Recent developments and applications of the continuum-discretized coupled-channels method

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Structural calculation and reaction observables

GEM calculation → Reaction calculation → observables $d\sigma/d\Omega$, $d\sigma/dE$

- Energy
- Wavefunction
  - g.s.
  - Excited state
  - Discretized continuum state

GEM gives not only energies, but also wave functions.

CDCC uses them as reaction input.

Outline

1 **CDCC basics** — Continuum discretization and coupled-channel equations

2 **^{6}He breakup** — Resonant and nonresonant continuum contributions

SO and T. Matsumoto, Phys. Rev. C **102**, 021602(R) (2020).

3 **^{9}C breakup** — Channel decomposition with CDCC + CSLS

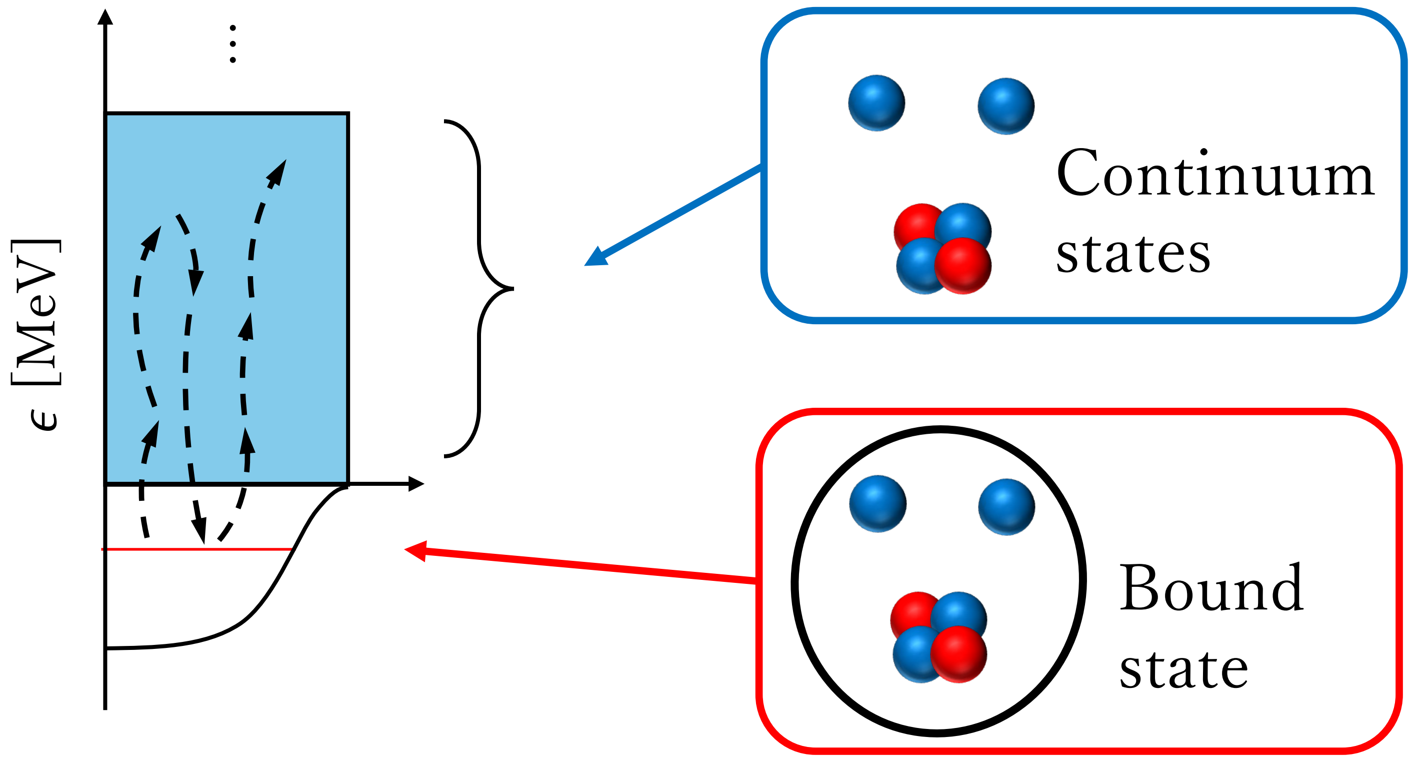
SO+, Prog. Theor. Exp. Phys. **2026**, ptag096 (2026).

4 **Future work: ^{7}Li + N** — More particles system

1. CDCC basics

Continuum-Discretized Coupled Channels method (CDCC)

Describe transitions of the projectile to continuum states during the reaction.

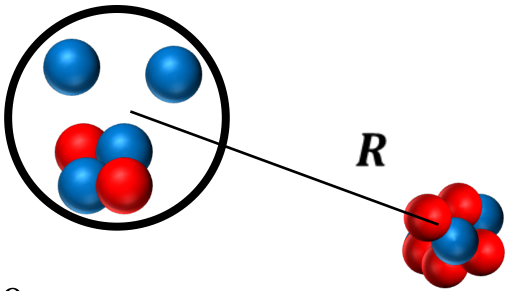


Solve coupled channel equations including various breakup states

Continuum-Discretized Coupled Channels method (CDCC)

- Schrödinger eq.

$$[K_R + U_n + U_n + U_\alpha + h_\mathrm{P} - E]\Psi(\boldsymbol{\xi}, \boldsymbol{R}) = 0$$



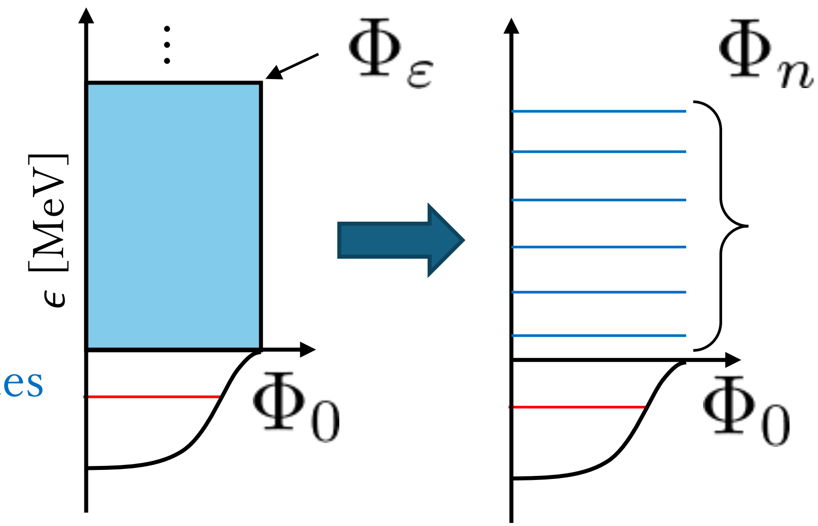
$\boldsymbol{\xi}$: Jacobi coordinate

- $\Psi(\boldsymbol{\xi}, \boldsymbol{R})$ is expanded with discretized eigen states, $\{\Phi_n\}$, of ^{6}He.

$$\Psi(\boldsymbol{\xi}, \boldsymbol{R}) = \Phi_0(\boldsymbol{\xi})\psi_0(\boldsymbol{R}) + \int d\epsilon\, \Phi_\epsilon(\boldsymbol{\xi})\psi_\epsilon(\boldsymbol{R})$$

Bound state Discretized continuum states

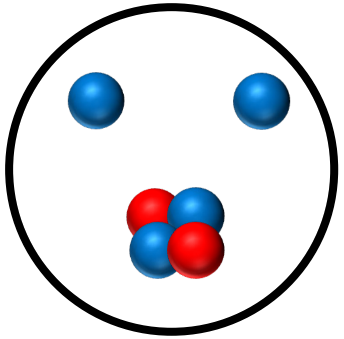
$$\Psi(\boldsymbol{\xi}, \boldsymbol{R}) = \boxed{\Phi_0(\boldsymbol{\xi})}\psi_0(\boldsymbol{R}) + \sum_n^{n_{\max}} \boxed{\Phi_n(\boldsymbol{\xi})}\psi_n(\boldsymbol{R})$$



Bound-bound, bound-continuum, and continuum-continuum transitions can be treated non-perturbatively.

