

# Probing pairing and its isospin modes at finite angular momentum

Bo Cederwall

KTH Royal Institute of Technology  
Stockholm University (affiliated)

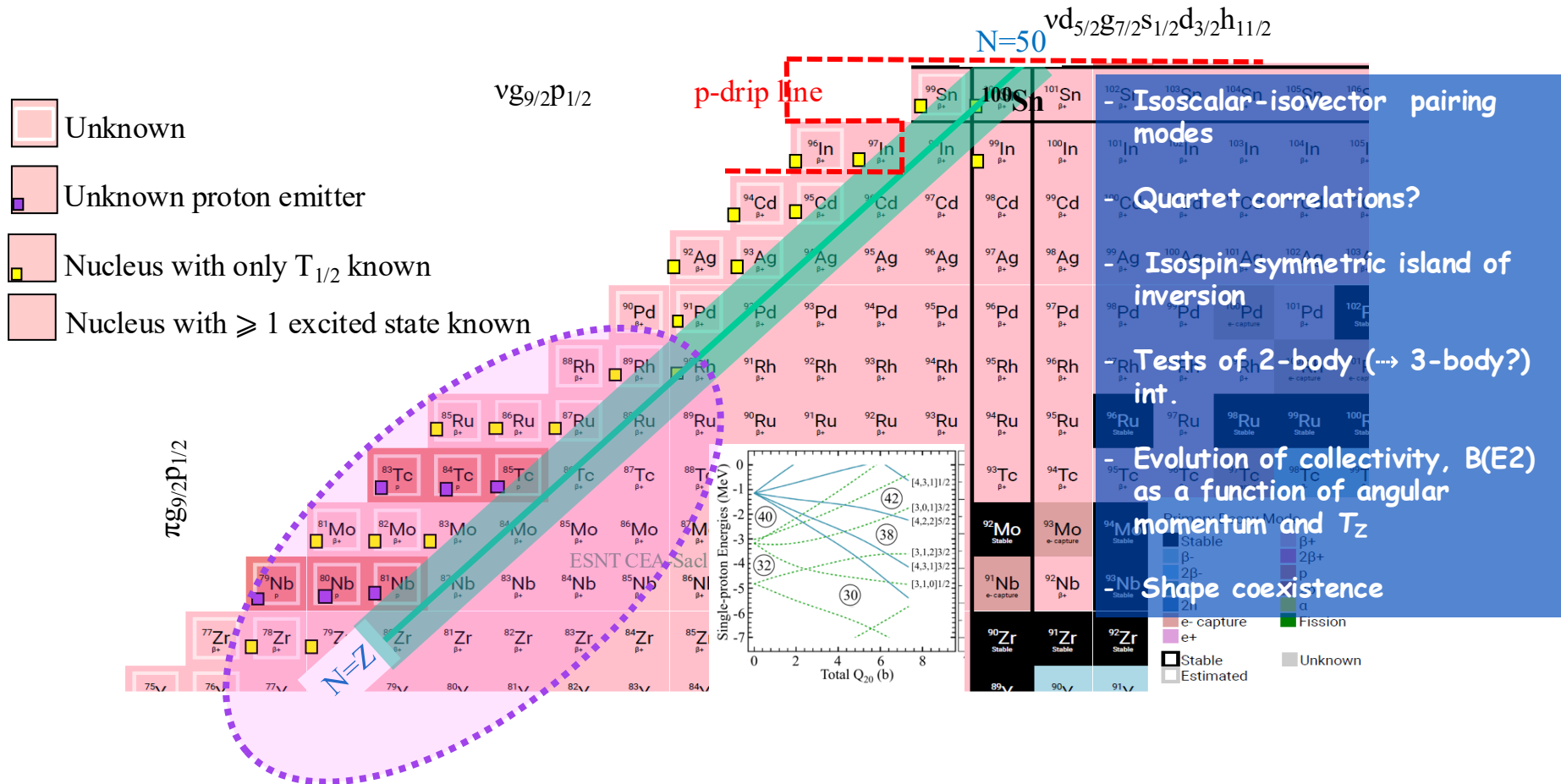
International Workshop  
on  
Many-body correlations, phase transitions and clustering

Espace de Structure et de réactions Nucléaires Théorique (ESNT), at CEA-Saclay, 7-11 September, 2026

*As we are approaching lunch - Le plat du jour (appetizers)*

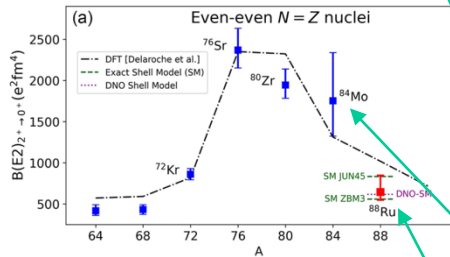
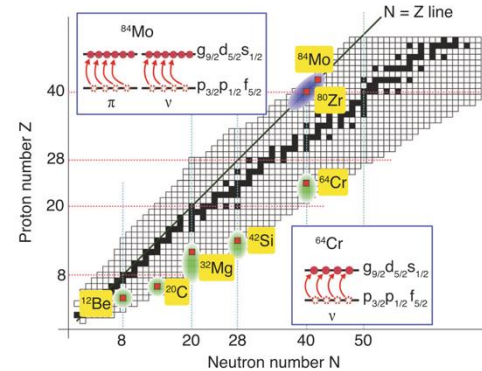
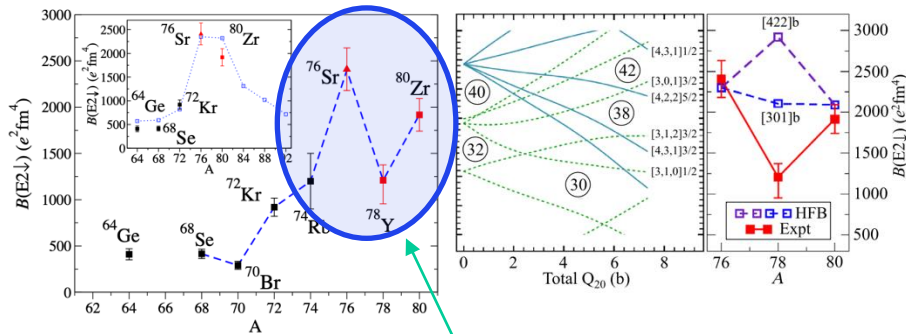
- Introduction, experimental perspective on the access to the heaviest  $N=Z$  nuclei
- Brief overview of experimental observables considered for probing np pairing.
- Evolution of structure, in particular pairing isospin modes with angular momentum along  $N=Z$ .
- Can we really “understand” delayed alignments in deformed  $N=Z$  nuclei e.g. in a simple CSM+generalized BCS picture including isoscalar pairing?
- Quartet condensate (spatially delocalized alpha-like spin-isospin correlations) – has it been overlooked in the experimental quest for  $T=0$  pairing in deformed  $N=Z$  nuclei?
- Could the Coriolis-Mottelson-Valatin effect as applied to alpha-like quartets explain the delayed alignments attributed to  $T=0$  pairing?
- A cranked QCM is proposed as a tool to overcome the computational burden of calculating intermediate-high spin collective states and determine if quartets could explain delayed alignments in deformed  $N=Z$  nuclei.
- Experimental outlook: probing the heaviest  $N \sim Z$  systems at finite angular momentum using lifetime measurements will be possible in the next years with AGATA and GRETA gamma-ray tracking arrays. Will this be the smoking gun for resolving the enigmatic  $T=0$  pairing issue?

# Physics we might learn from probing the structure of the heaviest $N=Z$ nuclei



# Isospin-symmetric island of inversion

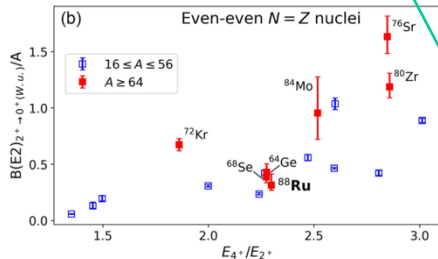
$B(E2; 2^+ \rightarrow 0^+)$  measurements with GRETINA@ FRIB



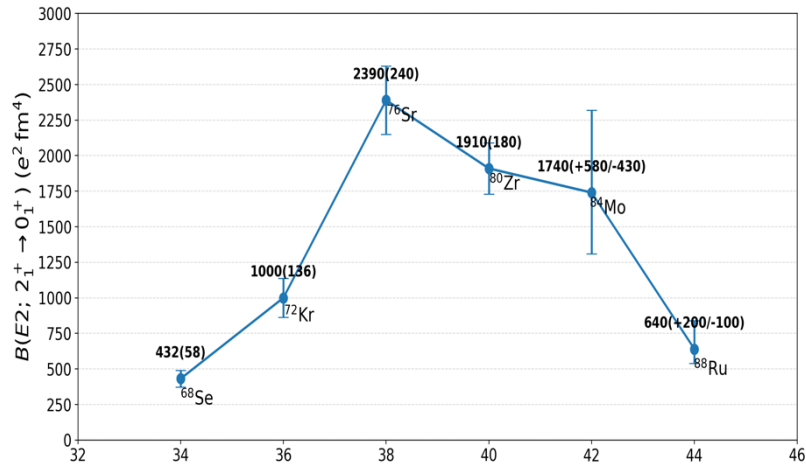
R.D.O. Llewellyn *et al.*, Establishing the Maximum Collectivity in Highly Deformed  $N = Z$  Nuclei, Phys. Rev. Lett. 124, 152501 (2020).  
<https://doi.org/10.1103/PhysRevLett.124.152501>

Ha, J., Recchia, F., Lenzi, S.M. *et al.* Abrupt structural transition in exotic molybdenum isotopes unveils an isospin-symmetric island of inversion. *Nat Commun* 16, 10631 (2025). <https://doi.org/10.1038/s41467-025-65621-2>

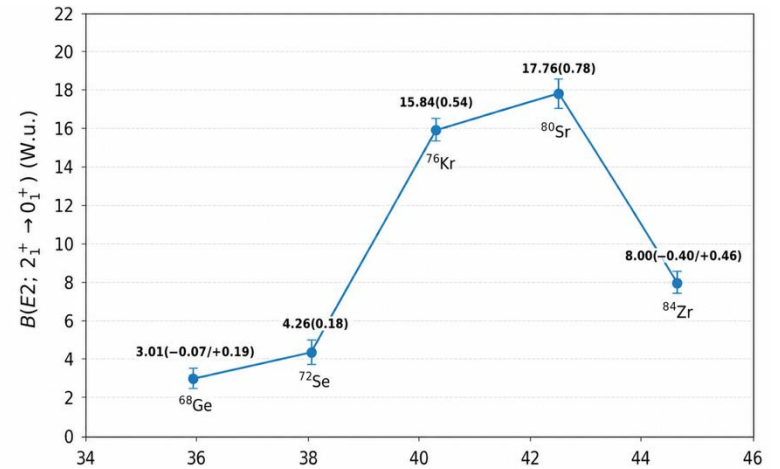
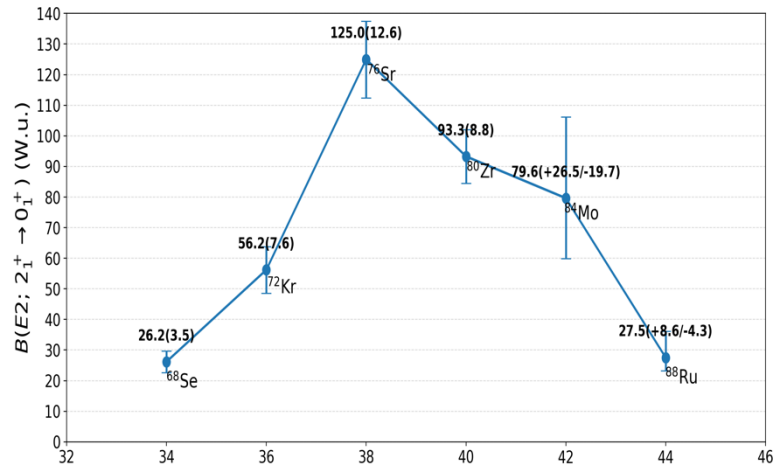
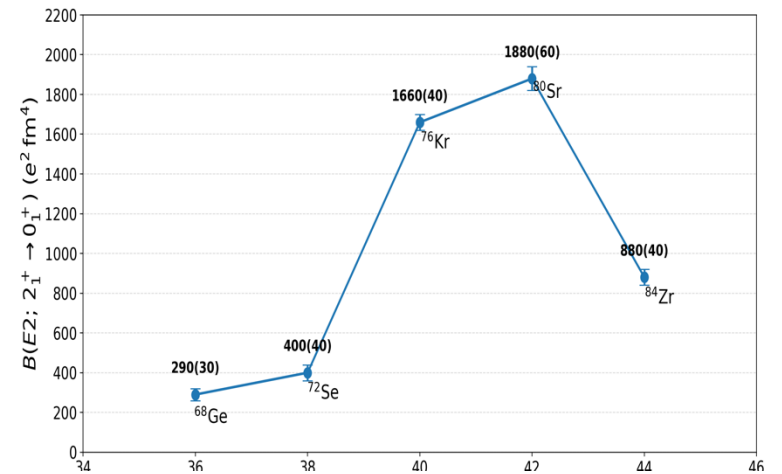
M.A. Bentley *et al.*, Collapse of nuclear collectivity along the  $N = Z$  line Phys. Rev. Lett. 136, 152501 (2026).  
<https://doi.org/10.1103/r6ns-ypw8>



Even-even  $N = Z$  nuclei



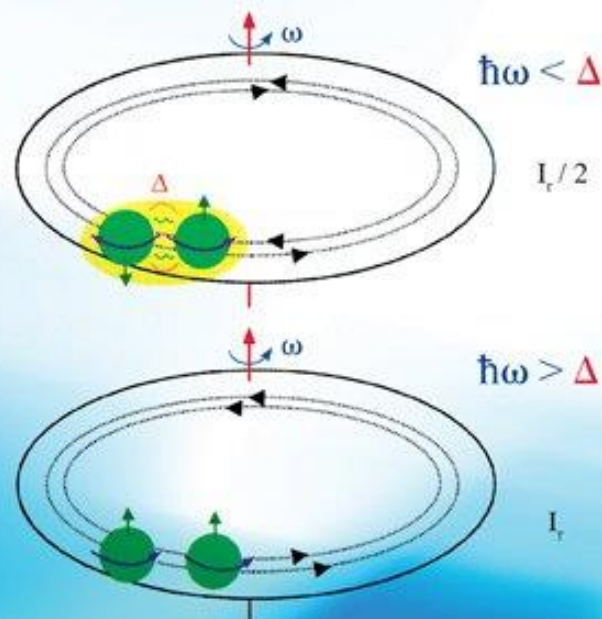
Even-even  $N = Z+4$  nuclei



11 Septen

# Fifty Years of Nuclear BCS

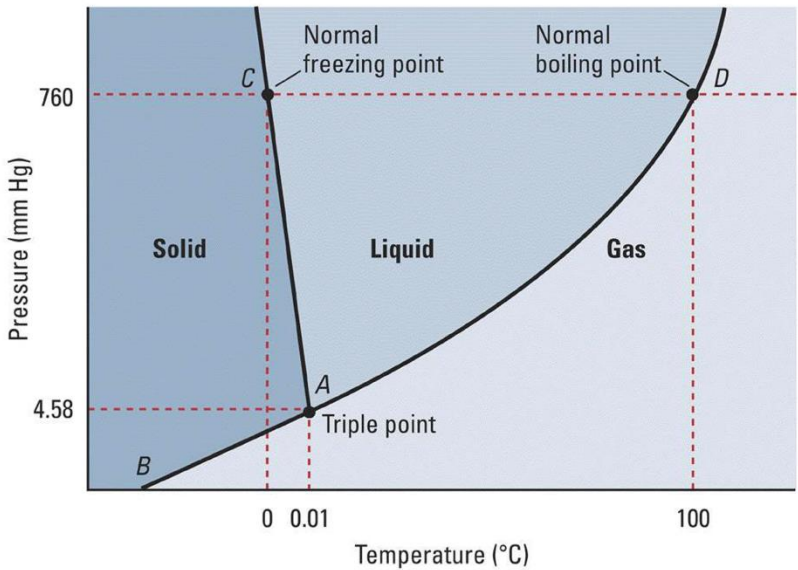
## Pairing in Finite Systems



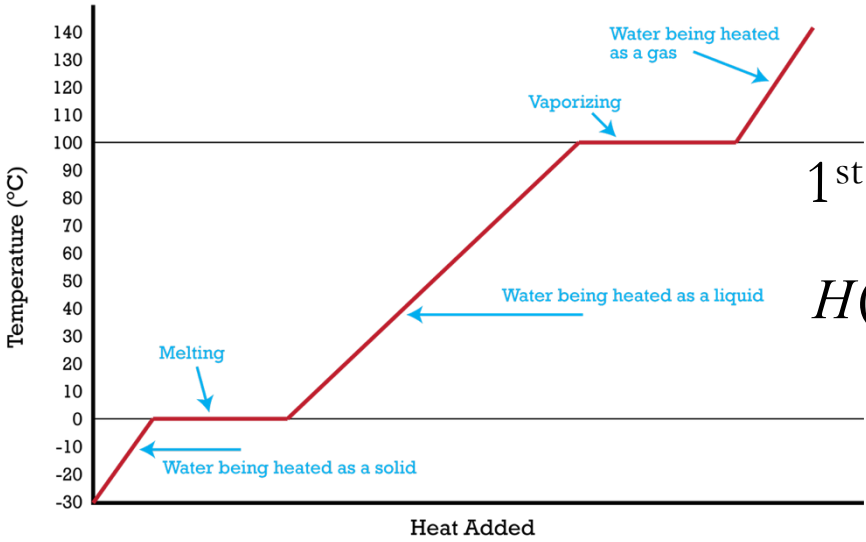
Ricardo A Broglia  
Vladimir Zelevinsky  
*editors*

# Phases of matter - phase transitions

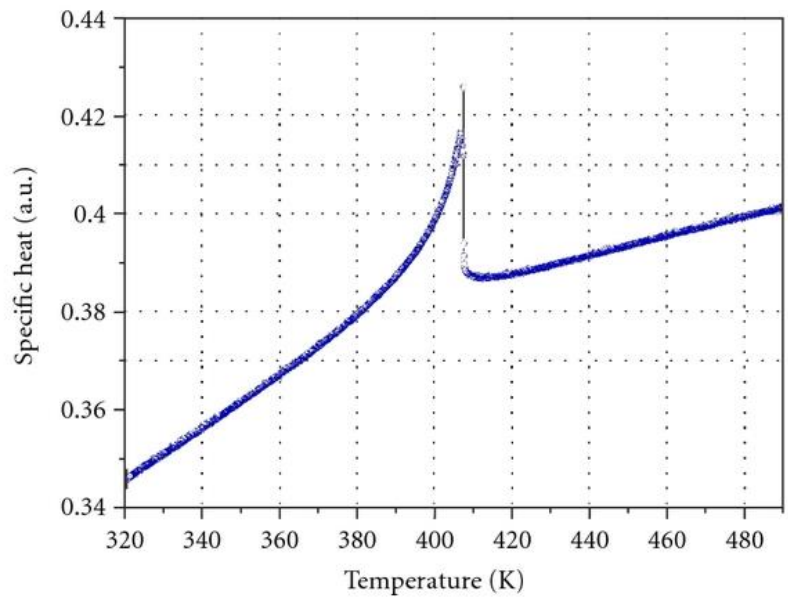
(a macroscopic phase is essentially a particular organization of many-body correlations generated by the interactions between the particles)



Heating Curve for Water at 1.00 atm Pressure



<https://socratic.org>



Temperature dependence of specific heat of BaTiO<sub>3</sub> single crystal around the paraelectric-ferroelectric phase transition point T<sub>c</sub>  
S. Grabovsky, M. Takesada, A. Onodera et al., Annual Meeting of Physical Society of Japan, March 2012

1<sup>st</sup> order phase transition:

$$H(\lambda) = (1 - \eta)H_0 + \eta H_1 \quad \eta - \text{"control" parameter}$$

$$\frac{d \langle H(\eta) \rangle}{d\eta} \text{ discontinuity at } \eta = \eta_{cr}$$

## Letters to the Editor

**P**UBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length and should be submitted in duplicate.

### Bound Electron Pairs in a Degenerate Fermi Gas\*

LEON N. COOPER

Physics Department, University of Illinois, Urbana, Illinois  
(Received September 21, 1956)

**I**T has been proposed that a metal would display superconducting properties at low temperatures if the one-electron energy spectrum had a volume-independent energy gap of order  $\Delta \simeq kT_c$ , between the ground state and the first excited state.<sup>1,2</sup> We should like to point out how, primarily as a result of the exclusion principle, such a situation could arise.

