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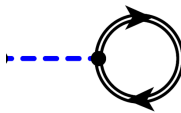
To which extent pairing originates from
the exchange of collective fluctuations?


Thanks to
F. Barranco (Sevilla Univ.)
A. Idini (Lund Univ.)
G. Potel (Sevilla Univ.)

Ab initio GSCGF calculations

$$\sum_{\beta} \begin{pmatrix} t_{\alpha\beta} - \mu\delta_{\alpha\beta} + \Sigma_{\alpha\beta}^{(\infty)11} + \widetilde{\Sigma}_{\alpha\beta}^{11}(\omega) & \Sigma_{\alpha\beta}^{(\infty)12} + \widetilde{\Sigma}_{\alpha\beta}^{12}(\omega) \\ \Sigma_{\alpha\beta}^{(\infty)21} + \widetilde{\Sigma}_{\alpha\beta}^{21}(\omega) & -t_{\alpha\beta} + \mu\delta_{\alpha\beta} + \Sigma_{\alpha\beta}^{(\infty)22} + \widetilde{\Sigma}_{\alpha\beta}^{22}(\omega) \end{pmatrix} \begin{pmatrix} \mathcal{U}_{\beta}^k \\ \mathcal{V}_{\beta}^k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}_{\alpha}^k \\ \mathcal{V}_{\alpha}^k \end{pmatrix}$$

First order

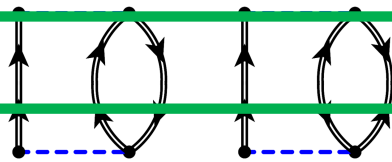
$$\Sigma_{\alpha\beta}^{(\infty)11} =$$


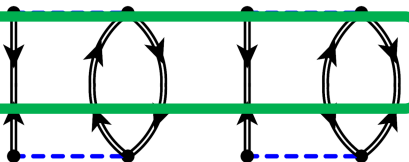
$$\Sigma_{\alpha\beta}^{(\infty)12} =$$


ω-dependent self-energies

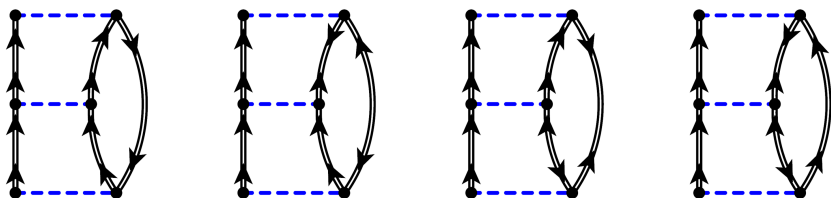
- Impact « m* » as just seen
- Also fragment qp strength
- Correct « \tilde{v} » as well... how?

Second order

$$\Sigma_{\alpha\beta}^{11(2)}(\omega) =$$


$$\Sigma_{\alpha\beta}^{12(2)}(\omega) =$$


Third order

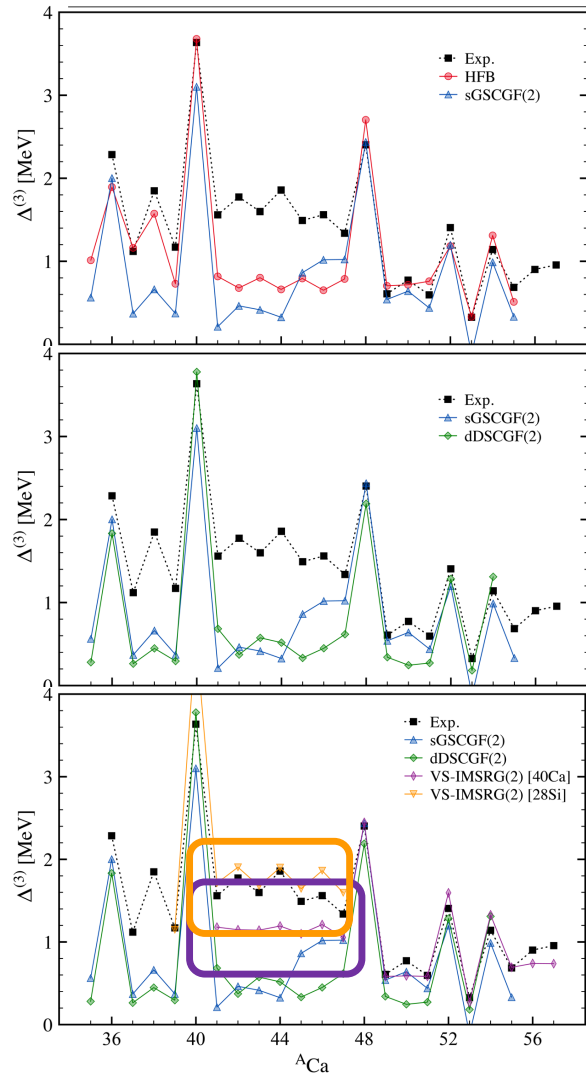
$$\Sigma_{\alpha\beta}^{11(3)}(\omega) =$$


+ 4 with hh intermediate states
+ 9 with ph intermediate states

$$\Sigma_{\alpha\beta}^{12(3)}(\omega) =$$

17 diagrams obtained by permuting direction of line as illustrated above

Ab initio sGSCGF(2) gaps in Ca isotopes



Somà et al., EPJA (2021)

Scallesi et al., unpublished

Scallesi et al., unpublished

Gaps are even often reduced compared to HFB

- Anti-pairing effect of fragmentation of strength wins
 - Insufficient m^* increase (as seen before)
 - Attractive (?) induced contribution to \tilde{v} not large enough
- More detailed analysis of the above components needed**

Solutions remain close to zero-pairing limit

- sGSCGF(2) close to dDSCGF(2) (no U(1) but SU(2) breaking)
- Does not benefit from U(1) breaking of reference state**
- $\Delta_{\alpha\beta}$ (or ΔN^2) constrained HFB unperturbed state to be explored

Pairing is in fact not gone anywhere!

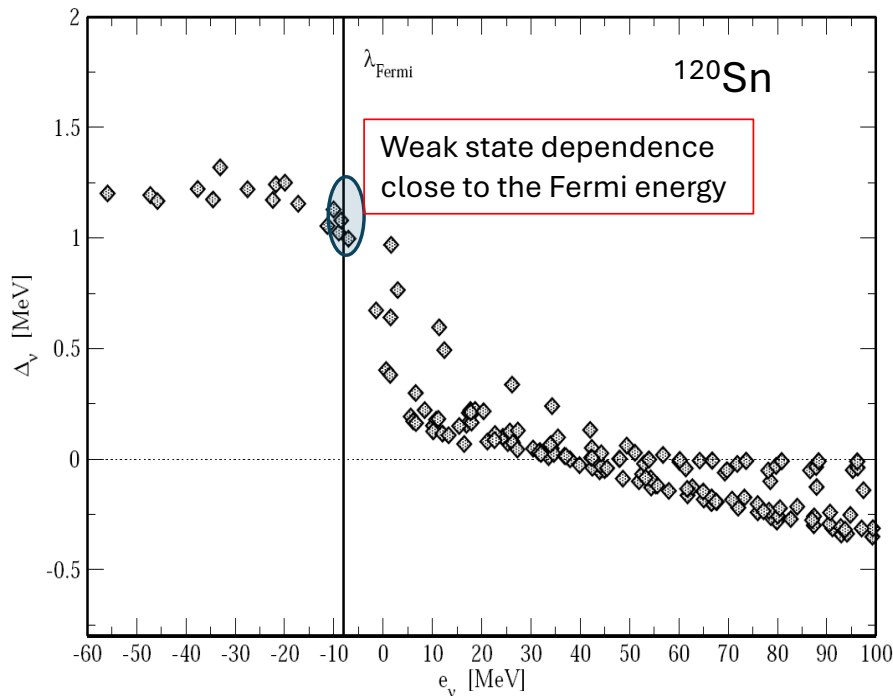
- VS-IMSRG(2)[^{40}Ca] improves to 70% of exp. $\Delta^{(3)}$
 - VS-IMSRG(2)[^{28}Si] reach 100%: fluctuation of ^{40}Ca core key
- Curvature and $\mathcal{I}_{\text{pair}}$ restored in VS-IMSRG(2)[^{28}Si]**
- $\mathcal{I}_{\text{pair}}$, i.e. the oscillation of Δ , scales with Δ [Ruike et al. arXiv:2504.11908](#)
- Not the Hamiltonian to be blamed (it seems*)**
- Collective fluctuations required in *polynomial* methods
- GSCGF[3] = ADC(3) = Tamm Dancoff (i.e. beyond 3rd order)**
- Derived formally [Barbieri, Duguet, Somà, PRC \(2022\)](#)
 - To be implemented numerically

HFB calculation with the bare pairing interaction in the SLy4 mean field

$$(\epsilon_{nlj} - \epsilon_F)U_{nlj}^q + \sum_{n'} \Delta_{nn'lj} V_{n'lj}^q = E_{lj}^q U_{nlj}^q$$

$$-(\epsilon_{nlj} - \epsilon_F)V_{nlj}^q + \sum_{n'} \Delta_{nn'lj} U_{n'lj}^q = E_{lj}^q V_{nlj}^q$$

1S_0 diagonal pairing gaps calculated with the bare Argonne potential and the SLy4 mean field



The value of the gap depends both on the value of the effective mass ($m_k \sim 0.7 m$) and on the detailed position of the orbital close to the Fermi energy

A calculation with the monopole force and $G_0 = 0.22$ MeV reproduces rather well the gaps obtained with the Argonne interaction close to the Fermi energy

A. Pastore et al, PRC 78 (2008) 024315

Semi-empirical HFB calculations

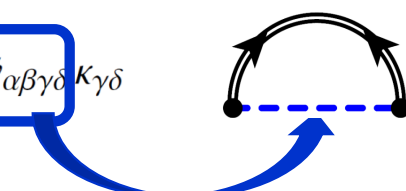
Effective mean field from, e.g. Skyrme EDF with $m^* \sim 0.7$

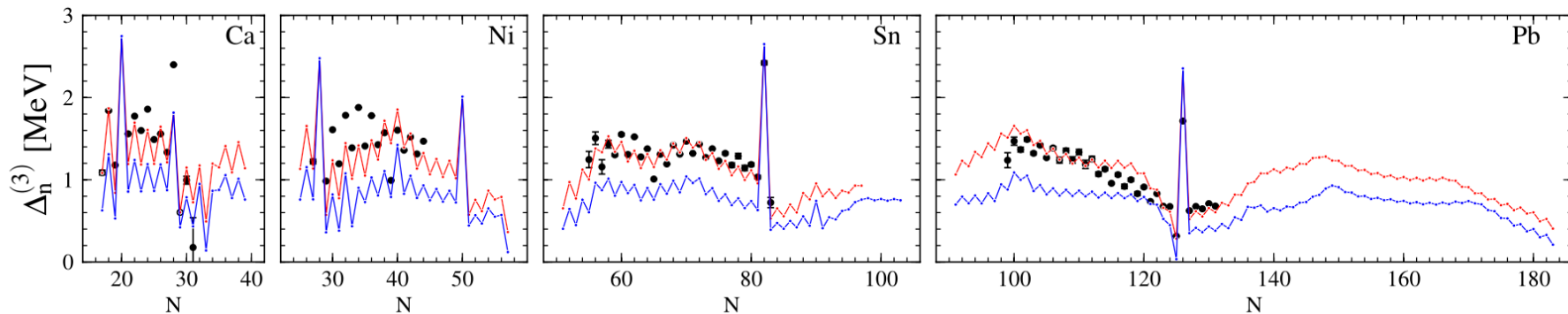
Bogoliubov field from realistic (low-k) 2N+3N interactions

$$\sum_{\beta} \begin{pmatrix} t_{\alpha\beta} - \mu\delta_{\alpha\beta} + \boxed{\Sigma_{\alpha\beta}^{(\infty)11}} + \cancel{\widetilde{\Sigma}_{\alpha\beta}^{11}(\omega)} & \boxed{\Sigma_{\alpha\beta}^{(\infty)12}} + \cancel{\widetilde{\Sigma}_{\alpha\beta}^{12}(\omega)} \\ \Sigma_{\alpha\beta}^{(\infty)21} + \widetilde{\Sigma}_{\alpha\beta}^{21}(\omega) & -t_{\alpha\beta} + \mu\delta_{\alpha\beta} + \Sigma_{\alpha\beta}^{(\infty)22} + \widetilde{\Sigma}_{\alpha\beta}^{22}(\omega) \end{pmatrix} \begin{pmatrix} u_{\beta}^k \\ v_{\beta}^k \end{pmatrix} = \omega_k \begin{pmatrix} u_{\alpha}^k \\ v_{\alpha}^k \end{pmatrix}$$

Static semi-effective HFB problem

➡ $\Sigma_{\alpha\beta}^{(\infty)12} = \frac{1}{2} \sum_{\gamma\delta} v_{\alpha\beta\gamma\delta} K_{\gamma\delta} + \frac{1}{2} \sum_{\gamma\delta\zeta\epsilon} w_{\alpha\beta\gamma\delta\zeta\epsilon} K_{\delta\zeta} \rho_{\epsilon\gamma} \equiv \frac{1}{2} \sum_{\gamma\delta} \boxed{\tilde{v}_{\alpha\beta\gamma\delta}} K_{\gamma\delta}$





Going beyond mean field and including the coupling between quasiparticles and vibrations in superfluid nuclei:

Can we obtain reasonable values of the pairing gaps starting from a bare interaction?

Can we keep the same level of agreement with experiment obtained with BCS theory with effective forces concerning the quasiparticle spectrum and the occupation factors?

