

Multimodal superfluidity of neutrons and neutron-rich nuclei

Dean Lee
Facility for Rare Isotope Beams
Michigan State University
Nuclear Lattice EFT Collaboration

Many-body correlations, phase transitions, and clustering
CEA Saclay, DPhN
September 8, 2026



Outline

Lattice effective field theory

Auxiliary field Monte Carlo

Wavefunction matching

Some nuclear structure results

Pinhole algorithm

More nuclear structure results

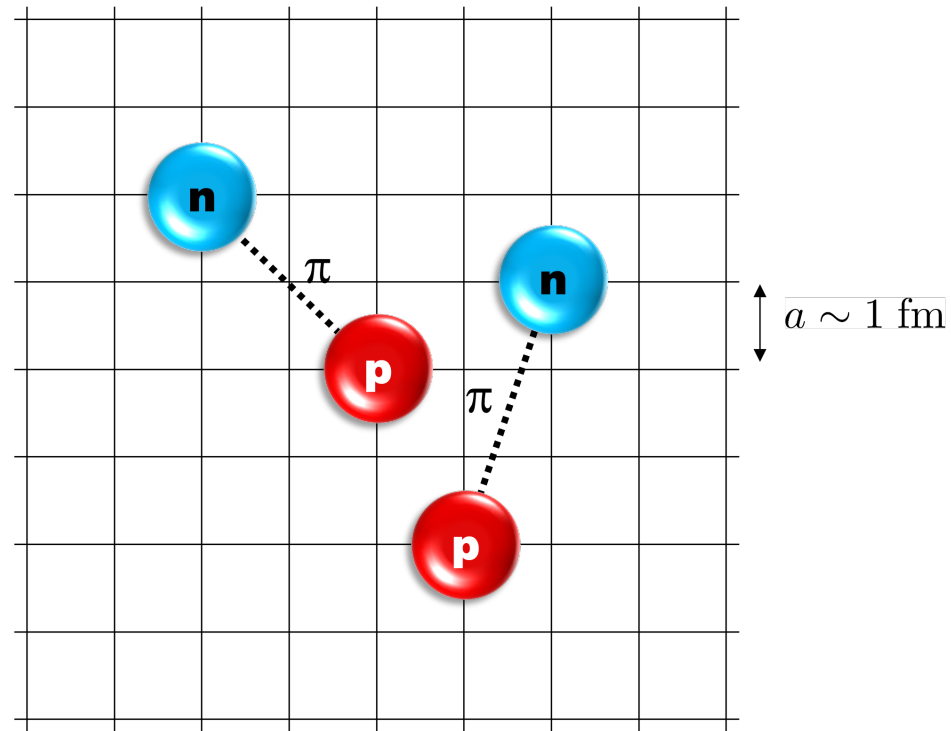
Superfluidity

Multimodal superfluidity

Experimental evidence in neutron-rich nuclei

Summary and outlook

Lattice effective field theory

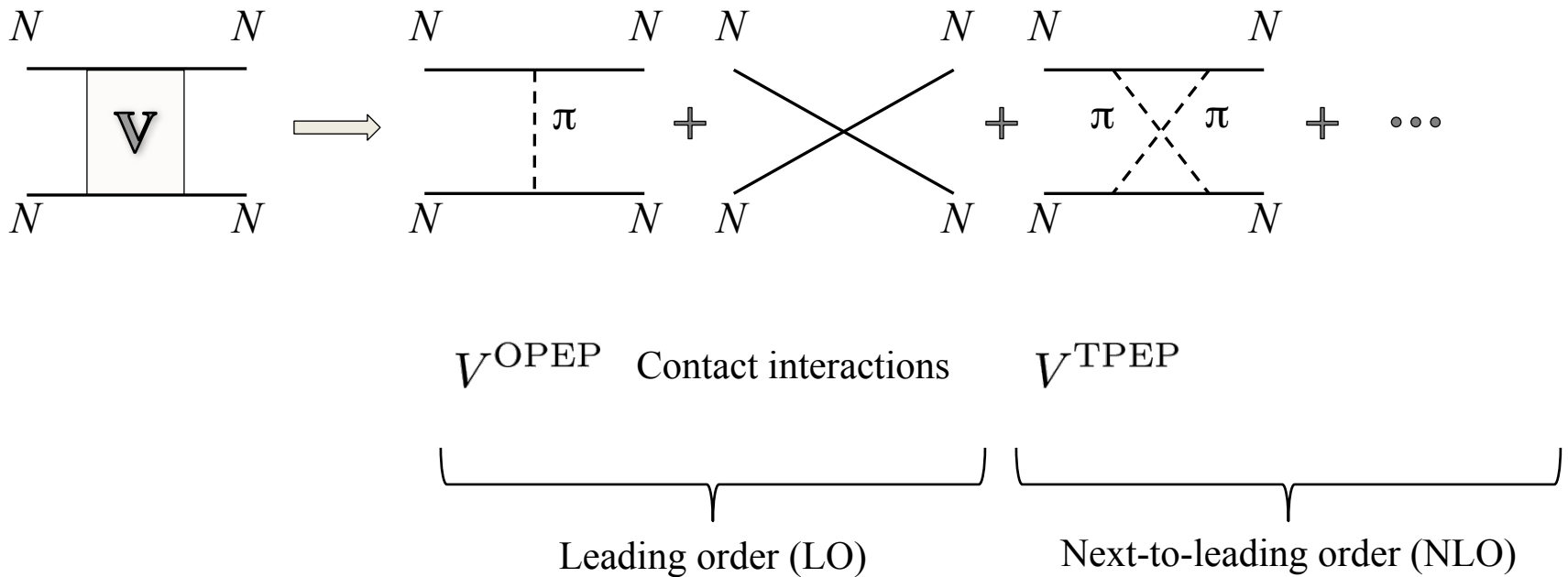


D.L., Annu. Rev. Nucl. Part. Sci. 75, 109 (2025)

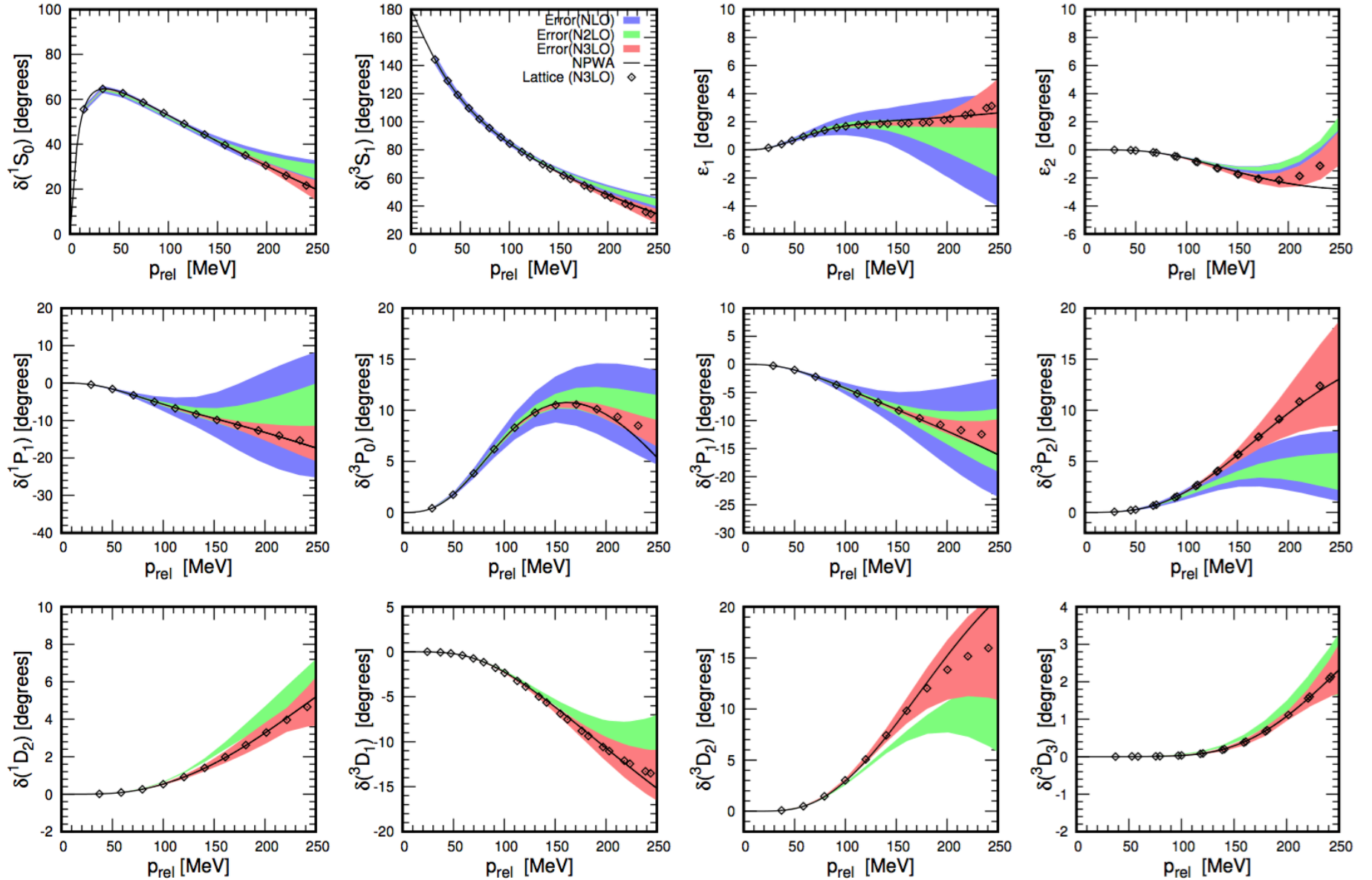
Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

Chiral effective field theory

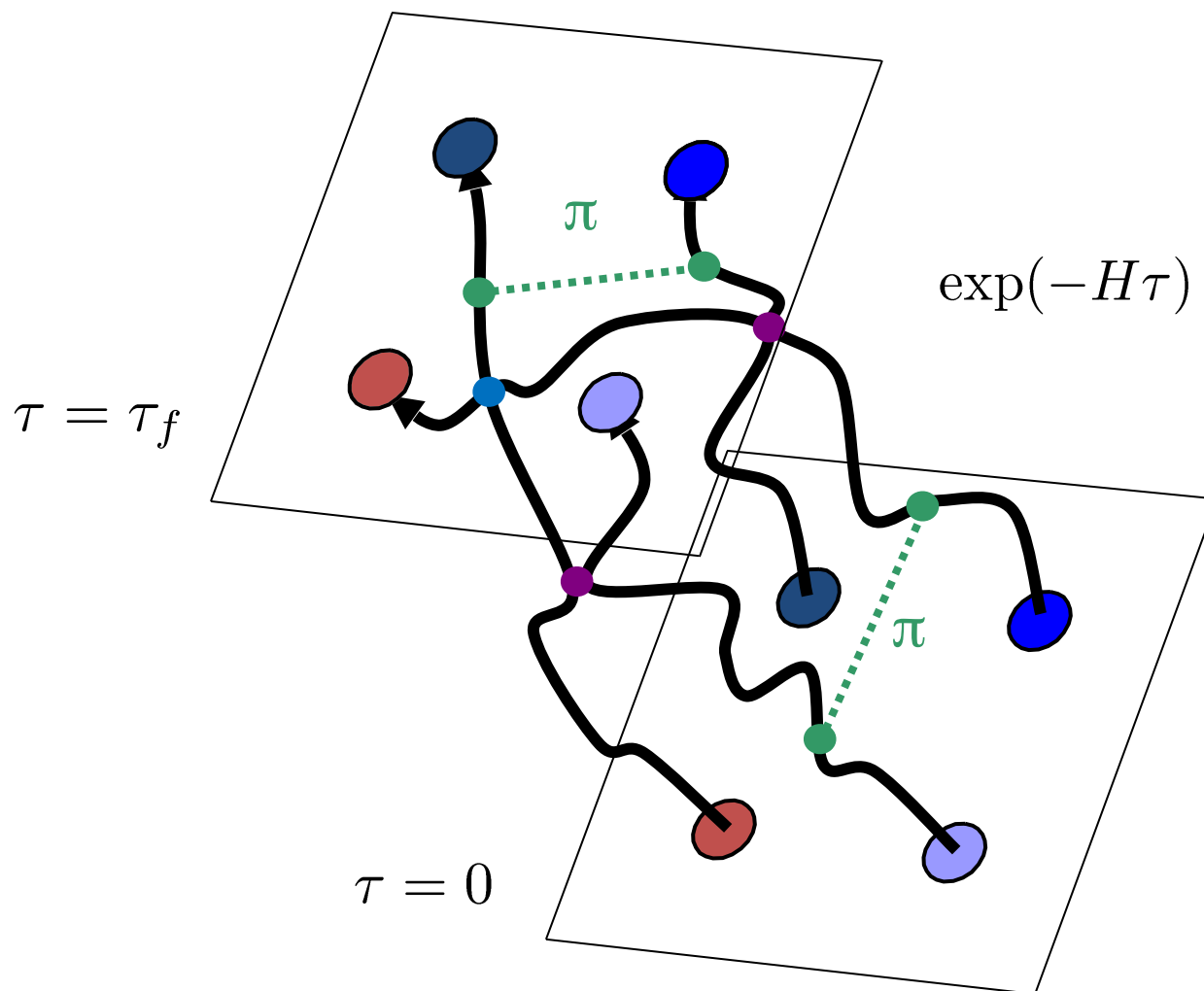
Construct the effective potential order by order



$$a = 1.315 \text{ fm}$$



Euclidean time projection



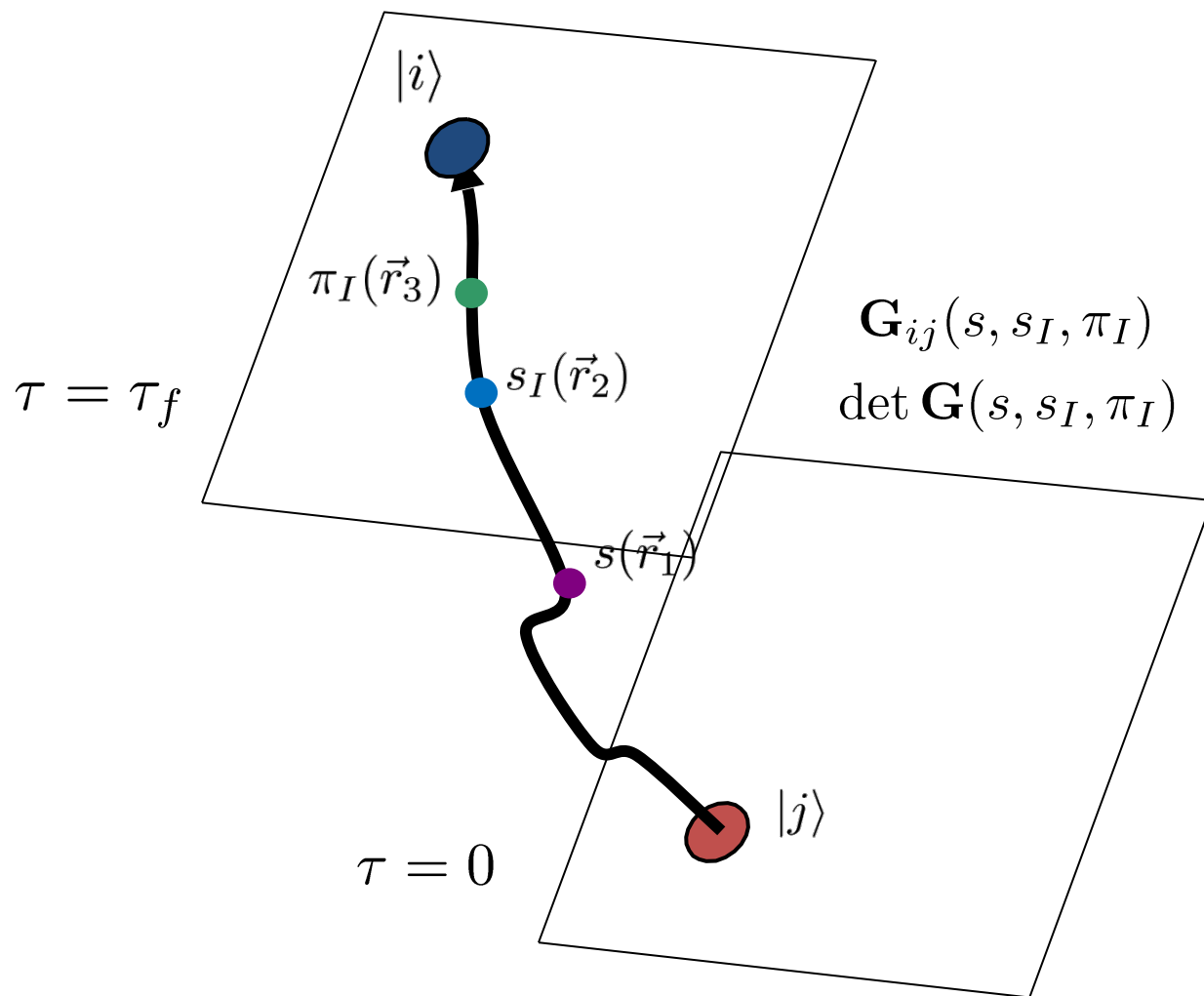
Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

