

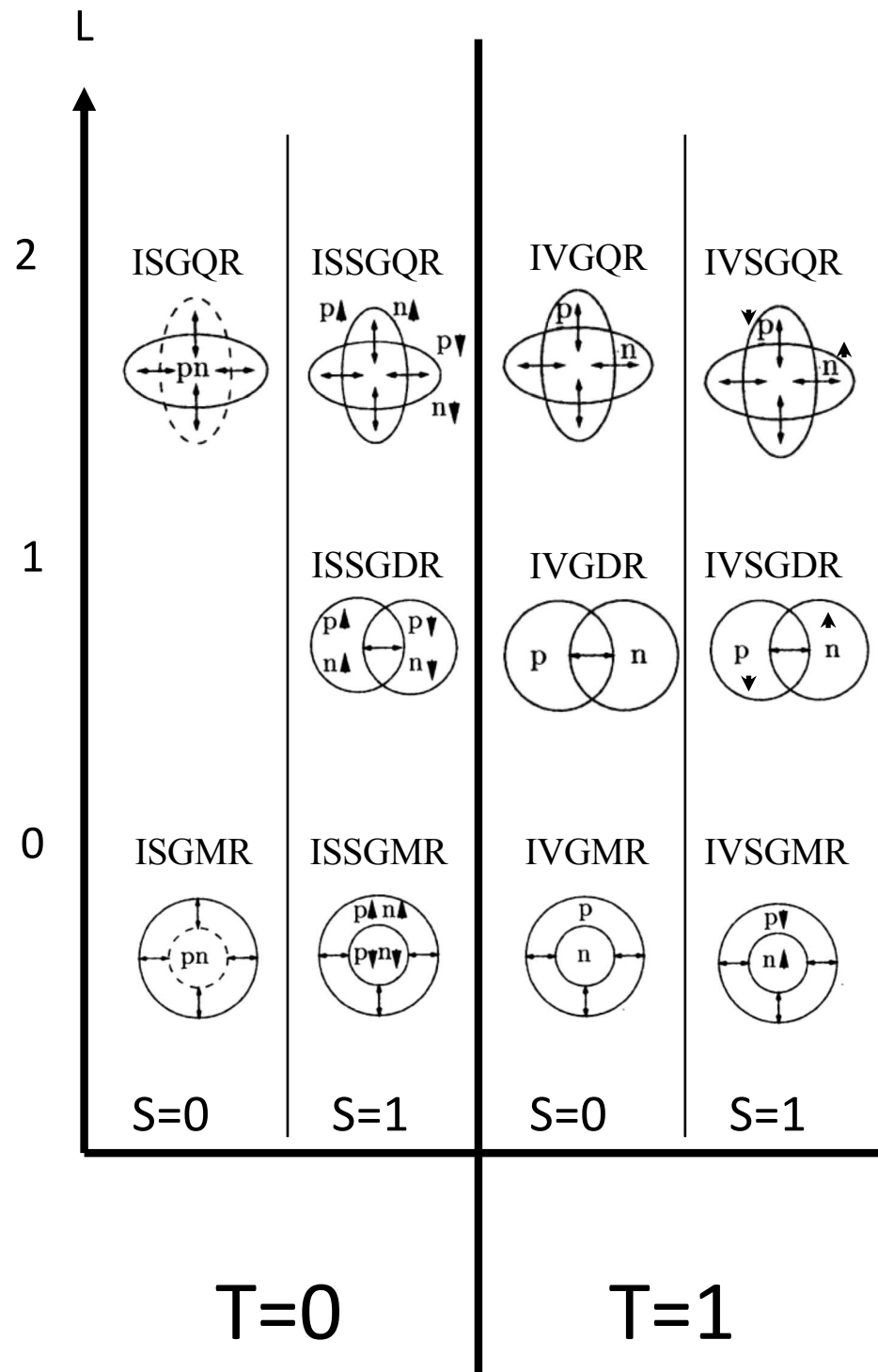
Fingerprint of superfluidity in EDF calculations on giant monopole resonances of mid-mass open-shell nuclei

- 1) The ISGMR: why and how (reminders) ?
- 2) Pairing and shell effects on the GMR using EDF ?
- 3) Low-energy component of GMR and pairing ?
- 4) Deformation effects on all that ?
- 5) Overview

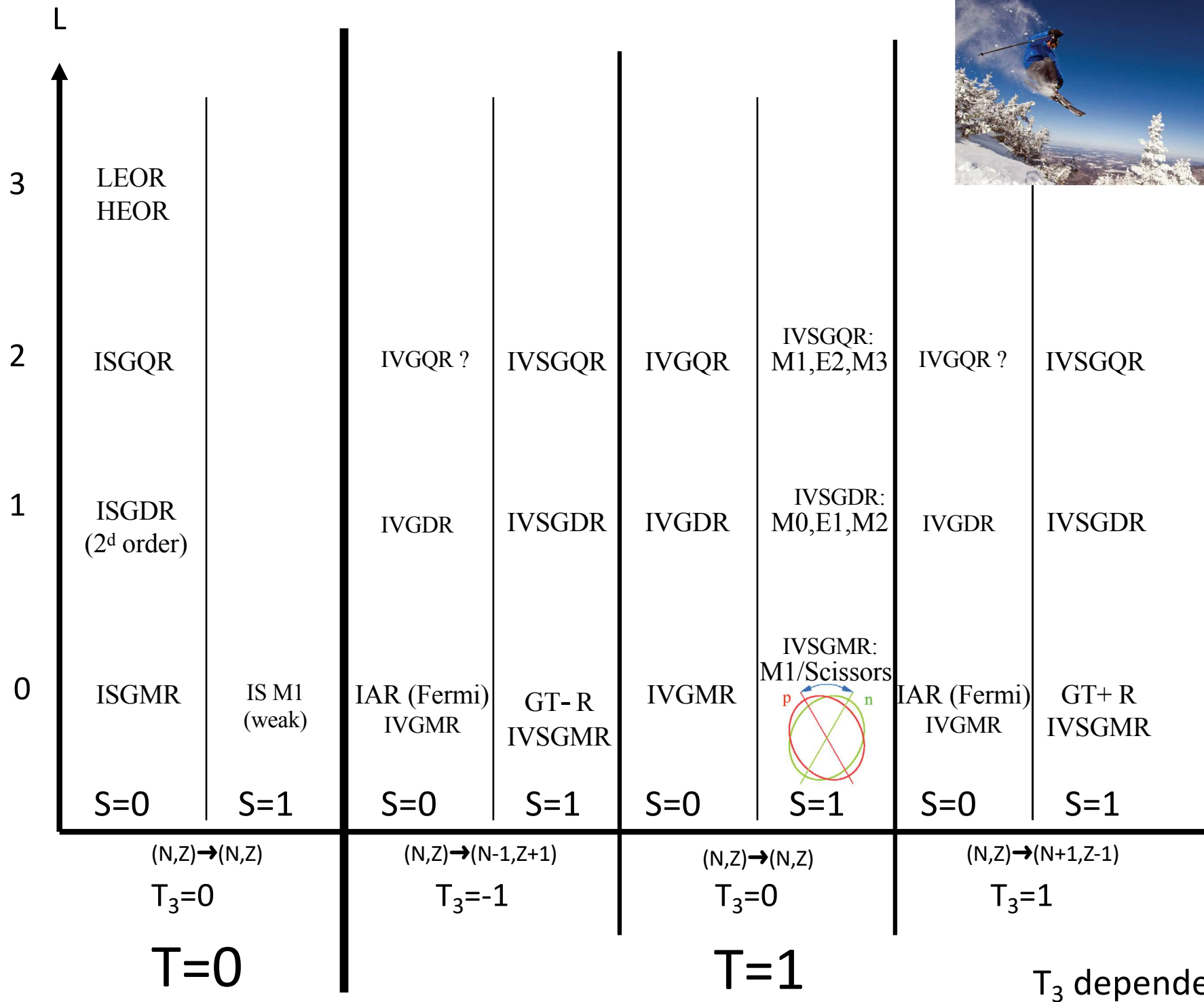
E. Khan



ESNT 13-21 May, 2025



Beginners



Experts

The ISGMR and the single-particle picture

Useful in EDF approaches

ISGMR
is
a $2\hbar\omega$ mode

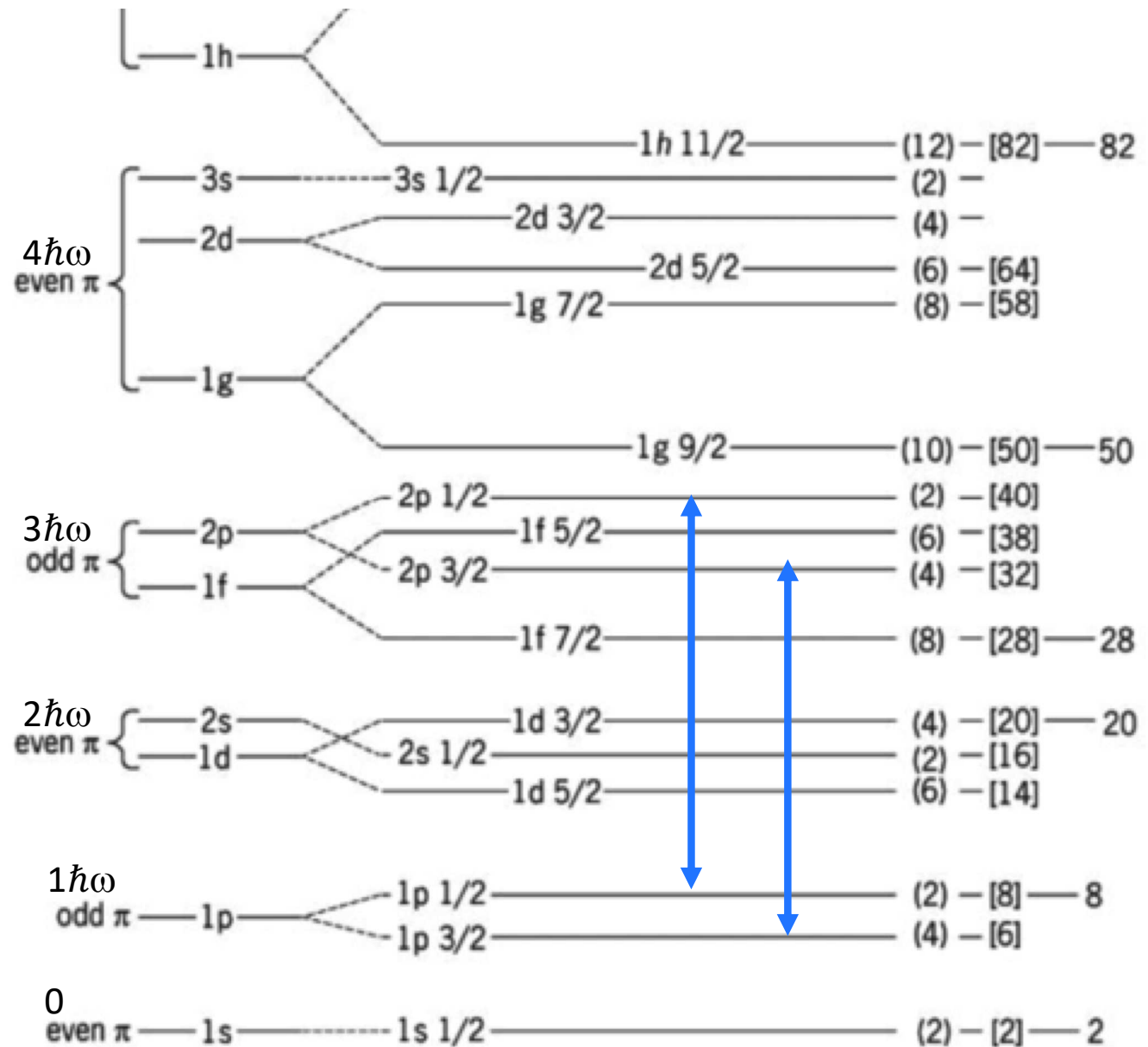
$$\vec{L}_R = \vec{l}_p + \vec{l}_h$$

$$\pi_R = \pi_p \cdot \pi_h$$

Non spin flip:

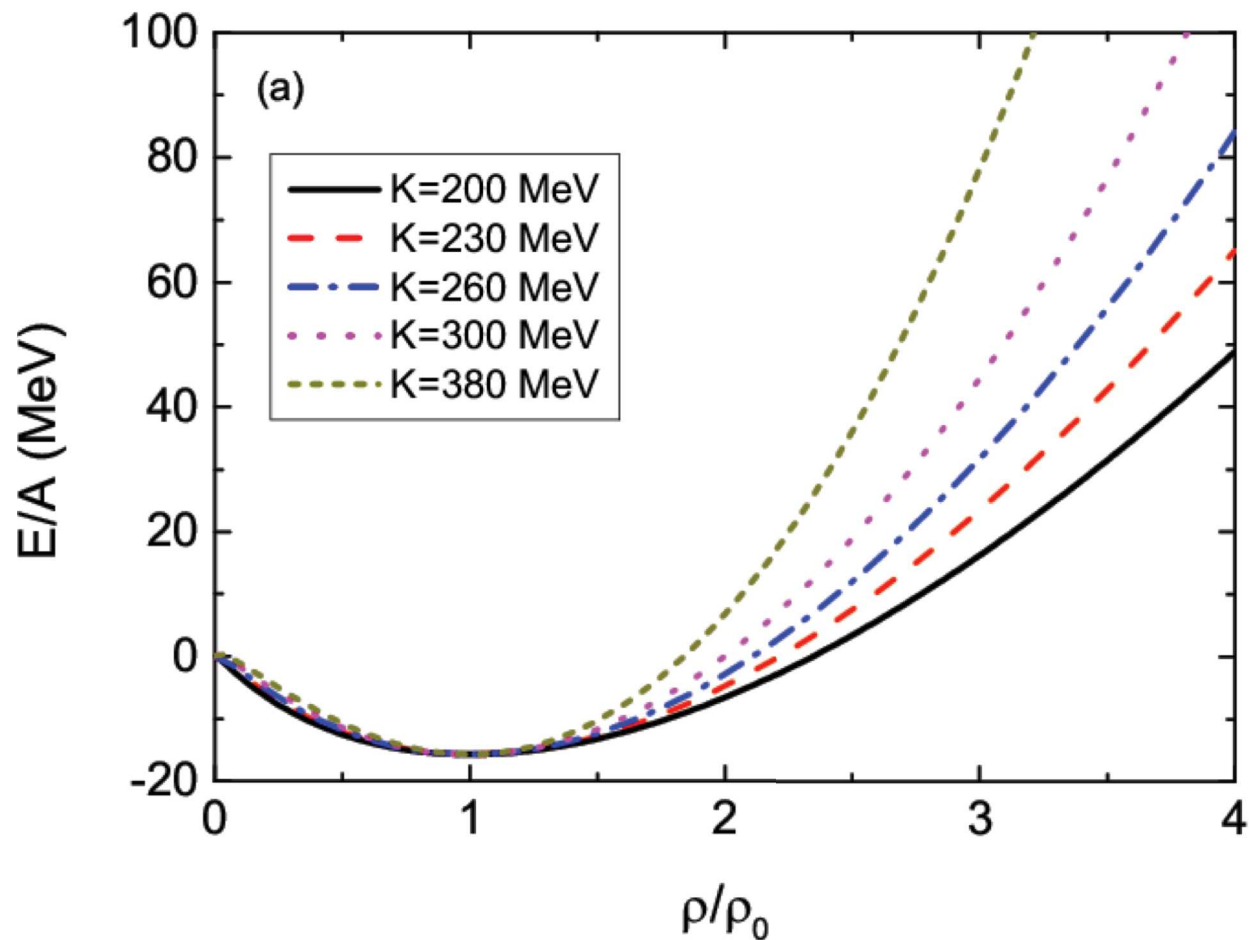
$$\vec{S}_R = \vec{0}$$

$$\begin{aligned}\vec{J}_R &= \vec{j}_p + \vec{j}_h \\ &= \vec{L}_R\end{aligned}$$




Why to study ISGMR ?