Consider a pair of electrons which interact above a quiescent Fermi sphere with an interaction of the kind that might be expected due to the phonon and the screened Coulomb fields. If there is a net attraction between the electrons, it turns out that they can form a bound state, though their total energy is larger than zero. The properties of a noninteracting system of such

$= (1/V) \exp[i(\mathbf{k}_1 \cdot \mathbf{r}_1 + \mathbf{k}_2 \cdot \mathbf{r}_2)]$  which satisfy periodic boundary conditions in a box of volume  $V$ , and where  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the coordinates of electron one and electron two. (One can use antisymmetric functions and obtain essentially the same results, but alternatively we can choose the electrons of opposite spin.) Defining relative and center-of-mass coordinates,  $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ ,  $\mathbf{r} = (\mathbf{r}_2 - \mathbf{r}_1)$ ,  $\mathbf{K} = (\mathbf{k}_1 + \mathbf{k}_2)$  and  $\mathbf{k} = \frac{1}{2}(\mathbf{k}_2 - \mathbf{k}_1)$ , and letting  $\mathcal{E}_K + \epsilon_k = (\hbar^2/m)(\frac{1}{4}K^2 + k^2)$ , the Schrödinger equation can be written

$$(\mathcal{E}_K + \epsilon_k - E)a_k + \sum_{\mathbf{k}'} a_{\mathbf{k}'} (\mathbf{k} | H_1 | \mathbf{k}') \times \delta(\mathbf{K} - \mathbf{K}') / \delta(0) = 0 \quad (1)$$

where

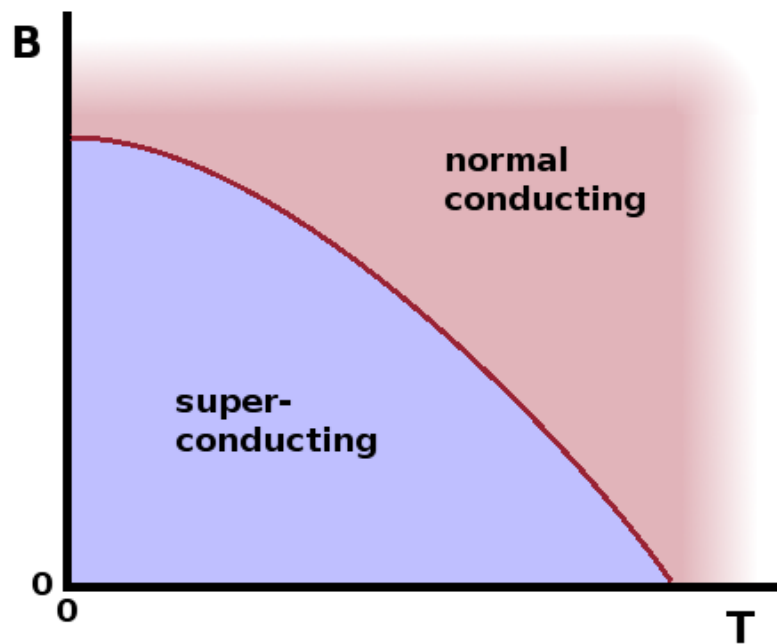
$$\begin{aligned} \Psi(\mathbf{R}, \mathbf{r}) &= (1/\sqrt{V}) e^{i\mathbf{K} \cdot \mathbf{R}} \chi(\mathbf{r}, K), \\ \chi(\mathbf{r}, K) &= \sum_{\mathbf{k}} (a_{\mathbf{k}}/\sqrt{V}) e^{i\mathbf{k} \cdot \mathbf{r}}, \end{aligned} \quad (2)$$

and

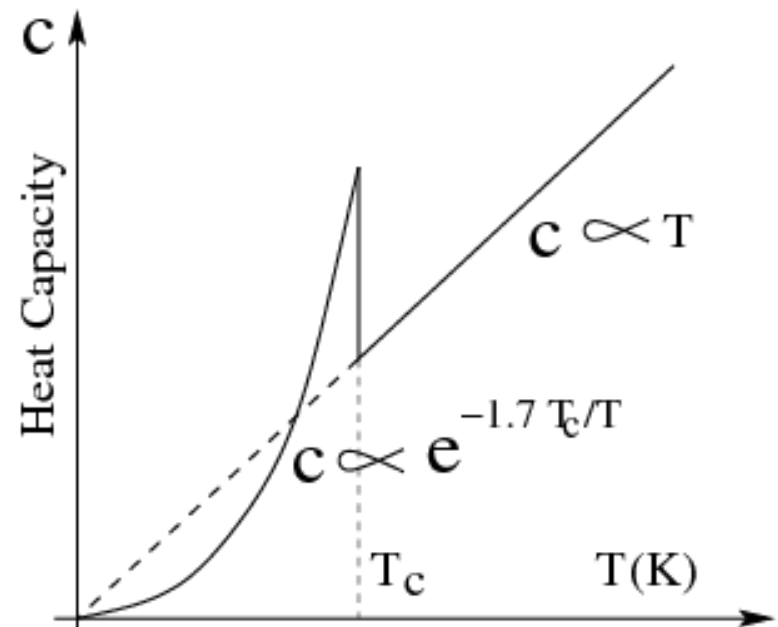
$$(\mathbf{k} | H_1 | \mathbf{k}') = \left( \frac{1}{V} \int d\mathbf{r} e^{-i\mathbf{k} \cdot \mathbf{r}} H_1 e^{i\mathbf{k}' \cdot \mathbf{r}} \right)_{0 \text{ phonons}}.$$

We have assumed translational invariance in the metal. The summation over  $\mathbf{k}'$  is limited by the exclusion principle to values of  $k_1$  and  $k_2$  larger than  $q_0$ , and by the delta function, which guarantees the conservation of the total momentum of the pair in a single scattering. The  $K$  dependence enters through the latter restriction.

Bardeen and Pines<sup>3</sup> and Fröhlich<sup>4</sup> have derived approximate formulas for the matrix element  $(\mathbf{k} | H_1 | \mathbf{k}')$ ; it is thought that the matrix elements for which the two electrons are confined to a thin energy shell near the Fermi surface,  $\epsilon_1 \simeq \epsilon_2 \simeq \epsilon_F$ , are the principal ones



<http://users.aber.ac.uk>



Can we observe something similar in atomic nuclei?

# THE PHYSICAL REVIEW

*A journal of experimental and theoretical physics established by E. L. Nichols in 1893*

SECOND SERIES, VOL. 114, NO. 6

JUNE 15, 1959

## Possible Superfluidity of a System of Strongly Interacting Fermions\*†

L. N. COOPER,‡ R. L. MILLS, AND A. M. SESSLER

*The Ohio State University, Columbus, Ohio*

(Received January 30, 1959)

The possible superfluidity of a system of strongly interacting fermions is investigated on the assumption that an adequate description of the system in its "normal" state is given by independent fermions in a momentum-dependent potential. On the basis of this assumption we have investigated whether a correlated wave function of the form used by Bardeen, Cooper, and Schrieffer minimizes the ground-state energy. The nonzero terms in the expectation value of the Hamiltonian contain the modified kinetic energy and the full two-body potential between the fermion pairs. An integral equation is obtained in configuration space for the correlation function between pairs. This integral equation is meaningful even for potentials with hard cores, and a nonzero solution implies the existence of a superfluid state. A variational method is devised which provides a criterion for superfluidity and a lower bound for the transition temperature into the superfluid state. We find that a repulsive hard core does not in principle forbid the existence of a superfluid state, but whereas in the absence of a hard core an attractive two-body potential always leads to a superfluid state at sufficiently low temperatures, in the presence of a repulsive core there appears to be a critical strength of attraction needed to form a superfluid state. When the variational principle is applied to liquid He<sup>3</sup> or to nuclear matter, it is found for a wide class of trial functions that the system does not become a superfluid.

### I. INTRODUCTION

THE fact that a system of fermions can become a superfluid is demonstrated by the observed behavior of the electron gas in many metals at low temperatures. It seems natural then to inquire whether or not other systems of fermions might display similar properties, and what the criterion for such behavior would be. This question is of particular interest because of recent conjectures<sup>1</sup> that nuclear matter might be superfluid in the sense that for an infinite medium there would be an energy gap between the ground state and the lowest single particle excitations. It has been further conjectured that this might show up for a finite nucleus as the explanation for the abnormally large single-particle excitation energy of even-even nuclei. Whether or not He<sup>3</sup>, the other well-known Fermi fluid, has a

superfluid phase at low temperatures has been a matter of concern since the discovery of the  $\lambda$  transition in He<sup>4</sup> and London's conjecture<sup>2</sup> that Bose statistics are crucial to the formation of the superfluid phase.

We have attempted to treat this question by taking over to an arbitrary system of fermions what appears to have worked very well for the electron gas. There the introduction of pair correlations into the wave function and the approximation that only pairs of given total spin and total momentum are strongly correlated was sufficient to account for the observed properties of the superconducting phase. In making this same assumption for an arbitrary system of fermions we have had to assume that in some sense the "normal" fluid could be described in an uncorrelated approximation. The situation in this regard, for a fermion system such as the nucleus, is much less clear than the corresponding situation in a metal where the lattice plays such a dominant rôle.

This basic conjecture of our procedure, the description of the normal fluid as a Fermi gas in a momentum-dependent potential, is discussed in Sec. II. In the third

\* Supported in part by the National Science Foundation.

† A preliminary account of this work was presented at the *Kamerlingh-Onnes Conference on Low-Temperature Physics, Leiden, Netherlands, June, 1958* (Suppl. Physica **24**, September, 1958).

‡ Present address: Brown University, Providence, Rhode Island.

<sup>1</sup> Bohr, Mottelson, and Pines, Phys. Rev. **110**, 936 (1958); C. De Dominicis and P. C. Martin, Bull. Am. Phys. Soc. Ser. II, **3**, 224 (1958).

<sup>2</sup> F. London, Nature **163**, 694 (1949).

# Thermal and electromagnetic properties of $^{166}\text{Er}$ and $^{167}\text{Er}$

E. Melby, M. Guttormsen, J. Rekstad, A. Schiller, and S. Siem

*Department of Physics, University of Oslo, N-0316 Oslo, Norway*

A. Voinov

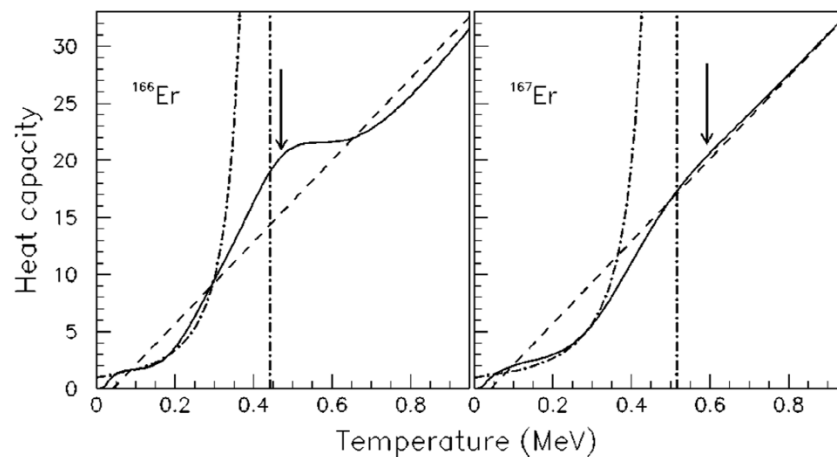
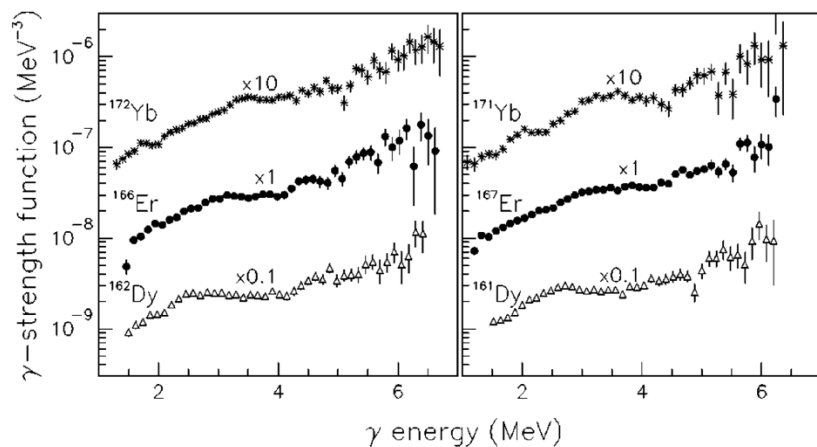
*Frank Laboratory of Neutron Physics, Joint Institute of Nuclear Research, RU-141980 Dubna, Moscow region, Russia*

(Received 30 October 2000; published 6 March 2001)

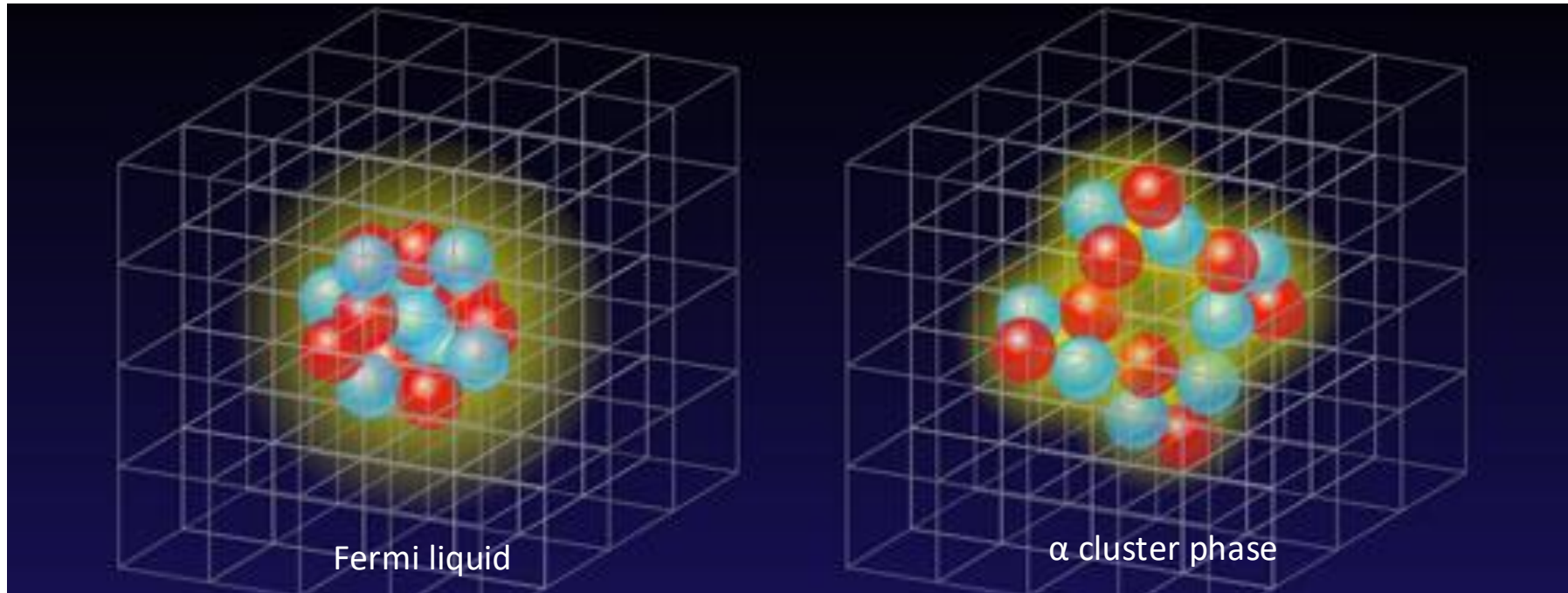
The primary  $\gamma$ -ray spectra of  $^{166}\text{Er}$  and  $^{167}\text{Er}$  are deduced from the  $(^3\text{He}, \alpha \gamma)$  reaction and the  $(^3\text{He}, ^3\text{He}' \gamma)$  reaction, respectively, enabling a simultaneous extraction of the level density and the  $\gamma$ -ray-strength function. Entropy, temperature, and heat capacity are deduced from the level density within the microcanonical and canonical ensembles, displaying signals of a phaselike transition from the pair-correlated ground state to an uncorrelated state at  $T_c \sim 0.5$  MeV. The  $\gamma$ -ray-strength function displays a bump around  $E_\gamma \sim 3$  MeV, interpreted as the pygmy resonance.

