Glauber-theory calculations using many-body wave functions

ESNT workshop: Towards high precision in multi-channel reactions of composite nuclei

2026.6.29-7.10

CEA Saclay, France

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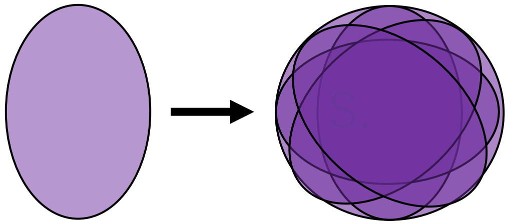
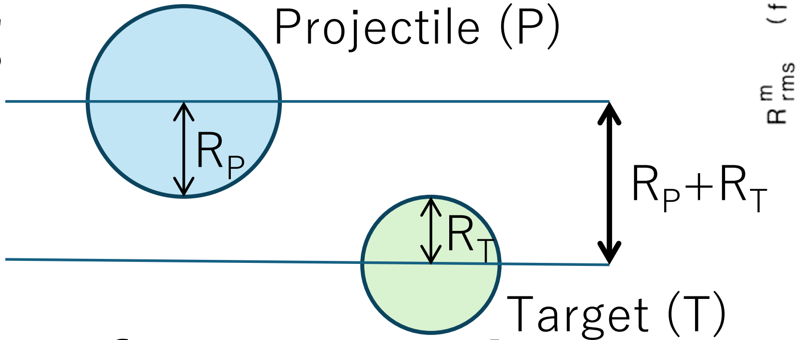
WH, Y. Suzuki, R. B. Wiringa, Phys. Rev. Lett. 136, 202501 (2026)
WH, Y. Suzuki, R. B. Wiringa, Phys. Rev. C 113, 064601 (2026)

Motivation

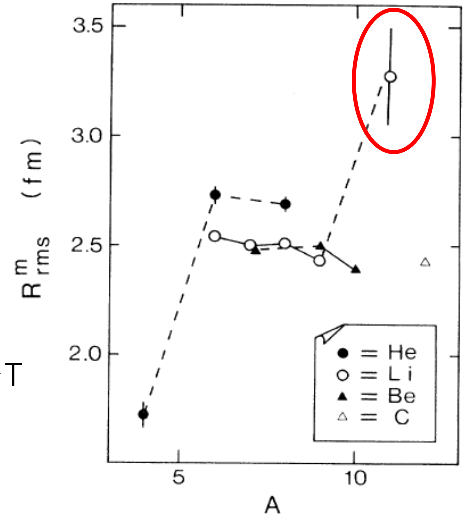
I. Tanihata et al., Phys. Rev. Lett. 55, 2676 (1985)
S. Bagchi et al., Phys. Rev. Lett. 124, 222504 (2020)
WH, T. Inakura, T. Nakatsukasa, Y. Suzuki, Phys. Rev. C 86, 024614 (2012)
M. Takechi et al., Phys. Rev. C 90, 061305(R) (2014)

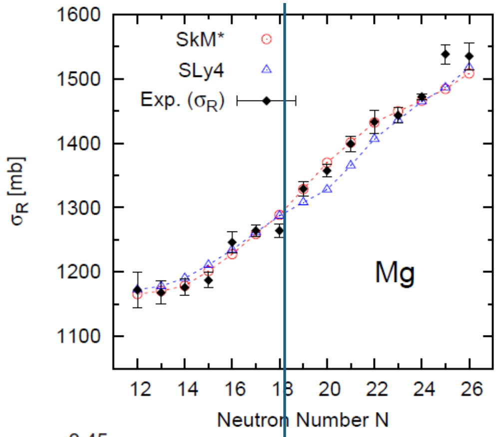


- Medium- to high-energy nucleus-nucleus scattering (∼100-1000 MeV/nucleon)
  - **Total reaction (interaction) cross section**
  - **Elastic scattering**



Total reaction (interaction) cross section $\sim \pi (R_P+R_T)^2$

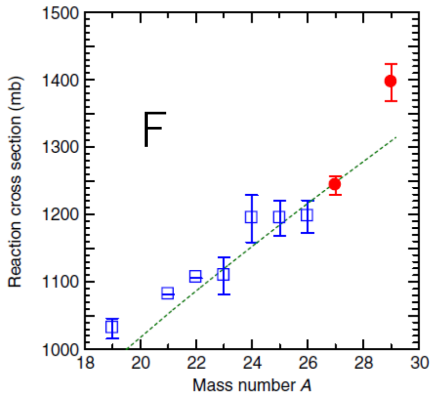


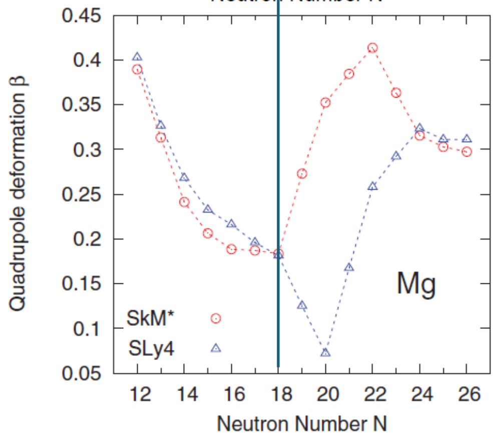


A probe of nuclear surface properties

- Halo/skin nuclei
- Spectroscopic information: shell evolution
- Deformation, core swelling, bubble, etc.

→ Bulk properties of nuclear matter





Motivation

- Glauber theory R. J. Glauber, in *Lectures in Theoretical Physics*, (Interscience, New York, 1959), Vol. 1, p. 315.
  - Microscopic high-energy reaction theory
    - Eikonal and adiabatic approximation
    - Starting from NN scattering phase-shift function

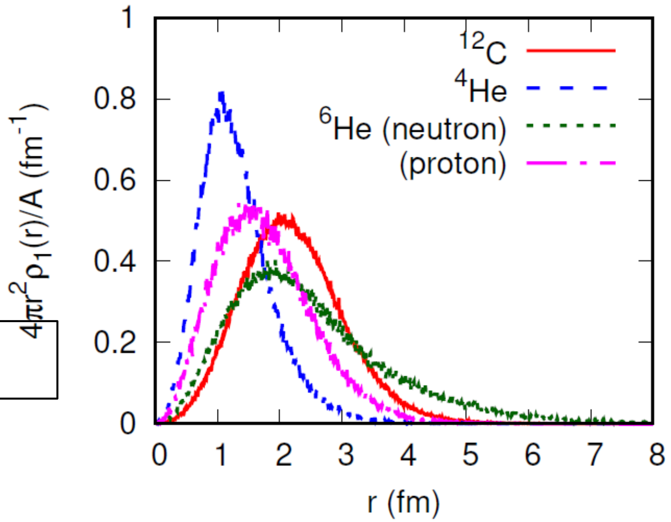
  - Direct relationship between many-nucleon wave function and scattering observables
    - “Original” Glauber-theory calculation is involved (e.g., multifold integral)
    - “Approximated” Glauber theory is often applied

Outline

- Complete evaluation of the Glauber's phase-shift function (psf) involving ^{12}C
  - ^{12}C: Standard target nucleus to extract nuclear size properties
  - Total reaction and elastic scattering cross sections can be evaluated **unambiguously**

  - Multiple integrals performed by a Monte Carlo integration
  - Input: Variational Monte Carlo (VMC) wave function
    - Previous work (^{12}C: cluster model wave function) K. Varga et al., Phys. Rev. C 66, 034611 (2002)
    - Realistic interaction: AV18+UIX R. B. Wiringa et al., Phys. Rev. C 51, 38 (1995)
    - This work: ^{12}C, ^{4}He, ^{6}He B. S. Pudliner et al., Phys. Rev. Lett. 74, 4396 (1995)
      - 40000 configurations are prepared

Nucleus	r_m	r_n	r_p	r_p (Expt.)
^{12}C	2.35(1)	2.35(1)	2.35(1)	2.326±0.002
^{4}He	1.44(1)	1.44(1)	1.44(1)	1.455±0.003
^{6}He	2.54(1)	2.80(1)	1.93(1)	1.92±0.01



The most accurate Glauber theory calculation involving ^{12}C

- Accuracy of the conventional approximations
  - OLA (leading order), 2nd order OLA, NTG

Glauber Theory

R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. 1, p. 315.

