# Microscopic Description of Hypernuclear Spectroscopy via Control Neural Networks


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ESNT project, Towards high precision in multi-channel reactions of composite nuclei

2026. 7. 2

# Outline

- Research Background & Motivation
- Theoretical Framework & Benchmarking

  **JT.**, Zheng Cheng, Changjian Yu, Mengjiao Lyu*, et al., Phys. Lett. B 855, 138816 (2024).
- High-excitation states & Parity-mixing

  **JT.**, Mengjiao Lyu*, Zheng Cheng, Masahiro Isaka, et al., Phys. Lett. B 862, 139338 (2025).
- Cluster-breaking effect & reconfiguration

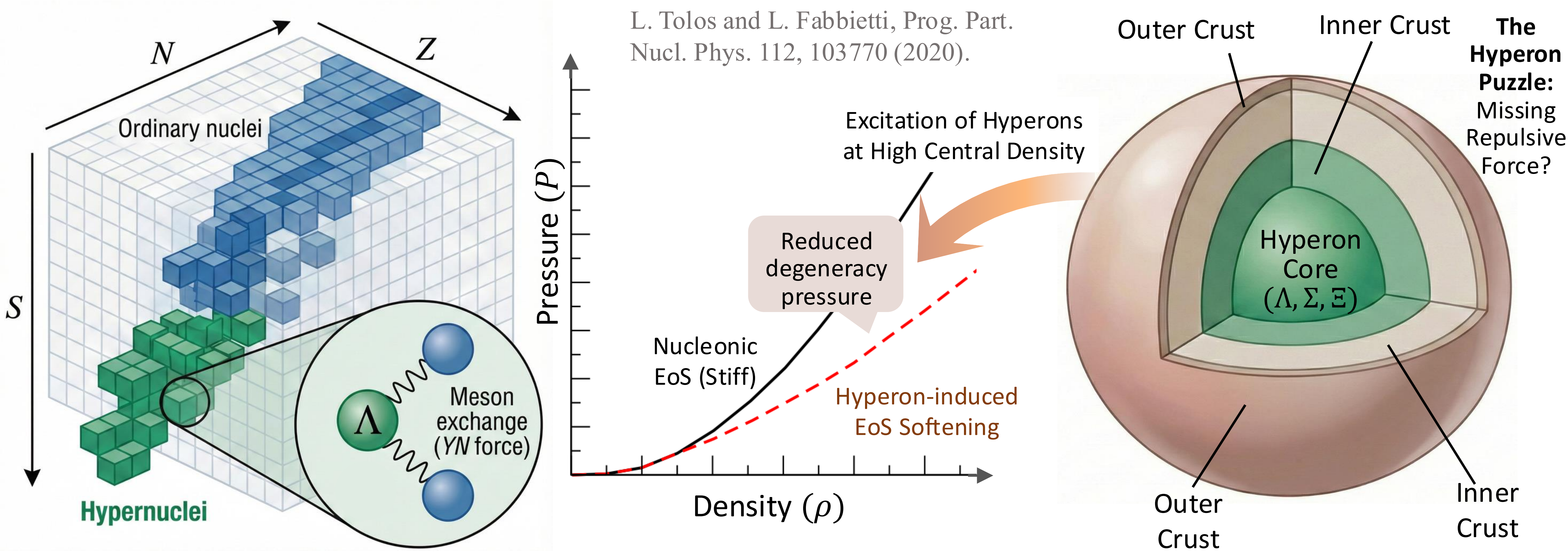
  **JT.**, Mengjiao Lyu*, Akinobu Dote, Zheng Cheng, et al., Phys. Rev. C under review (2026).

# Hypernuclear Physics: Bridging microscopic interaction to the Neutron star

**1. Microscopic Laboratory:**
The 3D Nuclear Chart & YN Interaction

**2. Physical Mechanism:**
EoS Softening at High Density

**3. Macroscopic Impact:**
The Neutron Star Hyperon Puzzle




**Microscopic Foundation:** Precise YN interaction data from hypernuclear spectroscopy

**EoS Softening:** Appearance of hyperons reduces internal pressure

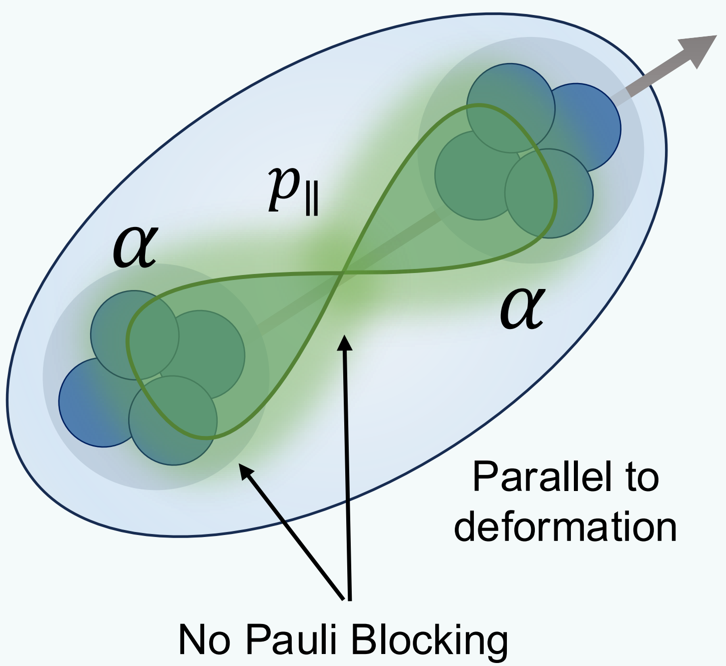
**Macroscopic Impact:** Current theoretical Eos with hyperons contradicts massive pulsar observations

# *Strangeness* gives rise to rich phenomena

E. Hiyama and T. Yamada, Prog. Part. Nucl. Phys. 63, 339 (2009).

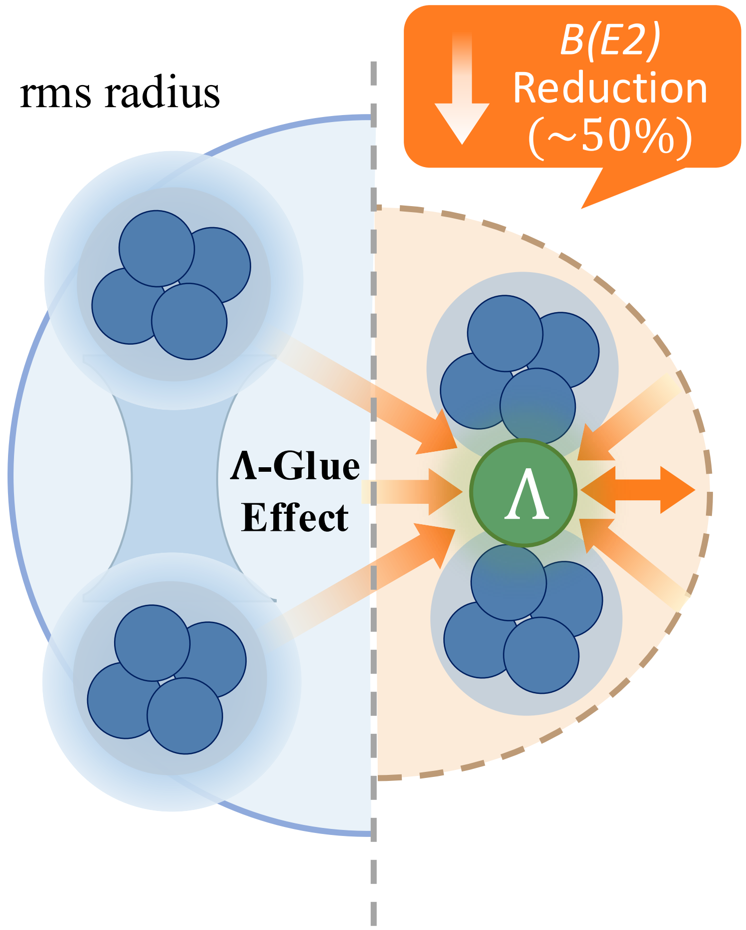
## 1. Genuine Hypernuclear Symmetry (Supersymmetry)

**Supersymmetric State**
$(\lambda\mu) = (50)$



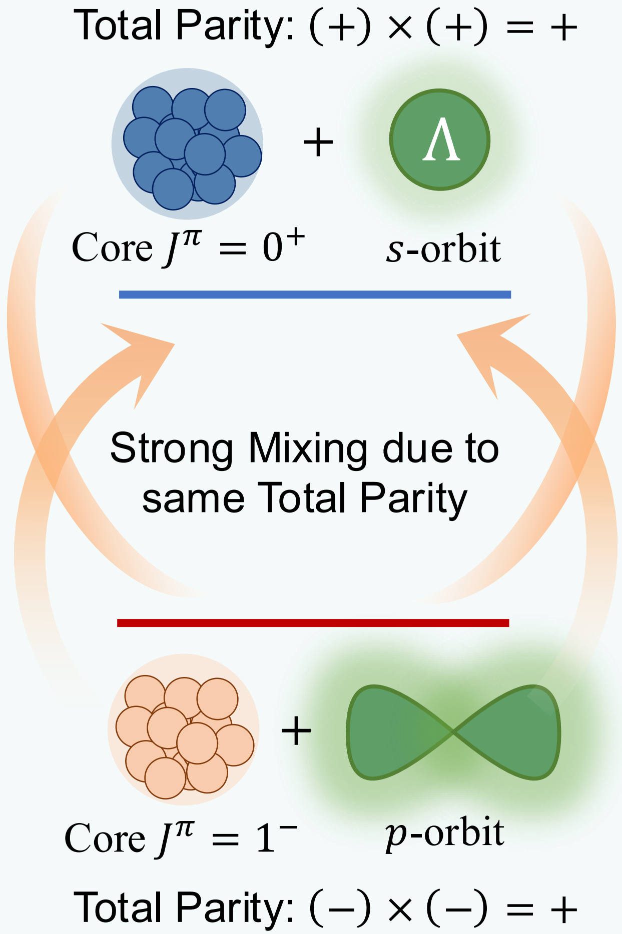

New spatial symmetry not found in ordinary Nuclei due to distinct particle nature

## 2. Λ-Glue Effect & Core Shrinkage




Contraction stabilized by Attractive $\Lambda - N$ interaction

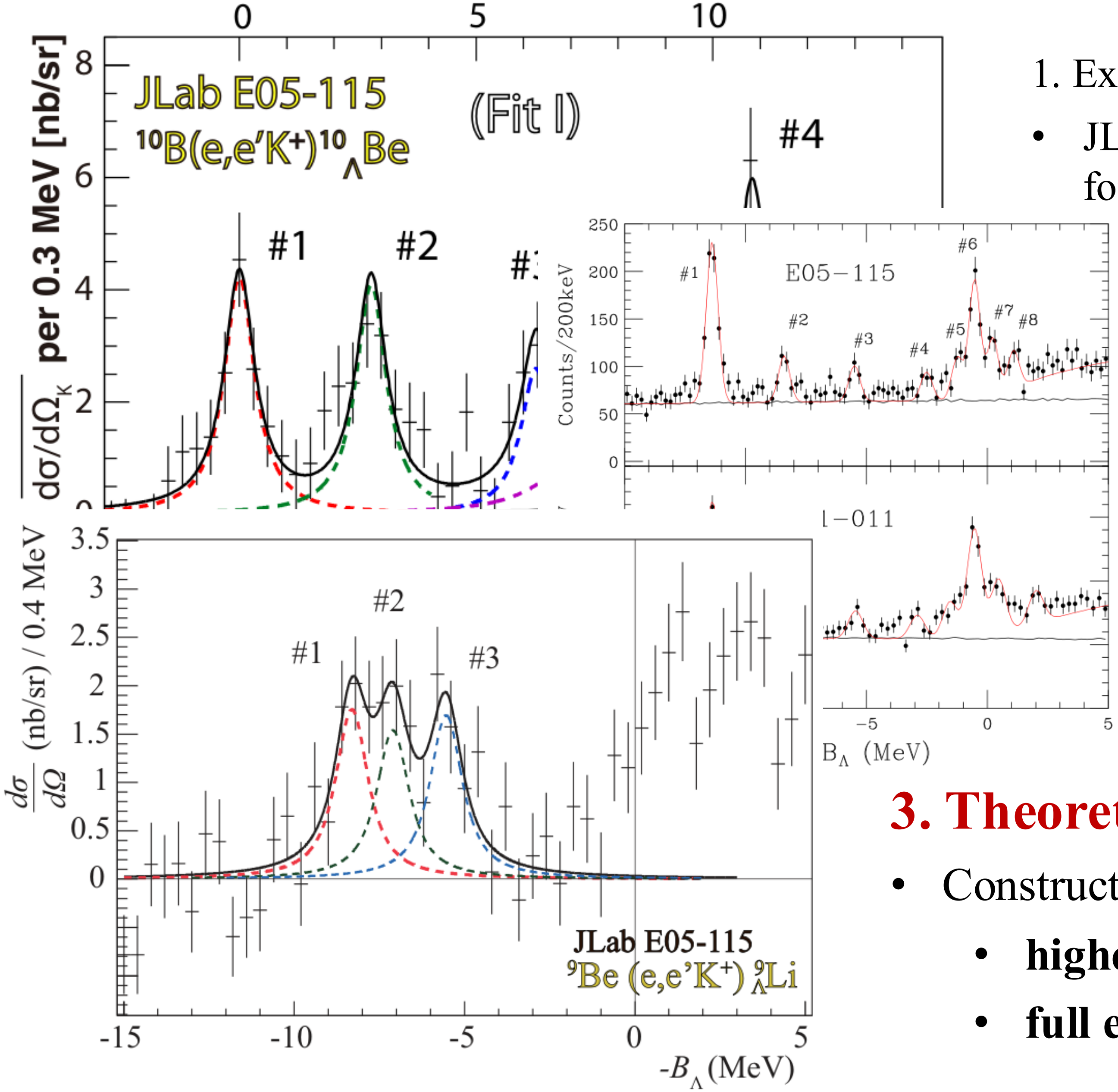
## 3. Parity Mixing & Intershell Coupling




Λ orbital energy gap matches core excitation, causing parity mixing

# Experimental Influx: High-Precision Spectroscopy

Jefferson Lab & J-PARC




1. Experimental Breakthrough

- JLab & J-PARC are delivering high-resolution (~ 500 keV) data for $p$-shell hypernuclei.