PHYSICAL REVIEW C **63** 044309

PHYSICAL REVIEW C **63** 044309



# Other exotic phases of nuclear matter - quartetting and $\alpha$ -particle condensation



PRL **117**, 132501 (2016)

 Selected for a **Viewpoint** in *Physics*  
PHYSICAL REVIEW LETTERS

week ending  
23 SEPTEMBER 2016



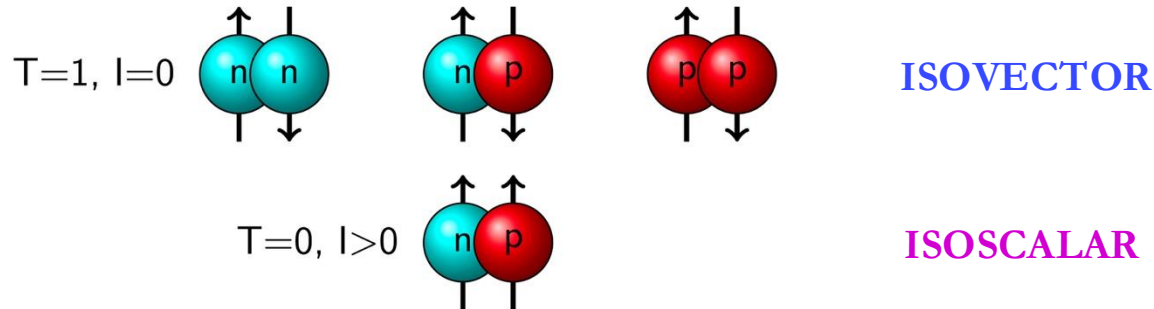
## Nuclear Binding Near a Quantum Phase Transition

Serdar Elhatisari,<sup>1</sup> Ning Li,<sup>2</sup> Alexander Rokash,<sup>3</sup> Jose Manuel Alarcón,<sup>1</sup> Dechuan Du,<sup>2</sup> Nico Klein,<sup>1</sup> Bing-nan Lu,<sup>2</sup>  
Ulf-G. Meißner,<sup>1,2,4</sup> Evgeny Epelbaum,<sup>3</sup> Hermann Krebs,<sup>3</sup> Timo A. Lähde,<sup>2</sup> Dean Lee,<sup>5</sup> and Gautam Rupak<sup>6</sup>

# Neutron-proton pairing in $N \approx Z$ nuclei - experimental probes

Isospin NN pairing modes in  $N \sim Z$  nuclei

- a unique feature of atomic nuclei due to the coupling of two Fermi liquids

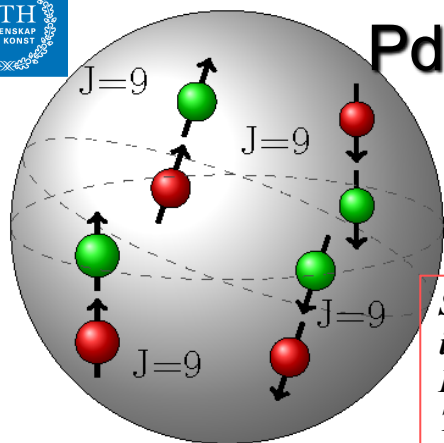


- Masses - binding energies in e-e and o-o nuclei indicate that  **$T=1$  *np* pairing** is dominant, no evidence for a  $T=0$  (deuteron-like) pair condensate **up to around  $A \approx 60$ . Need for accurate mass measurements in heavier  $N=Z$  systems**
  - *np* (deuteron) pair transfer reactions
  - GT Beta decay strengths
  - Charge radii
  - Other radioactive decay modes: (alpha) proton???
  - The above methods **probe ground-state or low-spin correlations**
  - Spectral properties of spherical  **$N=Z$**  nuclei near closed shells –deviations from classical s.p. behavior (deviations from seniority scheme, e.g.  $^{92}\text{Pd}$  – isoscalar spin-aligned coupling\*)
  - **Rotational properties (level structure, lifetimes) of deformed nuclei**
- We know the isoscalar effective NN interaction is strongly attractive but can it produce a correlated *np* pairing condensate?**

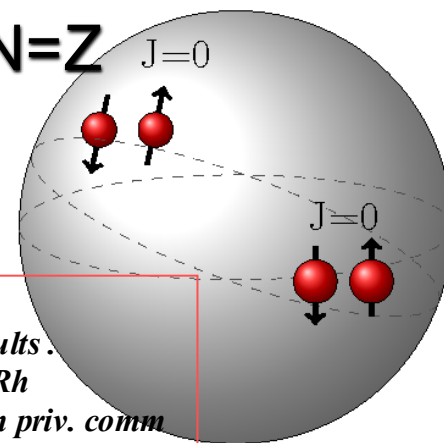
S. Frauendorf and A. Macchiavelli, Prog. Part. Nucl. Phys. 78, 24 (2014)

\* B. Cederwall et al., Nature 469, 68 (2011)

# Pd energy level systematics near N=Z - effects of np interactions



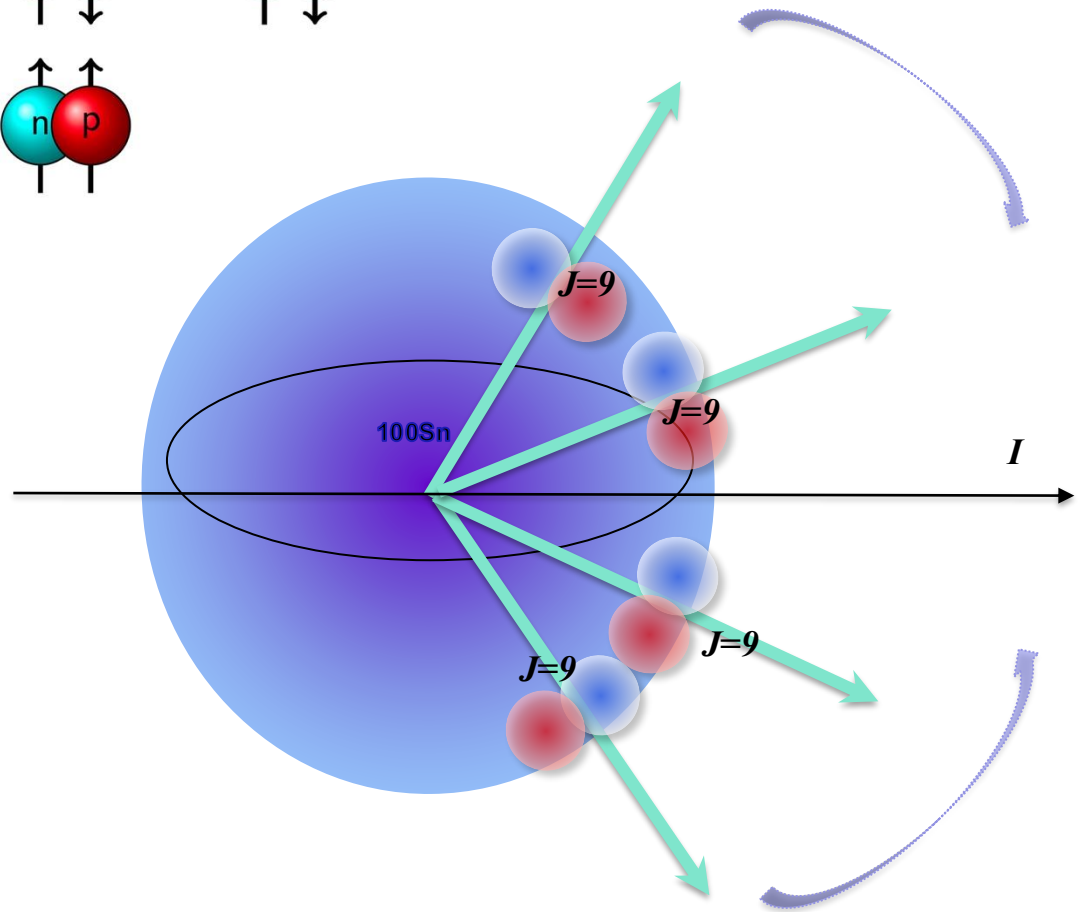
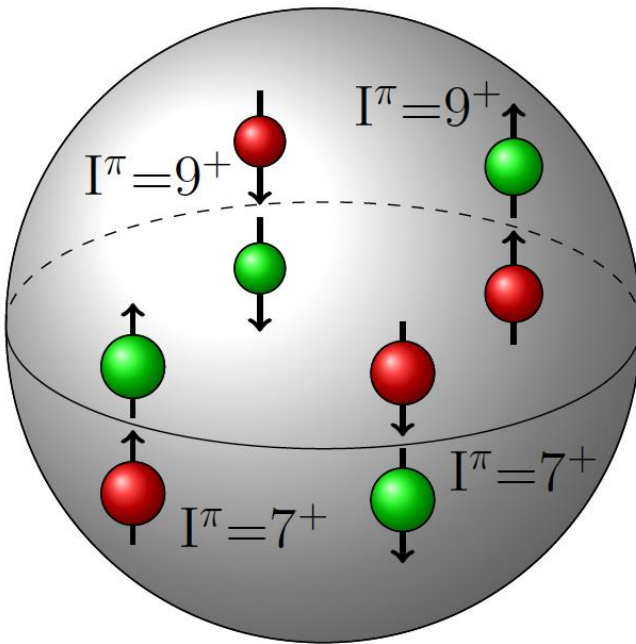
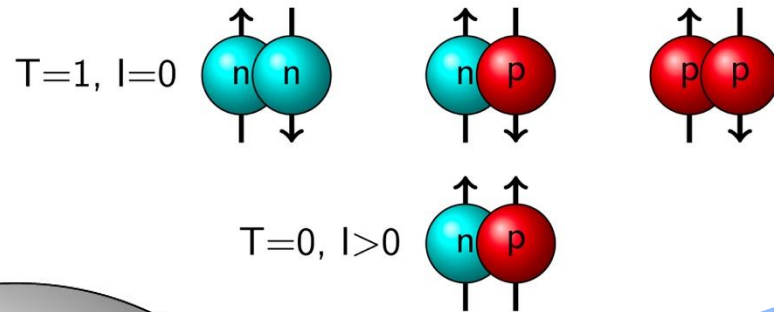
*Shell model calculations performed in several model spaces, i.e., 0g9/2, 0g9/2-1p1/2 and 0g9/2-1p1/2-0f5/2-1p3/2 with similar results. Int. parameters determined to reproduce exp energies in  $^{94,95}\text{Pd}$ ,  $^{93,94}\text{Rh}$  Taken from Lisetskiy et al, Phys.Rev.C70,044314(2004); B.A Brown priv. comm*



| $^{92}\text{Pd}$ |             |             |             |             | $^{94}\text{Pd}$ |             |             |             |             | $^{96}\text{Pd}$ |             |
|------------------|-------------|-------------|-------------|-------------|------------------|-------------|-------------|-------------|-------------|------------------|-------------|
| exp              | SM          | T=0         | T=1         | no np       | no np            | T=1         | T=0         | SM          | exp         | SM               | exp         |
| $0^+$ 0          | $0^+$ 0     | $0^+$ 0     | $0^+$ 0     | $0^+$ 0     | $0^+$ 0          | $0^+$ 0     | $0^+$ 0     | $0^+$ 0     | $0^+$ 0     | $0^+$ 0          | $0^+$ 0     |
| $2^+$ 874        | $2^+$ 878   | $2^+$ 797   | $2^+$ 1171  | $2^+$ 1417  | $2^+$ 1405       | $2^+$ 1199  | $2^+$ 864   | $2^+$ 861   | $2^+$ 814   | $2^+$ 1460       | $2^+$ 1415  |
| $4^+$ 1786       | $4^+$ 1708  | $4^+$ 1518  | $4^+$ 2079  | $4^+$ 2212  | $4^+$ 2223       | $4^+$ 2128  | $4^+$ 1709  | $4^+$ 1682  | $4^+$ 1720  | $4^+$ 2224       | $4^+$ 2099  |
| $6^+$ 2536       | $6^+$ 2466  | $6^+$ 2110  | $6^+$ 2600  | $6^+$ 2749  | $6^+$ 2635       | $6^+$ 2588  | $6^+$ 2374  | $6^+$ 2330  | $6^+$ 2380  | $6^+$ 2636       | $6^+$ 2530  |
| $8^+$ 3127       | $8^+$ 3127  | $8^+$ 2600  | $8^+$ 2600  | $8^+$ 2749  | $8^+$ 2635       | $8^+$ 2588  | $8^+$ 2792  | $8^+$ 2750  | $8^+$ 2704  | $8^+$ 2636       | $8^+$ 2530  |
| $10^+$ 4072      | $10^+$ 4072 | $10^+$ 4065 | $10^+$ 4052 | $10^+$ 4052 | $10^+$ 4052      | $10^+$ 4065 | $10^+$ 3862 | $10^+$ 3796 | $10^+$ 4131 | $10^+$ 3784      | $10^+$ 3784 |

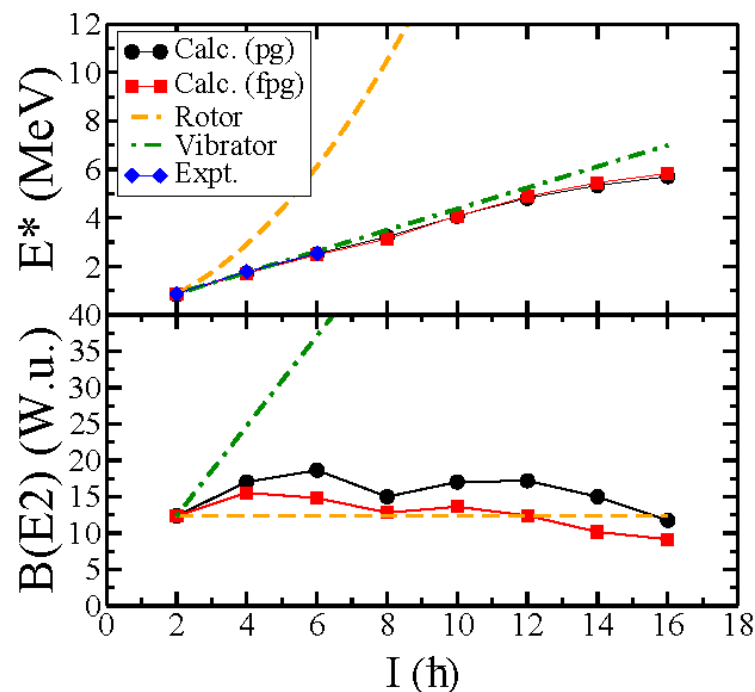
B. Cederwall et al., Nature **469**, 68 (2011)

# Spherical $N \sim Z$ systems: Generation of angular momentum in the isoscalar spin-aligned coupling scheme ( $^{92}\text{Pd}$ )



Isoscalar paired phase in  $^{92}\text{Pd}$ ?

B. Cederwall et al., Nature (London) 469, 68 (2011)

Angular momentum response of spectra and transition rates in the isoscalar spin-aligned coupling scheme ( $^{92}\text{Pd}$ )Upper: Shell model spectra of  $^{92}\text{Pd}$ 

calculated within the  $1p_{3/2}0f_{5/2}1p_{1/2}0g_{9/2}$  space [10] (fpg) and the  $1p_{1/2}0g_{9/2}$  space (pg).

Lower: B(E2; I  $\rightarrow$  I - 2) values in  $^{92}\text{Pd}$  calculated within the fpg and pg spaces. The two dashed

lines show the predictions of the geometric collective model normalized to the  $2^+_{1st}$  state

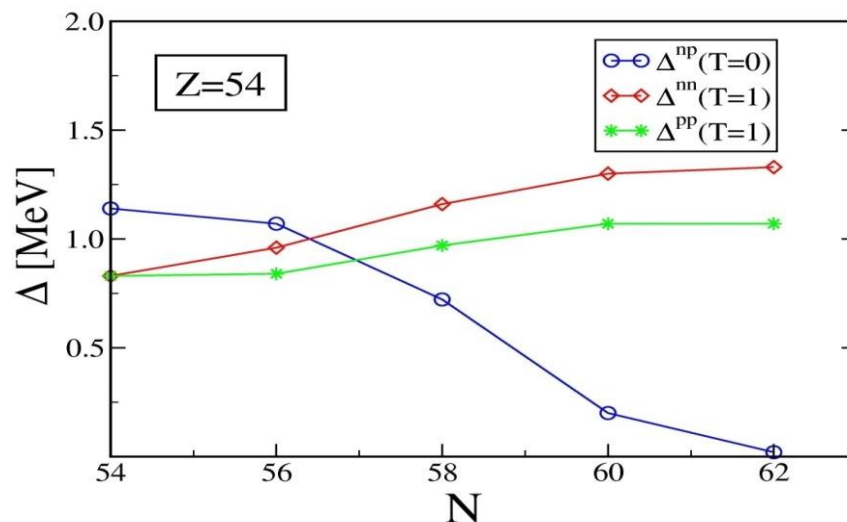
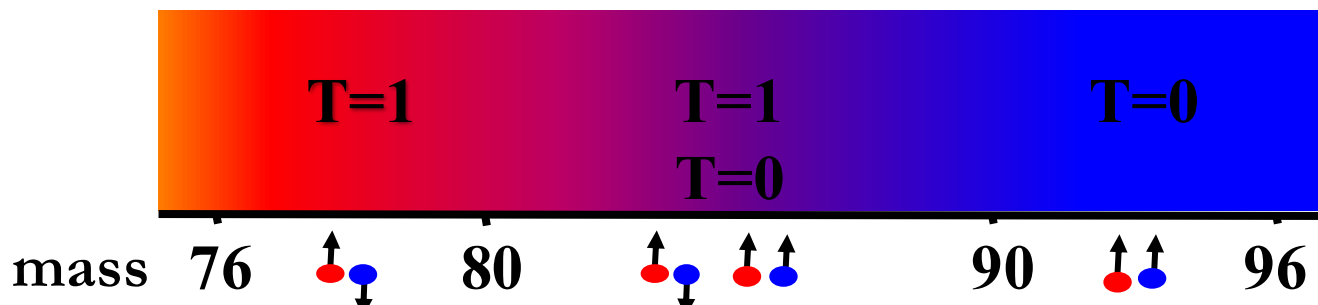
C. Qi, J. Blomqvist, T. Bäck, B. Cederwall, A. Johnson, R. J. Liotta, and R. Wyss,

PRC 84, 021301(R) (2011)

# Theory predictions for neutron-proton pairing in the heaviest *deformed* $N \sim Z$ nuclei

A. L. Goodman , PRC 60, 014311 (1999)

Isospin-generalized BCS-HFB predictions for ground states of e-e  $A = 76-96$ ,  $N = Z$  nuclei



W. Satula, R. Wyss,  
Phys. Rev. Lett. Vol. 86,  
4488 (2001)

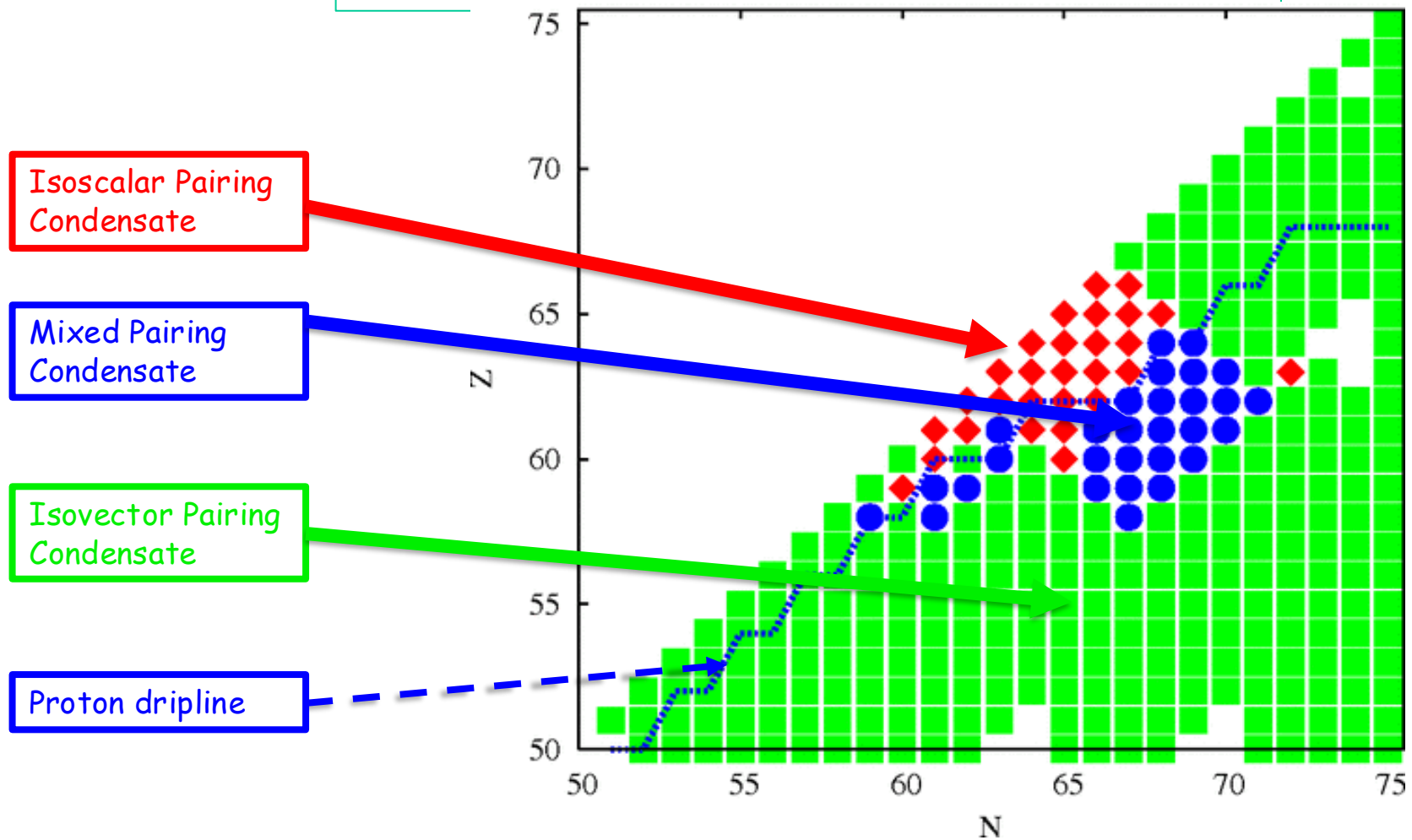
”Isocranking”: The isoscalar (np) pair gap is predicted to increase sharply as  $N \rightarrow Z$

