TRIUMF

Ab initio calculations of spin-singlet and spin-triplet pairing gaps in infinite nucleonic matter

Georgios Palkanoglou
TRIUMF

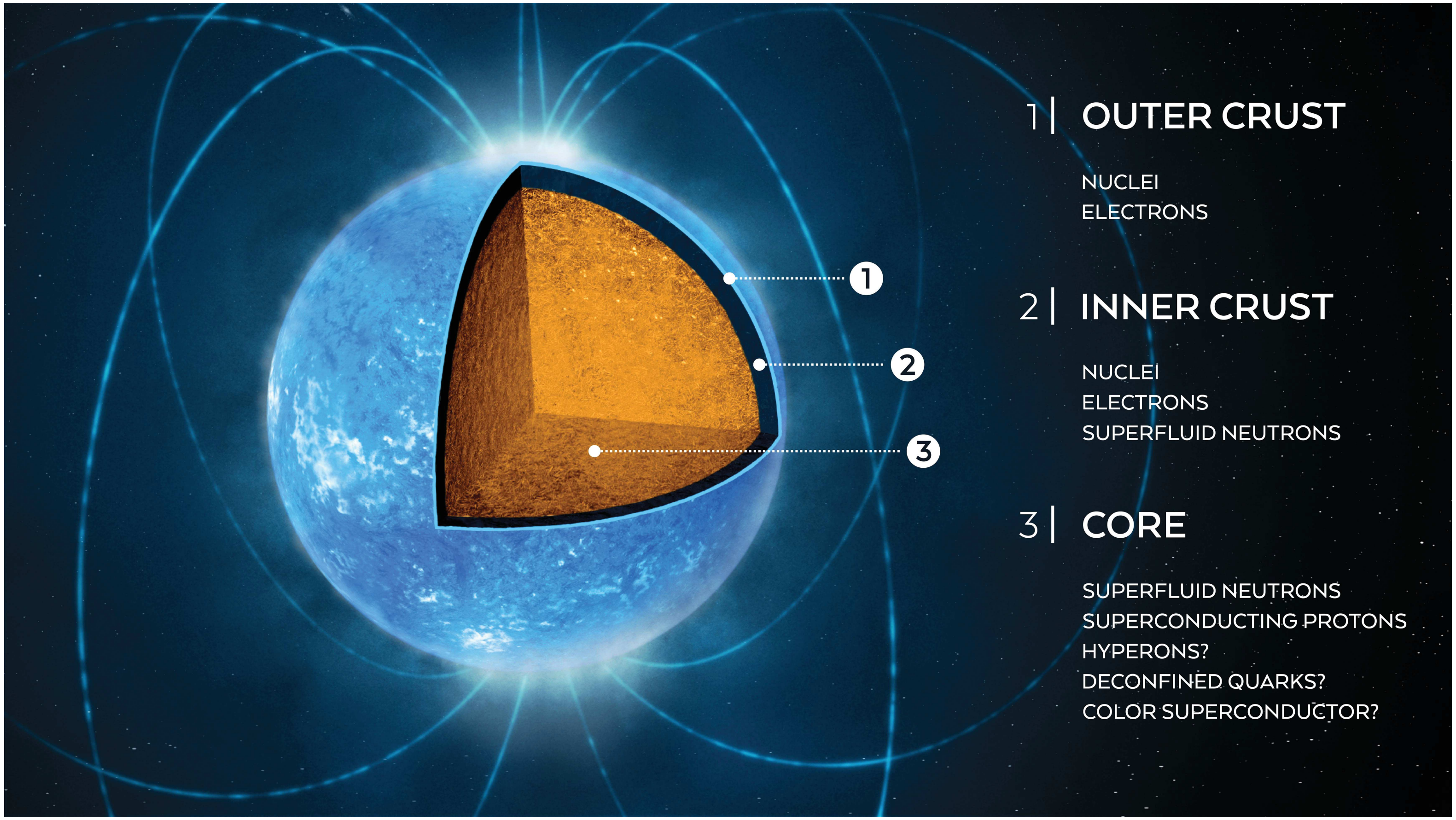
Introductory talk at ESNT

May 13, 2025

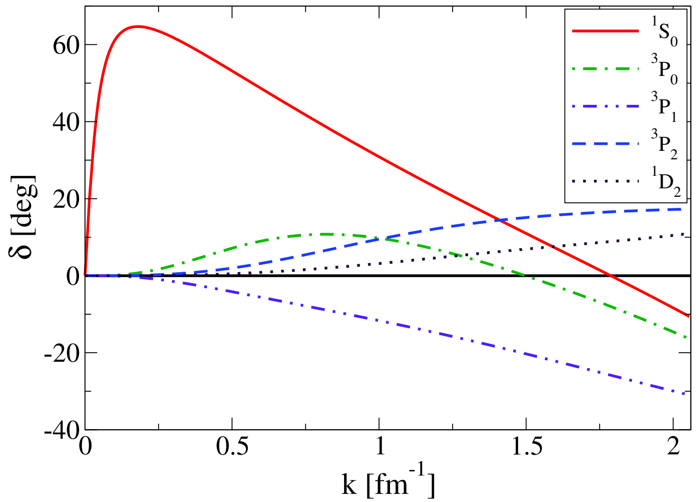
Discovery, accelerated

Outline

- Introduction
  - Definitions
  - Expectation from experiment
  - *Ab initio* techniques discussed
- Calculations of spin-singlet pairing gaps
  - **Neutrons:** Phenomenological analysis
  - **Neutrons:** Initial *ab initio* calculations
  - **Neutrons:** Validation from cold-atom experiments
  - **Neutrons:** Latest *ab initio* results
  - **Protons:** in passing
- Calculations of spin-triplet pairing gaps
  - **Neutrons:** Phenomenological analysis
  - **Neutrons:** *ab initio* calculations
- *Summary*


1
2
3
1 | OUTER CRUST
NUCLEI
ELECTRONS
2 | INNER CRUST
NUCLEI
ELECTRONS
SUPERFLUID NEUTRONS
3 | CORE
SUPERFLUID NEUTRONS
SUPERCONDUCTING PROTONS
HYPERONS?
DECONFINED QUARKS?
COLOR SUPERCONDUCTOR?

Definitions and expectations: PW analysis



From ***[arXiv:1406.6109]***

$$S = 0$$

$L = 0$: Strongly attractive up to $k_F \sim 1.7 \text{ fm}^{-1}$

$L = 2$: Weakly attractive at higher densities

$$S = 1$$

$\underset{(J=0)}{L=1}$: attractive up to $k_F \sim 1.5 \text{ fm}^{-1}$. Maybe not enough.

$\underset{(J=2)}{L=1}$: The most attractive channel after $k_F \sim 1.5 \text{ fm}^{-1}$. Large contribution from the spin-orbit force.

Pairing happens close to k_F so roughly translating k to k_F

$$\phi_{\text{pair}}(\mathbf{r}) = \sum_{L=\ldots} \phi_L(\mathbf{r})$$

Discovery, accelerated

What I will call *ab initio*

They should be

1. Systematically improvable
2. Controlled approximations
3. Any fitting involved done at lower energy scales / complexity

The ones that I will discuss and fit this

QMC: Quantum Monte Carlo (QMC; continuum / lattice)

+ (AF)DMC, AFQMC, dQMC, ...

SCGF: Self Consistent Green's Functions

+ SCGF, Gorkov SCGF, NC-SCGF, ...

Discovery, accelerated

TRIUMF

Two *ab initio* approaches

Quantum Monte Carlo

Solution via stochastic methods.

It needs a trial wavefunction that defines the nodal surface
For **fixed-node** nodal surface is frozen $\longrightarrow$ upper-limit for E_{gs}

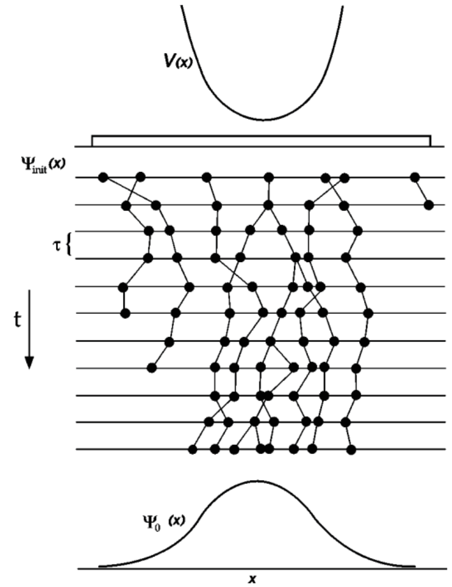
In canonical ensemble (fixed $\boxed{N}$)

[Foulkes *et al* Rev. Mod. Phys. **73**, 33 (2003)]

Based on ***diffusion***

$$\psi(\tau) = e^{-(H-E_0)\tau}\psi_{\text{init}} \quad \xrightarrow{\tau\to\infty} \quad c_0\psi_0$$

of the trial wavefunction ψ_{init}.



