

EXOTIC CLUSTERING OF LIGHT NUCLEI



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10 September 2026,
Many-body correlations, phase transitions,
and clustering: how to identify them?
CEA Saclay, France

Home > The European Physical Journal A > Article

Theoretical studies of α clustering in nuclei and beyond

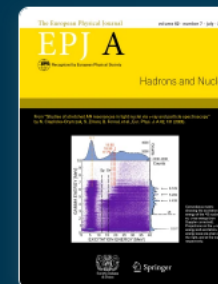
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The European Physical Journal A

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Abstract

This article comprises three sections after an introduction. Section 2 starts with a quick review of the results of *ab initio* no-core shell model calculations by Monte Carlo Shell Model on light nuclei. It is shown that α clustering arises in such first principles calculations for $^{8,10,12}\text{Be}$ and ^{12}C with the Daejeon16 and JISP16 interactions. The α clustering occurs even in well bound states such as the ground state of ^{12}C . The Hoyle state is shown to be dominated by α clustering in triangular configurations. The crossover between clustering and nuclear matter is demonstrated. As the ground and Hoyle states show strong deformations, they are also good cases to investigate rotational excitations. As

Part of a collection:

Clustering as a Window on the Hierarchical Structure of Quantum Systems

Sections

Figures

References

Abstract

Introduction

First-principle realization of α clustering and du...

A review on cluster model approaches

Cluster-shell competition and its modeling

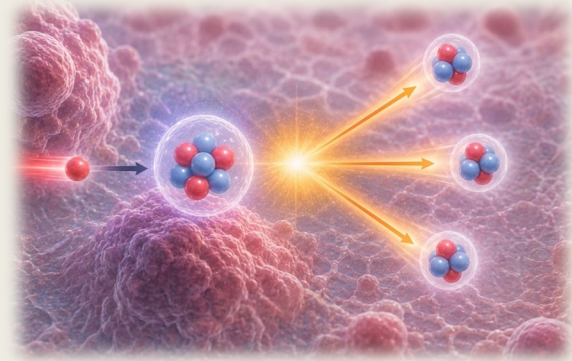
Remarks and prospects transcending sections

Data availability statement

Code availability statement

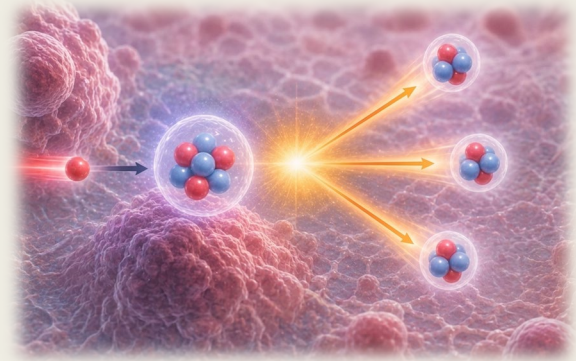
Contents

- 1: General description of cluster-shell competition
- 2: How we identify the cluster states?
- 3: Clustering of $^{12}\text{C}+^{12}\text{C}$ and astrophysical application
- 4: $^{11}\text{B}+p$ reaction and 3 alpha cluster structure in ^{12}C



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Nuclear structure

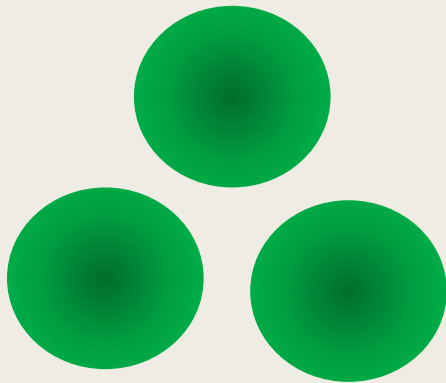
- Shell aspect
- Cluster aspect



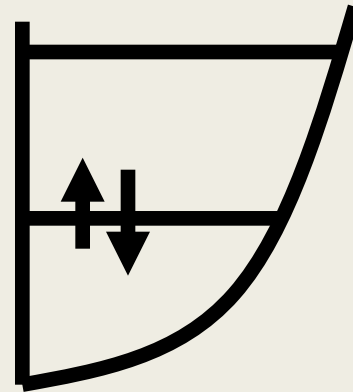
Our dream



Cluster model



Shell model



Spin-orbit interaction is a driving force
that breaks the clusters
and enhances the symmetry of jj-coupling shell model

Cluster side

shell-model side

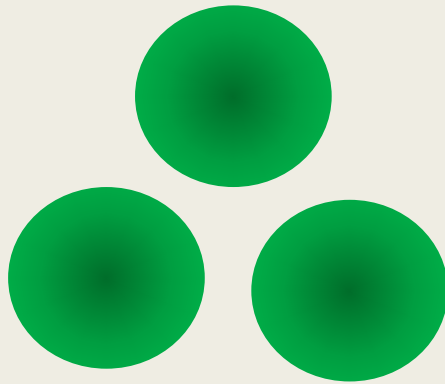


spin-orbit interaction

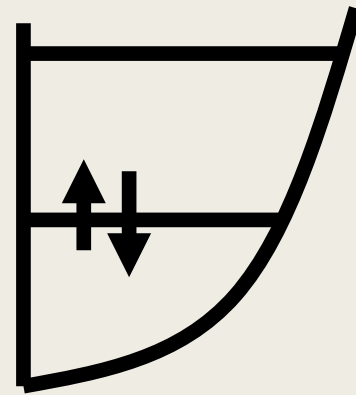


Combining these two features

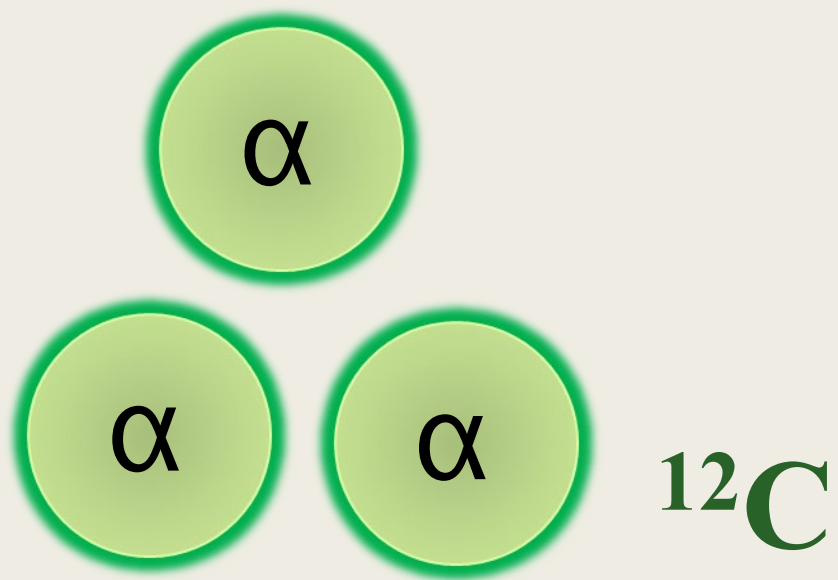
Cluster side

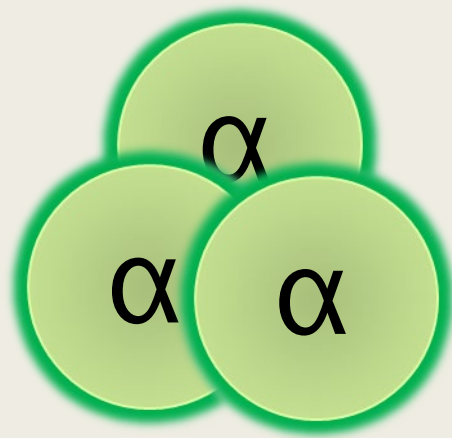


Shell model side

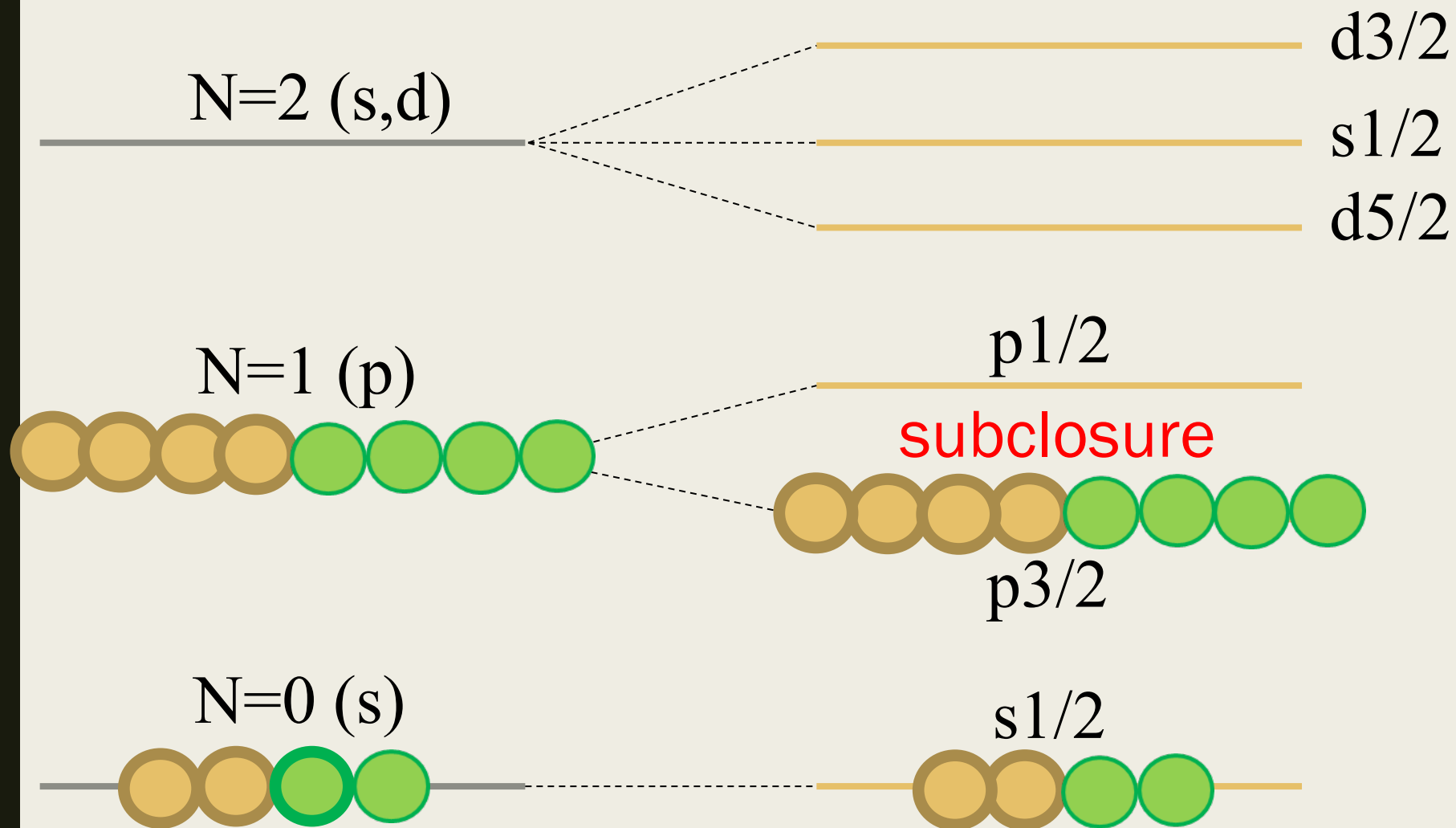


This talk





^{12}C



three-dimensional
harmonic oscillator

jj-coupling shell-model
with spin-orbit contribution

Conventional α cluster model (Brink model)

spatial part of the single particle wave function

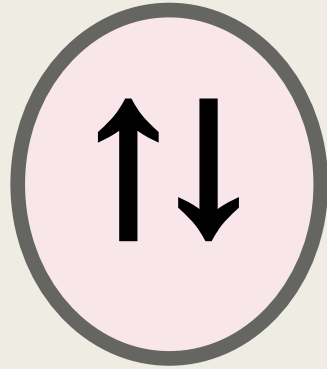
$$\exp[-\nu (\mathbf{r} - \mathbf{R})^2]$$

Locally shifted Gaussian form

$\mathbf{R}$: Gaussian center parameter

α cluster: 4 nucleons share the $\mathbf{R}$ value

No non-central interactions for “simple” cluster



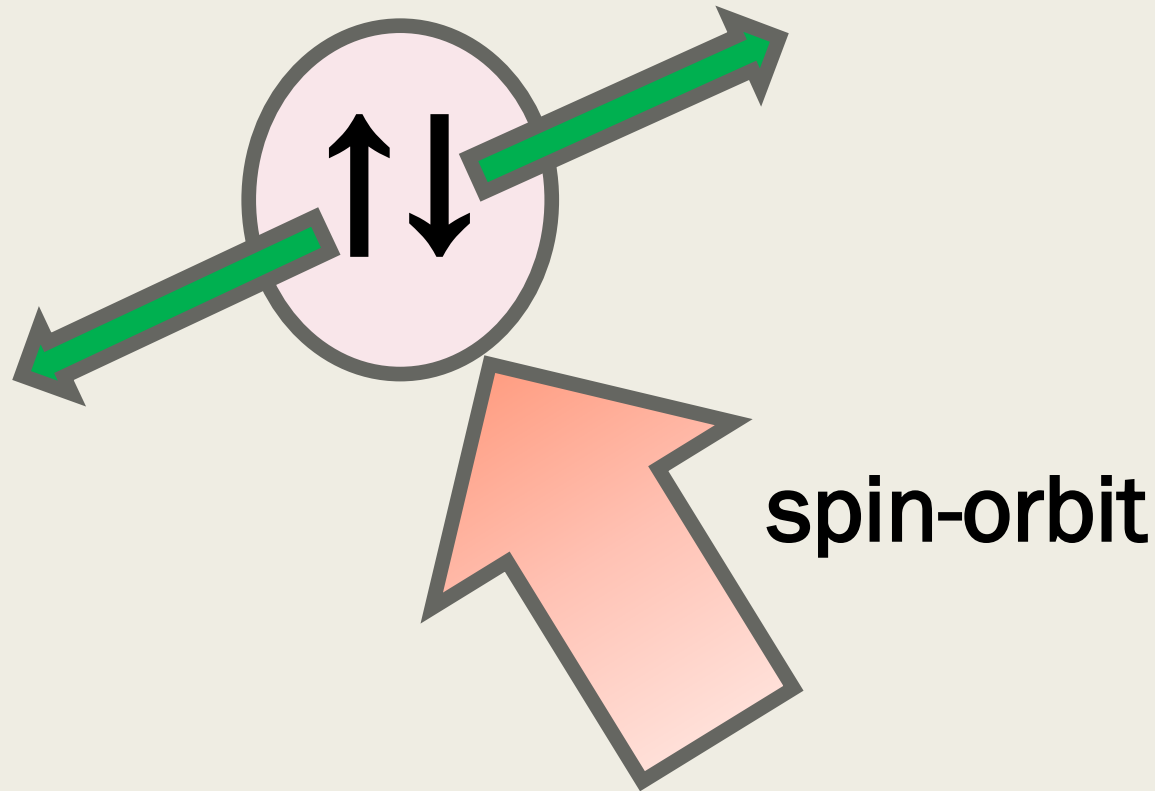
spatial wave functions
of the two nucleons
are the same
(isospin also)

spin part must be antisymmetric

$$\frac{1}{\sqrt{2}} \{ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \} = |00\rangle$$

$S=0$ (no non-central effect)

“Simple” cluster may be broken
by the spin-orbit interaction
when a path is opened



spin-orbit interaction
induces rotations in opposite directions

Antisymmetrized quasi-cluster model (AQCM)

spatial part of the single particle wave function

$$\exp[-\nu (\mathbf{r} - \mathbf{R})^2]$$

Four nucleons share the same $\mathbf{R}$ value
as in traditional cluster models

no spin-orbit effect

$$\mathbf{R} \rightarrow \mathbf{R} + i \mathbf{\Lambda} (\mathbf{e}_{\text{spin}} \times \mathbf{R})$$