- GR have large cross section: dominant excitation mode: many applications
- Among the easiest to detect ; provides information on nuclear structure
- Important links with the nuclear equation of state, such as the incompressibility



EoS: definitions

a.k.a: $a_v \simeq -16$ MeV a.k.a: $K_\infty \simeq 240$ MeV(?) Skewness

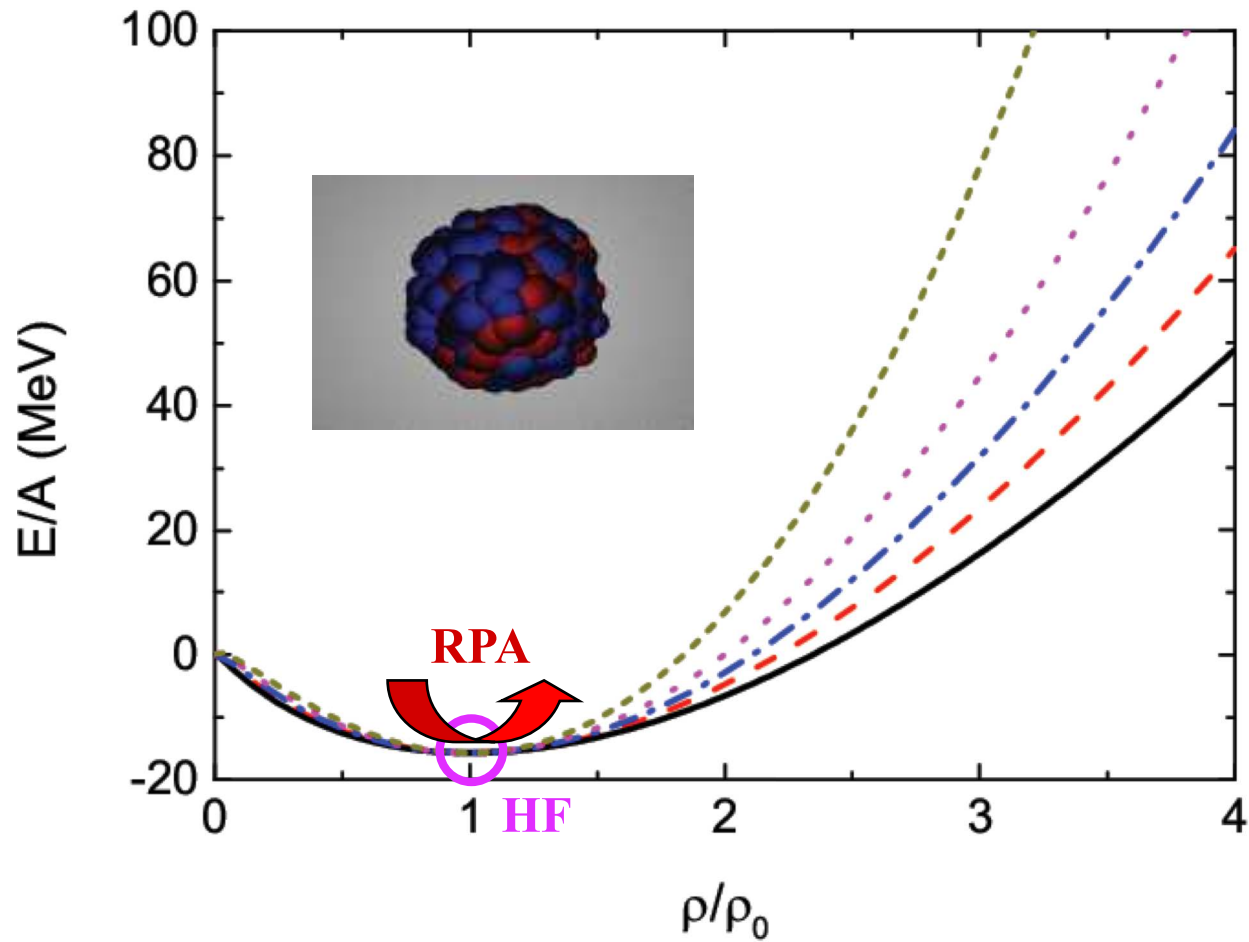


The diagram consists of three arrows pointing upwards and inwards from the text labels 'a.k.a: a_v ≃ -16 MeV', 'a.k.a: K_∞ ≃ 240 MeV(?)', and 'Skewness' towards a common point centered above the space between the middle and right labels.

- Generalisation to asymmetric nuclear matter:

$$\frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, 0) + S(\rho)\delta^2 + \dots$$

How to study the ISGMR ?



→ Small amplitude perturbations around the ground state

Energy density functional approach:

let's predict X, Y and E_{GR} by perturbing the g.s. density

1) **EDF** $E[\rho]: \langle Slat. | H | Slat. \rangle = \int E[\rho(\vec{r})] d\vec{r}$ with $\rho(r) = \sum_i \varphi_i^*(r) \varphi_i(r) = \sum_i |\varphi_i(r)|^2$

2) **HF**: $E[\rho + \delta\rho] \simeq E[\rho] + \frac{\partial E[\rho]}{\partial \rho} \delta\rho$

State of a single nucleon

$h[\rho]$, the Hartree-Fock field

3) **RPA**: $h[\rho + \delta\rho] \simeq h[\rho] + \frac{\partial h[\rho]}{\partial \rho} \delta\rho$ with $\frac{\partial h[\rho]}{\partial \rho} = \boxed{\frac{\partial^2 E[\rho]}{\partial \rho^2} \equiv V_{res}}$

The residual interaction

4) **Time evolution equation:**

$\hbar\omega\delta\rho$

$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \longrightarrow \boxed{i\hbar \frac{\partial \delta\rho}{\partial t} = [h, \delta\rho] + [V_{res} \delta\rho, \rho]}$ $\longrightarrow \boxed{\delta\rho(r) = \sum_{mi} (X_{mi} - Y_{mi}) \varphi_i^*(r) \varphi_m(r)}$

RPA equation

$E_{GR} = \hbar\omega$

In practice, the RPA equation can also look like this:
(matricial form)

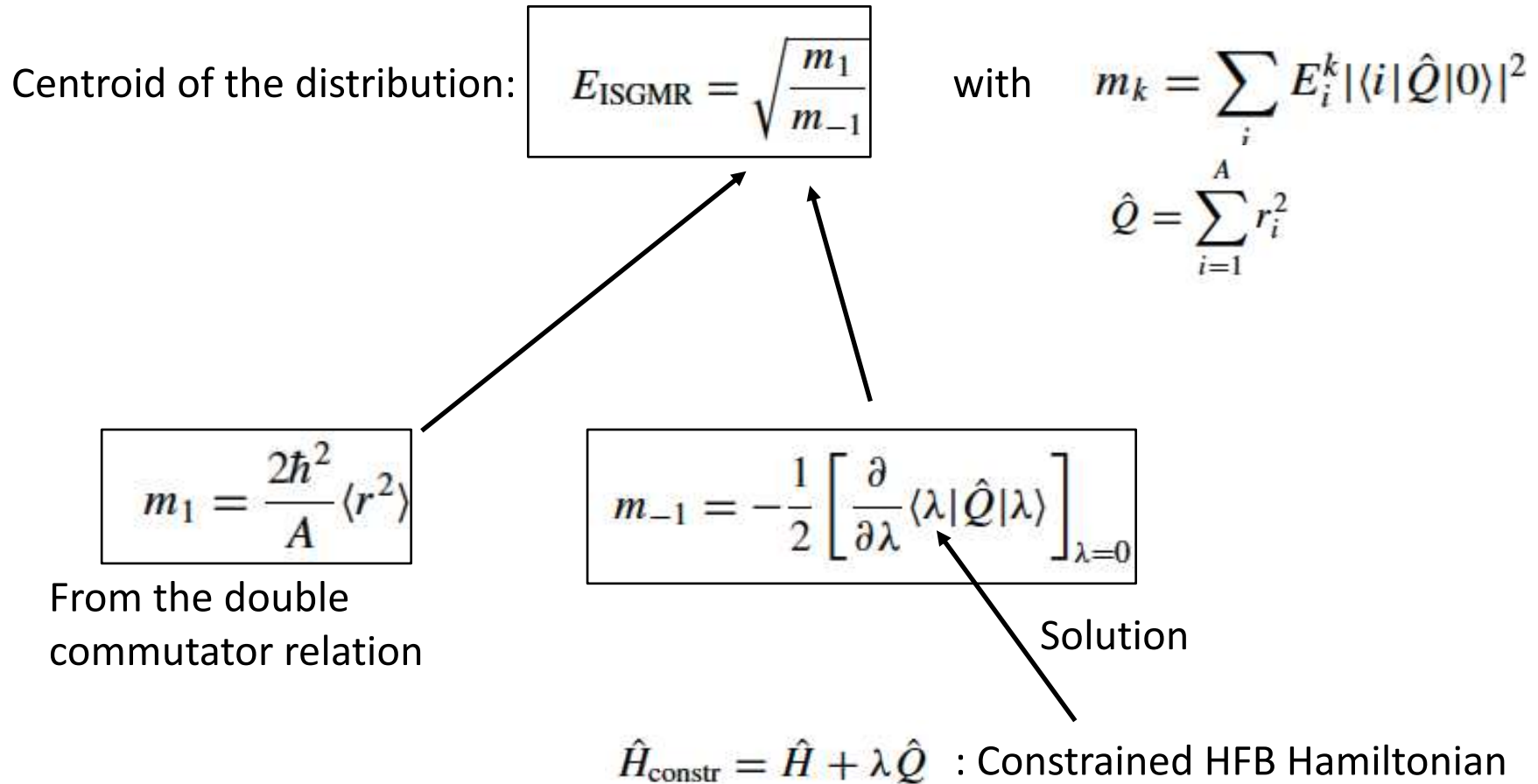
$$\begin{pmatrix} A & B \\ B^* & A \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \hbar\omega \begin{pmatrix} X^\nu \\ -Y^\nu \end{pmatrix}$$

with $A_{minj} = (\epsilon_m - \epsilon_i)\delta_{mn}\delta_{ij} + \frac{\partial^2 E[\rho]}{\partial \rho_{im} \partial \rho_{nj}}$

$$B_{minj} = \frac{\partial^2 E[\rho]}{\partial \rho_{im} \partial \rho_{jn}}$$

- It still shows how to get the X and Y from the residual interaction
- In the case of pairing, Quasiparticle-RPA (QRPA):
same structure with additional terms/dimensions (U,V, κ)

Another EDF-based way to predict E_{GMR} remaining at the HFB level

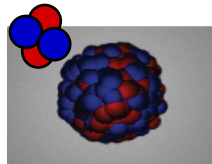


No need for predictions of excited states such as QPRA

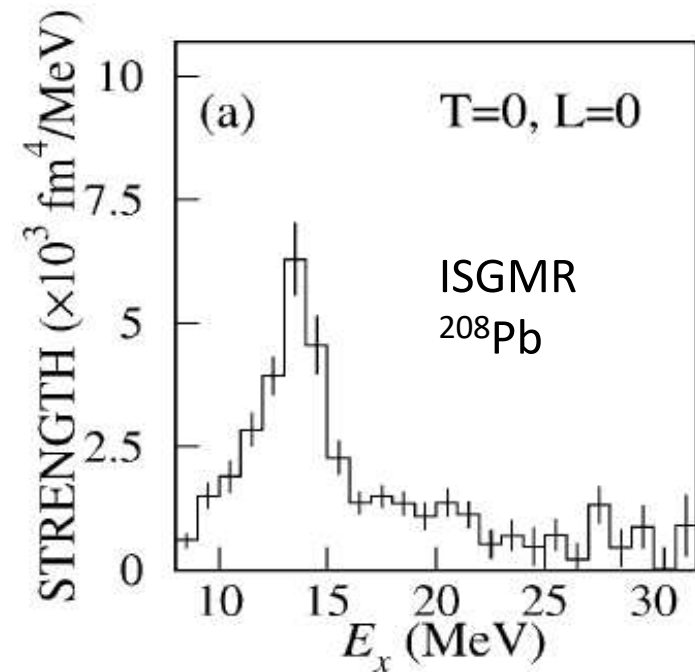
Method to determine K_∞ from E_{GMR}

- The nucleus exhibits a collective compression mode (how nice !):
the **Giant Monopole Resonance**

