Alternative signatures of neutron-proton pairing

P. Van Isacker, GANIL, France
A.O. Macchiavelli, Oak Ridge National Laboratory, USA

$T = 1$ and $T = 0$ pairing correlations
- General considerations
- Pair vibrations
- Transfer of $T = 1$, $T = 0$ and aligned np pairs

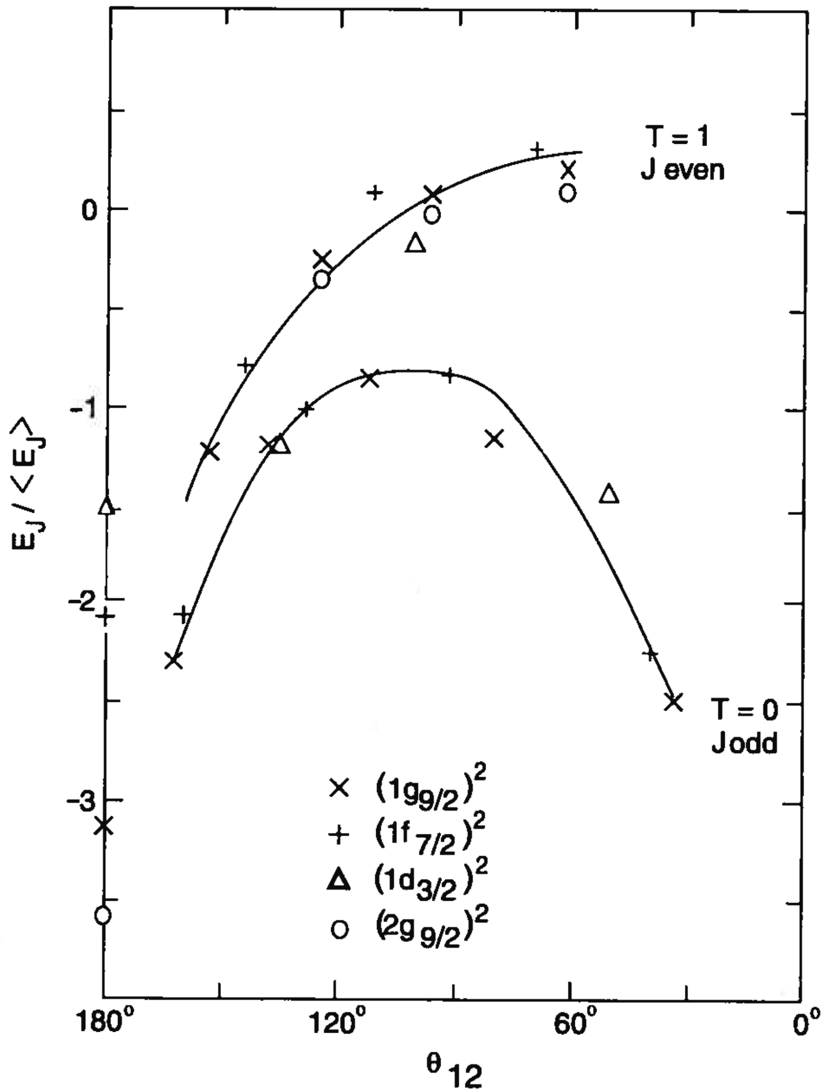
Deuteron cluster probability

Neutron-proton spin-spin correlations

Conclusion and outlook

ESNT, Saclay, 16 May 2025

Nucleon-nucleon interaction



J.P. Schiffer, Ann. Phys. (N.Y.) **66** (1971) 798

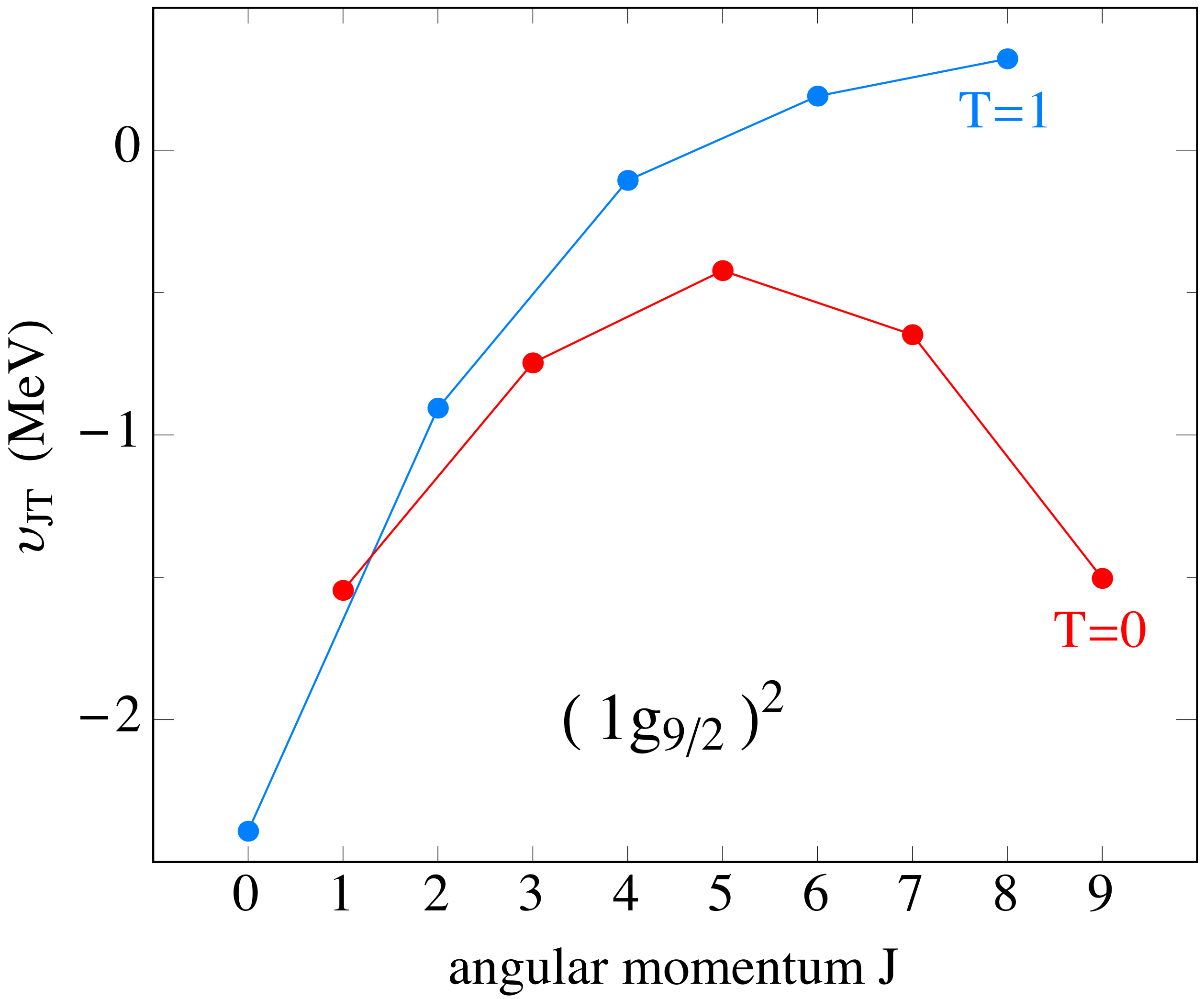
Pairing and other interactions

Pairing refers to the interaction between nucleons in 'time-reversed orbits':

- *isovector (or spin-singlet) pairing:* $J = 0$ & $T = 1$
- *isoscalar (or spin-triplet) pairing:* $J = 1$ & $T = 0$

Aligned np interaction (not pairing): $J = 2j$ & $T = 0$

Nucleon-nucleon interaction



Time-reversed orbits

A jj-coupled two-nucleon state with angular momentum $J=0$:

$$\left|j^2;J=0\right\rangle \propto \sum_{m_j}\left\langle jm_j\; j-m_j|00\right\rangle \left|jm_j\right\rangle \left|j-m_j\right\rangle \propto \sum_{m_j}\left|jm_j\right\rangle \left|j\bar{m}_j\right\rangle$$

What about $J=1$? Use LS-coupled states:

$$\left|(\ell s)^2;L=0,SM_S\right\rangle \propto \sum_{m_\ell m_s m'_s}\left\langle sm_s\; sm'_s|SM_S\right\rangle \left|\ell m_\ell;sm_s\right\rangle \left|\ell\bar{m}_\ell;sm'_s\right\rangle$$

The Pauli principle allows $(L=0,S=0,T=1)$ and $(L=0,S=1,T=0)$ $\Longrightarrow$ two pairing interactions.

A matter of strength

Relation between jj-coupled and LS-coupled matrix elements:

$$V\left(j^2;JT\right)=\sum_{LS}\begin{bmatrix}\ell & s & j\\ \ell & s & j\\ L & S & J\end{bmatrix}^2 V\left[\left(\ell s\right)^2;LST\right]$$

For pairing matrix elements:

$$V\left(j^2;J=0,T=1\right)\approx\frac{1}{2}V\left[\left(\ell s\right)^2;L=0,S=0,T=1\right]$$

$$V\left(j^2;J=1,T=0\right)\approx\frac{1}{6}V\left[\left(\ell s\right)^2;L=0,S=1,T=0\right]$$