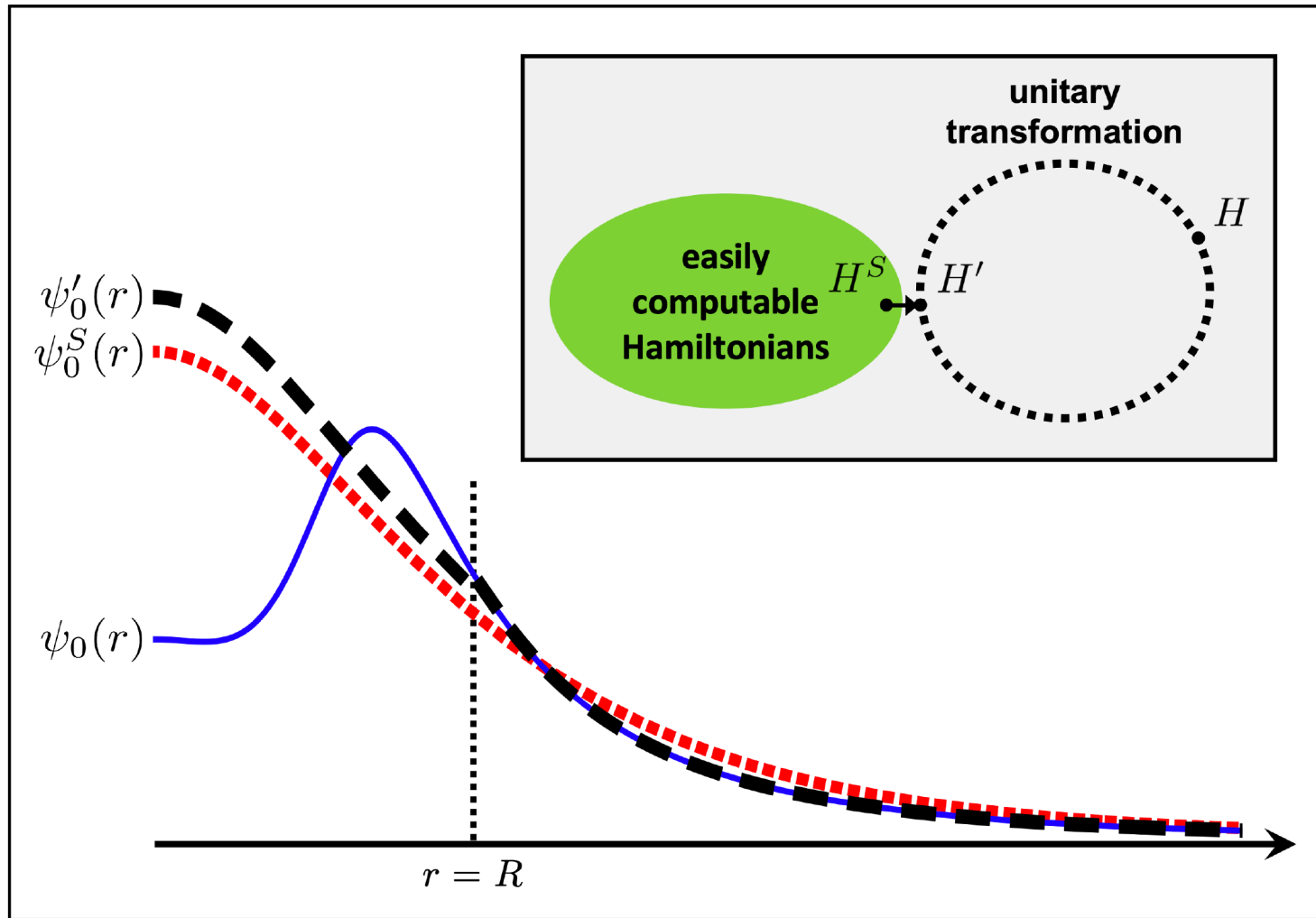
$$\exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] \quad \text{X} \quad (N^\dagger N)^2$$

$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[-\frac{1}{2} s^2 + \sqrt{-C} s (N^\dagger N) \right] \quad \rangle \quad s N^\dagger N$$

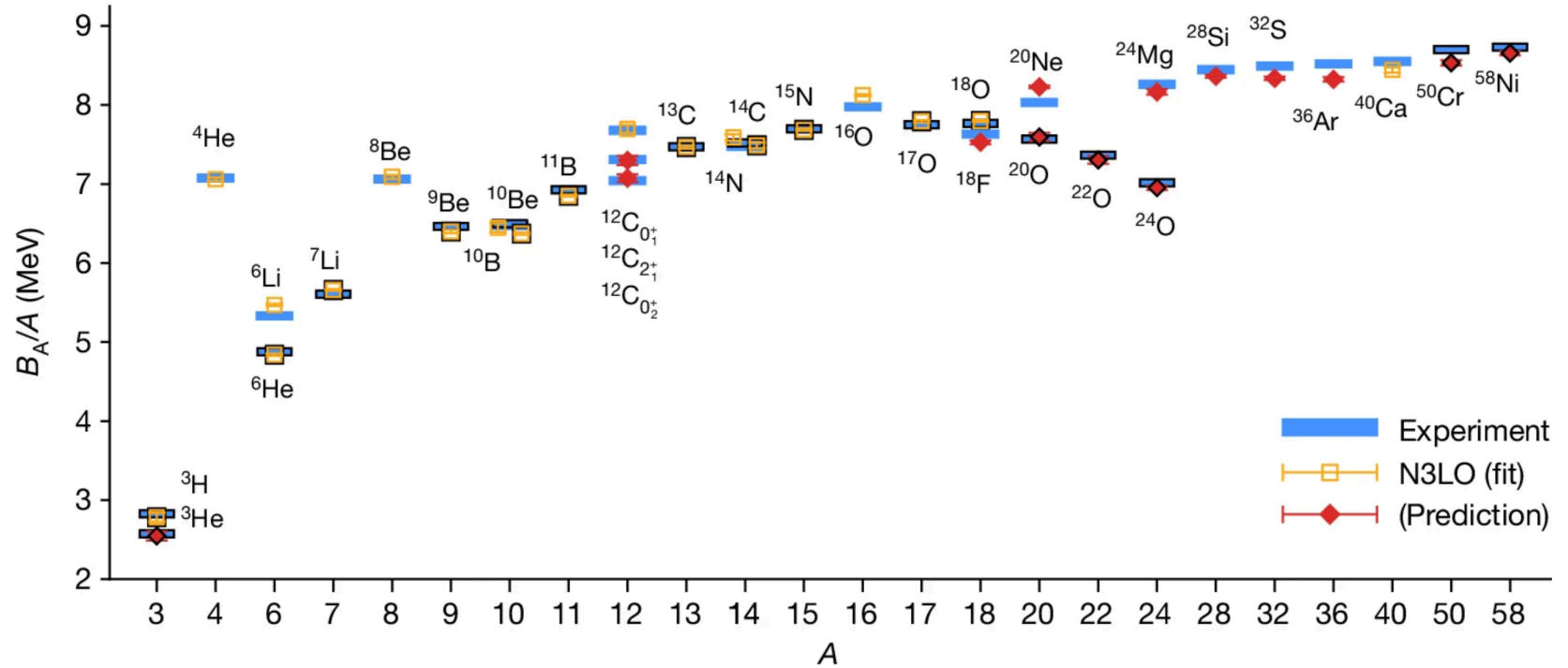
We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.



Wavefunction matching

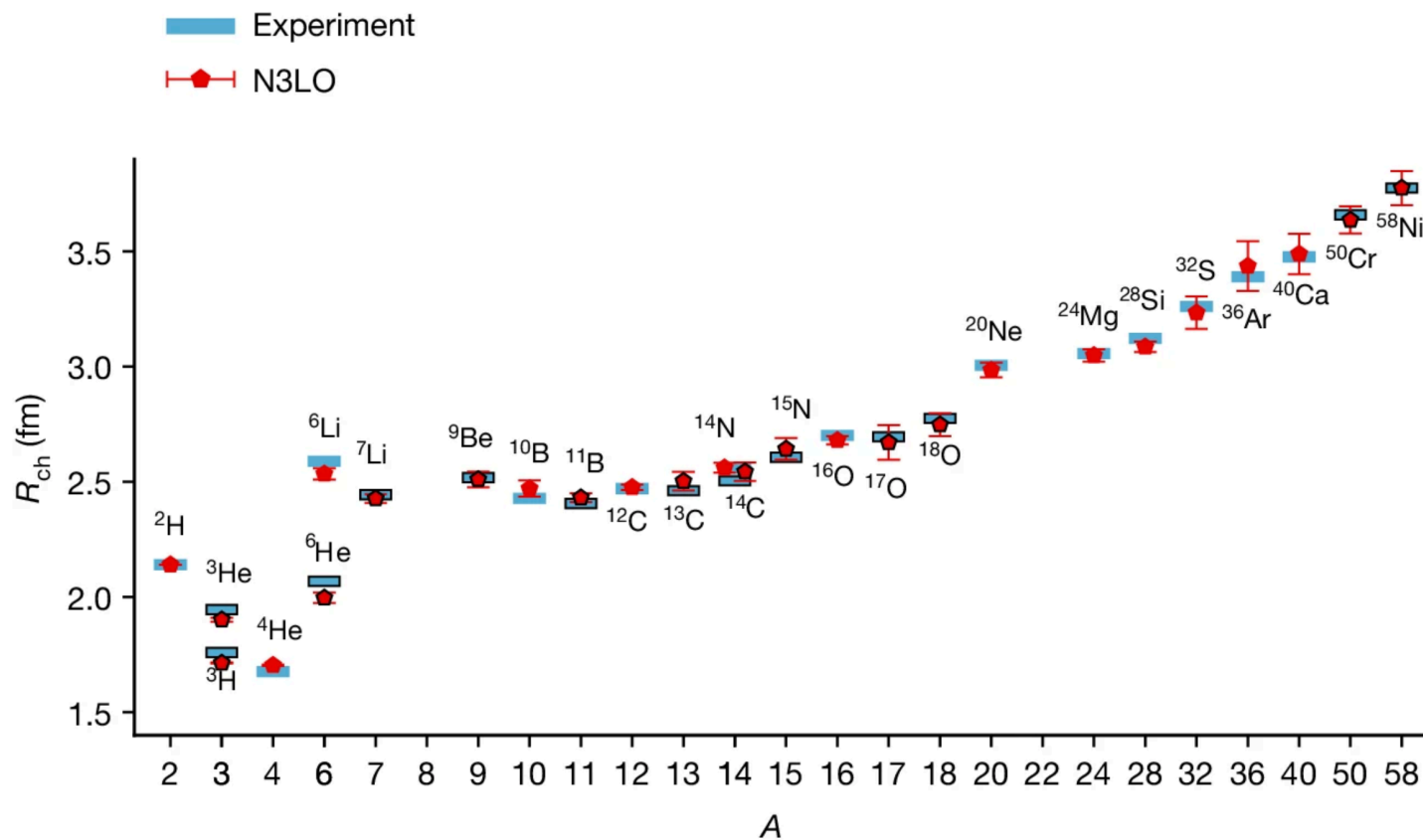


Binding energies



Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii

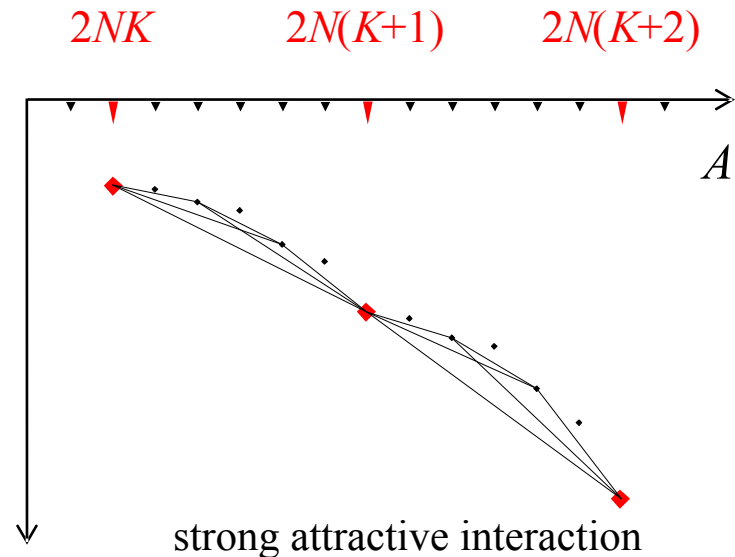
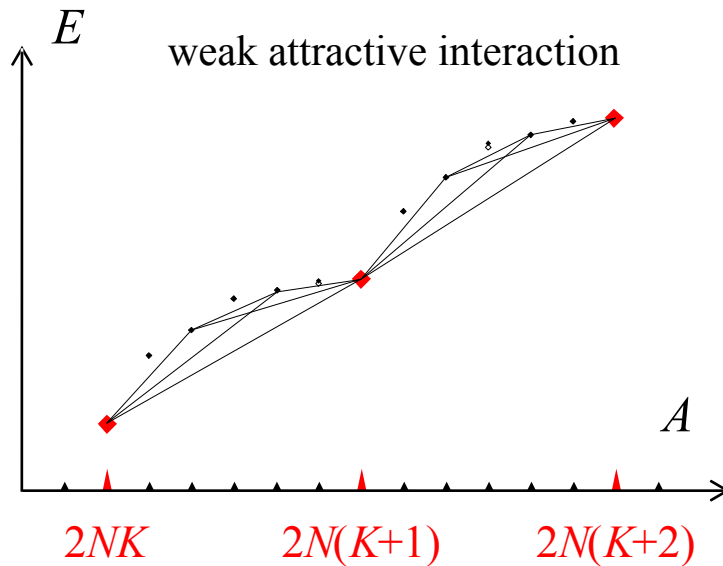


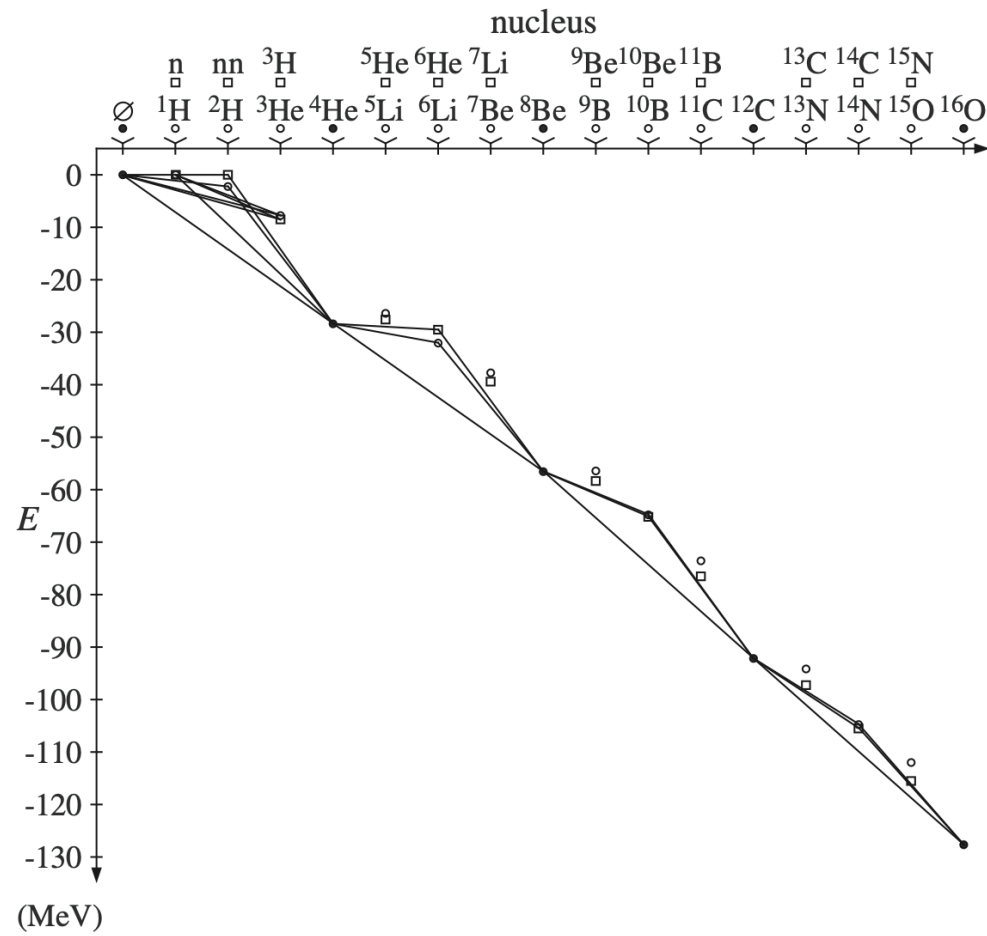
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Spectral convexity theorem

Consider any fermionic system with an $SU(2N)$ symmetry with purely attractive interactions. It must obey the $SU(2N)$ convexity bounds illustrated below, and the $SU(2N)$ symmetry is not spontaneously broken.