Inelastic α scattering



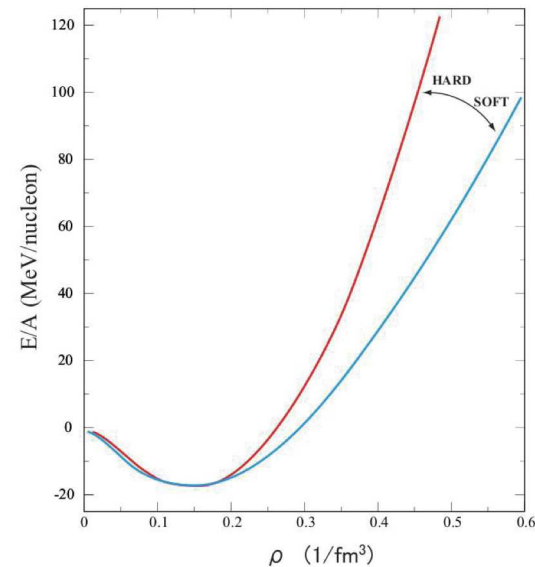
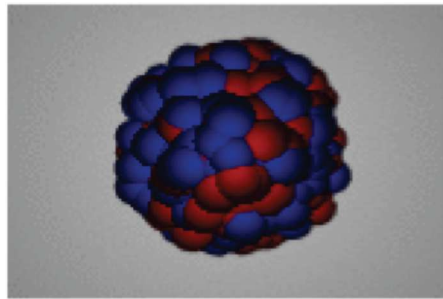
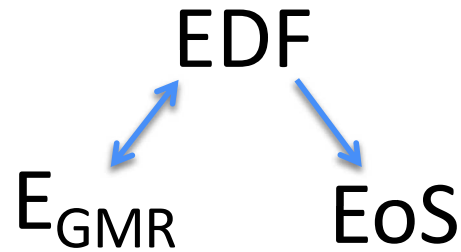
Nucleus (e.g. ^{208}Pb)



M. Uchida, H. Sakaguchi et al., PLB557(2003)12

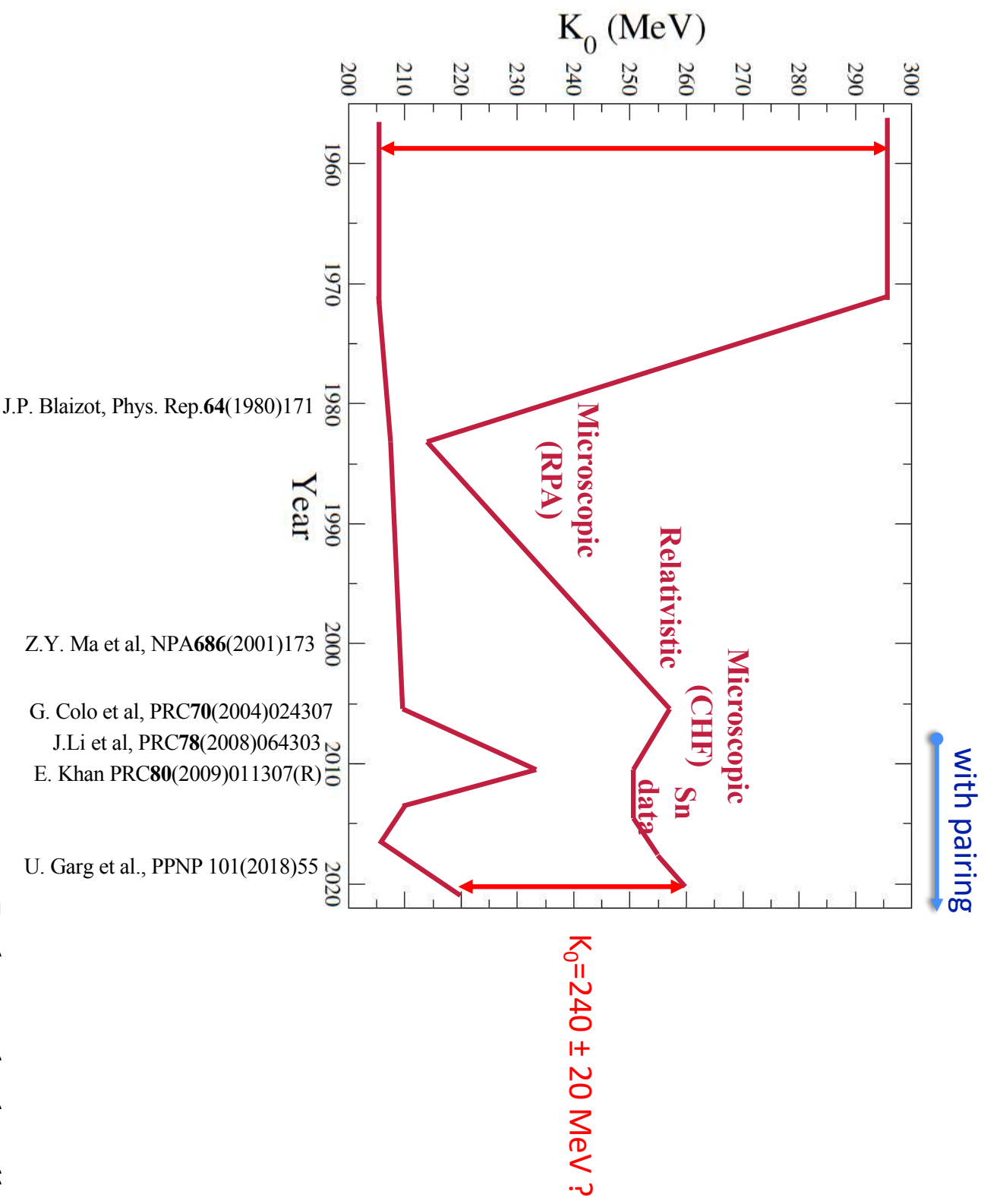
- How to link E_{GMR} to K_∞ ?

Microscopic method



- Nuclear structure models: from EDF to E_{GMR}
- Limitations : all the terms (EoS) have to be correctly predicted at once (indirect method)
- There also exists a macroscopic method based on LD expansion of incompressibility

Uncertainties on K_0



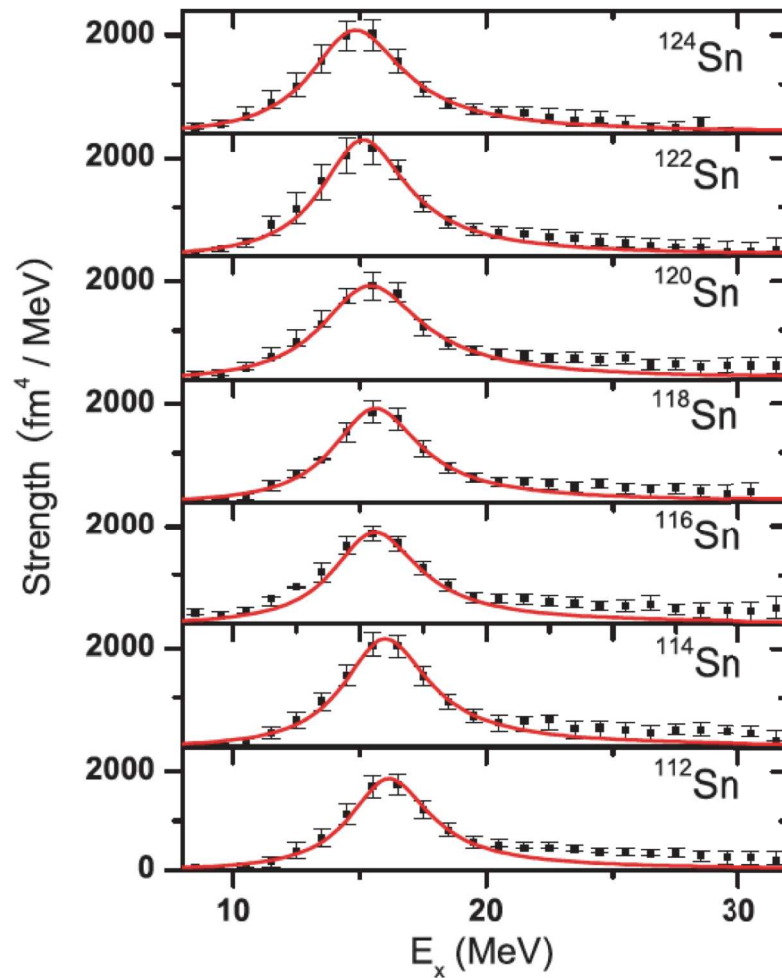
Is the method well defined ?

Incompressibilities

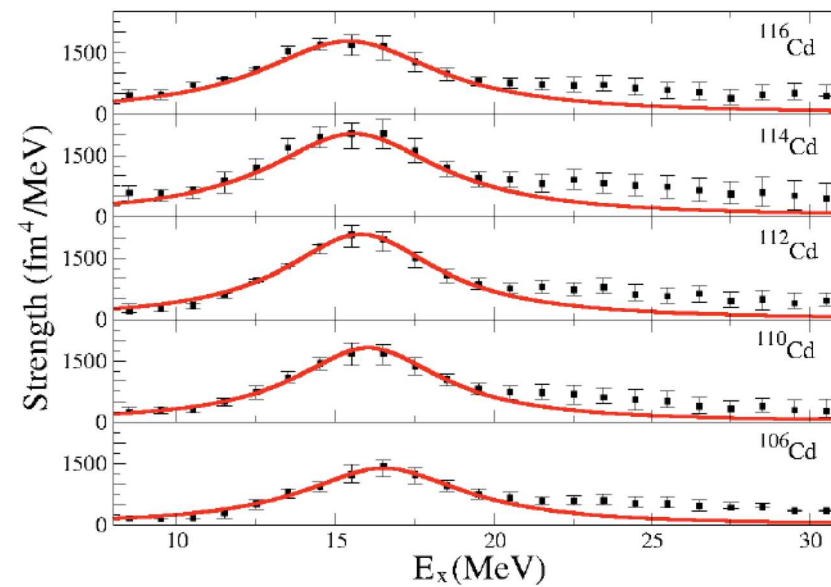
$$K_A \equiv 4\langle r^2 \rangle \frac{d^2 E/A}{d\langle r^2 \rangle^2} \bigg|_{\text{g.s.}}$$

$$K_0 = 9\rho_0^2 \left(\frac{\partial^2 E/A(\rho, 0)}{\partial \rho^2} \right)_{\rho=\rho_0}$$

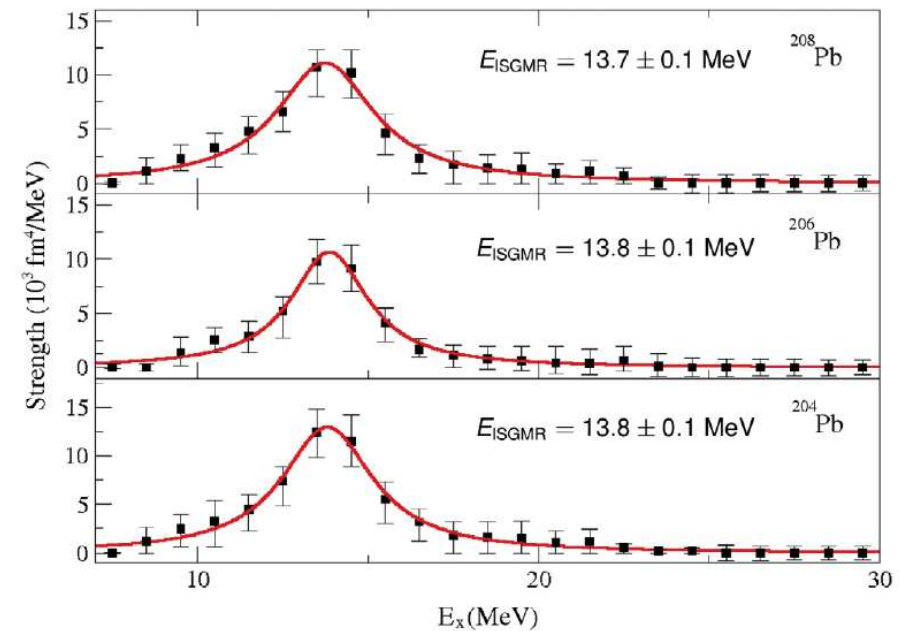
Measurements of the ISGMR



T. Li, U. Garg et al. PRC81(2010)034309



D. Patel, U. Garg, M. Fujiwara et al. PLB718(2012)447



D. Patel, (Ph.D. thesis), University of Notre Dame, 2014

Nuclear incompressibility vs. pairing

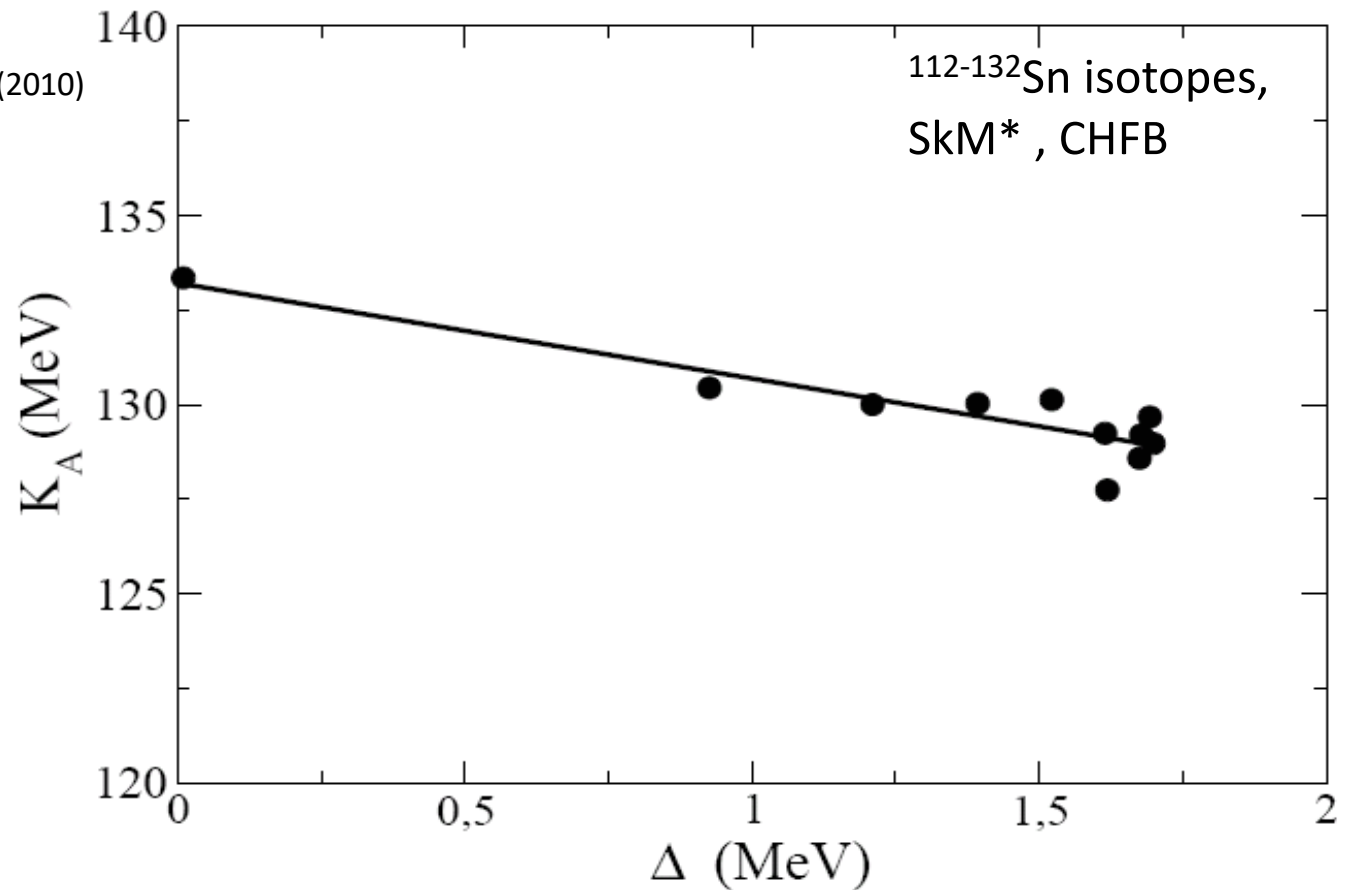
Civitarese et al. (1991)