# “Mixed-Spin Pairing Condensates in Heavy Nuclei”

A. Gezerlis, G. F. Bertsch, and Y. L. Luo

Phys. Rev. Lett. 106, 252502 (2011)

Note: this calculation for spherical systems

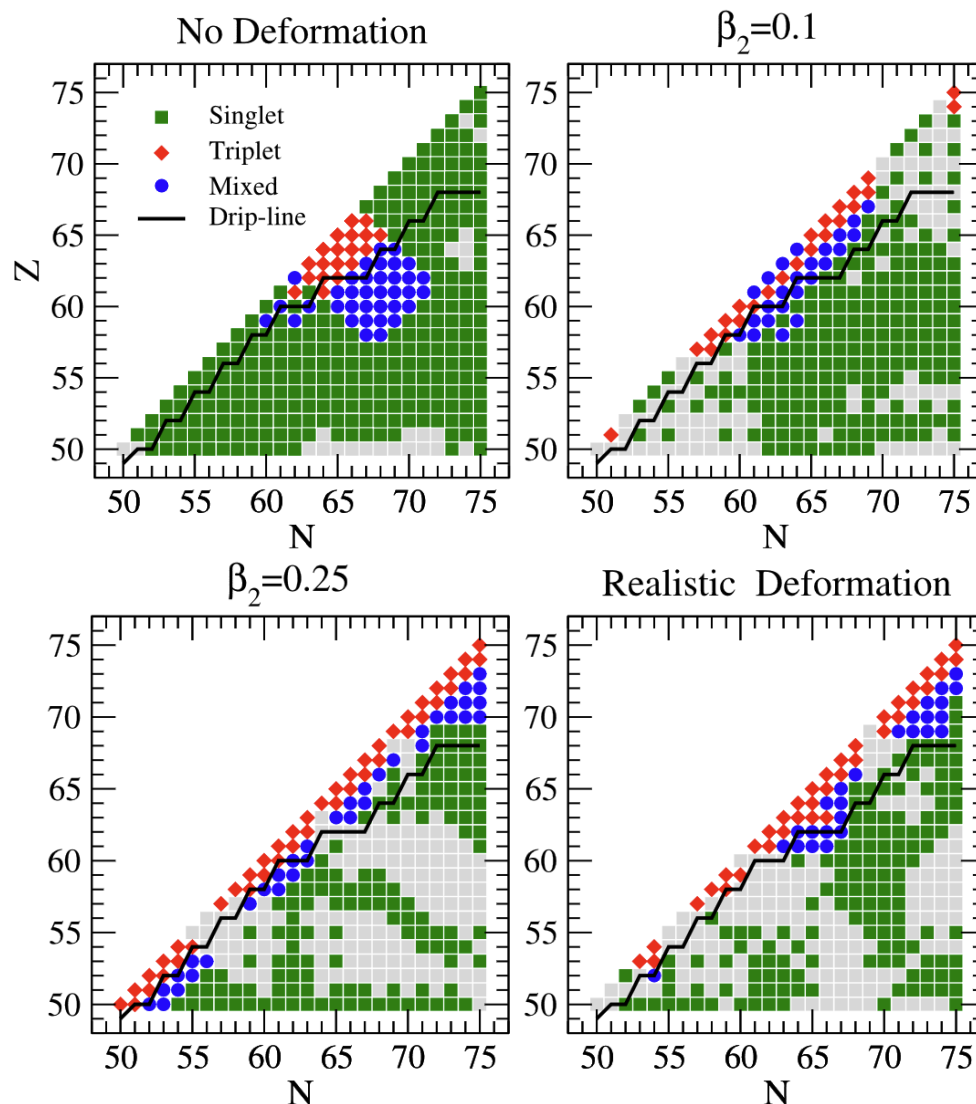


Note: predicted isoscalar pairing condensate coincides with region of maximum collectivity

# "Spin-Triplet Pairing in Heavy Nuclei Is Stable against Deformation"

G. Palkanoglou, M. Stuck, and A. Gezerlis,

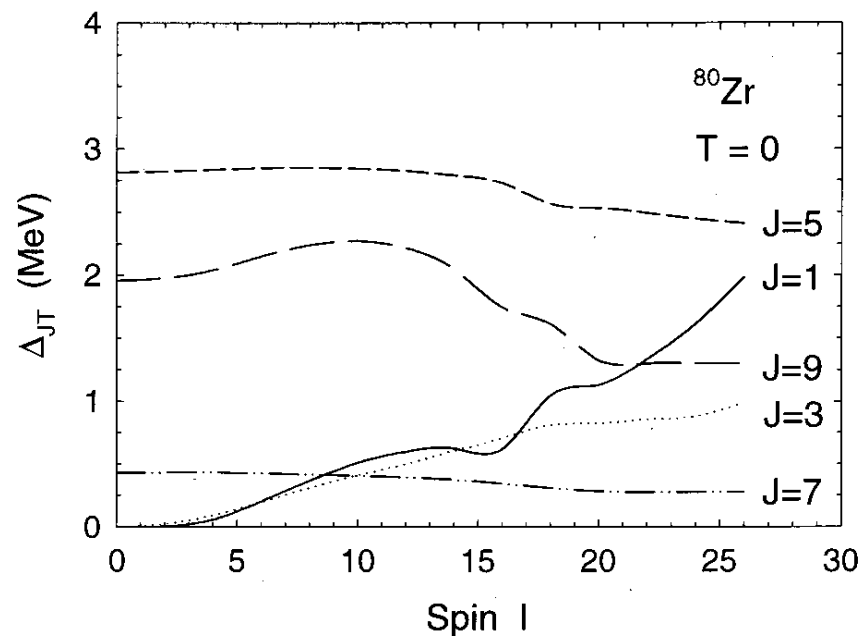
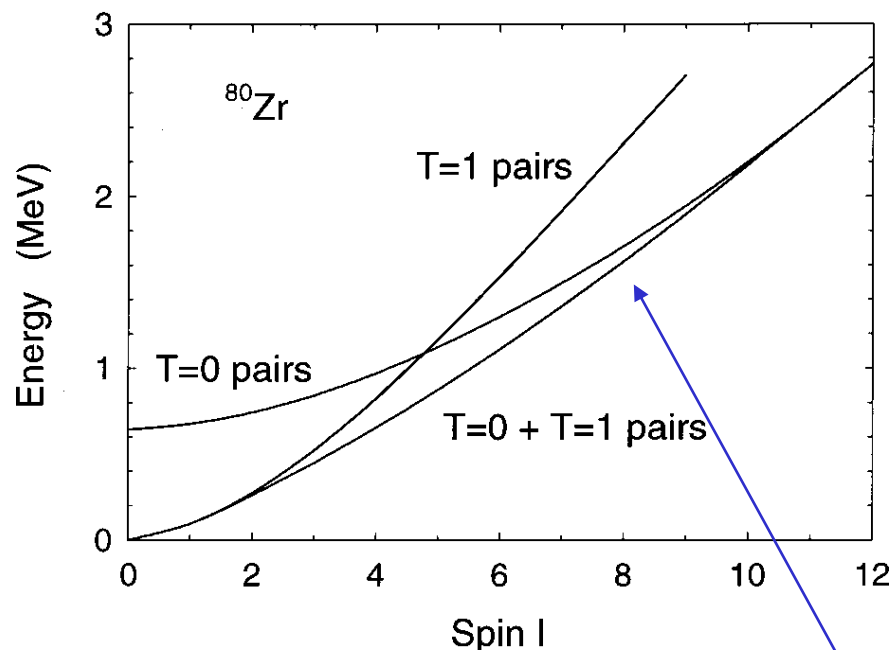
Phys. Rev. Lett. 134, 032501 (2025)



**Predicted isoscalar pairing condensate coincides with region of maximum collectivity!**

“ $T = 0$  and  $T = 1$  pairing in rotational states of the  $N = Z$  nucleus  $^{80}\text{Zr}$ ”

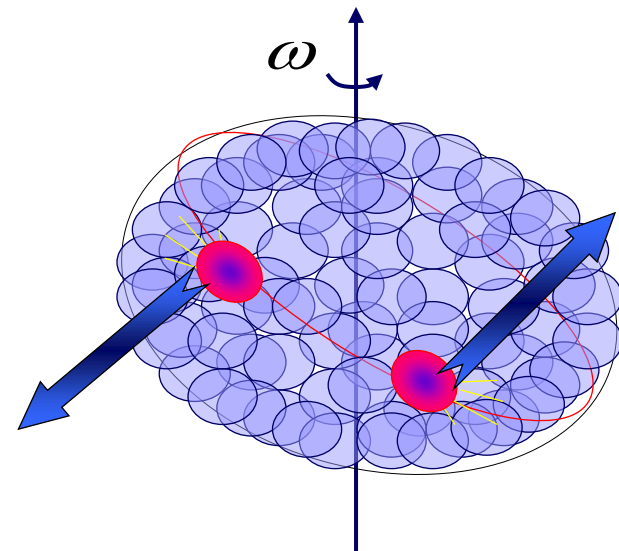
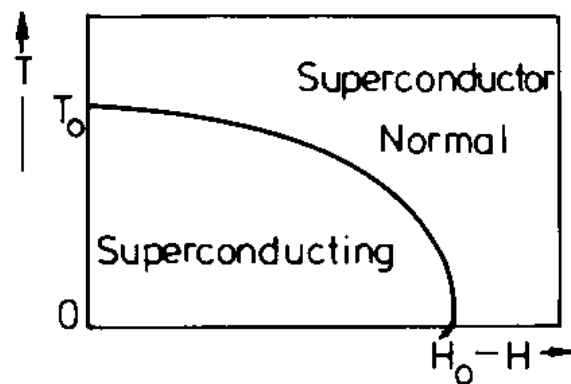
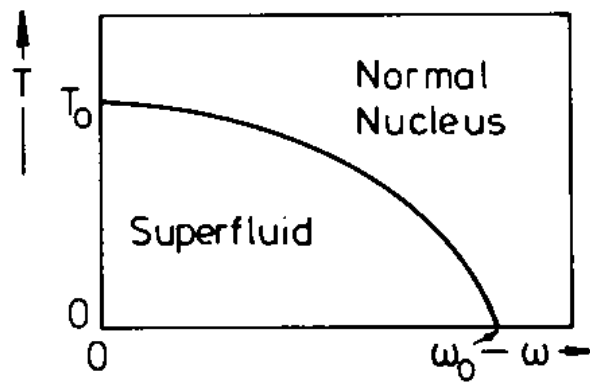
A. L. Goodman, PRC 63, 044325 (2001)


$$\Delta =$$

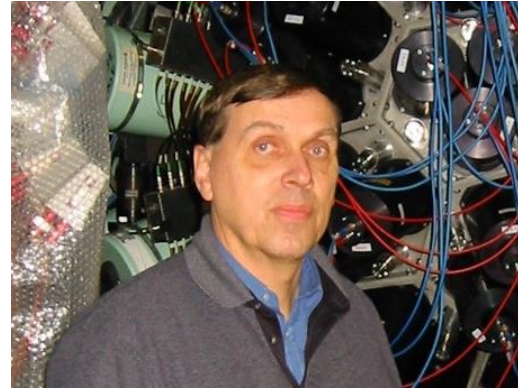
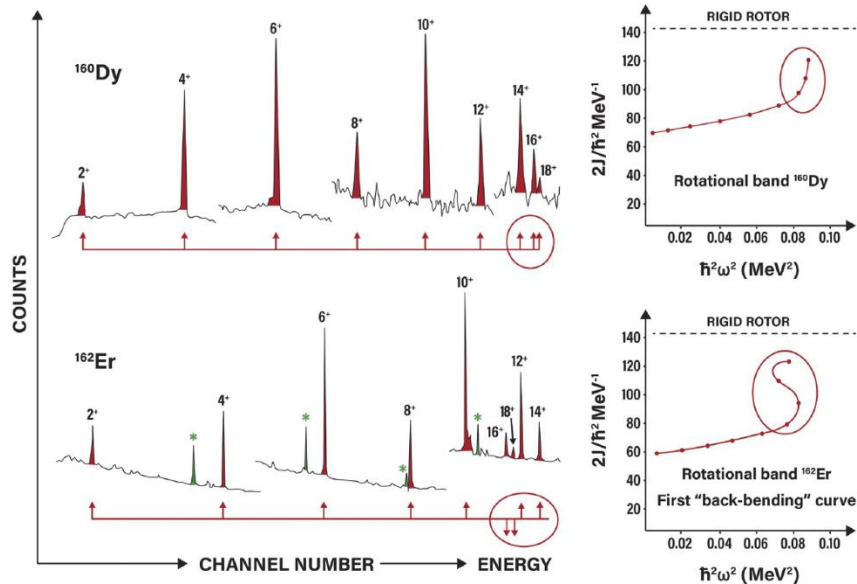
T=1 pairing is suppressed by the Coriolis effect in high-spin states

The  $T=0$  coupling is less affected by the Coriolis anti-pairing effect and will still be active and could become dominant as  $I$  increases

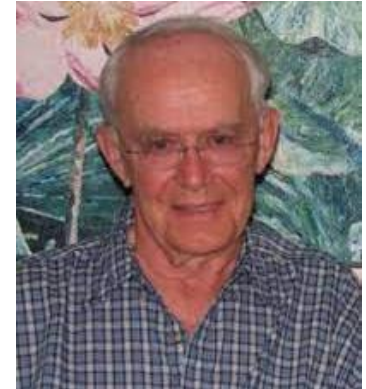
Similarly to a magnetic field on a superconductor, the rotational motion of a **deformed** nucleus counteracts the isovector-coupled ( $J=0$ ) superfluid phase by breaking time-reversed nn or pp pairs. (Coriolis anti-pairing – Mottelson Valatin effect)



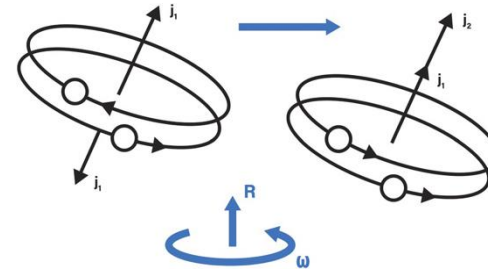
# Rotational alignments ("backbending") in deformed nuclei: are considered a sensitive probe of pairing strength



Arne Johnson  
1944-2023



Frank Stephens  
1931-2024



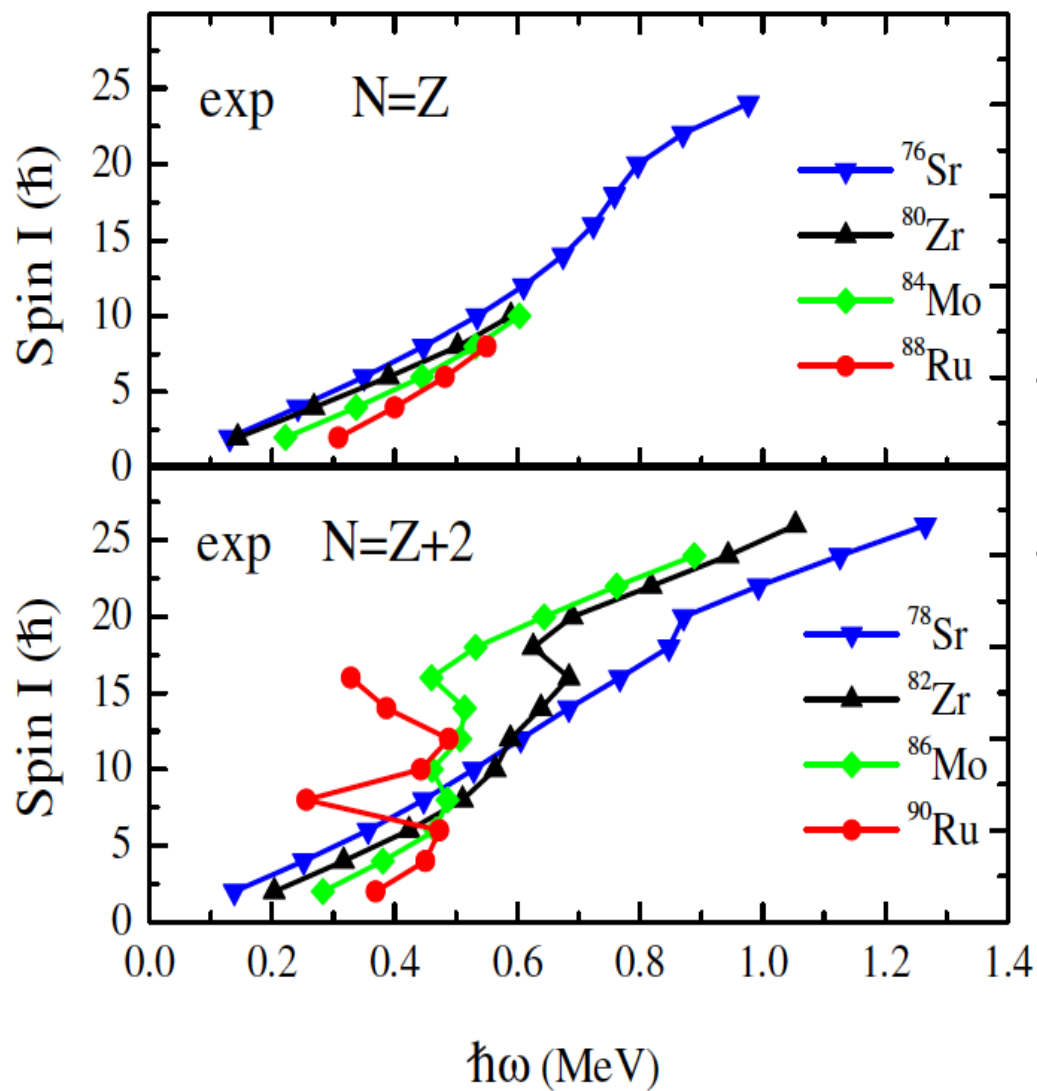
**Note: The “Backbending” effect (A. Johnson et al., 1971) is not a complete phase transition. In the finite nuclear system the paired and unpaired phases are mixed after the first alignment (“Stockholm, S-band”) (F.S. Stephens and R.S. Simon, 1972).**