2. Expanding the Energy Domain

- Spectroscopy now extends to High-excitation regions ($E_x \sim 10 - 15$ MeV)
- Very complicated structures should be described correctly
- Very large model space should be covered for convergence

**3. Theoretical Imperative; Main purpose of this work**

- Construct theoretical frameworks to cover
  - **higher mass systems** and
  - **full energy spectra**.

AI FOR SCIENCE

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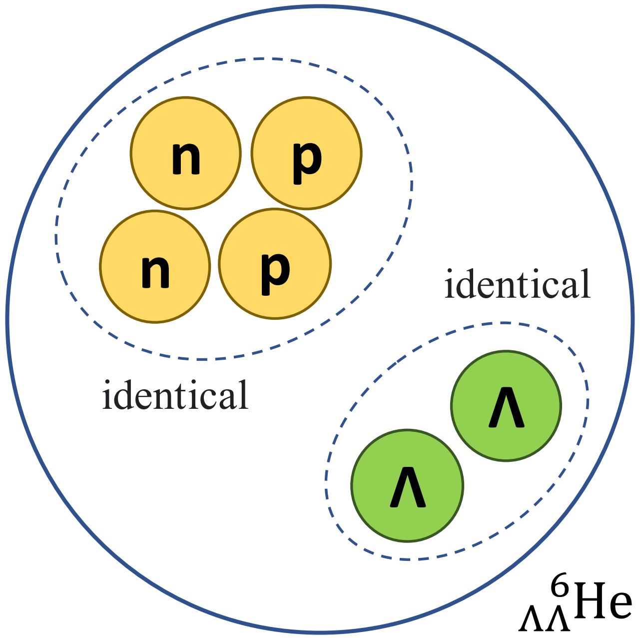
# Cluster model for hypernuclei

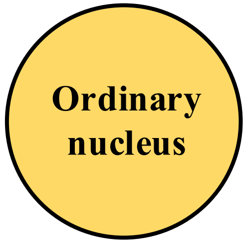
- Start from conventional cluster model, Brink-Block
- Introducing Λ particle wave function
- Treatment for identity of particles
- parity coupling can be naturally considered (GCM)




## *Conventional cluster model*

$$\left|\Psi_{\text{Brink}}\right\rangle \propto \mathcal{A}\left\{\prod_{i=1}^{n}\phi^{N}\left(\boldsymbol{r}_i^{N},\boldsymbol{Z}_i^{N}\right)\right\}$$

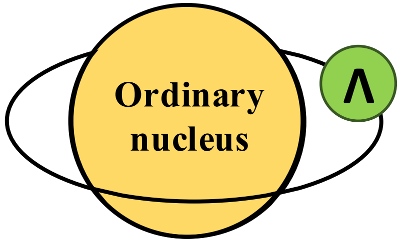
generated coordinates

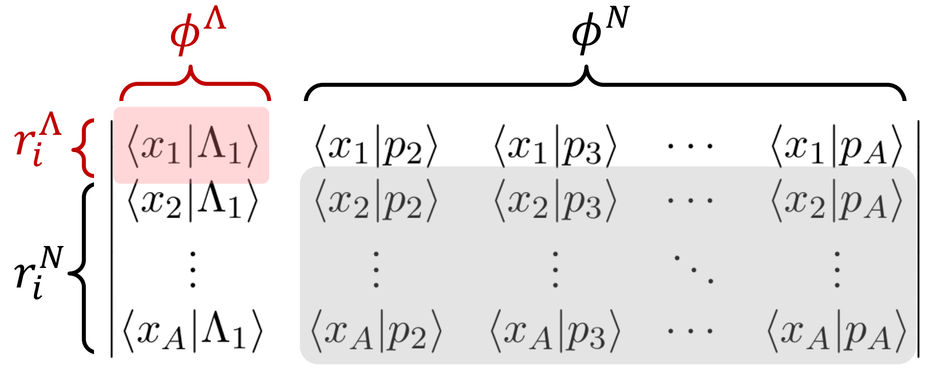



## *Extended version for hypernuclei*

$$\left|\Psi_{\text{Brink}}^{\text{hyper}}\right\rangle \propto \phi^{\Lambda}\left(\boldsymbol{r}_i^{\Lambda},\boldsymbol{Z}_i^{\Lambda}\right)\mathcal{A}\left\{\prod_{i=1}^{n}\phi^{N}\left(\boldsymbol{r}_i^{N},\boldsymbol{Z}_i^{N}\right)\right\}$$

