Kohn-Luttinger effects for superfluid nuclear matter
— Renormalization group approach

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Reference:
Y. Fujimoto, Phys. Rev. B 111, 184510 (2025), arXiv:2502.01169 [cond-mat.supr-con].

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Outline

1. Kohn-Luttinger superconductivity

2. BCS instability from renormalization group

3. Possible application to superfluid neutron matter

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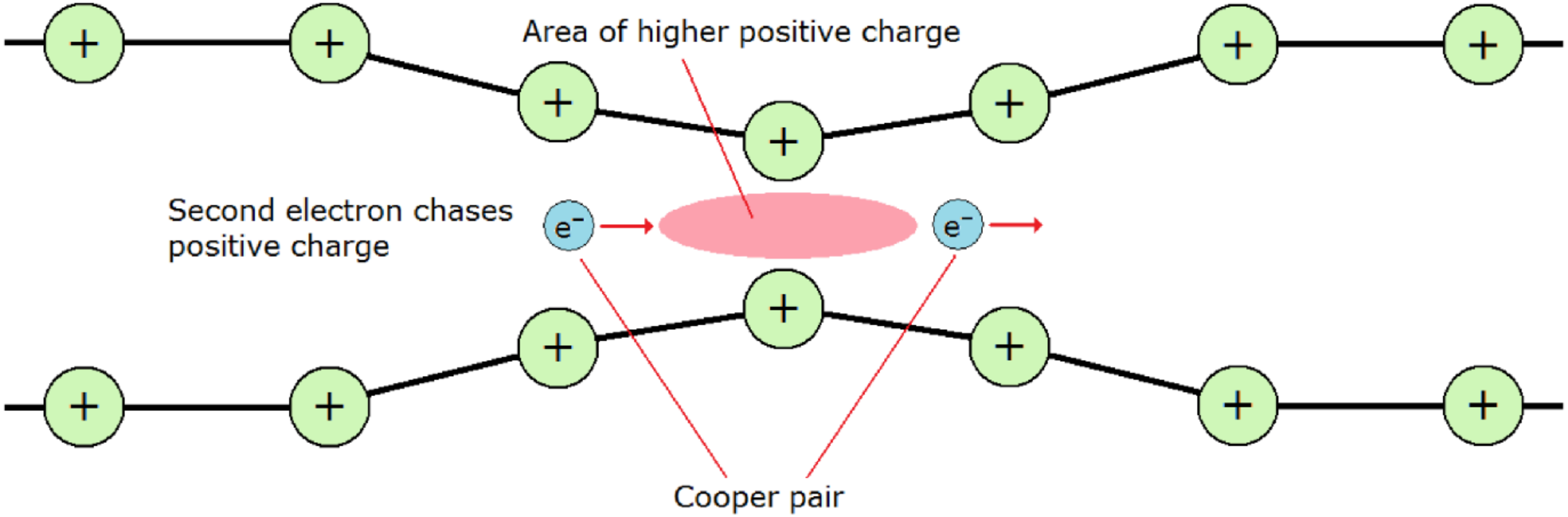
Cooper pairs in conventional superconductivity

Cooper (1956); Bardeen,Cooper,Schrieffer (1957)

BCS theory: superconductivity is induced by macroscopic Cooper pair condensate

Cooper pairing: arbitrary weak attraction is capable of producing bound state

Pairing mechanism in conventional superconductor:



Two electrons feels attraction via phonons, quanta of lattice vibration

Superconductivity from repulsive interaction?

Coulomb interaction is repulsive. Is Cooper pairing possible?

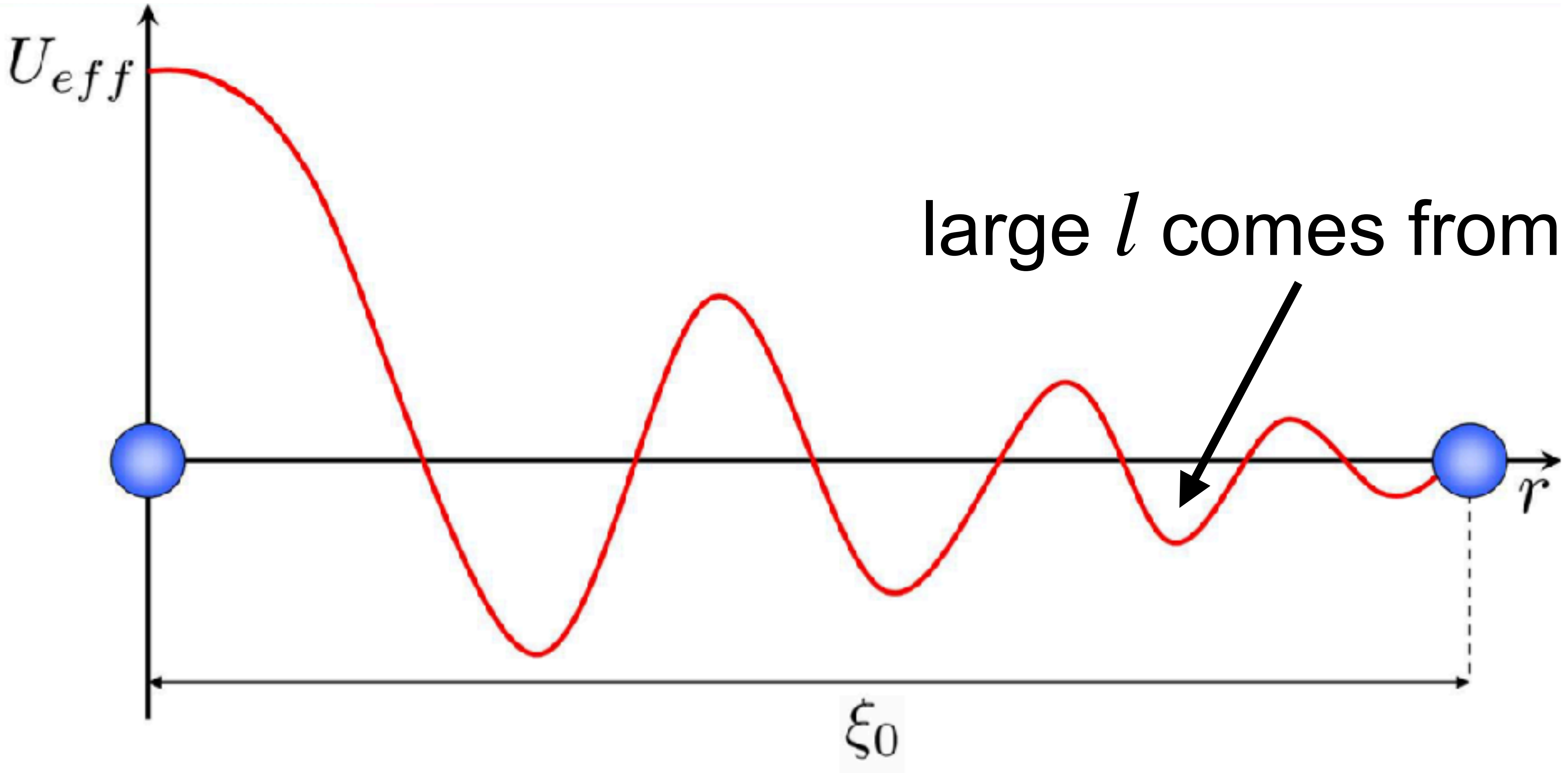
Pitaevskii; Brueckner,Soda, et al.; Emery,Sessler; Anderson,Morel; Balian,Werthamer, Leggett (1960s)

Yes. Pairing problem decouples between different angular momentum l

Sufficient to have an attraction for just a single l channel.