α cluster $\rightarrow$ quasi cluster

z axis



y axis



x axis



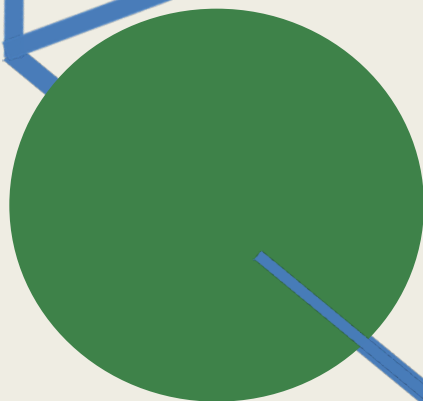
z axis



y axis



α cluster



x axis



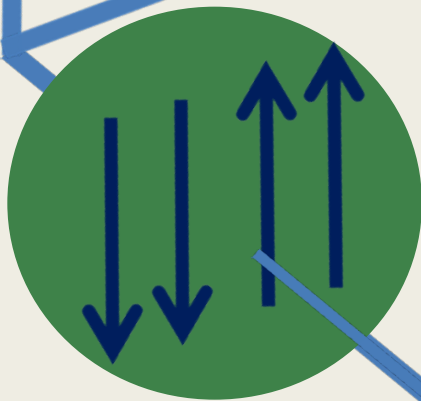
z axis



y axis



α cluster



x axis



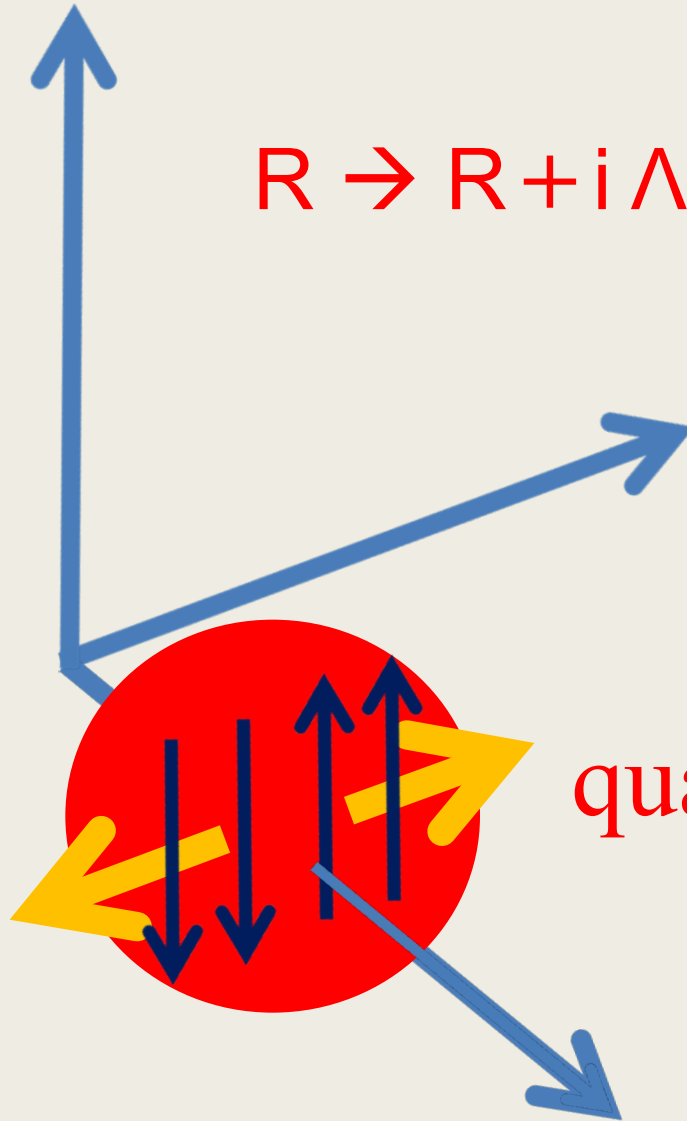
z axis

$$R \rightarrow R + i \Lambda (e_{\text{spin}} \times R)$$

y axis

quasi cluster

x axis



^8Be

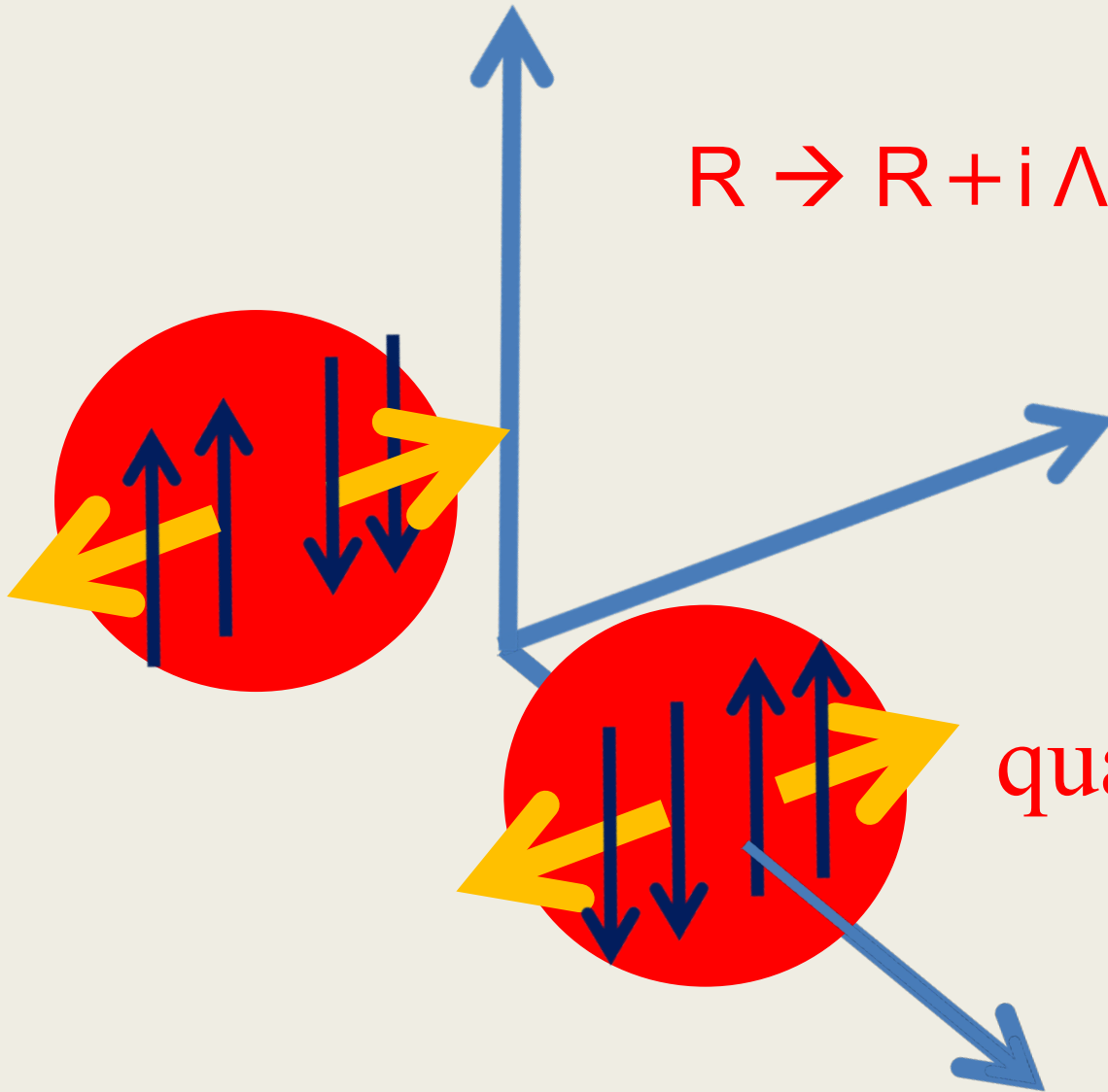
z axis

$$R \rightarrow R + i \Lambda (e_{\text{spin}} \times R)$$

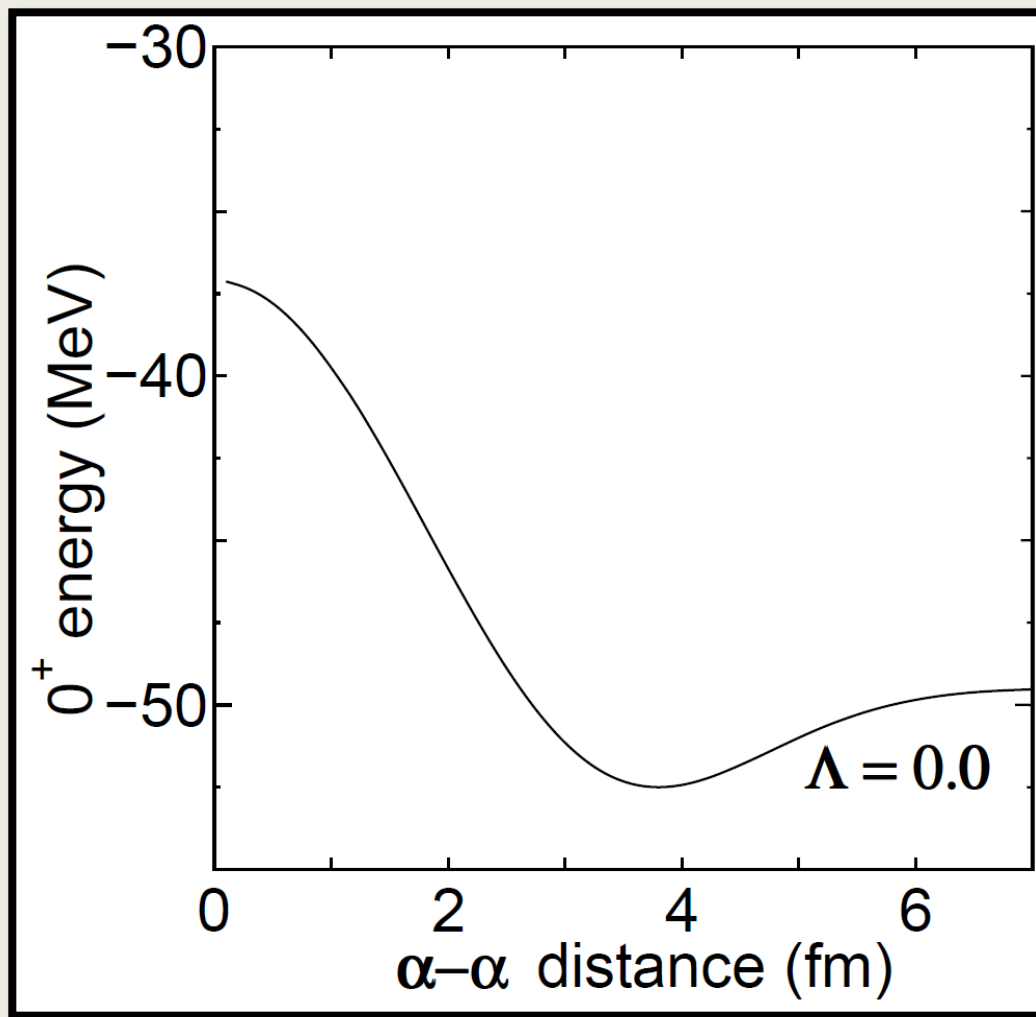
y axis

quasi cluster

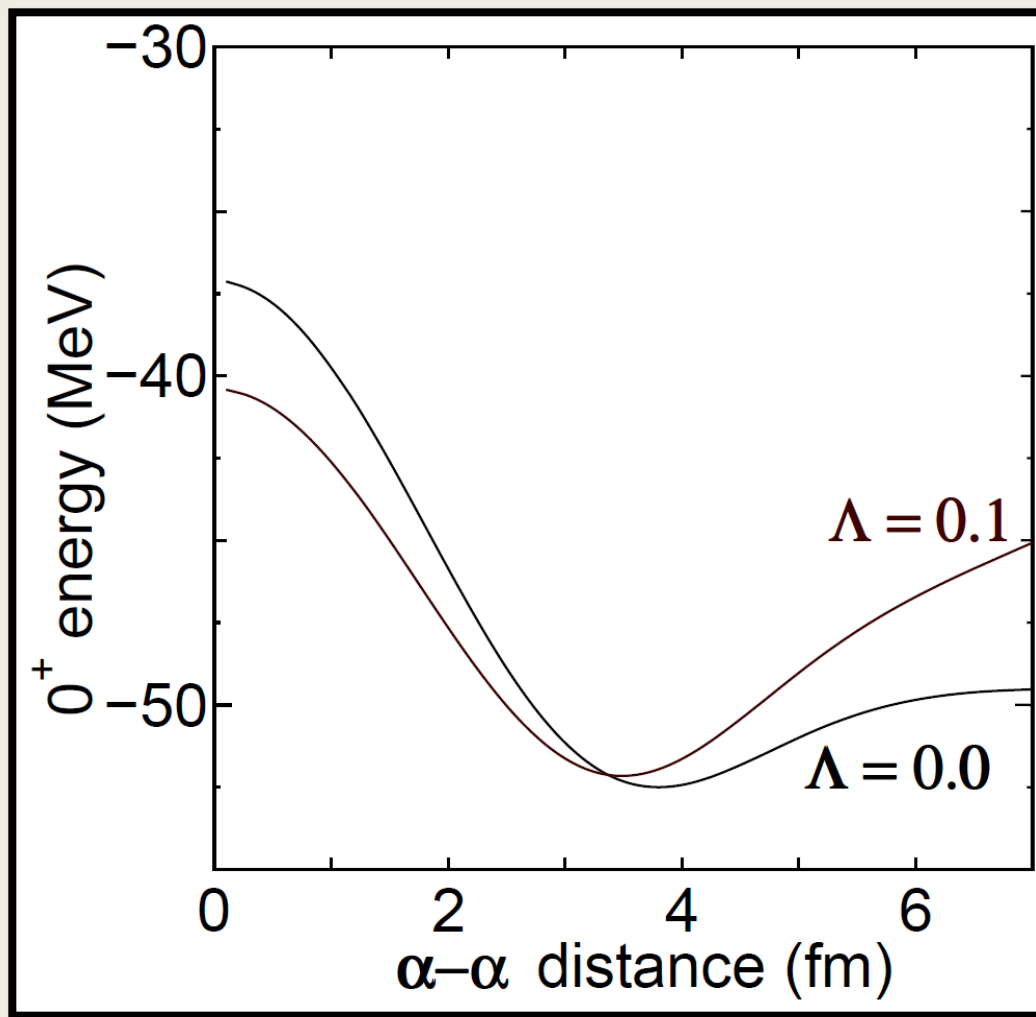
x axis



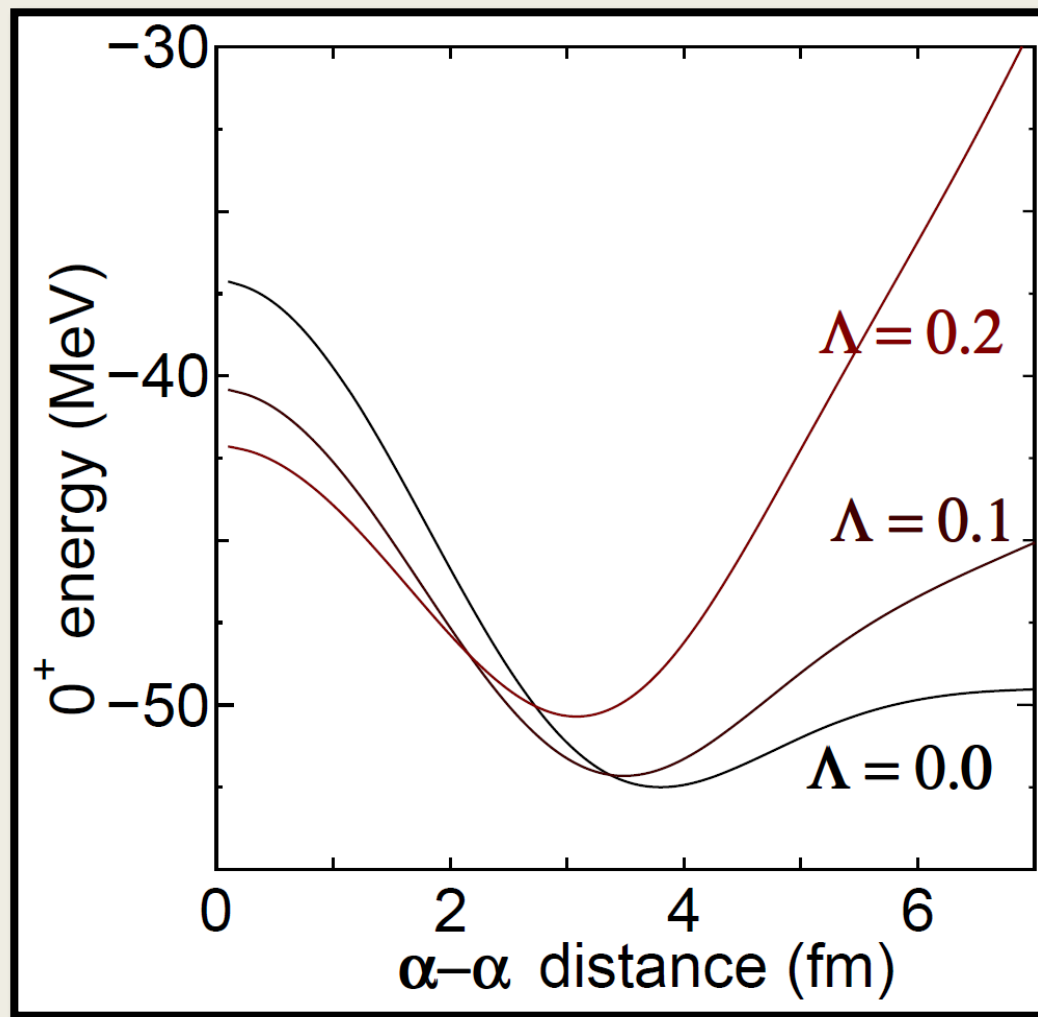
^8Be



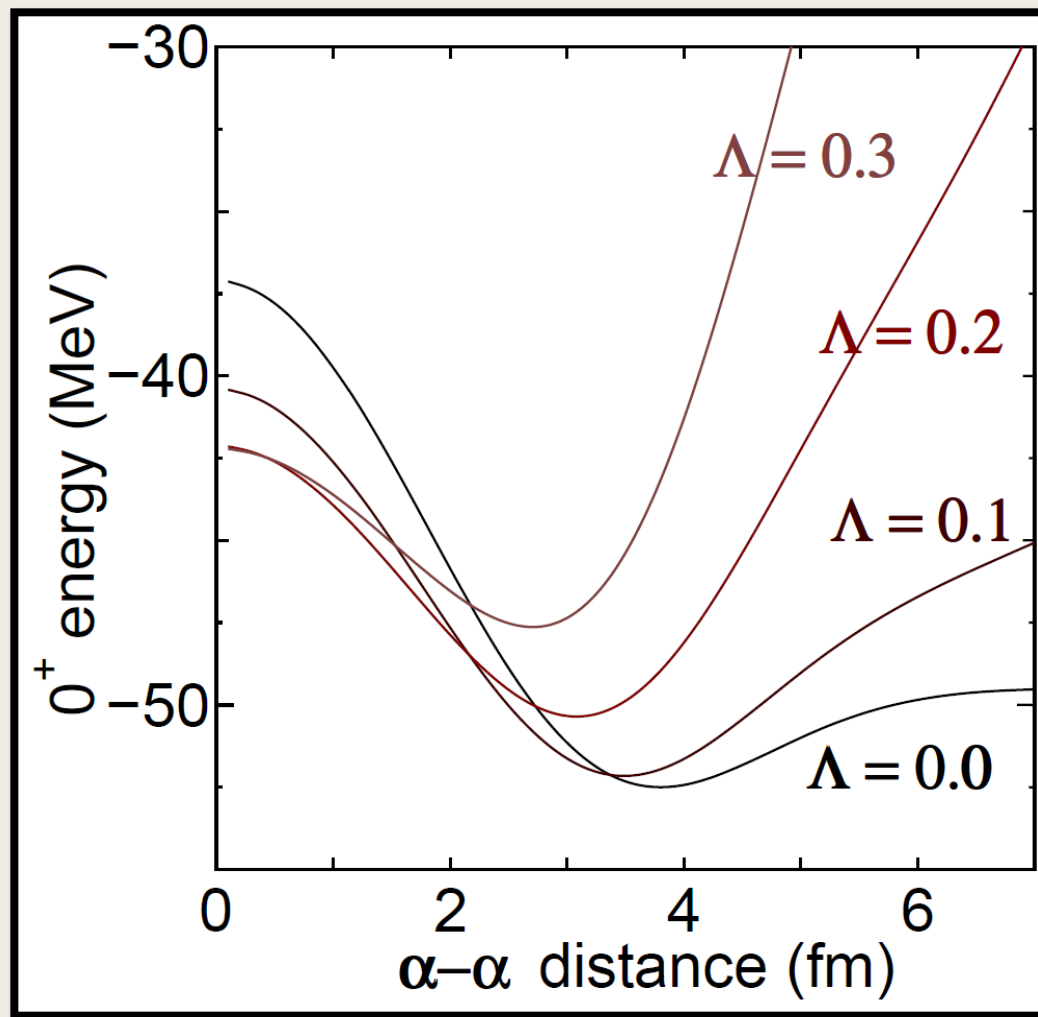
^8Be



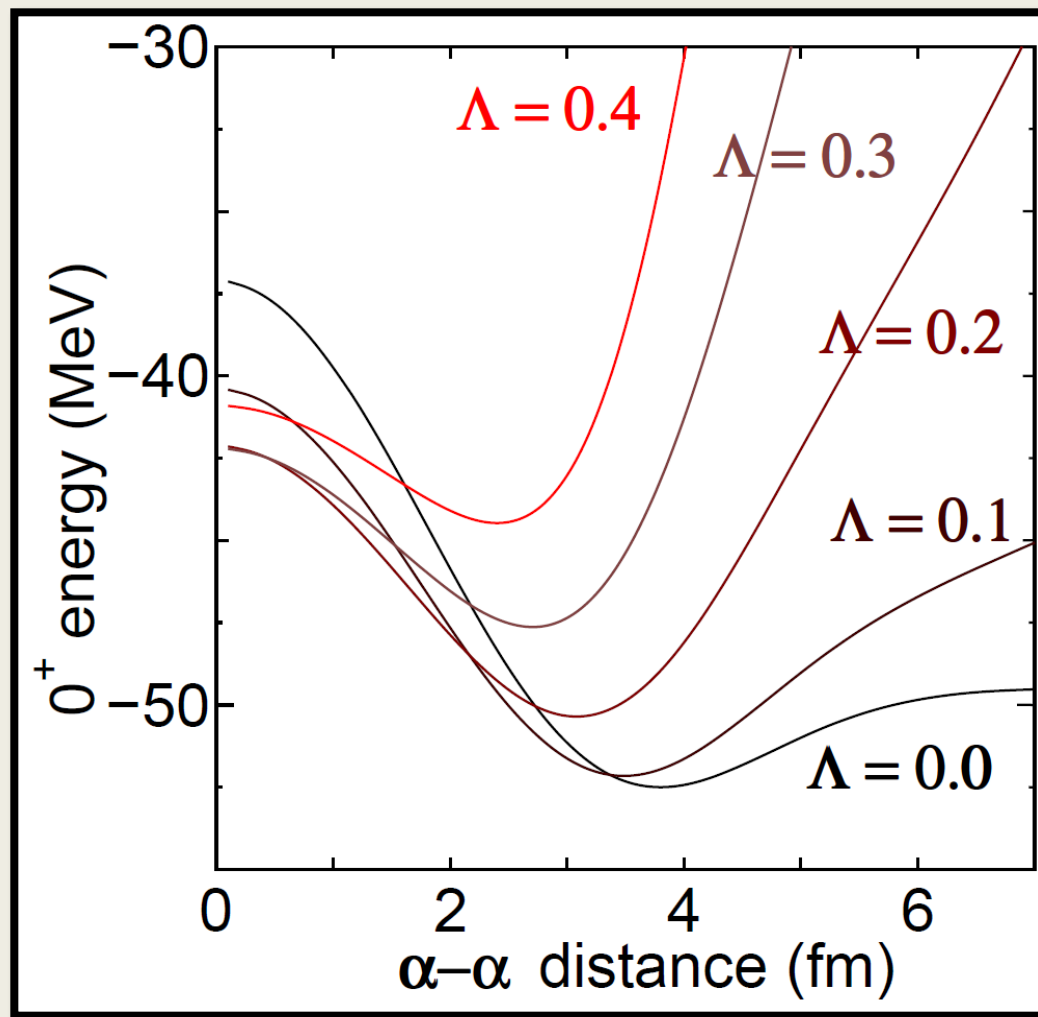
^8Be



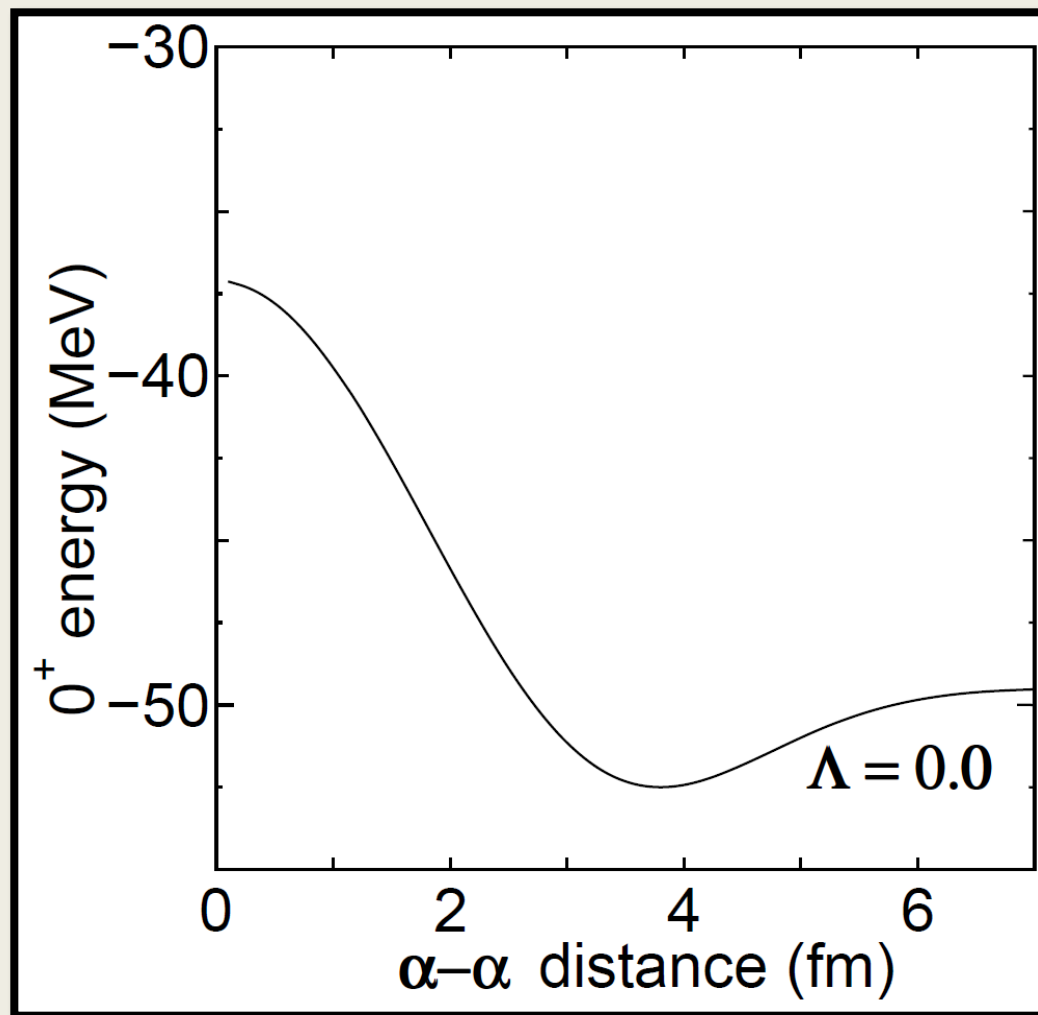
^8Be



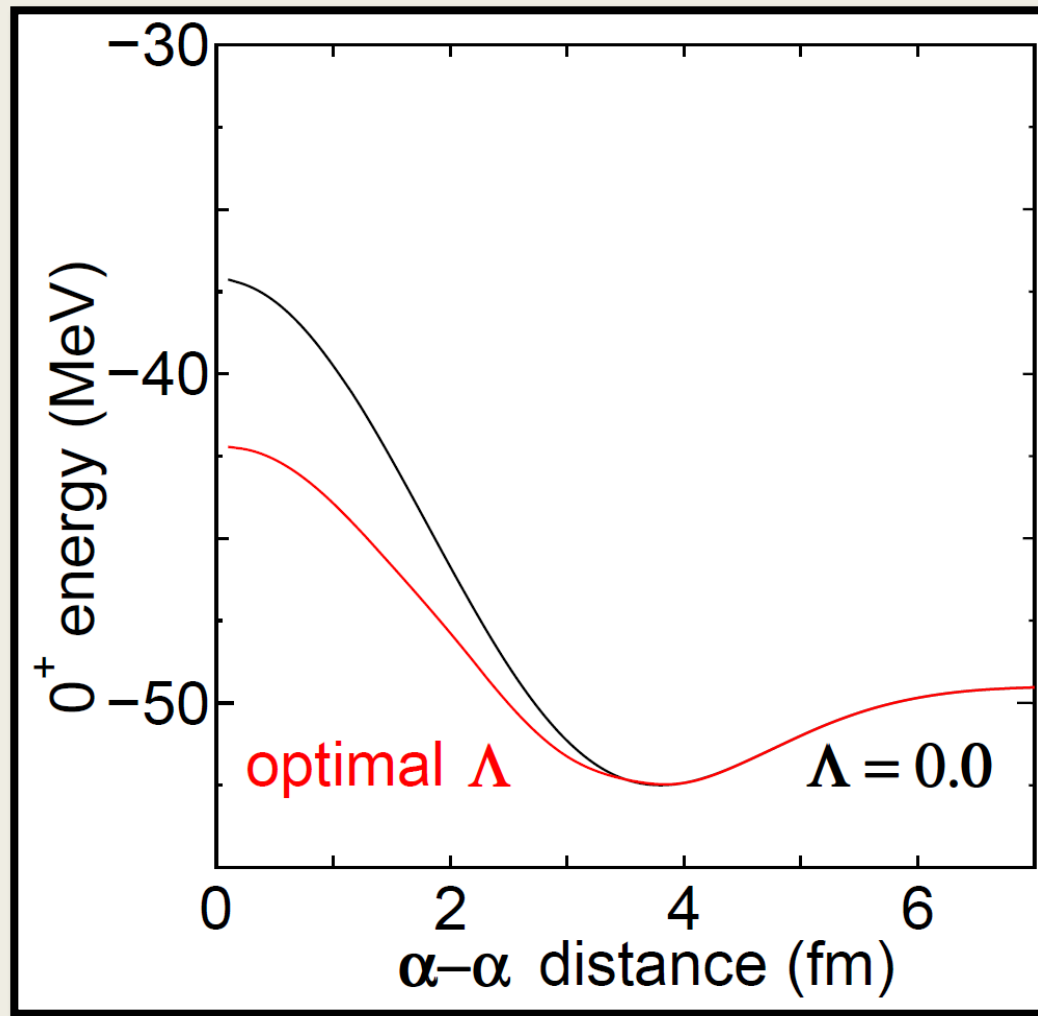
^8Be



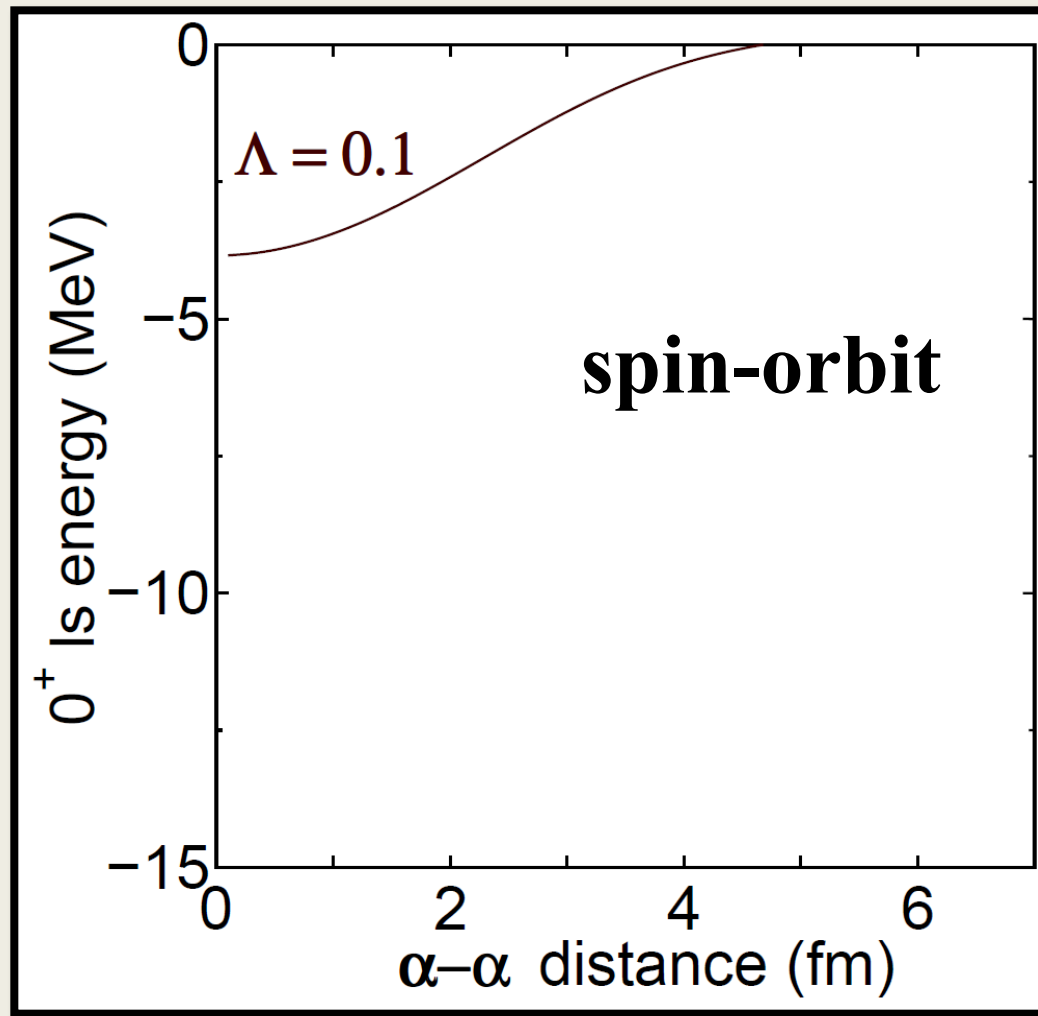
^8Be