Can we also reproduce the observed fragmentation of some valence orbitals, the electromagnetic decays, the multiplet splittings observed in the low-lying part of the spectrum?

Basic formalism for the particle-vibration coupling in superfluid nuclei close to the Fermi surface

Van der Sluys et al., NPA551(1993)210

Let us start from a BCS calculation leading to a set of quasiparticle energies E_a and quasiparticle amplitudes u_a, v_a .

Let us consider the coupling with 1qp-1phonon states $b\lambda\nu$ and $c\lambda\nu$.

The renormalized, fragmented quasiparticles are obtained from the diagonalization of the matrix

$$\begin{pmatrix} E_a & V(ab\lambda\nu) & V(ac\lambda\nu) & W(ab\lambda\nu) & W(ac\lambda\nu) & 0 \\ V(ab\lambda\nu) & \hbar\omega_{\lambda\nu} + E_b & 0 & 0 & 0 & W(ab\lambda\nu) \\ V(ac\lambda\nu) & 0 & \hbar\omega_{\lambda\nu} + E_c & 0 & 0 & W(ac\lambda\nu) \\ W(ab\lambda\nu) & 0 & 0 & -\hbar\omega_{\lambda\nu} - E_b & 0 & -V(ab\lambda\nu) \\ W(ac\lambda\nu) & 0 & 0 & 0 & -\hbar\omega_{\lambda\nu} - E_c & -V(ac\lambda\nu) \\ 0 & W(ab\lambda\nu) & W(ac\lambda\nu) & -V(ab\lambda\nu) & -V(ac\lambda\nu) & -E_a \end{pmatrix}$$

The phonons $\lambda\nu$ are calculated in QRPA, with forward and backward amplitudes X,Y.

The particle-vibration matrix elements V,W are given in terms of the antisymmetrized particle-particle and particle-hole matrix elements of the two-body interaction G and F

$$V(ab\lambda\nu) = \left[\frac{2\lambda + 1}{2j_a + 1} \right]^{1/2} \sum_{c \leq d} (1 + \delta_{cd})^{-1/2} \\ \times [X_{cd}(\lambda\nu)V(cd\lambda b; a) + (-1)^{j_a - j_b + \lambda} \\ \times Y_{cd}(\lambda\nu)V(cd\lambda a; b)]$$

$$V(cd\lambda b; a) = -(u_a v_b u_c u_d - v_a u_b v_c v_d)G(abcd\lambda) \\ + (u_a u_b u_c v_d - v_a v_b v_c u_d)F(abcd\lambda) \\ + (-1)^{j_c - j_d + \lambda} (u_a u_b v_c u_d - v_a v_b u_c v_d) \\ \times F(abdc\lambda),$$

$$W(ab\lambda\nu) = \left[\frac{2\lambda + 1}{2j_a + 1} \right]^{1/2} \sum_{c \leq d} (1 + \delta_{cd})^{-1/2} \\ \times [X_{cd}(\lambda\nu)R(abcd; \lambda) + Y_{cd}(\lambda\nu)Q(abcd; \lambda)]$$

$$Q(abcd; \lambda) = (u_a u_b u_c u_d + v_a v_b v_c v_d)G(abcd\lambda) \\ + (u_a v_b u_c v_d + v_a u_b v_c v_d)F(abcd\lambda) \\ + (-1)^{j_c - j_d + \lambda} (u_a v_b v_c u_d + v_a u_b u_c v_d) \\ \times F(abdc\lambda),$$

$$R(abcd; \lambda) = -(u_a u_b v_c v_d + v_a v_b u_c u_d)G(abcd\lambda) \\ + (u_a v_b v_c u_d + v_a u_b u_c v_d)F(abcd\lambda) \\ + (-1)^{j_c - j_d + \lambda} (u_a v_b u_c v_d + v_a u_b v_c u_d) \\ \times F(abdc\lambda),$$

From the diagonalization

$$\begin{pmatrix} E_a & V(ab\lambda\nu) & V(ac\lambda\nu) & W(ab\lambda\nu) & W(ac\lambda\nu) & 0 \\ V(ab\lambda\nu) & \hbar\omega_{\lambda\nu} + E_b & 0 & 0 & 0 & W(ab\lambda\nu) \\ V(ac\lambda\nu) & 0 & \hbar\omega_{\lambda\nu} + E_c & 0 & 0 & W(ac\lambda\nu) \\ W(ab\lambda\nu) & 0 & 0 & -\hbar\omega_{\lambda\nu} - E_b & 0 & -V(ab\lambda\nu) \\ W(ac\lambda\nu) & 0 & 0 & 0 & -\hbar\omega_{\lambda\nu} - E_c & -V(ac\lambda\nu) \\ 0 & W(ab\lambda\nu) & W(ac\lambda\nu) & -V(ab\lambda\nu) & -V(ac\lambda\nu) & -E_a \end{pmatrix} \begin{pmatrix} x_{a(n)} \\ C_{a(n),b,\lambda\nu} \\ C_{a(n),c,\lambda\nu} \\ -D_{a(n),b,\lambda\nu} \\ -D_{a(n),c,\lambda\nu} \\ -y_{a(n)} \end{pmatrix} = \tilde{E}_{a(n)} \begin{pmatrix} x_{a(n)} \\ C_{a(n),b,\lambda\nu} \\ C_{a(n),c,\lambda\nu} \\ -D_{a(n),b,\lambda\nu} \\ -D_{a(n),c,\lambda\nu} \\ -y_{a(n)} \end{pmatrix}.$$

one obtains the new quasiparticle amplitudes associated with a fragment a(n):

$$\tilde{u}_{a(n)} = x_{a(n)}u_a - y_{a(n)}v_a,$$

$$\tilde{v}_{a(n)} = x_{a(n)}v_a + y_{a(n)}u_a$$

as well as the components C,D on the complex 1qp-1phonon states.

$$x_{a(n)}^2 + \sum_{b,\lambda,\nu} [C_{a(n),b,\lambda\nu}^2] + y_{a(n)}^2 + \sum_{b,\lambda,\nu} [D_{a(n),b,\lambda\nu}^2] = 1.$$

The equivalent energy-dependent problem:

$$\begin{pmatrix} E_a + \Sigma_{a(n)}^{11\text{pho}} & \Sigma_{a(n)}^{12\text{pho}} \\ \Sigma_{a(n)}^{12\text{pho}} & -E_a + \Sigma_{a(n)}^{22\text{pho}} \end{pmatrix} \begin{pmatrix} x_{a(n)} \\ y_{a(n)} \end{pmatrix} = \tilde{E}_{a(n)} \begin{pmatrix} x_{a(n)} \\ y_{a(n)} \end{pmatrix}$$

$$\Sigma_{a(n)}^{11\text{pho}} = \sum_{b,m,\lambda,\nu} \frac{V^2[ab(m)\lambda\nu]}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}}$$

$$+ \sum_{b,m,\lambda,\nu} \frac{W^2[ab(m)\lambda\nu]}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}}$$

$$\Sigma_{a(n)}^{22\text{pho}} = \sum_{b,m,\lambda,\nu} \frac{W^2[ab(m)\lambda\nu]}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}}$$

$$+ \sum_{b,m,\lambda,\nu} \frac{V^2[ab(m)\lambda\nu]}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}}$$

$$\Sigma_{a(n)}^{12\text{pho}} = - \sum_{b,m,\lambda,\nu} V[ab(m)\lambda\nu] W[ab(m)\lambda\nu]$$

$$\times \left[\frac{1}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}} - \frac{1}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}} \right]$$

$$x_{a(n)}^2 + y_{a(n)}^2 - \frac{\partial \Sigma_{a(n)}^{11\text{pho}}}{\partial \tilde{E}_{a(n)}} x_{a(n)}^2 - \frac{\partial \Sigma_{a(n)}^{22\text{pho}}}{\partial \tilde{E}_{a(n)}} y_{a(n)}^2 - 2 \frac{\partial \Sigma_{a(n)}^{12\text{pho}}}{\partial \tilde{E}_{a(n)}} x_{a(n)} y_{a(n)} = 1.$$

In the non superfluid limit ($u_a, v_a = 0, 1$), the phonons are of (ph) or of (pp) type. The contribution from (ph) phonons reads

$$\Sigma_{a(n)}^{11\text{pho}} = \sum_{b,m,\lambda,\nu} \frac{V^2[ab(m)\lambda\nu]}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}} + \sum_{b,m,\lambda,\nu} \frac{W^2[ab(m)\lambda\nu]}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}}$$

$$V(ab\lambda\nu) \sim \sum_{ph} [X_{ph} + Y_{ph}](\lambda\nu) F(abph), \epsilon_b > E_F$$

$$W(ab\lambda\nu) \sim \sum_{ph} [X_{ph} + Y_{ph}](\lambda\nu) F(abph), \epsilon_b < E_F$$

A close connection with Faddeev RPA

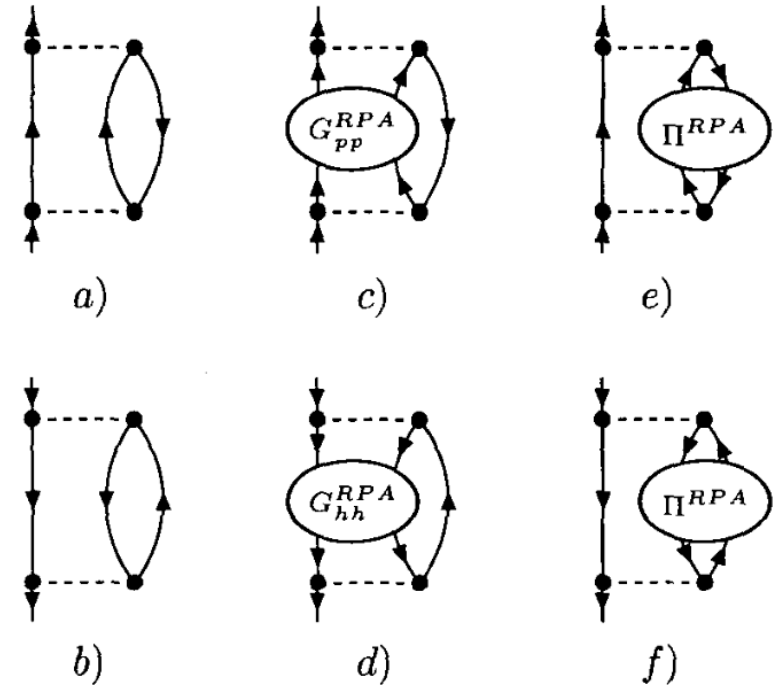
$$\Sigma^{RPA}(\alpha, \beta : E) = \frac{1}{2} \left\{ \sum_{\mu > F, n \neq 0} \frac{\Delta_{\alpha\mu}^{A+,n*} \Delta_{\beta\mu}^{A+,n}}{E - (\epsilon_\mu + (E_n^A - E_0^A)) + i\eta} + \sum_{\mu < F, m \neq 0} \frac{\Delta_{\alpha\mu}^{A-,m} \Delta_{\beta\mu}^{A-,m*}}{E - (\epsilon_\mu + (E_0^A - E_m^A)) - i\eta} \right\}$$

with

$$\Delta_{\alpha\mu}^{A+,n} = \sum_{\nu > F, \kappa < F; \nu < F, \kappa > F} \langle \alpha\kappa | G | \mu\nu \rangle R_{\nu\kappa}^{A,n}$$

and

$$\Delta_{\alpha\mu}^{A-,m} = \sum_{\nu > F, \kappa < F; \nu < F, \kappa > F} \langle \alpha\kappa | G | \mu\nu \rangle R_{\kappa\nu}^{A,m}.$$



The diagonalization can be rewritten in terms of the amplitudes $\tilde{u}_{a(n)}$ and $\tilde{v}_{a(n)}$ and takes on a form analogous to BCS:

$$\begin{pmatrix} \tilde{\epsilon}_{a(n)} - \epsilon_F & \tilde{\Delta}_{a(n)} \\ \tilde{\Delta}_{a(n)} & -(\tilde{\epsilon}_{a(n)} - \epsilon_F) \end{pmatrix} \begin{pmatrix} \tilde{u}_{a(n)} \\ \tilde{v}_{a(n)} \end{pmatrix} = \tilde{E}_{a(n)} \begin{pmatrix} \tilde{u}_{a(n)} \\ \tilde{v}_{a(n)} \end{pmatrix}$$

$$\begin{aligned} \tilde{\epsilon}_{a(n)} - \epsilon_F &= Z_{a(n)} [(\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{a(n)}^{\text{even}}] \\ \tilde{\Sigma}_{a(n)}^{\text{even}} &= \frac{\tilde{\Sigma}_{a(n)}^{11} - \tilde{\Sigma}_{a(n)}^{22}}{2} = (u_a^2 - v_a^2) \frac{\Sigma_{a(n)}^{11\text{pho}} - \Sigma_{a(n)}^{22\text{pho}}}{2} - 2u_a v_a \Sigma_{a(n)}^{12\text{pho}}. \end{aligned}$$

