The Trouble With Nuclear Physics

In fact the trouble in the recent past has been a surfeit of different *models* [of the nucleus], each of them successful in explaining the behavior of nuclei *in some situations*, and each in *apparent contradiction with other successful models* or with our ideas about nuclear forces.

Rudolph E. Peierls: "The Atomic Nucleus", *Scientific American* **200 (1959)**, no. 1, p. 75; emphasis added

Model: Capture *some* aspects with lots of data – no "fail" but "tuned". Cargo Cult mode.

Theory: Comprehensive, prescriptive, predictive, accurate, Explain-All-To-Some-Degree. – *Can fail.*

There is no principle built into the laws of Nature that says that theoretical physicists have to be happy.

S. Weinberg on NOVA: THE FABRIC OF THE COSMOS – Episode 3: Quantum Leap (2011)

Contact-EFT of Nuclear Interactions:
Basics, Features, Pitfalls, Bugs, Questions

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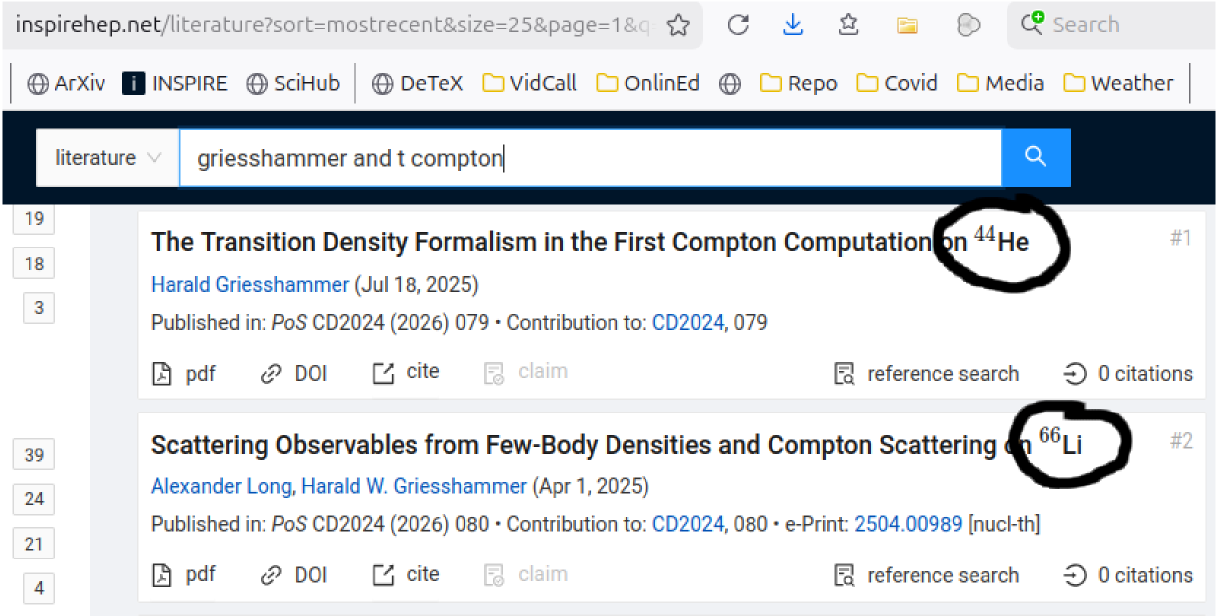
How to root Nuclear Physics in QCD?
Which constituents rule nucleons and nuclei at low energies?
How do nuclei react?
How does that serve our understanding?
How to plan effective experiments & test theory?

Biased Review of Community Effort.

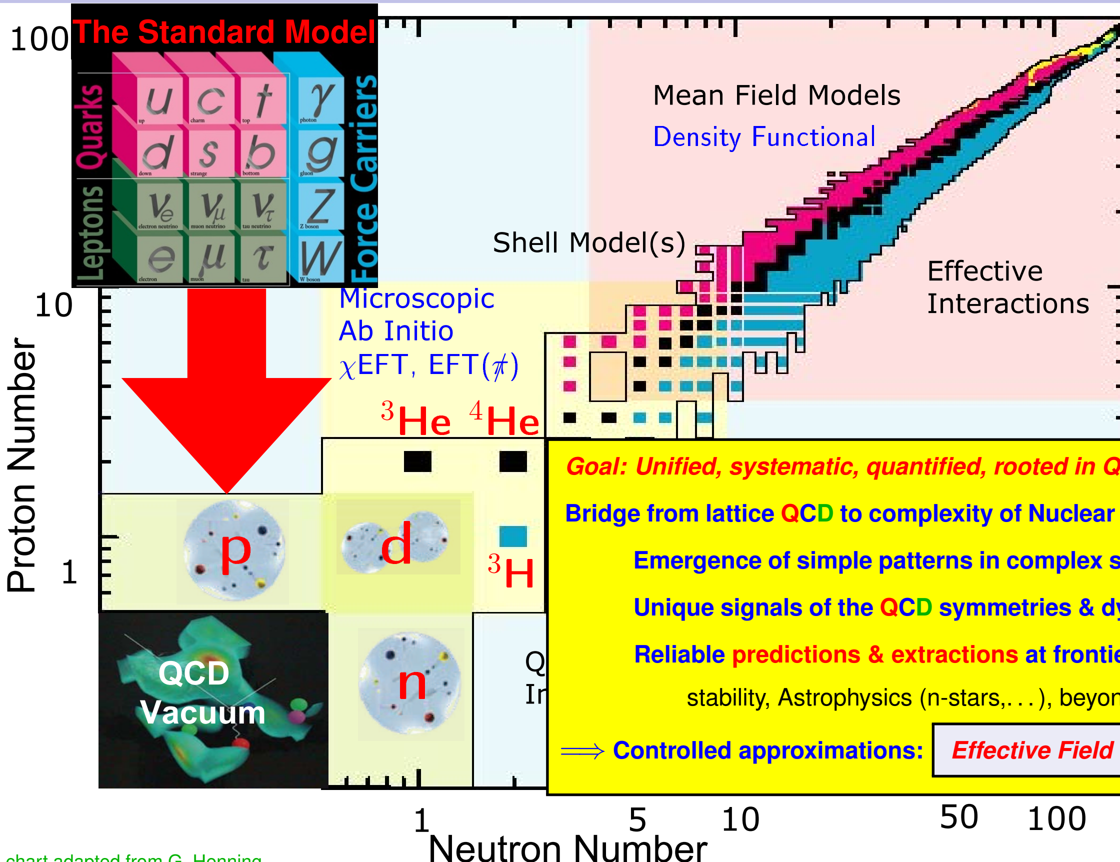
Despite What INSPIRE Says, This is Not A Pattern We See



http://inspirehep.net/literature?sort=mostrecent&size=25&page=1&q=griesshammerandtcompton&ui-citation-summary=true

