

"Nuclear Josephson effect, theoretical aspects and predictions"

F.Barranco¹, G.Potel¹ and E. Vigezzi^{1,2}

(1)Dept. Física Aplicada III, Univ. of Seville, Spain

(2)INFN, Milano, Italy

Many-body correlations, phase transitions, and
clustering: how to identify and quantify them?

CEA Saclay, DPhN, Orme des Merisiers, b. 703, room
135, F-91191 Gif-sur-Yvette



PID2023-146401NB-I00

SUMMARY

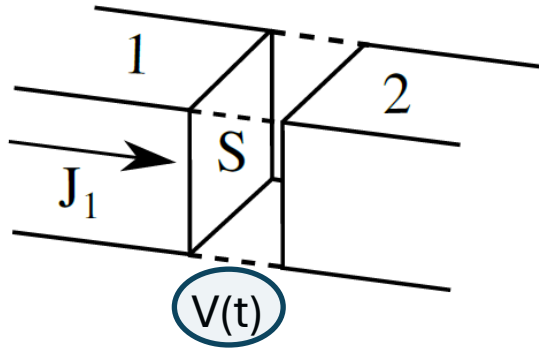
- Recalling the “Solid State” Josephson Junction DC and AC Supercurrents, and Electromagnetic Radiation (5m)
- * Historical revision of the Nuclear Josephson Effect in Transfer Reactions between Superfluid Nuclei: Theory and Experimental Data (10m)
- * More recent experiments: the LNL 2014 experiment: 2nd order DWBA analysis (10m)
- * Predicted Josephson-like Gamma Emission in Transfer Reactions (10m)
- * Other Predictions and Speculations

Recalling the “Solid State” Josephson Junction DC and AC Supercurrents, and Electromagnetic Radiation

(5m)

Josephson Junctions (DC and AC): In Condensed Matter

B. D. Josephson, Phys. Lett. 1, 251 (1962)



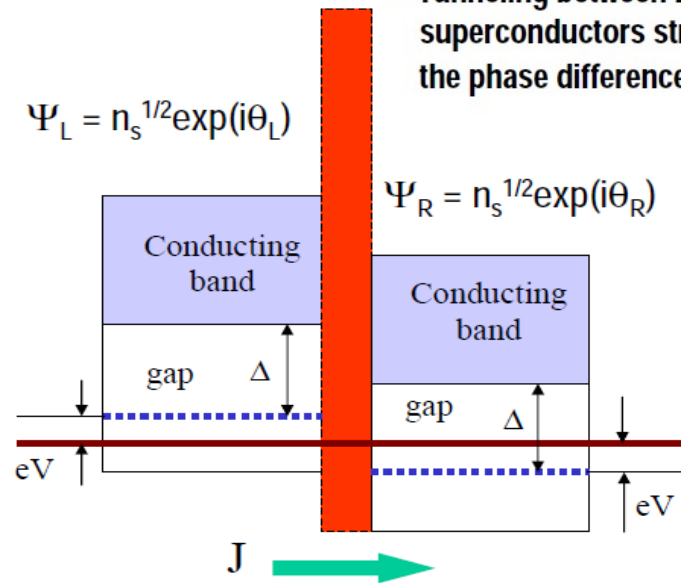
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \sqrt{n_A} e^{i\phi_A} \\ \sqrt{n_B} e^{i\phi_B} \end{pmatrix} = \begin{pmatrix} eV & K \\ K & -eV \end{pmatrix} \begin{pmatrix} \sqrt{n_A} e^{i\phi_A} \\ \sqrt{n_B} e^{i\phi_B} \end{pmatrix}$$



- Tunneling between 2 weakly coupled superconductors strongly depends on the phase difference: $\theta = \theta_L - \theta_R$

$$\dot{n}_A = \frac{2K\sqrt{n_A n_B}}{\hbar} \sin \varphi. \quad \varphi = \phi_B - \phi_A$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$



$$I(t) = I_c \sin(\varphi(t))$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$V=0 \Rightarrow$ DC
 $V=\text{const.} \Rightarrow$ AC

Because of the phase coherence, each superconductor behaves as a single-level quantum-mechanical system

2. Josephson Junctions (DC and AC): In Condensed Matter

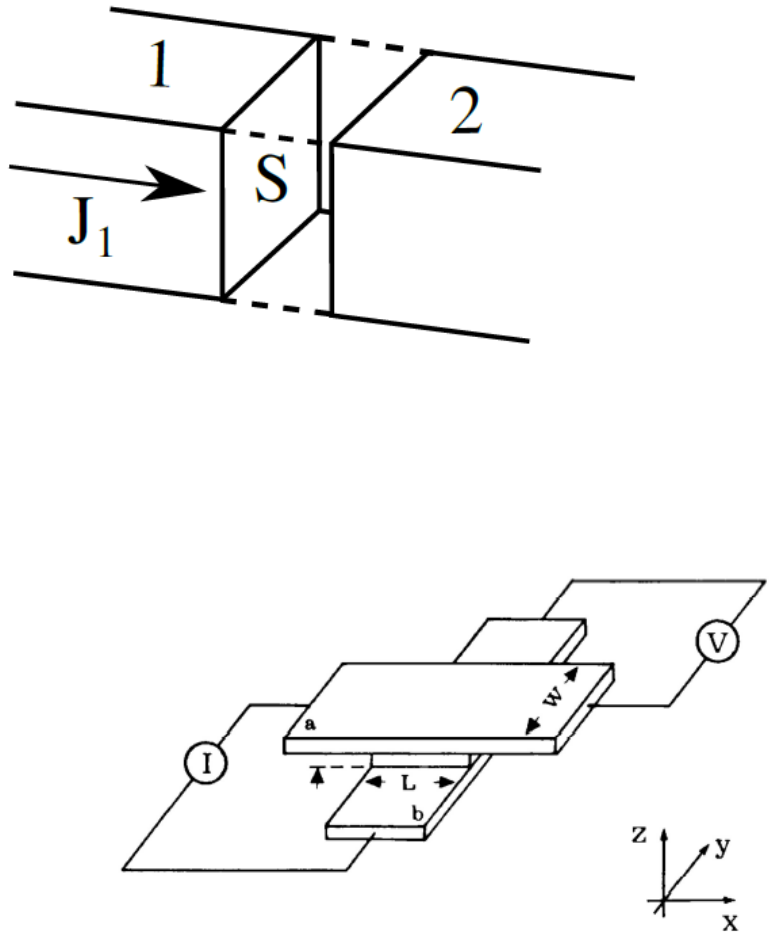


Figure 1.1 Tunneling junction of cross-type geometry. The dimensions are L and W; a and b are the two superconducting films.

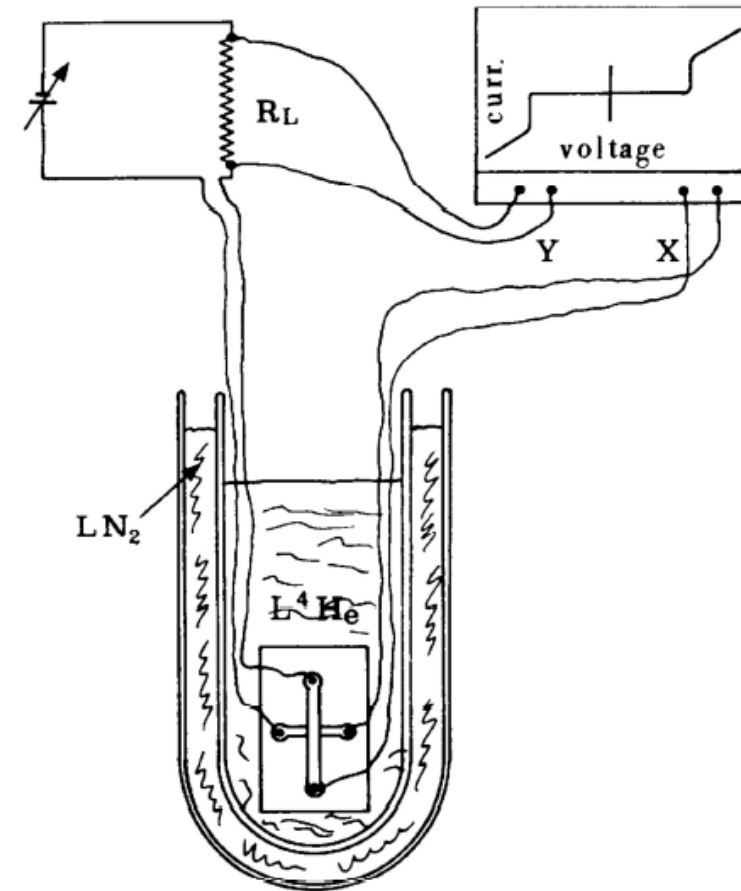


Figure 3.9 Schematic of the experimental apparatus used to measure the voltage-current characteristics of a junction. The inner dewar in which the sample is inserted is filled with liquid helium ($L^4\text{He}$), the outer dewar contains liquid nitrogen (LN_2).

3. Josephson Junctions (DC): A revision

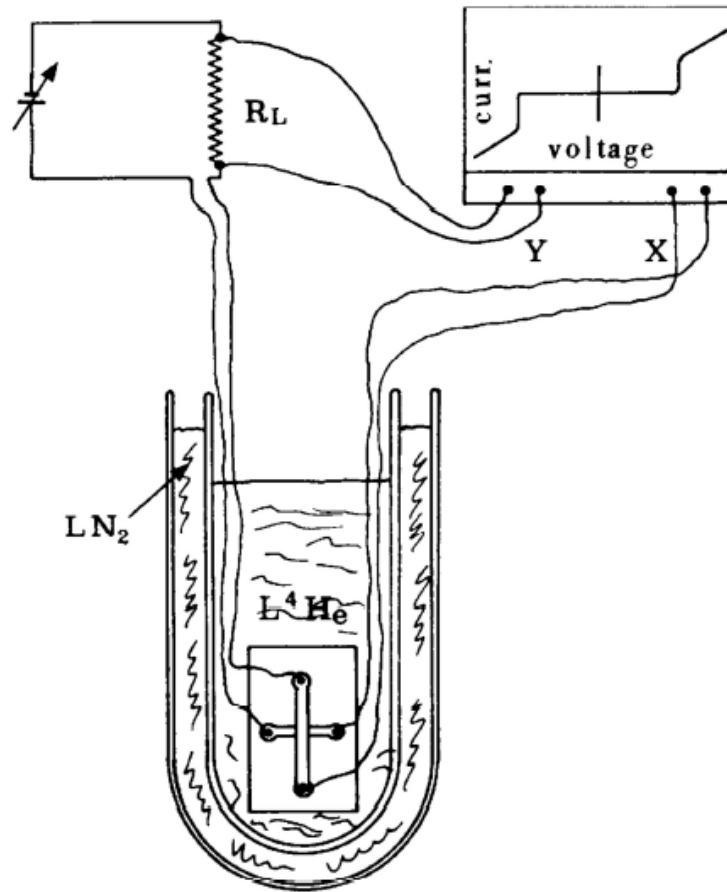


Figure 3.9 Schematic of the experimental apparatus used to measure the voltage-current characteristics of a junction. The inner dewar in which the sample is inserted is filled with liquid helium ($L^4\text{He}$), the outer dewar contains liquid nitrogen (LN_2).

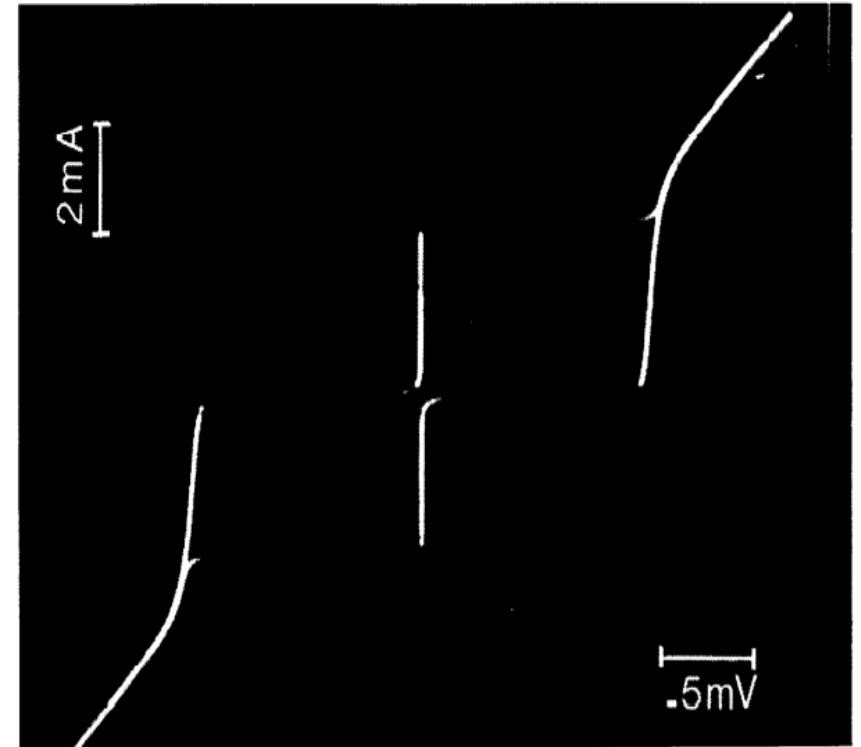


Figure 1.7 Typical voltage-current characteristic for a $\text{Sn-Sn}_x\text{O}_y\text{-Sn}$ Josephson junction at $T=1.52$ K. Horizontal scale: 0.5 mV/div; vertical scale: 2 mA/div.

Intrinsic Josephson Effects in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ Single Crystals

R. Kleiner, F. Steinmeyer, G. Kunkel, and P. Müller

Walther-Meissner-Institut, Walther-Meissner-Strasse 8, W-8046 Garching, Germany

(Received 21 August 1991; revised manuscript received 11 February 1992)

We have observed Josephson coupling between CuO double layers in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystals by direct measurements of ac and dc Josephson effects with current flow along the c axis. The results show that a small $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystal behaves like a series array of Josephson junctions which can exhibit mutual phase locking.

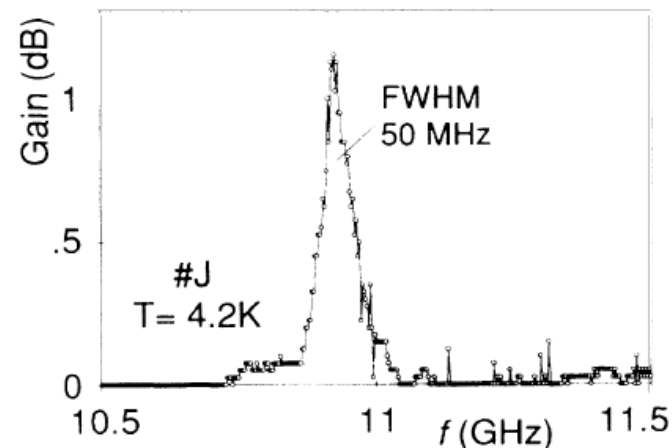


FIG. 6. Power spectrum of the microwave emission at a voltage where coherent emission occurs. Unity gain is set at -123.8 dBm. The measured points are connected by lines to guide the eye.

Emission of Coherent THz Radiation from Superconductors

L. Ozyuzer,^{1,2} A. E. Koshelev,² C. Kurter,^{2,3} N. Gopalsami,⁴ Q. Li,² M. Tachiki,⁵ K. Kadowaki,⁶ T. Yamamoto,⁶ H. Minami,⁶ H. Yamaguchi,⁶ T. Tachiki,⁷ K. E. Gray,² W.-K. Kwok,² U. Welp^{2*}

Compact solid-state sources of terahertz (THz) radiation are being sought for sensing, imaging, and spectroscopy applications across the physical and biological sciences. We demonstrate that coherent continuous-wave THz radiation of sizable power can be extracted from intrinsic Josephson junctions in the layered high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$. In analogy to a laser cavity, the excitation of an electromagnetic cavity resonance inside the sample generates a macroscopic coherent state in which a large number of junctions are synchronized to oscillate in phase. The emission power is found to increase as the square of the number of junctions reaching values of 0.5 microwatt at frequencies up to 0.85 THz, and persists up to ~50 kelvin. These results should stimulate the development of superconducting compact sources of THz radiation.

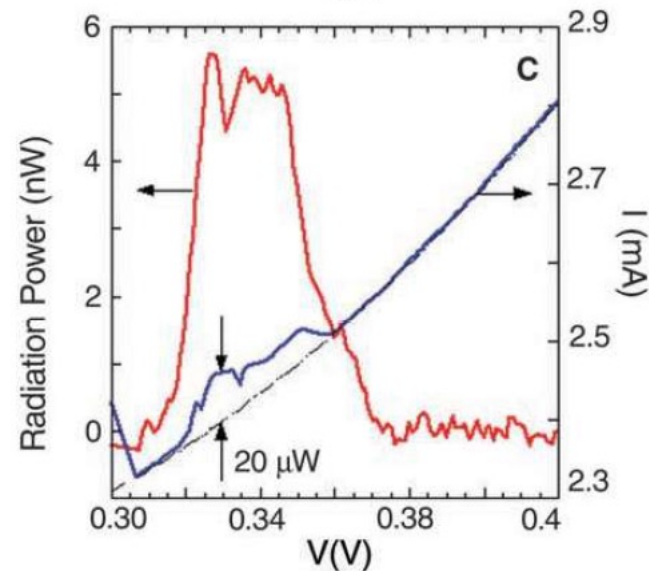


Fig. 4. Coherence of the emission. (A) Sequence of emission peaks acquired on successive voltage scans. The arrow labels the peak shown in detail in (C). (B) Radiation power in the peaks as a function of n_{rel}^2 . n_{rel} is the number of resistive intrinsic junctions normalized to the number at the peak with the highest bias. (C) On decreasing bias voltage, the entire emission peak labeled by an arrow in (A) can be scanned. These data indicate that about 20 μW are pumped into the in-phase cavity resonance.

Terahertz emission from mutually synchronized standalone $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ intrinsic-Josephson-junction stacks

Raphael Wieland^{1,*}, Olcay Kizilaslan^{1,2}, Nickolay Kinev³, Eric Dorsch¹, Stefan Guénon¹, Ziyu Song⁴, Zihan Wei⁵, Huabing Wang^{4,5}, Peiheng Wu^{4,5}, Dieter Koelle¹, Valery P. Koshelets³, and Reinhold Kleiner^{1,†}

¹Physikalisches Institut, Center for Quantum Science and LISA⁺, Universität Tübingen, 72076 Tübingen, Germany

²Department of Biomedical Engineering, Faculty of Engineering, Inonu University, Malatya, Turkey

³Kotel'nikov Institute of Radio Engineering and Electronics, Moscow, 125009, Russia

⁴Research Institute of Superconductor Electronics, Nanjing University, Nanjing, 210023, China

⁵Purple Mountain Laboratories, Nanjing, 211111, China

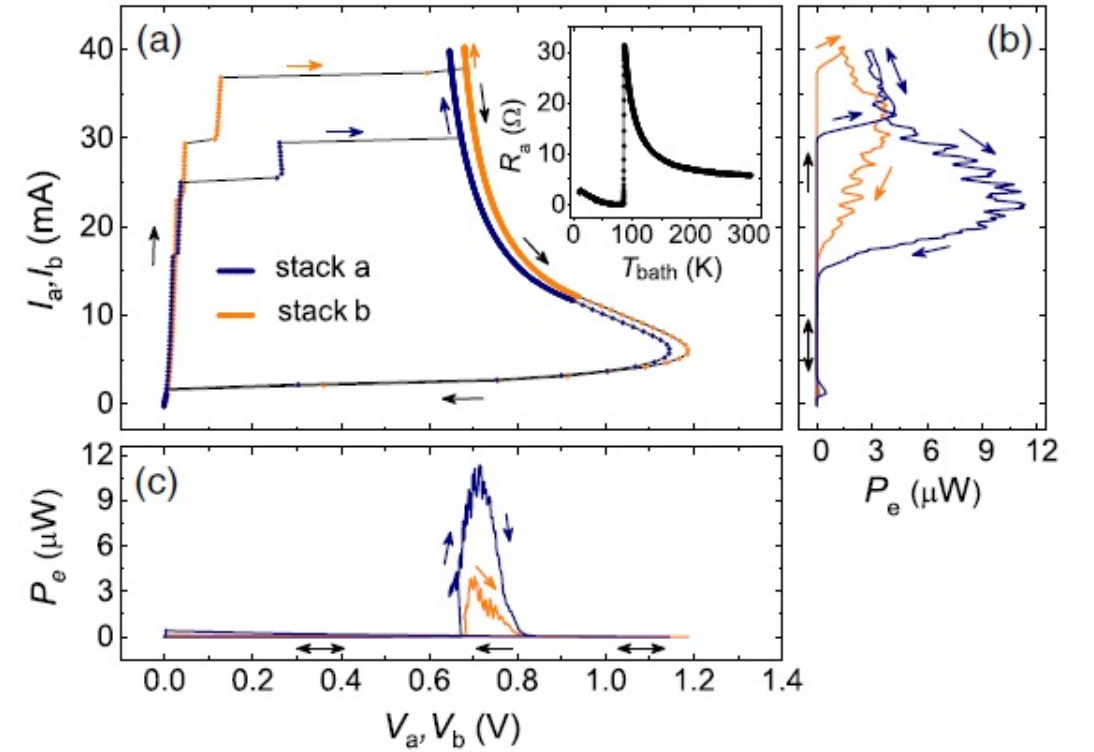
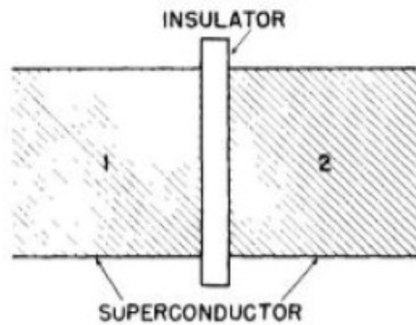


FIG. 4. (a) IVCs of stacks a and b, biased only one at a time, at $T_{\text{bath}} = 10$ K. For stack b, an Ohmic resistance of 2.3Ω has been subtracted from the data to remove the parasitic contribution from the wire connecting the sample to the room-temperature part of the setup. The inset shows resistance versus bath temperature for stack a. (b),(c) Emission power, as detected by the bolometer, as a function of (b) bias current and (c) voltage across the stacks. Emission measurements were done simultaneously with the corresponding IVC measurement. In all graphs, arrows indicate the direction of the current sweeps. Different colors are used to distinguish stack a and stack b.

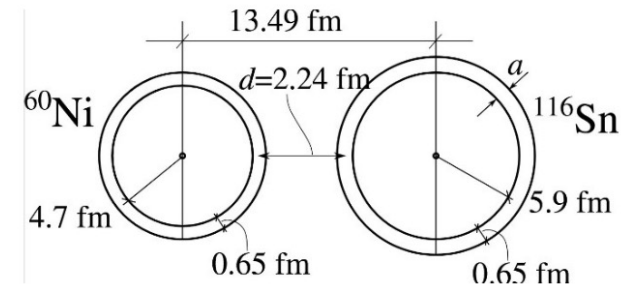
Historical revision of the Nuclear Josephson Effect in Transfer Reactions between Superfluid Nuclei: Theory and Experimental Data

(10m)

from macroscales to femtoscales



dielectric layer



surface-surface distance of nuclei at the distance of closest approach

Junction Dimensions and Surface Area

- **Area size:** $0.025 \text{ cm} \times 0.065 \text{ cm}$.
- **Total surface area:** $1.625 \times 10^{-3} \text{ cm}^2$ (or 0.1625 mm^2).
- **Electrode films:** Formed by overlapping evaporated layers of tin and lead films, each roughly 200 nm ($2000 \text{ \AA}$) thick. [Stanford University +1](#)

Insulator Thickness

- **Barrier type:** A very thin thermally or ambiently grown tin oxide layer (SnO_x).
- **Thickness:** Less than 2 nm ($< 20 \text{ \AA}$), which is thin enough to allow quantum mechanical tunneling of Cooper pairs. [Indian Academy of Sci... +2](#)

AN ANALOG OF THE JOSEPHSON EFFECT IN NUCLEAR TRANSFORMATIONS

V. I. GOL'DANSKIĬ and A. I. LARKIN

Institute of Chemical Physics, Academy of Science, U.S.S.R.

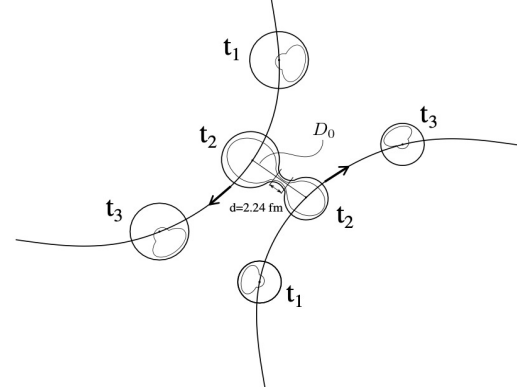
Submitted March 30, 1967

Zh. Eksp. Teor. Fiz. 53, 1032-1037 (September, 1967)

When nuclei are bombarded by heavy ions, various processes of nucleon tunneling through the potential barrier that separates the interacting nuclei at the smallest possible classical distance are observed. It is shown that nucleon pairing may give rise to a significant increase of the cross section for the transition of neutron or proton pairs, a phenomenon which in some respects is analogous to the Josephson effect in superconductors. Pairing is taken into account in the calculation of the probability for the excitation of various levels by one-nucleon exchange, which has been calculated earlier by Breit and Ebel^[1] without such corrections. The probability for two-nucleon exchange is determined. An expression is obtained for the two-proton radioactivity with account of any number of arbitrary levels, which goes over into the Galitskii-Chel'tsov formula^[2] in the limiting case of a single S level.

AN ANALOG OF THE JOSEPHSON EFFECT IN NUCLEAR TRANSFORMATIONS

V. I. GOL'DANSKIĬ and A. I. LARKIN



One-neutron transfer

Two-neutron transfer

$$M = \left\langle A_1 + 2, A_2 - 2 \left| \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' V(t) V(t') \right| A_1, A_2 \right\rangle$$

$$= \frac{1}{2} \sum_{\lambda_1 \lambda_2} \int \int dt dt' T_{\lambda_1 \lambda_2}(t) T_{-\lambda_1 - \lambda_2}(t') F_{\lambda_1}(t, t') F_{\lambda_2}(t', t). \quad (12)$$

Enhancement factor

The probability for one-nucleon transfer is given by the square of the modulus of the matrix element

$$W = \left| \int \langle A_1 + 1, A_2 - 1 | V(t) | A_1, A_2 \rangle dt \right|^2. \quad (9)$$

Let us substitute here $V(t)$ from (8) and use for the matrix elements of the operators a and a^+ the expressions obtained in the superconducting model of the nucleus^[5] for even-even nuclei:

$$|\langle a_\lambda \rangle|^2 = 1 - |\langle a_\lambda^+ \rangle|^2 = (E_\lambda - \varepsilon_\lambda) / 2E_\lambda. \quad (10)$$

$$\frac{\dot{W}_2}{W_1^2} = \frac{1}{\pi} \left| \frac{\omega_0 \Delta_1 \Delta_2}{4(E_{\lambda_1} + E_{\lambda_2})(E_{\lambda_1} + \varepsilon_{\lambda_1})(E_{\lambda_2} - \varepsilon_{\lambda_2})} \right.$$

$$\times \left. \int_0^\infty \exp\{iy - \omega_0^2 y^2 / 4(E_{\lambda_1} + E_{\lambda_2})\} dy \right|^2$$

$$\times \exp \left\{ - \frac{(M_{A_1} + M_{A_2} - M_{A_2-2} - M_{A_1+2})^2}{2\omega_0^2} \right\}$$

$$\times \exp \left\{ + \frac{2(M_{A_1+1} - M_{A_1} + M_{A_2-1} - M_{A_2} + E_1 + E_2)^2}{\omega_0^2} \right\}.$$

$$W_1 = 2\pi\omega_0^2 |T_{\lambda_1 \lambda_2}(0)|^2 \frac{E_{\lambda_1} + \varepsilon_{\lambda_1}}{2E_{\lambda_1}} \frac{E_{\lambda_2} - \varepsilon_{\lambda_2}}{2E_{\lambda_2}}$$

$$\times \exp \left\{ - \frac{(M_{A_1+1} - M_{A_1} + M_{A_2-1} - M_{A_2} + E_1 + E_2)^2}{\omega_0^2} \right\}.$$

6. CONCLUSION

The pairing of nucleons has a large effect on the probability for nuclear transitions. Besides the usual effect due to the difference in binding energy of the nucleons in even and odd nuclei, there exists an additional effect which enhances strongly the probability for two-proton radioactivity and for two-nucleon exchange. The point is that even if the nuclear levels are far apart, the model of a single j level may turn out to be ill-suited for a description of such transitions. Usually the upper level has a large angular momentum, and the probability for a transition from it is small because of the centrifugal barrier. The pairing has the

effect that the angular momentum of an individual nucleon is no longer well defined, and there is a significant probability that the nucleon is in a level with small angular momentum. Since the overlap integral depends strongly on the angular momentum, it may happen that the total transition rate is determined by the transition from a level with small l .

It should be kept in mind that formulas (14) and (19) describe the pairing effect only qualitatively. In order to obtain quantitative results, one must determine the factors in front of the exponentials in the overlap integrals and take account of the quantum mechanics of the motion of the colliding nuclei. In addition, several physical effects must also be considered. It is possible, for example, that the excitation of collective vibrational or rotational states plays an appreciable role. In classical terms, this corresponds to the appearance of a tidal wave during the collision of the nuclei, which may influence the value of the overlap integrals. In the case of far-away collisions one must possibly take account of the interaction of the nucleons in the sub-barrier region.

MULTIPLE PAIR TRANSFER IN REACTIONS BETWEEN HEAVY NUCLEI

K. DIETRICH

*Institute for Theoretical Physics of the University of Heidelberg and
Kernforschungszentrum Karlsruhe, Karlsruhe, Germany*

K. HARA

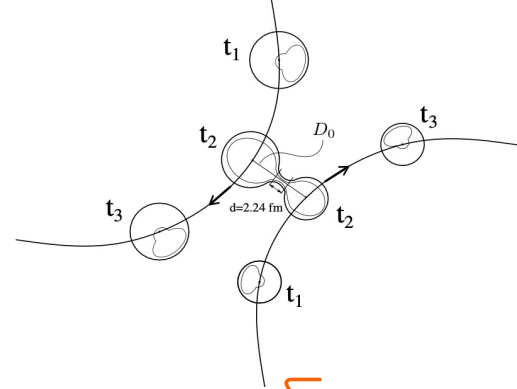
Institute for Theoretical Physics of the University of Heidelberg, Heidelberg, Germany

F. WELLER

Kernforschungszentrum Karlsruhe, Karlsruhe, Germany

Received 5 April 1971

Multiple pair transfer between superconducting nuclei below the Coulomb barrier is investigated in a semi-classical theory and is found to occur with appreciable probability. The dependence of the transfer probability on the energy approximately follows a simple rule.



$$p_{j_0+k}(+\infty; E, \theta) = |-i^k J_k(2a\varphi)|^2 = J_k^2(2a\varphi)$$

$$\varphi := - \int_{-\infty}^{+\infty} d\tau V(\tau) \cos \omega \tau$$

The results (9) and (11) are based on approximations which are valid for infinite systems, but obviously crude for finite systems. Nevertheless, it can be easily checked that the relative position of the maxima and minima of the transfer probabilities as well as their relative magnitude as a function of the energy is reasonably well reproduced by (11).

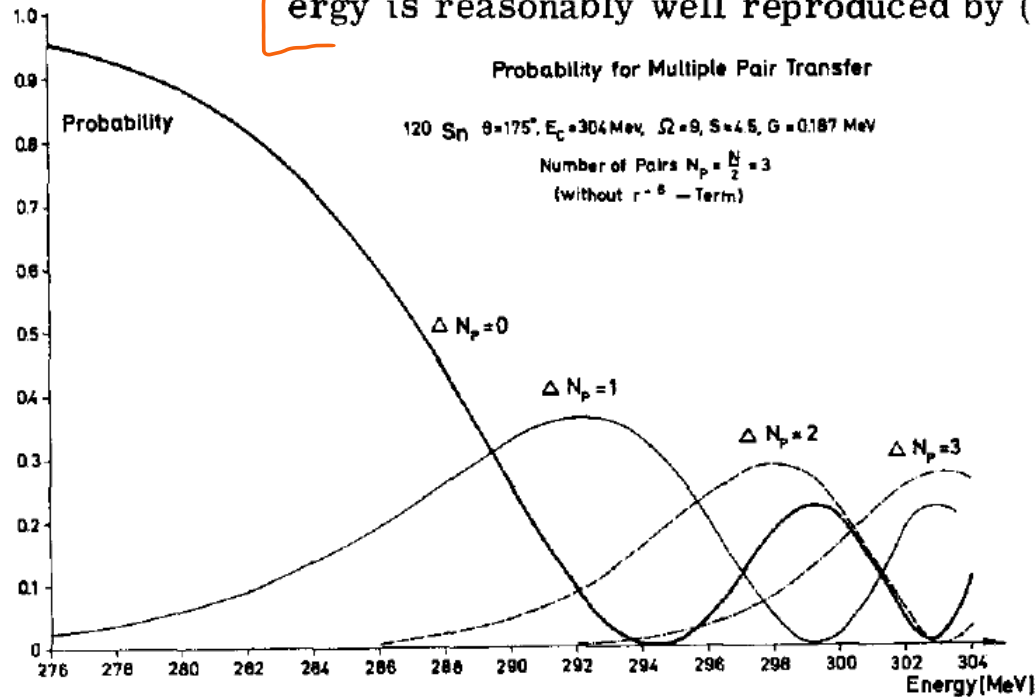


Fig.1. Scattering of ^{120}Sn on ^{120}Sn , transfer probability $p_{3,k}(+\infty; E, 175^\circ)$ for different pair transfer ($\Delta N_p = k$) as a function of the energy.

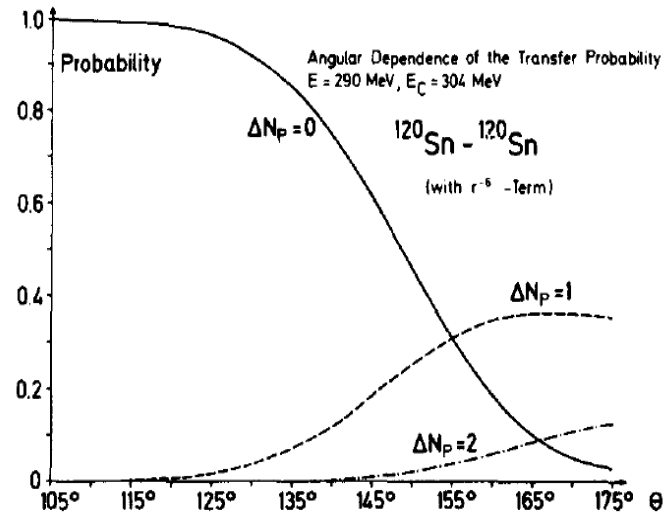


Fig.4. Scattering of ^{120}Sn on ^{120}Sn , transfer probability $p_{3,k}(\infty; 290\text{MeV}, \theta)$ as a function of angle at $E = 290\text{ MeV}$

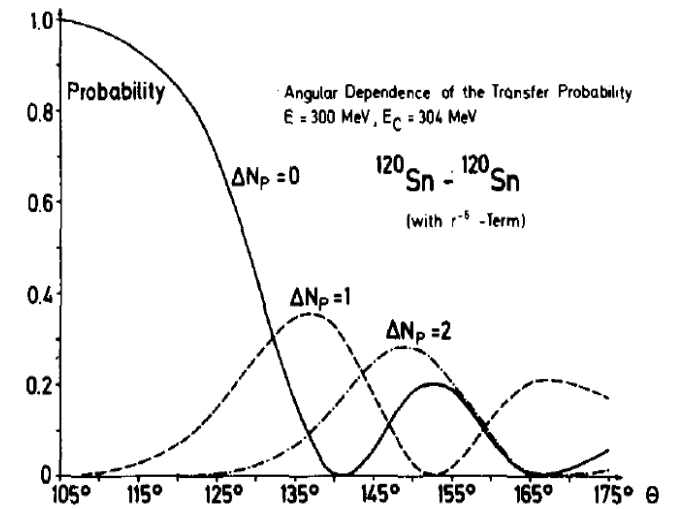


Fig.5. Same as 4, but for different energy $E = 300\text{ MeV}$: $p_{3,k}(\infty; 300\text{ MeV}, \theta)$

TRANSFER GAUGE ANGLE

(after D. Brink, Brink_Nuclear Physics at the Borderlines Proceedings of the Fourth International Summer School de la Rábida, La Rábida, Huelva, Spain, June 17–29, 1994.)

Consider two even nuclei A and B in their ground states in close proximity with fixed positions. The internal state of the system is denoted by $|0\rangle$. The ground states of

$$\Psi = \sum_N a_N |N\rangle, \quad H_t |N\rangle = -\frac{1}{2} J_0 (|N+1\rangle + |N-1\rangle)$$

$$\Psi = \sum_N e^{iN\phi} |N\rangle, \quad E = -J_0 \cos \phi.$$

$$f(\phi) = \frac{1}{\sqrt{2\pi}} \sum_N a_N e^{iN\phi}, \quad \text{where} \quad a_N = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{-iN\phi} f(\phi) d\phi, \quad N = -i \frac{\partial}{\partial \phi}.$$

Josephson equations

A more general hamiltonian has a term depending on the number of pairs moved as well as the transfer term (5.2)

$$H = H_t + \epsilon(N). \quad (5.6)$$

The term $\epsilon(N)$ is the total ground state energy of the two nuclei when N pairs have been transferred. The Schrödinger equation with the hamiltonian (5.6) can not be solved analytically for a general ϵN but the Heisenberg equations of motion for the operators ϕ and N have an interesting form. They are

$$i\hbar \frac{dN}{dt} = [N, H] = [N, H_t] = iJ_0 \frac{d \cos \phi}{d\phi}$$

$$i\hbar \frac{d\phi}{dt} = [\phi, H] = [\phi, \epsilon(N)] = i \frac{d\epsilon(N)}{dN}$$

and simplify to give

$$\frac{dN}{dt} = -(J_0/\hbar) \sin \phi, \quad \frac{d\phi}{dt} = \epsilon'(N)/\hbar. \quad (5.7)$$

These are Josephson's two relations. The first connects the Josephson tunnelling current with the gauge angle and the second gives the rate of change of ϕ in terms of the potential difference between A and B.

DC “Nuclear supercurrent”

The transfer amplitude J_0 is time dependent for pair transfer between heavy ions. It is small when the nuclei are far apart and a maximum at the point of closest approach. We can write a time dependent Schrödinger equation for the wave function $f(\phi)$ in the gauge representation as

$$i\hbar \frac{\partial f}{\partial t} = H_t f = -J_0(t) \cos(\phi) f, \quad (5.8)$$

where we have used the form of H_t in the gauge representation give in equation (5.3). It should be solved with the initial condition $N = 0$, ($f(\phi) = 1/\sqrt{2\pi}$ as $t \rightarrow -\infty$). The solution as $t \rightarrow +\infty$ is

$$f(\phi) = \frac{1}{\sqrt{2\pi}} e^{ix \cos \phi}, \quad \text{with} \quad x = \frac{1}{\hbar} \int_{-\infty}^{\infty} J_0(t) dt. \quad (5.9)$$

The amplitudes a_N are the Fourier coefficients in the expansion of $f(\phi)$ and are proportional to Bessel functions

$$a_N = i^N J_N(x).$$

Thus the probability that N pairs have been transferred after the collision is

$$P_N = |a_N|^2 = |J_N(x)|^2.$$

Unfortunately in the heavy ion case x is normally small so that the probability that $|N| > 1$ is very small.