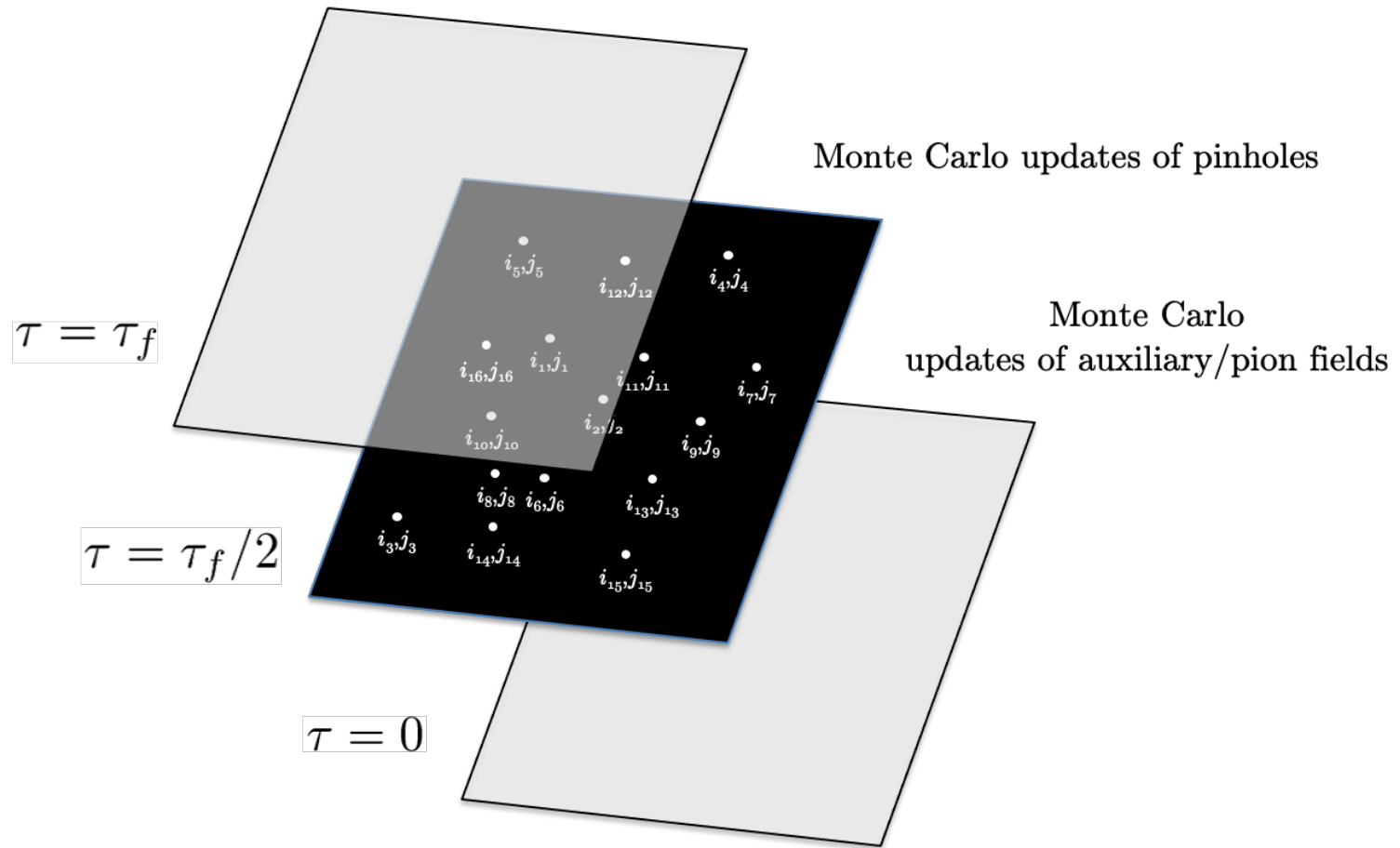
D.L., Phys. Rev. Lett. 98 (2007) 182501



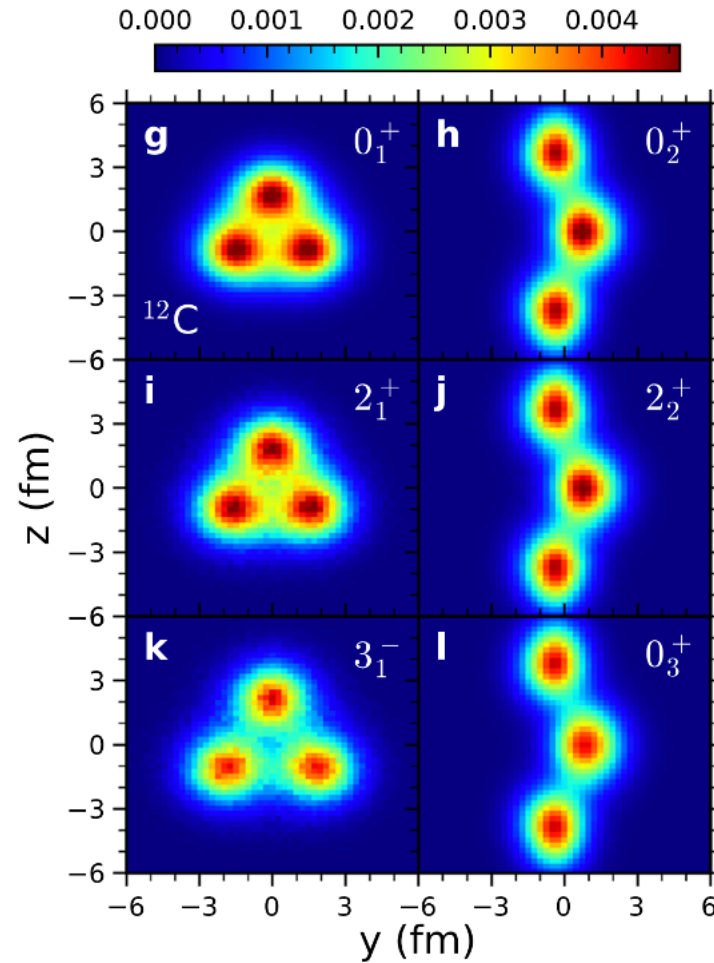


D.L., Phys. Rev. Lett. 98 (2007) 182501

Pinhole algorithm

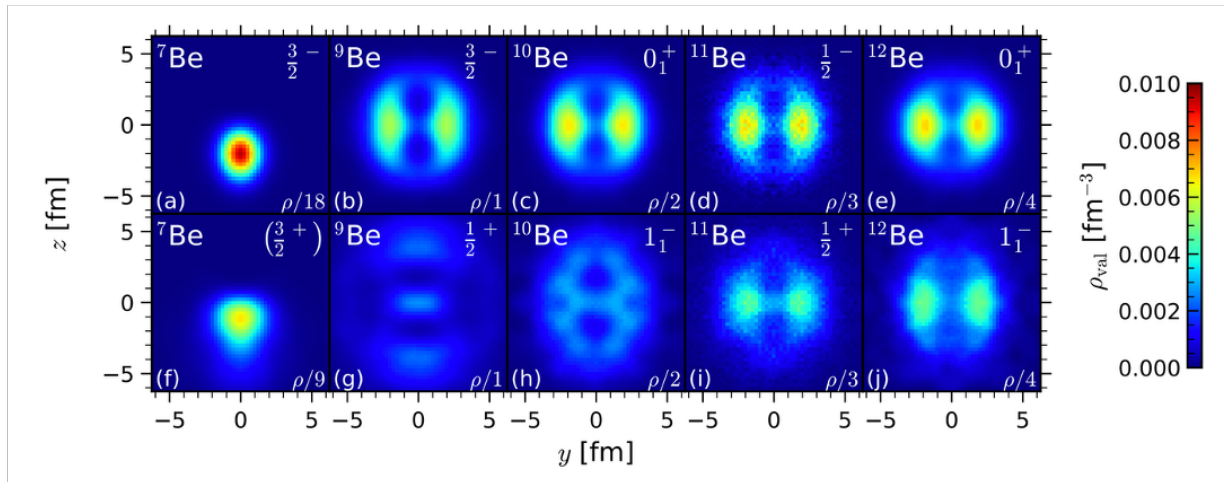
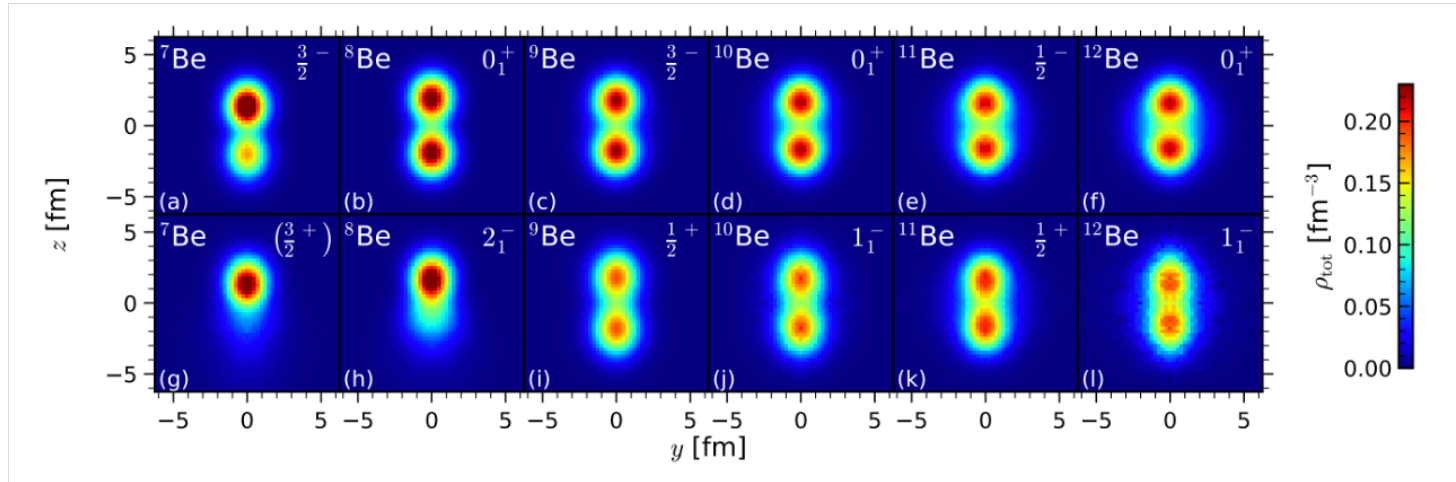


Emergent geometry of ^{12}C



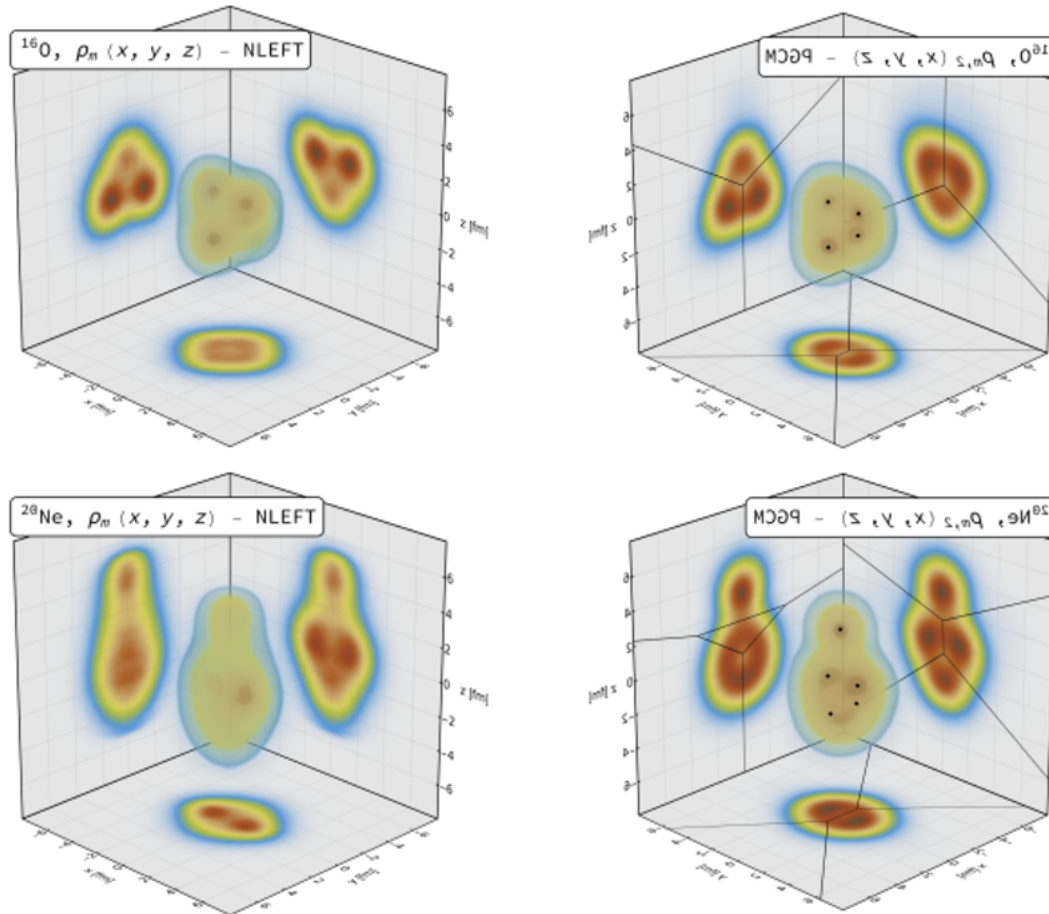
Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)