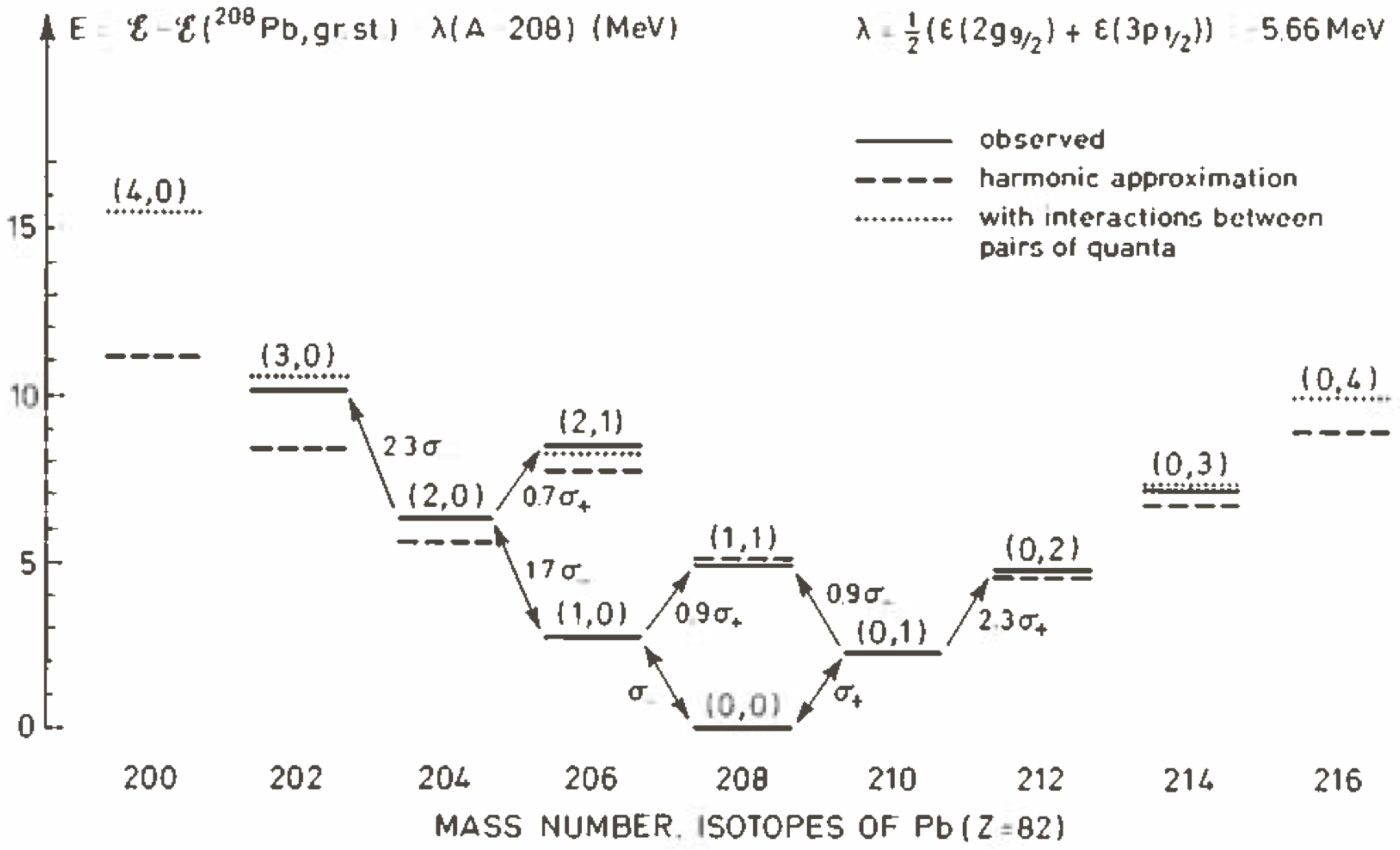
What is the question?

The question is not whether $T = 0$ interactions between nucleons exist or whether they are important. They do and they are.

The questions are

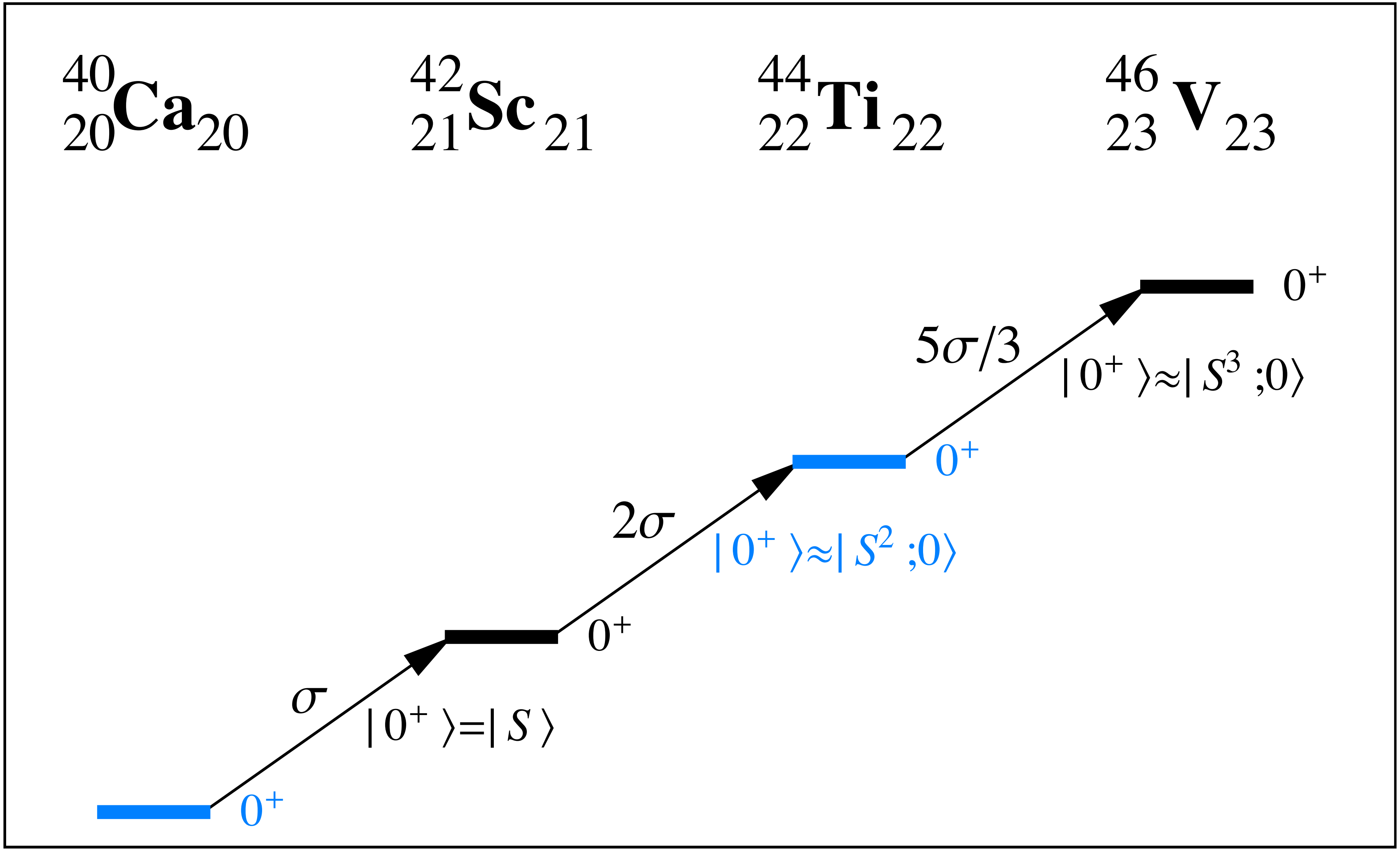
- *Do* $T = 1$ *pairing correlations exist?*
- *Do* $T = 0$ *pairing correlations exist?*
- *Are aligned* $T = 0$ *pairs important?*