A. Johnson, H. Ryde, and J. Sztarkier, Phys. Lett. B34, 605 (1971)

A. Johnson, H. Ryde, S. A. Hjorth, Nuc. Phys. A179, 753 (1972)

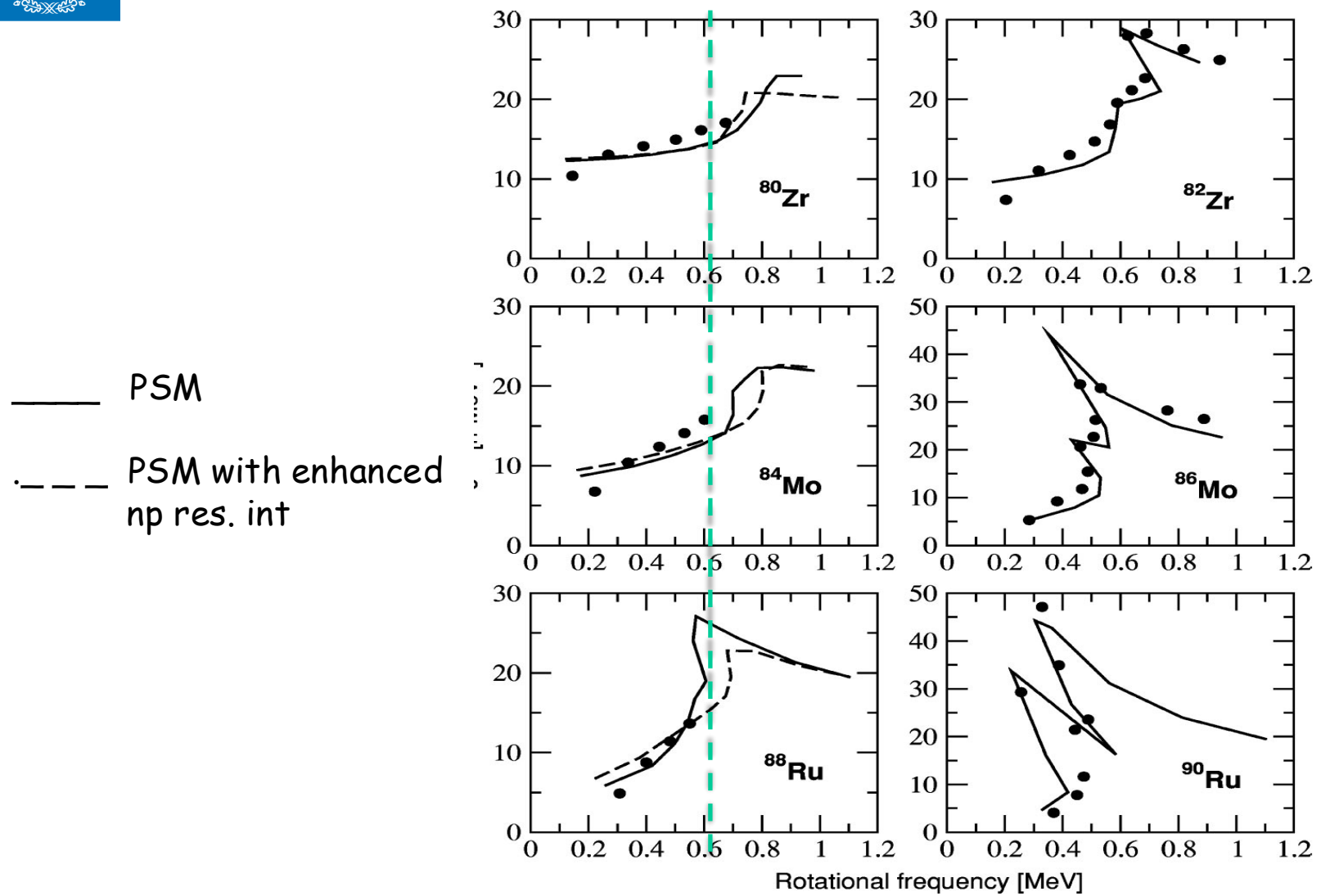
F. S. Stephens and R. S. Simon. Nuc. Phys. A183, 257 (1972)

# Experimental $I$ vs $\omega$ in deformed $N \sim Z$ nuclei (pre 2018)



- $N=Z$  nuclei consistently more collective than the corresponding  $N=Z+2$  isotopes
- Delayed or missing q.p. alignments

# Delayed (or absent) paired ( $T=1$ ) bandcrossings in deformed $N=Z$ nuclei?



N. Marginenan *et al.*, Phys. Rev. C 65, 051303R (2002)

# Experiment



PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About

Accepted Paper

Isospin properties of nuclear pair correlations from the level structure of the self-conjugate nucleus  $^{88}\text{Ru}$

Phys. Rev. Lett.

B. Cederwall et al.

Accepted 18 December 2019

PHYSICAL REVIEW C  
covering nuclear physics

Highlights Recent Accepted Collections Authors Referees Search Press About

Letter Open Access

Evidence for enhanced neutron-proton correlations from the level structure of the  $N = Z + 1$  nucleus  $^{87}_{43}\text{Tc}_{44}$

X. Liu et al.  
Phys. Rev. C **104**, L021302 – Published 20 August 2021

PHYSICAL REVIEW C  
covering nuclear physics

Highlights Recent Accepted Collections Authors Referees Search Press About Editorial Team

Open Access

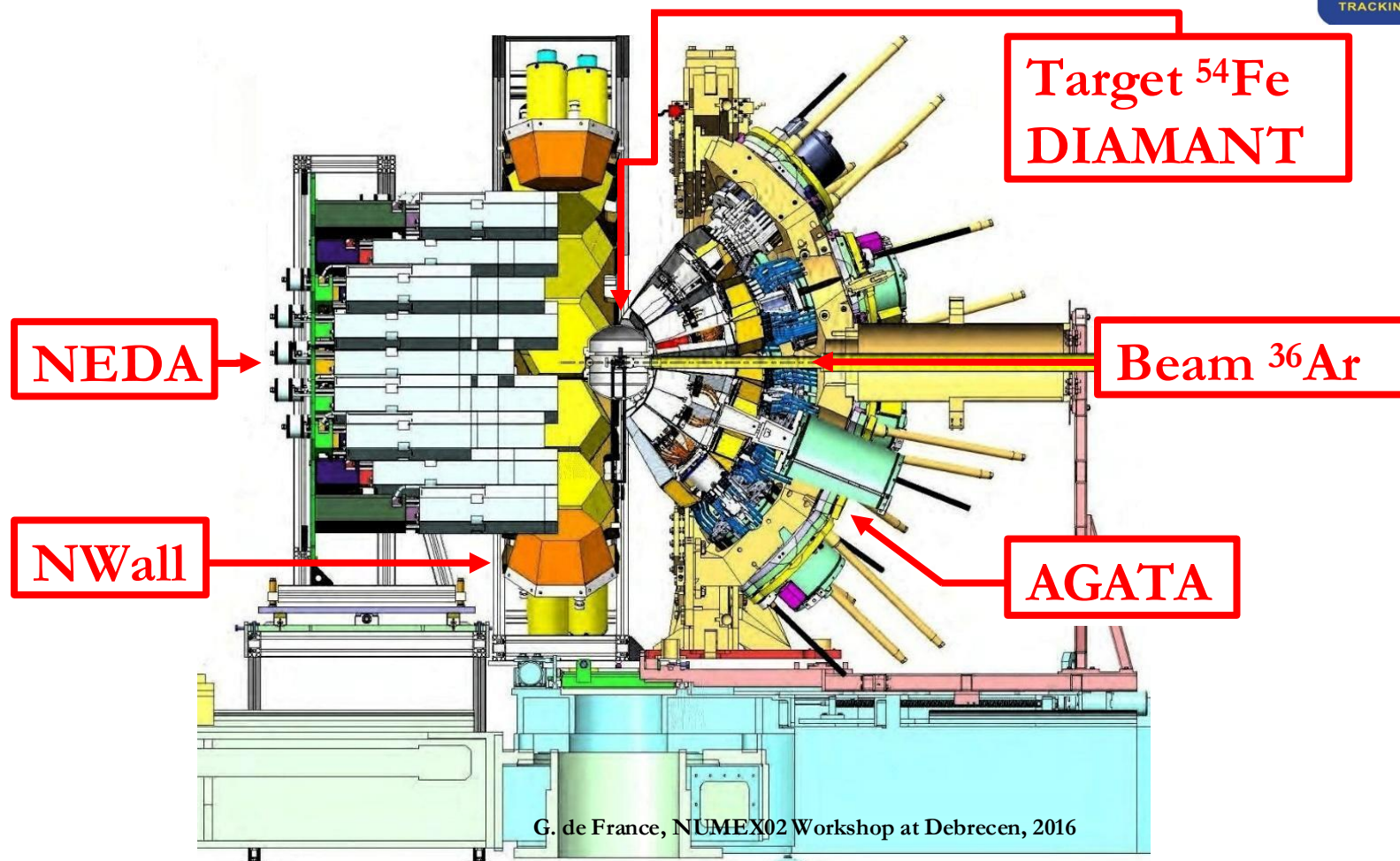
Access by Royal Institute of Technology Library Go Mobile

Evidence for spherical-oblate shape coexistence in  $^{87}\text{Tc}$

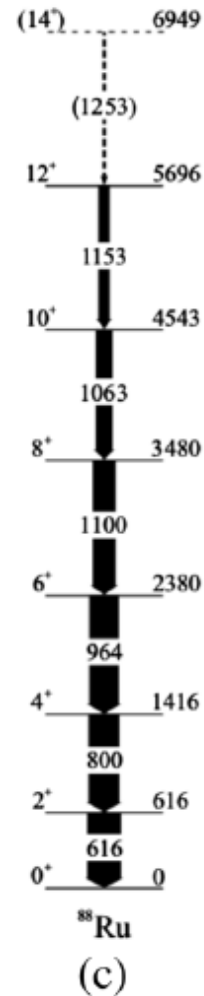
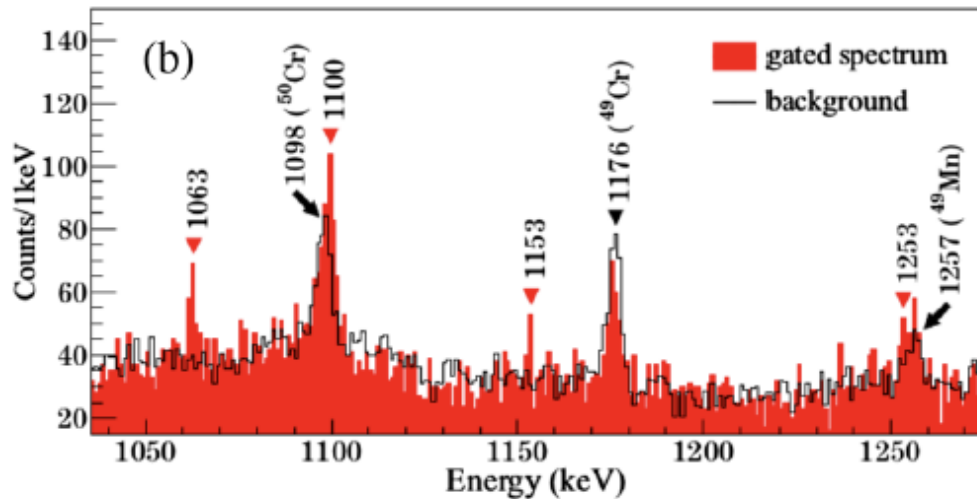
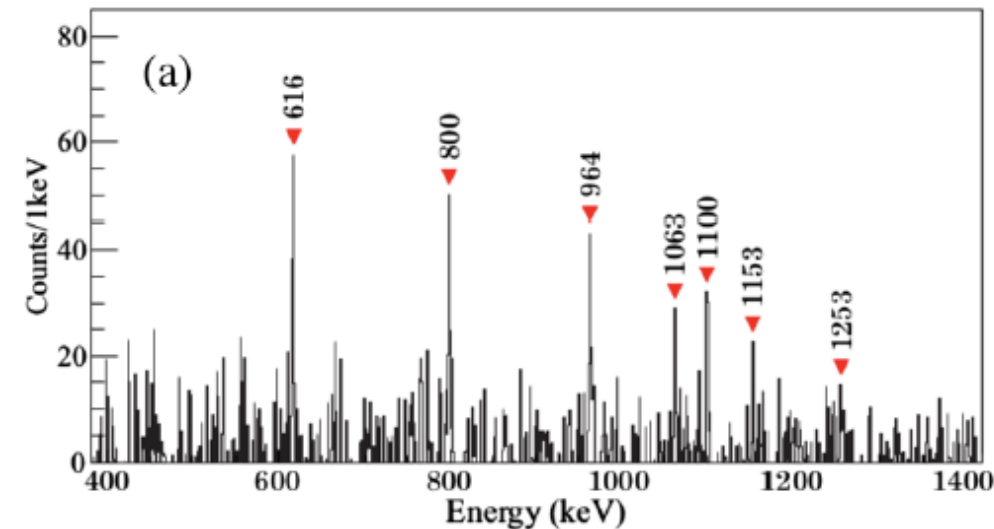
X. Liu et al.  
Phys. Rev. C **106**, 034304 – Published 9 September 2022

Twitter Facebook More

# Experimental Setup

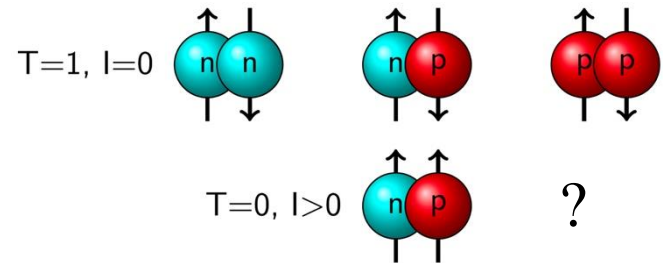
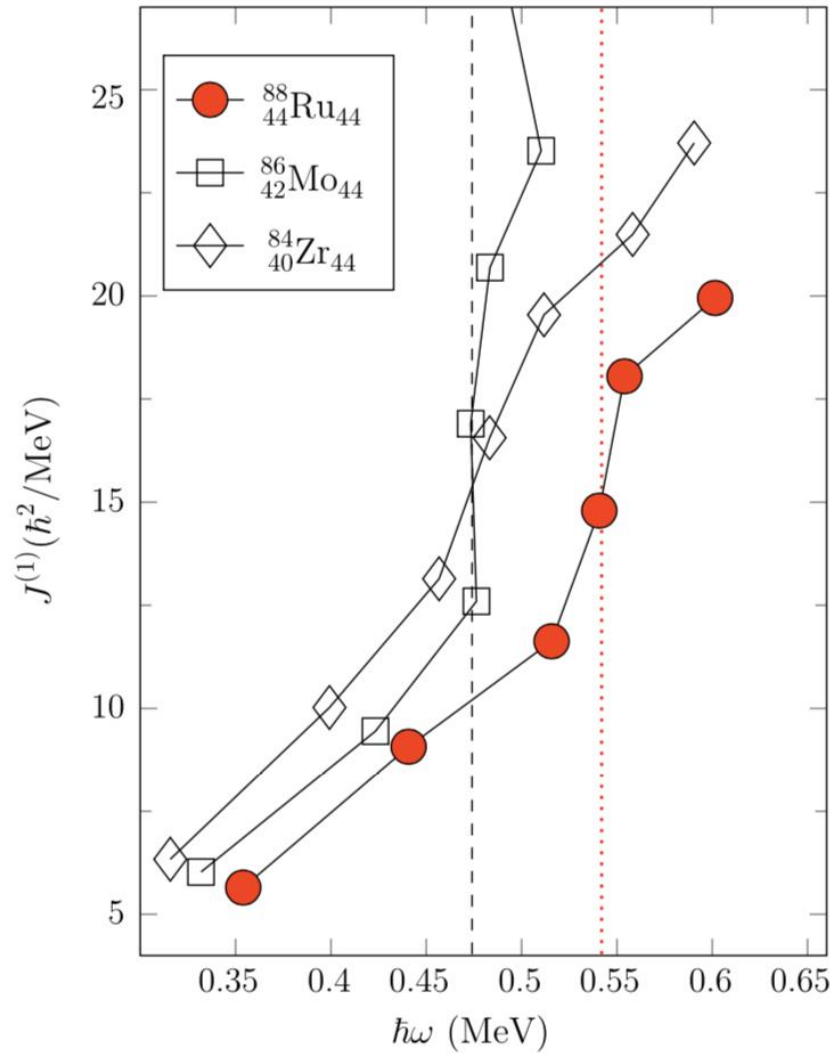


# Results: yrast band in $^{88}\text{Ru}$



Marginean et al.,  
Phys. Rev. C, 63, 031303(R), 2001.

# N=44 isotones



# LSSM predictions for $^{88}\text{Ru}$ with interaction including an explicit pairing term (PMMU)

PMMU:  $H = H_o + H_p + H_M + H_m$  :

$H_o$  = s.p,  $H_p$  = pairing

$H_M$  = multipole, contains QQ + OO components

$H_m$  = monopole term

$$H = H_0 + H_P + H_M + H_m^{MU},$$

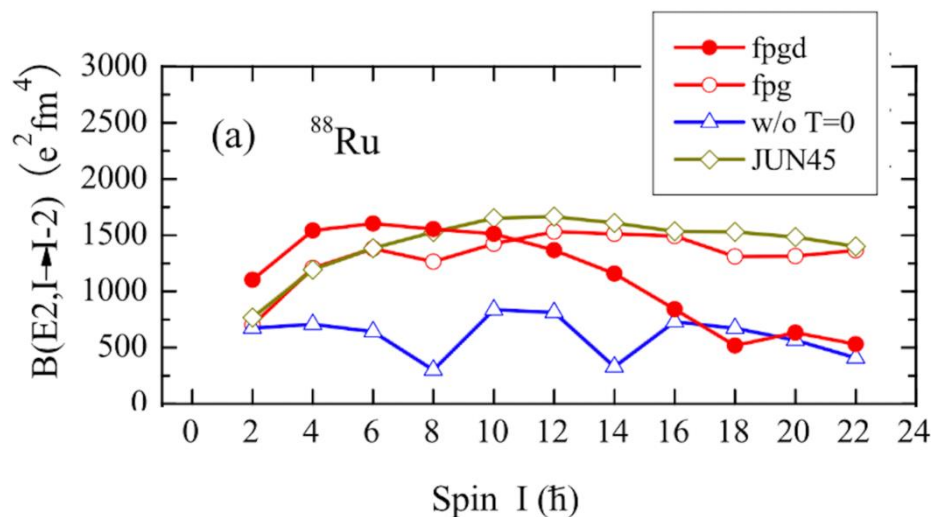
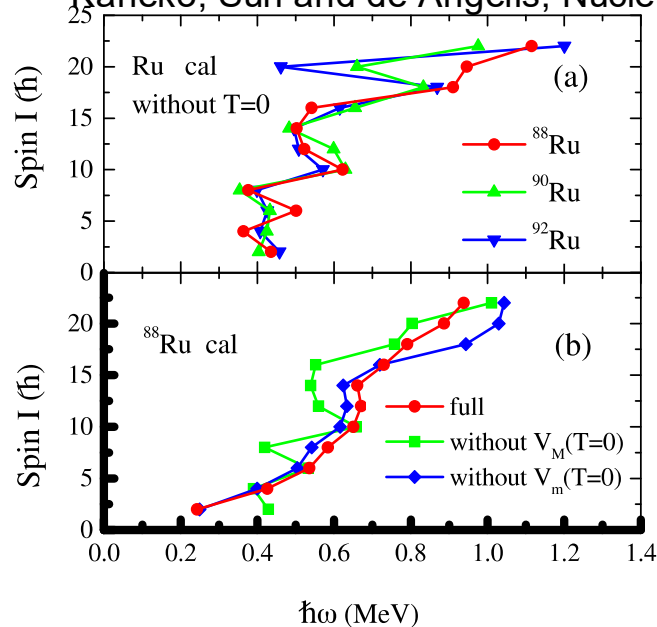
$$H_0 = \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}, \quad H_P = - \sum_{J=0,2} \frac{1}{2} g_J \sum_{MK} P_{JM1K}^{\dagger} P_{JM1K}$$

$$H_M = -\frac{1}{2} \chi_2 \sum_M : Q_{2M}^{\dagger} Q_{2M} : - \frac{1}{2} \chi_3 \sum_M : O_{3M}^{\dagger} O_{3M} :$$

$$H_m^{MU} = \sum_{a \leq b, T} V_m^{MU}(ab, T) \sum_{JMK} A_{JMTK}^{\dagger}(ab) A_{JMTK}(ab),$$