Intuitive explanation based on **Friedel oscillation**

Kohn,Luttinger (1965)



large l comes from large distance

$$U(r) = \frac{\cos(2k_F r)}{r^3}$$

At large r, screened interaction oscillates and occasionally becomes over-screened

Kohn-Luttinger (KL) superconductivity

Kohn,Luttinger (1965)

- Cooper pair formation: Attractive interaction necessary

- **Kohn-Luttinger mechanism**: Even when bare s-wave interaction is repulsive,
  induced interaction in higher partial wave l can be attractive

- KL mechanism based on perturbation theory (1-loop):

$$\Delta \sim \epsilon_{\mathrm{F}} \exp\left(-\# \, l^4\right)$$

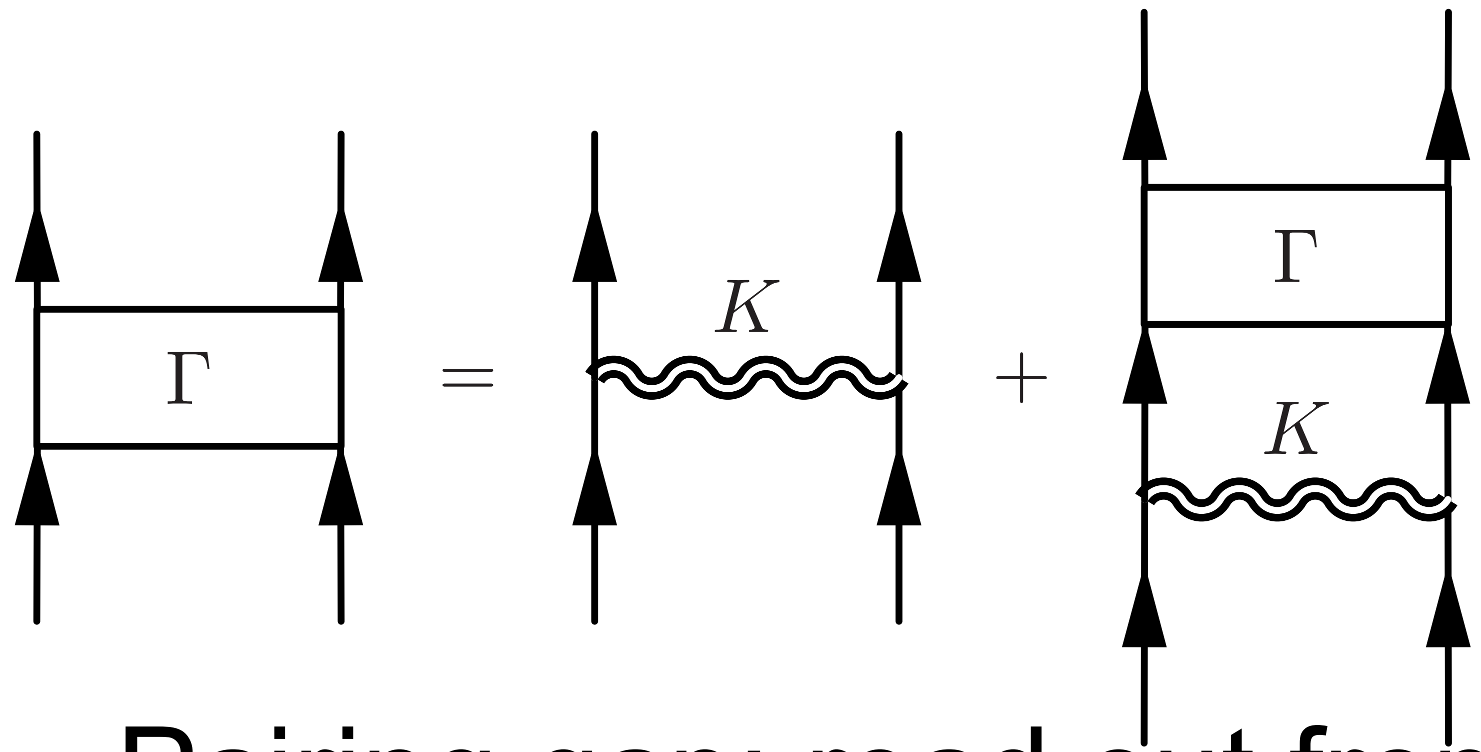
… very small.

Cooper instability always sets in,
but the gap is too small to be phenomenologically relevant

KL mechanism

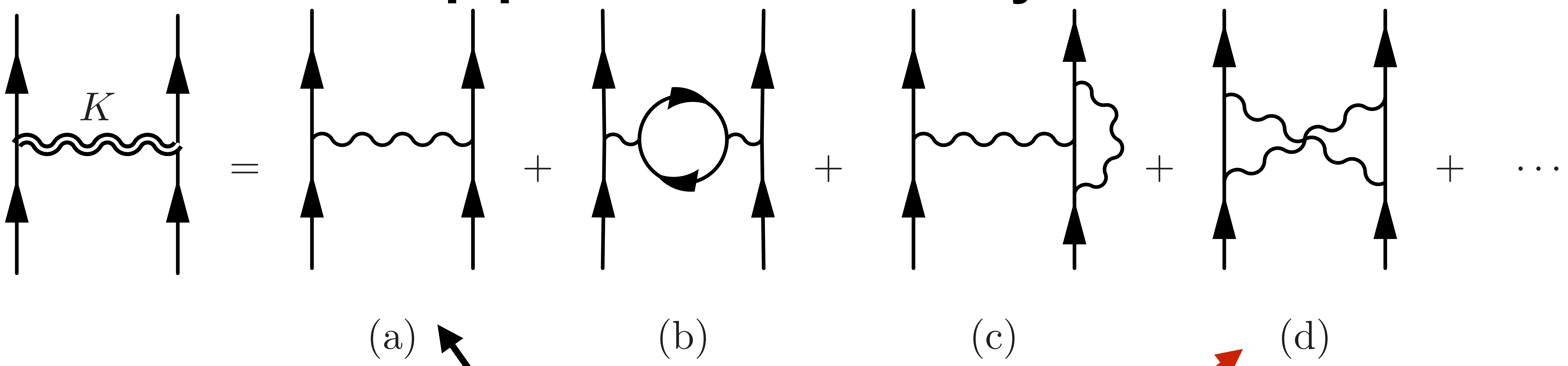
Kohn,Luttinger (1965)