Pairing correlations of nucleons and multi-nucleon transfer between heavy nuclei

To cite this article: W von Oertzen and A Vitturi 2001 *Rep. Prog. Phys.* **64** 1247

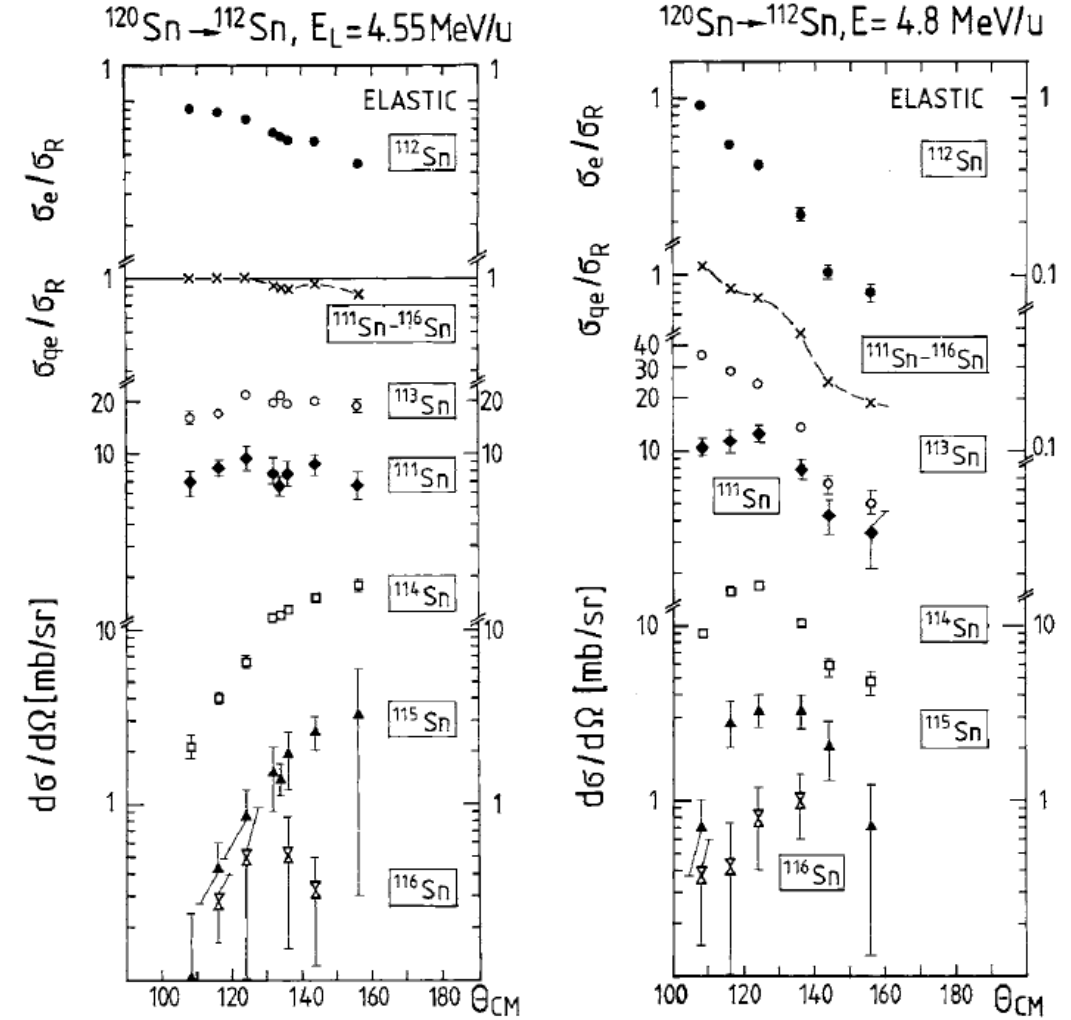
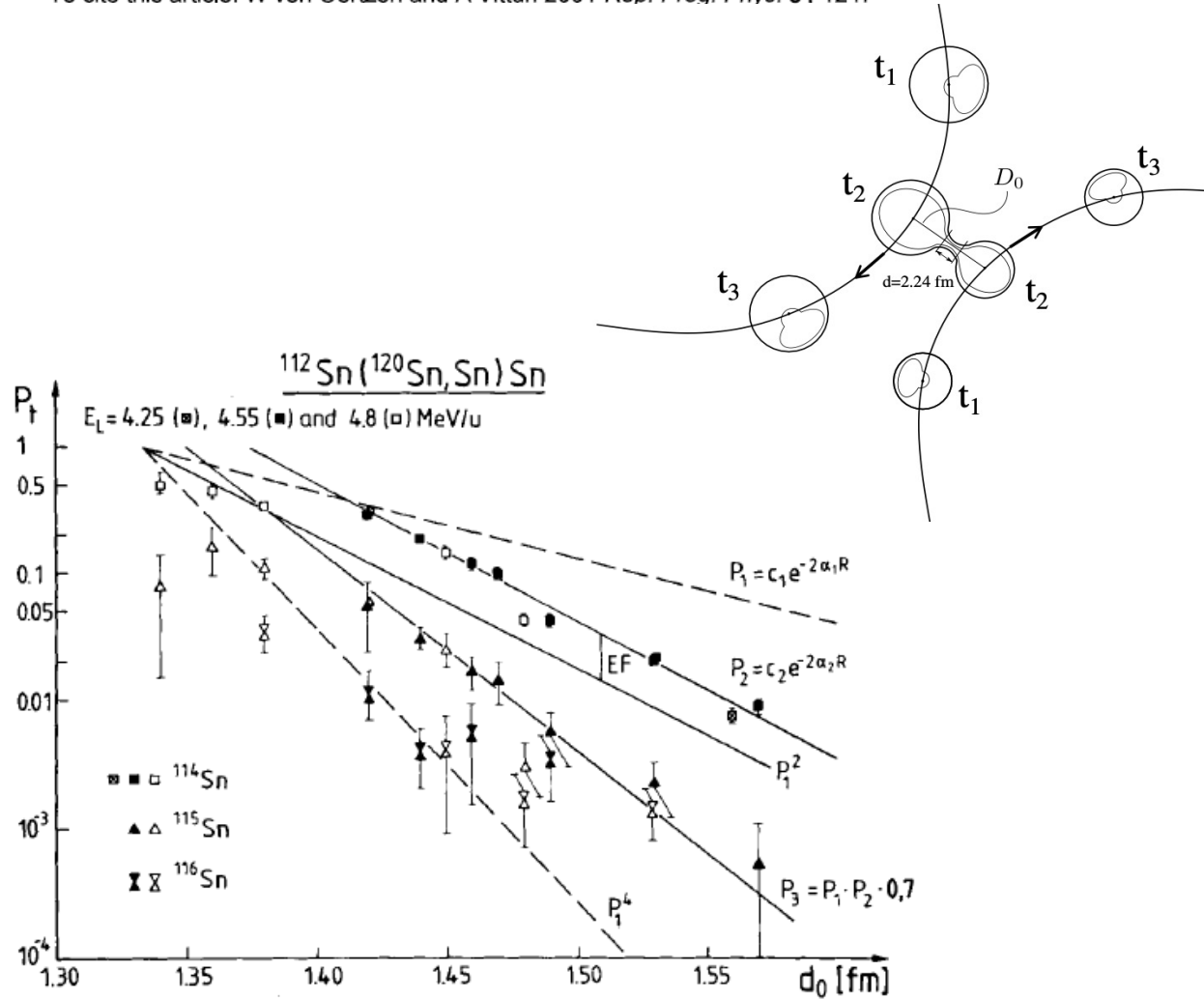
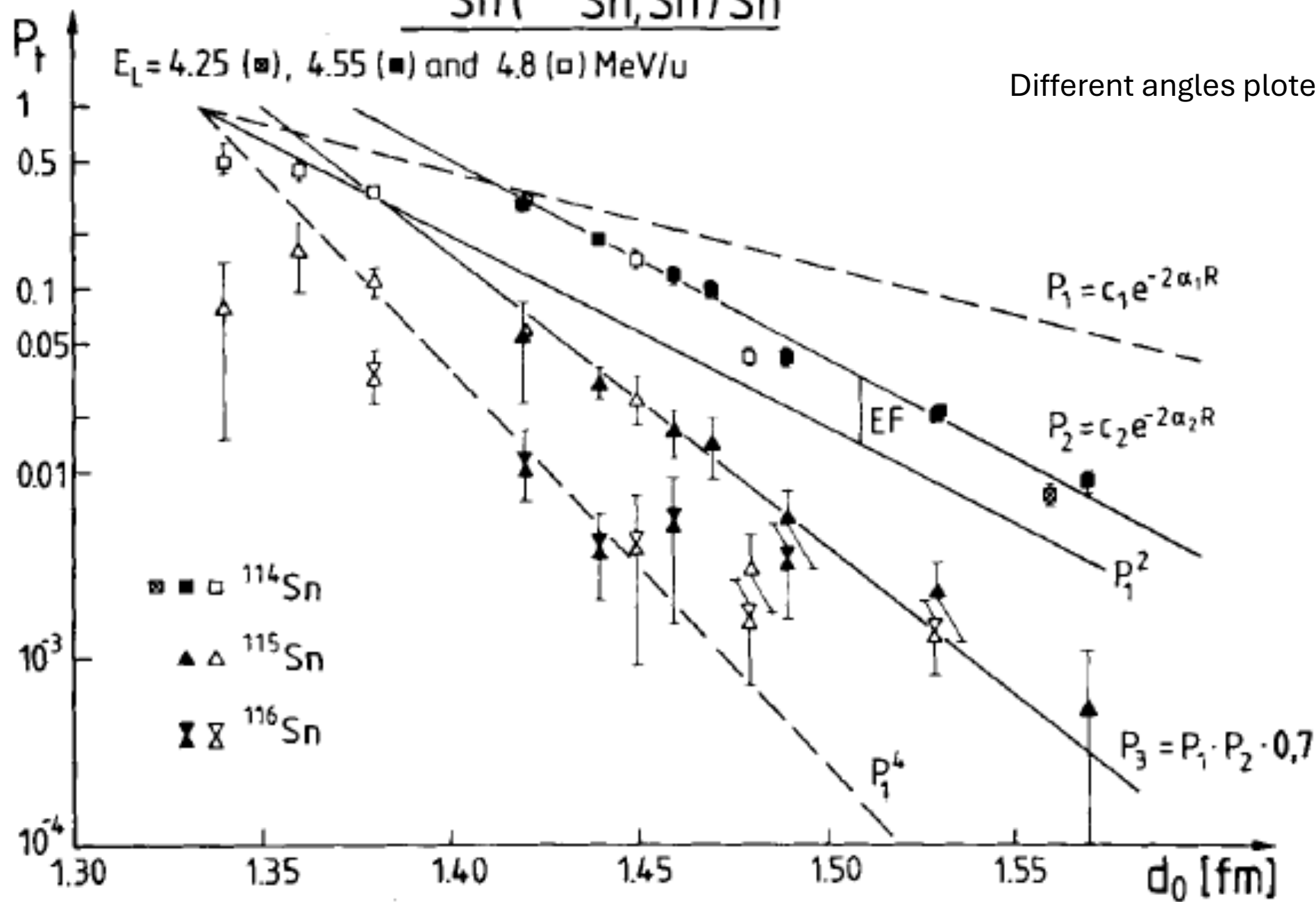


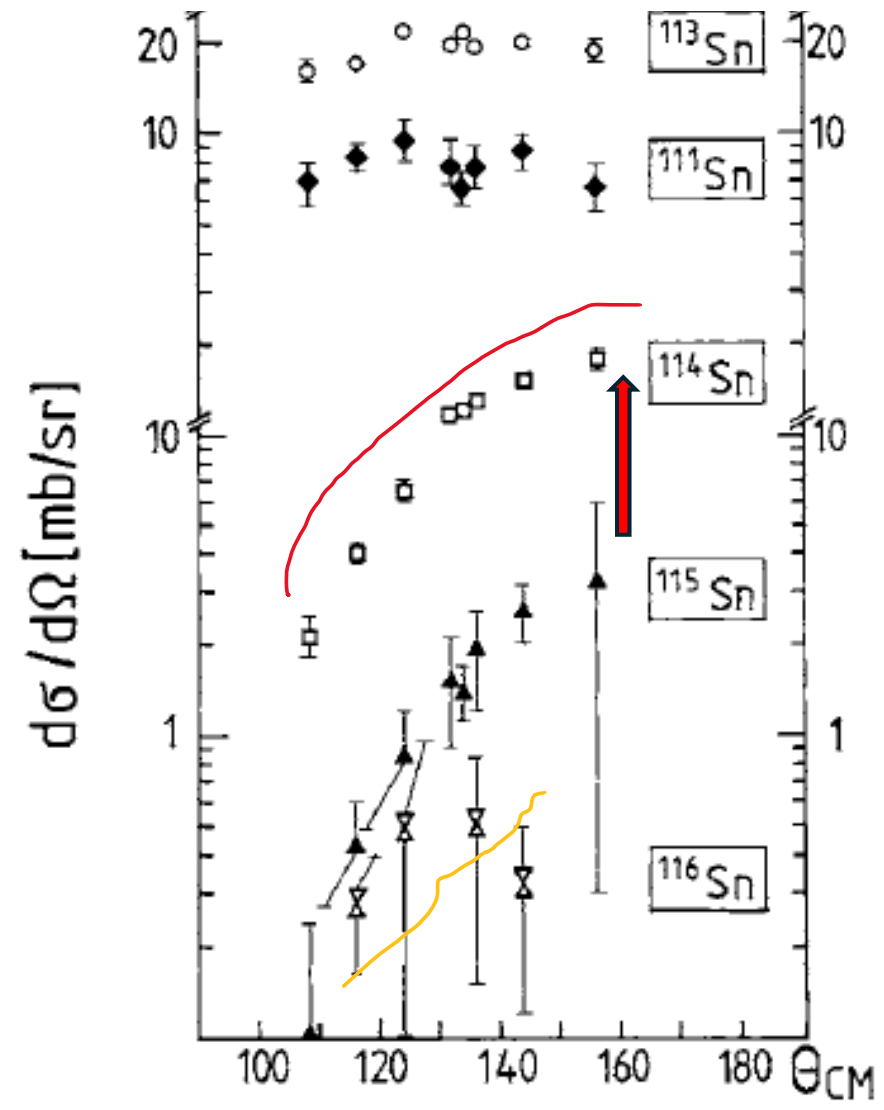
Figure 10. Angular distributions of elastic scattering as well as of neutron transfer (from one up to four neutrons) in the system $^{112}\text{Sn} + ^{120}\text{Sn}$ at two energies close to the Coulomb barrier. The quantity σ_{qe} is the sum of the elastic and transfer cross sections.

$^{112}\text{Sn}(^{120}\text{Sn}, \text{Sn})\text{Sn}$

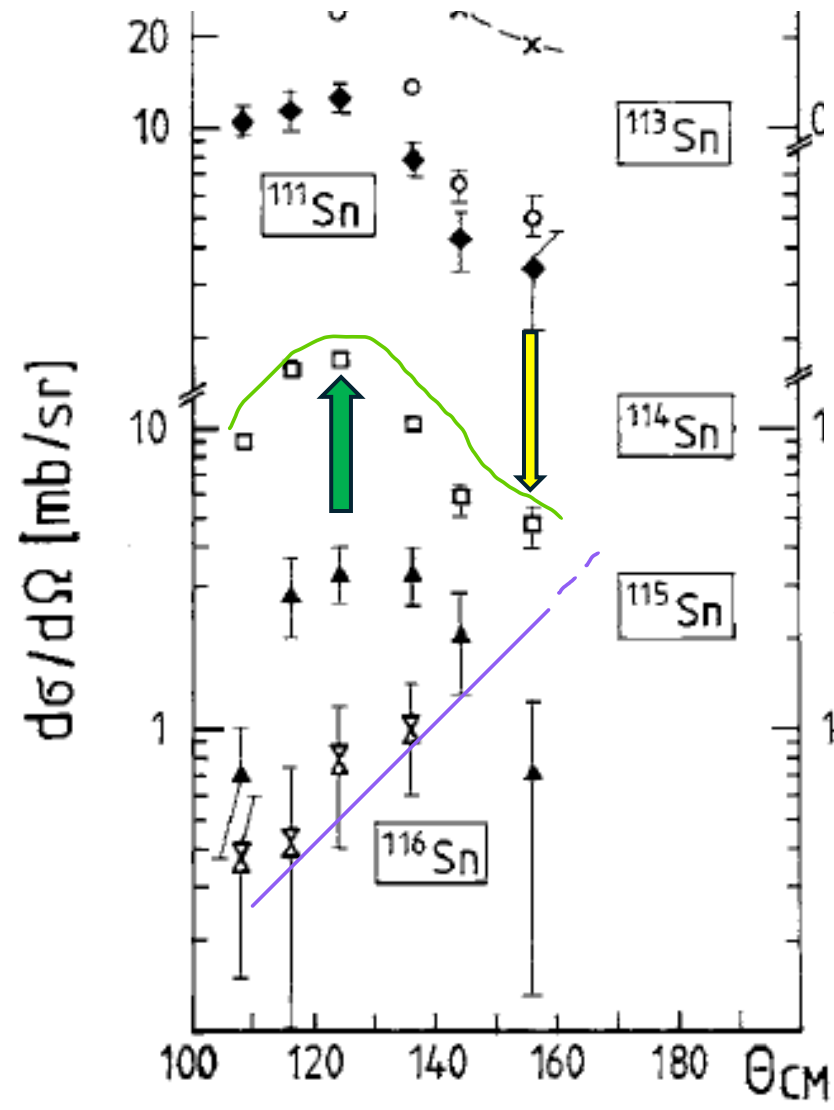
$E_L = 4.25$ (□), 4.55 (■) and 4.8 (○) MeV/u

Different angles plotted together?





Ecm aprox.255MeV



dEcm= 15MeV

Ecm approx.270MeV

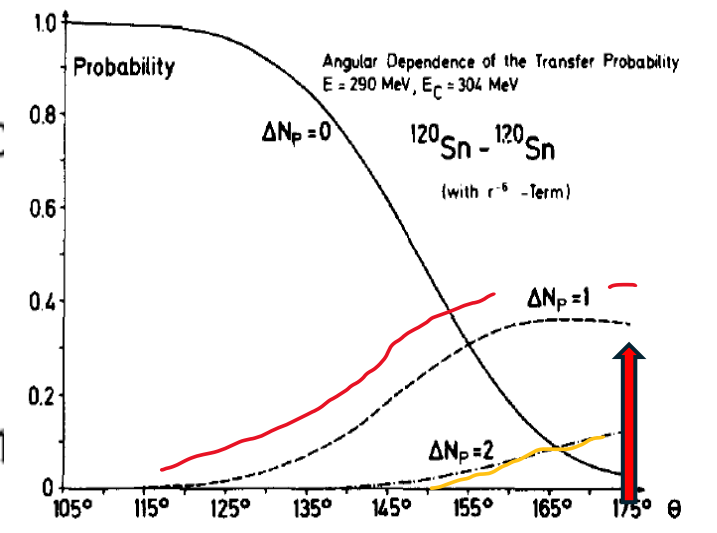


Fig.4. Scattering of ^{120}Sn on ^{120}Sn . transfer probability

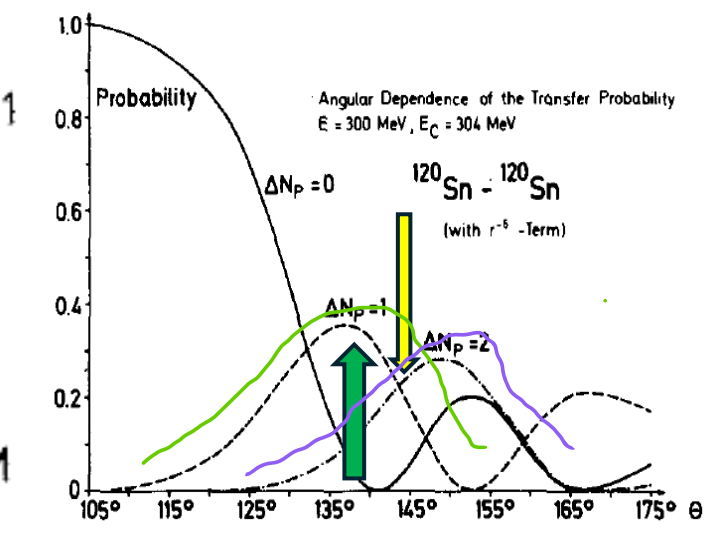


Fig.5. Same as 4, but for different energy $E = 300$ MeV:
 $p_{3,k}(\infty; 300 \text{ MeV}, \theta)$

More recent experiments: the LNL 2014 experiment:
2nd order DWBA analysis

(10m)

The Legnaro's $^{116}\text{Sn}+^{60}\text{Ni}$ 2NT and 1NT data at different E_{cm} :

PRL **113**, 052501 (2014)

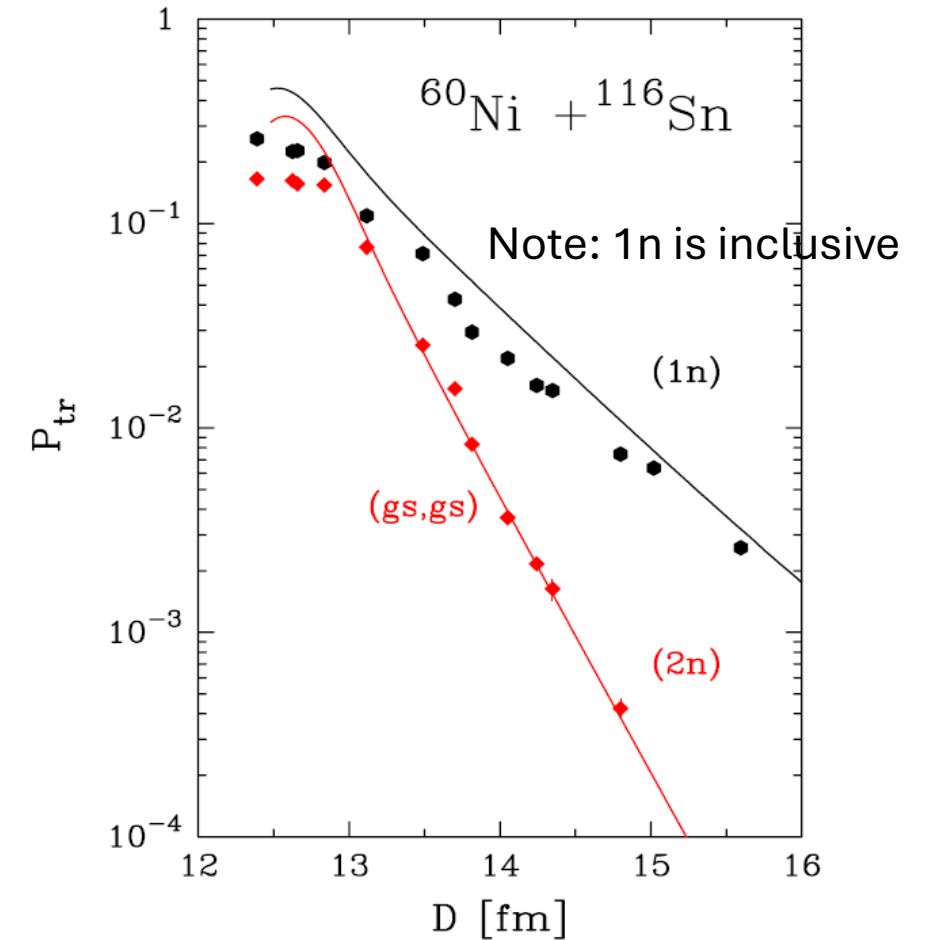
PHYSICAL REVIEW LETTERS

week ending
1 AUGUST 2014

Neutron Pair Transfer in $^{60}\text{Ni} + ^{116}\text{Sn}$ Far below the Coulomb Barrier

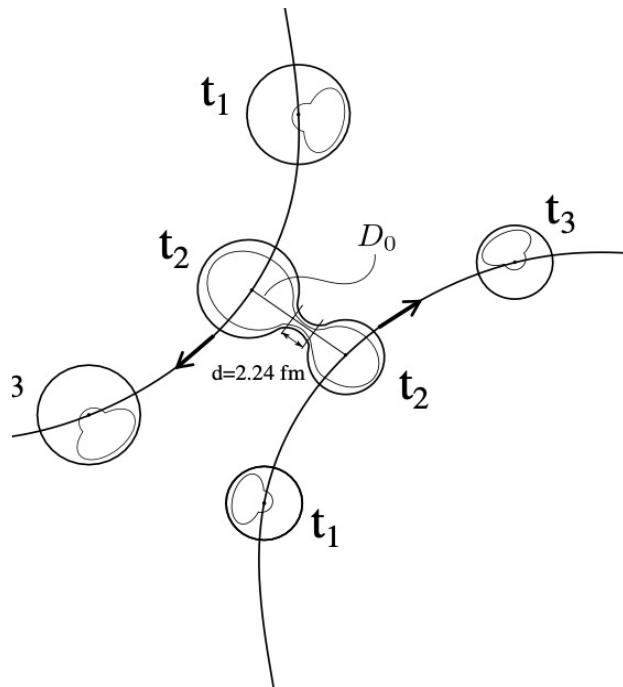
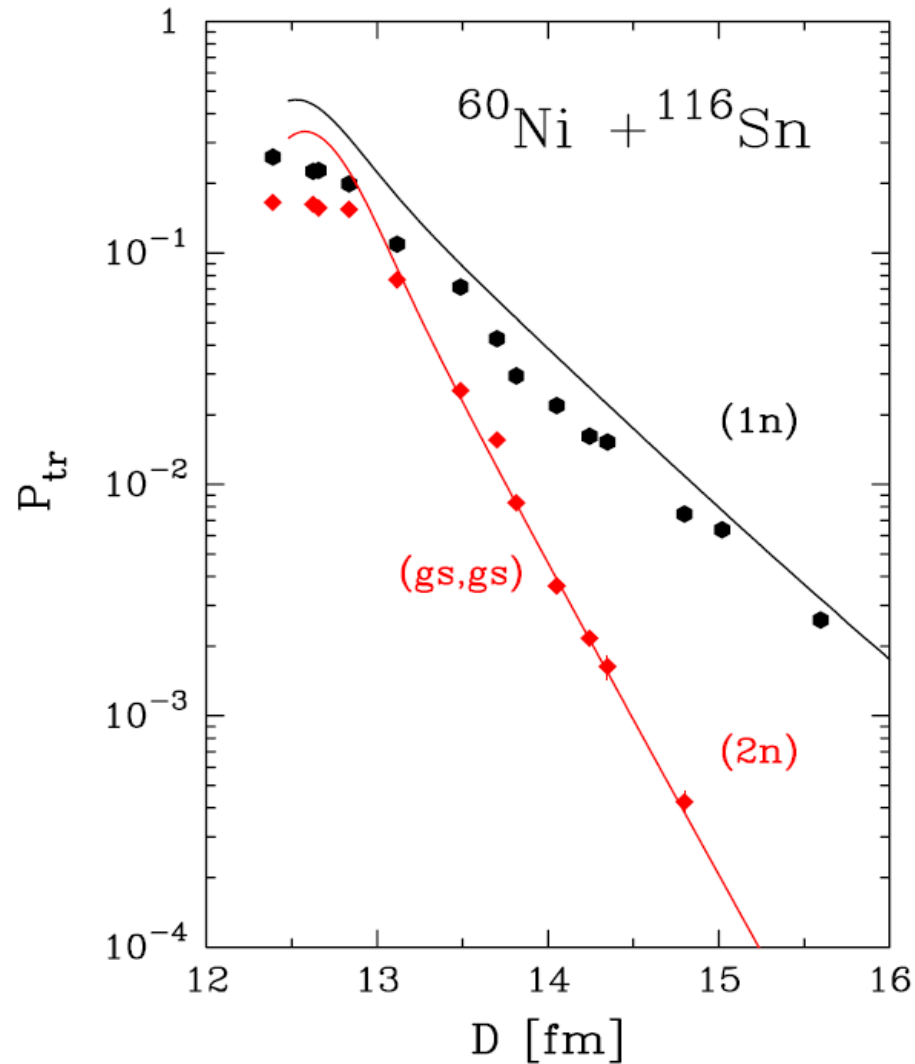
D. Montanari,¹ L. Corradi,² S. Szilner,³ G. Pollaro,⁴ E. Fioretto,² G. Montagnoli,¹ F. Scarlassara,¹ A. M. Stefanini,²
S. Courtin,⁵ A. Goasduff,^{5,6} F. Haas,⁵ D. Jelavić Malenica,³ C. Michelagnoli,² T. Mijatović,³ N. Soić,³
C. A. Ur,¹ and M. Varga Paitler⁷

Experimental (points) and microscopically calculated (lines) transfer probabilities for the one- (^{61}Ni) and two-neutron (^{62}Ni) pickup plotted as a function of the distance of closest approach D (the entrance channel Coulomb barrier is estimated to be at 12.13 fm [4]). We also report (top) the reduced distance $d_0 = D/(A_1^{1/3} + A_2^{1/3})$. The shown errors are only statistical and in most cases are smaller than the size of the symbol.



Semiclassical calculations in this paper

One- and two-neutron transfer probability between superfluid heavy ions as a function of the distance of closest approach



$D_0(\text{fm})$	$E_{cm} \text{ (MeV)}$	$(E_B - E_{cm}) \text{ (MeV)}$	$\left(\frac{4}{\pi}\right)^2 \left(\frac{\sigma_{2n}}{\sigma_{1n}}\right)$
13.12	158.63	-1.03	1.14
13.49	154.26	3.34	0.57
13.70	151.86	5.74	0.59
13.81	150.62	6.98	0.46
14.05	148.10	9.50	0.27
14.24	146.10	11.50	0.22
14.39	145.02	12.58	0.18

At $D_0 = 13.3 \text{ fm}$ $P_2 = P_1$!!!

D. Montanari et al, PRL 113 052501 (2014)

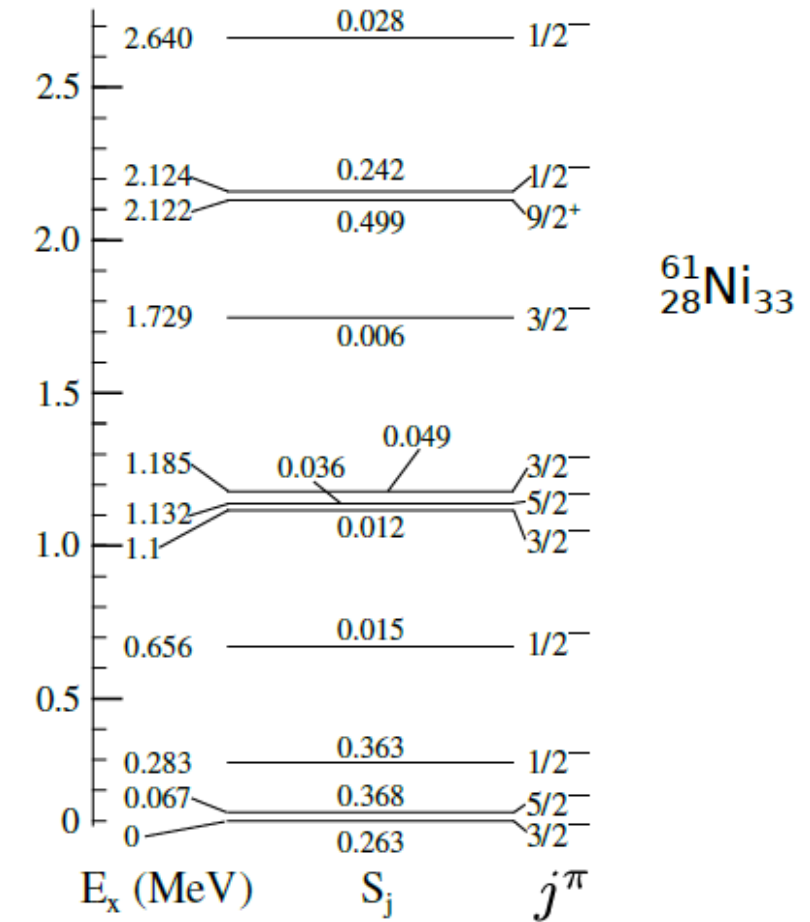
The Legnaro's $^{116}\text{Sn}+^{60}\text{Ni}$ 2NT and 1NT: BCS ground states and quasiparticles

	ϵ_{l_j}	E_{l_j}	$V_{l_j}^2$	B_{l_j}	E_{l_j}	$V_{l_j}^2$	B_{l_j}
$2d_{5/2}$	-11.51	2.35	0.874	0.658	2.35	0.874	0.575
$1g_{7/2}$	-10.86	1.92	0.790	0.910	1.92	0.789	0.816
$3s_{1/2}$	-9.70	1.56	0.487	0.483	1.57	0.484	0.500
$2d_{3/2}$	-9.51	1.58	0.428	0.661	1.58	0.424	0.699
$1h_{11/2}$	-8.12	2.25	0.140	0.770	2.26	0.139	0.847

Table 1: Properties of the five valence states calculated in ^{115}Sn , starting from the single-particle energies ϵ_{l_j} shown in [1]. The quasiparticle energies E_{l_j} , the occupation probabilities $V_{l_j}^2$ and the transfer spectroscopic amplitudes B_{l_j} shown in Table 1 of ref. [1] are shown in the second, third and fourth column. They are compared in the last three columns with the corresponding quantities recalculated here with a pairing coupling constant $G = 0.217$ MeV (25/A).

	ϵ_{l_j}	E_{l_j}	$U_{l_j}^2$	B_{l_j}	B_{l_j}
$2p_{3/2}$	-11.33	1.56	0.33	0.524	0.665
$2p_{1/2}$	-9.66	1.83	0.65	0.500	0.477
$1f_{5/2}$	-9.33	2.05	0.400	0.34	0.849
$1g_{9/2}$	-5.16	5.80	0.500	0.1	1.224

Table 2: Properties of the four valence states calculated in ^{61}Ni , starting from the single-particle energies ϵ_{l_j} shown in [1]. The quasiparticle energies E_{l_j} , the occupation probabilities $U_{l_j}^2$ and the transfer spectroscopic amplitudes B_{l_j} shown in Table 1 of ref. [1] are shown in the second, third and fourth column. The values of the spectroscopic are compared in the last column with the value obtained from the relation $B = UV\sqrt{j+1/2}$, using $V^2 = 1 - U^2$.

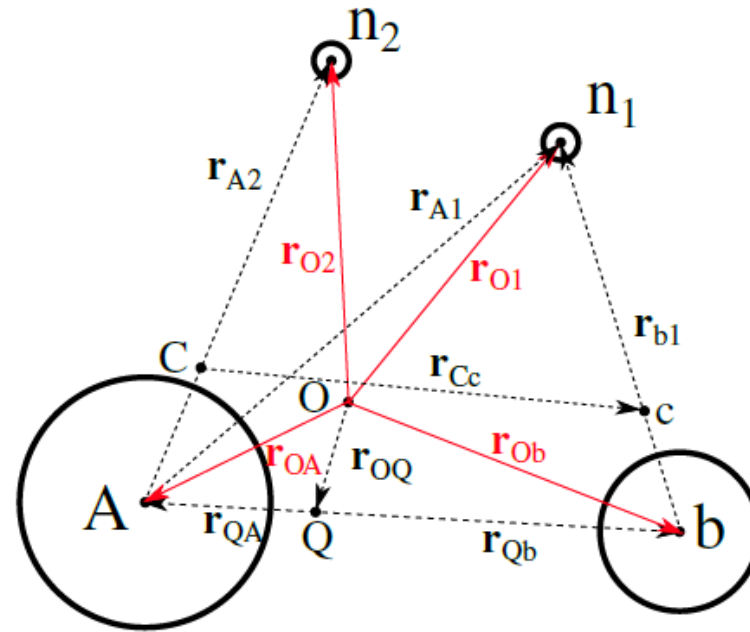


Second order DWBA: sequential transfer

$$T(\mathbf{k}_f, \mathbf{k}_i) = 2 \sum_{KM} \sum_{j_i, j_f} B_{j_f}^{(A)*} B_{j_i}^{(b)} \int \chi_f^*(\mathbf{r}_{Bb}, \mathbf{k}_f) \left[\phi_{j_f}^{(A)}(\mathbf{r}_{A_2}) \phi_{j_f}^{(A)}(\mathbf{r}_{A_1}) \right]_0^{0*} U^{(A)}(r_{b1}) \left[\phi_{j_f}^{(A)}(\mathbf{r}_{A_2}) \phi_{j_i}^{(b)}(\mathbf{r}_{b1}) \right]_M^K d\mathbf{r}_{Cc} d\mathbf{r}_{b1} d\mathbf{r}_{A_2} \\ \times \int G(\mathbf{r}_{Cc}, \mathbf{r}'_{Cc}) \left[\phi_{j_f}^{(A)}(\mathbf{r}'_{A_2}) \phi_{j_i}^{(b)}(\mathbf{r}'_{b1}) \right]_M^{K*} U^{(A)}(r'_{b2}) \left[\phi_{j_i}^{(b)}(\mathbf{r}'_{b2}) \phi_{j_i}^{(b)}(\mathbf{r}'_{b1}) \right]_0^0 \chi_i(\mathbf{r}'_{Aa}, \mathbf{k}_i) d\mathbf{r}'_{Cc} d\mathbf{r}'_{b1} d\mathbf{r}'_{A_2},$$

$$B_j = \sqrt{\frac{(2j+1)}{2}} U'_j V'_j,$$

$$|\Psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k c_k^\dagger c_{\bar{k}}^\dagger) |0\rangle$$



Nuclear Superfluidity: Sn(p,t): Occupations

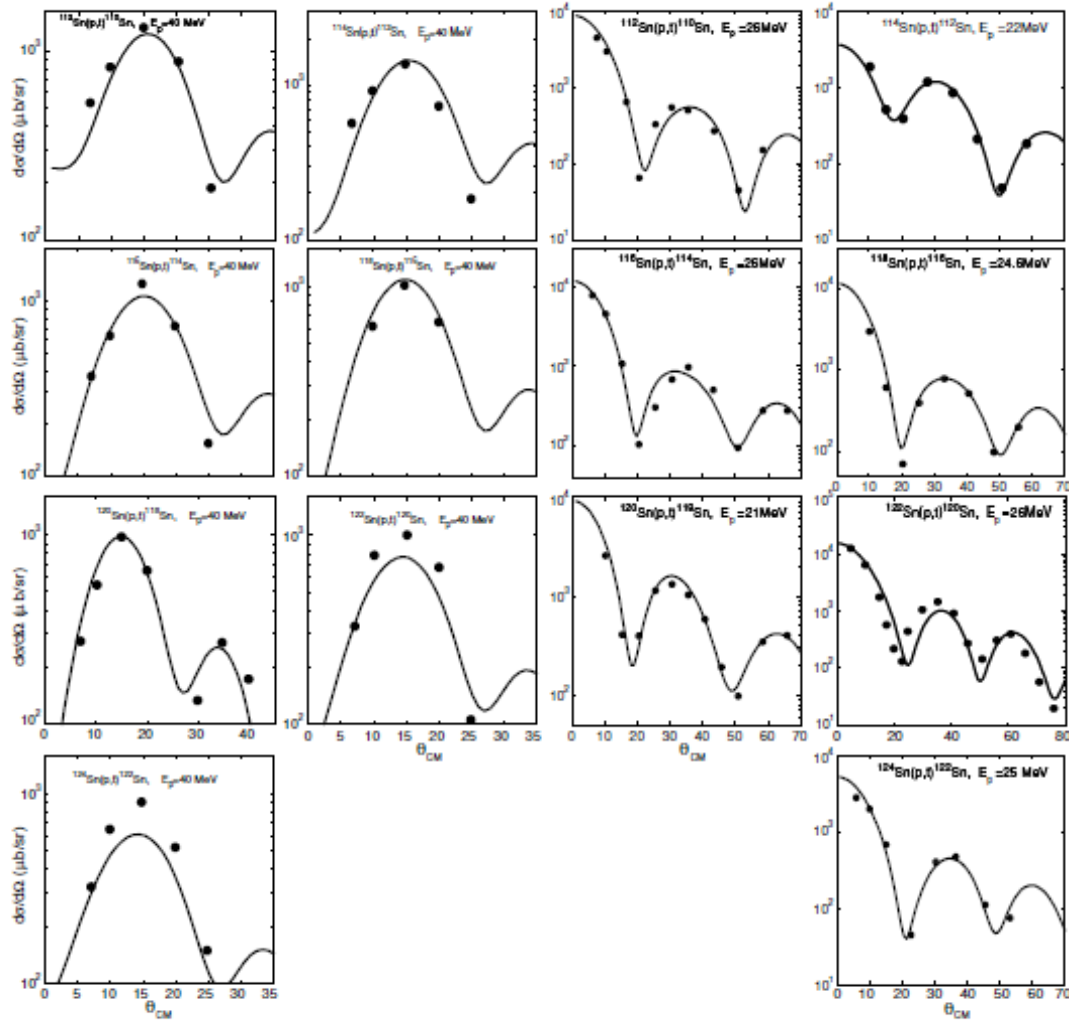


FIG. 7: Predicted absolute differential $A+2\text{Sn}(p,t)^A\text{Sn}(gs)$ cross sections for bombarding energies 21 MeV $\leq E_p \leq 26$ MeV, and $E_p = 40$ MeV in comparison with the experimental data (see [39–44] and [45] respectively).