- Efficient description of medium to high energy scattering
  - Eikonal approximation
  - Adiabatic approximation

$$\Psi(\boldsymbol{R}, \boldsymbol{r}_1^P, \ldots, \boldsymbol{r}_{A_P}^P, \boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{A_T}^T) \equiv e^{iKZ} \hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}_1^P, \ldots, \boldsymbol{r}_{A_P}^P, \boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{A_T}^T)$$

$$\left[vP_z + \frac{\boldsymbol{P}^2}{2M_{PT}} + (H_P - E_0^P) + (H_T - E_0^T) + \sum_{i\in P}^{A_P} \sum_{j\in T}^{A_T} V(\boldsymbol{R} + \boldsymbol{r}_i^P - \boldsymbol{r}_j^T) \right] \hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}_1^P, \ldots, \boldsymbol{r}_{A_P}^P, \boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{A_T}^T) = 0,$$

$$Z \to \infty \qquad \hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}_1^P, \ldots, \boldsymbol{r}_{A_P}^P, \boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{A_T}^T) = \underline{e^{i\chi(\boldsymbol{b})}}\, \Psi_0^P(\boldsymbol{r}_1^P, \ldots, \boldsymbol{r}_{A_P}^P) \Psi_0^T(\boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{A_T}^T).$$

- Essential quantity: Glauber's phase-shift function (psf)

$$e^{i\chi(\boldsymbol{b})} = \langle \Psi_0^P \Psi_0^T | \hat{\Psi} \rangle = \langle \Psi_0^P \Psi_0^T | \prod_{i\in P}^{A_P} \prod_{j\in T}^{A_T} e^{i\chi_{NN}(\boldsymbol{b} + \boldsymbol{s}_i^P - \boldsymbol{s}_j^T)} | \Psi_0^P \Psi_0^T \rangle \qquad \boldsymbol{r}_j = \boldsymbol{s}_j + z_j \hat{\boldsymbol{z}}$$

$e^{i\chi_{NN}}$:NN phase-shift function

- "Original" Glauber theory
  - Input: A-body densities ($|\Psi_0|^2$) of projectile and target
    → $3 \times (A_T + A_P)$ fold integral for each b **expensive!**
  - Optical limit approximation (OLA) often be used
    - Input: one-body density (3+3-fold integral)

Cross sections

Glauber's psf

$$e^{i\chi(\boldsymbol{b})} = \langle \Psi_0^P \Psi_0^T | \prod_{i\in P}^{A_P} \prod_{j\in T}^{A_T} e^{i\chi_{NN}(\boldsymbol{b}+\boldsymbol{s}_i^P-\boldsymbol{s}_j^T)} | \Psi_0^P \Psi_0^T \rangle$$

$$= \langle \Psi_0^P \Psi_0^T | \prod_{i=1}^{A_P} \prod_{j=1}^{A_T} [1-\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}_i^P-\boldsymbol{s}_j^T)] | \Psi_0^P \Psi_0^T \rangle$$

$e^{i\chi_{NN}(\boldsymbol{b})} \equiv 1-\Gamma_{NN}(\boldsymbol{b})$ Profile function $\Gamma_{NN}(\boldsymbol{b}) = \frac{1-i\alpha_{NN}}{4\pi\beta_{NN}}\sigma_{NN}^{\mathrm{tot}} \exp\left(-\frac{b^2}{2\beta_{NN}}\right)$

B. Abu-Ibrahim, WH, Y. Suzuki, A. Kohama, Phys. Rev. C 77, 034607 (2008).

Consistent with NN scattering properties for E<~300 MeV

Contribution of hadron productions at E>~300 MeV

NNN contribution negligible in the high-energy reaction processes

Total reaction cross section $\sigma_R = \int d\boldsymbol{b}\left(1-|e^{i\chi(\boldsymbol{b})}|^2\right)$

Elastic scattering cross section $\frac{d\sigma}{d\Omega}(\theta) = |F(\theta)|^2$

F_C: Rutherford scattering amplitude

Scattering amplitude $F(\theta) = F_C(\theta) + \frac{iK}{2\pi}\int d\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}+i\chi_C(\boldsymbol{b})}(1-e^{i\chi(\boldsymbol{b})})$

$$i\chi_C(\boldsymbol{b}) = 2i\eta \ln(Kb)$$
$$\eta = Z_P Z_T e^2/\hbar v$$

Variational Monte Carlo (VMC) method

- Hamiltonian

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} : Argonne v18 two-nucleon potential

V_{ijk} : Urbana IX three-nucleon potential

- Variational principle $E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$

Trial function

$$|\Psi_V\rangle = \mathcal{S} \prod_{i<j}^{A} \left[1 + U_{ij} + \sum_{k \neq i,j}^{A} \tilde{U}_{ijk} \right] |\Psi_J\rangle$$

Jastrow type wave function

$$|\Psi_J\rangle = \prod_{i<j} f_c(r_{ij}) |\Phi_A(J^\pi; TT_z)\rangle$$

Short-range correlation

Fully antisymmetrized wf (Uncorrelated)

Two-body correlator $U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O_{ij}^p$

Three-body correlator $\tilde{U}_{ijk} = -\epsilon \tilde{V}_{ijk}(\tilde{r}_{ij}, \tilde{r}_{jk}, \tilde{r}_{ki})$

Monte Carlo integration

A type of integration $\int p(x)g(x)dx,$ x : multiple variable

$\int p(x)dx = 1$ $p(x) \geq 0$ Probability distribution

$$\int p(x)g(x)dx \approx \frac{1}{M}\sum_{i=1}^{M} g(x_i),$$

$x_1, x_2, \ldots, x_M$ generated by Metropolis method

N. Metropolis et al., J. Chem. Phys. 21, 1087 (1953)

Glauber's psf

$$e^{i\chi(\boldsymbol{b})} = \int \cdots \int d\boldsymbol{r}_1^P \ldots d\boldsymbol{r}_{A_P}^P d\boldsymbol{r}_1^T \ldots d\boldsymbol{r}_{A_T}^T\, \rho_{A_P}^P(\boldsymbol{r}_1^P, \ldots, \boldsymbol{r}_{A_P}^P)\rho_{A_T}^T(\boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{A_T}^T) \prod_{i=1}^{A_P}\prod_{j=1}^{A_T}\left[1 - \Gamma_{NN}(\boldsymbol{b} + \boldsymbol{s}_i^P - \boldsymbol{s}_j^T)\right],$$

A-body density (Probability distribution)

$$\rho_A(\bar{\boldsymbol{r}}_1, \ldots, \bar{\boldsymbol{r}}_A) = \langle \Psi | \prod_{i=1}^{A} \delta(\boldsymbol{r}_i - \bar{\boldsymbol{r}}_i) | \Psi \rangle$$

VMC wave function

$$e^{i\chi(\boldsymbol{b})} = \frac{1}{M^P M^T}\sum_{k=1}^{M^P}\sum_{l=1}^{M^T}\left\{\prod_{i=1}^{A_P}\prod_{j=1}^{A_T}\left[1 - \Gamma_{NN}(\boldsymbol{b} + \boldsymbol{t}_{i,k}^P - \boldsymbol{t}_{j,l}^T)\right]\right\}$$

E.g., ^{12}C-^{12}C (40k*40k)
1.6 billion configs.
~60 days for each E

Coulomb interaction

- Colomb breakup (CBU) contribution

Total Coulomb phase

$$e^{i\chi_C^{\mathrm{tot}}(\boldsymbol{b},\{\boldsymbol{s}^P\},\{\boldsymbol{s}^T\})} \equiv \prod_{j=1}^{A_P}\prod_{k=1}^{A_T} e^{i\epsilon_j\epsilon_k\chi_C(\boldsymbol{b}_{jk})}.$$