Bethe-Salpeter equation for
summing the Cooper logarithms:



Pairing gap: read out from the singular point in Γ

Irreducible interaction in one-loop perturbation theory:



Partial wave expansion:

$$K(\theta) = \sum_l (2l+1) K_l P_l(\cos\theta)$$

$$K_l^{(a)} \sim e^{-l} \sim 0$$

$$K_l^{(b,c,d)} \sim \frac{(-1)^l}{l^4}$$

… attraction in odd l channel

Problem of one-loop analysis of KL mechanism

Fay,Layzer (1969); Kagan,Chubukov (1988); Efremov et al. (2000)

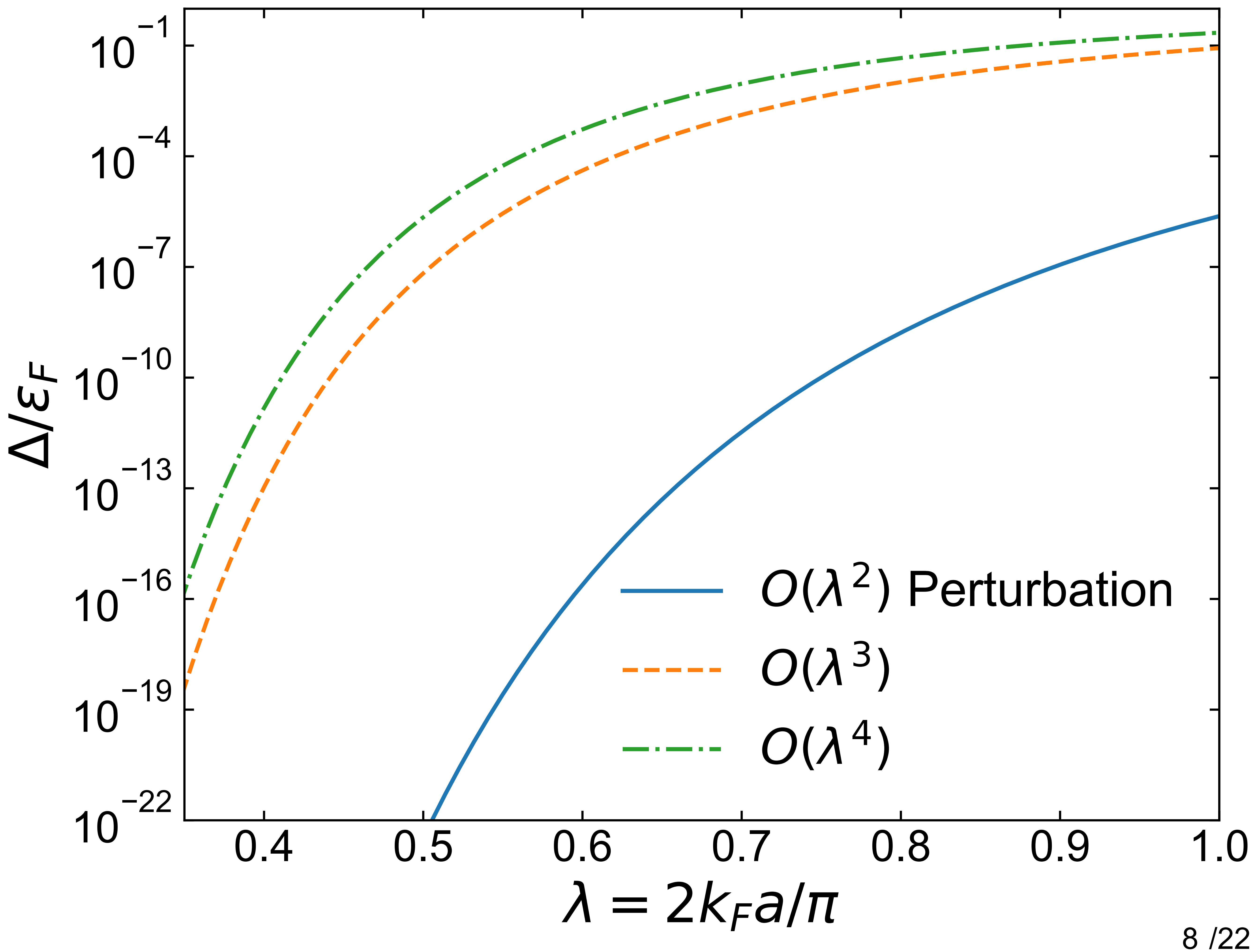
Example: Consider the $l = 1$ case with repulsive s-wave contact interaction

$$\mathscr{L}_{\text{int}} = -\frac{1}{2}\frac{4\pi a}{m}(\psi^\dagger_\sigma \psi_\sigma)^2$$

- Perturbative results up to 3-loop order are known

- Converges very poorly, 1-loop deviates largely from 2 & 3-loop results

- Subset of the subleading contributions has divergent integrand
  $\rightarrow$ it has to be resummed. Summation done by RG



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1. Kohn-Luttinger superconductivity

2. BCS instability from renormalization group

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Renormalization group analysis of KL mechanism

Kohn,Luttinger (1965)

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- **Kohn-Luttinger mechanism**: Even when bare s-wave interaction is repulsive,
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- KL mechanism based on perturbation theory (1-loop):

$$\Delta \sim \epsilon_{\mathrm{F}} \exp\left(-\# \, l^4\right)$$

Kohn,Luttinger (1965)

- From the reanalysis using RG, it turns out:

$$\Delta \sim \epsilon_{\mathrm{F}} \exp\left(-\# \, l\right)$$

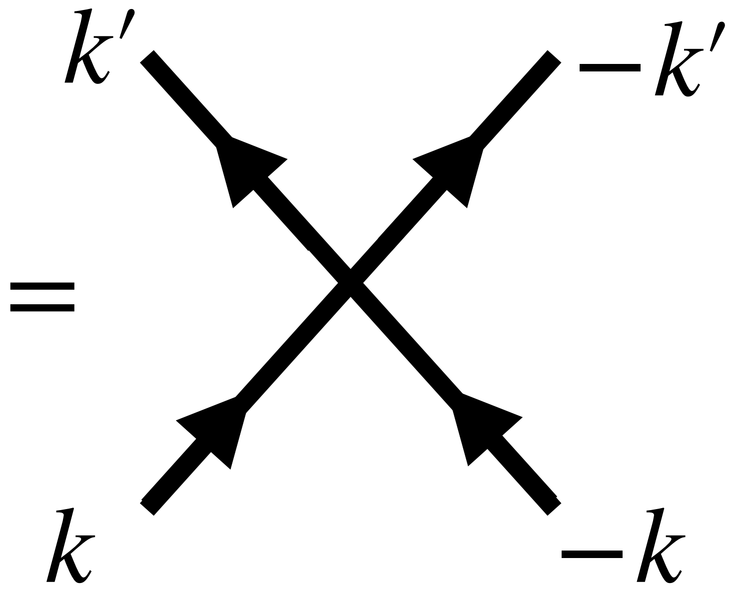
Fujimoto (2025)