K. Kaneko, T. Mizusaki, Y. Sun, and S. Tazaki, Phys. Rev. C89, 011302(R) (2014)

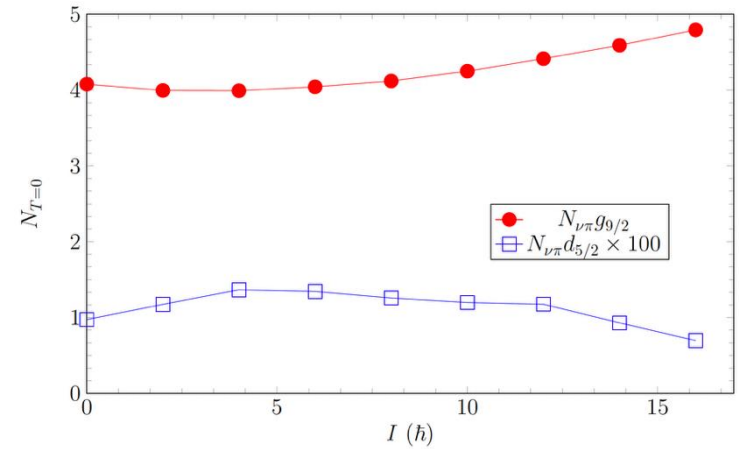
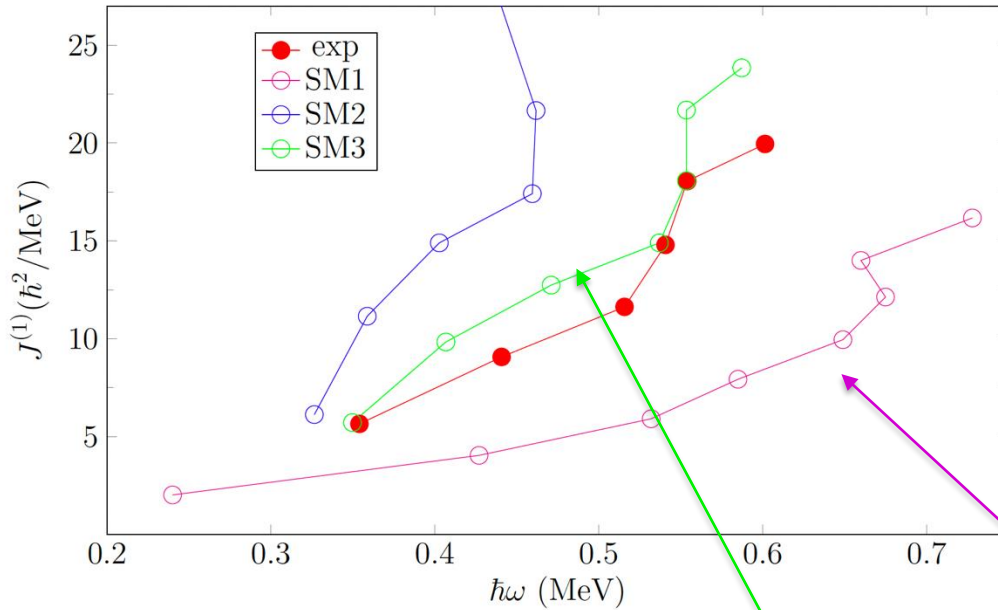
Kaneko, Sun and de Angelis, Nuclear Physics A957, 144-153 (2017)



effective charges  $e_p = 1.5e$  and  $e_n = 1.1e$

**Note:** Without the  $T = 0$   $np$  pairing, the smooth pattern in  $^{88}\text{Ru}$  disappears and it lacks rotational behaviour as for  $^{90,92}\text{Ru}$ . **The np QQ T=0 is most important in the PMMU for driving the collective strength**

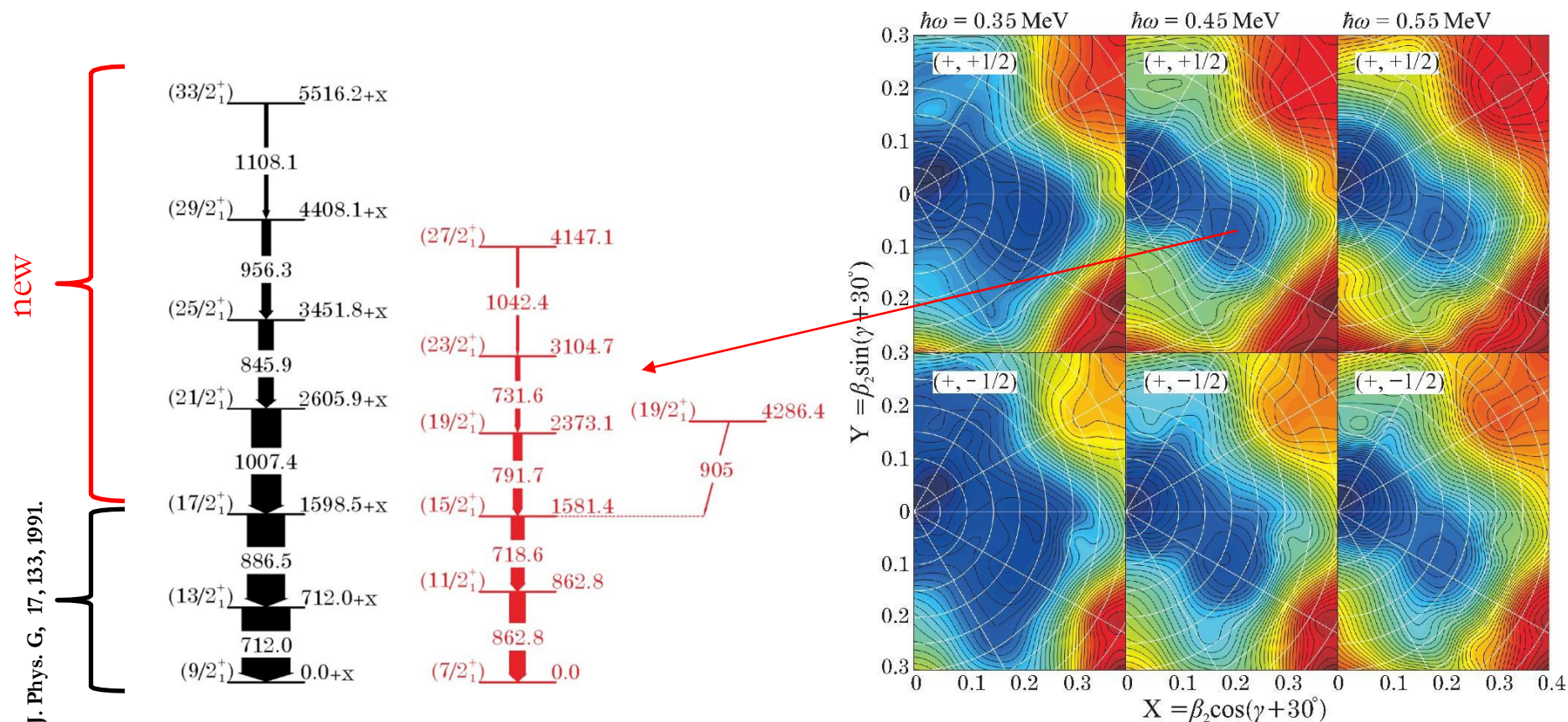
# LSSM calculations: JUN45 with (slightly) adjusted QQnp



PMMU

JUN45 with QQnp  $\rightarrow$  1.09 x QQnp\_0

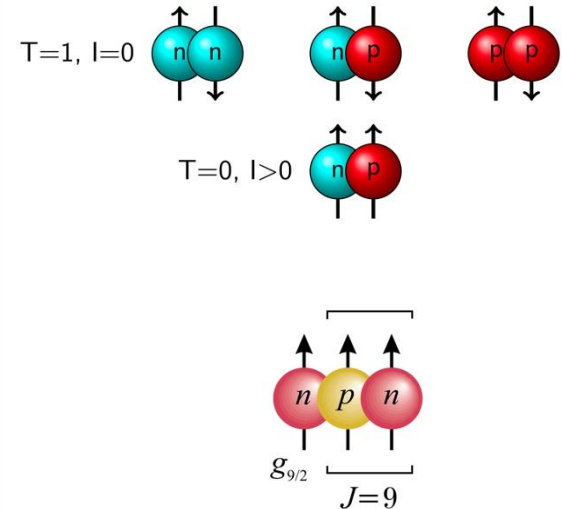
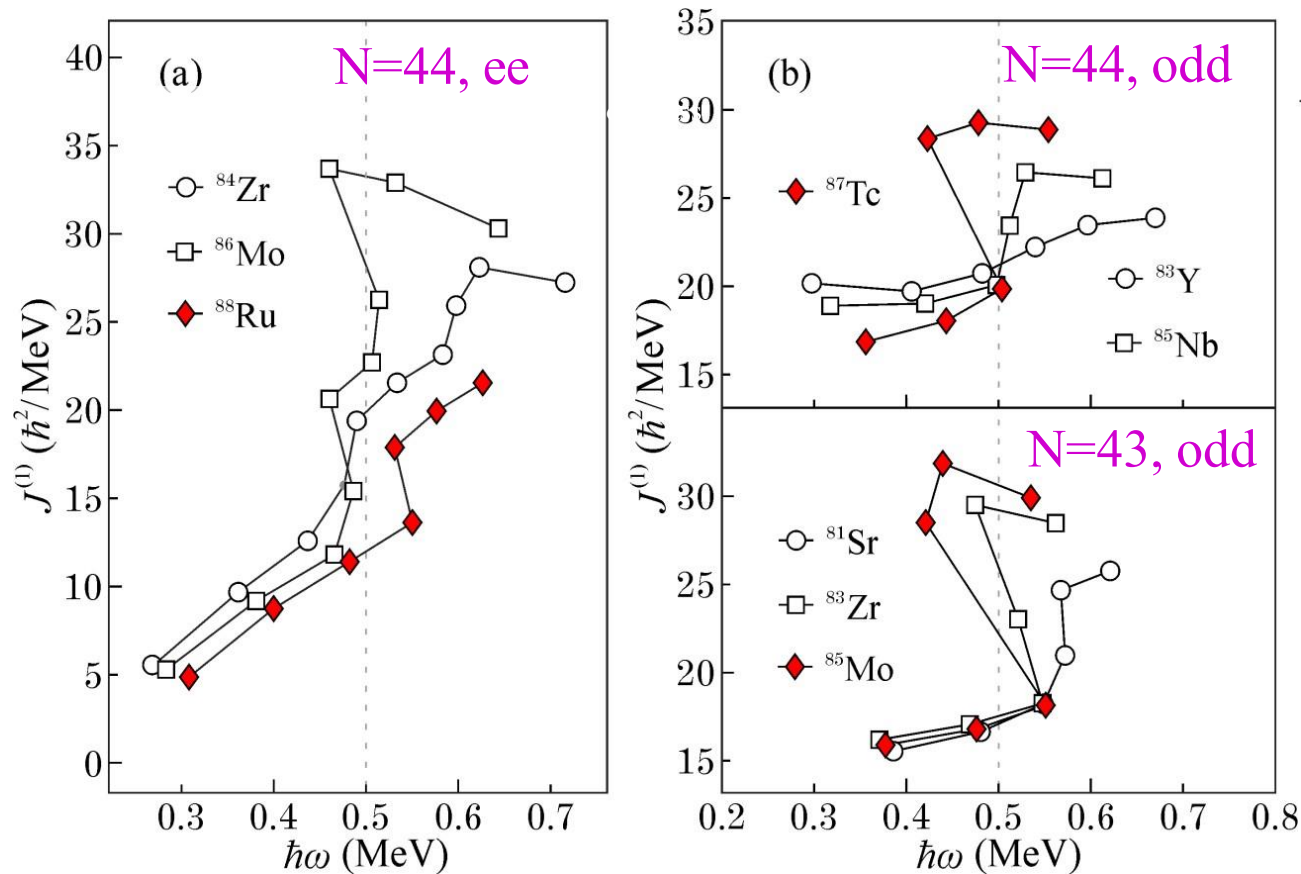
# Extension of the ground state ( $\pi g_{9/2}$ ) band and new oblate band in the $T_Z=1/2$ nucleus $^{87}\text{Tc}$



X. Liu, B. Cederwall, Ö. Aktas et al, PRC 104 L021302 (2021)

X. Liu, B. Cederwall, Ö. Aktas et al, PRC 106 034304 (2022)

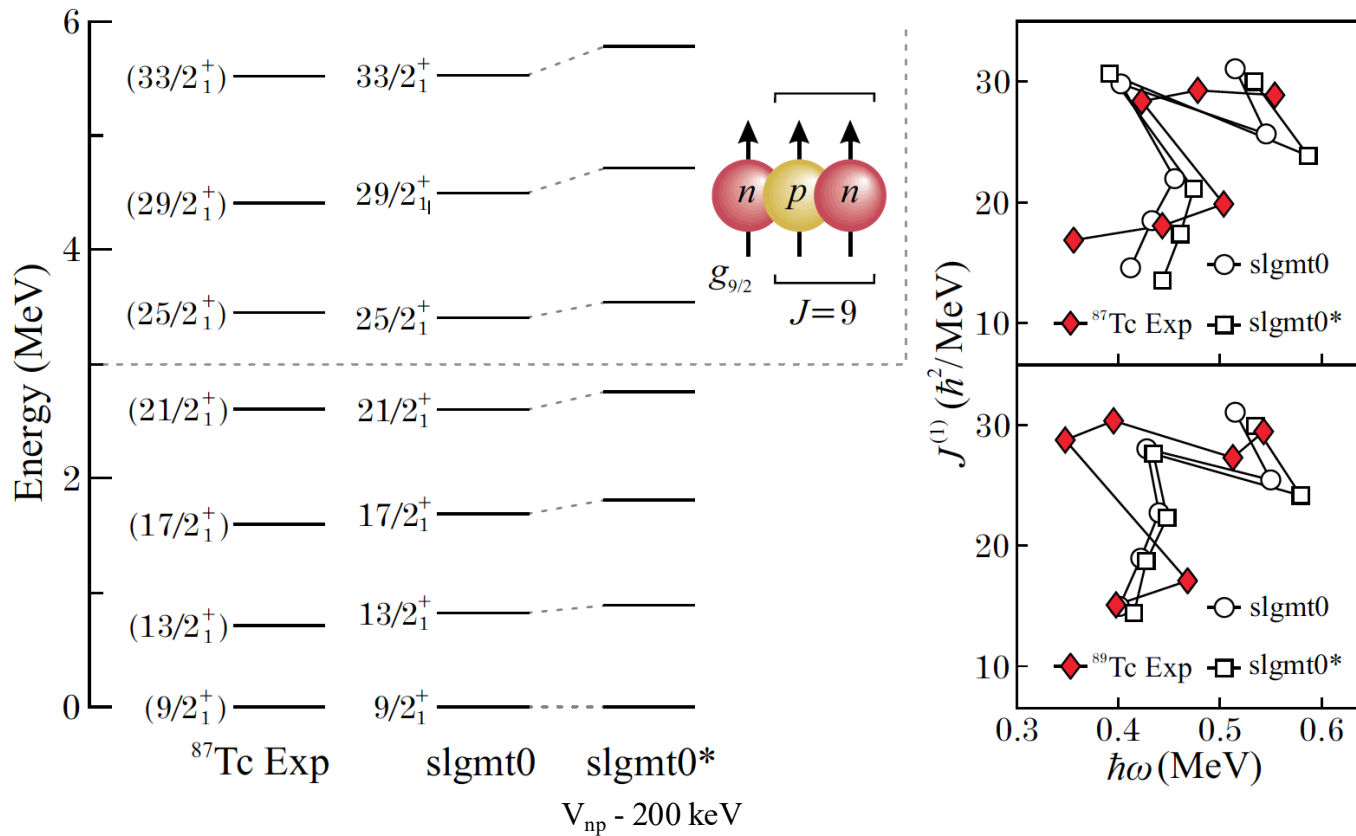
# Pairing isospin modes in deformed $N \sim Z$ nuclei – odd-even vs even-even



B. Cederwall et al, PRL 124 062501 (2020)

X. Liu et al, PRC 104 L021302 (2021)

# Shell model calculation for $^{87}\text{Tc}$ ( $T_Z=1/2$ ) – effect of spin-aligned np pairs ( $V_{np}$ )



X. Liu et al, PRC 104 L021302 (2021)

F. J. D. Serduke, R. D. Lawson, and D. H. Gloeckner, Nucl. Phys. A 256, 45 (1976)

Odd-mass  $N \sim Z$  ( $T_Z=1/2$ ) nuclei:

- Odd valence particle contributes constructively to isoscalar np pairing correlations "spin-dependent effective 3-N" interaction as opposed to "blocking" effect in pure isovector case?
- Increase of structural difference between 1qp and 3qp configuration reduces interaction strength in the band crossing?