1. The Goals of Modern Nuclear Physics

Nuclear/Hadronic Long-Range Plans: NSAC 2023, NAS 2012
NuPECC 2024, OECD 2008, STFC-NPA (UK) 2009



Goal: Unified, systematic, quantified, rooted in QCD.

Bridge from lattice QCD to complexity of Nuclear Physics:

Emergence of simple patterns in complex systems?

Unique signals of the QCD symmetries & dynamics;

Reliable predictions & extractions at frontiers:

stability, Astrophysics (n-stars,. . .), beyond-SM,. . .

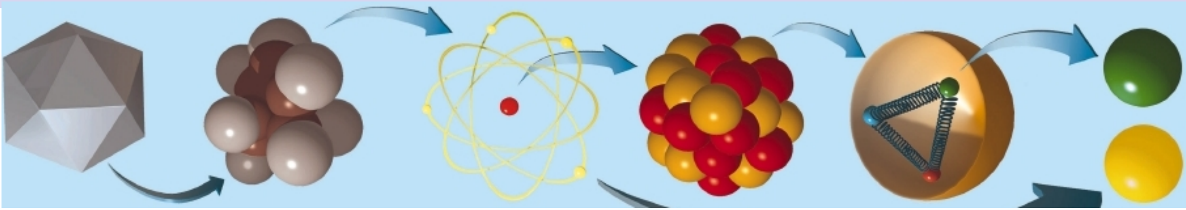
$\Longrightarrow$ **Controlled approximations:** ***Effective Field Theories***

chart adapted from G. Henning

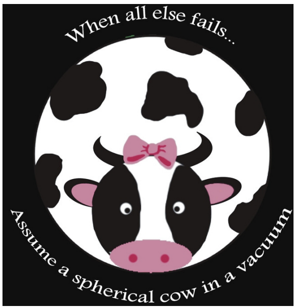
2. EFT Platitudes

(a) Why It Works: $\Delta x\,\Delta p \gtrsim \hbar$, or What You See Is What You Get

Weinberg: “folk lore theorem”



To probes with wavelength λ, object of size R appears	point-like for $\lambda \gg R$,	blurry for $\lambda \gtrsim R$,	composed for $\lambda \lesssim R$.



EFT Tenet: **Short-distance physics does not have to be right for a good calculation, because a low-energy process cannot probe details of the high-energy structure.**

$\Longrightarrow$ ***Effective Field Theories***

Identify those degrees of freedom and symmetries which are *appropriate* to resolve the *relevant* Physics at the scale of interest.

Systematic approximation of real world with **estimate of theoretical uncertainties.**

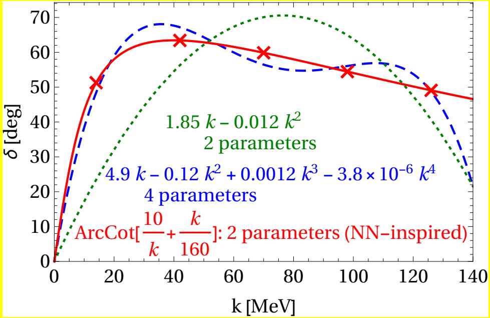
Information-Theory Aspect of the EFT Promise:

Encode information about unresolved short-range at given resolution and *at given order* into smallest number of independent CTs:

minimal set of parameters for *lossless compression*:

No simpler description at this accuracy.

$\Longrightarrow$ Falsifiability; robust predictions to uncover new Physics, Alternative Worlds, hidden symmetries (unitarity, large-N_c Schindler/Springer 2018-,...).



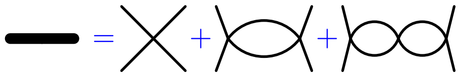
Anomalous NN scatt. lengths
$\Longrightarrow$ Separation of Scales: $Q = \frac{\text{target size } \approx m_\pi^{-1} \approx 1 \text{ fm}}{\text{typ. resolution } a \approx 5 \text{ fm}} \simeq \frac{1}{3\dots5}$

"Pion-less" EFT

2-Body Sector: Extend **Effective Range Expansion** to include external currents, relativity,...; well-understood.

2 "binding momenta"

LO (Q^{-1}, $\lesssim 30\%$) :



$= -\frac{4\pi}{M}\frac{1}{\gamma + ik}$

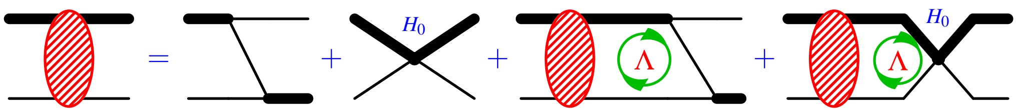
$\gamma_{s,t} = \frac{1}{a_{s,t}}$

NLO (Q^0, $\lesssim 10\%$) :

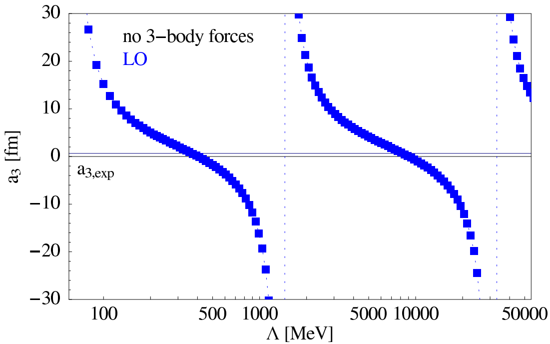
$= -\frac{4\pi}{M}\frac{1}{\gamma+ik}\frac{(k^2+\gamma^2)\,\rho}{2}\frac{1}{\gamma+ik}$

2 effective ranges $\rho_{s,t}$

3-Body Sector:



Thomas Effect (1935): *Without 3N interaction*, attractive $\frac{1}{r^2}$-potential $\Longrightarrow$ wave-function collapse to $r \to 0$.