$T = 1$ pair vibrations in Pb

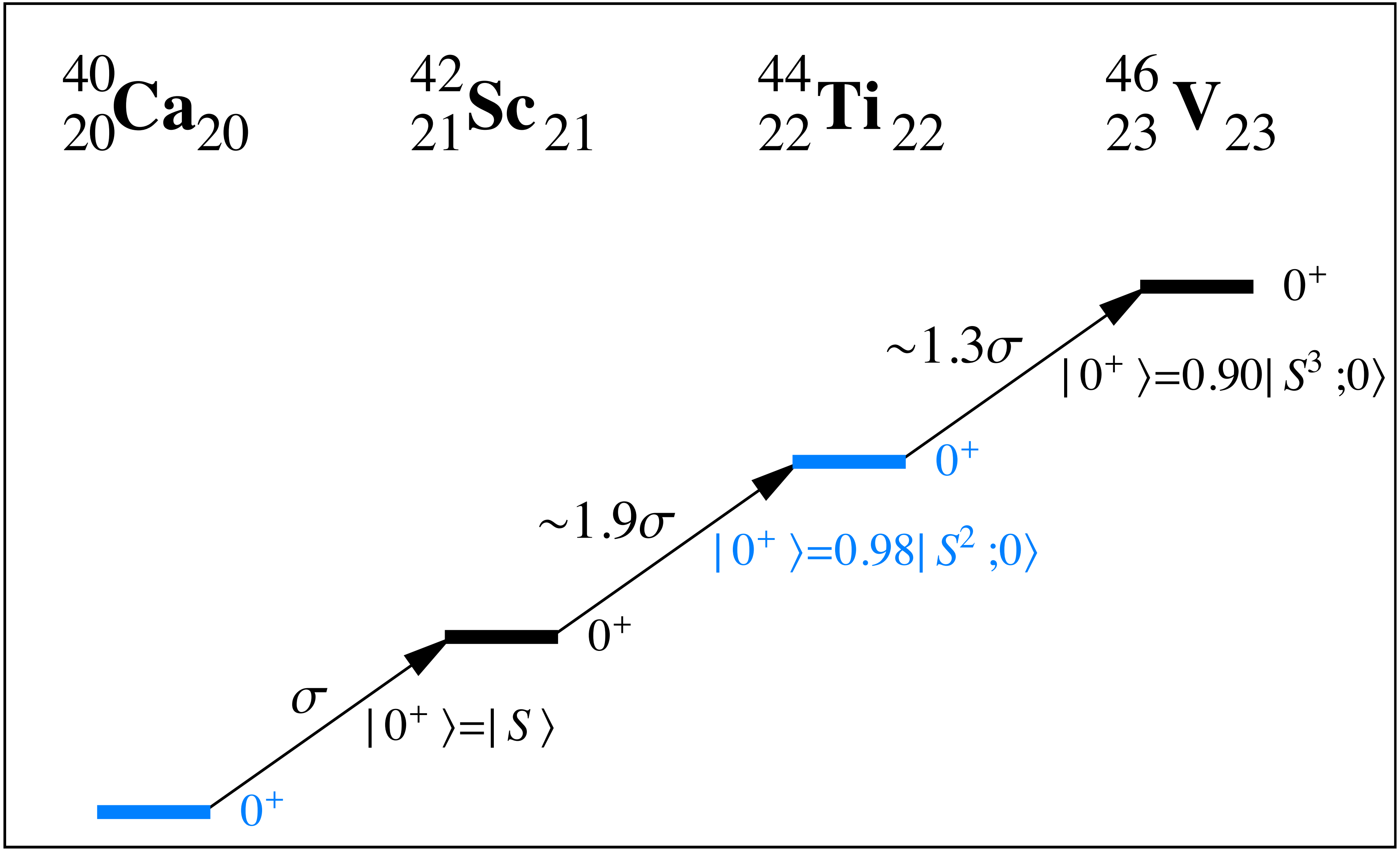


A. Bohr & B.R. Mottelson, Volume 2, page 646

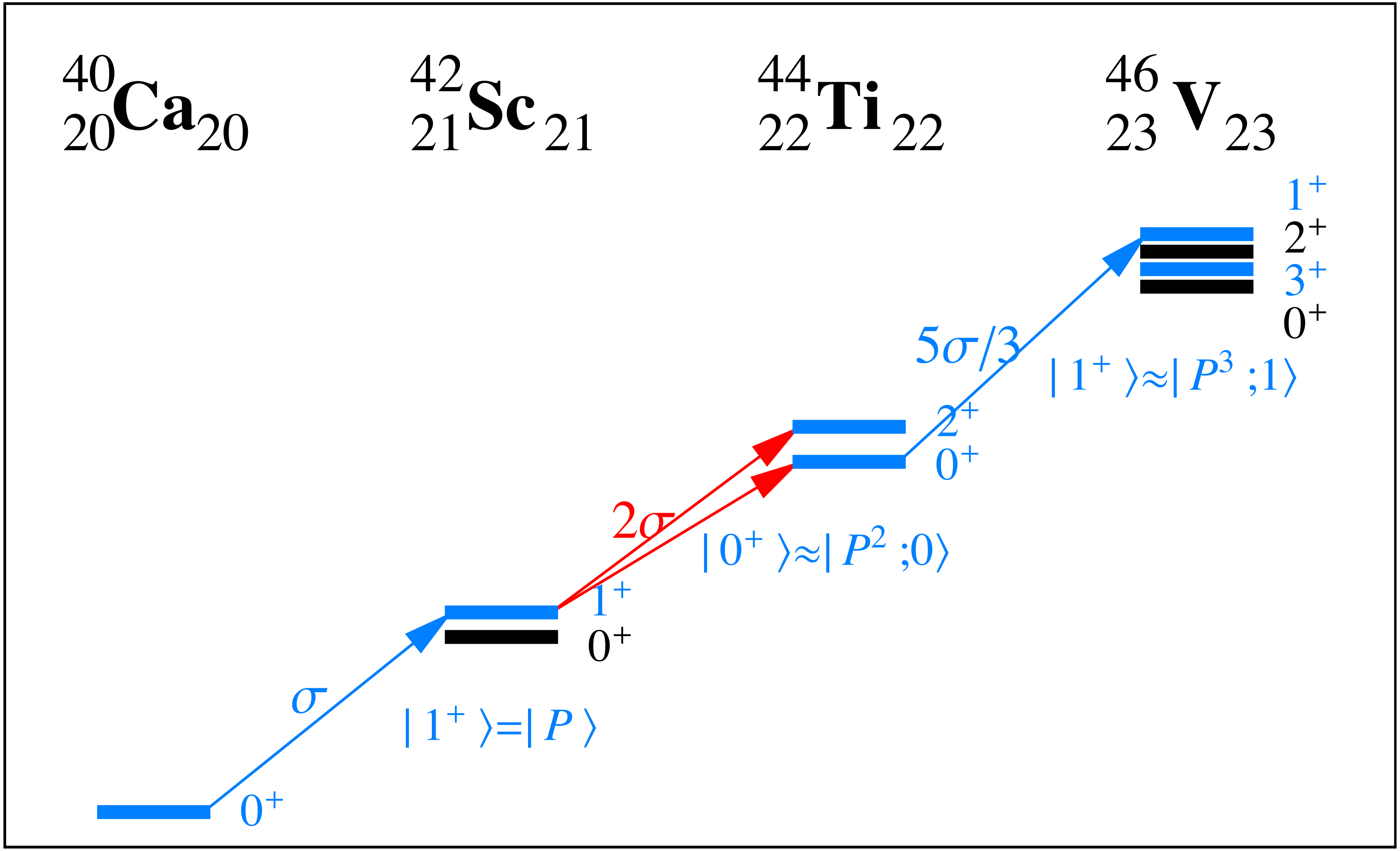
$T = 1$ pair vibrations in Ca–Sc–Ti–V?



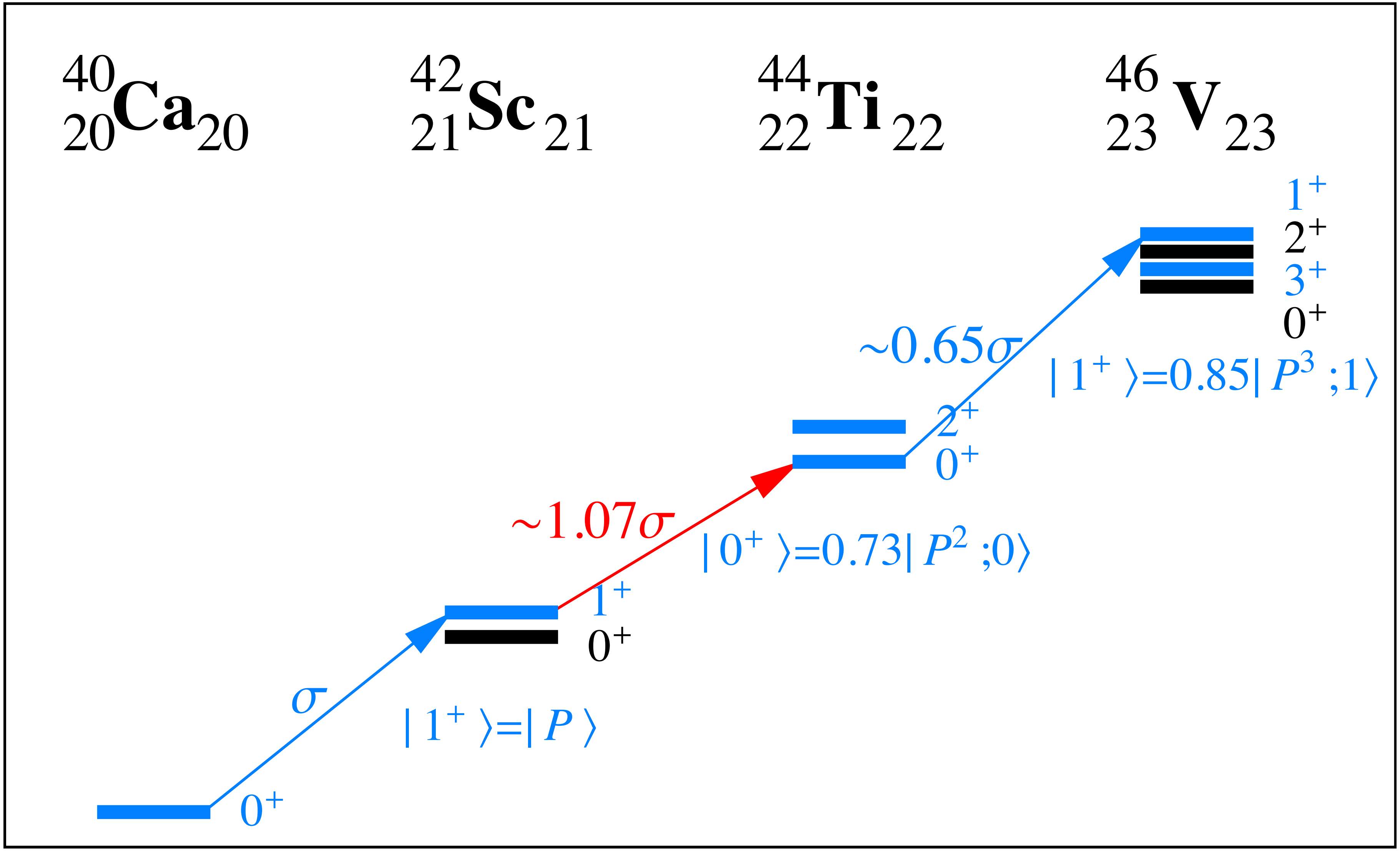
$T = 1$ pair vibrations in Ca–Sc–Ti–V?



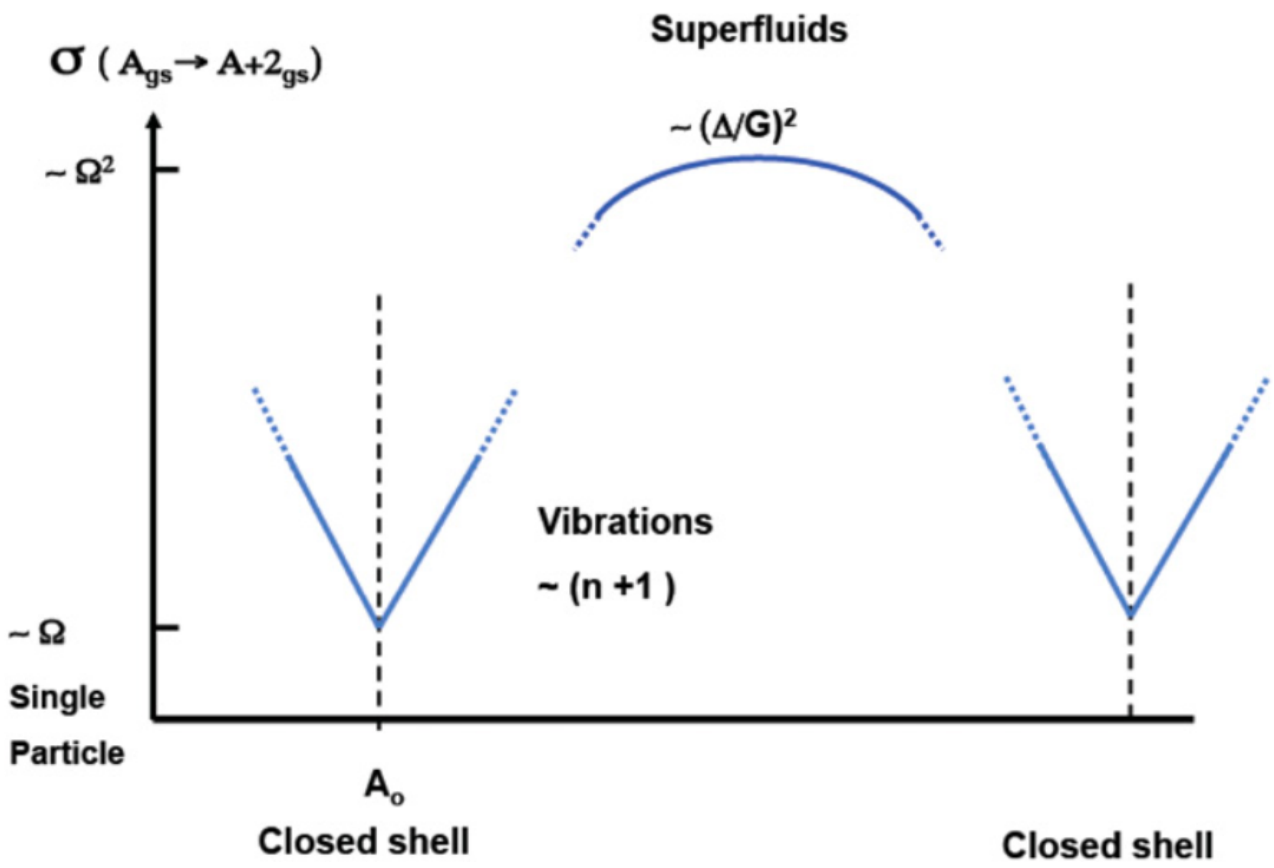
$T = 0$ pair vibrations in Ca-Sc-Ti-V?



$T = 0$ pair vibrations in Ca–Sc–Ti–V?