Introducing Λ particle wave function

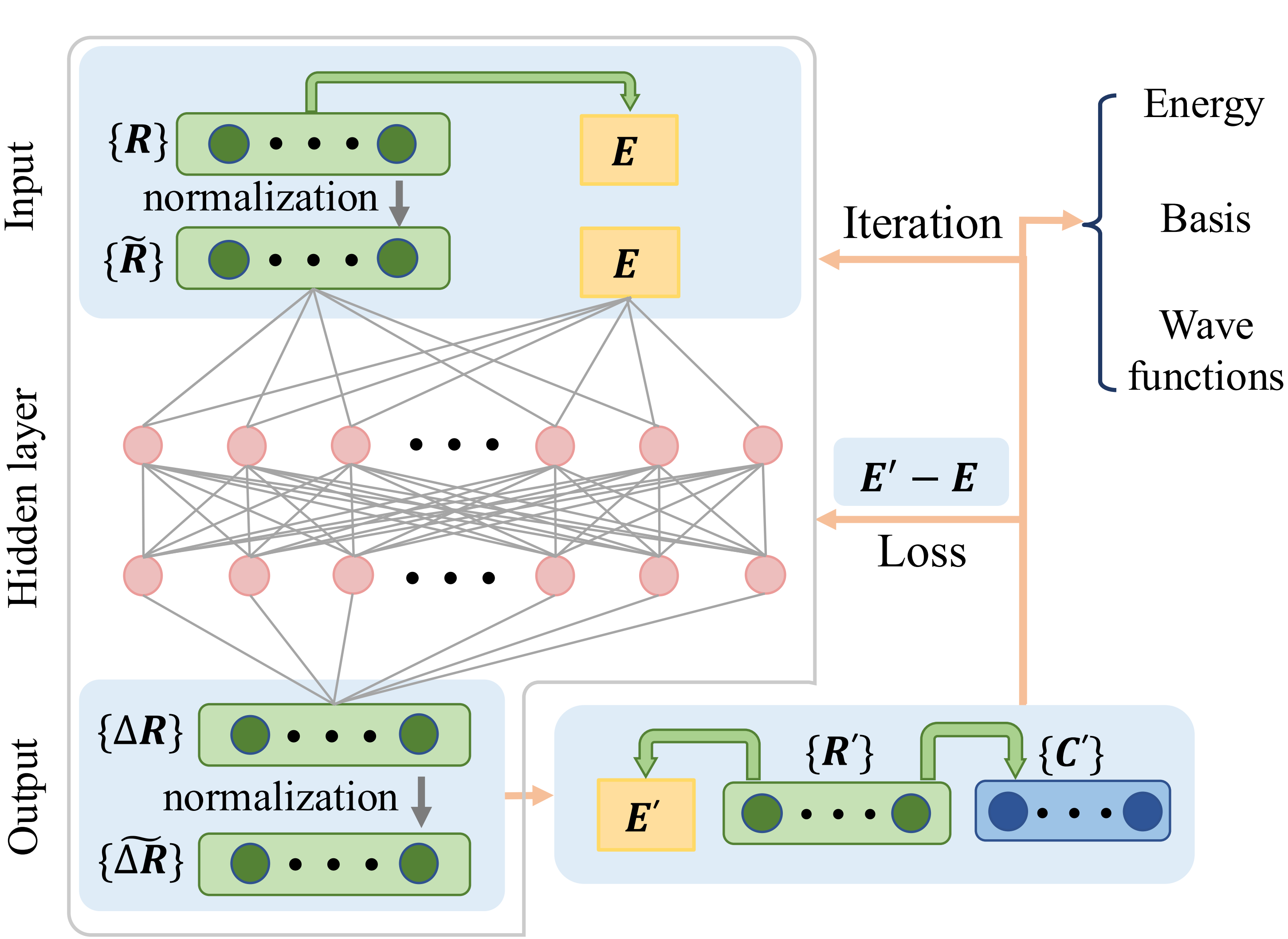


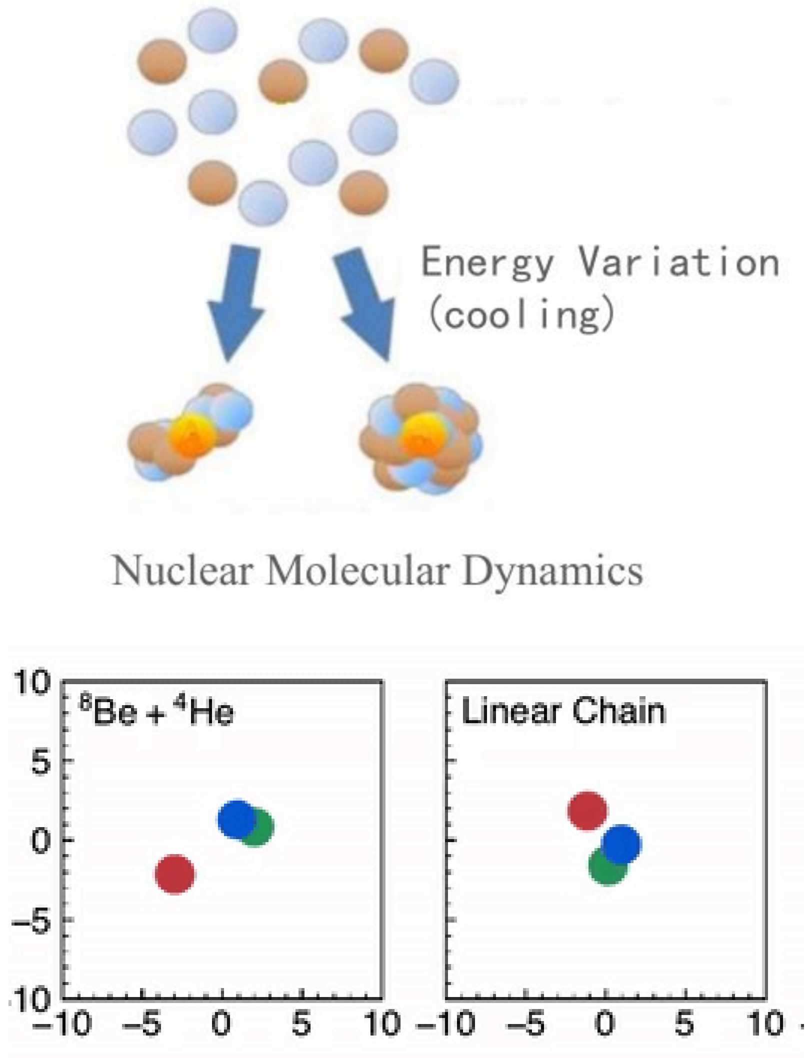




Submatrix Laplace expansion


[1] J. Tian, et al. Phys. Lett. B 855 (2024) 138816

# Neural network-guided Calculation

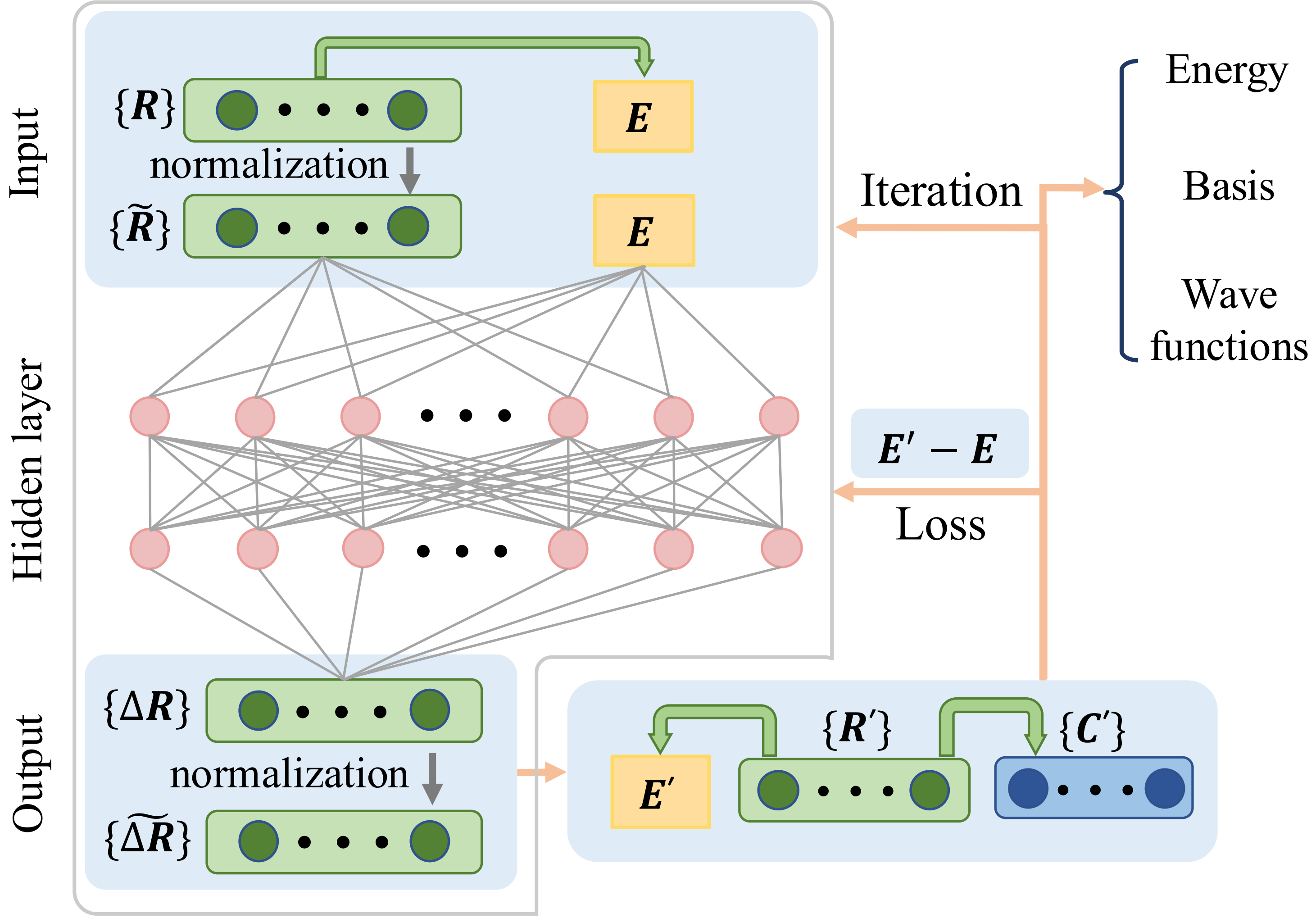


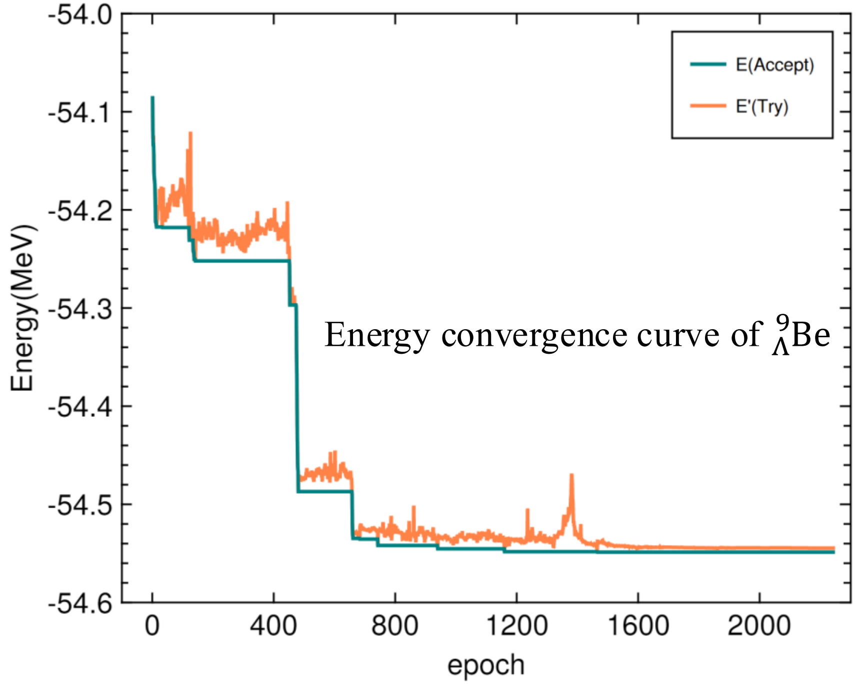




Multi-cool of $^{12}C$ guided by Ctrl.NN in ref [2]


[1] M. Myo, et al., Phys. Rev. C 108 (2023): 064314.
[2] Z. Cheng, et al., Phys. Lett. B 864 (2025) 139397

# Neural network-guided Calculation




**Orange curve**

- Neural networks try a new set of parameters
- Neural networks learn the properties of quantum states simultaneously.

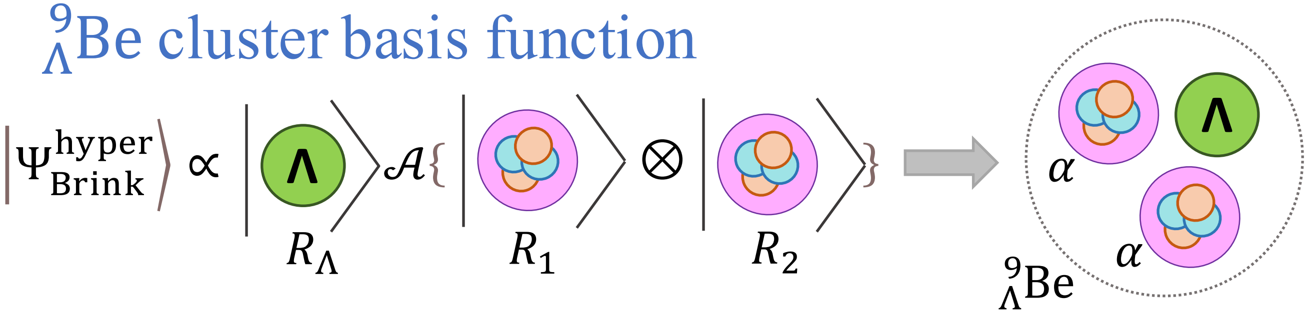
**Green curve**

- If energy is lower, Neural networks accept new parameters set, until energy curve converge.


[1] M. Myo, et al., Phys. Rev. C 108 (2023): 064314.
[2] Z. Cheng, et al., Phys. Lett. B 864 (2025) 139397

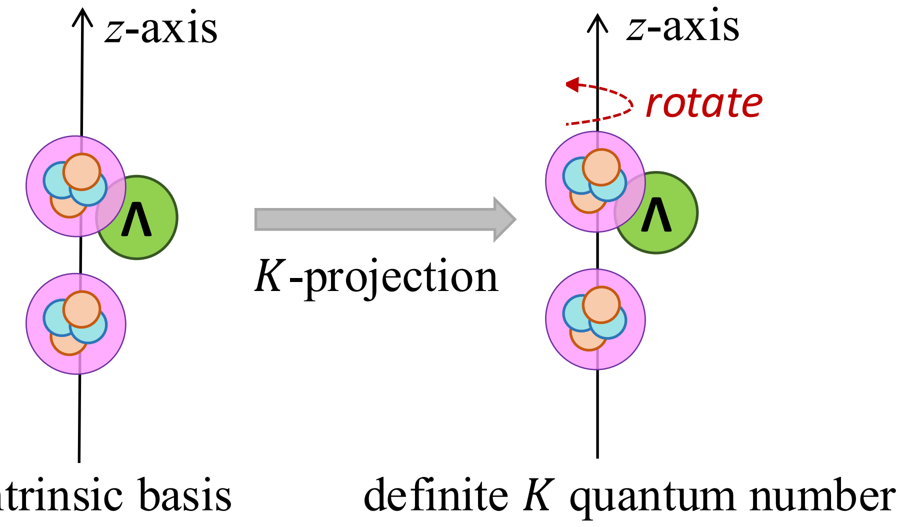
# Example: Framework of ${}^{9}_{\Lambda}\mathrm{Be}$

- ${}^{9}_{\Lambda}\mathrm{Be}$ cluster basis function




*Variation after K-projection*

- restore rotational symmetry about *z*-axis
- reduce the number of bases




Y. Shikata and Y. Kanada-En'yo, Prog. Theor. Exp. Phys. 073D01 (2020).

- Hamiltonian

$$H = T_N + T_\Lambda - T_G + V^{\mathrm{cent}}_{NN} + V^{ls}_{NN} + V_C + V^{\mathrm{cent}}_{\Lambda N}$$

No spurious center-of-mass energy

NN interaction: Volkov No.2 + G3RS LS force

ΛN interaction: YNG-JA & ESC14

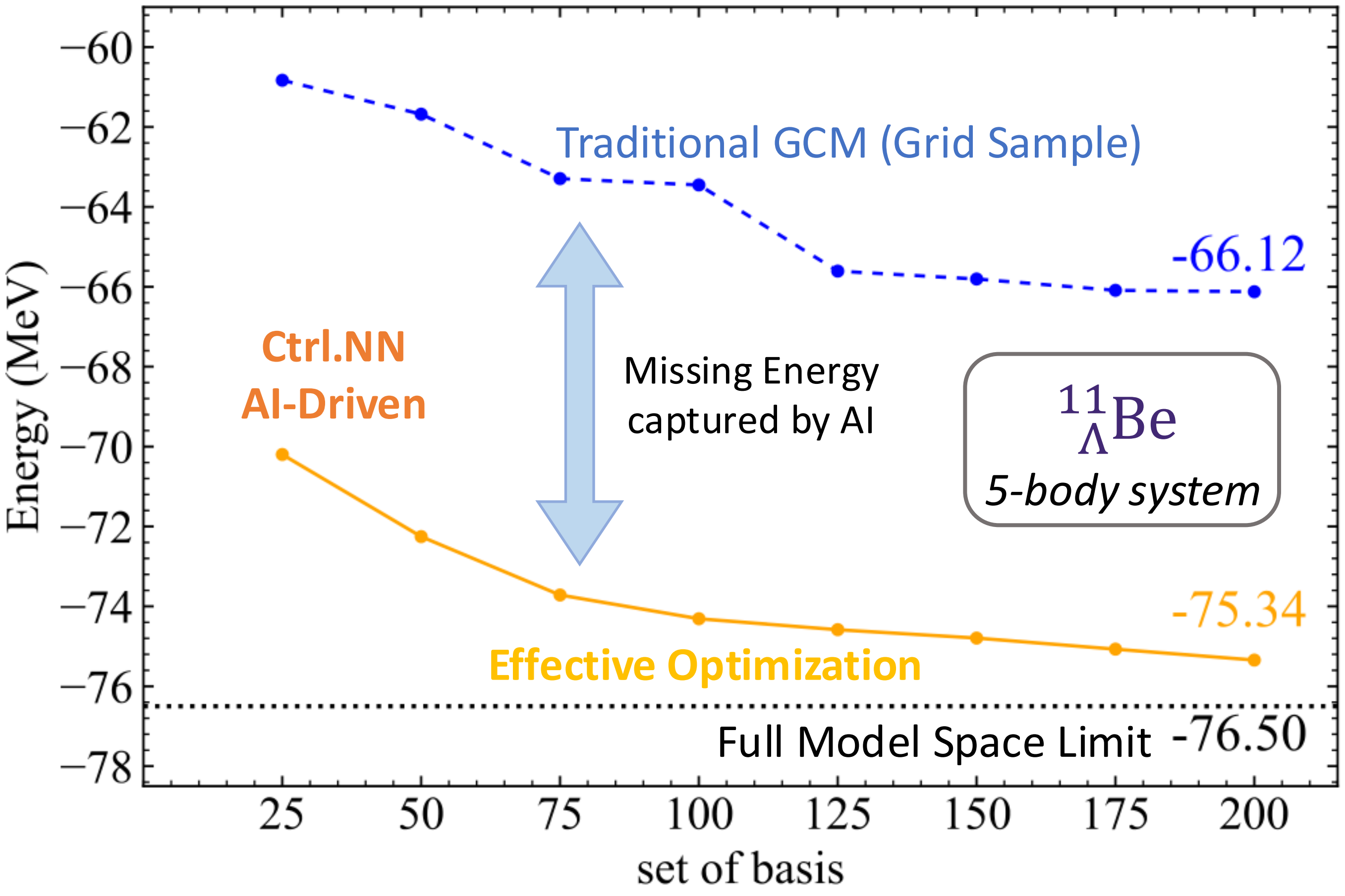
$$v^{(c)}(k_F, r) = \sum_{i=1}^{3}\left(a_i^{(c)} + b_i^{(c)} k_F + c_i^{(c)} k_F^2\right) e^{-r^2/\beta_i^2}$$

$$V^{\mathrm{cent}}_{\Lambda N} = \sum_{i=1}^{3}\left\{\left(v_0^{E(i)} + v_\sigma^{E(i)}\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\right)\hat{P}(E) + \left(v_0^{O(i)} + v_\sigma^{O(i)}\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\right)\hat{P}(O)\right\} e^{-r^2/\beta_i^2}$$

M. Isaka,, Phys. Rev. C 101 (2020) 024301.

# Benchmarking the Ctrl.NN Approach: Efficiency & Precision

| $B_\Lambda$ (MeV) | ${}^{9}_{\Lambda}$Be | ${}^{10}_{\Lambda}$Be | ${}^{11}_{\Lambda}$Be |
|---|---|---|---|
| $k_F$ | 0.963 | 0.763 | 0.763 |
| Ctrl.NN | 6.43 | 8.35 | 10.47 |
| Exp. [28][21][27] | 6.71 | 8.60 | 10.24 (assumed) |




- **Systematic reproduction**:
  Calculated $B_\Lambda$ values for ${}^{9-11}_{\Lambda}$Be align perfectly with experimental data
- **Reliable prediction**:
  Predicted $B_\Lambda$ for ${}^{11}_{\Lambda}$Be is consistent with the empirical assumption
- **Faster energy convergence :**
  Converges with < 100 bases (vs. GCM > 200).
- **Effective optimization:**
  Dynamic search of generator coordinates via Neural Networks.