Insensitivity to short-range details *requires*
k-indep. 3NI to absorb cutoff-dependence in observables.

By *Renormalisability*, not by data or values of $a_{s,t}$.

LO & NLO ($\lesssim 10\%$ accuracy): One new, dimensionful parameter Λ_*, scale set e.g. by ^{3}H binding energy.

Efimov effect 1971: Tower of 3N states $\frac{B^{(n)}}{B^{(n+1)}} \approx 22.7$.

(d) Ordering of Few-N Interactions in EFT($\not\pi$)

2N: Bethe 1949; Kaplan/Savage/Wise 1997; Chen/Rupak/S 1998;. . .
3N: Bedaque/Hammer/van Kolck 1999; B/Rupak/H/hg 2003; hg 2005
4N: Platter/. . . 2005; Bazak/Kirscher/König/. . . 2019

	LO ($\lesssim 30\%$?)	NLO ($\lesssim 10\%$??)	N^{2}LO ($\lesssim 4\%$???)	N^{3}LO ($\lesssim 2\%$????)
2N	$^1\mathrm{S}_0$, $^3\mathrm{S}_1$: $C_S(N^\dagger N)^2, C_T(N^\dagger\vec{\sigma}N)^2$ 2 param.	$^1\mathrm{S}_0$, $^3\mathrm{S}_1$: $p^2 C_2$ $+2$ param.	$^1\mathrm{S}_0$, $^3\mathrm{S}_1$: $p^4 C_4 \propto (p^2 C_2)^2$ $+0$ param.: driven by C_2	$^1\mathrm{S}_0$, $^3\mathrm{S}_1$: $p^4 C_4$ $+2$ param.
			$^3\mathrm{SD}_1$ mixing: $\vec{p}\otimes\vec{p}\; C_2^{SD}$ $+1$ param.	$^3\mathrm{P}_{0,1,2}$, $^1\mathrm{P}_1$ $\vec{p}\cdot\vec{p}'\, C_\mathrm{P}$ $+4$ param.
3N	$^2\mathrm{S}_{\frac{1}{2}}$: $\widehat{=}$ $H_0(N^\dagger N)^3$ $+1$ param.	—	$^2\mathrm{S}_{\frac{1}{2}}$: $p^2 H_2$ Wigner-inv. only $+1$ param.	N$^{3.1}$LO: $^2\mathrm{S}_\mathrm{WS}$-$^4\mathrm{D}_\mathrm{WA}$ N$^{3.5}$LO: $^2\mathrm{P}_\mathrm{WA}$, $^4\mathrm{P}$ **fractional promotion** $+$ up to 3 param.
4N	—	S: $G_0(N^\dagger N)^4$ $+1$ param.	**likely none??**	**unknown, likely ≥ 1: $p^2\, G_2$**
$\geq$5N	—	Pauli principle: $p^2\ (N^\dagger N)^5$ etc. believed to be very high order.		—
	total 3 param.	total 6 param.	total 8 param.	total 14-18 param.

Must add isospin breaking?!

(a) EFTs Can Go Wrong: Check Fundamental Building Blocks – Hidden Or Not

Expand observables as $\mathcal{O} = c_0 + c_1\,Q^1 + c_2\,Q^2 + \dots$

with $Q = \dfrac{\text{typ. momentum } p_{\text{typ.}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} < 1.$

– Incorrect usage: $p_{\text{typ.}} \nearrow \overline{\Lambda}_{\text{EFT}} \implies Q \not\ll 1$?

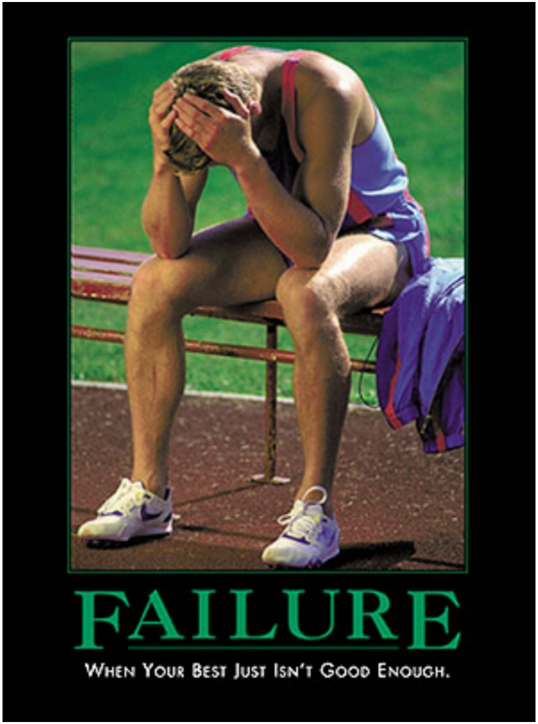
$\implies$ Each EFT points to a more-fundamental EFT.

"EFTs carry seed of own destruction." D. R. Phillips

The Assumptions:

- ***Workable*** **Separation of Scales**
- ***Appropriate*** **Degrees of Freedom**
- ***Relevant*** **Symmetries at These D.o.F's and Scales**
- ***Consistent*** **Ordering Scheme**

Different choices lead to different EFTs, even with same symmetries and degrees of freedom.



(b) EFT Uses Obvious and Hidden Symmetries *Relevant* at Given Scale

Gauge, Galilei $\longrightarrow$ Lorentz: relativistic corrections "should" be irrelevant.