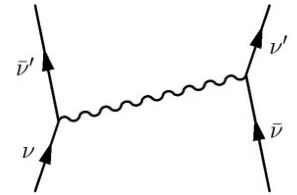
$$\begin{aligned} \tilde{\Delta}_{a(n)} &= Z_{a(n)} \tilde{\Sigma}_{a(n)}^{12} \\ &= Z_{a(n)} \left(\Delta_a^{\text{BCS}} + \tilde{\Sigma}_{a(n)}^{12,\text{pho}} \right) \equiv \tilde{\Delta}_{a(n)}^{\text{bare}} + \tilde{\Delta}_{a(n)}^{\text{pho}} \end{aligned}$$

$$\tilde{\Sigma}_{a(n)}^{12,\text{pho}} = - \sum_{b,m} \frac{(2j_b + 1)}{2} V_{\text{ind}}[a(n)b(m)] \tilde{u}_{b(m)} \tilde{v}_{b(m)},$$

$$V_{\text{ind}}[a(n)b(m)] = \sum_{\lambda,\nu} \frac{2V[ab(m)\lambda\nu]W[ab(m)\lambda\nu]}{2j_b + 1} \left[\frac{1}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}} - \frac{1}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}} \right]$$

$$Z_{a(n)} = \left(1 - \frac{\tilde{\Sigma}_{a(n)}^{11} - \tilde{\Sigma}_{a(-n)}^{11}}{2\tilde{E}_{a(n)}} \right)^{-1} \approx \left(1 - \frac{\partial \tilde{\Sigma}_{a(n)}^{11}}{\partial \tilde{E}_{a(n)}} \right)^{-1}$$

$$\tilde{\Sigma}_{a(n)}^{11} = u_a^2 \Sigma_{a(n)}^{11\text{pho}} + v_a^2 \Sigma_{a(n)}^{22\text{pho}} - 2u_a v_a \Sigma_{a(n)}^{12\text{pho}}$$



If we do not start from a previous BCS calculation,
but include directly the bare force (one step calculation)

$$\begin{pmatrix} E_a & V(ab\lambda\nu) & V(ac\lambda\nu) & W(ab\lambda\nu) & W(ac\lambda\nu) & \mp \Sigma_a^{12\text{bare}} \\ V(ab\lambda\nu) & \hbar\omega_{\lambda\nu} + E_b & 0 & 0 & 0 & W(ab\lambda\nu) \\ V(ac\lambda\nu) & 0 & \hbar\omega_{\lambda\nu} + E_c & 0 & 0 & W(ac\lambda\nu) \\ W(ab\lambda\nu) & 0 & 0 & -\hbar\omega_{\lambda\nu} - E_b & 0 & -V(ab\lambda\nu) \\ W(ac\lambda\nu) & 0 & 0 & 0 & -\hbar\omega_{\lambda\nu} - E_c & -V(ac\lambda\nu) \\ \mp \Sigma_a^{12\text{bare}} & W(ab\lambda\nu) & W(ac\lambda\nu) & -V(ab\lambda\nu) & -V(ac\lambda\nu) & -E_a \end{pmatrix} \begin{pmatrix} x_{a(n)} \\ C_{a(n),b,\lambda\nu} \\ C_{a(n),c,\lambda\nu} \\ -D_{a(n),b,\lambda\nu} \\ -D_{a(n),c,\lambda\nu} \\ -y_{a(n)} \end{pmatrix} = \tilde{E}_{a(n)} \begin{pmatrix} x_{a(n)} \\ C_{a(n),b,\lambda\nu} \\ C_{a(n),c,\lambda\nu} \\ -D_{a(n),b,\lambda\nu} \\ -D_{a(n),c,\lambda\nu} \\ -y_{a(n)} \end{pmatrix},$$

$$\tilde{\Delta}_{a(n)} = Z_{a(n)} \left(\Sigma_a^{12\text{bare}} + \tilde{\Sigma}_{a(n)}^{12,\text{pho}} \right) = \tilde{\Delta}_{a(n)}^{\text{bare}} + \tilde{\Delta}_{a(n)}^{\text{pho}}$$

$$\Sigma_a^{12\text{bare}} = - \sum_{b,m} V_{\text{bare}}[ab] \frac{(2j_b + 1)}{2} \tilde{u}_{b(m)} \tilde{v}_{b(m)},$$

$$\tilde{\Sigma}_{a(n)}^{12,\text{pho}} = - \sum_{b,m} \frac{(2j_b + 1)}{2} V_{\text{ind}}[a(n)b(m)] \tilde{u}_{b(m)} \tilde{v}_{b(m)},$$

$$\tilde{\Delta}_{a(n)} = -Z_{a(n)} \sum_{b,m} V_{\text{eff}}[a(n)b(m)] N_{b(m)} \frac{\tilde{\Delta}_{b(m)}}{2\tilde{E}_{b(m)}} \longrightarrow$$

$$V_{\text{eff}}[a(n)b(m)] = V_{\text{bare}}[ab] + V_{\text{ind}}[a(n)b(m)].$$

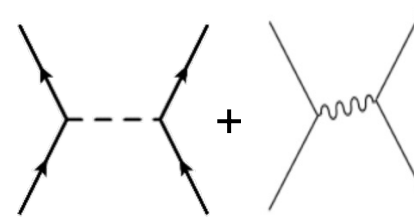
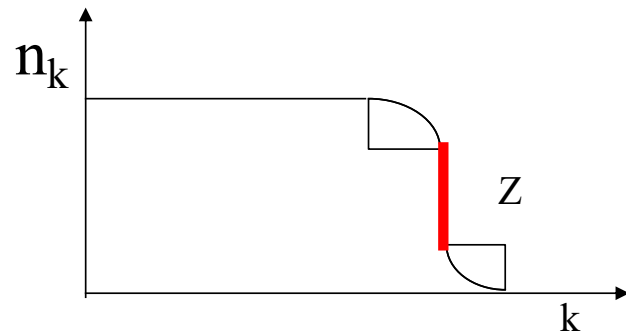
$$\Delta_k = -\frac{1}{2} \int d^3\vec{k}' \mathcal{V}_{k,k'} \frac{Z_k Z_{k'} \Delta_{k'}}{\sqrt{(\epsilon_{k'} - \epsilon_F)^2 + \Delta_{k'}^2}},$$

L.G.Cao et al., PRC 74 (2006) 064301

$$N_{a(n)} = x_{a(n)}^2 + y_{a(n)}^2 \approx x_{a(n)}^2 \approx \left(1 - \frac{\partial \Sigma_{a(n)}^{11}}{\partial \tilde{E}_{a(n)}} \right)^{-1} \approx Z_{a(n)}$$

$Z=1$ free Fermi gas

$Z<1$ correlated Fermi system



Quasiparticle
strength <1

Bare+Induced
interaction

$$\Delta_p = -\frac{1}{2} \int d^3 p' \frac{Z_p V_{pp'} Z_{p'}}{\sqrt{(\tilde{\epsilon}_{p'} - \epsilon_F)^2 + \Delta_{p'}^2}} \Delta_{p'}$$

Renormalized
s.p. energy

If the phonon $\lambda\nu$ is calculated in QRPA using the separable force

$$V(\vec{r}_1, \vec{r}_2) = - r_1 \frac{\partial U}{\partial r_1} r_2 \frac{\partial U}{\partial r_2} \sum_{\lambda\mu} \chi_\lambda Y_{\lambda\mu}^*(\theta_1) Y_{\lambda\mu}(\theta_2),$$

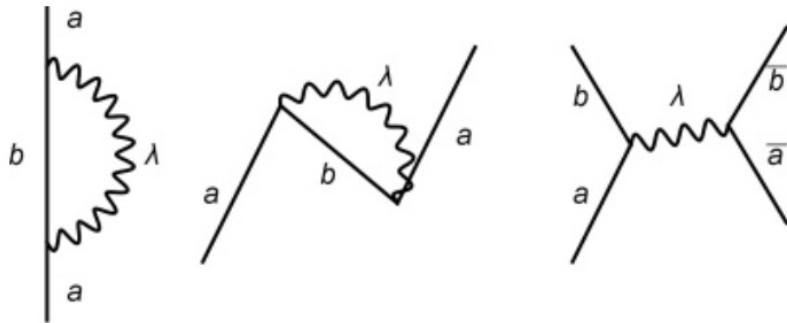
one finds

$$V(ab\lambda\nu) = h(ab\lambda\nu) (u_a u_b - v_a v_b)$$

$$W(ab\lambda\nu) = h(ab\lambda\nu) (u_a v_b + v_a u_b)$$

where $h(ab\lambda\nu)$ is the particle-vibration coupling vertex

$$h(ab\lambda\nu) = -(-1)^{j_a - j_b} \chi_\lambda \beta_{\lambda\nu} \langle a | r_1 \frac{\partial U}{\partial r_1} | b \rangle \langle j_b || Y_\lambda || j_a \rangle \left[\frac{1}{(2j_a + 1)(2\lambda + 1)} \right]^{1/2},$$



A. Idini et al:

PRC 85(2012) 014331

PRC 92 (2015) 031304(R)

PRC 96 (2017) 034606

Many-body effects in superfluid nuclei

6-5f Polarization Contributions to Effective Two-Particle Interactions

A. Bohr, B.R. Mottelson,
Nuclear Structure, Vol. II

In second order, the particle-vibration coupling gives rise to an interaction between two particles, which can be evaluated in a manner similar to the particle-phonon interaction considered in Sec. 6-5d. To illustrate the magnitude of the polarization force, we consider the limiting case in which the frequency of the exchanged phonon is large compared to the energy differences between the particle states. In this case, one can view the interaction as resulting from the static deformation (6-217) produced by the first particle acting on the second. Thus, for a mode of multipolarity λ , one obtains (see Eq. (6-68))

$$V_{\lambda}(1,2) = -\frac{2\lambda+1}{4\pi C_{\lambda}} k_{\lambda}(r_1)k_{\lambda}(r_2)P_{\lambda}(\cos\vartheta_{12}) \quad (6-228)$$

The polarization interaction resulting from the coupling to the low-frequency modes may be considerably larger than the bare force; since the frequencies of these modes may be comparable with the particle frequencies, it may be necessary to go beyond the static approximation (6-228), as in the evaluation of the particle-phonon interaction.

Outline of the various steps of the calculation:

1) Perform a QRPA calculation with a separable force. The coupling is tuned to reproduce the experimental values of the polarizability of low-lying modes.

Calculate the particle-vibration couplings with levels that reproduce the experimental energies. These values will be frozen in the rest of the calculation.

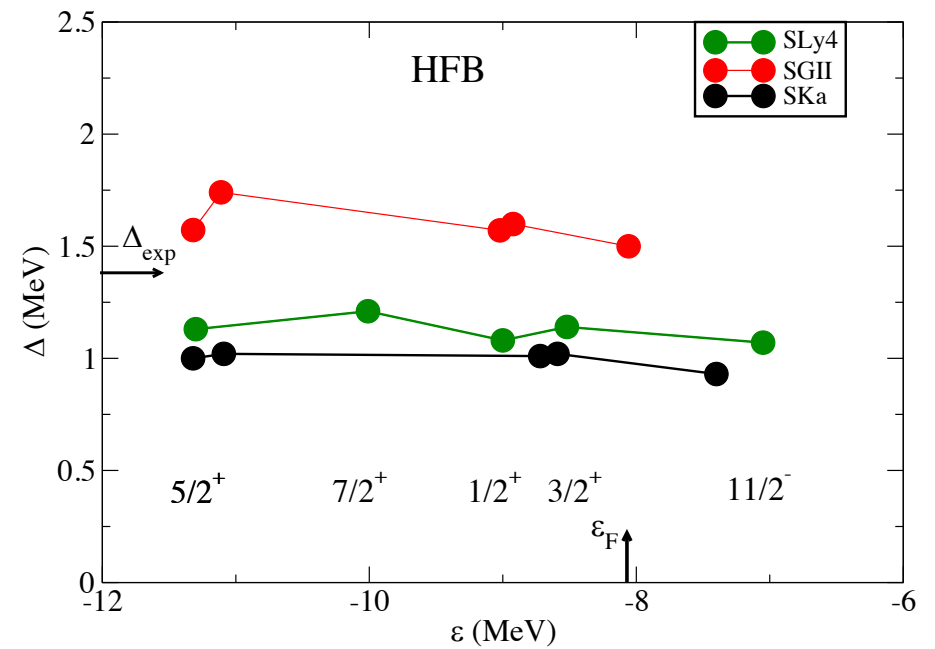
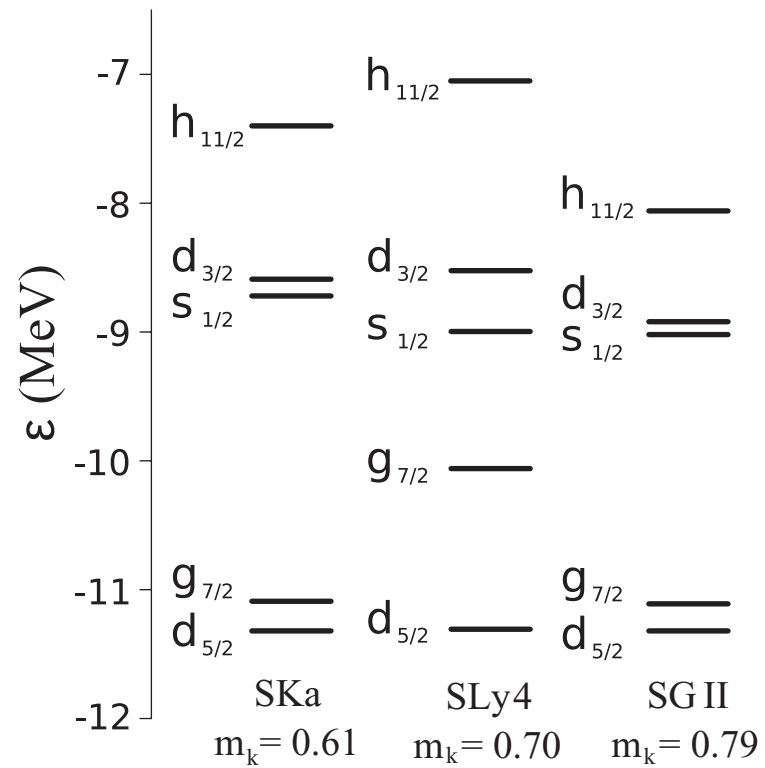
2) Perform a HF calculation with an effective force (SLy4)

3) Perform a BCS calculation with a bare pairing force (Argonne, Vlow-k) on the HF mean field to obtain quasiparticle and occupation factors of the orbitals close to the Fermi energy

4) Solve the Nambu-Gorkov equations with the particle-vibration couplings to obtain the dynamic self-energies. Iterate to convergence.