Li, Colò, Meng (2008)

Khan, Margueron, Colò, Hagino, Sagawa (2010)

Li-Gang Cao, Sagawa, Colò (2012)

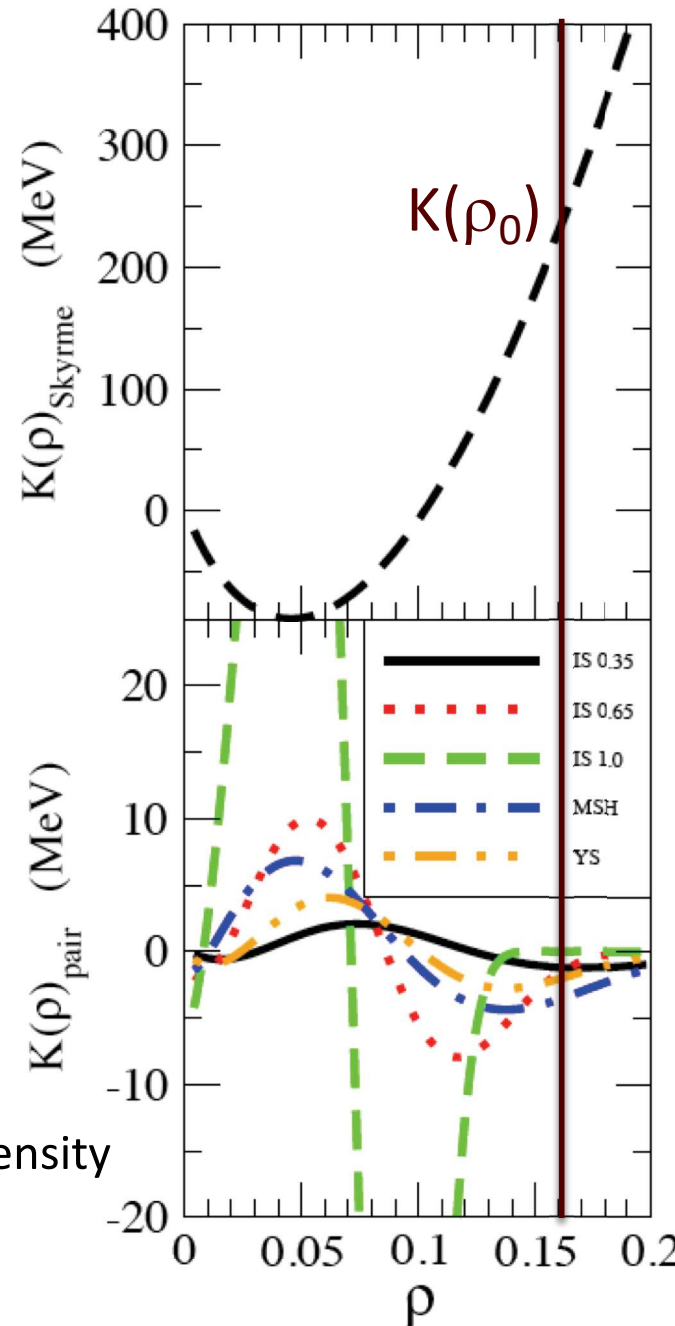
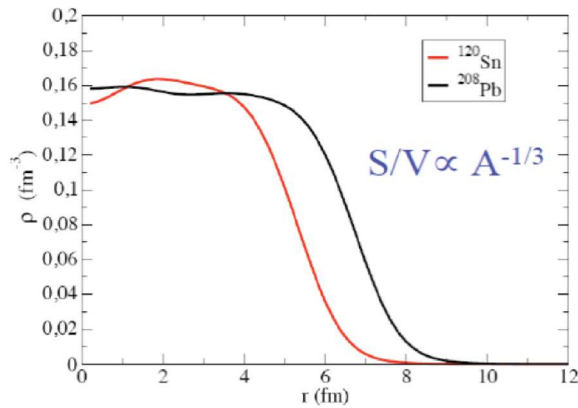
$$E_{\text{GMR}} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}}$$



- Cooper pairs favor compressibility
- Incompressibility of **superfluid** nuclear matter: $K_{\infty}(\Delta)$

→ Pairing has a small but non negligible effect on the GMR: softening

Origin of the pairing effect: density




Density dependence of K

Effect of pairing at low density

EoS: definitions

a.k.a: $a_v \simeq -16$ MeV a.k.a: $K_\infty \simeq 240$ MeV(?) Skewness

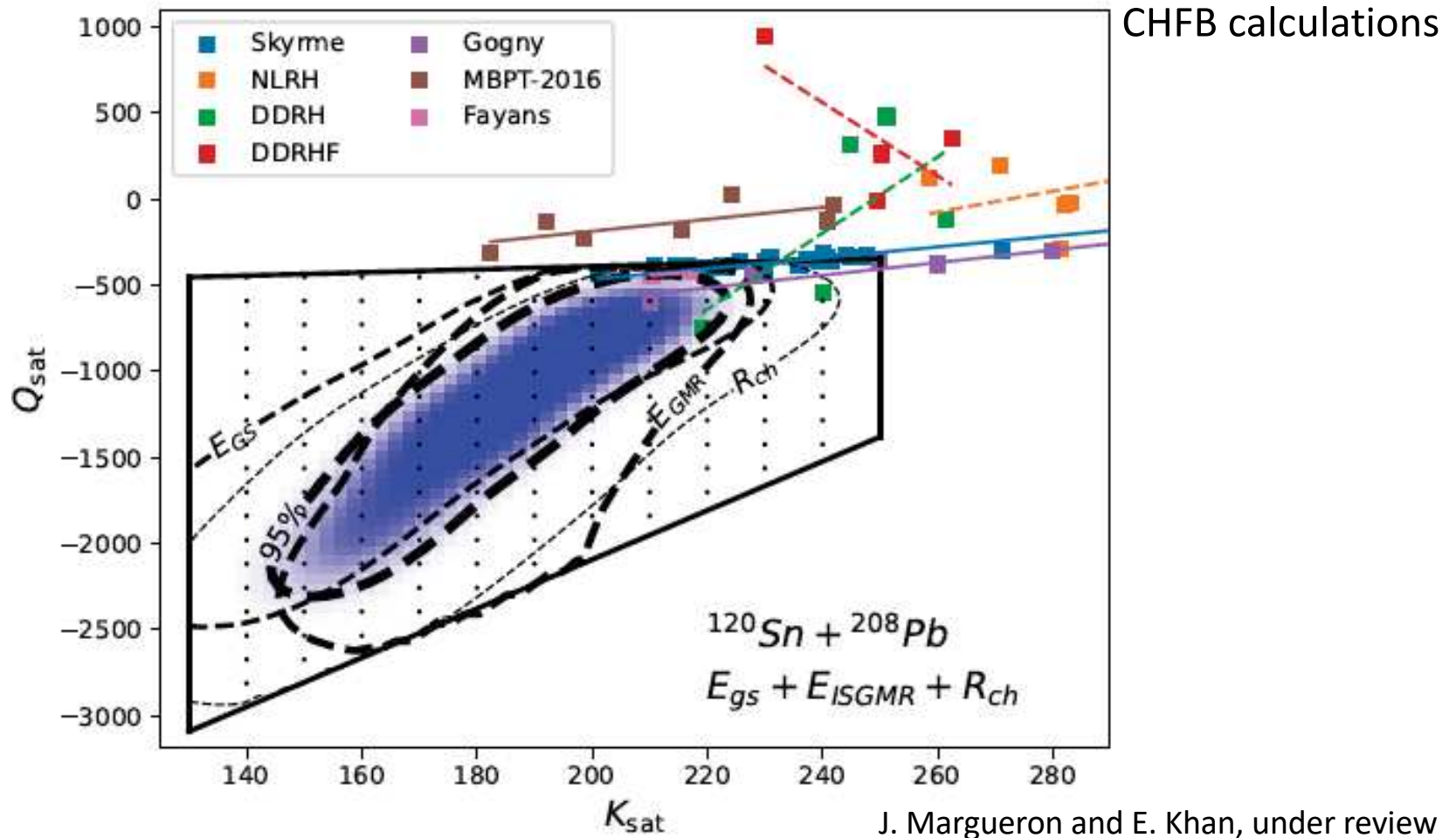


The diagram consists of three terms arranged horizontally. Above the space between the first and second terms, and above the space between the second and third terms, there are arrows pointing upwards and outwards from a central point, suggesting a relationship or commonality between the three terms.

- Generalisation to asymmetric nuclear matter:

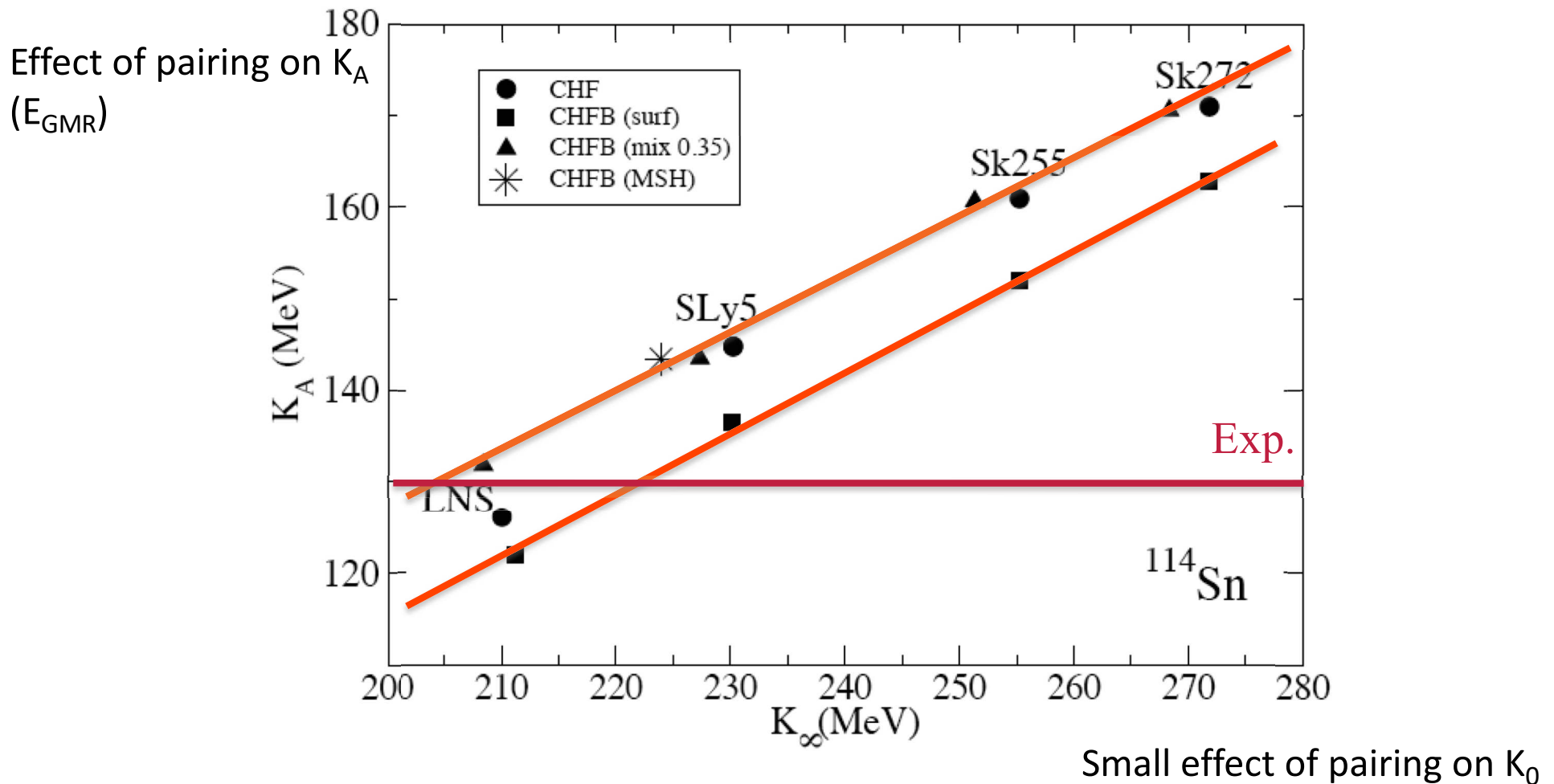
$$\frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, 0) + S(\rho)\delta^2 + \dots$$

