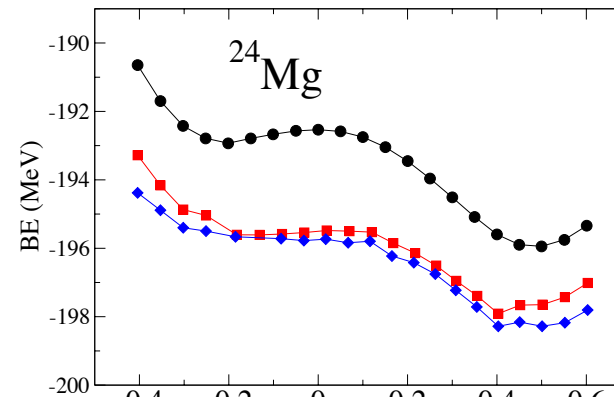
Pairing energies



Quadrupole deformation

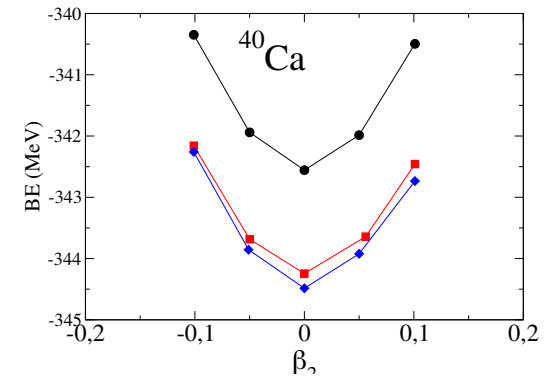
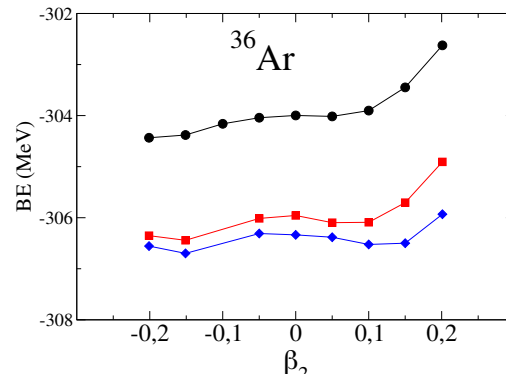
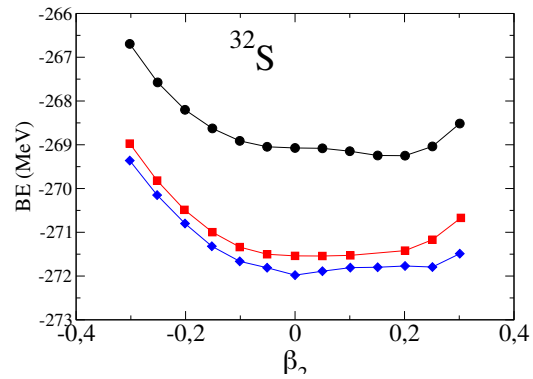
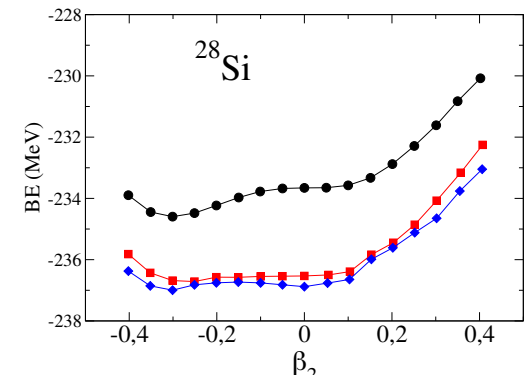
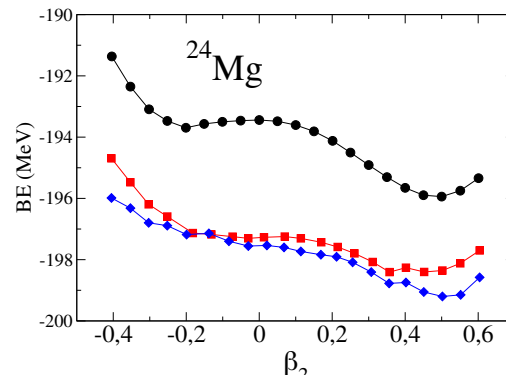
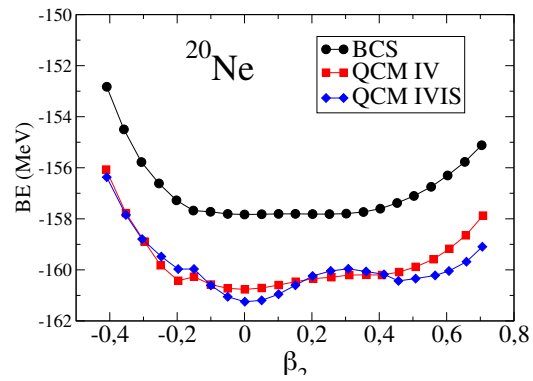
A	QCM	FRDM	Exp
20	0.474	0.36	0.720
24	0.478	0.39	0.606
28	-0.308	-0.36	0.412
32	0.193	0.22	0.314
36	-0.18	-0.26	0.353

BE versus deformation



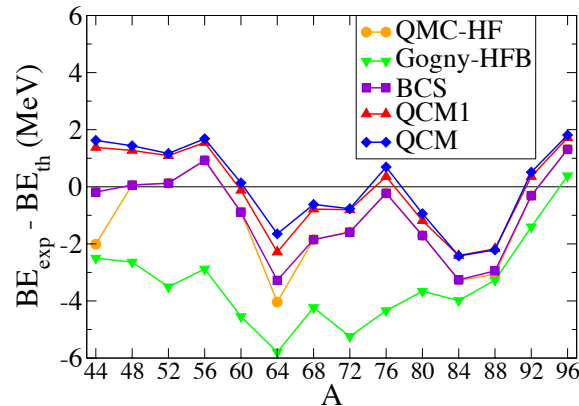
T=1 & T=0 pairing coexist and compete in all nuclei

Binding energy function of deformation: sd-shell nuclei

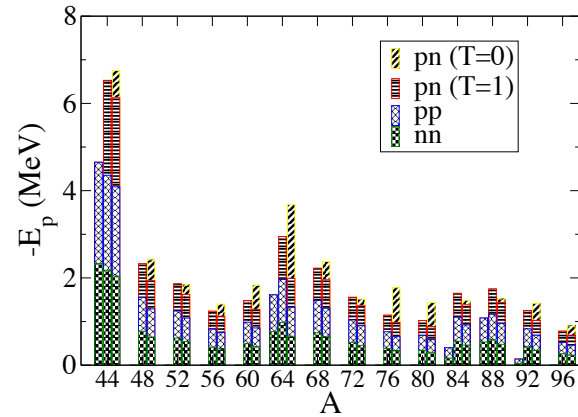


QMC+QCM calculations for N=Z nuclei: pfg-shell

Binding energy residuals



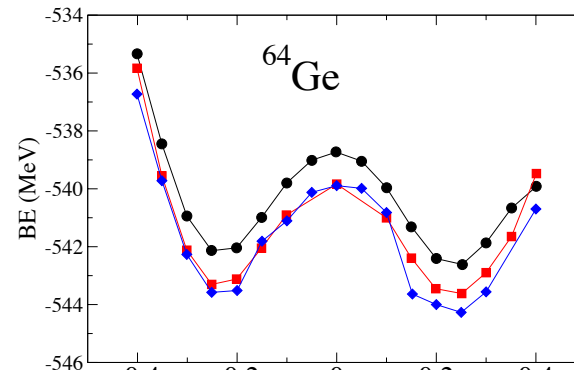
Pairing energies



Quadrupole deformation

A	QCM	FRDM	Exp
44	0.000	0.00	0.288
48	0.291	0.23	0.351
52	0.230	0.12	0.229
56	0.004	0.00	0.127
60	0.228	0.16	-
64	0.238	0.21	0.258
68	-0.259	0.23	0.242