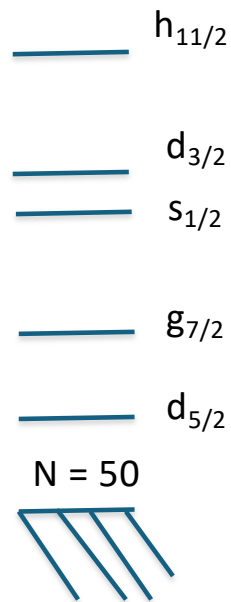
Approximation: only the levels close to the Fermi energy are considered.
Diagonal self-energy.

Starting point: the value of the 'bare' 1S_0 pairing gap

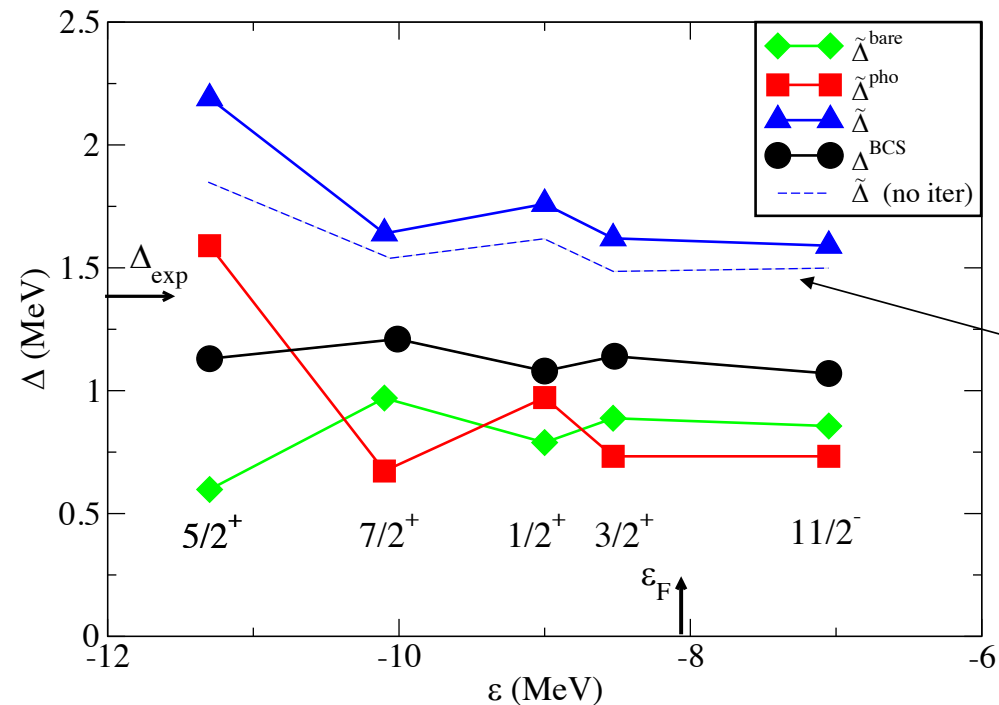


Renormalization of BCS pairing gap for states close to the Fermi energy (single node approximation, SLy4 mean field)

^{120}Sn

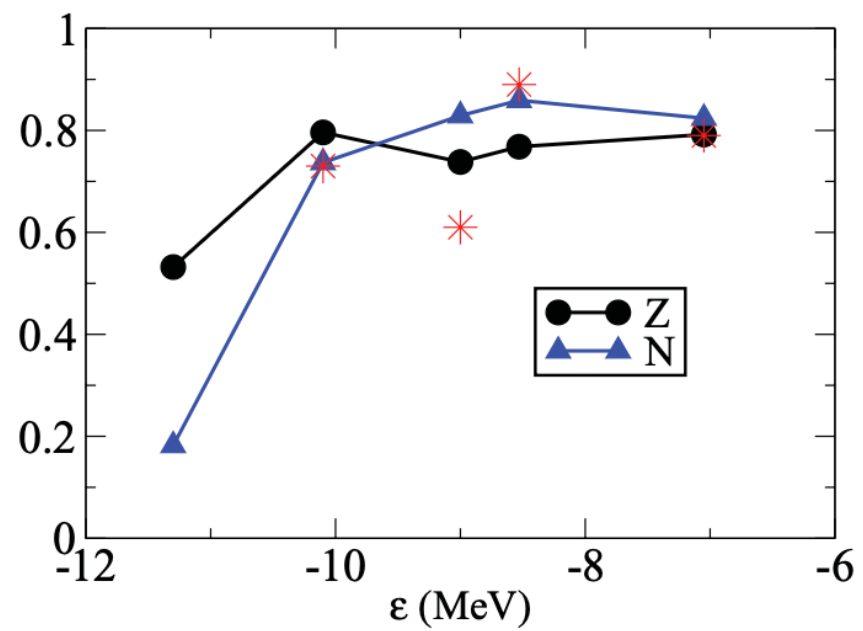
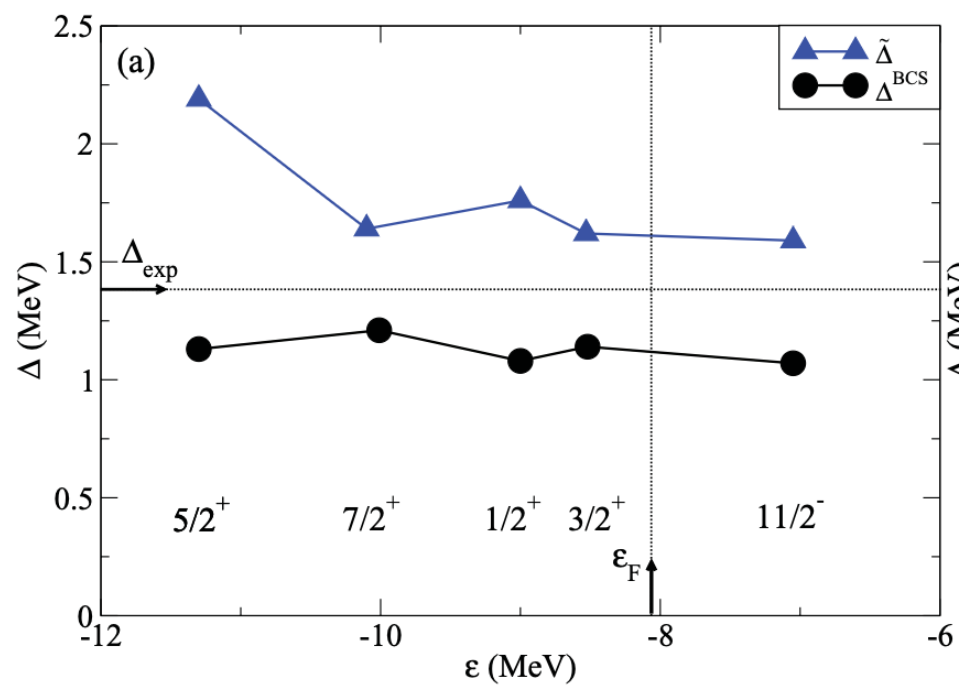


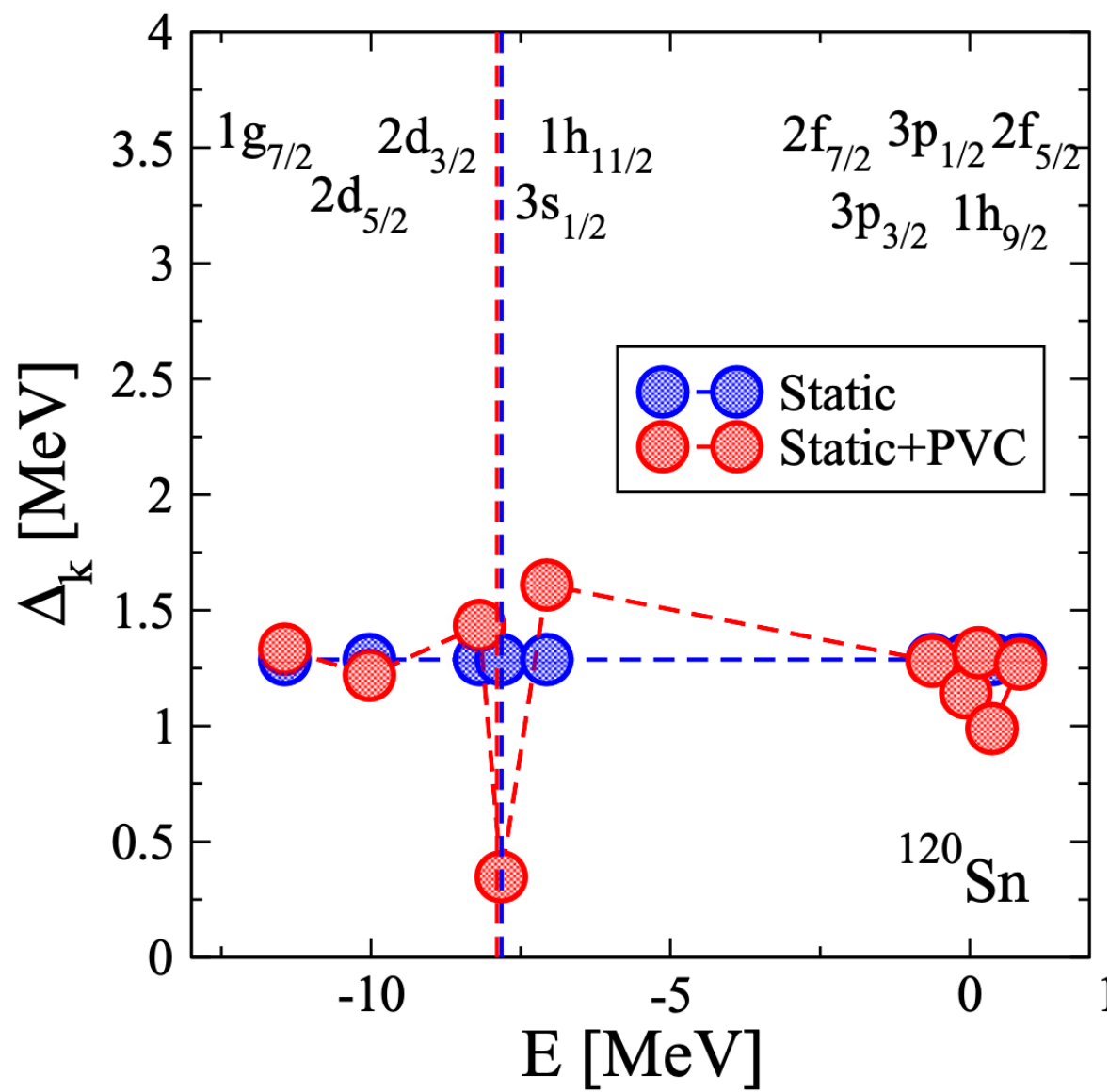
$$\begin{aligned}\tilde{\Delta}_{a(n)} &= Z_{a(n)} \tilde{\Sigma}_{a(n)}^{12} \\ &= Z_{a(n)} \left(\Delta_a^{\text{BCS}} + \tilde{\Sigma}_{a(n)}^{12, \text{pho}} \right) \equiv \tilde{\Delta}_{a(n)}^{\text{bare}} + \tilde{\Delta}_{a(n)}^{\text{pho}}.\end{aligned}$$