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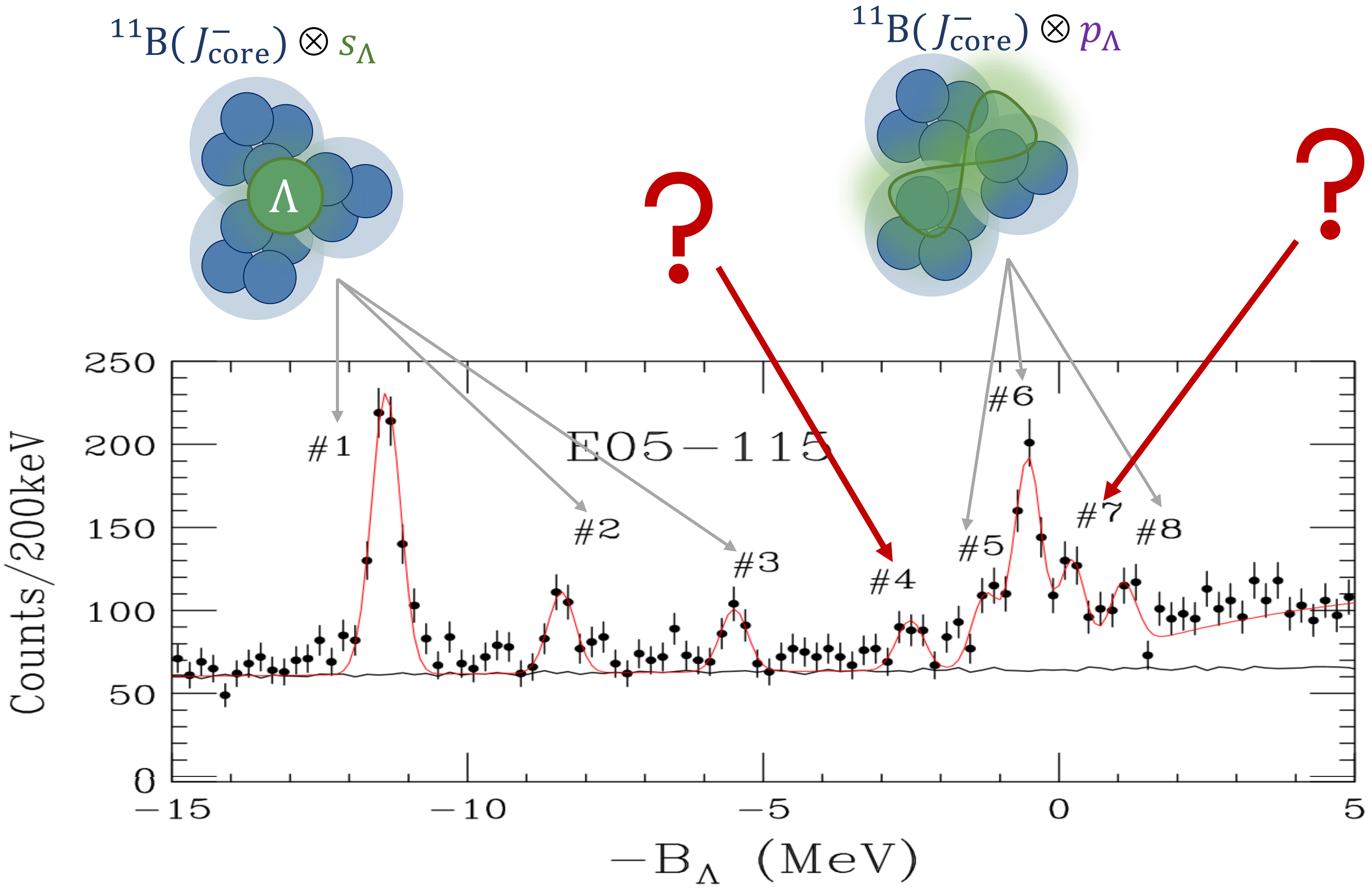
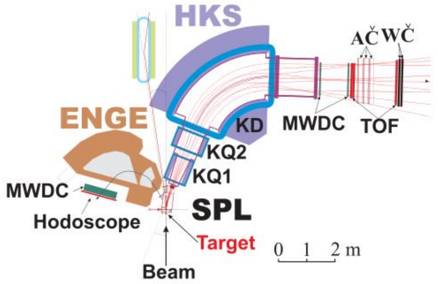
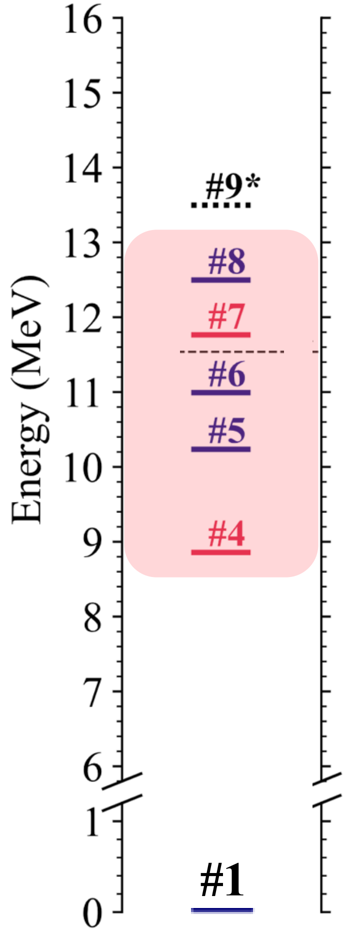
# Experiments of $^{12}_{\Lambda}$B hypernucleus: a gateway to *"parity coupling"*



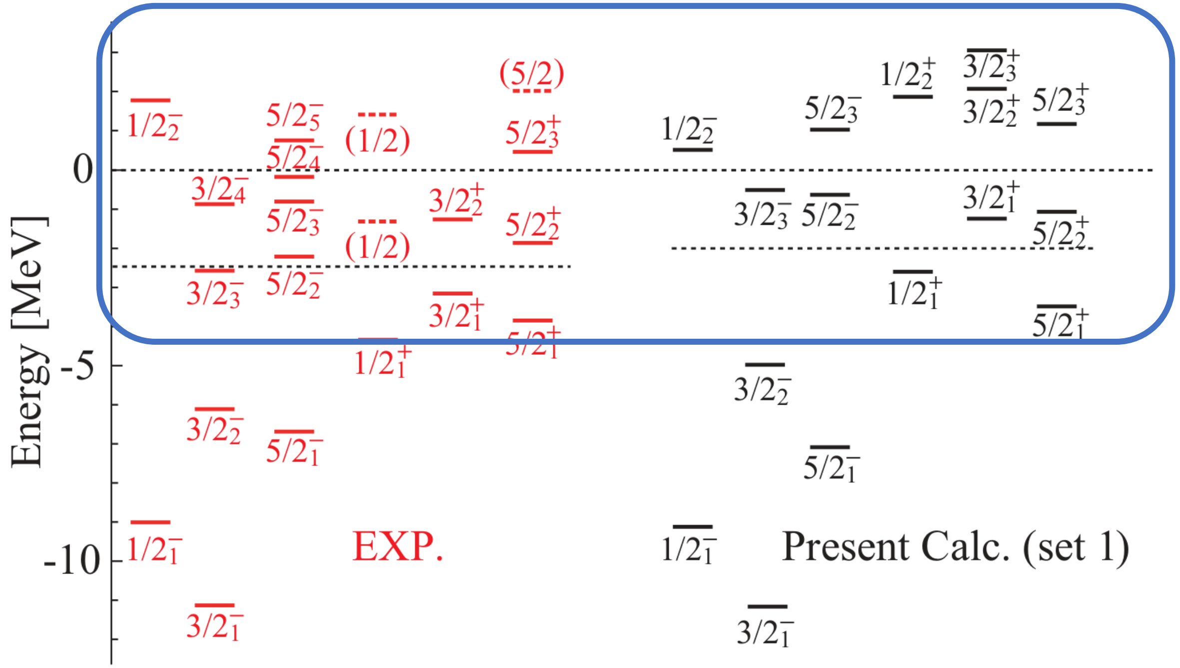

(*e, e′ K*+) reaction


[1] L. Tang, et al. Phys. Rev. C 90, 034320 (2014).
[2] M. Iodice, et al., Phys. Rev. Lett. 99, 052501 (2007).

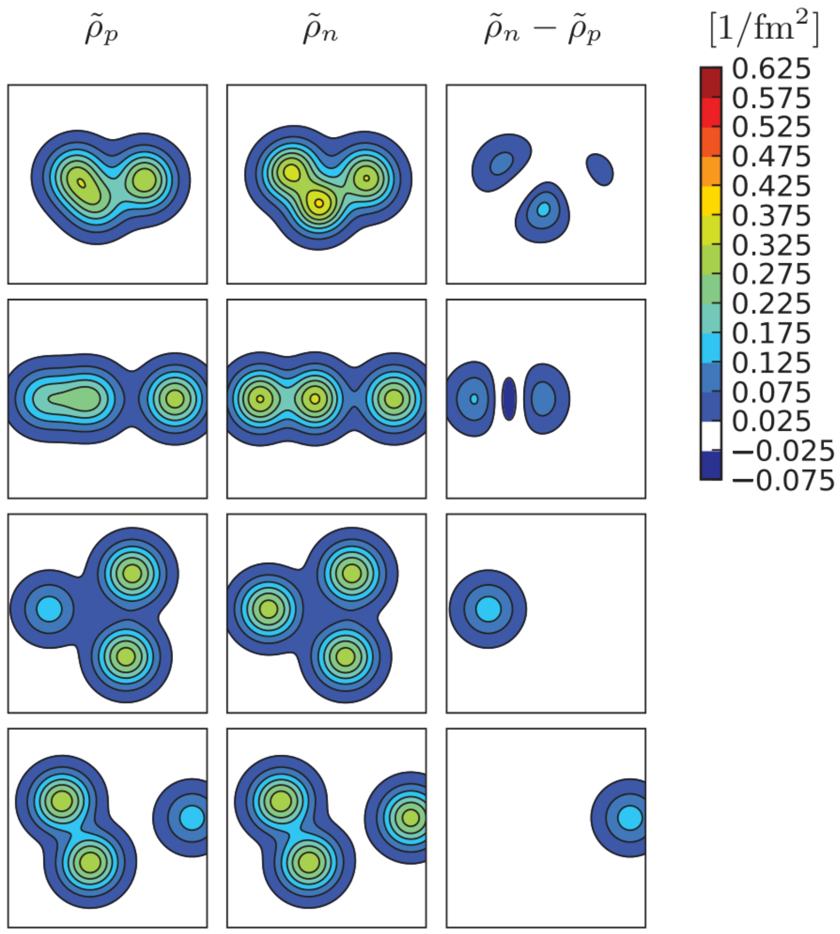
# Positive states of $^{11}$B core nucleus

*Around α + α + t threshold*

- Clustering feature is developed
- Also, excitation region of $p_\Lambda$
- parity coupling is likely to appear




Spectrum of $^{11}$B




Density distributions for the positive-parity states of $^{11}$B within AMD

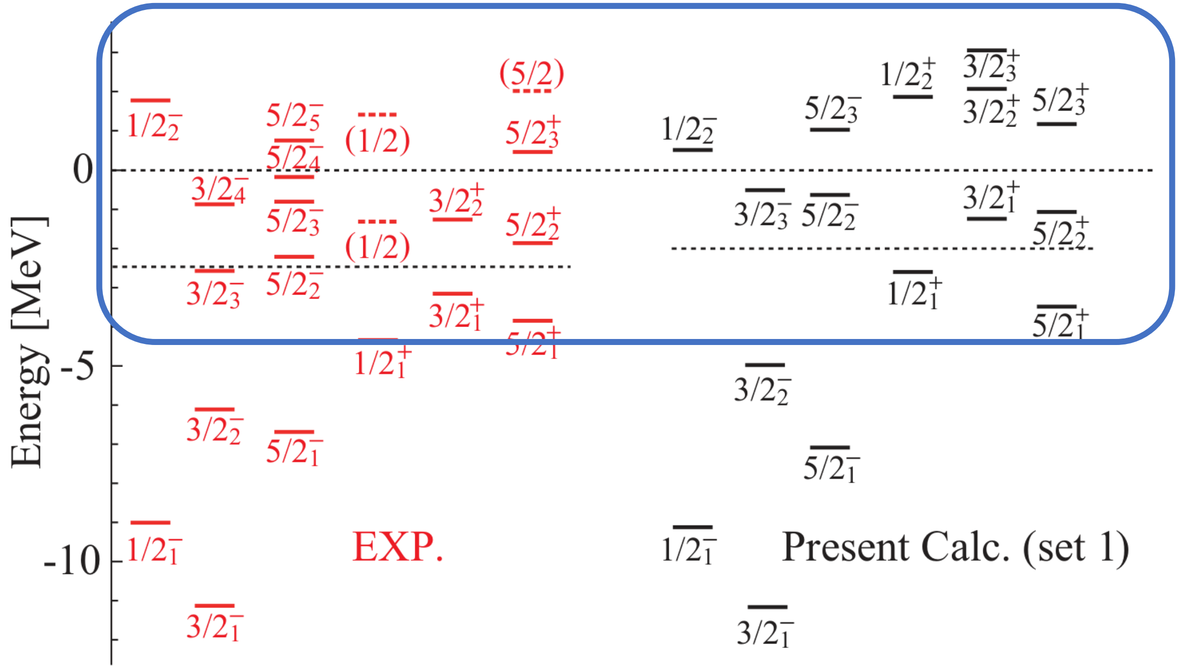

[1] B. Zhou, M. Kimura. Phys. Rev. C, 2018, 98(5): 054323.
[2] T. Suhara, Y. Kanada-En'yo. Phys. Rev. C, 2012, 85(5): 054320.

# Positive states of $^{11}$B core nucleus

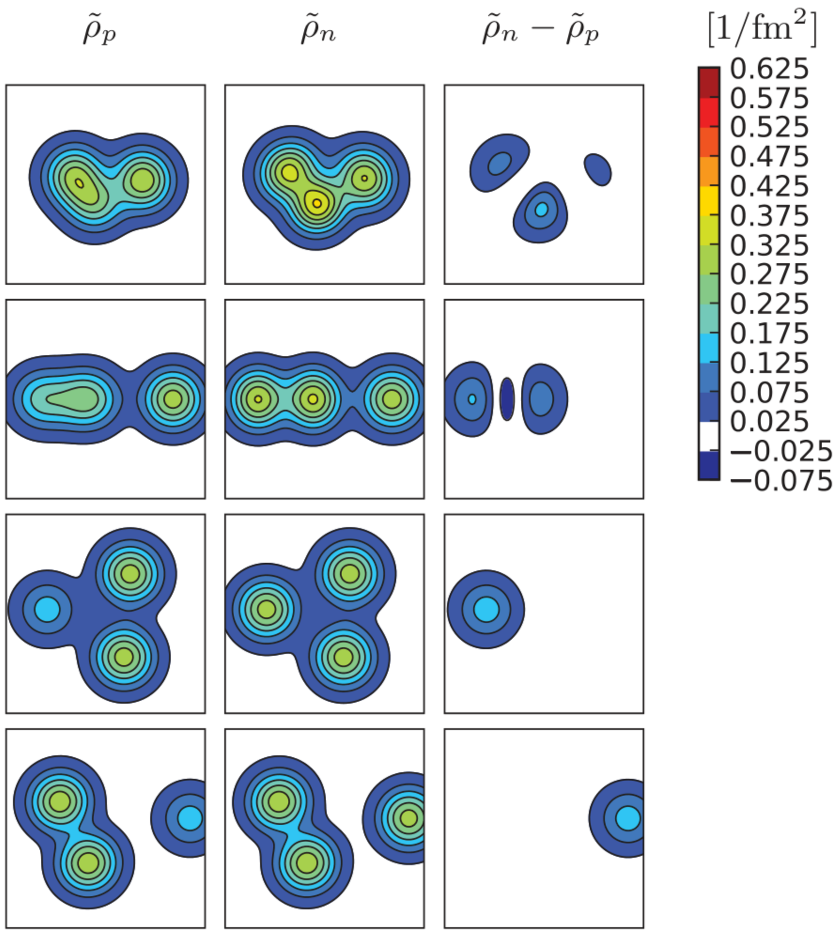
*Around $\alpha + \alpha + t$ threshold*