Isospin weakly broken: ${}^1\mathrm{S}_0\ a_{\mathrm{nn}} \neq a_{\mathrm{np}} \neq a_{\mathrm{pp}}, \ \dfrac{M_{\mathrm{n}}-M_{\mathrm{p}}}{M_{\mathrm{n}}+M_{\mathrm{p}}} \approx 0.3\%.$ $\Longrightarrow$ Relevant in "sensitive" processes?!

A Hidden Symmetry: Wigner-$\mathrm{SU}(4)$

of Combined Spin-Isospin Rotations

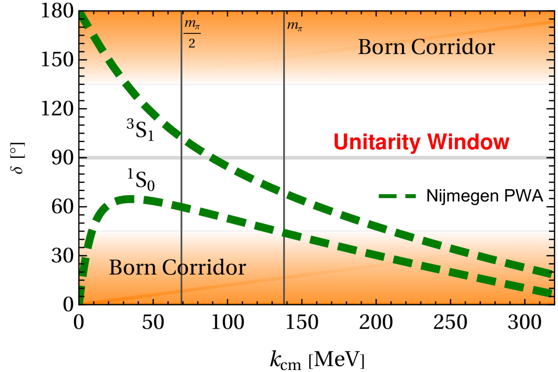
$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix} \to U \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

Wigner, Hund 1937
for heavy nuclei
cf. Mehen/Stewart/Wise 1999

NN: ${}^1\mathrm{S}_0$-${}^3\mathrm{S}_1$ Wigner-super-sextuplet if $a({}^3\mathrm{S}_1) = a({}^1\mathrm{S}_0)$: $\quad \dfrac{4\pi}{M}\dfrac{1}{-\frac{1}{a} - \mathrm{i}k} = A_{\mathrm{NN}}({}^3\mathrm{S}_1) = A_{\mathrm{NN}}({}^1\mathrm{S}_0)$

EFT($\not\pi$): The only possible LO, NLO, N^{2}LO 3NI $(p^0,p^2)(\mathrm{N}^\dagger\mathrm{N})^3$ & 4NI $(p^0,p^2)(\mathrm{N}^\dagger\mathrm{N})^4$ are Wigner-invariant.

Theorists love Unitarity Limit as Nontrivial Fixed Point
characterised by high symmetry:
Wigner-$\mathrm{SU}(4)+$ scale-invariance
close to FP protected in renormalisation.
Nature: Small breaking $\dfrac{1/a({}^3\mathrm{S}_1) - 1/a({}^1\mathrm{S}_0)}{m_\pi \sim 1/\rho} \approx 0.3$
in perturbation where phase shifts large.



(c) Some Literature: Mathematical Perturbation Theory and Applications

Incomplete, biased, self-centred.

[1] C. M. Bender and S. A. Orszag: Advanced Mathematica Methods for Scientists and Engineers. McGraw-Hill 1978.

[2] J. M. Murdock: Perturbations: Theory and Methods. Wiley 1991.

[3] F. W. Byron and R. W. Fuller: Mathematics of Classical and Quantum Physics. Dover 1992.

[4] H. W. Grießhammer, *Technical Note: Strict Perturbation Theory for Nonperturbative Algorithms*, distributed in response to the ECT* workshop New Ideas in Constraining Nuclear Forces 2018, (being) updated July 2026, available from hgrie@gwu.edu .

[5] J. Vanasse, Phys. Rev. C **88** (2013) no.4, 044001 [arXiv:1305.0283].

[6] Y. P. Teng and H. W. Grießhammer, Eur. Phys. J. A **61** (2025) no.9, 211 [arXiv:2410.09653].

[7] A. J. Andis, S. Lyu, B. Long and S. König, [arXiv:2512.12823].

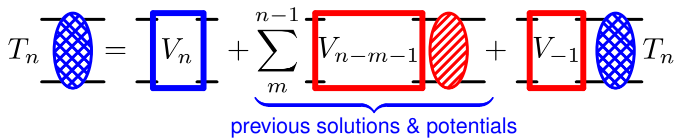
(d) Perturbations About Non-Perturbative LO

Corrections in $Q \ll 1$ by "strict perturbation" about LO *amplitude* (Distorted-Wave Born; efficient).

Vanasse [1305.0283]; hg notes 2018-21; Shi/Long/…[2205.02000]; Thim/Ekström/Forssén [2402.15325]; Andis/Liu/Long/König [2512.12823];…

Claim: Every perturbative solution recycles the LO method, just with different ingredients. ($\rightarrow$ Friday)

Example: iterative N^{n}LO in NN Lippmann-Schwinger



Pragmatic Features:

– **Operator Structures only**: $V_n = \sum_i C_i \widehat{V}_{ni} \implies$ Coefficients C_i separate in $T_n = \sum_i C_i T_{ni}$, T_{ni} solved using $\widehat{V}_{ni}$.

$\implies$ Tames increasingly complex operator structures spin$\otimes$isospin$\otimes$position. $\implies$ Less CPU time (emulators?!).

$\implies$ **Linear fit** of higher-order parameters, re-fitting lower orders as $C_n = C_n^{\text{LO}} + C_n^{\text{NLO}} + C_n^{\text{N}^2\text{LO}} + \dots$.

$\oplus$ **Order-by-Order for free**. $\ominus$ **Low-order solution feeds into high-order problem.** $\implies$ **Numerical stability?!**

Contrast to Popular "Partially-Resummed Perturbation": Power-count (un-physical) V_{NN} & iterate. Weinberg 1990

$$T = \frac{V_{\text{LO}} + V_{\text{NLO}} + \dots}{1 - (V_{\text{LO}} + V_{\text{NLO}} + \dots)\, G_{NN}}, \qquad \text{e.g. } \mathcal{A}_{\text{NN}} \propto \frac{1}{\gamma - \frac{\rho}{2}k^2 - v_2 k^4 \dots + ik}$$

$\implies$ Mixes orders. – Nonlinear parameter fits. – New effort at each order.

$\implies$ Changes UV, obscures PC in observables, less control over convergence. $\implies$ Renormalisation Tests??