Sn: 112 to 124

DWBA

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} |T^{(1)} + T_{succ}^{(2)} - T_{NO}^{(2)}|^2$$

$$T^{(1)} = 2 \sum_{l_i, j_i} \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{tA} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB})$$

$$\times v(\mathbf{r}_{p1}) \phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) \chi_{tA}^{(+)}(\mathbf{r}_{tA}).$$

$$T_{succ}^{(2)} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\sigma_1 \sigma_2} \sum_{\sigma'_1 \sigma'_2} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1})$$

$$\times \phi_d(\mathbf{r}_{p1}) \varphi_{l_f, j_f, m_f}^{A+1}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{dF} d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} G(\mathbf{r}_{dF}, \mathbf{r}'_{dF})$$

$$\times \phi_d(\mathbf{r}'_{p1})^* \varphi_{l_f, j_f, m_f}^{A+1*}(\mathbf{r}'_{A2}) \frac{2\mu_{dF}}{\hbar^2} v(\mathbf{r}'_{p2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi_{tA}^{(+)}(\mathbf{r}'_{tA})$$

$$T_{NO}^{(2)} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\sigma_1 \sigma_2} \sum_{\sigma'_1 \sigma'_2} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1})$$

$$\times \phi_d(\mathbf{r}_{p1}) \varphi_{l_f, j_f, m_f}^{A+1}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} d\mathbf{r}'_{dF}$$

$$\times \phi_d(\mathbf{r}'_{p1})^* \varphi_{l_f, j_f, m_f}^{A+1*}(\mathbf{r}'_{A2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi_{tA}^{(+)}(\mathbf{r}'_{tA})$$

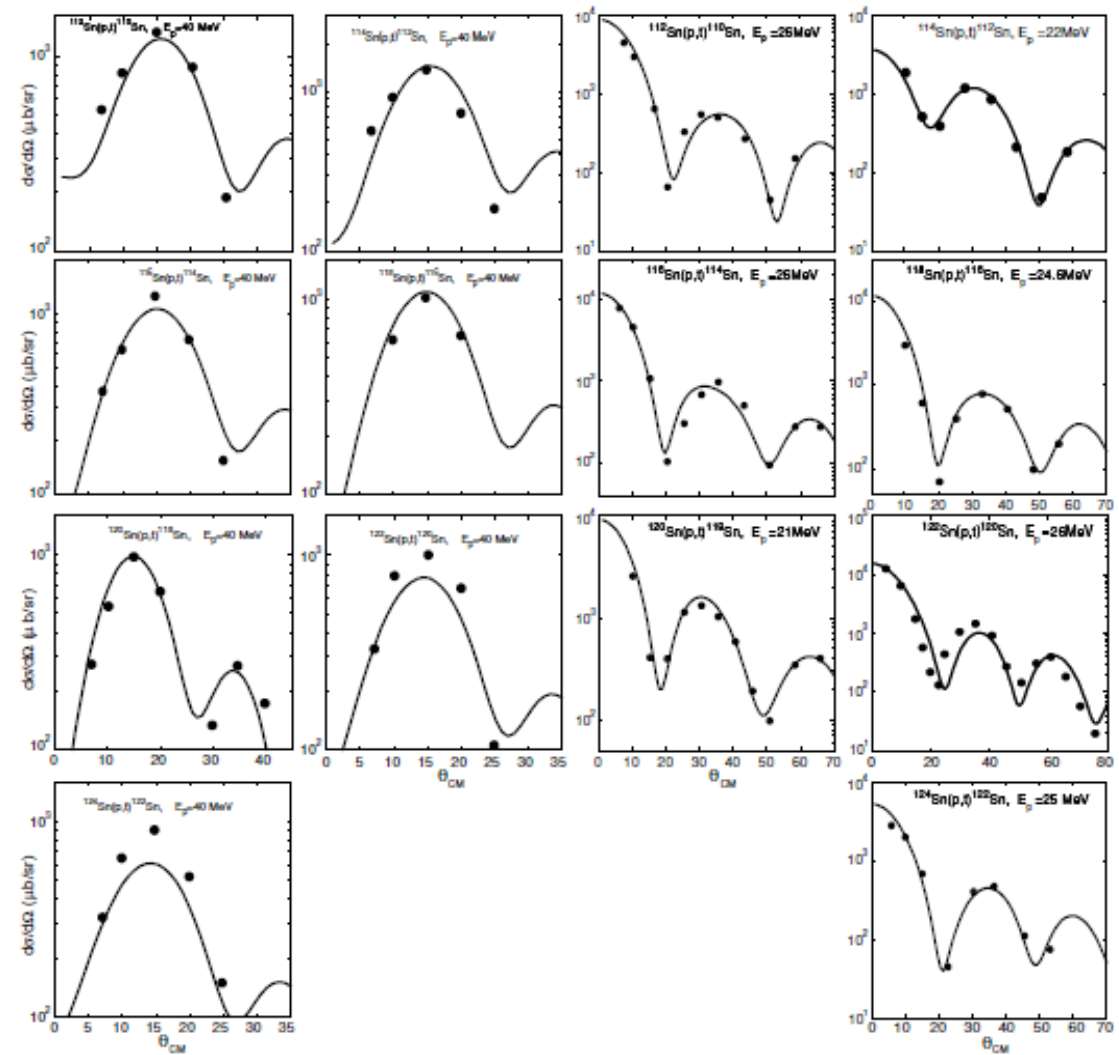
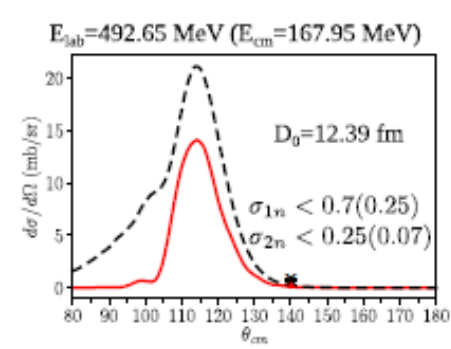
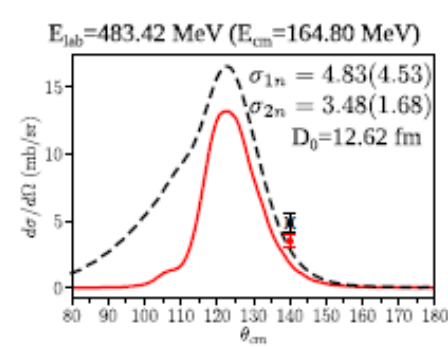
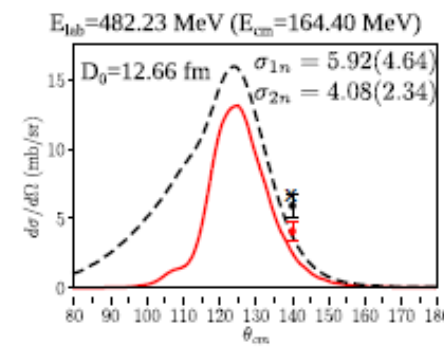
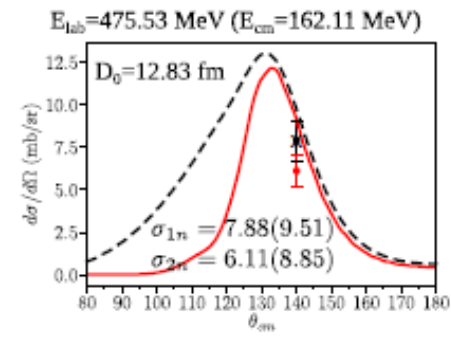
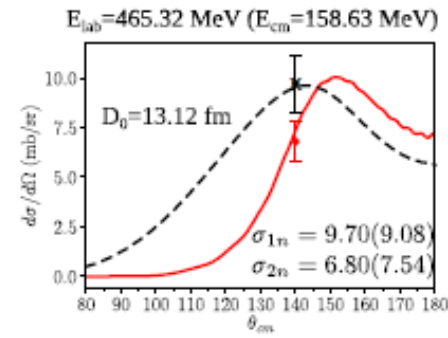
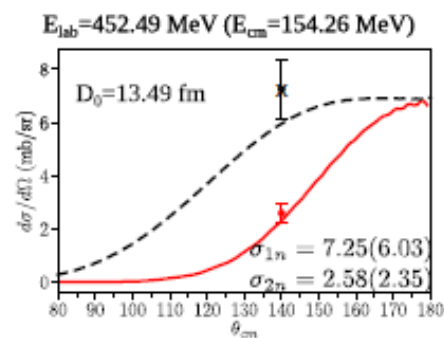
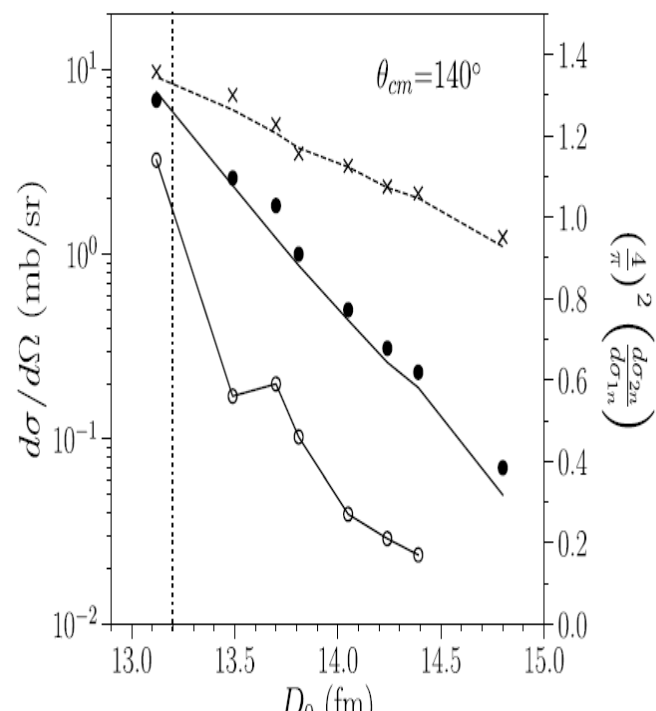
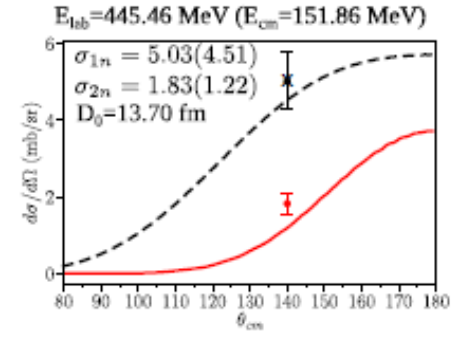
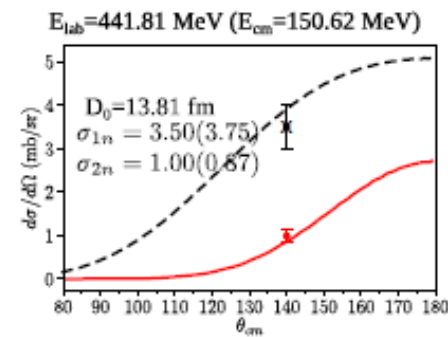
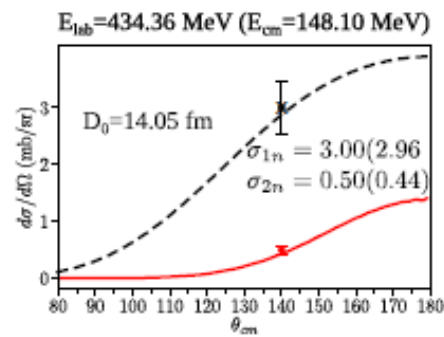
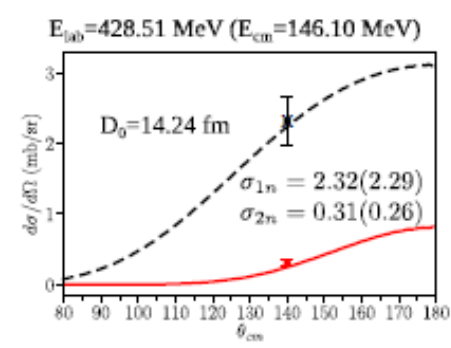
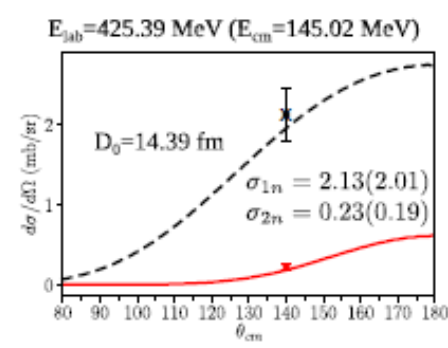
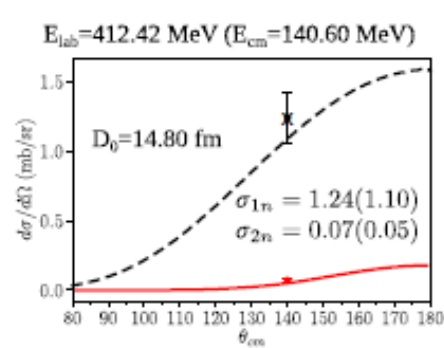
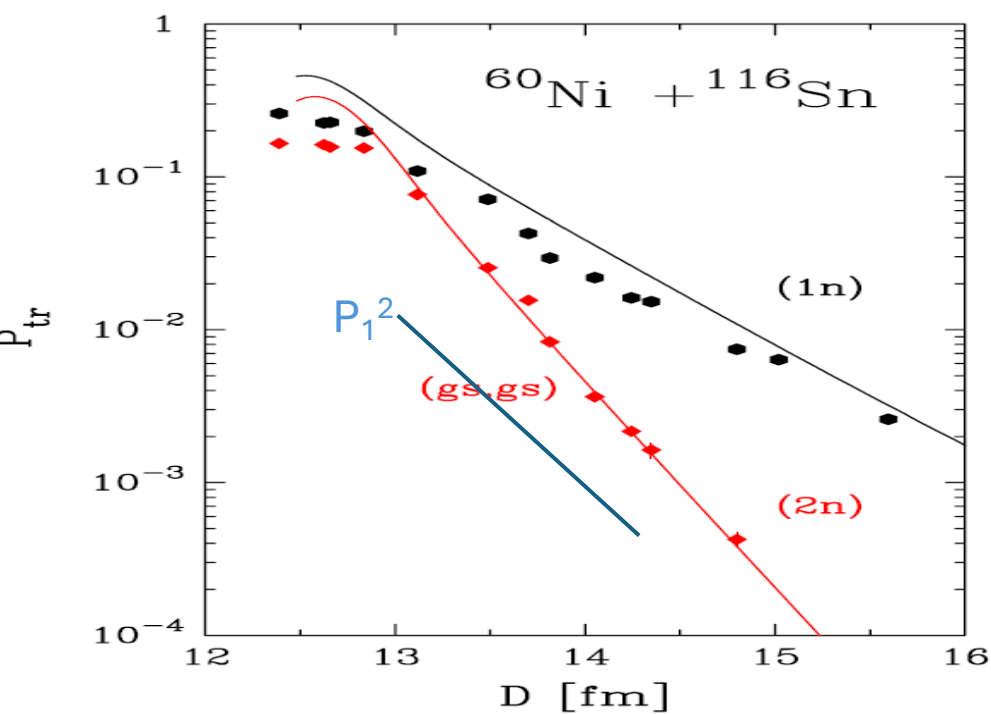
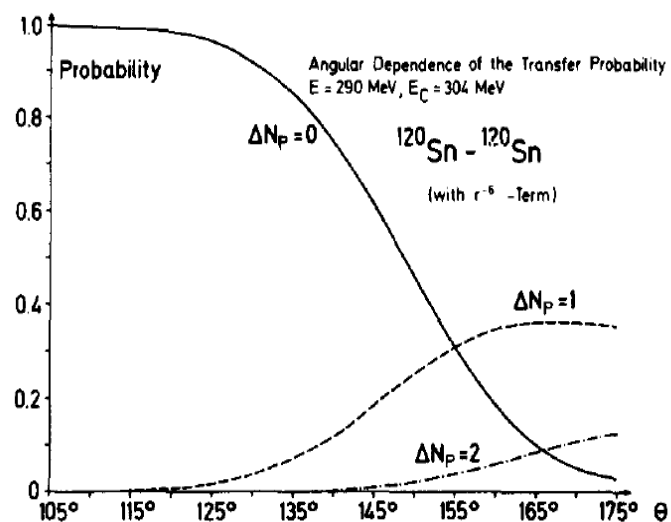
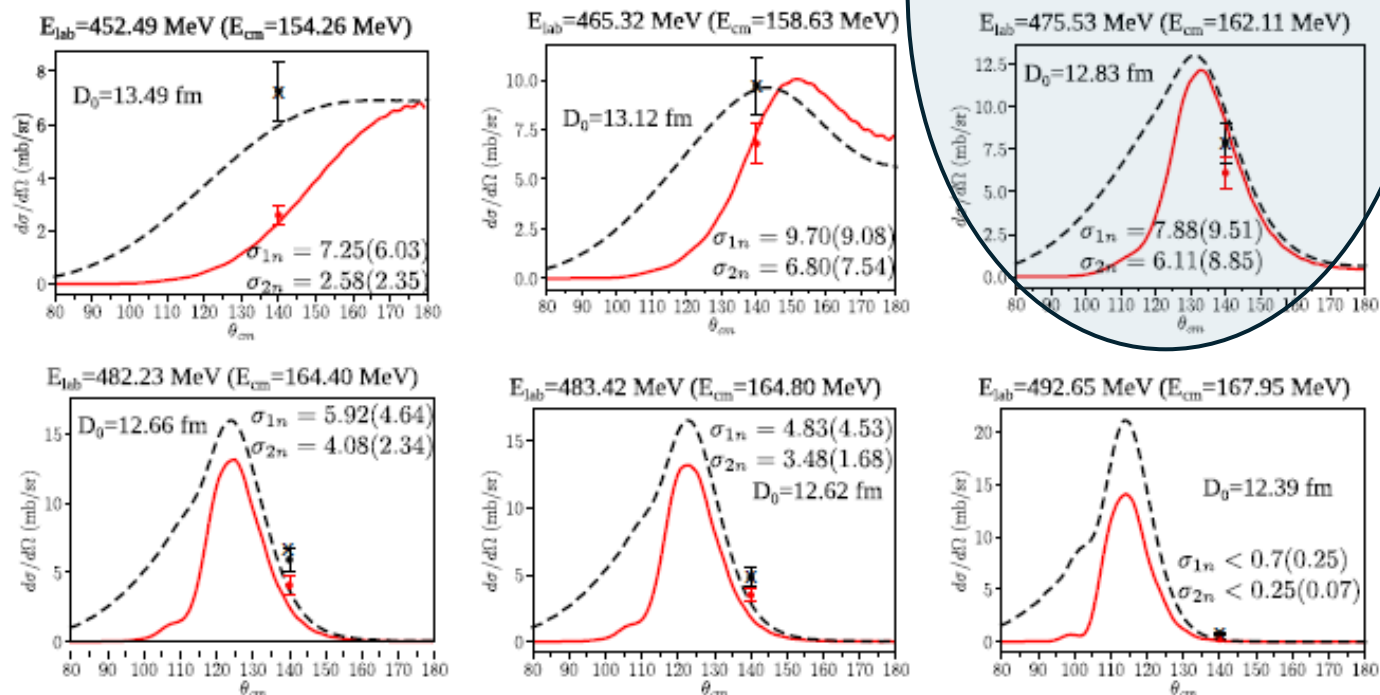
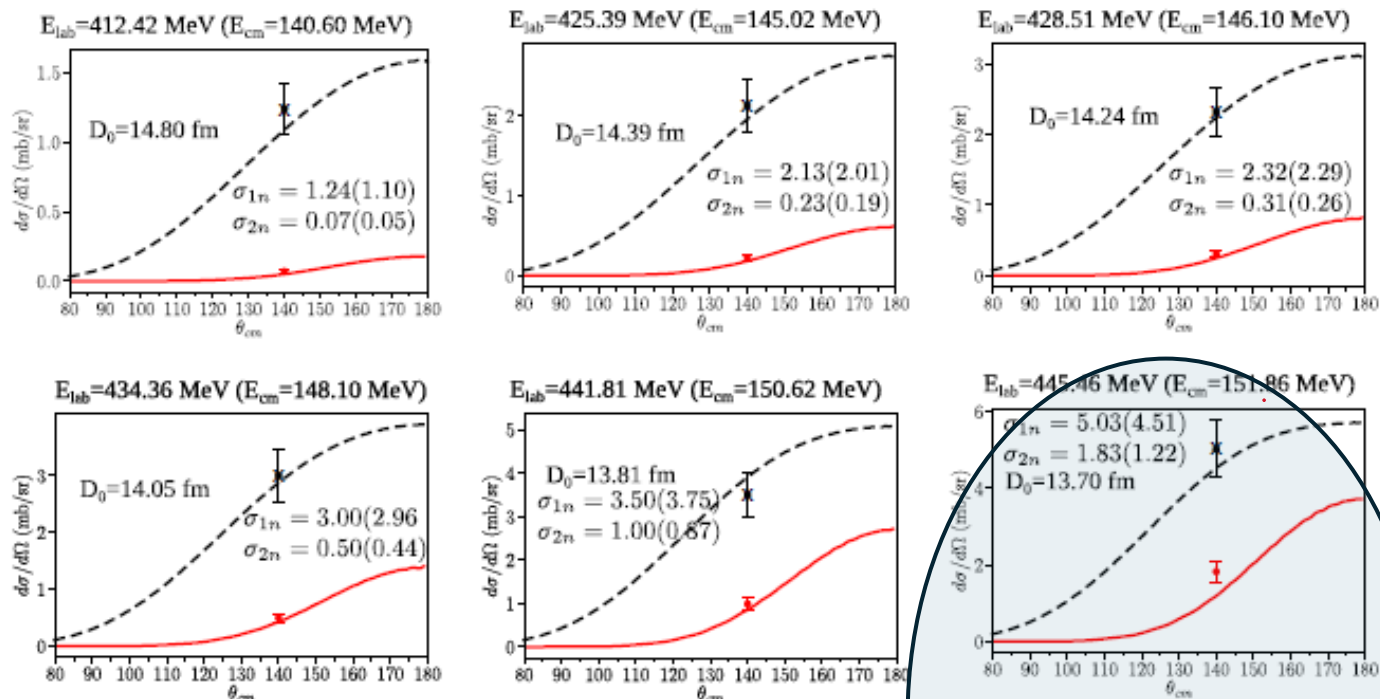


FIG. 7: Predicted absolute differential $^{A+2}\text{Sn}(p,t)^A\text{Sn}(gs)$ cross sections for bombarding energies $21 \text{ MeV} \leq E_p \leq 26 \text{ MeV}$, and $E_p = 40 \text{ MeV}$ in comparison with the experimental data (see [39–44] and [45] respectively).



Fig.4. Scattering of ^{120}Sn on ^{120}Sn . transfer probabilityFig.5. Same as 4, but for different energy $E = 300 \text{ MeV}$:
 $p_{3,k}(\infty; 300 \text{ MeV}, \theta)$

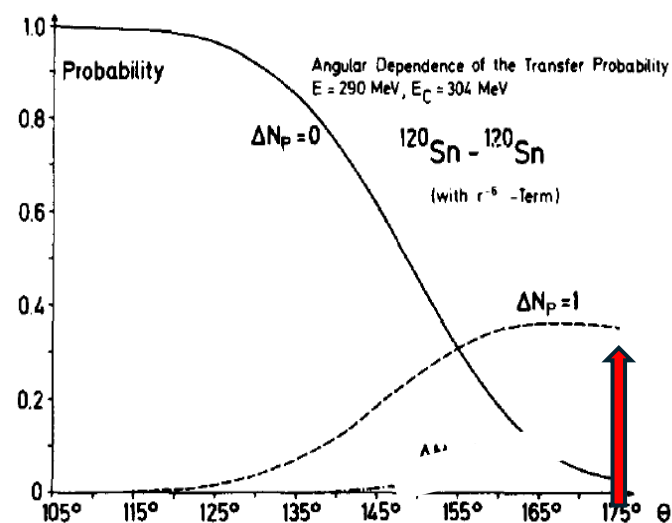


Fig.4. Scattering of ^{120}Sn on ^{120}Sn . transfer probability
 i

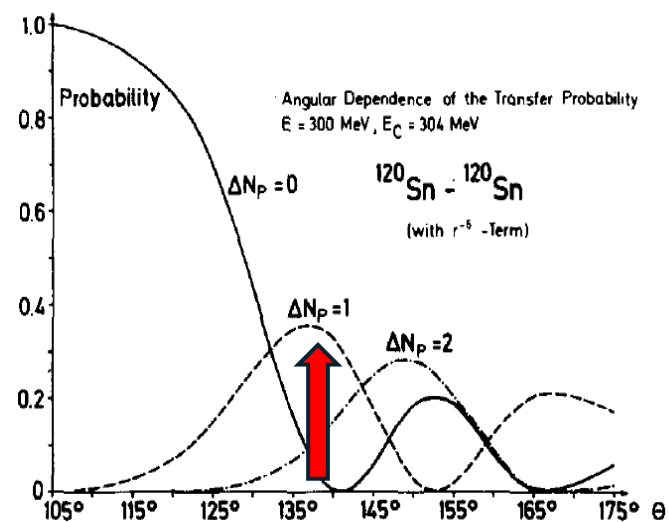
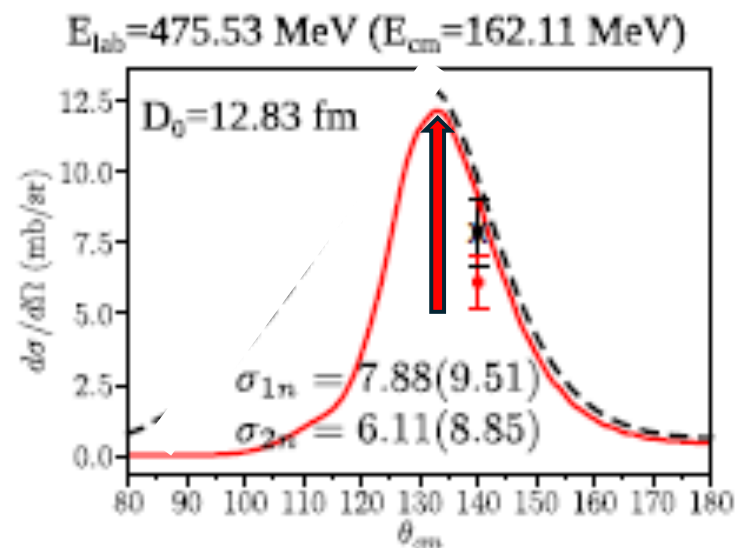
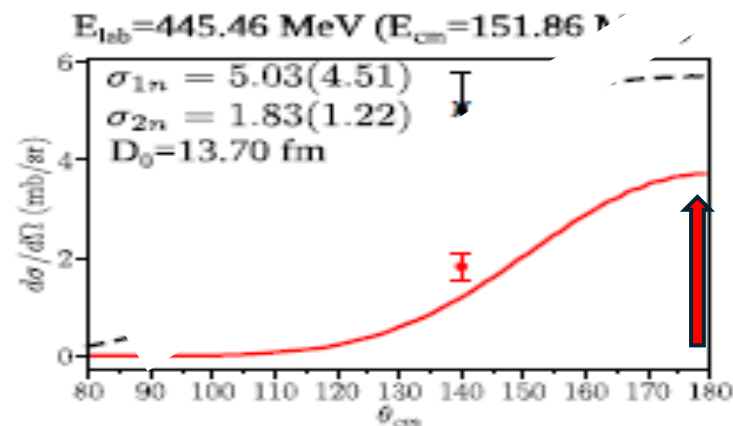
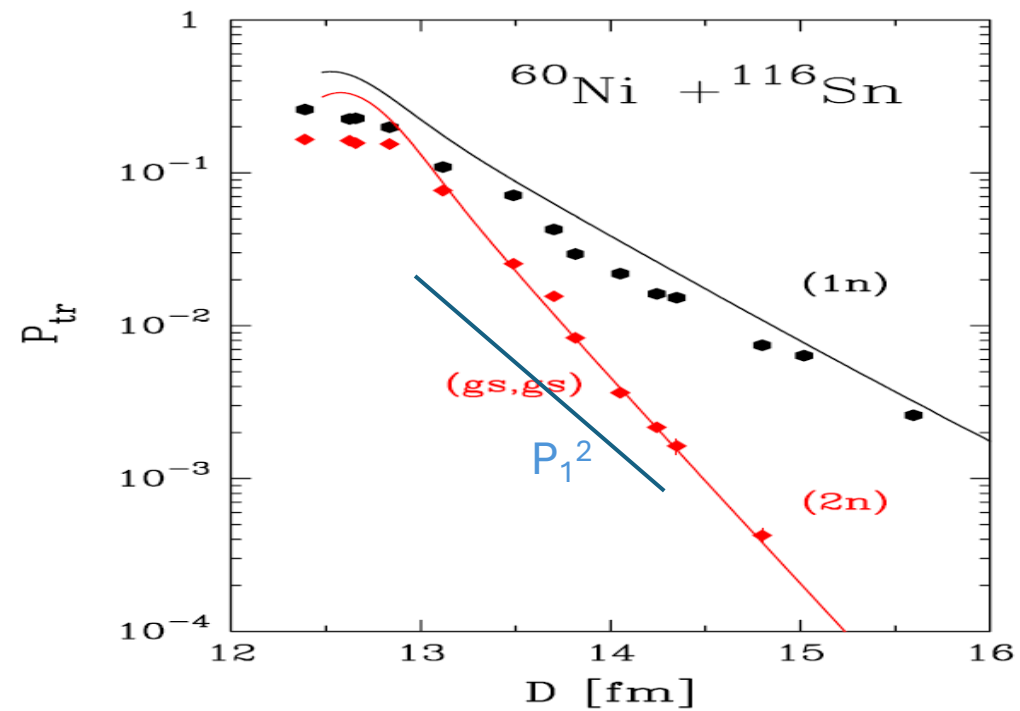
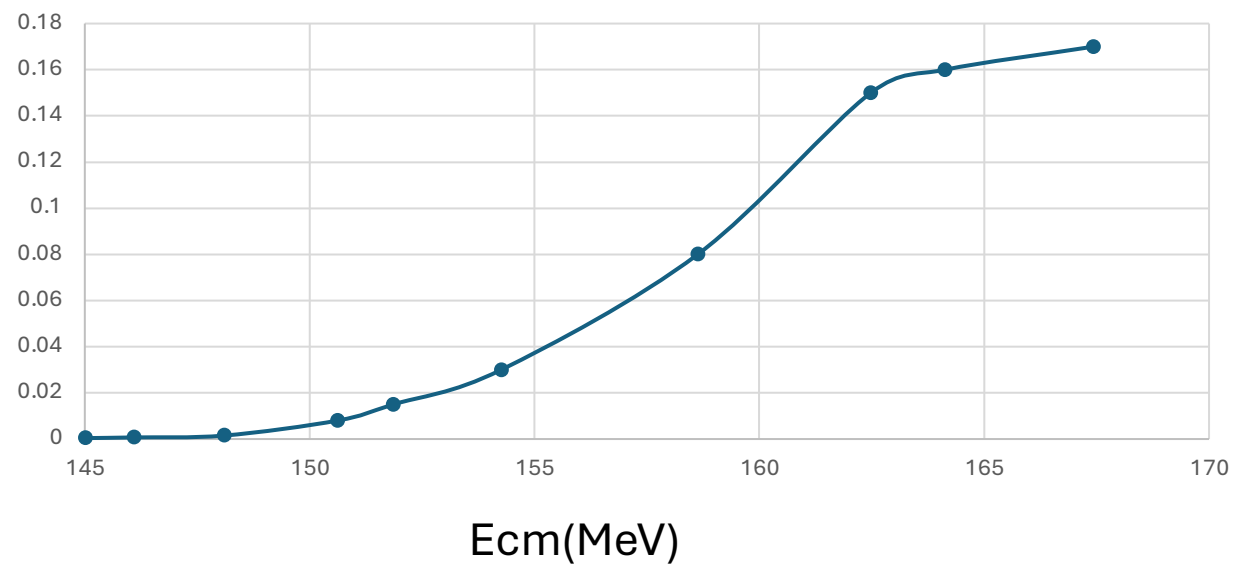
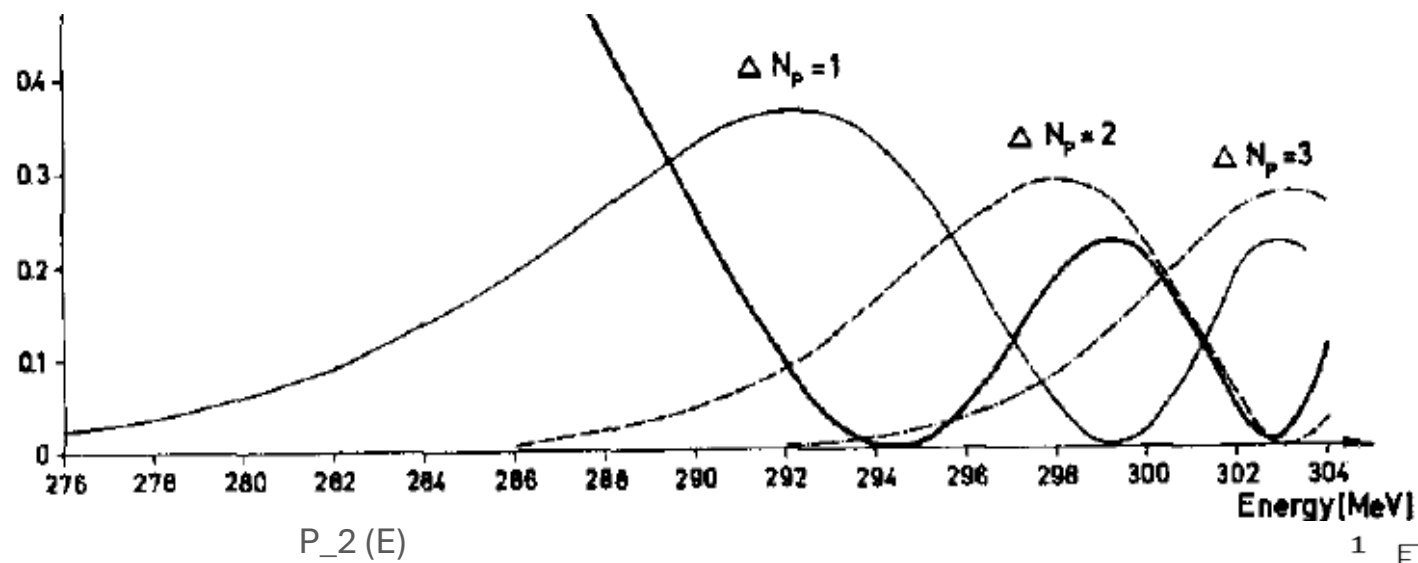
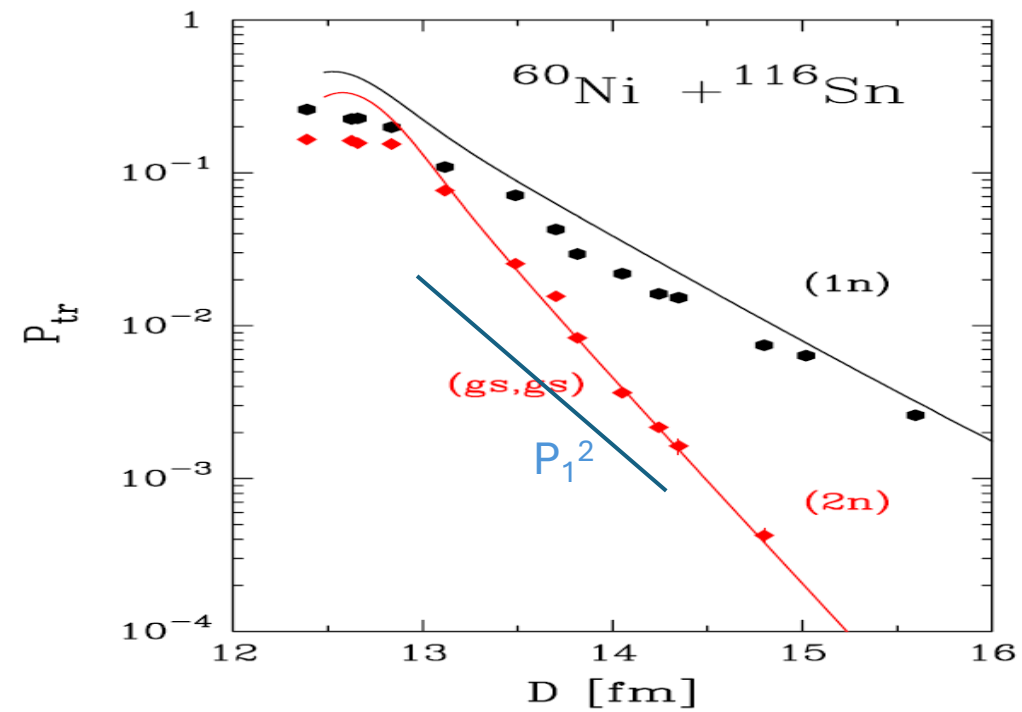
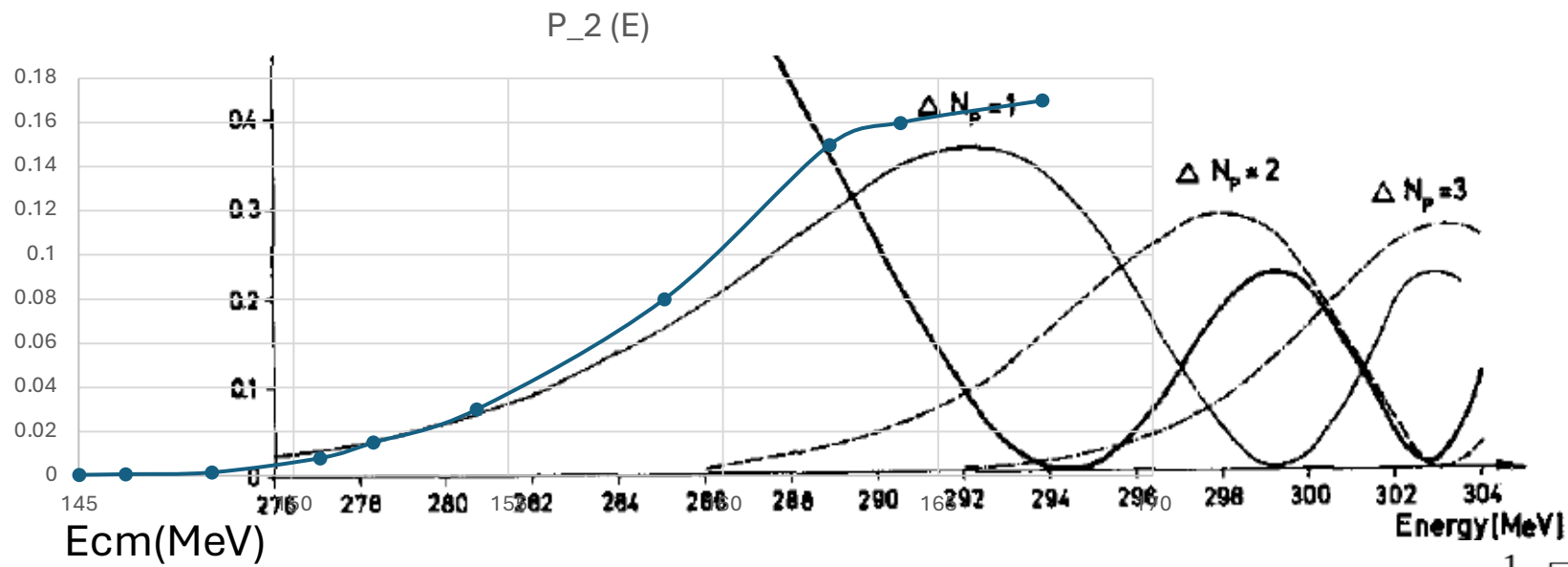