- Clustering feature is developed
- Also, excitation region of $p_\Lambda$
- parity coupling is likely to appear

***High-excitation brings Enormous model space!***




Spectrum of $^{11}$B

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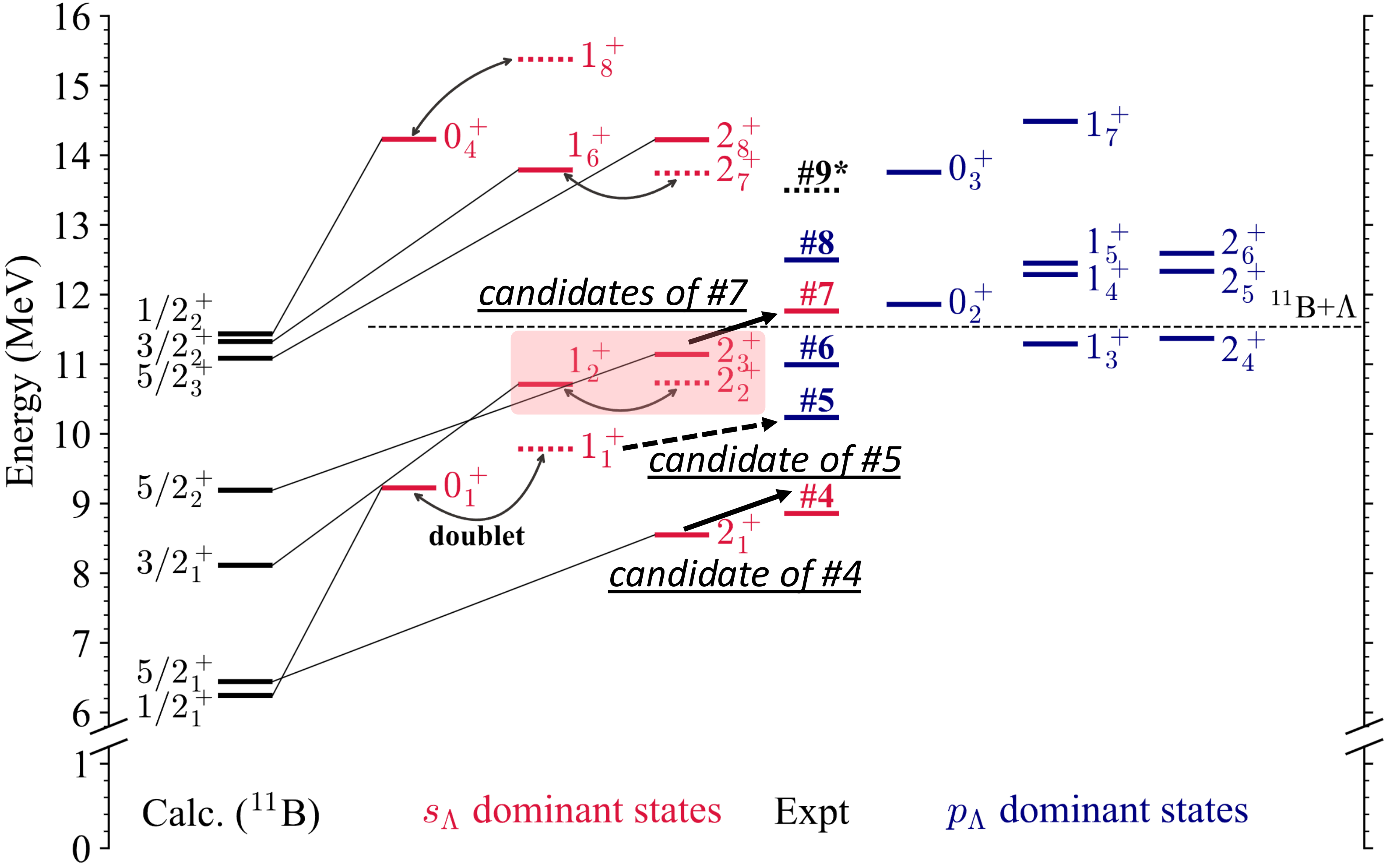
# Classification of spectrum and prediction for peaks

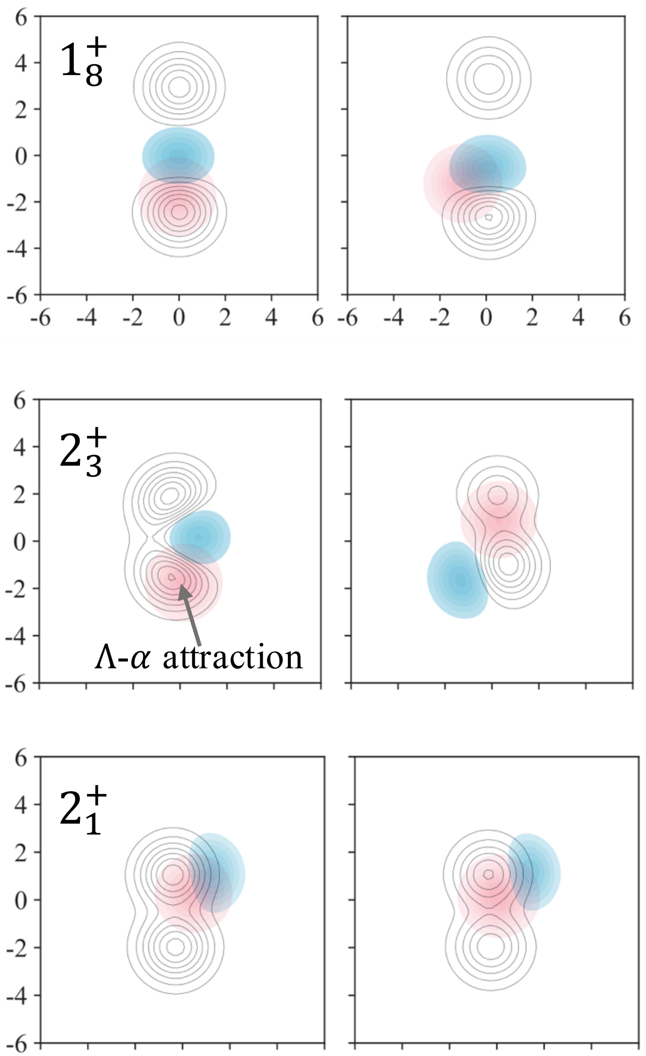
- Dotted line for $j_>$ in doublet states
- Conveniently corresponded to those of core nucleus states
- Energy spacing is reliable to predict the candidates of experimental peaks

| Peak | Possible candidates |
|---|---|
| #4 ($s_\Lambda$) | $2_1^+$ |
| #5 (contradict) | $1_1^+$ |
| #6 ($p_\Lambda$) | $1_3^+, 2_4^+$ |
| #7 ($s_\Lambda$) | $1_2^+, 2_2^+, 2_3^+$ |
| #8 ($p_\Lambda$) | $1_5^+, 2_6^+$ |



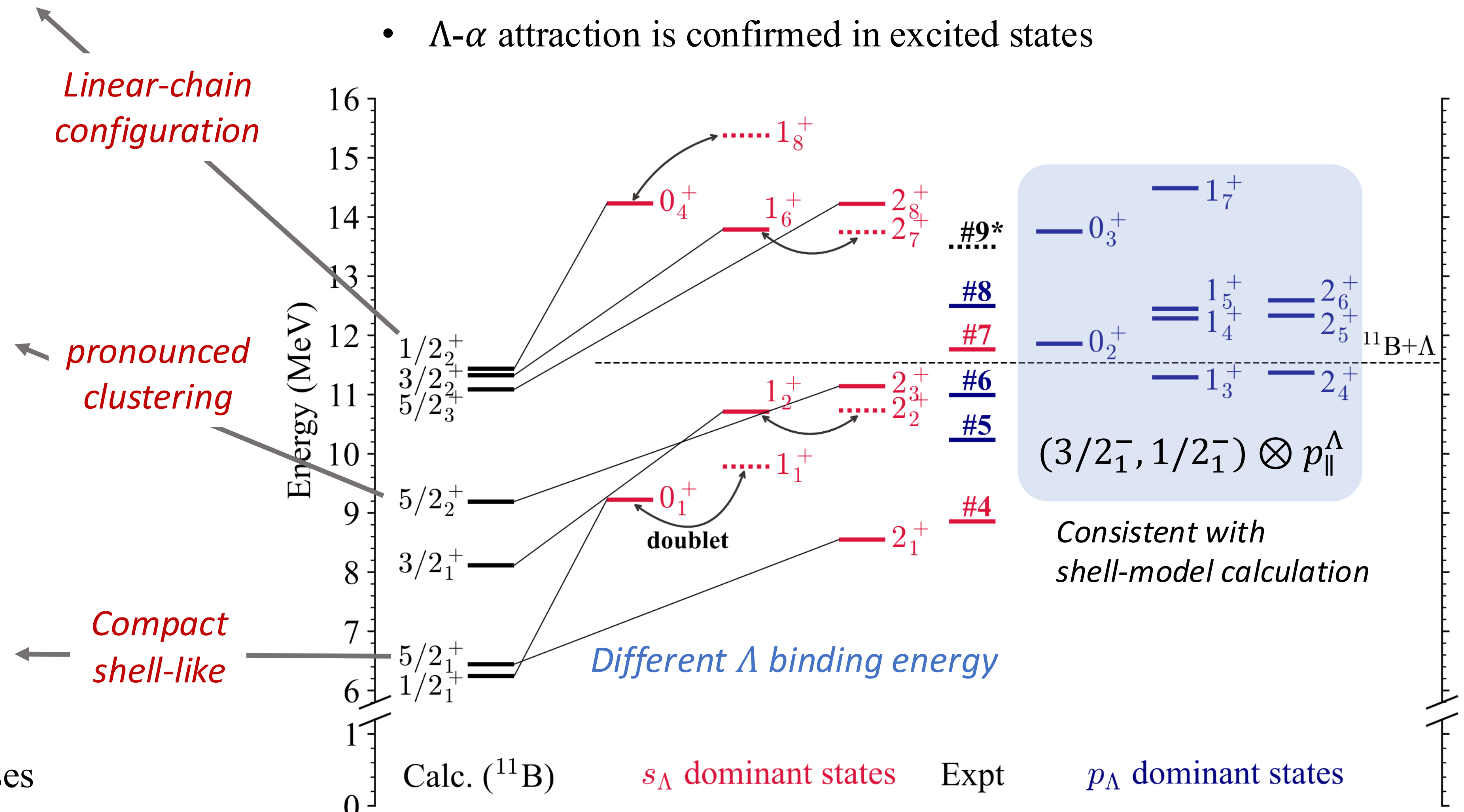
# Structures of $^{12}_{\Lambda}$B states

- Linear-chain configuration is reserved, but $\alpha$-$t$-$\alpha$ is preferred
- A novel isosceles triangle configuration appears in $2_3^+$
- Compact shell-like configuration of $2_1^+$, like $5/2_1^+$($^{11}$B )
- $\Lambda$-$\alpha$ attraction is confirmed in excited states



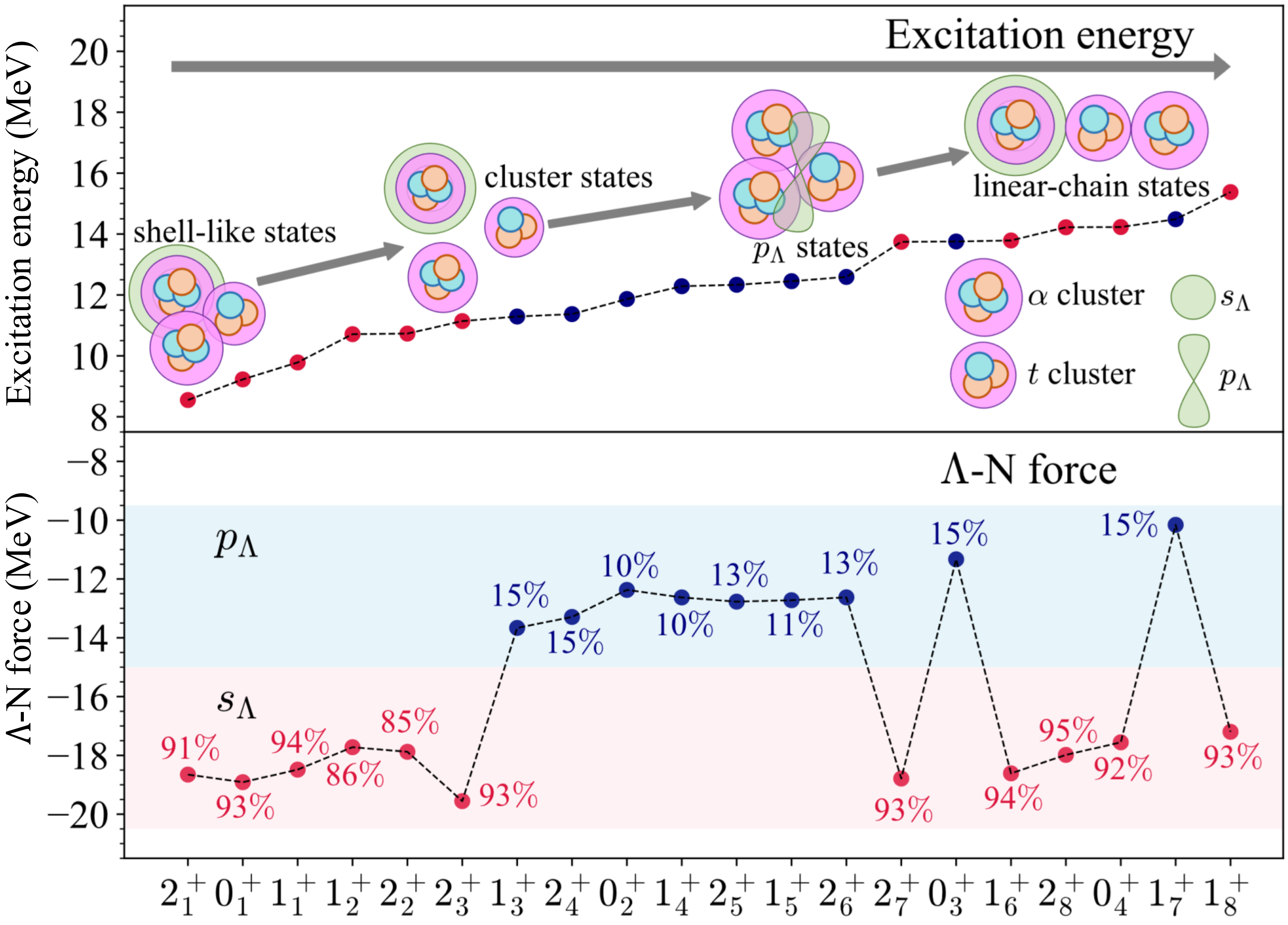

Density distributions of main bases

$t$ cluster, $\Lambda$ particle



# Structural evolution of ${}^{12}_{\Lambda}B$

- By the excitation energy increasing, ${}^{12}_{\Lambda}B$ undergoes significantly structural change



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# Cluster model for the entire spectrum?