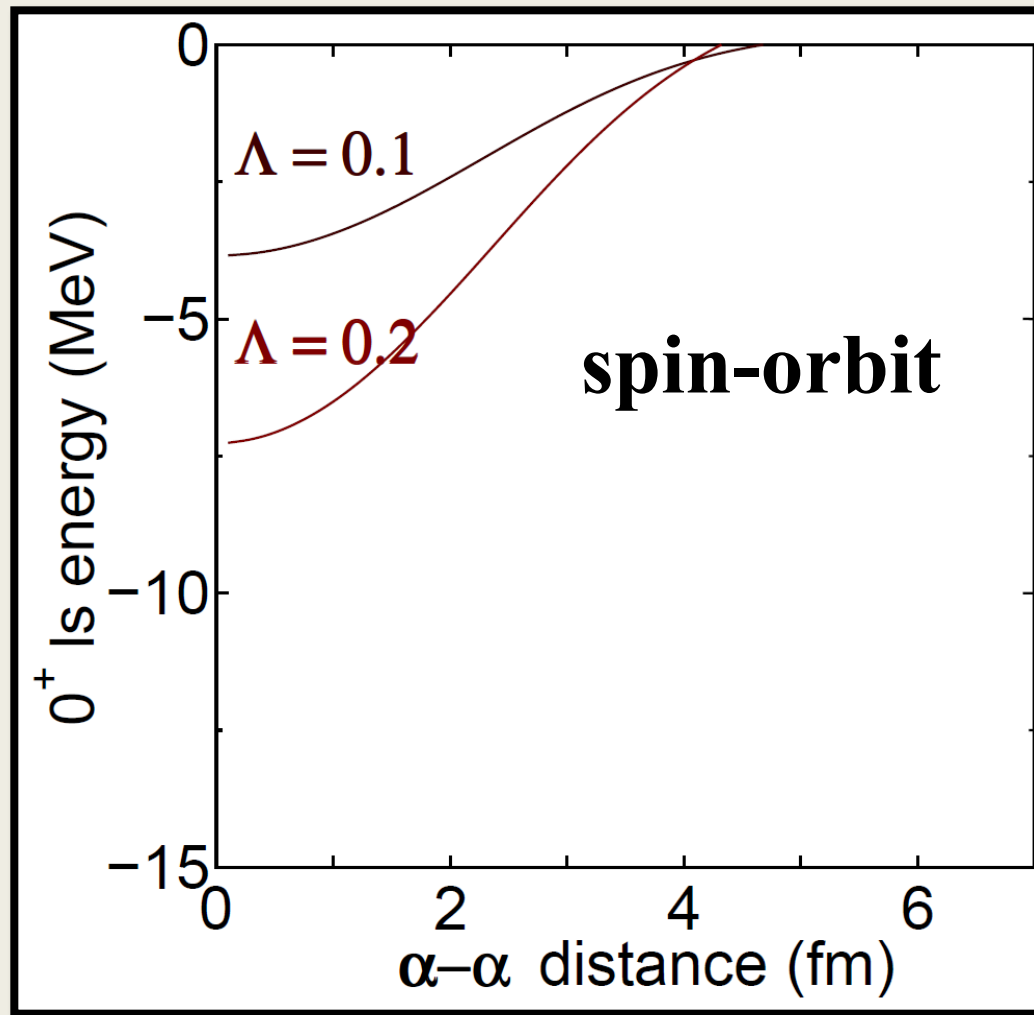
^8Be



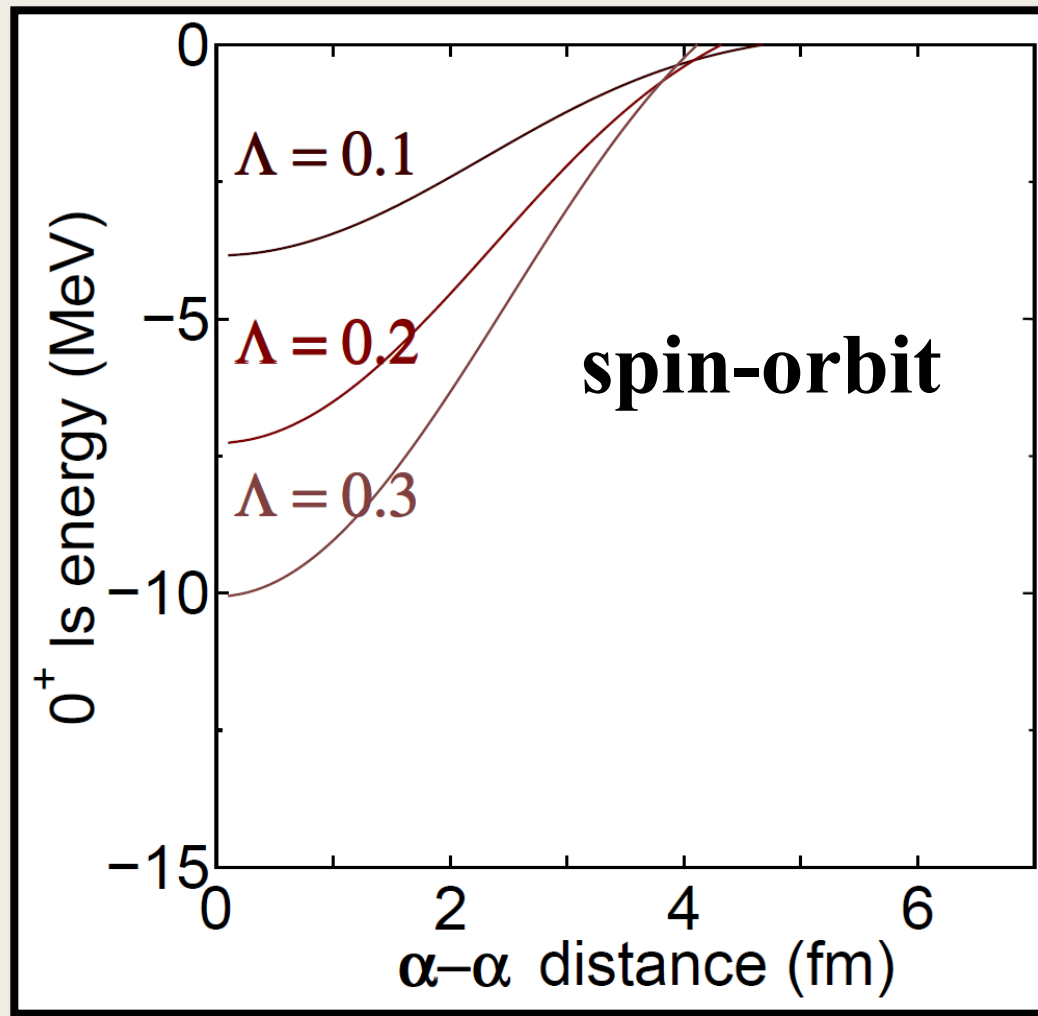
^8Be



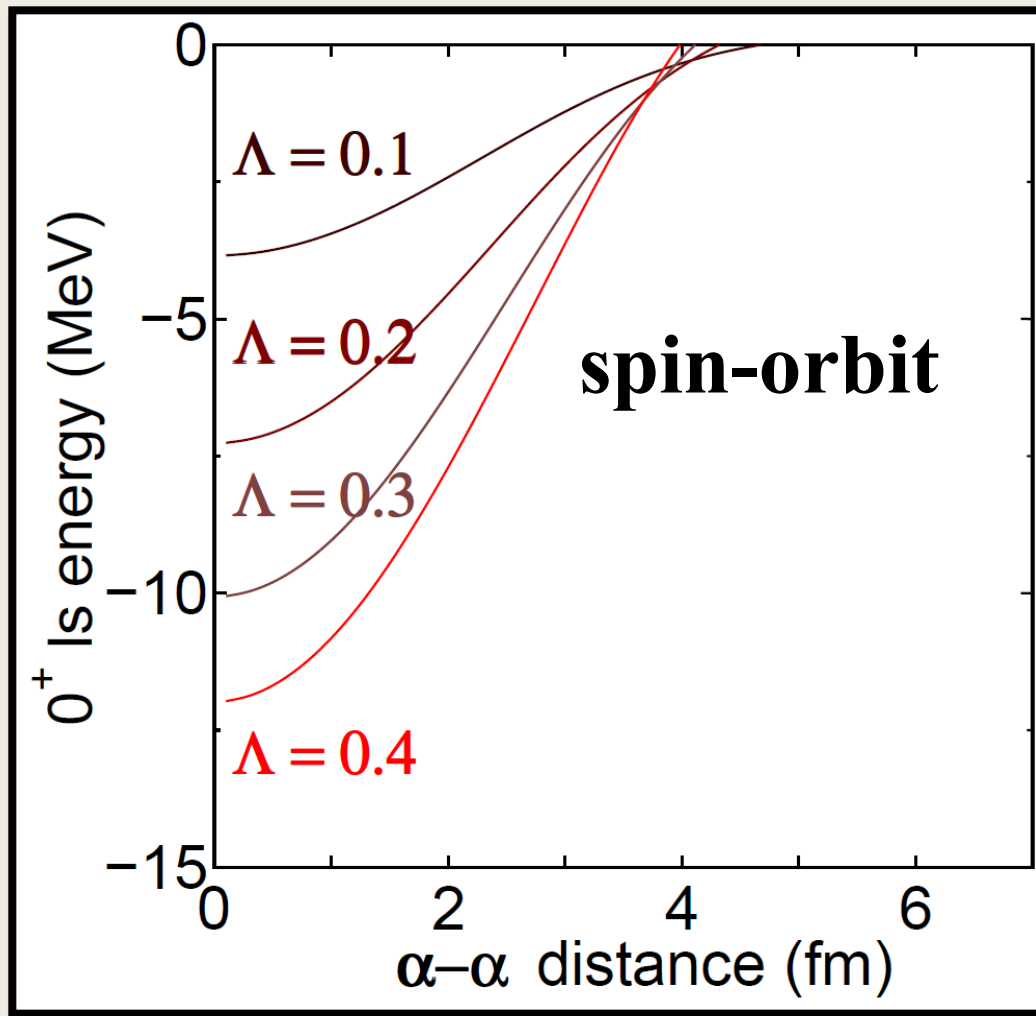
^8Be



^8Be



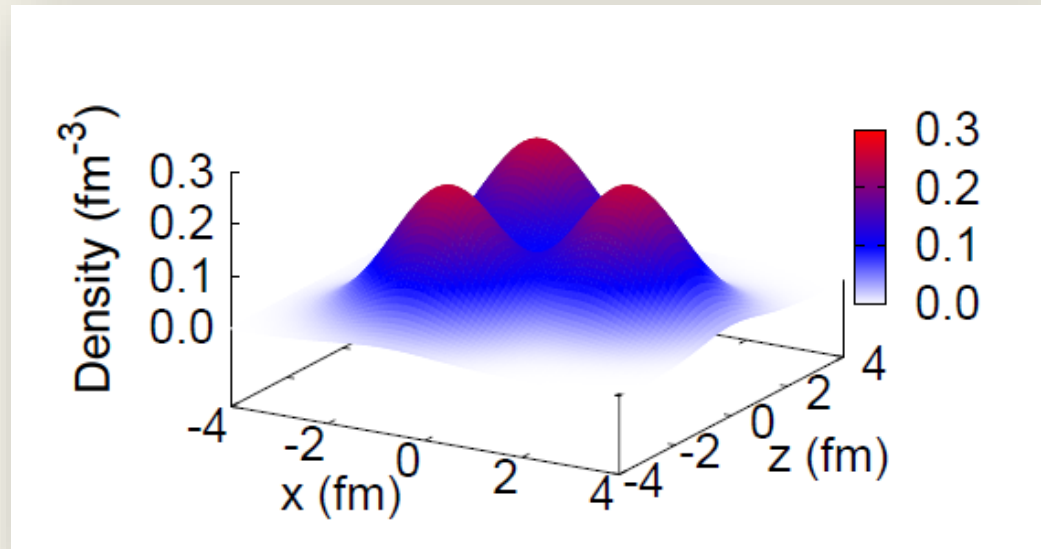
^8Be



^{12}C L: relative distance between clusters

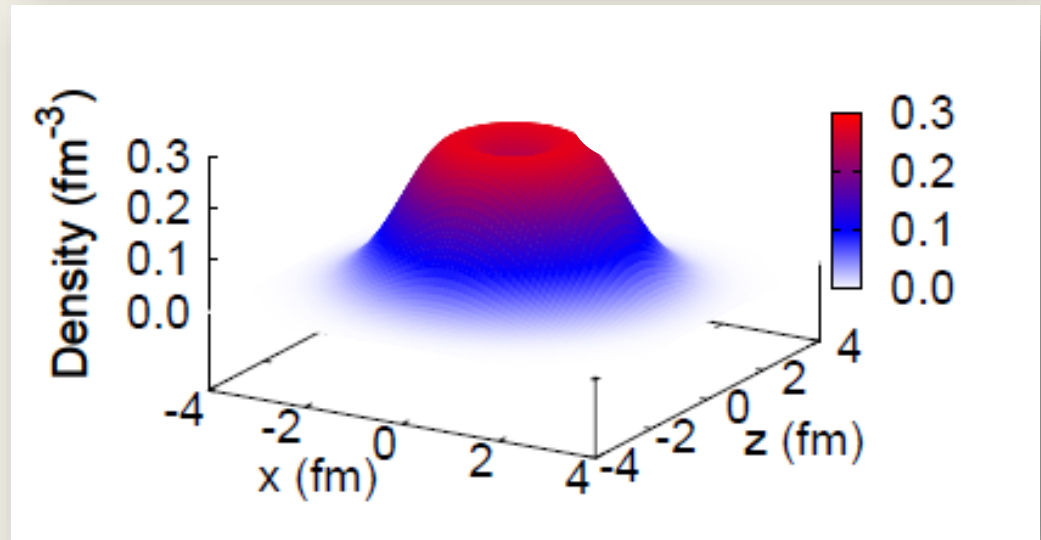
$$L = 3 \text{ fm}$$

$$\Lambda = 0$$



$$L = 0.01 \text{ fm}$$

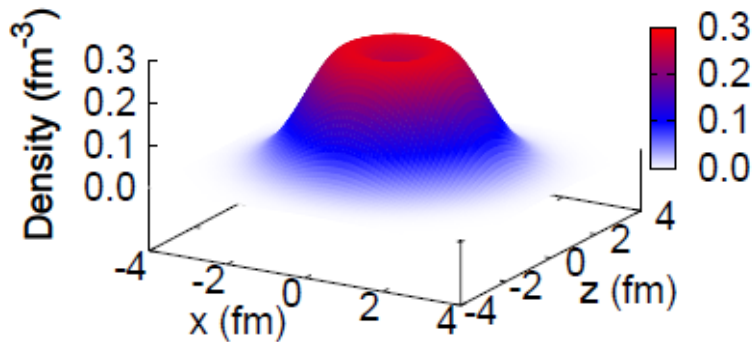
$$\Lambda = 0$$



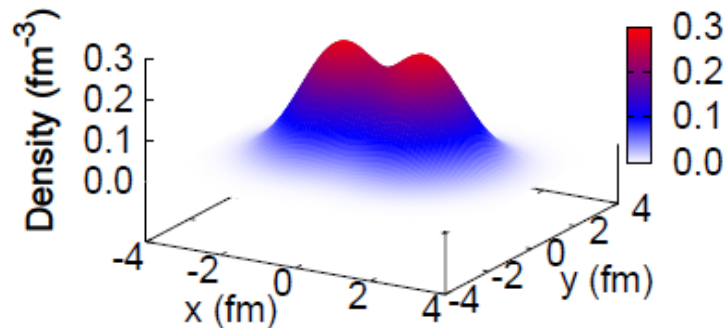
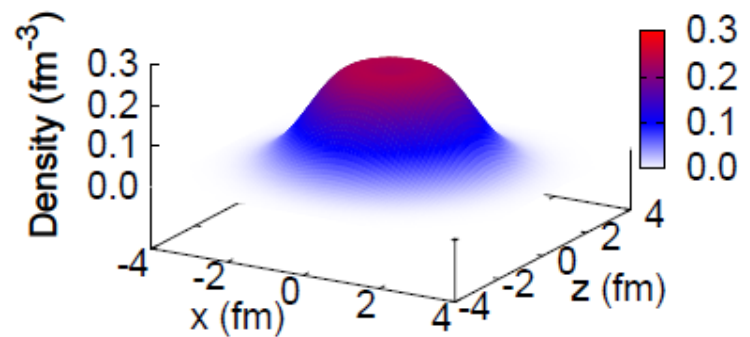
^{12}C $L = 0.01$ fm

$\Lambda = 0$

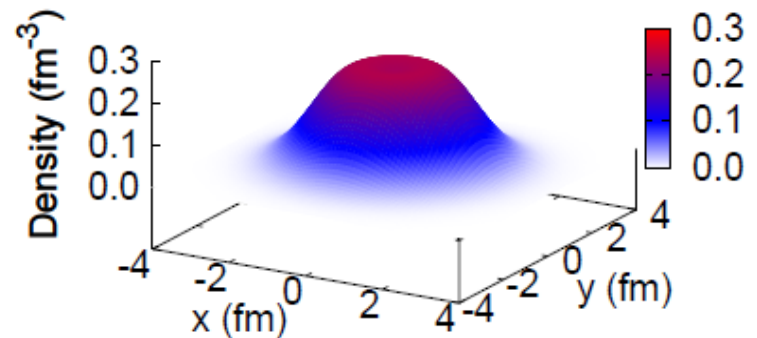
$\Lambda = 1$

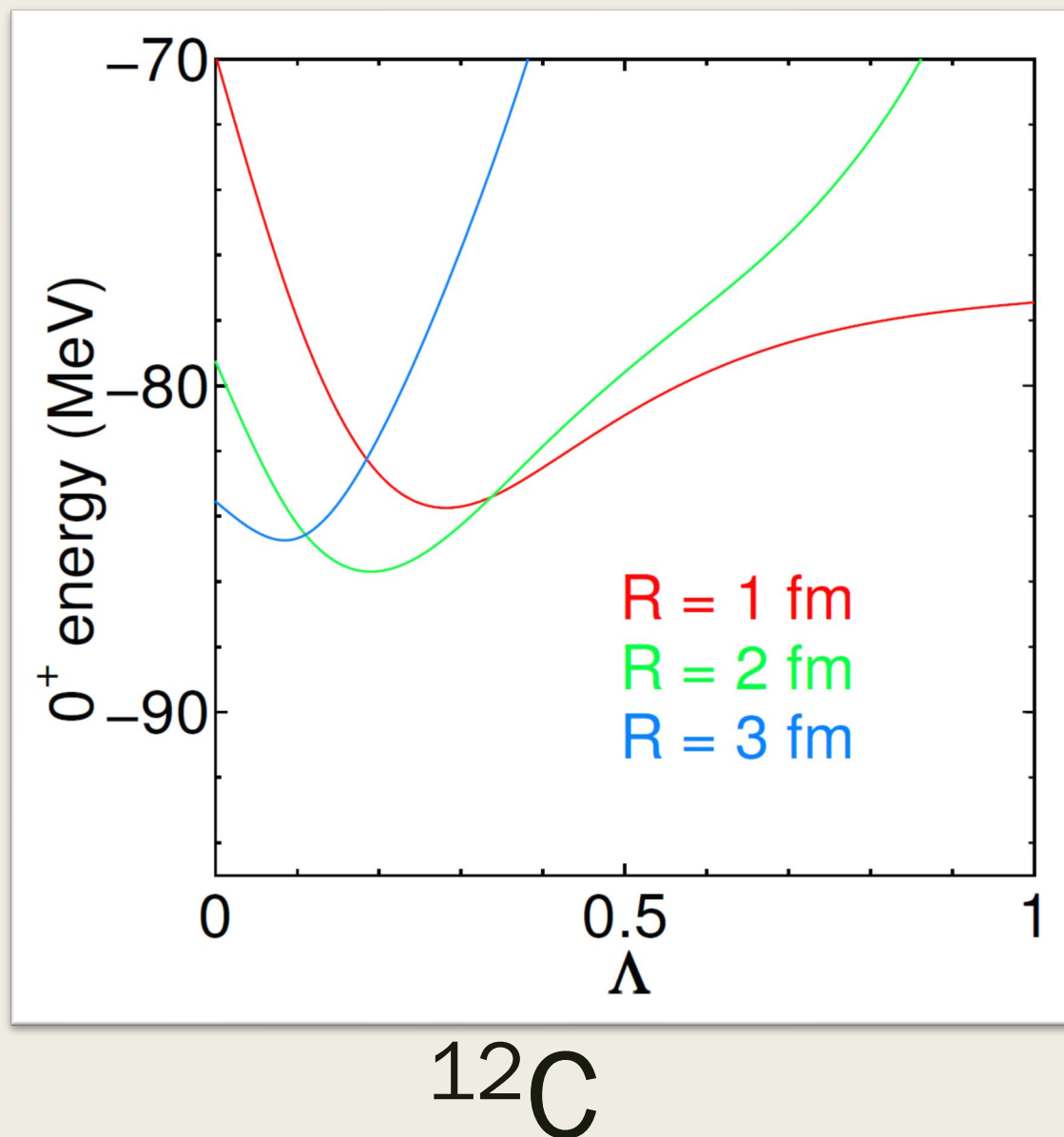


xz



xy



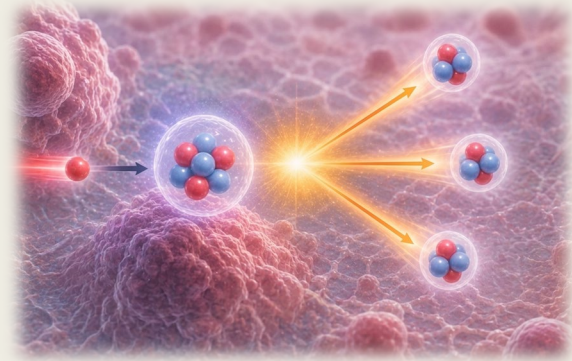


Summary for this part

- ^8Be – outside of the interaction range of the spin-orbit interaction, which breaks the alpha clusters.
- ^{12}C – inside the interaction range of the spin-orbit interaction owing to the additional one alpha cluster to ^8Be .