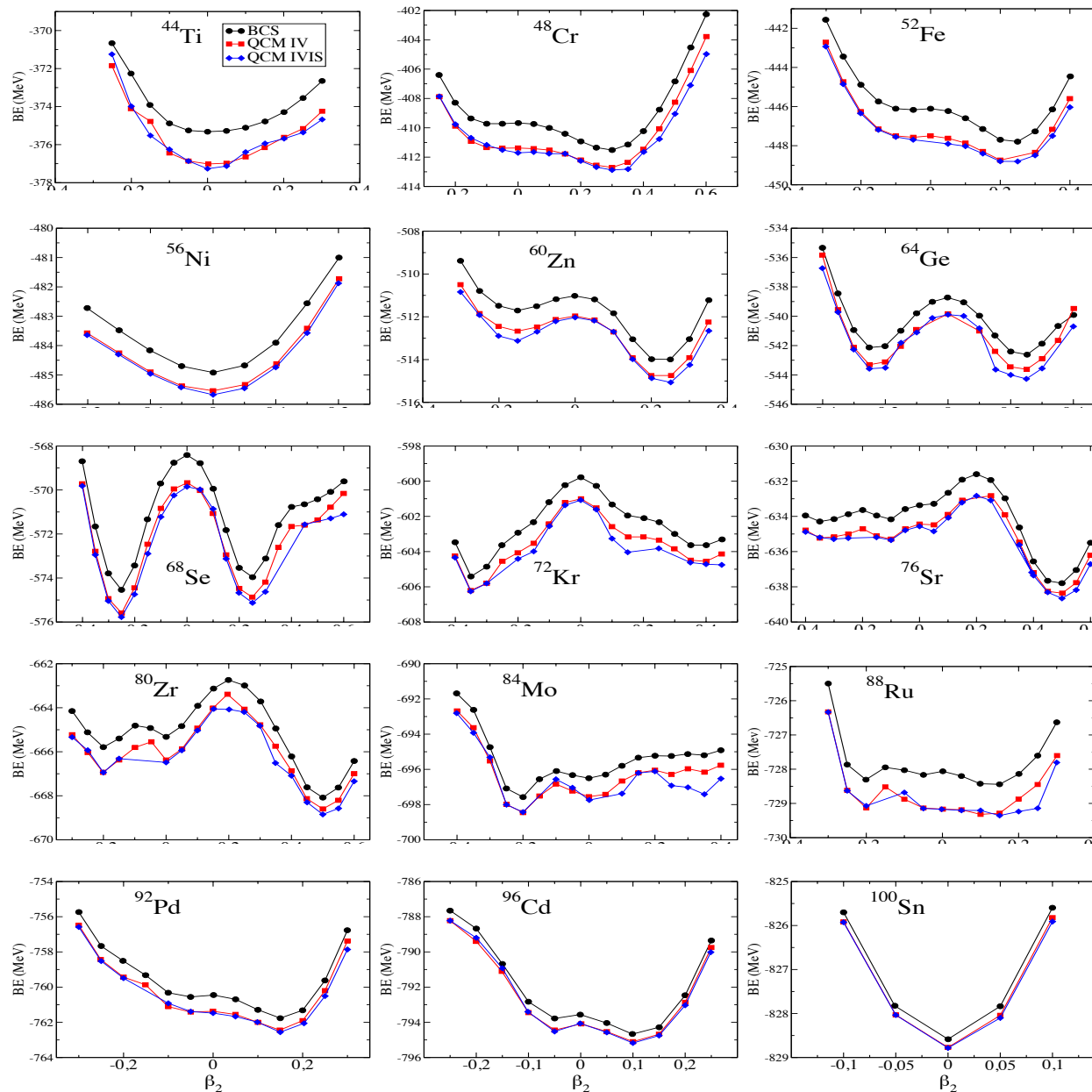
BE versus deformations



T=1 & T=0 pairing coexist in all nuclei

T=1 has dominant contribution

Binding energies function of deformation: pfg-shell nuclei



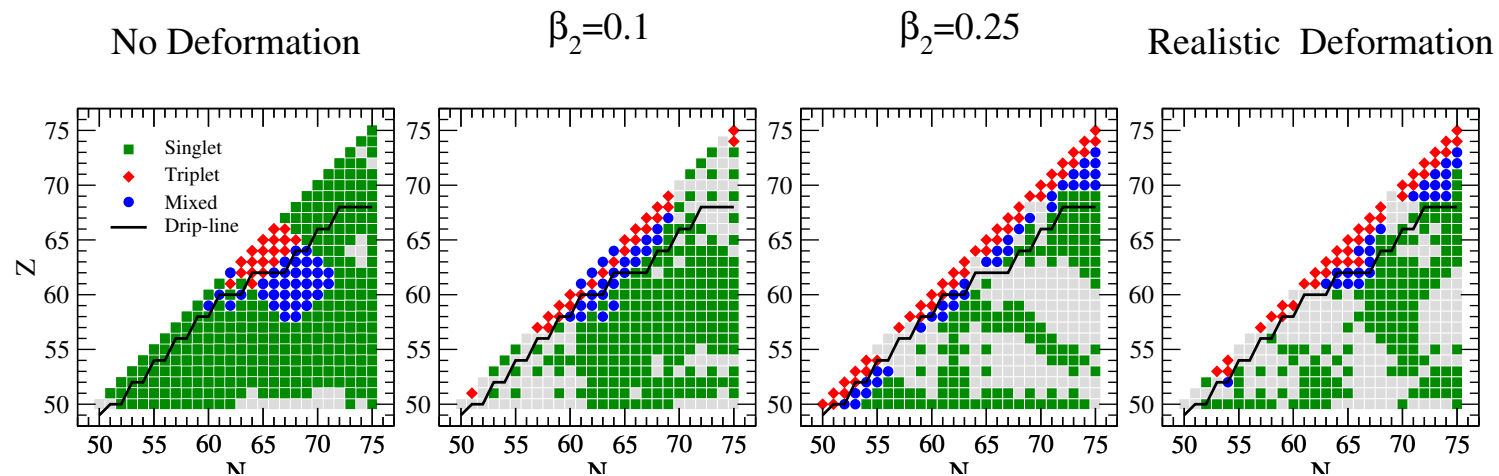
Isoscalar pairing in heavy nuclei above ^{100}Sn ?

HFB versus QMC+QCM calculations

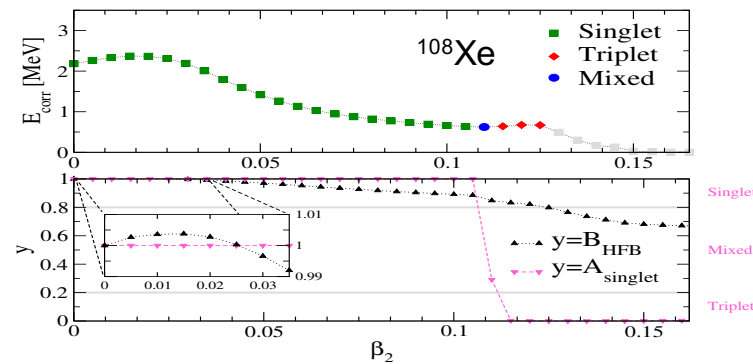
HFB calculations with proton-neutron pairing

A. Gezerlis, G. Bertsch, PRL (2011); G. Palkanoglou, M. Stuck, A. Gezerlis, PRL (2025)

- mean-field: Woods-Saxon + spin-orbit
- pairing force $V = V_0 \delta(\mathbf{r}-\mathbf{r}')$, $V^{T=0} = w V^{T=1}$ $w=1.38$



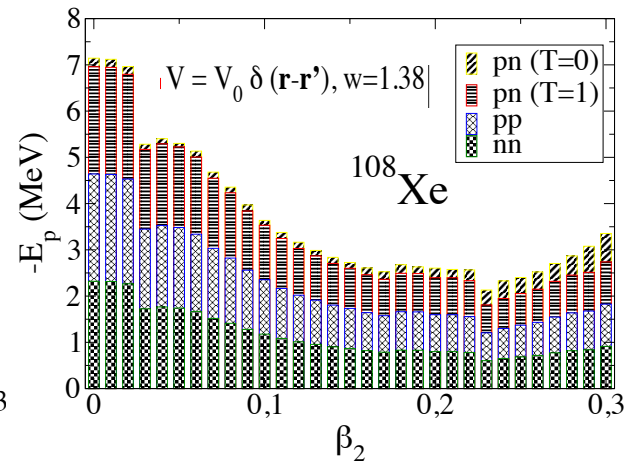
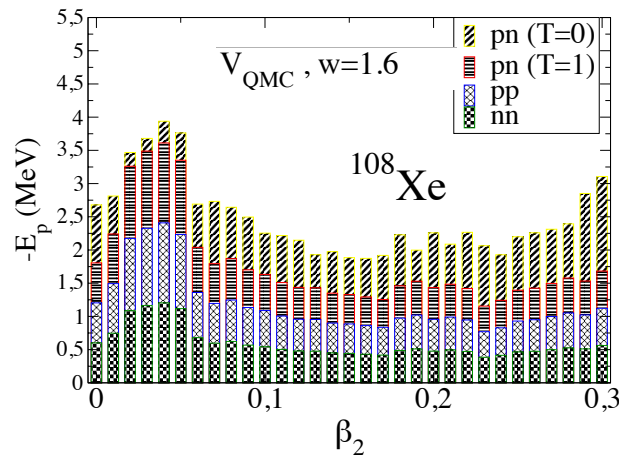
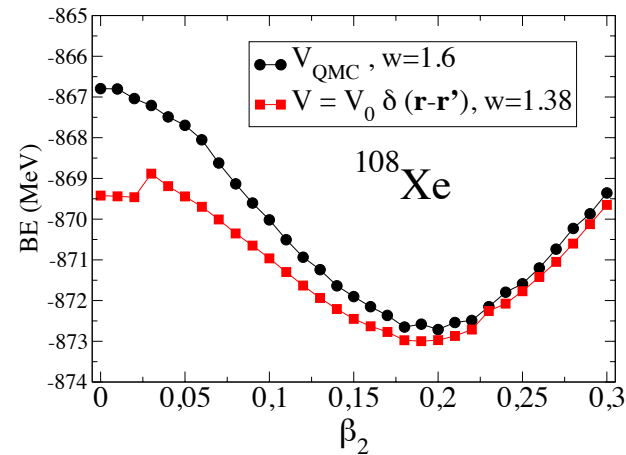
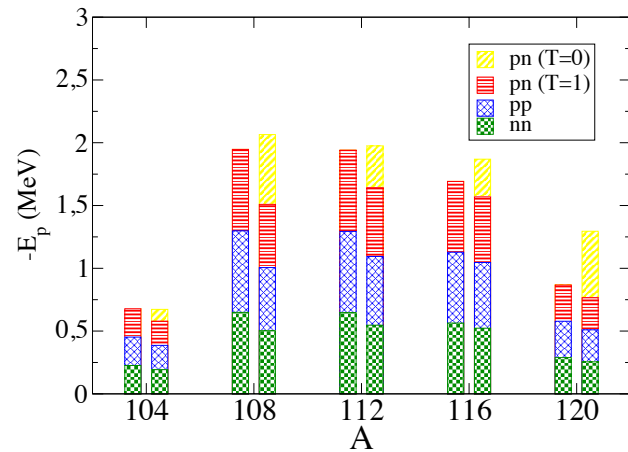
Gezerlis & Bertsch, PRL(2011)



fast transition to triplet pairing
around $\beta_2=0.12$

QMC+QCM results for $A > 100$ nuclei

Th. Popa, N. S., D. Gambacurta, J. Phys. G, in press



no dominant $T=0$ pairing phase in QMC+QCM calculations

Quartet correlations for general two-body interactions

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii', jj', J', T'} V_{JT}(ii'; jj') [A_{ii' J' T'}^+ A_{jj' J' T'}]^{J=0, T=0}$$

sd-shell: USDB

pf-shell: KB3G

Quartet correlations for general forces

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii', jj', J', T'} V_{JT}(ii'; jj') [A_{ii' J' T'}^+ A_{jj' J' T'}]^{J=0, T=0}$$

$$|QCM\rangle = |Q^{+n_q}\rangle \quad Q^+ = \sum_{ii', jj', JT} x_{ii', jj'} [A_{ii' JT}^+ A_{jj' JT}^+]^{0,0}$$

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$\langle SM QCM\rangle$
^{20}Ne	24.77	24.77	1
^{24}Mg	55.70	53.04 (4.77%)	0.85
^{28}Si	88.75	86.52 (2.52%)	0.86
^{32}S	122.51	122.02 (0.40%)	0.98

$\langle Q H Q\rangle$	$\langle Q V Q\rangle$
E_Q	
-40.47	(-28.74)
-39.96	(-27.53)
-37.58	(-23.46)
-37.54	(-23.33)

BE(α)=28.4MeV

$$E(n_q) = n_q \times E(1) + \frac{n_q(n_q - 1)}{2} \times V(n_q),$$

$$V(2)=-3.5, V(3)=-4.1; V(4)=-3.34 \text{ MeV} \quad (V>0 \text{ for } V_p !)$$

quartets acts as weakly interacting building blocks

Excited quartet condensate in ^{32}S ?