Extending the (K_0, Q_0) possibilities



$K_0=170$ MeV is favored when exploring the density dependence of the EoS

Pairing effects on the incompressibility

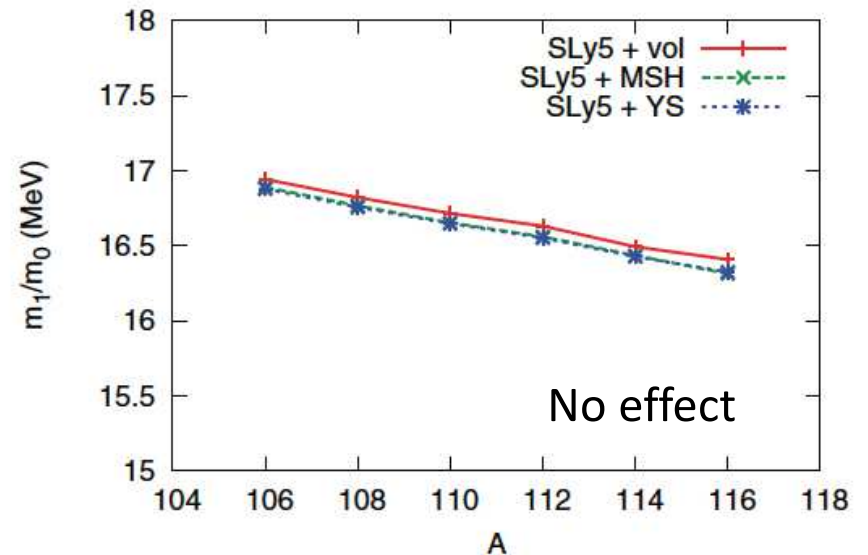
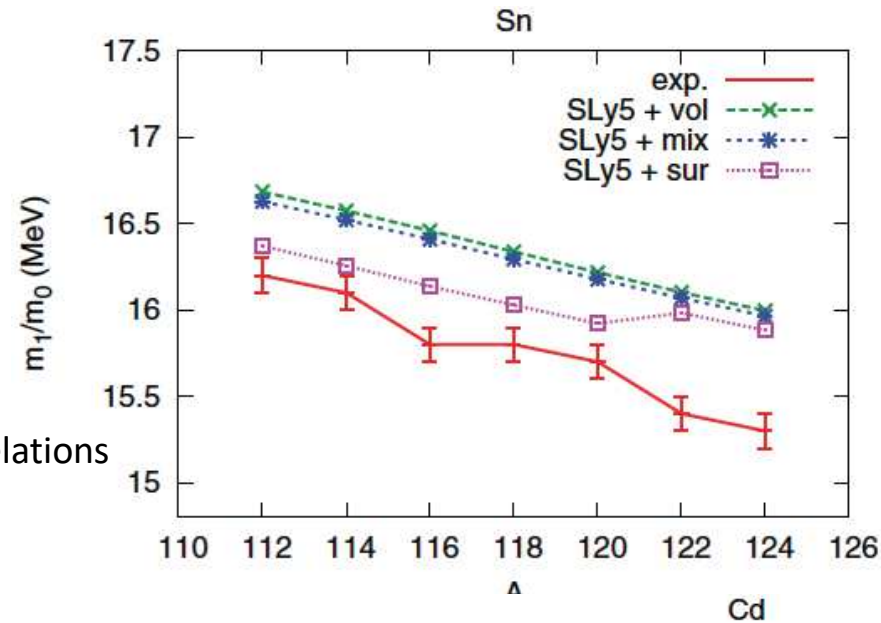


Pairing effects on the incompressibility

$$v(\mathbf{r}, \mathbf{r}') = V_0 \left[1 - \eta \left(\frac{\rho}{\rho_0} \right)^\gamma \right] \delta(\mathbf{r} - \mathbf{r}')$$

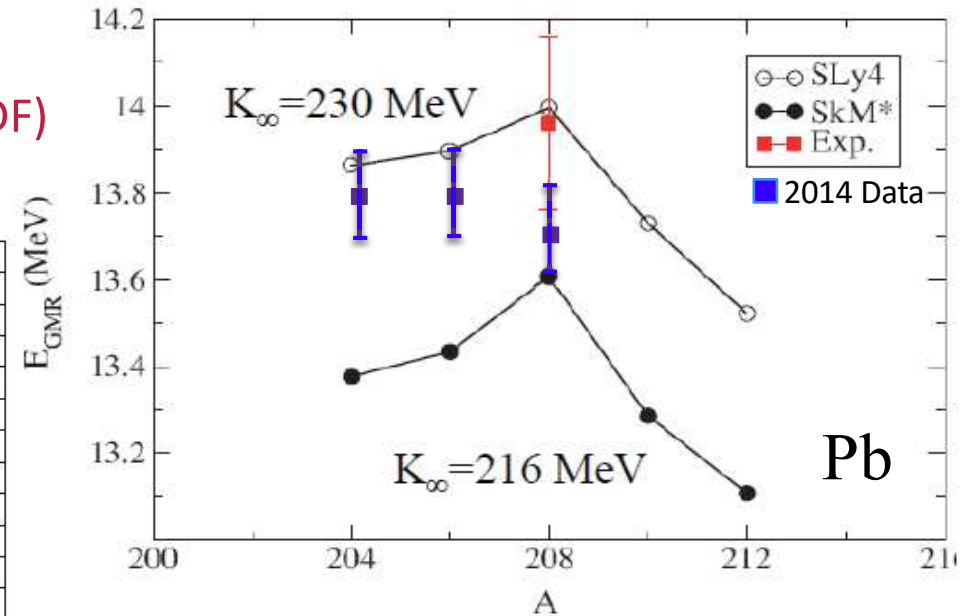
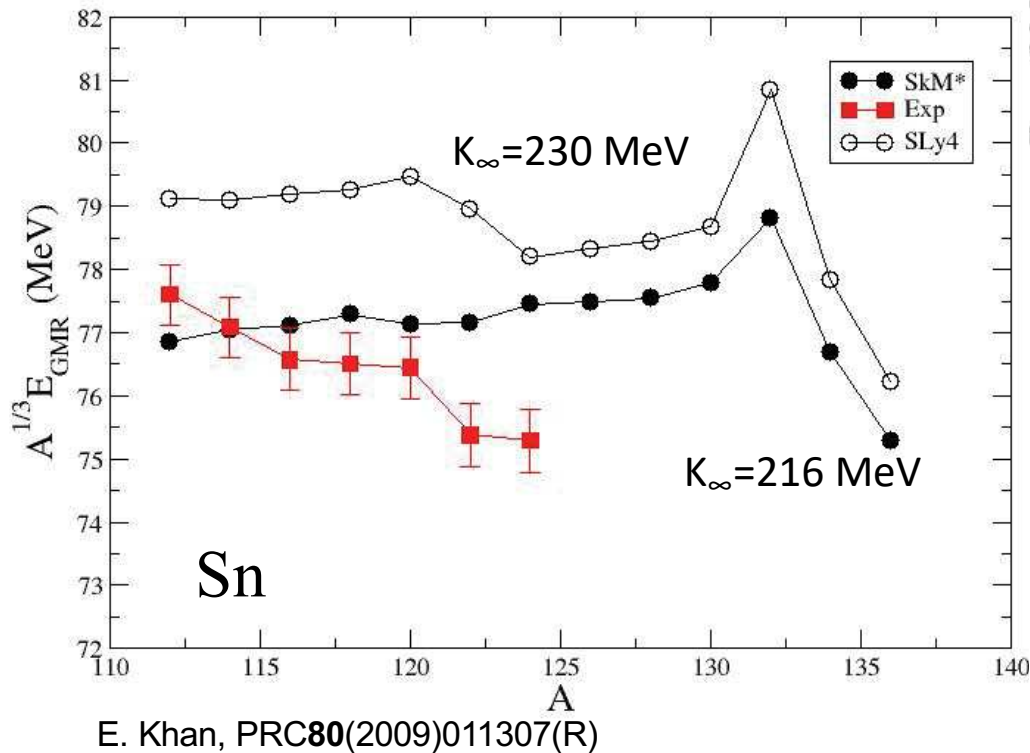
Softening due to larger pairing correlations

$$v_{\text{pair}}^{\text{MSH}}(\mathbf{r}, \mathbf{r}') = V_0 \left[1 - (1 - \delta)\eta_s \left(\frac{\rho}{\rho_0} \right)^{\alpha_s} - \delta\eta_n \left(\frac{\rho}{\rho_0} \right)^{\alpha_n} \right] \delta(\mathbf{r} - \mathbf{r}')$$

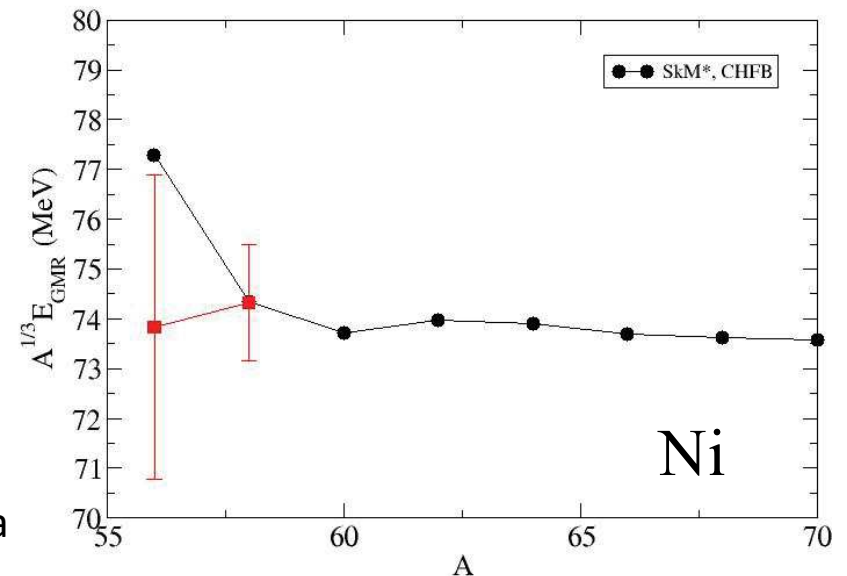


Shell effects on the GMR ?

- Pairing \Rightarrow shell effects on the GMR value ?
- Doubly magic nucleus : increase of the GMR (EDF)



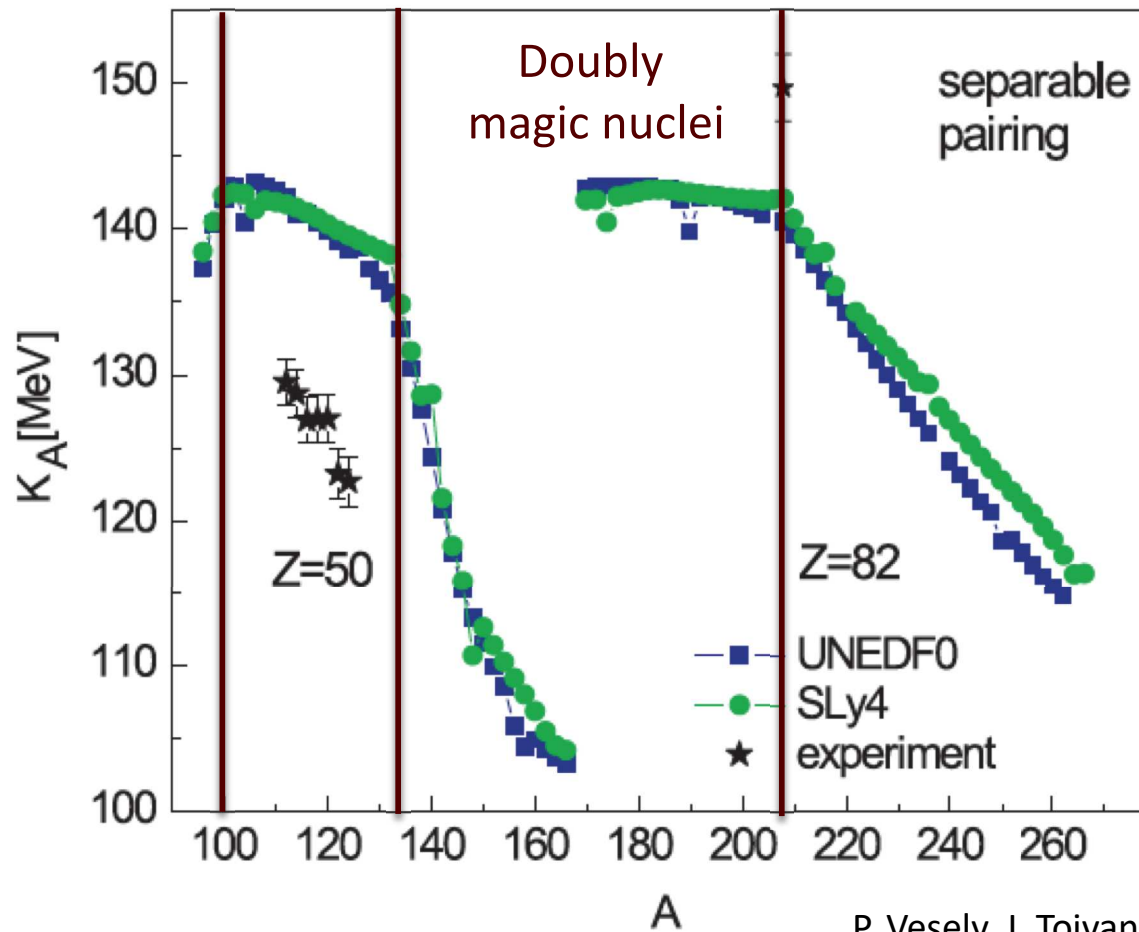
E. Khan, PRC80(2009)057302



M. Vandebrouck, E. Khan et al.,

→ No doubly magic effect on the GMR from the Pb data

Pb/Sn issue



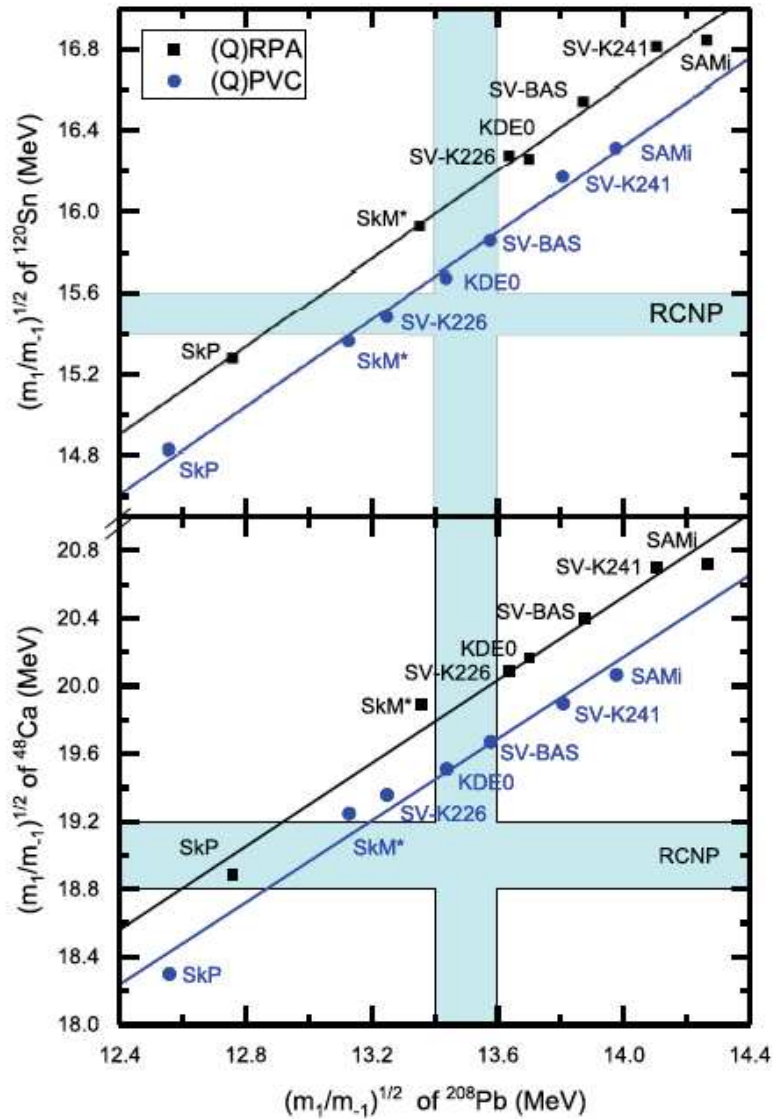
P. Vesely, J. Toivanen, B.G. Carlsson, J. Dobaczewski,
N. Michel, A. Pastore, PRC86(2012)024303

→ shell effects (MEM ?)