Fig.5. Same as 4, but for different energy $E = 300 \text{ MeV}$:
 $p_{3,k}(\infty; 300 \text{ MeV}, \theta)$



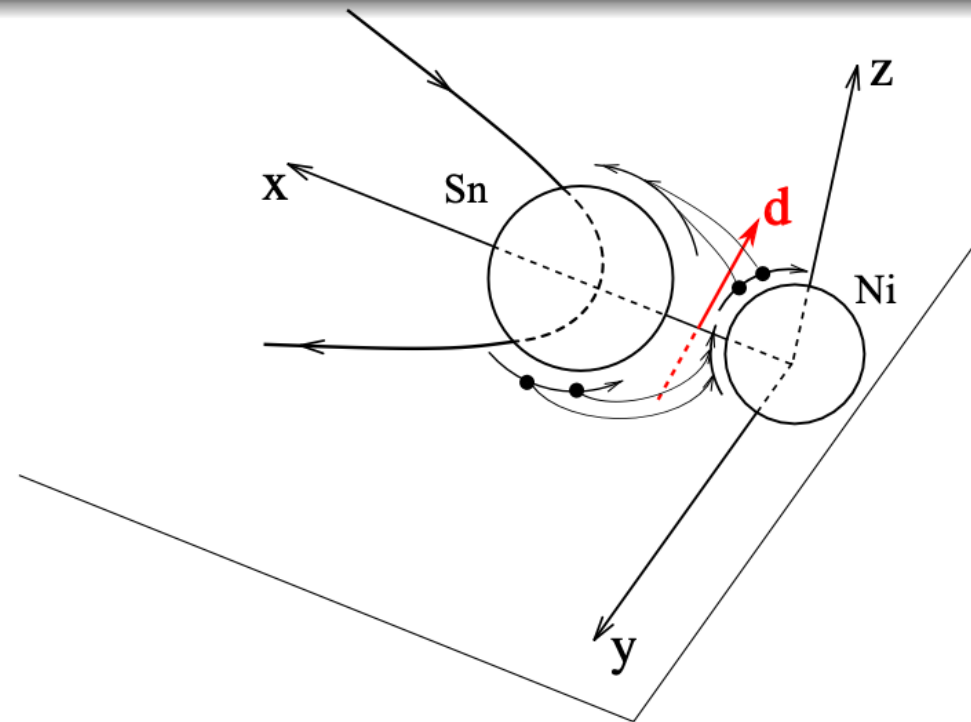




Predicted Josephson-like Gamma Emission in Transfer Reactions

(10m)

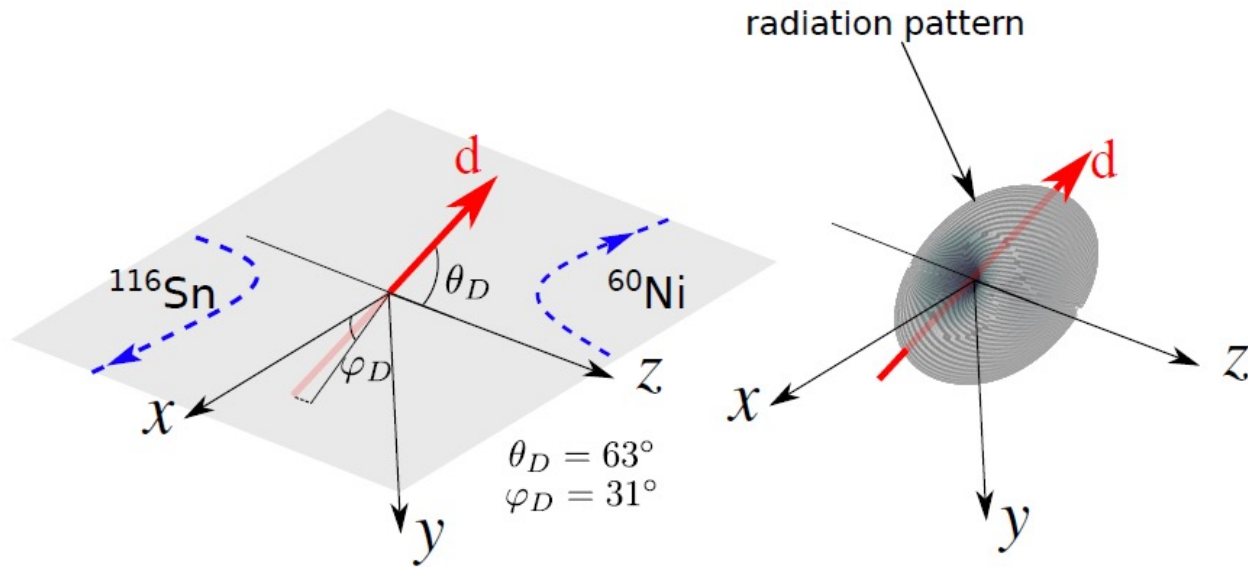
Charge rearrangement/recoil during the 2n-transfer



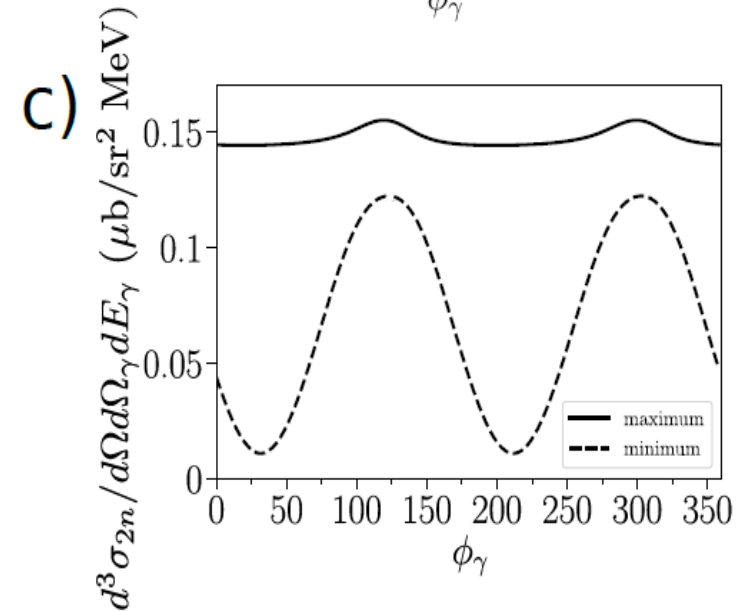
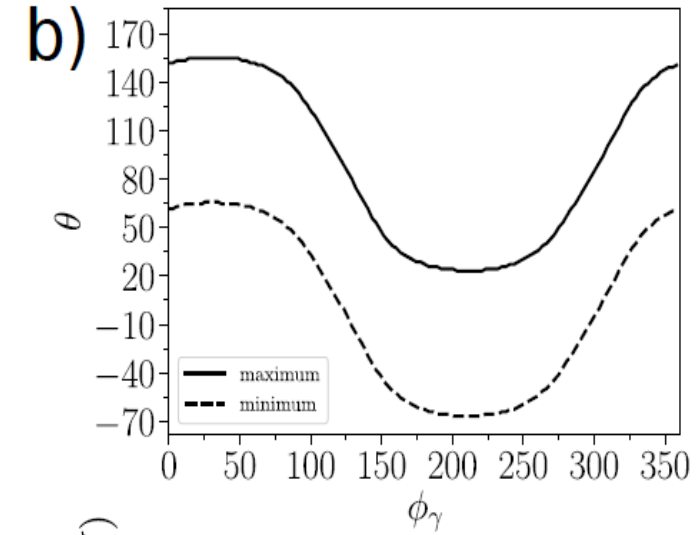
- Neutrons have effective charge
 $e_{eff} = -e(Z_1 + Z_2)/(A_1 + A_2) = -0.44e$.
- An oscillating^(*) electric dipole $\vec{d}$ is induced during the 2n transfer process.
- The dipole emits electromagnetic radiation like an antenna during a very short time $\tau_{coll} \sim 10^{-21}$ s.

(*) Off diagonal

Gamma emission: predicted angular distribution

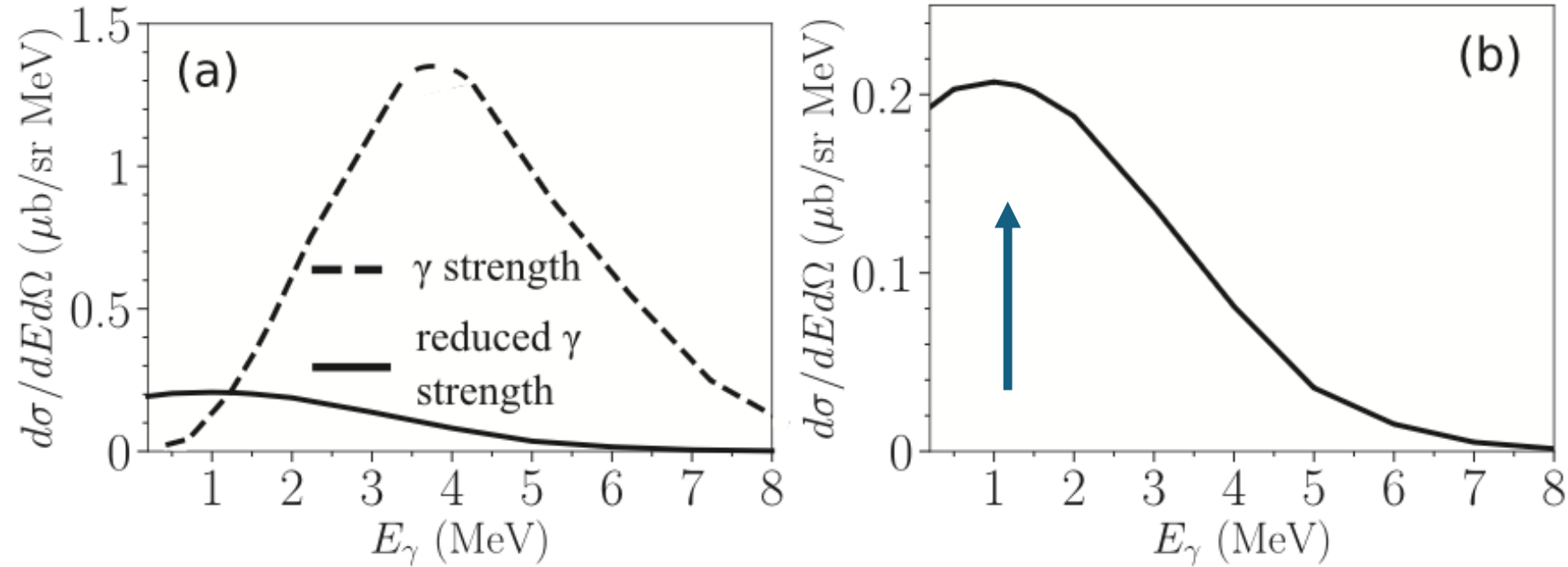


- The angular radiation pattern reflects the orientation of the dipole, providing novel insight into the reaction mechanism.
- The polarization of the emitted photons can also provide valuable information.



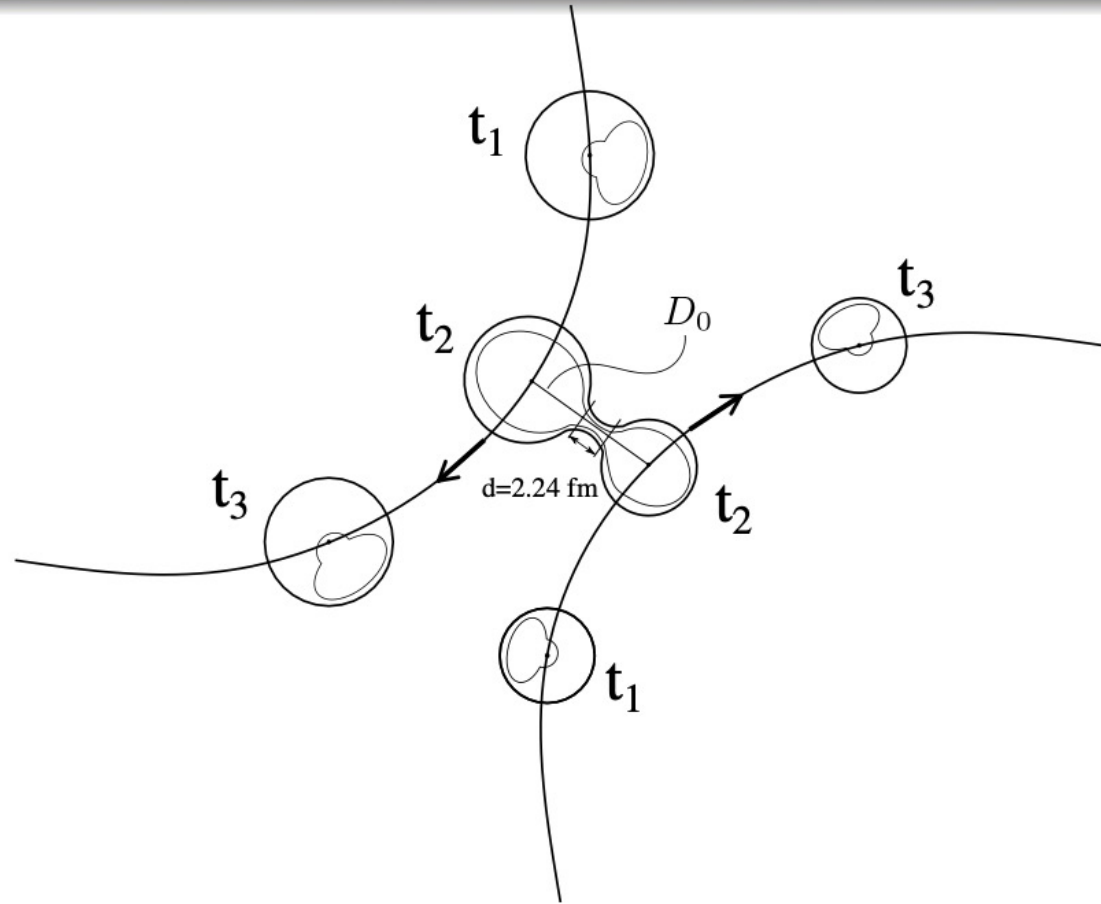
$$\frac{d^2\sigma}{d\Omega dE_\gamma} = \left(\frac{\mu_i \mu_f}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} \right) \left(\frac{8\pi}{3} \frac{E_\gamma^2}{(\hbar c)^3} \right) |T_{m_\gamma}(\mathbf{k}_f, \mathbf{k}_i)|^2 \\ \times \delta[E_\gamma + E_f - (E_i + Q)],$$

$$E_\gamma^{\max} = Q_{2n} = 1.3 \text{ MeV}$$



G.Potel et al,
 PRC 103 L021601 (2021)
 PRC 105 L061602 (2022)
 arXiv:2206.0531

Transient Josephson junction



transient Josephson junction of width D_0

A Josephson junction of width D_0 between the two superfluid nuclei is established during the very short collision time $\tau_{coll} \approx t_3 - t_1 \approx 10^{-21} \text{ s}$.

① centroid: 4 MeV

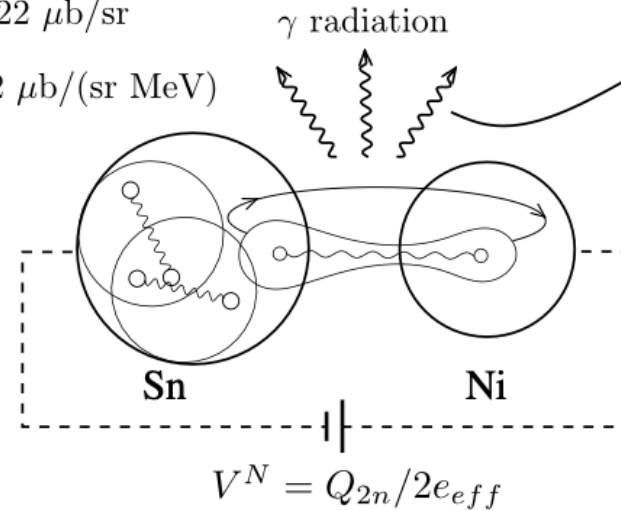
② width: 4 MeV

③ integral: 5.22 $\mu\text{b}/\text{sr}$

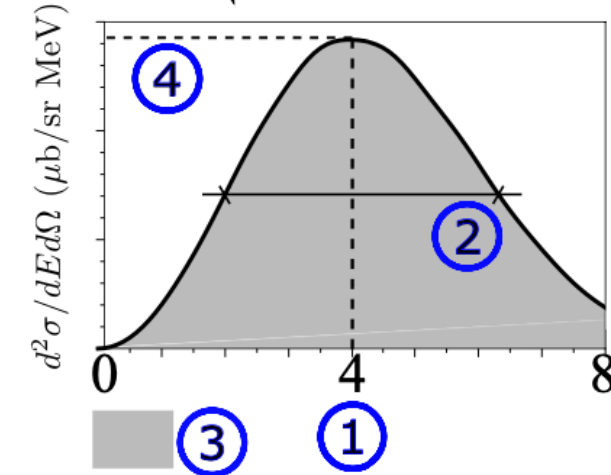
④ max. : 1.42 $\mu\text{b}/(\text{sr MeV})$

Quantum entanglement in nuclear Cooper-pair tunneling with γ rays

G. Potel¹, F. Barranco², E. Vigezzi³, and R. A. Broglia^{4,5}



nuclear superconducting “circuit”



calculated γ -strength function

- The Q -value of the reaction acts as a “battery”, providing an equivalent potential V^N .
- The finite collision time, as well as recoil effects, provides a width ($\Delta E \sim \hbar/\tau_{coll}$).

Original proposal approved by
the LNL PAC → experiment
performed in 2023

PIAVE-ALPI ACCELERATOR

Search for a Josephson-like effect in the $^{116}\text{Sn}+^{60}\text{Ni}$ system

PRISMA + AGATA experiment

Spokesperson(s): L. Corradi, S. Szilner

(GLIMOS: L. Corradi)

L. Corradi¹, E. Fioretto¹, F. Galtarossa¹, A. Goasduff¹, A. Gottardo¹, A. M. Stefanini¹, J. J. Valiente-Dobón¹, M. Balogh¹, D. Brugnara¹, G. de Angelis¹, A. Ertoprak¹, B. Gongora¹, T. Marchi¹, D.R. Napoli¹, J. Pellumaj¹, R.M. Pérez-Vidal¹, M. Sedlak¹, L. Zago¹, I. Zanon¹, G. Montagnoli², D. Mengoni², M. del Fabbro², F. Scarlassara², P. Aguilera², D. Bazzacco², R. Escudeiro², S.M. Lenzi², R. Menegazzo², S. Pigliapoco², F. Recchia², K. Rezyunkina², G. Zhang², S. Szilner³, J. Diklič³, D. Jelavić Malenica³, T. Mijatović³, M. Milin⁴, G. Benzoni⁵, S. Bottoni^{6,5}, A. Bracco^{6,5}, F. Camera^{6,5}, F. Crespi^{6,5}, R. Depalo^{6,5}, E. Gamba^{6,5}, S. Leoni^{6,5}, B. Million⁵, O. Wieland⁵, M. Caamano⁷, Y. Ayyad⁷, F. Barranco⁸, G. Pollarolo⁹, G. Potel¹⁰, E. Vigezzi⁵, R. A. Broglia^{11,6}

¹ *Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Legnaro, Legnaro, Italy.*

² *Dipartimento di Fisica, Università di Padova, and Istituto Nazionale di Fisica Nucleare, Padova, Italy*

³ *Ruder Bošković Institute, Croatia*

⁴ *University of Zagreb, Croatia*

⁵ *Istituto Nazionale di Fisica Nucleare, Milano, Italy*

⁶ *Dipartimento di Fisica, Università degli Studi di Milano*

⁷ *Universidade de Santiago de Compostela, Spain*

⁸ *Departamento de Física Aplicada III, Universidad de Sevilla, Spain*

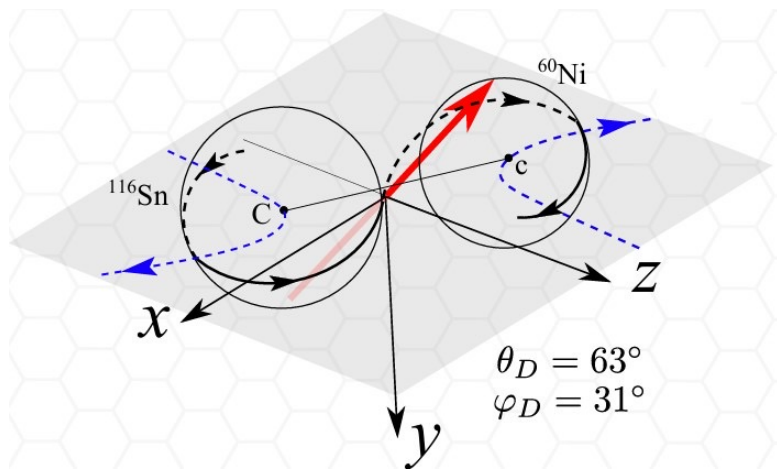
⁹ *Dip. di Fisica Teorica, Università di Torino, and Istituto Nazionale di Fisica Nucleare, Torino, Italy*

¹⁰ *Lawrence Livermore National Laboratory, Livermore, California, USA*

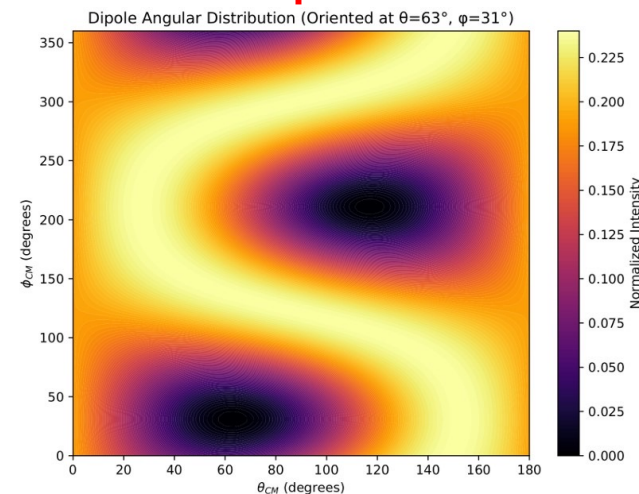
¹¹ *The Niels Bohr Institute, University of Copenhagen, Denmark*

Josephson dipole gamma ray angular distributions

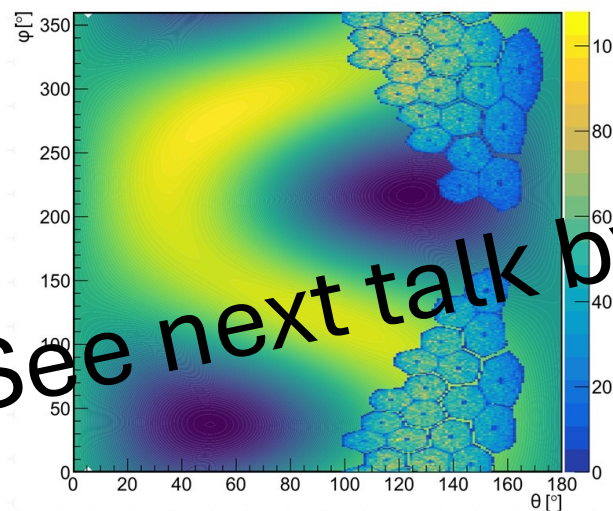
scheme of the reaction



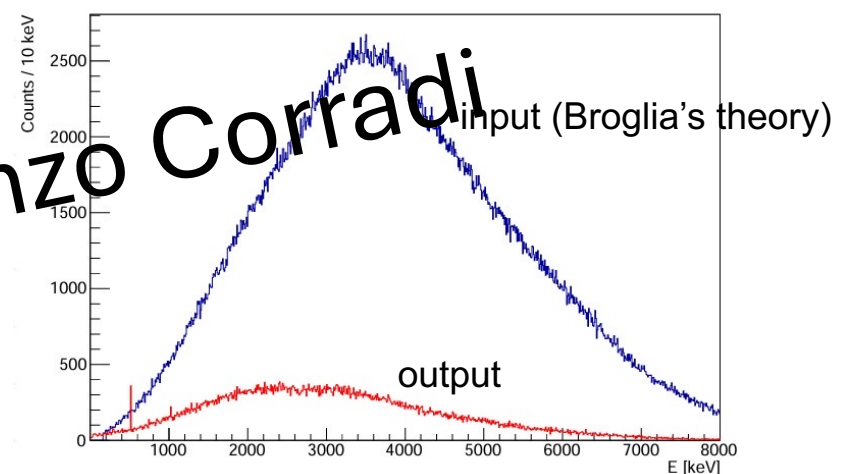
theoretical predictions



overlap with AGATA geometry

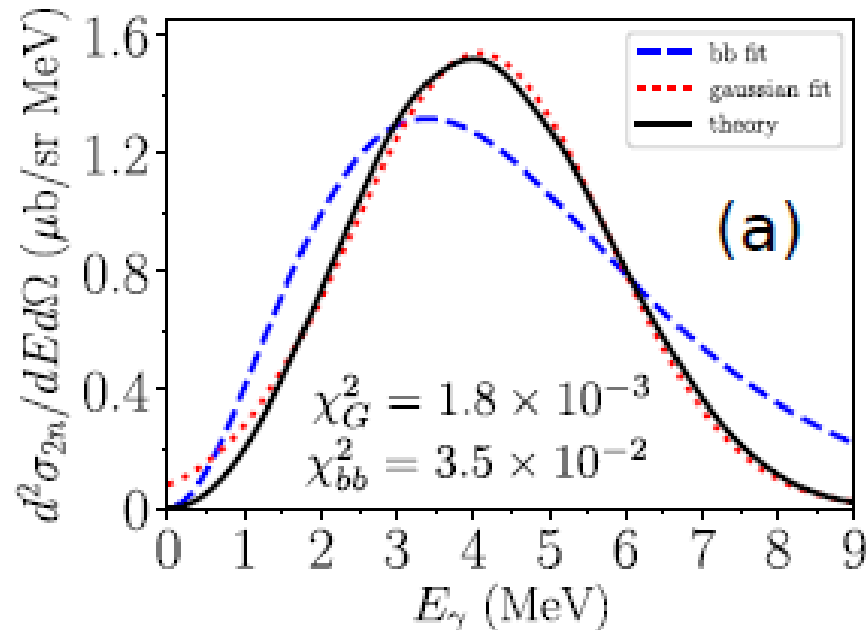


simulations



Gamma strength: 2NT versus 1NT channel

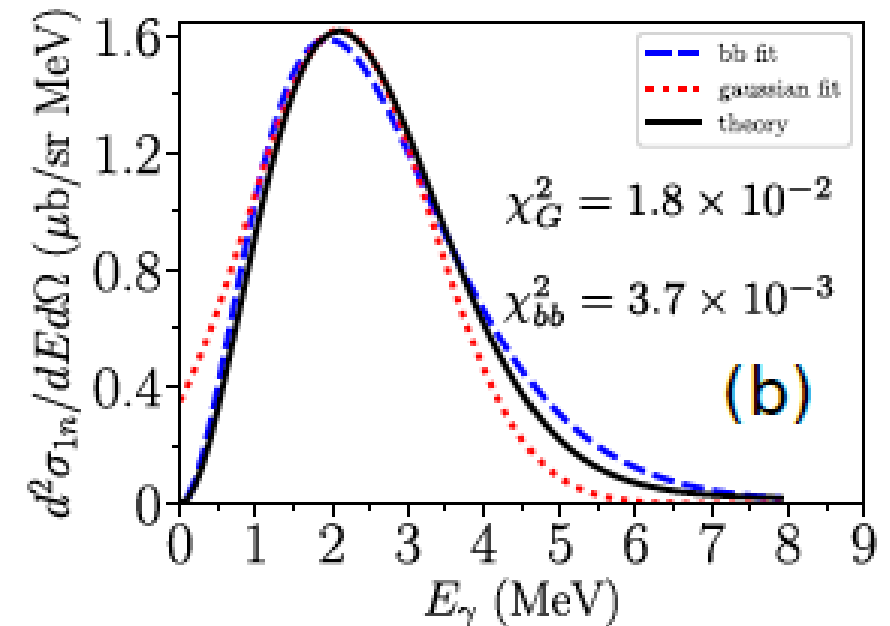
2n-transfer



Gaussian: $\sigma = 1.67$ MeV, $E_0 = 4.08$ MeV

black body: $T = 1.20$ MeV

1n-transfer



Gaussian: $\sigma = 1.2$ MeV, $E_0 = 2.1$ MeV

black body: $T = 0.69$ MeV

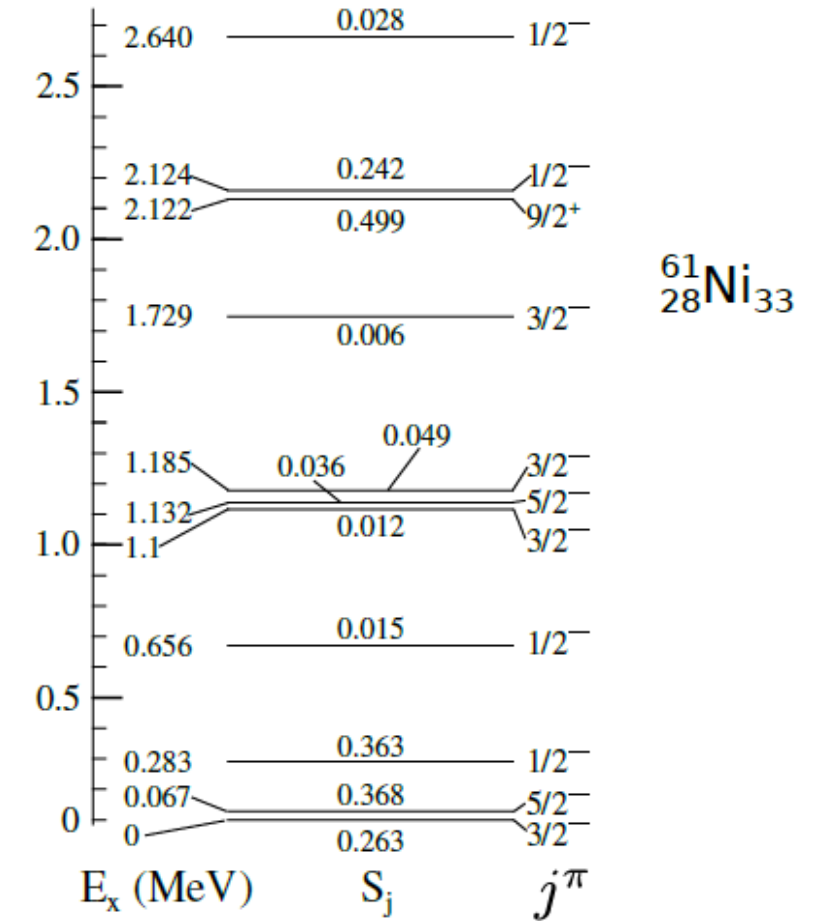
The Legnaro's $^{116}\text{Sn}+^{60}\text{Ni}$ 2NT and 1NT: BCS ground states and quasiparticles

	ϵ_{l_j}	E_{l_j}	$V_{l_j}^2$	B_{l_j}	E_{l_j}	$V_{l_j}^2$	B_{l_j}
$2d_{5/2}$	-11.51	2.35	0.874	0.658	2.35	0.874	0.575
$1g_{7/2}$	-10.86	1.92	0.790	0.910	1.92	0.789	0.816
$3s_{1/2}$	-9.70	1.56	0.487	0.483	1.57	0.484	0.500
$2d_{3/2}$	-9.51	1.58	0.428	0.661	1.58	0.424	0.699
$1h_{11/2}$	-8.12	2.25	0.140	0.770	2.26	0.139	0.847

Table 1: Properties of the five valence states calculated in ^{115}Sn , starting from the single-particle energies ϵ_{l_j} shown in [1]. The quasiparticle energies E_{l_j} , the occupation probabilities $V_{l_j}^2$ and the transfer spectroscopic amplitudes B_{l_j} shown in Table 1 of ref. [1] are shown in the second, third and fourth column. They are compared in the last three columns with the corresponding quantities recalculated here with a pairing coupling constant $G = 0.217$ MeV (25/A).

	ϵ_{l_j}	E_{l_j}	$U_{l_j}^2$	B_{l_j}	B_{l_j}
$2p_{3/2}$	-11.33	1.56	0.33	0.524	0.665
$2p_{1/2}$	-9.66	1.83	0.65	0.500	0.477
$1f_{5/2}$	-9.33	2.05	0.400	0.34	0.849
$1g_{9/2}$	-5.16	5.80	0.500	0.1	1.224

Table 2: Properties of the four valence states calculated in ^{61}Ni , starting from the single-particle energies ϵ_{l_j} shown in [1]. The quasiparticle energies E_{l_j} , the occupation probabilities $U_{l_j}^2$ and the transfer spectroscopic amplitudes B_{l_j} shown in Table 1 of ref. [1] are shown in the second, third and fourth column. The values of the spectroscopic are compared in the last column with the value obtained from the relation $B = UV\sqrt{j+1/2}$, using $V^2 = 1 - U^2$.



Other Predictions and Speculations

Measuring the Coherence Length : Velocity of the Transferred Cooper and (critical) Depairing velocity .

$$v_c = \frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$

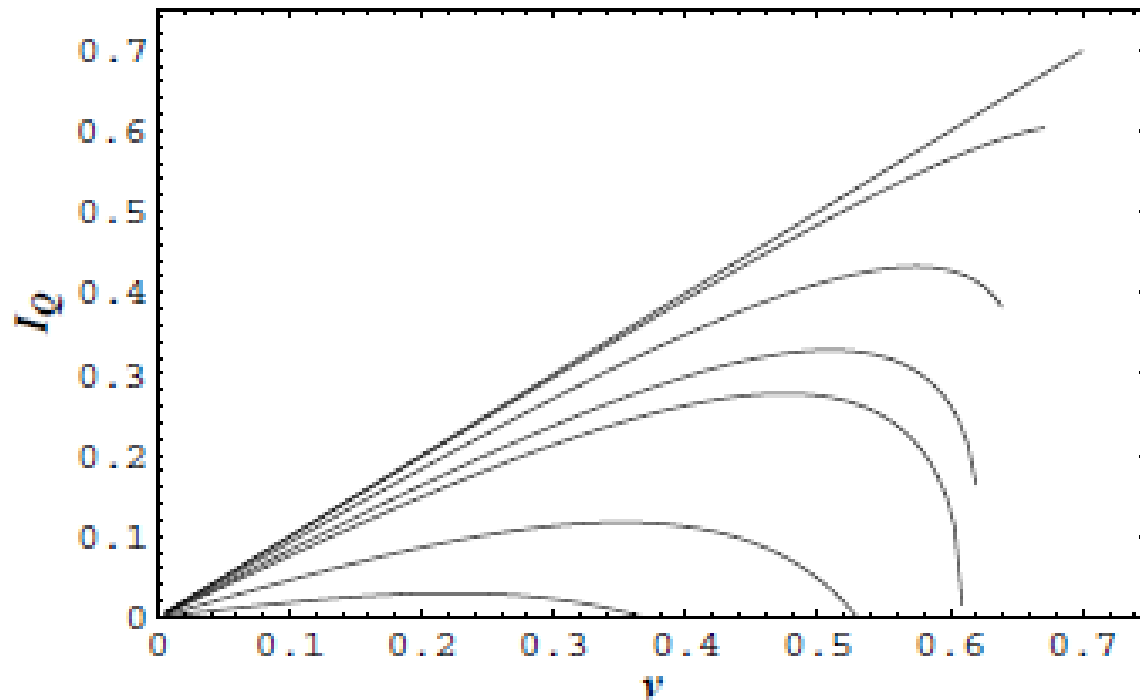


FIG. 5: Supercurrent I_Q (in units of I_Q^0) vs. superfluid velocity v (in unit of v_L) for various temperatures. From top to bottom: $T = (0.1, 0.25, 0.4, 0.5, 0.556, 0.75, 0.9) T_c^0$. The curves terminate at the critical velocities $v_c(T)$ appropriate to these temperatures. The maximum supercurrent for a particular curve determines the value of the critical current at that temperature.