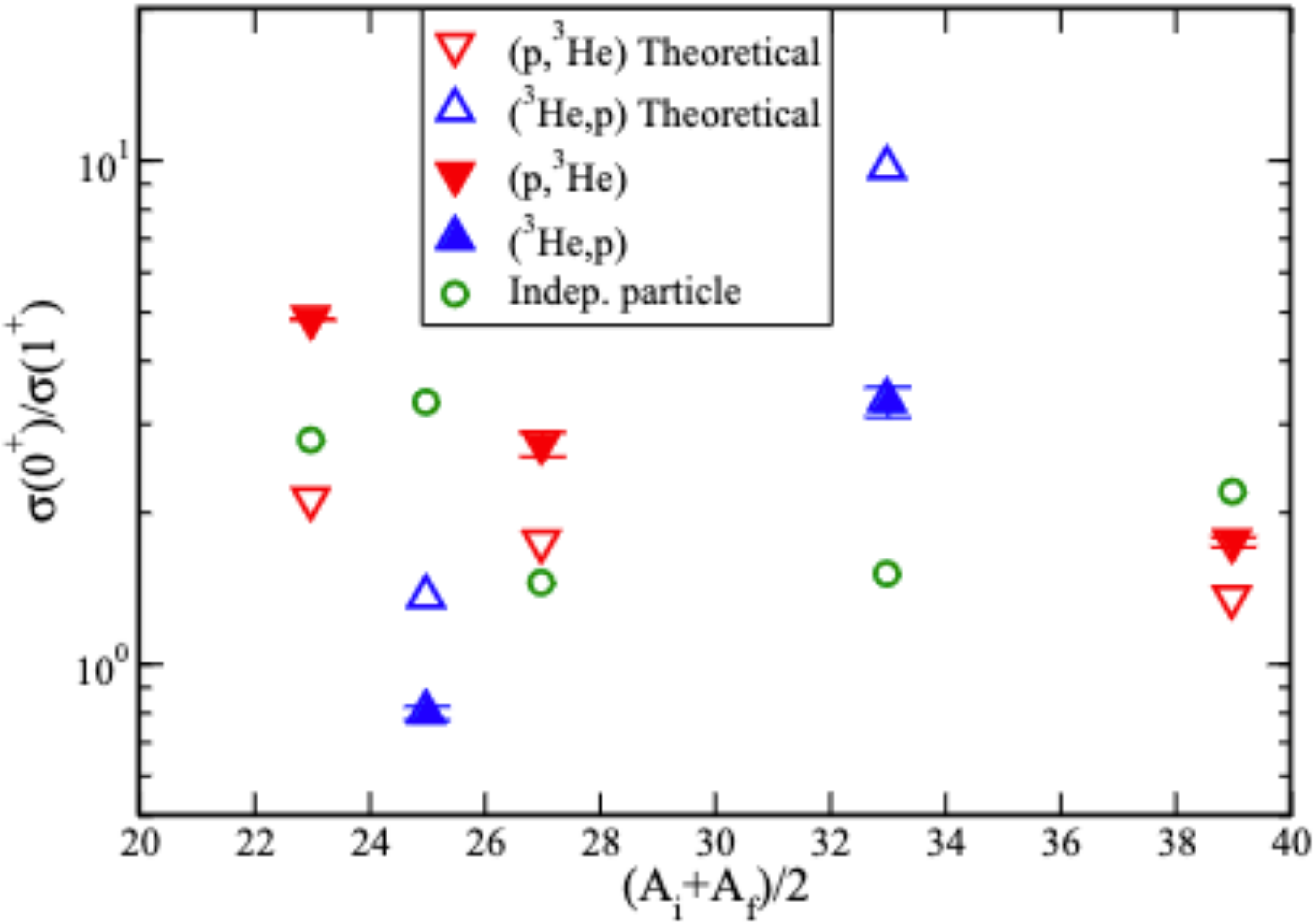


From vibrations to rotations



S. Frauendorf, A.O. Macchiavelli, Prog. Part. Nucl. Phys. **78** (2014) 24

(p,^{3}He) and (^{3}He,p) in *sd* shell



Y. Ayyad *et al.*, Phys. Rev. C **96** (2017) 021303R

(d,^{4}He) and (p,^{3}He) in $f_{7/2}$ shell

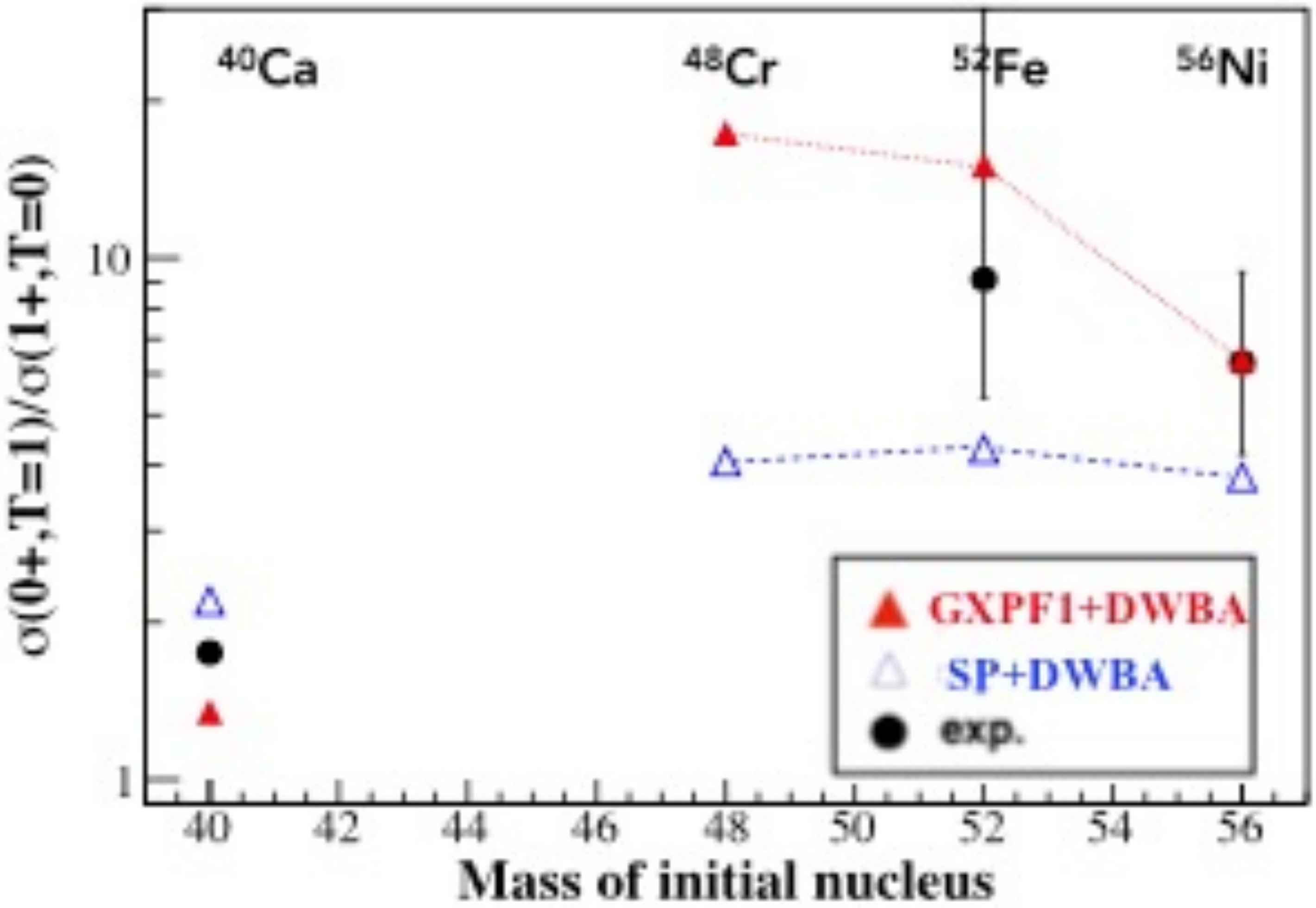
GANIL
PROPOSAL FOR EXPERIMENT

PAC date :	EXP # (Do not fill in):
November, 2011	E

TITLE : Study of n-p pairing through two-nucleon transfer reactions Is it a follow up experiment? [Yes/No]: If yes, experiment number: E		
Spokespersons (if several, please use capital letters to indicate the name of the contact person): MARLENE ASSIE (IPNO), L. Pollacco (SPhN Saclay), W. Catford (Surrey).		
Address of the CONTACT person: IPN Orsay 15 rue Georges Clémenceau, 91406 ORSAY cedex		
Phone:+33 1 69 15 79 18	Fax:+33 1 69 15 64 70	E mail: assie@ipno.in2p3.fr
Address of the backup person:		
Phone:	Fax:	E mail:
Please indicate here other participants with their affiliation (laboratory, university …) D. Beaumel, Y. Blumenfeld, F. Hammache, N. de Séréville, S. Franchoo, E. Khan, J.A. Scarpaci, L. Lefrebvre, M. Vandebrouck, P. Morfouace, A. Matta, D. Suzuki with theoretical support from N.V. Giai, J. Margueron, P. Schuck, M. Grasso (IPN Orsay) N. Alamanos, A. Drouart, A. Gillibert, A. Obertelli, A. Corsi, V. Lapoux, L. Nalpas, C. Simenel, Ch. Theisen (SPhN Saclay) G. De France, P. van Isacker, F. de Oliveira, C. Stodel, B. Bastin, L. Caceres, O. Kamalou, J-C Thomas (GANIL) R. Lemmon (Daresbury); M. Chartier (Liverpool) M. Labiche, R. Chapman (Paisley) L. Corradi, G. deAngelis (Legnaro); W. von Oertzen (HMI Berlin) N. Keeley (Poland); B. Fernandez-Dominguez (Spain)		
Short abstract: We propose to investigate neutron-proton pairing in the f7/2 shell through the measurement of ^{48}Cr(d,alpha), ^{48}Cr(d,^{3}He), ^{56}Ni(d,alpha) and ^{56}Ni(d,^{3}He) at 30 A. MeV. The beam is produced by fragmentation and selected with the LISE spectrometer. The experimental set-up will consist of MUST2, BTD, TIARA and 4 EXOGAM clovers.		

