

Building renormalizable very low-energy theory for few-body nuclear processes and bound states

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Towards high precision in multi-channel reactions of composite nuclei

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Thanks to my collaborators...

Betzalel Bazak, Nir Barnea, Mirko Bagnarol, Johannes Kirscher, Lorenzo Contessi, Ubirajara van Kolck, Rimantas Lazauskas, Jaume Carbonell, Matúš Rojik, Sourav Mondal

Introduction

Analyzing power A_y in low-energy N - d and p - ^3He elastic scattering

(A. Margaryan et al. Phys. Rev. C 93 (2016) 054001; L. Girlanda, Phys. Rev. C 99 (2019) 054003)

Isoscalar monopole resonance of ^4He (the first ^4He excited state)

(S. Bacca et al., Phys. Rev. Lett. 110 (2013) 042503; S. Kegel et al. Phys. Rev. Lett. 130 (2023) 152502)

→ discrepancy between experimental and theoretically predicted monopole transition form factor

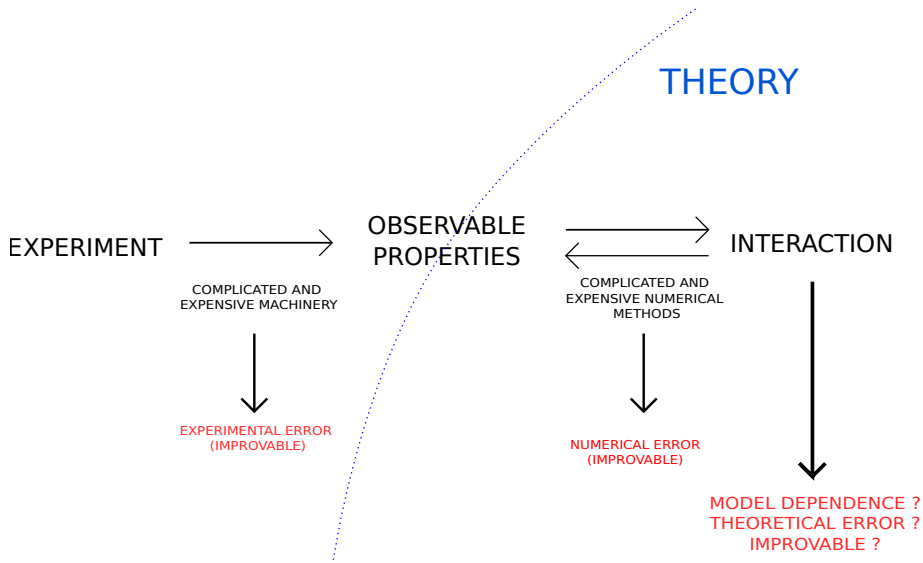
Splitting between $^2P_{3/2}$ and $^2P_{1/2}$ partial waves in $^4\text{He} + n$

(R. Lazauskas, Phys. Rev C 97 (2018) 044002; A. M. Shirokov et al., Phys. Rev. C 98 (2018) 044624)

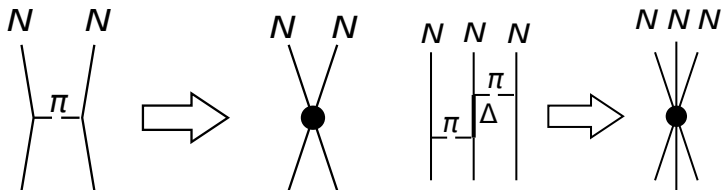
Big Bang Nucleosynthesis reactions

→ astrophysical S -factors of $d(d, p)^3\text{H}$ and $d(d, n)^3\text{He}$ reactions at very low energies (100 keV)

Introduction



$\nexists$ EFT - basic idea



Baryonic EFT :

→ no pionic degrees of freedom

$\not\sim$ EFT

LO



$$\delta(\mathbf{r}_{12}), \delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})$$

NLO



$$\overleftarrow{\nabla}_{\mathbf{r}_{12}}^2 \delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12}) \overrightarrow{\nabla}_{\mathbf{r}_{12}}^2, \delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})\delta(\mathbf{r}_{34})$$

 N^2LO 

$S - D$ tensor ($T = 0$), momentum dep. 3-body

 $N^3LO \quad \dots$

$(\nabla_{\mathbf{r}_1} \cdot \nabla_{\mathbf{r}_2})\delta(\mathbf{r}_{12})$, LS , tensor ($T = 1$), more 4-body ?

$\not\Lambda$ EFT

Where do we stand ?

- NLO $\not\Lambda$ EFT using 6 experimental constraints
 (a, r) of $NN(^1S_0)$ and $NN(^3S_1)$, $B(^3\text{H})$, $B(^4\text{He})$
 + 3 additional at NLO with non-perturbative Coulomb interaction
 (a, r) of $pp(^1S_0)$ and $B(^3\text{He})$
- perturbative treatment of higher-order corrections
 (development of both **interaction potentials** and **numerical few-body techniques**)

What do we want to study ?

- behavior of $\not\Lambda$ EFT predictions with increasing Λ
- comparison with experimental results

→ techniques developed for $\not\Lambda$ EFT can be also used in other renormalizable EFTs that demand perturbative corrections
 (nuclear EFTs with perturbative pions, RG-invariant versions of χ EFT, ...)