Beryllium isotopes



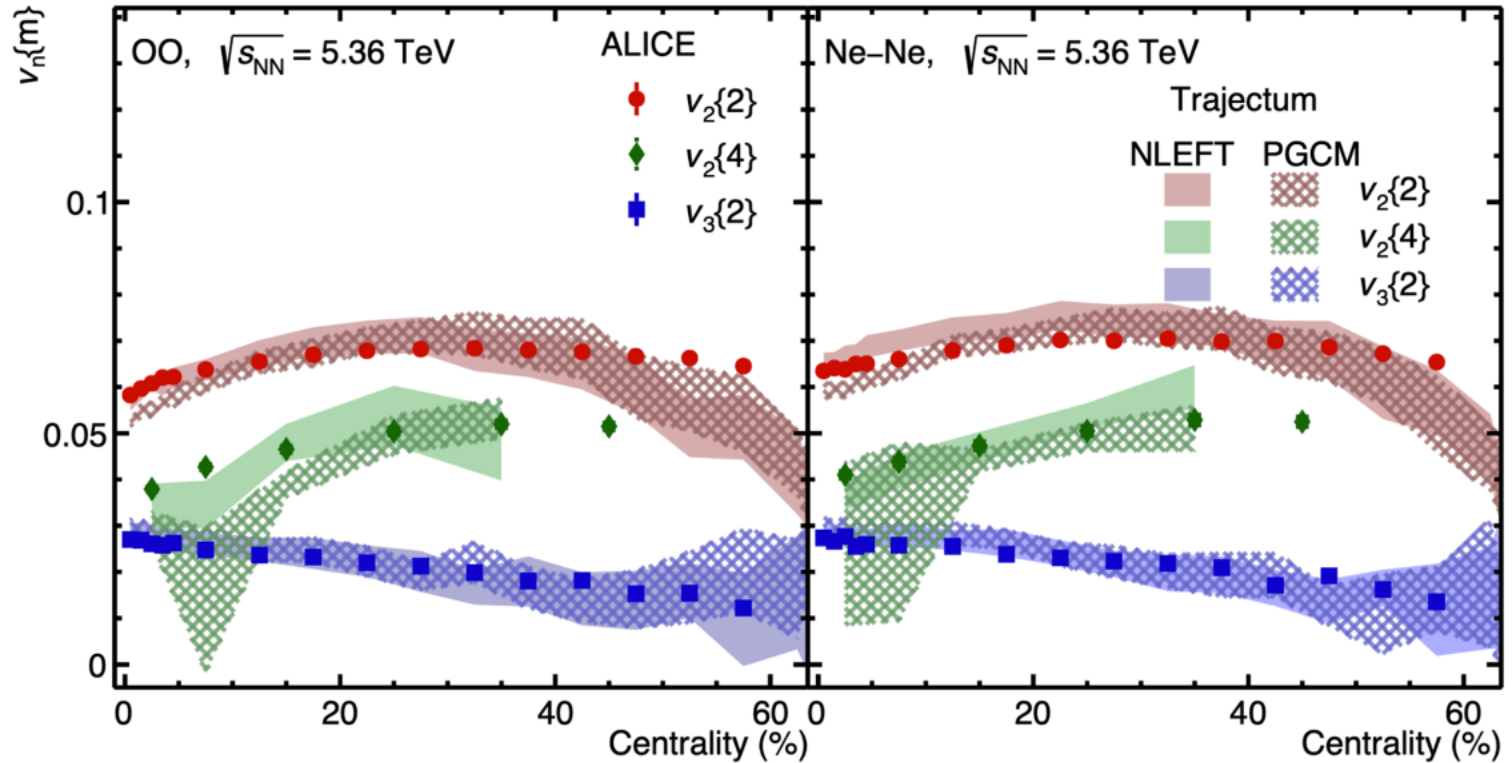
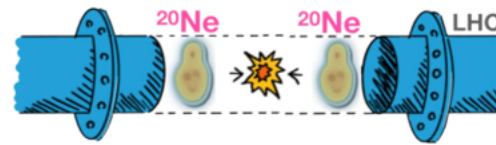
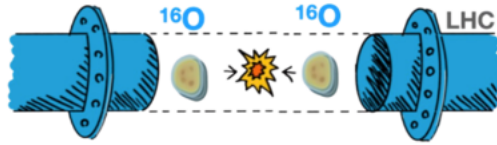
Shen, Elhatisari, D.L., Meißner, Ren, PRL 134, 162503 (2025)

Structure of ^{16}O and ^{20}Ne



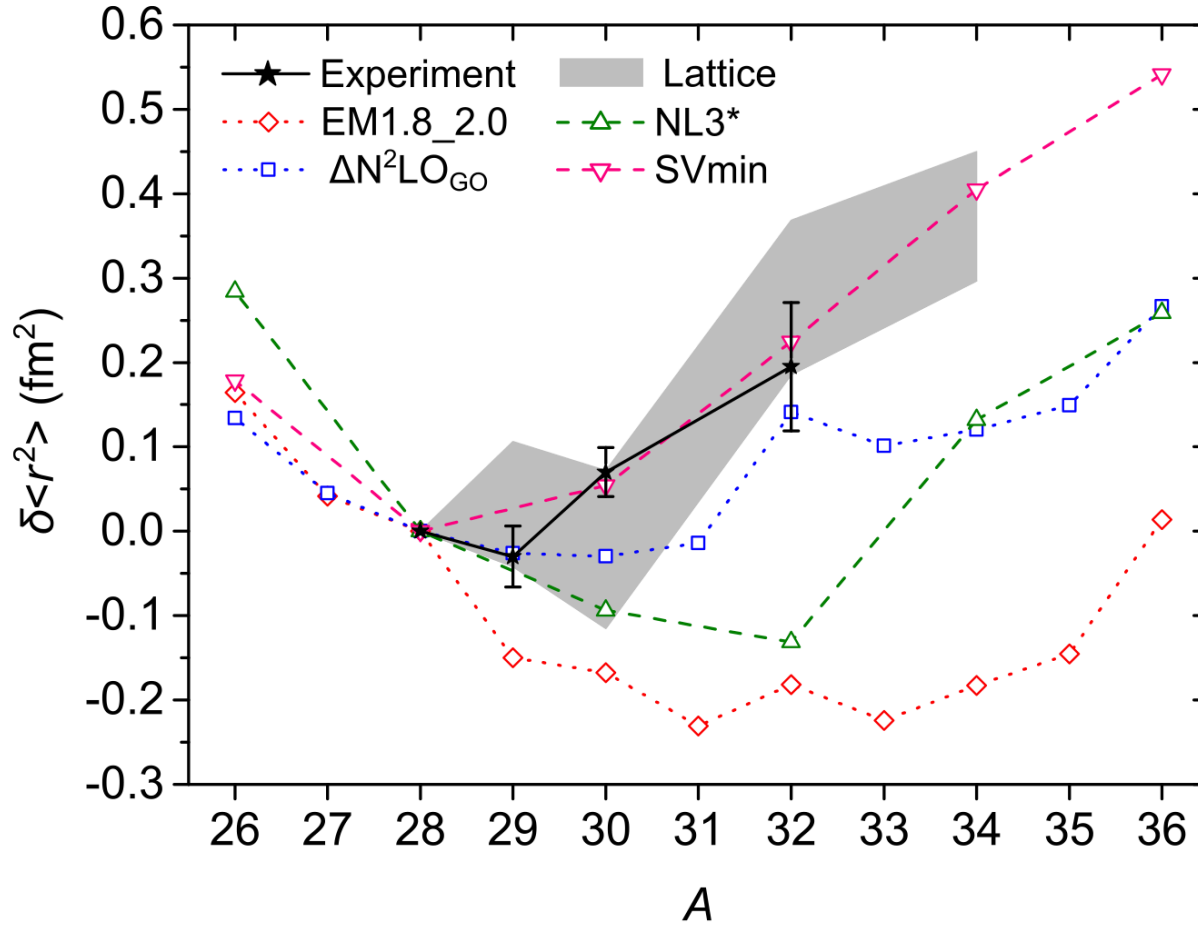
Giacalone et al., Phys. Rev. Lett. 135, 012302 (2025)

Giacalone et al., Phys. Rev. Lett. 134, 082301 (2025)



ALICE Collab., "Evidence of nuclear geometry-driven anisotropic flow in OO and Ne-Ne collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV", <https://arxiv.org/abs/2509.06428>

Charge radii of silicon isotopes



König et al., Phys. Rev. Lett. 133, 059901 (2024)

Magnetic moments

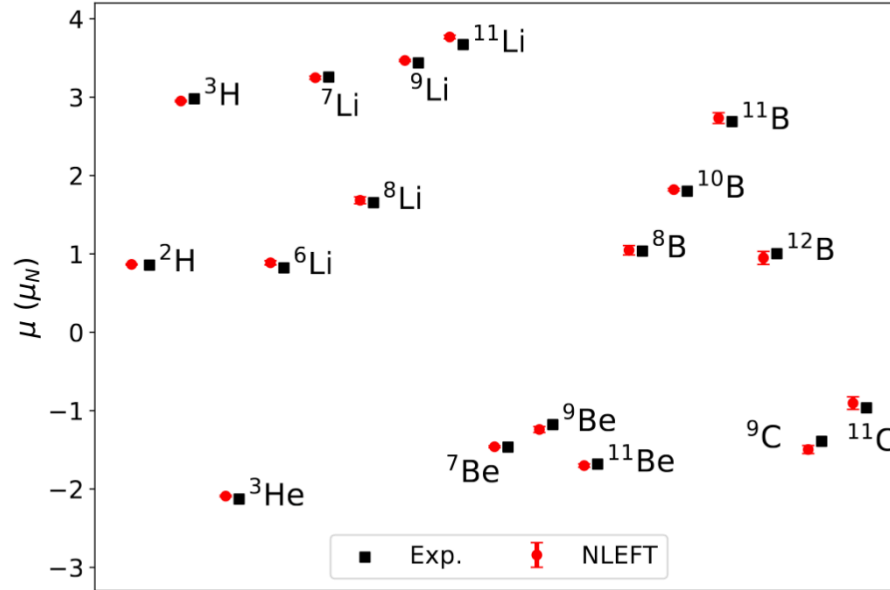


Figure 1. Global comparison between the full NLEFT results and experimental results, denoted by the red and black points respectively, for the ground state magnetic moments of selected $A \leq 12$ nuclei. The error bar denotes theoretical uncertainty.

Neutron and nuclear matter

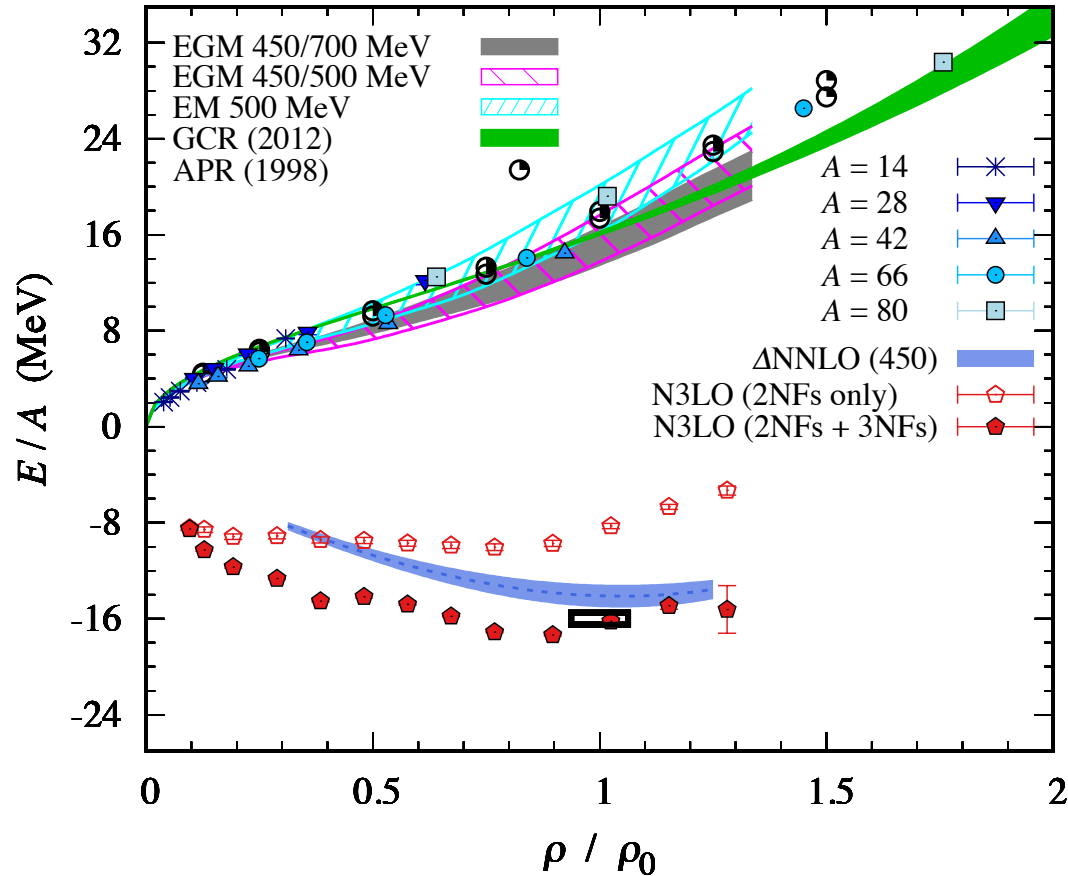
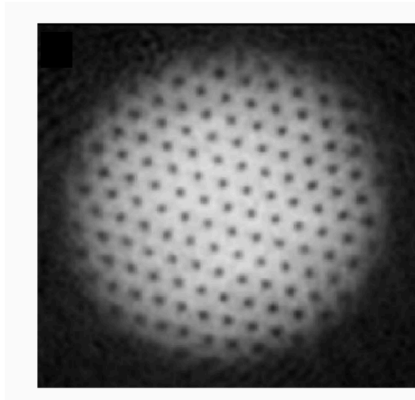


Figure adapted from Tews, Krüger, Hebeler, Schwenk, Phys. Rev. Lett. 110, 032504 (2013)

Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Superfluid condensation

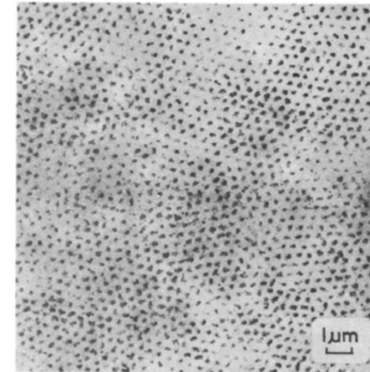
BEC Theory



Ketterle, Zwierlein,
Ultracold Fermi Gases (2008)



BCS Theory



Essmann, Träuble,
Physics Letters A 27, 3 (1968)

Superfluid condensation

Bosonic superfluidity

$$\langle \Psi_0 | a^\dagger(\mathbf{r}) a(\mathbf{0}) | \Psi_0 \rangle$$

Fermionic superfluidity (S-wave)

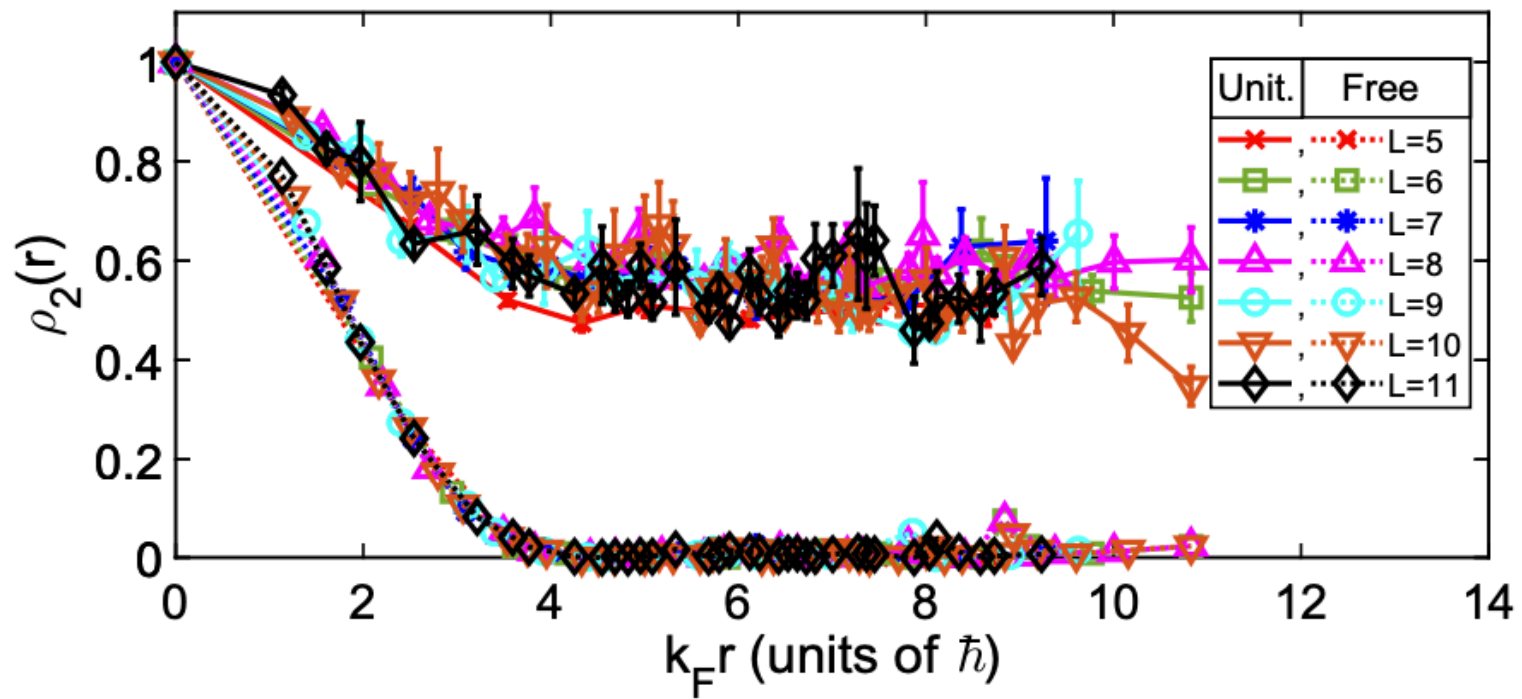
$$\langle \Psi_0 | a^\dagger_\downarrow(\mathbf{r}) a^\dagger_\uparrow(\mathbf{r}) a_\uparrow(\mathbf{0}) a_\downarrow(\mathbf{0}) | \Psi_0 \rangle$$