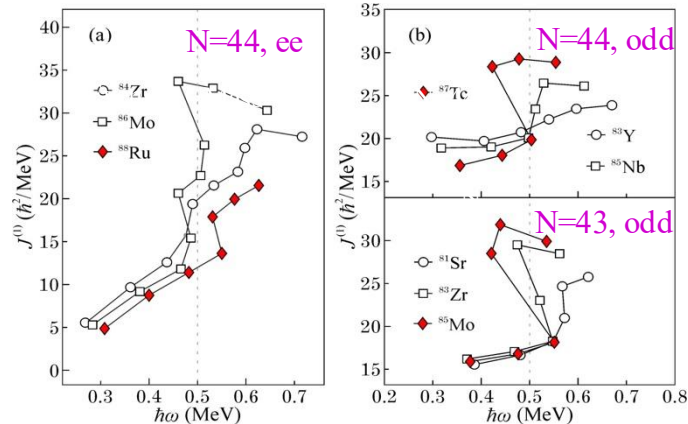
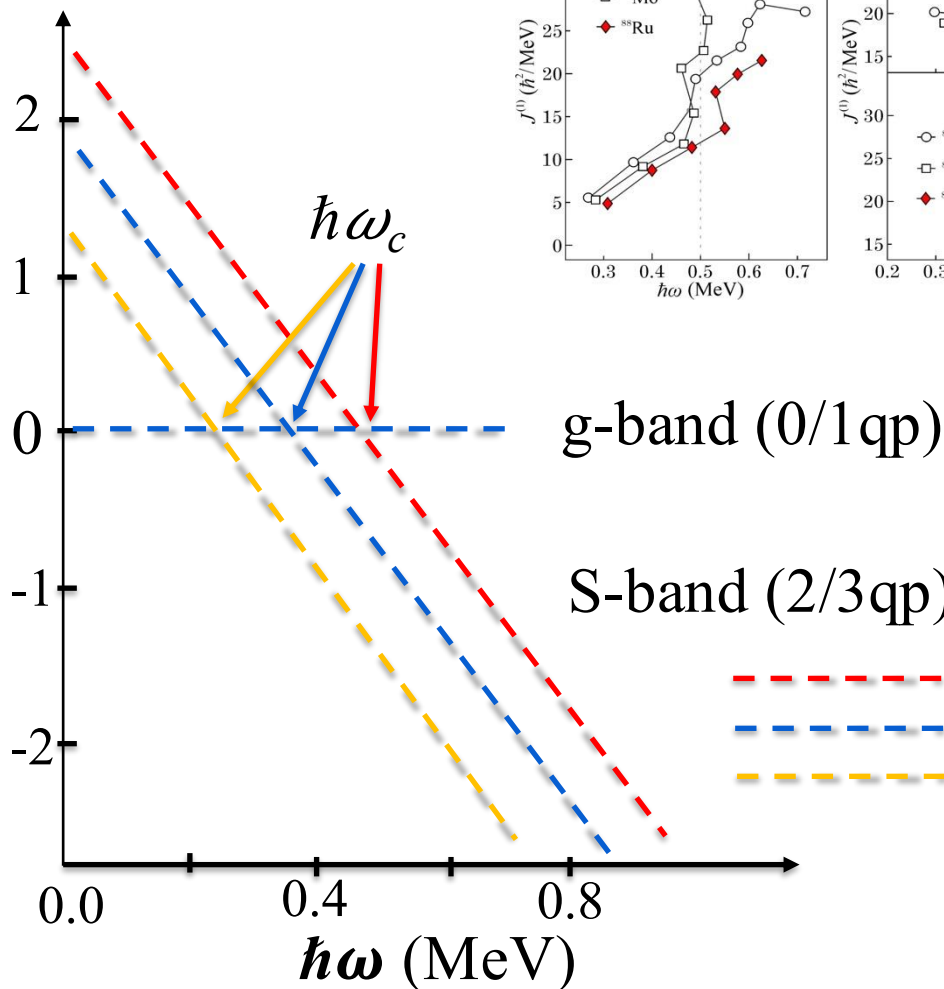
# Influence of pairing isospin modes in deformed $N \sim Z$

– odd-A vs even-even cases (naïve picture)

$$h' = h_0 + h_{pair} - \omega j_x$$

$e'$  (MeV)

iv pairing gap "2 $\Delta$ "



- $iv + is$  pairing,  $ee$
- $iv$  pairing
- $iv + is$  pairing,  $odd$

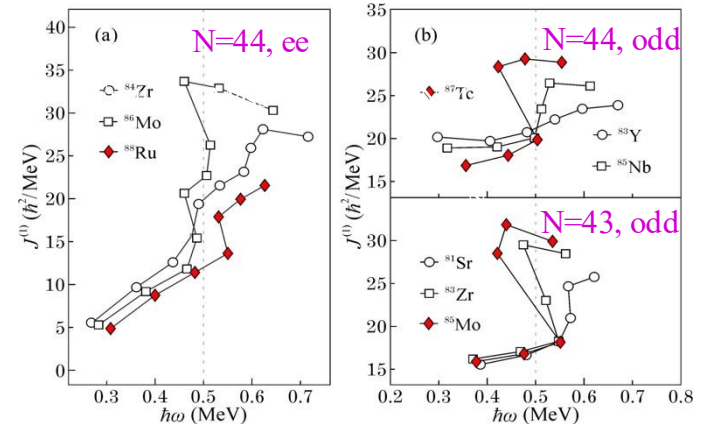
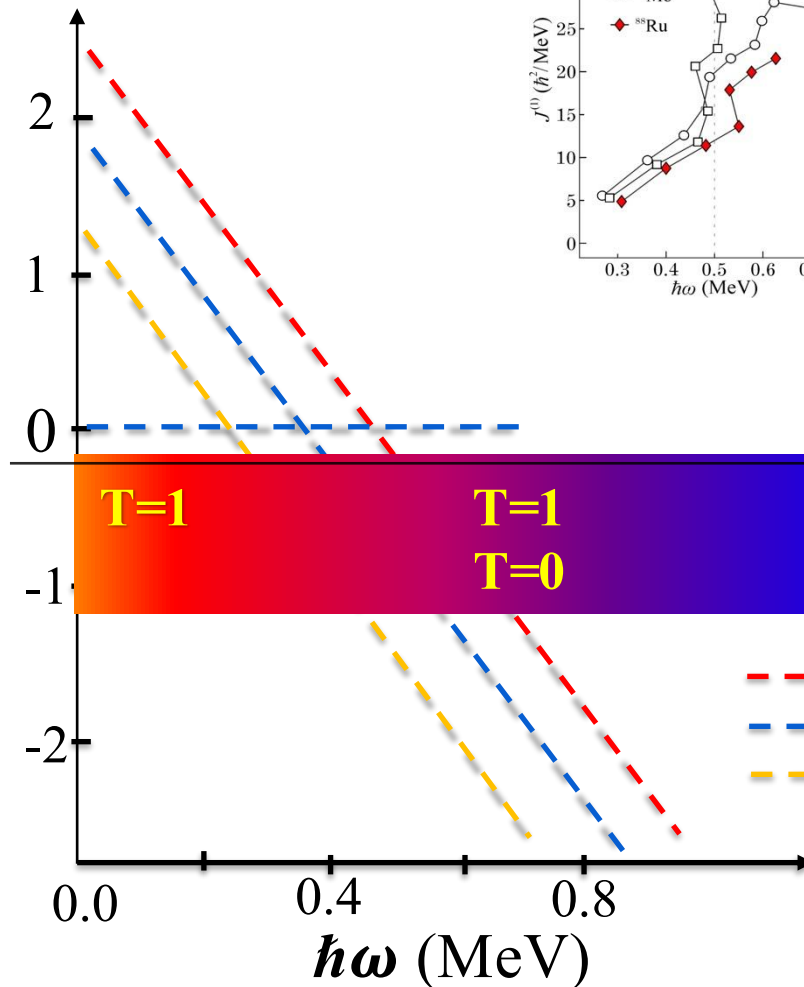
# Influence of pairing isospin modes in deformed $N \sim Z$

– odd-A vs even-even cases (naïve picture)

$$h' = h_0 + h_{pair} - \omega j_x$$

$e'$  (MeV)

iv pairing gap "2 $\Delta$ "



- $iv + is$  pairing,  $ee$
- $iv$  pairing
- $iv + is$  pairing,  $odd$

# Quartet Condensation Model

Isovector-pair quartets for N=Z nuclei — Sandulescu, Sambataro, Negrea, Dukelsky, Johnson et al.  
See talk by N. Sandulescu later in the program (Tuesday).



- Quartet = two isovector ( $T=1, J=0$ ) pairs coupled to total  $T=0, J=0$  — “alpha-like” in isospin/angular-momentum quantum numbers, not real-space clustering
- Ground state of an N=Z nucleus = condensate of  $N_Q = A/4$  identical quartets
- Benchmarked against exact shell-model diagonalization: correlation-energy errors typically below 1%
- Extended to deformed (Skyrme-HF / Nilsson) bases, **to isoscalar ( $T=0, J=1$ ) pairing**, and to N>Z systems (quartets + excess-neutron pair condensate)
- So far demonstrated mainly for ground states and limited angular momentum: exact diagonalization / angular-momentum projection in the quartet space grows combinatorially costly with  $I$ .

## Isovector pairing Hamiltonian

$$\hat{H} = \sum_i \epsilon_i N_i - \sum_{i,j,\tau} V_{ij} P_{i,\tau}^\dagger P_{j,\tau}$$

( $\tau = pp, nn, pn$  channels)

## Isovector pair operators ( $T=1, J=0$ )

$$P_{i,+1}^\dagger (pp), P_{i,-1}^\dagger (nn), P_{i,0}^\dagger (pn)$$

## Quartet operator ( $T=0, J=0$ )

$$A_i^\dagger \propto P_{i,+1}^\dagger P_{i,-1}^\dagger - (P_{i,0}^\dagger)^2$$

## Quartet condensate ansatz

$$|QCM\rangle = N (\sum_i x_i A_i^\dagger)^{N_Q} |0\rangle, \quad N_Q = A/4$$

( $\{x_i\}$  fixed variationally; particle number,  $J$  and  $T$  exactly conserved)

**Key references:** N. Sandulescu, D. Negrea, J. Dukelsky, C. W. Johnson, Phys. Rev. C 85, 061303(R) (2012) • M. Sambataro, N. Sandulescu, Phys. Rev. C 88, 061303(R) (2013) • D. Negrea, N. Sandulescu, Phys. Rev. C 90, 024322 (2014) • M. Sambataro, N. Sandulescu, Phys. Lett. B 827, 136987 (2022) [band-like structures in deformed N=Z nuclei]

Standard description of rotational bands for decades:  
Cranked Deformed Shell Model (Nilsson/WS)+ BCS (isovector pairing)

Configuration interaction (shell model) description of collective rotation possible in some cases, e.g.  $^{48}\text{Cr}$

**Physical Review Letters**

Highlights Recent Accepted Collections Authors Referees Press About Editorial Team RSS

## Backbending Mechanism of $^{48}\text{Cr}$

Kenji Hara<sup>1</sup>, Yang Sun<sup>2</sup>, and Takahiro Mizusaki<sup>3</sup>

Show more

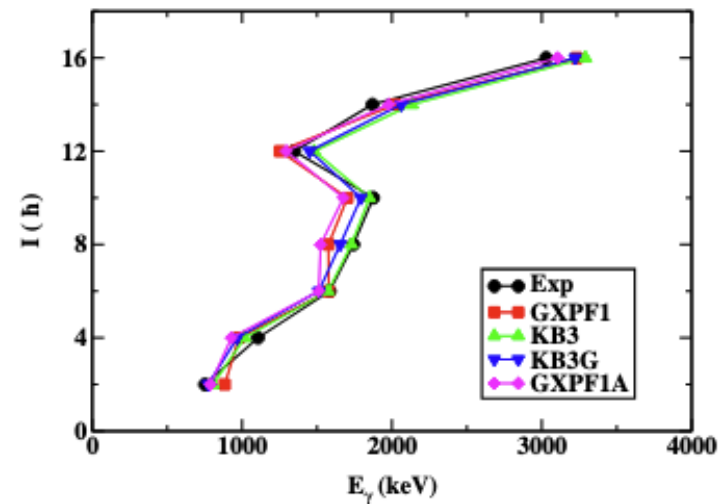
Phys. Rev. Lett. **83**, 1922 – Published 6 September, 1999

DOI: <https://doi.org/10.1103/PhysRevLett.83.1922>

Am score 0 Citations 53 Show metrics

**Abstract**

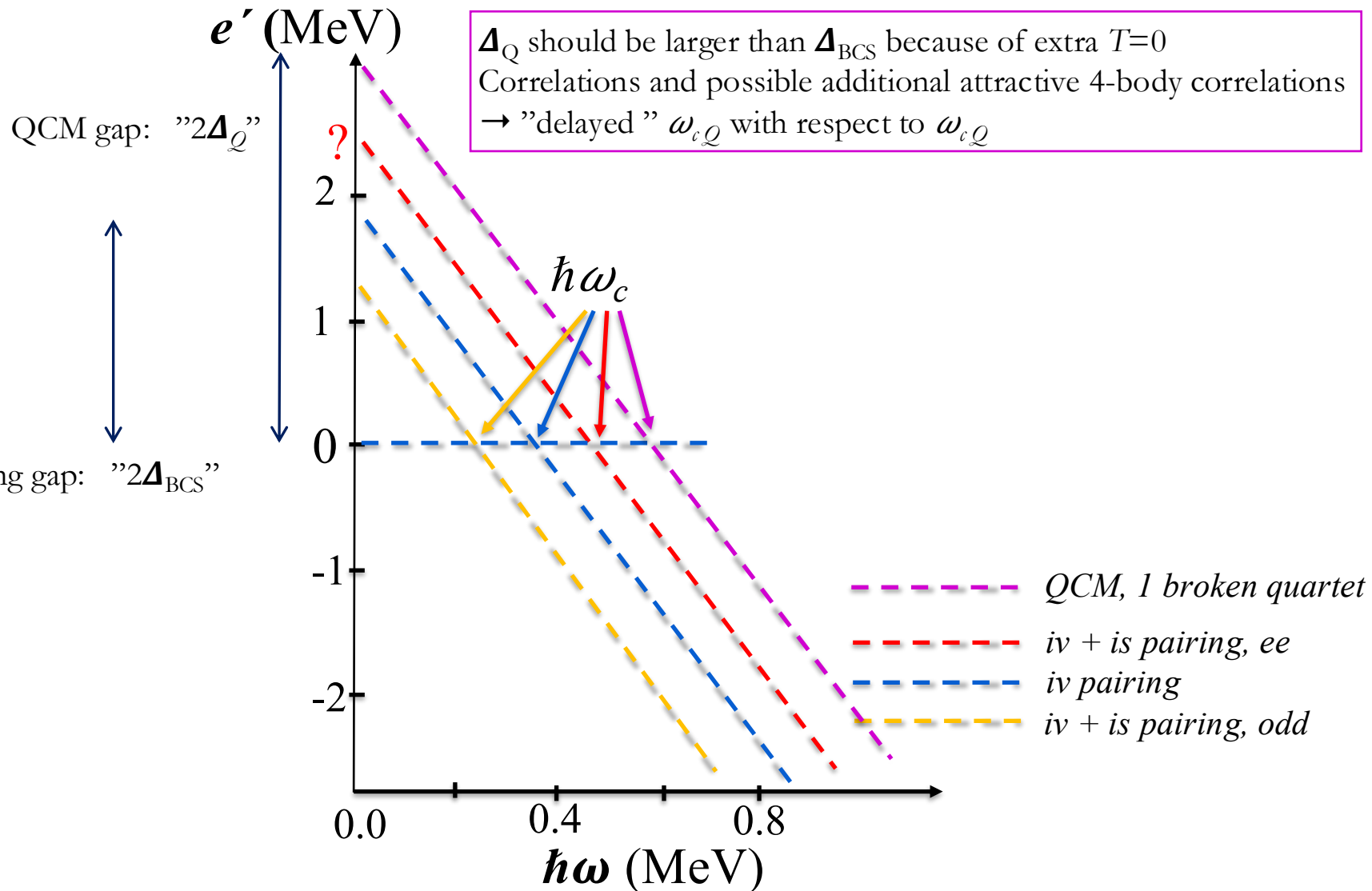
The mechanism of backbending in  $^{48}\text{Cr}$  is investigated in terms of the projected shell model and the generator coordinate method. It is shown that both methods are reasonable shell model truncation schemes. These two quite different quantum mechanical approaches lead to a similar conclusion that the backbending is due to a band crossing involving an excited band which is built on simultaneously broken neutron and proton pairs in the "intruder" subshell  $f_{7/2}$ . It is pointed out that this type of band crossing is usually known to cause the second backbending in rare-earth nuclei.



Y.R., R.P.S., Proc. DAE Symp. on Nucl. Phys. 66 (2022)

Could a **Cranked** Quartet Condensation Model "explain"  
naturally the delayed alignments observed in even-even  $N=Z$  nuclei?

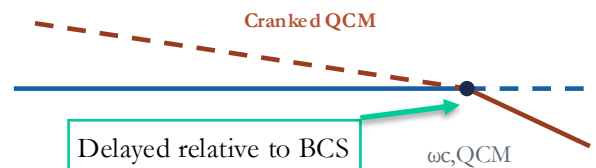
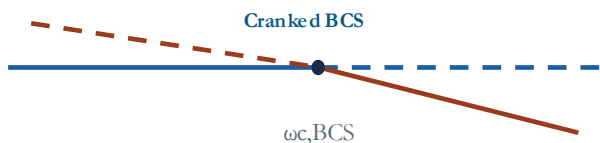
$$h' = h_{0Q} - \omega j_x$$



# Cranked QCM: A new approach to rotating $N=Z$ nuclei

Extending the (static) quartet condensate to a rotating mean field — both models start from the same deformed single-particle basis and the same cranking term  $h'(\omega) = h_0 - \omega J_x$

|                                   | Cranked Shell Model (HFB / BCS)                                            | Cranked QCM                                                                      |
|-----------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| Correlated building block         | Cooper pair (2-body)                                                       | Isovector quartet: two $T=1, J=0$ pairs $\rightarrow T=0, J=0$ (4-body)          |
| Ground-state ansatz               | BCS/HFB quasiparticle vacuum $ BCS\rangle$                                 | $ QCM(\omega)\rangle = N (\sum_i x_i(\omega) A_i^\dagger(\omega)) N_q  0\rangle$ |
| Ground-band routhian $E'(\omega)$ | Flat at leading order; weak Mottelson–Valatin softening at higher $\omega$ | Flat at leading order ( $J_x = 0$ exactly for an intact quartet)                 |
| Backbending mechanism             | Break one Cooper pair; align the two freed quasiparticles                  | Break one isovector pair inside a quartet; align the two freed quasiparticles    |
| Relevant gap                      | $\Delta$ (pairing gap)                                                     | $\Delta_Q = \Delta + \delta E(\text{quartet}) > \Delta$                          |
| Predicted crossing frequency      | $\omega_c \approx \Delta / \hbar x$                                        | $\omega_c \approx \Delta_Q / \hbar x$ — delayed relative to BCS                  |
| Status                            | Standard (Bengtsson–Frauendorf, 1979 onward); implemented in many codes    | Proposed in this talk — not published                                            |

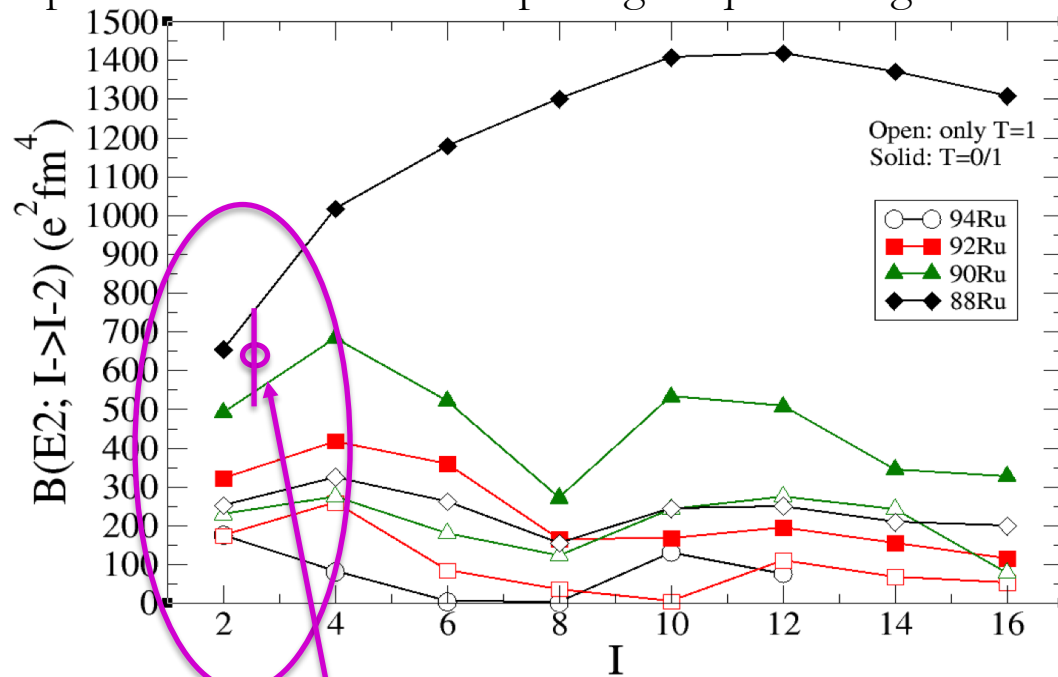


*If the same slope for the aligned band in both panels — the larger gap  $\Delta_Q$  alone delays the crossing.*

*This cranking construction is a proposed extension of the published (static) QCM framework — no cranked-QCM calculation has been published to date.*

LSSM calculations using the JUN45 interaction in the fpg shell show lifetimes are a key indicator for  $T=0$  correlations as a driver of collectivity.