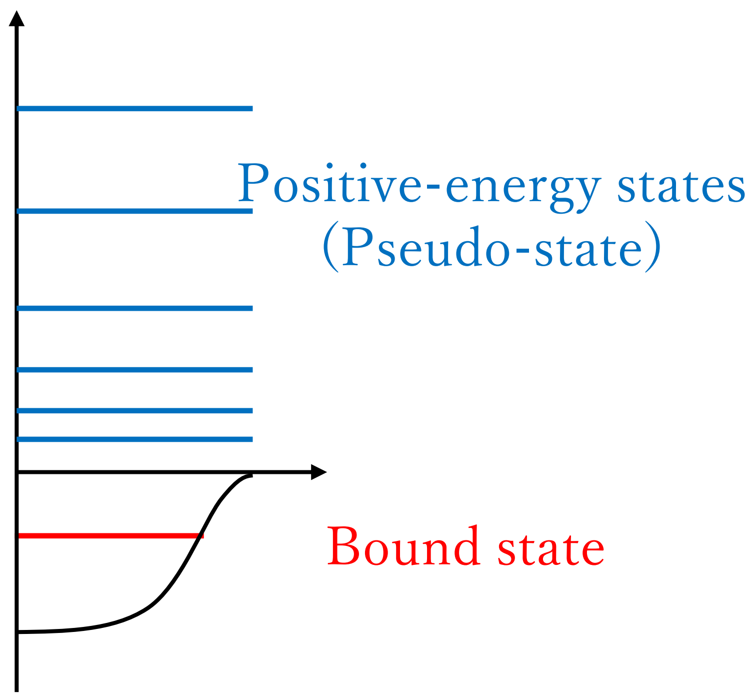
Continuum-state discretization

^{6}He continuum states are described by the Pseudo-state method.



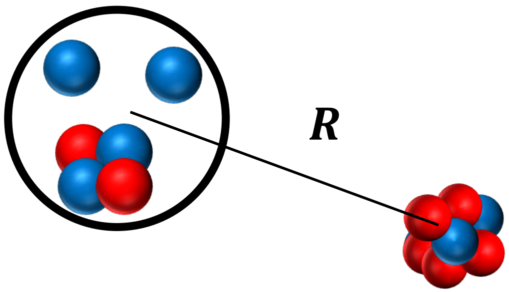
$$h_{\mathrm{P}} = K_x + K_y + V_{nn} + V_{n\alpha} + V_{n\alpha}$$

Diagonalizing Hamiltonian of ^{6}He with GEM.



Pseudo states are included as discretized continuum states in the CDCC calculation.

Continuum-Discretized Coupled Channels method (CDCC)



Total wave function

$$\Psi(\boldsymbol{\xi}, \boldsymbol{R}) = \Phi_0(\boldsymbol{\xi})\psi_0(\boldsymbol{R}) + \sum_{n}^{n_{\max}} \Phi_n(\boldsymbol{\xi})\psi_n(\boldsymbol{R})$$

Coupled-Channels equation

$$\langle \Phi_{n'} | H - E | \Psi \rangle = 0$$

$$[K_R + U_{nn}(\boldsymbol{R}) \ + (E - \epsilon_n)]\psi_n(\boldsymbol{R}) = -\sum_{n' \neq n} U_{nn'}\psi_{n'}(\boldsymbol{R})$$

Coupling potential

$$U_{nn'}(\boldsymbol{R}) = \int d\boldsymbol{\xi}\, \Phi_n(\boldsymbol{\xi}) U(\boldsymbol{\xi}, \boldsymbol{R}) \Phi_{n'}(\boldsymbol{\xi})$$

Cross section

$$\frac{d^2\sigma}{d\epsilon d\Omega}$$

ϵ : Internal energy of the emitted ^{6}He after the reaction.

Ω: Scattering angle of ^{6}He.

Continuum-Discretized Coupled Channels method (CDCC)