Fermionic superfluidity (P-wave)

$$\langle \Psi_0 | a^\dagger_\uparrow(\mathbf{r}) a^\dagger_\uparrow(\mathbf{r} + \Delta\mathbf{r}) a_\uparrow(\Delta\mathbf{r}) a_\uparrow(\mathbf{0}) | \Psi_0 \rangle$$

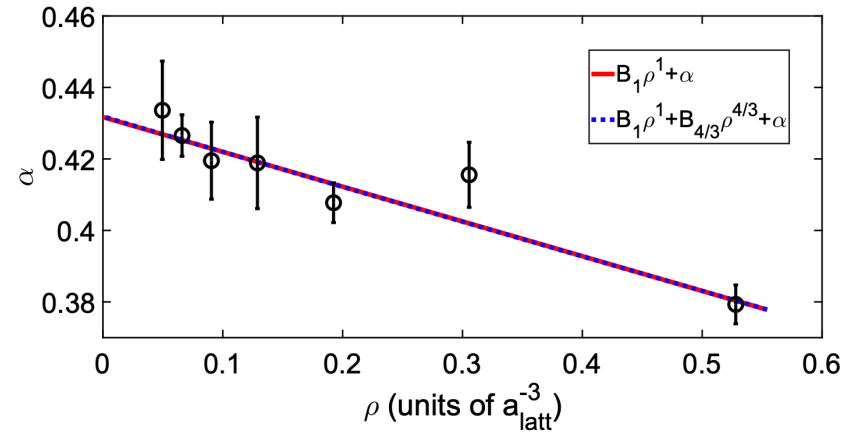
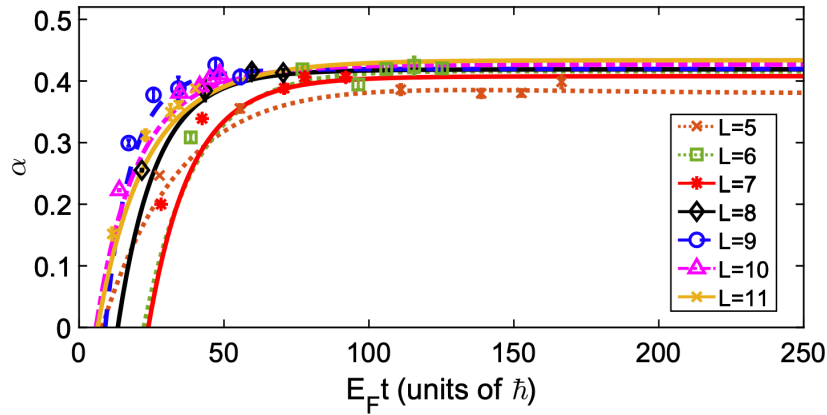
We can also perform calculations in momentum space. But we need to compute cumulants to obtain irreducible contributions only.

Unitary limit



He, Li, Lu, D.L., Phys. Rev. A 101, 063615 (2020)

Unitary limit



condensate fraction = 0.43(2)

He, Li, Lu, D.L., Phys. Rev. A 101, 063615 (2020)

^6Li experiments: 0.46(7) [1, 2] and 0.47(7) [3]

[1] Zwierlein, Stan, Schunck, Raupach, Kerman, Ketterle, PRL 92, 120403 (2004).

[2] Zwierlein, Schunck, Stan, Raupach, Ketterle, PRL 94, 180401 (2005).

[3] Kwon, Pace, Panza, Inguscio, Zwerger, Zaccanti, Scazza, Roati, Science 369, 84 (2020).

Can a many-body system have S-wave and P-wave superfluid condensation at the same time?

arXiv:2602.17611

Evidence for Multimodal Superfluidity of Neutrons

**Yuan-Zhuo Ma^{1,2}, Georgios Palkanoglou^{3,4}, Joseph Carlson⁵, Stefano Gandolfi⁵,
Alexandros Gezerlis⁴, Gabriel Given^{1,2}, Ashe Hicks^{1,2}, Dean Lee^{1,2}, Kevin E. Schmidt⁶, and
Jiabin Yu⁷**

¹Facility for Rare Isotope Beams, Michigan State University, East Lansing, Michigan 48824

²Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824

³TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

⁴Department of Physics, University of Guelph, Guelph, ON N1G 2W1, Canada

⁵Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

⁶Department of Physics, Arizona State University, Tempe, Arizona 85287, USA

⁷Department of Physics and Quantum Theory Project, University of Florida, Gainesville, FL 32611, USA

Generalized attractive extended Hubbard models

We consider generalized attractive extended (GAE) Hubbard models for two-component fermions in 1, 2, 3 dimensions

$$H = H_{\text{free}} + \frac{1}{2}C_2 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2$$

$$\tilde{\rho}(\mathbf{n}) = \sum_{j=\uparrow,\downarrow} \tilde{a}_j^\dagger(\mathbf{n}) \tilde{a}_j(\mathbf{n}) + s_L \sum_{|\mathbf{n}-\mathbf{n}'|=1} \sum_{j=\uparrow,\downarrow} \tilde{a}_j^\dagger(\mathbf{n}') \tilde{a}_j(\mathbf{n}')$$

$$\tilde{a}_j(\mathbf{n}) = a_j(\mathbf{n}) + s_{NL} \sum_{|\mathbf{n}-\mathbf{n}'|=1} a_j(\mathbf{n}')$$

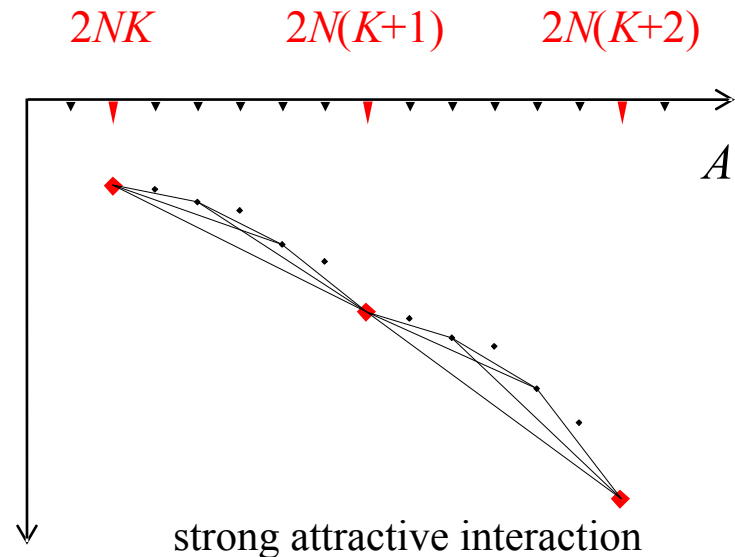
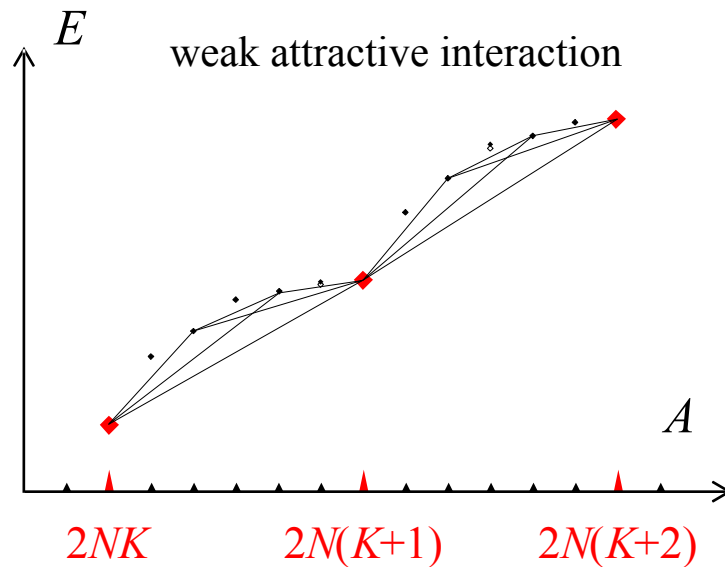
This interaction produces both S-wave and P-wave attraction. The symmetry group is

$$G = \text{U}(1) \times \text{SU}(2)_{\text{spin}} \times \text{SO}(3)_{\text{orb}}$$

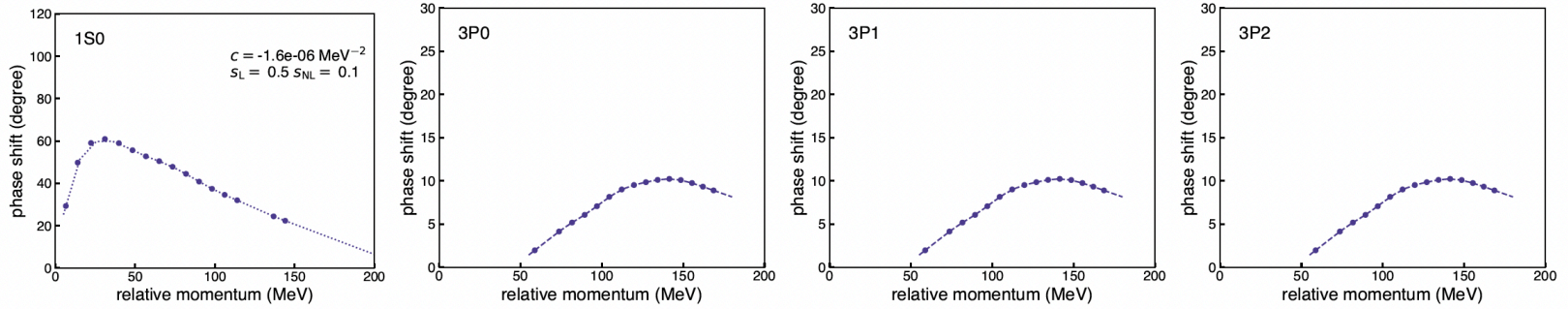
Spectral convexity theorem

Consider any fermionic system with an $SU(2N)$ symmetry with purely attractive interactions. It must obey the $SU(2N)$ convexity bounds illustrated below, and the $SU(2N)$ symmetry is not spontaneously broken.

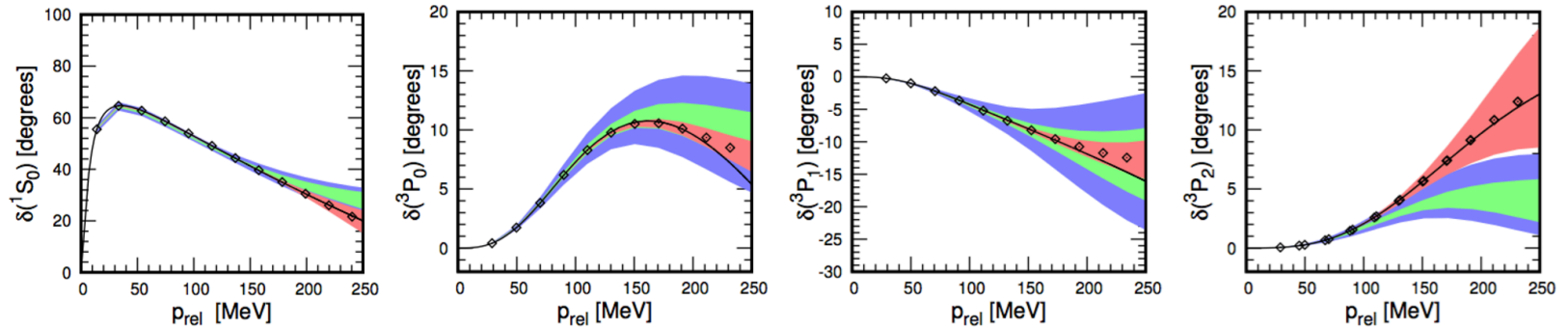
D.L., Phys. Rev. Lett. 98 (2007) 182501



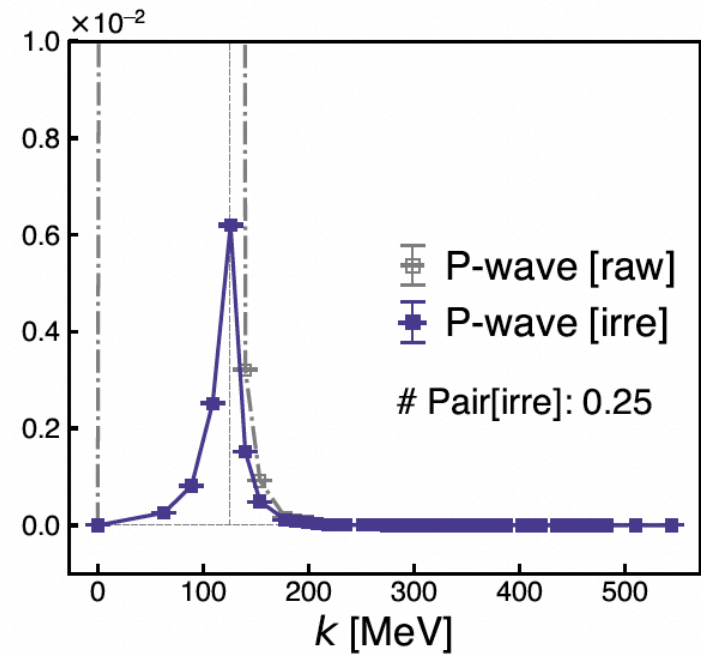
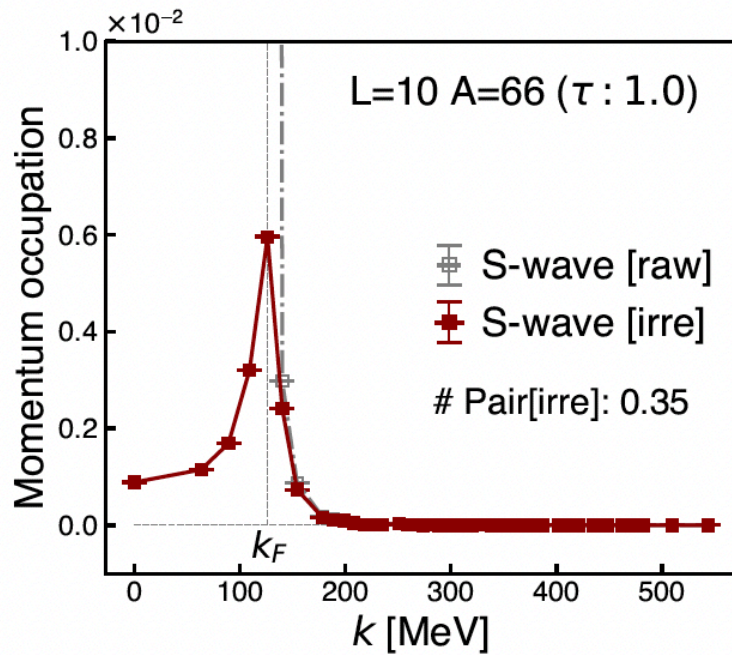
3D GAE Hubbard model