Note: we'll focus a lot on continuum QMC

Discovery, accelerated

Two *ab initio* approaches

Self-Consistent Green's Functions

Solution via determining the right one-body Green's function.

It needs a class of diagrams for the self-energy

In grand-canonical ensemble (TL)

[Rios, Front. Phys., vol. **8**, 387 (2020)]

Based on determining the ***spectral function*** $\mathcal{A}(\omega)$

$$\mathcal{G}_k(z) = \int \frac{d\omega}{2\pi} \frac{\mathcal{A}_k(\omega)}{z-\omega}$$

from the self-energy's Dyson Eq.

$$\mathcal{A}_k(\omega) = \frac{-2\mathrm{Im}\Sigma_k(\omega)}{\left[\omega - \frac{k^2}{2m} - \mathrm{Re}\Sigma_k(\omega)\right]^2 + \left[\mathrm{Im}\Sigma_k(\omega)\right]^2}$$

Physical quantities derived from $\mathcal{A}_k(\omega)$ [or $\Sigma_k(\omega)$].

Spin-Singlet

TRIUMF

Phenomenological estimation

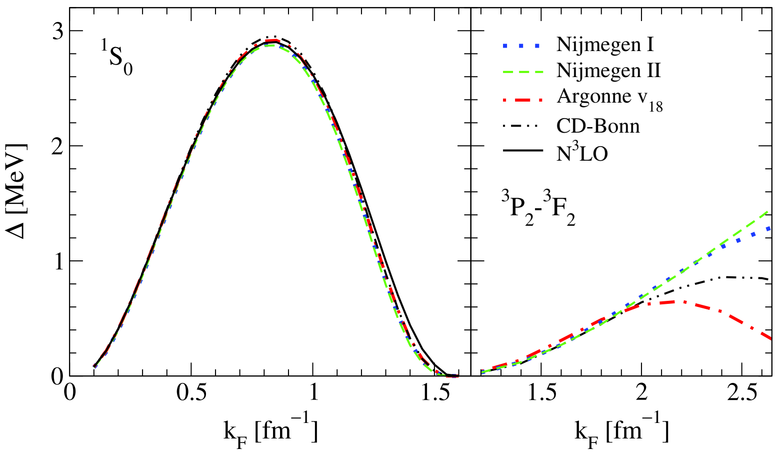
The simplest model: uncorr. pairs with $L = 0$

$$|\psi\rangle \propto \exp\left[\sum_{\mathbf{k}} g_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}\right] |0\rangle = \sum_N A_N \left(g_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}\right)^{N/2} |0\rangle$$

with g minimizing the energy.

$$\Delta(k) = -\frac{1}{\pi}\int dp p^2 V(k,p)\Delta(k)/E(k) \ .$$

$$\Delta = \min E_{\rm qp}(\mathbf{k}; k_F) = \Delta(k_F)$$



[Gezerlis *et al*, arxiv:1406.6109]

Expectations:

- Maximum of around $3\ \mathrm{MeV}$
- Independent of interaction

Discovery, accelerated

TRIUMF

Challenges for an *ab initio* approach

Precision:

1. *How to extract gap?:* OES, $\Delta(k_F;\Lambda)$,...
2. *Which correlations are important?* How many superfluid. corr. do I need? Yes

$\star$**3.** *Precision*: $\sim \%$ is essential. (Also in experimental measurements)

$\star$**4.** *FSE:* an extrapolation to the TL needs large $\boxed{N}$

“Could be worse”

$\star$**3.** Precision: In terrestrial superconductors higher precision is needed Δ/E_F is much smaller

$\star$**4.** FSE: driven by range of interaction

Discovery, accelerated

TRIUMF

One challenge at a time: 1. Recipe

"What is a (model independent) recipe for the pairing gap?"

- Minimum of qp-excitation: $\Delta^{(\mathrm{qp})} = \mathrm{min}_{\mathbf{k}} E_{\mathrm{qp}}(\mathbf{k})$
  - + Clear connection to pairing
  - + Model-specific
- The gap-function (or *pairing potential / anom. self energy*) at k_F: $\Delta^{(\mathrm{BCS})} = \Delta(\mathbf{k}_{\mathrm{F}})$
  - + Accessible mainly to Green's function's approaches (BCS, SCGF, *etc*)
- Odd-Even staggering: $\Delta^{(\mathrm{OES})} = E(N) - \frac{1}{2}\left[E(N-1) + E(N+1)\right]$
  - + Accessible to all
  - + "*Does it measure pairing?*"

Discovery, accelerated

TRIUMF

1. Recipe

$$\Delta^{(3)}(N) = E(N) - \frac{1}{2}\left[E(N+1) + E(N-1)\right]$$

OES has a **long history** in **nuclei**.

- A. Bohr, B.R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).
- H. Häkkinen, J. Kolehmainen, M. Koskinen, P. O. Lipas, and M. Manninen, Phys. Rev. Lett. **78**, 1034 (1996).
- W. Satuła, J. Dobaczewski, and W. Nazarewicz, Phys. Rev. Lett. **81**, 3599 (1998).
- M. Bender, K. Rutz, P.-G. Reinhard and J.A. Maruhn, Eur. Phys. J. A**8**, 59 (2000).
- Duguet, Bonche, Heenen, and Meyer, Phys. Rev. C **65**, 014311 (2001).
- ...

*What about **matter** (in a box)?*

Shell effects: **nuclei** $>$ $\boxed{\text{neutrons}}$ Because nn's finite $r_e = 2.7$ fm.
OES(3) is enough: A. Gezerlis, *et al*, arXiv:1406.6109 (2014); GP, *et al*, Phys. Rev. C **102**, 064324 (2020).

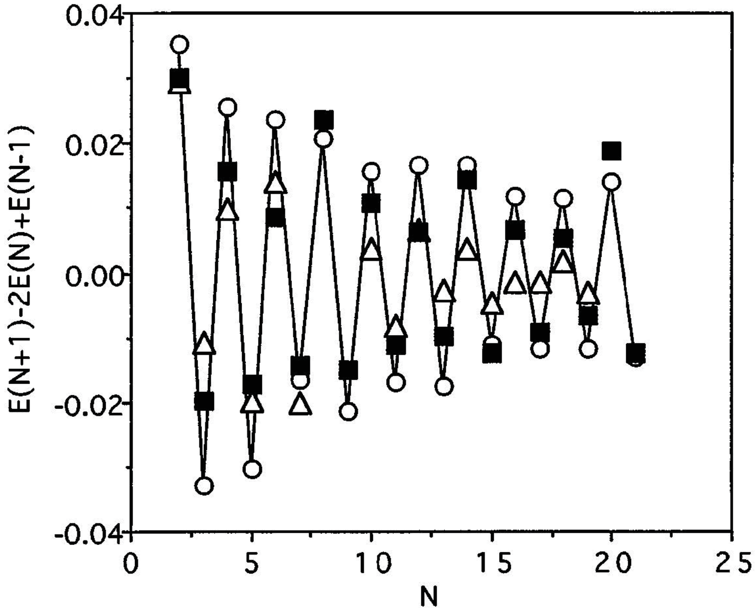
Fixed volume / fixed density?
Doesn't matter: S. Gandolfi, *et al*, Phys. Rev. Lett **101**, 132501 (2008).