- ground state $|QCM\rangle = |Q^{+n_q}\rangle \rightarrow Q^+ = \sum_{ii',jj',JT} x_{ii',jj'} [A_{ii',JT}^+ A_{jj',JT}^+]^{0,0}$
- excited state $|\overline{QCM}\rangle = |\overline{Q}^{+n_q}\rangle \rightarrow \overline{Q}^+ = \sum_{ii',jj',JT} \overline{x}_{ii',jj'} [A_{ii',JT}^+ A_{jj',JT}^+]^{0,0}$

	$E_{0_1^+}(SM)$	$E_{0_1^+}(QCM)$	$\langle SM QCM\rangle$
^{32}S	-178.98 (3.46)	-178.04 (3.92)	0.95

experimental energy : 3.778 MeV

Ground and ground-state band with intrinsic quartets

- intrinsic quartet $Q_g^\dagger = \alpha_{g0}(q_g^\dagger)_{00} + \alpha_{g2}(q_g^\dagger)_{20} + \alpha_{g4}(q_g^\dagger)_{40}.$

$$(q_g^\dagger)_{J0} = \sum_{\substack{i_1 j_1 J_1 \\ i_2 j_2 J_2 \\ T}} q_{i_1 j_1 J_1, i_2 j_2 J_2, T}^{(g)} \left[[a_{i_1}^\dagger a_{j_1}^\dagger]_{J_1 T'} [a_{i_2}^\dagger a_{j_2}^\dagger]_{J_2 T'} \right]_{J, T=0; M=0}.$$
- condensate of intrinsic quartet $|\Theta_g\rangle = (Q_g^\dagger)^n |0\rangle$
- projected states $\bar{P}_J |\Theta_g\rangle \quad J=0, 2, 4, 6, \dots$

Example: ^{24}Mg

$$Q_g^\dagger = 0.74259(q_g^\dagger)_{00} + 0.65203(q_g^\dagger)_{20} + 0.15301(q_g^\dagger)_{40},$$

ground state energy

$E_{\text{intr}}(g) = -85.19401 \text{ MeV.}$	$E_{\text{proj}}(0_1^+) = -86.2533 \text{ MeV.}$	$E_{\text{SM}}(0_1^+) = -87.10445 \text{ MeV,}$
---	--	---

error in correlation energy: 1.4%

$$\langle 0_{\text{SM}} | 0_{\text{proj}} \rangle = 0.960$$

Ground- state band for ^{24}Mg

$$|\Theta_g\rangle = (Q_g^+)^n|0\rangle$$

$$P_J|\Theta_g\rangle \quad J=0, 2, 4, 6$$

J^π	E_x^{proj} (MeV)	E_x^{SM} (MeV)	E_x^{exp} (MeV)	$\langle J_{\text{SM}} J_{\text{proj}}\rangle$
0^+	0.0000	0.000	0.00	0.960
2^+	1.2210	1.502	1.369	0.975
4^+	4.3072	4.112	4.123	0.959
6^+	8.6066	8.269	8.113	

Transition	$P_J \Theta_g\rangle$	SM	Exp
$2_1^+ \rightarrow 0_1^+$	90.8	95	88(4)
$4_1^+ \rightarrow 2_1^+$	125.3	124	160(16)
$6_1^+ \rightarrow 4_1^+$	118.1	140	115

N.S, in preparation

Band-like structures in N=Z nuclei based on quartets

- “Intrinsic” ground state: *quartet condensate*

$$|\Theta_g\rangle = (Q_g^+)^n |0\rangle, \quad Q_g^+ = \sum_J \alpha_{g,J} (q_g^+)_{J0} \quad J=0,2,4$$

ground state and ground state band

$$\hat{P}_J |\Theta_g\rangle \quad J=0,2,4,6 \dots$$

- Intrinsic excited states: *one - broken - quartet states*

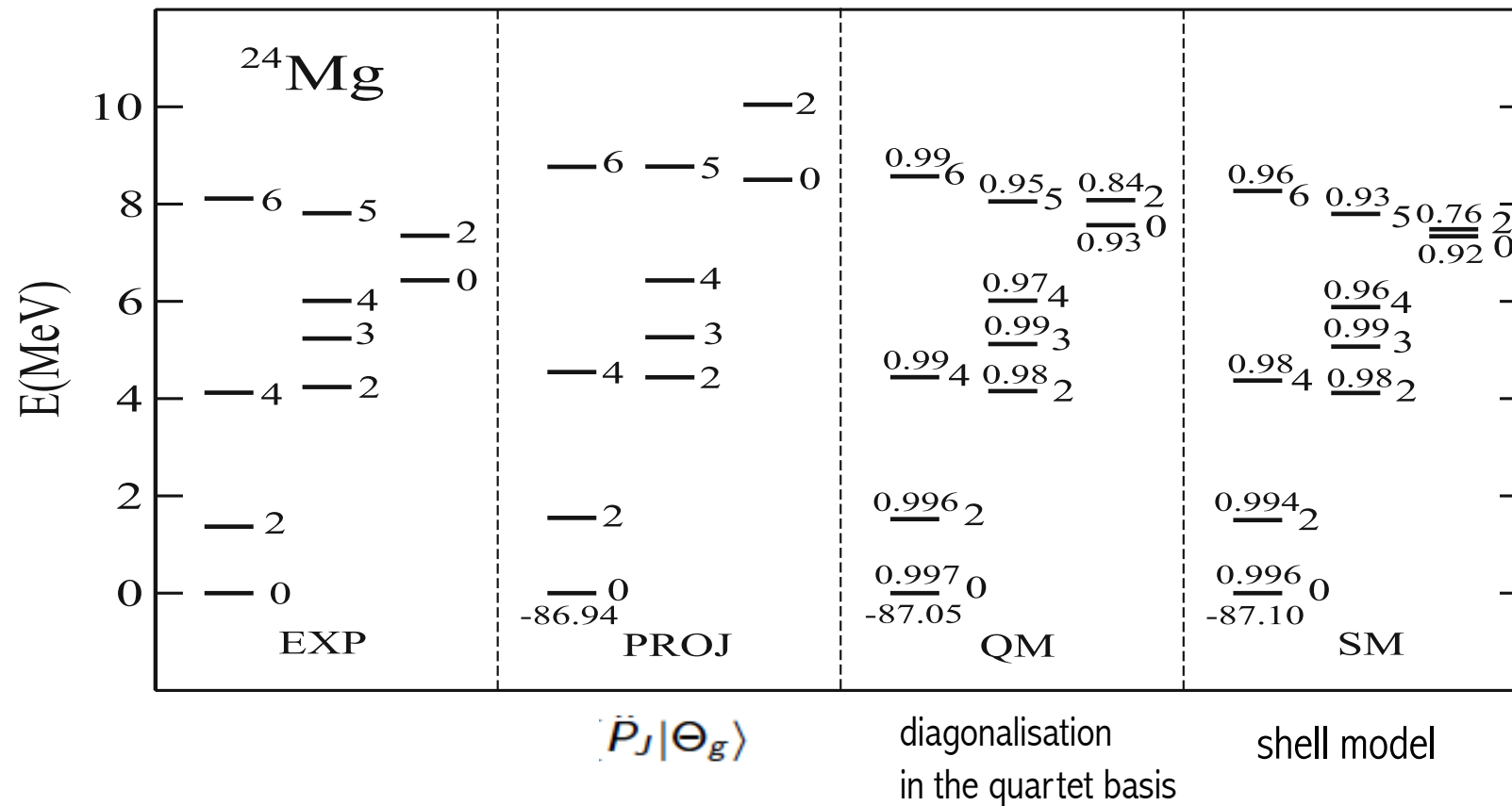
$$|\Theta_k\rangle = Q_k^\dagger (Q_g^\dagger)^{(n-1)} |0\rangle, \quad Q_k^\dagger = \sum_J \alpha_{k,J} (q_k^\dagger)_{Jk}$$

$$k = 0 \rightarrow \text{“}\beta\text{”}, \quad k = 2 \rightarrow \text{“}\gamma\text{”}, \dots$$