- Below the threshold, alpha clusters are broken due to antisymmetrization and non-central forces
- Starting from cluster model, breaking is described by paired momentum excitation


nature communications

Article

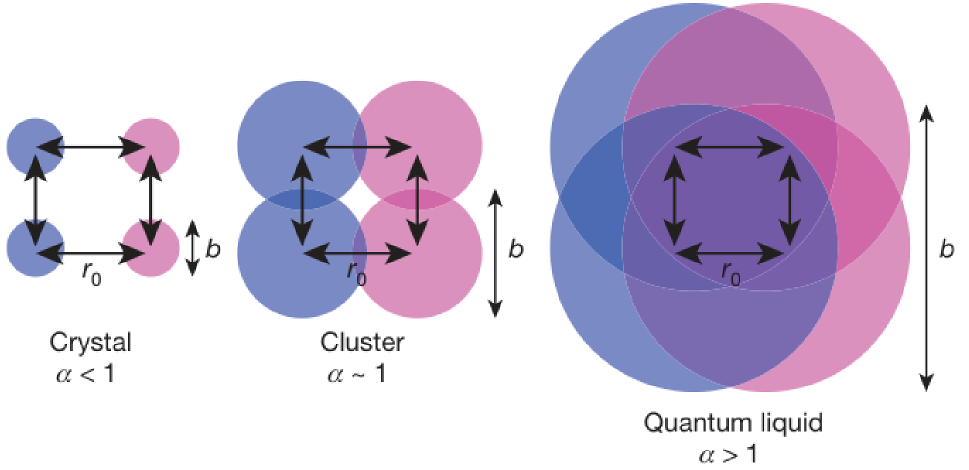
https://doi.org/10.1038/s41467-023-43816-9

**The 5α condensate state in $^{20}$Ne**

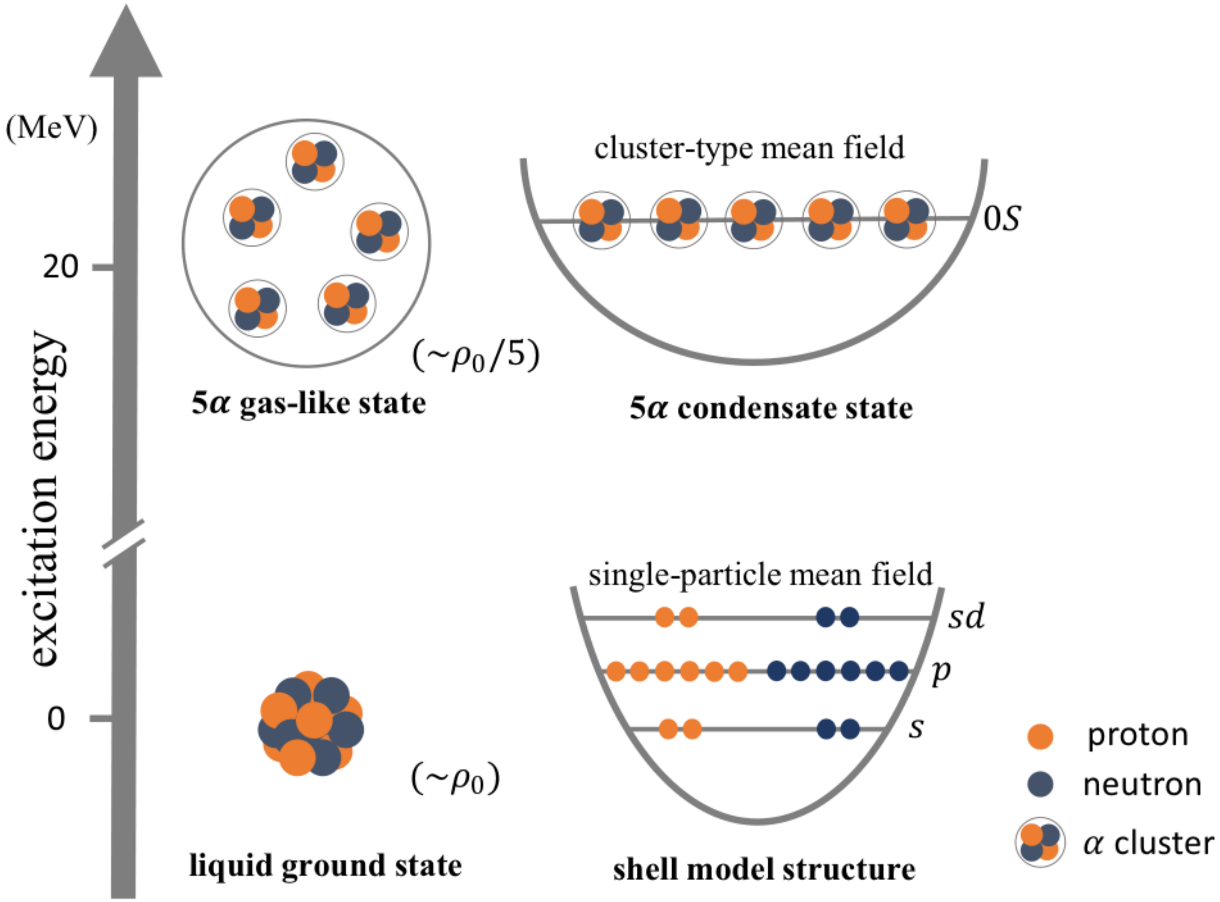
Received: 16 May 2023

Accepted: 21 November 2023

Bo Zhou[1,2], Yasuro Funaki[3], Hisashi Horiuchi[4], Yu-Gang Ma[1,2], Gerd Röpke[5], Peter Schuck[6], Akihiro Tohsaki[4] & Taiichi Yamada[3]




Transition from a crystalline to a quantum liquid phase
J.-P. Ebran et al., Nature 487, 341 (2012)




B. Zhou, et al. Nat. Commun. 14, 8206 (2023).

# Cluster model for the entire spectrum?

- Cluster formation near n-$\alpha$ threshold
- Below the threshold, alpha clusters are broken due to antisymmetrization and non-central forces
- Starting from cluster model, breaking is described by paired momentum excitation

◆ *Conventional cluster model*

$$|\Psi_\alpha\rangle = \mathcal{A}\left\{\prod_{i=1}^{4}\phi^N(\boldsymbol{r}_i^N, \boldsymbol{Z}^\alpha)\right\}$$

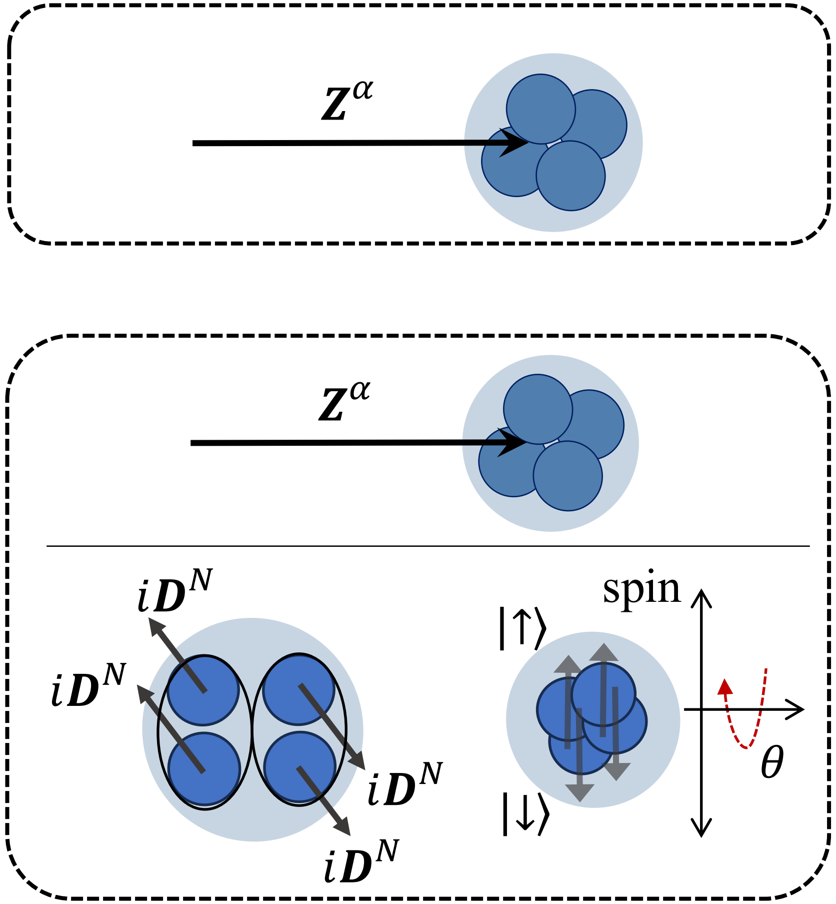
$\boldsymbol{Z}^\alpha$ is a real number

◆ *Extended for cluster breaking*

$$|\Psi_\alpha\rangle = \mathcal{A}\left\{\prod_{i=1}^{4}\phi^N(\boldsymbol{r}_i^N, \boldsymbol{\xi}^\alpha)\right\}$$

$$\boldsymbol{\xi}^\alpha = \boldsymbol{Z}^\alpha \underbrace{\pm i\boldsymbol{D}^N}_{\text{Imaginary part}}$$

$$\langle\phi^N|\hat{\boldsymbol{p}}_i|\phi^N\rangle \propto \mathrm{Im}[\boldsymbol{\xi}]$$




**Variational parameters**

Coordinates: $\{\boldsymbol{R}\}$

coordinates

momentum

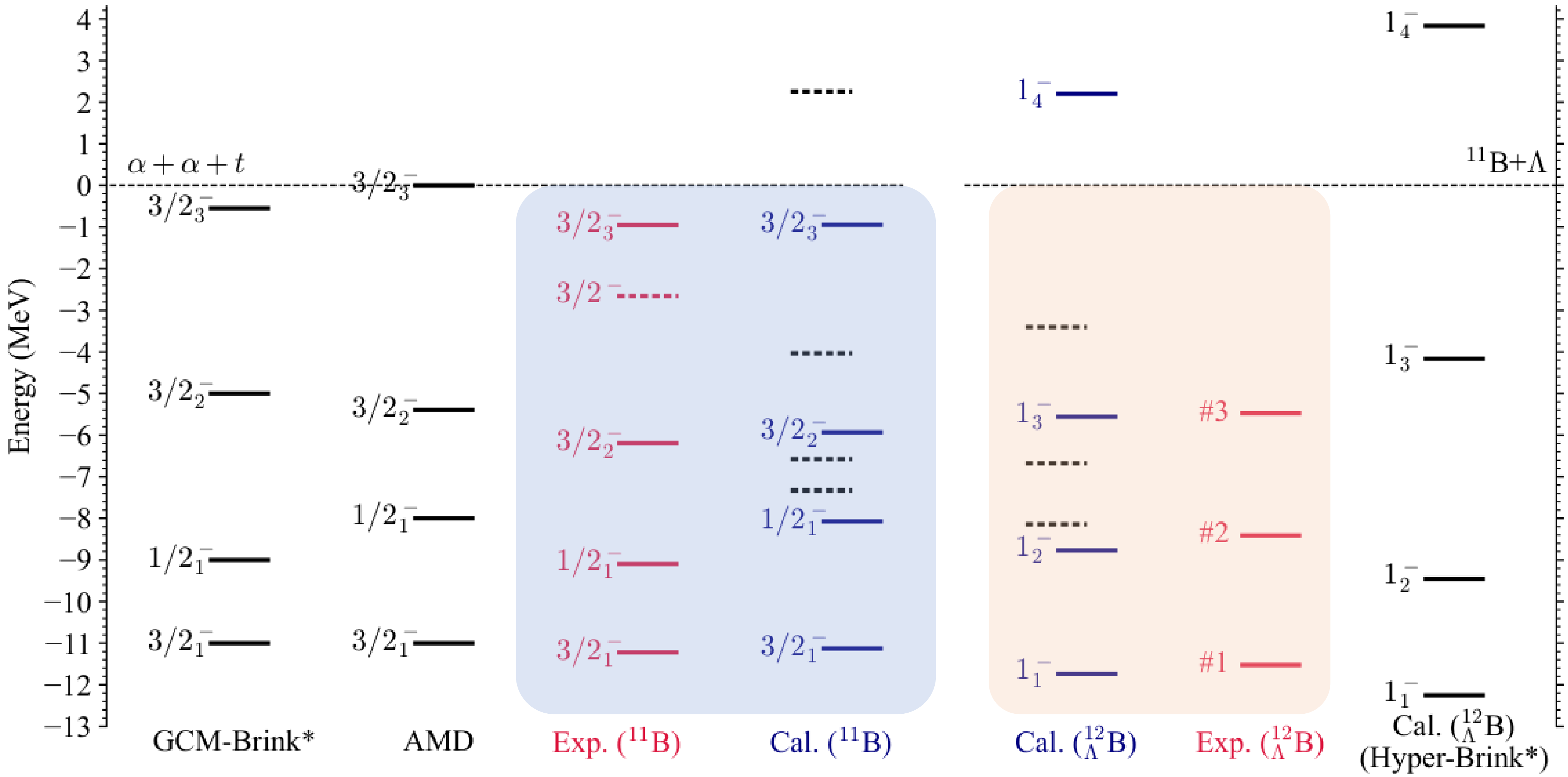
spin direction

$\{\boldsymbol{R}, \boldsymbol{D}^N, \theta\}$

*input of Ctrl.NN*

# Spectrum of negative parity states

- $\alpha$ and $t$ cluster are breaking
- triton and $\Lambda$ spins are fixed
- $\alpha$ cluster spin is free (keep anti-parallel)