RG approach to BCS instability

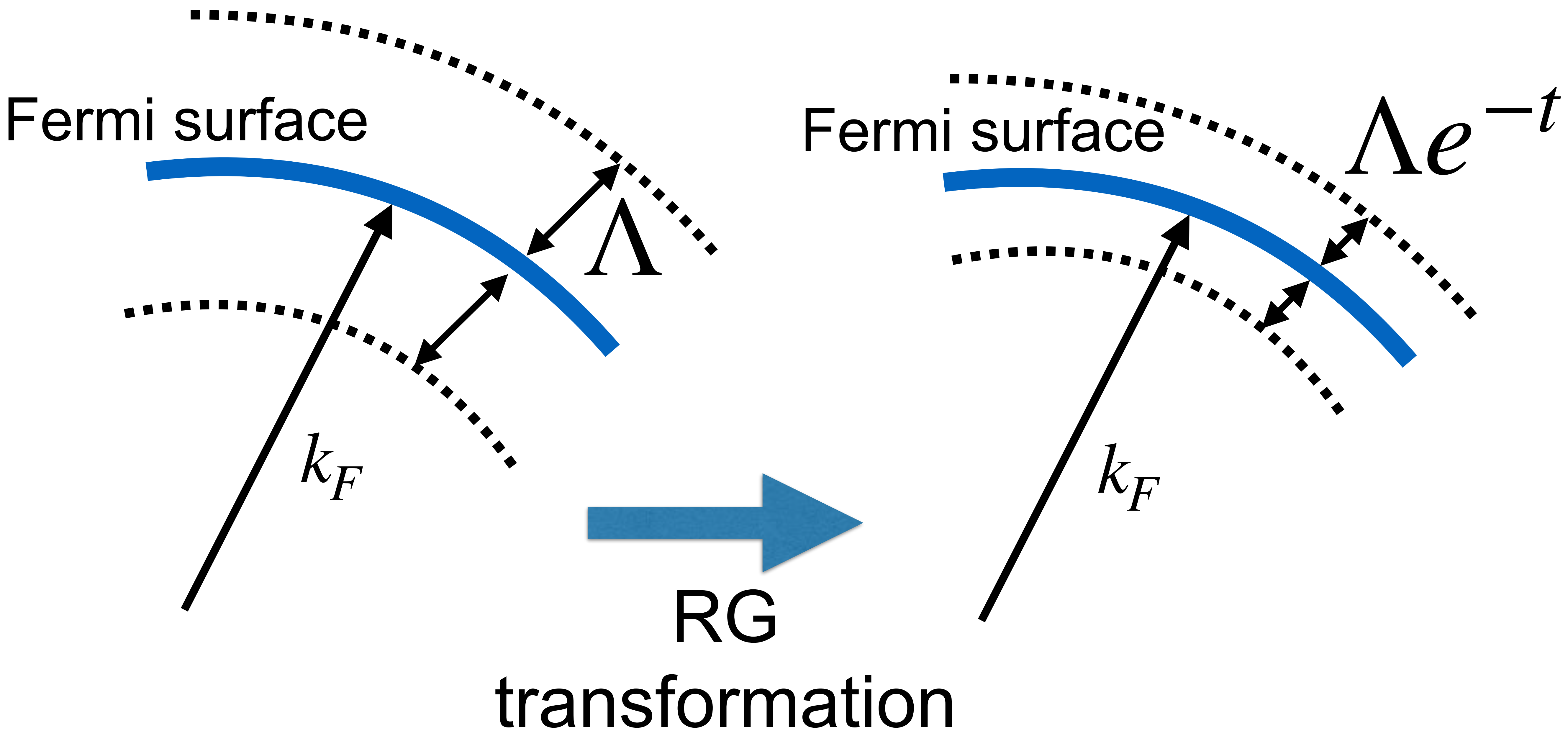
Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

Consider an EFT with the UV cutoff $|l| < \Lambda$, **(**$l = k - k_F$**)**

$$S_{\rm int} = \prod_{i=1}^{4} \int_{|l|<\Lambda} \frac{d^4 k_i}{(2\pi)^4} V(l_1, l_2, l_3, l_4) \bar{\psi}(l_4) \bar{\psi}(l_3) \psi(l_2) \psi(l_1)$$

$V =$ 

RG transformation near the Fermi surface:



Slow mode: $\psi_< = \psi(l), \; 0 < |l| < \Lambda e^{-t}$

Fast mode: $\psi_> = \psi(l), \; \Lambda e^{-t} < |l| < \Lambda$

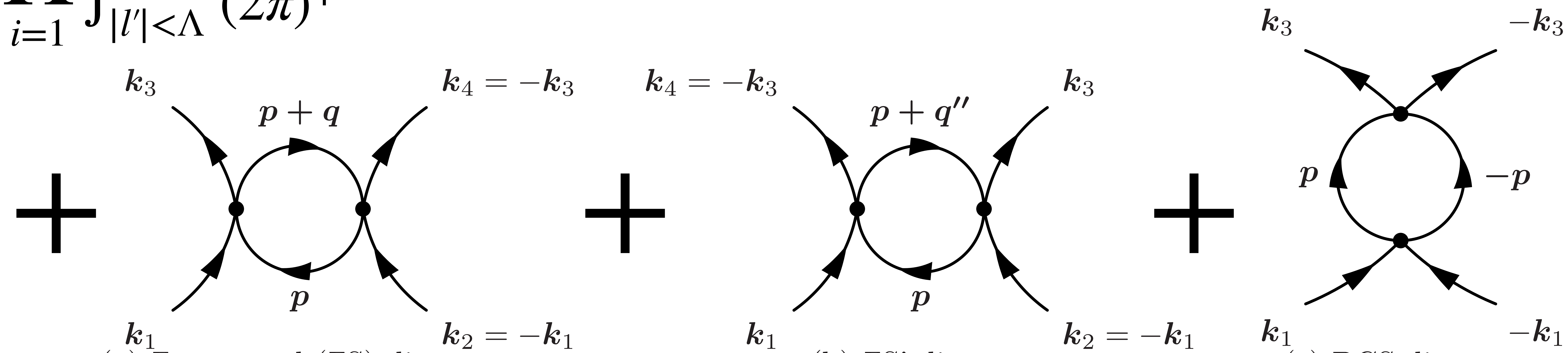
1. Integrate out the fast modes
   i.e., reduced the cutoff as $\Lambda \to \Lambda e^{-t}$
2. Introduce rescaled momenta:
   $l' = l\, e^{t}$ (l' goes up to Λ)
3. Rewrite in terms of rescaled field:
   $\psi'(l') = e^{-3t/2} \psi_<(l' e^{-t})$