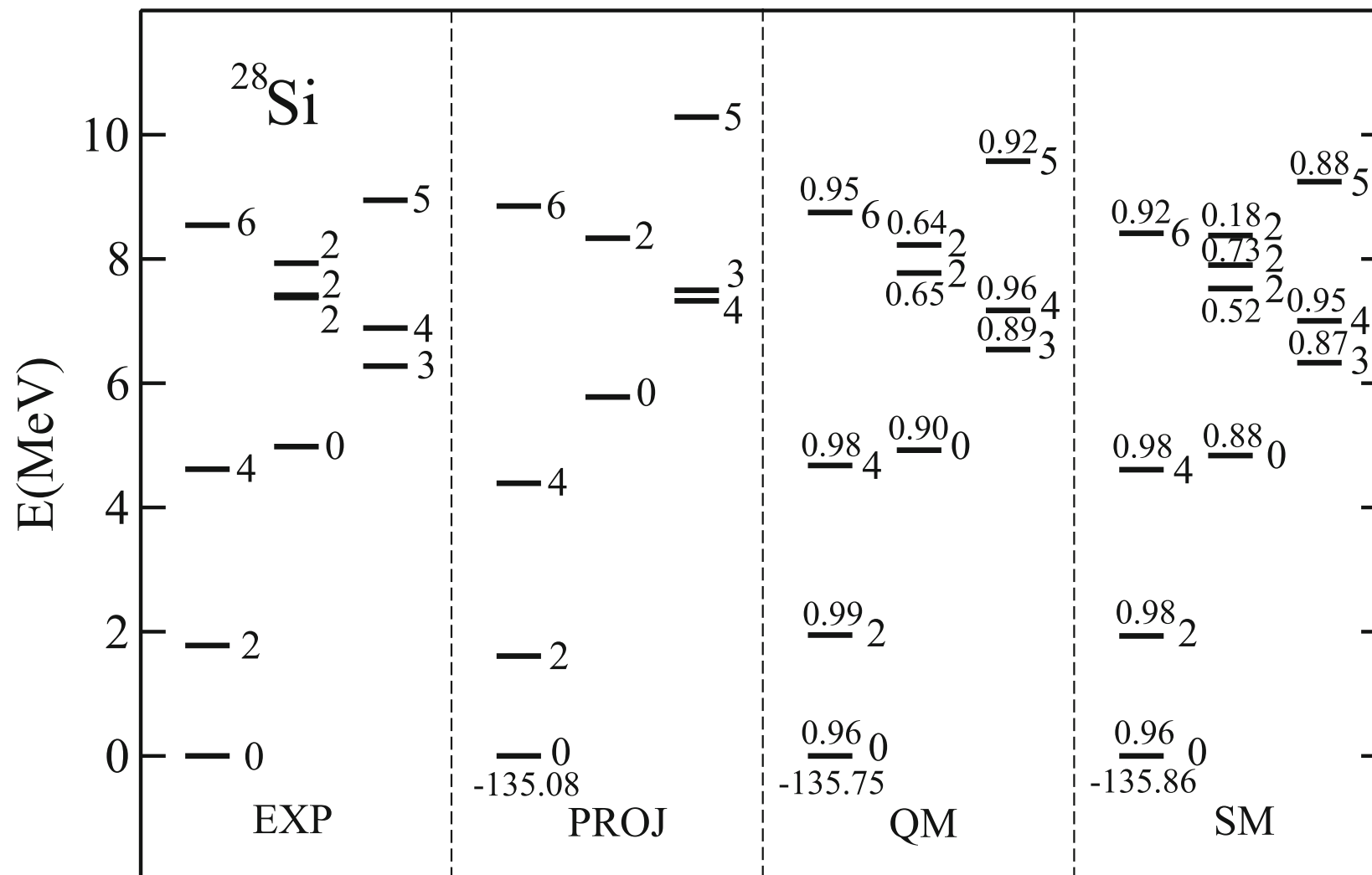
band structures

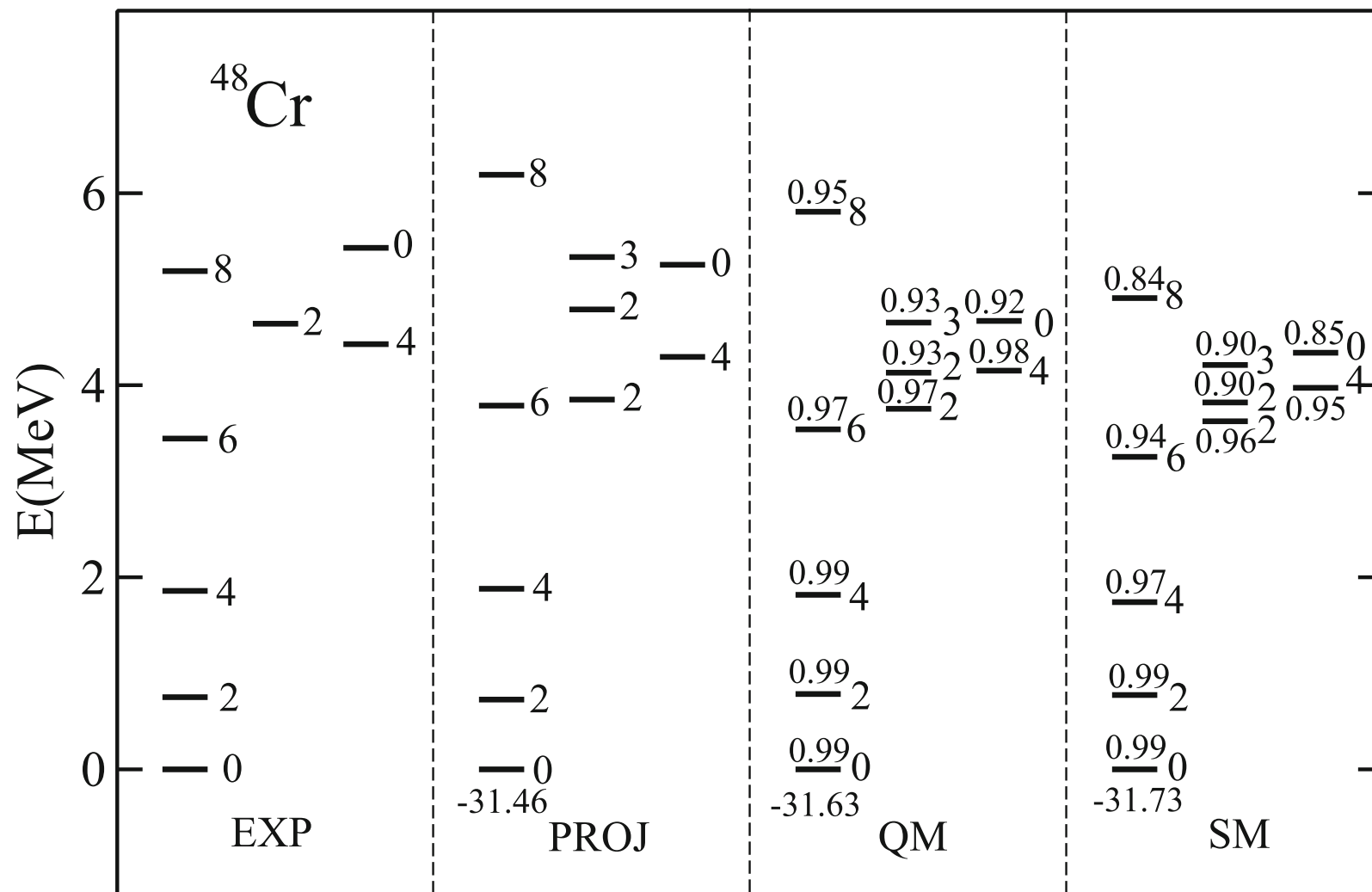
$$\hat{P}_J |\Theta_k\rangle$$

Band structures generated from quartet states: ^{24}Mg



- band-like structures similar to experimental bands
- ground state energy very close to exact SM
- large overlaps with the shell-model exact states





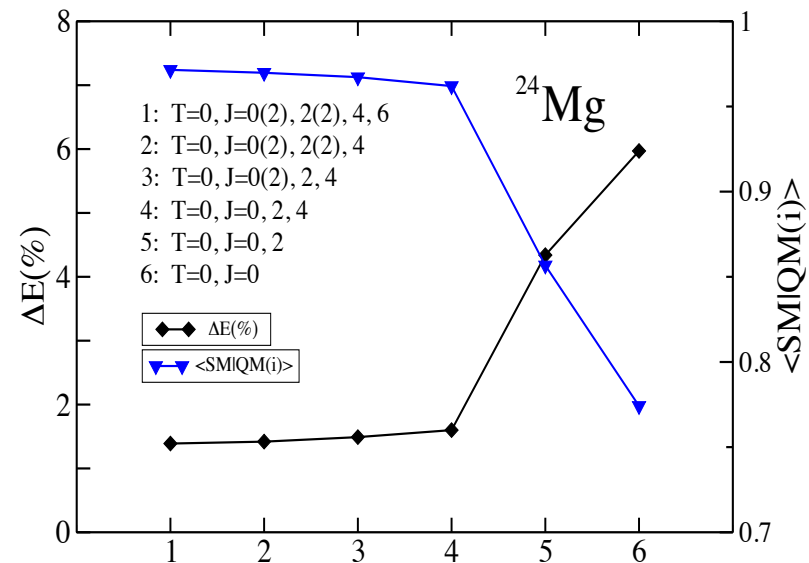
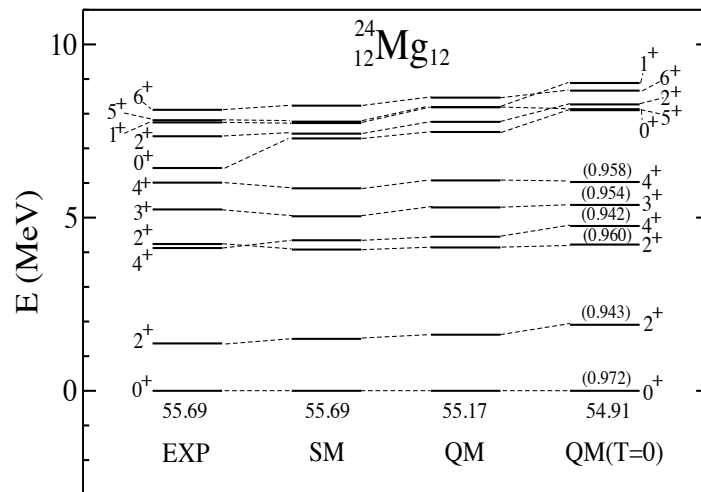
Four-body correlations in N=Z **and** N>Z nuclei

diagonalisation in a quartet basis

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii', jj', J', T'} V_{JT} (ii'; jj') [A_{ii' J' T'}^+ A_{jj' J' T'}]^{J=0, T=0}$$

$$|QM; J\rangle = [Q_{J_1}^{(\nu_1)+} Q_{J_2}^{(\nu_2)+}]^J |core\rangle$$

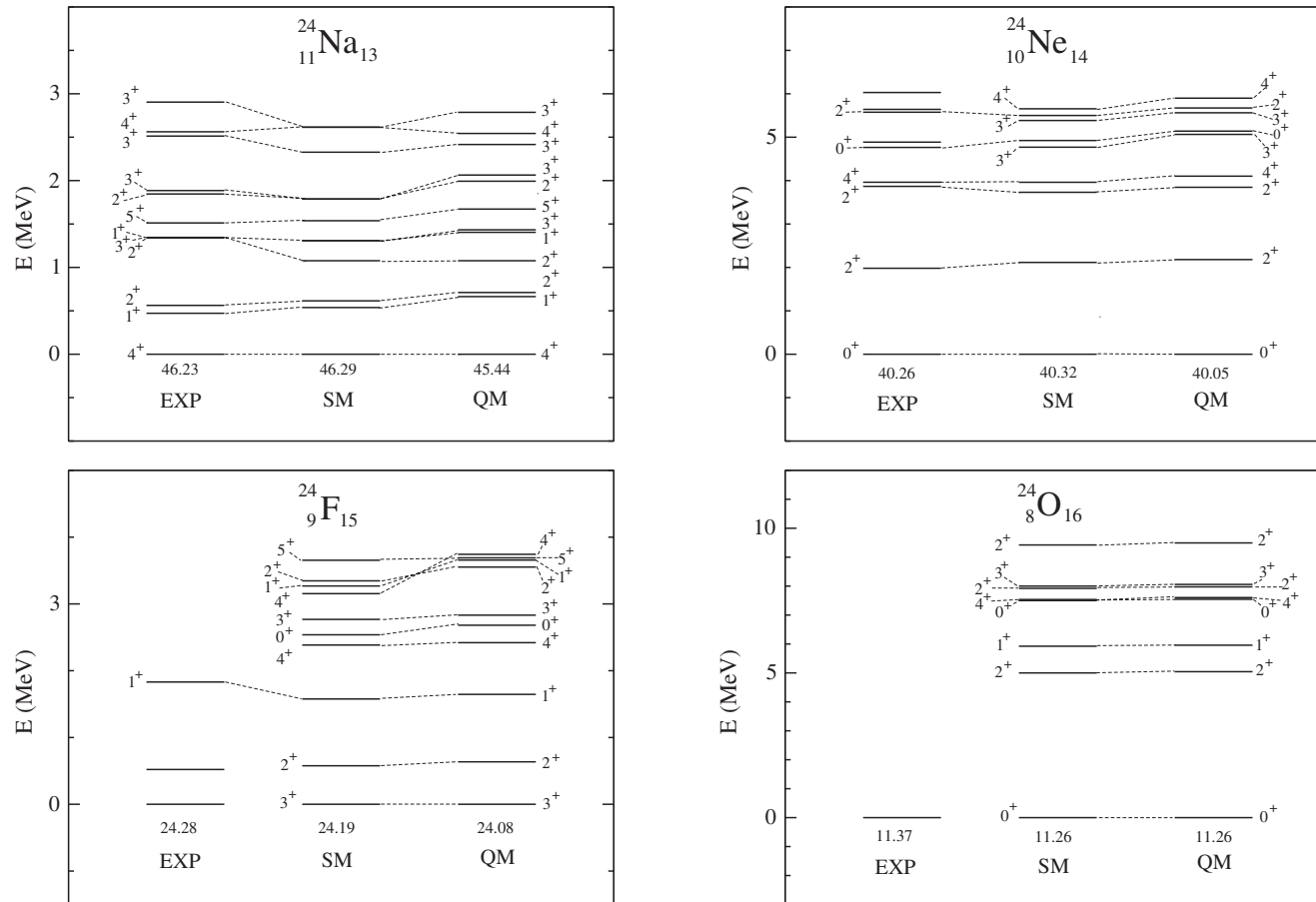
$$Q_{JT}^{(d)+} = \sum_{ii', jj'} x_{ii', jj'}^{(d)} [A_{ii' J_1 T_1}^+ A_{jj' J_2 T_2}^+]^{J, T}$$



all low-lying states can be well described by T=0, J=0, 2, 4 quartets !

T=0, J=0 quartets give the dominant contribution in the ground state !