Coulomb phase

$$\chi_C(b) = -\eta\int_{-\infty}^{\infty} dz\frac{1}{\sqrt{b^2+z^2}},$$

Colomb potential between P and T

$$V_C(\boldsymbol{R}) = \left\langle \sum_{i=1}^{A_P}\sum_{j=1}^{A_T}\frac{\epsilon_i\epsilon_j e^2}{|\boldsymbol{R}+\boldsymbol{r}_i^P-\boldsymbol{r}_j^T|}\right\rangle$$

$$\approx \frac{1}{N_PN_T}\sum_{k=1}^{N_P}\sum_{l=1}^{N_T}\sum_{i=1}^{A_P}\sum_{j=1}^{A_T}\frac{\epsilon_i\epsilon_j e^2}{|\boldsymbol{R}+\bar{\boldsymbol{r}}_{i,k}^P-\bar{\boldsymbol{r}}_{j,l}^T|},$$

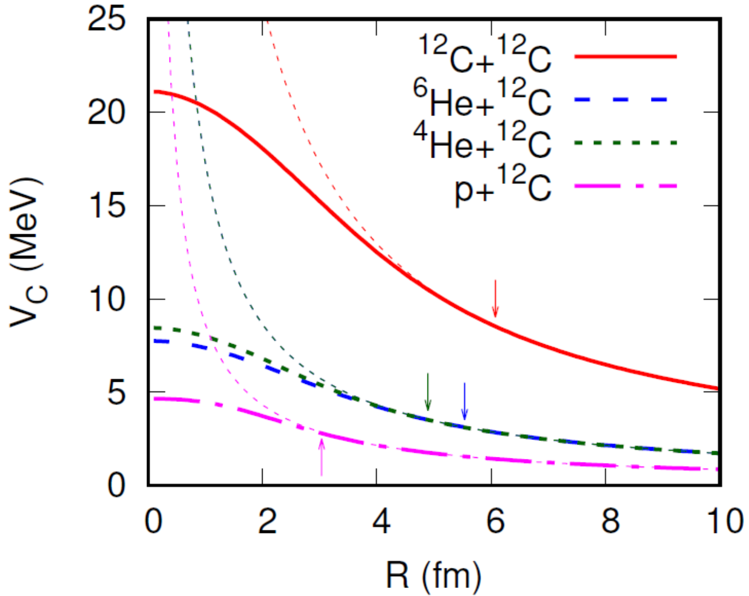
$$\chi_C^{\mathrm{tot}}(\boldsymbol{b},\{\boldsymbol{s}^P\},\{\boldsymbol{s}^T\}) = \chi_C^{\mathrm{point}}(b) + \Delta\chi_C(\boldsymbol{b},\{\boldsymbol{s}^P\},\{\boldsymbol{s}^T\}),$$

$$\Delta\chi_C(\boldsymbol{b},\{\boldsymbol{s}^P\},\{\boldsymbol{s}^T\}) = 2\eta\sum_{j=1}^{A_P}\sum_{k=1}^{A_T}\epsilon_j\epsilon_k\ln\frac{|\boldsymbol{b}_{jk}|}{b}$$

Divergent behavior at b→∞

CBU occurs at b less than $b_C \sim R_P+R_T$

$$e^{i\Delta\chi_C(\boldsymbol{b},\{\boldsymbol{s}^P\},\{\boldsymbol{s}^T\}} \equiv \prod_{j=1}^{A_P}\prod_{k=1}^{A_T}\left(\frac{|\boldsymbol{b}_{jk}|}{b}\right)^{2i\eta\epsilon_j\epsilon_k}\Theta(b_C-b)+\Theta(b-b_C).$$