$^{11}$B spectrum

$^{12}_{\Lambda}$B spectrum

# How about cluster breaking level?

Onebody LS operator: $\hat{\mathcal{O}}^{LS} = \boldsymbol{ls}/\hbar^2$

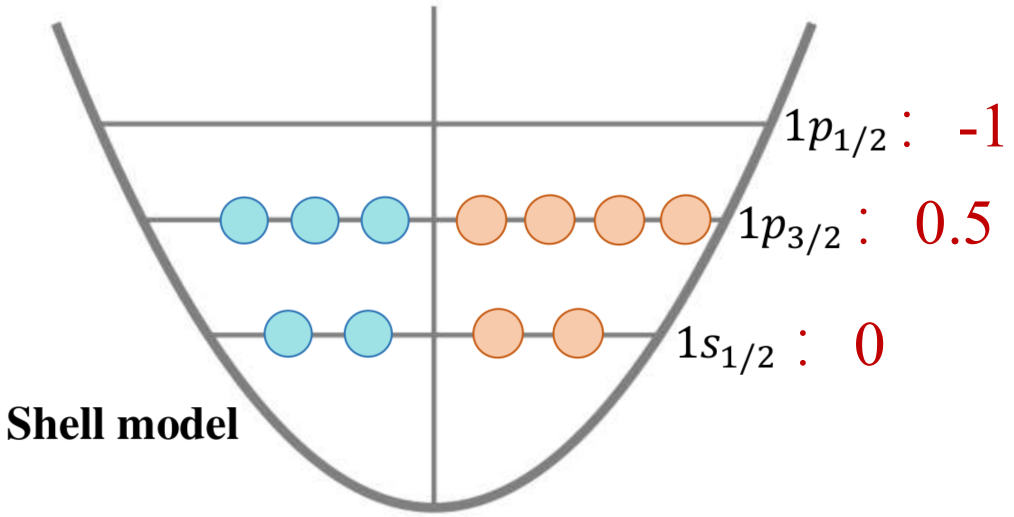
Onebody LS operator :

| $^{11}$B | | $^{12}_{\Lambda}$B (nuclear part) | |
|---|---|---|---|
| $3/2^-_1$ | **1.92** | $1^-_1(3/2^-_1 \otimes s_\Lambda)$ | **1.74** |
| $1/2^-_1$ | 0.75 | $1^-_2(1/2^-_1 \otimes s_\Lambda)$ | 0.87 |
| $3/2^-_2$ | 0.22 | $1^-_3(3/2^-_2 \otimes s_\Lambda)$ | 0.25 |
| $3/2^-_3$ | **0.99** | $1^-_4(3/2^-_3 \otimes s_\Lambda)$ | **0.86** |

Rms radius:

| $^{11}$B | | $^{12}_{\Lambda}$B (nuclear part) | |
|---|---|---|---|
| $3/2^-_1$ | 2.30 | $1^-_1(3/2^-_1 \otimes s_\Lambda)$ | 2.28 |
| $1/2^-_1$ | 2.44 | $1^-_2(1/2^-_1 \otimes s_\Lambda)$ | 2.33 |
| $3/2^-_2$ | 2.50 | $1^-_3(3/2^-_2 \otimes s_\Lambda)$ | 2.37 |
| $3/2^-_3$ | 2.78 | $1^-_4(3/2^-_3 \otimes s_\Lambda)$ | 2.54 |

Onebody LS expectation value of particles




$p_{3/2}$ subshell closure ⟷ $\hat{\mathcal{O}}^{LS} = 0.5 * 7 = 3.5$

Pure cluster state ⟷ $\hat{\mathcal{O}}^{LS} = 0$

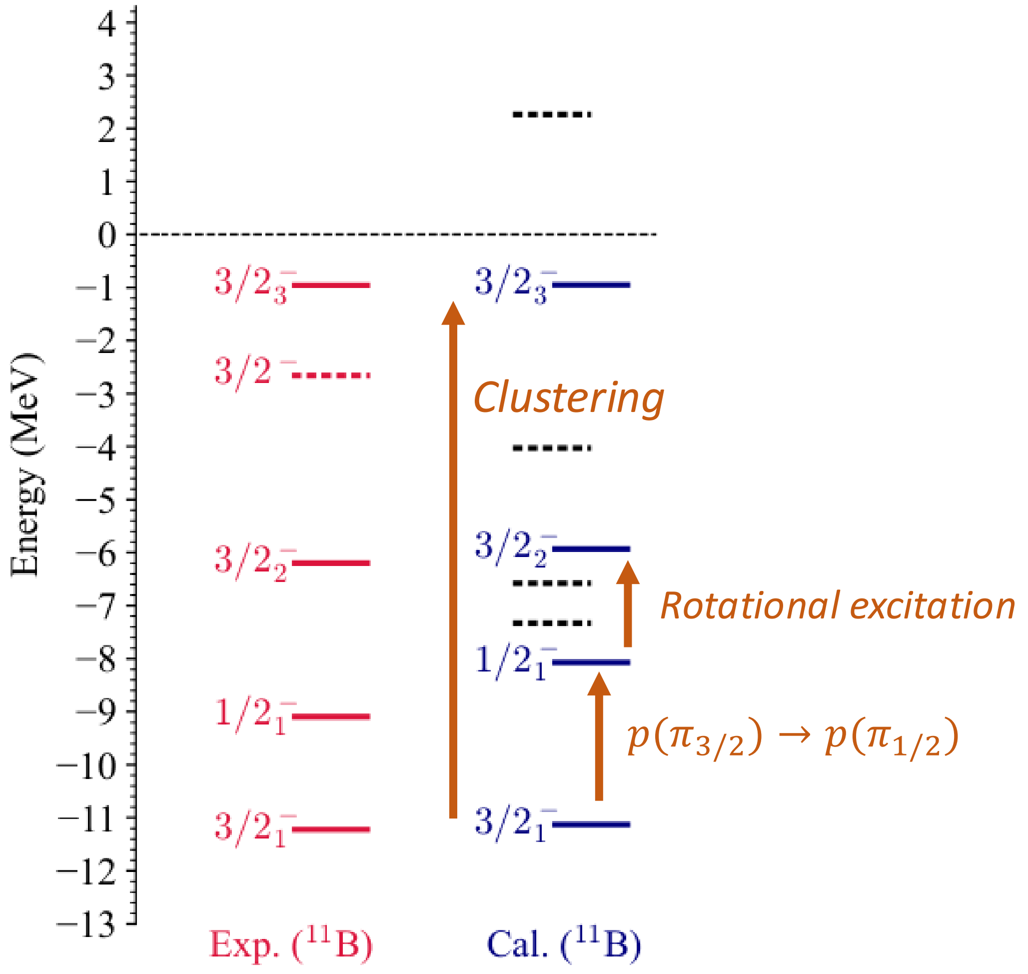
# How about the excitation modes?

Onebody LS operator and Rms radius :

| $^{11}$B | LS | radius | $^{12}_{\Lambda}$B (nucl.) | LS | radius |
|---|---|---|---|---|---|
| $3/2^-_1$ | **1.92** | **2.30** | $1^-_1(3/2^-_1 \otimes s_\Lambda)$ | **1.74** | **2.28** |
| $1/2^-_1$ | 0.75 | 2.44 | $1^-_2(1/2^-_1 \otimes s_\Lambda)$ | 0.87 | 2.33 |
| $3/2^-_2$ | 0.22 | 2.50 | $1^-_3(3/2^-_2 \otimes s_\Lambda)$ | 0.25 | 2.37 |
| $3/2^-_3$ | **0.99** | **2.78** | $1^-_4(3/2^-_3 \otimes s_\Lambda)$ | **0.86** | **2.54** |

**$B(E2)$: Possible probe to assess the Cluster-breaking**

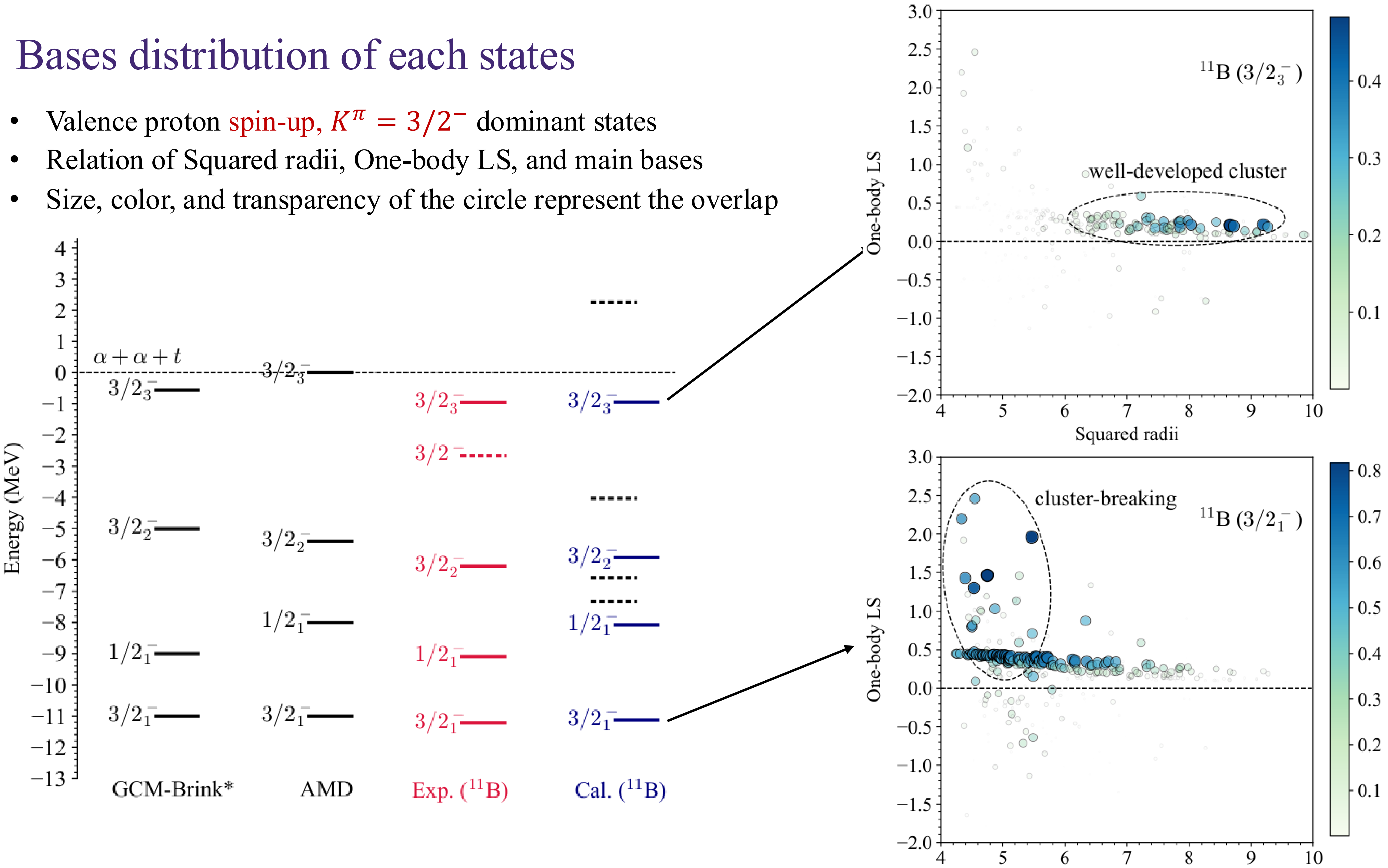
| $B(E2)$ | $^{11}$B: $3/2^-_1 \to 3/2^-_3$ | $^{12}_{\Lambda}$B: $1^-_1 \to 1^-_4$ |
|---|---|---|
| Cluster | 1.55 | 1.485 |
| | **different** | |
| +breaking | 0.84 | 1.14 |
| Cluster [1] | 1.5 | —— |
| AMD [2] | 0.84 | —— |




- $1/2^-_1$: valance proton is excited from $1p_{3/2}$ to $1p_{1/2}$
- $3/2^-_2$: rotational excitation of $1/2^-_1$
- $3/2^-_3$: well-developed clustering