Quartets in $N > Z$ nuclei



Quartets built not only by $(2p, 2n)$ but also by $(1p, 3n)$ and 4 neutrons !

How to find the fingerprints of the quartet condensation in $N=Z$ nuclei ?

Long-range correlations of superfluidity-type and density matrix

Yang's criterion (1962)

n-body long-range correlations $\longleftrightarrow$ a large eigenvalue of n-body density matrix
 “off-diagonal long-range order”

Example: pair condensation

$$\rho^{(2)}(r_1, r_2; r'_1, r'_2) = \lambda_0 \phi_0^*(r_1, r_2) \phi_0(r'_1, r'_2) + \sum_{n>0} \lambda_n \phi_n^*(r_1, r_2) \phi_n(r'_1, r'_2)$$

- long-range correlations: coherent dominant mode small contribution

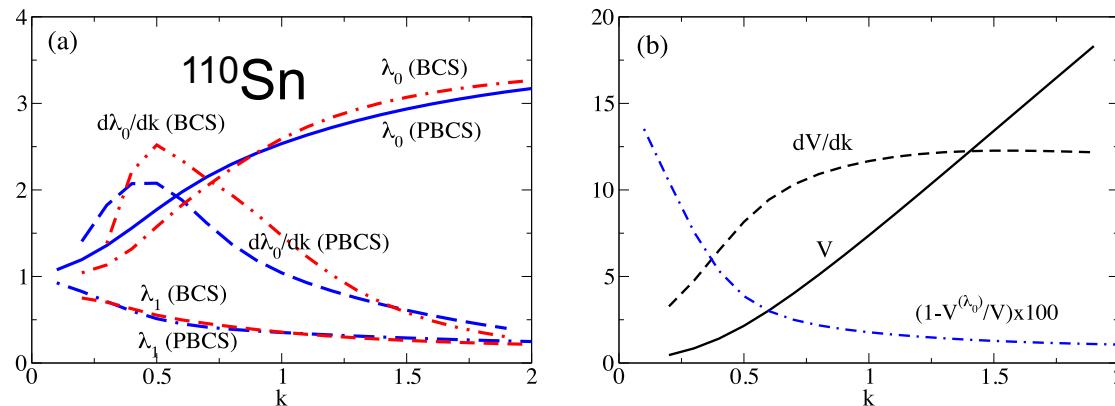
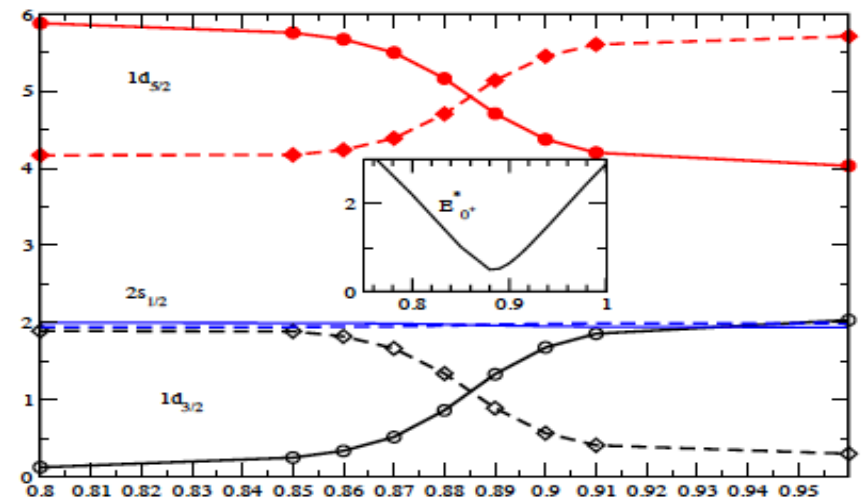
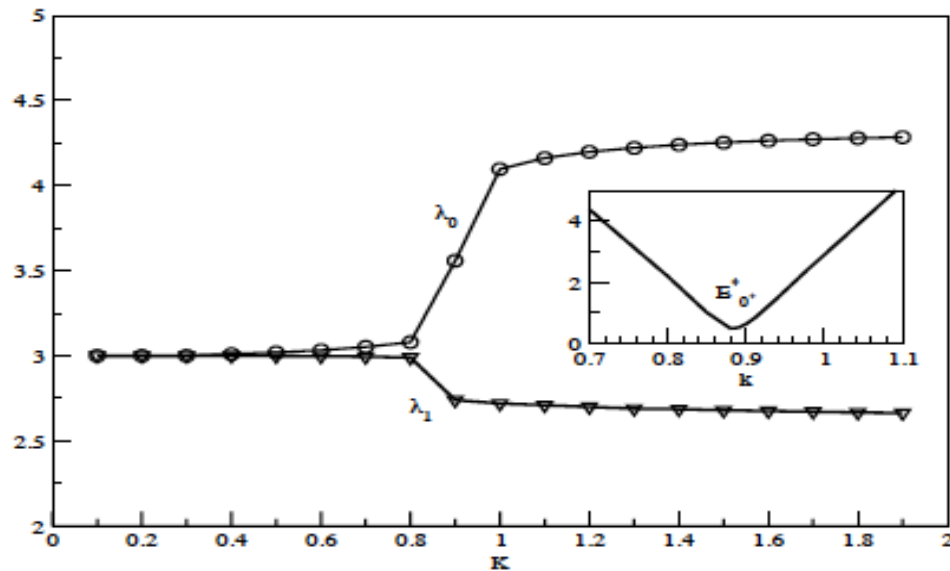


Fig. 3. BCS and PBCS results for ^{110}Sn . (a) The largest (λ_0) and the second largest (λ_1) eigenvalues of two-body density matrix as a function of the scaling factor k . (b) The average of the pairing force, V , its derivative to k and the quantity $(1 - V^{(\lambda_0)}/V) \times 100$ calculated in the BCS approximation.

Eigenvalues of 2-body density matrix for $J=1$ pairing



behaviour related to levels crossing

no indication of pair condensation !

4-body density for general two-body forces: sd-shell nuclei

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii', jj', J', T'} V_{JT}(ii'; jj') [A_{ii' J' T'}^+ A_{jj' J' T'}]^{J=0, T=0}$$

$$\rho_{i,j}^{(4)} = \langle SM | q_i^+ q_j | SM \rangle \quad q_i^+ = (a_{i_1}^+ a_{i_2}^+ a_{i_3}^+ a_{i_4}^+)^{T=0}$$

Largest 5 eigenvalues for sd-shell nuclei

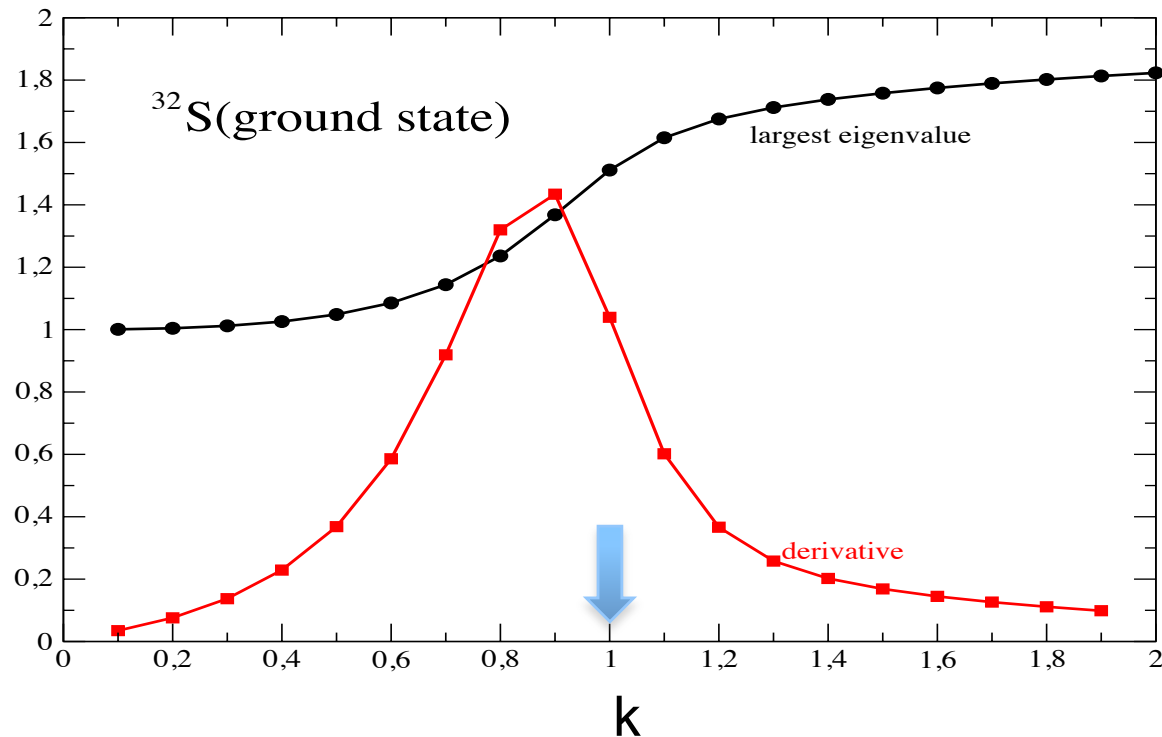
²⁴ Mg	1.18	0.15	0.03	0.29	0.01
²⁸ Si	1.19	0.47	0.27	0.20	0.12
³² S	1.51	0.83	0.74	0.59	0.53

indication of long-range quartet correlations

M. Sambataro and N. S., in preparation

Evolution of the largest eigenvalue of 4-body density matrix: ^{32}S

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + k \sum_{ii', jj', J', T'} V_{JT}(ii'; jj') [A_{ii'J'T'}^+ A_{jj'J'T'}]^{J=0, T=0}$$

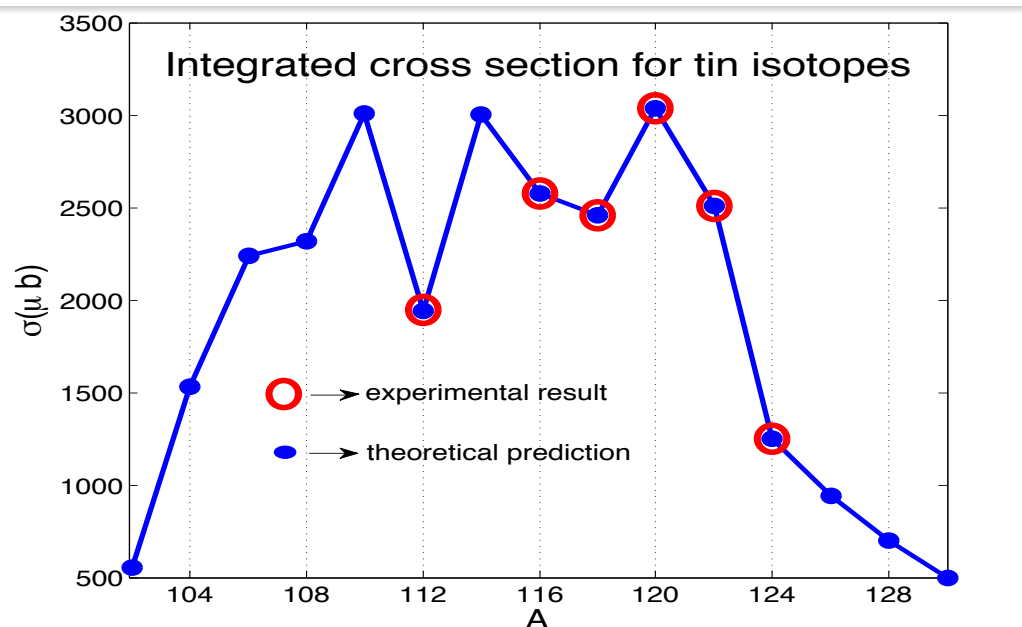


Indication of a fast transition towards a quartet condensate

What experimental data could probe the quartet correlations/condensation ?

