Towards high precision in multi-channel reactions of composite nuclei

"Always aim to convince and not primarily to impress."

— Jürgen Habermas

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Espace de Structure et de réactions Nucléaires Théorique (ESNT)

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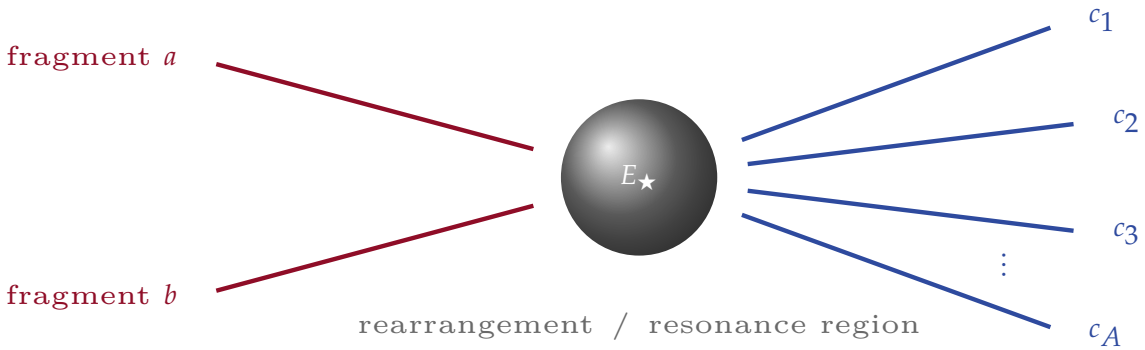
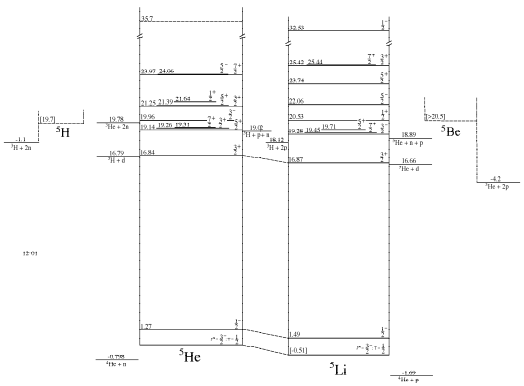


$$d + t \longrightarrow n + {}^4\mathrm{He}$$

via $^5\mathrm{He}^*$, broad $\frac{3}{2}^+$ at 16.84 MeV

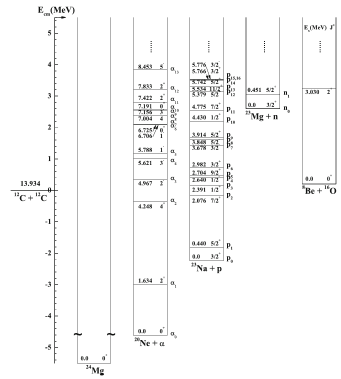


NCSM, RGM, GFMC, SVM+R-matrix,&c.

$$\Psi = \Psi_{\text{compound}} + \Psi_{\text{direct}} = \sum_\mu a_\mu \Phi_\mu + \mathscr{A}\left[\Phi_d \Phi_t \chi(r)\right]$$

$$^{12}\mathrm{C} + {}^{12}\mathrm{C} \longrightarrow {}^{20}\mathrm{Ne} + \alpha \,/\, {}^{23}\mathrm{Na} + p$$

via $^{24}\mathrm{Mg}^*$ ($E_x \approx 14\,\mathrm{MeV} + E_{\mathrm{c.m.}}$)



double-folding + coupled channels

(Glauber / CDCC)

$$\left[T_R + V_{cc}(R) - (E - \epsilon_c)\right]\chi_c(R) = -\sum_{c' \neq c} V_{cc'}(R)\,\chi_{c'}(R)$$