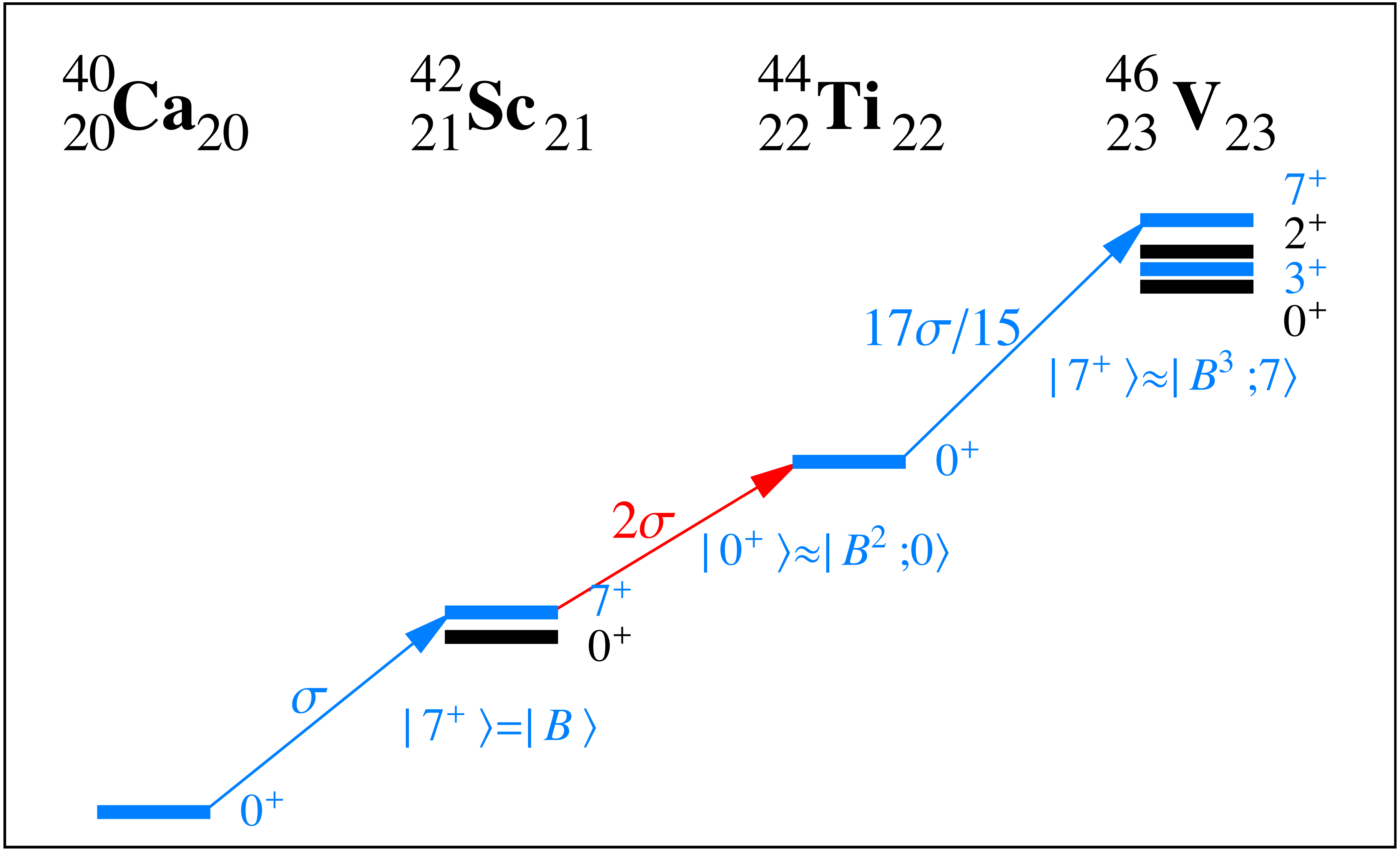
M. Assié *et al.*

(d,^{4}He) and (p,^{3}He) in $f_{7/2}$ shell

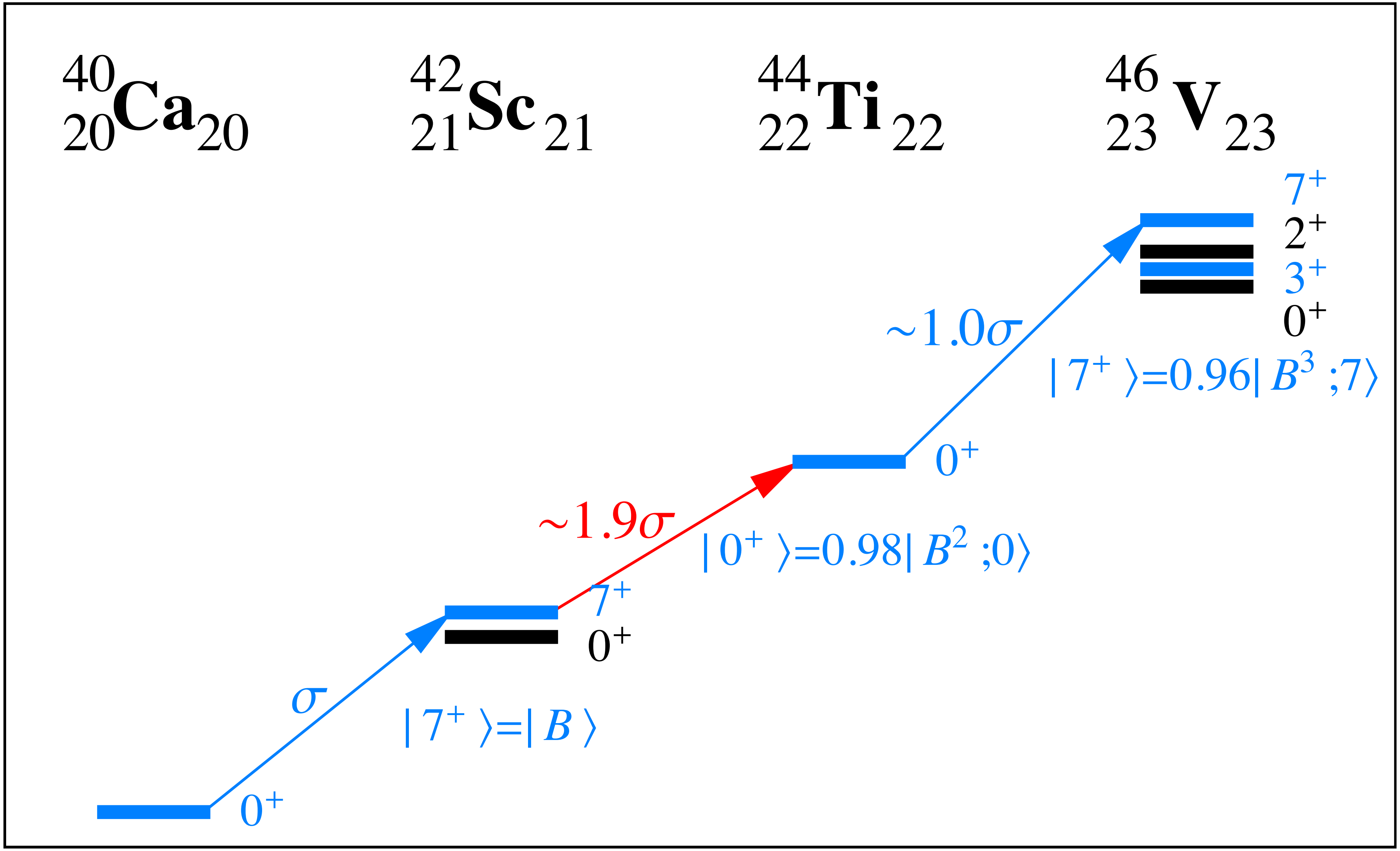


B. Le Crom *et al.*, Phys Lett. B **829** (2022) 137057

$T = 0$ aligned pairs in Ca–Sc–Ti–V?



$T = 0$ aligned pairs in Ca–Sc–Ti–V?



(d,^{4}He) transfer

What is the role of the spin-aligned neutron-proton pairs in the *fp*-shell ?

Study of the reactions ^{52}Fe(d,^{4}He) and ^{56}Ni(d,^{4}He)

[1]M. Assié, [2]G. de France, [3]G. de Angelis, [1]D. Beaumel, [1]Y. Blumenfeld, [2]E. Clément, [4]B. Fernandez-Dominguez, [3]F. Galtarossa, [1]V. Girard-Alcindor, [3]A. Gottardo, [1]F. Hammache, [1]H. Jacob, [5]N. Keeley, [4]A.O. Macchiavelli, [3]D. Mengoni, [1]O. Nasr, [1]N. de Séréville, [1]I. Stefan,[2]T. Roger, [7]F. Rotaru, [7]M. Stanoiu, [2]O. Sorlin, [2]J-C. Thomas, [2]P. van Isacker

[1] *IJCLab Orsay,[2]GANIL, [3]LNL, INFN-University of Padova, [4] University of Santiago di Compostela, [5]National centre for nuclear physics, Warsaw, [6]ORNL, [7]NIPNE Romania*

Deuteron clustering in shell model

How to quantify the probability of formation of a deuteron in a shell-model state?

This probability is obtained by summing over all np pairs with the *intrinsic* structure of a deuteron.

Put all quanta of the pair in the centre-of-mass coordinate and none in the relative degree of freedom.

Deuteron cluster probability

Deuteron projection operator in isospin formalism:

$$\hat{P}_d = \sum_{i<j} \hat{P}_d(i,j)$$

Expectation value of $\hat{P}_d$ in $|\Omega^n \alpha \Gamma_n\rangle$:

$$\langle \Omega^n \alpha \Gamma_n | \hat{P}_d | \Omega^n \alpha \Gamma_n \rangle = \frac{n(n-1)}{2} \sum_{\Gamma_{n-2}} [\Omega^2 \Gamma_d, \Omega^{n-2} \Gamma_{n-2} | \} \Omega^n \alpha \Gamma_n]^2 \left(\tilde{a}^{n_a l n_b l}_{\mathcal{N}000, \Gamma_d} \right)^2$$

Deuteron cluster probability

Deuteron projection operator in np formalism:

$$\hat{P}_d = \sum_{i\in\nu}\sum_{j\in\pi}\hat{P}_d(i,j)$$

Expect value of $\hat{P}_d$ in $|\Omega^n\Gamma_n\rangle \equiv |\Omega_\nu^{n_\nu}\Gamma_{n_\nu},\Omega_\pi^{n_\pi}\Gamma_{n_\pi};\Gamma_n\rangle$:

$$\langle\Omega^n\Gamma_n|\hat{P}_d|\Omega^n\Gamma_n\rangle$$

$$= n_\nu n_\pi \sum_{\Gamma_{n_\nu-1}\Gamma_{n_\pi-1}} \begin{bmatrix} \Gamma_{1_\nu} & \Gamma_{n_\nu-1} & \Gamma_{n_\nu} \\ \Gamma_{1_\pi} & \Gamma_{n_\pi-1} & \Gamma_{n_\pi} \\ \Gamma_d & \Gamma_{n-2} & \Gamma_n \end{bmatrix}^2 \left(a^{n_\nu l n_\pi l}_{\mathcal{N}000,\Gamma_d}\right)^2$$

$$\times\left[\Omega_\nu n_\nu l,\Omega_\nu^{n_\nu-1}\Gamma_{n_\nu-1}|\}\Omega_\nu^{n_\nu}\Gamma_{n_\nu}\right]^2\left[\Omega_\pi n_\pi l,\Omega_\pi^{n_\pi-1}\Gamma_{n_\pi-1}|\}\Omega_\pi^{n_\pi}\Gamma_{n_\pi}\right]^2$$