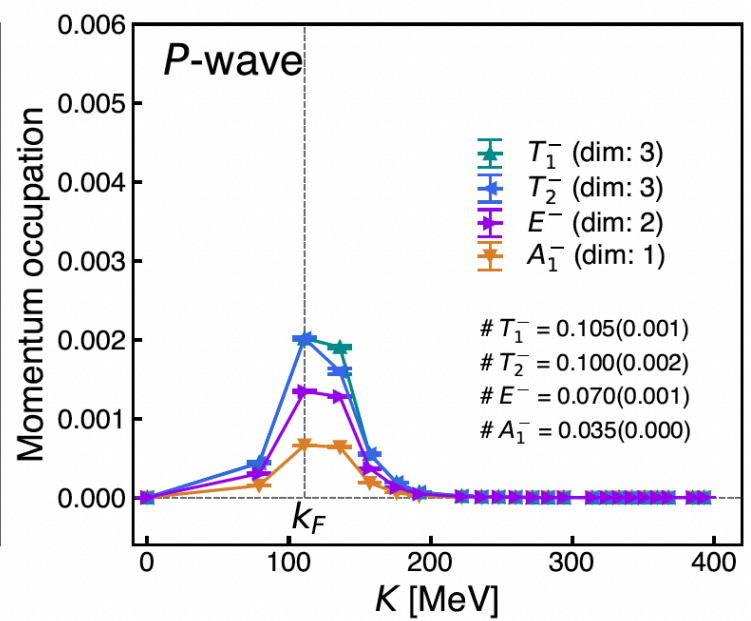
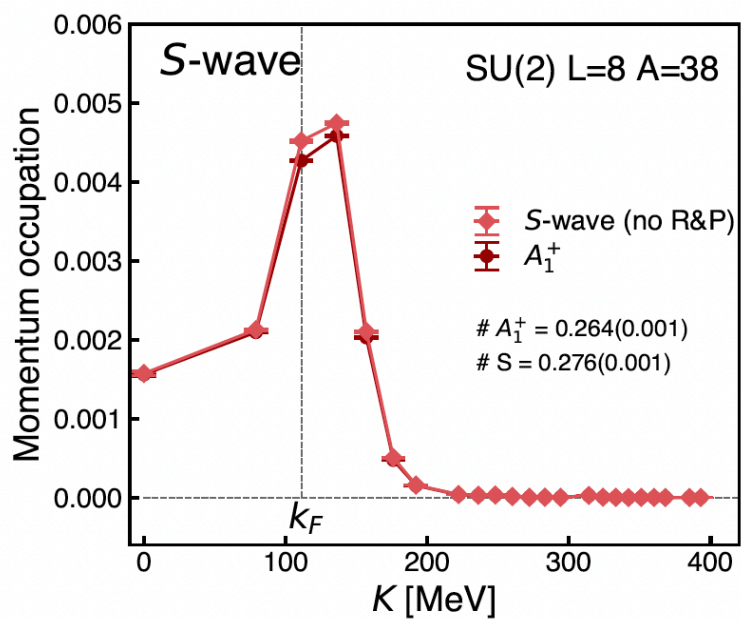


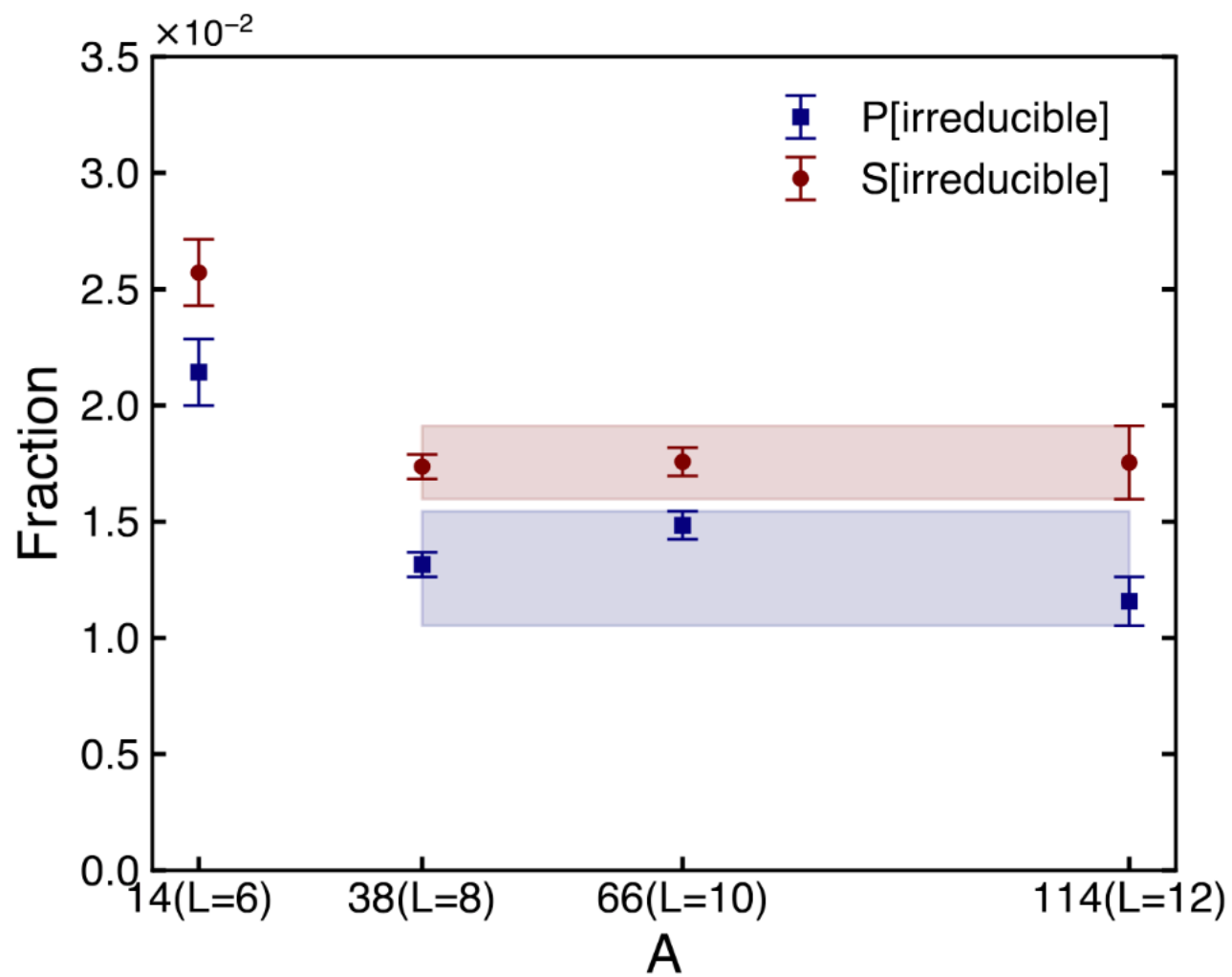
Comparison with N3LO chiral



GAE Hubbard model



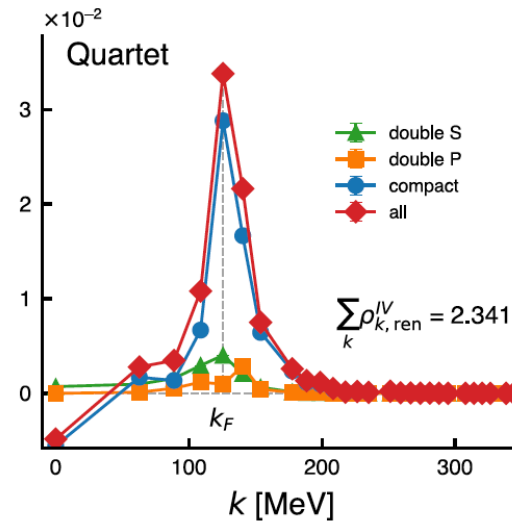
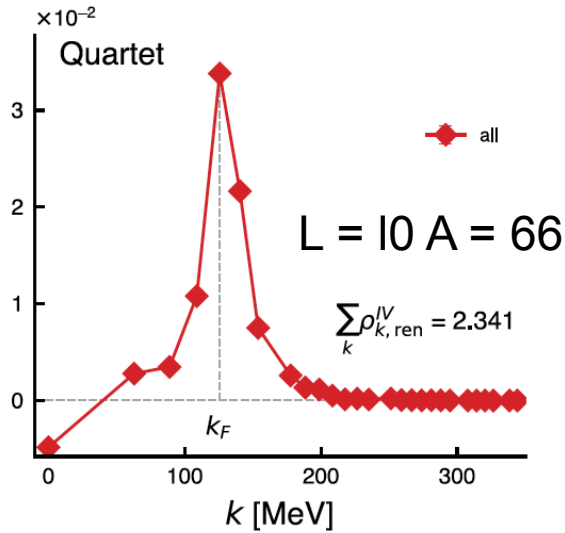




How does the $SU(2)$ spin symmetry remain unbroken if there is a P-wave condensate?

Why are the S-wave and P-wave condensates so small?

GAE Hubbard model

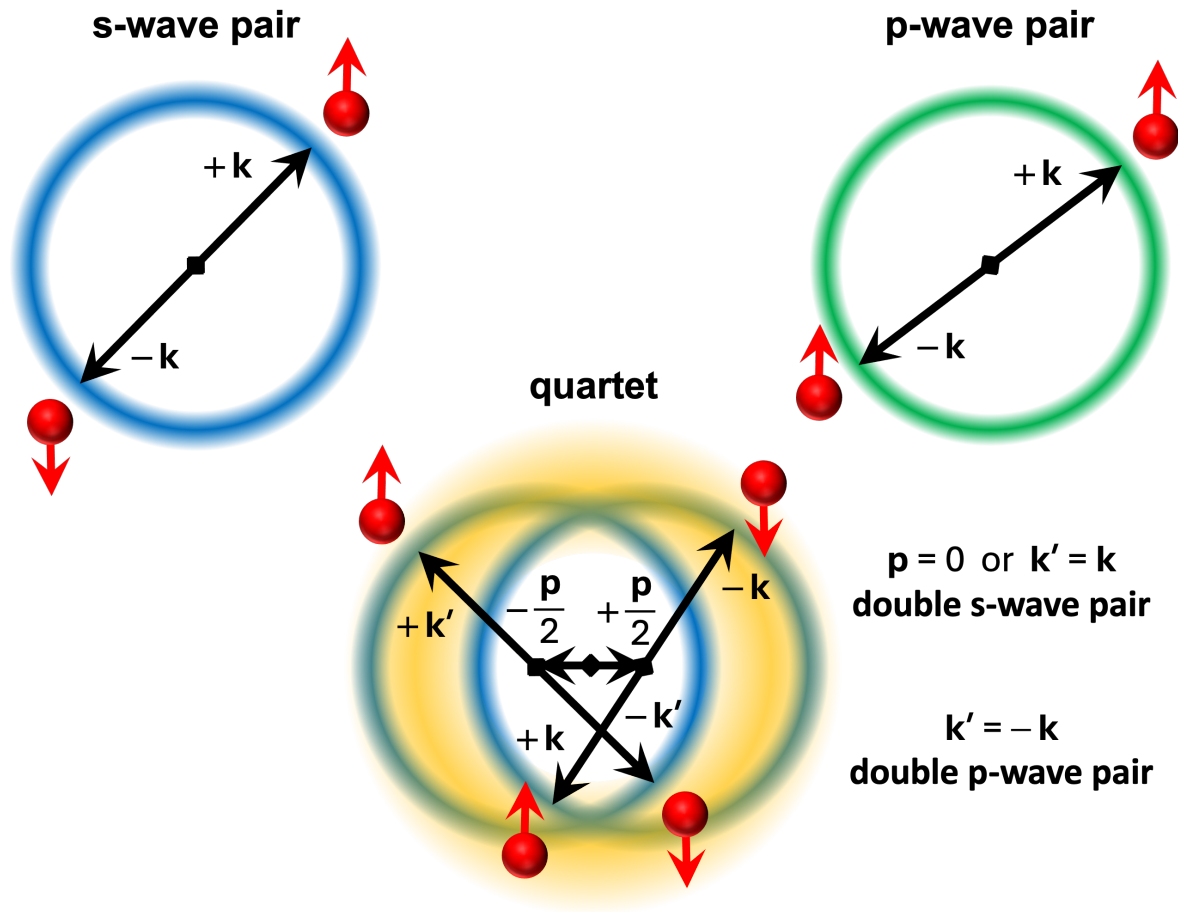


Double S-wave pairs	1.8(4)%
Double P-wave pairs	1.3(3)%
Compact quartets	10(3)%

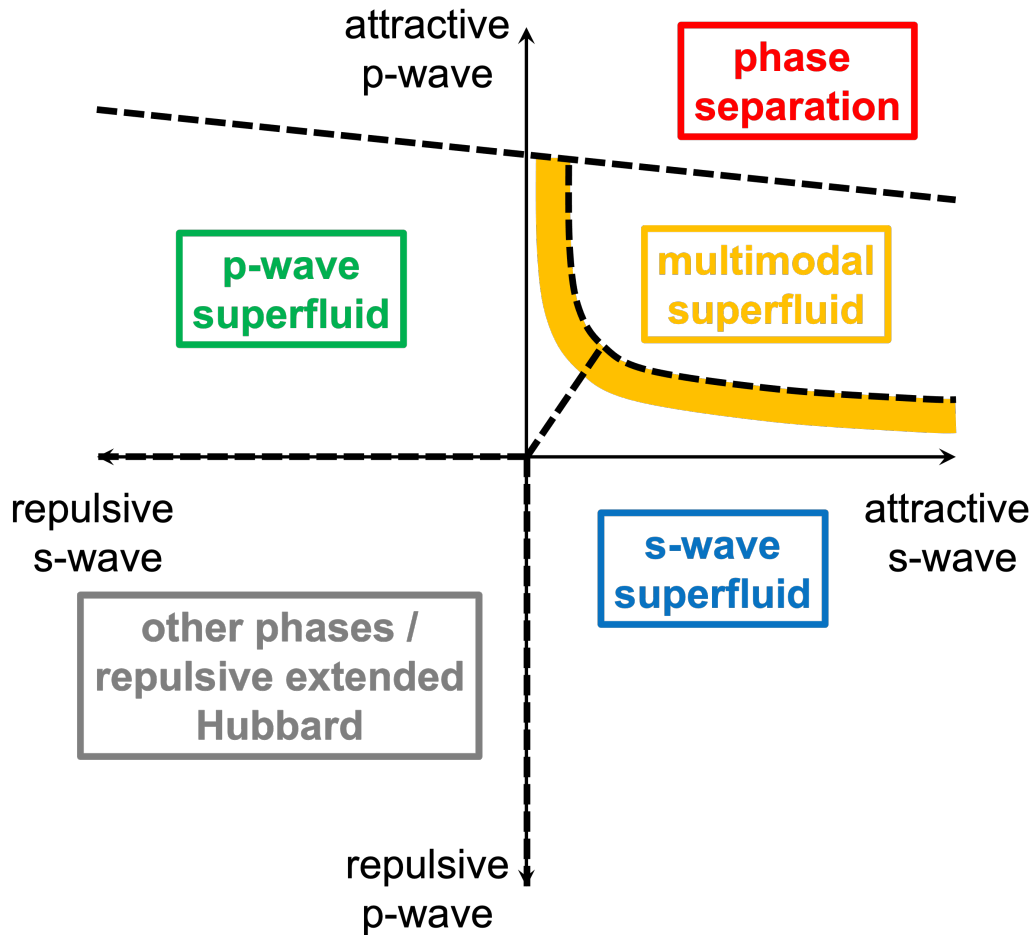
Table S9. SU(2) lattice energies for $L^3 = 10^3$. Lattice SU(2) energies calculated with $c = -1.6 \times 10^{-6} \text{ MeV}^{-2}$, $s_L = 0.5$, and $s_{NL} = 0.1$. For $A = 66$, the density is $\rho = 8.5898 \times 10^{-3} \text{ fm}^{-3}$, corresponding to a Fermi momentum $k_F = 125.66 \text{ MeV}$.