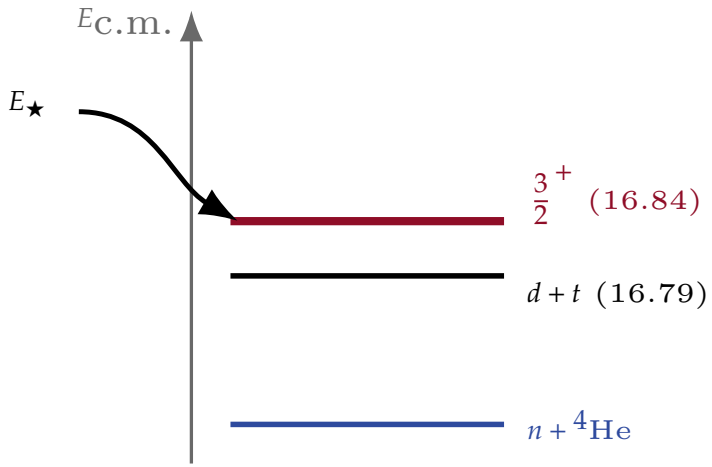
identical fermions: α, nucleons

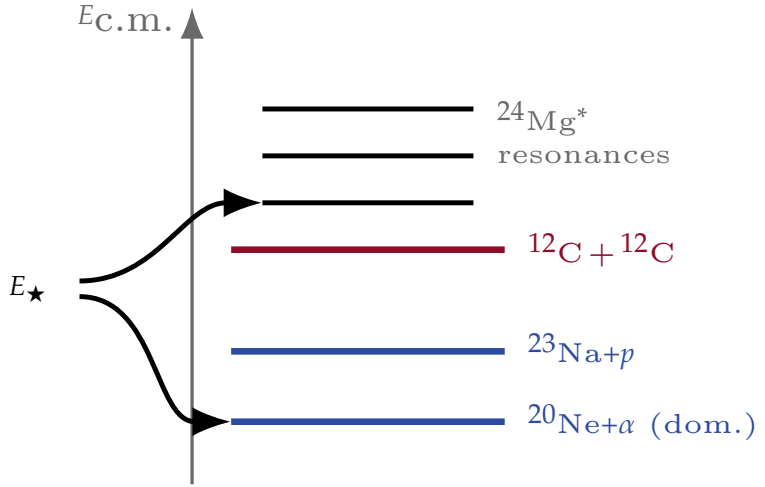
potential with long- and short-range, perturbative and non-perturbative components

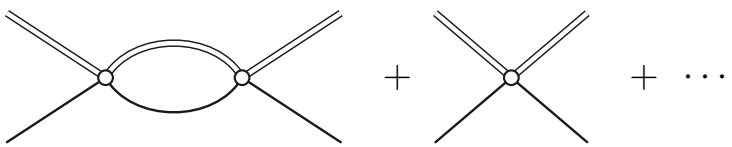
defined energy interval of interest: $E_\star^{(\mathrm{res/rearr})} \lesssim E_{\mathrm{c.m.}} \ll E_{\mathrm{exc}}^{(d,t,\alpha,N)}$

Hupin *et al.*, Nat. Commun. **10**, 351 (2019) | Horiuchi *et al.*
Taniguchi & Kimura, PLB **823**, 136790 (2021) Hiyama & Kamimura,Front. Phys. **13**, 132101 (2018)