Revisiting the critical velocity of a clean one-dimensional superconductor

Tzu-Chieh Wei

*Institute for Quantum Computing and Department of Physics and Astronomy,
University of Waterloo, Waterloo, ON N2L 3G1, Canada**

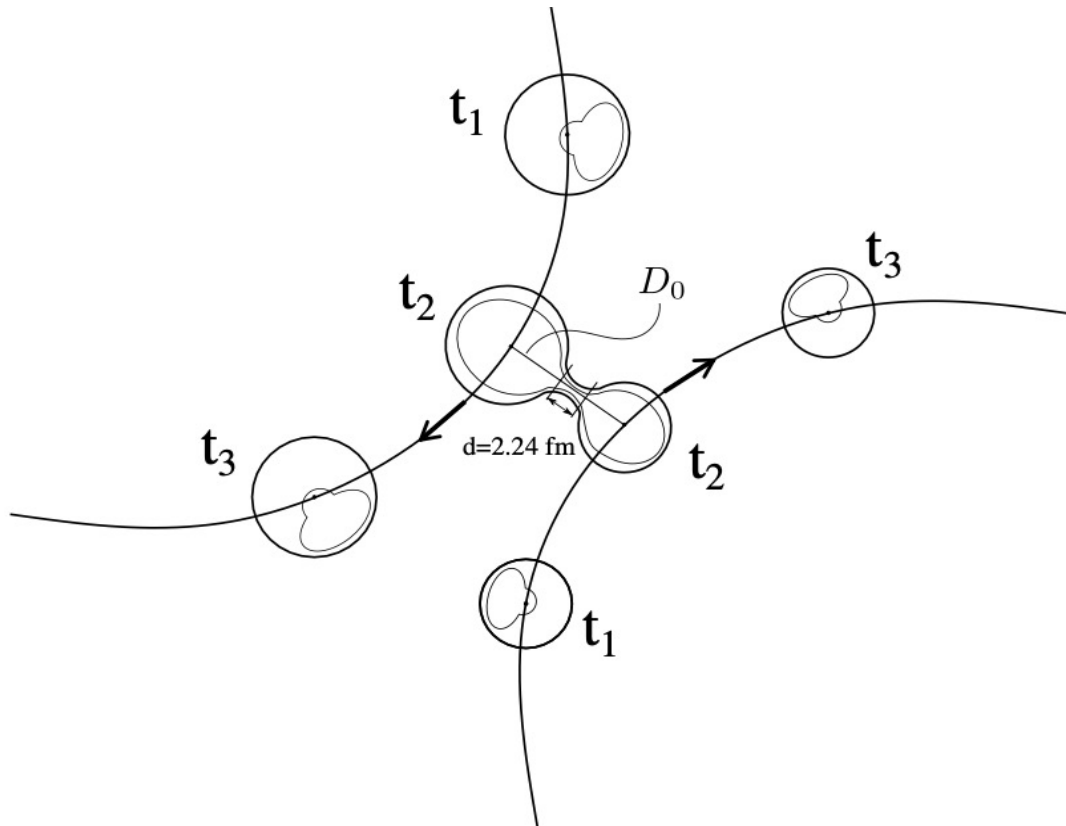
Paul M. Goldbart

*Department of Physics, Institute for Condensed Matter Theory,
and Frederick Seitz Materials Research Laboratory,
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, U.S.A.*

(Dated: April 15, 2009)

Critical depairing velocity

$$v_c = \frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$



$$v = \frac{(D_0) - (R_A + R_b + a)}{\tau_{coll}}$$

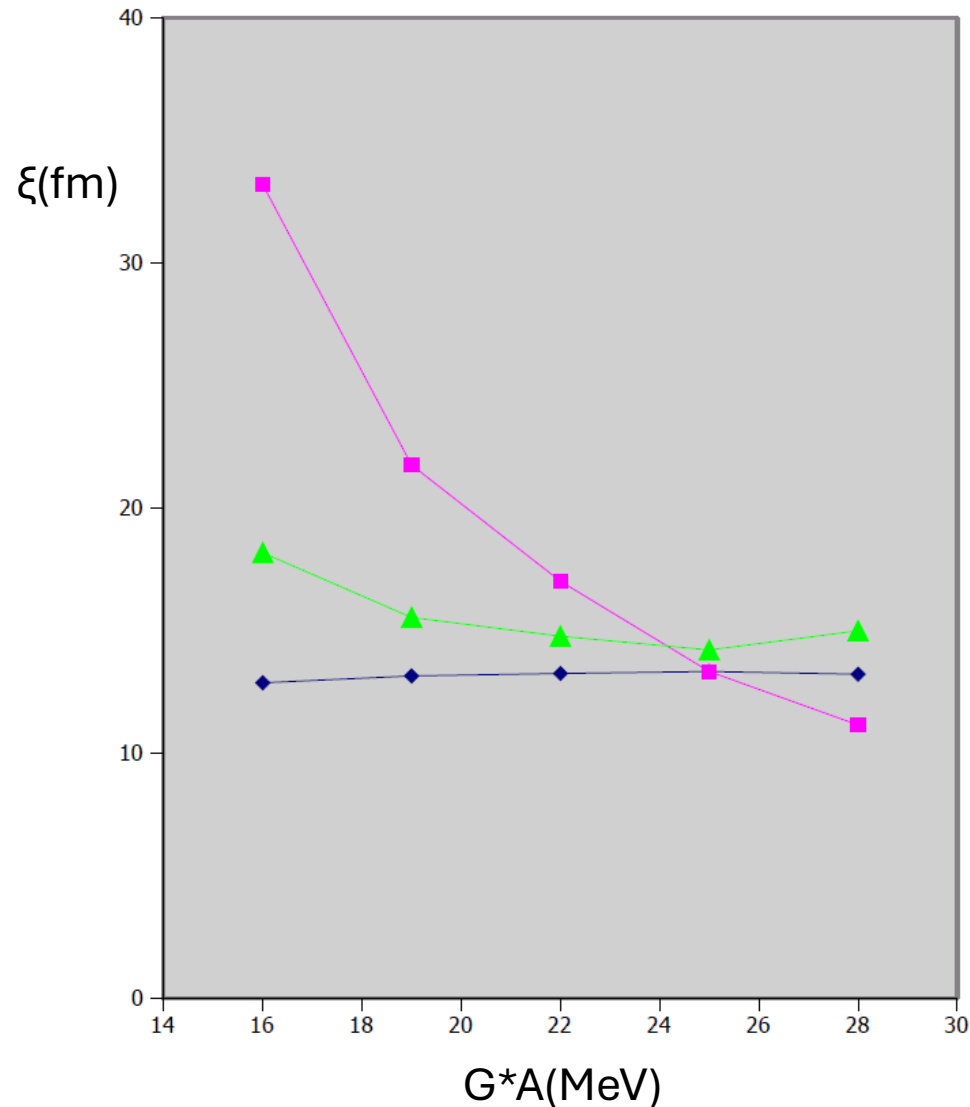
$$v < v_c \rightarrow P_2 > P_1$$

$$v = v_c \rightarrow P_2 \sim P_1$$

$$v > v_c \rightarrow P_2 < P_1$$

Velocity of the Transferred (nuclear) Cooper Pair.

Simulated data (DWBA calc's) as a function of pairing interaction.



Magenta curve

$$\frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$

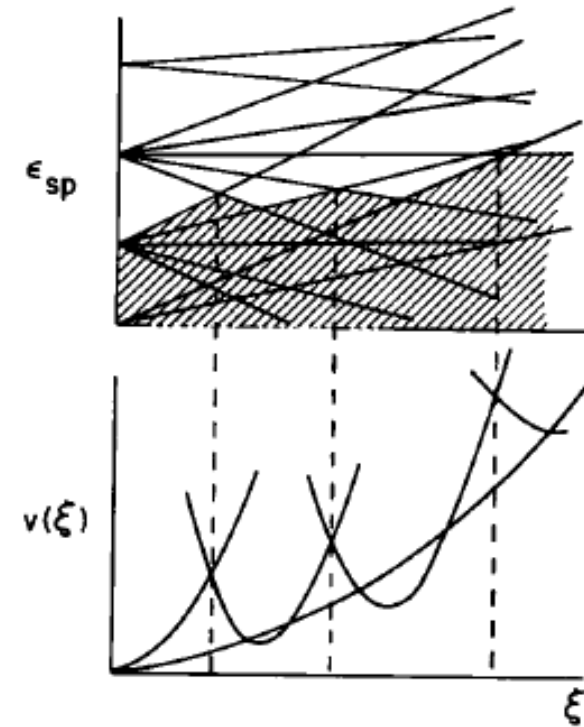
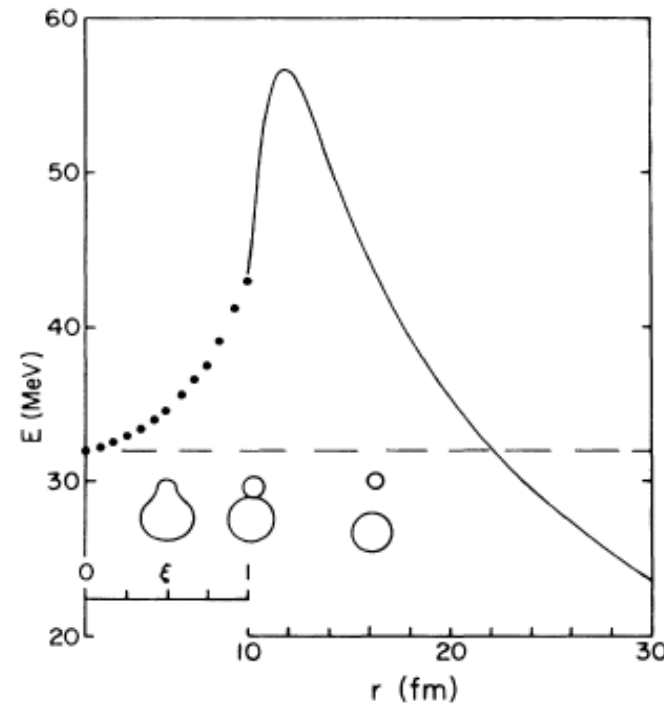
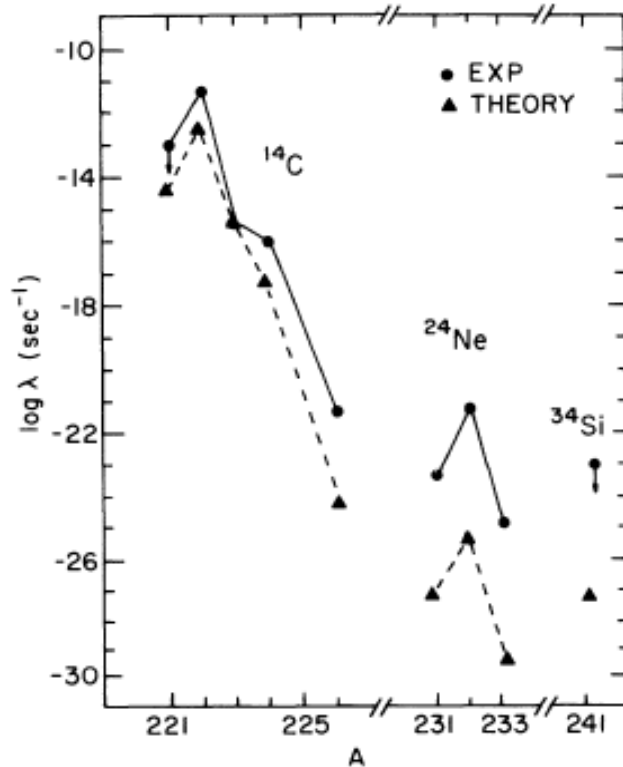
Green curve

$$\frac{(D_0)_c - (R^{(A)} + R^{(b)} + a)}{\tau_{coll}} = \frac{1}{m} \frac{\hbar}{\xi}$$

Some de-pairing velocity effect seems to be present in DWBA

About clustering:

Exotic Decay (^{14}C , ^{24}Ne ,... Spontaneous emisión) as a Self-consistent Josephson Effect



$$\left(-\frac{\hbar^2}{2D} \frac{d^2 \Phi(\xi)}{d\xi^2} + V(\xi) \right) \Phi_\alpha(\xi) = E \Phi_\alpha(\xi).$$

$$D = \hbar^2 \frac{2G}{\Delta_\nu^2 + \Delta_\pi^2} \left(\frac{dn}{d\xi} \right)^2.$$

Barranco et al. PRL60, 507 and Nucl.Phys.A 512 (1990)253

THANKS FOR YOUR ATTENTION

Collaborators:

E. Vigezzi (Milano)

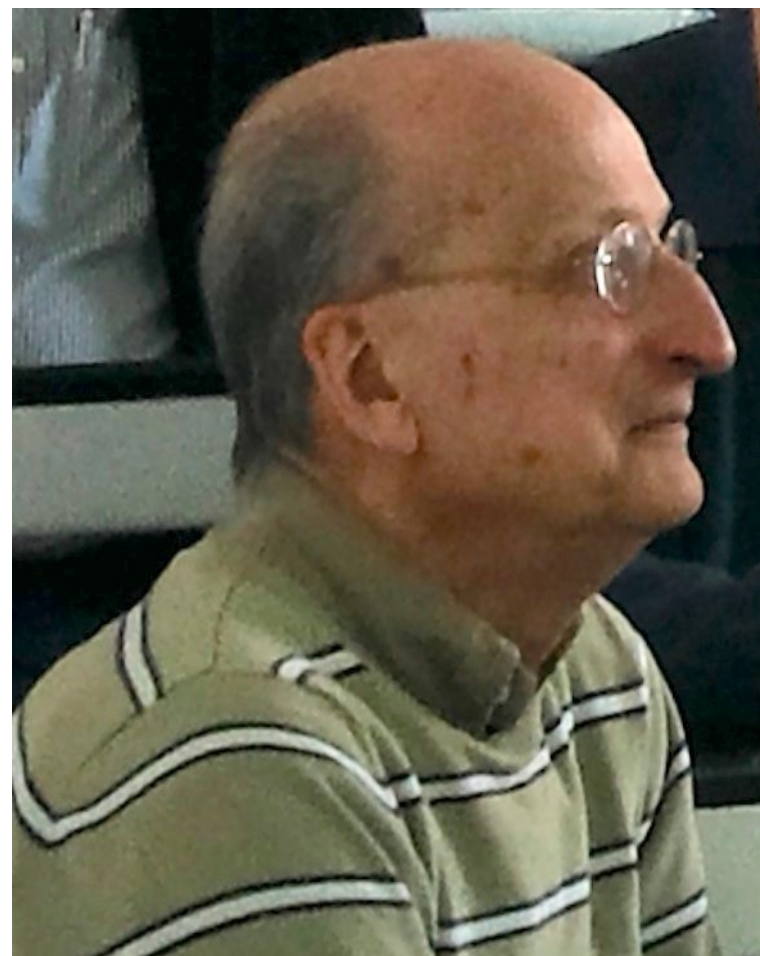
G. Potel (Sevilla)

A legacy of Riccardo Broglia

The Scientific Activity of Riccardo Broglia

Francisco Barranco, Thomas Døssing, Guido Tiana, and Enrico Vigezzi

Proc. of the International School of Physics
“Enrico Fermi” Course 213 Springer



Back-up slides.....

TRANSFER GAUGE ANGLE

(after D. Brink, Brink_Nuclear Physics at the Borderlines Proceedings of the Fourth International Summer School de la Rábida, La Rábida, Huelva, Spain, June 17–29, 1994.)

Consider two even nuclei A and B in their ground states in close proximity with fixed positions. The internal state of the system is denoted by $|0\rangle$. The ground states of

$$\Psi = \sum_N a_N |N\rangle, \quad H_t |N\rangle = -\frac{1}{2} J_0 (|N+1\rangle + |N-1\rangle)$$

$$\Psi = \sum_N e^{iN\phi} |N\rangle, \quad E = -J_0 \cos \phi.$$

$$f(\phi) = \frac{1}{\sqrt{2\pi}} \sum_N a_N e^{iN\phi}, \quad \text{where} \quad a_N = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{-iN\phi} f(\phi) d\phi, \quad N = -i \frac{\partial}{\partial \phi}.$$

Josephson equations

A more general hamiltonian has a term depending on the number of pairs moved as well as the transfer term (5.2)

$$H = H_t + \epsilon(N). \tag{5.6}$$

The term $\epsilon(N)$ is the total ground state energy of the two nuclei when N pairs have been transferred. The Schrödinger equation with the hamiltonian (5.6) can not be solved analytically for a general ϵN but the Heisenberg equations of motion for the operators ϕ and N have an interesting form. They are

$$\begin{aligned} i\hbar \frac{dN}{dt} &= [N, H] = [N, H_t] = iJ_0 \frac{d \cos \phi}{d\phi} \\ i\hbar \frac{d\phi}{dt} &= [\phi, H] = [\phi, \epsilon(N)] = i \frac{d\epsilon(N)}{dN} \end{aligned}$$

and simplify to give

$$\frac{dN}{dt} = -(J_0/\hbar) \sin \phi, \qquad \frac{d\phi}{dt} = \epsilon'(N)/\hbar. \tag{5.7}$$

These are Josephson's two relations. The first connects the Josephson tunnelling current with the gauge angle and the second gives the rate of change of ϕ in terms of the potential difference between A and B.

DC “Nuclear supercurrent”

The transfer ampiltude J_0 is time dependent for pair transfer between heavy ions. It is small when the nuclei are far appart and a maximum at the point of closest approach. We can write a time dependent Schrödinger equation for the wave function $f(\phi)$ in the gauge representation as

$$i\hbar \frac{\partial f}{\partial t} = H_t f = -J_0(t) \cos(\phi) f, \quad (5.8)$$

where we have used the form of H_t in the gauge representation give in equation (5.3). It should be solved with the initial condition $N = 0$, ($f(\phi) = 1/\sqrt{2\pi}$ as $t \rightarrow -\infty$). The solution as $t \rightarrow +\infty$ is

$$f(\phi) = \frac{1}{\sqrt{2\pi}} e^{ix \cos \phi}, \quad \text{with} \quad x = \frac{1}{\hbar} \int_{-\infty}^{\infty} J_0(t) dt. \quad (5.9)$$

The amplitudes a_N are the Fourier coefficients in the expansion of $f(\phi)$ and are proportional to Bessel functions

$$a_N = i^N J_N(x).$$

Thus the probability that N pairs have been transferred after the collision is

$$P_N = |a_N|^2 = |J_N(x)|^2.$$

Unfortunately in the heavy ion case x is normally small so that the probability that $|N| > 1$ is very small.

EXTENDED pp-RPA

$$\begin{pmatrix} A_{pp'p''p'''} & B_{pp'h''h'''} \\ B_{p''p'''hh'} & A_{hh'h''h'''} \end{pmatrix} \begin{pmatrix} X_{p''p'''} \\ Y_{h''h'''} \end{pmatrix} = \begin{pmatrix} E \\ ? \end{pmatrix} \begin{pmatrix} X_{pp'} \\ Y_{hh'} \end{pmatrix}$$

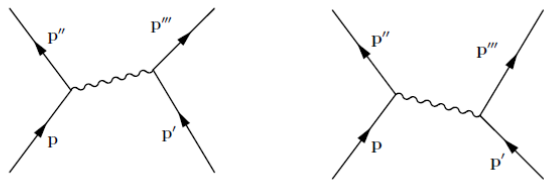
PVC

$$h_{pp''\lambda\nu} = \text{diagram}$$

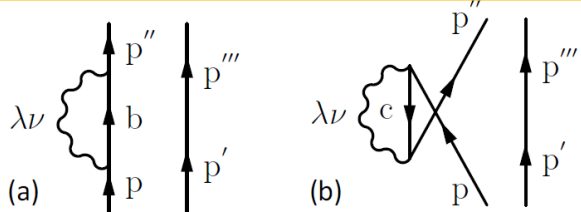
Includes **Self-energy** and **Induced Interaction** <->

$$A_{pp'p''p'''} = [(\epsilon_p + \epsilon_{p'}) + \text{PVC} \Sigma_{pp''(p')}(E)\delta_{p'p'''} + \Sigma_{p'p''(p)}(E)\delta_{pp''}] + V_{pp'p''p'''}^{bare} + \text{PVC} V_{pp'p''p'''}^{ind}(E) + Exch(p, p') N_{pp'p''p'''}]$$

$$B_{pp'hh'} = [V_{pp'hh'}^{bare} + \text{PVC} V_{pp'hh'}^{ind}(E) + Exch(p, p')] N_{pp'p''p'''}]$$



$$V_{pp'p''p'''}^{ind} = \sum_{\lambda\nu} \left[\frac{h_{pp''\lambda\nu} h_{p''p'\lambda\nu}}{E - (\epsilon_{p''}^{emp} + \epsilon_{p'}^{emp} + \hbar\omega_{\lambda\nu})} + \frac{h_{p''p\lambda\nu} h_{p'p'''\lambda\nu}}{E - (\epsilon_p^{emp} + \epsilon_{p'''}^{emp} + \hbar\omega_{\lambda\nu})} \right]$$

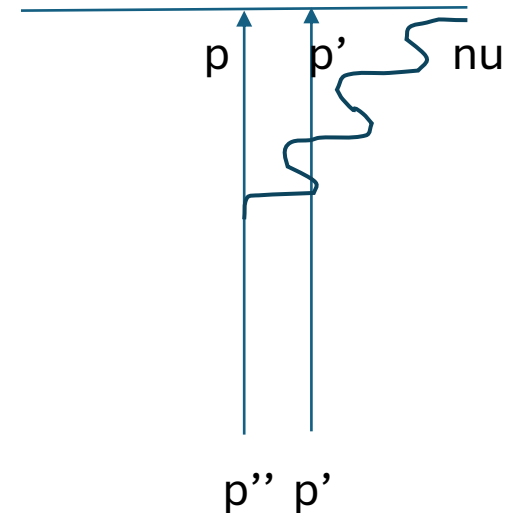
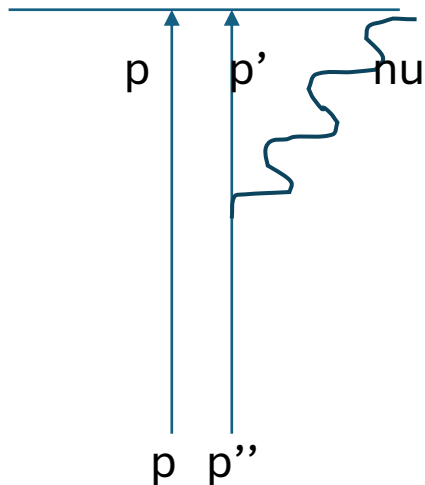


$$\Sigma_{pp''(p')}(E) = \sum_{b, \epsilon_b > \epsilon_F} \frac{h_{pb\lambda\nu} h_{p''b\lambda\nu}}{E - (\epsilon_b^{emp} + \epsilon_{p'}^{emp} + \hbar\omega_{\lambda\nu})} + \sum_{c, \epsilon_c < \epsilon_F} \frac{h_{pc\lambda\nu} h_{p''c\lambda\nu}}{E - \epsilon_c^{emp} - \epsilon_{p'}^{emp} + \hbar\omega_{\lambda\nu}} \quad (6)$$

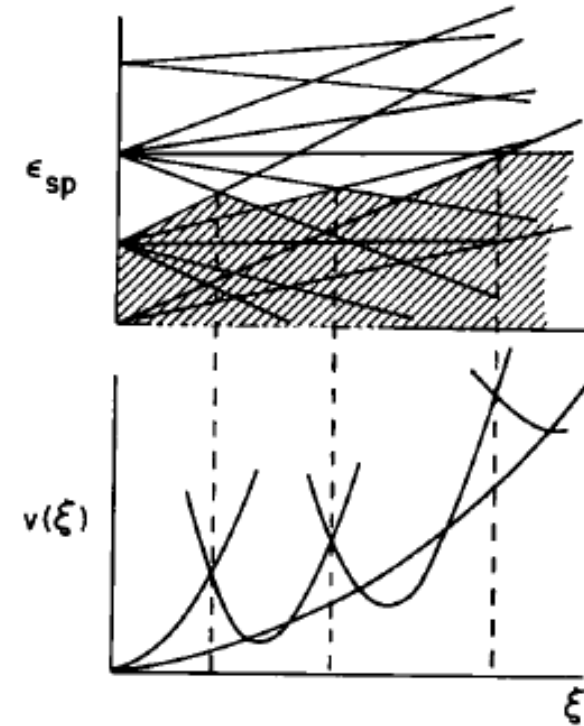
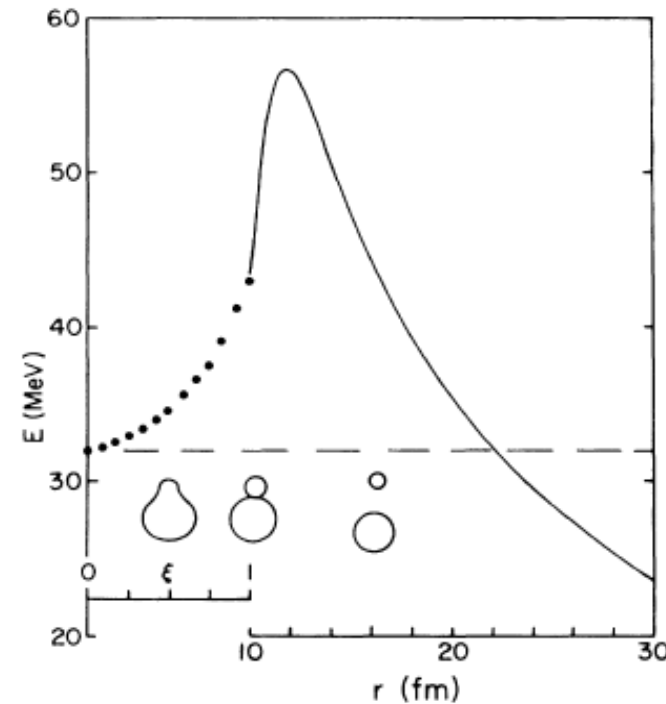
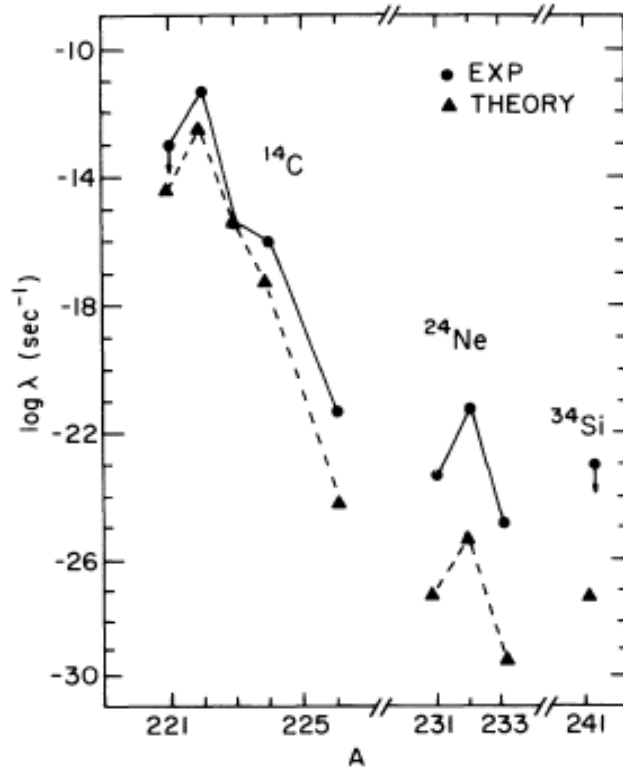
The energy dependence of Self-energy and Vind
“hides” the amplitudes on the intermediate states:

HIDDEN AMPLITUDES

$$X_{pp'\nu} = \sum_{p''} X_{pp''} \frac{h_{p''p'\nu}}{E - (e_p + e_{p'} + \hbar\omega_\nu)} + \sum_{p''} X_{p''p'} \frac{h_{p''p\nu} (-)^{(j_p - j_{p'} - \lambda)}}{E - (e_p + e_{p'} + \hbar\omega_\nu)}$$



1. Nuclear Superfluidity: Inertial Mass in Exotic Decay (^{14}C , ^{24}Ne ,... (internal 2-nucleon transfer steps: hopping model)



$$\left(-\frac{\hbar^2}{2D} \frac{d^2 \Phi(\xi)}{d\xi^2} + V(\xi) \right) \Phi_\alpha(\xi) = E \Phi_\alpha(\xi).$$

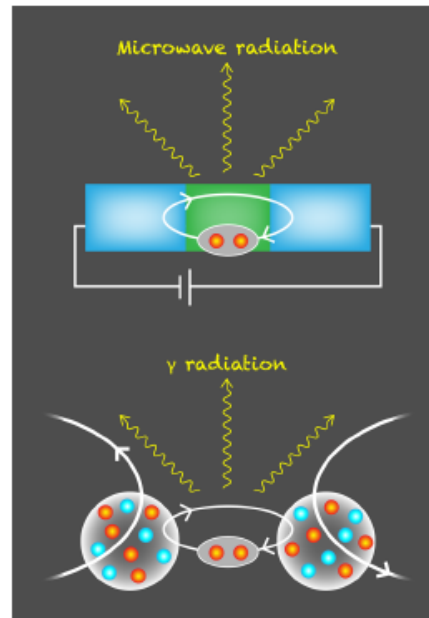
$$D = \hbar^2 \frac{2G}{\Delta_\nu^2 + \Delta_\pi^2} \left(\frac{dn}{d\xi} \right)^2.$$

Barranco et al. PRL60, 507 and Nucl.Phys.A 512 (1990)253

The Tiniest Superfluid Circuit in Nature

A new analysis of heavy-ion collision experiments uncovers evidence that two colliding nuclei behave like a Josephson junction—a device in which Cooper pairs tunnel through a barrier between two superfluids.

By **Piotr Magierski**



2. Josephson Junctions (DC and AC): In Condensed Matter

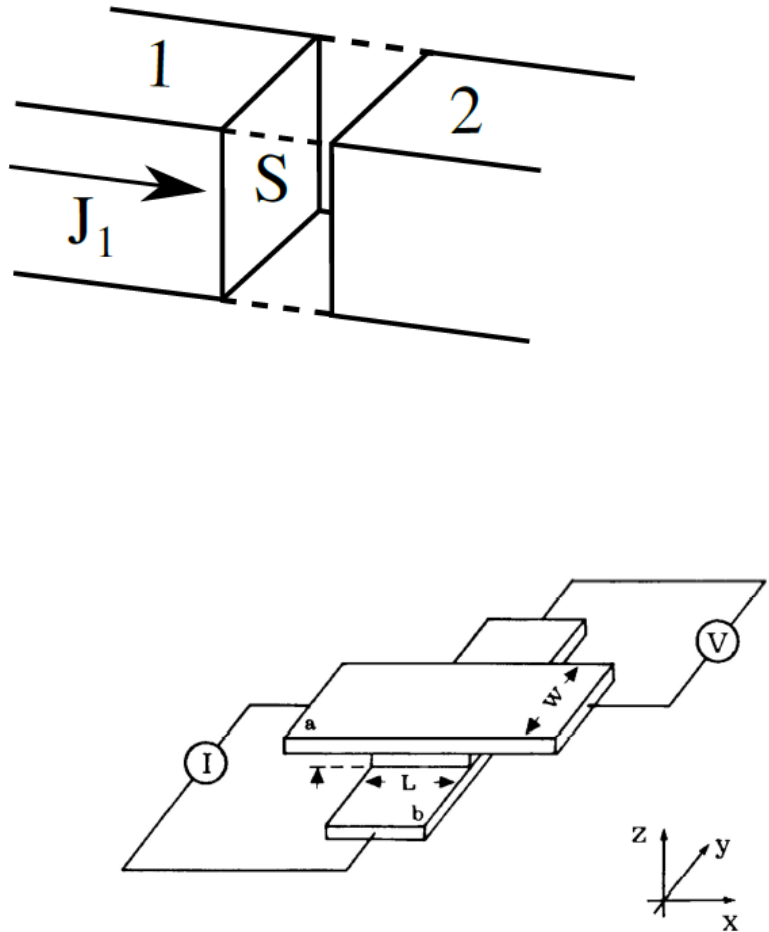


Figure 1.1 Tunneling junction of cross-type geometry. The dimensions are L and W ; a and b are the two superconducting films.

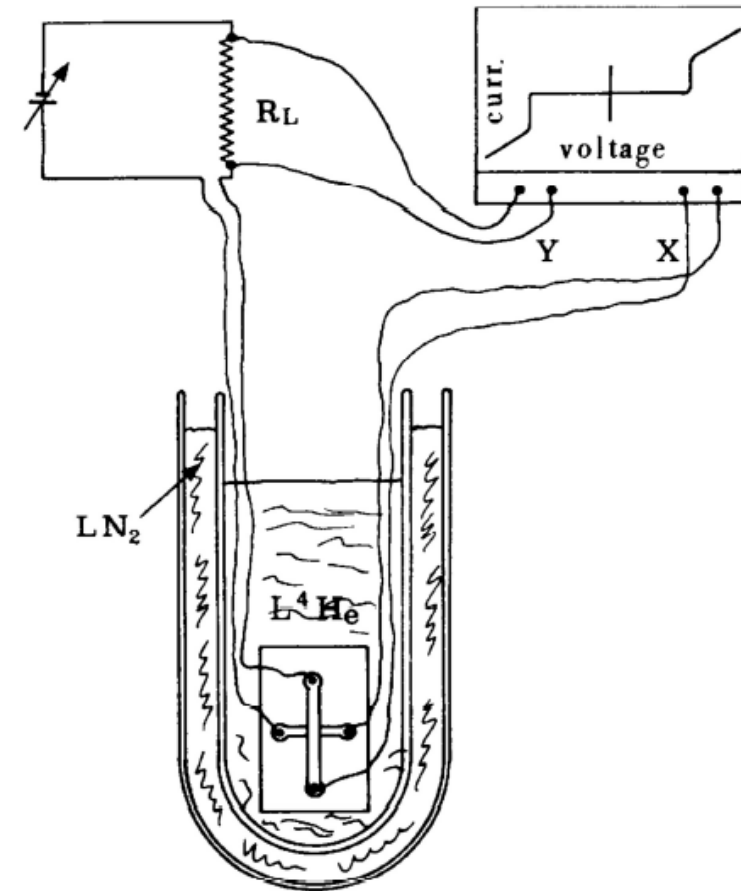


Figure 3.9 Schematic of the experimental apparatus used to measure the voltage-current characteristics of a junction. The inner dewar in which the sample is inserted is filled with liquid helium ($L^4\text{He}$), the outer dewar contains liquid nitrogen (LN_2).

$$a(= b + 2) + A \rightarrow f(= b + 1) + F(= A + 1) \rightarrow b + B(= A + 2) \quad (116)$$

Making use of the notation $\alpha \equiv (a, A)$, $\gamma \equiv (f, F)$ and $\beta \equiv (b + B)$. one can write the successive transfer amplitude in the semiclassical approximation ([51] p. 306 Eq. (23)), as

$$(a_\beta(t))_{succ} = \left(\frac{1}{i\hbar}\right)^2 \sum_{\gamma \neq \beta} \int_{-\infty}^t dt' \langle \Psi_\beta | V_\beta - \langle V_\beta \rangle | \Psi_\gamma \rangle_{\mathbf{R}_{\beta,\gamma}(t')} \\ \times \int_{-\infty}^{t'} dt'' \langle \Psi_\gamma | V_\alpha - \langle V_\alpha \rangle | \Psi_\alpha \rangle_{\mathbf{R}_{\gamma,\alpha}(t'')}, \quad (117)$$

where the quantal *cm* coordinate $\mathbf{r}_{\beta\gamma} = (\mathbf{r}_\beta + \mathbf{r}_\gamma)/2$ should be identified with the average classical coordinate, i.e. $\mathbf{r}_{\beta\gamma} \rightarrow \mathbf{R}_{\beta\gamma} = (\mathbf{R}_\beta + \mathbf{R}_\gamma)/2$ which, together with $\mathbf{v}_\beta = \dot{\mathbf{R}}_\beta$ and similar for the channel γ , are assumed to describe the motion of the centers of the wavepackets, and satisfy the classical equations $m_\beta \dot{\mathbf{v}}_\beta = -\nabla_\beta U(\mathbf{R}_\beta)$. The functions Ψ describe the structure of the nuclei and, e.g. $\Psi_\alpha = e^{-i\frac{E_\alpha}{\hbar}t} \psi_\alpha$ while $\psi_\alpha = \psi^a(\xi_a) \psi^A(\xi_A)$ is the product of the intrinsic BCS wavefunctions describing the structure of nuclei a and A , ξ being the corresponding intrinsic variables. One can rewrite (117) as,

$$(a_\beta(t))_{succ} = \left(\frac{1}{i\hbar}\right)^2 \sum_{\gamma \neq \beta} \int_{-\infty}^t dt' \langle \Psi_\beta | V_\beta - \langle V_\beta \rangle | \psi_\gamma \rangle_{\mathbf{R}_{\beta,\gamma}(t')} \\ \times \int_{-\infty}^{t'} dt'' \langle \psi_\gamma | V_\alpha - \langle V_\alpha \rangle | \Psi_\alpha \rangle_{\mathbf{R}_{\gamma,\alpha}(t'')} e^{i\frac{E_\gamma}{\hbar}(t''-t')}, \quad (118)$$

Assuming the quasiparticle excitation energies E_γ to be much larger than the reaction Q -value, the periodic functions in (119) oscillate so rapidly, that the outcome of the integration in (117) amounts to nothing unless $t' \approx t''$. In other words and further assuming that the matrix elements (formfactors) are smooth functions of time along the trajectories of relative motion one can write $\sum_{\gamma \neq \beta} \exp\left(i\frac{E_\gamma}{\hbar}(t'' - t')\right) \approx \frac{1}{\Delta E} \int dE \exp\left(i\frac{E}{\hbar}(t'' - t')\right) \approx \frac{\hbar}{\Delta E} \times \delta(t'' - t')$, where $1/\Delta E$ is the average density of levels of the two-quasiparticle states. Making use of Eq. (9), one can write Eq. (118) as,

$$(a_\beta(t))_{succ} = \left(\frac{1}{i\hbar}\right)^2 \int_{-\infty}^t dt' \langle \psi_\beta | (V_\beta - \langle V_\beta \rangle) | \psi_\gamma \rangle_{\mathbf{R}_{\beta,\gamma}(t')} \\ \times \frac{\hbar}{\Delta E} \mathbf{R}_{\gamma,\alpha}(t') \langle \psi_\gamma | V_\alpha - \langle V_\alpha \rangle | \psi_\alpha \rangle e^{i\frac{Q_{2n}}{\hbar}t'}, \quad (120)$$

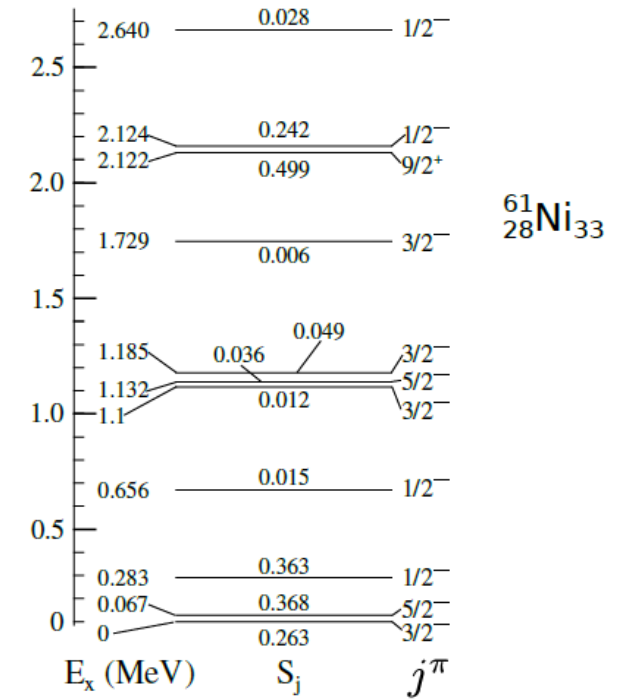
4. The Legnaro's $^{116}\text{Sn}+^{60}\text{Ni}$ 2NT and 1NT: BCS ground states and quasiparticles

	ϵ_{l_j}	E_{l_j}	$V_{l_j}^2$	B_{l_j}	E_{l_j}	$V_{l_j}^2$	B_{l_j}
$2d_{5/2}$	-11.51	2.35	0.874	0.658	2.35	0.874	0.575
$1g_{7/2}$	-10.86	1.92	0.790	0.910	1.92	0.789	0.816
$3s_{1/2}$	-9.70	1.56	0.487	0.483	1.57	0.484	0.500
$2d_{3/2}$	-9.51	1.58	0.428	0.661	1.58	0.424	0.699
$1h_{11/2}$	-8.12	2.25	0.140	0.770	2.26	0.139	0.847

Table 1: Properties of the five valence states calculated in ^{115}Sn , starting from the single-particle energies ϵ_{l_j} shown in [1]. The quasiparticle energies E_{l_j} , the occupation probabilities $V_{l_j}^2$ and the transfer spectroscopic amplitudes B_{l_j} shown in Table 1 of ref. [1] are shown in the second, third and fourth column. They are compared in the last three columns with the corresponding quantities recalculated here with a pairing coupling constant $G = 0.217$ MeV (25/A).

	ϵ_{l_j}	E_{l_j}	$U_{l_j}^2$	B_{l_j}	B_{l_j}
$2p_{3/2}$	-11.33	1.56	0.33	0.524	0.665
$2p_{1/2}$	-9.66	1.83	0.65	0.500	0.477
$1f_{5/2}$	-9.33	2.05	0.400	0.34	0.849
$1g_{9/2}$	-5.16	5.80	0.500	0.1	1.224

Table 2: Properties of the four valence states calculated in ^{61}Ni , starting from the single-particle energies ϵ_{l_j} shown in [1]. The quasiparticle energies E_{l_j} , the occupation probabilities $U_{l_j}^2$ and the transfer spectroscopic amplitudes B_{l_j} shown in Table 1 of ref. [1] are shown in the second, third and fourth column. The values of the spectroscopic are compared in the last column with the value obtained from the relation $B = UV\sqrt{j+1/2}$, using $V^2 = 1 - U^2$.