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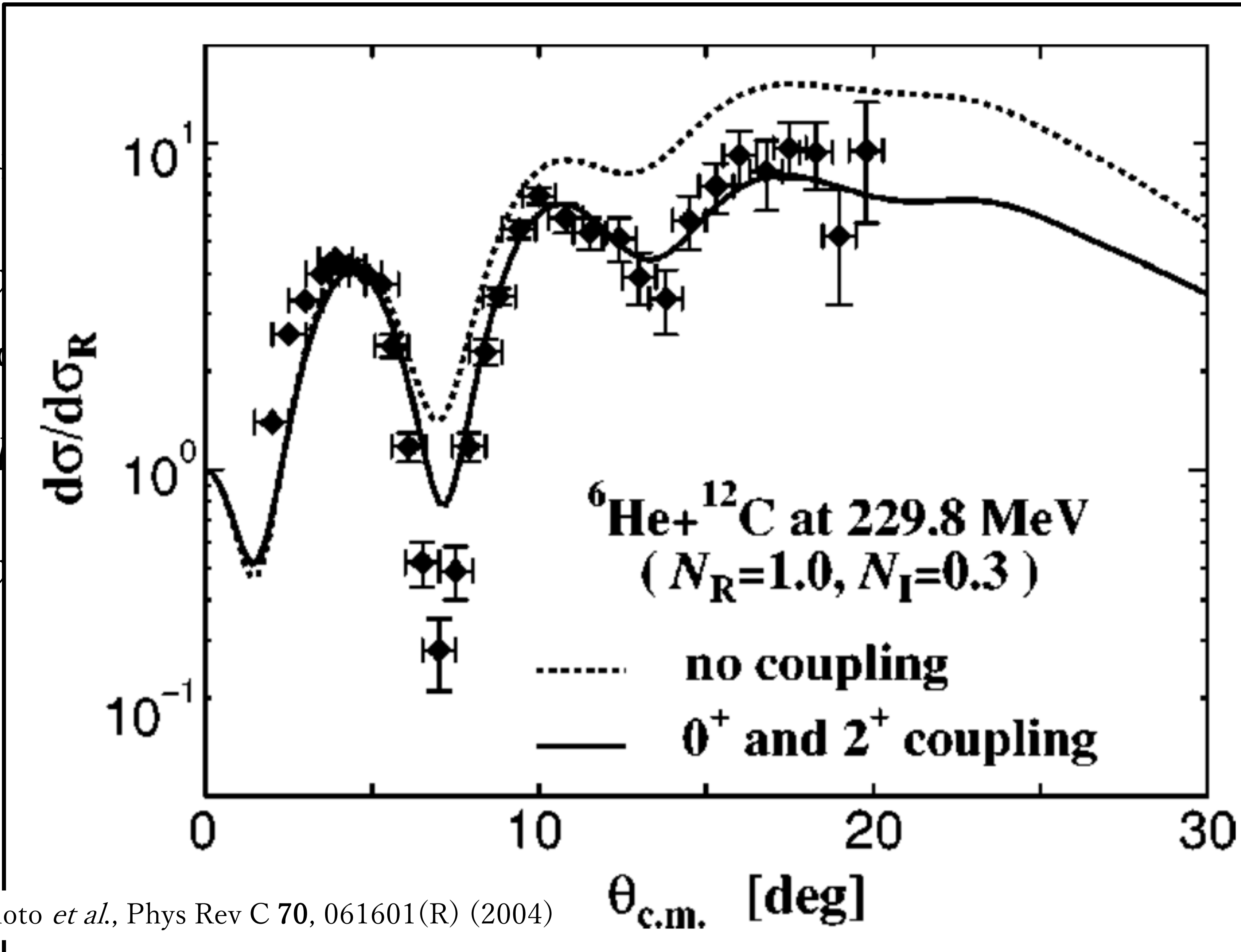
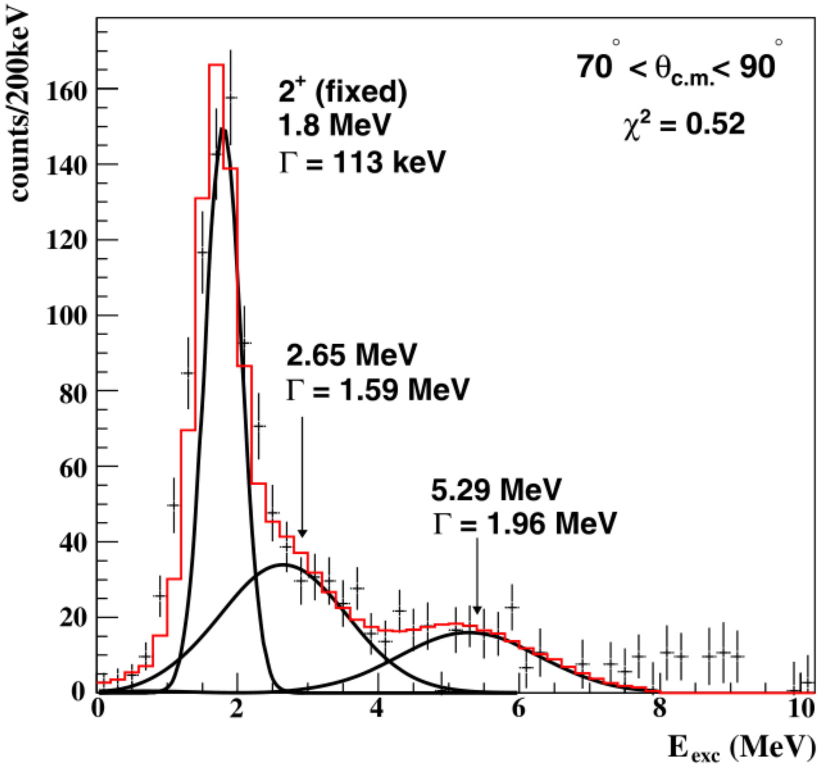


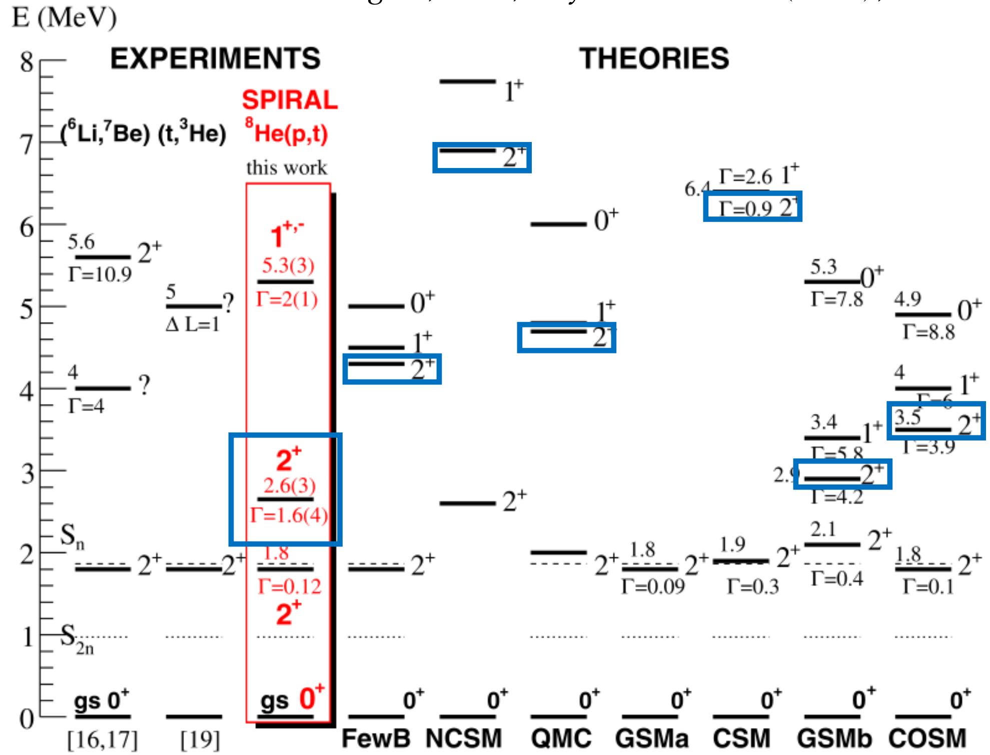
T. Matsumoto *et al.*, Phys Rev C **70**, 061601(R) (2004)

2. Example I: ^{6}He breakup

Study of 2_2^+ via $^8He(p,t)^6He$



X. Mougeot, et al., Phys. Lett. B **718** (2012), 441.



The 2_2^+ was discovered with excitation energy 2.65 MeV and width 1.59 MeV.

- These values are lower and narrower than those obtained by theoretical calculations.

The 2_2^+ was obtained by the Breit-Wigner fitting.

- Detailed studies treating resonant and non-resonant states are needed.

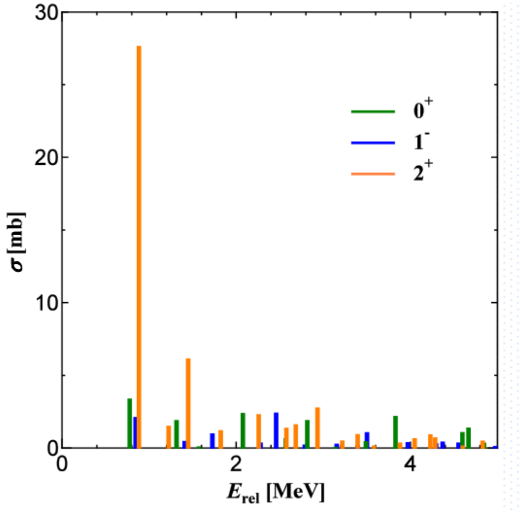
Broad resonances in ^{6}He

- ^{6}He is a two-neutron halo nucleus, described as $\alpha + n + n$.
- Breakup spectra contain both resonant and nonresonant continuum contributions.
- A broad resonance may not appear as a simple Breit-Wigner peak.