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We can prepare shell- and cluster-model wave functions based on AQCM on the same footing

If we prepare shell- and cluster-model wave functions, both reproducing the observed charge radius, we could see the difference in the proton elastic scattering. He can connect AQCM to his Glaubar-model calculation.

Application to ^{12}C

S-type (jj-coupling shell model)

We take the small-distance limit of α clusters.

Λ is set to 1.

The Gaussian size parameter ν for the single particles is adjusted to reproduce the experimental charge radius.

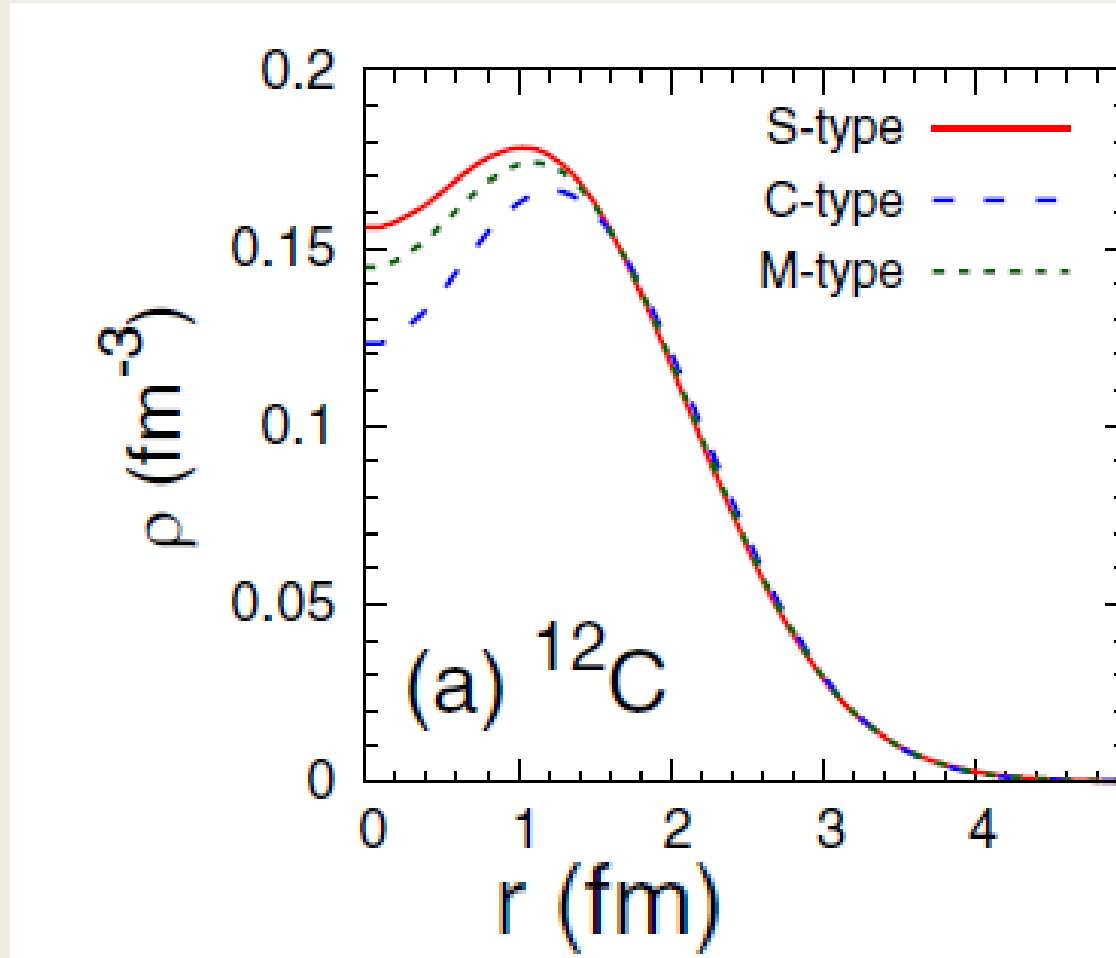
C-type (α -cluster model)

We prepare three “free” α particles.

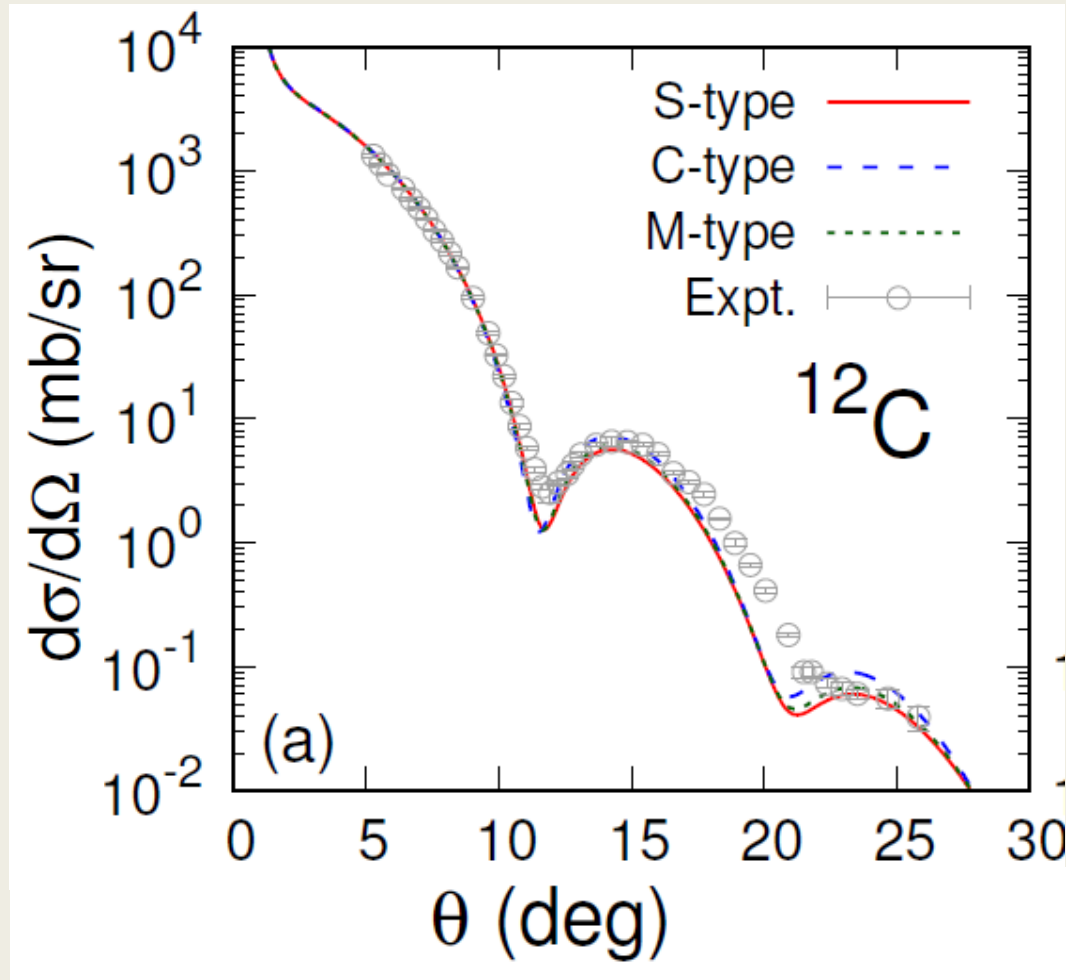
Λ is set to 0.

The distances between three α particles with an equilateral triangular configuration are adjusted to reproduce the experimental charge radius.