RG approach to BCS instability

Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

Renormalized effective action (only in terms of the slow modes):

$$S'_{\text{int}} = \prod_{i=1}^{4} \int_{|l'|<\Lambda} \frac{d^4 k'_i}{(2\pi)^4} V(l'_1 e^{-t}, l_2 e^{-t}, l_3 e^{-t}, l_4 e^{-t}) \bar{\psi}'(l'_4)\bar{\psi}'(l'_3)\psi'(l'_2)\psi'(l'_1)$$



(a) Zero sound (ZS) diagram

(b) ZS' diagram

(c) BCS diagram

RG approach to BCS instability

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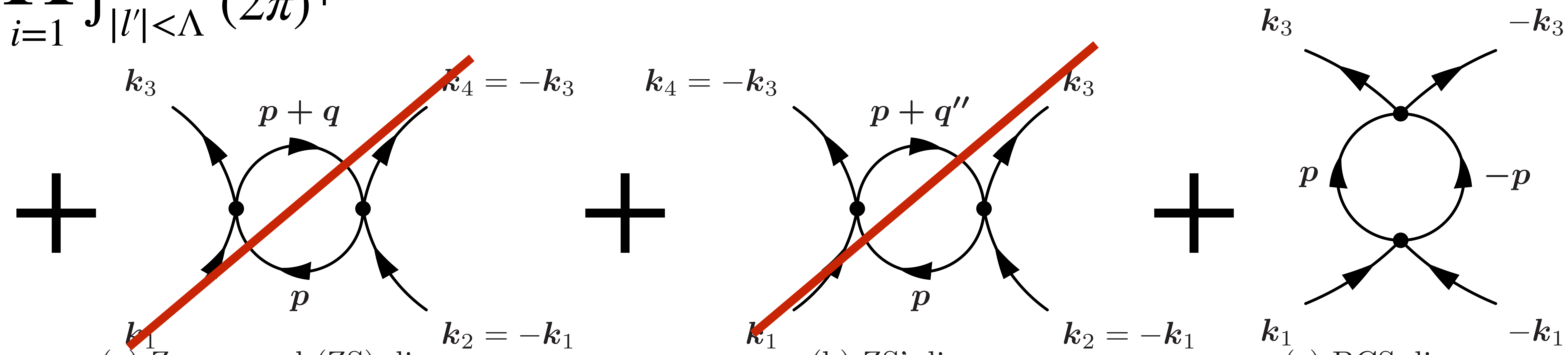
Renormalized effective action (only in terms of the slow modes):

$$S'_{\text{int}} = \prod_{i=1}^{4} \int_{|l'|<\Lambda} \frac{d^4 k_i'}{(2\pi)^4} V(l_1' e^{-t}, l_2 e^{-t}, l_3 e^{-t}, l_4 e^{-t}) \bar{\psi}'(l_4') \bar{\psi}'(l_3') \psi'(l_2') \psi'(l_1')$$



(a) Zero sound (ZS) diagram (b) ZS' diagram (c) BCS diagram

RG equation:

$$\frac{dV_l(t)}{dt} = -V_l^2(t)$$

Partial wave expansion:

$$V(\theta) = \sum_l (2l+1) V_l P_l(\cos\theta)$$

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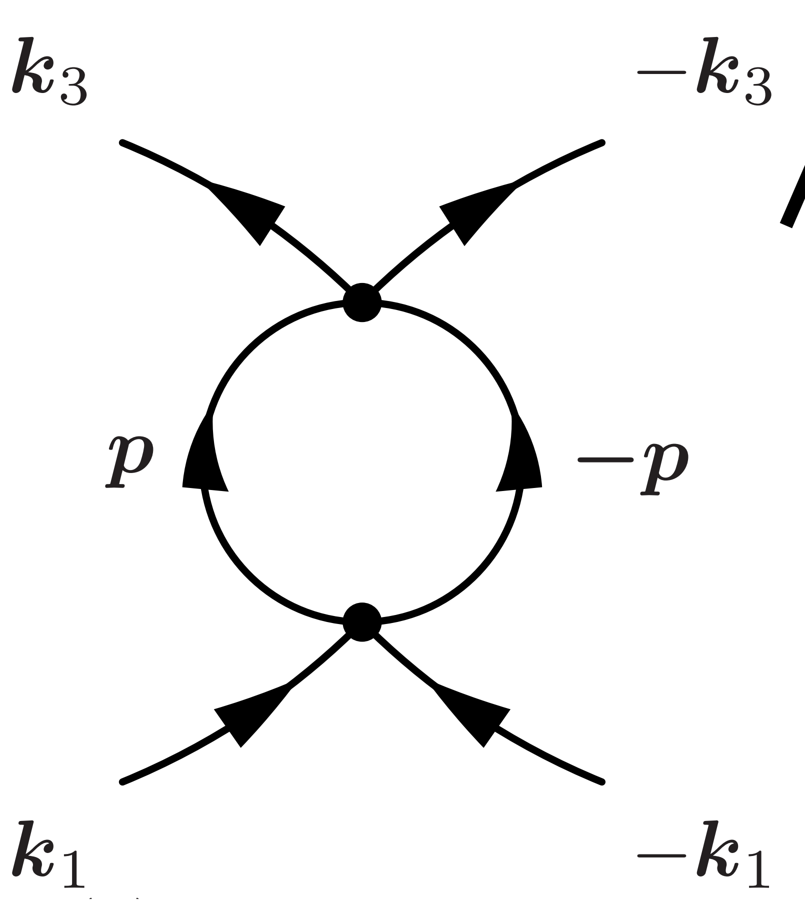


Solution:

$$V_l(t) = \frac{V_l(t=0)}{1 + V_l(t=0)\,t}$$

… singular at $t = -1/V_l(0)$ when $V_l(0) < 0$

(attractive interaction)

- Singularity → Break down of the Fermi liquid picture
  Manifestation of the BCS instability
- Pairing gap Δ = Energy scale Λ at the BCS instability
- From the scale parameter: $t = -\ln(\Lambda/\epsilon_F)$