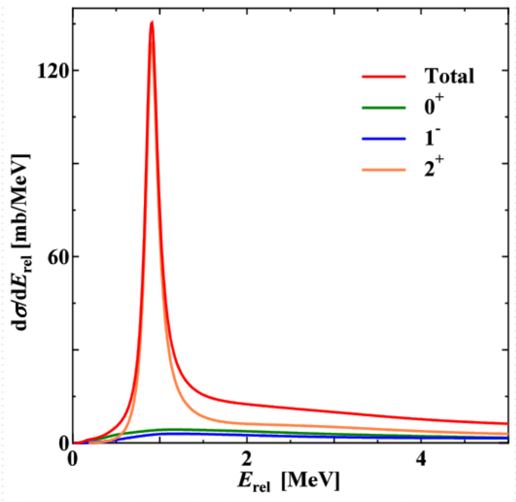
Structure calculation → Reaction calculation four-body CDCC → breakup observables $d\sigma/dE$

Smoothing of the cross section

- The calculated cross section is discrete in energy of ^{6}He.



Smoothing



- Continuous T matrix and discrete T matrix

$$T_\epsilon = \sum_n \underline{\langle \Phi_\epsilon | \Phi_n \rangle} T_n$$

Smoothing factor

Smoothing procedure

CDCC + CSM T. Matsumoto, *et al.*, Phys. Rev. C **82** (2010), 051602(R)

$$\frac{d^2\sigma}{d\epsilon d\Omega} = -\frac{1}{\pi}\mathrm{Im}\sum_{nn'} T_{n'}^{\dagger}\langle\Phi_{n'}|\frac{1}{\epsilon - h_{^6\mathrm{He}} + i\eta}|\Phi_n\rangle T_n$$

Green's function with complex scaling method

$$\frac{1}{\epsilon - h_{^6\mathrm{He}} + i\eta} = U^{-1}(\theta)\frac{1}{\epsilon - h_{^6\mathrm{He}} + i\eta}U(\theta) = \sum_{\nu} U^{-1}(\theta)|\Phi_\nu^\theta\rangle\frac{1}{\epsilon - \epsilon_\nu^\theta}\langle\tilde{\Phi}_\nu^\theta|U(\theta)$$

Double differential cross section

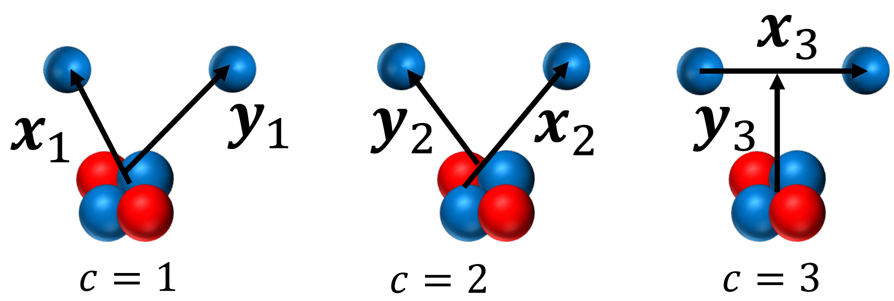
$$\frac{d^2\sigma}{d\epsilon d\Omega} = \sum_{\nu} -\frac{1}{\pi}\mathrm{Im}\frac{\left[\sum_{n'} T_{n'}^{\dagger}\langle\Phi_{n'}|U^{-1}(\theta)|\Phi_\nu^\theta\rangle\right]\left[\sum_n\langle\tilde{\Phi}_\nu^\theta|U(\theta)|\Phi_n\rangle T_n\right]}{\epsilon - \epsilon_\nu^\theta}$$

Complex Scaling Method (CSM)

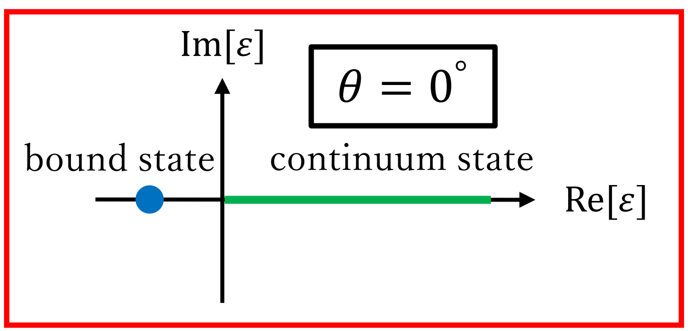
- Complex rotation of the internal coordinate of ^{6}He.

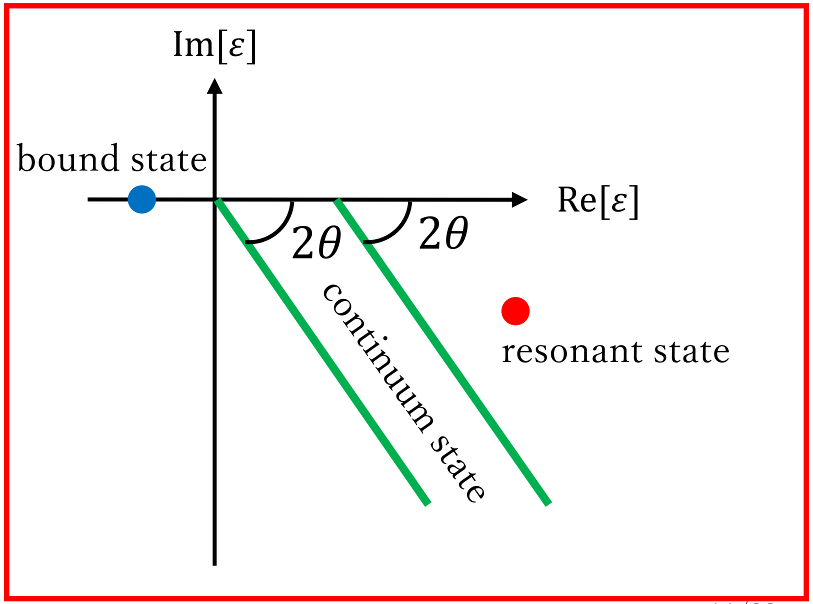
$$U(\theta)\boldsymbol{x}U^{-1}(\theta) = \boldsymbol{x}e^{i\theta}$$

$$U(\theta)\boldsymbol{y}U^{-1}(\theta) = \boldsymbol{y}e^{i\theta}$$



Under the transformation, we solve the Schrödinger eq. of ^{6}He with GEM.