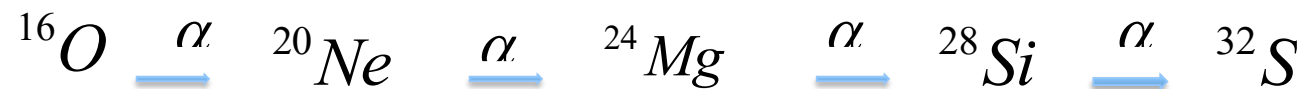
Probing quartet correlations in N=Z nuclei by a transfer ?

- fingerprints of pair condensation: *pair transfer on a chain of isotopes*

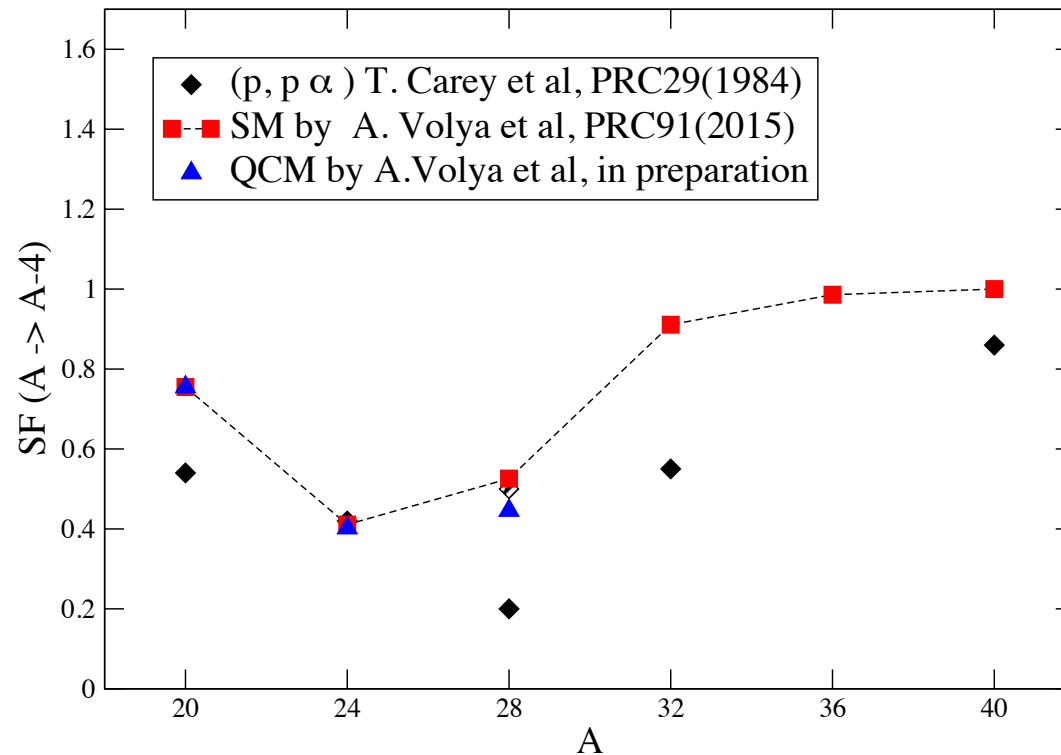


G. Potel et al, PRL107 (2011)

- fingerprints of quartet condensation : *alpha particle transfer along N=Z line ?*



Spectroscopic factors for α transfer



quartet and shell model wave functions provides similar results !

not a clear plateau region: *sd-shell is too small*

transfer a quartet from ^{20}Ne instead from ^6Li ?

Probing quartet correlations/condensation : Josephson-like effect ?

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PHYSICAL REVIEW LETTERS

27 JANUARY 1975

Systematics of the (d , ${}^6\text{Li}$) Reaction and α Clustering in Heavy Nuclei*

F. D. Becchetti, L. T. Chua, J. Jänecke, and A. M. VanderMolen

Cyclotron Laboratory, Physics Department, The University of Michigan, Ann Arbor, Michigan 48105

(Received 5 August 1974)

Data for the α -particle pickup reaction (d , ${}^6\text{Li}$) have been obtained at 35-MeV bombarding energy for even-even nuclei from ${}^{12}\text{C}$ to ${}^{238}\text{U}$. The cross sections for the transitions to the ground states decrease approximately as $1/A_t^3$ where A_t is the target mass. α -particle transfer probabilities have been extracted from the data and are found to be substantially enhanced in heavy nuclei away from shell closures, particularly for deformed nuclei near $A \approx 150$. α -particle correlations appear to be related to two-nucleon pairing effects.

to those observed in the Josephson effect in ordinary superconductors. Such an effect might also be observed in the alpha-transfer between alpha-superconducting nuclei.”

Summary and Conclusions

- proton-neutron pairing is described accurately by an α -like condensate
quartets appears naturally from the isospin-spin conservation
- proton-neutron pairing contributes significantly to the binding energies
Quark-meson-coupling + QCM calculations: no $T=0$ phase for $A>100$
- band-structures in $N=Z$ nuclei generated from intrinsic quartet condensates !

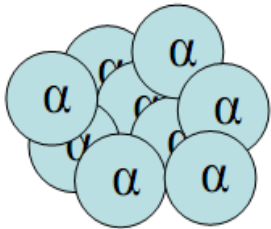
Perspectives

- Probing α -like condensation by α -transfer in heavier $N=Z$ nuclei ?
- Unified description of α -quarteting and α -clustering/condensation ?

Alpha-like quartetting versus alpha clustering

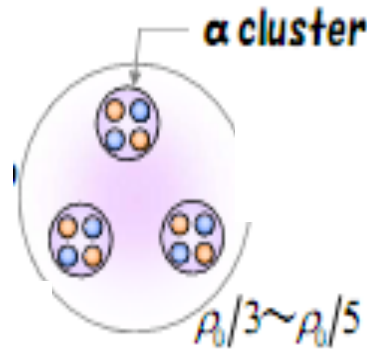
unified microscopic treatment ?

Quartetting



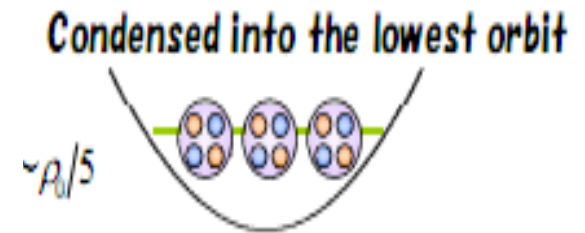
ground state

Alpha-clustering



excited states

Alpha condensation ?



Hoyle state in ^{12}C ?

transition through the collectivity of the quartet

$$| QCM \rangle = Q^{+n_q} | - \rangle$$

Thank you for your attention !

Isvector-isoscalar pairing and quartetting for N>Z nuclei

nuclei with $N-Z=2n_N$

ansatz

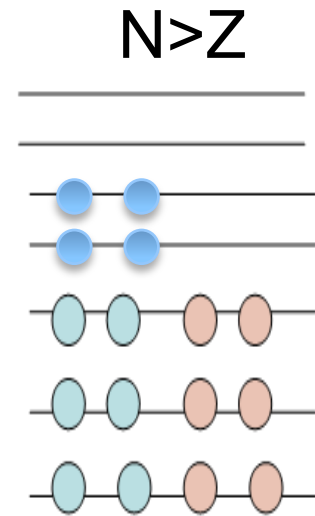
- all protons are correlated in alpha-like quartets
- neutrons in excess form a pair condensate

$$|QCM\rangle = (\tilde{\Gamma}_{\nu\nu}^+)^{n_N} (Q_{T=1}^+ + \Delta_0^{+2})^{n_q} | - \rangle$$

$$Q_{T=1}^+ = 2\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+ - \Gamma_{\nu\pi}^{+2} \quad \Delta_0^+ = \sum y_i D_{i,0}^+$$

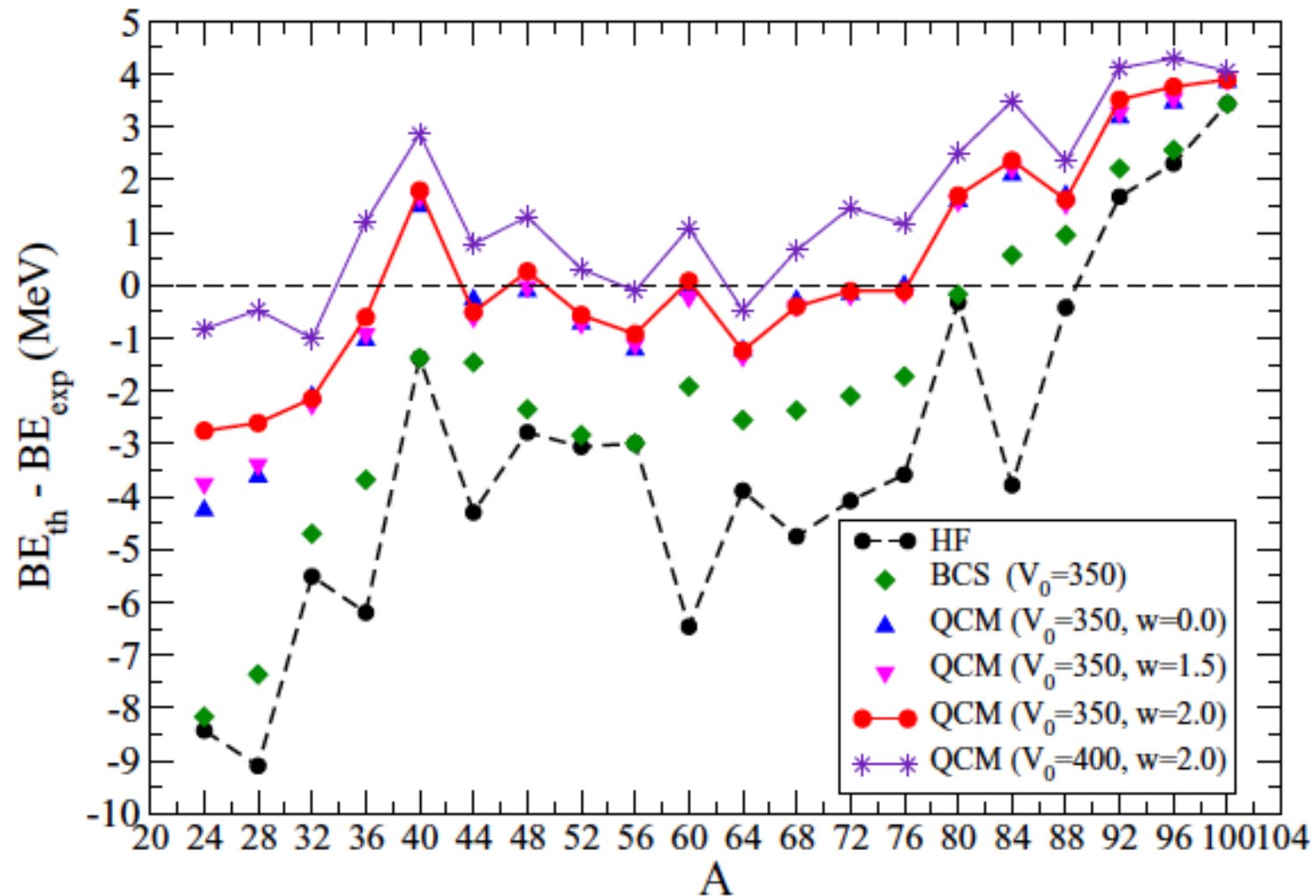
errors for correlation energies smaller than 1%

pn pairing and quartet correlations survive in N > Z nuclei !