- breakdown scale $M = m_\pi$
- EFT tailored to describe nuclear processes/properties at very low typical momenta
- simplicity, clearer separation of different EFT orders, theoretical errors

Regularization/Renormalization

$$C \delta(\mathbf{r}_{ij}) \rightarrow C(\lambda) \left(\frac{\lambda}{2\sqrt{\pi}} \right)^3 e^{\frac{-\lambda^2 r_{ij}^2}{4}}$$
$$D \delta(\mathbf{r}_{ij})\delta(\mathbf{r}_{jk}) \rightarrow D(\lambda) \left(\frac{\lambda}{2\sqrt{\pi}} \right)^6 e^{\frac{-\lambda^2 (r_{ij}^2 + r_{jk}^2)}{4}}$$

- $C(\lambda), D(\lambda)$ are low energy constants (LECs) tuned to reproduce two-body resp. three-body observables for each λ
- required (RG invariance for $\lambda \gg M$)
→ all observable will become λ independent when $\lambda \rightarrow \infty$

$$O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \dots$$

⚡EFT potential up to NLO

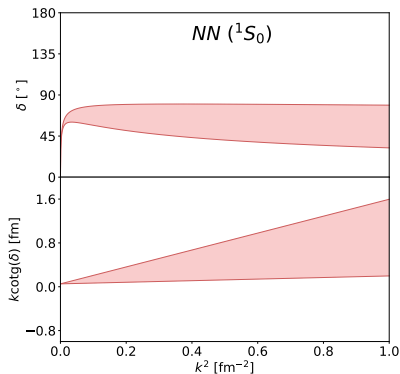
Leading order potential (3 LECs; iterated) :

$$V^{(\text{LO})} = \sum_{i < j} \left[C_0^{(0)}(\lambda) P_{ij}^{T=1, S=0} + C_1^{(0)}(\lambda) P_{ij}^{T=0, S=1} \right] e^{-\frac{\lambda^2}{4} r_{ij}^2} \\ + D_0^{(0)}(\lambda) \sum_{i < j < k} \mathcal{Q}_{ijk}^{T=1/2, S=1/2} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{ij}^2 + r_{jk}^2)}$$

Next-to-leading order potential (6 LECs; 1st order PT) :

$$V^{(\text{NLO})} = \sum_{i < j} \left[C_0^{(1)}(\lambda) P_{ij}^{T=1, S=0} + C_1^{(1)}(\lambda) P_{ij}^{T=0, S=1} \right] e^{-\frac{\lambda^2}{4} r_{ij}^2} \\ + \sum_{i < j} \left[C_2^{(1)}(\lambda) P_{ij}^{T=1, S=0} + C_3^{(1)}(\lambda) P_{ij}^{T=0, S=1} \right] (\mathbf{k}^2 + \mathbf{q}^2) e^{-\frac{\lambda^2}{4} r_{ij}^2} \\ + D_0^{(1)}(\lambda) \sum_{i < j < k} \mathcal{Q}_{ijk}^{T=1/2, S=1/2} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{ij}^2 + r_{jk}^2)} \\ + E_0^{(1)}(\lambda) \sum_{i < j < k < l} \mathcal{Q}_{ijkl}^{T=0, S=0} e^{-\frac{\lambda^2}{4} (r_{ij}^2 + r_{ik}^2 + r_{il}^2 + r_{jk}^2 + r_{jl}^2 + r_{kl}^2)}$$

LO ∇ EFT



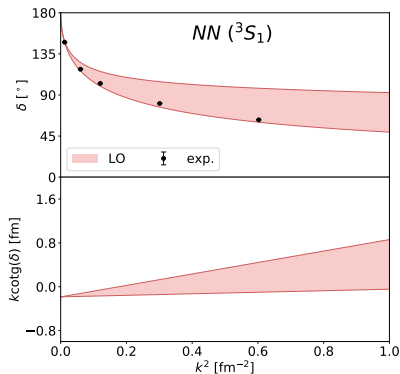
Leading order (LO) :

(exp. constraints)

$$a_{NN}^0 (a_{nn}^0) = -18.95(40) \text{ fm}$$

$$a_{NN}^1 (a_{np}^1) = 5.419(7) \text{ fm}$$

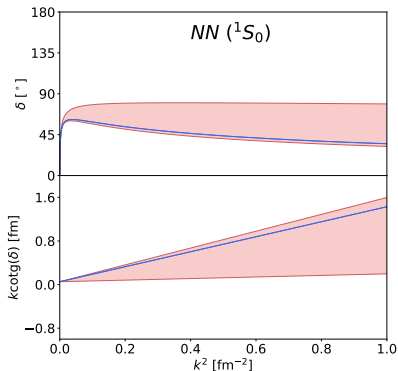
$$B(^3\text{H}) = 8.482 \text{ MeV}$$



Effective range expansion :

$$k \cotg(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \dots$$

NLO $\not\pi$ EFT



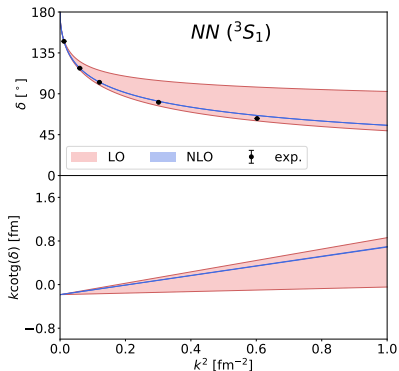
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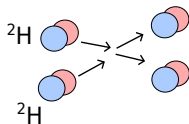
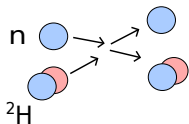
Next-to-leading order (NLO) :