Discovery, accelerated

TRIUMF

1. Recipe

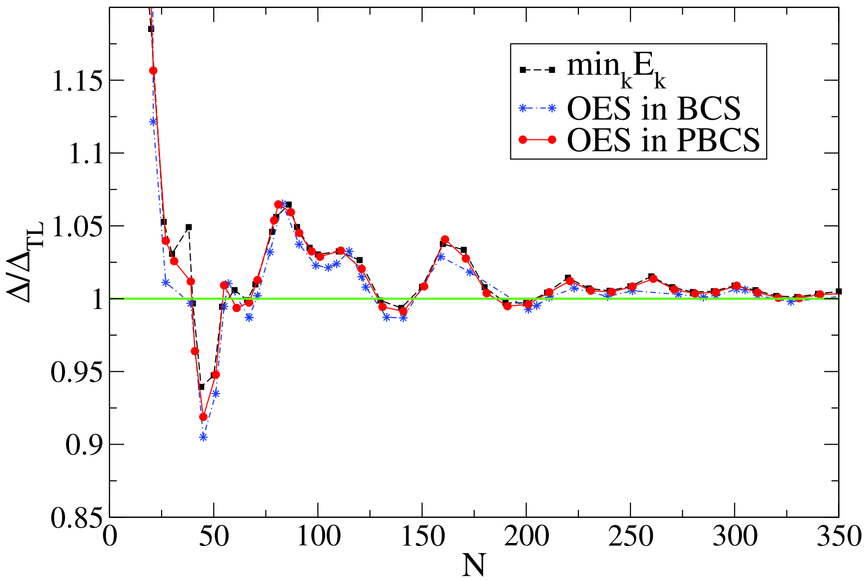
Small clusters:
universal OES from deformation



■ OES in atomic clusters (pair BE $\sim 10^{-5}$)
–◯– OES in nuclei (pair BE $\sim 10^{-2}$)
△ OES in the electron gas

Häkkinen *et al*, Phys. Rev. Lett. **78**, 1034 (1996)

Many neutrons:
OES measures pairing



■ BCS quasiparticle gap

$\ast$, ● OES

GP *et al*, Phys. Rev. C **102**, 064324 (2020)

Discovery, accelerated

2. Add. Correlations

Aren't Slater Determinants (SDs) enough ?

$$\psi_{\text{init}} = \langle \text{SD}|\{\mathbf{r}\}_N\rangle = \det\begin{pmatrix} \phi_{\mathbf{k}_1}(\mathbf{r}_{1\uparrow}) & \cdots & \phi_{\mathbf{k}_N}(\mathbf{r}_{1\uparrow}) \\ \phi_{\mathbf{k}_1}(\mathbf{r}_{2\uparrow}) & \cdots & \phi_{\mathbf{k}_N}(\mathbf{r}_{2\uparrow}) \\ \vdots & \vdots & \vdots \\ \phi_{\mathbf{k}_1}(\mathbf{r}_{\frac{N}{2}\uparrow}) & \cdots & \phi_{\mathbf{k}_N}(\mathbf{r}_{\frac{N}{2}\uparrow}) \end{pmatrix} \det\begin{pmatrix} \phi_{\mathbf{k}_1}(\mathbf{r}_{1\downarrow}) & \cdots & \phi_{\mathbf{k}_N}(\mathbf{r}_{1\downarrow}) \\ \phi_{\mathbf{k}_1}(\mathbf{r}_{2\downarrow}) & \cdots & \phi_{\mathbf{k}_N}(\mathbf{r}_{2\downarrow}) \\ \vdots & \vdots & \vdots \\ \phi_{\mathbf{k}_1}(\mathbf{r}_{\frac{N}{2}\downarrow}) & \cdots & \phi_{\mathbf{k}_N}(\mathbf{r}_{\frac{N}{2}\downarrow}) \end{pmatrix}$$

No, SDs block pair exchanges

$$\begin{array}{rcccc} & \uparrow_1\downarrow_1 \quad \leftarrow\delta r\rightarrow \quad \uparrow_2\downarrow_2 & & \uparrow_2\downarrow_2 \quad \leftarrow\ \delta r\rightarrow \quad \uparrow_1\downarrow_1 \\ \hat{F}_x\ : & x=0 & & x=1 \\ \mathbf{r}_{i\uparrow}=\mathbf{r}_{i\downarrow}\ : & \psi=\psi_\uparrow^2>0 & \psi=0 & \psi=\psi_\uparrow^2>0 \\ \mathbf{r}_{i\uparrow}=\mathbf{r}_{i\downarrow}+\delta\mathbf{r}\ : & \psi_\uparrow\psi_\downarrow>0 & \boxed{\psi_\uparrow<0\ \psi_\downarrow>0} & \psi_\uparrow\psi_\downarrow>0 \end{array}$$

For three pair exchanges: BCS/SD $\sim$ 1.7 more

J. Carlson *et al*, Phys. Rev. Lett. **91**, 050401 (2003)

TRIUMF

2. Add. Correlations

Make SDs with pairing: the BCS determinant (or *antisymmetrized product of geminals*):

$$\langle \mathrm{BCS} | \{\mathbf{r}\}_N \rangle = \psi_{\mathrm{BCS}}(\mathbf{r}_{1\uparrow}, \ldots, \mathbf{r}_{\frac{N}{2}\downarrow}, \mathbf{r}_{1\downarrow}, \ldots, \mathbf{r}_{\frac{N}{2}\uparrow}) = \det \begin{pmatrix} \phi(\mathbf{r}_{1\uparrow 1\downarrow}) & \phi(\mathbf{r}_{1\uparrow 2\downarrow}) & \cdots & \phi(\mathbf{r}_{1\uparrow \frac{N}{2}\downarrow}) \\ \phi(\mathbf{r}_{2\uparrow 1\downarrow}) & \phi(\mathbf{r}_{2\uparrow 2\downarrow}) & \cdots & \phi(\mathbf{r}_{2\uparrow \frac{N}{2}\downarrow}) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(\mathbf{r}_{\frac{N}{2}\uparrow 1\downarrow}) & \phi(\mathbf{r}_{\frac{N}{2}\uparrow 2\downarrow}) & \cdots & \phi(\mathbf{r}_{\frac{N}{2}\uparrow \frac{N}{2}\downarrow}) \end{pmatrix}$$

where the geminals

$$\phi(\mathbf{r}_{n\uparrow m\downarrow}) \text{ antisymm in } n\uparrow \longleftrightarrow m\downarrow \quad \Longrightarrow \quad \boxed{\text{only } \textbf{singlet} \text{ and } \textbf{central interactions}}$$

Blatt, Progr. Theoret. Phys. (Kyoto) **23**, 447 (1960)

Coleman, J. of Math. Phys. **6**, 1425 (1965)

Bouchaud *et al*, J. Phys. Paris **49**, 553, (1988)

Carlson *et al*, Phys. Rev. Lett. 91, 050401 (2003)

Gezerlis *et al*, Phys. Rev. C **77**, 032801(R) (2008)

⋮

Discovery, accelerated

2. Add. Correlations

More general pair wavefunction: the "Pfaffian"

$$\langle \text{Gen BCS} | \{\mathbf{r}\}_N \rangle = \psi(\mathbf{r}_{1\uparrow}, \dots, \mathbf{r}_{\frac{N}{2}\downarrow}, \mathbf{r}_{1\downarrow}, \dots, \mathbf{r}_{\frac{N}{2}\downarrow}) = \mathrm{pf}\begin{pmatrix} \Phi_{\uparrow\uparrow} & \Phi_{\uparrow\downarrow} & \varphi_{\uparrow} \\ -\Phi^T_{\uparrow\downarrow} & \Phi_{\downarrow\downarrow} & \varphi_{\downarrow} \\ -\varphi^T_{\uparrow} & \varphi^T_{\downarrow} & 0 \end{pmatrix}$$

where φ sp orbital and the pair orbitals

$$\phi(\mathbf{r}_i s_{z_i}, \mathbf{r}_j s_{z_j}) = \sum_{SS_z} \phi_{SS_z}(\mathbf{r}_i s_{z_i}; \mathbf{r}_j s_{z_j}) \iff \boxed{\text{can handle } \textbf{all spin} \text{ pairing and } \textbf{tensor interactions}}$$

Bajdich *et al*, Phys. Rev. B **77**, 115112 (2008).

Gandolfi *et al*, Phys. Rev. Lett. **99**, 022507 (2007)

Gandolfi *et al*, Condens. Matter **7**, 19 (2022)

One challenge at a time: 3. FSE

Finite Size Effects (**FSE**): *deviations of intensive quantities from their Thermodynamic Limit* (TL)

$$N/N_{\mathrm{corr}} \gg 1\ , \quad V/x \gg 1\ , \quad N/V = \mathrm{const.}$$

Generally speaking their amplitude depends on the density and the range of the interaction

$$R_{\mathrm{int}}\rho^{1/3} \sim k_F r_e$$

Compare: **cold alkali atoms** $\sim 10^{-2} \ll$ **neutrons** ~ 1.