Binding energy residuals for N=Z nuclei: Skyrme+QCM

D. Negrea, N.S , D. Gambacurta, PRC105 (2022)



Binding energies residuals for N>Z nuclei

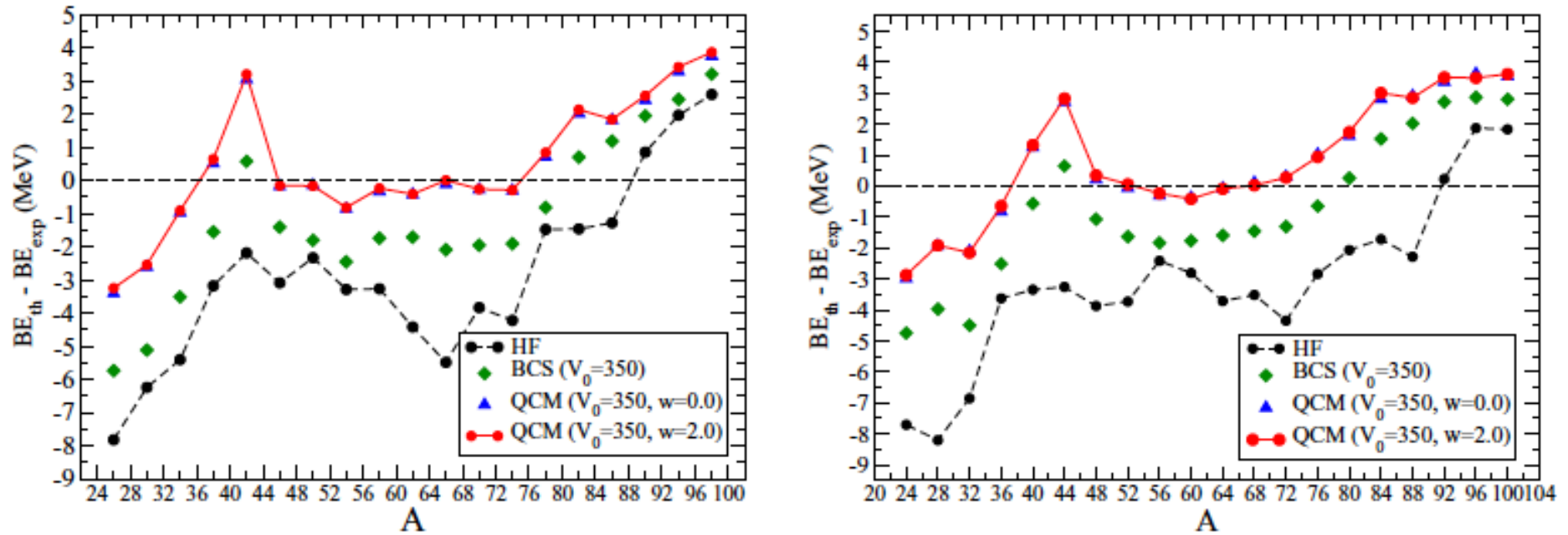
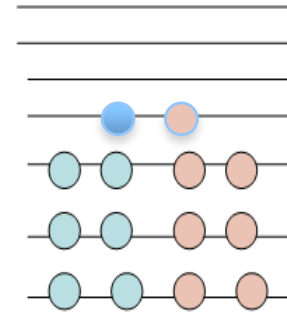


FIG. 5. The residuals, function of $A = N + Z$, for (left) $N = Z + 2$ and (right) $N = Z + 4$ nuclei.

$$|QCM\rangle = (\tilde{\Gamma}_{\nu\nu}^+)^{n_N} (Q_{T=1}^+ + \Delta_0^{+2})^{n_q} |-\rangle$$

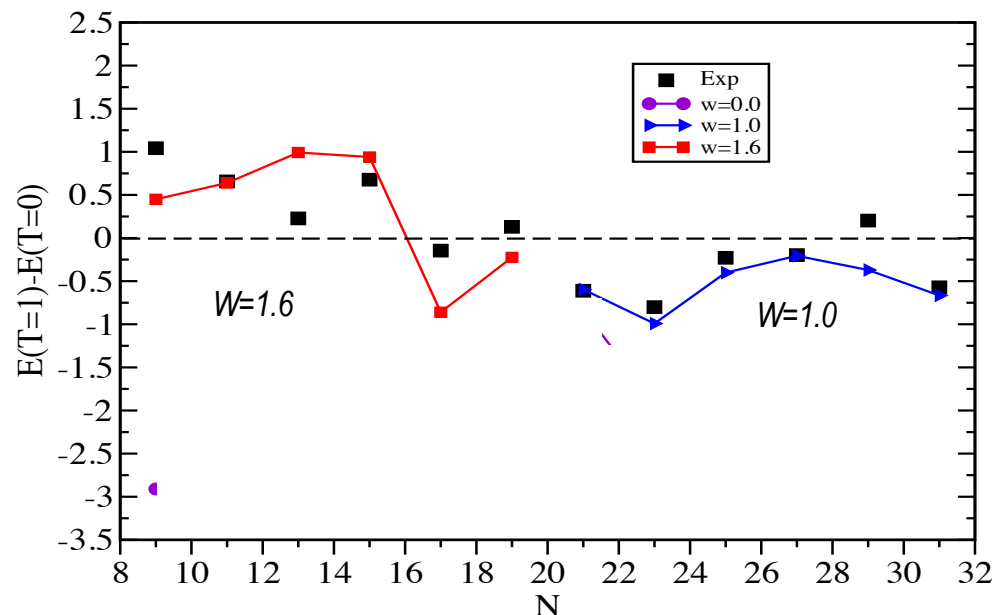
Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P_{i,t}^+ P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D_{i,0}^+ D_{j,0}$$



T=1 state $|iv; QCM \rangle = \tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q} | - \rangle$

T=0 state $|is; QCM \rangle = \tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q} | - \rangle$



$$V_{\text{paring}}^{T=\{0,1\}} = v_0^{T=\{0,1\}} \delta(r_1 - r_2) \hat{p}_{S=\{0,1\}} \quad w = \frac{V_0^{T=0}}{V_0^{T=1}}$$

what we can learn about
the structure of the states ?

The structure of lowest T=0 and T=1 states

T=0 ground state

		Exact	$\tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q}$	$\tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+)^{n_q}$	$(\Delta_{\nu\pi}^+)^{2n_q+1}$	$\tilde{\Delta}_{\nu\pi}^+ (\Gamma_{\nu\pi}^{+2})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

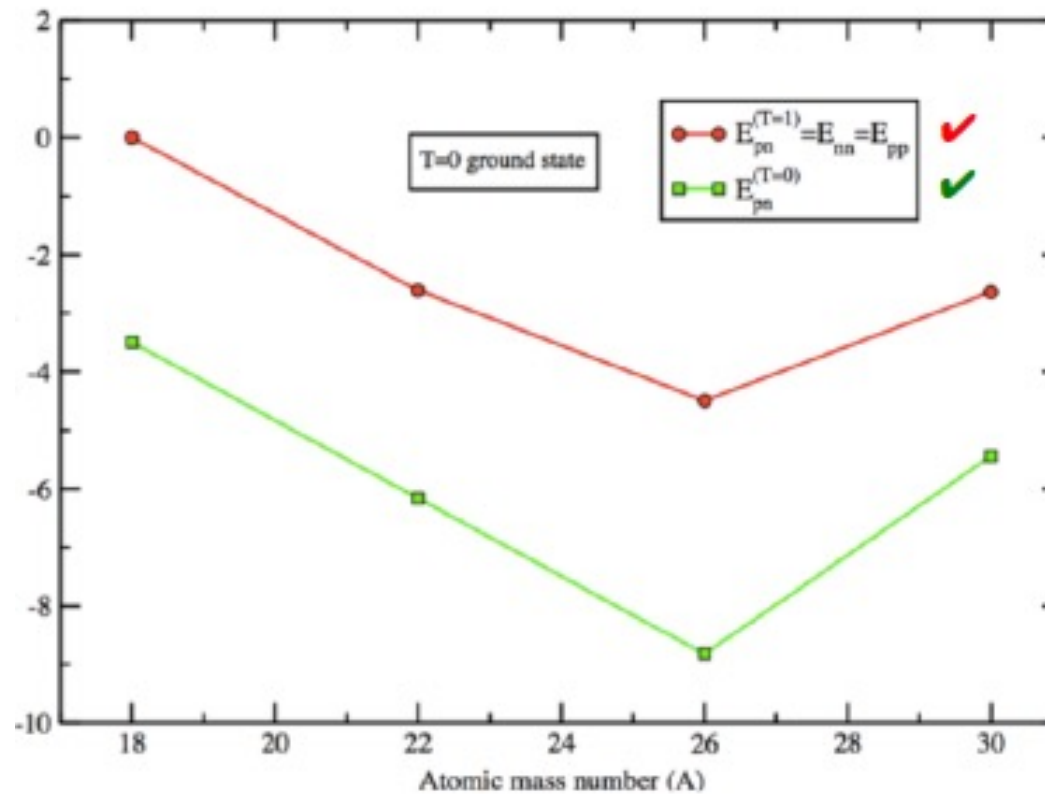
T=1 ground state

		Exact	$\tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q}$	$\tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+)^{n_q}$	$\tilde{\Gamma}_{\nu\pi}^+ (\Delta_{\nu\pi}^{+2})^{n_q}$	$(\Gamma_{\nu\pi}^+)^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

conclusion

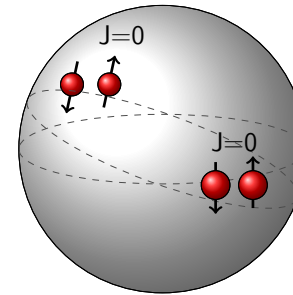
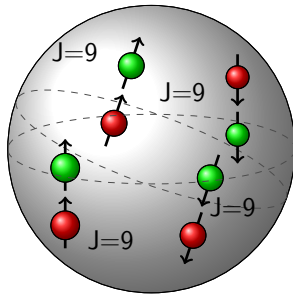
isovector correlations are stronger in both T=0 and T=1 low-lying states

Average value of pairing interactions



the extra $T=0$ pairing energy comes from the odd pn pair !