(exp. constraints)

$$r_{NN}^0 (r_{nn}^0) = 2.75(11) \text{ fm}$$

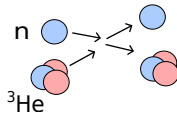
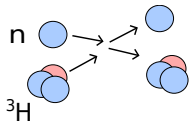
$$r_{NN}^1 (r_{np}^1) = 1.753(8) \text{ fm}$$

$$B(^4\text{He}) = 28.296 \text{ MeV}$$



Few-Body scattering

(elastic; no Coulomb interaction)



Correlated Gaussian basis

(K. Varga et al., NPA571 (1994) 447, K. Varga, Y. Suzuki, PRC52 (1995) 2885)

$$H\psi = E\psi, \quad \psi = \sum_{i=0}^N c_i \varphi^i$$

Basis states

- antisymmetrized correlated Gaussians
- here shown for $L = 0$; Global Vector representation for higher L s
(Y. Suzuki *et al.*, Few Body Syst 42 (2008) 33)

$$\varphi_{SM_S TM_T}^i(\mathbf{x}, A_i) = \mathcal{A}\{G_{A_i}(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_{A_i}(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A_i\mathbf{x}}$$

- Jacobi coordinates $\mathbf{x}$, A_i symmetric positive definite matrix of $\frac{N(N-1)}{2}$ real parameters, spin χ_{SM_S} and isospin η_{TM_T} parts

optimization of variational basis in a **random trial and error procedure**

Few-body scattering: BERW formula

→ assumption of short-range potential with range $R \ll b_{\text{HO}} = \sqrt{\frac{1}{\mu\omega}}$

Quantization condition (BERW formula)

$$(-)^{l+1} \left(\sqrt{4\mu\omega} \right)^{2l+1} \frac{\Gamma(3/4 + l/2 - \epsilon_n/2\omega)}{\Gamma(1/4 + l/2 - \epsilon_n/2\omega)} = k^{2l+1} \cotg(\delta), \quad k = \sqrt{2\mu\epsilon_n}$$

(A. Suzuki, Phys. Rev. A 80 (2009) 033601, T. Bush Found. of Phys. 28 (1998) 4)

LO :

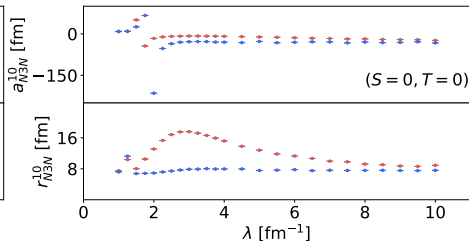
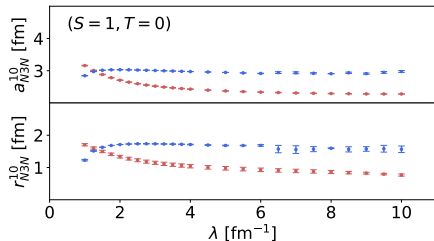
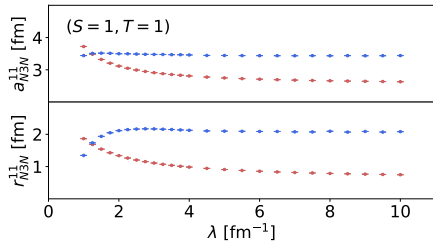
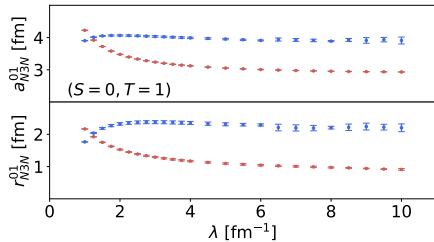
$$H(\omega) = T_k + V_\lambda^{(\text{LO})} + V_{\text{HO}}(\omega) \longrightarrow H(\omega)\psi_n = \epsilon_n\psi_n \xrightarrow{\text{BERW}} k \cotg(\delta)$$

NLO :

$$\epsilon_n^{(\text{NLO})} = \epsilon_n + \langle \psi_n | V_\lambda^{(\text{NLO})} | \psi_n \rangle \xrightarrow{\text{BERW}} k \cotg(\delta^{(\text{NLO})})$$

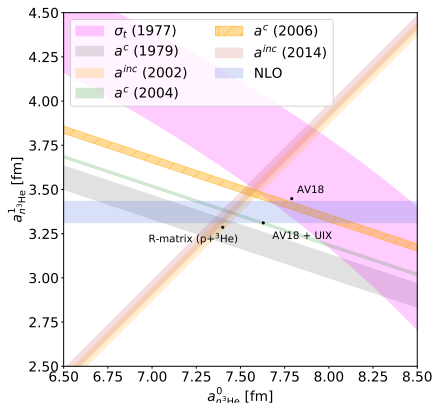
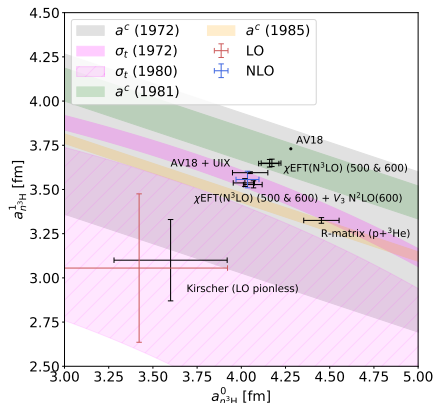
$n + {}^3\text{H}$ and $n + {}^3\text{He}$ scattering