Unphysical poles around $\overline{\Lambda}_{\text{EFT}}$, e.g. NN N^{3}LO: $[170 \pm 100\text{i}]$ MeV with neg. residues. $\implies$ Causality wrong.

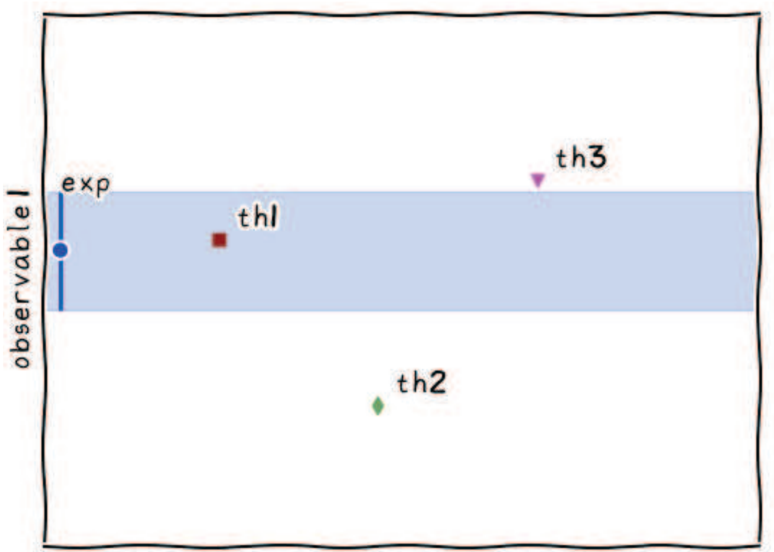
$\implies$ Limited to small cutoff variation range $\Lambda \approx \overline{\Lambda}_{\text{EFT}} \pm 20\%$, numerics more difficult, beware of missing CTs!

Works *under assumption that* expansion indeed small, e.g. $1 + x + x^2 + x^3 - \dfrac{1}{1-x} = x^4 + \mathcal{O}(x^5)$ *if* $|x| < 1$.

Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

Need procedure which is established, economical, reproducible: room to argue about "error on the error".

"Double-Blind" Theory Errors: Assess with pretense of no/very limited data.



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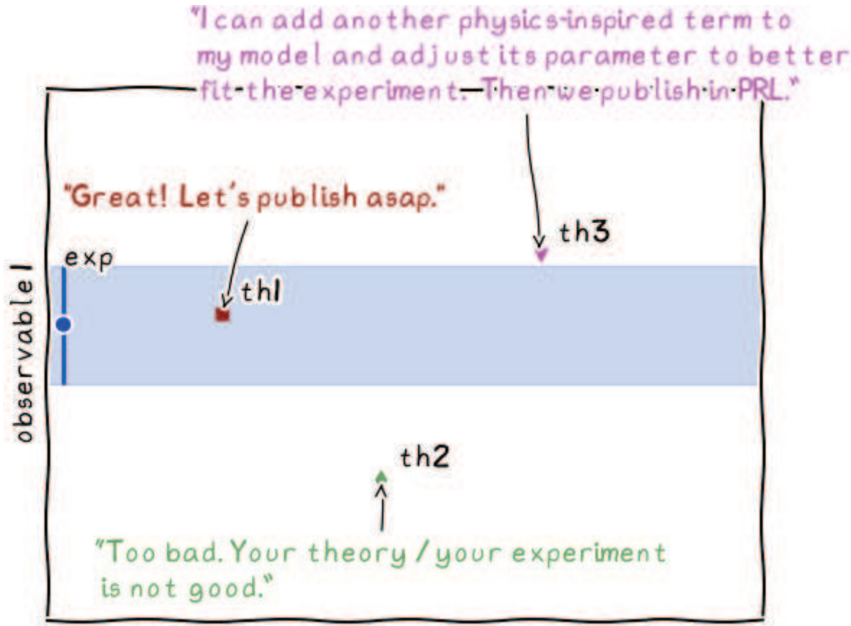
favourite theorist reaction until ca. 2020



(e) Why Theory Error Bars Are Relevant

Ch. Forssén, idea: V. Nazarewicz

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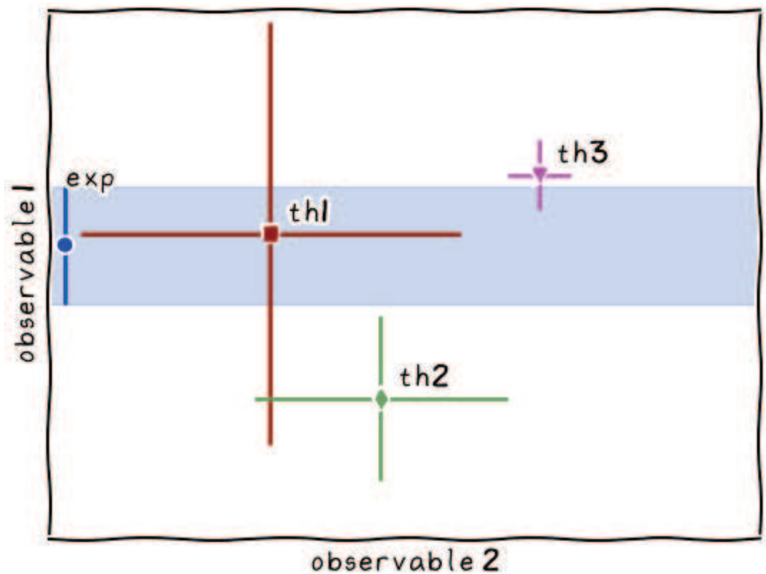
theories with uncertainties $\Longrightarrow$ falsifiable