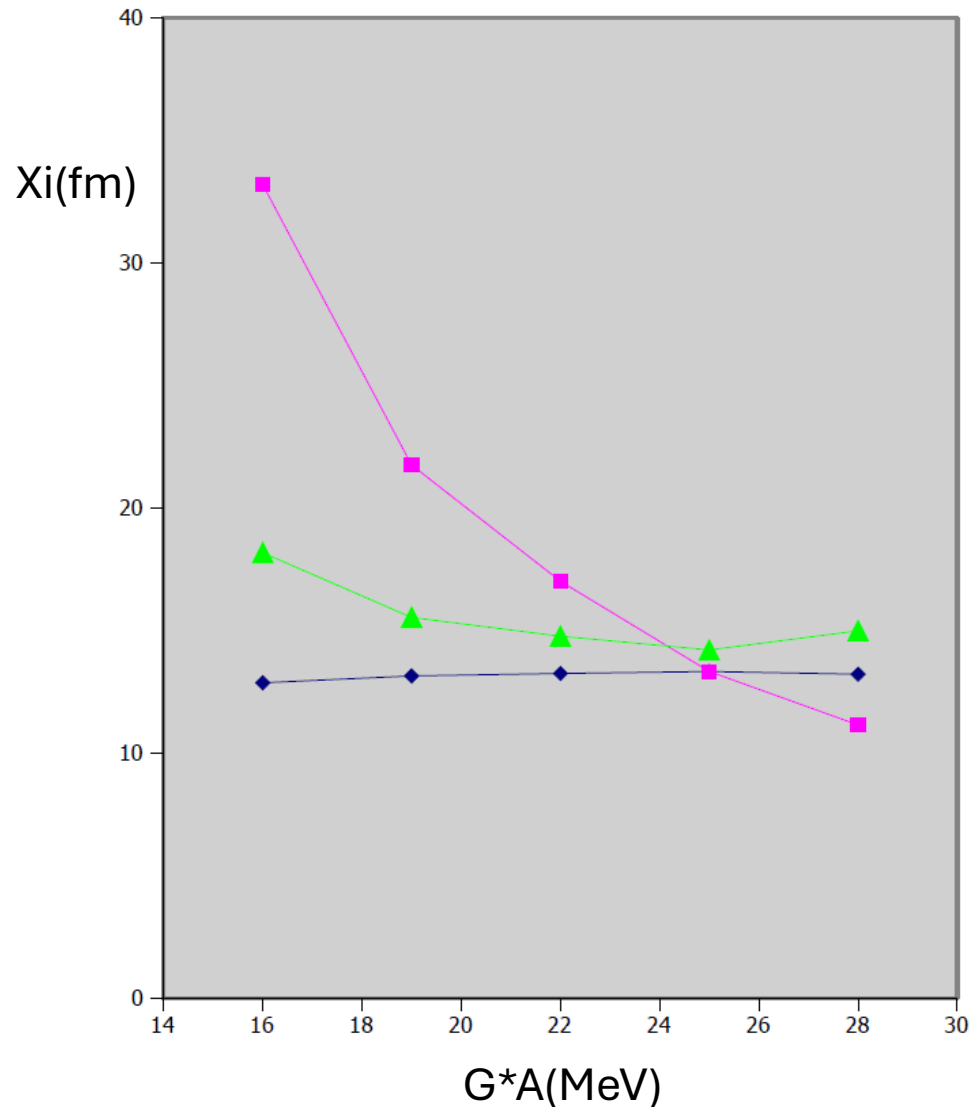
A new set of calculations was carried out, using slightly modified inputs.

nl_j	ϵ_{nl_j} (MeV)	E_{nl_j} (MeV)	$U_{nl_j}^2$
$2p_{3/2}$	-7.82	1.05	0.44
$2p_{1/2}$	-6.26	1.76	0.90
$1f_{5/2}$	-7.04	1.22	0.24
$1g_{9/2}$	-3.30	4.51	0.99

Table 3: Quasiparticle energies and amplitudes obtained in a BCS calculation for ^{60}Ni with a pairing constant $G = 0.331$ MeV ($= 19.9/A$), following ref. [6].

6. Velocity of the Transferred (nuclear) Cooper Pair.

6.2 Simulated data (DWBA calc's) as a function of pairing interaction.



$$\frac{(D_0)_c - (R^{(A)} + R^{(b)} + a)}{\tau_{coll}} = \frac{1}{m} \frac{\hbar}{\xi}$$

Some de-pairing velocity effect seems to be present in DWBA

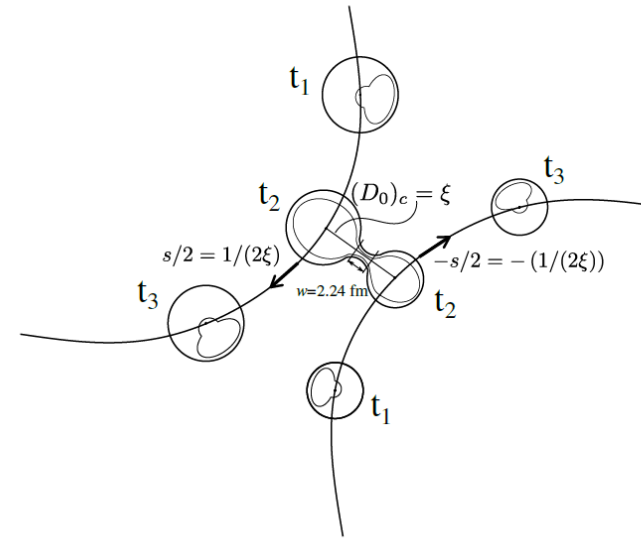
4. The Legnaro's $^{116}\text{Sn}+^{60}\text{Ni}$ 2NT and 1NT data at different E_{cm} :

4.1 Semiclassical analysis.

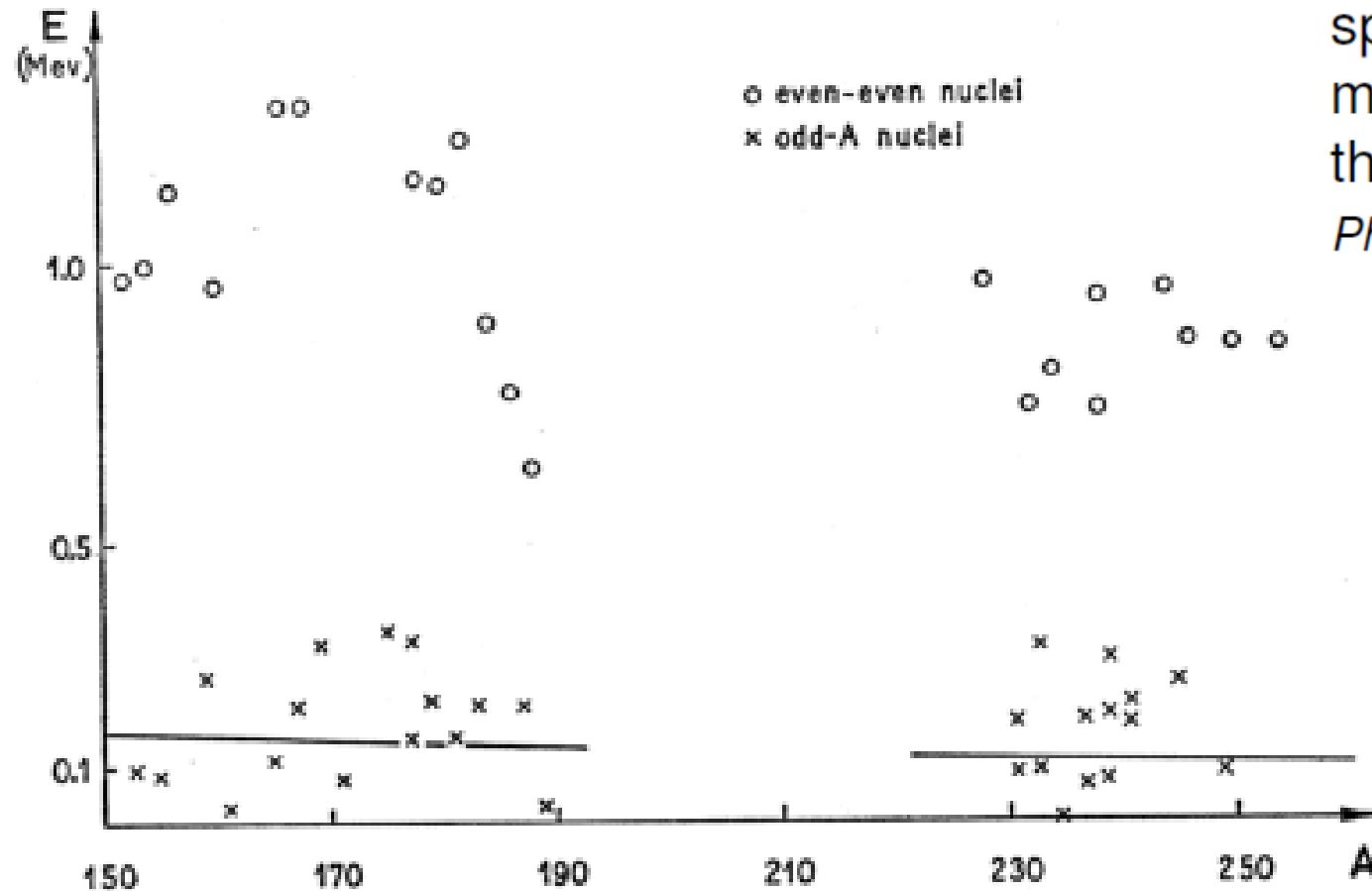
In heavy ion collisions below the Coulomb barrier with typical values of the ratio $D_0/\lambda \approx 10^2$ between the distance of closest approach and the reduced de Broglie wavelength, Cooper pair tunneling can be described in terms of the semiclassical second order transfer amplitude (see [51] p. 306 Eq. (23)),

$$(a_\beta)_{\text{succ}} \approx \left(\frac{1}{i\hbar} \right)^2 \times \sum_{Ff \neq Bb} \int_{-\infty}^{\infty} dt \langle \Psi_{Bb} | V_{Ff} - \langle V_{Ff} \rangle | \Psi_{Ff} \rangle \mathbf{R}_{Bb,Ff}(t) e^{i \frac{(E_{Bb} - E_{Ff})}{\hbar} t} \times \int_{-\infty}^t dt' \langle \Psi_{Ff} | V_{Aa} - \langle V_{Aa} \rangle | \Psi_{Aa} \rangle \mathbf{R}_{Aa,Ff}(t') e^{i \frac{(E_{Ff} - E_{Aa})}{\hbar} t'}.$$

$$B_j = \sqrt{\frac{(2j+1)}{2}} U'_j V'_j,$$



2. Nuclear Superfluidity: The origin.



Bohr, Mottelson, and Pines speculated that nuclear pairing might explain the **energy gap** in the excitation spectra of nuclei.

Phys. Rev. 110, 936 (1958)

2. Nuclear superfluidity: BCS ground state

$$|BCS\rangle = \prod_k \left(U'_k + e^{-2i\phi} V'_k a_{\mathbf{k}+\mathbf{s},\uparrow}^\dagger a_{-\mathbf{k}+\mathbf{s},\downarrow}^\dagger \right) |0\rangle.$$

$$\begin{aligned} \alpha_v^\dagger &= u_v a_v^\dagger - v_v a_{\bar{v}} \\ \alpha_{\bar{v}}^\dagger &= u_v a_{\bar{v}}^\dagger + v_v a_v \\ \alpha_v &= u_v a_v - v_v a_{\bar{v}}^\dagger \\ \alpha_{\bar{v}} &= u_v a_{\bar{v}} + v_v a_v^\dagger \end{aligned} \quad \begin{aligned} a_v^\dagger &= u_v \alpha_v^\dagger + v_v \alpha_{\bar{v}} \\ a_{\bar{v}}^\dagger &= u_v \alpha_{\bar{v}}^\dagger - v_v \alpha_v \\ a_v &= u_v \alpha_v + v_v \alpha_{\bar{v}}^\dagger \\ a_{\bar{v}} &= u_v \alpha_{\bar{v}} - v_v \alpha_v^\dagger. \end{aligned}$$

$$\begin{aligned} H' &= H - \lambda n \\ &= \sum_{v>0} (\varepsilon_v^{(0)} - \lambda)(a_v^\dagger a_v + a_{\bar{v}}^\dagger a_{\bar{v}}) - G \sum_{\mu, v>0} a_\mu^\dagger a_\mu^\dagger a_{\bar{v}} a_v, \end{aligned}$$

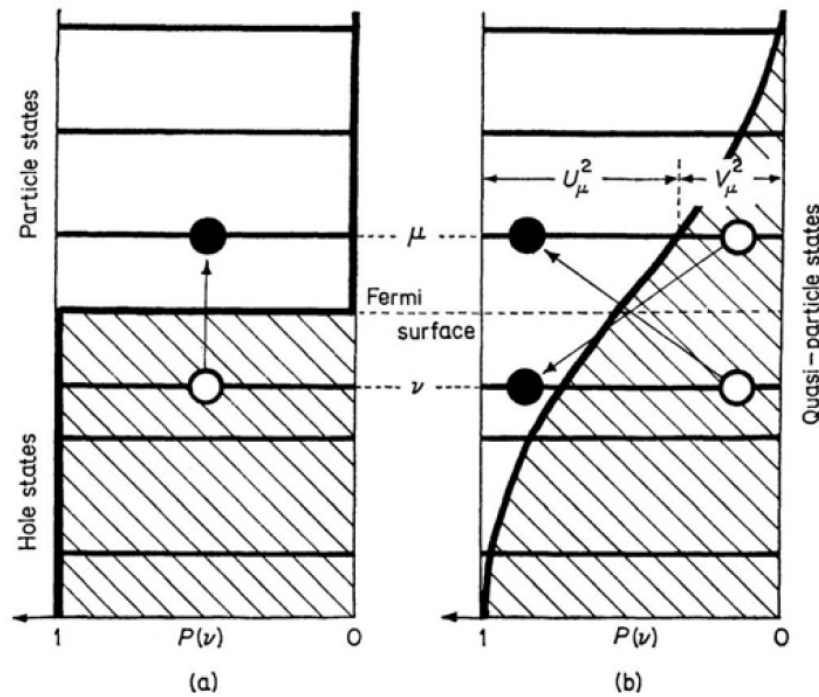
$$\sum_{v>0} \left\{ 1 - \frac{\varepsilon_v - \lambda}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\} = N.$$

$$\frac{G}{2} \sum_{v>0} \frac{1}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} = 1.$$

$$u_v^2 = \frac{1}{2} \left\{ 1 + \frac{\varepsilon_v - \lambda}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\}$$

$$v_v^2 = \frac{1}{2} \left\{ 1 - \frac{\varepsilon_v - \lambda}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\}.$$

2. Nuclear Superfluidity: Occupations.



In figure 11.4 are shown experimental values of v_j^2 obtained by Cohen and Price (1961) from (d,p) and (d,t) experiments on the Sn

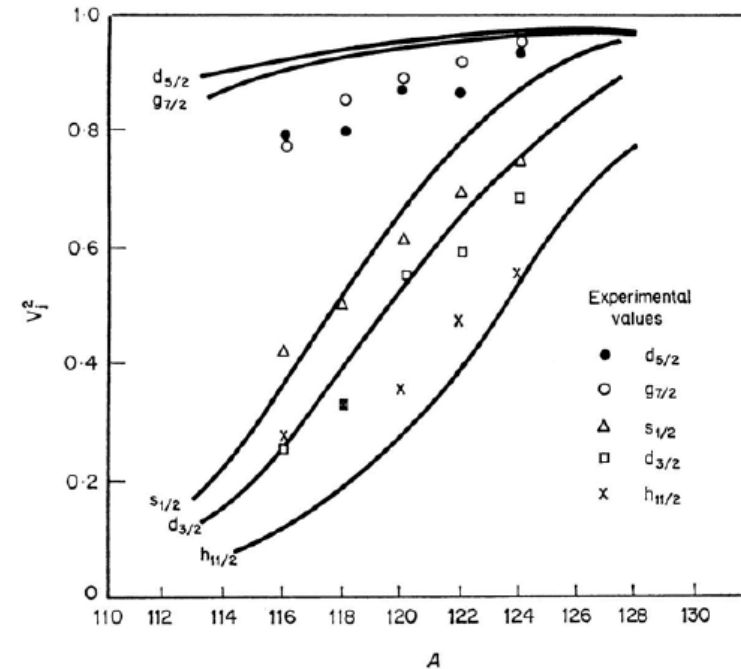


Figure 11.4. Experimental values of v_j^2 obtained by Cohen and Price (1961) from (d,p) and (d,t) experiments on the Sn isotopes, compared to the theoretical values of Kisslinger and Sorensen (1960).

2. Nuclear Superfluidity: Sn(p,t): Occupations

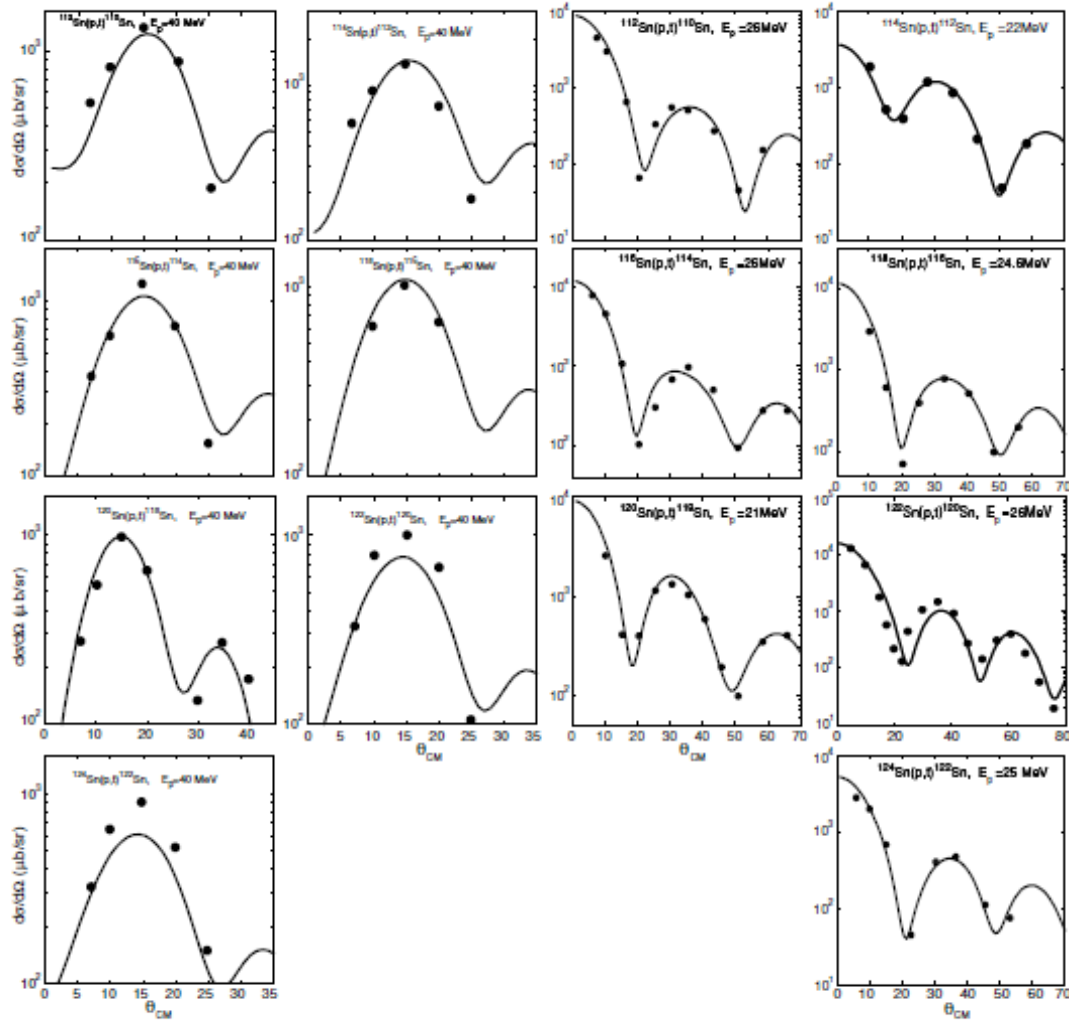


FIG. 7: Predicted absolute differential $A+2\text{Sn}(p,t)^A\text{Sn}(gs)$ cross sections for bombarding energies 21 MeV

$\leq E_p \leq 26$ MeV, and $E_p = 40$ MeV in comparison with the experimental data (see [39–44] and [45]

respectively).

Sn: 112 to 124

DWBA

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} |T^{(1)} + T_{succ}^{(2)} - T_{NO}^{(2)}|^2$$

$$T^{(1)} = 2 \sum_{l_i, j_i} \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{tA} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB})$$

$$\times v(\mathbf{r}_{p1}) \phi_t(\mathbf{r}_{p1}, \mathbf{r}_{p2}) \chi_{tA}^{(+)}(\mathbf{r}_{tA}).$$

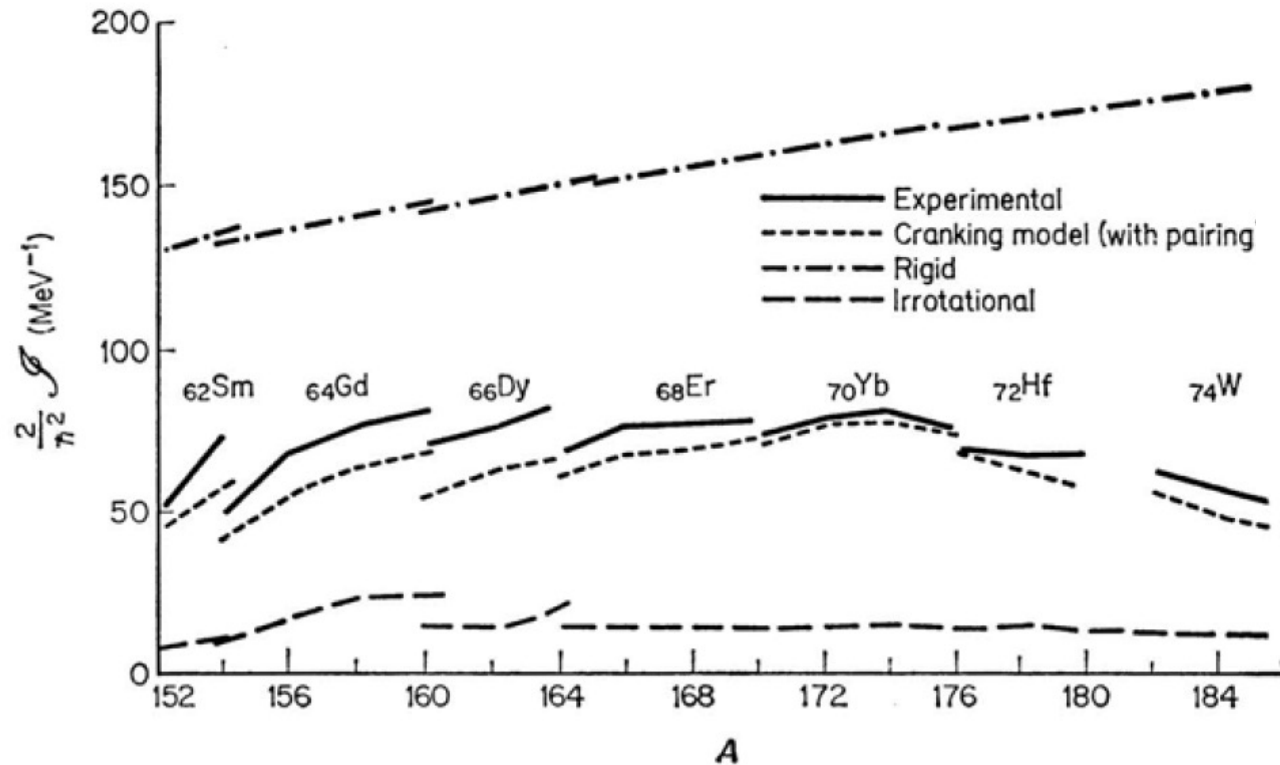
$$T_{succ}^{(2)} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1}) \\ \times \phi_d(\mathbf{r}_{p1}) \varphi_{l_f, j_f, m_f}^{A+1}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{dF} d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} G(\mathbf{r}_{dF}, \mathbf{r}'_{dF}) \\ \times \phi_d(\mathbf{r}'_{p1})^* \varphi_{l_f, j_f, m_f}^{A+1*}(\mathbf{r}'_{A2}) \frac{2\mu_{dF}}{\hbar^2} v(\mathbf{r}'_{p2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi_{tA}^{(+)}(\mathbf{r}'_{tA})$$

$$T_{NO}^{(2)} = 2 \sum_{l_i, j_i} \sum_{l_f, j_f, m_f} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{dF} d\mathbf{r}_{p1} d\mathbf{r}_{A2} [\phi_{l_i, j_i}^{A+2}(\mathbf{r}_{A1}, \sigma_1, \mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{pB}^{(-)*}(\mathbf{r}_{pB}) v(\mathbf{r}_{p1})$$

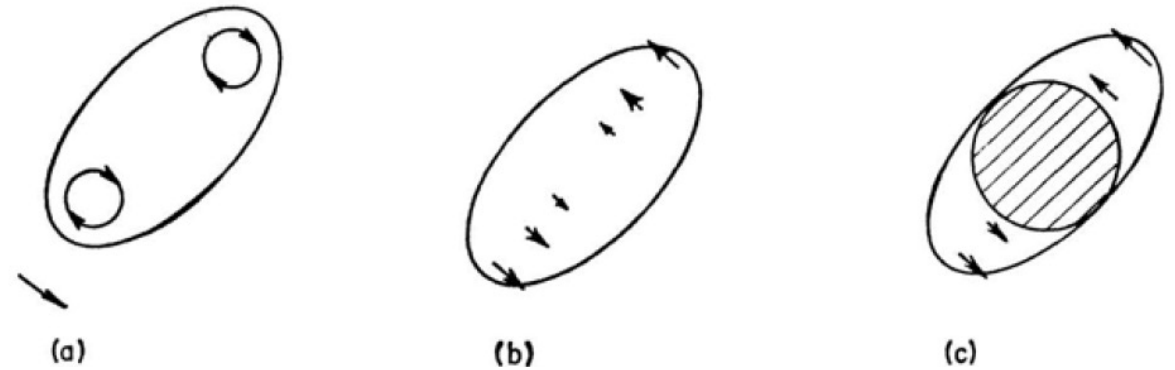
$$\times \phi_d(\mathbf{r}_{p1}) \varphi_{l_f, j_f, m_f}^{A+1}(\mathbf{r}_{A2}) \int d\mathbf{r}'_{p1} d\mathbf{r}'_{A2} d\mathbf{r}'_{dF}$$

$$\times \phi_d(\mathbf{r}'_{p1})^* \varphi_{l_f, j_f, m_f}^{A+1*}(\mathbf{r}'_{A2}) \phi_d(\mathbf{r}'_{p1}) \phi_d(\mathbf{r}'_{p2}) \chi_{tA}^{(+)}(\mathbf{r}'_{tA})$$

2. Nuclear Superfluidity: Moments of Inertia.



Irrotational, Rotational and
Two Fluids picture

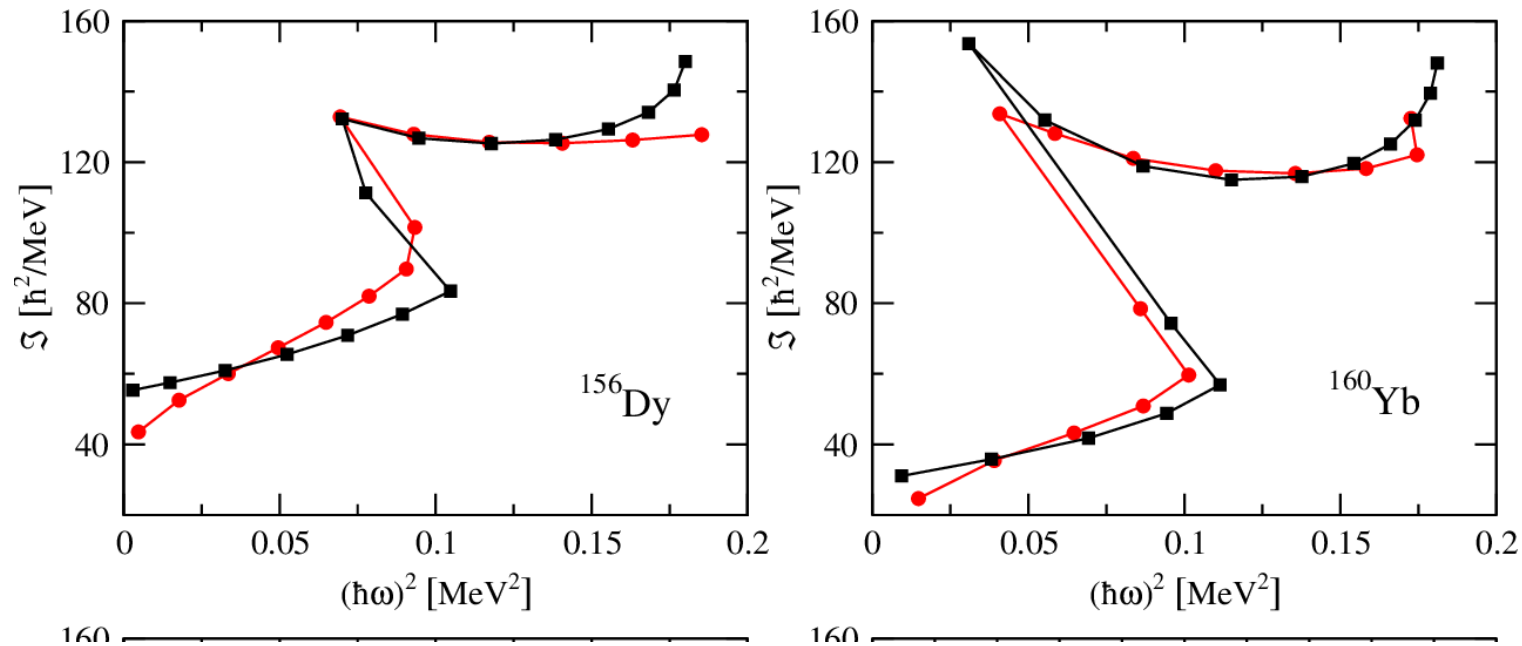


D.Rowe, Nucl. Coll. Motion, Dover.

$$\mathcal{J} = 2\hbar^2 \sum_{\mu\nu} \frac{|\langle \mu | J_x | \nu \rangle|^2 (U_\mu V_\nu - V_\mu U_\nu)^2}{E_\mu + E_\nu}$$

2. Nuclear Superfluidity: Moment of Inertia-2

Mottelson and Valatin [12] argued that there is a close formal correspondence between equations of motion in a **constant magnetic field and in a rotating reference system**. They suggested that critical magnetic field phenomena in superconductors should



[12] B. R. Mottelson and J. G. Valatin, *Phys. Rev. Lett.* 5 (1960) 511.

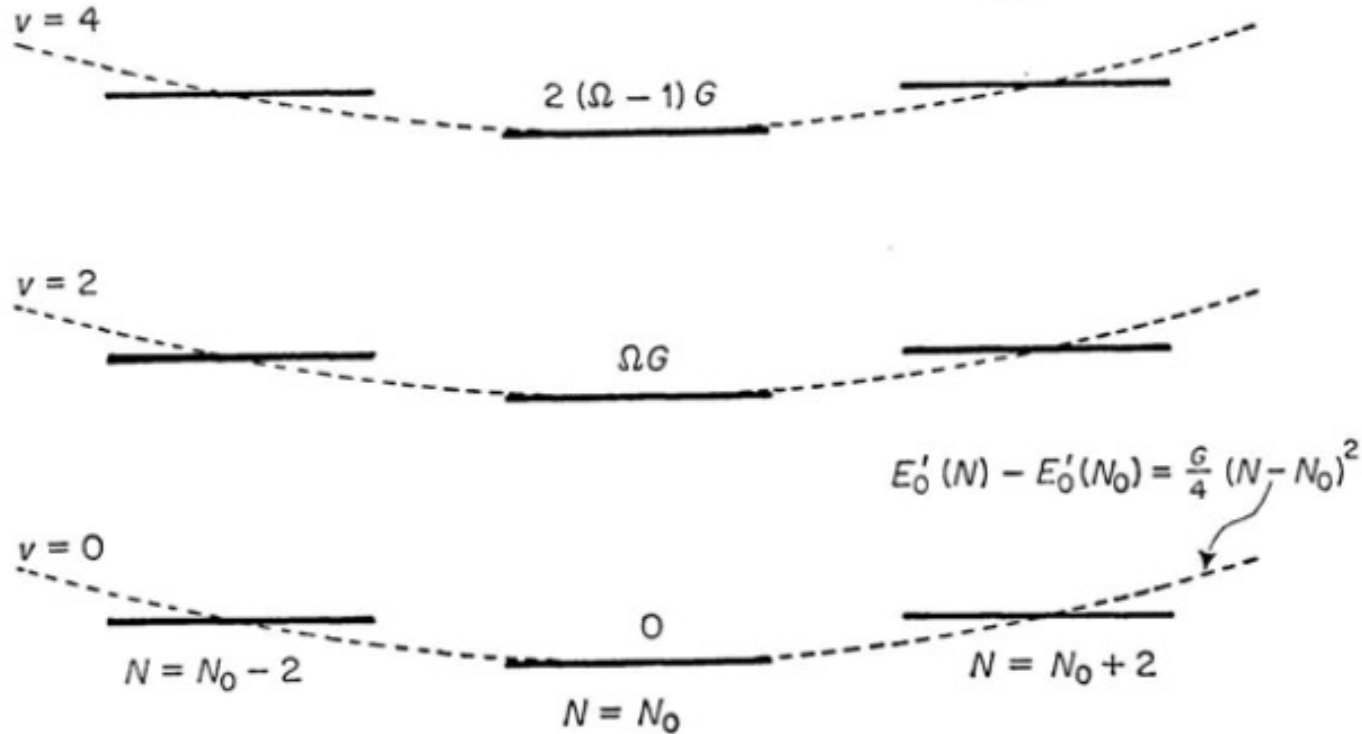
A lesson: Nuclear superfluidity is not just Occupation Numbers.

It exhibits strong similarities with macroscopic superfluidity/superconductivity

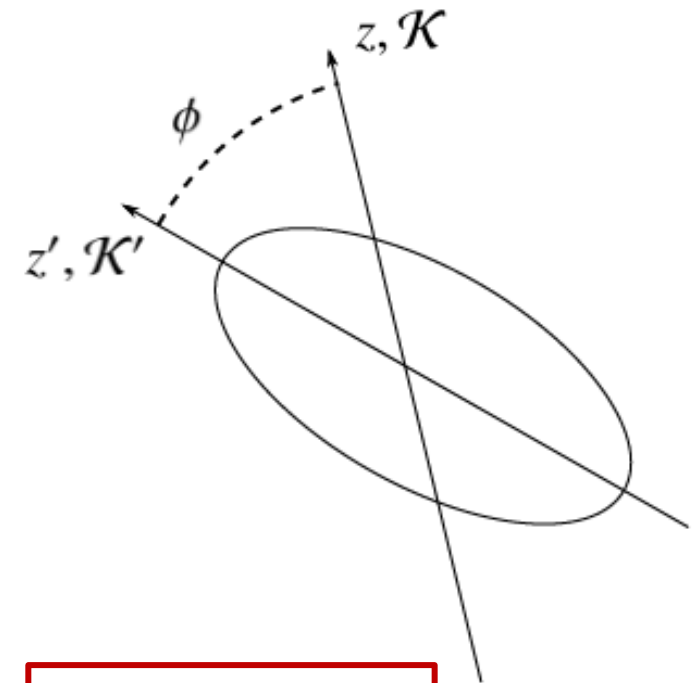
2. Nuclear Superfluidity: the Gauge Angle

$$H' = H - \lambda n$$

$$= \sum_{\nu > 0} (\epsilon_{\nu}^{(0)} - \lambda)(a_{\nu}^{\dagger} a_{\nu} + a_{\bar{\nu}}^{\dagger} a_{\bar{\nu}}) - G \sum_{\mu, \nu > 0} a_{\mu}^{\dagger} a_{\bar{\mu}}^{\dagger} a_{\bar{\nu}} a_{\nu}$$



$$|BCS\rangle = \prod_k (U'_k + e^{-2i\phi} V'_k a_{\mathbf{k}+\mathbf{s},\uparrow}^{\dagger} a_{-\mathbf{k}+\mathbf{s},\downarrow}^{\dagger}) |0\rangle.$$



$$N = -i \frac{\partial}{\partial \phi}.$$

Figure 11.2. Exact energy spectrum for the Hamiltonian $H' = H - \lambda(N_0)n$.

What is the phase coherence?



Incoherent (normal) crowd:
each electron for itself



Phase-coherent (superconducting) condensate
of electrons

2. Josephson Junctions (DC and AC): In Condensed Matter

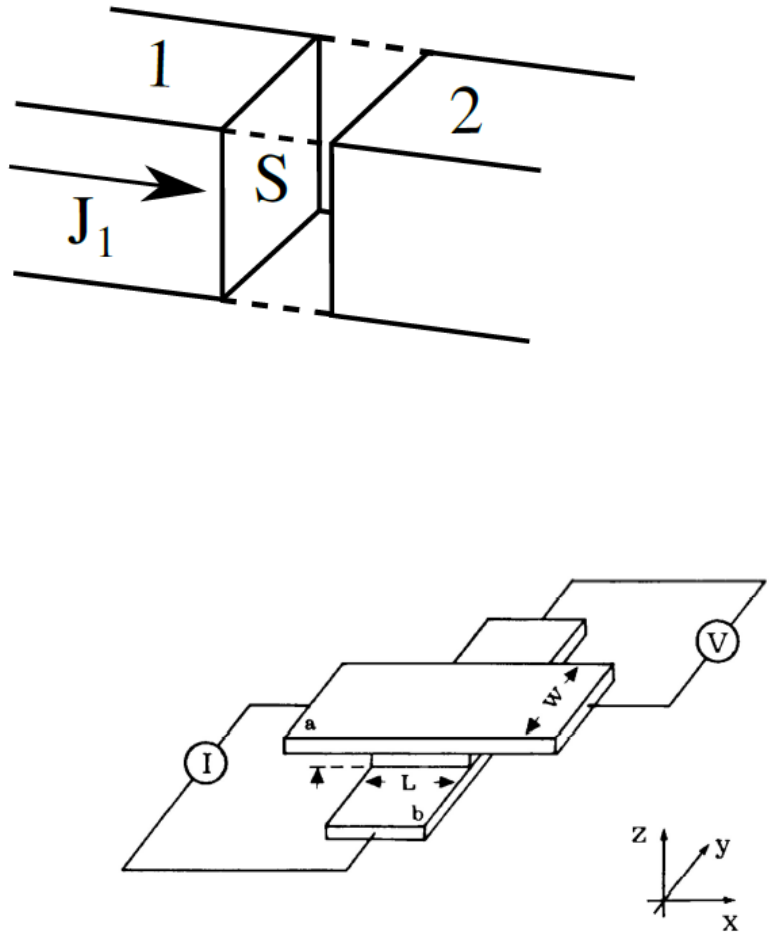


Figure 1.1 Tunneling junction of cross-type geometry. The dimensions are L and W; a and b are the two superconducting films.