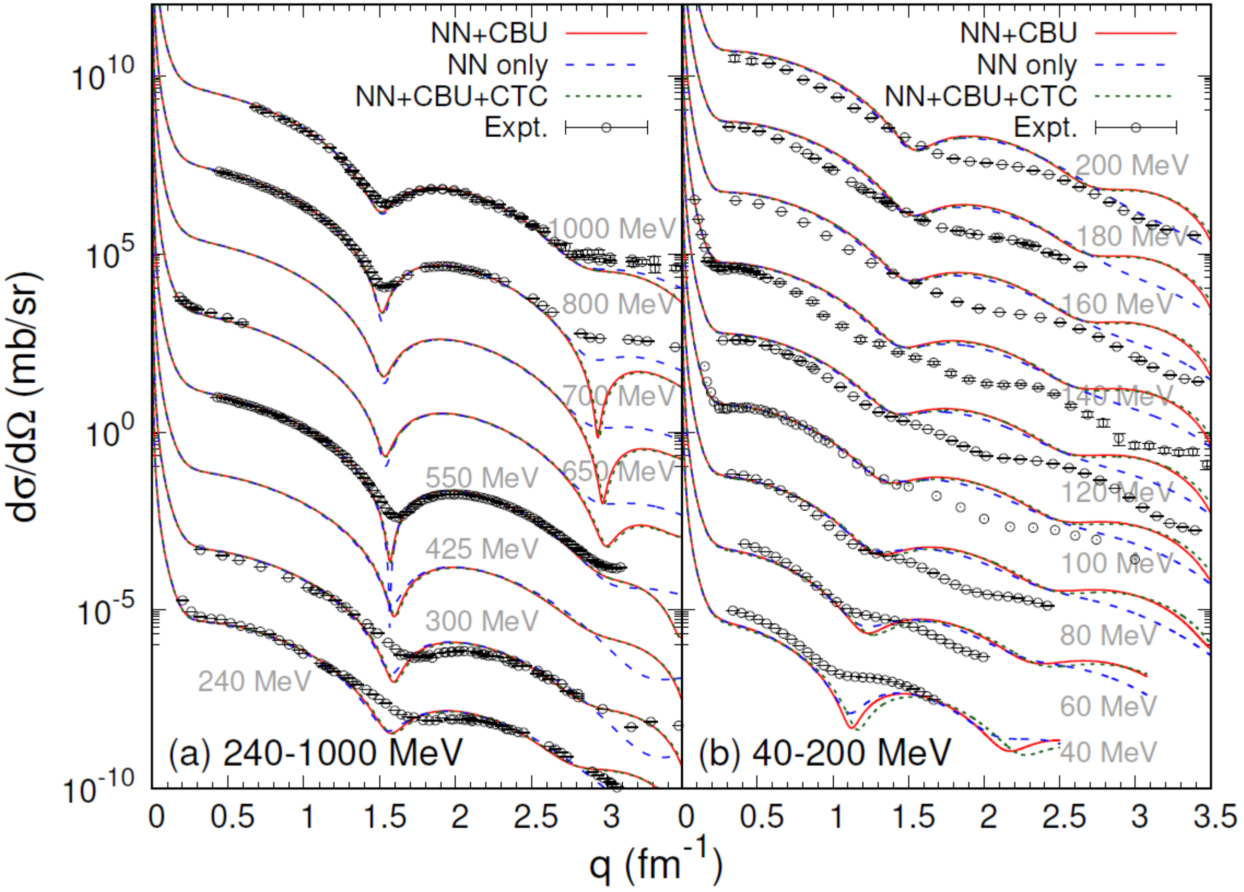
- Coulomb trajectory correction

A. Vitturi, F. Zardi, Phys. Rev. C 36, 1404 (1987)

$$Kb \to \sqrt{(Kb)^2+\eta_{PT}^2}+\eta_{PT}.$$

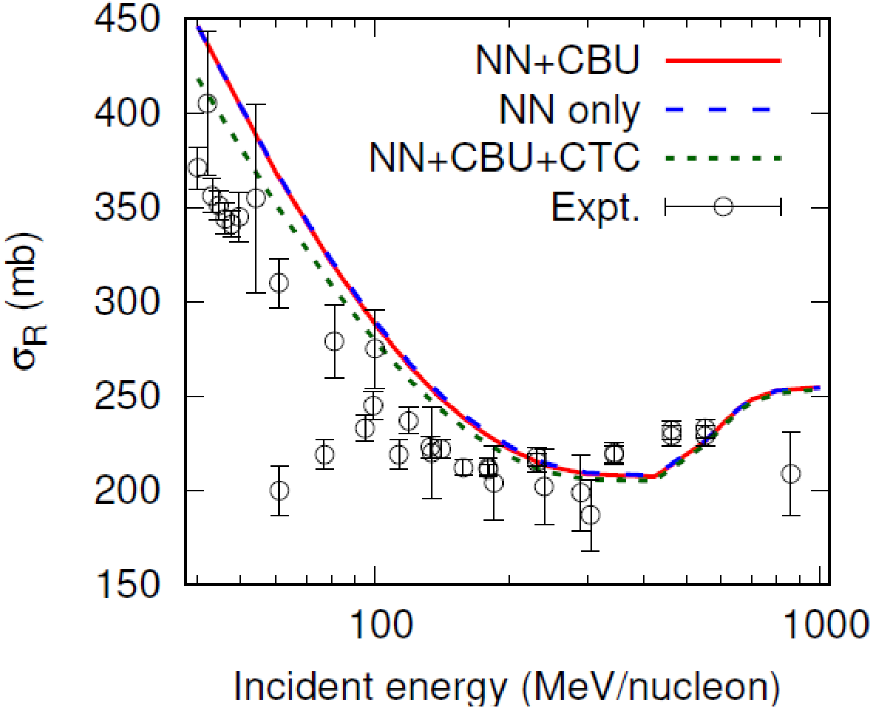
Proton-^{12}C scattering

Elastic scattering cross sections



Perfect agreement Fair agreement

Total reaction cross sections



Eikonal condition: Ka>>1, q/K<<1

a: interaction radius

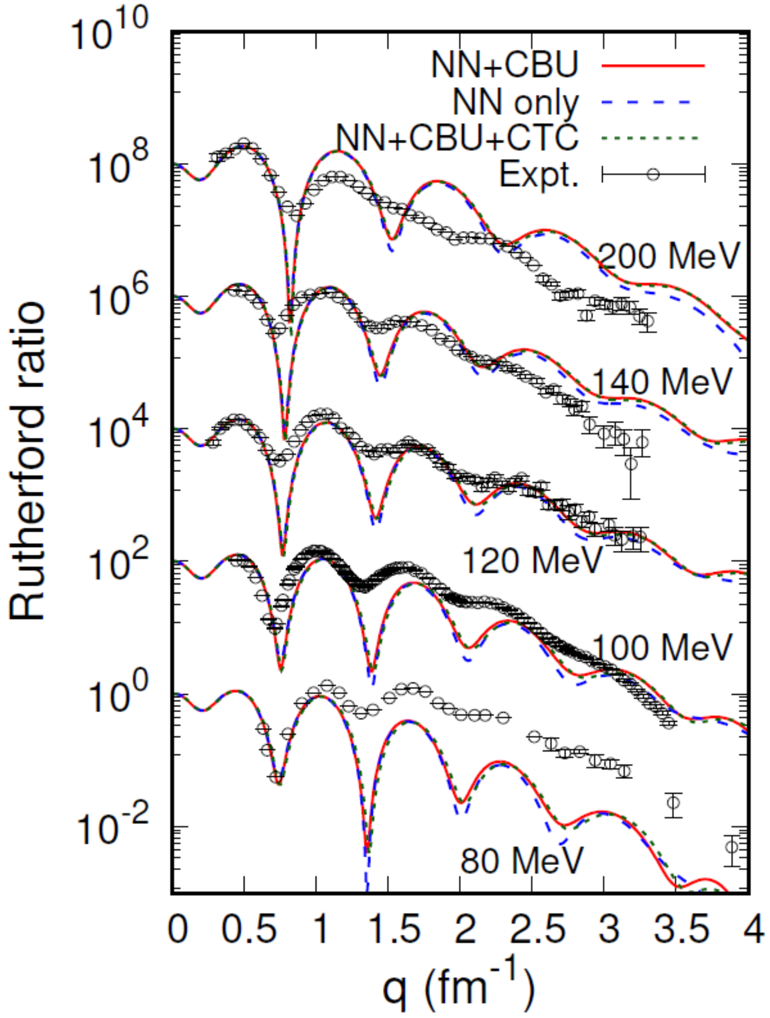
K: Relative wave number

→ Safe for E/A> 200 MeV

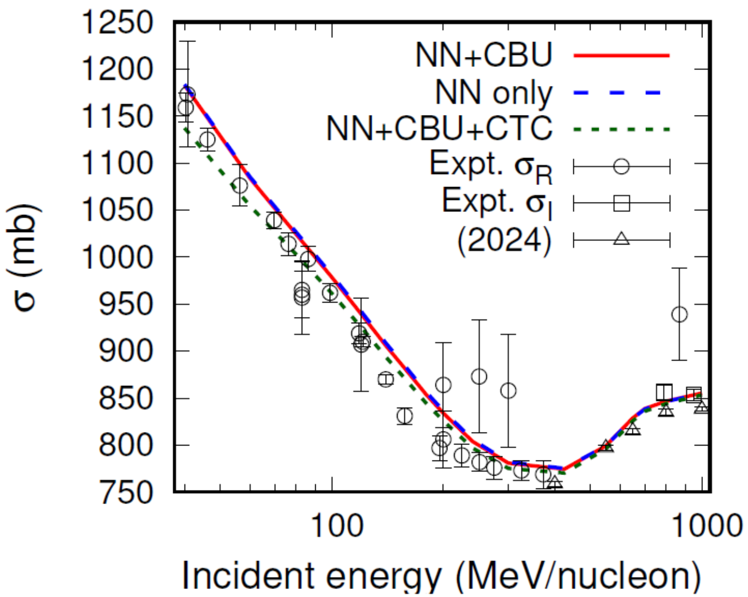
K～3 fm^{-1}, a～3 fm

^{12}C-^{12}C scattering

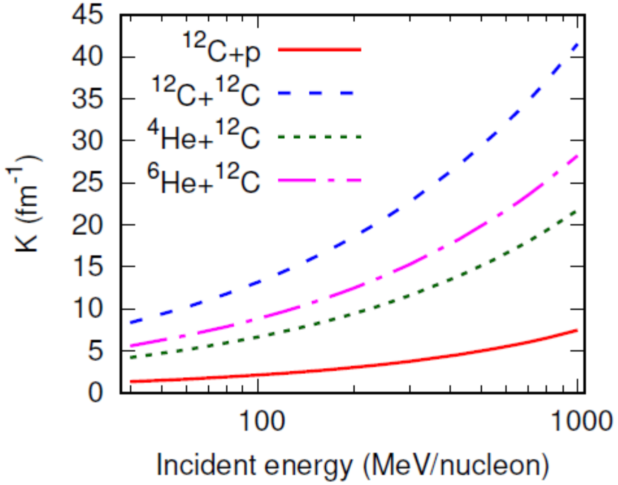
Elastic scattering cross sections



Total reaction cross sections

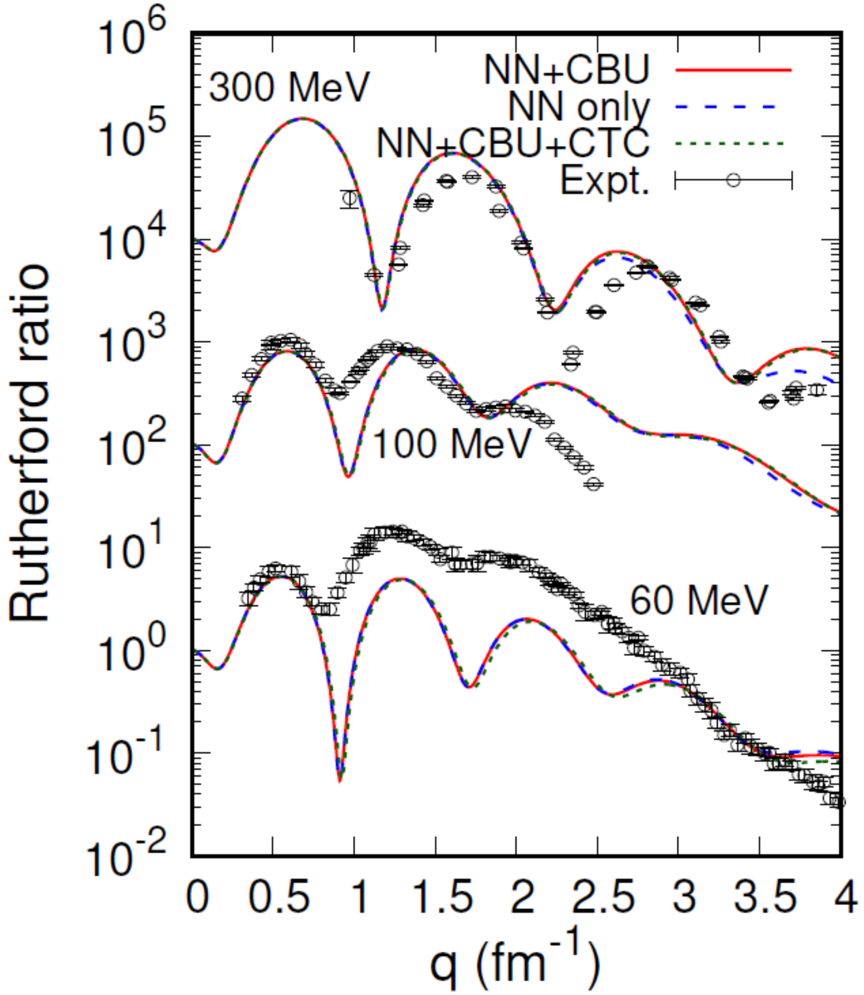


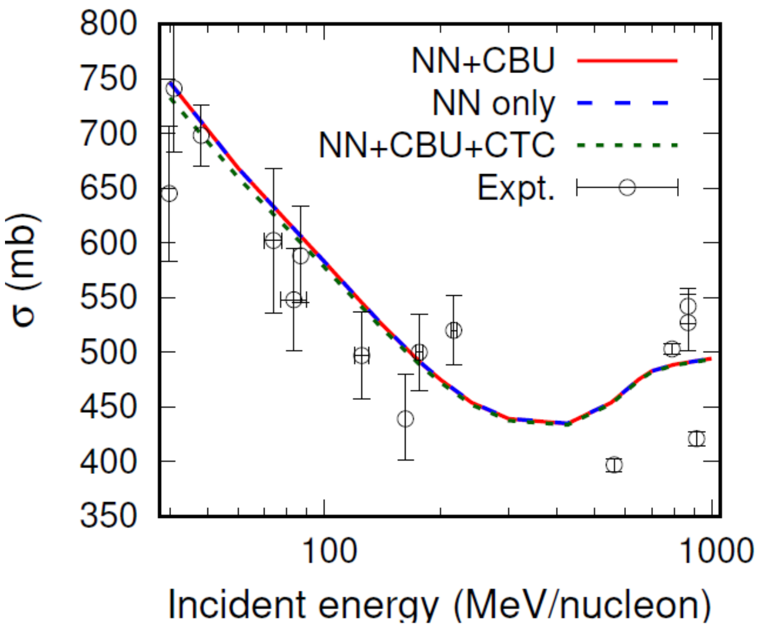
Relative wave numbers



- **Eikonal conditions always fulfilled**
  K: large, a～6 fm
- σ_R :Perfect
- Elastic scattering: Good for E/A > 100 MeV
  Limited reproduction for E/A=80 MeV
  - Fermi energy for two nuclei ～60 MeV

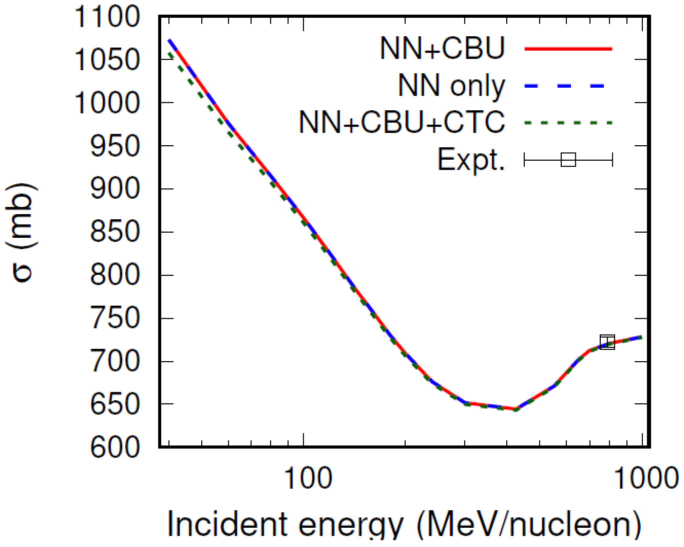
^{4}He-^{12}C scattering

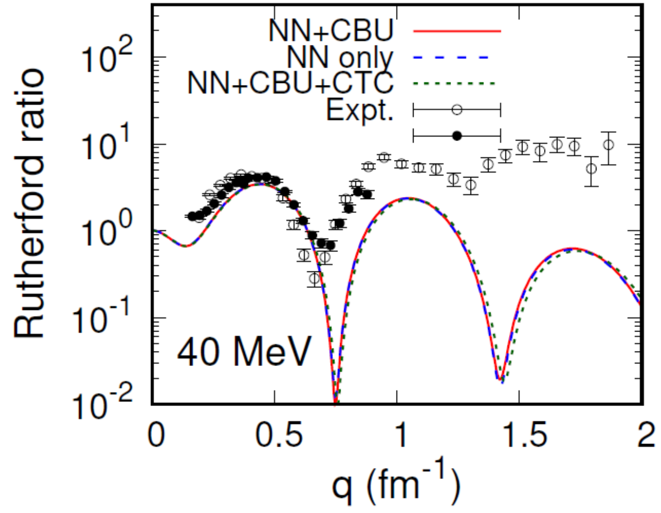




- **Eikonal conditions fulfilled**
- σ_R:Good (Expt. scattered)