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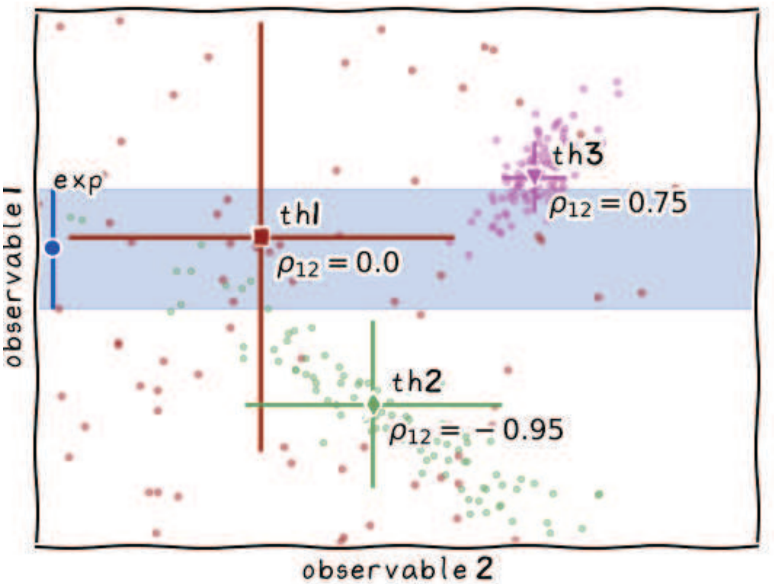
mindful of correlations:

theory 1: low accuracy, imprecise
theory 2: mild tension with data of observable 1
theory 3: high-ish accuracy, strong tension with **theory 2**



(f) From Orders To Accuracies: Testing Assumptions, Building Confidence

Workable Separation of Scales with *Appropriate* Degrees of Freedom

Order in small, dimension-less expansion parameter $Q = \dfrac{\text{typ. low momenta } p_{\text{low}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} \ll 1.$

If convergence fails, wrong degrees of freedom possible: Yes, We Don't Have Pions!

$\Longrightarrow$ What is expansion parameter? Folk Lore: $Q_{\not\pi} \approx \dfrac{1}{3} \overset{???}{\ll} 1$, and *increases* with p_{low}!

Suggestions of typ. low momenta p_{low} in A-body system/$A_1 + A_2$-processes:

– "Equi-distribute momenta between nucleons" $\gamma_A \sim \sqrt{\dfrac{2M_{\text{N}} B_A}{A}} \approx \begin{cases} 70\ \text{MeV} & A=3 \\ 110\ \text{MeV} & A=4 \\ 120\ \text{MeV} & A\to\infty \end{cases}$ König/hg/Hammer/van Kolck [1607.04623]

– "Resonances of Valence Nucleons": $\gamma_A \sim \sqrt{2M_{\text{N}} B_A^{\text{min}}}$.

– "Exchange momentum": $\gamma_A \sim |\vec{q}|$: typical momentum exchange.

Suggestions of breakdown scale $\overline{\Lambda}_{\not\pi}$

– Strict analyticity: One-Pion Exchange cut starts at $\dfrac{m_\pi}{2} \approx 70\ \text{MeV}$.

– Pragmatic "somehow pion Physics": m_π.

– Result-informed: EFT($\not\pi$) works better than one expects, even up to $k \sim 200\ \text{MeV}$. SWAVE PLOTS

Order-by-Order Convergence: Bayesian Uncertainty Estimates with Reasonable Assumptions

EFT($\not\pi$) N^{2}LO total NN and Nd cross sections: $\overline{\Lambda}_{\not\pi} \approx [140 \pm 30]\ \text{MeV} \approx m_\pi$. Ekström/Platter [2507.08700]

(g) Cutoff-Dependence: Curse and Blessing

Which order 2NI/3NI/4NI?: Naïve Dimensional Analysis, or earlier if needed for Renormalisation-Group Invariance.

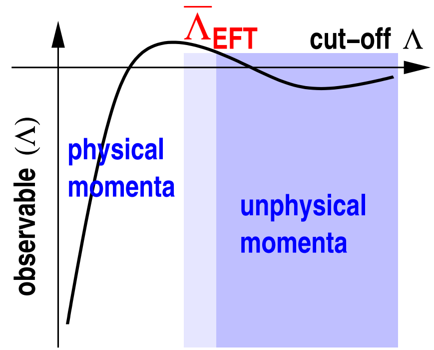
Few-Nucleon EFTs: Analytic results rare (even in EFT($\not\pi$)).

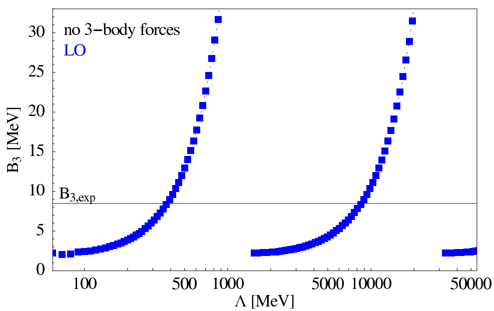
$\Longrightarrow$ ***Unphysical* cutoff parameter/regulator** Λ: if in momentum/coordinate space, often confused with

$\Longleftrightarrow$ ***Physical* breakdown scale** $\overline{\Lambda}_{\text{EFT}}$: $Q = \dfrac{p_{\text{typ}}}{\overline{\Lambda}_{\text{EFT}}} < 1$: scale of new Physics.

"Cutoff Democracy": Any cutoff $\Lambda \gtrsim \overline{\Lambda}_{\not\pi}$ is equally valid as it probes momenta beyond range where description makes sense.

"Be insensitive to momenta $q \gtrsim \overline{\Lambda}_{\text{EFT}}$ at which EFT is not supposed to work/unphysical."





$\Longrightarrow$ Scan in $\Lambda \in [\overline{\Lambda}_{\text{EFT}}; \infty[$ estimates higher-order corrections (lower bound, often too small).

3NI needed at LO: Efimov effect!