Still *could be worse*:

a) strong interactions smear the dramatic shell effects (i.e., s.p. is unlikely)

b) Nuclear forces short-ranged $r_e = 2.7\ \mathrm{fm}$, so dilute NM is OK.

$$\boxed{\textbf{Careful: FSE}\ \text{worse for pairing gap than for the energy}}$$

Box $L^3 \implies \delta k_n \propto \frac{1}{L} \implies \boxed{E(N+1)}$ can't access lowest exc.

Other recipes suffer the same [e.g., $\min_{\mathbf{k}} E_{\mathrm{qp}(\mathbf{k})}$]

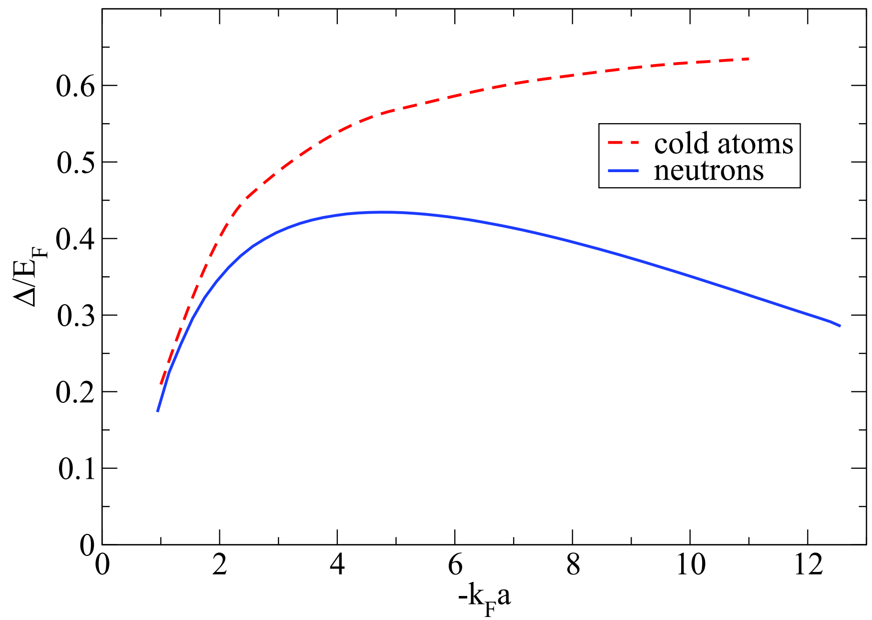
Discovery, accelerated

TRIUMF

3. FSE

Extra scale: size of long-range order (pair-size):

$$\delta x \sim \frac{E_F}{\Delta k_F} \approx 6.5 \text{ fm} < L(66) = 16 \text{ fm} \quad \text{for } \boxed{\text{neutrons}} \; at \; k_F = 0.8$$



Gezerlis *et al*, Phys. Rev. C **77**, 032801(R) (2008)

Gandolfi *et al*, Condens. Matter **7**, 19 (2022)

Discovery, accelerated

TRIUMF

4. Precision

What precision is needed to calculate the pairing gap, a fraction of the total energy:

$$\Delta(N) = E(N) - \frac{1}{2}\left[E(N+1) + E(N-1)\right]$$

Some scaling:

- $E(N) \sim NE_F$ **fermions**
- $\Delta \sim E_F$ **strongly paired fermions**
- **precision needed** $\sim \Delta/E(N) = \Delta/NE_F \approx 1/N \sim$ a few %

Note: in terrestrial superconductors $\Delta/E \sim 10^{-3}$. ($\Delta \sim \mathrm{meV}$).

Discovery, accelerated

TRIUMF

Neutrons and (cold) atoms

Neutron Matter

- In neutron stars and heavy nuclei
- Close to unitary gas $a = -18.5\ \mathrm{fm}$ (plus small effective range $r_e/a \approx 0.15$)
- MeV scale

Cold atoms

- Experimentally accessible
- Tunable s-wave (now and p-wave) interactions
- peV scale

Similar E/E_F and Δ/E_F

Discovery, accelerated

QMC calculations

Cold atoms: $k_{\mathrm{F}} a \gg 1\ ;\quad k_{\mathrm{F}} r_{\mathrm{e}} \ll 1$ (*unitarity*)

$$\xi = E/E_{\mathrm{F}}\ ,\quad \eta = \Delta/E_{\mathrm{F}}$$

Ref.	**Method**	**Wf**	**Pot**	ξ	η
2003[1]	DMC	BCS	PT	$0.44(1)$	0.54
2005[2]	DMC	BCS	PT	$0.42(1)$	$0.50(5)$
2008[3]	PIMC	–	HAP	$0.37(5)$	0.5
2008[4]	DMC	BCS	PT	$0.40(1)$	$0.45(5)$
2011[5]	DMC/DFT	BCS	PT	$0.383(1)$	$0.87(2)$
2008[6]	**EXP**	–	–	0.43(3)	0.44(3)

1 Carlson *et al*, Phys. Rev. Lett. **91**, 050401 (2003); Chang et al, Phys. Rev. A **70**, 043602 (2004)

2 Carlson *et al*, PRL **95**, 060401 (2005)

3 Magierski *et al*, Phys. Rev. Lett. **103**, 210403 (2008)

4 Carlson *et al*, PRL **100**, 150403 (2008)

5 Gezerlis *et al*, Phys. Rev. C **77**, 032801(R) (2008)

6 A. Schirotzek *et al*, Phys. Rev. Lett. **101**, 140403 (2008)

TRIUMF

QMC calculations

Neutrons

$$k_F(\rho_0 = 0.16 \text{ fm}^{-3}) \approx 1.7 \text{ fm}^{-1}$$

Ref.	**Method**	**Wf**	**Pot. 2N**	**Pot. 3N**	Max k_F [fm^{-1}]	Notes
2004[1]	GFMC+	BCS	AV8	–	1.05	$N = 11 - 15$
2005[2]	AFDMC	CBF-Pfaff.	AV8	– / UIX	0.8	$N = 13 - 17$
2007[3]	dQMC	–	s-$\cancel{\pi}$EFT (NLO)	–	0.6	from c. fract.
2008[4]	AFDMC	CBF-Pfaff.	AV8	UIX	0.8	$N = 64 - 68$
2008[5]	DMC	v-BCS	AV4	–	0.54	$N = 66$
2022[6]	AFDMC	v-Pfaff	AV8 / N2LO	UIX / N2LO	1.7	$N = 47$

[1] Chang *et al*, Nucl. Phys. A **746**, 215-221 (2004)
[2] Fabrocini *et al*, Phys. Rev. Lett. **95** 192501 (2005)
[3] Abe *et al*, Phys. Rev. C **79**, 054002 (2009)
[4] Gandolfi *et al*, Phys. Rev. Lett. **101**, 132501 (2008)
[5] Gezerlis *et al*, Phys. Rev. C **77**, 032801(R) (2008)
[6] Gandolfi *et al*, Condens. Matter **7** 19 (2022)

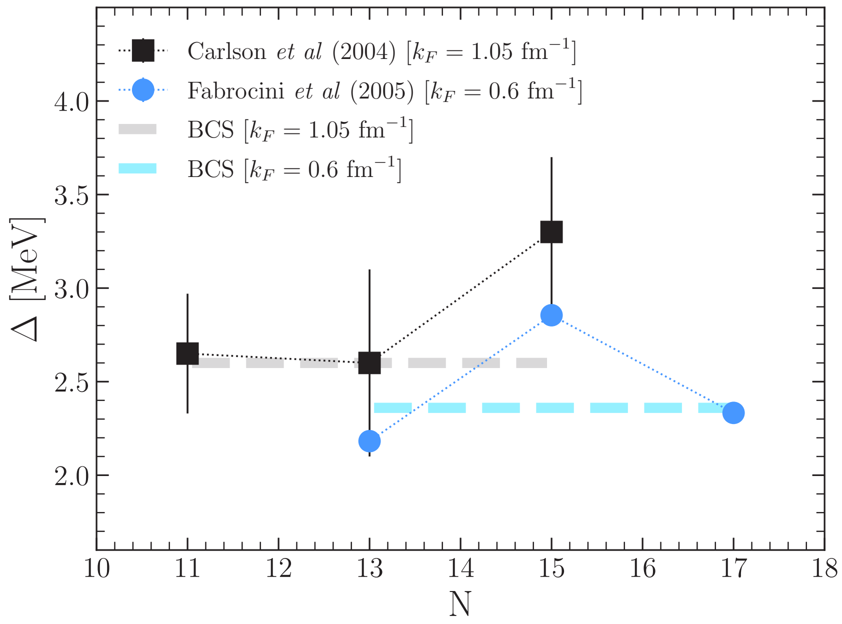
TRIUMF

Analysis

Ref.	Method	Wf	Pot. 2N	Pot. 3N	Max k_{F} $[\mathrm{fm}^{-1}]$	Notes
2004[1]	GFMC+	BCS	AV8	–	1.05	$N = 11 - 15$
2005[2]	AFDMC	CBF-Pfaff.	AV8	– / UIX	0.8	$N = 13 - 17$

[1] Chang *et al*, Nucl. Phys. A **746**, 215-221 (2004) $\longleftarrow$ **reported "*preliminary*"**

[2] Fabrocini *et al*, Phys. Rev. Lett. **95** 192501 (2005)



Correcting for **FSE** (from PBCS)

$\Delta(14) = 1.15\Delta_{\mathrm{TL}}$

we recover the 2008[4] value.