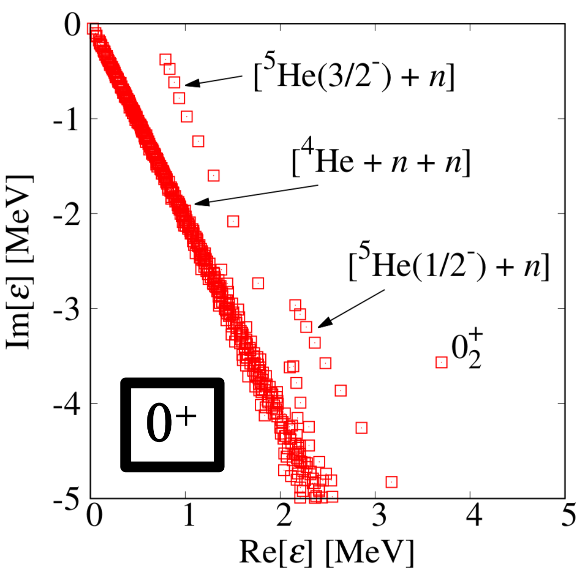


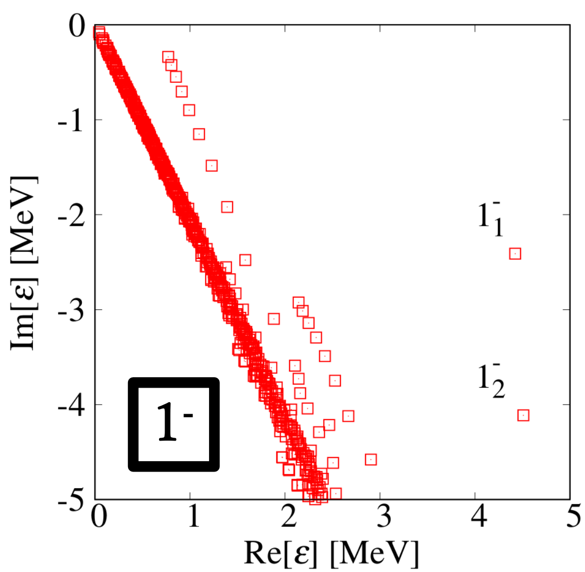


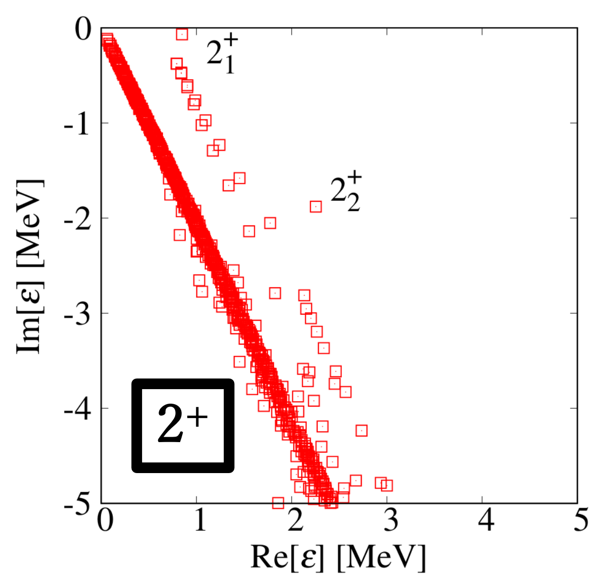
Complex energy states appear.

T. Myo+, PTP **99**, 801 (1998)

Results of CSM







	Calc.		T. Myo, *et al.* [1]	
J^π	E_r	Γ	E_r	Γ
2^+_1	0.85	0.14	0.81	0.13
2^+_2	2.25	3.75	2.35	4.22
0^+_2	3.70	7.13	3.69	9.14
1^-_1	4.42	4.82		
1^-_2	4.51	8.23		

[1] T. Myo, *et al.*, Phys. Rev. C **63** (2001), 054313

Smoothing procedure

CDCC + CSM   T. Matsumoto, *et al.*, Phys. Rev. C **82** (2010), 051602(R)

$$\frac{d^2\sigma}{d\epsilon d\Omega} = -\frac{1}{\pi}\mathrm{Im}\sum_{nn'} T^{\dagger}_{n'}\langle\Phi_{n'}|\frac{1}{\epsilon - h_{^6\mathrm{He}} + i\eta}|\Phi_n\rangle T_n$$

Green's function with complex scaling method

$$\frac{1}{\epsilon - h_{^6\mathrm{He}} + i\eta} = U^{-1}(\theta)\frac{1}{\epsilon - h_{^6\mathrm{He}} + i\eta}U(\theta) = \sum_{\nu} U^{-1}(\theta)|\Phi^{\theta}_{\nu}\rangle\frac{1}{\epsilon - \epsilon^{\theta}_{\nu}}\langle\tilde{\Phi}^{\theta}_{\nu}|U(\theta)$$

Double differential cross section

$\frac{d^2\sigma_\nu}{d\epsilon d\Omega}$ Cross section for transitions to states obtained with CSM.

$$\frac{d^2\sigma}{d\epsilon d\Omega} = \sum_{\nu}\boxed{-\frac{1}{\pi}\mathrm{Im}\frac{\left[\sum_{n'} T^{\dagger}_{n'}\langle\Phi_{n'}|U^{-1}(\theta)|\Phi^{\theta}_{\nu}\rangle\right]\left[\sum_{n}\langle\tilde{\Phi}^{\theta}_{\nu}|U(\theta)|\Phi_n\rangle T_n\right]}{\epsilon - \epsilon^{\theta}_{\nu}}}$$

The resonant contribution can be separated.

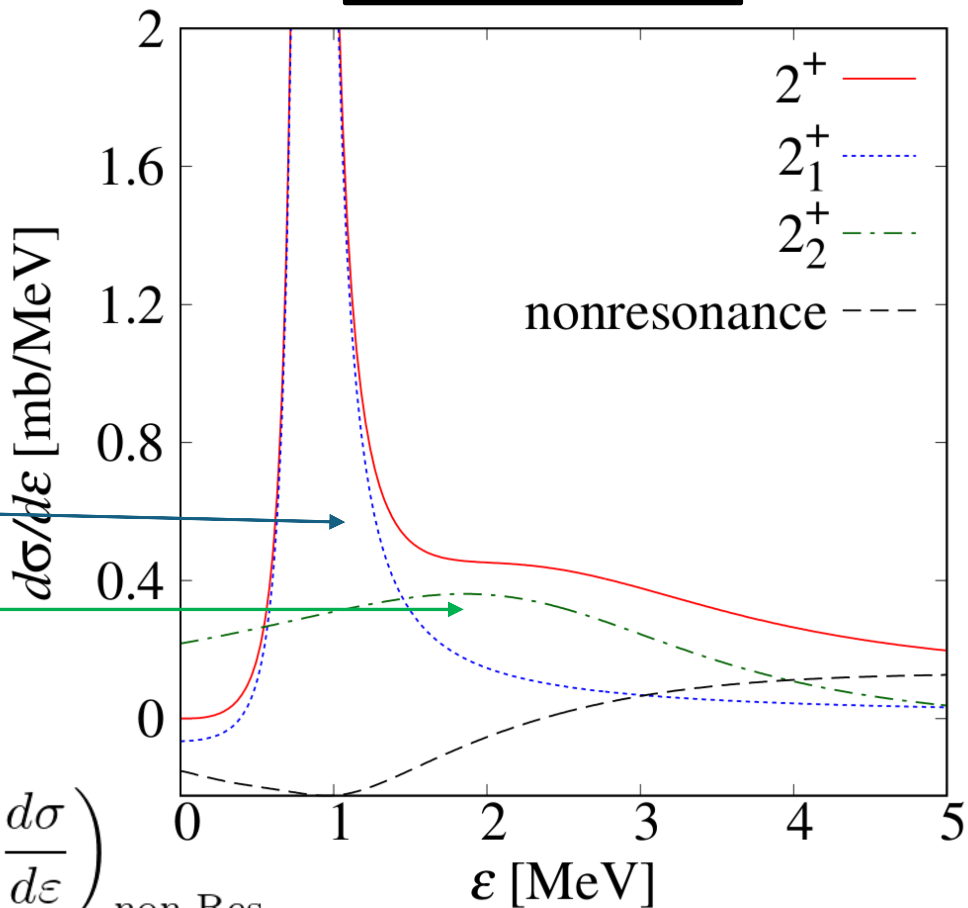
Energy spectrum (2^+)