Spin-aligned J=9 pairs in ^{92}Pd ?

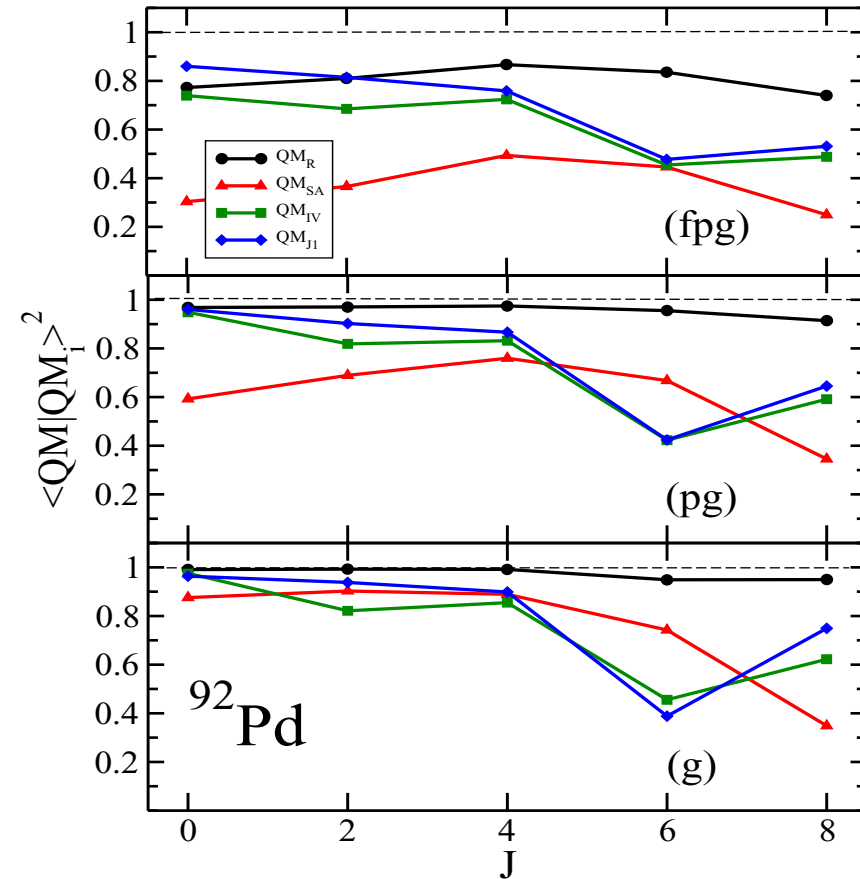
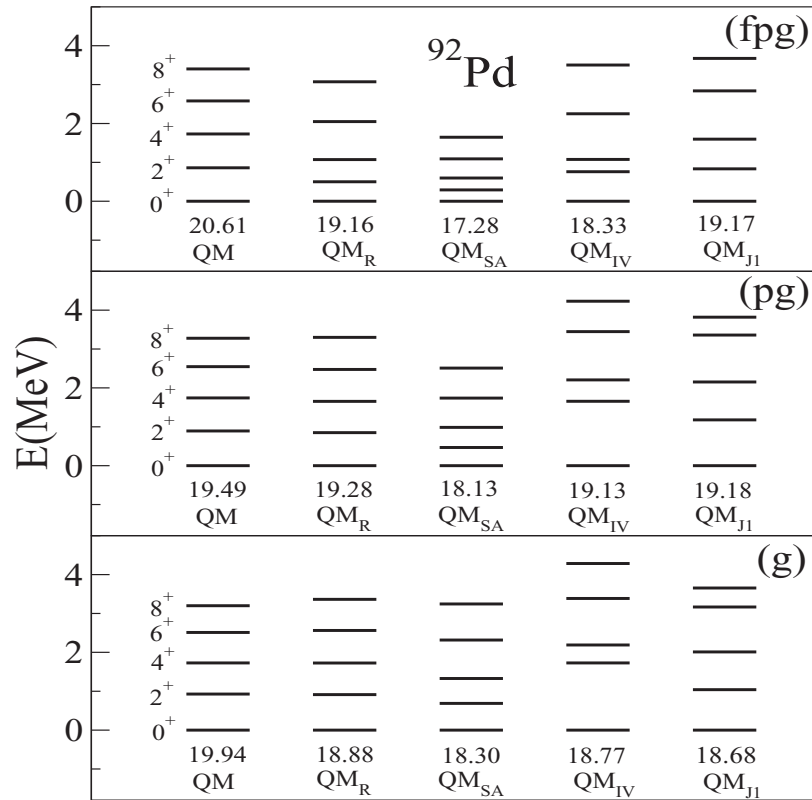


	10^+ 4072		10^+ 4065	10^+ 4052	10^+ 4052	10^+ 4065		10^+ 3862	10^+ 3796		10^+ 4131	
												10^+ 3784
		8^+ 3127	10^+ 3257				10^+ 3257					
(6^+) 2536	6^+ 2466	8^+ 2600	8^+ 2749	8^+ 2633	8^+ 2635	8^+ 2588	8^+ 2792	8^+ 2750	8^+ 2704	8^+ 2636	8^+ 2530	
		6^+ 2110	4^+ 2079	4^+ 2212	6^+ 2223	6^+ 2128	6^+ 2374	6^+ 2330	6^+ 2380	6^+ 2224	6^+ 2099	
(4^+) 1786	4^+ 1708	4^+ 1518		2^+ 1417	4^+ 1709	4^+ 1682	4^+ 1720			8.2		
	20				2^+ 1405	2^+ 1199		13				
(2^+) 874	2^+ 878	2^+ 797	2^+ 1171				2^+ 864	2^+ 861	2^+ 814	2^+ 1460	2^+ 1415	
	15							11		7.5		
0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	0^+ 0	
^{92}Pd exp	^{92}Pd SM	^{92}Pd T=0	^{92}Pd T=1	^{92}Pd no np	^{94}Pd no np	^{94}Pd T=1	^{94}Pd T=0	^{94}Pd SM	^{94}Pd exp	^{96}Pd SM	^{96}Pd exp	

B. Cederwall et al, Nature 469 (2011)68

Role of spin-aligned pairs in ^{92}Pd

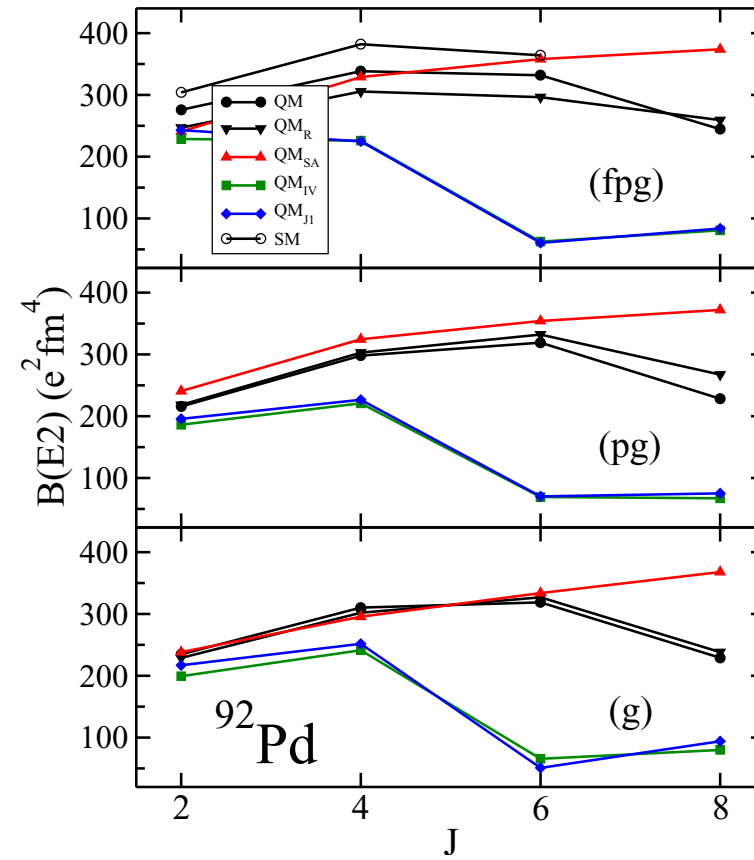
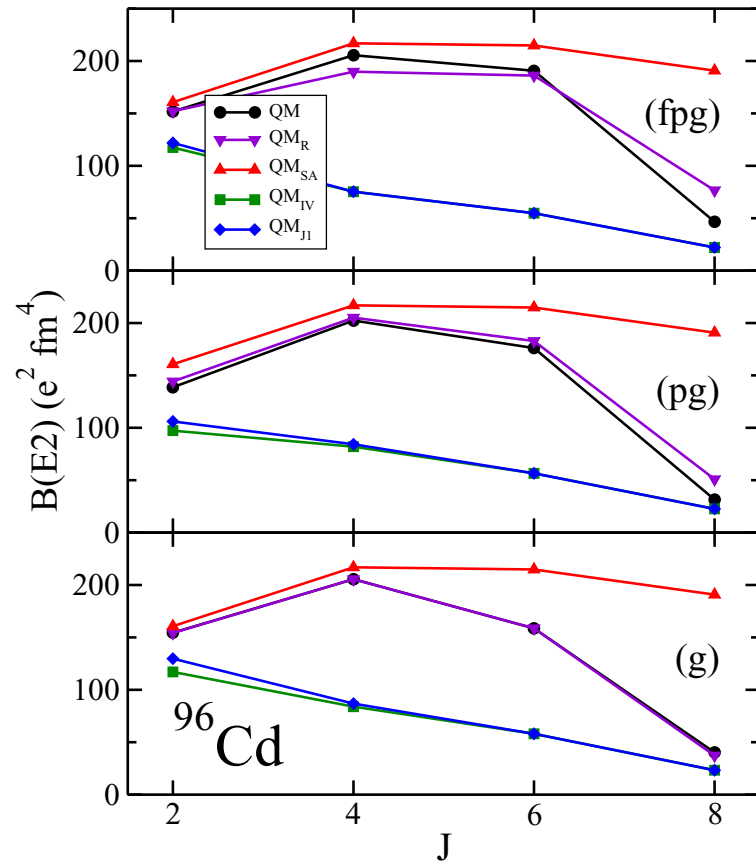
$$Q_{\alpha, JM, TT_z}^+ = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2}^{(\alpha)} \times [[a_{i_1}^+ a_{j_1}^+]^{J_1 T_1} [a_{i_2}^+ a_{j_2}^+]^{J_2 T_2}]_{MT_z}^{JT}, \quad [Q_{\alpha_1, J', T'}^+ \otimes Q_{\alpha_2, J'', T''}^+]^{J, T},$$



the structure of ^{92}Pd is **not** dominated by J=9 pairs

ground state is mainly built by J=0 and J=1 pairs

B(E2)'s in ^{96}Cd and ^{92}Pd



M. Sambataro and N. Sandulescu, PRC91 (2015)