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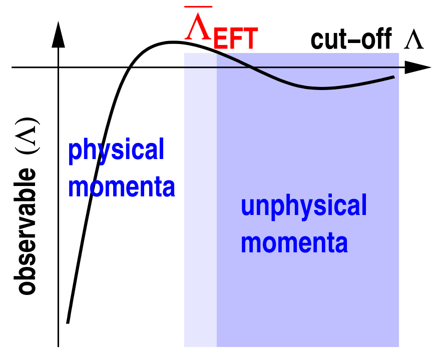

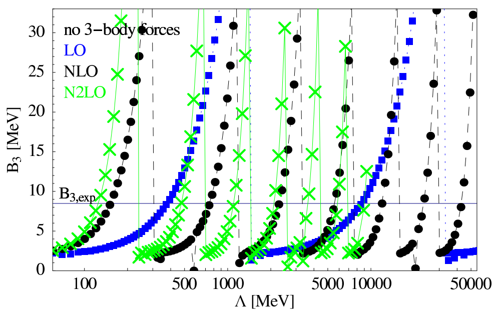

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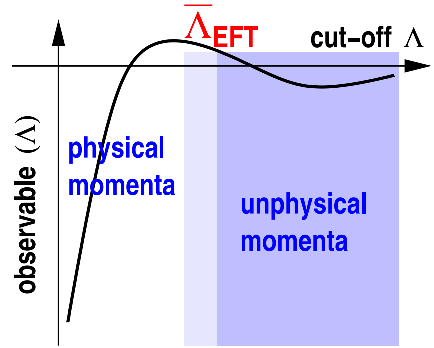

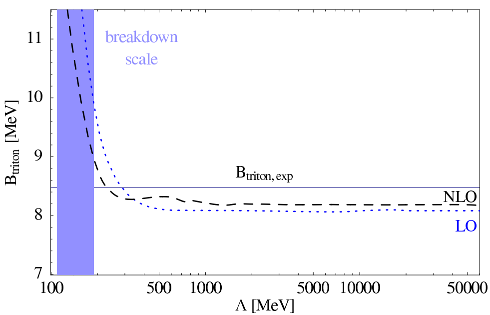

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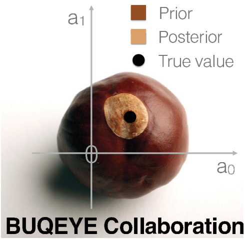
(h) Quantifying Theory Truncation Uncertainties: “This Is The (Bayesian) Way”

N^{k-1}LO Observable as series: $\mathcal{O}^{(k)} = c_0 + c_1\delta^1 + c_2\delta^2 + \cdots + \text{unknown} \times \delta^k . \Longrightarrow \mathcal{O}^{(k)} \pm \delta^k \max\{|c_i|\}$?



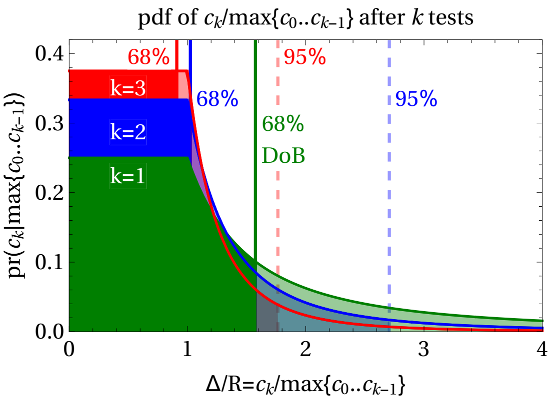
No infinite sampling pool; data fixed; more data changes confidence.

Call upon the Reverend Bayes for probabilistic interpretation!

New information increases level of confidence.

$\Longrightarrow$ **Smaller corrections, more reliable uncertainties.**

Priors clearly state your assumptions – including *Naturalness.*



Priors: all c_n “equally likely”, “any upper bound” $\bar{c}$.

order	in $\pm c_{\max}$	$\Delta^{(k)}(68\%)$	$\Delta^{(k)}(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\, c_{\max}$	$11 c_{\max} = 7\,\Delta_{68}^{(1)}$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\, c_{\max}$	$2.7 c_{\max} = 2.6\,\Delta_{68}^{(2)}$
N^{2}LO	$\frac{3}{4} = 75\%$	$0.9\, c_{\max}$	$1.8 c_{\max} = 1.9\,\Delta_{68}^{(3)}$
N^{k-1}LO k terms	$\frac{k}{k+1}$	$0.68\,\frac{k+1}{k}\, c_{\max} (k \geq 2)$	
Gauß	68.27%	$1.0\, c_{\max}$	$2.0\,\Delta_{68}^{(k)}$

Laplace’s Law of Succession: $\text{pdf}(c_k > c_{\max}) = \dfrac{1}{k+1}$.

$\Longrightarrow$ **Quantified theory truncation uncertainties as 68% *DoB interval*** $[\mathcal{O}^{(k)} - \Delta_{68}^{(k)}; \mathcal{O}^{(k)} + \Delta_{68}]$.

Posterior pdf *not Gauß’ian:* Plateau & “fat” (power-law) tail.

(i) Parameterising and Calibrating Theory

Parameters $\widehat{=}$ interaction strengths $\widehat{=}$ Low-Energy Coefficients encode short-distance Physics.

$\Longrightarrow$ Compute from underlying theory (QCD) or determine from data.

Any determination ok, as long as within range of validity of EFT. All converge to same result with increasing order.

Also used for technical issue: bound systems which are unbound at LO. $\to$ Martin, Matúš

Define your Goal (cf. yesterday):

Ab-initio "explanation":
Nuclear Physics "Bottom Up" from NN system,
as little few-N input as possible (3NI, 4NI,...).

Best prediction of few-N observables:
Calibrate on bound/scatt. $A \geq 4$ systems
to avoid "trivial" correlations in observables.

Example: Asymptotic deuteron wave-function

$$\Psi(r \to \infty) = \sqrt{\frac{\gamma Z}{2\pi}}\,\frac{\mathrm{e}^{-\gamma r}}{r}, \qquad \gamma = \sqrt{M_\mathrm{N} B_2}$$ "binding momentum", eff. range ρ.

wave-function norm $\widehat{=}$ residue of deuteron pole:

$$Z = 1.690(3) = \frac{1}{1-\gamma\rho} = 1 + \gamma\rho + (\gamma\rho)^2 + (\gamma\rho)^3 + \cdots = 1 + 0.409 + 0.167 + 0.068 + \ldots$$

Z for deuteron properties: Correct strength; strong correlation to charge radius,..., but NN scatt. converges slowly.