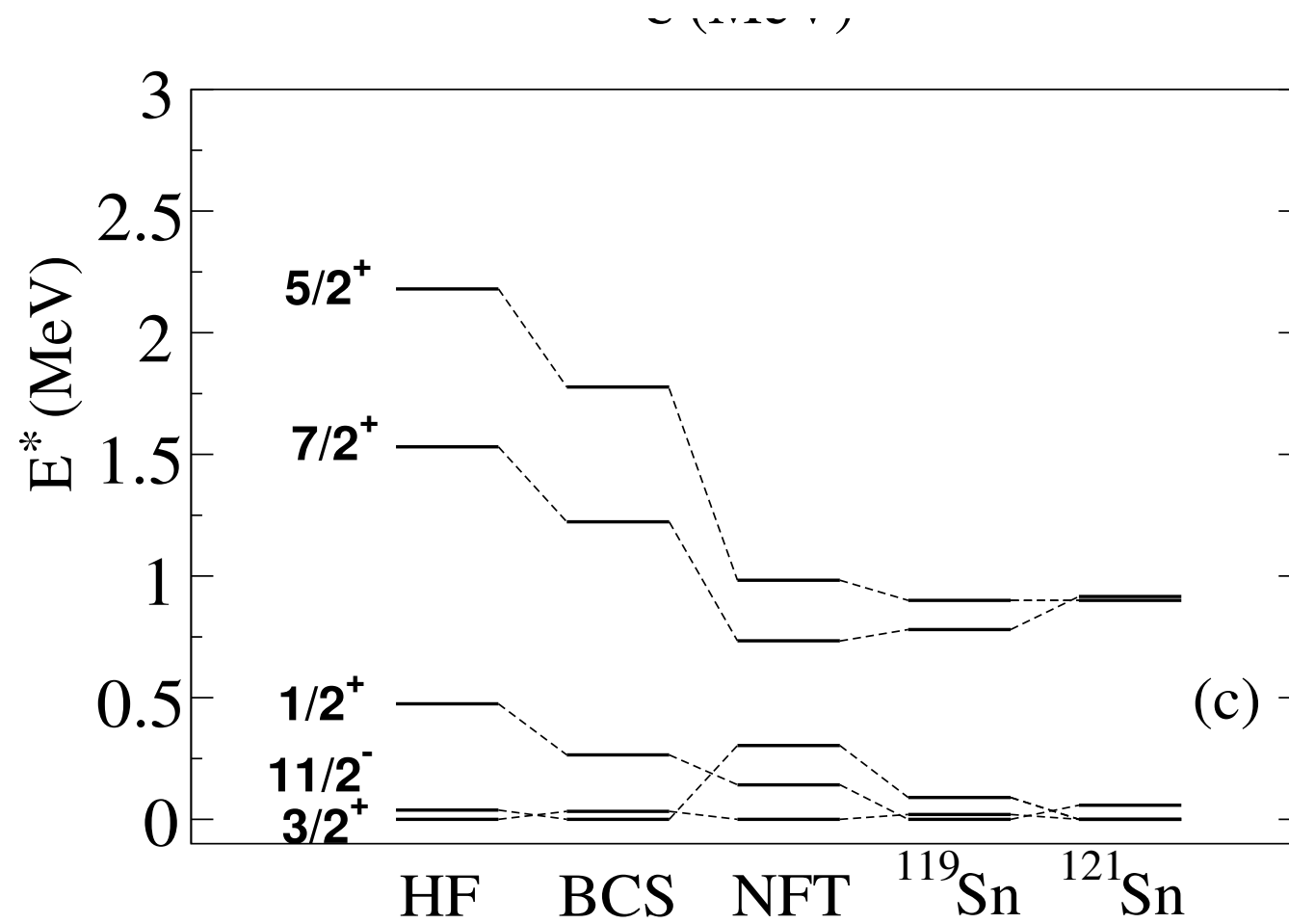
1st iteration

Experimental phonons were used for natural parity modes with $\lambda=2,3,4,5$

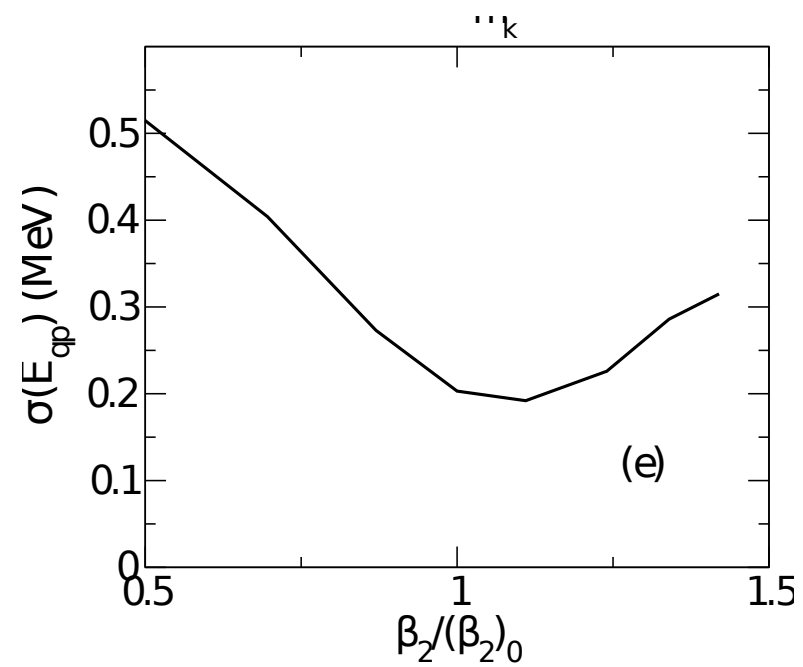
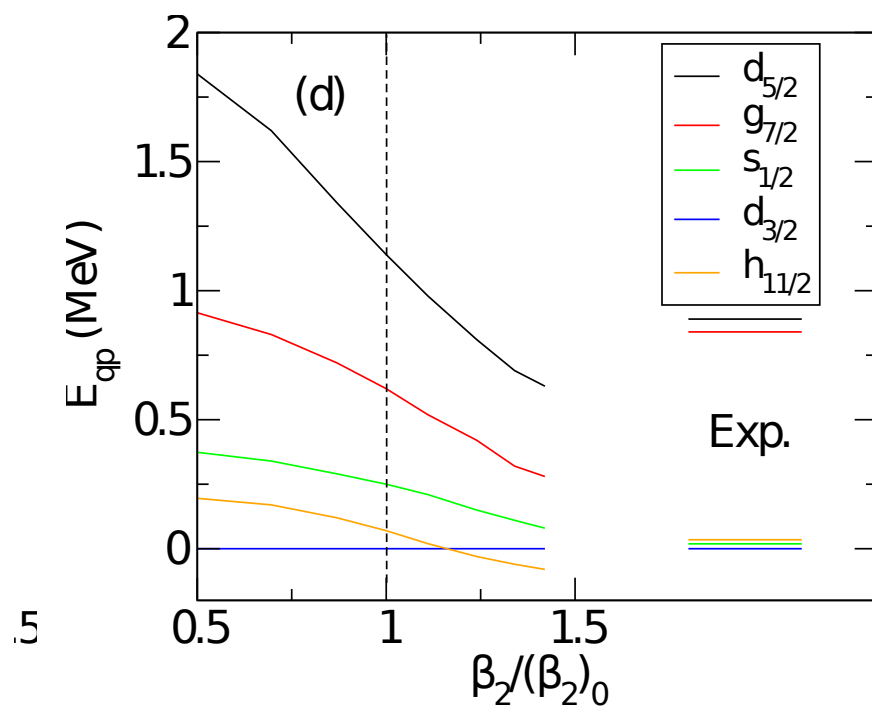




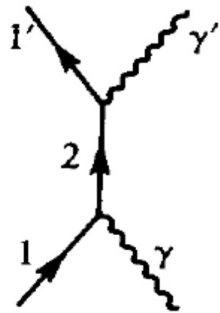
E. Litvinova and P. Schuck,
PRC 102 (2020) 034310



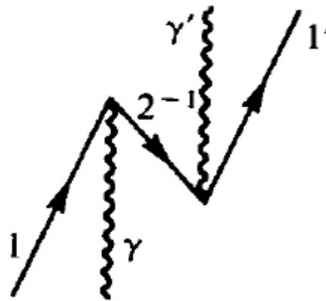
Quasiparticle spectrum



A specific test of particle-phonon interaction: multiplet splitting

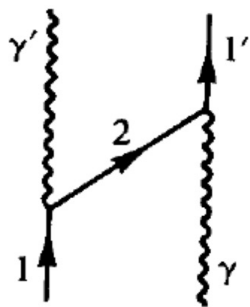


(a)

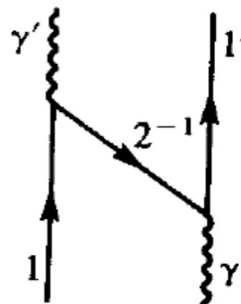


(b)

Already included in the diagonalization



(c)

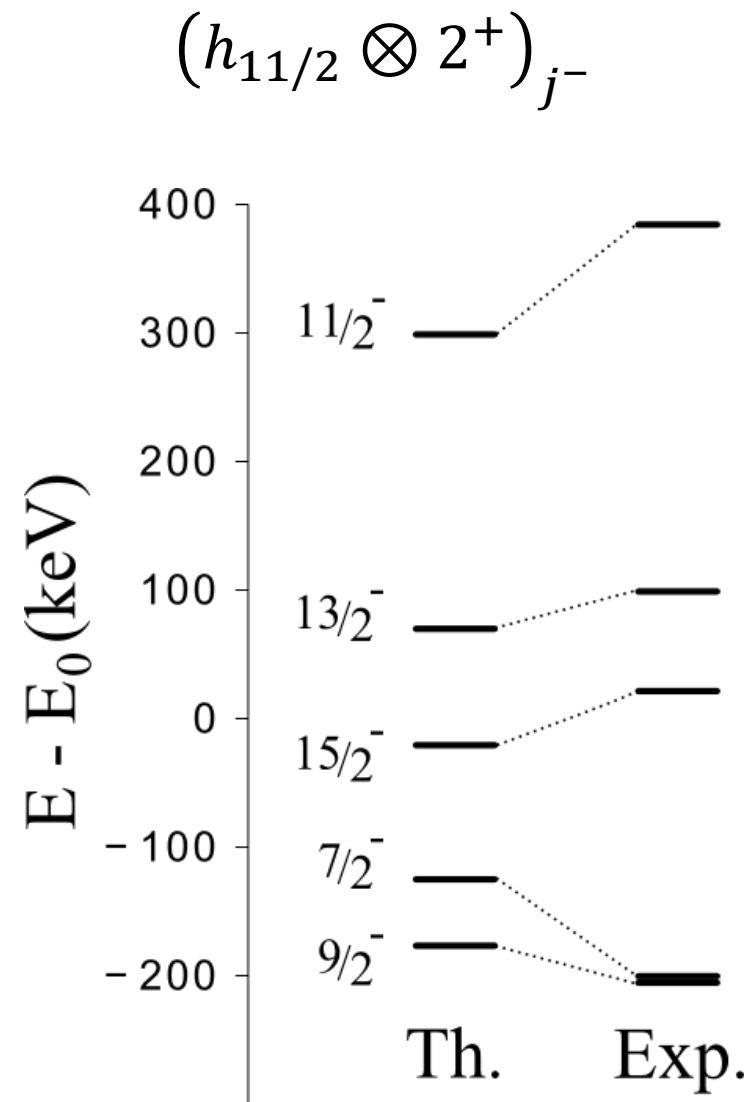


(d)

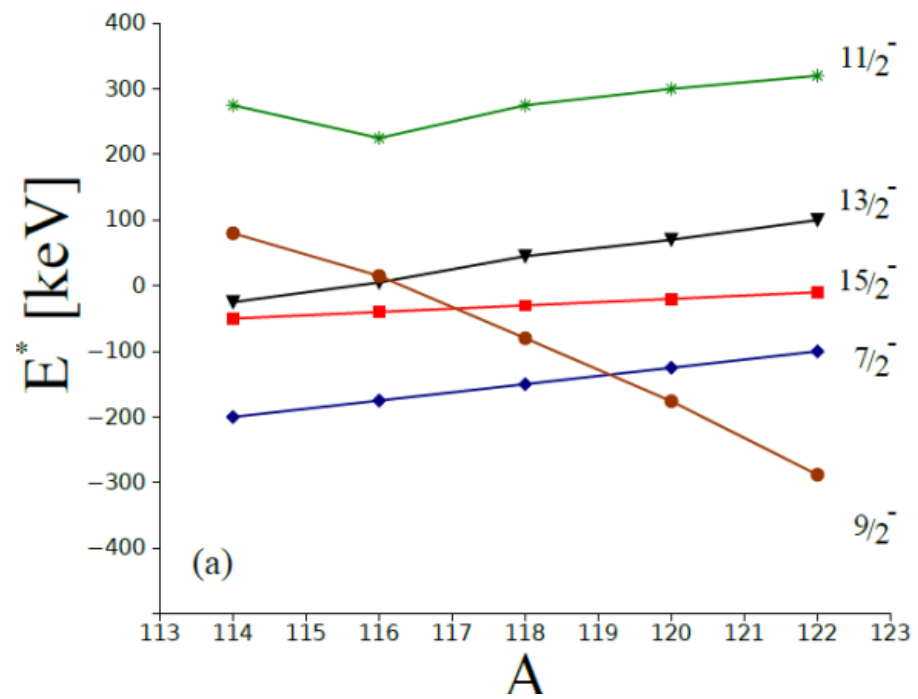
Include two-phonon states; calculated perturbatively

^{120}Sn (th.)
 $^{119-121}\text{Sn}$ (exp., ave)

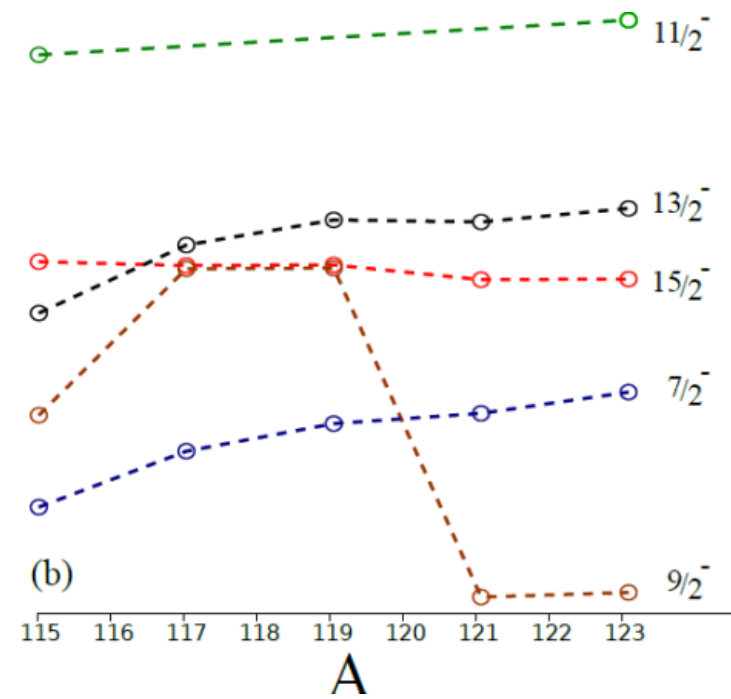
Important notice:
the splitting is insensitive
(within 20 keV)
to the assumed mean field
(SGII, SkM*, SLy4)