(M. Schäfer, B. Bazak, Phys. Rev. C 107, 2023, 064001)



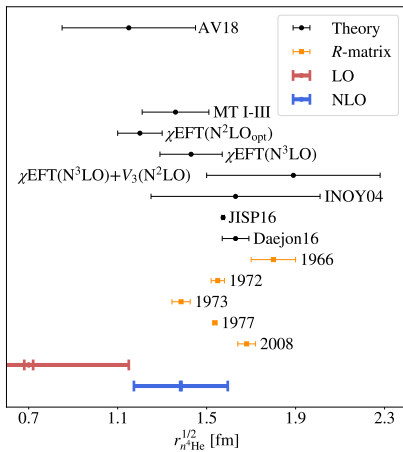
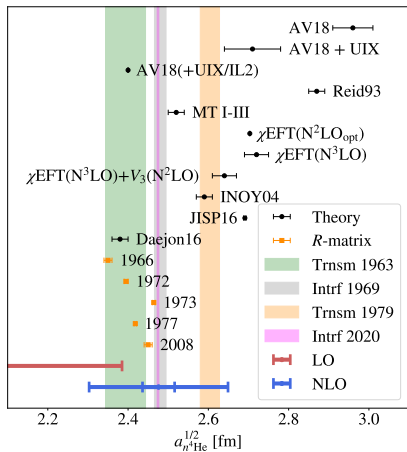
Experiment & Theory : $n + {}^3\text{H}$ and $n + {}^3\text{He}$ scattering lengths

(M. Schäfer, B. Bazak, Phys. Rev. C 107, 2023, 064001)

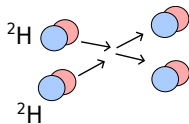
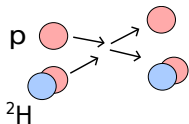


(Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002)

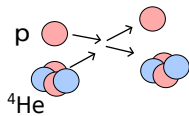
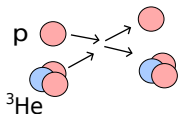
$n + {}^4\text{He}$ scattering (M. Bagnarol et al., Phys. Lett. B 844, 2023, 138078)



For references to all theoretical results, see (Phys. Lett. B 844 (2023) 138078).



Few-Body scattering with nonperturbative Coulomb interaction



~~EFT~~ potential with nonperturbative Coulomb interaction

- new pp 2-body cnt. force at LO
(Kong, Ravndal, Nucl. Phys. A 665 (2000) 137; Rupak, Kong, Nucl. Phys. A 717 (2003) 73)
- new ppn 3-body cnt. force at NLO
(J. Vanasse *et al.*, Phys. Rev. C 89 (2014) 064003; S. König *et al.*, J. Phys. G 42 (2015) 045101)

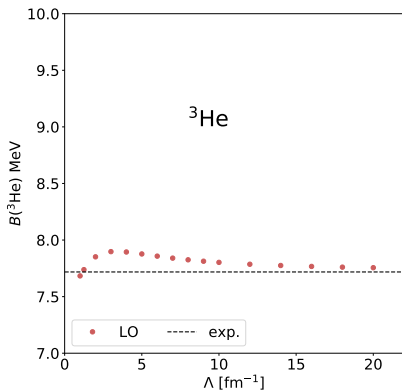
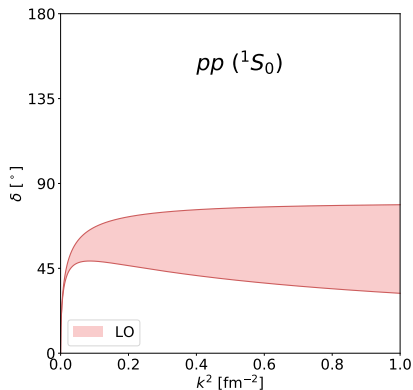
Additional leading order potential terms (**1 LECs; iterated**) :

$$V_C^{(\text{LO})} = \sum_{i < j} C_2^{(0)}(\lambda) P_{ij}^{T=1, S=0; pp} e^{-\frac{\lambda^2}{4} r_{ij}^2} + \frac{e^2}{r}$$

Additional next-to-leading order potential terms (**3 LECs; 1st order PT**) :

$$\begin{aligned} V_C^{(\text{NLO})} = & \sum_{i < j} P_{ij}^{T=1, S=0; pp} \left[C_4^{(1)}(\lambda) + C_5^{(1)}(\lambda)(\mathbf{k}^2 + \mathbf{q}^2) \right] e^{-\frac{\lambda^2}{4} r_{ij}^2} \\ & + D_0^{(1)}(\lambda) \sum_{i < j < k} Q_{ijk}^{T=1/2, S=1/2; ppn} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4} (r_{ij}^2 + r_{jk}^2)} \end{aligned}$$