ρ for NN scattering; but deuteron properties converge slowly and strongly correlated.

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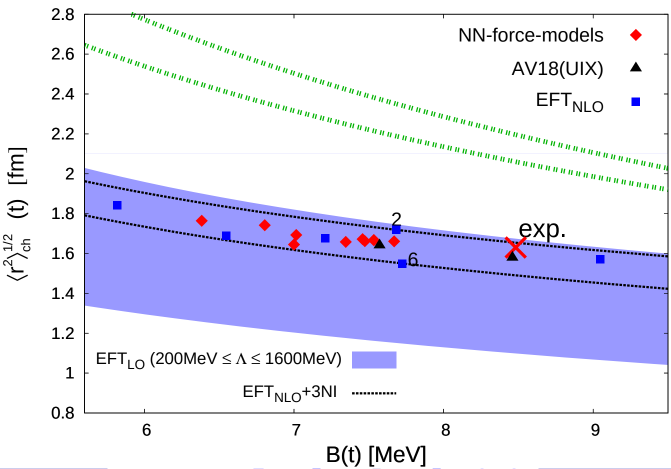
Example Correlation ^{3}H binding–charge radius from Efimov effect in EFT($\not\pi$) Kirscher/hg/Hofmann/Shukla 2009



(j) Ordering of Few-N Interactions in EFT($\not\pi$)

2N: Bethe 1949; Kaplan/Savage/Wise 1997; Chen/Rupak/S 1998;. . .
3N: Bedaque/Hammer/van Kolck 1999; B/Rupak/H/hg 2003; hg 2005
4N: Platter/. . . 2005; Bazak/Kirscher/König/. . . 2019

	LO ($\lesssim 30\%$?)	NLO ($\lesssim 10\%$??)	N^{2}LO ($\lesssim 4\%$???)	N^{3}LO ($\lesssim 2\%$????)
2N	${}^1\mathrm{S}_0$, ${}^3\mathrm{S}_1$: $C_S(N^\dagger N)^2, C_T(N^\dagger \vec{\sigma} N)^2$ 2 param.	${}^1\mathrm{S}_0$, ${}^3\mathrm{S}_1$: $p^2 C_2$ $+2$ param.	${}^1\mathrm{S}_0$, ${}^3\mathrm{S}_1$: $p^4 C_4 \propto (p^2 C_2)^2$ $+0$ param.: driven by C_2	${}^1\mathrm{S}_0$, ${}^3\mathrm{S}_1$: $p^4 C_4$ $+2$ param.
			${}^3\mathrm{SD}_1$ mixing: $\vec{p} \otimes \vec{p}\; C_2^{SD}$ $+1$ param.	${}^3\mathrm{P}_{0,1,2}$, ${}^1\mathrm{P}_1$ $\vec{p} \cdot \vec{p}'\, C_\mathrm{P}$ $+4$ param.
3N	${}^2\mathrm{S}_{\frac{1}{2}}$: $\widehat{=}$ $H_0(N^\dagger N)^3$ $+1$ param.	—	${}^2\mathrm{S}_{\frac{1}{2}}$: $p^2 H_2$ Wigner-inv. only $+1$ param.	N$^{3.1}$LO: ${}^2\mathrm{S}_{\mathrm{WS}}$-${}^4\mathrm{D}_{\mathrm{WA}}$ N$^{3.5}$LO: ${}^2\mathrm{P}_{\mathrm{WA}}$, ${}^4\mathrm{P}$ **fractional promotion** $+$ up to 3 param.
4N	—	S: $G_0(N^\dagger N)^4$ $+1$ param.	**likely none??**	**unknown, likely ≥ 1: $p^2\, G_2$**
$\geq$5N	—	Pauli principle: $p^2\; (N^\dagger N)^5$ etc. believed to be very high order.		—
	total 3 param.	total 6 param.	total 8 param.	total 14-18 param.

Must add isospin breaking?!

4. Concluding Follies: Propositions

What Is The Goal?: Reproduction vs. prediction.

A convincing theory describes convincingly (within quantified uncertainties) the known systems & reactions to build trust in predictions of somewhat related but un-explored/un-explorable systems/reactions.

Motivation: Reactions on composite nuclei important to understand nucleo-synthesis, stellar evolution, terrestrial fusion,... usually in region inaccessible to lab experiments.

Expectations and Correlations: Use e.g. $d + {}^3\text{H} \to n + {}^4\text{He}$ as benchmark: If not predicted, then at least establish correlation between ${}^5\text{He}^*$ resonance position & width.

Invest in credible assessment of theoretical truncation uncertainties:

Order-by-order convergence, cutoff-dependence,... $\Longrightarrow$ Strict Perturbation after LO.

Relevance: Match data uncertainties, predict measurements with likely biggest impact on unknowns, indirect signals of resonance,...

Apply same techniques to ${}^{12}\text{C}+{}^{12}\text{C} \to {}^{24}\text{Mg}^* \to {}^{20}\text{Ne}+{}^4\text{He}$ or ${}^{23}\text{Ne}+\text{p}$:

Predict or at least identify the likely most-attainable, highest-impact measurements for future planning.

Question of Rôle of Wigner-$SU(4)$ Symmetry and Broken Isospin Symmetry...

Question of Scales in Composite Systems:

e.g. $E({}^5\text{He}) - E(dt) \approx 0.05 \text{ MeV} \lesssim T = E_{\text{cm}} \ll B_2 = 2.2 \text{ MeV} \ll E_{\text{excitation}}({}^3\text{H}) \ll E({}^4\text{He}) = 24 \text{ MeV}$:
No simple, *a-prior* answer which is (most) relevant.

Bayesian *a-posteriori* analysis of order-b-order convergence can reveal $Q = \dfrac{p_{\text{typ}}}{\overline{\Lambda}_{\not\pi}} \Longrightarrow p_{\text{typ}}, \overline{\Lambda}_{\not\pi}$.

The efficient person gets the job done right. The effective person gets the right job done.