A	E_{SU2} (MeV)	2Δ (MeV)	$4\Delta_Q$ (MeV)
66	84.88 (11)		
67	88.27 (13)		
68 $_{\uparrow\downarrow}$	89.07 (19)	2.59 (34)	
68 $_{\uparrow\uparrow}$	90.65 (21)	1.01 (35)	
70	92.10 (28)		1.17 (48)

Multimodal superfluidity

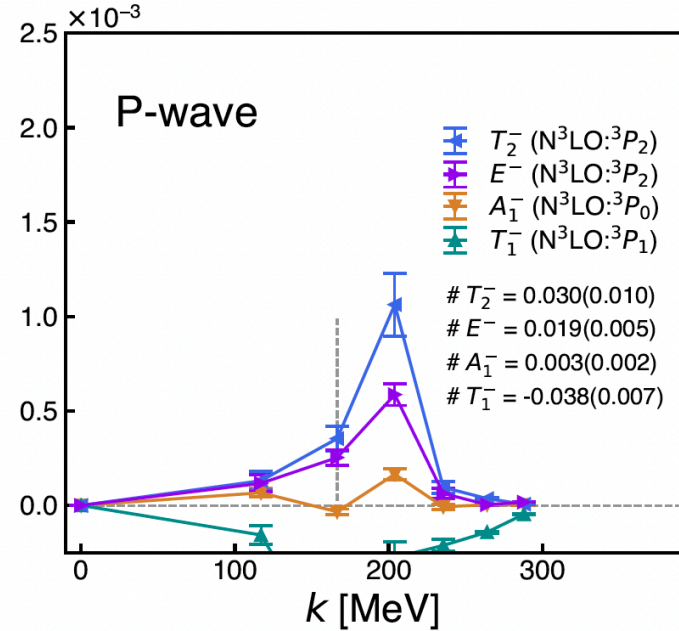
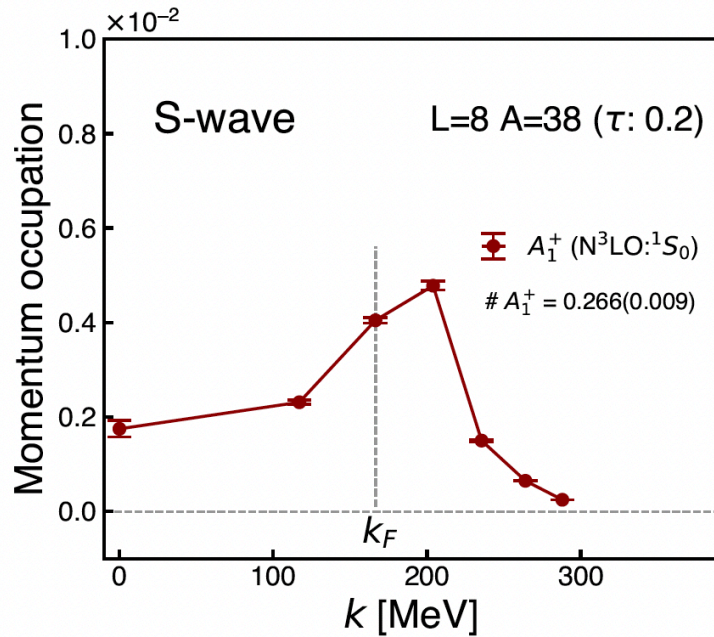


Quantum phase diagram



Neutron matter with N3LO interactions

$$k_F = 167 \text{ MeV}$$

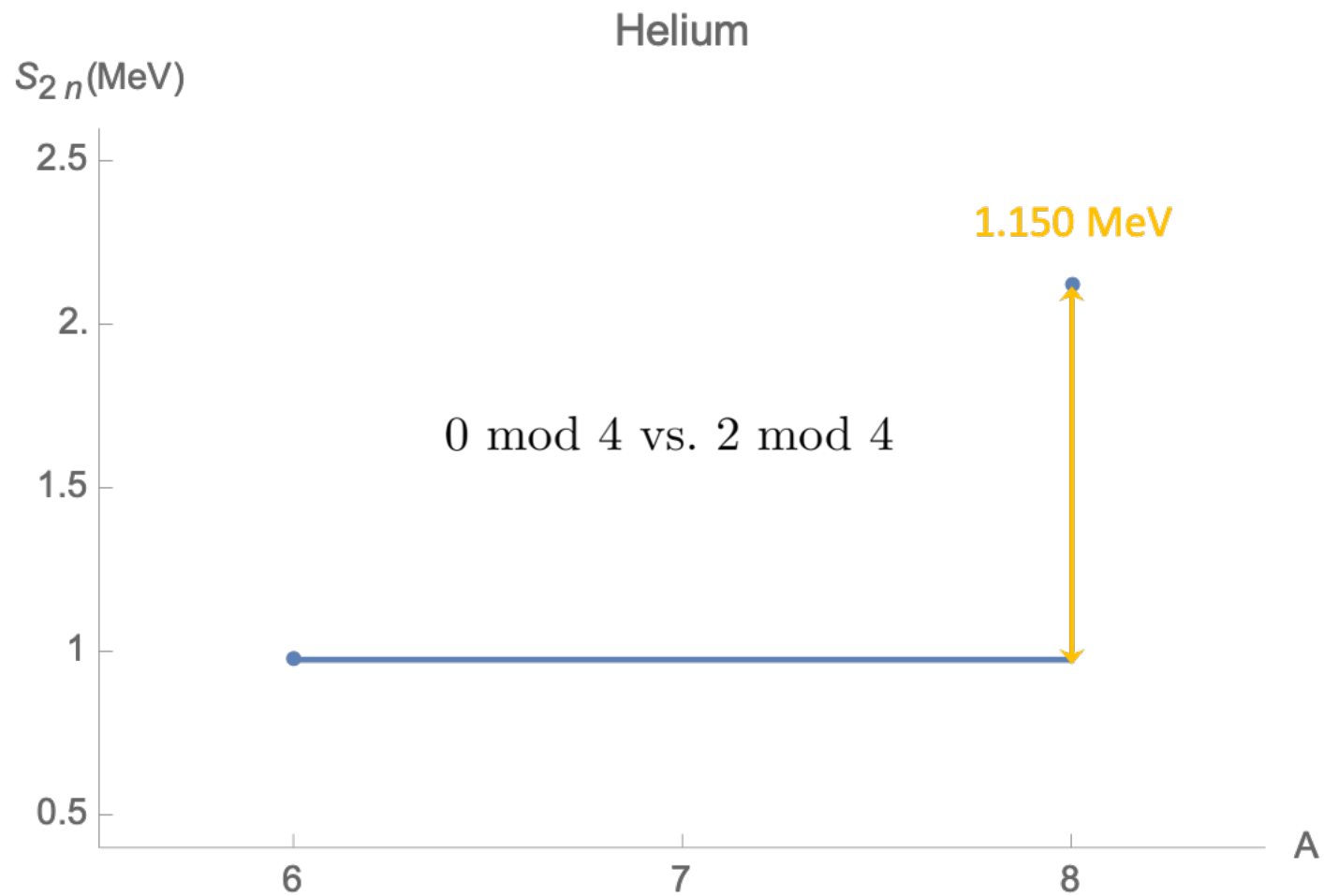


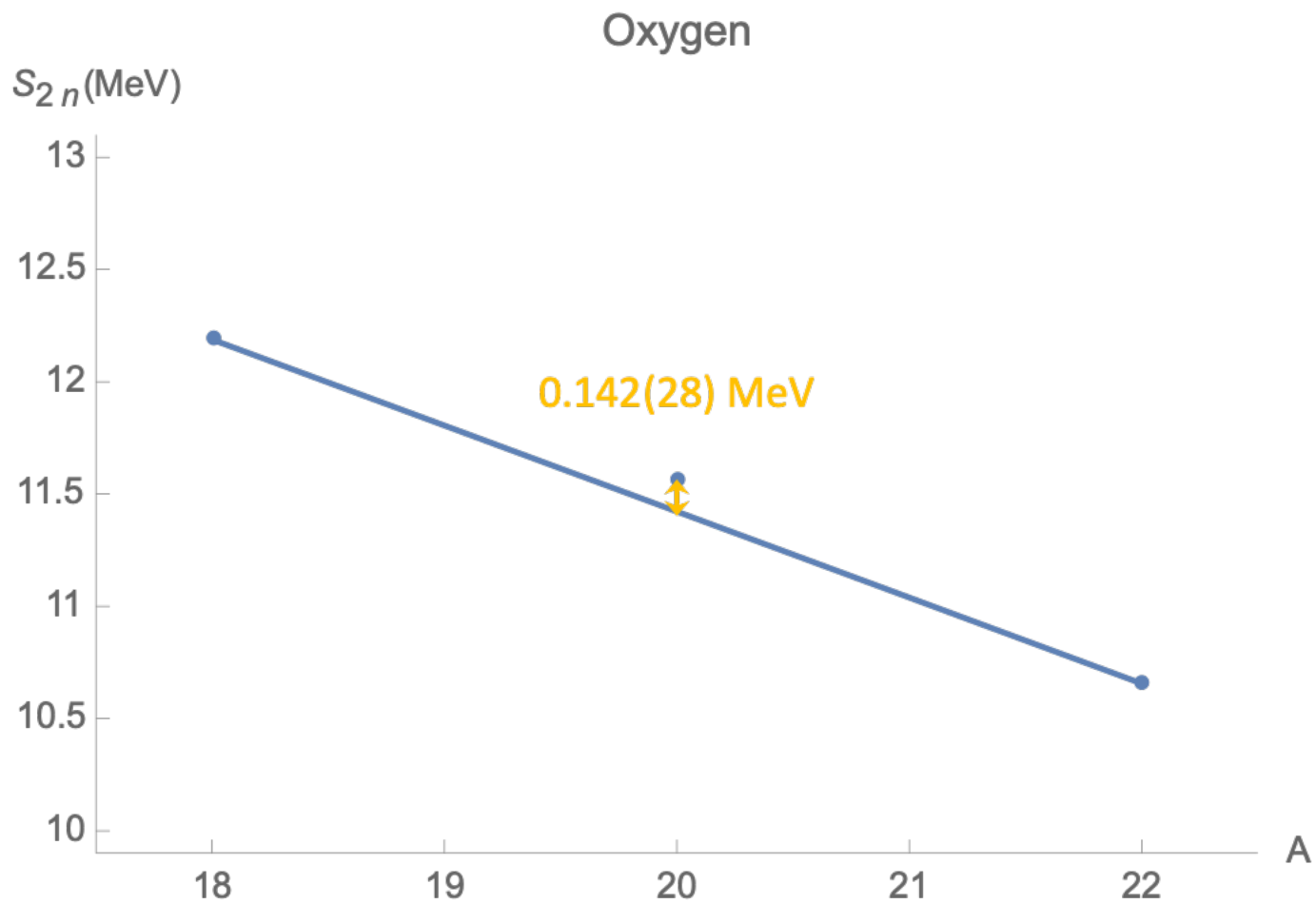
a. Neutron matter (chiral N³LO)		
Density (fm ⁻³)		2Δ _S (MeV)
0.033		5.58(92)
0.085		2.61(76)
Density (fm ⁻³)	channel	2Δ _P (MeV)
0.033	³ P ₀	1.25(86)
0.033	³ P ₁	0.59(113)
0.033	³ P ₂ [T ₂ ⁻]	1.36(86)
0.033	³ P ₂ [E ⁻]	2.16(90)
0.085	³ P ₀	1.95(95)
0.085	³ P ₁	-0.24(81)
0.085	³ P ₂ [T ₂ ⁻]	2.10(80)
0.085	³ P ₂ [E ⁻]	2.03(87)
Density (fm ⁻³)		4Δ _Q (MeV)
0.033		3.90(167)
0.085		2.01(86)

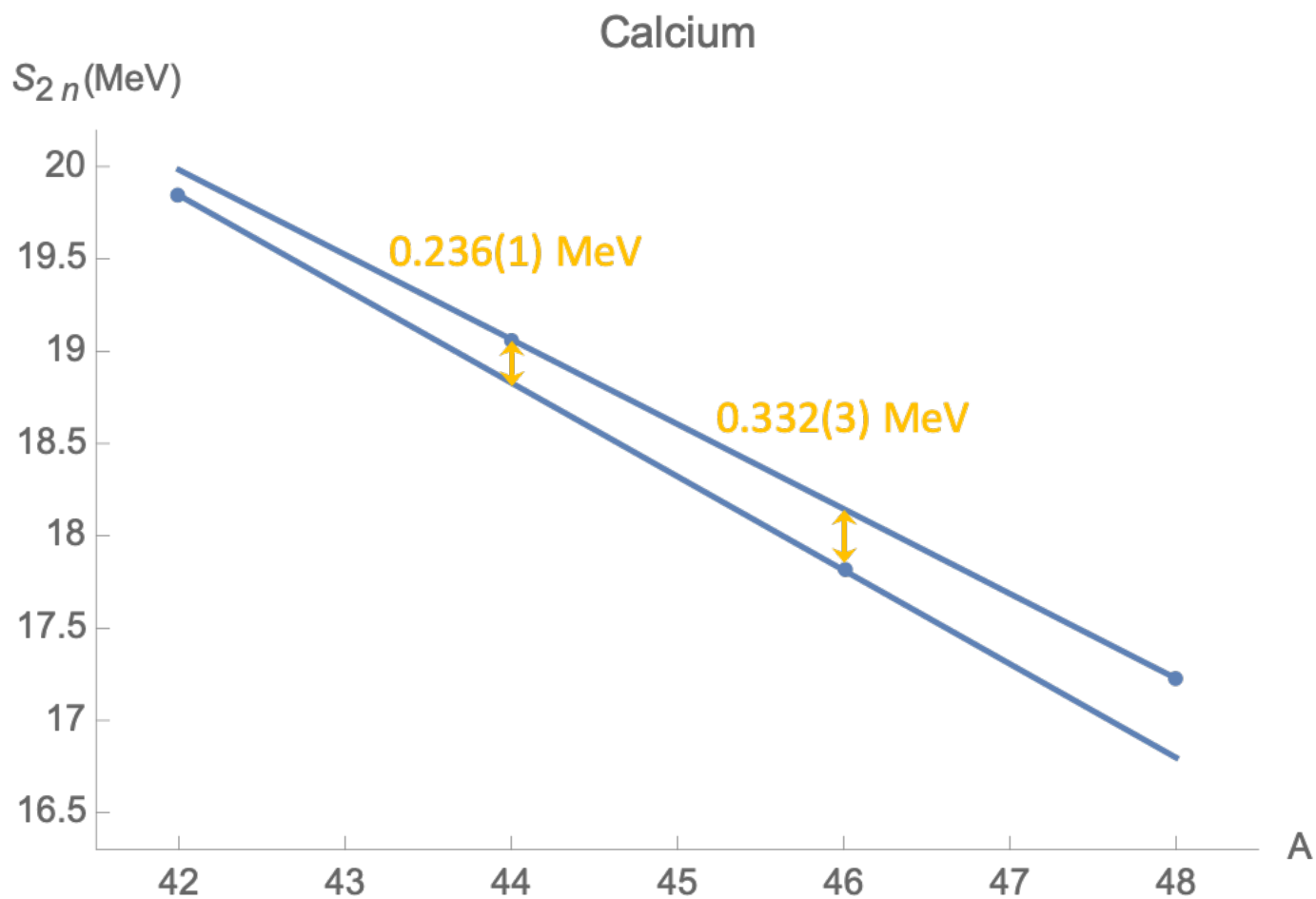
Experimental evidence

S-wave pair binding in MeV

Nuclei	Predominant Orbitals	$2\Delta_S$
$S_n\{^{17}_8\text{O}_9, ^{18}_8\text{O}_{10}\}+$	$(1d_{5/2})^2$	3.902
$\frac{1}{2}S_n\{^{17}_8\text{O}_9, ^{18}_8\text{O}_{10}, ^{19}_8\text{O}_{11}\}-$	$(1d_{5/2})^2$	3.996(1)
$\frac{1}{4}S_n\{^{17}_8\text{O}_9, ^{18}_8\text{O}_{10}, ^{19}_8\text{O}_{11}, ^{20}_8\text{O}_{12}\}+$	$(1d_{5/2})^2$	3.934(2)
$S_n\{^{41}_{20}\text{Ca}_{21}, ^{42}_{20}\text{Ca}_{22}\}+$	$(1f_{7/2})^2$	3.118
$\frac{1}{2}S_n\{^{41}_{20}\text{Ca}_{21}, ^{42}_{20}\text{Ca}_{22}, ^{43}_{20}\text{Ca}_{23}\}-$	$(1f_{7/2})^2$	3.333
$\frac{1}{2}S_n\{^{41}_{20}\text{Ca}_{21}, ^{42}_{20}\text{Ca}_{22}, ^{43}_{20}\text{Ca}_{23}, ^{44}_{20}\text{Ca}_{24}\}-$	$(1f_{7/2})^2$	3.353
$S_n\{^{209}_{82}\text{Pb}_{127}, ^{210}_{82}\text{Pb}_{128}\}+$	$(2g_{9/2})^2$	1.248(2)
$\frac{1}{2}S_n\{^{209}_{82}\text{Pb}_{127}, ^{210}_{82}\text{Pb}_{128}, ^{211}_{82}\text{Pb}_{129}\}-$	$(2g_{9/2})^2$	1.299(2)
$\frac{1}{4}S_n\{^{209}_{82}\text{Pb}_{127}, ^{210}_{82}\text{Pb}_{128}, ^{211}_{82}\text{Pb}_{129}, ^{212}_{82}\text{Pb}_{130}\}+$	$(2g_{9/2})^2$	1.309(2)