N.B.: Pairing not enough to explain Pb/Sn difference on K_∞ (solution from 2p-2h)

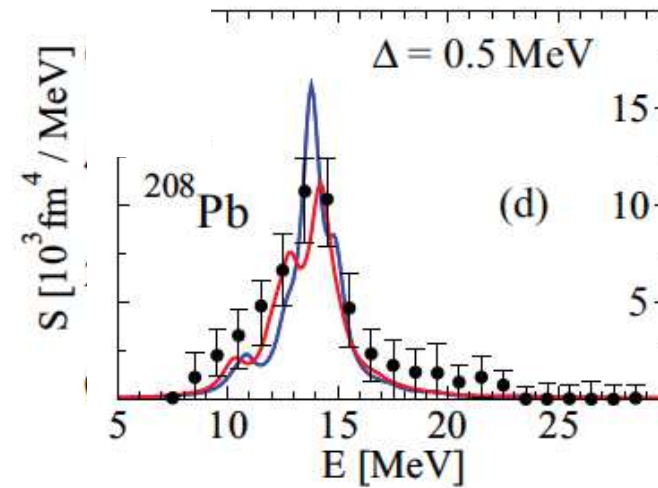
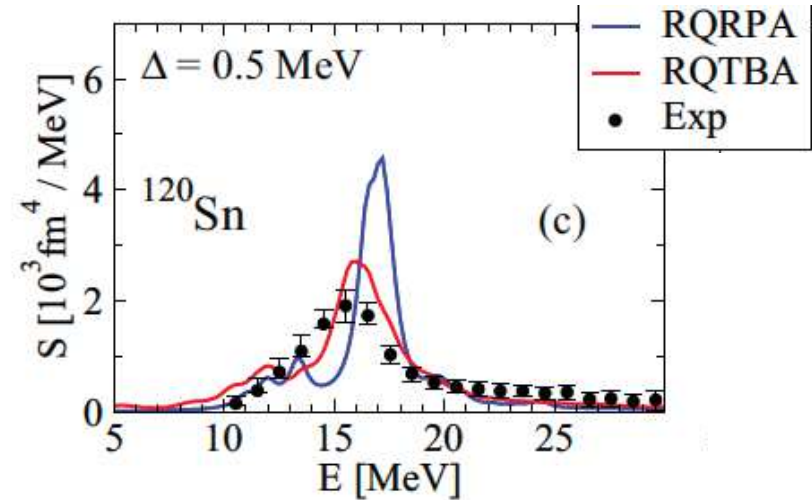
QRPA+2p-2h solution

QRPA+PVC



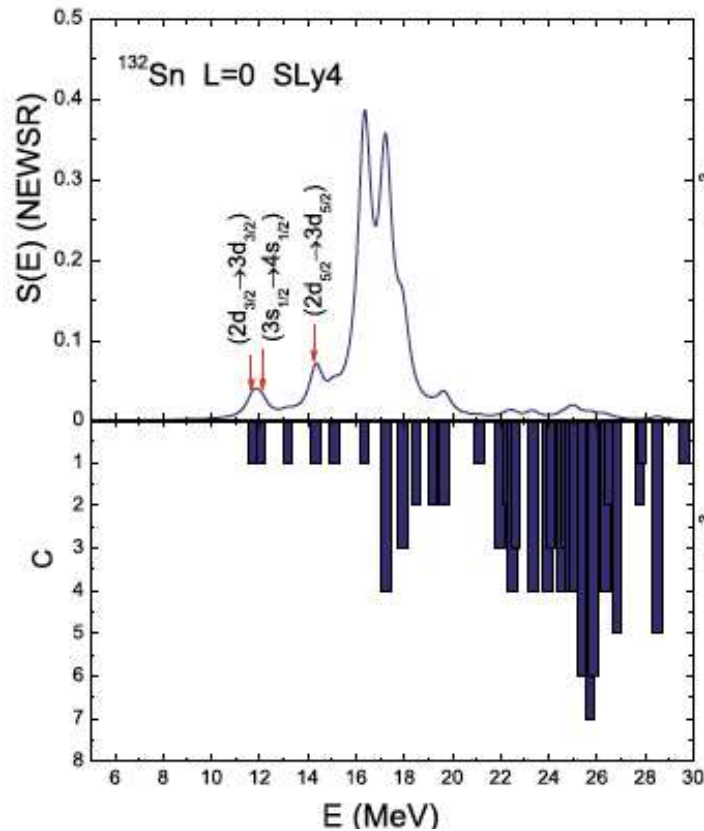
Z.Z Li, Y.F. Niu and G. Colò, PRL131(2023)082501

RQTBA = RQRPA + PVC

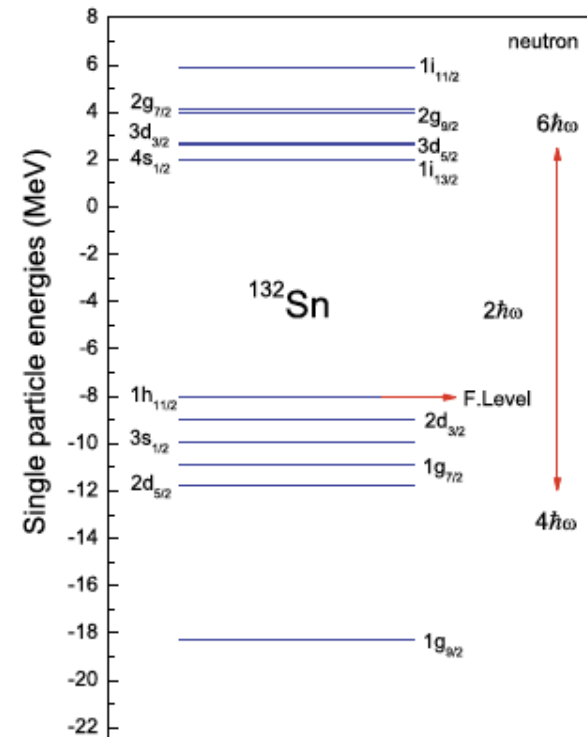


E. Litvinova, PRC107(2023)L041302

The soft monopole mode is non-collective

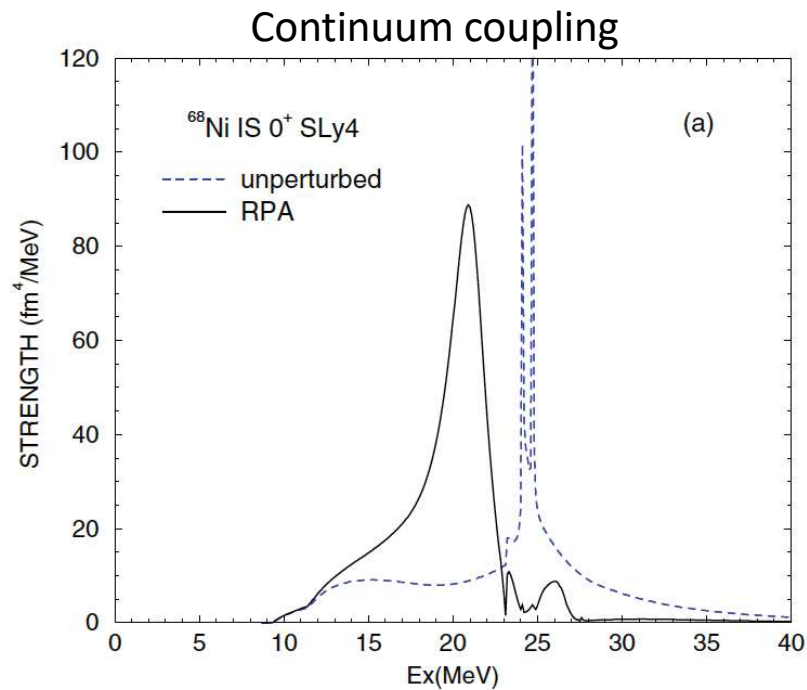


E. Yuksel et al, EPJA 49, 124 (2013)

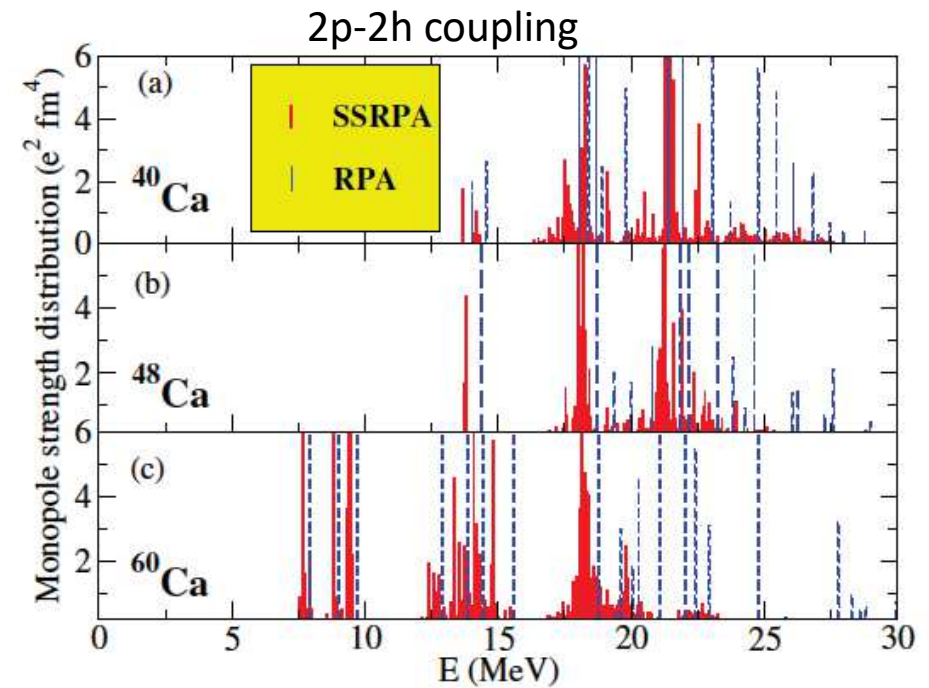


Probe for the single-particle spectrum ?