$$\rightarrow \Delta = \epsilon_F \exp\left(-\frac{1}{|V_l(0)|}\right)$$

Gap in BCS approximation.
Ladder summation via RG eq

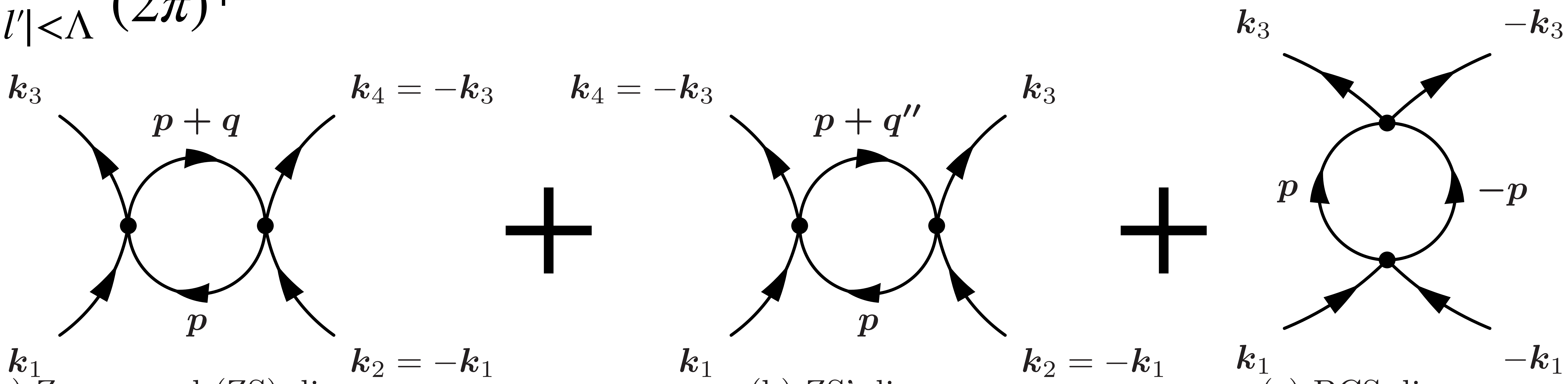
Modified RG equation

Shankar (1993); Fujimoto (2025)

Renormalized effective action (only in terms of the slow modes):

$$S'_{\rm int} = \prod_{i=1}^{4} \int_{|l'|<\Lambda} \frac{d^4k'_i}{(2\pi)^4} V(l'_1 e^{-t}, l_2 e^{-t}, l_3 e^{-t}, l_4 e^{-t})\bar{\psi}'(l'_4)\bar{\psi}'(l'_3)\psi'(l'_2)\psi'(l'_1)$$

$+ \quad + \quad +$



(a) Zero sound (ZS) diagram (b) ZS' diagram (c) BCS diagram

RG equation:

$$\frac{dV_l(t)}{dt} = -\frac{V^2(\theta=\pi)\ln 2}{2l+1}e^{-t} - V_l^2(t)$$

These particle-hole loop diagrams should also be included.

Dominated at $\theta = \pi$

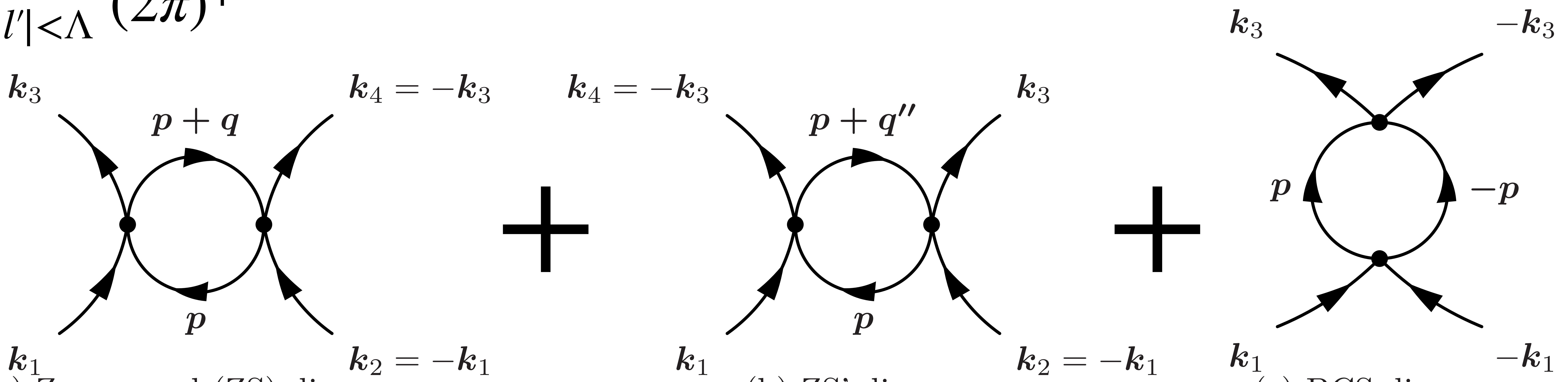
(back to back scattering)

Modified RG equation

Shankar (1993); Fujimoto (2025)

Renormalized effective action (only in terms of the slow modes):

$$S'_{\rm int} = \prod_{i=1}^{4} \int_{|l'|<\Lambda} \frac{d^4 k'_i}{(2\pi)^4} V(l'_1 e^{-t}, l_2 e^{-t}, l_3 e^{-t}, l_4 e^{-t}) \bar{\psi}'(l'_4)\bar{\psi}'(l'_3)\psi'(l'_2)\psi'(l'_1)$$

$+$ 

(a) Zero sound (ZS) diagram $+$ (b) ZS' diagram $+$ (c) BCS diagram

RG equation:

$$\frac{dV_l(t)}{dt} = -\frac{V^2(\theta=\pi)\ln 2}{2l+1} e^{-t} - V_l^2(t)$$

Inclusion of this term changes Δ as:

$$\Delta \sim \epsilon_{\rm F} \exp\left(-\# \, l\right)$$

The modified RG equation and its solution

Shankar (1993); Fujimoto (2025)

$$\frac{dV_l(t)}{dt} = \underline{-\frac{V^2(\pi)\ln 2}{2l+1}e^{-t}} - V_l^2(t)$$

Irrelevant. Renormalizes to 0 when $t \to \infty$.
Does not affect the location of the fixed point.
However, the BCS instability is hit before the RG flow reaches the fixed point
Thus, the location of the BCS instability is affected by this irrelevant term

Solution: $V_\ell(t) = c_\ell e^{-t/2}\frac{\mathcal{X}}{\mathcal{Y}}$,

$$\mathcal{X} = \left[V_\ell(0)J_0(c_\ell) - c_\ell J_1(c_\ell)\right]Y_1\left(e^{-t/2}c_\ell\right) - \left[V_\ell(0)Y_0(c_\ell) - c_\ell Y_1(c_\ell)\right]J_1\left(e^{-t/2}c_\ell\right),$$

$$\mathcal{Y} = \left[V_\ell(0)J_0(c_\ell) - c_\ell J_1(c_\ell)\right]Y_0\left(e^{-t/2}c_\ell\right) - \left[V_\ell(0)Y_0(c_\ell) - c_\ell Y_1(c_\ell)\right]J_0\left(e^{-t/2}c_\ell\right),$$

$$c_\ell = \sqrt{2V^2(\pi)\ln 2/(2\ell+1)}$$

The gap read out from the solution: $\ln\left(\frac{\Delta}{\epsilon_F}\right) = \frac{1}{V_l(t=0) - \frac{V^2(\pi)\ln 2}{2l+1}} \to \Delta \sim \epsilon_{\mathrm{F}} e^{-\# l}$