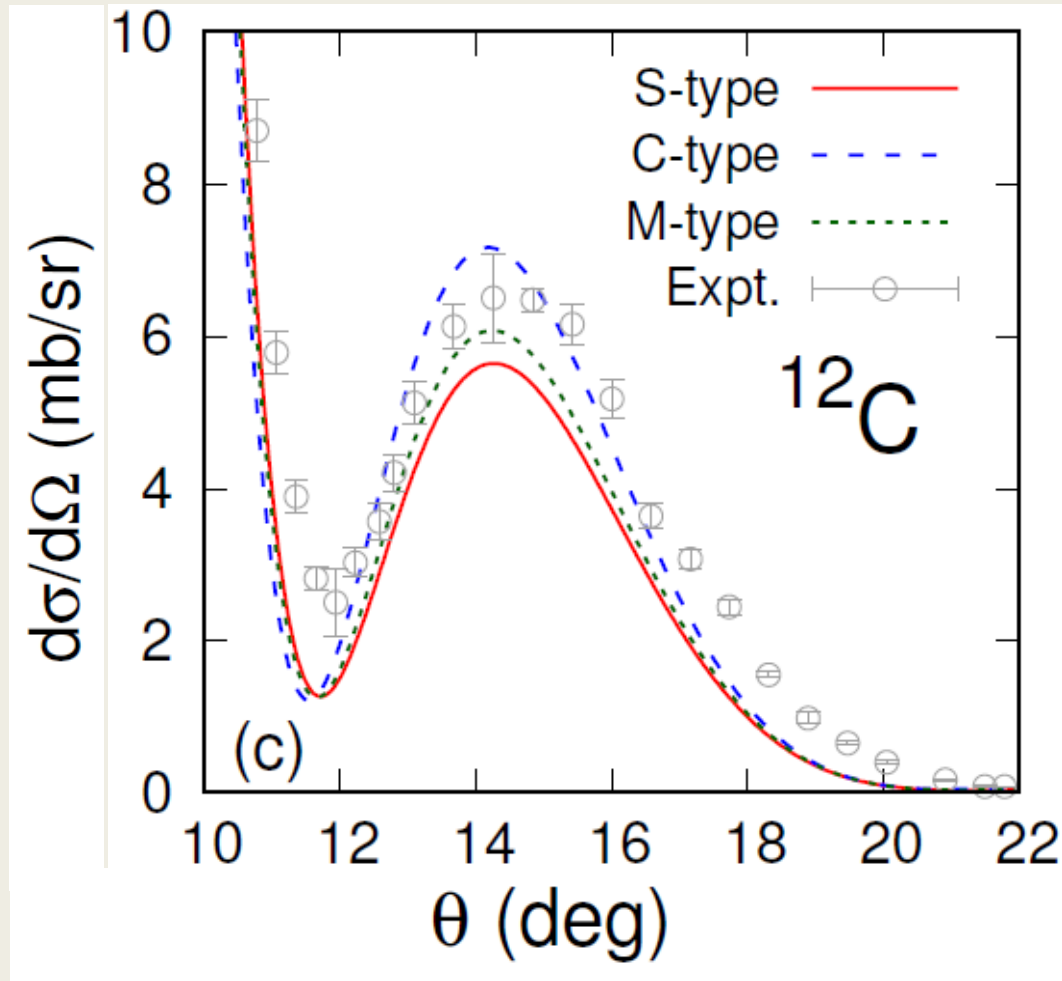
^{12}C ground state is shell or cluster? (density distribution)



^{12}C ground state is shell or cluster? (proton elastic scattering, 1000 MeV)



^{12}C ground state is shell or cluster? (proton elastic scattering, 1000 MeV)



Application to ^{16}O

S-type (jj-coupling shell model)

We take the small-distance limit of α clusters with a tetrahedron configuration.

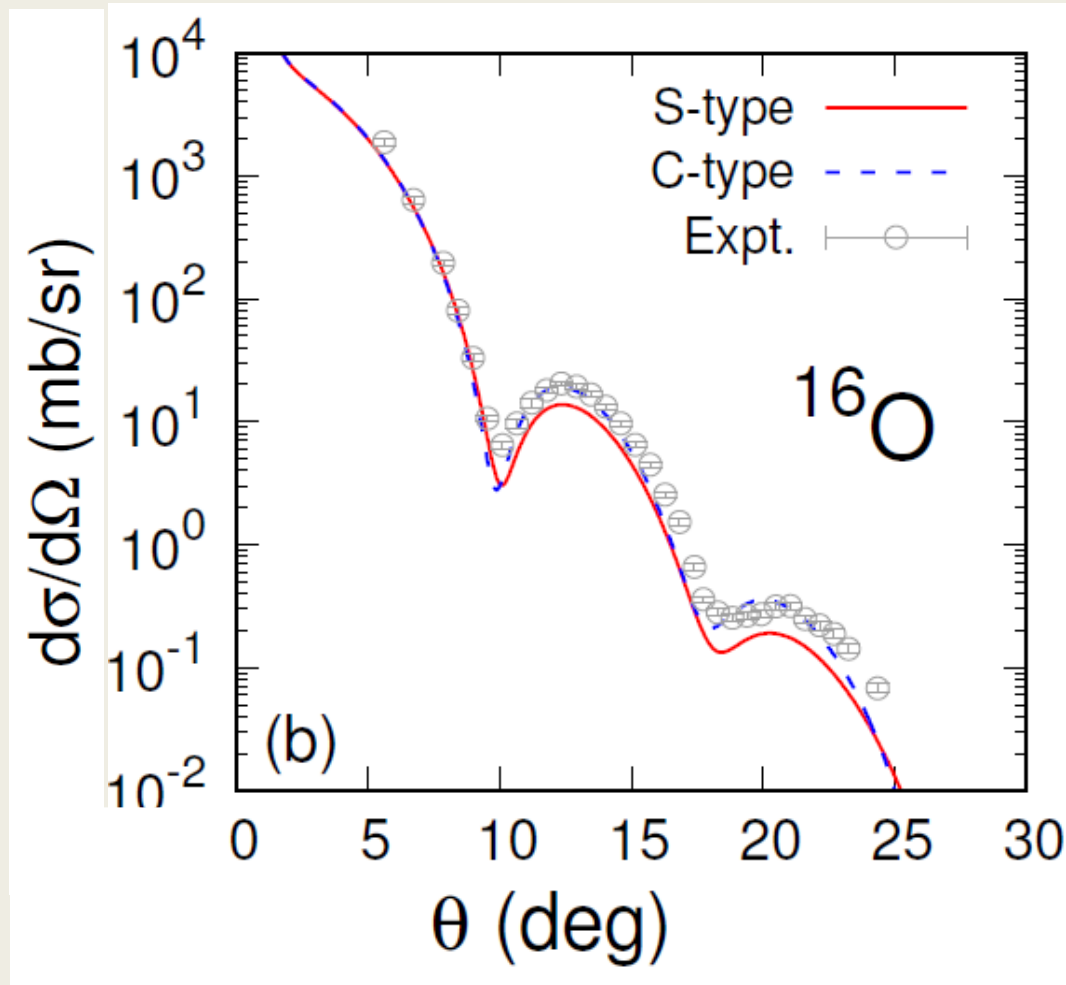
The Gaussian size parameter ν for the single particles is adjusted to reproduce the experimental charge radius.

C-type (α -cluster model)

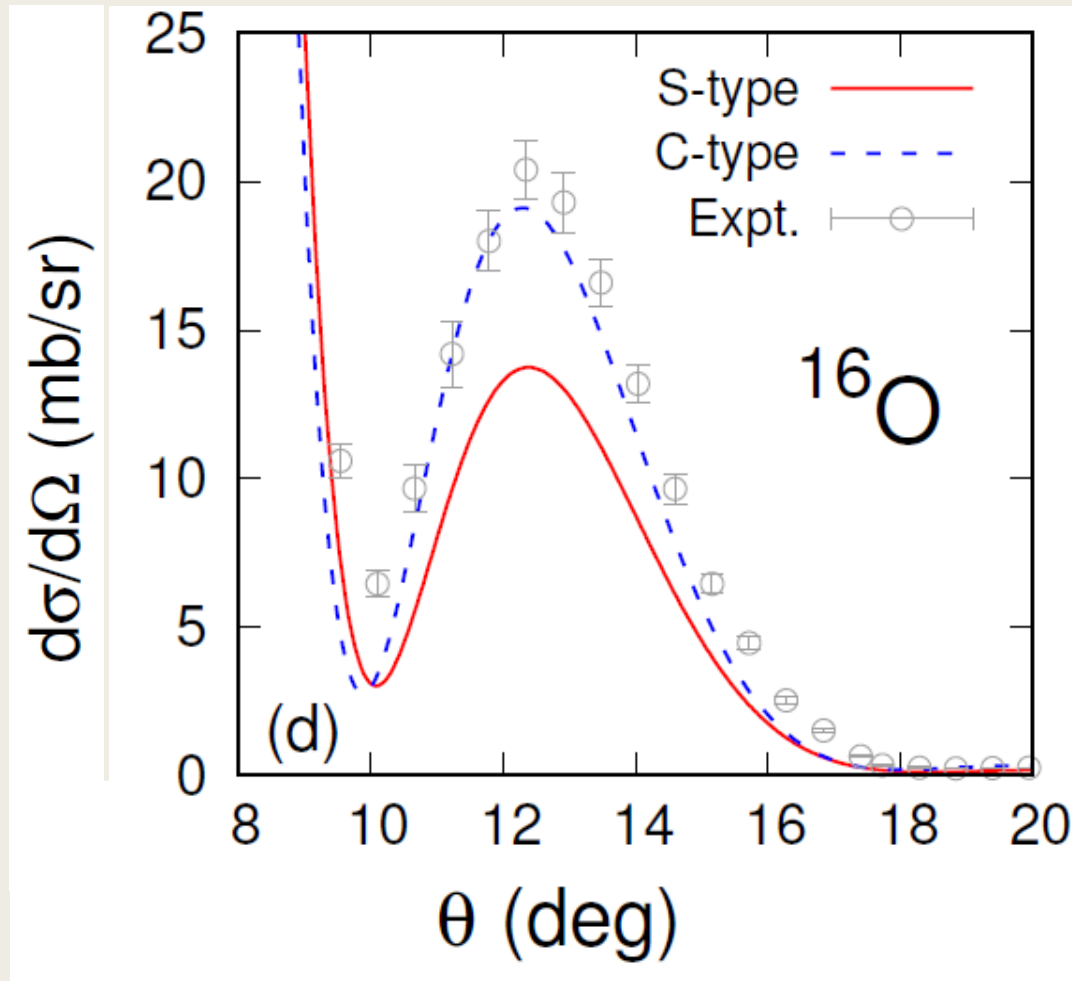
We prepare four “free” α particles.

The distances between four α particles with a tetrahedron configuration are adjusted to reproduce the experimental charge radius.

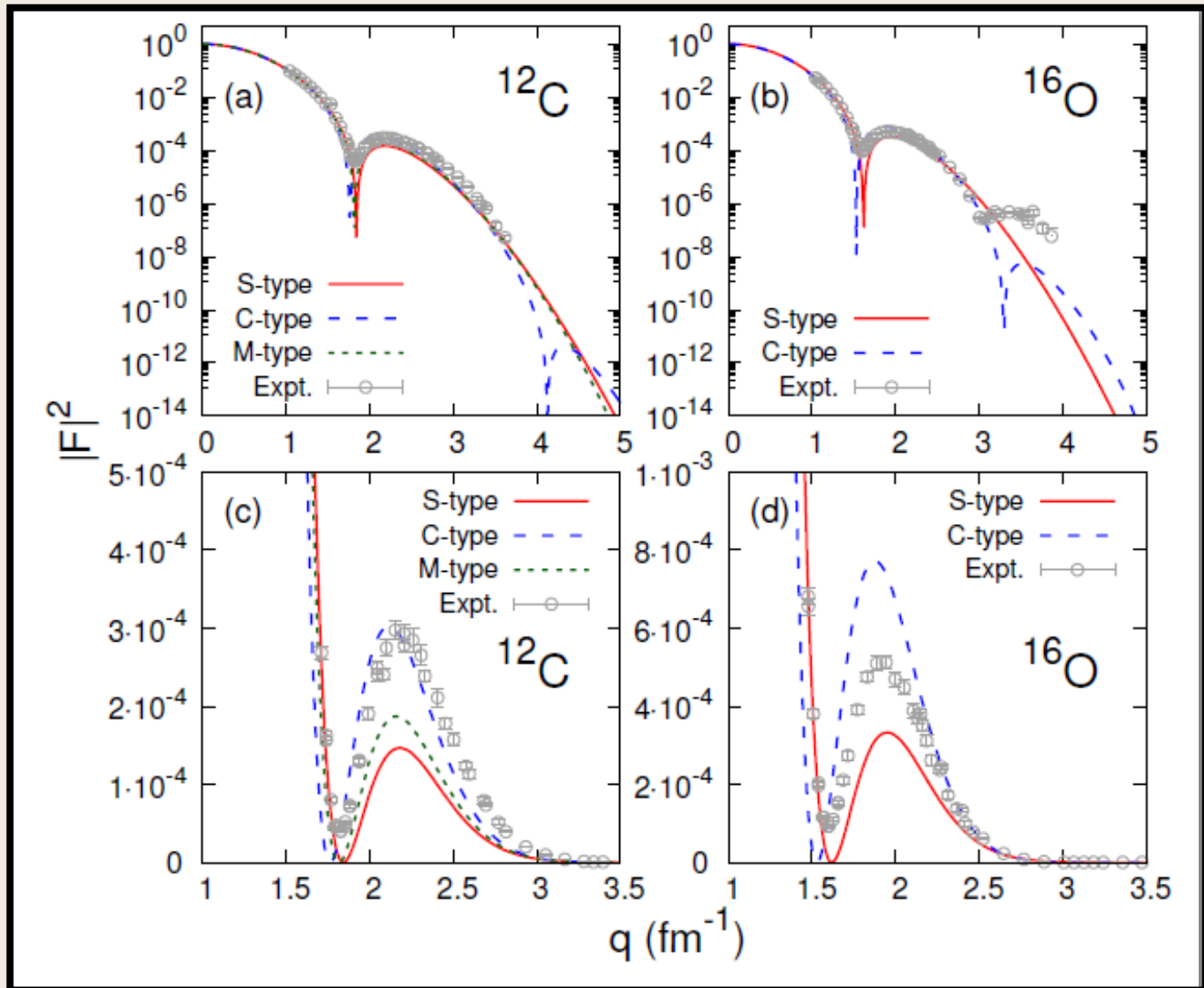
^{16}O ground state is shell or cluster? (proton elastic scattering, 1000 MeV)



^{16}O ground state is shell or cluster? (proton elastic scattering, 1000 MeV)



Elastic charge form factor



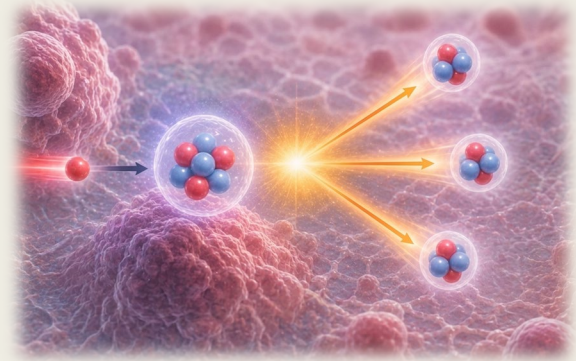
W. Horiuchi and N. Itagaki, Phys. Rev. C **107**, L021304 (2023)

Summary for this part

- ^{12}C – according to the proton elastic scattering, the surface region is rather cluster like than the shell model like, but the mixture of these two best reproduces the experiment.
- ^{16}O – tetrahedron configuration of four alpha clusters?

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$^{12}\text{C}+^{12}\text{C}$ reaction

- Famous for molecular resonance states
- Astrophysically important (X-ray superburst, Type-Ia supernova explosion, etc.)
- Subbarrier fusion reaction (a few MeV above the threshold) is of great interest.
- Coulomb barrier height is ~ 5 MeV at the relative distance of 10 fm.

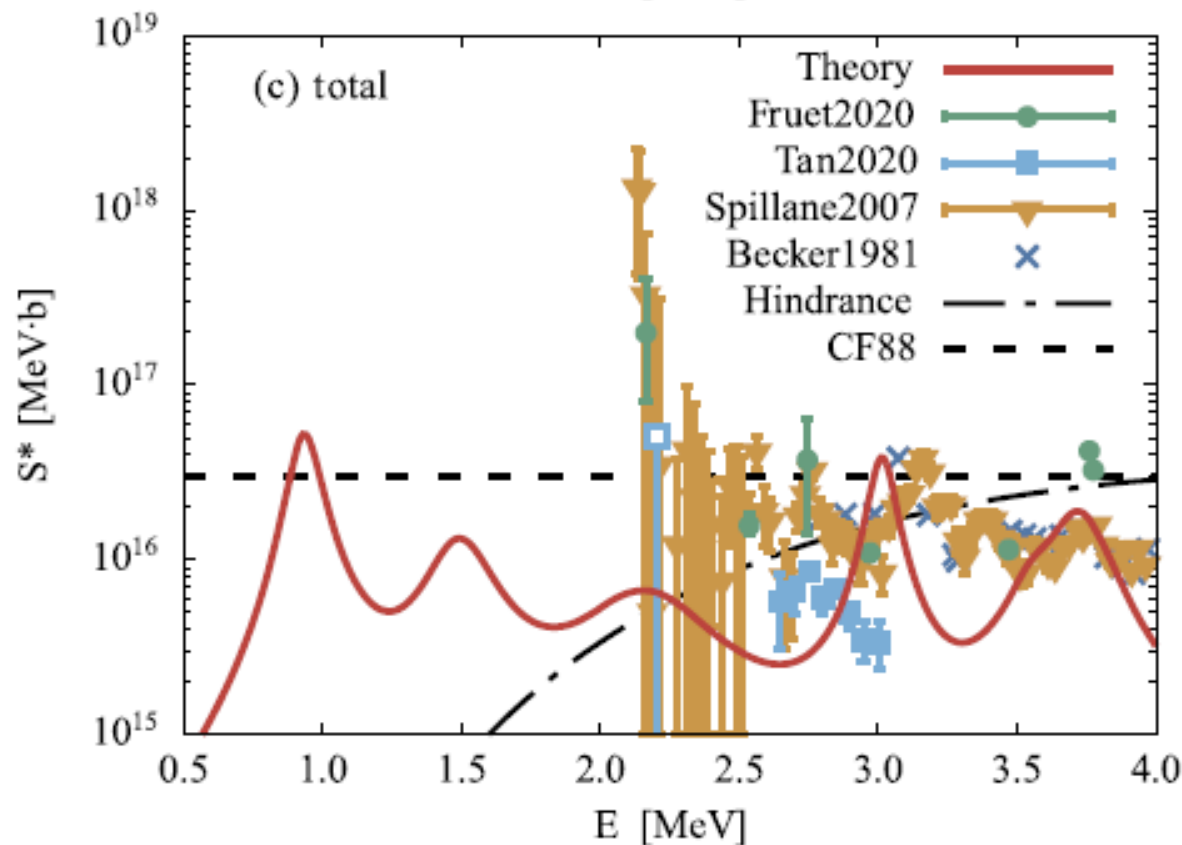


Fig. 2. Calculated and observed [14,16,17,38] modified astrophysical S^* -factors in the (a) p_1 and (b) α_1 channels. The calculated S^* -factors in the p_0 and α_0 channels are also shown for the comparison. Panel (c) compares the calculated and observed total S^* -factors with the evaluations [7,8]. Open symbols indicate upper limits.

Yasutaka Taniguchi, Masaaki Kimura,
Phys. Lett. **B 823**, (2021) 136790

Table 1

The properties of the $J^\pi = 0^+$ and 2^+ resonances obtained by using the D1M* and SkM* density functionals. Only the resonances with energies (E_R) lower than 4 MeV are listed. Γ is a total decay width, and θ_C^2 is a dimensionless reduced width amplitude for the $^{12}\text{C} + ^{12}\text{C}$ channel in percent at channel radius $a = 6$ fm (asterisked numbers are for $a = 7$ fm). Branching ratios larger than 10^{-2} are also listed. $M(\text{IS}\lambda \uparrow)$ is the matrix element of the isoscalar monopole (quadrupole) transitions from the ground state of ^{24}Mg to the $J^\pi = 0^+$ (2^+) resonances.

DF	J^π	E_R [MeV]	Γ [MeV]	θ_C^2 [%]	branching ratio						$M(\text{IS}\lambda \uparrow)$ [W.u.]
					α_0	α_1	α_2	p_0	p_1	p_2	
D1M*	2^+	0.40	0.14	0.6*	.31	.24		.19	.25		1.37
	0^+	0.93	0.47	4.8	.93	.04		.01			0.39
	2^+	1.06	0.10	1.1*	.54	.03		.27	.14		2.34
	2^+	2.10	0.45	0.6*	.40	.40		.02	.15		3.89
	0^+	2.33	0.63	8.8	.58	.38			.03		0.74
	2^+	2.45	0.18	0.2*	.08	.67		.09	.13		0.96
	2^+	3.21	0.67	5.8	.42	.46	.01	.03	.06	.01	3.68
	0^+	3.84	0.73	8.4	.26	.65	.02	.01	.04		1.07
	2^+	4.29	0.79	7.8	.30	.31	.16	.12	.07	.01	1.90
SkM*	2^+	2.42	0.36	1.2*	.01	.13		.53	.27	.05	3.23
	2^+	3.30	0.38	1.8	.20	.20		.20	.36	.01	1.42
	0^+	3.49	0.32	11.1	.65			.06	.27		0.94
	2^+	4.27	2.41	0.5*				.68	.26	.04	1.83

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Phys. Lett. **B 823**, (2021) 136790

Motivation 1

- For the cross section, presence of resonance states around the corresponding energy region is quite important.
- In the preceding work, Skyme and Gogny interactions were adopted and the results were compared.
- How would the results change when the interaction developed for the cluster studies are adopted?
- Especially, how the range of the interaction affects the potential+Coulomb energy surface?