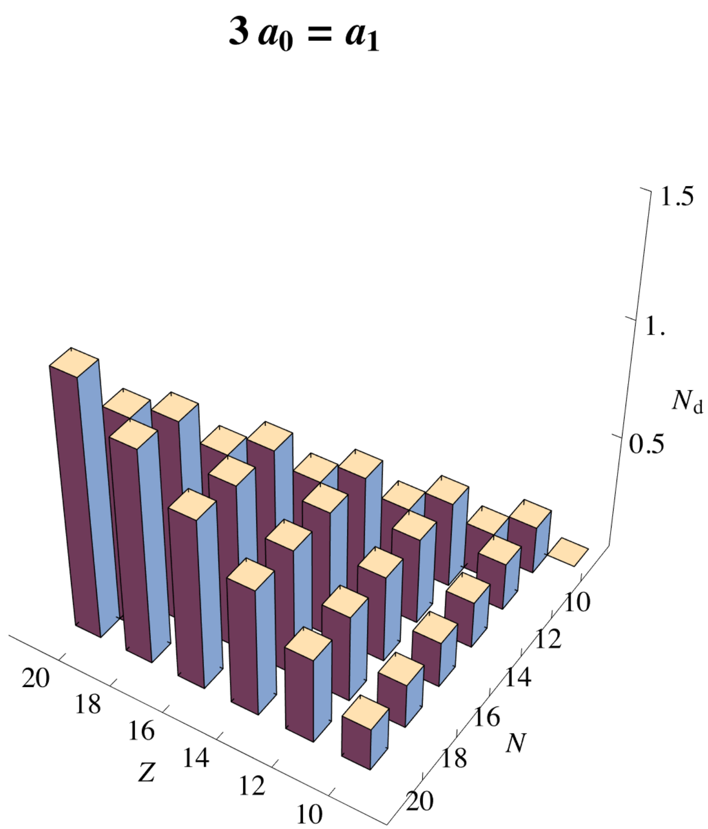
Example: the *sd* shell

How does the number of deuteron clusters N_d change with neutron and proton numbers?

Consider the ground state of even-even and odd-odd $N = Z$ nuclei in the sd shell.

Three choices of interaction:

- ***isovector*** $>$ ***isoscalar***
- *isovector* $=$ *isoscalar [SU(4)]*
- *isovector* $<$ *isoscalar*



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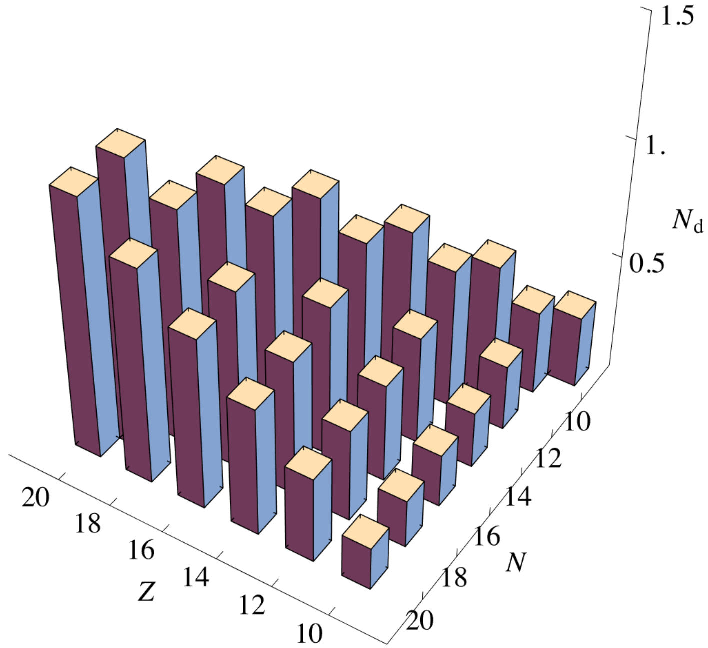
$a_0 = a_1$



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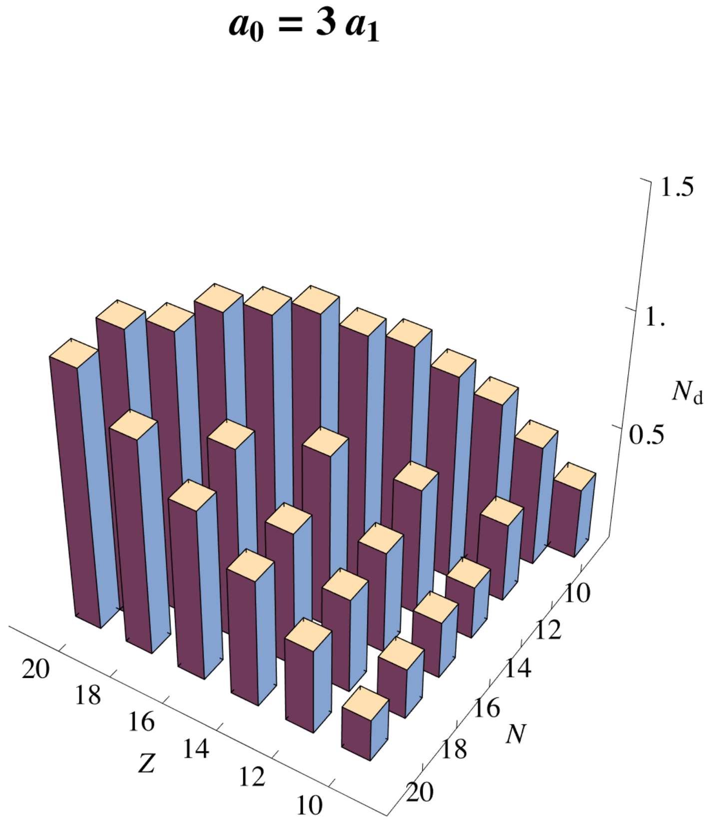

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The question is

- *Do* $T = 1$ *pairing correlations exist?*
- *Do* $T = 0$ *pairing correlations exist?*

The Osaka experiments

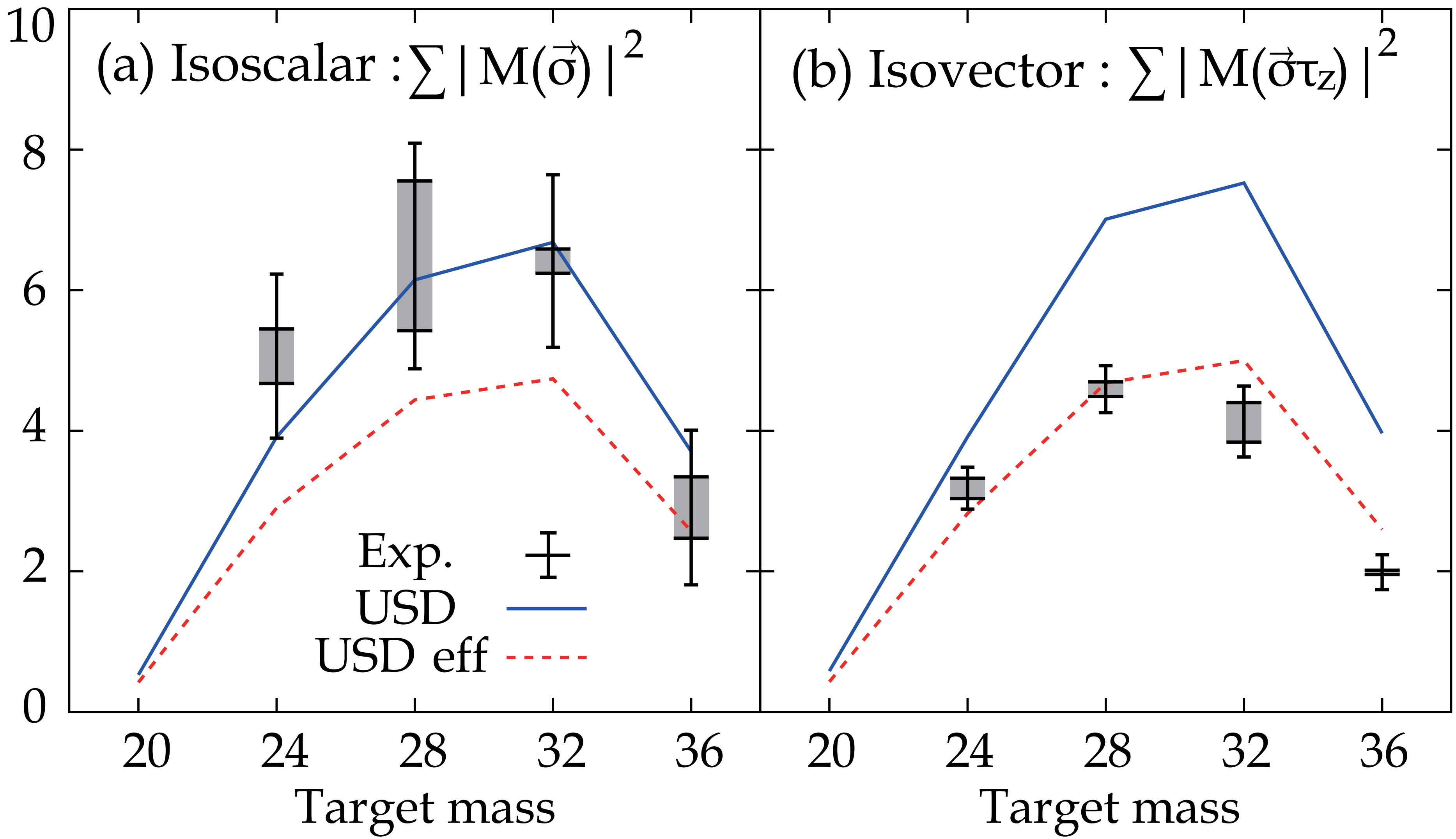
- Proton inelastic scattering at $E_p = 295\ \mathrm{MeV}$ on sd-shell nuclei.
- Isoscalar and isovector spin-M1 transition strengths up to $E_x = 16\ \mathrm{MeV}$.

$$S(\boldsymbol{\sigma}) = \sum_f \langle 1_f^+ || \Sigma_k \boldsymbol{\sigma}_k || 0_1^+ \rangle^2$$

$$S(\boldsymbol{\sigma}\tau_z) = \sum_f \langle 1_f^+ || \Sigma_k \boldsymbol{\sigma}_k \tau_{z,k} || 0_1^+ \rangle^2$$