$$(V_l(t=0) \sim (-1)^l/l^4)$$

$l = 1$ example: perturbation theory vs RG

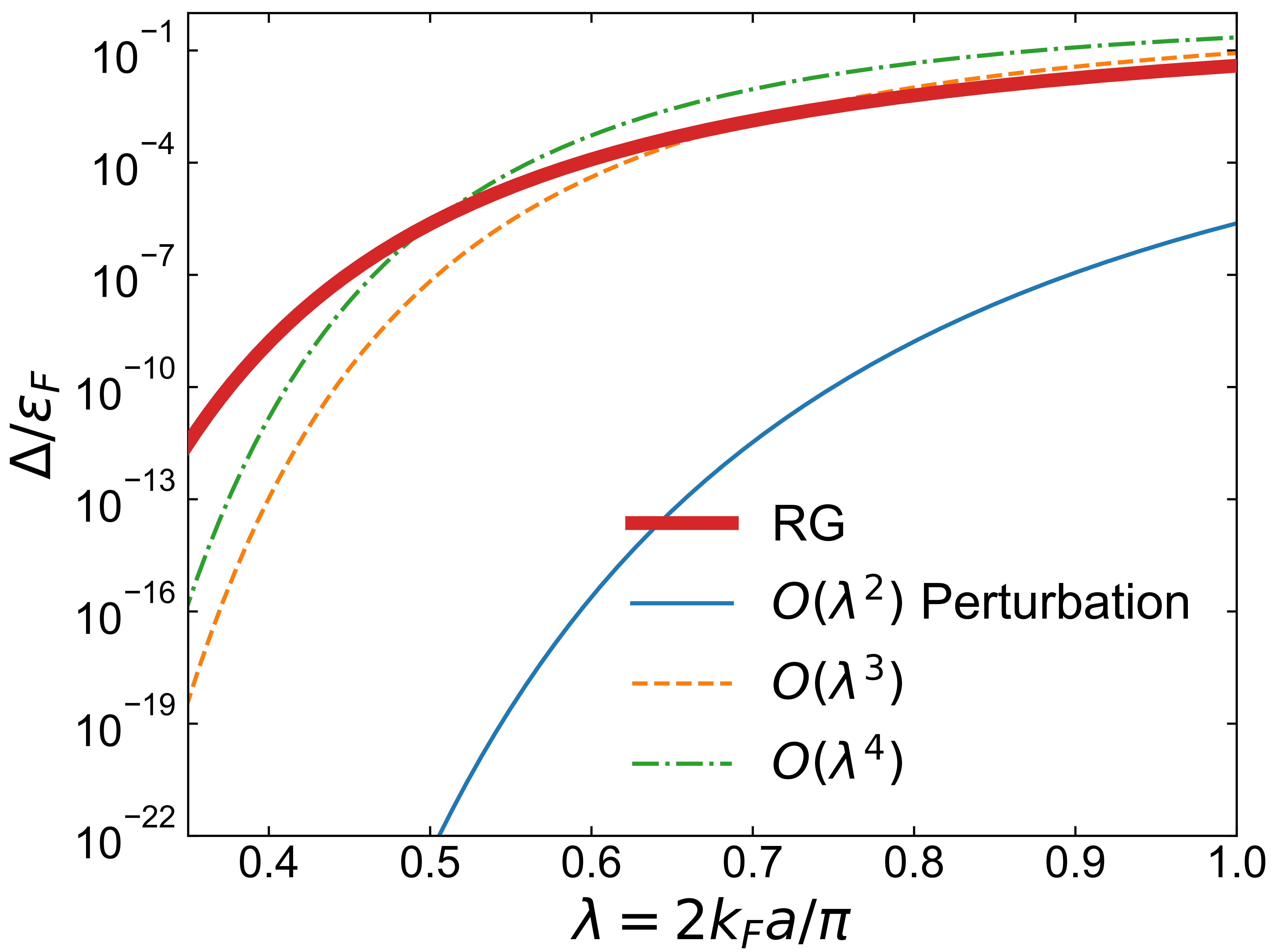
Efremov et al. (2000); Fujimoto (2025)

Example: Consider the $l = 1$ case with repulsive s-wave contact interaction

$$\mathscr{L}_{\text{int}} = -\frac{1}{2}\frac{4\pi a}{m}(\psi_\sigma^\dagger \psi_\sigma)^2$$

- RG is capable of reproducing 2-loop results at relatively large λ
  $\rightarrow$ Subset of the sub-leading contributions are resummed

- RG results predicts $\ln \Delta \sim -l$
  So, 2-loop result should also predict $\ln \Delta \sim -l$ instead of $-l^4$



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Motivation - Emerging picture of neutron star EoS

Figure based on method by Altiparmak,Ecker,Rezzolla (2022)

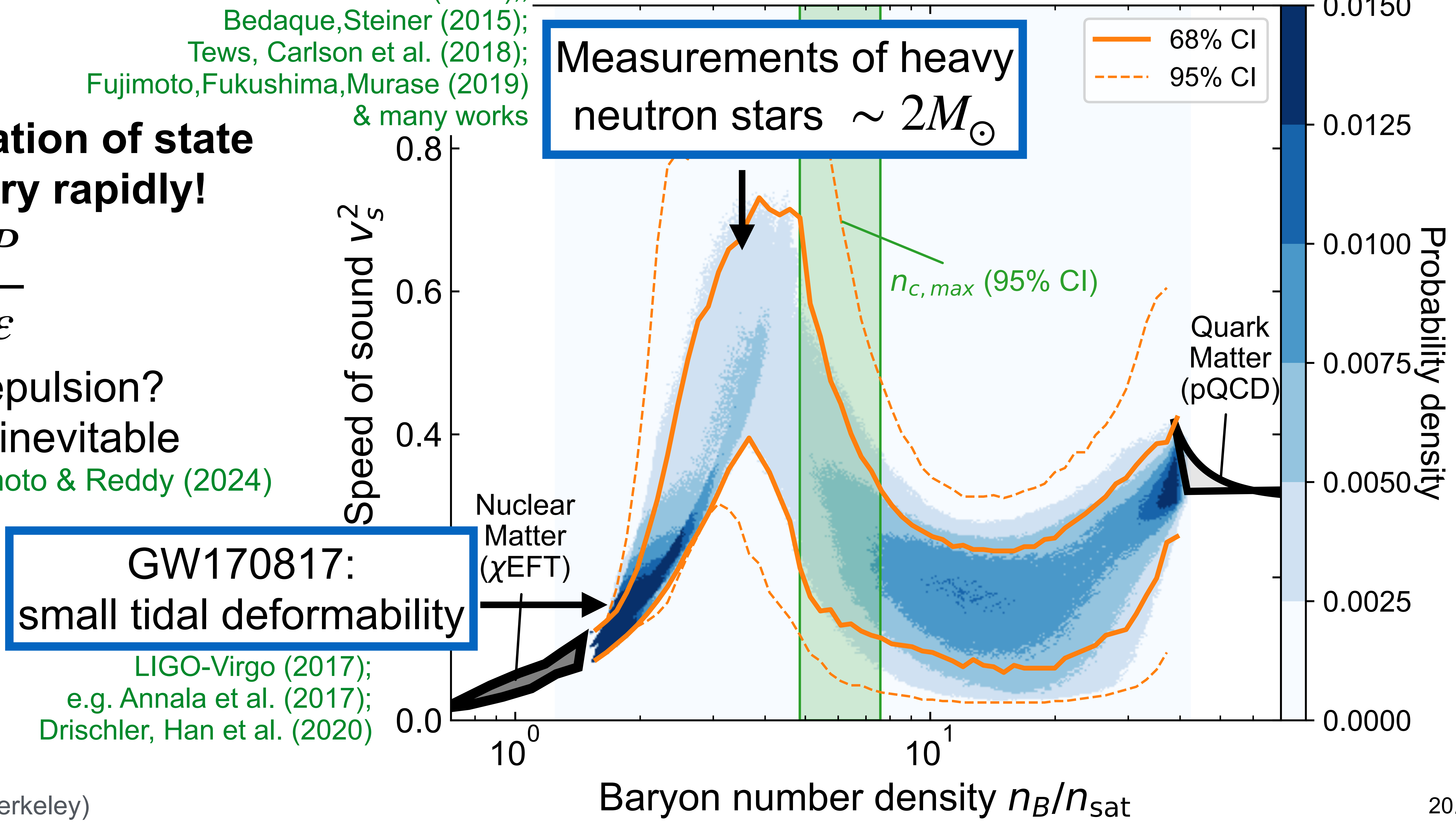
Demorest et al. (2010);;
Bedaque,Steiner (2015);
Tews, Carlson et al. (2018);
Fujimoto,Fukushima,Murase (2019)
& many works