Tohsaki interaction (effective N-N)

$$\mathcal{H} = -\frac{\hbar^2}{2M} \sum_i \nabla_i^2 - T_G + \sum_{i < j} V_{ij}^C + \sum_{i < j} V_{ij}^{(2)} \\ + \sum_{i < j < k} V_{ijk}^{(3)},$$

A. Tohsaki, Phys. Rev. C **49**, 1814 (1994).

$$V_{ij}^{(2)} = \sum_n v_n^{(2)} \exp \left\{ - \left(\frac{r_{ij}}{r_n^{(2)}} \right)^2 \right\} (W_n^{(2)} + M_n^{(2)} P_{ij})$$

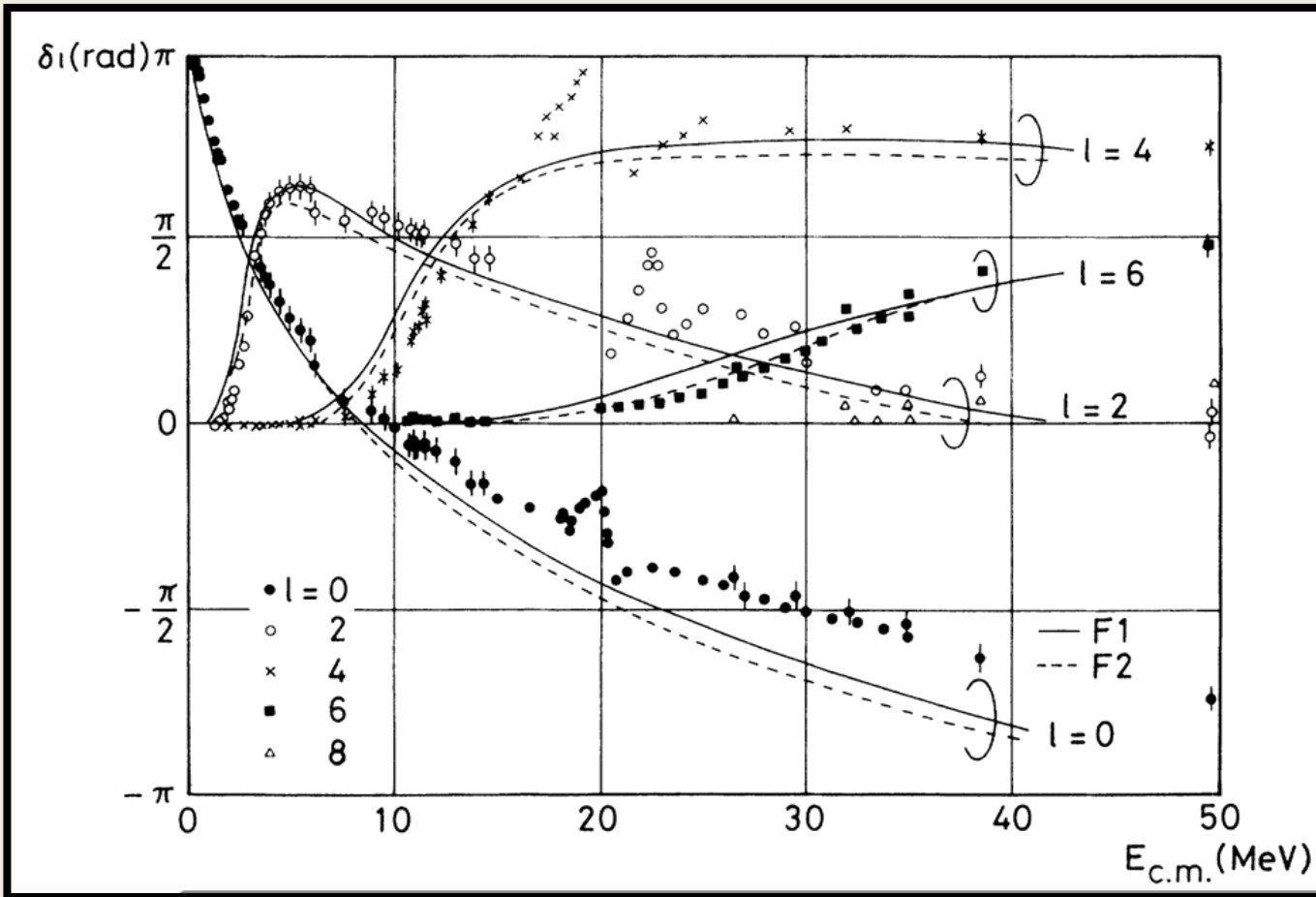
$$V_{ijk}^{(3)} = \sum_n v_n^{(3)} \exp \left\{ - \left(\frac{r_{ij}}{r_n^{(3)}} \right)^2 - \left(\frac{r_{jk}}{r_n^{(3)}} \right)^2 \right\} \\ \times (W_n^{(3)} + M_n^{(3)} P_{ij})(W_n^{(3)} + M_n^{(3)} P_{jk}),$$

TABLE I. Parameter set for the EINF 1 proposed (F1 force).

n	$r_n^{(k)}$ (fm)	$v_n^{(2)}$ (MeV)	$M_n^{(2)}$	$v_n^{(3)}$ (MeV)	$M_n^{(3)}$
1	2.5	-5.00	0.750	-0.31	0.0
2	1.8	-43.51	0.462	7.73	0.0
3	0.7	60.38	0.522	219.00	1.909

A. Tohsaki, Phys. Rev. C **49**, 1814 (1994).

Alpha-alpha Phase shift

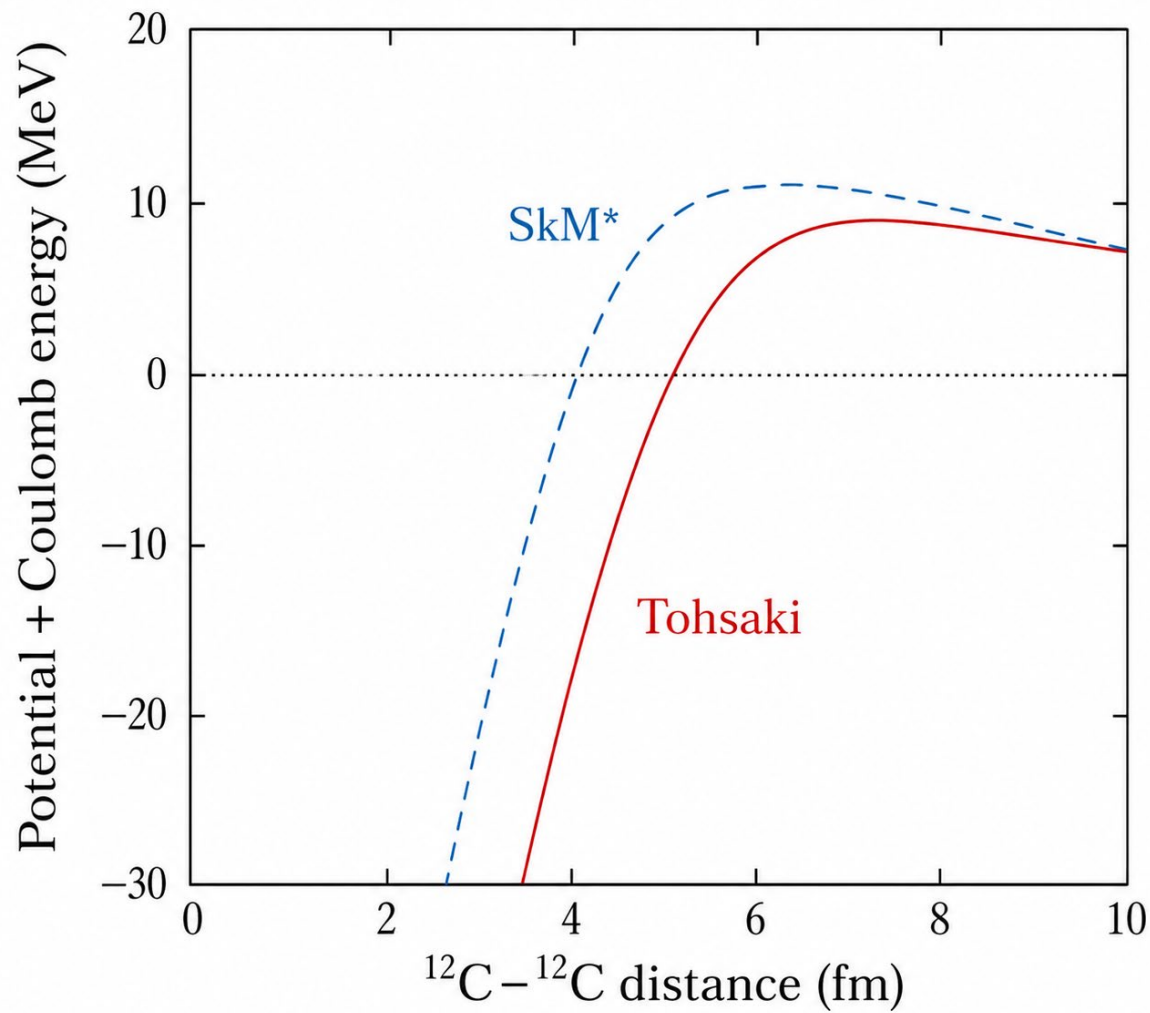


A. Tohsaki, Phys. Rev. C **49**, 1814 (1994).

TABLE III. Physical quantities for double closed shell nuclei together with nuclear matter.

		F1	F2	BB	SII	Expt.
α	E_{bin} (MeV)	27.5	27.0	27.4	26.0	28.3
	a (fm ⁻²)	0.50	0.50	0.50	0.41	0.51
¹⁶ O	E_{bin} (MeV)	123.0	124.0	93.0	121.4	127.6
	a (fm ⁻²)	0.35	0.35	0.31	0.32	0.34
	E_{mp} (MeV)	23.3	27.7	23.2	32.7	26.4
⁴⁰ Ca	E_{bin} (MeV)	334.0	340.2	250.8	325.5	342.1
	a (fm ⁻²)	0.26	0.27	0.25	0.26	0.27
	E_{mp} (MeV)	20.8	24.1	19.0	27.0	23.5
NM	E/A (MeV)	17.0	16.7	15.7	16.0	16.0
	k_f (fm ⁻¹)	1.27	1.29	1.45	1.30	1.30
	K (MeV)	309	332	193	342	300

A. Tohsaki, Phys. Rev. C **49**, 1814 (1994).



Projected
to 0^+

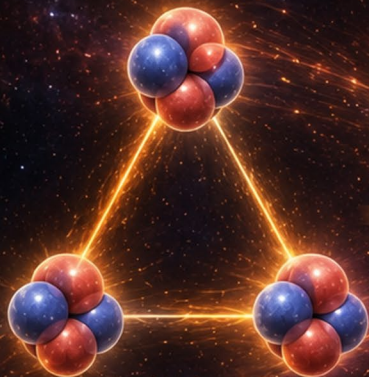
Motivation 2

- If the ^{12}C nucleus has spherical shape with $J^\pi = 0^+$, the projection of the angular momentum of the total system is equivalent to the projection of relative angular momentum.

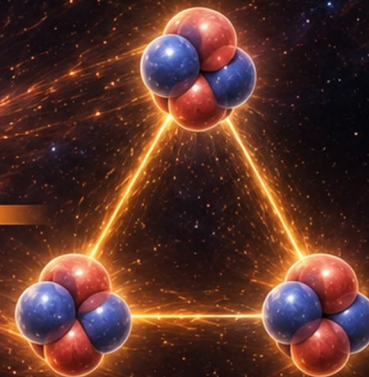
$$\left[\text{---} \right]^{J^\pi} = \left[\text{---}^{J^\pi} \right]$$

The diagram consists of two horizontal rows. The top row shows two gray circles connected by a horizontal line, enclosed in large square brackets. A superscript J^π is placed to the upper right of the closing bracket. The bottom row shows two identical gray circles connected by a horizontal line, also enclosed in large square brackets. A superscript J^π is placed above the connecting line. An equals sign is positioned to the left of the bottom row's opening bracket.

α 粒子
(ヘリウム4原子核)

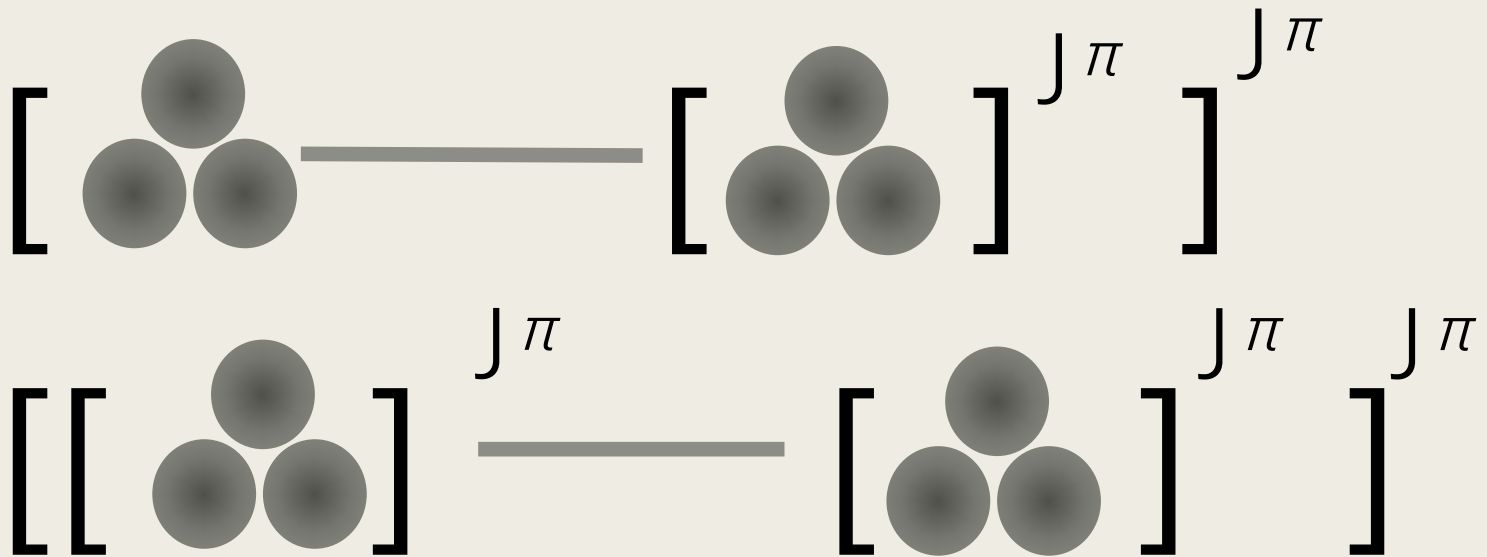


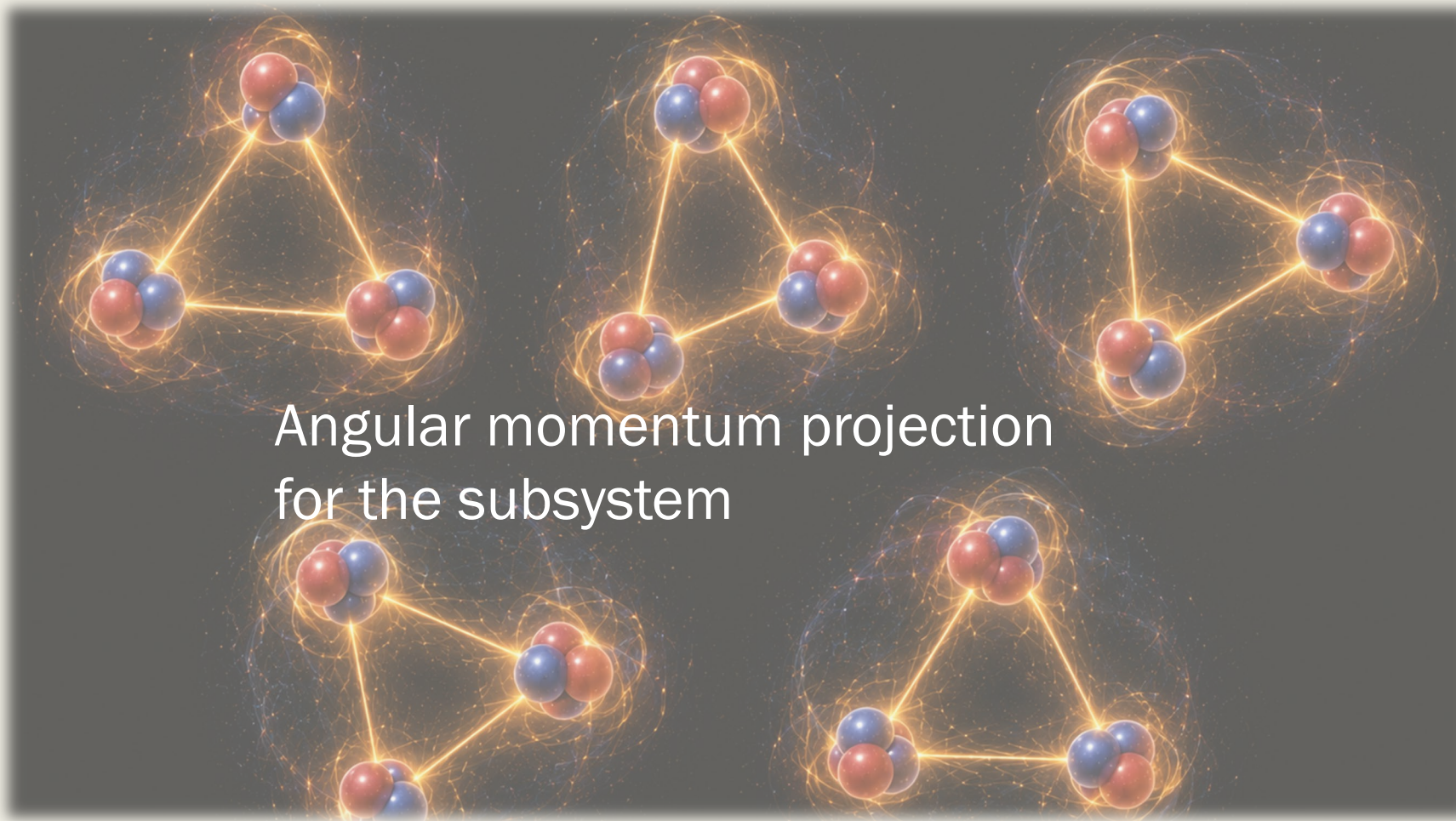
α 粒子
(ヘリウム4原子核)



Motivation 2

- If ^{12}C nucleus is treated as 3 alpha clusters, double projection or triple projection is needed.
- How much is the double or triple projection effect?



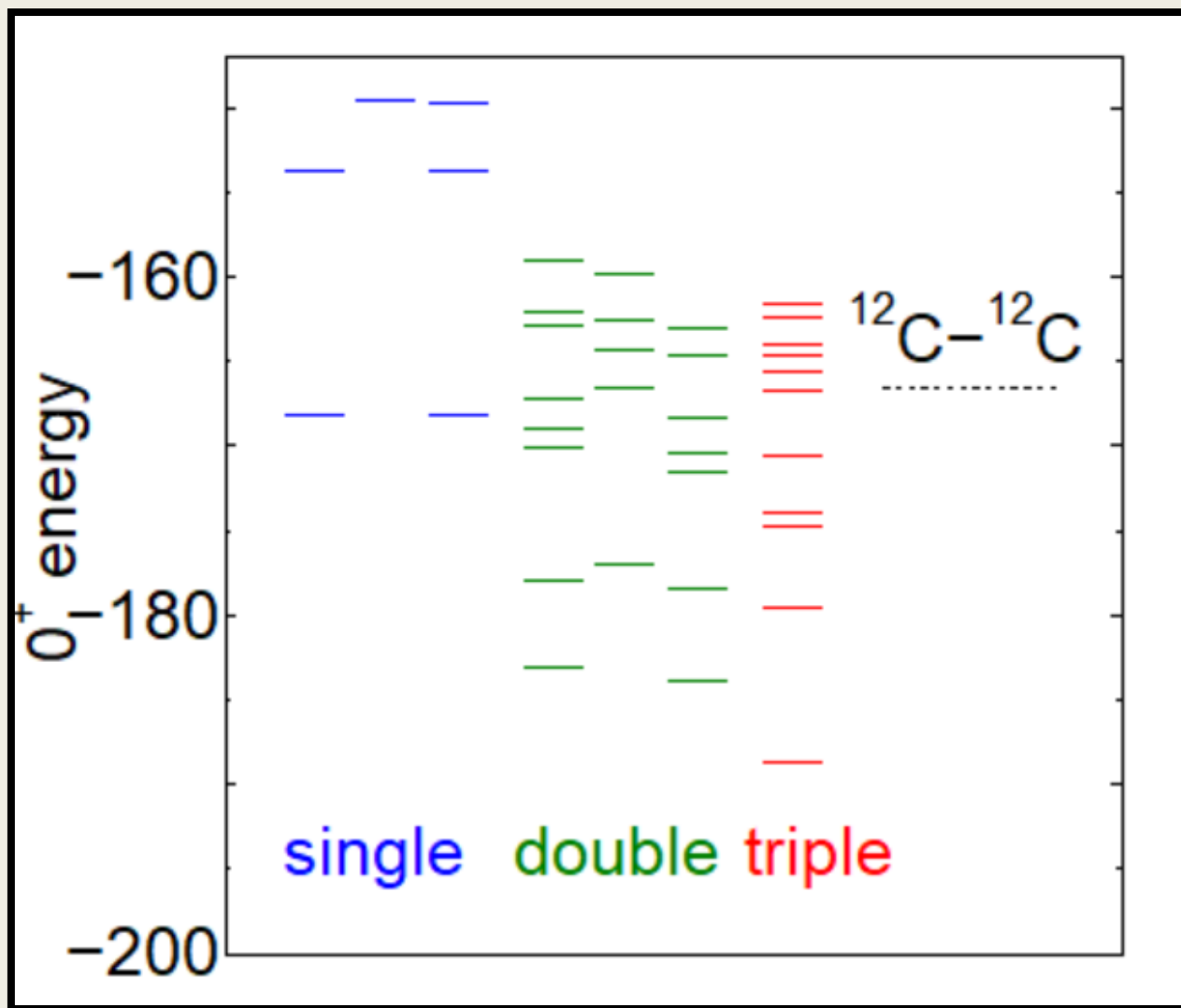


Angular momentum projection
for the subsystem

The image displays five distinct configurations of a three-body system, each represented by three clusters of red and blue spheres (likely nucleons) connected by glowing yellow lines. These clusters are enclosed within a complex, golden, web-like field. The configurations are arranged in two rows: three in the top row and two in the bottom row. Each configuration illustrates a different state or projection of angular momentum for the subsystem, showing how the relative positions and orientations of the three clusters change.