^{6}He + p @41MeV/A

$$\frac{d\sigma}{d\varepsilon} = \int_{20^\circ \le \theta \le 30^\circ} \frac{d^2\sigma}{d\varepsilon d\Omega} d\Omega$$

	Calc.	
I^π	E_r	Γ
2_1^+	0.85	0.14
2_2^+	2.25	3.75

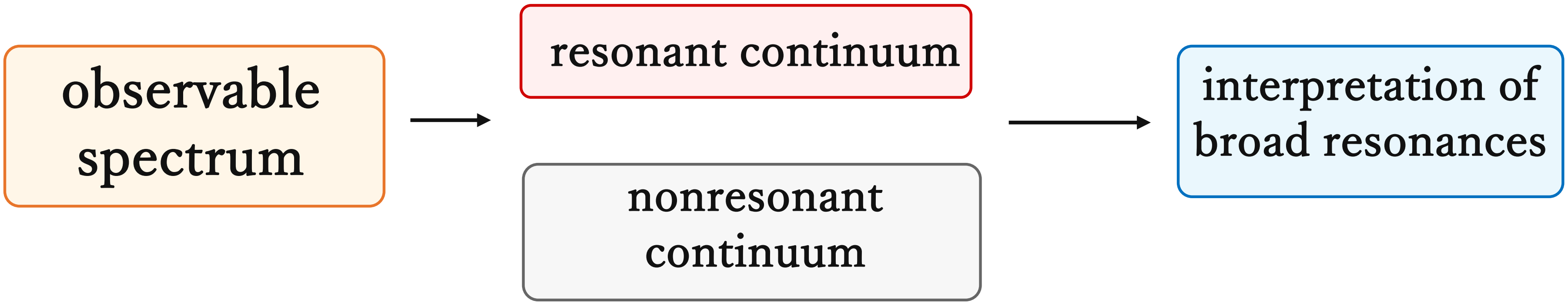
2$^+$-continuum



$$\left(\frac{d\sigma}{d\varepsilon}\right)_{2^+} = \left(\frac{d\sigma}{d\varepsilon}\right)_{2_1^+} + \left(\frac{d\sigma}{d\varepsilon}\right)_{2_2^+} + \left(\frac{d\sigma}{d\varepsilon}\right)_{\text{non Res.}}$$

- We separate cross section to components for $2_1^+, 2_2^+$, non-res.
- The shoulder peak comes from 2_2^+ in 2~3 MeV.

What ^{6}He teaches us



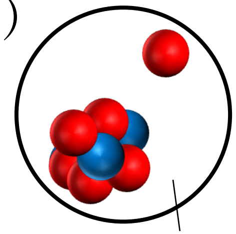
- Reaction data do not isolate a resonance automatically.
- The resonant energy and decay width calculated with CSM are connected to the peak structure of the reaction observables.

3. Example II: ^{9}C breakup

Analysis with the three-body reaction model

- Three-body continuum-discretized coupled channels method (Three-body CDCC)

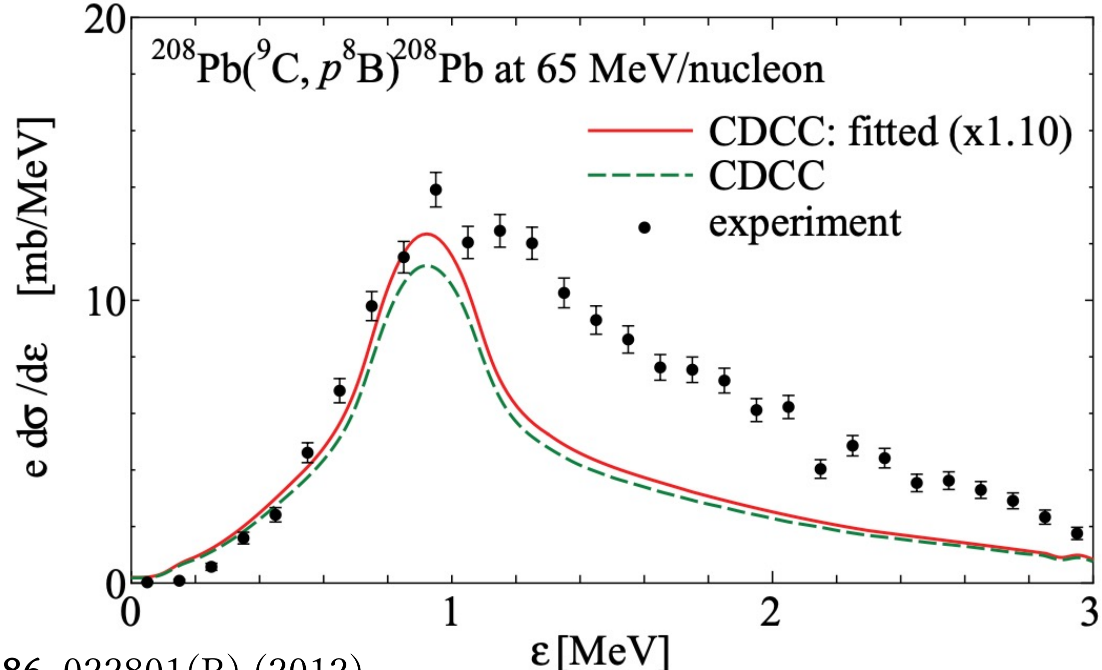
^{9}C$(= p + {}^{8}$B$)$



^{8}B is treated as **inert**.

- ^{8}B
  - ✓ Proton halo nucleus
  - ✓ $S_p = 0.137$ MeV

A breakup of ^{8}B should be taken into account in a reaction analysis.