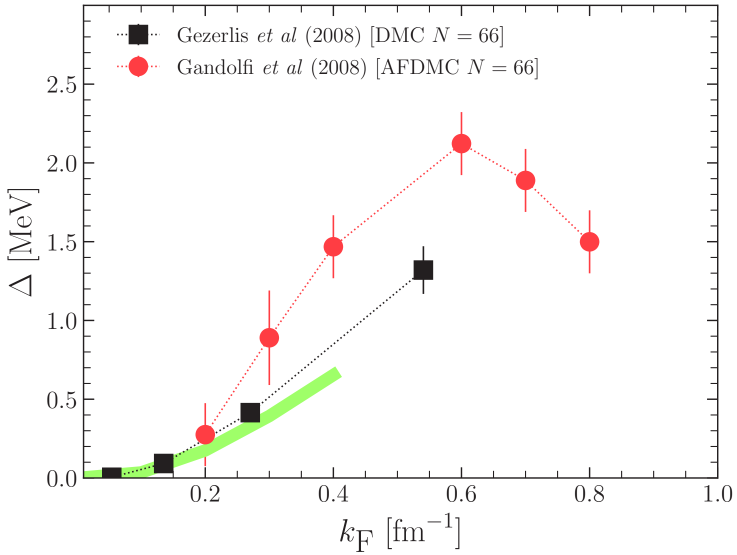
Discovery, accelerated

Analysis

Ref.	Method	Wf	Pot. 2N	Pot. 3N	Max k_F [fm^{-1}]	Notes
2008[4]	AFDMC	CBF-Pfaff.	AV8	UIX	0.8	$N = 64 - 68$
2008[5]	DMC	v-BCS	AV4	–	0.54	$N = 66$

[4] Gandolfi, *et al*, Phys. Rev. Lett. **101**, 132501 (2008)

[5] Gezerlis, Phys. Rev. C **77**, 032801(R) (2008)



Wavefunction or interaction (tensor, 3NF, etc.)?

DMC(2008[5]) with the CBF-Pfaff and AV4 reproduced AFDMC(2008[4])

Also CBF-Pfaff does **not** reproduce the Gor'kov and Melik-Barkhudarov limit

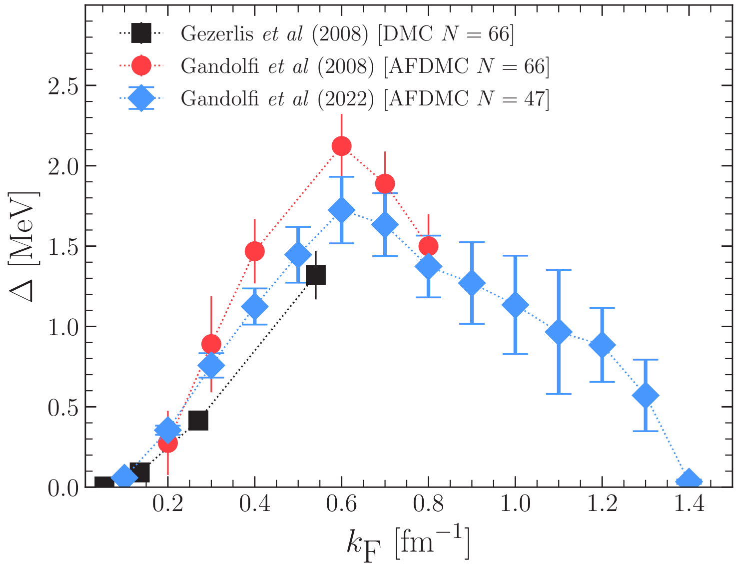
Analysis

Ref.	Method	Wf	Pot. 2N	Pot. 3N	Max k_F $[\mathrm{fm}^{-1}]$	Notes
2008[4]	AFDMC	CBF-Pfaff.	AV8	UIX	0.8	$N = 64 - 68$
2008[5]	DMC	v-BCS	AV4	–	0.54	$N = 66$
2022[6]	AFDMC	v-Pfaff	AV8 / N2LO	UIX / N2LO	1.7	$N = 47$

[4] Gandolfi *et al*, Phys. Rev. Lett. **101**, 132501 (2008)

[5] Gezerlis *et al*, Phys. Rev. C **77**, 032801(R) (2008)

[6] Gandolfi, *et al*, Condens. Matter **7** 19 (2022)



Extra features:

- Closure at $k_F = 1.4\ \mathrm{fm}^{-1}$
- Error quantification of **FSE**: $\sim 5\%$

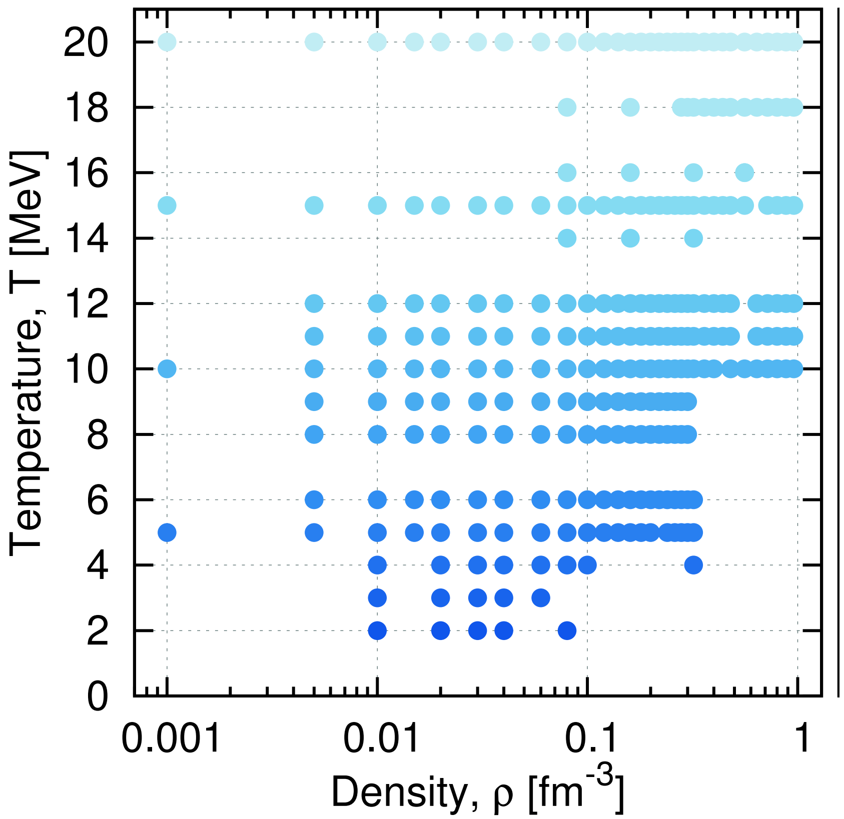
Conclusions from QMC:

- In medium effects: $\sim 50\%$ suppression
- 3NF: at intermediate densities and suppressing

TRIUMF

Challenge for SCGF

Calculation of non-superfluid **spectral functions** $\mathcal{A}^N$ at $T < T_0$ need extrapolation



Rios *et al*, Low Temp Phys **189** 234-249 (2017)

Approaches that handle superfluid $\mathcal{A}$ have been developed:

Gorkov SCGF

Somà *et al*, Phys. Rev. C **84**, 064317 (2011)