$E_\star$: INTERMEDIATE STATES WITH SIGNIFICANT IMPACT ON ASYMPTOTIC OBSERVABLES



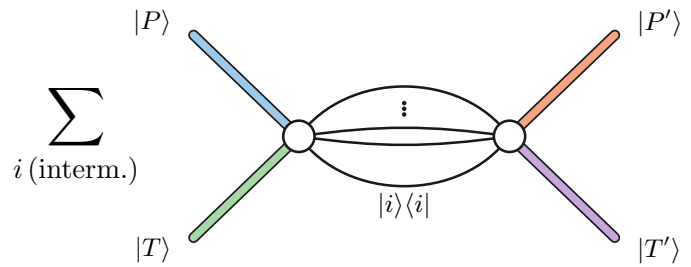




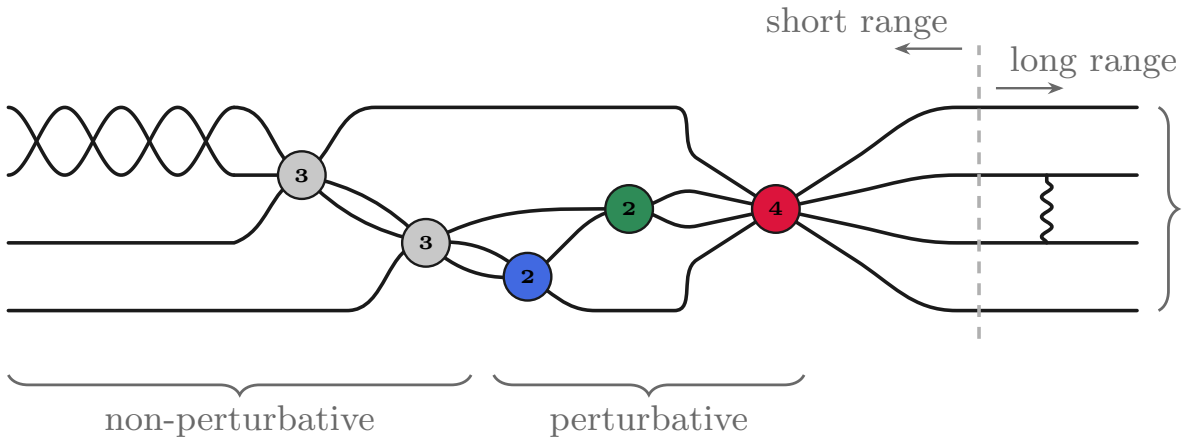
Intermediate states = the in/out fragments only

⇒ insufficient for the relevant reactions.

e.g., the $\frac{3}{2}^+$ resonance in ^{5}He is not a d+t state.

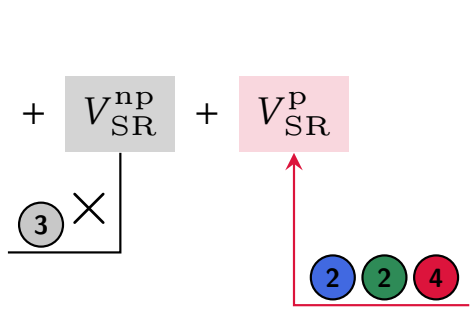


In $\sum_i |i\rangle\langle i|$ both the *interaction* **and** *solution algorithm* are active.



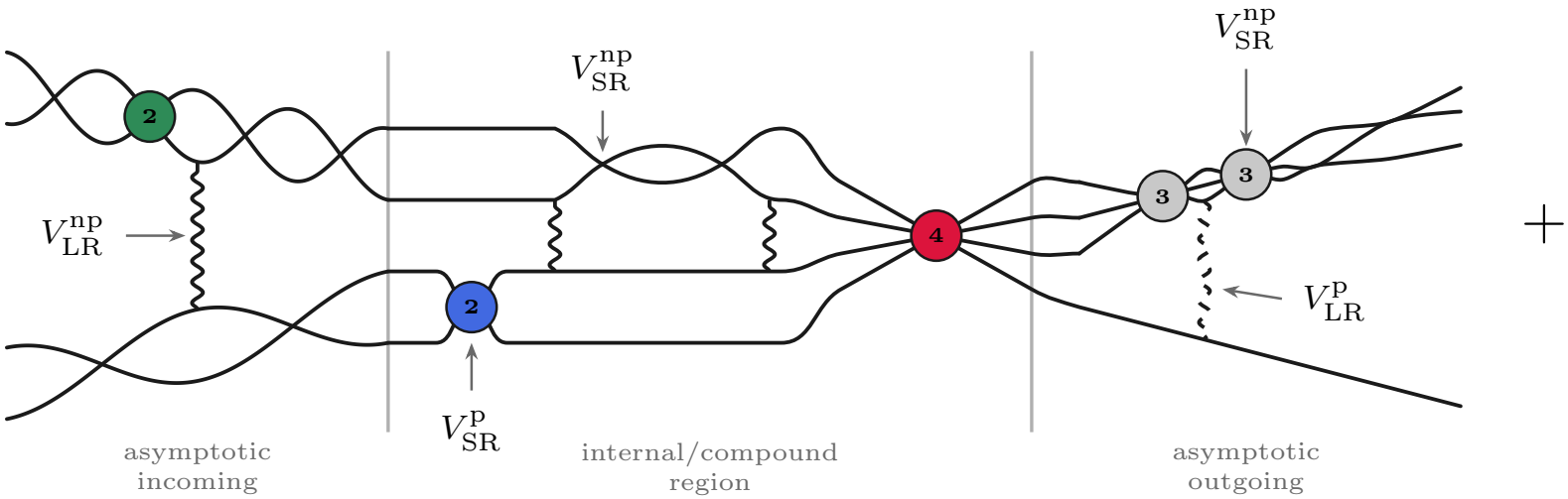
Few-body potential for non-relativistic applications