- Elastic scattering: Good for E/A > 100 MeV

^{6}He-^{12}C scattering





Conventional approximations

Glauber's psf

$$e^{\chi(\boldsymbol{b})} \equiv G(\boldsymbol{b};\lambda=1) = \left\langle \Psi_0^P \Psi_0^T \right| \mathcal{O}(\boldsymbol{b};\lambda=1) \left| \Psi_0^P \Psi_0^T \right\rangle$$

Multiple scattering operator $\mathcal{O}(\boldsymbol{b};\lambda) = \prod_{i=1}^{A_P} \prod_{j=1}^{A_T} \left[1 - \lambda \Gamma_{NN}(\boldsymbol{b} + \boldsymbol{s}_i^P - \boldsymbol{s}_j^T)\right]$

Binomial (Moment) expansion → slow convergence B. Abu-Ibrahim et al., Nucl. Phys. A 657, 391 (1999)

$$G(\boldsymbol{b};\lambda) = \sum_{n=0}^{A_P A_T} \lambda^n \mu_n$$

$$\mu_0 = 1, \qquad \mu_1 = -\left\langle \Psi_0^P \Psi_0^T \right| \sum_{i=1}^{A_P} \sum_{j=1}^{A_T} \Gamma_{NN}(\boldsymbol{b} + \boldsymbol{s}_i^P - \boldsymbol{s}_j^T) \left| \Psi_0^P \Psi_0^T \right\rangle,$$

$$\mu_2 = \left\langle \Psi_0^P \Psi_0^T \right| \sum_{i=1}^{A_P} \sum_{j=1}^{A_T} \sum_{k=1}^{A_P} \sum_{l=1}^{A_T} \Gamma_{NN}(\boldsymbol{b} + \boldsymbol{s}_i^P - \boldsymbol{s}_j^T) \Gamma_{NN}(\boldsymbol{b} + \boldsymbol{s}_k^P - \boldsymbol{s}_l^T) \left| \Psi_0^P \Psi_0^T \right\rangle$$