Nambu Covariant SCGF

M.Drissi *et al*, Ann. Phys. **469**, 169729 (2024)

Discovery, accelerated

TRIUMF

SCGF+ Calculations

Neutrons

$$k_F(\rho_0 = 0.16\ \mathrm{fm}^{-3}) \approx 1.7\ \mathrm{fm}^{-1}$$

Ref.	**Pot. 2N**	**Pot. 3N**	Gaps	Notes
2004[1]	Gogny D1	–	Peak 1.5 MeV; 3NF effects	lowest app. to SE and vertex
2016[2]	CDBonn / AV18 / N3LO	N2LO	Peak 1.8 MeV; robust	LRC+SRC
2017[3]	N3LO(414,450,500)	N2LO	Peak $2.$ MeV; robust	SRC

[1] Shen *et al*, Phys. Rev. C **67**, 061302(R) (2003) $\longleftarrow$ ***reported exploratory***

[2] Ding *et al*, Phys. Rev. C **94**, 025802 (2016)

[3] Rios *et al*, Low Temp Phys **189** 234-249 (2017)

Consensus:

– Suppression of $\sim 50\%$

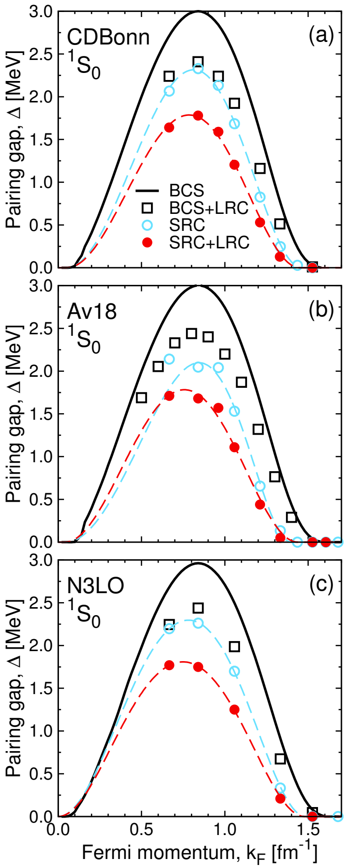
– Closure at $k_F \approx 1.5\mathrm{fm}^{-1}$

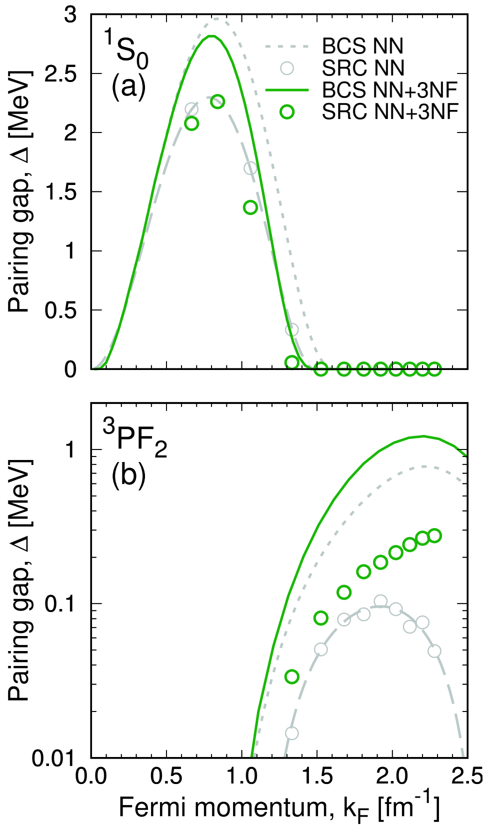
– Minimal regulator effects; phase shifts are enough

Discovery, accelerated

TRIUMF

SCGF calculations





Ding, *et al*, *Phys. Rev. C* ***94****, 025802 (2016)*

Consensus:

- Suppression of $\sim 50\%$
- Closure at $k_\mathrm{F} \approx 1.5\mathrm{fm}^{-1}$
- Minimal regulator effects; phase shifts are enough

Protons in passing

Protons

in beta-equilibrium

$$k_{\mathrm{F}}^{\mathrm{p}}(x) = k_{\mathrm{F}}^{\mathrm{n}} \left[x/(1-x)\right]^{1/3} \ , \quad x = n_p/n \ : \ \text{proton fraction}$$

Main difficulties for calc.:

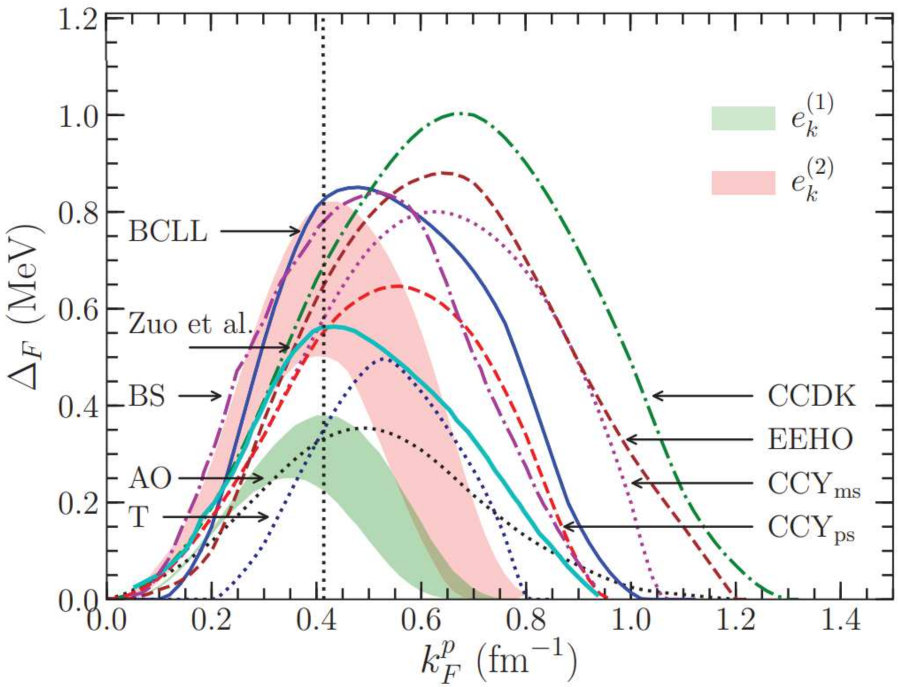
- Strong coupling to neutron background ($\sim 95\%$ neutrons).
- $m_p^* < m_n^* \quad \longrightarrow$ suppressed proton gaps
  Sjoberg, Nucl. Phys. A **265**, 511 (1976)
  M. Baldo *et al*, Phys. Rev. C **75**, 025802 (2007)
- amp. 3NF repulsion $\longrightarrow$ *more* suppressed of proton gaps
- Unclear induced interactions