Method

- Each ^{12}C nucleus is represented as equilateral triangular shape of three alpha clusters with the relative distance of 2.5 fm.
- For the double and triple projections, the orientations of three alpha clusters are determined by generating random numbers for the Euler angles, and each Slater determinants with fixed orientation is treated as independent basis states.
- The basis states with different ^{12}C - ^{12}C distance and orientation of each ^{12}C are superposed based on the generator coordinate method (GCM).
- Angular momentum projection is single projection.
- ^{12}C - ^{12}C distance is restricted within 6 fm to avoid continuum solution.



Breit-Wigner formula

$$\sigma(E) = \frac{\pi \hbar^2 (2J + 1)}{\mu E} \frac{\Gamma_I \Gamma_{A_1 + A_2}}{(E - E_R)^2 + \Gamma^2/4},$$

Modified Astrophysical S-factor

$$S^*(E) = E \sigma(E) \exp(2\pi \eta + 0.46 \text{ MeV}^{-1} E),$$

J.R. Patterson, H. Winkler, C.S. Zaidins, *Astrophys. J.* **157** (1969) 36

Width

$$\Gamma = 2P_l(a)\gamma^2(a),$$

Penetrability

$$P_l(a) = \frac{ka}{F_l^2(ka) + G_l^2(ka)},$$

Reduced Width Amplitude

$$\gamma^2(a) = \frac{\hbar^2}{2\mu a} [ay_l(a)]^2$$

Useful approximation

$$|ay_l(a)| \approx \frac{1}{\sqrt{2}} \left(\frac{2\gamma}{\pi} \right)^{1/4} |\langle \Phi | \Phi_{\text{BB}}^{J\pi}(S_k = a) \rangle| \equiv ay^{\text{app}}(a).$$

$$\gamma = \nu A_1 A_2 / A$$

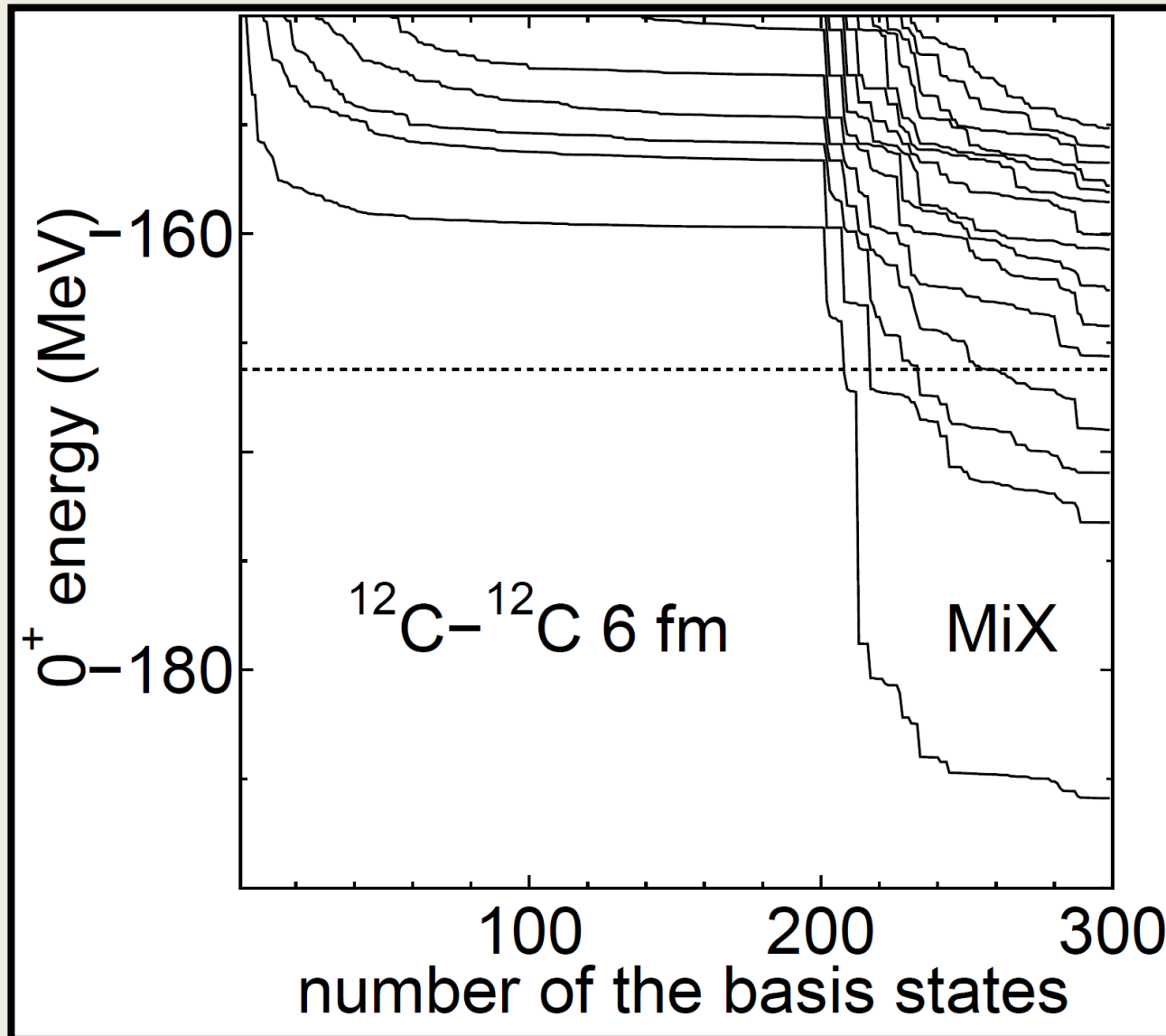
Overlap between the solved state and the model space
at the channel radius

Y. Kanada-En'yo , T. Suhara , Y. Taniguchi,
Prog. Theor. Exp. Phys. **2014**, 073D02 (2014).

GCM 0^+ energy convergence

Squared
overlap
with
 $^{12}\text{C}-^{12}\text{C}$ 6 fm

0.45
0.00
0.05
0.00
0.06
0.01
0.00
0.00

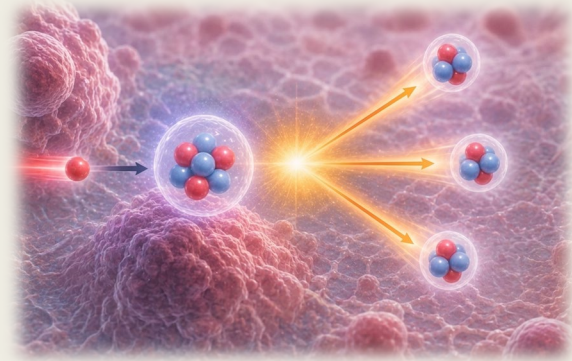


Summary for this this part

- We have investigated the ^{12}C - ^{12}C structure of ^{24}Mg with an interaction (Tohsaki interaction) developed for the cluster study.
- The finite range effect of this interaction gives significant change for the energy curve (potential+Coulomb) compared with the previous SkM* result.
- The treatment of 3alpha structure for each ^{12}C requires the double and triple projections of the angular momentum, which has significant effect.
- States around the threshold has certain components of the $^{12}\text{C}(0^+)$ - $^{12}\text{C}(0^+)$ channel; however, due to the Coulomb interaction, the cluster state with the relative distance of 6 fm has strong overlap with the state well above the threshold (mixing of ^{20}Ne +alpha channel may be essential for the states around threshold).

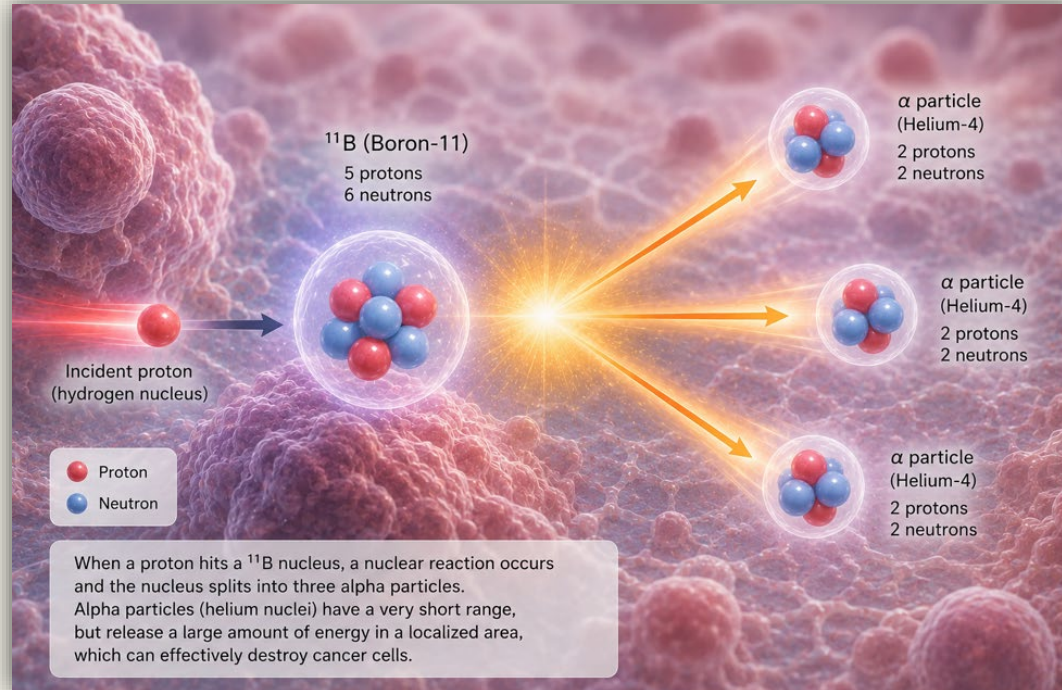
Contents

- 1: General description of cluster-shell competition
- 2: How we identify the cluster states?
- 3: Clustering of $^{12}\text{C}+^{12}\text{C}$ and astrophysical application
- 4: $^{11}\text{B}+p$ reaction and 3 alpha cluster structure in ^{12}C



Clustering in ^{11}B is important for the emission of 3 alpha's when a proton is injected

- Proton Boron Capture Therapy (PBCT)
- $^{11}\text{B} + \text{p} \rightarrow ^{12}\text{C}^* \rightarrow 3 \alpha$ may destroy cancer cells

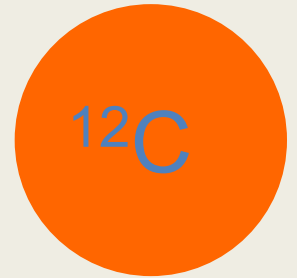


^{12}C threshold energies

$$^{11}\text{B} + \text{p} \rightarrow 15.96 \text{ MeV}$$

$$^8\text{Be} + \alpha \rightarrow 7.38 \text{ MeV}$$

$$^8\text{Be} \rightarrow \alpha + \alpha$$



Energy release of $\sim 8.5 \text{ MeV}$ is quite large compared with $^{10}\text{B} + \text{n} \rightarrow ^7\text{Li} + \alpha + 2.8 \text{ MeV}$

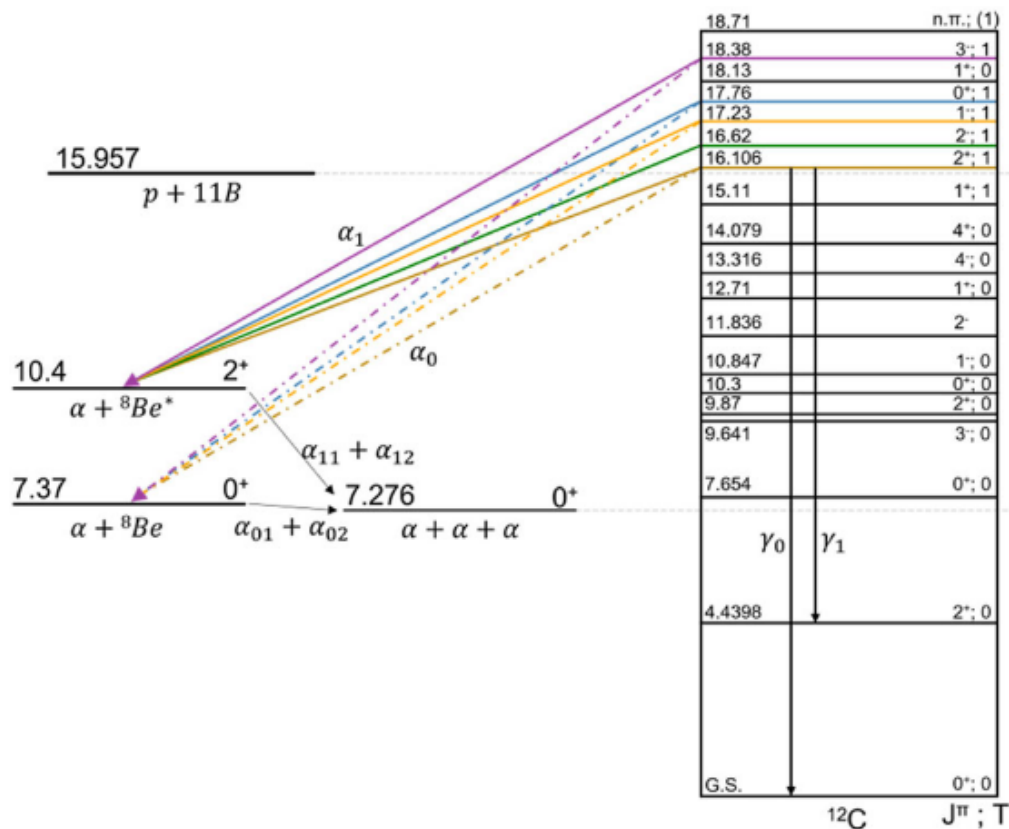


Fig. 1 The reaction scheme for five excited states of ^{12}C is shown. The arrows indicate the possible decay channel (α_0 or α_1) according to the conservation of angular momentum and parity

Mazzucconi, D., Bellotti, E.S., Vavassori, D. et al.

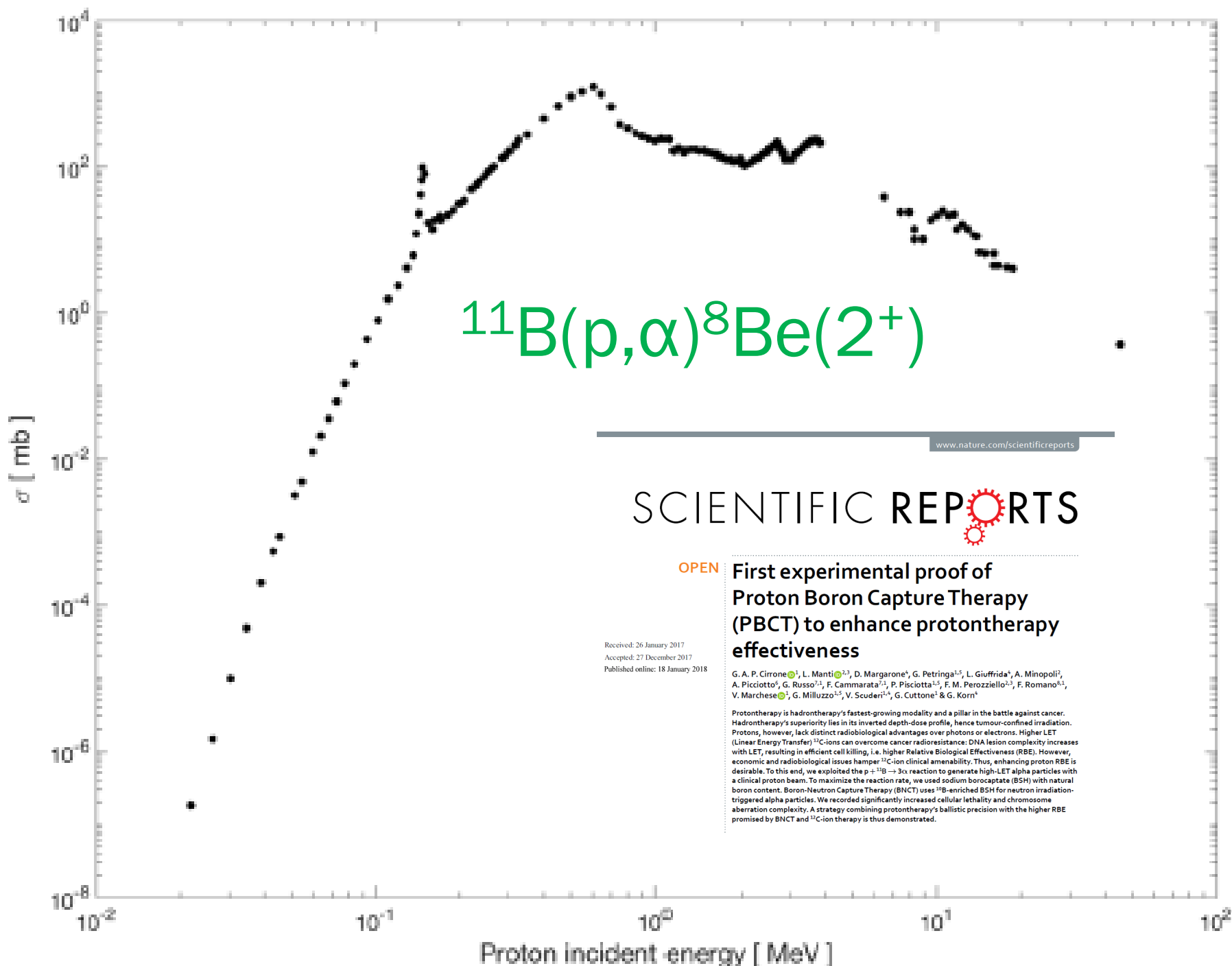
Evaluation of the p-11B reaction cross section through a silicon telescope in the 0.3–4.7 MeV range.

Eur. Phys. J. A **61**, 114 (2025).