Cumulant expansion

$$\ln G(\lambda) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \kappa_n$$

$$\kappa_0 = 0, \quad \kappa_1 = \mu_1, \quad \kappa_2 = 2\mu_2 - \mu_1^2,$$

Optical Limit Approximation (OLA)

$$\mu_1 = -\iint d\boldsymbol{r}^P d\boldsymbol{r}^T \left[\rho_1^P(\boldsymbol{r}^P)\rho_1^T(\boldsymbol{r}^T)\Gamma_{NN}(\boldsymbol{b} + \boldsymbol{s}^P - \boldsymbol{s}^T)\right]$$

$$G(\lambda=1) = e^{\ln G(\lambda=1)} = \exp\left(\kappa_1 + \frac{1}{2}\kappa_2 + \dots\right) = \exp\left[\mu_1 + \left(\mu_2 - \frac{1}{2}\mu_1^2\right) + \dots\right]$$

"Average" Conventional OLA "Variance" 2nd order OLA

Cumulant expansion

- Cumulant expansion

$$\ln G(\boldsymbol{b};\lambda) = \sum_{n=1}^{\infty} \frac{\lambda^n}{n!}\kappa_n(\boldsymbol{b}).$$

$\kappa_1(\boldsymbol{b}) = \mu_1(\boldsymbol{b}),$ "average"

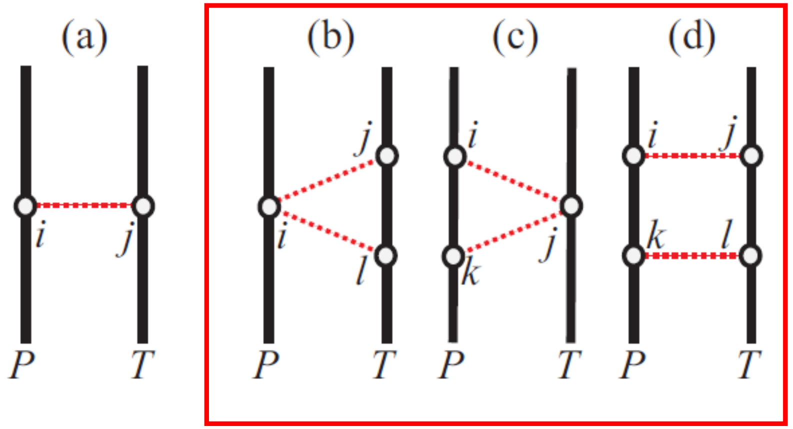
$\kappa_2(\boldsymbol{b}) = 2\mu_2(\boldsymbol{b}) - \mu_1(\boldsymbol{b})^2,$ "variance"

$\kappa_3(\boldsymbol{b}) = 6\mu_3(\boldsymbol{b}) - 6\mu_2(\boldsymbol{b})\mu_1(\boldsymbol{b}) + 2\mu_1(\boldsymbol{b})^3$ "kurtosis"

$$\mu_1(\boldsymbol{b}) = -\left\langle \sum_{i=1}^{A_P}\sum_{j=1}^{A_T}\Gamma_{NN}(\boldsymbol{b}_{ij})\right\rangle = -\iint d\boldsymbol{r}\,d\boldsymbol{r}\rho_1^P(\boldsymbol{r})\rho_1^T(\boldsymbol{r}')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}-\boldsymbol{s}'),$$

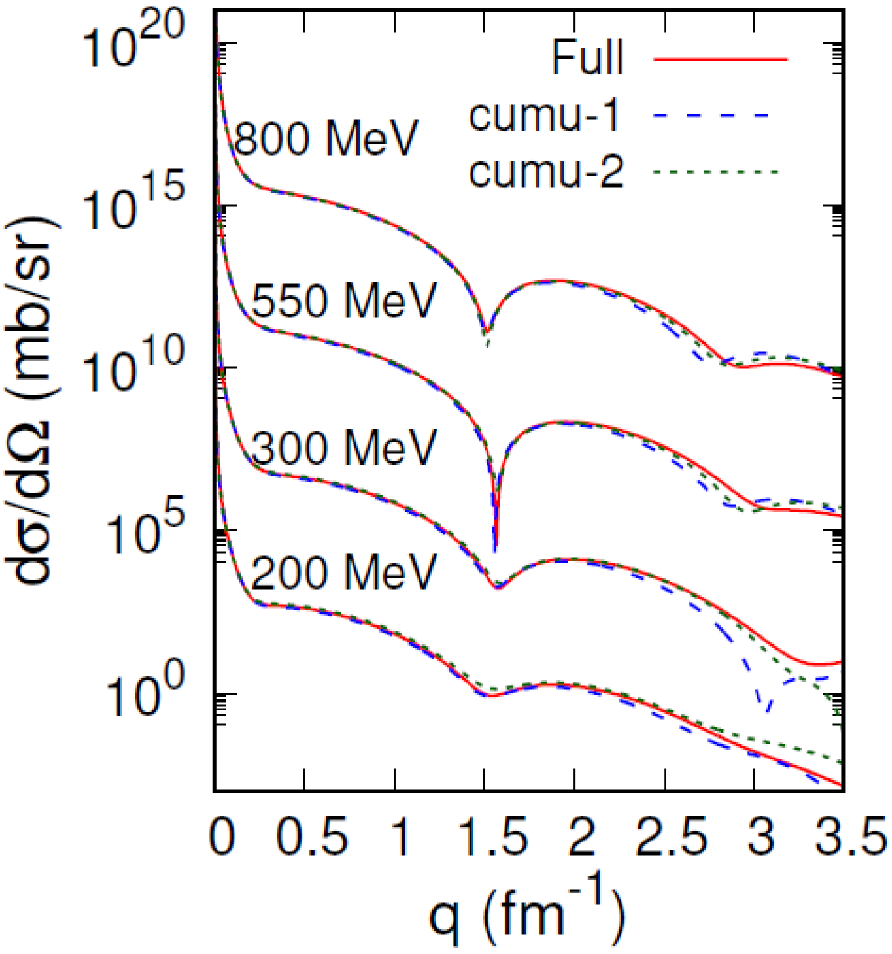
One-body density

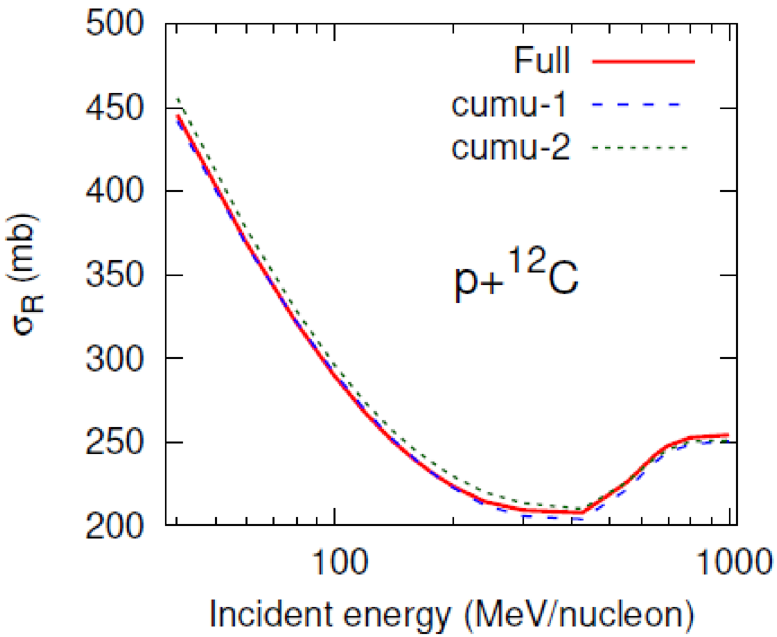
$$\begin{aligned}\mu_2(\boldsymbol{b}) =& \left\langle \sum_{i=1}^{A_P}\sum_{1\le j<l}^{A_T}\Gamma_{NN}(\boldsymbol{b}_{ij})\Gamma_{NN}(\boldsymbol{b}_{il})\right\rangle + \left\langle \sum_{1\le i<k}^{A_P}\sum_{j=1}^{A_T}\Gamma_{NN}(\boldsymbol{b}_{ij})\Gamma_{NN}(\boldsymbol{b}_{kj})\right\rangle + 2\left\langle \sum_{1\le i<k}^{A_P}\sum_{1\le j<l}^{A_T}\Gamma_{NN}(\boldsymbol{b}_{ij})\Gamma_{NN}(\boldsymbol{b}_{kl})\right\rangle \\ =& \iiint d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{r}''\rho_1^P(\boldsymbol{r})\rho_2^T(\boldsymbol{r}',\boldsymbol{r}'')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}-\boldsymbol{s}')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}-\boldsymbol{s}'') \\ &+ \iiint d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{r}''\rho_2^P(\boldsymbol{r},\boldsymbol{r}')\rho_1^T(\boldsymbol{r}'')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}-\boldsymbol{s}'')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}'-\boldsymbol{s}'') \\ &+ 2\iiiint d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{r}''\,d\boldsymbol{r}'''\rho_2^P(\boldsymbol{r},\boldsymbol{r}')\rho_2^T(\boldsymbol{r}'',\boldsymbol{r}''')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}-\boldsymbol{s}'')\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}'-\boldsymbol{s}''').\end{aligned}$$