LO $\neq$ EFT with nonperturbative Coulomb interaction

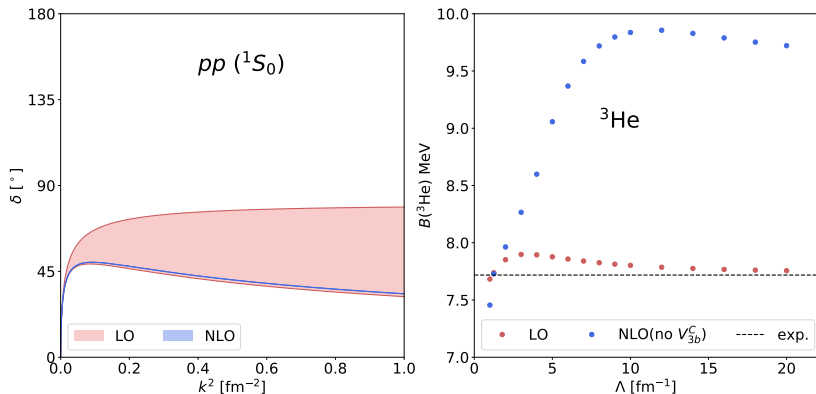


Additional LO parameters :

$$a_{pp}(^1S_0) = -7.8063(26) \text{ fm}$$

$$e^2/(\hbar c)^2 = 1.44 \text{ MeV}\cdot\text{fm}$$

NLO $\neq$ EFT with nonperturbative Coulomb interaction



Additional NLO parameters :

$$r_{pp}(^1S_0) = 2.794(14) \text{ fm}$$

$$B(^3\text{He}) = 7.718 \text{ MeV}$$

BERW formula for charged projectile and target

(M. Bagnarol et al., Phys. Lett. B 861 (2025) 139230)

Quantization condition for the HO trap energy spectrum

$$\det \left[\cot(\delta(E)) - \mathcal{F}^{\text{HO}}(E) \right] = 0$$

$$\mathcal{F}^{\text{HO}}(E) = \left(2\mu k^{2l+1} C_l^2(\eta) \right)^{-1} \lim_{r, r' \rightarrow 0} \frac{\Re \left[G_l^C(r, r'; E) \right] - G_l^{C, \omega}(r, r'; E)}{(rr')^l}$$

(P. Guo, Phys. Rev. C 103 (2021) 064611; H. Zhang et al. Phys. Lett. B 850 (2024) 138490)

$$[20\text{pt}] \quad C_l(\eta) = \frac{2^l}{(2l+1)!} |\Gamma(l+1+i\eta)| e^{-\frac{\pi}{2}\eta}; \quad \eta = q_1 q_2 \mu / k$$

$G_l^C(r, r'; E)$... Coulomb Green function

$G_l^{C, \omega}(r, r'; E)$... Coulomb + HO potential Green function

BERW formula for charged projectile and target

(M. Bagnarol et al., Phys. Lett. B 861 (2025) 139230)

→ developed new code to which connects $(E, \omega) \leftrightarrow (\delta, \omega)$

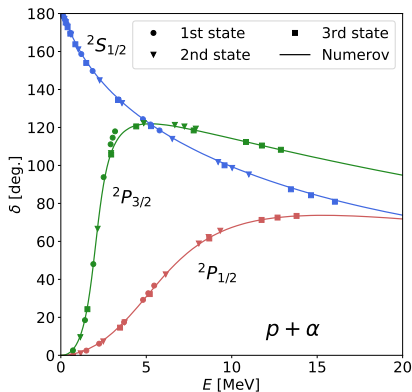
Limit approximation:

$$\frac{\Re [G_l^C(r_{\min}, r_{\min}, E)] - G_l^{C, \omega}(r_{\min}, r_{\min}, E)}{(r_{\min})^{2l}}$$

Numerical solution of Dyson equations:

$$\begin{aligned} G_l^C(r, r') &= G_l^{\text{free}}(r, r') + \\ &+ \int_0^{+\infty} dr'' G_l^{\text{free}}(r, r'') r''^2 V_C(r'') G_l^C(r'', r') \\ G_l^{C, \omega}(r, r') &= G_l^{\omega}(r, r') + \\ &+ \int_0^{+\infty} dr'' G_l^{\omega}(r, r'') r''^2 V_C(r'') G_l^{C, \omega}(r'', r') \end{aligned}$$

→ code tested on s - and p -wave $p + \alpha$ scattering

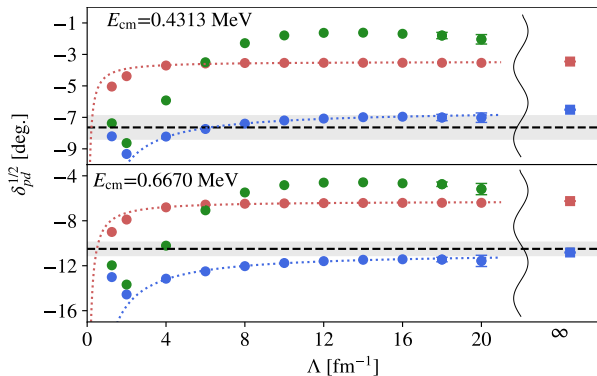


s -wave $p + d$ scattering

(M. Rojik et al., arXiv:2507.16250 [nucl-th] 2025)

• experimental phase shifts from pd PSA

(M. H. Wood et al., Phys. Rev. C 65 (2002) 0334002)



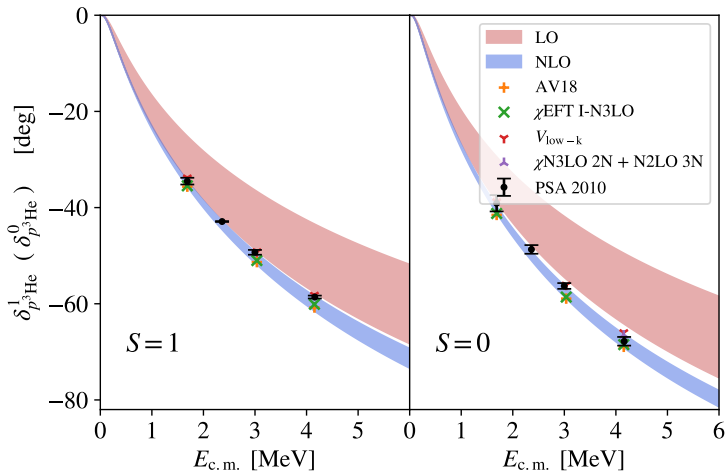
Inclusion of the NLO ppn force leads to a larger cutoff dependence in this channel. (J. Vanasse et al., Phys. Rev. C 89 (2014) 064003)

s -wave $p+{}^3\text{He}$ scattering

(M. Rojik et al., arXiv:2507.16250 [nucl-th] 2025)