Protons in passing

Protons

in beta-equilibrium

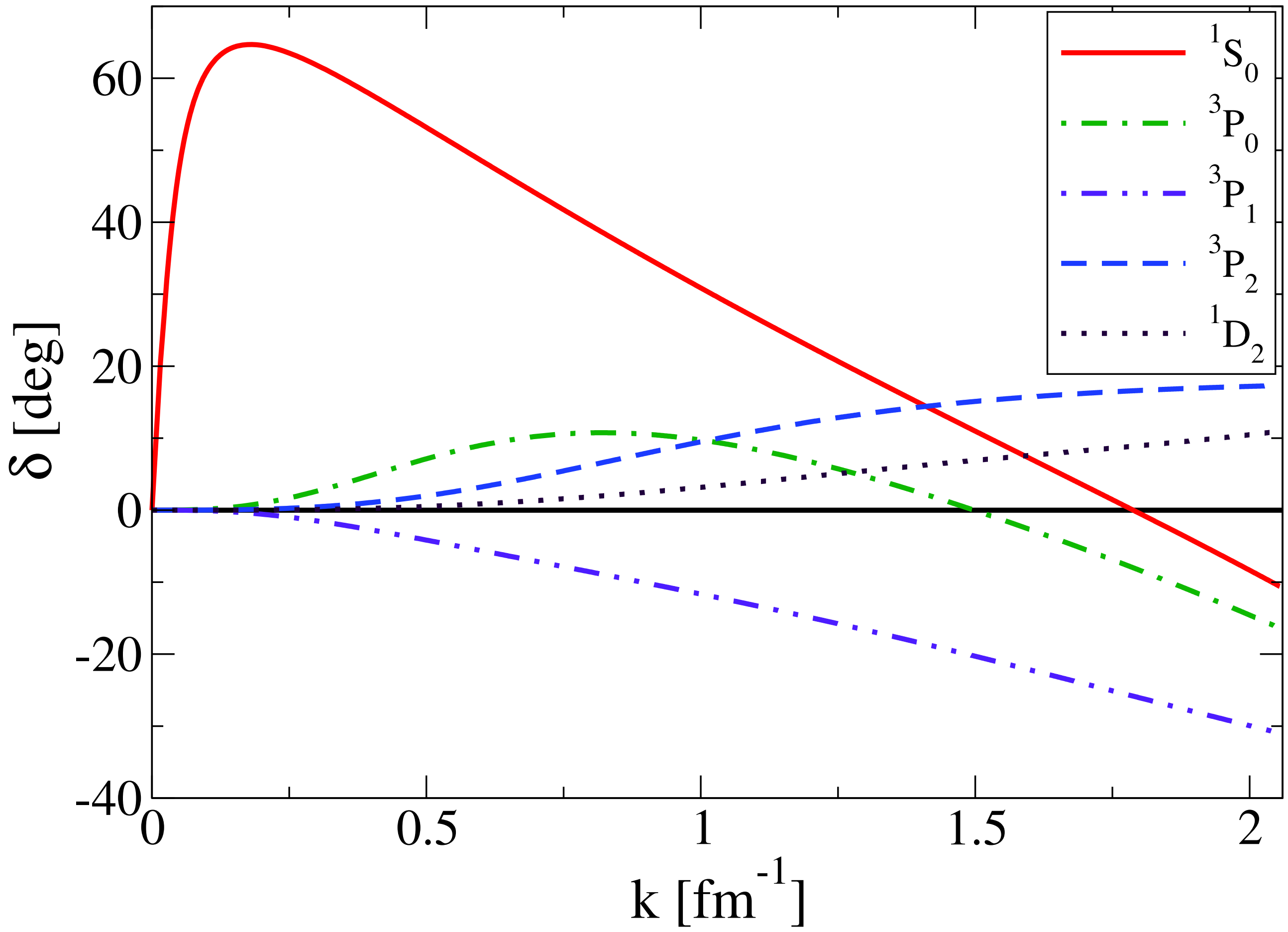
$$k_{\mathrm{F}}^{\mathrm{p}}(x) = k_{\mathrm{F}}^{\mathrm{n}}\left[x/(1-x)\right]^{1/3} \quad , \quad x = n_p/n \ : \ \text{proton fraction}$$



Lim *et al*, Phys. Rev C **103**, 025807 (2021)

Spin-Triplet

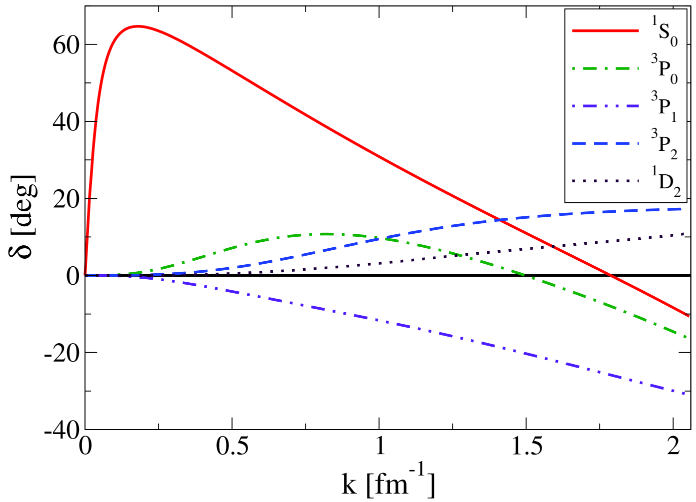
Expectations from Experiment



[Gezerlis *et al*, arXiv:1406.6109]

Discovery, accelerated

Definitions and expectations: Phenomenology



[Gezerlis *et al*, arXiv:1406.6109]

$$S = 0$$

$L = 0$: Strongly attractive up to $k_F \sim 1.7\ \mathrm{fm}^{-1}$

$L = 2$: Weakly attractive at higher densities

$$S = 1$$

$\underset{(J=0)}{L=1}$: attractive up to $k_F \sim 1.5\ \mathrm{fm}^{-1}$. Maybe not enough.

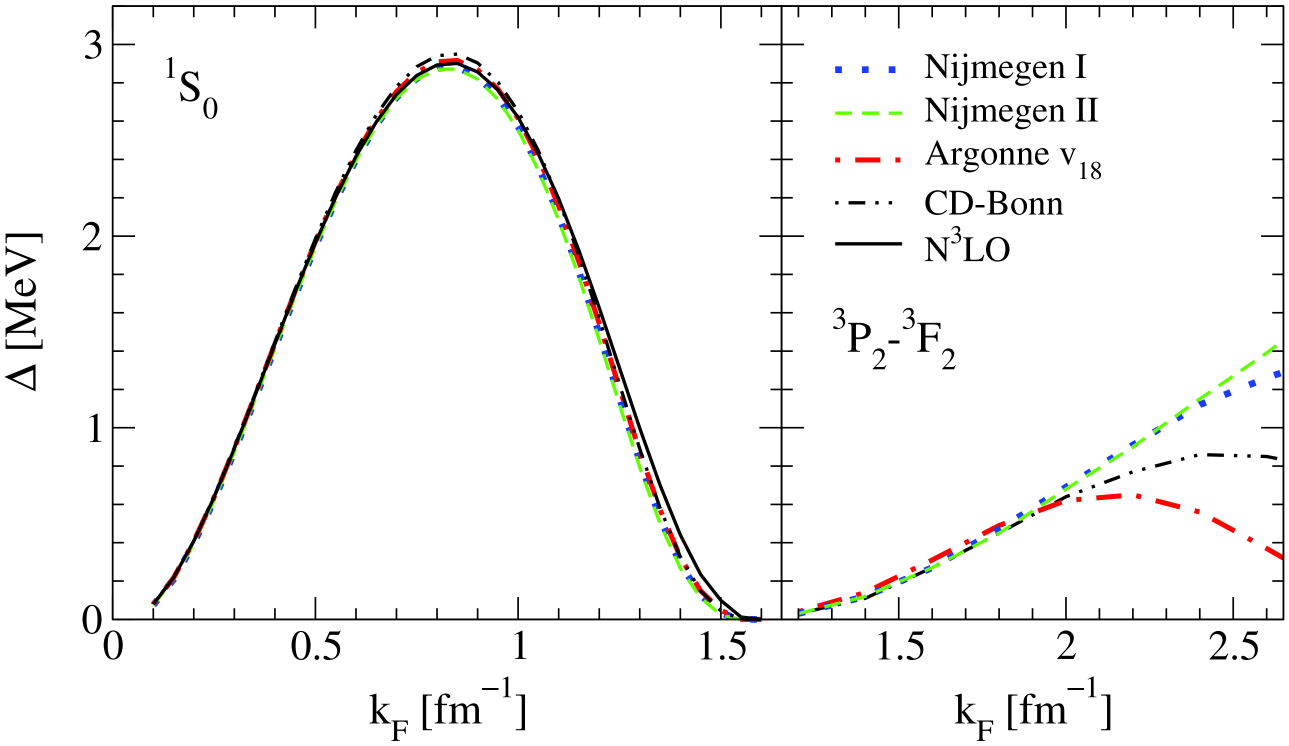
$\underset{(J=2)}{L=1}$: The most attractive channel after $k_F \sim 1.5\ \mathrm{fm}^{-1}$. Large contribution from the spin-orbit force.