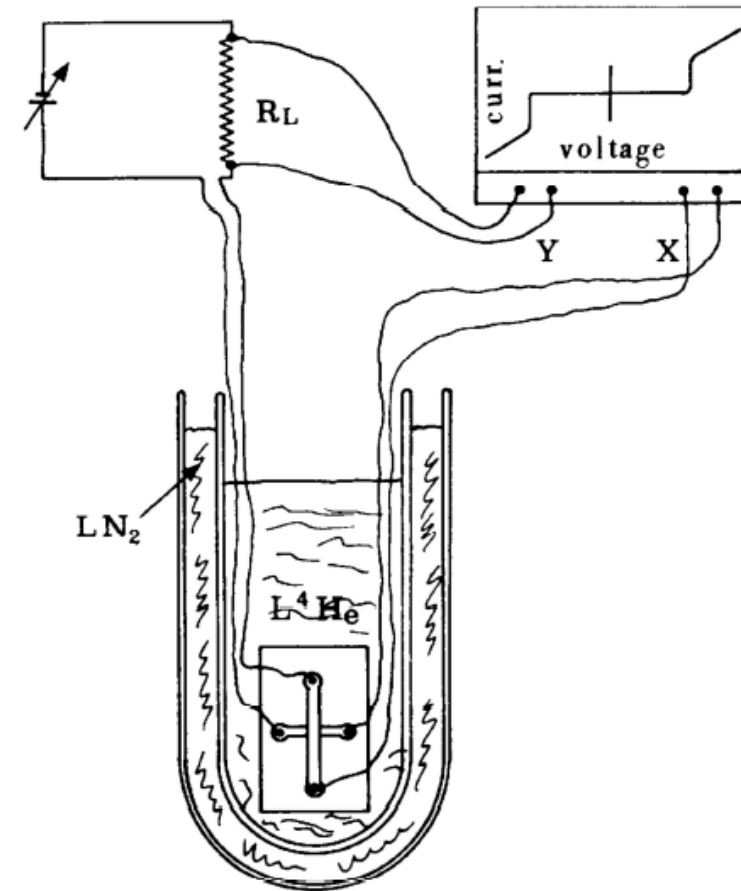


Figure 3.9 Schematic of the experimental apparatus used to measure the voltage-current characteristics of a junction. The inner dewar in which the sample is inserted is filled with liquid helium ($L^4\text{He}$), the outer dewar contains liquid nitrogen (LN_2).

3. Josephson Junctions (DC): A revision

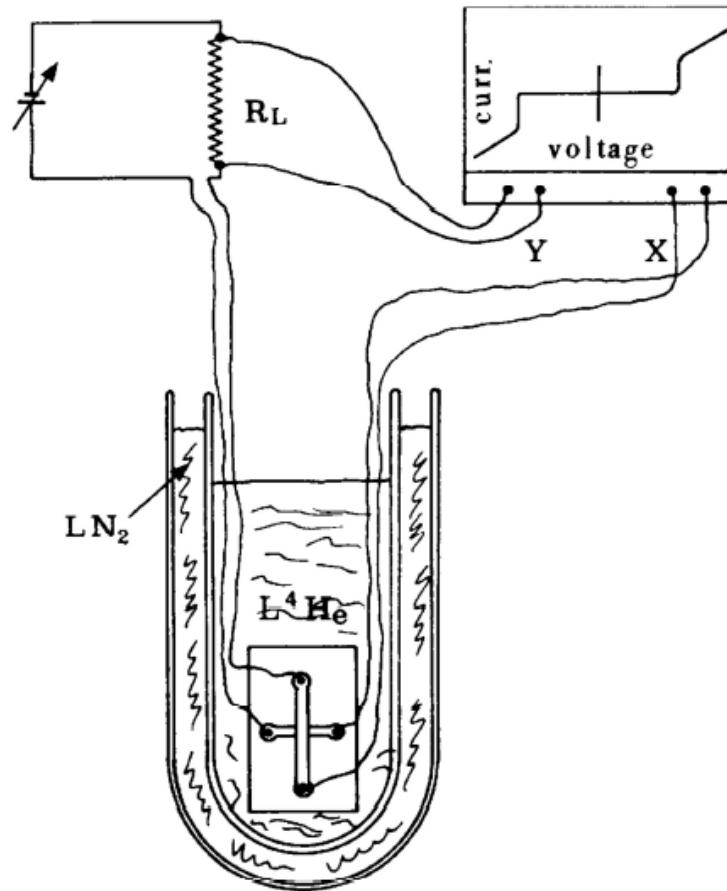


Figure 3.9 Schematic of the experimental apparatus used to measure the voltage-current characteristics of a junction. The inner dewar in which the sample is inserted is filled with liquid helium ($L^4\text{He}$), the outer dewar contains liquid nitrogen (LN_2).

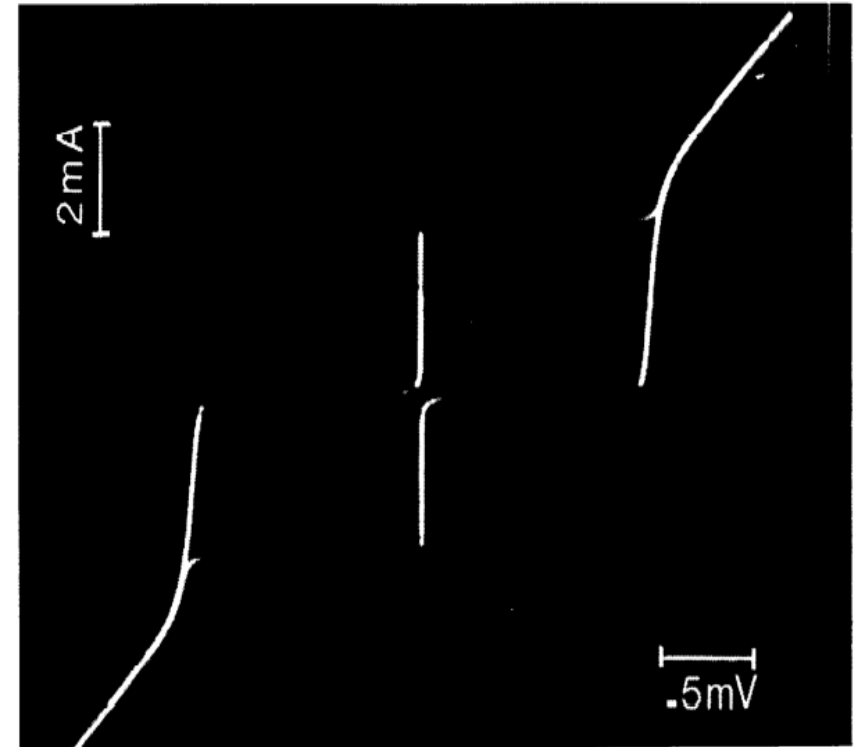
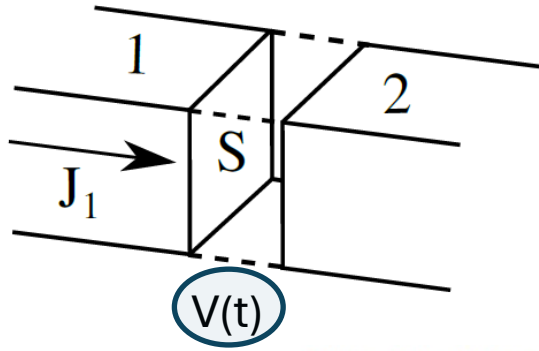


Figure 1.7 Typical voltage-current characteristic for a $\text{Sn-Sn}_x\text{O}_y\text{-Sn}$ Josephson junction at $T=1.52$ K. Horizontal scale: 0.5 mV/div; vertical scale: 2 mA/div.

3. Josephson Junctions (DC and AC): In Condensed Matter



$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \sqrt{n_A} e^{i\phi_A} \\ \sqrt{n_B} e^{i\phi_B} \end{pmatrix} = \begin{pmatrix} eV & K \\ K & -eV \end{pmatrix} \begin{pmatrix} \sqrt{n_A} e^{i\phi_A} \\ \sqrt{n_B} e^{i\phi_B} \end{pmatrix}$$

- Tunneling between 2 weakly coupled superconductors strongly depends on the phase difference: $\theta = \theta_L - \theta_R$



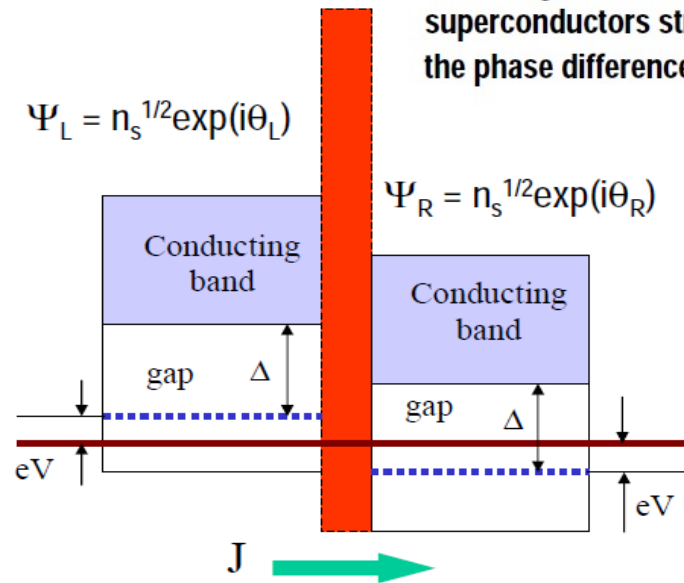
$$\dot{n}_A = \frac{2K\sqrt{n_A n_B}}{\hbar} \sin \varphi.$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$$I(t) = I_c \sin(\varphi(t))$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$$\varphi = \phi_B - \phi_A$$



Because of the phase coherence, each superconductor behaves as a single-level quantum-mechanical system

3. Josephson Junctions (DC and AC): A revision.

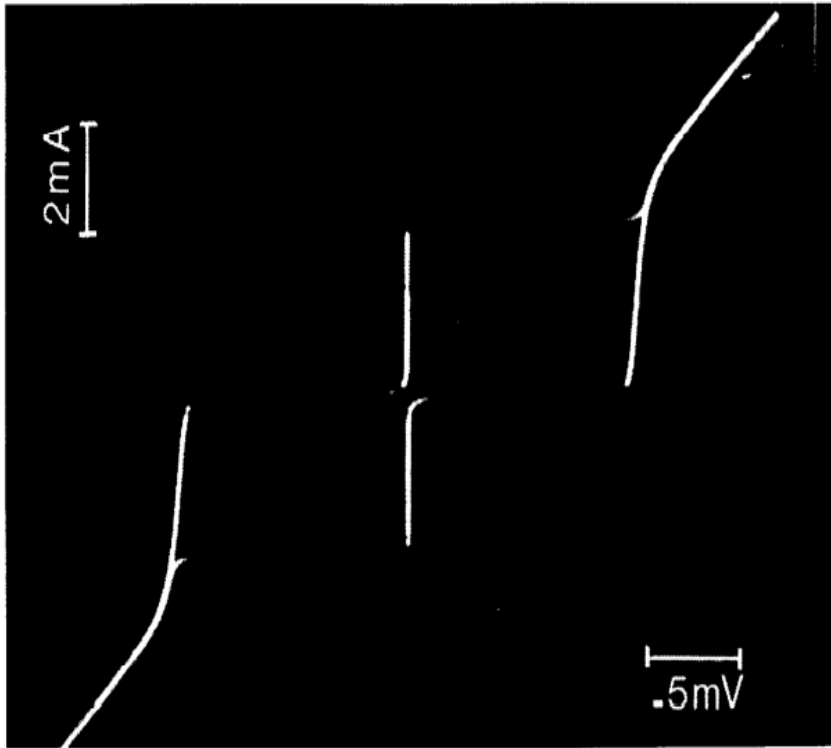
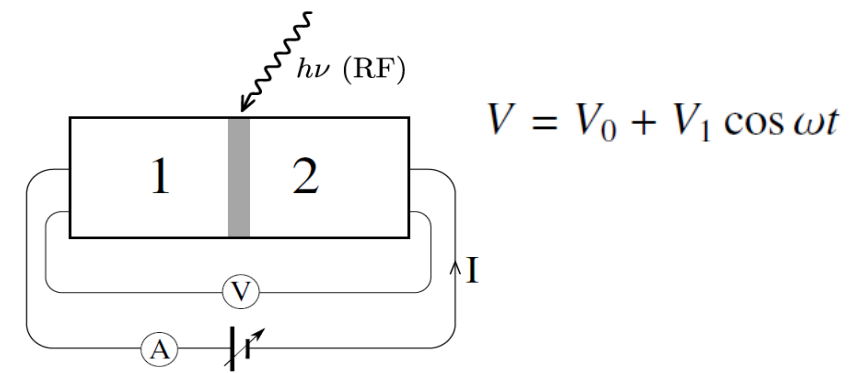


Figure 1.7 Typical voltage-current characteristic for a Sn-Sn_xO_y-Sn Josephson junction at $T = 1.52$ K. Horizontal scale: 0.5 mV/div; vertical scale: 2 mA/div.

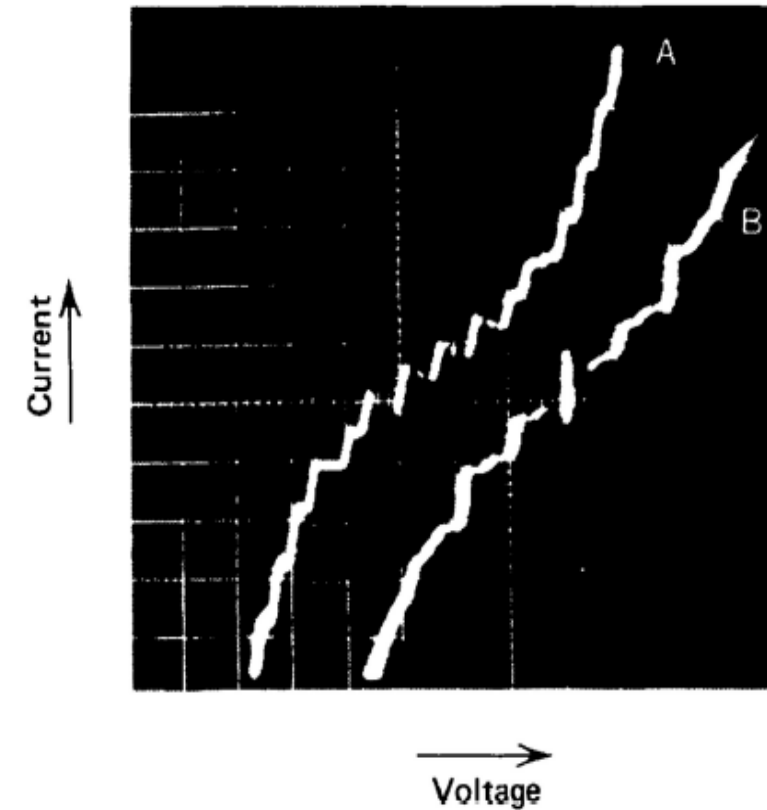
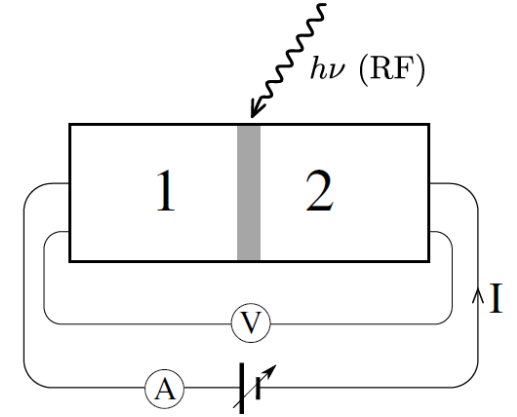
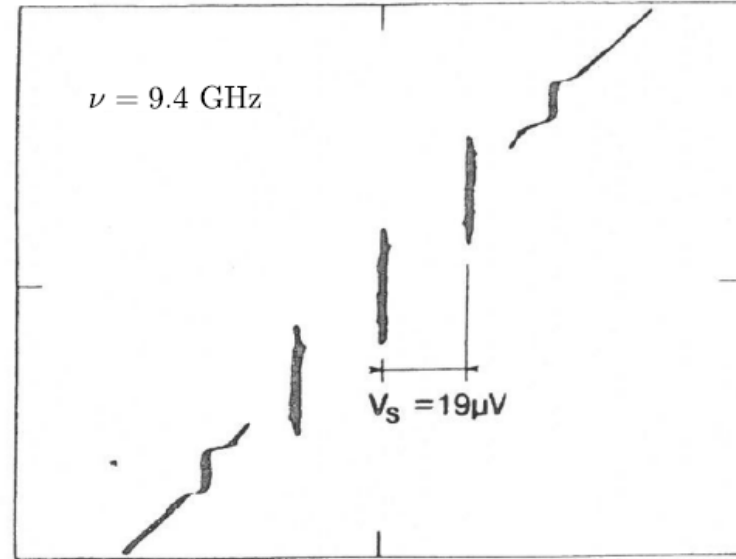


Figure 1.8 Microwave power at 9300 Mc/sec(A) and 24850 Mc/sec(B) produces many zero slope regions spaced at $h\nu/2e$ or $h\nu/e$. For A, $h\nu/e = 38.5$; for B, 103 μ V. For A, horizontal scale is 58.8 μ V/cm and vertical scale is 67 nA/cm; for B, horizontal scale is 50 μ V/cm and vertical scale is 50 μ A/cm. (After Shaprio 1963.)

3. Josephson Junctions (DC and AC): A revision.



$$V = V_0 + V_1 \cos \omega t$$

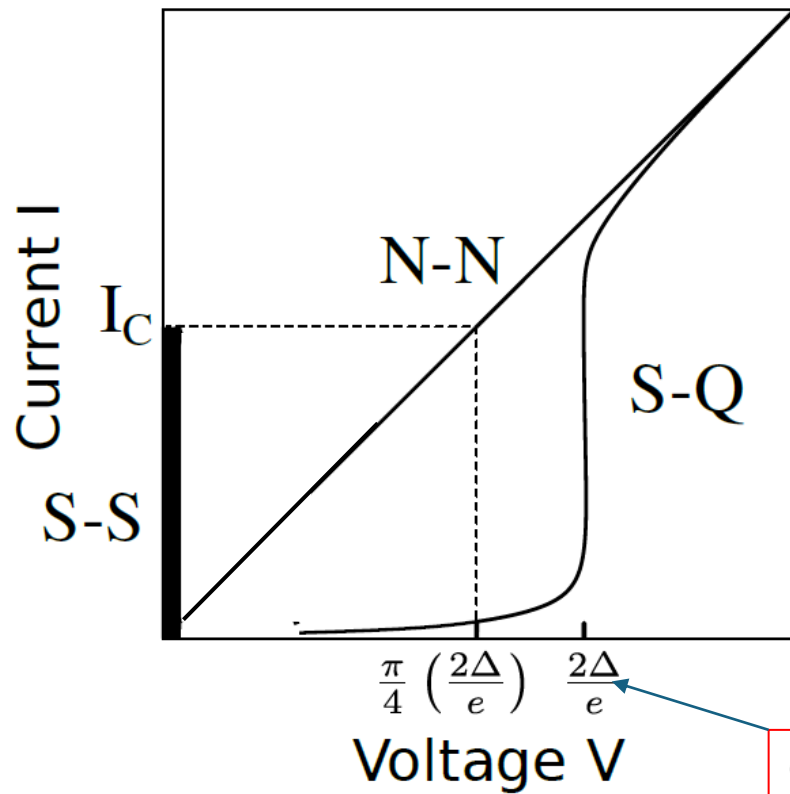
$$\varphi(t) = \varphi_0 + n\omega t + a \sin(\omega t)$$

$$I(t) = I_c \sum_{m=-\infty}^{\infty} J_m(a) \sin(\varphi_0 + (n + m)\omega t)$$

Oscilloscope presentation of current-versus-voltage characteristics of a tunnel junction at 4.2

K formed by an Al tip on a $(\text{La}_{0.925}\text{Sr}_{0.075})_2\text{CuO}_4$ sample ([82]). Steps induced by incident microwave radiation at 9.4 GHz, implying $V_0 = V_S = kK_J\nu = k \times 2.07 \times 10^{-12} \text{ mV s} \times 9.4 \times 10^9 \text{ s}^{-1} = 19.4 \mu\text{V} \times k$ ($k = 0, \pm 1, \pm 2, \dots$).

3. Josephson Junctions (DC and AC): A revision.



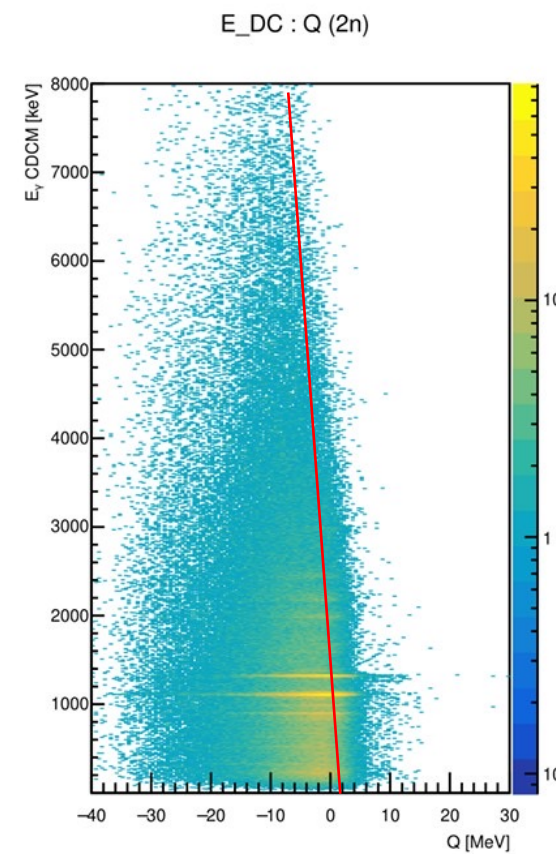
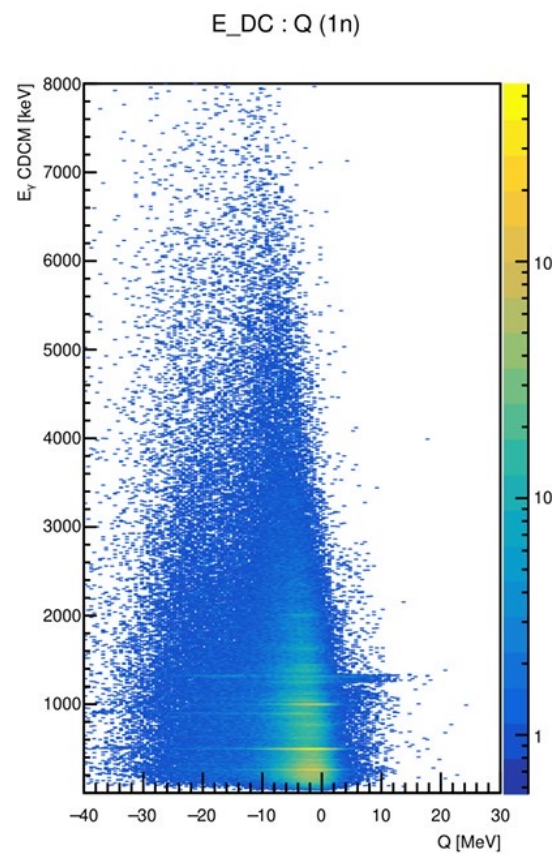
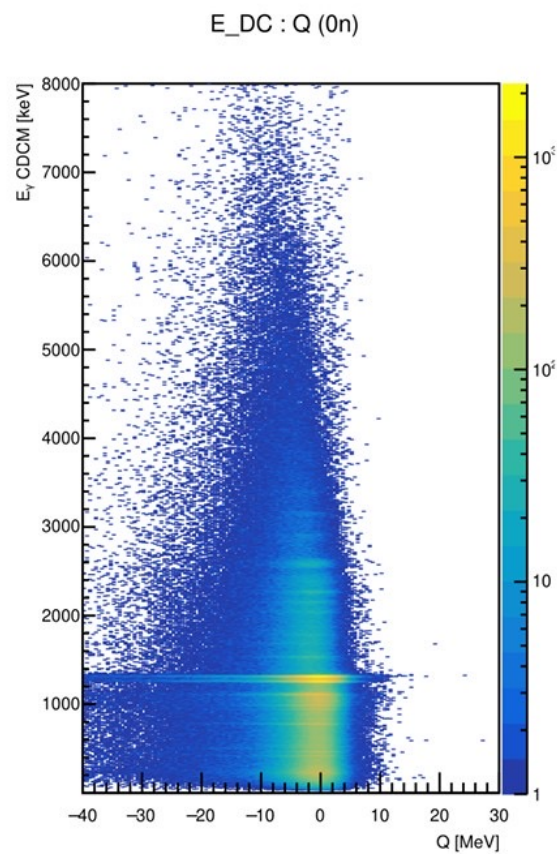
$$J = J_c \sin \left(\phi_{rel}(0) - \left(\frac{2eV}{\hbar} t \right) \right)$$

$$\omega_J = \frac{2eV}{\hbar}.$$

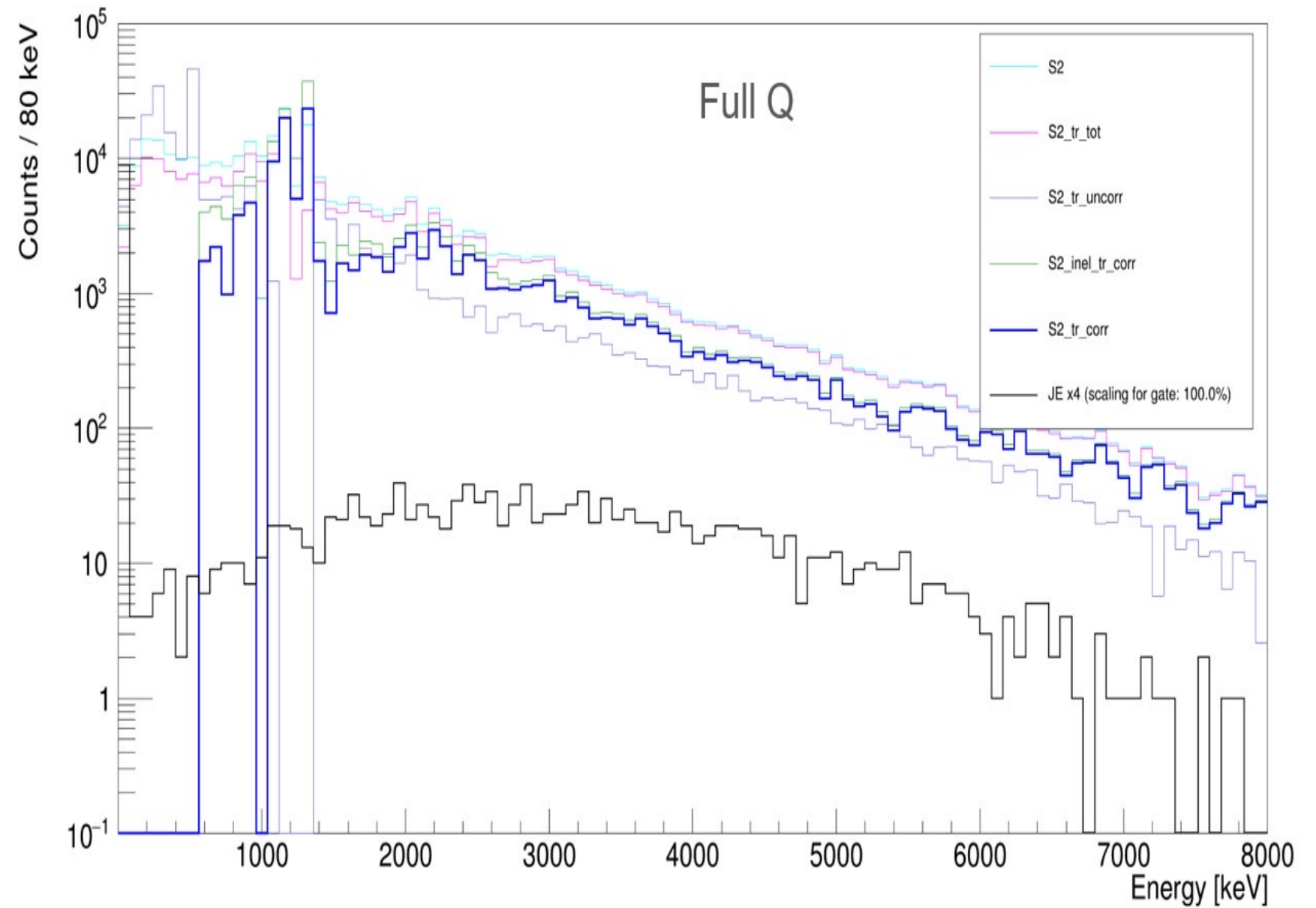
Results: $E_{\gamma}^{CM} - Q$



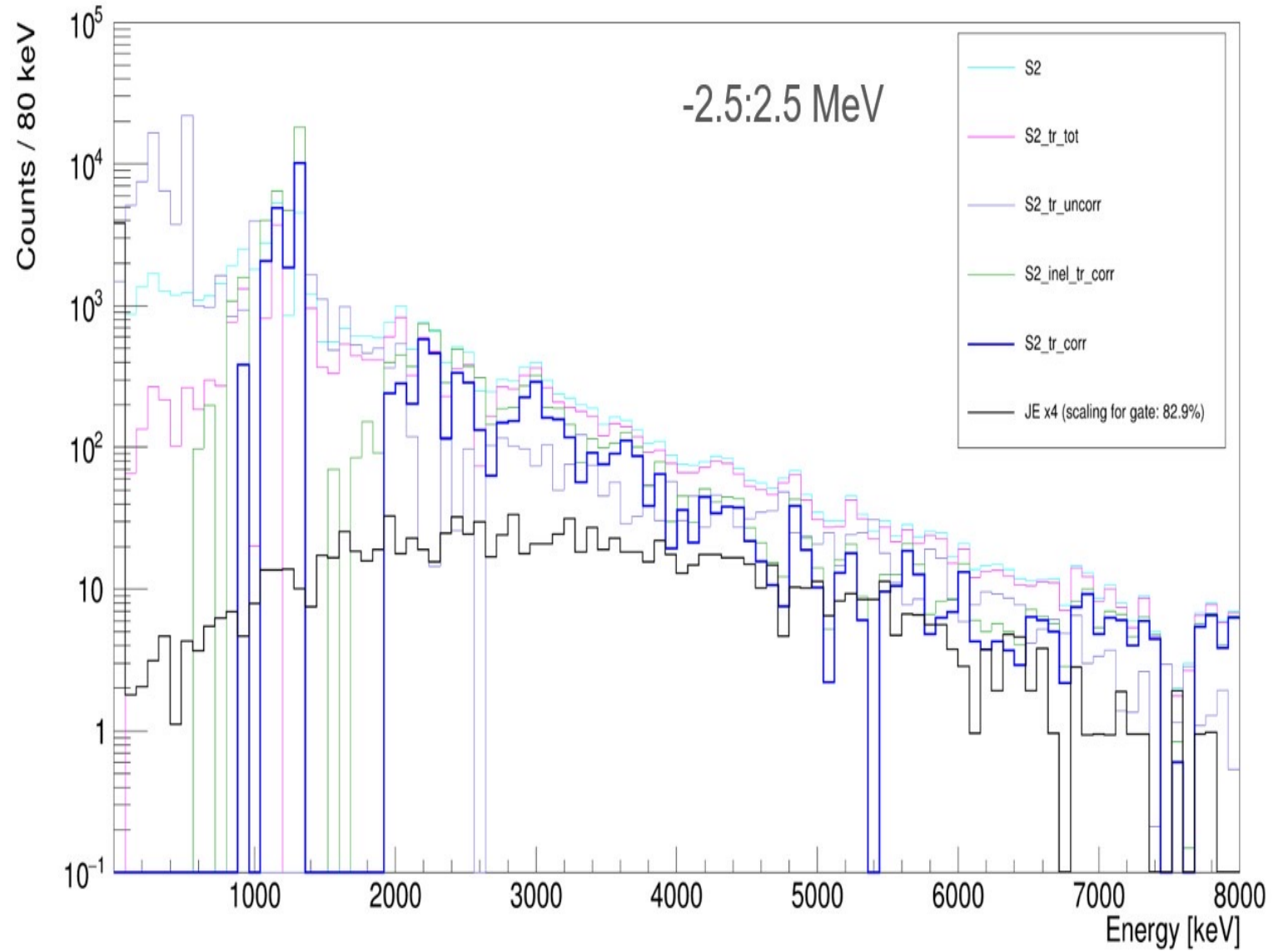
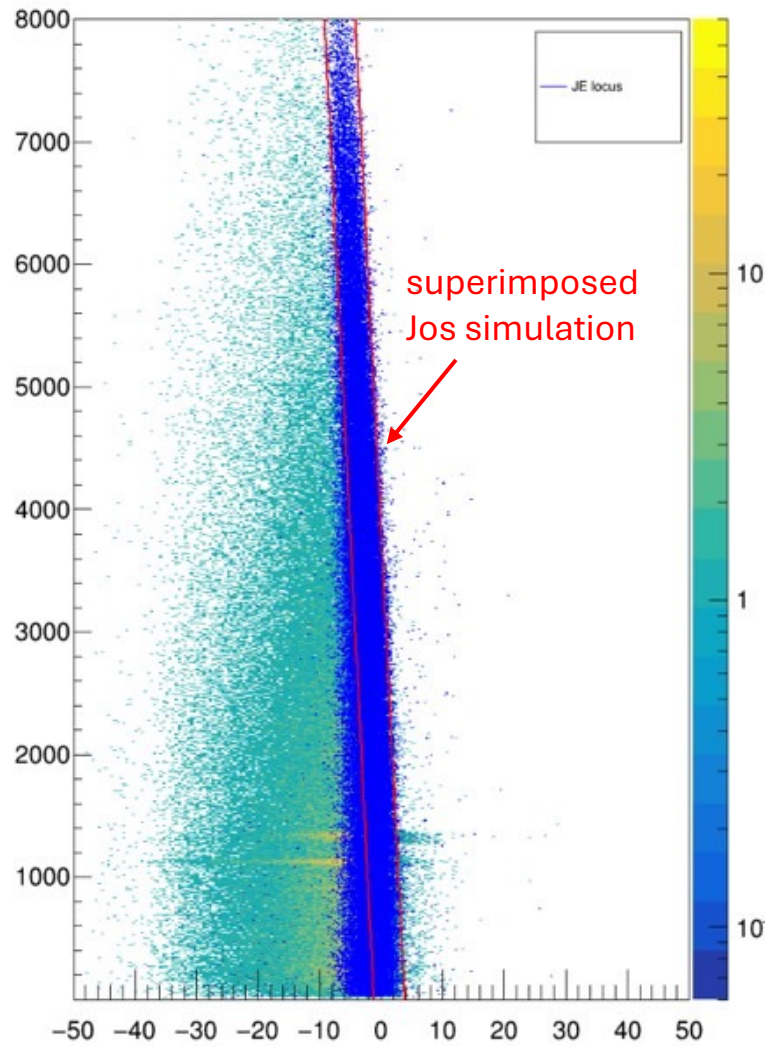
UNIVERSITÀ
DEGLI STUDI
DI PADOVA



Full Q data vs. prediction



Gating on Q-Egamma matrixes



Gracias por su atención!!

Velocity of the Transferred Cooper and (critical) Depairing velocity .

$$v_c = \frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$

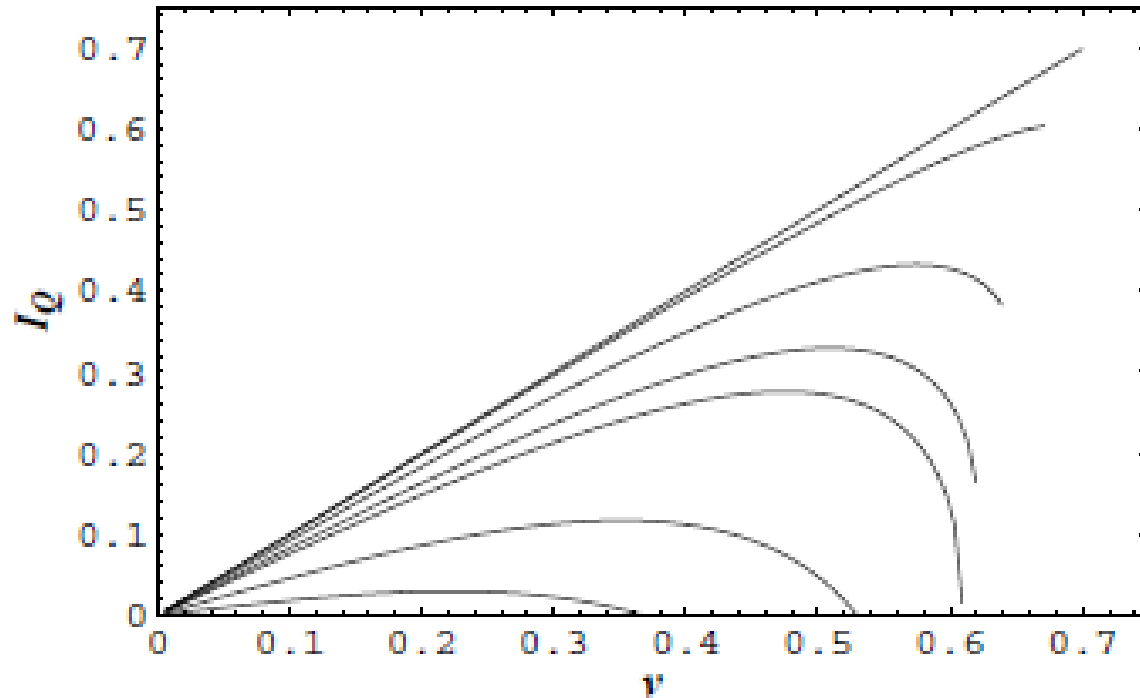


FIG. 5: Supercurrent I_Q (in units of I_Q^0) vs. superfluid velocity v (in unit of v_L) for various temperatures. From top to bottom: $T = (0.1, 0.25, 0.4, 0.5, 0.556, 0.75, 0.9) T_c^0$. The curves terminate at the critical velocities $v_c(T)$ appropriate to these temperatures. The maximum supercurrent for a particular curve determines the value of the critical current at that temperature.