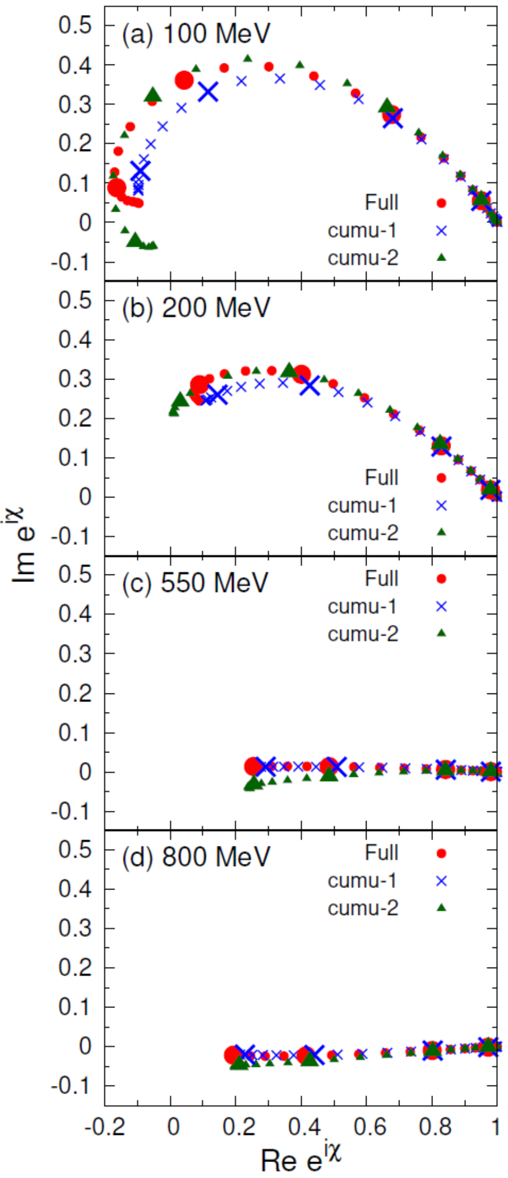


One- and **two**-body densities

Proton-^{12}C scattering

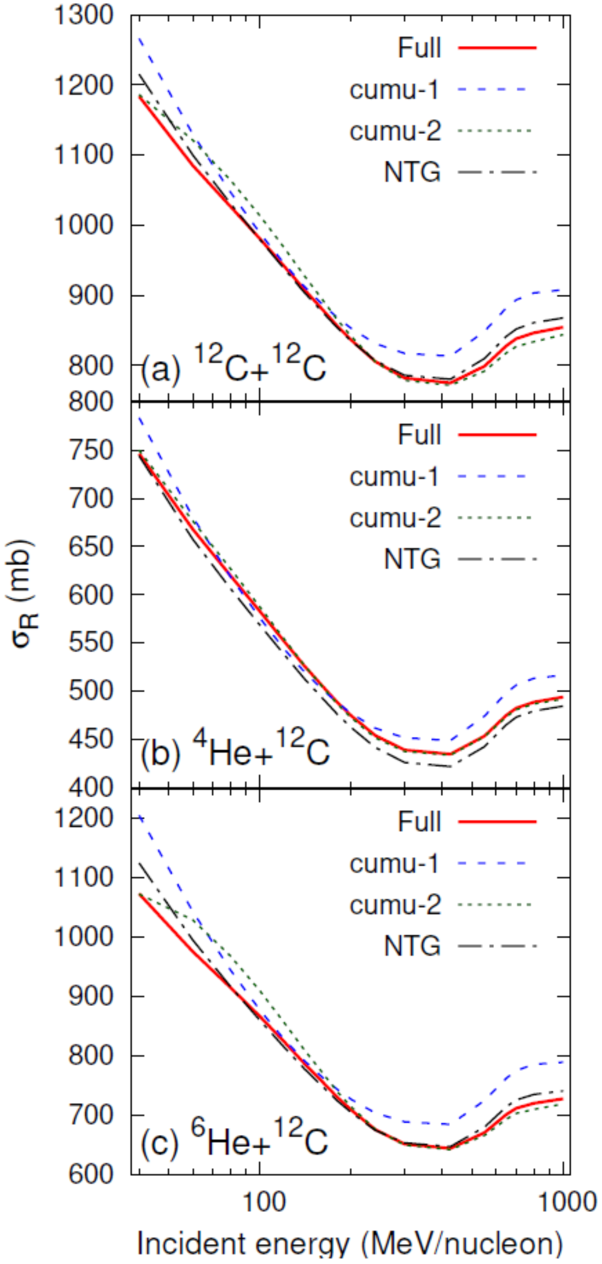


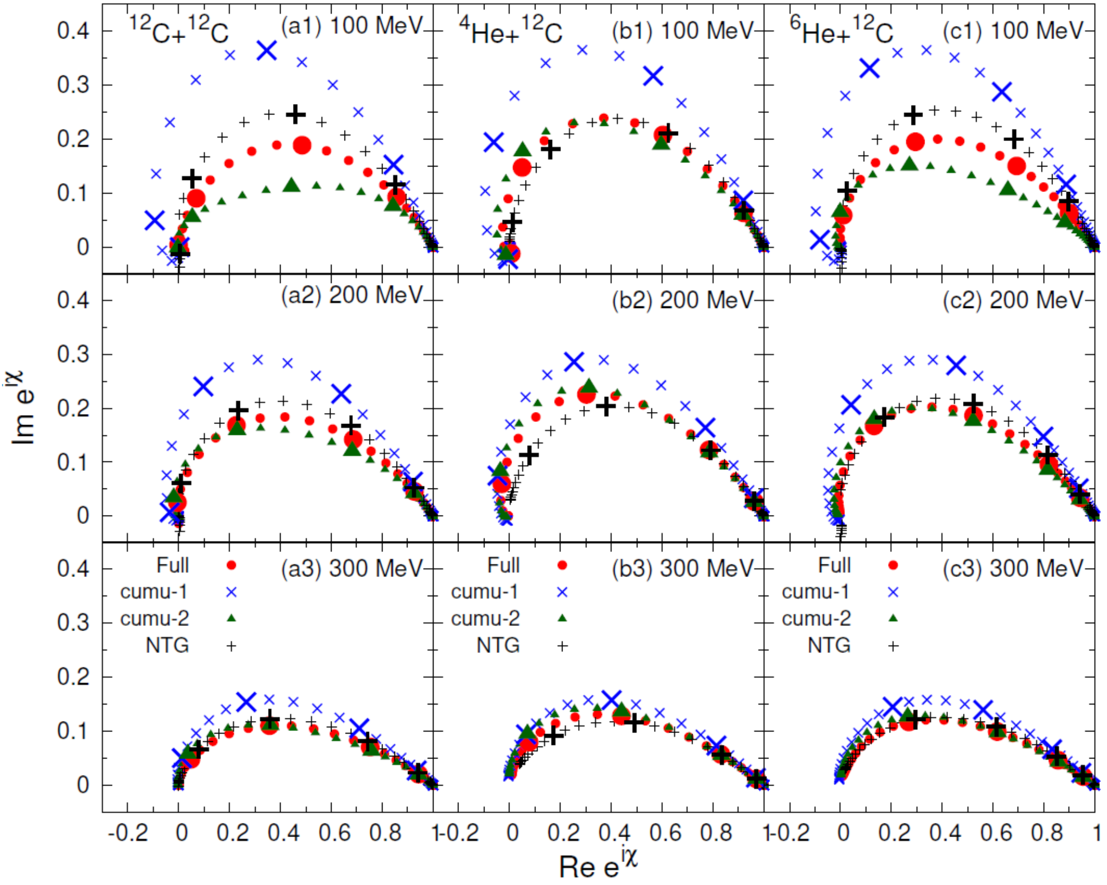




Fluctuation of the psf is not so large
→ Converged already with cumu-1
(conventional OLA)

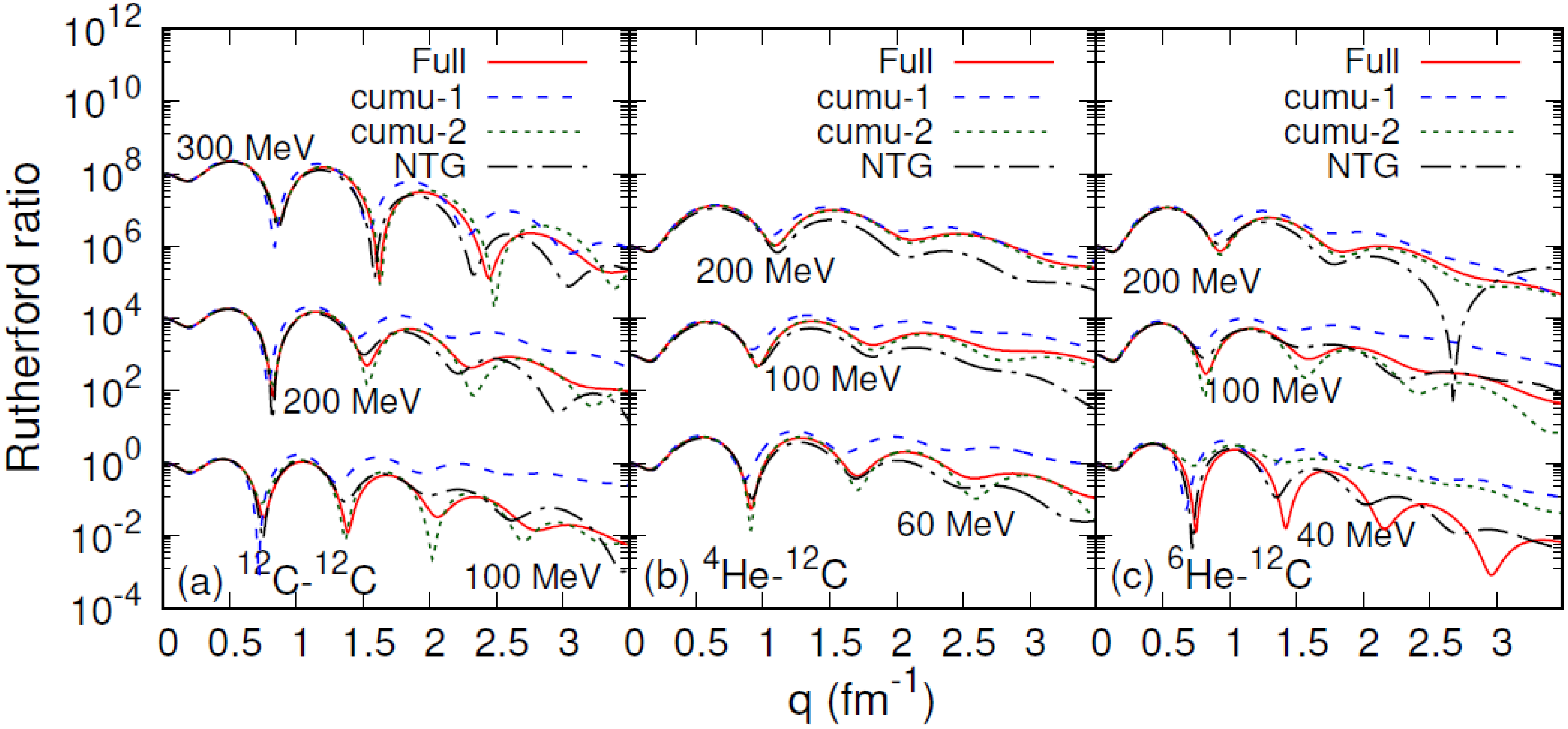
Nucleus-nucleus case: σ_R





- Conventional OLA tends to overestimate the full Glauber results
- 2nd OLA offers converged results for typical nuclear systems

Nucleus-nucleus case: Elastic scattering



- Conventional OLA tends to overestimate the full Glauber results at backward angles
- 2nd OLA offers converged results

Remark on conventional approximation II

- Product ansatz $$\rho_A(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_A)=\prod_{i=1}^{A}\bar{\rho}(\boldsymbol{r}_i), \qquad \bar{\rho}(\boldsymbol{r}) = \rho_1(\boldsymbol{r})/A.$$

R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. 1, p. 315.

- Glauber's psf

$$e^{i\chi(\boldsymbol{b})}=\int\cdots\int d\boldsymbol{r}_1^P\ldots d\boldsymbol{r}_{A_P}^P d\boldsymbol{r}_1^T\ldots d\boldsymbol{r}_{A_T}^T\,\rho_{A_P}^P(\boldsymbol{r}_1^P,\ldots,\boldsymbol{r}_{A_P}^P)\rho_{A_T}^T(\boldsymbol{r}_1^T,\ldots,\boldsymbol{r}_{A_T}^T)\prod_{i=1}^{A_P}\prod_{j=1}^{A_T}\left[1-\Gamma_{NN}(\boldsymbol{b}+\boldsymbol{s}_i^P-\boldsymbol{s}_j^T)\right],$$