• experimental data from $p^3\text{He}$ phase-shift analysis

(T. V. Daniels et al., Phys. Rev. C 82 (2010) 034002)



Perturbative corrections to few-body observables

(S. Mondal et al., arXiv:2509.17366 [nucl-th] 2025)

Numerically feasible methods to calculate perturbative corrections to few-body observables

$$\langle \hat{O} \rangle = O^{(0)} + \delta O^{(1)} + \delta O^{(2)} + \dots$$

Introducing multipliers:

$$\hat{H}[\alpha, \beta] |\psi\rangle = (\hat{T} + \hat{V}^{(0)} + \alpha \cdot \hat{V}^{(1)} + \beta \cdot \hat{O}) |\psi\rangle = E |\psi\rangle$$

→ solving Schrödinger equation on the (α, β) grid

→ energies and wave functions are functions of α and β

One-multiplier method:

$$\langle \hat{O} \rangle \stackrel{(I)}{=} O_n(\alpha; \beta = 0) \stackrel{(II)}{=} O_n^{(0)} + \sum_{i=1} \alpha^i \delta O_n^{(i)}$$

Perturbative corrections to few-body observables

(S. Mondal et al., arXiv:2509.17366 [nucl-th] 2025)

Introducing multipliers:

$$\hat{H}[\alpha, \beta] |\psi\rangle = (\hat{T} + \hat{V}^{(0)} + \alpha \cdot \hat{V}^{(1)} + \beta \cdot \hat{O}) |\psi\rangle = E |\psi\rangle$$

Two-multiplier method:

$$E_n \stackrel{(I)}{=} E_n(\alpha, \beta) \stackrel{(II)}{=} E_n^{(0,0)} + \sum_{\substack{i,j=0 \\ i \neq j=0}}^N \alpha^i \beta^j \delta E_n^{(i,j)}$$

→ applying Feynman-Hellmann theorem

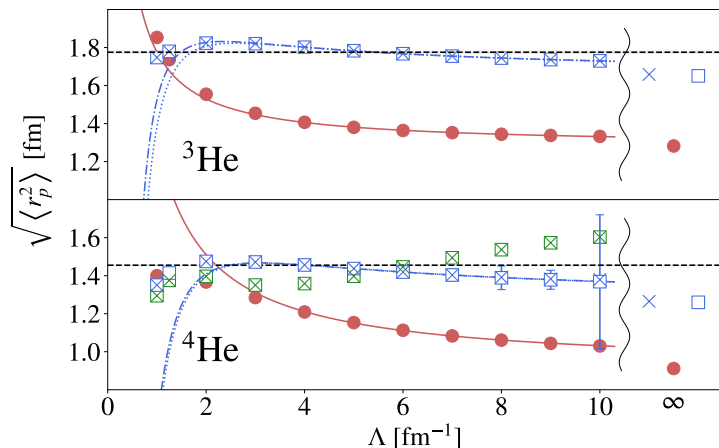
$$\frac{\partial E_n(\alpha, \beta)}{\partial \beta} = \left\langle \frac{\partial \hat{H}[\alpha, \beta]}{\partial \beta} \right\rangle = \hat{O}$$

relates the expansion coefficients linear in β , $\delta E(i, 1)$, to the i th perturbation order of the observable

$$O_n^{(0)} + \sum_{i=1}^N \alpha^i \delta O_n^{(i)} = \delta E_n^{(0,1)} + \sum_{i=1}^N \alpha^i \delta E_n^{(i,1)}$$

Perturbative corrections to few-body observables

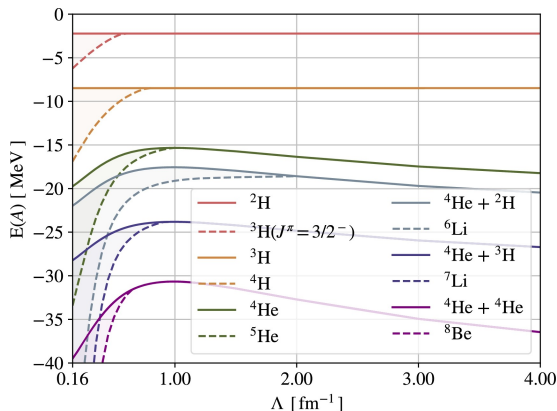
(S. Mondal et al., arXiv:2509.17366 [nucl-th] 2025)



crosses - one-multiplier method
squares - two-multipliers method

LO $\neq$ EFT calculations of several p -shell nuclei up to ^8Be

(M. Schäfer, L. Contessi, J. Kirscher, and J. Mareš, Phys. Lett. B 816 (2021) 136194)



I. Stetcu, B. R. Barrett, and U. van Kolck, Phys. Lett. B 653 (2007) 358

L. Contessi *et al.*, Phys. Lett. B 772 (2017) 839

W. G. Dawkins, J. Carlson, U. van Kolck, and A. Gezerlis, Phys. Rev. Lett. 124 (2020) 143402

Intermezzo

Conjecture based on the numerical evidence :

Beyond s -shell nuclear systems break in the zero-range limit into p , n , ${}^2\text{H}$, ${}^3\text{H}/{}^3\text{He}$, and ${}^4\text{He}$. (free nucleon(s) + spherically-symmetric fragments)

Option 1 :

LO shallow continuum pole; RG invariant; can be mutated into a bound state with perturbative insertions of sub-leading operators.