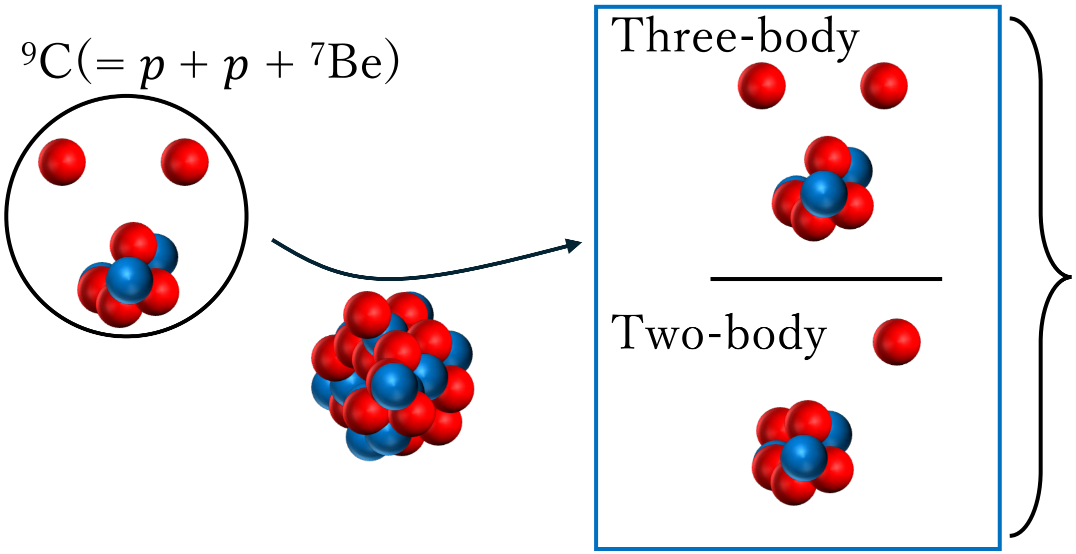
T. Fukui+, PRC **86**, 022801(R) (2012).

Analysis with the Four-body reaction model

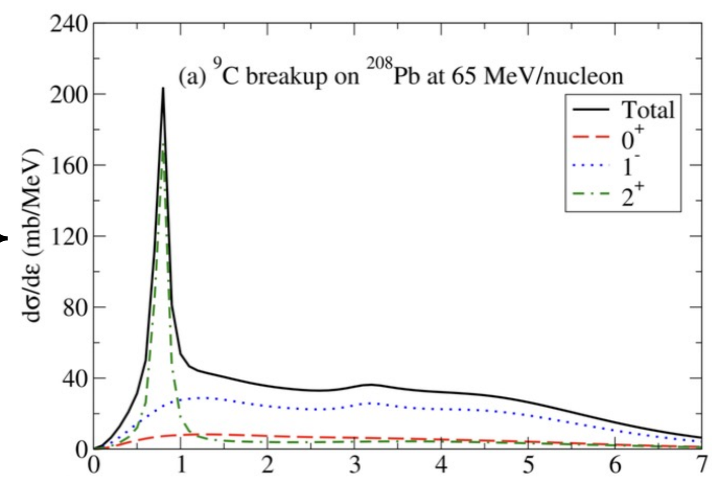
- **Four-body CDCC**
  - The reaction system is treated as $p + p + {}^{7}\mathrm{Be} + {}^{208}\mathrm{Pb}$.
  - The breakup cross section contains the two following channels:

$$ {}^{9}\mathrm{C} + {}^{208}\mathrm{Pb} \to p + {}^{8}\mathrm{B} + {}^{208}\mathrm{Pb} $$

$$ {}^{9}\mathrm{C} + {}^{208}\mathrm{Pb} \to p + p + {}^{7}\mathrm{Be} + {}^{208}\mathrm{Pb} $$



J. Singh+, PRC **104**, 034612 (2021).



- To compare with the experiment, we want to see the contribution of the two-body ($p + {}^{8}\mathrm{B}$) channel.
  - Separating contributions from each channel is difficult!

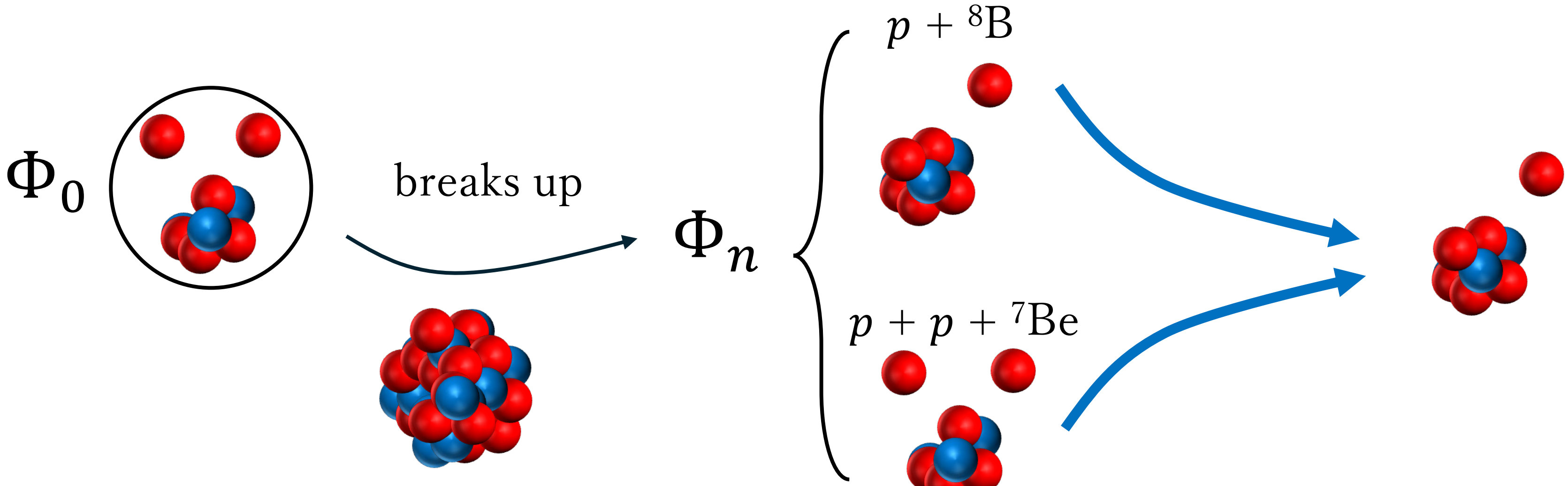
Cross section obtained with the four-body CDCC

- T matrix that the p+^{8}B channel is set to the final state.

$$T_\varepsilon = \sum_n \left\langle \underline{\Phi_\varepsilon^{(-)}} \middle| \Phi_n \right\rangle T_n^{\mathrm{4bCDCC}}$$

T_n^{4bCDCC}:
T matrix describing transition to a pseudo state Φ_n.

Three-body scattering w.f. of the $\boldsymbol{p}$ + $\boldsymbol{p}$ + ^{7}Be system, which the $\boldsymbol{p}$ + ^{8}B channel is set to the final state.



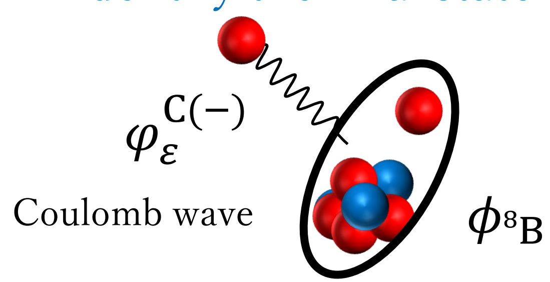
Smoothing of cross section

- Complex-Scaled solutions of Lippmann-Schwinger eq. (CSLS) Y. Kikuchi+, Phys. Rev. C **88**, 021602(R) (2013).