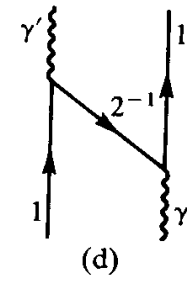
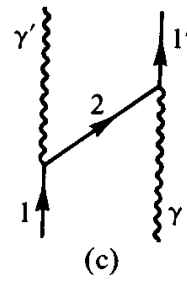
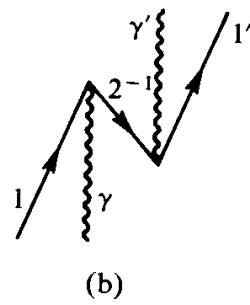
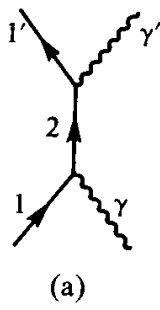
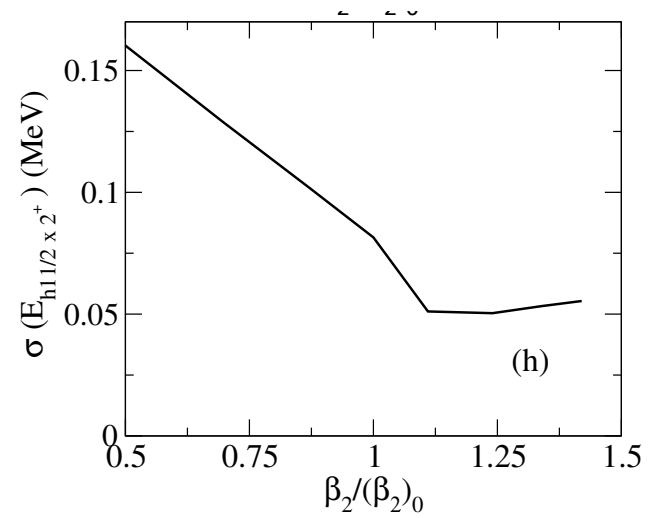
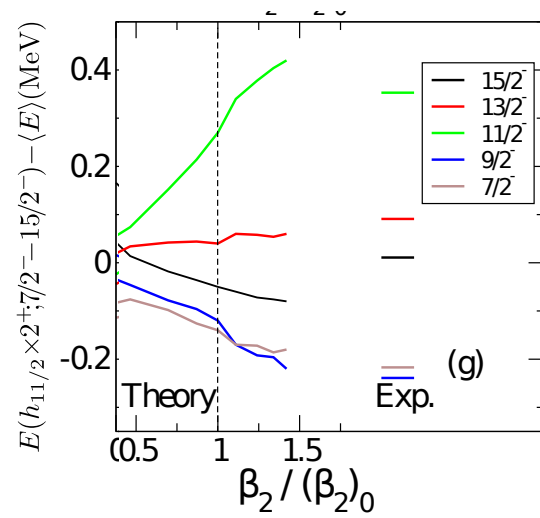
Theory

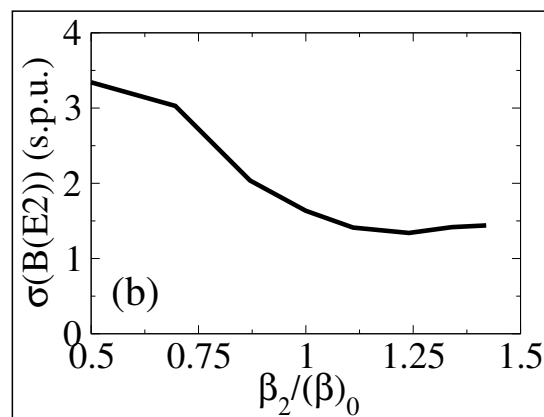


Experiment

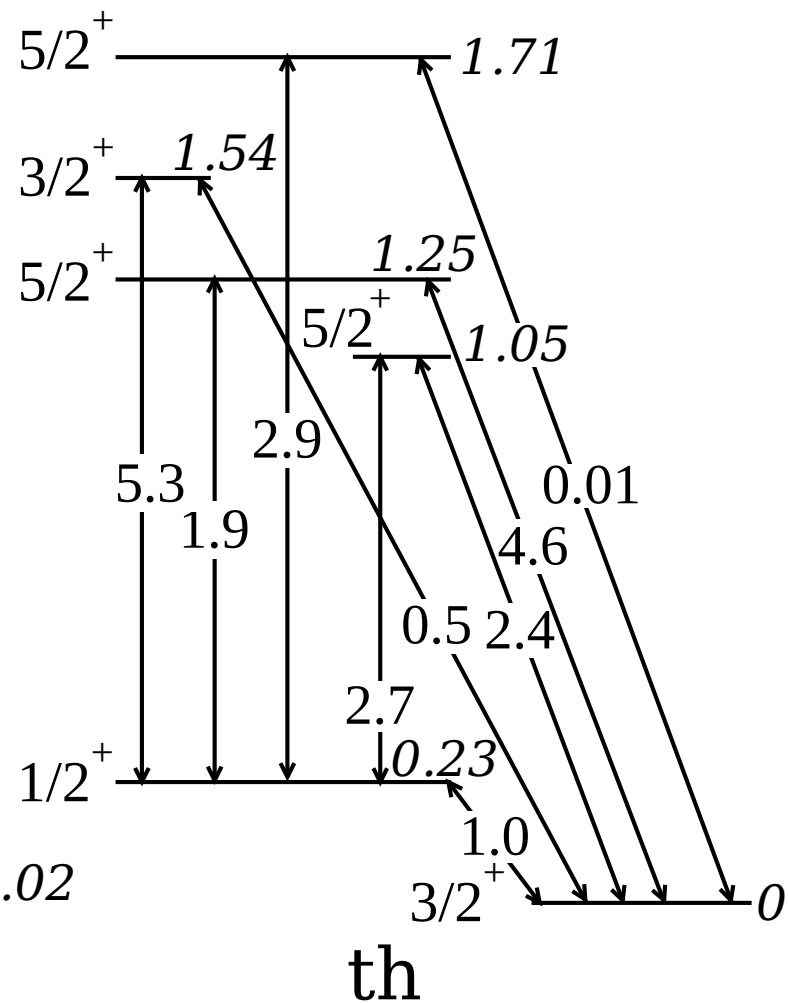
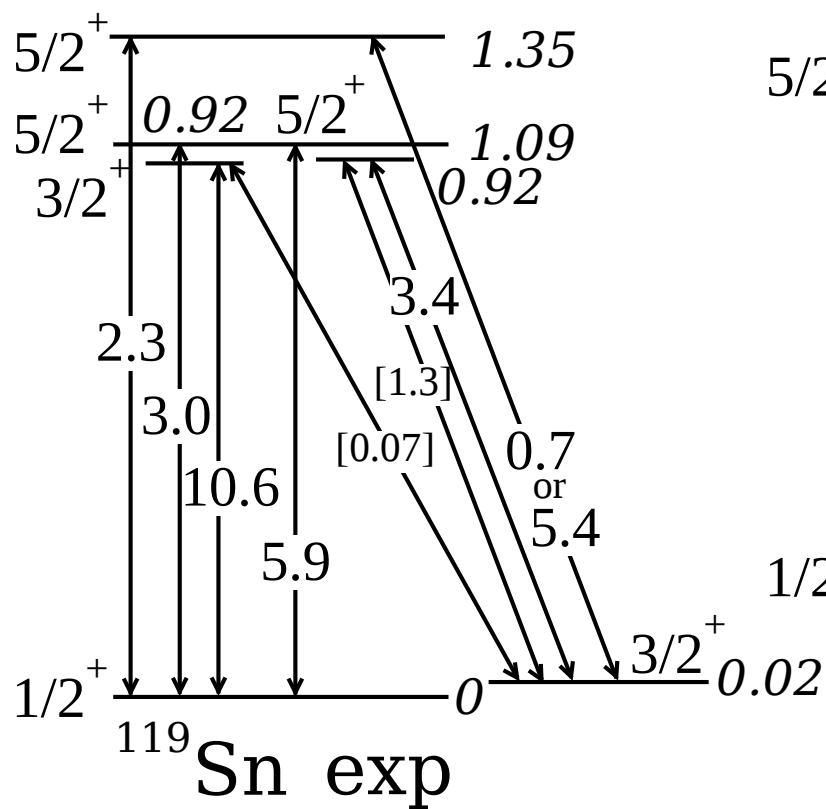