Escape and spreading width effects on the soft monopole mode

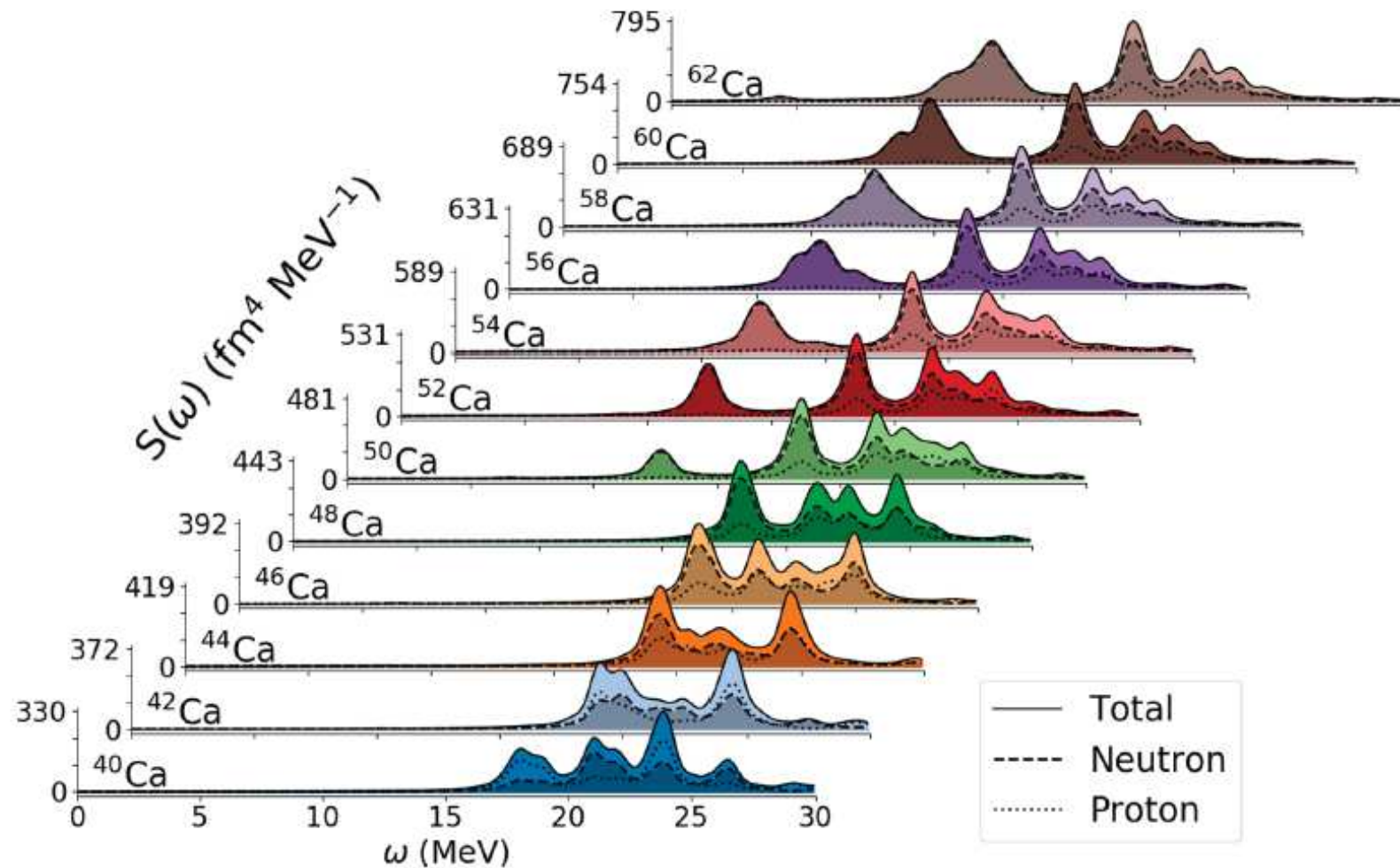


I. Hamamoto et al., PRC 90, 031302(R)(2014)



D. Gambacurta et al., Phys.Rev.C 100, 014317 (2019)

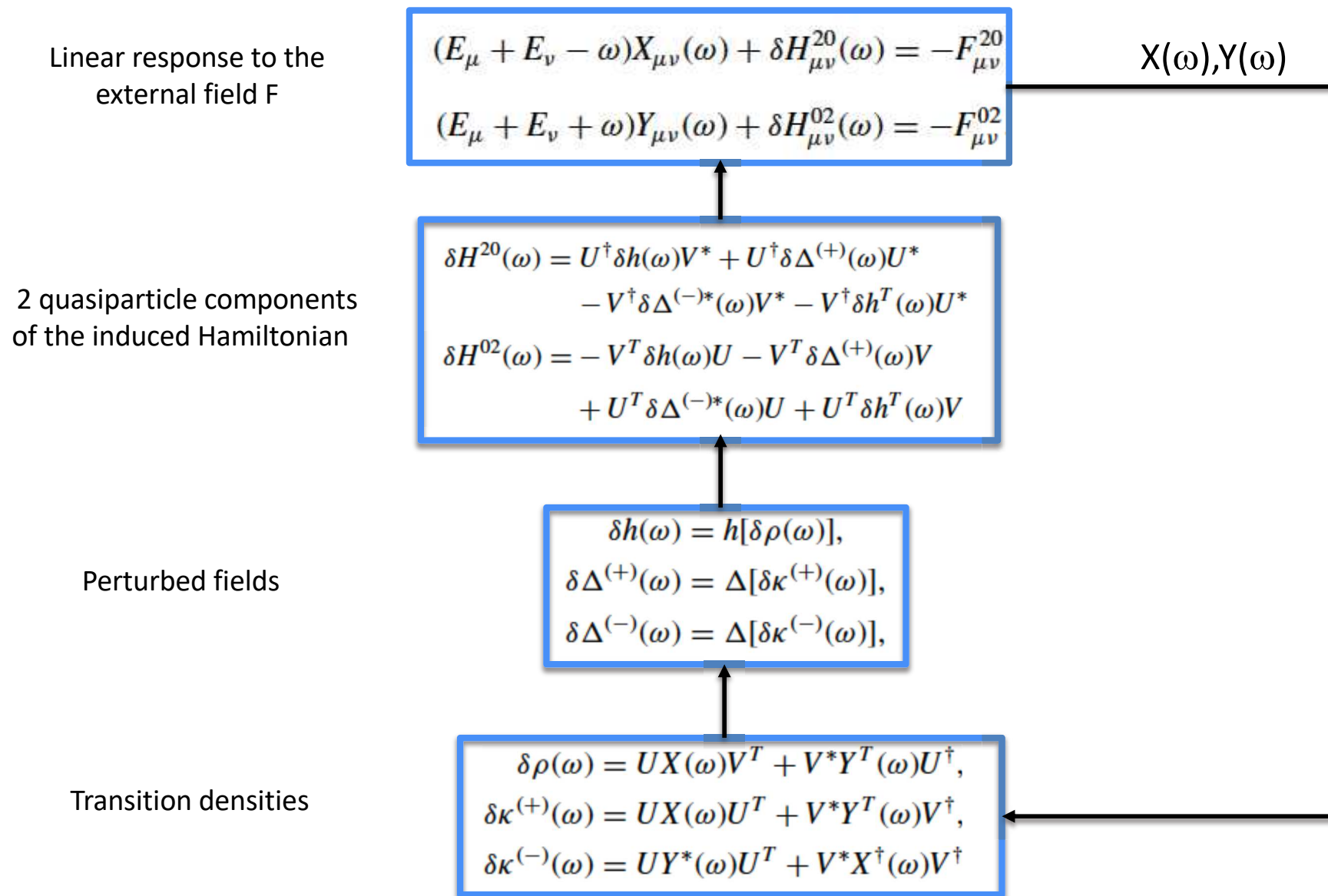
Increase of the soft monopole mode with n excess



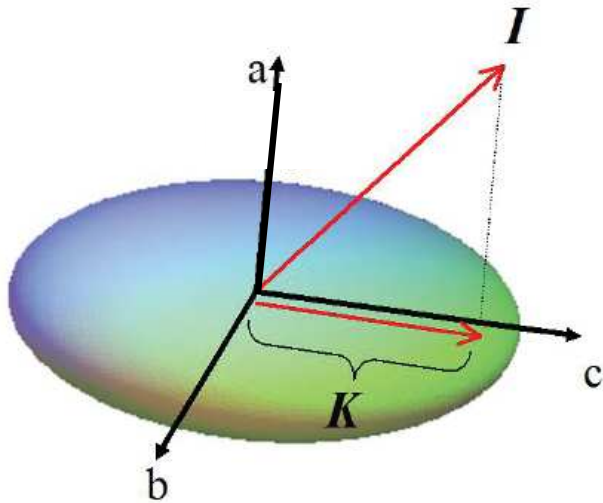
F. Mercier et al., PRC 105, 034343 (2022)

Incompressibility of n-rich matter at low density ?

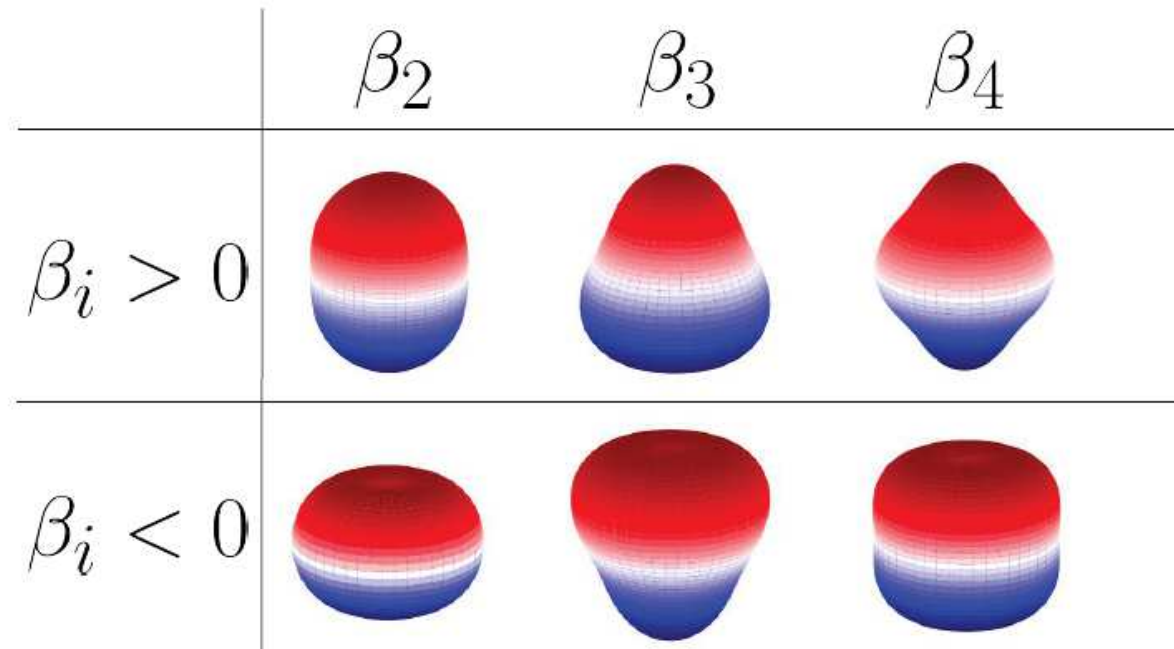
The Quasiparticle Finite Amplitude Method (QFAM)



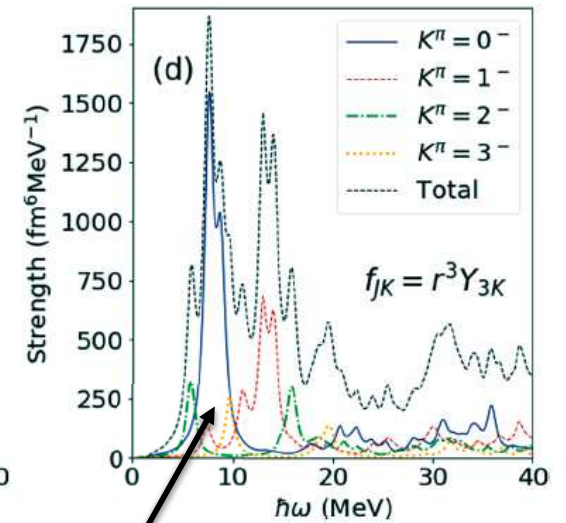
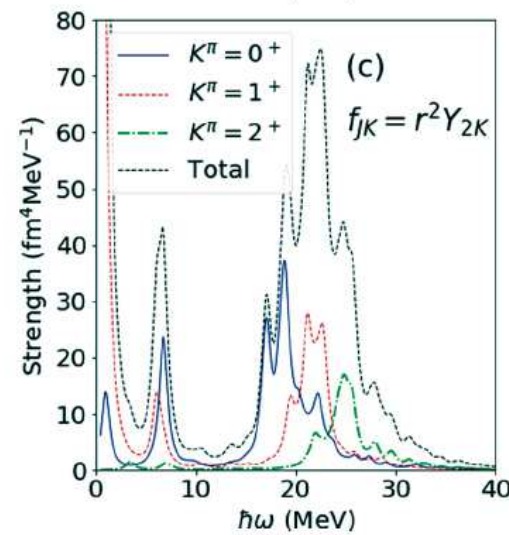
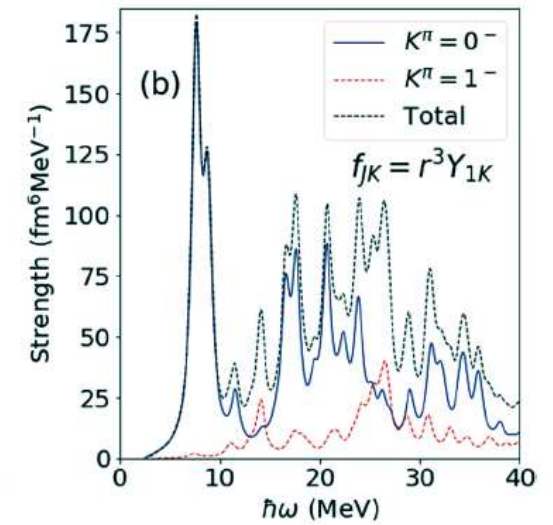
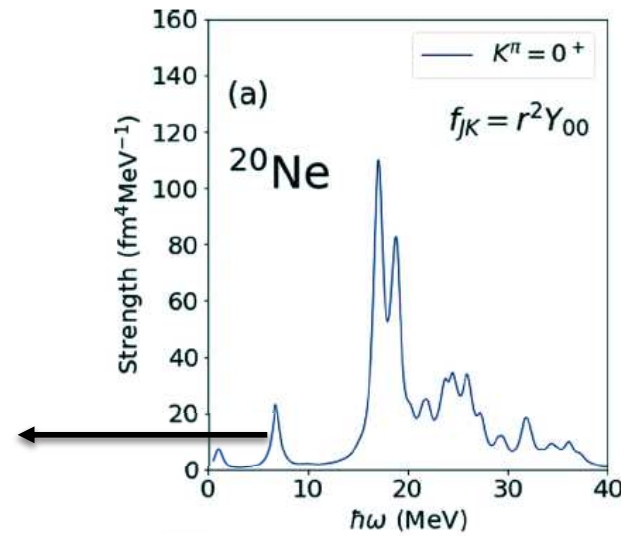
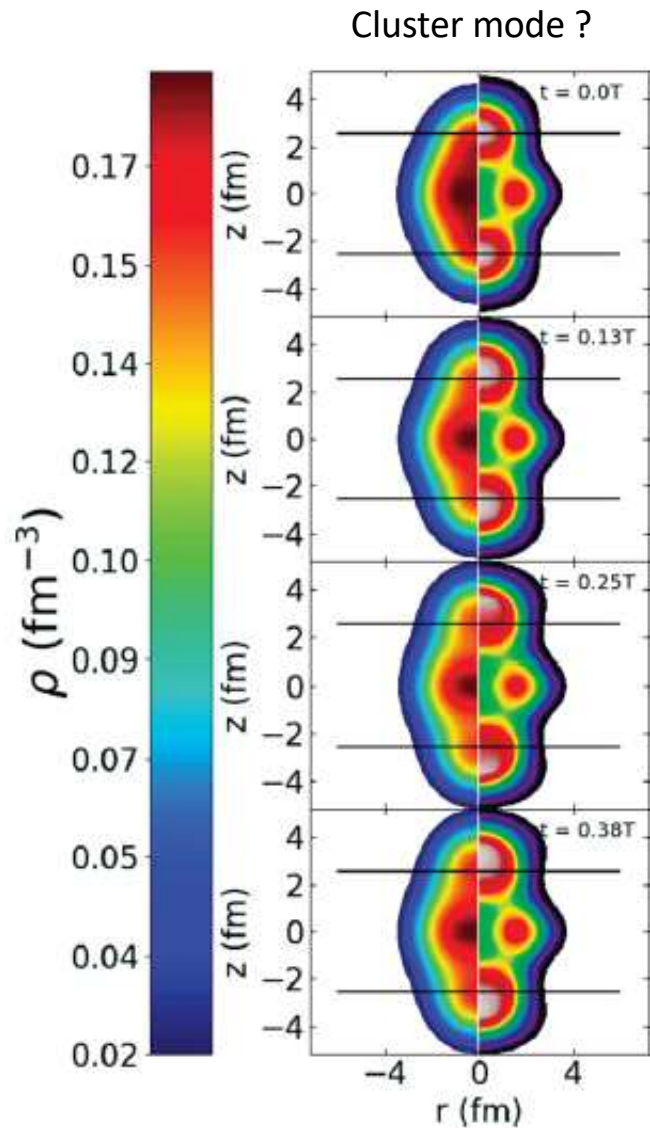
Reminder on deformations