  - Lippmann-Schwinger eq. of the $p + p + {}^{7}\mathrm{Be}$ system

$$\langle \Phi_{\varepsilon}^{(-)} | = \langle \varphi_{\varepsilon}^{\mathrm{C}(-)} \phi_{^{8}\mathrm{B}} | + \lim_{\eta\to 0} \langle \varphi_{\varepsilon}^{\mathrm{C}(-)} \phi_{^{8}\mathrm{B}} | V \frac{1}{\varepsilon - h_{^{9}\mathrm{C}} + i\eta}$$

Identify the final state



  - Three-body scattering w.f. can be calculated with GEM+CSM.

$$\boxed{\text{Four-body CDCC + CSLS enables us to decompose the cross section.}}$$

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Naively, direct numerical calculation is difficult.

Complex scaling method (CSM)

$$\frac{1}{\varepsilon - h_{^9\mathrm{C}} + i\eta} = \sum_\nu U^{-\theta} |\Phi_\nu^\theta\rangle \frac{1}{\varepsilon - \varepsilon_\nu^\theta} \langle \tilde{\Phi}_\nu^\theta | U^\theta$$

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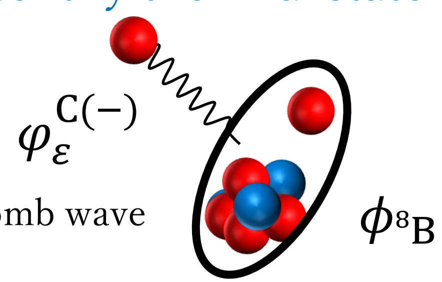

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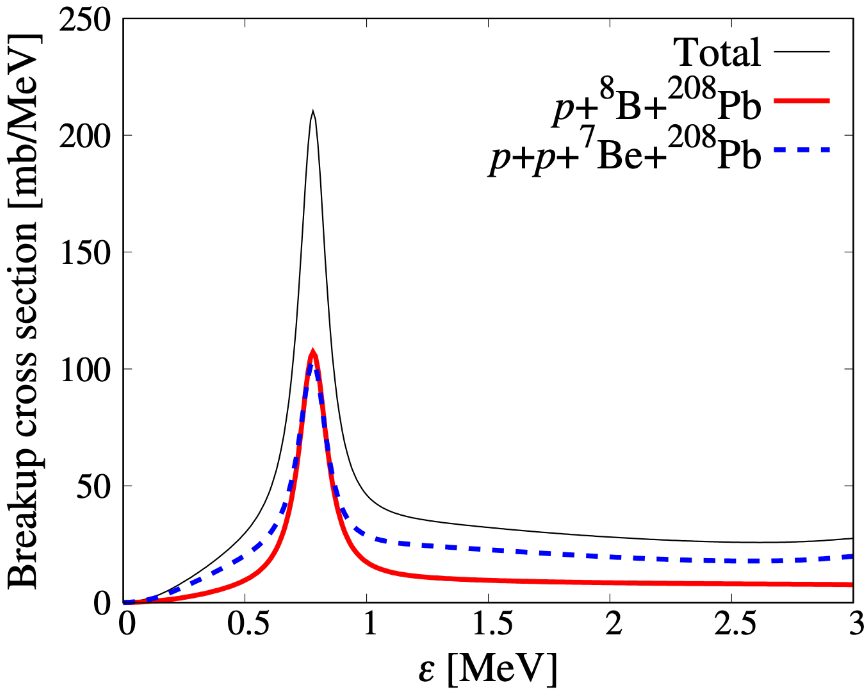
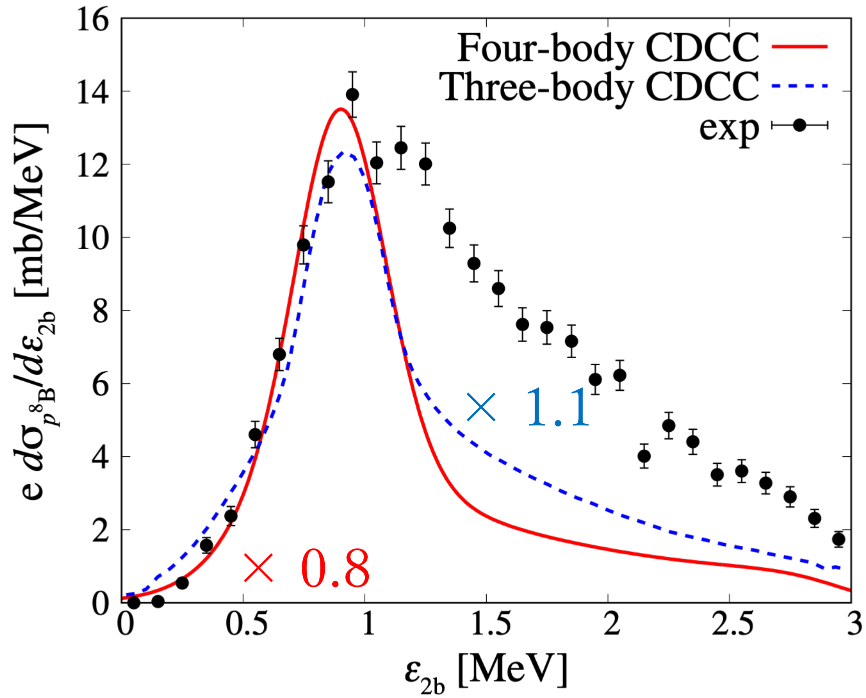
- <u>Three-body scattering w.f. can be calculated with GEM+CSM.</u>

Four-body CDCC + CSLS enables us to decompose the cross section.

^{9}C+^{208}Pb @ 65 MeV/A





- Decomposing the breakup cross section into the $\boldsymbol{p}$ + ^{8}B and $\boldsymbol{p}$ + $\boldsymbol{p}$ + ^{7}Be components.

Instead of Summary

1 **CDCC basics** Continuum discretization and coupled-channel equations

2 **^{6}He breakup** Resonant and nonresonant continuum contributions

SO and T. Matsumoto, Phys. Rev. C **102**, 021602(R) (2020).

3 **^{9}C breakup** Channel decomposition with CDCC + CSLS

SO+, Prog. Theor. Exp. Phys. **2026**, ptag096 (2026).

4 **Perspective: ^{7}Li + N** More particles system

5. Limitations and questions

CDCC as part of a larger reaction framework

What CDCC can do well

- Treat breakup and continuum coupling non-perturbatively.
- Use GEM bound and pseudo-state wave functions as reaction channels.
- Calculate elastic, inelastic, and breakup observables.
- Combine with CSM/CSLS to smooth and decompose continuum spectra.
- Provide a practical bridge between structure calculations and experiments.

Limitations of CDCC

- Reaction observables are affected by FSI and are not direct images of wave functions.
- Rearrangement channels are not fully included in standard CDCC.
- Target excitation and compound-nucleus-like processes are not included.
- Nonlocal or energy-dependent interactions are technically challenging.