Pairing happens close to k_F so roughly translating k to k_F

$$\phi_{\mathrm{pair}}(\mathbf{r}) = \sum_{L=\dots} \phi_L(\mathbf{r})$$

Discovery, accelerated

Expectations from Phenomenology

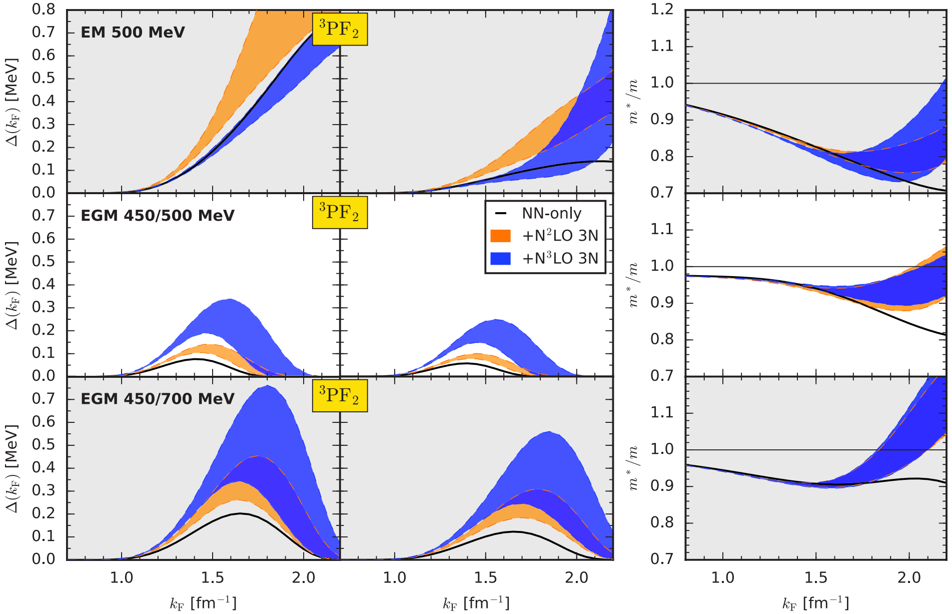


[Gezerlis *et al*, arXiv:1406.6109]

Discovery, accelerated

TRIUMF

Expectations from Phenomenology



Free vs HF spectrum; bands from 3NF params.

Large χEFT expansion parameter

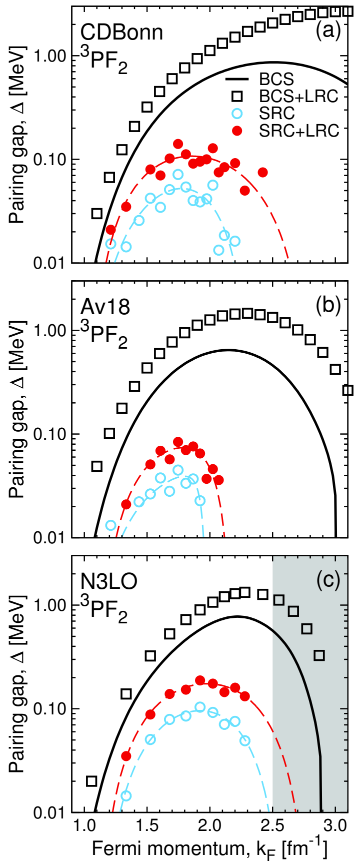
$$Q(2.0\ \mathrm{fm}^{-1}) = \begin{cases} 0.66 & \text{for } R_0 = 0.8, 0.9, 1.0\ \mathrm{fm}\,, \\ 0.79 & \text{for } R_0 = 1.1\ \mathrm{fm}\,, \text{ and} \\ 0.99 & \text{for } R_0 = 1.2\ \mathrm{fm}\,. \end{cases}$$

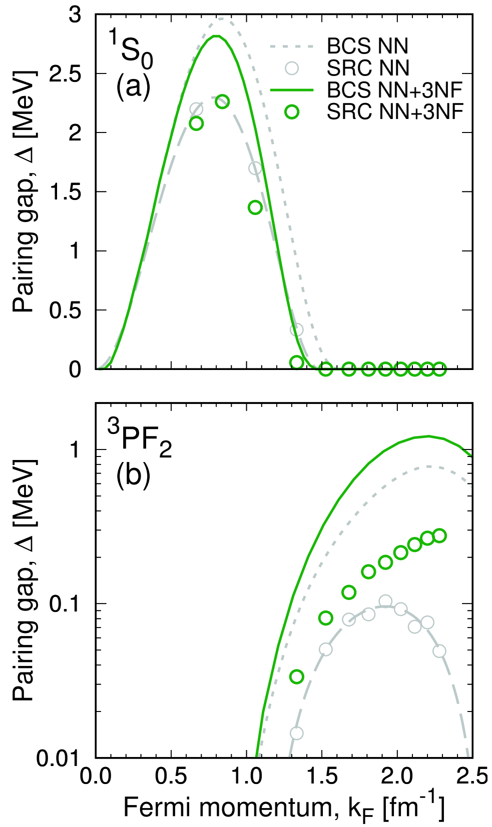
Expect: enhancement from 3NF and large regulator effects

Drischler *et al*, Phys. Rev. C **95**, 024302 (2017)

Discovery, accelerated

SCGF calculations





Features:

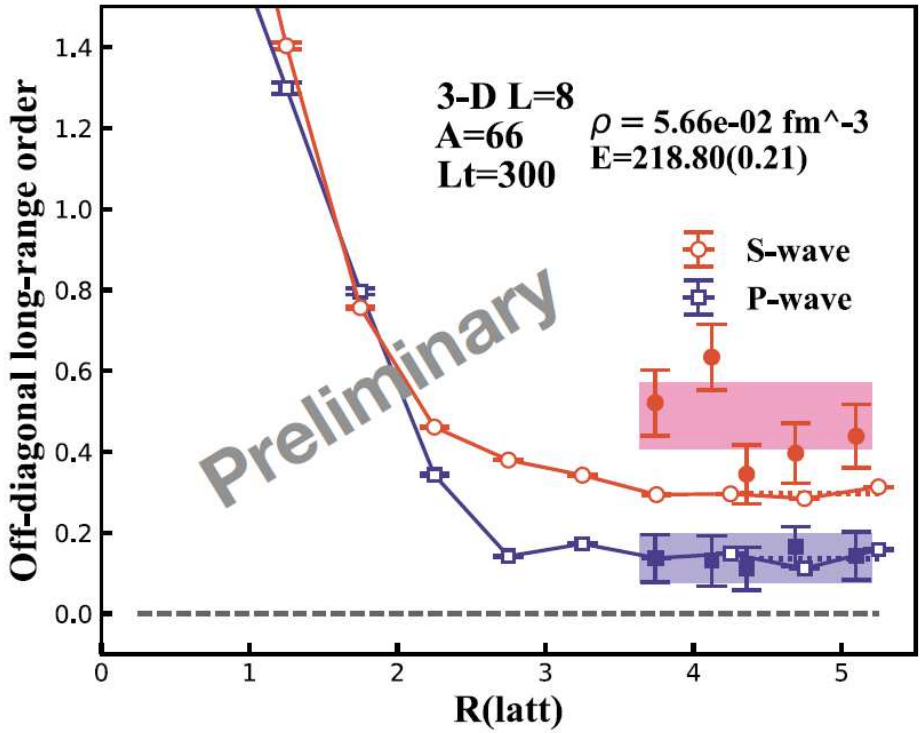
- peak at ~ 0.1 MeV
- 3NFs enhance
- closure uncertain: regulator effects at high densities

Ding, *et al, Phys. Rev. C* ***94****, 025802 (2016)*

Discovery, accelerated

Prelim. QMC calculations

Off-diagonal Long Range Order with chiral N3LO interaction



Y. Ma, GP, J. Carlson, S. Gandolfi, A. Gezerlis, G. Given, A. Hicks, D. Lee, K. E. Schmidt, and J. YU, in preparation

Slide by Yuanzhuo Ma from MSU

Discovery, accelerated

Summary

- Neutron spin-singlet: emergent consensus
- Proton spin-singlet: currently no agreement
- Neutron triplet: phenomenological expectations are verified; current χ EFT interactions insufficient
- Evidence of coexistence of spin-triplet and spin-singlet pairing

More detail

1. A systematic study of protons in a dense neutron background is needed
2. Cooperation of methods for spin-triplet neutron pairing gaps will help nail-down details.

TRIUMF

Thank you.

Ab initio calculations mentioned

Chang *et al*, Nucl. Phys. A **746**, 215-221 (2004)

Fabrocini *et al*, Phys. Rev. Lett. **95** 192501 (2005)

Abe *et al*, Phys. Rev. C **79**, 054002 (2009)

Gezerlis *et al*, Phys. Rev. C **77**, 032801(R) (2008)

Gandolfi *et al*, Phys. Rev. Lett. **101**, 132501 (2008)

Shen *et al*, Phys. Rev. C **67**, 061302(R) (2003)

Ding *et al*, Phys. Rev. C **94**, 025802 (2016)

Gandolfi *et al*, Condens. Matter **7** 19 (2022)

Discovery, accelerated

TRIUMF

Thank you.

Discovery, accelerated