Experimental evidence

Quartet binding in MeV

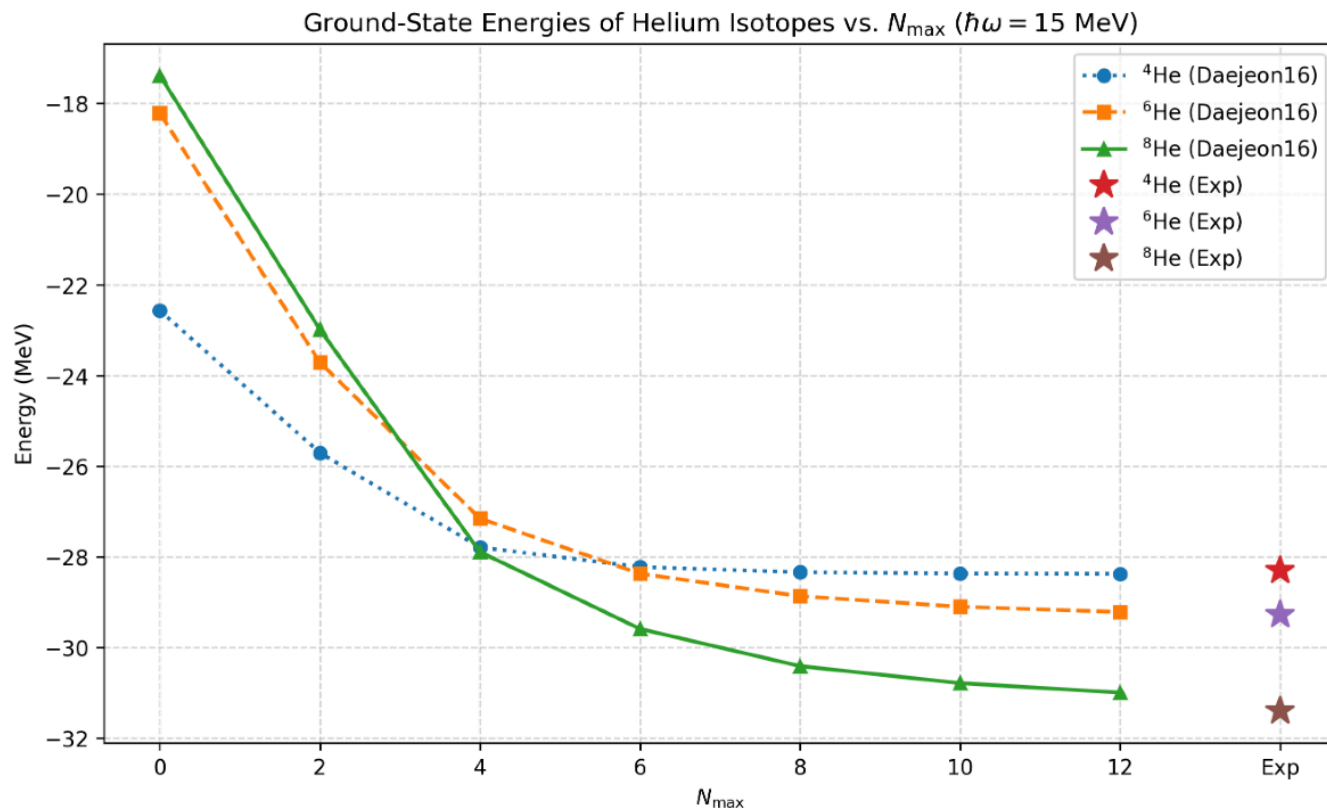
Nuclei	Predominant Orbitals	$4\Delta_Q$
$S_{2n}\{^6\text{He}_4, ^8\text{He}_4\}+$	$(1p_{3/2})^4$	1.150
$\frac{1}{2}S_{2n}\{^{18}_8\text{O}_{10}, ^{20}_8\text{O}_{12}, ^{22}_8\text{O}_{14}\}-$	$(1d_{5/2})^4$	0.142(28)
$\frac{1}{2}S_{2n}\{^{42}_{20}\text{Ca}_{22}, ^{44}_{20}\text{Ca}_{24}, ^{46}_{20}\text{Ca}_{26}\}-$	$(1f_{7/2})^4$	0.236(1)
$\frac{1}{2}S_{2n}\{^{44}_{20}\text{Ca}_{24}, ^{46}_{20}\text{Ca}_{26}, ^{48}_{20}\text{Ca}_{28}\}+$	$(1f_{7/2})^4$	0.332(3)
$\frac{1}{2}S_{2n}\{^{106}_{50}\text{Sn}_{56}, ^{108}_{50}\text{Sn}_{58}, ^{110}_{50}\text{Sn}_{60}\}+$	combination	0.033(11)
$\frac{1}{2}S_{2n}\{^{108}_{50}\text{Sn}_{58}, ^{110}_{50}\text{Sn}_{60}, ^{112}_{50}\text{Sn}_{62}\}-$	combination	0.007(17)
$\frac{1}{2}S_{2n}\{^{110}_{50}\text{Sn}_{60}, ^{112}_{50}\text{Sn}_{62}, ^{114}_{50}\text{Sn}_{64}\}+$	combination	0.025(16)
$\frac{1}{2}S_{2n}\{^{112}_{50}\text{Sn}_{62}, ^{114}_{50}\text{Sn}_{64}, ^{116}_{50}\text{Sn}_{66}\}-$	combination	0.015(7)
$\frac{1}{2}S_{2n}\{^{114}_{50}\text{Sn}_{64}, ^{116}_{50}\text{Sn}_{66}, ^{118}_{50}\text{Sn}_{68}\}+$	combination	0.050(30)
$\frac{1}{2}S_{2n}\{^{194}_{82}\text{Pb}_{112}, ^{196}_{82}\text{Pb}_{114}, ^{198}_{82}\text{Pb}_{116}\}+$	combination	0.047(22)
$\frac{1}{2}S_{2n}\{^{196}_{82}\text{Pb}_{114}, ^{198}_{82}\text{Pb}_{116}, ^{200}_{82}\text{Pb}_{118}\}-$	combination	0.057(16)
$\frac{1}{2}S_{2n}\{^{198}_{82}\text{Pb}_{116}, ^{200}_{82}\text{Pb}_{118}, ^{202}_{82}\text{Pb}_{120}\}+$	combination	0.021(15)
$\frac{1}{2}S_{2n}\{^{200}_{82}\text{Pb}_{118}, ^{202}_{82}\text{Pb}_{120}, ^{204}_{82}\text{Pb}_{122}\}-$	combination	0.013(13)
$\frac{1}{2}S_{2n}\{^{202}_{82}\text{Pb}_{120}, ^{204}_{82}\text{Pb}_{122}, ^{206}_{82}\text{Pb}_{124}\}+$	combination	0.014(7)
$\frac{1}{2}S_{2n}\{^{204}_{82}\text{Pb}_{122}, ^{206}_{82}\text{Pb}_{124}, ^{208}_{82}\text{Pb}_{126}\}-$	combination	0.110(2)
$\frac{1}{2}S_{2n}\{^{210}_{82}\text{Pb}_{128}, ^{212}_{82}\text{Pb}_{130}, ^{214}_{82}\text{Pb}_{132}\}-$	$(2g_{9/2})^4$	0.014(2)
$\frac{1}{2}S_{2n}\{^{212}_{84}\text{Po}_{128}, ^{214}_{84}\text{Po}_{130}, ^{216}_{84}\text{Po}_{132}\}-$	$(2g_{9/2})^4$	0.019(1)
$\frac{1}{2}S_{2n}\{^{214}_{84}\text{Po}_{130}, ^{216}_{84}\text{Po}_{132}, ^{218}_{84}\text{Po}_{134}\}+$	$(2g_{9/2})^4$	0.015(2)
$\frac{1}{2}S_{2n}\{^{214}_{86}\text{Rn}_{128}, ^{216}_{86}\text{Rn}_{130}, ^{218}_{86}\text{Rn}_{132}\}-$	$(2g_{9/2})^4$	0.079(12)

Experimental evidence

P-wave pair binding in MeV

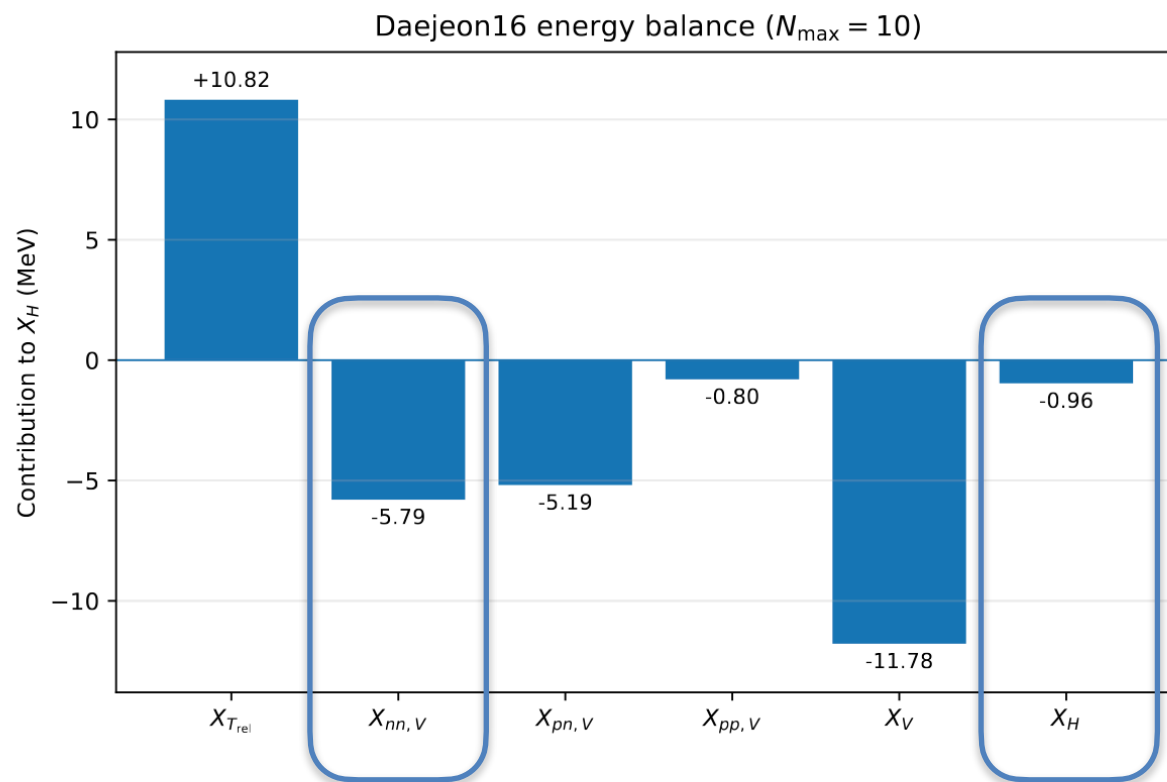
Nuclei	Predominant Orbitals	$2\Delta_p$
$S_n\{^{17}_8\text{O}_9(\frac{3}{2}^+), ^{18}_8\text{O}_{10}(1^+)\}_+$	$1d_{5/2} \otimes 1d_{3/2}$	0.172(12)
$\frac{1}{2}S_n\{^{55}_{26}\text{Fe}_{29}(\frac{5}{2}^-), ^{56}_{26}\text{Fe}_{30}(1^+), ^{57}_{26}\text{Fe}_{31}(\frac{5}{2}^-)\}_-$	$2p_{3/2} \otimes 1f_{5/2}$	0.139(3)
$\frac{1}{2}S_n\{^{57}_{28}\text{Ni}_{29}(\frac{5}{2}^-), ^{58}_{28}\text{Ni}_{30}(1^+), ^{59}_{28}\text{Ni}_{31}(\frac{5}{2}^-)\}_-$	$2p_{3/2} \otimes 1f_{5/2}$	0.245(1)

Neutron quartet structure of ^8He



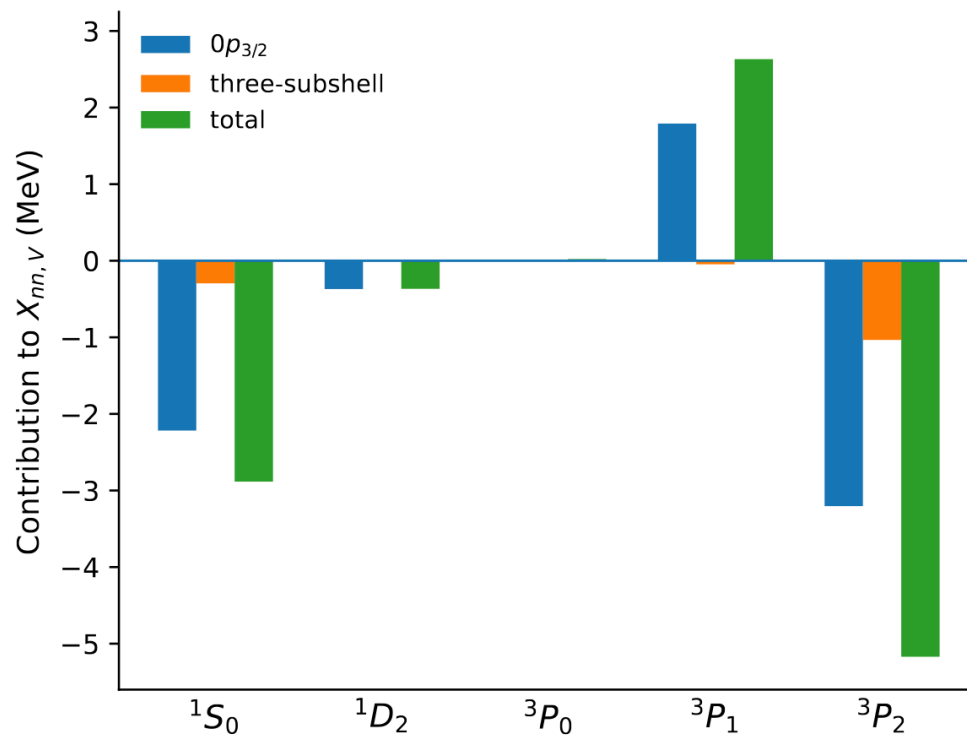
Neutron quartet structure of ^8He

$$X_f = f(^8\text{He}) - 2f(^6\text{He}) + f(^4\text{He})$$



Neutron quartet structure of ^8He

$$X_f = f(^8\text{He}) - 2f(^6\text{He}) + f(^4\text{He})$$



Summary and outlook

We reviewed the framework of nuclear lattice effective field theory, detailing essential computational methods such as auxiliary-field Monte Carlo, wavefunction matching, and the pinhole algorithm. Our results covered properties of light and medium-mass nuclei, with a focus on nuclear clustering, charge radii, and magnetic moments. Furthermore, we introduced theoretical evidence for a novel phase of matter—multimodal superfluidity in neutron matter—together with experimental evidence in neutron-rich nuclei.