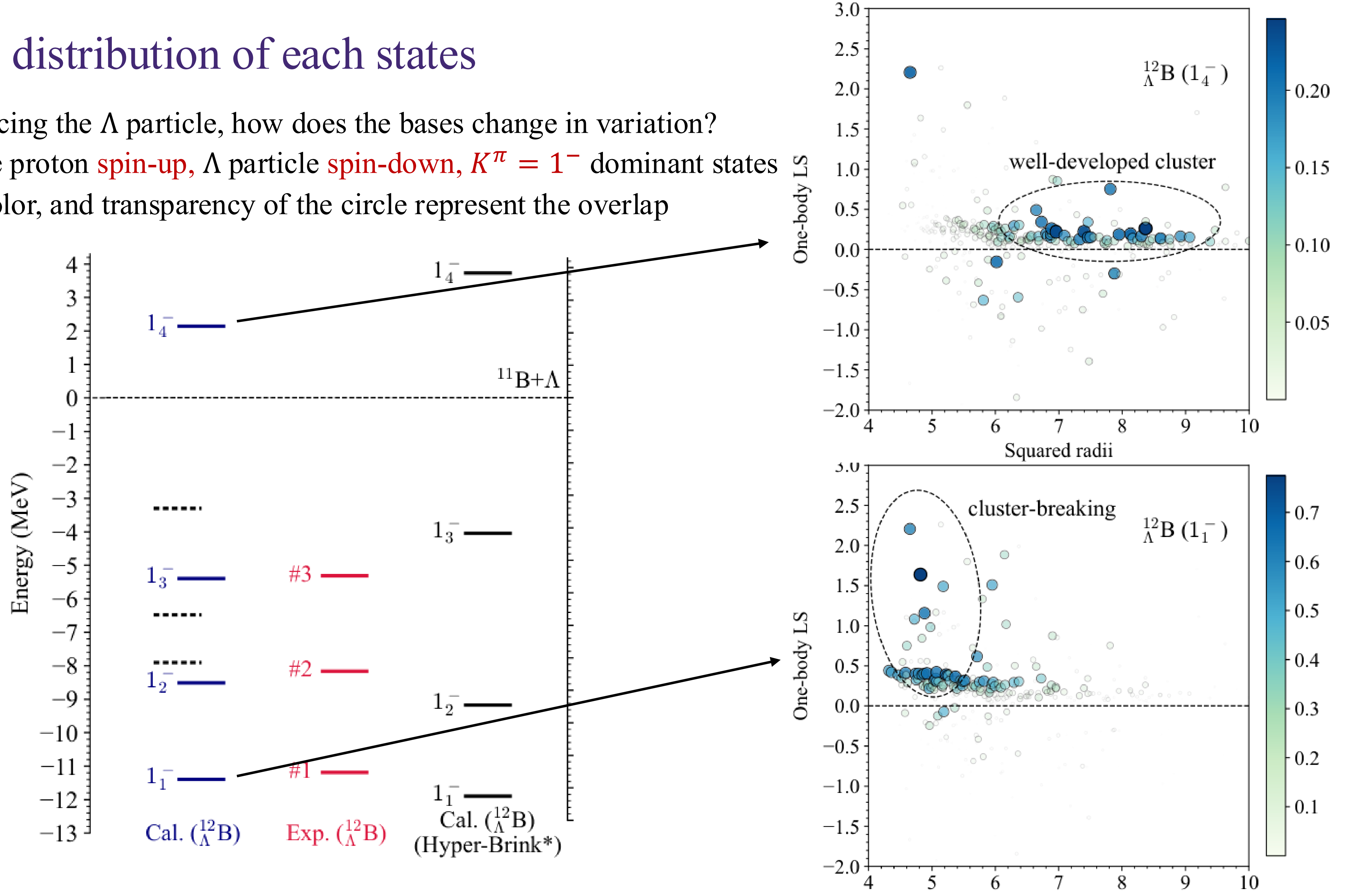
# Bases distribution of each states

- Valence proton spin-up, $K^\pi = 3/2^-$ dominant states
- Relation of Squared radii, One-body LS, and main bases
- Size, color, and transparency of the circle represent the overlap



# Bases distribution of each states

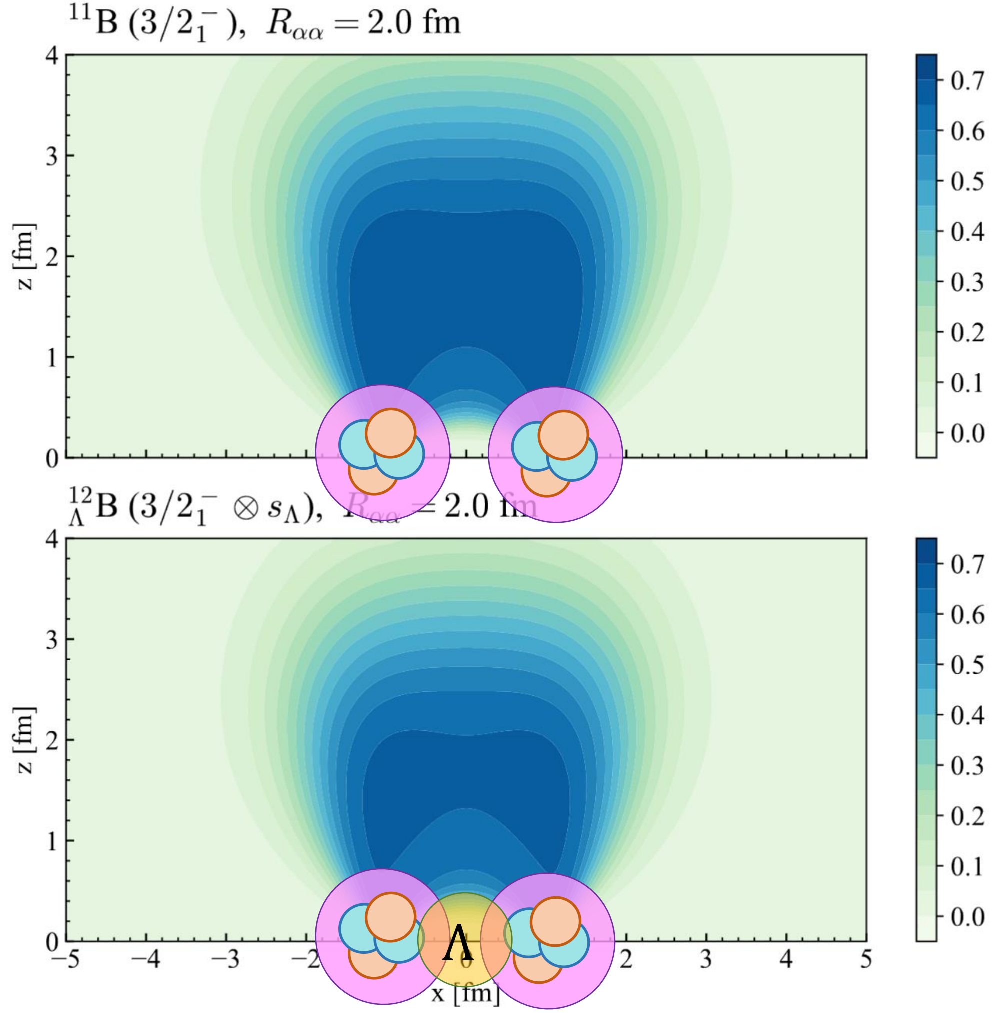
- Introducing the Λ particle, how does the bases change in variation?
- Valence proton spin-up, Λ particle spin-down, $K^{\pi} = 1^{-}$ dominant states
- Size, color, and transparency of the circle represent the overlap

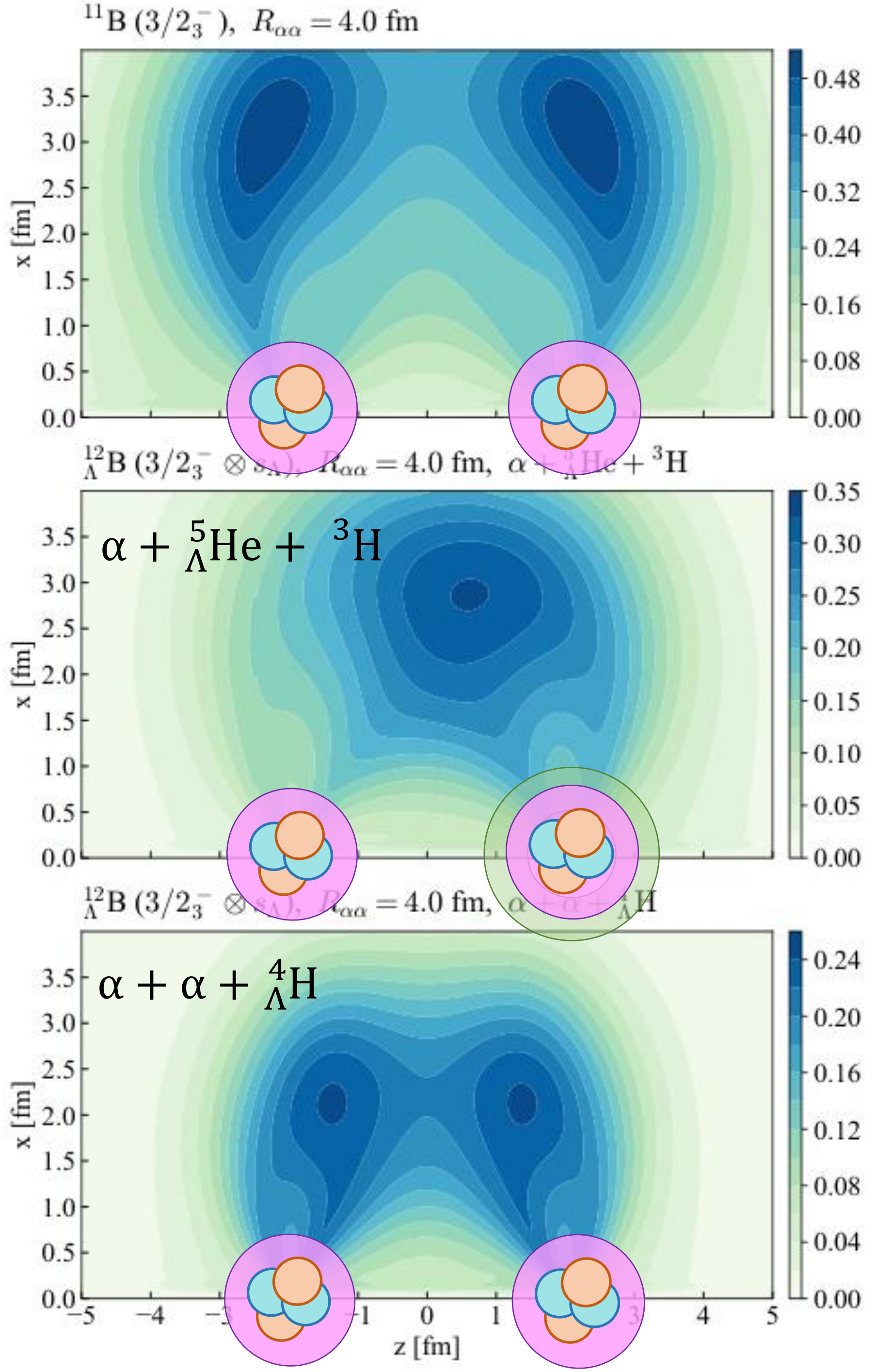


# How does the cluster distribution change?

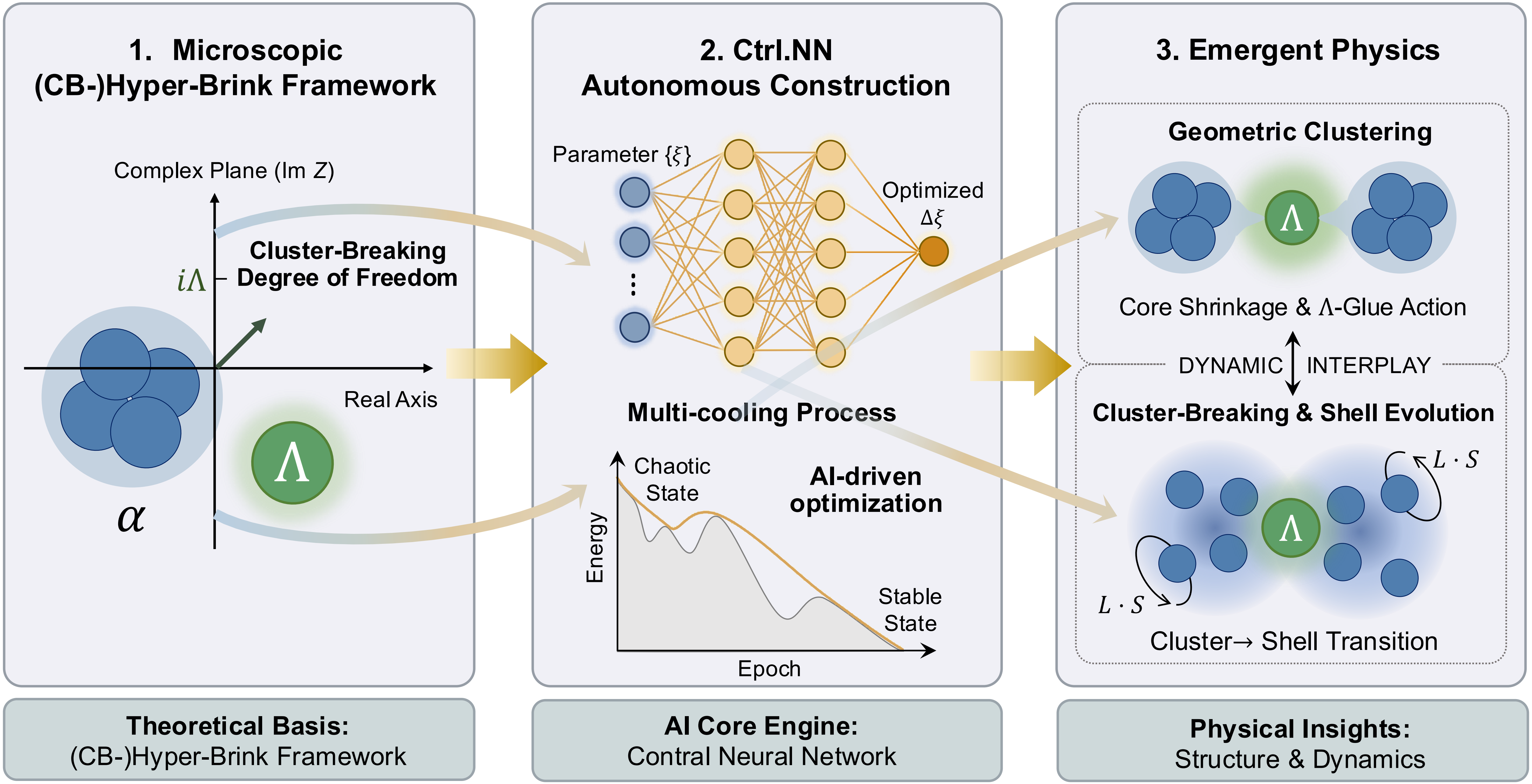
- $\alpha$ clusters fix on Z-axis, $\Lambda$ fix on center of coordinate
- Triton is in x-z plane

Calculate overlap between reference basis and total w.f.



# Summary and Outlook



# Summary and Outlook

1. We already calculated energy level and corresponding wave functions within cluster/AMD model.

   The next step is to calculate reaction observables and compare with data

2. Cluster/AMD model could be optimized by AI technique,

   but it is difficult to deal with complex correlation of realistic strong force

   → Is it possible to optimize the powerful few-body method by AI technique?

   → Is it possible to conduct the fine-tuning for parameters of strong force by AI technique?