Revisiting the critical velocity of a clean one-dimensional superconductor

Tzu-Chieh Wei

*Institute for Quantum Computing and Department of Physics and Astronomy,
University of Waterloo, Waterloo, ON N2L 3G1, Canada**

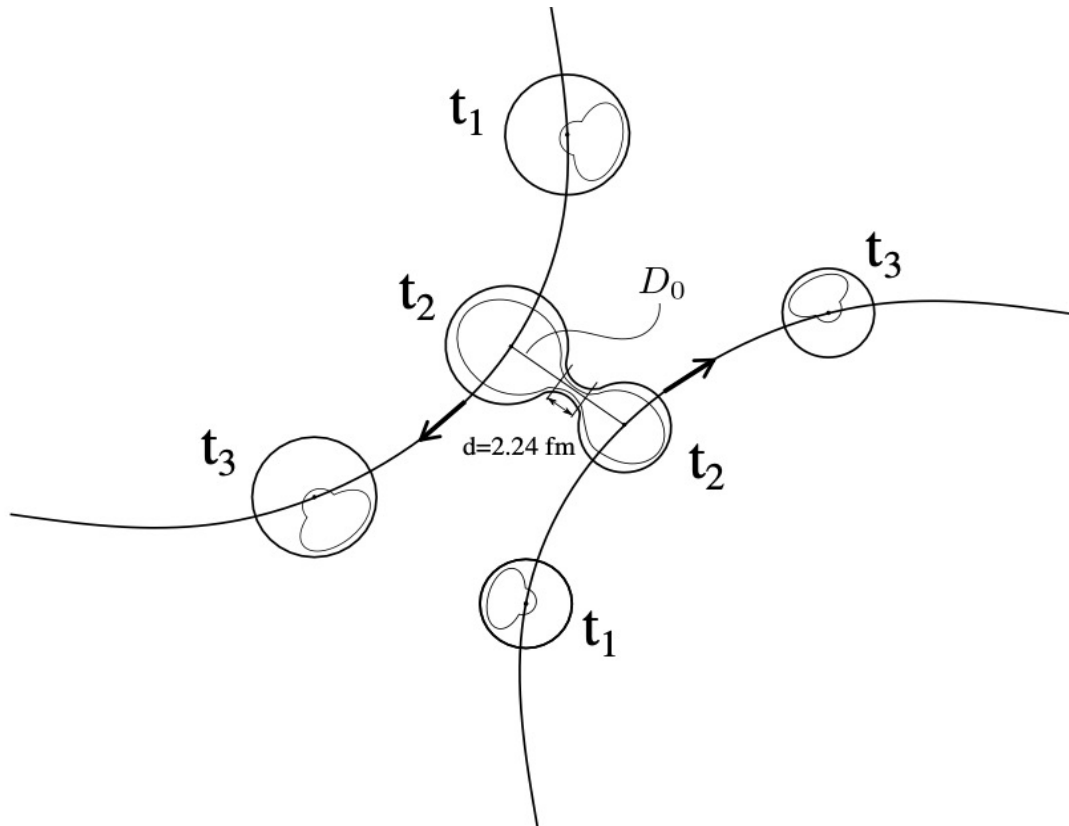
Paul M. Goldbart

*Department of Physics, Institute for Condensed Matter Theory,
and Frederick Seitz Materials Research Laboratory,
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, U.S.A.*

(Dated: April 15, 2009)

Critical depairing velocity

$$v_c = \frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$



$$v = \frac{(D_0) - (R_A + R_b + a)}{\tau_{coll}}$$

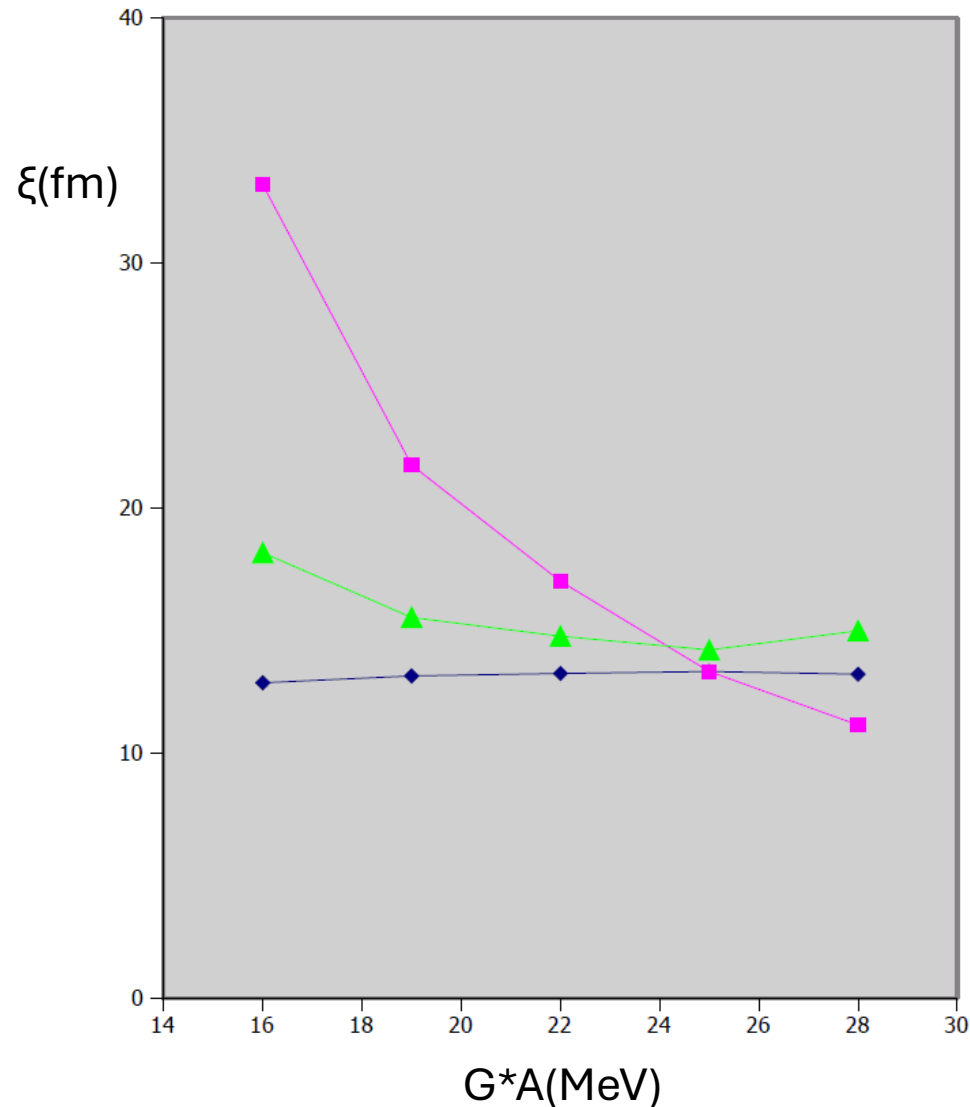
$$v < v_c \rightarrow P_2 > P_1$$

$$v = v_c \rightarrow P_2 \sim P_1$$

$$v > v_c \rightarrow P_2 < P_1$$

Velocity of the Transferred (nuclear) Cooper Pair.

Simulated data (DWBA calc's) as a function of pairing interaction.



Magenta curve

$$\frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$

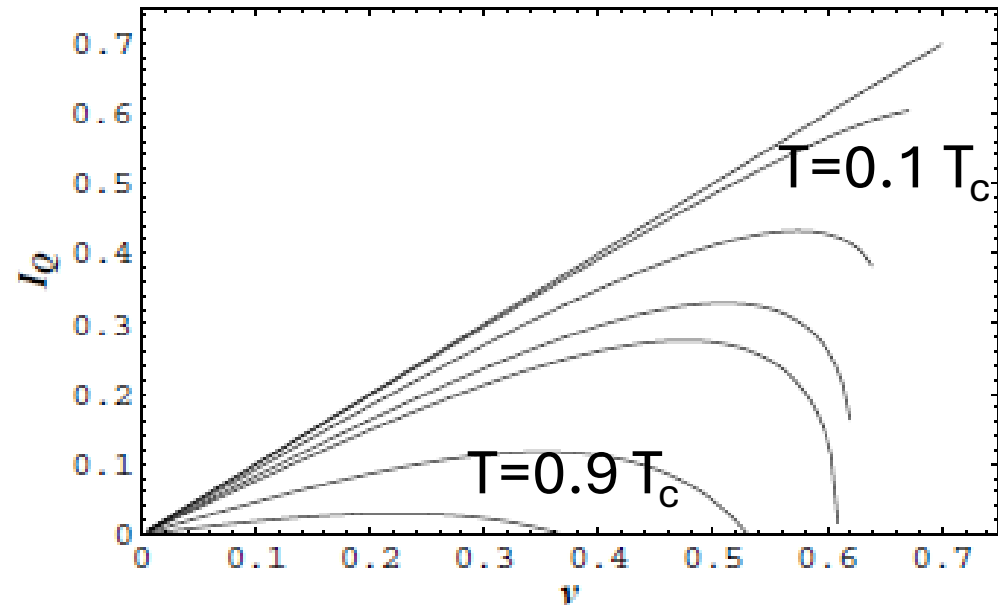
Green curve

$$\frac{(D_0)_c - (R^{(A)} + R^{(b)} + a)}{\tau_{coll}} = \frac{1}{m} \frac{\hbar}{\xi}$$

Some de-pairing velocity effect seems to be present in DWBA

Critical depairing velocity

$$v_c = \frac{2\Delta}{mv_F} = \frac{\hbar}{m\xi}$$



$$v = \frac{(D_0)_c - (R_A + R_b + a)}{\tau_{coll}}$$

$$v < v_c \rightarrow P_2 > P_1$$

$$v = v_c \rightarrow P_2 \sim P_1$$

$$v > v_c \rightarrow P_2 < P_1$$

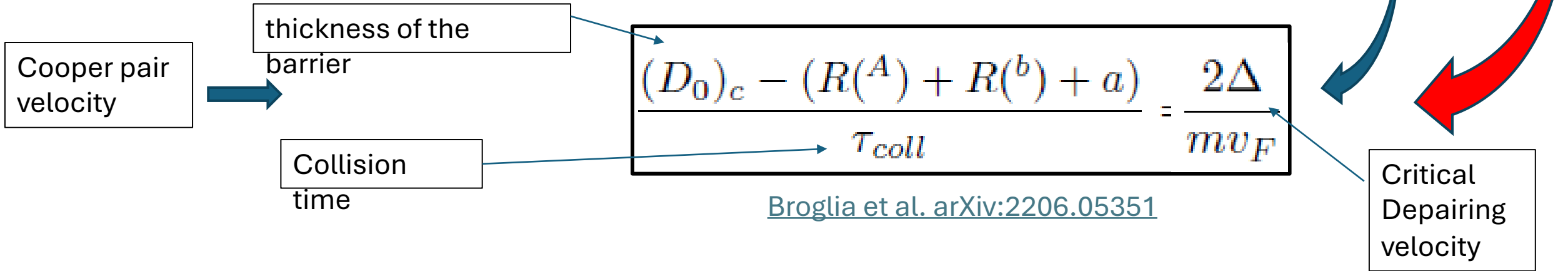
6. Velocity of the Transferred (nuclear) Cooper Pair larger than the critical depairing velocities leads to $P2 < P1$

If the Cooper-pair velocity between the nuclei is larger than v_{crit}

Cooper pairs breaking is most likely $\rightarrow P1 > P2$ (Do > Do_crit)

Conversely if $P2 > P1$ Cooper pairs velocity is smaller than v_{crit} (Do < Do_crit)

When $P2 = P1$ one can see that in fact both velocities are equal (Critical situation)



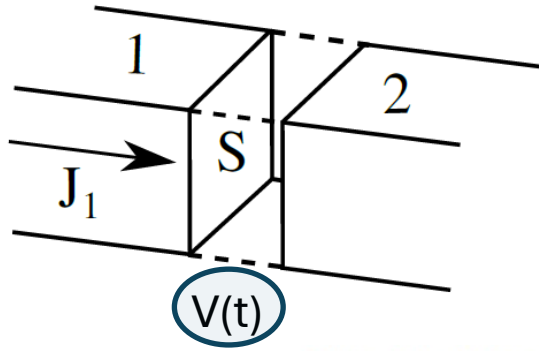
The Scientific Activity of Ricardo Broglia



Francisco Barranco, Thomas Døssing, Guido Tiana, and Enrico Vigezzi

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2026
F. Barranco et al. (eds.), *Proceedings of the International School
of Physics “Enrico Fermi”—Course 213*, Springer Proceedings in Physics 353,
https://doi.org/10.1007/978-3-030-31388-4_1

3. Josephson Junctions (DC and AC): In Condensed Matter



$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \sqrt{n_A} e^{i\phi_A} \\ \sqrt{n_B} e^{i\phi_B} \end{pmatrix} = \begin{pmatrix} eV & K \\ K & -eV \end{pmatrix} \begin{pmatrix} \sqrt{n_A} e^{i\phi_A} \\ \sqrt{n_B} e^{i\phi_B} \end{pmatrix}$$

- Tunneling between 2 weakly coupled superconductors strongly depends on the phase difference: $\theta = \theta_L - \theta_R$



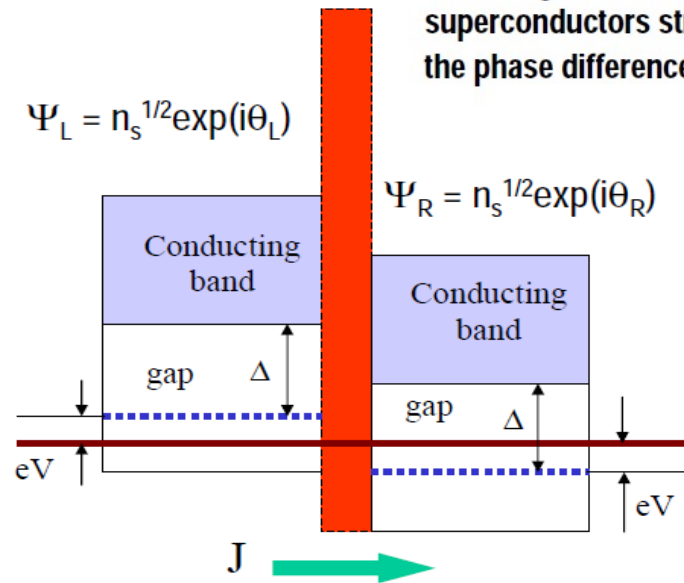
$$\dot{n}_A = \frac{2K\sqrt{n_A n_B}}{\hbar} \sin \varphi.$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$$I(t) = I_c \sin(\varphi(t))$$

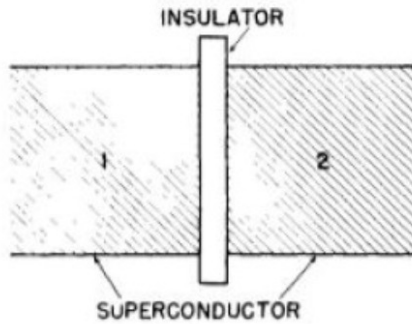
$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$$\varphi = \phi_B - \phi_A$$

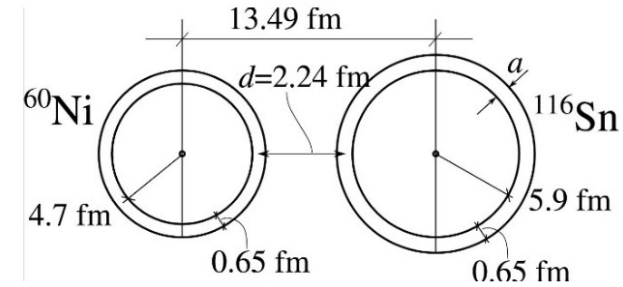


Because of the phase coherence, each superconductor behaves as a single-level quantum-mechanical system

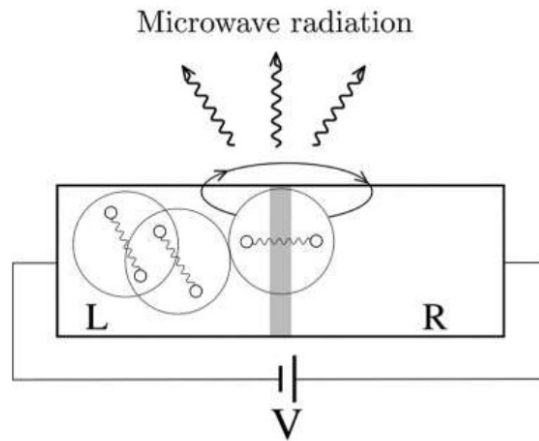
from macroscales to femtoscales



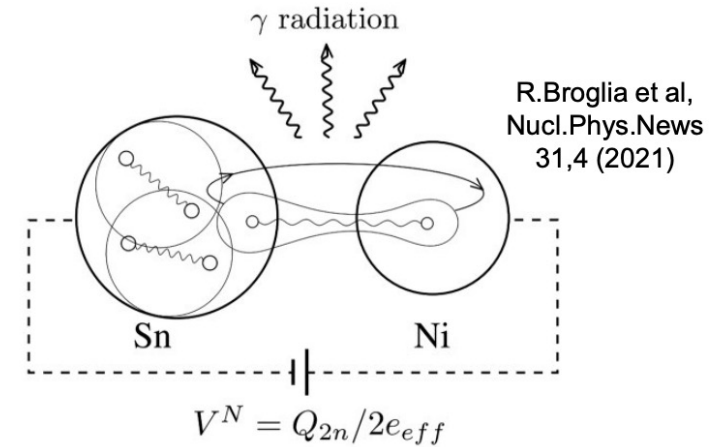
dielectric layer



surface-surface distance of nuclei at the distance of closest approach



applied voltage



reaction Q-value divided by the effective charge of the tunneling Cooper pair

One-body operators

$$\langle \Psi_{\text{BCS}} | \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} | \Psi_{\text{BCS}} \rangle$$

. Because the wave function is a product of independent $\mathbf{k}$ -states, the expectation value for a specific state $\mathbf{k}$ only depends on its own term in the product. Let's isolate state $\mathbf{k}$:

$$\langle \Psi_{\mathbf{k}} | \xi_{\mathbf{k}} (c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\uparrow} + c_{-\mathbf{k}\downarrow}^\dagger c_{-\mathbf{k}\downarrow}) | \Psi_{\mathbf{k}} \rangle$$

where $|\Psi_{\mathbf{k}}\rangle = (u_{\mathbf{k}} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger) |0\rangle$.

Since $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are chosen to be real numbers, this yields:

$$\langle K_{\text{kin}} \rangle = 2 \sum_{\mathbf{k}} \xi_{\mathbf{k}} v_{\mathbf{k}}^2$$

Two-body operators

First we evaluate the general two body term: $\langle \Psi_{\text{BCS}} | c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} | \Psi_{\text{BCS}} \rangle$. This operator destroys a pair at $\mathbf{k}'$ and creates one at $\mathbf{k}$. Because all $\mathbf{k}$ -modes are independent in the product, this term factors cleanly into two separate pieces (assuming $\mathbf{k} \neq \mathbf{k}'$):

$$\langle \Psi_{\text{BCS}} | c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} | \Psi_{\text{BCS}} \rangle = \left(\langle \Psi_{\mathbf{k}} | c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger | \Psi_{\mathbf{k}} \rangle \right) \cdot \left(\langle \Psi_{\mathbf{k}'} | c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} | \Psi_{\mathbf{k}'} \rangle \right)$$

Let's compute the right piece first:

$$c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} (u_{\mathbf{k}'} + v_{\mathbf{k}'} c_{\mathbf{k}'\uparrow}^\dagger c_{-\mathbf{k}'\downarrow}^\dagger) |0\rangle = v_{\mathbf{k}'} |0\rangle$$

Taking the inner product with $\langle \Psi_{\mathbf{k}'} | = \langle 0 | (u_{\mathbf{k}'} + v_{\mathbf{k}'} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow})$ yields:

$$\langle 0 | (u_{\mathbf{k}'} + \dots) v_{\mathbf{k}'} |0\rangle = u_{\mathbf{k}'} v_{\mathbf{k}'}$$

By symmetry, the left piece is also $u_{\mathbf{k}} v_{\mathbf{k}}$. Therefore, the potential energy expectation value simplifies to:

$$\langle E_{\text{pair}} \rangle = -G \sum_{\mathbf{k}, \mathbf{k}'} u_{\mathbf{k}} v_{\mathbf{k}} u_{\mathbf{k}'} v_{\mathbf{k}'}$$

$$(a_\beta(t))_{succ} = \left(\frac{1}{i\hbar}\right)^2 \sum_{\gamma \neq \beta} \int_{-\infty}^t dt' \langle \Psi_\beta | V_\beta - \langle V_\beta \rangle | \psi_\gamma \rangle_{\mathbf{R}_{\beta,\gamma}(t')} \\ \times \int_{-\infty}^{t'} dt'' \langle \psi_\gamma | V_\alpha - \langle V_\alpha \rangle | \Psi_\alpha \rangle_{\mathbf{R}_{\gamma,\alpha}(t'')} e^{i\frac{E_\gamma}{\hbar}(t''-t')}, \quad (120)$$

where

$$\exp\left(i\frac{E_\gamma}{\hbar}(t''-t')\right) = \cos\left(\frac{E_\gamma}{\hbar}(t''-t')\right) + i \sin\left(\frac{E_\gamma}{\hbar}(t''-t')\right). \quad (121)$$

Assuming the quasiparticle excitation energies E_γ to be much larger than the reaction Q -value, the periodic functions in (121) oscillate so rapidly, that the outcome of the integration in (119) amounts to nothing unless $t' \approx t''$. In other words and further assuming that the matrix elements (form-factors) are smooth functions of time along the trajectories of relative motion one can write $\sum_{\gamma \neq \beta} \exp\left(i\frac{E_\gamma}{\hbar}(t''-t')\right) \approx \frac{1}{\Delta E} \int dE \exp\left(i\frac{E}{\hbar}(t''-t')\right) \approx \frac{\hbar}{\Delta E} \times \delta(t''-t')$, where $1/\Delta E$ is the average density of levels of the two-quasiparticle states. Making use of Eq. (9), one can write Eq. (120) as,

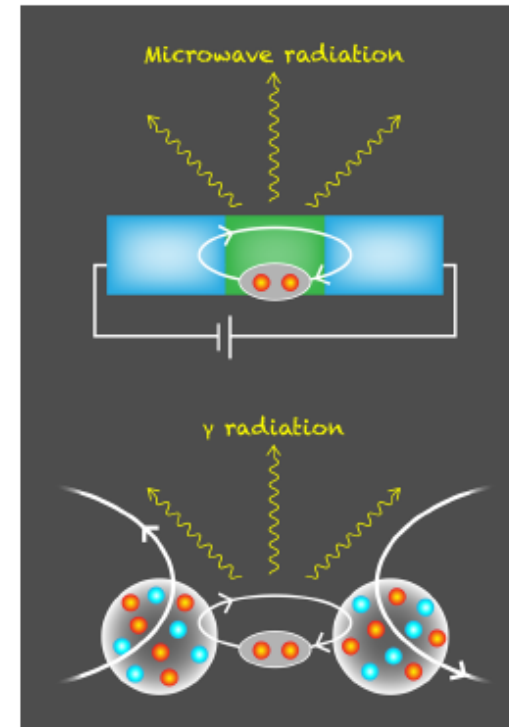
$$(a_\beta(t))_{succ} = \left(\frac{1}{i\hbar}\right)^2 \int_{-\infty}^t dt' \langle \psi_\beta | (V_\beta - \langle V_\beta \rangle) | \psi_\gamma \rangle_{\mathbf{R}_{\beta,\gamma}(t')} \\ \times \frac{\hbar}{\Delta E} \mathbf{R}_{\gamma,\alpha}(t') \langle \psi_\gamma | V_\alpha - \langle V_\alpha \rangle | \psi_\alpha \rangle e^{i\frac{Q_{2n}}{\hbar}t'}, \quad (122)$$

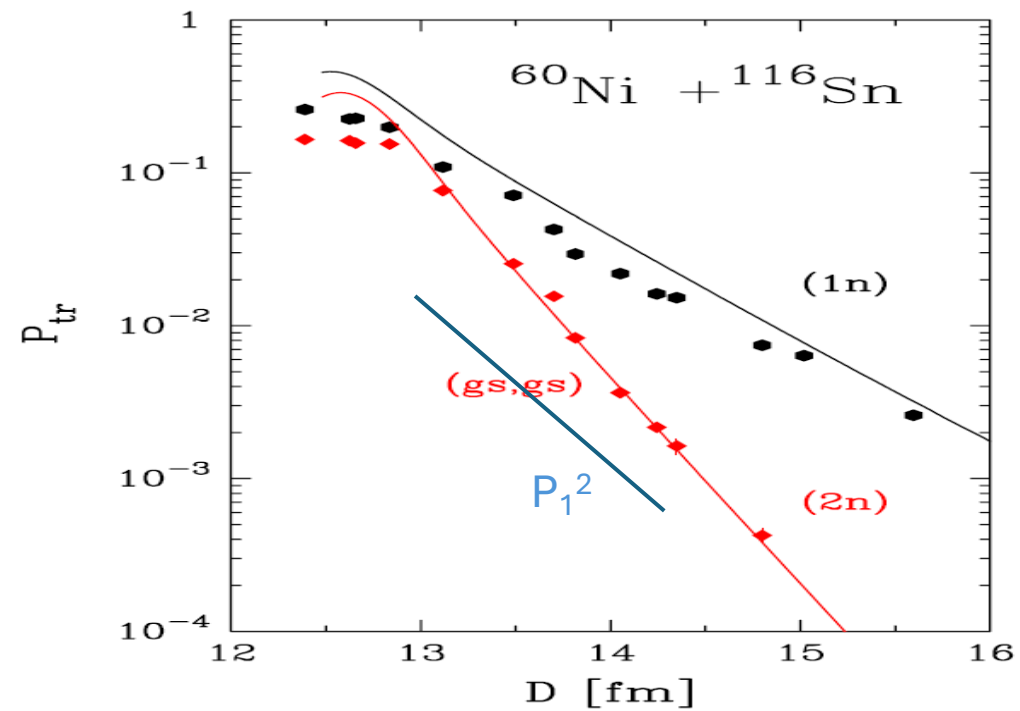
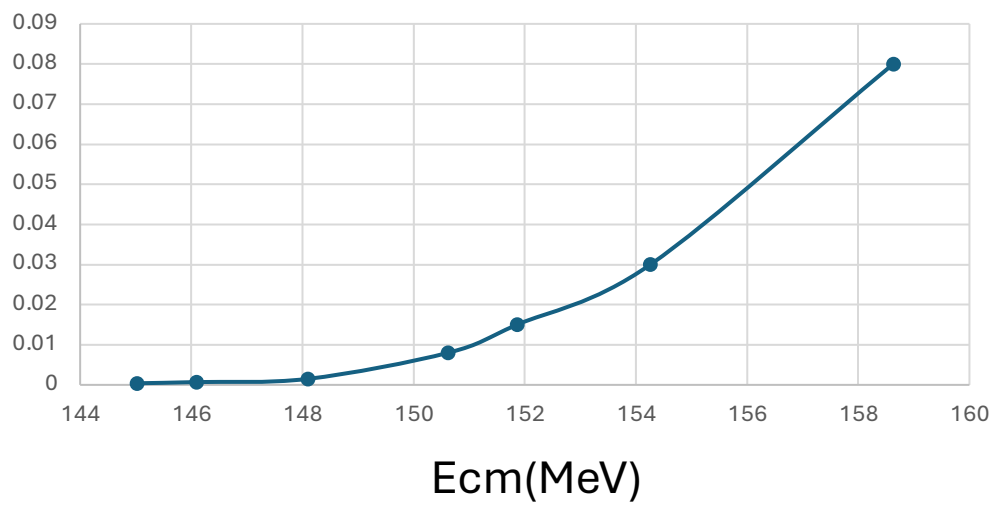
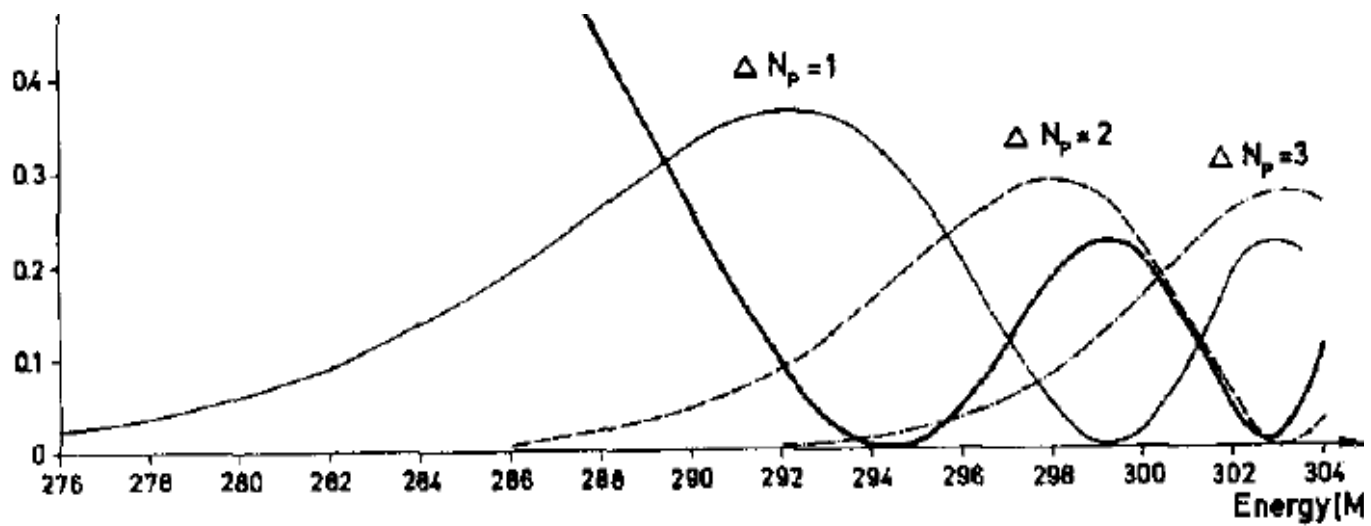
where $(Q_{2n}/\hbar) = \omega_J^N$ is the nuclear parallel to the angular Josephson frequency, while $Q_{2n}/(2 \times e_{eff}) = V^N$ is the nuclear equivalent to the weak link direct current bias. The quantity e_{eff} is the effective charge of a nucleon which, for the reaction $^{116}_{50}\text{Sn} + ^{60}_{28}\text{Ni} \rightarrow ^{114}_{50}\text{Sn} + ^{62}_{28}\text{Ni}$ is $e_{eff} = -e((50+28)/(114+62)) = -e \times 0.44$. It is of notice that the fact that (119), and thus (122) has the right gauge phasing, is in keeping with the fact that the two-particle reaction mechanism at the basis of (120) has been worked out fully quantum mechanically in a particle conserving, gauge invariant representation.

The Tiniest Superfluid Circuit in Nature

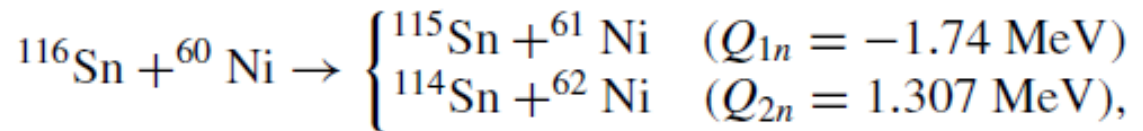
A new analysis of heavy-ion collision experiments uncovers evidence that two colliding nuclei behave like a Josephson junction—a device in which Cooper pairs tunnel through a barrier between two superfluids.

By Piotr Magierski





4. The Legnaro's $^{116}\text{Sn}+^{60}\text{Ni}$ 2NT and 1NT data at different E_{cm} : DWBA results

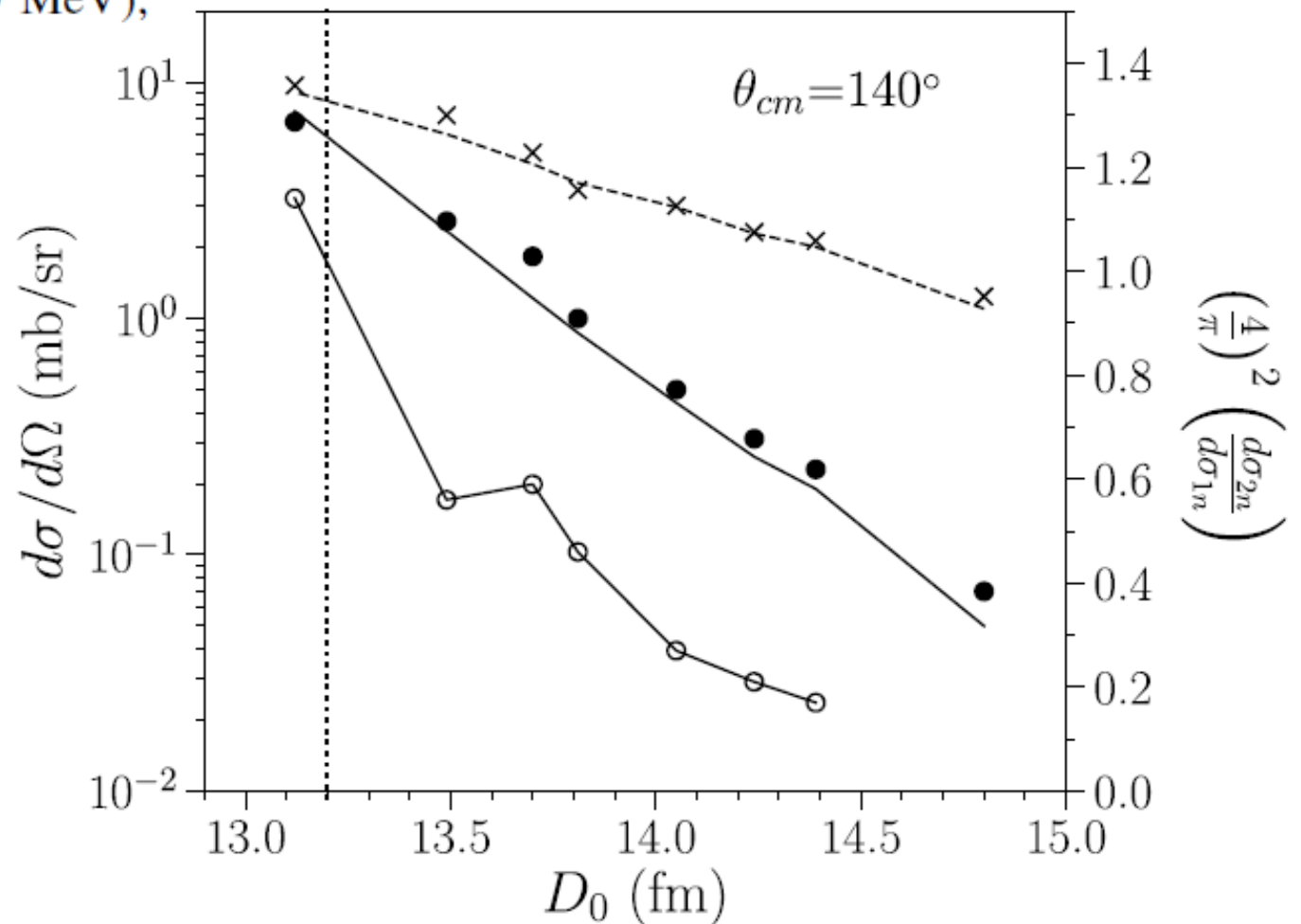


MAIN INGREDIENTS

Montanari's Opt. Potentials (Pollarollo)

$G=25/A \text{ MeV}$

s-p levels from WS potentials



Nuclear superfluidity: BCS ground state

$$|BCS\rangle = \prod_k \left(U'_k + e^{-2i\phi} V'_k a_{\mathbf{k}+\mathbf{s},\uparrow}^\dagger a_{-\mathbf{k}+\mathbf{s},\downarrow}^\dagger \right) |0\rangle.$$

$$\begin{aligned} \alpha_v^\dagger &= u_v a_v^\dagger - v_v a_{\bar{v}} \\ \alpha_{\bar{v}}^\dagger &= u_v a_{\bar{v}}^\dagger + v_v a_v \\ \alpha_v &= u_v a_v - v_v a_{\bar{v}}^\dagger \\ \alpha_{\bar{v}} &= u_v a_{\bar{v}} + v_v a_v^\dagger \end{aligned} \quad \begin{aligned} a_v^\dagger &= u_v \alpha_v^\dagger + v_v \alpha_{\bar{v}} \\ a_{\bar{v}}^\dagger &= u_v \alpha_{\bar{v}}^\dagger - v_v \alpha_v \\ a_v &= u_v \alpha_v + v_v \alpha_{\bar{v}}^\dagger \\ a_{\bar{v}} &= u_v \alpha_{\bar{v}} - v_v \alpha_v^\dagger. \end{aligned}$$

$$\begin{aligned} H' &= H - \lambda n \\ &= \sum_{v>0} (\varepsilon_v^{(0)} - \lambda)(a_v^\dagger a_v + a_{\bar{v}}^\dagger a_{\bar{v}}) - G \sum_{\mu, v>0} a_\mu^\dagger a_\mu^\dagger a_{\bar{v}} a_v, \end{aligned}$$

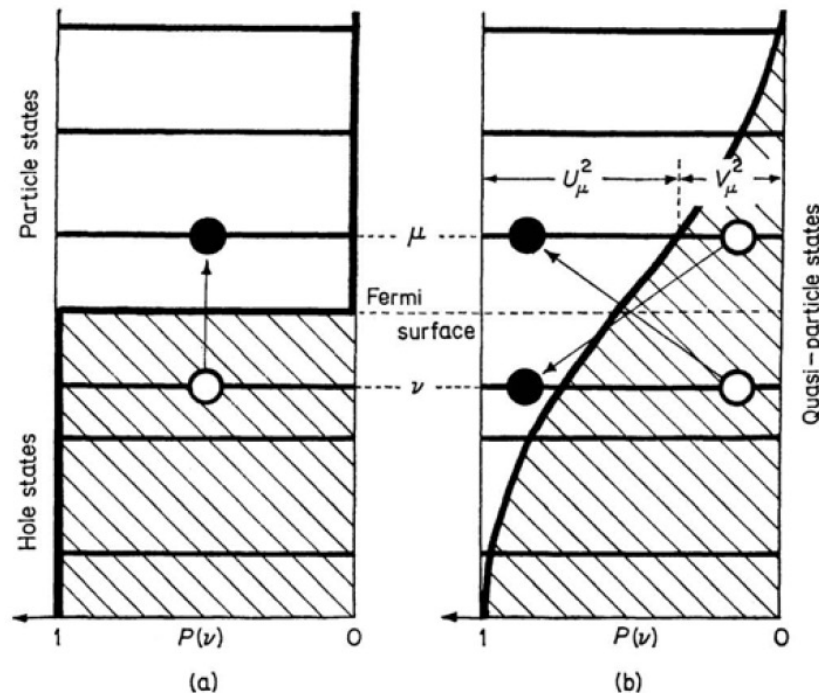
$$\sum_{v>0} \left\{ 1 - \frac{\varepsilon_v - \lambda}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\} = N.$$

$$\frac{G}{2} \sum_{v>0} \frac{1}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} = 1.$$

$$u_v^2 = \frac{1}{2} \left\{ 1 + \frac{\varepsilon_v - \lambda}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\}$$

$$v_v^2 = \frac{1}{2} \left\{ 1 - \frac{\varepsilon_v - \lambda}{[(\varepsilon_v - \lambda)^2 + \Delta^2]^{\frac{1}{2}}} \right\}.$$

2. Nuclear Superfluidity: Occupations.



In figure 11.4 are shown experimental values of v_j^2 obtained by Cohen and Price (1961) from (d,p) and (d,t) experiments on the Sn

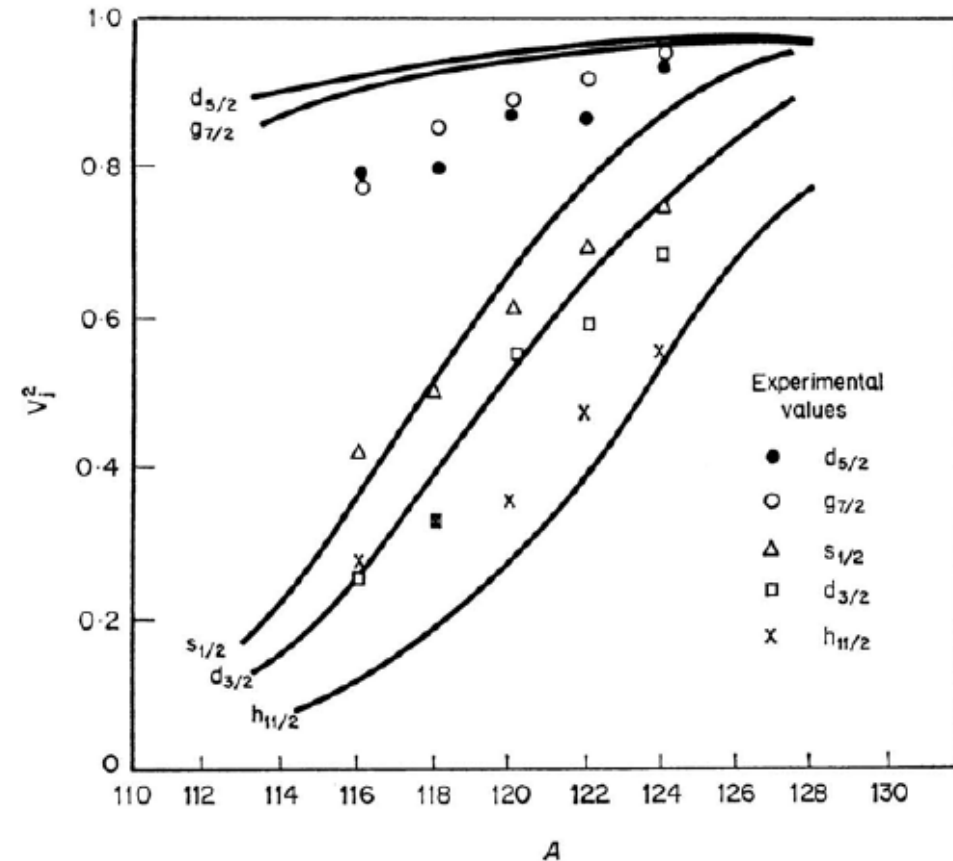


Figure 11.4. Experimental values of v_j^2 obtained by Cohen and Price (1961) from (d,p) and (d,t) experiments on the Sn isotopes, compared to the theoretical values of Kisslinger and Sorensen (1960).