$$e^{i\chi(\boldsymbol{b})}=\left(1-\frac{1}{A_PA_T}\iint d\boldsymbol{r}d\boldsymbol{r}'\rho_1^P(\boldsymbol{r})\rho_1^T(\boldsymbol{r}')\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s}+\boldsymbol{s}')\right)^{A_PA_T}$$

$(A_PA_T\to$Large$)$ $$\approx\exp\left(-\iint d\boldsymbol{r}d\boldsymbol{r}'\rho_1^P(\boldsymbol{r})\rho_1^T(\boldsymbol{r}')\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s}+\boldsymbol{s}')\right),$$ → OLA

Nothing better than OLA!

Summary

WH, Y. Suzuki, R. B. Wiringa, Phys. Rev. Lett. 136, 202501 (2026)
WH, Y. Suzuki, R. B. Wiringa, Phys. Rev. C 113, 064601 (2026)

Complete Glauber-theory cal. using the most accurate wave function of ^{12}C

- Solving Glauber's original equation without approximation
  - NN interactions consistent with the NN scattering properties
  - Multiple integration in the Glauber psf is performed by a Monte Carlo technique
- Total reaction and elastic scattering cross sections can be evaluated unambiguously within the Glauber theory
  - Accuracy of the conventional approximations

The Glauber theory works well for ～100-1000 MeV/nucleon

Accurate expt. data → Precise determination of the nuclear size properties

- Conventional OLA can be used for proton scattering (One-body density)
- For nucleus-nucleus scattering, the second order OLA offers a good approximation (Two-body density)