Option 2 :

Identical as above; nonperturbative mechanism required to move the pole into a bound-state region.

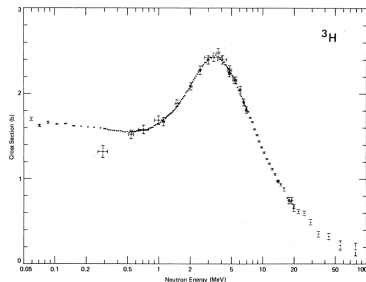
Option 3 :

No LO RG invariant continuum pole in the EFT's convergence radius; must be created nonperturbatively by modifying the LO.

^4H resonances

Experimental evidence :

- $n^3\text{H}$ elastic cross-sections
(Ann. Phys. 74 (1972) 250; Phys. Rev. C 22 (1980) 384)
- π^- absorption experiments
(Phys. Lett. B 103 (1981) 409; Nucl. Phys. A 531 (1991) 613)
- transfer reactions
(Phys. Rev. C 33 (1986) 2204; Nucl. Phys. A 719 (2003) C229)
- extracted resonance positions significantly differ



Theory :

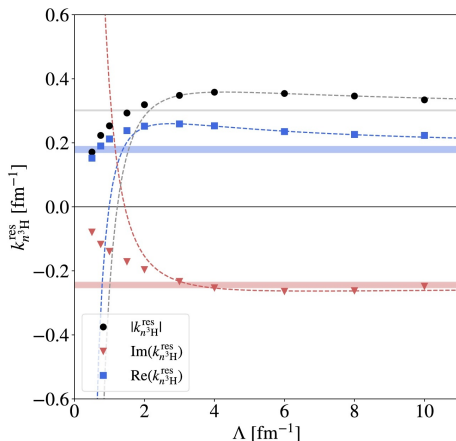
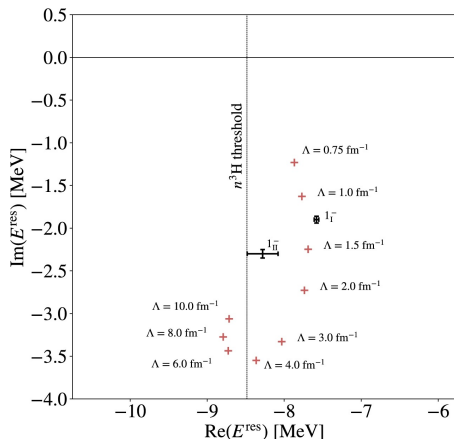
- RGM with complex scaling
(Phys. Rev. C 68 (2003) 034303)
- Spin-dipole strength functions
(Phys. Rev. C 87 (2013) 034001)
- Faddeev-Yakubovsky equations
(Phys. Lett. B 791 (2019) 335)
- No-core Gamov Shell Model
(Phys. Rev. C 104 (2021) 024319)

J^π	E_r [MeV]	Γ [MeV]
2^-	1.15(5)	3.97(7)
1^-_I	0.90(3)	3.80(8)
0^-	0.78(15)	7.7(7)
1^-_{II}	0.2(2)	4.6(1)

INOY; (Phys. Lett. B 791 (2019) 335)

Emergence of ${}^4\text{H}$ $J^\pi = 1^-$ resonance in LO $\nrightarrow$ EFT

(L. Contessi, M. Schäfer, J. Kirscher, R. Lazauskas, J. Carbonell, Phys. Lett. B 840 (2023) 137840)

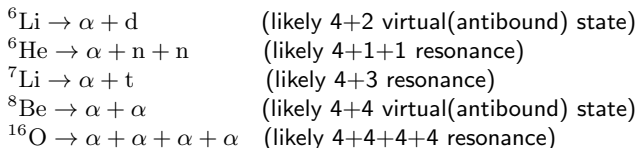


$$k_{\text{res}} = \sqrt{2\mu\tilde{E}_{\text{res}}}, \quad \tilde{E}_{\text{res}} = E_{\text{res}} - E_t$$

Summary

- Beyond s -shell nuclear systems break in the zero-range limit into p , n , ${}^2\text{H}$, ${}^3\text{H}/{}^3\text{He}$, and ${}^4\text{He}$ (conjecture based on the numerical evidence)
- Suprisingly, the same type of instability for ${}^6\text{Li}$ and ${}^{16}\text{O}$ found using RG-invariant power-counting scheme at LO in χEFT (Phys. Rev. C 103 (2021) 054304)
- Numerical evidence of **stabilization of ${}^4\text{H } J^\pi = 1^-$ resonance pole with increasing momentum cutoff**

→ We can speculate that in heavier systems:



Improved action on (no only) nuclear EFTs

L. Contessi, M. Schäfer, A. Gnech, A. Lovato, U. van Kolck, arXiv:2505.09299 [nucl-th]

L. Contessi, M. Schäfer, U. Van Kolck, Phys.Rev.A 109 (2024) 022814

L. Contessi, M. P. Valderrama, U. Van Kolck, Phys Lett B 856 (2024) 138903

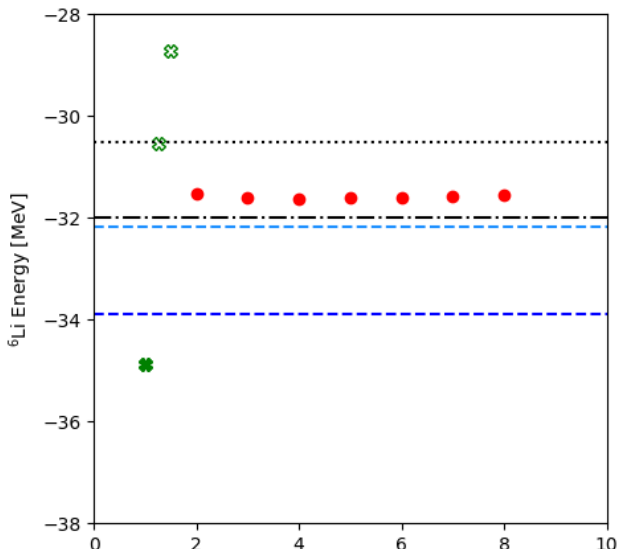
Starting at LO with a certain "fake" finite range to sustain the bound states

$$\begin{aligned}
 \tilde{V}_{2B}^{(0)}(x) &= \tilde{C}_{0,s}^{(0)} \sum_{i < j} \mathcal{P}_{ij,s} \delta_{(xR_s)-1}(\vec{r}_{ij}) \\
 &+ \tilde{C}_{0,t}^{(0)} \sum_{i < j} \mathcal{P}_{ij,t} \delta_{(xR_t)-1}(\vec{r}_{ij}), \\
 \tilde{V}_{3B}^{(0)}(x) &= \tilde{D}^{(0)} \sum_{i < j < k} \sum_{\text{cyc}} \delta_{(xR_3)-1}(\vec{r}_{ij}) \delta_{(xR_3)-1}(\vec{r}_{jk}).
 \end{aligned}$$

- higher-order perturbative corrections remain unchanged
- "fake" range must be sufficiently small so that it is reabsorbed at higher-orders
- power-counting remains unchanged

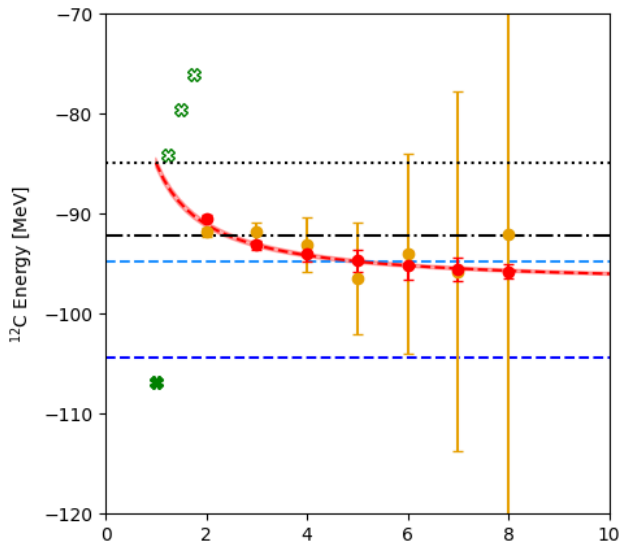
Improved action: ${}^6\text{Li}$

L. Contessi, M. Schäfer, A. Gnech, A. Lovato, U. van Kolck, arXiv:2505.09299 [nucl-th]



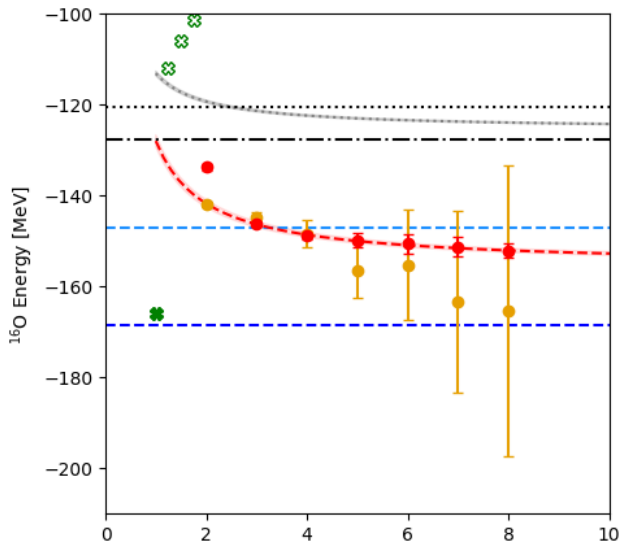
Improved action: ^{12}C

L. Contessi, M. Schäfer, A. Gnech, A. Lovato, U. van Kolck, arXiv:2505.09299 [nucl-th]



Improved action: ^{16}O

L. Contessi, M. Schäfer, A. Gnech, A. Lovato, U. van Kolck, arXiv:2505.09299 [nucl-th]



Questions

How to assess *a priori* typical momenta in nuclear reactions ? (too high for EFT to converge with increasing order)

Which experimental data one should choose to fit parameters (LECs) of the EFT?

On the few-body level the cutoff can not be always varied up to arbitrary high value due to numerical reasons. How we should then assess renormalizability of higher-order EFT corrections to establish the power-counting?

How to accurately evaluate higher-order perturbative corrections in few-body nuclear reactions? Which reaction technique is the most convenient to use?