<https://doi.org/10.1140/epja/s10050-025-01589-3>

#	Nuclide	E_x [keV]	J^π _{order}	Band	$T_{1/2}$ or Width	$T_{1/2}$ [s]	Decay modes BR [%]	Isospin	μ [μ_N]	Q [b]	Additional data	
1	$^{12}_6\text{C}_6$	0	0+		STABLE			0			g factor	2.0010415963
2	$^{12}_6\text{C}_6$	4439.82 27	2+		10.8×10^{-3} eV 6	42.2E-15 23	IT 100	0		6 3		
3	$^{12}_6\text{C}_6$	7654.07 19	0+ 2		9.3 eV 9	49E-18 5	$\alpha \approx 100$ IT 4.16×10^{-2}	0				
4	$^{12}_6\text{C}_6$	9641 5	3-		46 keV 3	9.9E-21 6	$\alpha \approx 100$ IT $< 4.1 \times 10^{-5}$	0				
5	$^{12}_6\text{C}_6$	9870 60	2+ 2		850 keV 85	54E-24 5	$\alpha \approx 100$ IT $\approx 7.1 \times 10^{-6}$	0				
6	$^{12}_6\text{C}_6$	9930 ? 30	0+ 3		2710 keV 80	16E-24 5						
7	$^{12}_6\text{C}_6$	10.3×10^3 ? 3	(0+)		3.0 MeV 7	15E-24 4	$\alpha \approx 100$	0				
8	$^{12}_6\text{C}_6$	10847 4	1-		273 keV 5	1.67E-21 3	$\alpha \approx 100$	0				
9	$^{12}_6\text{C}_6$	11836 4	2-		230 keV 8	1.98E-21 7	$\alpha \approx 100$ IT > 0					
10	$^{12}_6\text{C}_6$	12400 ?	(5+,4-,6-,7+)				$\alpha \approx 100$					
11	$^{12}_6\text{C}_6$	12710 6	1+		18.1 eV 28	25E-18 4	α 97.8 IT 2.2	0			Width	$17.7^{28}_{\times 10^Q 12}$
12	$^{12}_6\text{C}_6$	13.3×10^3 ? 2	4+		1.7 MeV 2	27E-24 3		0				
13	$^{12}_6\text{C}_6$	13316 20	4-		360 keV 43	1.27E-21 15	$\alpha \approx 100$ IT > 0	0				
14	$^{12}_6\text{C}_6$	14079 5	4+ 2		272 keV 6	1.68E-21 4	$\alpha \approx 100$	0				
15	$^{12}_6\text{C}_6$	15110 3	1+ 2		43.6 eV 10	10.46E-18 24	IT 95.9 α 4.1	1			Width	$1.8^{4}_{41.8 12}$
16	$^{12}_6\text{C}_6$	15440 40	(2+)		1.77 MeV 20	26E-24 3	$\alpha \approx 100$	0				
17	$^{12}_6\text{C}_6$	16106.0 8	2+ 3		5.3 keV 2	86E-21 3	α 99.3 p 0.41 IT 0.27	1			Width	$14.4^{17}_{21.5 33}$ 5.26 2
18	$^{12}_6\text{C}_6$	16620 50	2- 2		280 keV 28	1.63E-21 16	$\alpha \approx 50$ p ≈ 50 IT 2.9×10^{-3}	1			Width	$\times 10^Q$ $\times 10^Q$ $\times 10^Q$
19	$^{12}_6\text{C}_6$	17230	1- 2		1.15 MeV	396.72E-24	p 87 α 13 IT 4.3×10^{-3}	1			Width	$\times 10^Q$ $\times 10^Q$ AP
20	$^{12}_6\text{C}_6$	17760 20	0+ 4		96 keV 5	4.8E-21 2	p 82 α 17.4 IT $< 10^{-3}$	1				

#	Nuclide	E_x [keV]	J^π _{order}	Band	$T_{1/2}$ or Width	$T_{1/2}$ [s]	Decay modes BR [%]	Isospin	μ [μ_N]	Q [b]	Additional data
1	$^{12}_6\text{C}_6$	0	0+		STABLE			0			g factor 2.0010415963
2	$^{12}_6\text{C}_6$	4439.82 27	2+		10.8×10^{-3} eV 6	42.2E-15 23	IT 100	0		6 3	
3	$^{12}_6\text{C}_6$	7654.07 19	0+ 2		9.3 eV 9	49E-18 5	$\alpha \approx 100$ IT 4.16×10^{-2}	0			
4	$^{12}_6\text{C}_6$	9641 5	3-		46 keV 3	9.9E-21 6	$\alpha \approx 100$ IT $< 4.1 \times 10^{-5}$	0			
5	$^{12}_6\text{C}_6$	9870 60	2+ 2		850 keV 85	54E-24 5	$\alpha \approx 100$ IT $\approx 7.1 \times 10^{-6}$	0			
6	$^{12}_6\text{C}_6$	9930 ? 30	0+ 3		2710 keV 80	16E-24 5					
7	$^{12}_6\text{C}_6$	$10.3 \times 10^3 ? 3$	(0+)		3.0 MeV 7	15E-24 4	$\alpha \approx 100$	0			
8	$^{12}_6\text{C}_6$	10847 4	1-		273 keV 5	1.67E-21 3	$\alpha \approx 100$	0			
9	$^{12}_6\text{C}_6$	11836 4	2-		230 keV 8	1.98E-21 7	$\alpha \approx 100$ IT > 0				
10	$^{12}_6\text{C}_6$	12400 ?	(5+,4-,6-,7+)				$\alpha \approx 100$				
11	$^{12}_6\text{C}_6$	12710 6	1+		18.1 eV 28	25E-18 4	α 97.8 IT 2.2	0			Width 17.7 28 $\times 10^Q$ 12
12	$^{12}_6\text{C}_6$	$13.3 \times 10^3 ? 2$	4+		1.7 MeV 2	27E-24 3		0			
13	$^{12}_6\text{C}_6$	13316 20	4-		360 keV 43	1.27E-21 15	$\alpha \approx 100$ IT > 0	0			
14	$^{12}_6\text{C}_6$	14079 5	4+ 2		272 keV 6	1.68E-21 4	$\alpha \approx 100$	0			
15	$^{12}_6\text{C}_6$	15110 3	1+ 2		43.6 eV 10	10.46E-18 24	IT 95.9 α 4.1	1			Width 1.8 4 41.8 12
16	$^{12}_6\text{C}_6$	15440 40	(2+)		1.77 MeV 20	26E-24 3	$\alpha \approx 100$	0			
17	$^{12}_6\text{C}_6$	16106.0 8	2+ 3		5.3 keV 2	86E-21 3	α 99.3 p 0.41 IT 0.27	1			Width 14.4 17 21.5 33 5.26 2
18	$^{12}_6\text{C}_6$	16620 50	2- 2		280 keV 28	1.63E-21 16	$\alpha \approx 50$ p ≈ 50 IT 2.9×10^{-3}	1			Width $\times 10^Q$ $\times 10^Q$ $\times 10^Q$
19	$^{12}_6\text{C}_6$	17230	1- 2		1.15 MeV	396.72E-24	p 87 α 13 IT 4.3×10^{-3}	1			Width $\times 10^Q$ $\times 10^Q$ AP
20	$^{12}_6\text{C}_6$	17760 20	0+ 4		96 keV 5	4.8E-21 2	p 82 α 17.4 IT $< 10^{-3}$	1			



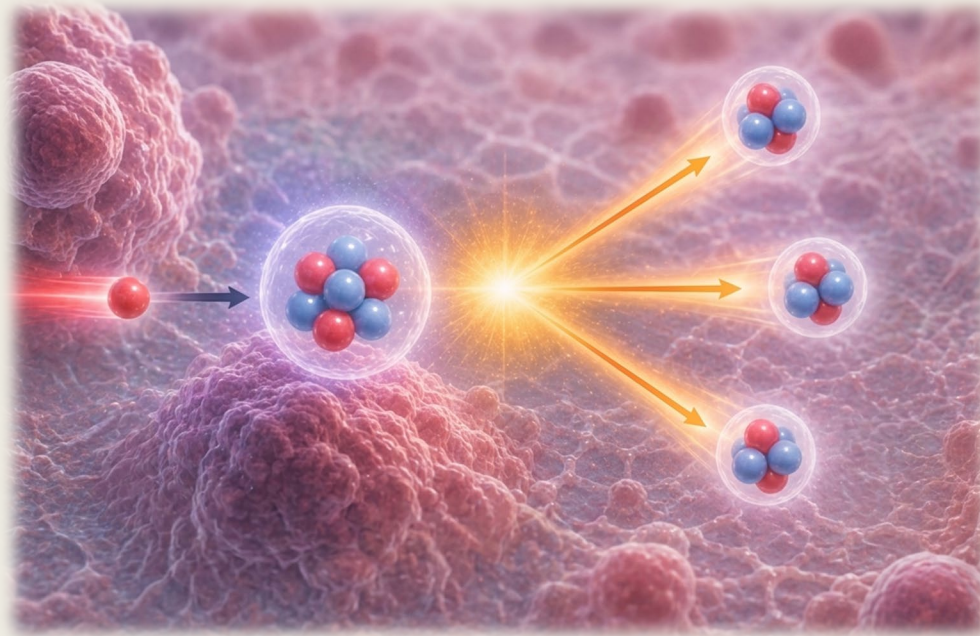
Experimental situation for the 2^+ state of ^{12}C at 16.11 MeV

Decay Widths Comparison

Decay Channel	Γ_x (Conventional Value)	Γ_x (Latest Updated Value)
Total Width Γ_{tot}	$5.3 \pm 0.2 \text{ keV}$	$5.3 \pm 0.2 \text{ keV}$
α_0 (Be_{gs})	$290 \pm 45 \text{ eV}$	$270 \pm 30 \text{ eV}$
α_1 (Be^*)	$6.3 \pm 5 \text{ keV}$	$5.0 \pm 0.2 \text{ keV}$
Proton Width Γ_p	$21.7 \pm 1.8 \text{ eV}$	$37 \pm 7 \text{ eV}$
γ (gs)	$0.59 \pm 11 \text{ eV}$	$0.35 \pm 4 \text{ eV}$
γ (4.44 MeV)	$12.8 \pm 1.7 \text{ eV}$	$10.5 \pm 1.6 \text{ eV}$

K. L. Laursen, H.O.U. Fynbo, O. S. Kirsebom, K. S. Madsbøll, and Riisager, Eur. Phys. J. **A52** 370 (2016).

Isospin mixing?



How much percent of the isospin mixing should be assumed to account for the observed alpha width?

α_1 width of ^{12}C 16.11 MeV (2^+) state

$^{12}\text{C}(2^+, T=1) \rightarrow ^8\text{Be}(2^+) + \alpha$

- Initial state: 2^+ , Final state: 2^+ .
- Emitted α carries $L = 0$ (s-wave).
- No centrifugal barrier appears.
- α emission is strongly enhanced.
- Assumed amplitude between $^{12}\text{C}(2^+)$ and $^8\text{Be}(2^+) + \alpha$ cluster state is 0.04.
- $\Gamma_{\alpha 1} = 5.22$ keV compared with the experimental value of $\Gamma_{\alpha 1} \approx 5.0$ keV.

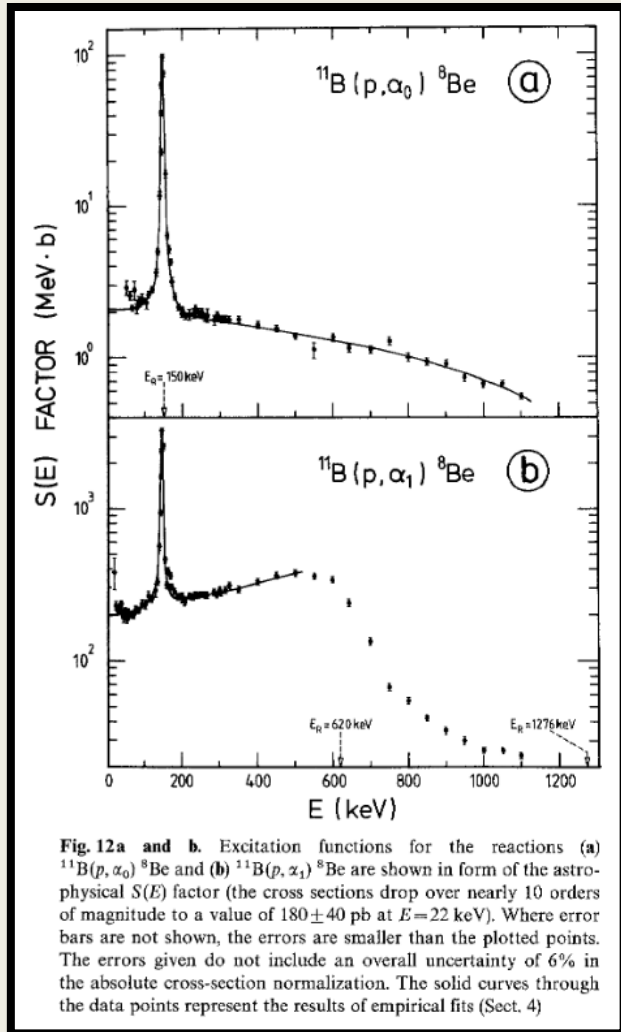
~0.1% $T = 0$ component is required

- A little large isospin-mixing is required?
- According to the ab initio calculation based on the Green's Function Monte Carlo, the isospin mixing is the order of 10^{-5} in ${}^4\text{He}$ and 10^{-4} in light nuclei.
- Microscopic calculation to analyze the isospin mixing is going on.

Let us remove the Coulomb penetration effect to see the physics

- Let us convert the cross section to the astrophysical S factor.

Astrophysical S factor



Contribution of the subthreshold
T=0 cluster state (15.44 MeV)?

Three alpha states of ^{12}C not observed yet contribute?

- ^{11}B ground state is $3/2^-$ and a proton has $1/2^+$.
- Most probable angular momentum for the relative motion is 0^+ .
- 3α state is considered to have natural parity, 1^- .
- Another possibility is the contribution of the subthreshold state at 15.44 MeV, which is (2^+) .

How we select important basis states for non-bound states?

PHYSICAL REVIEW C **105**, 024314 (2022)

Optimization of basis functions for multiconfiguration mixing using the replica exchange Monte Carlo method and its application to ^{12}C

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¹*Digital & SaaS Sherpa AI, SIGMAXYZ Inc., 4-1-28 Toranomon, Minato-ku, Tokyo 105-0001, Japan*

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(Received 25 October 2021; accepted 27 January 2022; published 16 February 2022)

Background: To calculate excited states in quantum many-body systems, multiconfiguration mixing has often been employed. However, it remains unclear how to choose important Slater determinants from a huge model space.

Purpose: We propose an efficient method as the replica exchange Monte Carlo (RXMC) method to sample important Slater determinants and optimize and analyze the results obtained. As an application, we investigate the ground and excited states of ^{12}C based on the Bloch-Brink α cluster model and show the detailed structure of the obtained states.

Methods: The RXMC method enables us to efficiently sample Slater determinants following the Boltzmann distribution on the multidimensional potential-energy surface (PES) under a given model space. To analyze the excited states obtained, we embed sampled basis functions onto the PES calculated with the β - γ constraint method and discuss the main component in each state.

Results: The RXMC method efficiently performs the samplings with a temperature parameter of $T_L = 2.5$ MeV in ^{12}C . We obtain the gas-like state with a wide density distribution in the tail part in the second 0^+ state. We also obtain the linear-chain-like states with the bending and stretching vibrational modes in the third and fourth 0^+ states, respectively. In the fifth 0^+ state, the main component of the basis functions contains expanded equilateral-triangle configurations.

Conclusions: The second 0^+ gas-like state emerges at the local minimum in the PES, which is the beginning of the valley structure connected to the linear-chain breakup channel. The third and fourth linear-chain-like states emerge in this valley structure. We conclude that the RXMC method is a powerful method to calculate the excited states of nuclei, which would be applied to much more complicated nuclear fission dynamics in heavier nuclei.

DOI: [10.1103/PhysRevC.105.024314](https://doi.org/10.1103/PhysRevC.105.024314)

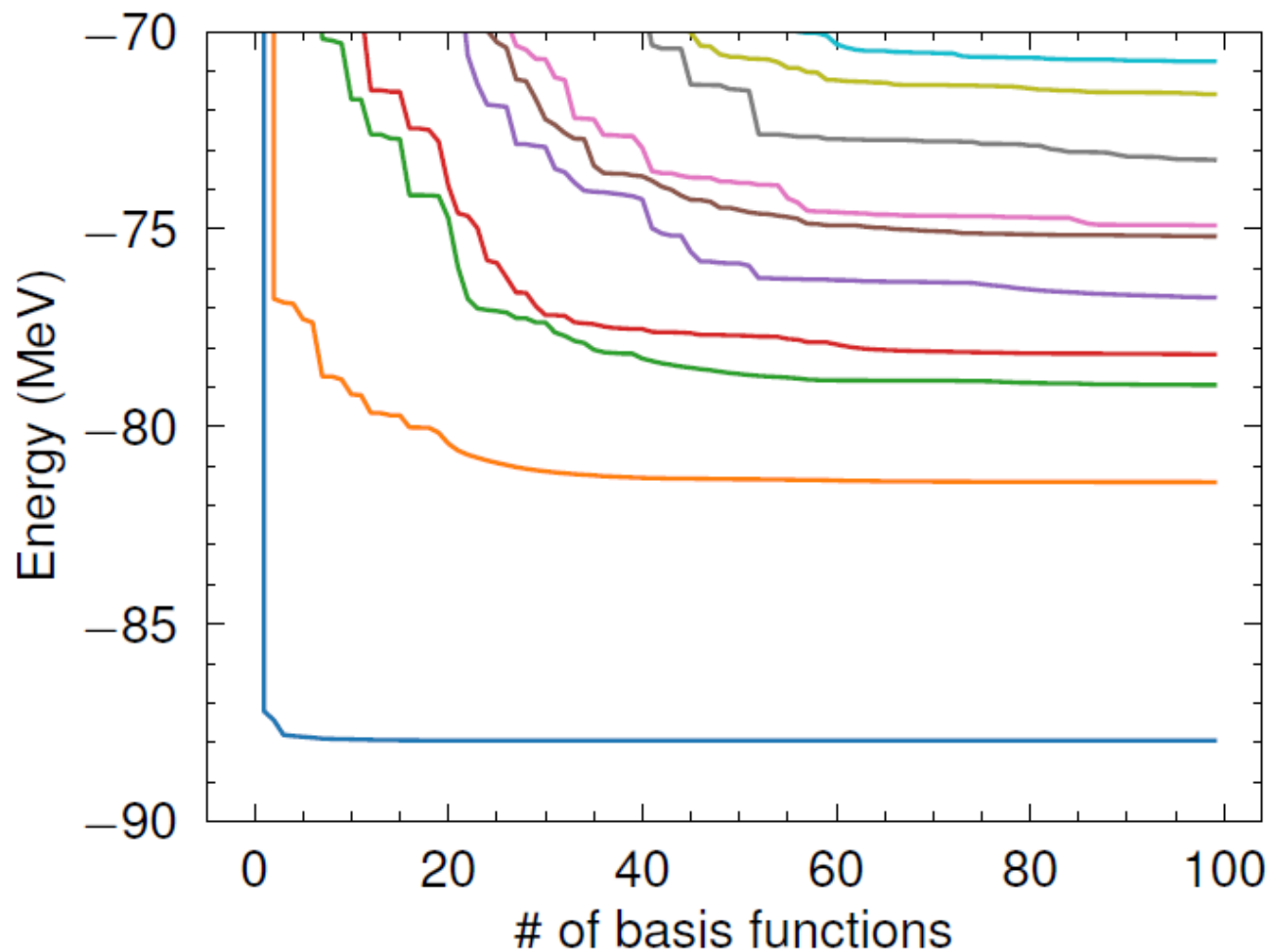


FIG. 7. Convergence of energies for the 0^+ states of ^{12}C as a function of the number of basis functions sampled with $T_L = 2.5$ MeV.

Volkov No.2 (M=0.6)

angular momentum 0^+

-87.933 -81.211 -77.765 -77.627 -74.891 -73.375

angular momentum 2^+

-85.556 -78.488 -75.740 -74.562 -73.175 -70.372

angular momentum 4^+

-79.546 -74.772 -73.277 -71.599 -70.265 -67.848

angular momentum 1^-

-77.758 -75.584 -72.055 -70.990 -70.105 -68.247

angular momentum 3^-

-80.361 -76.075 -73.458 -71.872 -70.147 -67.908

Tohsaki No.1

angular momentum 0^+

-86.6382 -81.3150 -78.2113 -77.7273 -75.8842

angular momentum 2^+

-84.0520 -78.4091 -76.1162 -74.3722 -74.2053

angular momentum 4^+

-77.5305 -74.8277 -73.7488 -72.0577 -70.7400

angular momentum 1^-

-77.7504 -75.8838 -72.4256 -71.9665 -70.4307

angular momentum 3^-

-80.3999 -76.0137 -73.6767 -72.4220 -70.9014

Summary for this part

- For the cancer treatment, 3 alpha states of ^{12}C above the $^{11}\text{B}+p$ threshold (16.0 MeV) are important.
- However, the states observed until now is T=1 state that are isobaric analogue states of ^{12}B and ^{12}N .
- It is found that the ^{12}C must have rather large isospin mixing of $\sim 0.1\%$ to account for the observed α -decay width.
- We investigated whether unobserved T=0 states around this energy region.
- These state may have large decay width and serve as the background of the cross section, which could be more important than individual peaks.

Thank you very much