H. Matsubara *et al.*, Phys. Rev. Lett. **115** (2015) 102501

The experimental result



H. Matsubara *et al.*, Phys. Rev. Lett. **115** (2015) 102501
H. Sagawa, T. Suzuki, Phys. Rev. C **97** (2018) 054333

Spin-spin correlations $\langle \boldsymbol{S}_n \cdot \boldsymbol{S}_p \rangle$

Neutron and proton spin operators

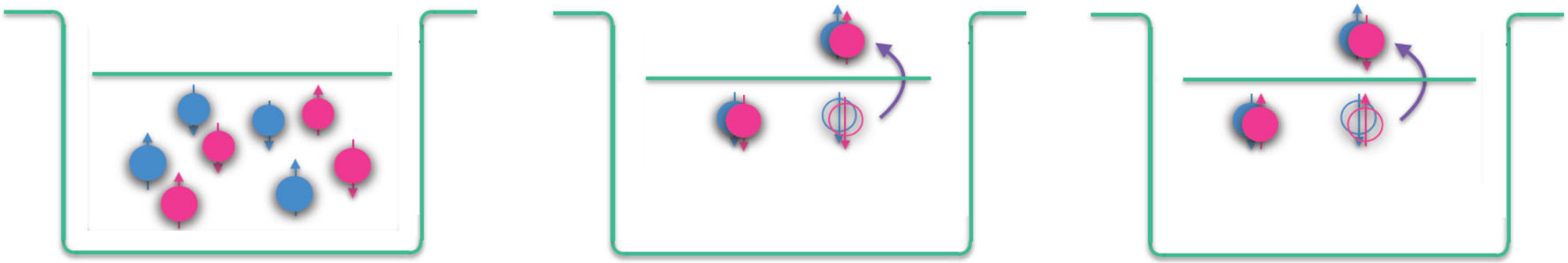
$$\boldsymbol{S}_n = \sum_{k \in n} \boldsymbol{\sigma}_k , \qquad \boldsymbol{S}_p = \sum_{k \in p} \boldsymbol{\sigma}_k$$

Relation between M1 transitions and neutron-proton spin-spin correlations:

$$\langle 0_1^+ | \boldsymbol{S}_n \cdot \boldsymbol{S}_p | 0_1^+ \rangle \approx \frac{1}{16} \{ S(\boldsymbol{\sigma}) - S(\boldsymbol{\sigma} \tau_z) \}$$

$\langle \boldsymbol{S}_n \cdot \boldsymbol{S}_p \rangle$: a pairing probe

$$\left\langle \vec{s}_n \cdot \vec{s}_p \right\rangle = \begin{cases} +\dfrac{1}{4} & \text{for IS } np \text{ pair (deuteron)} \quad \text{statistical weight} = 3 \\ -\dfrac{3}{4} & \text{for IV } np \text{ pair} \quad \text{statistical weight} = 1 \end{cases}$$



IS np pairing

IV np pairing

$\left\langle \vec{S}_n \cdot \vec{S}_p \right\rangle = 0$

$\left\langle \vec{S}_n \cdot \vec{S}_p \right\rangle > 0$

$\left\langle \vec{S}_n \cdot \vec{S}_p \right\rangle < 0$

Unperturbed ground state

Correlated ground state

$\langle \boldsymbol{S}_n \cdot \boldsymbol{S}_p \rangle$: a pairing probe

Some simple results:

LS-coupling

$$\langle (l_n, \tfrac{1}{2})(l_p, \tfrac{1}{2}); LS | S_n \cdot S_p | (l_n, \tfrac{1}{2})(l_p, \tfrac{1}{2}); LS \rangle =$$

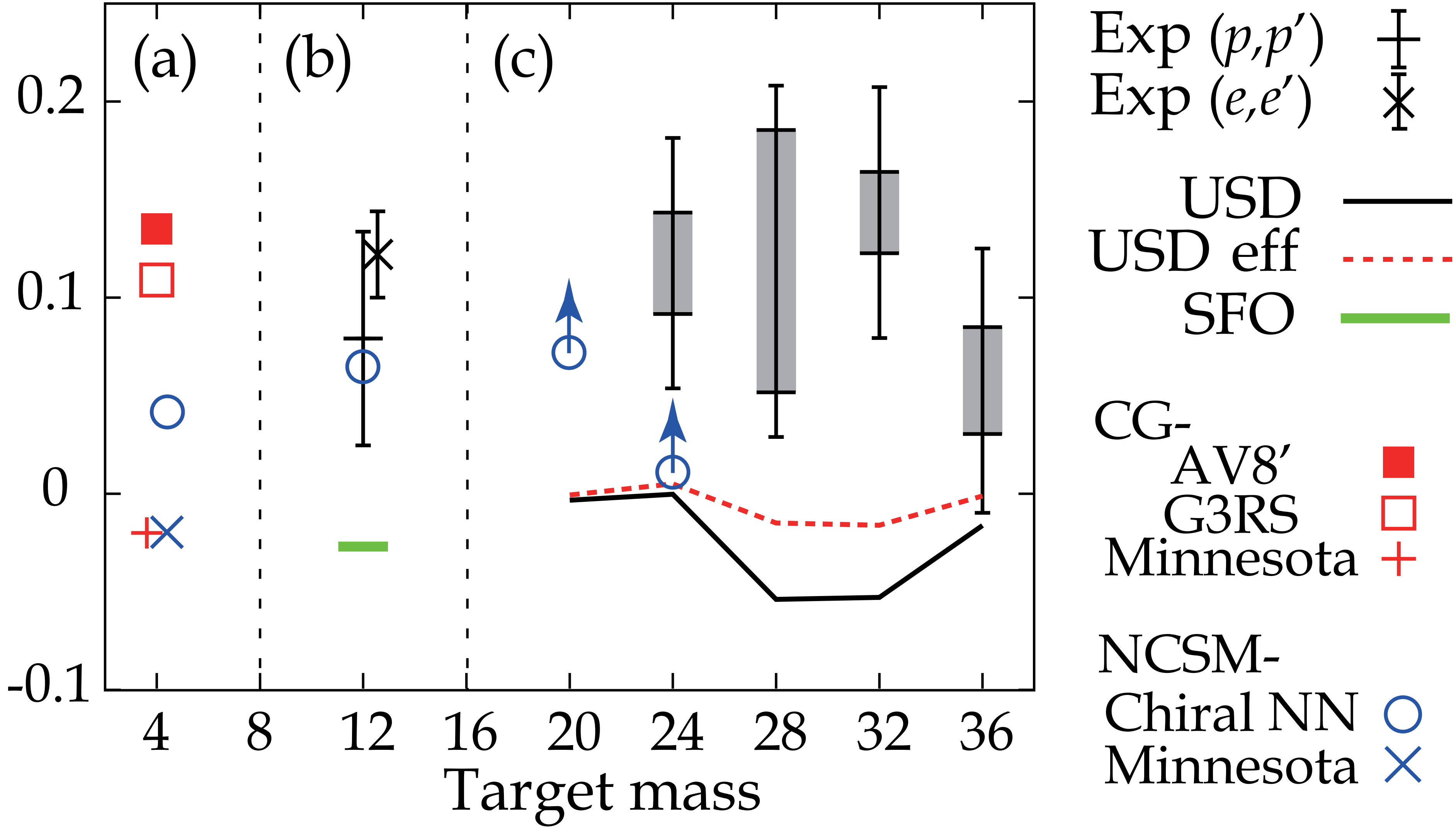
$$= \frac{2}{3}(-)^{S+1} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & S \end{Bmatrix} = \begin{cases} +\frac{1}{4}, & S = 1, T = 0 \\ -\frac{3}{4}, & S = 0, T = 1 \end{cases}$$

jj-coupling:

$$\langle j_n j_p; J | S_n \cdot S_p | j_n j_p; J \rangle = \frac{J(J+1) - 2j(j+1)}{2(2l+1)^2}$$

P. Van Isacker, A.O. Macchiavelli, Eur. J. Phys. **57** (2021) 178

Spin-spin correlations $\langle \boldsymbol{S}_n \cdot \boldsymbol{S}_p \rangle$



Schematic shell-model calculations

Single-l shell (with $j = l \pm \frac{1}{2}$) and a SDI with isoscalar and isovector strengths:

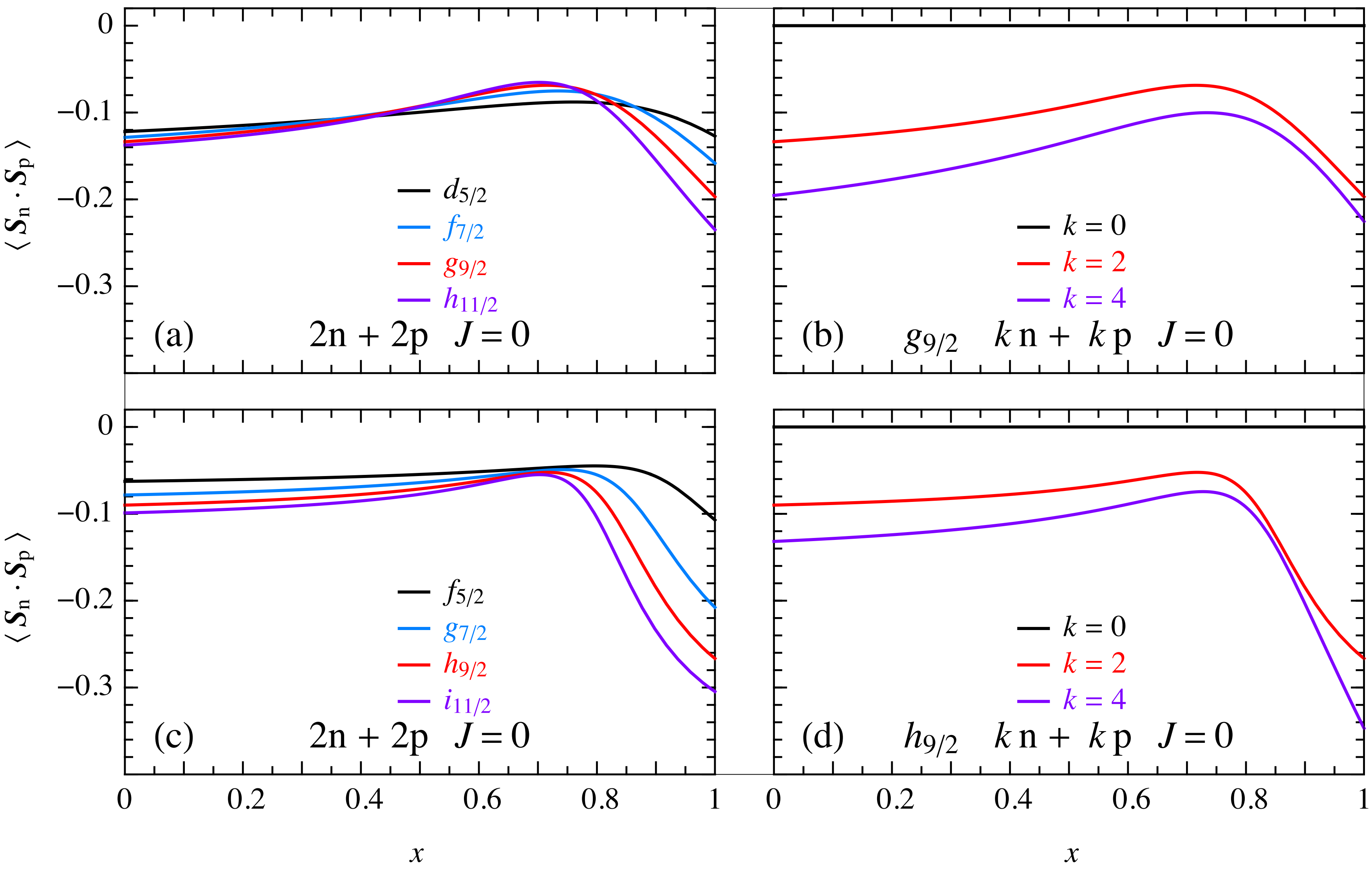
$$\hat{H} = \sum_{\pm} \varepsilon_{\pm} \hat{n}_{\pm} - \sum_{T=0,1} a_T \sum_{i<j} \delta(\boldsymbol{r}_i - \boldsymbol{r}_j)\, \delta(\boldsymbol{r}_i - \boldsymbol{R}_0)$$