$h_{11/2} \times 2^+$ multiplet

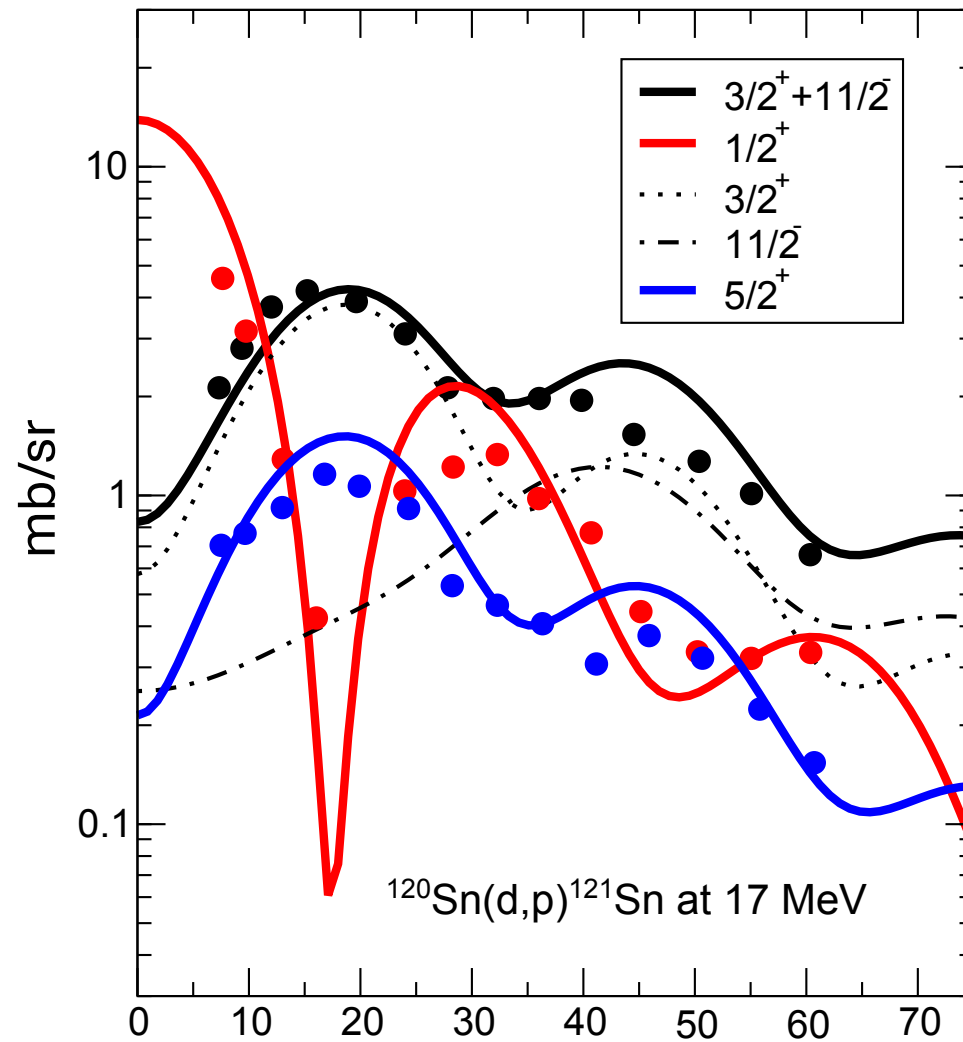




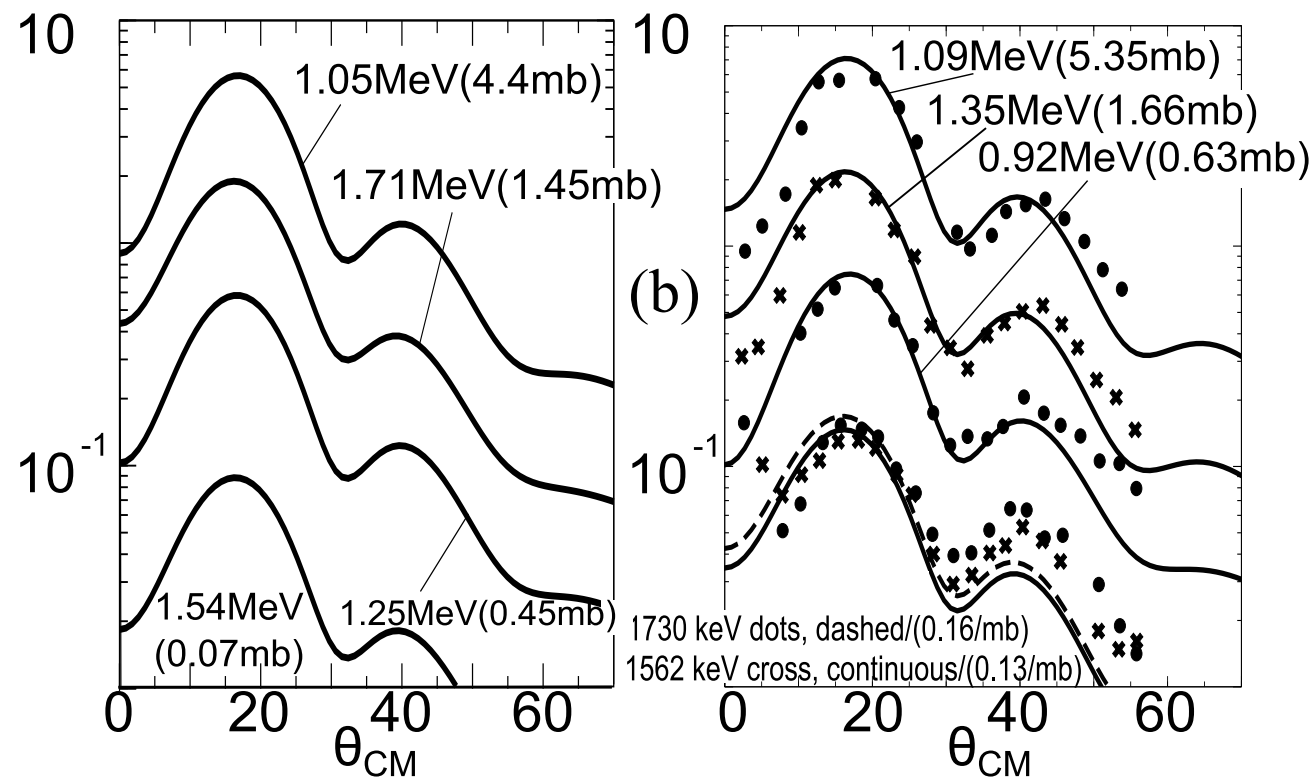
B(E2) transition strength



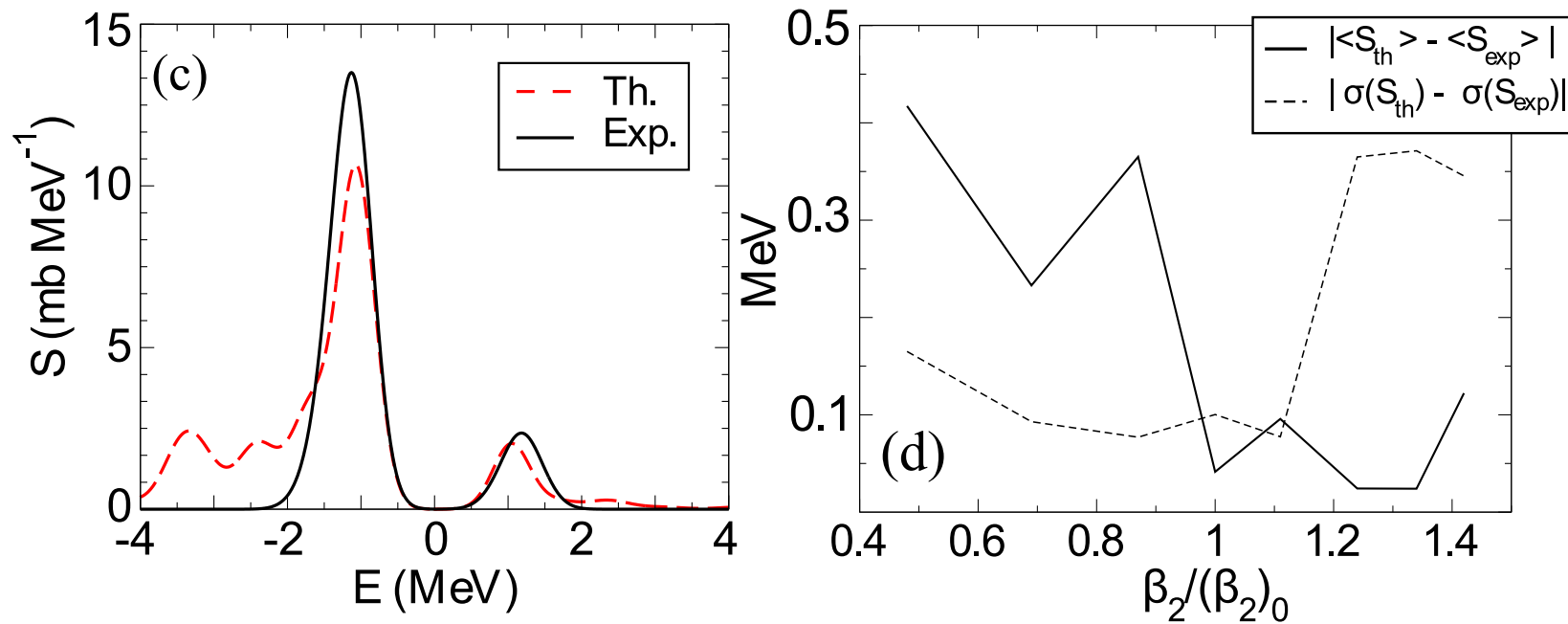
One-neutron transfer reactions to quasiparticle states using calculated occupation factors



Analysis of the transfer to the $5/2^+$ states (fragmented $d_{5/2}$ strength)

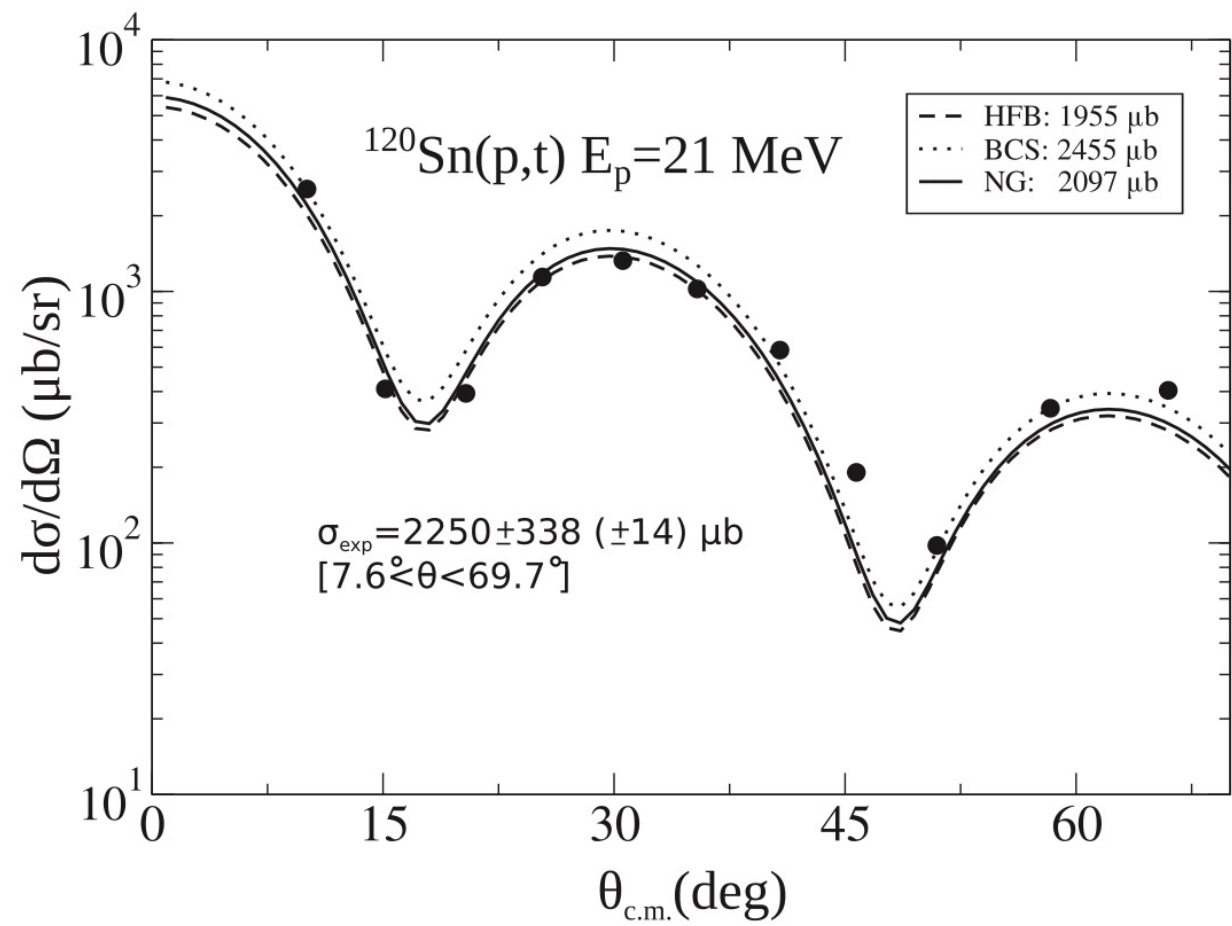


Analysis of the transfer to the $5/2^+$ states (fragmented $d_{5/2}$ strength)



Spectroscopic properties
of the main fragments

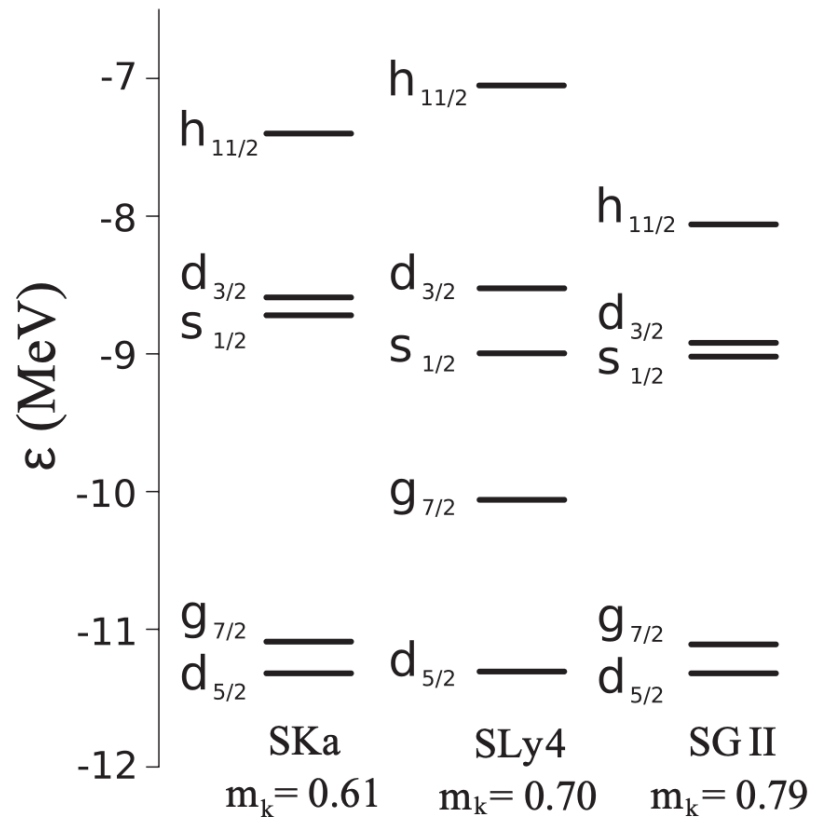
a	ϵ_a	$\tilde{\epsilon}_{a(n)}$	n	$\tilde{E}_{a(n)}$	$\tilde{u}_{a(n)}^2$	$\tilde{v}_{a(n)}^2$	$N_{a(n)}$	$Z_{a(n)}$	$\tilde{\Delta}_{a(n)}$
$d_{5/2}$	-10.7	-9.4	1	2.55	0.06	0.28	0.34	0.60	1.96
		-9.9	2	2.75	0.01	0.10	0.11		1.80
		-10.5	3	3.19	0.01	0.10	0.11		1.68
		-10.6	4	3.36	0.01	0.07	0.08		1.88
		-11.2	5	3.95	0.01	0.07	0.08		1.97
		-12.4	6	4.77	0.0	0.07	0.07		-1.29
		-12.7	7	4.98	0.0	0.09	0.09		-0.61
$g_{7/2}$	-10.1	-9.3	1	2.10	0.09	0.59	0.68	0.78	1.43
		-10.6	2	2.83	0.00	0.08	0.08		0.34
		-9.9	3	3.20	0.00	0.0	0.0		-2.40
		-11.2	4	3.50	0.00	0.11	0.11		0.97
$s_{1/2}$	-9.0	-8.4	1	1.80	0.26	0.53	0.79	0.72	1.69
		-10.4	2	2.84	0.00	0.04	0.04		-1.03
		-10.1	3	3.20	0.00	0.0	0.0		-2.20
		-12.4	4	4.64	0.00	0.07	0.07		-0.46
$d_{3/2}$	-8.5	-7.9	1	1.48	0.38	0.46	0.84	0.76	1.48
		-7.5	2	2.75	0.0	0.00	0.0		-2.73
		-8.8	3	3.06	0.0	0.01	0.01		-2.88
		-11.3	4	3.49	0.0	0.05	0.05		-0.14
$h_{11/2}$	-7.1	-7.2	1	1.64	0.57	0.26	0.83	0.79	1.52
		-4.7	2	3.08	0.09	0.00	0.09		0.08
		-9.6	3	3.97	0.00	0.00	0.0		3.54