KTH nuclear physics group



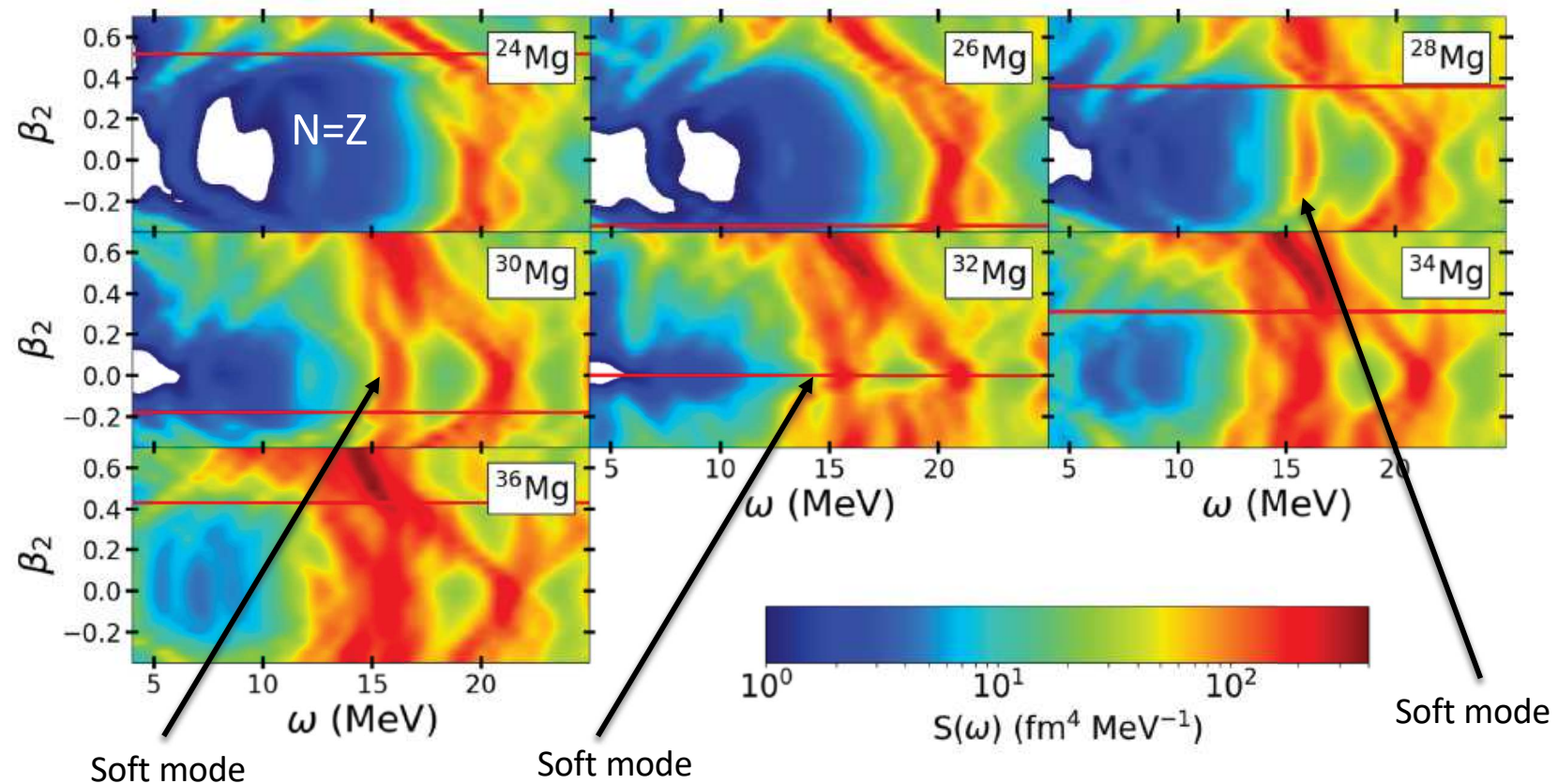
Cluster modes in N=Z nuclei



F. Mercier et al., PRC 103, 024303 (2021)

Cluster mode ?

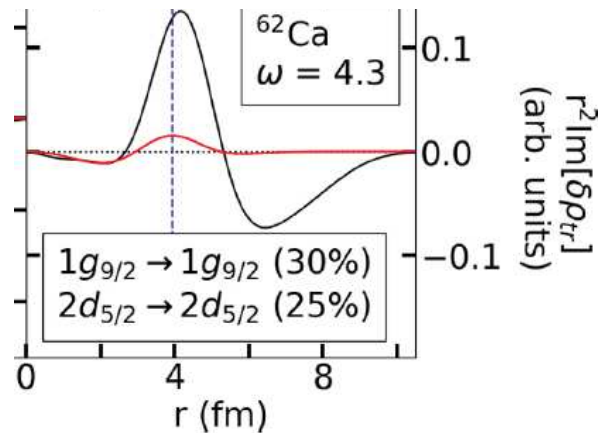
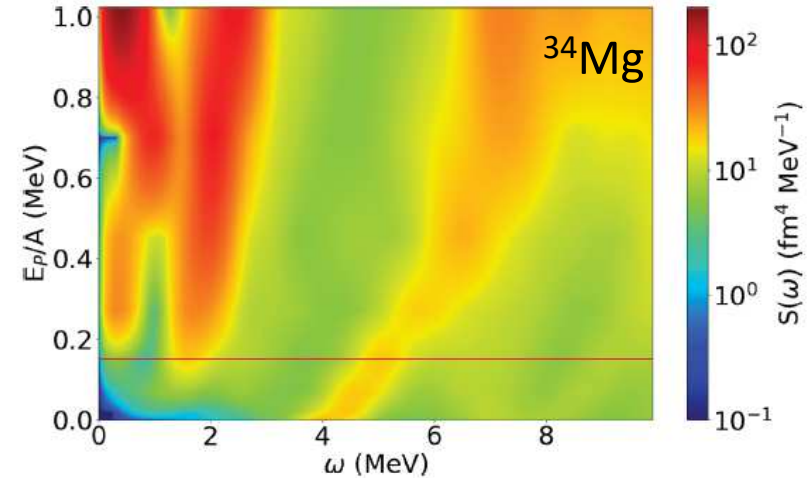
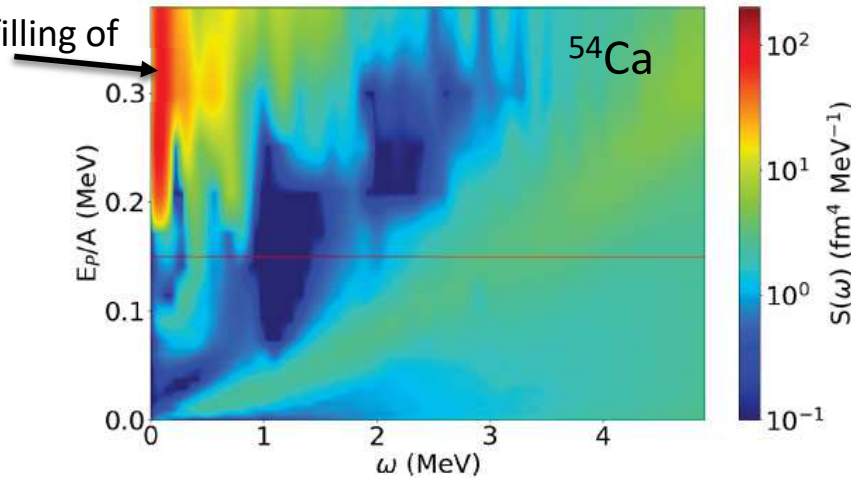
Impact of neutron excess and deformation on the monopole response



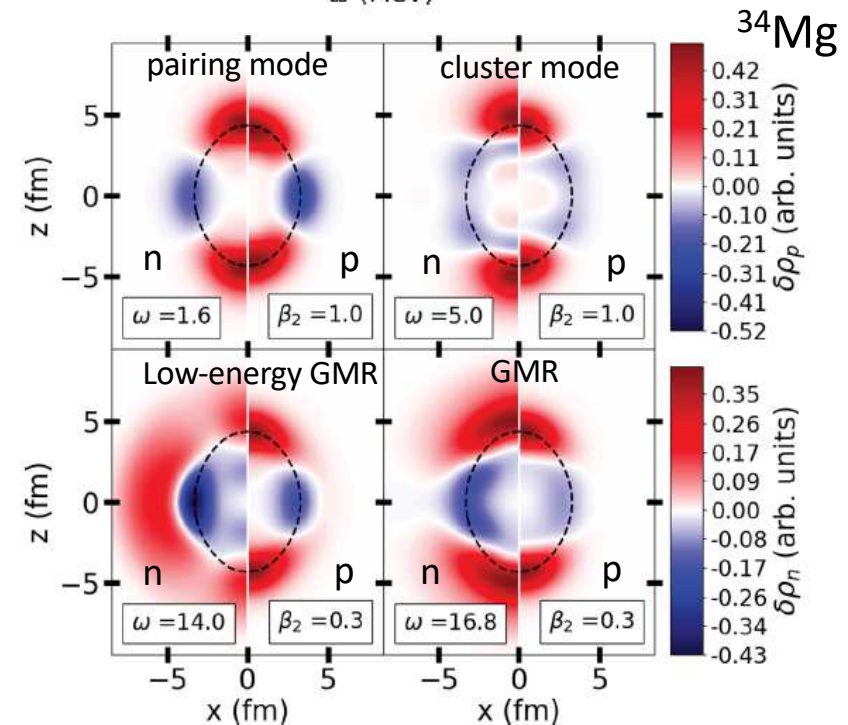
F. Mercier et al., PRC 105, 034343 (2022)

Impact of the pairing effect on low-energy L=0 modes

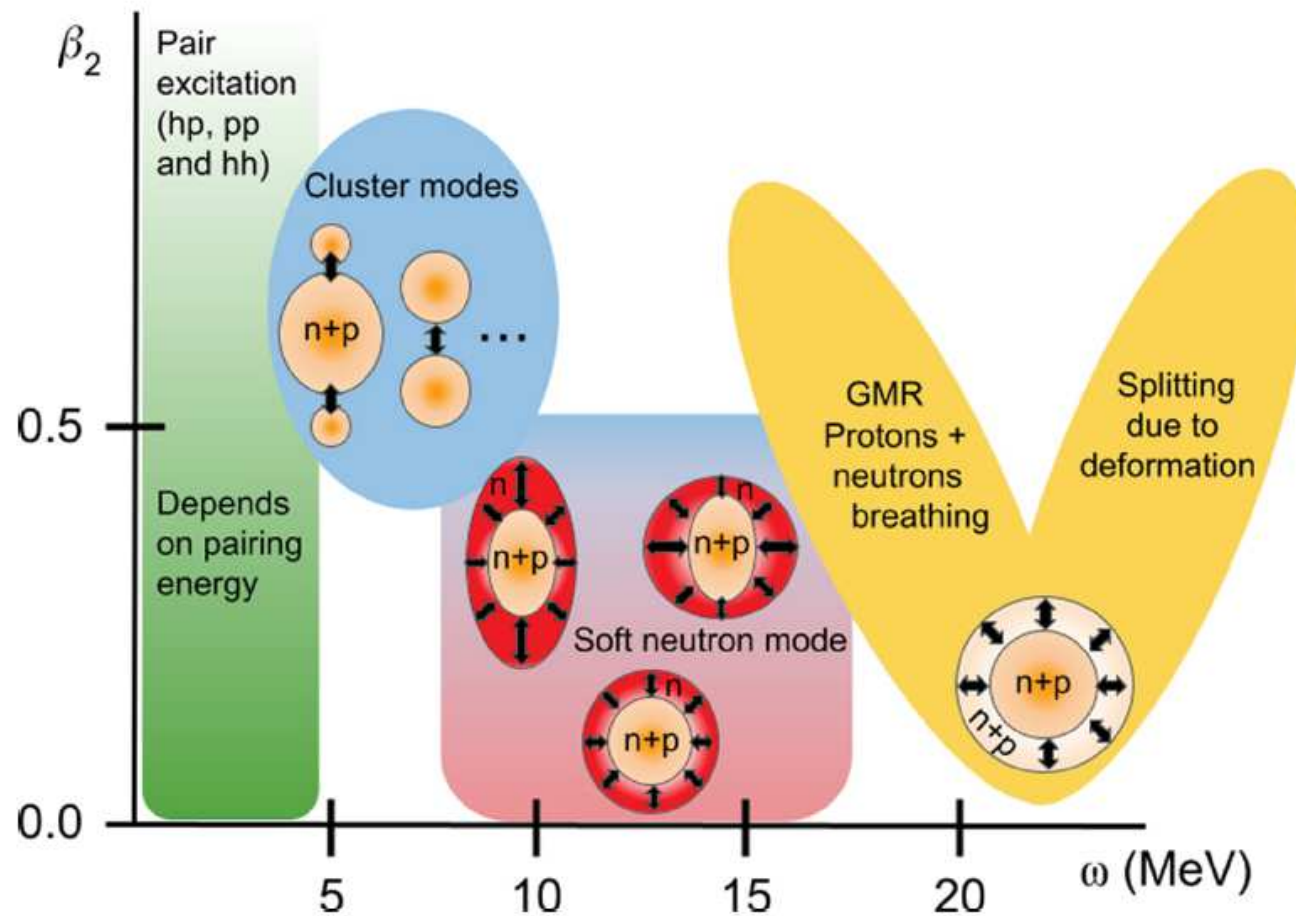
Pairing mode due to the partial filling of 2p1/2



L=0 mode due to pair formations among a given s.p. state



IS monopole excitations in deformed nuclei



Summary

- EDF- based methods to consider pairing effects in the ISGMR: CHFB (EGMR only), QRPA (+PVC) (strength), QFAM (deformation)
- EDF predicts large increase of EGMR in doubly magic nuclei, but not observed in data on Pb chain
- Pairing effects non-negligible on EGMR along isotopic chain, due to low-densities (surface) effects: small softening of the GMR
- To describe both Sn and Pb data, 2p-2h-like calculations are necessary
- Interplay of pairing and deformation in low energy modes