Are lifetimes the smoking gun for the existence of exotic phenomena such as  $T=0$  pairing or quartetting in  $N=Z$  nuclei?



M.A. Bentley *et al.*, Phys. Rev. Lett. 136, 152501 (2026).  
<https://doi.org/10.1103/r6ns-ypw8>

LSSM calc. JUN45 interaction, fpg model space effective charges  $e_p = 1.5e$  and  $e_n = 0.8e$   
 Chong Qi, priv. comm

Experimental opportunities to study excited states in  $N=Z$  nuclei (in particular lifetime measurements) will increase dramatically in the next years

Istituto Nazionale di Fisica Nucleare | Laboratori Nazionali di Legnaro

GANI  
laboratoire commun CEA/DRF  
spiral2  
CNRS/IN2P3



ESNT CEA-Saclay 7-11 September, 2026 /BC

## Summary

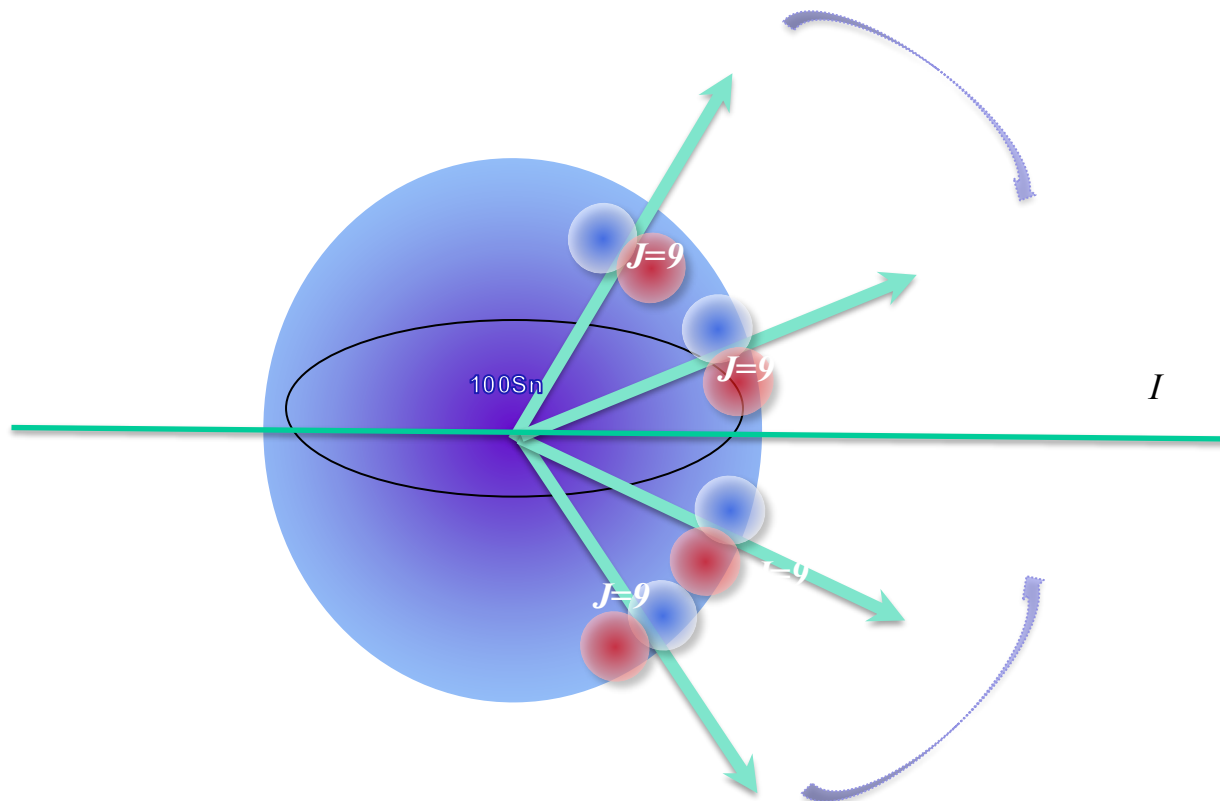
- Intermediate-angular momentum levels in the heaviest deformed  $N \sim Z$  nuclei  $^{88}\text{Ru}$  and  $^{87}\text{Tc}$  have been observed with AGATA + ancillary detectors (NEDA/NWALL, DIAMANT)
- In  $^{88}\text{Ru}$ , the rotational frequency for the configuration change between the ground-state and qp-aligned structures indicates a "delay" wrt standard calculations and expt data in neighboring  $T_z=1$  isotones, in agreement with SM predictions with enhanced isoscalar np QQ correlations or as contributions from spin-aligned isoscalar np pairs .
- For, the  $T_z=1/2$  isotone,  $^{87}\text{Tc}$ , we observe an **earlier** band crossing frequency compared with the neighboring  $N=44$  and  $N=43$  odd-mass nuclei for the yrast band built on the  $\pi g_{9/2}$  ground state
- IV band crossing frequencies for heavy deformed  $N \sim Z$  nuclei show opposite behavior as  $T_z \rightarrow 0$  for ee and oe systems- a new signature for enhanced IS pair correlations in these  $N \sim Z$  systems? But can we really "understand" delayed alignments in deformed  $N=Z$  nuclei e.g. in a simple CSM+generalized BCS picture including isoscalar pairing?

- Quartet condensate (spatially delocalized alpha-like spin-isospin correlations) – has it been overlooked in the experimental quest for  $T=0$  pairing in deformed  $N=Z$  nuclei? Note talk by N. Sandulescu later in the program (Tuesday).
- Could the Coriolis-Mottelson-Valatin effect as applied to alpha-like quartets explain the delayed alignments attributed to  $T=0$  pairing?
- A **cranked quartet condensation model** is proposed as a tool to overcome the computational burden of calculating intermediate-high spin collective states and to determine if quartets could explain delayed alignments in deformed  $N=Z$  nuclei.
- Could a **cranked QCM** explain effects ascribed to isoscalar pairing, including shifts in q.p. alignments? (Quartets could probably also coexist with IV and IS spin-isospin pairing modes).
- Experimental outlook: probing the heaviest  $N \sim Z$  systems at finite angular momentum using lifetime measurements will be possible in the next years with AGATA and GRETA gamma-ray tracking arrays. Will this provide the smoking gun for resolving the enigmatic  $T=0$  pairing issue or for new phenomena, including quartetting?

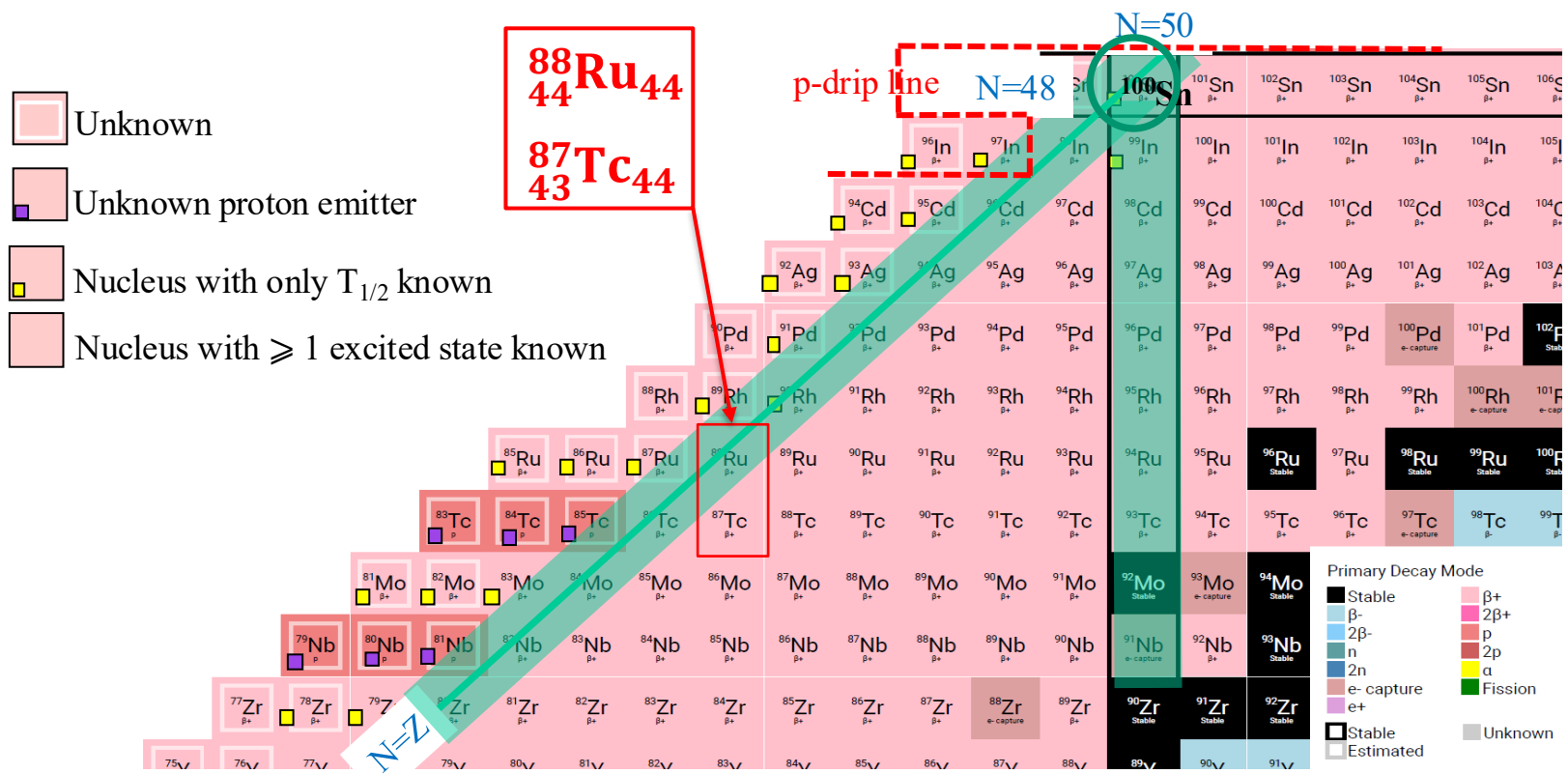
Thank You

Backup

Spherical  $N \sim Z$  systems: Generation of angular momentum in the isoscalar  
spin-aligned coupling scheme ( $^{92}\text{Pd}$ )  
(not a np pairing **condensate** in the traditional sense)



Beam:  $^{36}\text{Ar}$  (115 MeV, 5 ~ 10 pnA)  
 Target:  $^{54}\text{Fe}$  (99.58%, 6 mg/cm<sup>2</sup>)  
 Main contaminations:  $^{16}\text{O}$  from oxidation,  $^{56}\text{Fe}$  (0.40%)  
 Fusion-evaporation reactions of interest:  
 $^{36}\text{Ar}(^{54}\text{Fe}, 2n)^{88}\text{Ru}$   
 $^{36}\text{Ar}(^{54}\text{Fe}, 2n1p)^{87}\text{Tc}$



# LSSM predictions for $^{88}\text{Ru}$ with interaction including an explicit pairing term (PMMU)

PMMU:  $H = H_o + H_p + H_M + H_m$  :

$H_o$  = s.p,  $H_p$  = pairing

$H_M$  = multipole, contains QQ + OO components

$H_m$  = monopole term

$$H = H_0 + H_P + H_M + H_m^{MU},$$

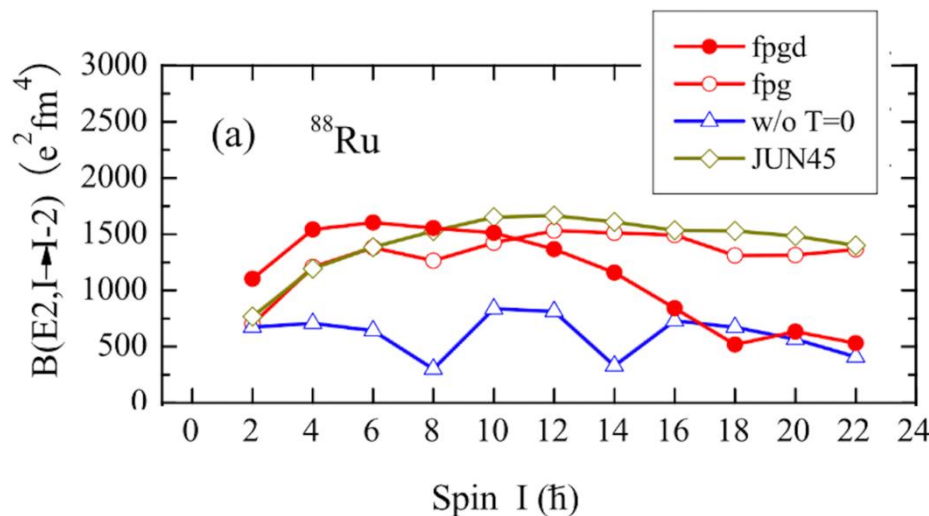
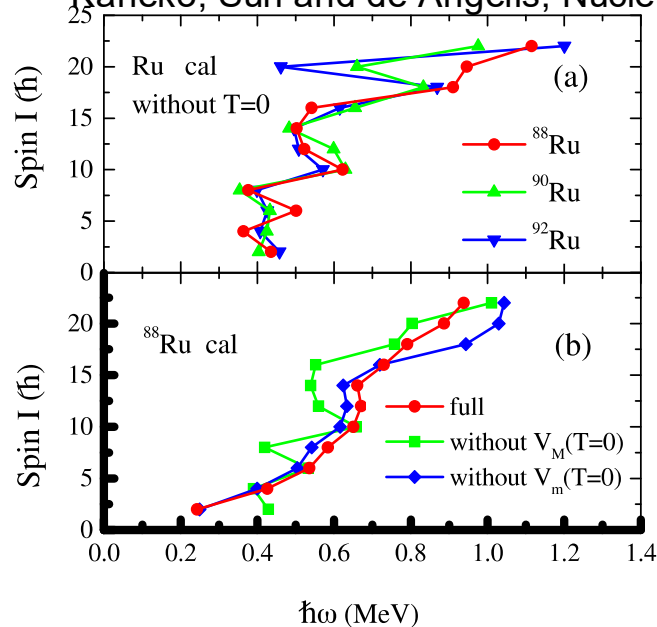
$$H_0 = \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}, \quad H_P = - \sum_{J=0,2} \frac{1}{2} g_J \sum_{M\kappa} P_{JM1\kappa}^{\dagger} P_{JM1\kappa}$$

$$H_M = -\frac{1}{2} \chi_2 \sum_M : Q_{2M}^{\dagger} Q_{2M} : - \frac{1}{2} \chi_3 \sum_M : O_{3M}^{\dagger} O_{3M} :$$

$$H_m^{MU} = \sum_{a \leq b, T} V_m^{MU}(ab, T) \sum_{JMK} A_{JMTK}^{\dagger}(ab) A_{JMTK}(ab),$$

K. Kaneko, T. Mizusaki, Y. Sun, and S. Tazaki, Phys. Rev. C89, 011302(R) (2014)

Kaneko, Sun and de Angelis, Nuclear Physics A957, 144-153 (2017)

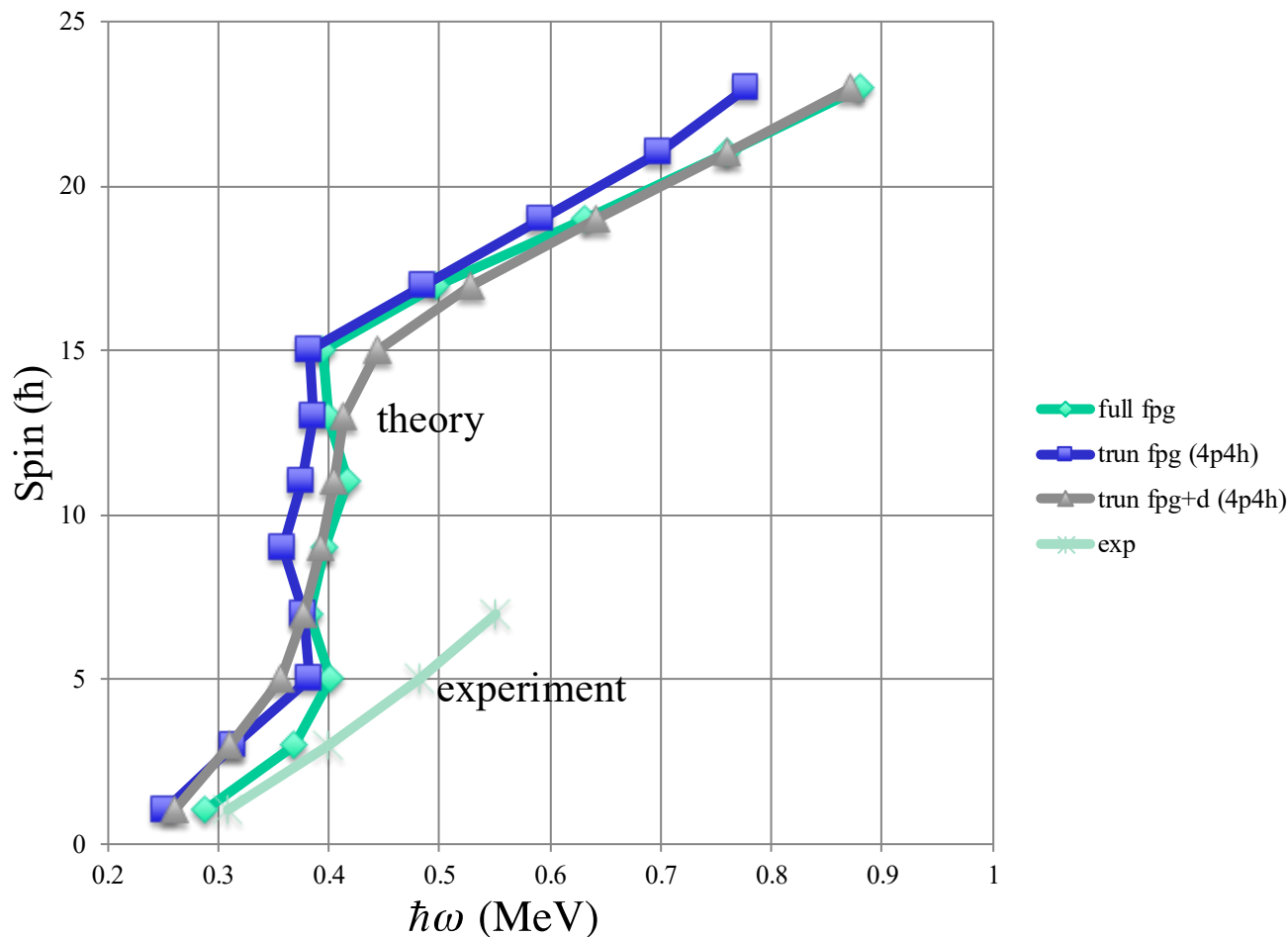


effective charges  $e_p = 1.5e$  and  $e_n = 1.1e$

**Note:** Without the  $T = 0$   $np$  pairing, the smooth pattern in  $^{88}\text{Ru}$  disappears and it lacks rotational behaviour as for  $^{90,92}\text{Ru}$ . **The np QQ T=0 is most important in the PMMU for driving the collective strength**

# Predictions for $^{88}\text{Ru}$

"Standard" SM calculations\* for  $^{88}\text{Ru}$  predict similar "alignment" of angular momentum



\* F.G. Moradi et al., Phys. Rev. C89, 014301 (2014)