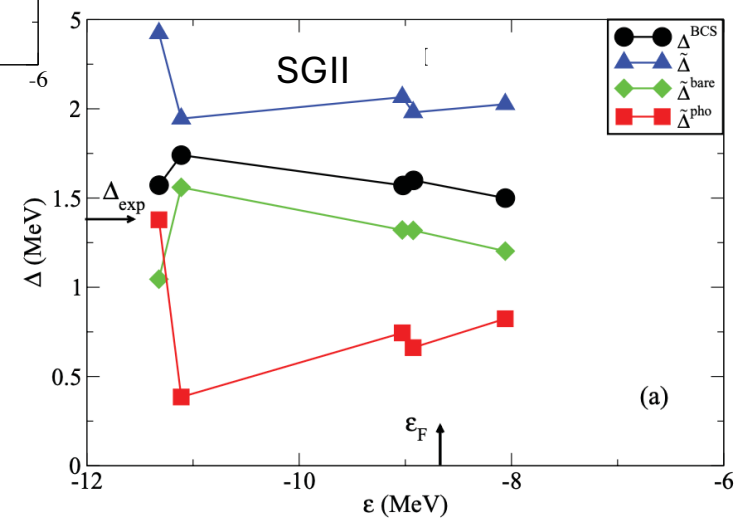
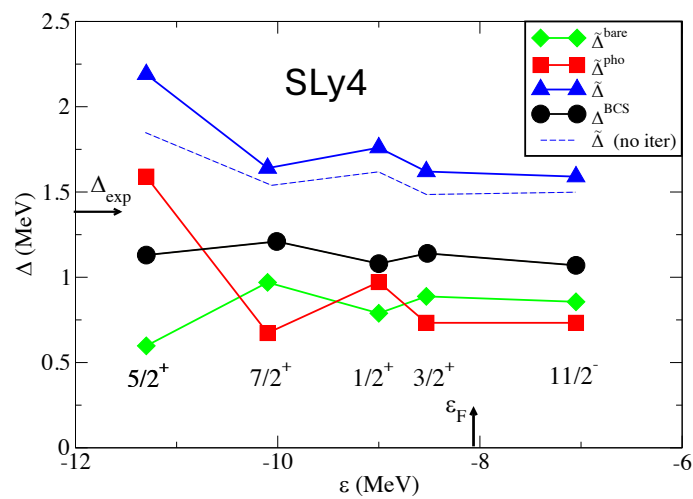
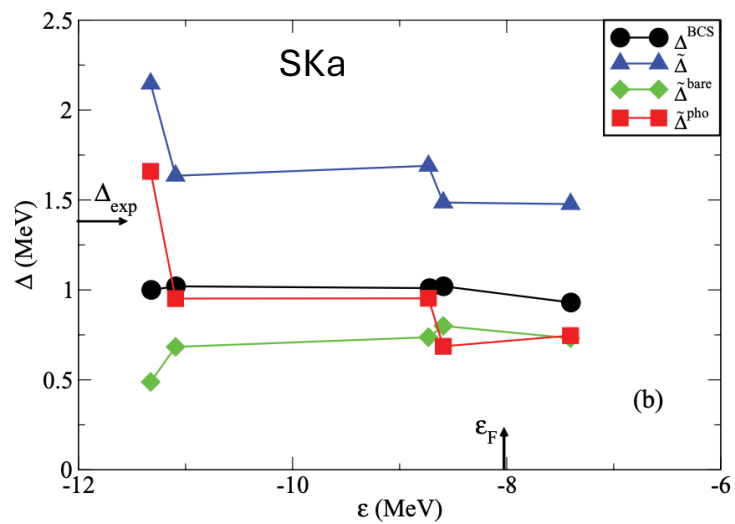


Error on the observables

Observables	SLy4
E_{qp}	190 (19%) keV
Mult. splitt.	50 (7%) keV
$d_{5/2}$ strength (centr.)	200 (20%) keV
$d_{5/2}$ strength (width)	160 (20%) keV
$B(E2)$	1.4 (14%) s.p.u.
$\sigma_{2n}(p,t)$	153 (7%) μb

Dependence on the mean field





The contribution from non natural parity modes to the induced interaction
is difficult to quantify

$$v_{\text{ph}}(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') \{ [F_0 + F'_0 \vec{\tau} \cdot \vec{\tau}'] \\ + [(G_0 + G'_0 \vec{\tau} \cdot \vec{\tau}') \vec{\sigma} \cdot \vec{\sigma}'] \}.$$

$$\langle v' m' v' \bar{m}' | v_{\text{ind}} | v m v \bar{m} \rangle = \sum_{J^\pi M i} \frac{2(f + g)_{vm; J^\pi M i}^{v' m'} (f - g)_{vm; J^\pi M i}^{v' m'}}{E_0 - E_{\text{int}}},$$

$$f_{vm; J^\pi M i}^{v' m'} = i^{l-l'} \langle j' m' | (i)^J Y_{JM} | j m \rangle \\ \times \int dr \varphi_v [(F_0 + F'_0) \delta \rho_{J^\pi n}^i + (F_0 - F'_0) \delta \rho_{J^\pi p}^i] \varphi_v,$$

$$g_{vm J^\pi M i}^{v' m'} = \sum_{L=J-1}^{J+1} i^{l-l'} \langle j' m' | (i)^L [Y_L \times \sigma]_{JM} | j m \rangle \\ \times \int dr \varphi_{v'} [(G_0 + G'_0) \delta \rho_{J^\pi L n}^i \\ + (G_0 - G'_0) \delta \rho_{J^\pi L p}^i] \varphi_v,$$

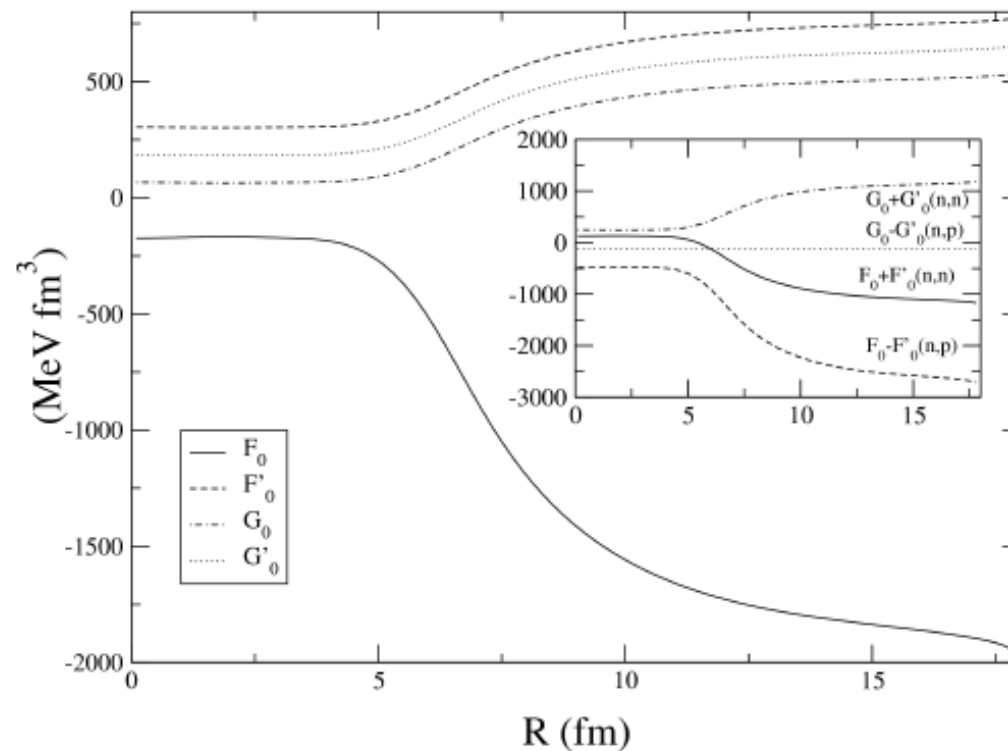
Why such a difference with neutron matter?

The proton-neutron interaction in the particle vibration coupling plays an essential role. If we cancel it, a net repulsive effect is obtained for the induced interaction.



Strong difference between induced interaction in neutron and nuclear matter

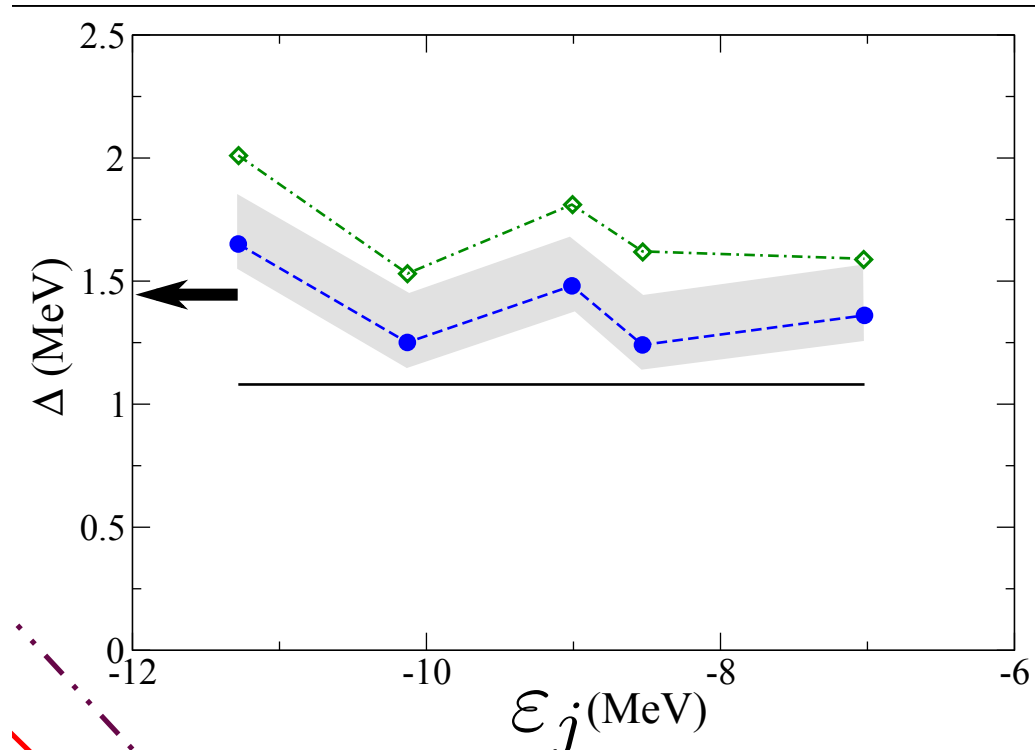
Landau parameters
of SkM* force
in ^{120}Sn



Coupling to abnormal parity phonons



Reduction of the gap by about 200 keV



What about the induced interaction in deformed nuclei?

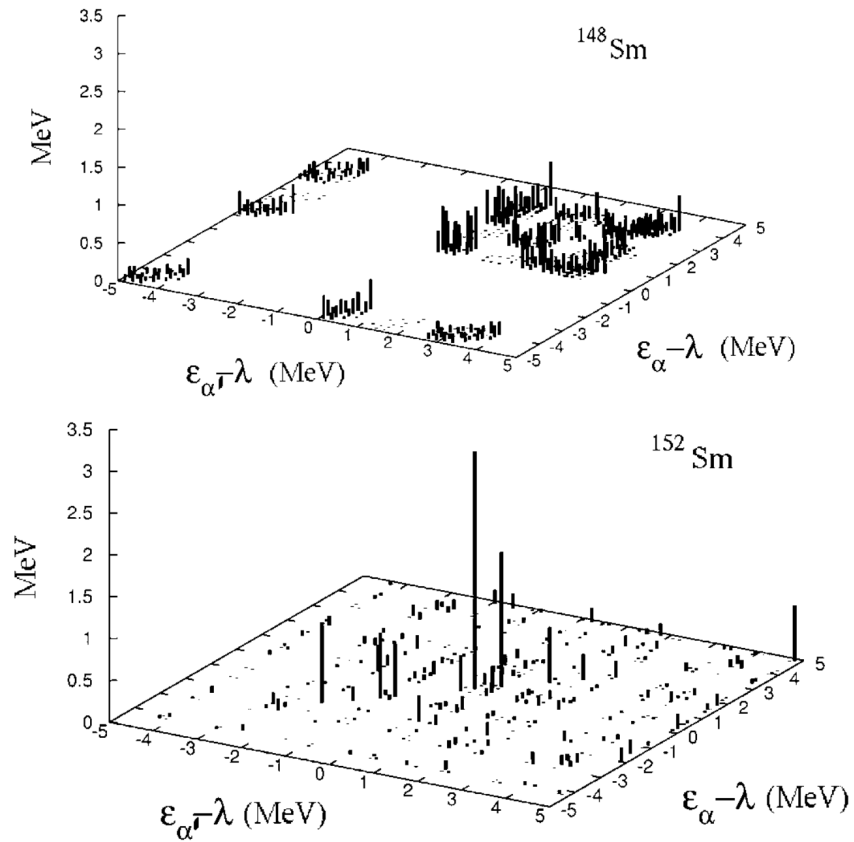


Figure 5. The induced pairing matrix elements $G_{\alpha\alpha'}^{\text{ind}}$ (cf equation (10)) for the spherical nucleus ^{148}Sm and the deformed nucleus ^{152}Sm , plotted as a function of $\epsilon_{\alpha} - \lambda$ (x -axis) and $\epsilon_{\alpha'} - \lambda$ (y -axis), where λ is the Fermi energy.

ˆ P. Donati et al., J. Phys. G 31 (2005) 295

CONCLUSIONS

The coupling of quasiparticle to the collective vibrations of the core induces a pairing interaction that enhances the bare pairing interaction in ^{120}Sn .

This effect is counteracted to some extent by the fragmentation of the quasiparticle strength, but in the end the gap is increased as compared to the BCS result.

While this mechanism does not depend on the assumed mean field, the final absolute value of the gap does.

One can obtain a rather comprehensive description of several phenomena taking place around the Fermi energy, starting from a “reasonable” mean field and adding a particle-vibration strength derived from the experimental deformation parameters of the quadrupole modes.

It is a completely open question, whether one can draw some parallel with the findings from ab initio calculations.