Nuclear equation of state stiffens very rapidly!

$$v_s^2 = \frac{dP}{d\varepsilon}$$

driven by repulsion?
→ KL effect inevitable

Kumamoto & Reddy (2024)



LIGO-Virgo (2017);
e.g. Annala et al. (2017);
Drischler, Han et al. (2020)

KL effect in neutron star - sketchy analysis

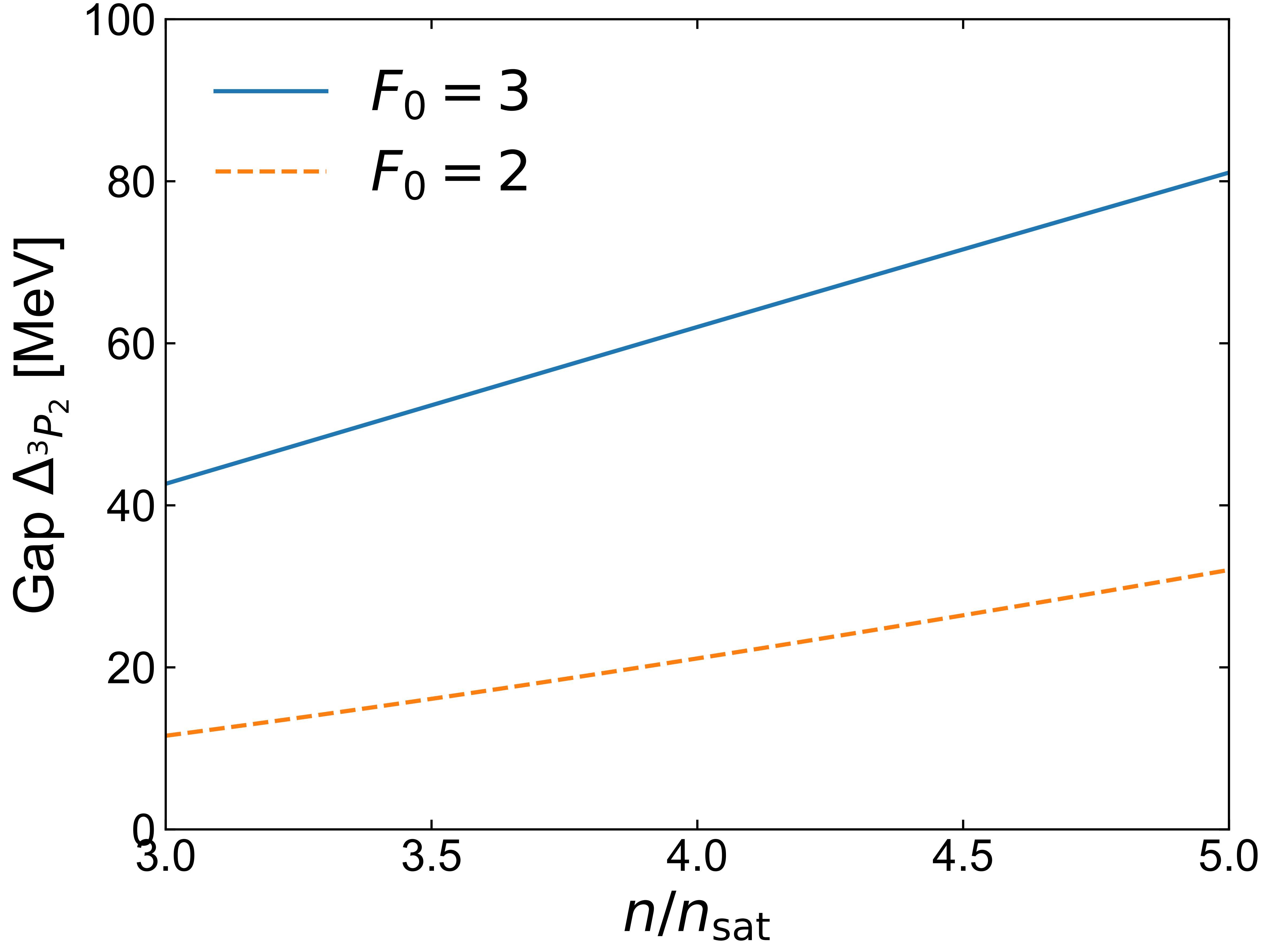
Based on: Kumamoto & Reddy (2024)

- Suppose interaction described by vector-vector coupling:

  $$L_{\text{int}} = -G_V(\bar{\psi}\gamma^\mu\psi)^2$$

- G_V is tuned so that the Landau Fermi liquid parameter is $F_0 \sim 2-3$.
  Friman & Weise (2019)

- The resulting KL gap in ${}^3P_2$ channel is O(10) MeV
  → e.g., suppression in neutrino emission



Summary & Outlook

- 1-loop analysis of the KL effect is insufficient due to bad convergence. Confirmed in the $l = 1$ case: 2- & 3-loop contribution to the KL effect is large

- The KL pairing gap from the RG should scale as $\ln \Delta \sim -l$ instead of $\ln \Delta \sim -l^4$ as in the original KL 1-loop argument

- 1-loop RG is capable of reproducing the 2-loop perturbative results (at least in the p-wave case).

- Repulsive interaction in neutron star seems to lead to large pairing inevitably

- More sophisticated analysis based on the ab initio method?