Spin-spin correlation $\langle \boldsymbol{S}_n \cdot \boldsymbol{S}_p \rangle$ is always negative in the ground state of even-even nuclei!

$$x = \frac{a_0}{a_0 + a_1}, \qquad y = \frac{\Delta\varepsilon/(a_0 + a_1)}{5 + |\Delta\varepsilon/(a_0 + a_1)|}$$

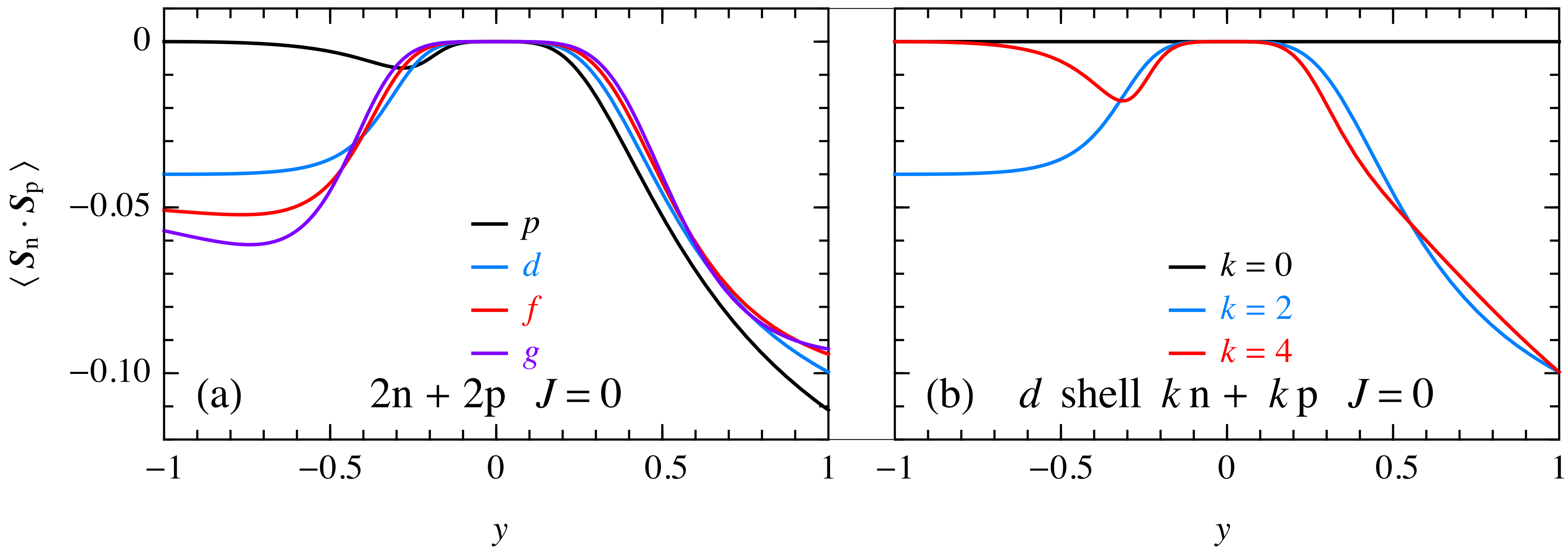
P. Van Isacker, A.O. Macchiavelli, Eur. J. Phys. **57** (2021) 178

Examples



0
−0.1
−0.2
−0.3
$\langle S_n \cdot S_p \rangle$
$d_{5/2}$
$f_{7/2}$
$g_{9/2}$
$h_{11/2}$
(a) 2n + 2p $J = 0$
$k = 0$
$k = 2$
$k = 4$
(b) $g_{9/2}$ k n + k p $J = 0$
$f_{5/2}$
$g_{7/2}$
$h_{9/2}$
$i_{11/2}$
(c) 2n + 2p $J = 0$
(d) $h_{9/2}$ k n + k p $J = 0$
0 0.2 0.4 0.6 0.8 1
x

Examples



0
−0.05
−0.10
⟨ $S_n \cdot S_p$ ⟩
p
d
f
g
(a) 2n + 2p $J = 0$
$k = 0$
$k = 2$
$k = 4$
(b) d shell k n + k p $J = 0$
−1
−0.5
0
0.5
1
y

(p,p') and (d,d') on $^{44,46}Ti$

Research Proposal
PHYSICAL SCIENCES, SSC FACILITY
iThemba Laboratory for Accelerator Based Sciences

1 Title of Proposed Experiment

Ground state neutron-proton spin-spin correlations studied by (p,p') and (d,d') scattering

2 Date

August 16th, 2019

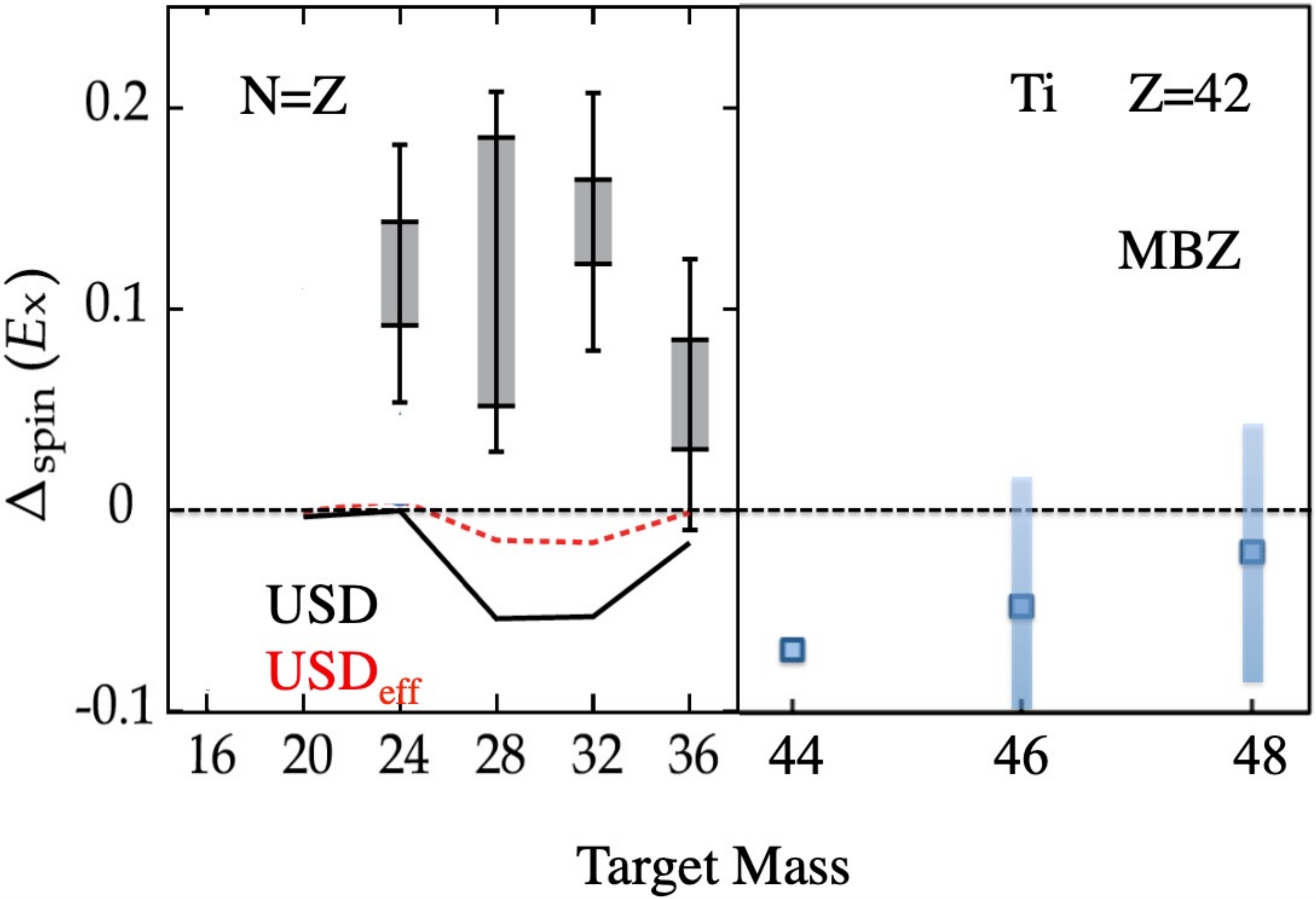
3 Members of the Collaboration

Spokesperson: A. O. Macchiavelli[a1]

H. L. Crawford [a], C. M. Campbell[a], R. M. Clark[a], M. Cromaz[a], P. Fallon[a], I. Y. Lee[a], C. Morse[a], C. Santamaria[a], L. Pellegri[b,c], R. Neveling[c], M. Wiedeking[c], P. Adsley[c,b], L. M. Donaldson [c], R. V. F. Janssens[d], H. Matsubara[e], P. von Neumann-Cosel[f], F.D. Smit[c], and A. Tamii[g]

Theoretical support: P. Van Isacker[h]

(p,p') and (d,d') on $^{44,46}Ti$



Conclusion

Isovector (isoscalar) spin-M1 transitions are (not) quenched.

This observation has no simple explanation in the shell model.

Outlook

Experiments:

Investigate odd-odd N=Z nuclei;

Go slightly off N=Z line.

Theory:

More realistic Hamiltonian (shell mixing, tensor force,...)

Revisit isoscalar pairing?

High-momentum correlated nucleon pairs?