$$V = \sum_i \hat{t}_i + V^{\mathrm{p}}_{\mathrm{LR}} + V^{\mathrm{np}}_{\mathrm{LR}} + V^{\mathrm{np}}_{\mathrm{SR}} + V^{\mathrm{p}}_{\mathrm{SR}}$$



from a field-theoretical Hamiltonian density linking high- and low-energy scales

$$H_{\mathrm{eff}} = \sum_i \psi_i^\dagger \left(-\frac{\nabla^2}{2m} \right) \psi_i + \tilde{C}_1 (\psi^\dagger \psi)^2 + \tilde{C}_3 (\psi^\dagger \overleftrightarrow{\nabla} \psi) \cdot (\psi^\dagger \overleftrightarrow{\nabla} \psi) + \ldots + \tilde{D}_0 (\psi^\dagger \psi)^3 + \tilde{E}_0 (\psi^\dagger \psi)^4$$



(i) power counting: each vertex carries an order $v(n)$ set by its dimension & scaling

(ii) order N: sum *all* initial→final processes with $\sum_{\text{vertices}} v(n) \leqslant N$

(iii) convergence to arbitrary accuracy: $\Delta^{(N+1)} = \Delta^{(N)} \cdot \epsilon\,, \qquad \epsilon = \dfrac{\text{small scale}}{\text{large scale}} < 1$

Central challenge: Implementation

One possible approach:

1. choose **basis-expansion** scheme ⇒ accurate fragment **wave functions**
   (considering, at first, all iterations of all potential components)
2. **solve** the multi-channel scattering problem
   (again, without regards to perturbations, but with appropriate compound structures expanded)
   and map out the **functional dependence** of this solution on the strengths of all perturbative terms contributing to order N accuracy
3. adapt a **multi-dimensional power series** to this functional dependence ⇒ order-N accuracy

implementation:

1. efficient representation of V

$$V_{\mathrm{SR}}^{\mathrm{n/np}}(r) = C_1\,\delta_a^{(3)}(r) + a^2 C_2\,\nabla^2\delta_a^{(3)}(r) + a^4\Big[\,C_3\Big(\frac{L(L+1)}{a^4} + \frac{2}{a^2}\Big(\frac{1}{2}\overleftrightarrow{\nabla}\Big)^2\delta_a^{(3)}(r)\Big) + C_4\,\nabla^4\delta_a^{(3)}(r) - C_5\,\nabla^2\delta_a^{(3)}(r)\Big(\frac{1}{2}\overleftrightarrow{\nabla}\Big)^2\Big]$$

$$\leftrightarrow\quad V_{\mathrm{SR}}^{\mathrm{n/np}}(K,Q) = C_1' + C_2'\,K^2 + C_3'\,(K\times Q)^2 + C_4'\,K^4 + C_5'\,K^2Q^2$$

2. callibration of the potential

informed selection of input data to optimize convergence and minimize uncertainty already at low orders